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Estimation of Mechanical Properties of Soft Tissue Subjected to Dynamic Impact

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ESTIMATION OF MECHANICAL PROPERTIES OF SOFT TISSUE SUBJECTED TO DYNAMIC IMPACT

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This study attempted to estimate the damping properties of human tissue by using spring and damper system as a model. Data of impacting human tissue at the deltoid area was used to obtain a continuous, second order system to represent the mass-spring-damper system. A discrete ARMA(2,1) model was fitted using the data obtained from experiments in which the deceleration of a pendulum impacting human shoulders in the area of the deltoid muscle was measured. The data of the deceleration was integrated twice to obtain estimates of displacement. The integration was done until the maximum displacement occurred at zero velocity.

An ARMA (2, 1) model was then fitted on the displacement data using the Data-Dependent-System (DDS) technique. The results were then converted to a continuous second order autoregressive model A(2) using the concept of Green's Function and the auto covariance. Utilizing the principles of a mass-spring-damper system enabled the estimation of the spring constant (K) and damper constant (C) for each trail of the experiment.

Estimates for both constants were found to be highly correlated with the mass of the impacting pendulum. Explanations for this string relationship were investigated.

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CHAPTER 1 INTRODUCTION

1.1 Introduction

How people interact with their environment is a topical issue and one of increasing importance. One form of physical interaction which is understood poorly, even by professionals, is concerned with human tissue response to stimuli of impact. This is important, for instance, when determining how human tissue acts when subjected to impacts. Understanding how human tissue responds to impact stimuli in terms of biomechanical stand point would provide a great knowledge of how to treat the effects resulting from impacts human tissue which occurs in many different sectors ranging from industry to healthcare and sports.

There is a little knowledge about the effect of stimuli resulting from impact on human tissue. In spite of the fact that the impact on human tissue is witnessed in a wide range of industries, healthcare treatment, and sports. The effect of impact on human tissue is still to be addressed in order to extract scientific knowledge that can participate in finding a good recipe of remedy the negative effects and to take necessary precautions to avoid undesirable outcomes.

This study was dedicated to extract damping properties of human tissue. The focus was on extracting spring and damper constants of the system presumed to represent human tissue.

Data of impacting human tissue at the deltoid area has been used to obtain a continuous second order system which represents Mass-spring-damper system.

A Discrete model ARMA(2, 1) was fitted on the data obtained from experiments in which the discrete data of deceleration of a pendulum hit of human shoulder. The data of the deceleration was integrated twice to obtain the data of the displacement. The integration was done until the maximum displacement is occurred (velocity = 0). ARMA (2, 1) was fitted on the displacement data using DDS (Data-Dependent-System) technique. This was converted to a continuous second order autoregressive model A(2) with using the concept of Green's Function and the auto covariance. A(2) can be used to obtain the physical characteristics of data extracted from the experiments.

The external pendulum force can be represented by Z (t) which causes the displacement obtained X (t) by integrating the experimental deceleration twice. The elastic resistance of human shoulder is represented by the term $\omega_n^2 x(t)$. And the damping characteristics can be represented by the term$2\zeta\omega_n \frac{dx(t)}{dt}$.

Utilizing the principles of a mass-spring-damper system enabled the estimation of the spring constant (K) and damper constant (C) for each trail of the experiment. Correlation and stepwise regression analyses were used to study the trends of the numerical estimates obtained of damping characteristics K and C.

1.2 History of the research on this topic

Application of Mass-Spring-Damper on human structure is a new topic and the problems in practice have recently emerged, there are not many publications available. The human body acts as a mass-spring-damper system was also noted by many investigators, however, very few studies have attempted to extract such model and physics from a data derived from experiments on human body.

1.4 Pertinent past research

A study conducted by (Luciana, 2006) in which they tried to relate Mass-Spring-Damper System to human body by conducting a simulation. Their results were somewhat general that said this system can be applied on human body but numerical results were not provided.

In another study, the human body was treated as a combination of Mass-Spring-Damper System. Again, numerical values were not collected. (Ji and Bell, 2008).

CHAPTER 2 BACKGROUND AND LITERATURE REVIEW

2.1 Summary of the previous research

Subjects were exposed to minor impacts to the deltoid area of the shoulder administered by a pendulum specifically designed for this research. Energy, Velocity, and size of the striking object were the independent variables. The general experimental design was 3 x 3 x 3 factorial where design with gender and subject nested under gender included as blocking factors. Energy of the impact had 3 levels, Velocity of the impact had 3 levels, and size of the impacting object had 3 levels. The physical characteristics of each impact were recorded and the perception of the severity of the impact was collected. The physical measurements consisted of Force vs. time and Acceleration vs. time recordings were taken simultaneously as the pendulum impacted the subject. The pendulum position during impact (measured at 0.01 second of the impact time) was derived from the Acceleration vs. time data.(Alkhaledi, 2010).

Energy, Velocity, and Mass might all be considered the independent variables in this study but Energy, Velocity, and Mass cannot be independently assigned values. Once two of them were assigned a value, the third was determined. In this research Energy and Velocity were treated as the independent variables and Mass was determined to accommodate those values. (Alkhaledi, 2010).

Energy (E) was set at the 3 levels of 0.75, 1.125, and 1.5 joules. Velocity (V) was set at the 3 levels of 1.25, 1.5, and 1.75 m/s. These were consistent with those levels investigated by Wiley (2007). To achieve the desired Energy and Velocity levels, Mass (M) was adjusted. This required 9 levels of Mass. The impacting objects were wooden balls mounted on the front of the pendulum and had the 3 levels of 0.0254, 0.0318, and 0.0381 meter in diameter. (Alkhaledi, 2010).

