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Particle Tracks in Emulsion

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A new theory of track formation in emulsion accounts for the tracks of charged particles on the basis of a theory developed earlier for the response of biological molecules and NaI(Tl) to energetic heavy ions. The probability that an emulsion grain will remain undeveloped when exposed to δ rays depositing a mean energy $\bar{E}$ is assumed to be $e^{-\bar{E}/E_0}$, where E_0 is the dose at which $1/e$ (37%) of the emulsion grains remain undeveloped, as in the one-or-more-hit cumulative Poisson distribution. The parameter E_0 incorporates variations in emulsion properties and processing conditions. Calculation of the spatial distribution of the ionization energy deposited by δ rays is combined with the assumed emulsion response to yield the spatial distribution of developed grains about the path of the charged particle. Calculations are in agreement with experimental data for grain counts (up to the relativistic rise), blackness profiles, and track width.

I. INTRODUCTION

In earlier work, the inactivation of dry enzymes and viruses by heavy ion bombardment,[1] and the response of NaI(Tl) to heavy ions,[2] were treated by assuming a one-hit response to the dosage of ionization energy deposited in the vicinity of the ion's path by δ rays. This treatment has now been extended to the theory of track formation in emulsion, to correlate several aspects of track structure. Calculations now yield theoretical predictions of grain counts, the blackness variation in a particle track with distance from the ion's path, and the track width. The parameters required in the theory are (1) the characteristic dose for sensitization of 63% of the undeveloped emulsion grains, (2) the number of undeveloped grains per unit volume, (3) the size of an undeveloped grain, and (4) the size of a developed grain. Grain count and track width data set narrow limits on the choice of the first of these parameters, while other measurements set the values of other parameters. From the theory it is possible to estimate the precision with which Z and β can be determined from emulsion measurements.

II. SPATIAL DISTRIBUTION OF DEVELOPED GRAINS

If G is the number of grains per unit volume in the undeveloped emulsion, and P is the probability that a grain will be sensitized and developed, then the number of developed grains per unit volume is PG. The geometric distribution of developed grains lies at the basis of the analysis of track structure.

We take the probability for the development of an emulsion grain to depend on the mean dose $\bar{E}$ of ionization energy, deposited by δ rays, to which the grain is exposed. If E_0 is the characteristic dose for sensitization and development of 63% of the emulsion grains in a uniform exposure, then

$$P=1-e^{-\bar{E}/E_0}. \tag{2.1}$$

The assumed one-hit nature of the photographic response is consistent with the observed response of emulsion to x rays.[3]

Aside from obvious geometric differences, the response of the emulsion to heavy particles and γ rays must be intimately related, for both interactions are predominently through secondary electrons. In small subvolumes near the ion's path, we assume that the response is as if the subvolume were part of a larger system uniformly irradiated with γ rays to the same dosage.

The spatial distribution of grains about a particle track may then be determined from knowledge of $\bar{E}(t)$, where t is the distance from the ion's path.

To find $\bar{E}(t)$, the mean dose averaged over an undeveloped grain, we must first find $E(t)$, the point distribution in dose. As in earlier work, this function is found by computation, from the δ-ray distribution formula, an assumed angular distribution of the ejected δ rays, and electron energy dissipation data. The emulsion is approximated as a homogeneous medium of appropriate composition.

In the present work, earlier computations[4,5] have been modified by the assumption that the angular distribution of the electrons ejected by passing ions follows classical kinematics (for agreement of calculated and measured track-width profiles, Sec. V), and is given by

$$\cos^2\theta=\omega/\omega_m, \tag{2.2}$$

where initially free electrons at rest are ejected with energy ω at angle θ to the ion's path, and

$$\omega_m=2mc^2\beta^2\gamma^2, \tag{2.3}$$

with ω_m the maximum energy delivered to an electron of mass m by an ion moving at speed βc, and

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† Present address: H. H. Wills Physics Laboratory, University of Bristol, Bristol, England.
[1] J. J. Butts and R. Katz, Radiation Res. **30**, 855 (1967).
[2] R. Katz and E. J. Kobetich, Phys. Rev. **170**, 397 (1968).
[3] G. M. Corney, in *The Theory of the Photographic Process*, edited by C. E. K. Mees and T. H. James (The Macmillan Co., New York, 1966), 3rd ed.
[4] E. J. Kobetich and R. Katz, Phys. Rev. **170**, 391 (1968).
[5] E. J. Kobetich and R. Katz, Phys. Rev. **170**, 405 (1968).

$\gamma=(1-\beta^2)^{-1/2}$. The earlier computation has also been modified by use of an improved algorithm for the computation of the energy dissipation of normally incident electron beams.[6] To shorten the computation, a single ionization potential ($\bar{I}=320$ eV) has been used to represent the emulsion. For incident ions only the leading term in the δ-ray distribution formula

$$dn/d\omega=2\pi Nz^2e^4/(mv^2\omega^2) \tag{2.4}$$

is used. The formula gives the number $dn/d\omega$ of δ rays having energies between ω and $\omega+d\omega$ liberated from matter having N free electrons/cm^3 by a passing ion of effective charge ze.

For electron bombardment, the δ-ray distribution formula of Møller[7] is used. Here

$$\frac{dn}{d\omega}=\frac{2\pi Ne^4}{mc^2\beta^2}\left[\left(\frac{T}{\omega(T-\omega)}-\frac{1}{T}\right)^2+\frac{1-2\gamma}{\gamma^2T}\left(\frac{T}{\omega(T-\omega)}+\frac{1}{T}\right)\right], \tag{2.5}$$

where T is the kinetic energy of the incident electron and $0<\omega<T/2$, since the electron of lower energy is defined to be the δ ray. The rays arising from electron collisions are assumed to be ejected normally, for kinematics requires the angle between two colliding particles of equal mass to be 90°, and the incident electron is taken to continue undeflected.

To treat the case of electrons initially bound to parent atoms with mean ionization potential $\bar{I}$, ω is interpreted as the total energy transferred to the δ ray, whose kinetic energy w is given by

$$\omega=w+\bar{I}. \tag{2.6}$$

When these changes are incorporated into the earlier computational structure, the result may be represented symbolically, in the notation used previously,[4] as

$$E(t)=-(2\pi t)^{-1}\int_{w_1}^{w_2}\frac{d}{dt}\times\left\{W(t,w,\theta)\eta(t,w,\theta)\frac{dn}{d\omega}\right\}d\omega, \tag{2.7}$$

where w_1 and w_2 are the kinetic energies of δ rays which just reach the cylinder of radius t when ejected at angles consistent with Eq. (2.2). For simplicity in the calculation, the contribution of straggling[6] electrons is neglected.

For use with emulsion, $\bar{E}(t)$ is calculated for a sphere of radius a_0, centered at t, whose AgBr content in the homogeneous emulsion approximates that of an undeveloped grain in the real emulsion. We take $a_0=0.12\,\mu$ for $K.5$ emulsion, and $a_0=0.2\,\mu$ for $G.5$ emulsion.

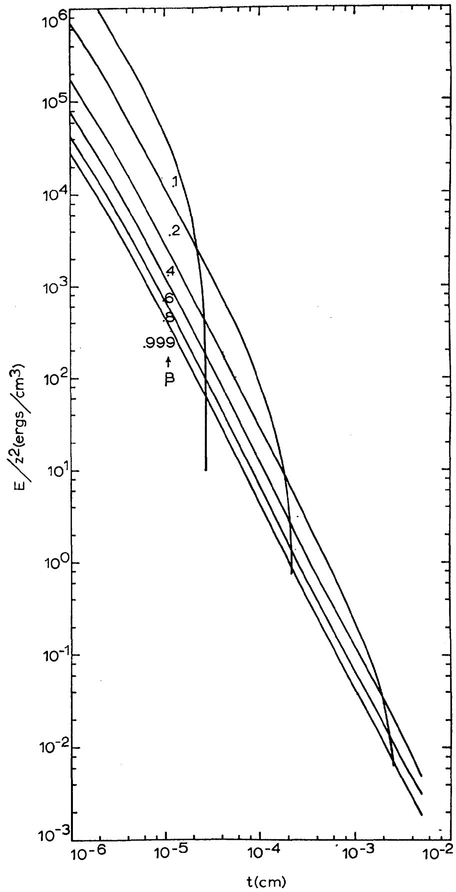

FIG. 1. Point distribution of the energy deposition by δ rays, divided by the square of the effective charge $E(t)/z^2$, as a function of the distance t from the path of an ion moving at speed βc, calculated with the angular distribution arising from classical kinematics.

Plots of $E(t)$ and, $\bar{E}(t)$ $a_0=0.2\,\mu$, for incident ions, and $E(t)$ for incident electrons, are shown in Figs. 1–3, respectively.

The rapid drop in $\bar{E}$ at $t>a_0$ accounts for the linear structure of the tracks of lightly ionizing particles.

The importance of the use of $\bar{E}$ for events close to the ion's path is illustrated in Fig. 4, where $\bar{E}/E$ is plotted as a function of t/a_0. As anticipated in an earlier analysis,[1] small error is made in neglecting to average over the sensitive volume for $t/a_0>2$. This neglect has been called the "point-target" approximation, valid for those bombardments where the response of the medium is saturated close to the ion's path.

Note that there are small differences in $E(t)$ for ion and for electron bombardment, at speeds and distances where the kinematic limit on the energy of the most energetic δ ray is not a contributing factor. Within these limits, the response of the medium must be the same for all singly charged particles.

[6] E. J. Kobetich and R. Katz, Nucl. Instr. Methods **71**, 226 (1969).
[7] C. Møller, Ann. Physik **14**, 531 (1932).