The controlled variables were: location of impact on the shoulder, room temperature, and relative humidity. The deltoid muscles was chosen because it is not socially sensitive area to uncover and show during testing, it is easy to access and conduct testing on it and since this study was concerned about impact testing, the deltoid has a relatively uniform skin, muscle and bone tissues. Location of impact was controlled by selecting one location for the impact area on each shoulder and marking it with a ball point pen. Subjects were asked to maintain the mark between sessions. Room temperature and humidity were controlled by doing all the experiments in the same location. (Alkhaledi, 2010).

The subjects used in this research were recruited from a pool of volunteers from the University of Nebraska- Lincoln and the general population of Lincoln, Nebraska. A random sample of five males and five females subjects were recruited. The Subject's ages ranged from 21 to 35 years old.

The apparatus consisted of a data collection system, a pendulum with specifically designed weights, a load cell, an accelerometer. (Alkhaledi, 2010).

2.2 Modeling Methodology

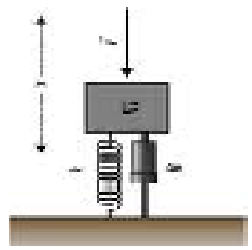

Figure 2.1: Mass-Spring-Damper System

Mass-Spring-Damper system showed in figure 2.1 is assumed by this research to represent human tissue. This system has a resistance against the movement. This resistance is represented by both spring resistance and damper resistance. Spring resistance is measured by the spring constant K and damper resistance is measured by the damper constant C.

To acquire these coefficients a recorded data of acceleration obtained when a pendulum hits the deltoid. **Deltoid muscle** is the muscle forming the rounded contour of the shoulder. This data was mathematically integrated twice in favor of obtaining the data of deformation. The deformation data indicates how the Mass-Spring-Damper system reacted when the movement occurred. A discreet model of ARMA was applied to this discreet data and later was converted to a continuous model A(n) which represents a second order differential equation system that is considered as Mass-Spring-Damper system.

The second order differential equation model of Mass-Spring-Damper was analyzed to obtain the natural frequency and the damping ratio.

2.3 Discrete second order autoregressive model ARMA(Pandit and Wu, 1983).

Mathematically, ARMA discrete model has the following general form (Pandit and Wu, 1983)

$$x_t = \Phi_1 x_{t-1} + \Phi_2 x_{t-2} + \cdots + \Phi_n x_{t-n} + a_t - \theta_1 a_{t-1} - \cdots - \theta_{n-1} a_{t-n+1} \quad (1)$$

Where

x_{t-1} = response discrepancy from the overall mean of all observations of sample points at time

t (i=1,2,……..,*n*),

Φ_i = autoregressive coefficient of lag *i* (lag being the number of observation interval from the

present observation towards the past observations),

a_{t-j} = disturbance or shock at time *t-j* (j=1, 2,…..,*m*),

θ_j= moving average coefficient of lag *j*.

Equation (1) is denoted by autoregressive moving average model (ARMA) or order *n, n-1*; i.e. ARMA (*n, n-1).*

The a_t is the white noise represented by the properties

$$\mathrm{E}(a_t) = 0, \quad \mathrm{E}(a_t a_{t-1})\delta_k\, \sigma_a^2$$

$$\delta_k = \begin{cases} 0 & k \neq 0 \\ 1 & k = 0 \end{cases} \tag{2}$$

Where E is the expectation operator and δ_k is the Kronecker delta.

Equation (1) can also be considered as the representation of a system governed with an *n*th order stochastic differential equation, with a white noise forcing function, sampled at uniform interval Δ. Then the characteristic roots of the difference equation model defined by

$$(1 - \Phi_1\, B - \Phi_2\; B^2 - \cdots - \Phi_n\, B^n) = (1 - \lambda_1 B)(1 - \lambda_2\, B) \ldots (1 - \lambda_n\, B) \tag{3}$$

Where

B= backshift operator $Bx_t = x_{t-1}$.

2.4 Second order autoregressive moving average model discrete ARMA(2,1)

This model which later will be used to obtain the variables ω_n which denotes the corresponding natural frequency and ζ which denotes the corresponding damping ratio. The formula of ARMA (2, 1) can be written where *n=2*

$$x_t - \Phi_1\, x_{i-1} - \Phi_2\, x_{i-2} = a_t - \Theta_1\, a_{t-1} \tag{4}$$

And for the autoregressive part

$$(1 - \Phi_1\, B - \Phi_2\; B^2) = (1 - \lambda_1 B)(1 - \lambda_2\, B) \tag{5}$$

That is,

$$\lambda_1 + \lambda_2 = \Phi_1$$

$$\lambda_1 \lambda_2 = -\Phi_2$$

Where λ_1 and λ_2 are the characteristic roots of the second order linear difference equation) given by

$$\lambda^2 - \Phi_1 \lambda - \Phi_2 = 0$$

$$\lambda_1, \lambda_2 = \frac{1}{2}(\Phi_1 \pm \sqrt{\Phi^2 + 4\Phi_2} \qquad (6)$$

The variables can be obtained using DDS technique.

2.5 The concept of the Green's function

The difference equation model are characterized by Green's Function (impulse response function). (Pandit and Wu, 1983)

The Green's Function can be obtained for ARMA(2,1) by two methods; implicit and explicit.