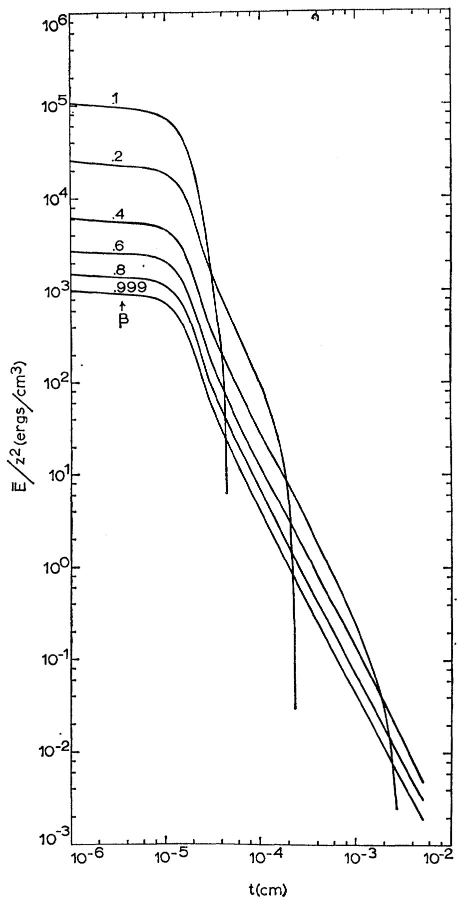

FIG. 2. Distribution of the average dose delivered by δ rays to spheres of radius $0.2\,\mu$, divided by the square of the effective charge $\bar{E}(t)/z^2$, as a function of the distance t from the ion's path.

To distances where ω_m is not an important limitation, the energy deposition is clearly as β^{-2}, when the angular distribution is independent of β, as in earlier work. The present work yields a dependence as $\beta^{-2+\Delta}$, where $-0.05<\Delta<+0.05$. The variation of E with t is as t^{-2} in the interval 0.1–10 μ, but the exponent drops to 1.8 in the interval 0.01–0.1 μ, and to as much as 1.91, varying with β, in the interval 5–50 μ. For many purposes it is sufficient to make the approximation

$$E(t) \propto z^2\beta^{-2}t^{-2} \tag{2.8}$$

for both ions and electrons.

Similarly, close to the ion's path, and at $\beta>0.1$,

$$\bar{E}(t)=f(t)z^2\beta^{-2}. \tag{2.9}$$

All calculations have been made for incident particles of effective charge ze moving at $v=\beta c$. The calculations are transcribed to particles of atomic number Z and range R as in earlier work.[5]

The principal differences between the present theory of track structure and our earlier studies of the width of heavy ion tracks in emulsion arise from (1) the use of the one-hit response for grain sensitization, (2) the use of the average dose $\bar{E}$, and (3) the classical angular distribution. These differences have prompted a revision in our conception of the observed track width. As presently constituted, the theory accounts for linear track structure as well as for extended track structure, through a model whose architecture is identical with that used for radiobiology and scintillation counters.

III. LINEAR TRACK STRUCTURE

Since such linear track measurements as blob and gap counts can be reduced to grain counts by statistical analysis,[8,9] the present discussion is limited to a theory of grain counts.

Following the theory of one-hit processes developed earlier,[1] the total number of grains N made developable by a passing ion in a short distance l along the path of the ion is

$$N=Gl\int_0^\infty 2\pi t dt(1-e^{-\bar{E}(t)/E_0})=Gl\sigma. \tag{3.1}$$

In grain, gap, and blob counts, observer judgements as to which grains belong to the track and which to background are involved. We postulate that the observer counts only those grains whose centers lie within a distance τ of the particle's path, and therefore reduce the upper limit of the integral in Eq. (3.1) from ∞ to τ, to obtain the measured grain count per unit length N_τ/l given by

$$N_\tau/l=G\sigma_\tau. \tag{3.2}$$

The value of τ is found from the density G of undeveloped grains, and the measured saturation grain

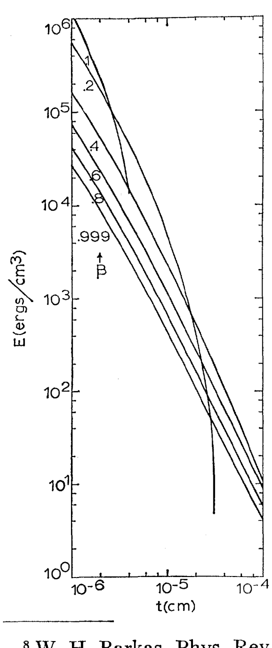

FIG. 3. Point distribution of the energy deposition by δ rays from electrons moving at speed βc.

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[8] W. H. Barkas, Phys. Rev. **124**, 897 (1961).
[9] R. L. Gluckstern, Nucl. Instr. Methods **45**, 166 (1966).

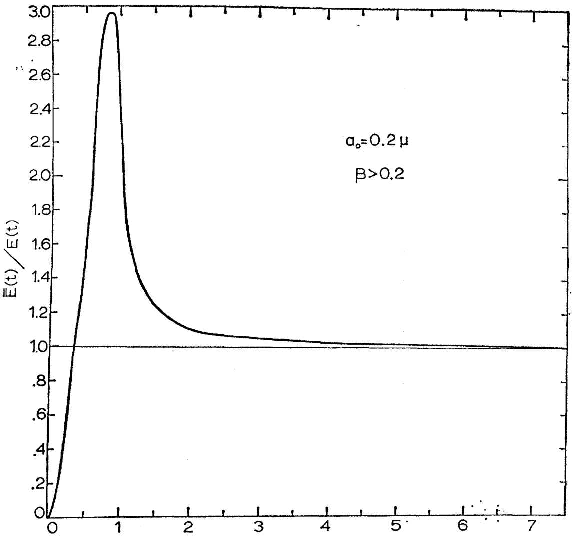

FIG. 4. At $\beta > 0.2$, where the kinematic limit on the energy of the most energetic δ ray is not significant, the ratio $\bar{E}/E$ at fixed t is the same for all β. The plot of $\bar{E}/E$ as a function of $t/(a_0 = 0.2\ \mu)$ underlines the validity of the point-target approximation[1,2] at $t \geq 2a_0$.

count, for at saturation $P \longrightarrow 1$, so that

$$(\sigma_\tau)_{\text{sat}} = \pi\tau^2. \tag{3.3}$$

The grain density in $K.5$ emulsion is given as a function of particle velocity by Patrick and Barkas.[10] For this emulsion, $G = 10^{14}\ \text{cm}^{-3}$, and the saturation grain count is 700 grains/100 μ. These data yield $\tau = 0.15\ \mu$, about 0.7 the diameter (0.20 μ) of an undeveloped $K.5$ grain. In Fig. 5, the data of Patrick and Barkas are superimposed on a curve calculated from Eq. (3.2) with $E_0 = 50\ 000\ \text{erg/cm}^{-3}$.

The grain density in $G.5$ emulsion is given as a function of Z^2 for relativistic ions, and as a function of proton range, by Fowler and Perkins,[11] for two stacks in which the grain density at minimum ionization is 180 and 270/mm, representing the limits of normally developed $G.5$ emulsion. In these stacks the saturation grain count is 500/100 μ. For this emulsion $G = 4.9 \times 10^{13}\ \text{cm}^{-3}$, so that $\tau = 0.18\ \mu$, about 0.7 the diameter (0.27 μ) of an undeveloped $G.5$ grain.

Data relating grain density to Z^2 are compared to curves calculated from Eq. (3.2) in Fig. 6, at $\beta = 0.95$, with assigned values of $E_0 = 11\ 000$ and $19\ 000\ \text{erg/cm}^{-3}$ corresponding to the emulsion stacks for which grain count at minimum ionization was 270 and 180/mm, respectively. Additional data from underdeveloped emulsion[12] are compared to a calculated curve at $E_0 = 55\ 000\ \text{erg/cm}^{-3}$.

Experimental grain counts in proton tracks, relative to grain count at minimum ionization, are compared with calculated curves at the two values of E_0, above, for normally processed emulsion, in Fig. 7. Calculated curves are normalized relative to each other and relative to experimental data at a residual proton range of 15 cm, for the theory does not yield minimum ionization.

As might be expected, the characteristic dose E_0 is high for underdeveloped emulsion.

These values of E_0 and a_0 imply that 63% of the emulsion grains of $K.5$ emulsion are developed when the average energy deposited is 230 eV/grain. Corresponding values for $G.5$ emulsion are 230 and 400 eV/grain, for normal processing, while underdevelopment leads to a value of 1000 eV/grain.

Expressing the integral of Eq. (3.2) as

$$\pi\tau^2(1 - e^{-\bar{E}(t')/E_0}),$$

where $0 < t' < \tau$, and making use of Eqs. (2.9) and (3.3) we find

$$N_\tau/l = (N_\tau/l)_{\text{sat}}[1 - e^{(-f(t')z^2/(\beta^2 E_0))}]. \tag{3.4}$$

Fitting Eq. (3.4) to Figs. 5–7 leads to the values $f(t') = 605$ and $975\ \text{erg/cm}^{-3}$, with corresponding values of $t' = 0.125$, and 0.09 μ, for $G.5$ and $K.5$ emulsion, respectively. Through Eq. (3.4) grain counts may be used to normalize other emulsion measurements.

The present calculations give no indication of a relativistic rise. Neither the leading term of the δ-ray distribution formula, nor the complete formula, nor the fully rigorous classical angular distribution [to which Eq. (2.2) is an approximation] yield a relativistic rise in grain count.

Since grain counts have been correlated with the restricted specific energy loss, the ratio of $(dE/dx)_{\text{restricted}}$ in AgBr[13] to σ_τ, in $G.5$ emulsion, is plotted in Fig. 8, as

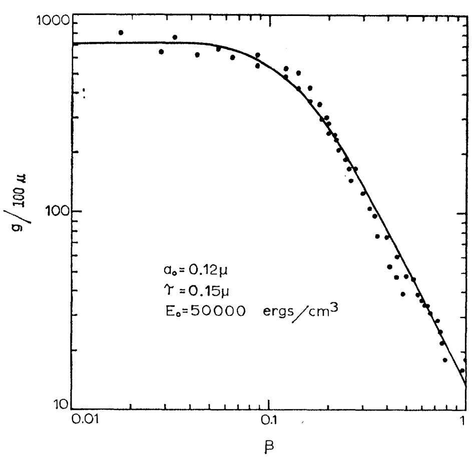

FIG. 5. Measured grain count for singly charged particles in $K.5$ emulsion (Ref. 10) shown against a curve calculated from Eq. (3.2).