<u>The implicit method:</u>

$$x_t - \Phi_1 x_{t-1} - \Phi_2 x_{t-2} = a_t - \theta_1 a_{t-1}$$

$$(1 - \Phi_1 B - \Phi_2 B^2) x_t = (1 - \theta_1 B) a_t \qquad (7)$$

the formula of the Green's Function is obtained for the AR (1) and then we compare the coefficients

$$x_t = \sum_{j=0}^{\infty} G_j a_{t-j} = (\sum_{j=0}^{\infty} G_j B^j) a_t$$

Substituting this in Eq (7),

$$(1 - \Phi_1 B - \Phi_2 B^2)(\sum_{j=0}^{\infty} G_j B^j) a_t = (1 - \theta_1 B) a_t$$

Since a_t's are orthogonal, this gives the operator identity

$$(1-\Phi_1\ B-\ \Phi_2\ \ B^2\ )\ (G_0+G_1B+\ G_2B^2+\ldots\ldots)\equiv(1-B)a_t$$

If the coefficients of equal powers of B are equated

0: $G_0 = 1$

1: $G_1 - \Phi_1\ = -\theta_1 \Rightarrow G_1 = \Phi_1\ - \theta_1$

2: $G_2 - \Phi_1\ G_1 - \Phi_2\ = 0 \Rightarrow G_2 = \Phi_1^2 - \Phi_1\ \theta_1 + \Phi_2$

And

$G_j = \Phi_1\ G_{j-1} + \Phi_2\ G_{j-2,} \qquad j \geq 3$

That is,

$$(1-\Phi_1\ B-\Phi_2\ B^2)\ G_j=0 \quad j\geq 2 \tag{8}$$

In fact, it is more interesting in obtaining the explicit formula of the Green's Function, which will be used later on.

<u>The explicit method:</u>

$$(1-\Phi_1\ B-\ \Phi_2\ \ B^2\ )=(1-\lambda_1 B)(1-\lambda_2\ B)$$

$$x_t\ -\Phi_1\ x_{t-1}-\Phi_2\ x_{t-2}=\ a_t-\theta_1\ a_{t-1}$$

$$(1-\Phi_1\ B-\ \Phi_2\ \ B^2\ )\ x_t=(1-\theta_1 B)\ a_t$$

$$x_t=\frac{(1-\theta_1 B)}{(1-\Phi_1\ B-\ \Phi_2\ \ B^2\ )}\ a_t$$

Using the concept of partial fractions,

$$x_t=\left[\frac{(1-\frac{\theta_1}{\lambda_1})}{(1-\frac{\lambda_2}{\lambda_1})}\cdot\frac{1}{(1-\lambda_1 B)}+\frac{(1-\frac{\theta_1}{\lambda_2})}{(1-\frac{\lambda_1}{\lambda_2})}\cdot\frac{1}{(1-\lambda_2 B)}\right]a_t$$

$$=\left[\frac{(\lambda_1-\theta_1)}{(\lambda_1-\lambda_2\ )}\cdot\frac{1}{(1-\lambda_1 B)}+\frac{(\lambda_2-\theta_1)}{(\lambda_2-\lambda_1\ )}\cdot\frac{1}{(1-\lambda_2 B)}\right]a_t$$

$$G_j = \sum_{j=0}^{\infty} \left[\frac{(\lambda_1 - \theta_1)}{(\lambda_1 - \lambda_2\,)} \,.\, \lambda_1^j + \frac{(\lambda_2 - \theta_1)}{(\lambda_2 - \lambda_1\,)} \,.\, \lambda_2^j \right] a_{t-1}$$

Let

$$g_1 = \frac{(\lambda_1 - \theta_1)}{(\lambda_1 - \lambda_2\,)}$$

$$g_2 = \frac{(\lambda_2 - \theta_1)}{(\lambda_2 - \lambda_1\,)}$$

Then

$$G_j = g_1 \,.\, \lambda_1^j + g_2 \,.\, \lambda_2^j \tag{9}$$

In general formula of ARMA(*n, n-1*), the Green's Function can be written

$$G_j = g_1 \,.\, \lambda_1^j + g_2 \,.\, \lambda_2^j + \cdots + g_n \,.\, \lambda_n^j$$

Where distinctλ_i^j's can be calculated using

$$g_k = \frac{\lambda_k^{n-1} - \lambda_k^{n-2}\theta_1 - \lambda_k^{n-3}\theta_2 - \cdots - \theta_{n-1}}{\prod_{\substack{i=1 \\ i \neq k}}^{n} (\lambda_k - \lambda_{i1}} \tag{10}$$

2.6 Auto covariance function of ARMA (2, 1)

To obtain the expression for γ_k the Green's Function obtained Eq. (9) is used

$$G_j = g_1 \,.\, \lambda_1^j + g_2 \,.\, \lambda_2^j$$

Hence,

$$\gamma_k = E(x_t x_{t-k})$$

$$\gamma_k = E\left[\left(\sum_{i=0}^{\infty} G_j a_{t-I} \right) \left(\sum_{i=0}^{\infty} G_j a_{t-(k+j)} \right) \right]$$

$$\gamma_k = \left(\sum_{i=0}^{\infty} G_{j+j} G_j\right)\sigma_a^2$$

$$\gamma_k = \sigma_a^2 \sum_{i=0}^{\infty} \left(g_1 \,.\, \lambda_1^{k+j} + g_2 \,.\, \lambda_2^{k+j}\right)\left(g_1 \,.\, \lambda_1^{j} + g_2 \,.\, \lambda_2^{j}\right)$$

$$\gamma_k = \sigma_a^2 \sum_{i=0}^{\infty} \left[g_1^2 \,.\, \lambda_1^k \lambda_1^{2j} + g_2^2 \,.\, \lambda_2^k \lambda_2^{2j} + g_1\, g_2 . \lambda_1^j \lambda_2^j (\lambda_1^k + \lambda_2^k)\right]$$

$$\gamma_k = \sigma_a^2 \left[\frac{g_1^2}{(1-\lambda_1^2)} \,.\, \lambda_1^k + \frac{g_2^2}{(1-\lambda_2^2)} \,.\, \lambda_2^k + \frac{g_1\, g_2}{(1-\lambda_1 \lambda_2} (\lambda_1^k + \lambda_2^k)\right]$$

$$\gamma_k = \sigma_a^2 \left(\frac{g_1^2}{(1-\lambda_1^2)} + \frac{g_1\, g_2}{(1-\lambda_1 \lambda_2)}\right) . \lambda_1^k + \sigma_a^2 \left(\frac{g_2^2}{(1-\lambda_2^2)} + \frac{g_1\, g_2}{(1-\lambda_1 \lambda_2)}\right) . \lambda_2^k$$