[10] J. W. Patrick and W. H. Barkas, Nuovo Cimento Suppl. **32**, 1 (1962).

[11] P. H. Fowler and D. H. Perkins, Phil. Mag. **46**, 587 (1955).

[12] C. F. Powell, P. H. Fowler, and D. H. Perkins, *The Study of Elementary Particles by the Photographic Method* (Pergamon Press Inc., New York, 1959).

[13] W. H. Barkas, *Nuclear Research Emulsions* (Academic Press Inc., New York, 1963), Vol. I.

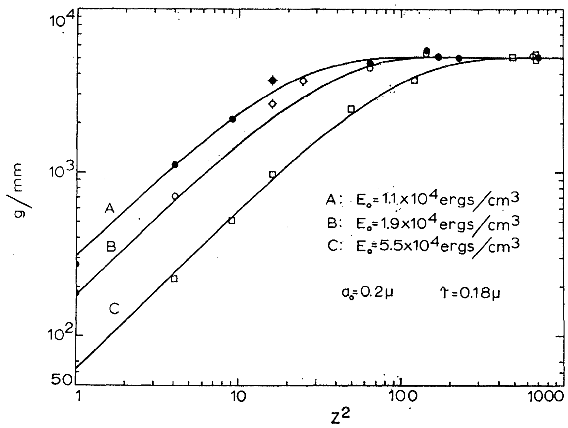

FIG. 6. Grain counts for relativistic ions in $G.5$ emulsion[11,12] as a function of Z^2, shown against curves calculated from Eq. (3.2) at $\beta=0.95$. Solid circles (A) and hollow circles (B) are from normally developed emulsions whose grain count at minimum ionization is 270 and 180/mm, respectively. Hollow squares (C) are from underdeveloped emulsion. Points flagged with a cross, at $Z^2=16$ and 25 would fit the theoretical curves if experimentally assigned values of Z were raised to $Z+1$.

a function of β. It seems clear from the figure that both σ_T and the restricted energy loss cannot be good parameters for describing the grain count over a wide range in β.

IV. MICRODENSITOMETRY OF PARTICLE TRACKS

The blackness profile of a particle track is determined by the distribution of developed emulsion grains about the particle's path, through the statistically based relationship

$$B=\alpha NA/2.3\,, \tag{4.1}$$

where $\alpha(<1)$ is a parameter intended to accomodate light scattering in the emulsion, N is the number of grains per unit area projected onto a plane perpendicular

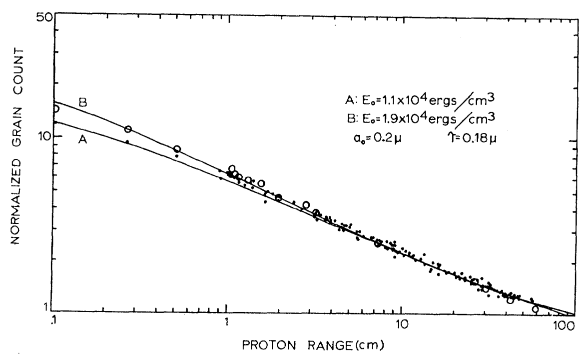

FIG. 7. Grain counts for protons (Ref. 11) relative to grain count at minimum ionization, as a function of proton range in $G.5$ emulsion, shown against curves calculated from Eq. (3.2). The graphic conventions follow Fig. 6. The curves are normalized to the data at a residual range of 15 cm, where solid and hollow circles cross, for the theory does not yield minimum ionization. Note that curves A and B are distinctly separated, consistent with the data.

to the path of the light beam, and A is the cross-sectional area of a developed grain. The point to point variation in blackness must be averaged over the slit width of the densitometer.

Let us assume that the light beam passes parallel to the y axis, through a track lying along the z axis. The photometer slit is parallel to the track, of width $2\Delta x$, and height $2\Delta z$, and makes its traverse by displacement along the x axis. The track lies at the center of an emulsion whose surfaces are at $\pm Y$. Then $t^2=x^2+y^2$.

As observed with a narrow slit, the blackness B is given by

$$B(x)=(\alpha GA/2.3)\int_{-Y}^{Y}(1-e^{-\bar{E}(t)/E_0})dy. \tag{4.2}$$

To find the average blackness $\bar{B}(x)$ across a densitometer slit centered at x, we note that

$$\bar{B}(x)=\log_{10}\left(\int_{x-\Delta x}^{x+\Delta x}dx\Big/\int_{x-\Delta x}^{x+\Delta x}10^{-B(x)}dx\right),$$
$$\bar{B}(x)=\log_{10}\left(2\Delta x\Big/\int_{x-\Delta x}^{x+\Delta x}10^{-B(x)}dx\right). \tag{4.3}$$

There are substantial optical differences between a microscope densitometer and the beam of parallel light implicit in Eq. (4.2). At best, we expect that the calculated density profiles are in relative agreement with experiment, and that the optical problem may be accommodated by adjustment of α.