Where

$$g_1 = \frac{(\lambda_1 - \theta_1)}{(\lambda_1 - \lambda_2\,)}$$

$$g_2 = \frac{(\lambda_2 - \theta_1)}{(\lambda_2 - \lambda_1\,)}$$

$$\gamma_k = d_1\, \lambda_1^k + d_2 \,.\, \lambda_2^k \tag{11}$$

Where

$$d_1 = \sigma_a^2 g_1 \left(\frac{g_1}{(1-\lambda_1^2)} + \frac{g_2}{(1-\lambda_1 \lambda_2)}\right)$$

$$d_2 = \sigma_a^2 g_2 \left(\frac{g_2}{(1-\lambda_2^2)} + \frac{g_1}{(1-\lambda_1 \lambda_2)}\right)$$

In particular,

$$\gamma_0 = d_1 + d_2 \tag{12}$$

2.7 Second order autoregressive continuous system A(2)

To formulate a continuous time second order autoregressive system, the start is with the homogeneous part.

$$(D^2 + \alpha_1 D + \alpha_0)x(t) = 0 \tag{13}$$

Where

$$\alpha_1 = 2\zeta\omega_n$$

$$\alpha_0 = \omega_n^2$$

Where

$$\omega_n^2 = \frac{K}{M}$$

$$\zeta = \frac{C}{2\sqrt{KM}}$$

Where K is the spring constant and C is the damping constant of the second order system of damped spring mass system.

The A (2) system equation is obtained by introducing the forcing function Z (*t*) as

$$(D^2 + \alpha_1 D + \alpha_0)x(t) = \mathrm{Z}(t) \tag{14}$$

$$\mathrm{E}[\mathrm{Z}(t)] = 0$$

$$\mathrm{E}[\mathrm{Z}(t)\mathrm{Z}(t-u)] = \sigma_z^2\delta(\mathrm{u})$$

Z (*t*) is defined by a covariance function with the Dirac delta $\delta(\mathrm{u})$. The property of Dirac delta is used to transmit from the discrete to the continuous system.

2.8 Dirac delta function and its properties

$$\delta(\mathrm{u})\begin{cases} = \infty, & \mathrm{u} = 0 \\ 0, & \mathrm{u} \neq 0 \end{cases}$$

And $$\int_{-\tau}^{\tau} \delta(\mathrm{u})\, du = 1$$

Also, $$\int_{-\infty}^{\infty} f(t-u)\, \delta(\mathrm{u}) du = f(t) = \int_{-\infty}^{\infty} f(u) \delta(\mathrm{t}-\mathrm{u}) du \qquad (15)$$

This function, called unit impulse, has played an important role in the analysis of physical systems since its use by Dirac in his work on quantum mechanics. Delta function has a relation with unit step function

$$s(t)\begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

The derivative of the unit step function is zero everywhere except at t=0, where it approaches infinity because of a discontinuity. The Dirac delta function is also zero everywhere except at t= 0, where it approaches infinity. Hence, the delta function can be considered as the derivative of the unit function, that is,

$$\delta(\mathrm{u}) = \frac{\mathrm{d}}{\mathrm{du}}\, s(t)$$

the integral of the step function is also defined as

$$r(t) = \int_{-\infty}^{t} s(u)\, du = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

This function is referred to as a 'unit ramp.' This is a continuous function, and it is easy to see that further integral of the unit ramp will also be continuous.

2.9 The Green's Function of the A(2) system

The non-homogeneous formula can be written

$$(D^2 + \alpha_1 D + \alpha_0)x(t) = \mathrm{Z}(t)$$

It can be rewritten

$$x(t) = (D^2 + \alpha_1 D + \alpha_0)^{-1}\,\mathrm{Z}(t)$$

$$x(t) = \int_0^{\infty} G(v)\,\mathrm{Z}(t-v)\mathrm{dv}$$

$$x(t) = \int_{-\infty}^{t} G(t-v)\,\mathrm{Z}(v)\mathrm{dv} \qquad (16)$$

For any continuous time forcing function$\mathrm{Z}(v)$.

Using the concept of delta function that the Green's Function $G(t)$ is defined by equivalent relations

$$(D^2 + \alpha_1 D + \alpha_0)G(t) = \delta(\mathrm{t}) \qquad (17)$$

$$G(t) = \int_0^{\infty} G(v)\,\delta(t-v)\mathrm{dv}$$

The non-homogeneous equation (17) can be solved by reducing it to a homogeneous equation with initial conditions. The initial conditions can be obtained by considering the continuity behavior of $G(t)$ and its derivative from equation (17). Since the delta function input is zero up to time t=0,

$$\acute{G}(t) = G(t) = 1 \qquad t < 0$$

At t = 0, $\acute{\acute{G}}(t)$, the second derivative of $G(t)$, contains the same discontinuity as that of a delta function. Therefore, $\acute{G}(t)$, which is the integral of $\acute{\acute{G}}(t)$, contains the same discontinuity as that of the integral of the delta function , which is a unit step function. Hence,