For a track inclined at angle θ to the emulsion plane, the number of grains per unit area projected onto a plane perpendicular to the light beam becomes $N\sec\theta$.

Calculations have been made from Eq. (4.3) for heavy particle tracks in $G.5$ emulsion, using values of the measurement parameters given by Fowler[14] for his measurements of such tracks.

The choice of E_0 is determined by the grain count at minimum ionization in the experimental emulsions, approximately 220/mm, nearly midway between the stacks discussed in the preceding section. Parameters used are $E_0=15\,000$ erg/cm^{-3}, $2\Delta x=5\,\mu$, $\beta=0.95$, $2Y=400\,\mu$, $\theta=47°$, and $\alpha=0.09$.

The experimental data of Fowler[14] are compared to blackness profiles calculated for $Z=26$, 80, 90, 100, 110, and 120 ions in Fig. 9. The data represent the average of measurements of 151 iron tracks whose average inclination is 47°, and measurements of two heavy tracks whose inclination angles are 45° and 48°, identified by Fowler[14] as $Z\simeq 83$ and 105, respectively. Both data and calculations are normalized by subtraction of the logarithmic average of the blackness at 60, 80, and 100 μ from the ion's path.

The calculated profiles indicate the quality of the agreement between theory and experiment, and the

[14] P. H. Fowler (private communication).

possible resolution of the method. No reassignment of Z is here intended, for this can only be done from knowledge of E_0 and β.

At large distances from the ion's path, the exponential in Eq. (4.2) may be expanded to yield

$$B(x)=\frac{\alpha GA}{2.3E_0}\int_{-Y}^{Y}E(t)dy, \tag{4.4}$$

so that we obtain the approximation, from Eq. (2.8),

$$B(x)\propto(\alpha GA/1.15)z^2E_0^{-1}\beta^{-2}x^{-1}\tan^{-1}(Y/x). \tag{4.5}$$

In a densitometer with a narrow slit, measuring at large fixed distances from the ion's path, the measurement of $B(x)$ serves to determine z, through the approximation

$$z\propto B^{1/2}E_0^{1/2}\beta. \tag{4.6}$$

The implications of Eq. (4.6) for the uncertainty in arising from uncertainty in B, E_0, or β are self-evident. To obtain a statistical uncertainty in E_0 of 1% requires that 10 000 grains be counted. This raises the question, how large an area of emulsion can be considered to have constant E_0?

How is it that the average energy deposition can describe grain formation at large distances from the ion's path, since grain formation is clearly correlated to the tracks of a few isolated δ rays? From Eq. (2.1), if E is the energy deposited by a δ ray in a volume V containing GV grains, then (at low dosage), the mean number of grains formed is $(E/E_0V)GV=EG/E_0$, independent of V. When track measurements average over a volume of emulsion initially containing many undeveloped grains, the correlation of grains to the tracks of energetic δ rays is not significant. The one-hit process is a weak test of randomness. The photometer is blind to the fact that the grains are not randomly deposited.

V. TRACK WIDTH

Long, flat, ending tracks of heavy primary cosmic rays in Ilford $G.5$ emulsion have been photographed (3500×) in 50 μ segments, whose mean width was determined by tracing the profile of the track core manually, and measuring the area within a segment profile with a planimeter.

In the ending 300 μ of track, and as the thin-down region is approached, observers consistently trace around the well-developed core boundary. At greater ranges, where the track is more diffuse, though still well defined and not obscured by overlapping, out of focus grain images, the result of tracing yields a measured width which corresponds to the track diameter at which approximately 40% of the available grains are developed.

Accordingly, we take the measured width beyond 300 μ to correspond to the calculated profile for which $P=0.4$. At $E_0=12\,000$ erg/cm^{-3}, this probability occurs

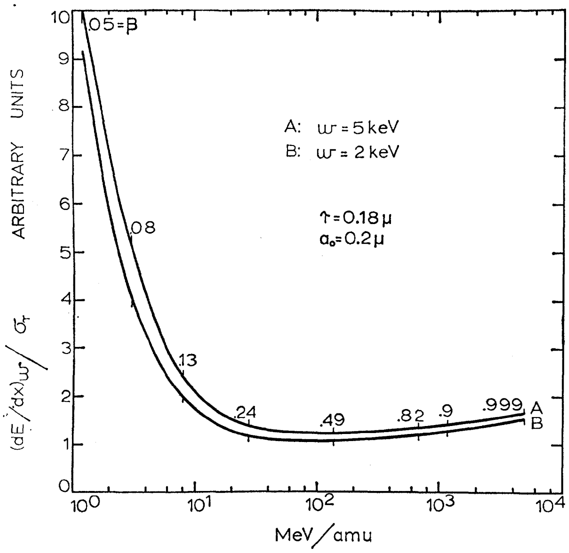

FIG. 8. Ratio of the restricted specific energy loss in AgBr (Ref. 13) (at two values of the maximum allowed δ-ray energy) to σ_τ in $G.5$ emulsion.

at 6100 erg/cm^{-3}. In earlier work[5] we have taken the nominal energy dosage to form a track edge to be 6000 erg/cm^{-3}.