$$\acute{G}(0) = 1$$

Similarly, $G(t)$, the integral of $\acute{G}(t)$, behaves at t = 0 like the integral of the step fnction , which is the ramp *r(t)*. Therefore, like *r(t)*, $G(t)$ is continuous at t = 0 and hence

$$G(0) = 0$$

Thus, the non-homogeneous equation (17) is equivalent to the homogeneous equation

$$(D^2 + \alpha_1 D + \alpha_0)x(t) = \mathrm{Z}(t) \tag{18}$$

With initial conditions

$$G(0) = 0, \quad \acute{G}(0) = 1$$

Since the solution of the homogeneous equation is

$$G(t) = c_{1e^{\mu_1 t}} + c_{2e^{\mu_2 t}}$$

Where

$$(D^2 + \alpha_1 D + \alpha_0) = (D - \mu_1)(D - \mu_2) = 0$$

That is,

$$\mu_1, \mu_2 = \frac{1}{2}(-\alpha_1 \pm \sqrt{{\alpha_1}^2 + 4\alpha_0} = \omega_n(-\zeta \pm \sqrt{\zeta^2 - 1} \tag{19}$$

Substituting the initial conditions,

$$c_1 + c_2 = 0$$

$$c_1\mu_1 + c_2\mu_2 = 1$$

and this gives

$$G(t) = \begin{cases} \frac{e^{\mu_1 t} - e^{\mu_2 t}}{\mu_1 - \mu_2} & t \geq 0 \\ 0 & other\ wise \end{cases} \quad (20)$$

2.10 Auto covariance of the A(2)

Substituting $G(t)$ in Eq.(16),

$$\int_{-\infty}^{t} \frac{e^{\mu_1 t} - e^{\mu_2 t}}{\mu_1 - \mu_2} \mathrm{Z}(v)\mathrm{dv}$$

and the covariance function of the A(2) can be obtained

$$\gamma(s) = \sigma_z^2 \int_0^\infty G(v)G(v+s)dv$$

$$\gamma(s) = \frac{\sigma_z^2}{2\mu_1\mu_2(\mu_1^2 - \mu_2^2)}(\mu_2 e^{\mu_1 t} - \mu_1 e^{\mu_2 t})$$

$$\gamma(0) = -\frac{\sigma_z^2}{2\mu_1\mu_2(\mu_1 + \mu_2)} = -\frac{\sigma_z^2}{4\zeta\omega_n^3}$$

2.11 Obtaining the continuous model A(2) from the discrete model ARMA(2,1)

At the sampled points s = KΔ, k = 1,2,…….., the auto covariance $\gamma(s)$ equals

$$\gamma_k = \gamma(k\Delta)$$

$$= \frac{\sigma_z^2}{2\mu_1\mu_2(\mu_1^2 - \mu_2^2)}(\mu_2 e^{\mu_1 k\Delta} - \mu_1 e^{\mu_2 k\Delta})$$

$$= d_1\, \lambda_1^k + d_2\, .\lambda_2^k$$

Where

$$d_1 = \frac{\sigma_z^2}{2\mu_1(\mu_1^2 - \mu_{2)}^2}$$

$$d_2 = \frac{\sigma_z^2}{2\mu_2(\mu_1^2 - \mu_{2)}^2}$$

$$\lambda_1 = e^{\mu_1 \Delta}$$

$$\lambda_2 = e^{\mu_2 \Delta}$$

Hence,

$$\mu_1 = \frac{1}{\Delta} \ln \lambda_1$$

$$\mu_2 = \frac{1}{\Delta} \ln \lambda_2$$

When

$$\Phi^2 + 4\Phi_2 \geq 0$$

$$\zeta = \sqrt{\frac{[\ln(-\Phi_2)]^2}{[\ln(-\Phi_2)]^2 - 4\left[\cosh^{-1}\left(\frac{\Phi_1}{2\sqrt{-\Phi_2}}\right)\right]^2}}$$

$$\omega_n = \frac{1}{\Delta}\sqrt{\frac{[\ln(-\Phi_2)]^2}{4} - \left[\cosh^{-1}\left(\frac{\Phi_1}{2\sqrt{-\Phi_2}}\right)\right]^2}$$

When

$$\Phi^2 + 4\Phi_2 < 0$$

$$\zeta = \sqrt{\frac{[\ln(-\Phi_2)]^2}{[\ln(-\Phi_2)]^2 + 4\left[\cos^{-1}\left(\frac{\Phi_1}{2\sqrt{-\Phi_2}}\right)\right]^2}}$$

$$\omega_n = \frac{1}{\Delta}\sqrt{\frac{[\ln(-\Phi_2)]^2}{4} + \left[\cos^{-1}\left(\frac{\Phi_1}{2\sqrt{-\Phi_2}}\right)\right]^2}$$

In case of $\Phi^2 + 4\Phi_2 < 0$ the transformation of the parameters is nonunique because they involve the $\cos^{-1}$that has multiple values. Therefore, to obtain the parameters the following is followed.

$$a = -\frac{\ln(-\Phi_2)}{2\Delta}$$

$$b = \frac{-\ln(-\Phi_2)}{2\Delta}\sqrt{-(\Phi^2 + 4\Phi_2)}\left[\frac{2\Phi_1 - (1-\Phi_2)(\theta_1 + \frac{1}{\theta_1})}{2(1-\Phi_2^2) - \Phi_1(1+\Phi_2)(\theta_1 + \frac{1}{\theta_1})}\right]$$

$$\omega_n = \sqrt{(a^2 + b^2}$$

$$\zeta = \frac{a}{\omega_n}$$

2.12 Mass-spring-damper system analysis

By Newton's law we get the equation of the motion

$$f(t) - kx(t) - c\frac{dx(t)}{dt} = M\frac{d^2x(t)}{dt^2}$$

Or

$$\frac{d^2x(t)}{dt^2} + \frac{C}{M}\frac{dx(t)}{dt} + \frac{K}{M}x(t) = \frac{f(t)}{M}$$

Let

$$\omega_n^2 = \frac{K}{M}$$

$$\zeta = \frac{C}{2\sqrt{KM}}$$

ω_n is called the natural frequency and ζ the damping ration of the spring –mass-damper system. It can be rewritten the equation like

$$\frac{d^2x(t)}{dt^2} + 2\zeta\omega_n\frac{dx(t)}{dt} + \omega_n^2 x(t) = \frac{f(t)}{M}$$

This equation represents the forced vibration of a one-degree-of-freedom system subjected to the forcing function$f(t)$.

Using the Green's function obtained from A(2) allows us to solve the non-homogeneous equation with an arbitrary forcing function by expressing the solution as a convolution. The solution of the general second order non-homogeneous equation can be expressed as

$$x(t) = \int_0^\infty \frac{e^{\mu_1 t} - e^{\mu_2 t}}{\mu_1 - \mu_2} . \frac{1}{M} f(t - v)dv$$

when $\frac{f(t)}{M} = \delta(t)$,we have $x(t) = G(t)$ by the property of the delta function.

In our case, the importance is obtaining the system variables $\omega_n$and ζ and then the constant of the spring and the damper. This can be done by utilizing the analysis obtained from the continuous model A (2).

Obtaining a continuous model such as A(2) from the data by conditional regression has been used in many studies (Pandit and Wu, 1983). And there are many examples of physical systems for which the A (2) model can be conjectured considering the physics of a system. One such example is the Brownian motion of a particle suspended in a fluid.

This particle is constantly bombarded by the molecules in the liquid and traverses a path known as the Brownian motion, which can be represented by the A (2) system (Pandit and Wu 1983). The 'purely random' force due to the fluctuations in the number of molecular collisions can be represented by Z (t), which is the forcing function causing the Brownian displacement X (t). The frictional forces opposing the motion are proportional to the velocity and can be represented as a damping force by the term $2\zeta\omega_n \frac{dx(t)}{dt}$. Similarly, the elastic forces acting on the particle and proportional to the displacement are represented by the term $\omega_n^2 x(t)$.

Another example (Pandit, 2001), however, occurs in electrical circuits. The thermal motion of the electrons produces fluctuations n the current and voltage that are called 'thermal noise.' This thermal noise can be very closely represented by Z (t). When such a noise passes through a circuit containing a resistance, a capacitance, and an inductance, the output fluctuations can be represented by an A(2) system. Also, (Pandit 1985) stated in a paper that the DDS analysis of the voltage and current signal can be related to the characteristics of the power supply and the physics of the electro discharge phenomena.

Consequently, our assumption has a high level of reliability and viability to obtain the estimates of physical-mechanical characteristics of human tissue which would reveal hidden facts that can be used in the sector of ergonomics to make industries safer and more reliable.

2.13 The assumptions of this research

The estimation methodology used in this research functions under a number of assumptions.

- The first is that there is a valid deformation curve. The deformation curve is used to extract the variables of spring and damper constants. In order to get a valid deformation curve a further set of assumptions was made.
- The first was that the impacted material is homogenous.
- The reaction duo to the impact is the same throughout its depth.
- The impacted material must be stationary.
- It is assumed that the impacted tissue is healthy and the subject has the same health condition in all trails.
- The impacted material has the same consistency for each trial.
- The impacted material was of sufficient thickness that the penetration of the pendulum is constrained only by that material.
- The modeling methodology functions under the assumptions that the model used which consists of a parallel set of one damper and one spring is appropriate .

CHAPTER 3 STATEMENT OF THE PROBLEM

This research utilized data collected where humans were impacted with different energies and velocities. A continuous second order system that represents Mass-spring-damper system by fitting a discrete model ARMA(2,1) was applied to the data obtained from experiments in which the a discrete data of deceleration of a pendulum was recorded as it hit a human shoulder. The data of the deceleration was integrated twice to obtain the data of the displacement. The integration was done until the maximum displacement occurred (velocity = 0). The ARMA (2, 1) model was fitted using the displacement data using the DDS (Data-Dependent-System) technique. It was then converted to a continuous second order autoregressive model A(2) using the concept of Green's Function and the auto covariance. The A(2) model was used to obtain the physical characteristics of data extracted from the experiments. (Pandit 1985).

OBJECTIVES:

1 - Estimate the dampening constants for the deltoid area of human subjects based on impact to that area of the human body.

2 - Estimate the average, range, variance, and approximate distributions the estimates of those constants.

3 - Investigate the effects of different energies, velocities, and masses of the impacting objects on the estimates of those constants.

4 - Investigate the effects of differing body compositions on the estimates of those constants.

5 – Evaluate the possible causes of the relationships between the estimates and the physical characteristics of the impacts.

CHAPTER 4 METHOD

4.1 Data used

The data used in this research was derived from research conducted at the Department of Industrial and Management Systems Engineering at the University of Nebraska-Lincoln by a Ph.D. student ,(Alkhaledi, 2010), that was presented in the background above.

In this research ninety trails have been analyzed. Nine combinations of energy and velocity for ten subjects were used. The mass was determined for each trail according to the set of Energy and Velocity. The acceleration data during the impact was used. A discreet data of the displacement could be obtained by integrating the data of the acceleration two times. Once, the data of the displacement was obtained, extracting the damping characteristics could be done by using the method explained in the background above.