To compare theory to experiment, track profiles have been calculated for $E_0=12\,000$ erg/cm^{-3}, and $P=0.4$, for residual ranges from 10^2 to 10 μ^6, from $\bar{E}(\beta,z,t)$. These profiles, for $G.5$ emulsion are displayed in Fig. 10, for $2\leq Z\leq 130$, using the mass of the most abundant isotope, or the expected mass of the most stable isotope.[15]

Measurements of individual tracks are compared to theoretical profiles in Fig. 11, while the average widths

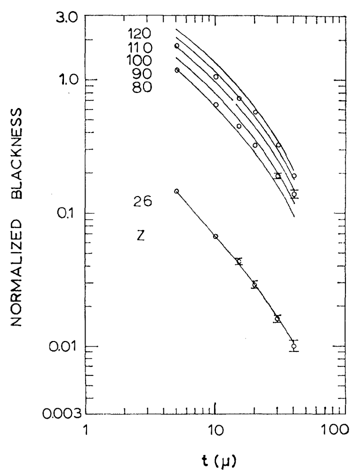

FIG. 9. Blackness variation as a function of the distance from the ion's path. Calculations from Eq. (4.3) are compared to experimental data, for tracks identified (Ref. 14) as $Z=26$, 83, and 105.

[15] A. P. Arya, *Fundamentals of Nuclear Physics* (Allyn and Bacon, Inc., Boston, 1966).

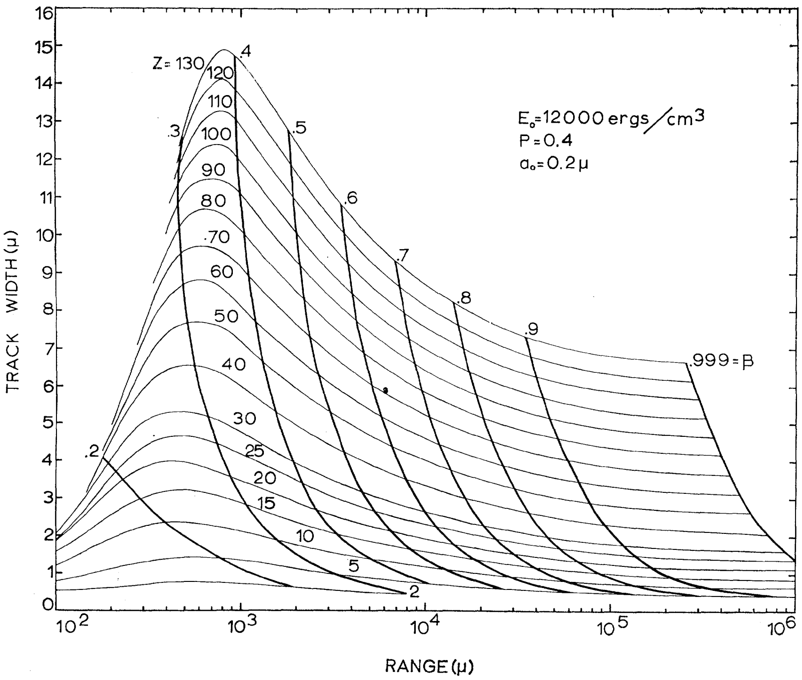

FIG. 10. Calculated track width for $G.5$ emulsion for a spectrum of Z, as a function of residual range.

of groups of tracks of similar width are compared to theory in Fig. 12. As before, the principal calibration arises from the abundance of heavy tracks, identified as iron, and shown in Fig. 12 between profiles for $Z=22$ and 30.

The basis for the present choice of the classical angular distribution is the somewhat better agreement of measured track profiles with theory at residual ranges between 300 μ and 2 cm. The difference in theoretical track diameter profiles between the present and earlier work [where the result was identified as the diameter of the sensitized cylinder, and the angular distribution was $f(\theta)=5\cos^4\theta$] is less than 5% for $\beta>0.6$, and all Z.

Below $\beta=0.15$, the present conceptual structure suggests that measured track widths will disagree increasingly with profiles calculated at $P=0.4$, and that the discrepancy will approximate a developed grain diameter near the stopping end, as determined by the method of measurement, where the width is essentially determined by the grain diameter and the range of the most energetic δ ray. Observer judgements as to the location of the track "edge" have a different quality in the thin-down region than at higher ranges.

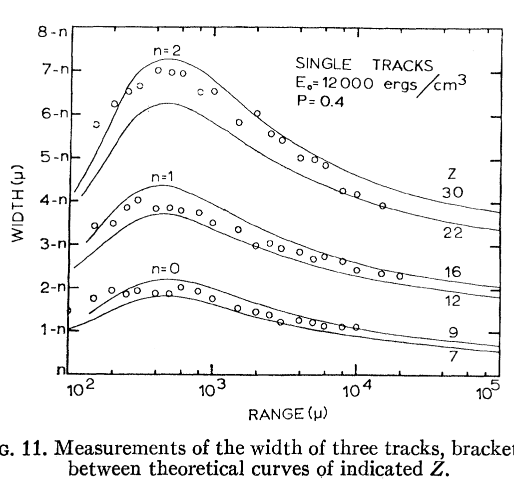

FIG. 11. Measurements of the width of three tracks, bracketed between theoretical curves of indicated Z.