It was assumed that the external pendulum force can be represented by Z (t) which causes the displacement obtained X (t) by integrating the experimental deceleration twice. The elastic resistance of human shoulder is represented by the term$\omega_n^2 x(t)$. And the damping characteristics can be represented by the term$2\zeta\omega_n \frac{dx(t)}{dt}$. Moreover, the composition of the tissue of the shoulder was assumed to be homogenous.

4.2 Summary of the procedures

1- Data of acceleration obtained by the previous research was integrated twice in order to obtain the displacement data. The integration process was done utilizing the concept that the integration represents the area under the curve. Integrating the acceleration curve resulted in a velocity curve. Once the velocity curve was obtained, the concept of the area under the curve was repeated in order to obtain the displacement curve.

2- The principles of time series analysis were applied to the displacement data in order to obtain a discrete second order autoregressive model. This was done by running the data of displacement in the DDS program by which the variables of a discrete second order autoregressive model were obtained - ARMA (2.1).

3- The discrete second order autoregressive model was converted into a continuous second order autoregressive model A(2). This was done by using the assumption of equal auto covariance, and the concept of the Green's Function. As a result, a continuous second order autoregressive model was obtained.

4- When A(2) was obtained the principles of Mass-spring-damper system analysis can be applied. Consequently, physical characteristics of both damping ratio and natural frequency were acquired. The breakup of the damping ratio and natural frequency is the spring constant K and the damper constant C.

5- For each set of Energy, velocity, and mass the spring constant and damper constant were estimated.

6- Trends of these variables were studied under different levels of external impact stimuli. These external stimuli were the Energy of the impact, Velocity of the impact and the mass of the impacting pendulum.

7- Correlation analysis was done for the whole data of the estimate.

8- Correlation analysis was repeated for the average of the estimate.

9- Regression analysis was done to find significant variables of estimate of spring constant, damper constant.

CHAPTER 5 RESULTS

5.1 Estimates

Estimates of K and C were made for 9 trials for 10 individual subjects giving 90 data points. The individual estimates grouped by trial number are contained in Tables 5.1 through 5.9.

Table 5.1: K and C estimates from Trail Number 7 data.

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
7	1	0.75	1.25	0.667	15.597	1.662
7	2	0.75	1.25	0.667	15.606	1.753
7	3	0.75	1.25	0.667	15.628	1.677
7	4	0.75	1.25	0.667	16.196	1.666
7	5	0.75	1.25	0.667	15.611	1.810
7	6	0.75	1.25	0.667	16.725	0.591
7	7	0.75	1.25	0.667	16.892	0.508
7	8	0.75	1.25	0.667	16.183	0.909
7	9	0.75	1.25	0.667	15.626	1.873
7	10	0.75	1.25	0.667	15.689	1.470

Table 5.2: K and C estimates from Trail Number 8

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
8	1	0.75	1.5	0.489	11.613	1.239
8	2	0.75	1.5	0.489	11.684	1.044
8	3	0.75	1.5	0.489	12.555	0.373
8	4	0.75	1.5	0.489	11.583	1.313
8	5	0.75	1.5	0.489	11.623	1.164
8	6	0.75	1.5	0.489	11.987	0.710
8	7	0.75	1.5	0.489	11.606	1.251
8	8	0.75	1.5	0.489	11.593	1.485
8	9	0.75	1.5	0.489	11.728	0.978
8	10	0.75	1.5	0.489	12.622	0.332

Table 5.3: K and C estimates from Trail Number 9

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
9	1	0.75	1.75	0.375	8.918	0.855
9	2	0.75	1.75	0.375	8.832	0.898
9	3	0.75	1.75	0.375	9.862	0.142
9	4	0.75	1.75	0.375	9.959	1.086
9	5	0.75	1.75	0.375	9.661	0.217
9	6	0.75	1.75	0.375	9.788	1.003
9	7	0.75	1.75	0.375	9.135	1.553
9	8	0.75	1.75	0.375	9.227	1.642
9	9	0.75	1.75	0.375	9.736	0.202
9	10	0.75	1.75	0.375	9.016	1.434

Table 5.4: K and C estimates from Trail Number 16

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
16	1	1.125	1.25	1	23.650	2.837
16	2	1.125	1.25	1	23.728	2.356
16	3	1.125	1.25	1	24.197	3.961
16	4	1.125	1.25	1	23.638	2.746
16	5	1.125	1.25	1	23.654	3.004
16	6	1.125	1.25	1	24.009	1.839
16	7	1.125	1.25	1	23.695	2.437
16	8	1.125	1.25	1	23.639	2.876
16	9	1.125	1.25	1	23.631	2.749
16	10	1.125	1.25	1	23.682	2.497

Table 5.5: K and C estimates from Trail Number 17

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
17	1	1.125	1.5	0.734	18.213	0.910
17	2	1.125	1.5	0.734	17.853	1.172
17	3	1.125	1.5	0.734	17.583	2.664
17	4	1.125	1.5	0.734	18.498	1.123
17	5	1.125	1.5	0.734	17.445	1.715
17	6	1.125	1.5	0.734	18.438	0.736
17	7	1.125	1.5	0.734	18.256	0.888
17	8	1.125	1.5	0.734	19.422	4.029
17	9	1.125	1.5	0.734	17.482	1.633
17	10	1.125	1.5	0.734	17.487	1.609

Table 5.6: K and C estimates from Trail Number 1

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
18	1	1.125	1.75	0.562	15.291	0.343
18	2	1.125	1.75	0.562	13.374	1.279
18	3	1.125	1.75	0.562	13.321	1.332
18	4	1.125	1.75	0.562	13.592	2.208
18	5	1.125	1.75	0.562	13.662	0.915
18	6	1.125	1.75	0.562	14.407	0.430
18	7	1.125	1.75	0.562	13.434	1.177
18	8	1.125	1.75	0.562	15.129	0.121
18	9	1.125	1.75	0.562	13.374	1.279
18	10	1.125	1.75	0.562	13.337	1.727