According to present calculations, the track width W, in μ, is given by the expression

$$W=\frac{3.92Z}{\beta\{E_0[-\ln(1-P)]\}^{1/2}}+0.12\,\mu \tag{5.1}$$

when E_0 is expressed in erg/cm^{-3}, for flat tracks fulfilling the conditions that $10<Z<25$, $\beta>0.4$; $25<Z<100$, $\beta>0.6$; $100<Z<130$, $\beta>0.7$, to an accuracy of 0.1 μ, or better.

To accomodate inclined tracks, we note that the observer measures W_{apparent} at his customary P_{apparent}, but for tracks inclined at small angle θ to the emulsion plane

$$P_{\text{apparent}}\cos\theta=P; \tag{5.2}$$

that is, the observer finds a track core which corresponds to a lower true value of P than for flat tracks, and so measures an inclined track to be wider than a flat one.

Steeply dipping tracks present special problems, for observation of their width is affected by light scattering, the depth of field of the microscope objective, and so on.

The present calculation indicates that the track width is linear in Z, above $Z=10$. For lower Z, say in the relativistic region, the tracks begin to gap, as described in Fig. 6, and so the track "width" measured by photometric opacity must display a nonlinear region initially proportional to Z^2, as observed by Kristiansson *et al.*[16]

The determination of Z by measurement of W is sensitive to β and E_0 in the same way as its determination by blackness measurement.

VI. DISCUSSION

While uncertainties remain in the detailed computation of the spatial distribution of ionization energy, associated with the extrapolation of electron energy dissipation data and the angular distribution of the ejected δ rays, the quality of the agreement between theory and experiment implies that the conceptual structure of the present theory of particle tracks in emulsion is on firm ground. The dosage of deposited ionization energy appears to be a good parameter. The response of the emulsion is one-hit to dose, and the characteristic dose E_0 is suitable for describing the combined effect of emulsion and processing variations.

The theory accounts for the characteristic appearance of tracks in emulsions. In a sensitive emulsion, where E_0 is low, the track of a heavy ion will appear broad, while in an insensitive emulsion, where E_0 is high, the same ion will make a track which consists of a series of isolated grains. Such behavior has been observed with 400-MeV argon ions in a series of emulsions.[13]

Neither the specific energy loss of the incident particle, nor the restricted energy loss can be expected to describe particle tracks over a wide variety of emulsion sensitivities or particle speeds, for these parameters contain no knowledge of the spatial distribution of the ionization energy within the medium.

We may think of grain production much as the radiation biophysicist thinks of relative biological effectiveness (RBE). The efficiency with which a charged particle produces developed silver, per unit of expended energy (in relation to silver production by γ rays) is the photographic analog of RBE. For a one-hit process, RBE<1, because of saturation effects near the ion's path. As shown earlier for biological processes,[1] it is only in the limit of low specific energy loss, there called linear energy transfer (LET), and high E_0, there called the D-37 dose, that the RBE approaches 1. In precisely the same way, it is only for lightly ionizing particles and insensitive emulsion that the mass of developed silver, the grain density, is a good measure of the specific energy loss.

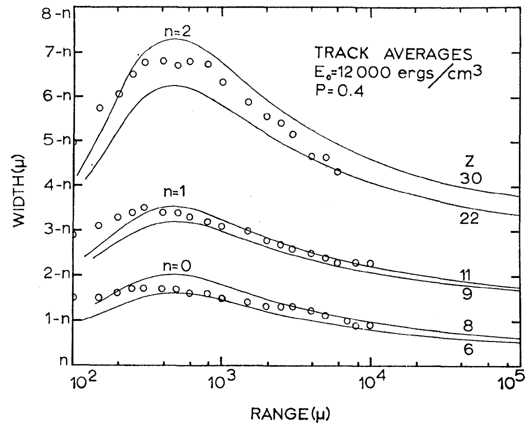

FIG. 12. The average measured widths of three groups of tracks, bracketed between theoretical profiles of indicated Z.

The same set of parameters, the same conceptual structure, the same neglect of direct excitation by the passing ion, and the same attribution of all observed effects to the average energy deposition by δ rays accounts for both the linear and the extended structure of particle tracks in emulsion. From Fig. 1 we see that the energy deposited by δ rays exceeds 3×10^4 erg/cm^{-3} at 10^{-6} cm, rising to about 10^6 erg/cm^{-3} at 10^{-7} cm. This dosage is itself sufficient to saturate detector response close to the ion's path. It is perhaps for this reason that the neglect of direct excitations in the present theory generates no difficulty even when the ion passes through the emulsion grain, as in the theory of grain counts.

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[16] K. Kristiansson, O. Mathiesen, and A. Stenman, Arkiv Fysik **23**, 479 (1963).