Table 5.7: K and C estimates from Trail Number 25

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
25	1	1.5	1.25	1.333	31.550	3.435
25	2	1.5	1.25	1.333	31.560	3.361
25	3	1.5	1.25	1.333	31.644	3.130
25	4	1.5	1.25	1.333	31.510	3.779
25	5	1.5	1.25	1.333	31.498	3.698
25	6	1.5	1.25	1.333	32.172	2.909
25	7	1.5	1.25	1.333	31.518	3.583
25	8	1.5	1.25	1.333	31.779	4.626
25	9	1.5	1.25	1.333	31.504	3.767
25	10	1.5	1.25	1.333	31.579	3.280

Table 5.8: K and C estimates from Trail Number 26

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
26	1	1.5	1.5	0.979	23.434	1.973
26	2	1.5	1.5	0.979	23.560	3.361
26	3	1.5	1.5	0.979	23.389	2.055
26	4	1.5	1.5	0.979	23.164	2.695
26	5	1.5	1.5	0.979	23.195	2.479
26	6	1.5	1.5	0.979	23.164	2.767
26	7	1.5	1.5	0.979	23.477	1.917
26	8	1.5	1.5	0.979	23.765	1.607
26	9	1.5	1.5	0.979	23.296	2.179
26	10	1.5	1.5	0.979	24.483	1.902

Table 5.9: K and C estimates from Trail Number 27

Trail	Subject No	Energy	Velocity	Mass	Spring constant K	Damper Constant C
27	1	1.5	1.75	0.75	18.313	1.197
27	2	1.5	1.75	0.75	19.554	3.968
27	3	1.5	1.75	0.75	18.313	1.013
27	4	1.5	1.75	0.75	18.383	3.197
27	5	1.5	1.75	0.75	17.950	1.131
27	6	1.5	1.75	0.75	18.427	1.041
27	7	1.5	1.75	0.75	17.194	1.814
27	8	1.5	1.75	0.75	17.756	2.250
27	9	1.5	1.75	0.75	17.230	1.504
27	10	1.5	1.75	0.75	18.310	1.130

Overall the estimated values of the spring constant, K, vary from 8.832 to 32.172 with a range equal 23.339 for the 90 trails. Damper constant, C, values fluctuate from 0.121 to 4.626 with a range equal 4.500 for the 90 trails.

5.2 Average, Standard Deviation, and Range by trail

The averages, range, standard deviation of the estimates for K and C for each trail are given in the Tables 5.10 and 5.11.

Table 5.10: Average data with regard to Spring Constant K

Trail	Energy	Velocity	Mass	Average of Spring constant K	Std Dev K	Range K
7	0.75	1.25	0.666	15.975	0.497	1.294
8	0.75	1.5	0.489	11.859	0.402	1.038
9	0.75	1.75	0.375	9.413	0.429	1.126
16	1.125	1.25	1.000	23.75	0.192	0.565
17	1.125	1.5	0.734	18.068	1.977	1.977
18	1.125	1.75	0.562	13.892	0.765	1.970
25	1.5	1.25	1.333	31.631	0.208	0.673
26	1.5	1.5	0.979	23.493	0.3968	1.319
27	1.5	1.75	0.750	18.143	0.677	2.360

Table 5.11: Average data with regard to Damper Constant C

Trail	Energy	Velocity	Mass	Average of Damper Constant C	Std Dev C	Range C
7	0.75	1.25	0.666	1.392	0.519	1.364
8	0.75	1.5	0.489	0.989	0.395	1.152
9	0.75	1.75	0.375	0.903	0.561	1.943
16	1.125	1.25	1.000	2.730	0.547	3.105
17	1.125	1.5	0.734	1.648	1.009	3.292
18	1.125	1.75	0.562	1.081	0.645	2.086
25	1.5	1.25	1.333	3.557	0.470	1.716
26	1.5	1.5	0.979	2.293	0.526	1.753
27	1.5	1.75	0.750	1.825	1.021	2.955

The K values averaged over subjects for each trial varied from 9.413 to 31.631 with a range of 22.217 as compared to that of the individual values of 23.339.

The C values averaged over subjects for each trial varied from 0.903 to 3.557 with a range of 2.6535 as compared to that of the individual values of 4.626.

Note that the range for the average of the trial estimates for K is not considerably less than the overall range for the individual values. It is different for C where the range for the averages is almost half that of the individual values.

5.3 Correlation Analyses

The correlation values for K and C for all of the data are contained in Tables 5.12 and 5.13. Additionally, the absolute values are graphed in descending values in Figures 5.1 and 5.2. For all variables considered statistically significant with K, correlations values ranged from + 0.997 for Mass down to - 0.024 for Skin Fold thickness. Meanwhile, Mass was the only one variable deemed statistically significant with C and its correlation is + 0.787.

Table 5.12: Correlation(Row data) with Spring Constant K

Variable	**Correlation with Spring Constant K**	**P-Value**
Mass	**0.997**	**0.000**
Mass Seq	0.980	0.000
Energy * Mass	0.959	0.000
Velocity * Mass	0.933	0.000
Energy* Mass *Velocity	0.866	0.000
Energy	0.752	0.000
Energy Seq	0.748	0.000
Velocity	- 0.625	0.000
Velocity Seq	- 0.620	0.000
Energy * Velocity	0.369	0.000
Skin Fold	- 0.024	0.818
Skin Fold+ Muscle Thickness	- 0.015	0.887
Muscle Thickness	- 0.010	0.912

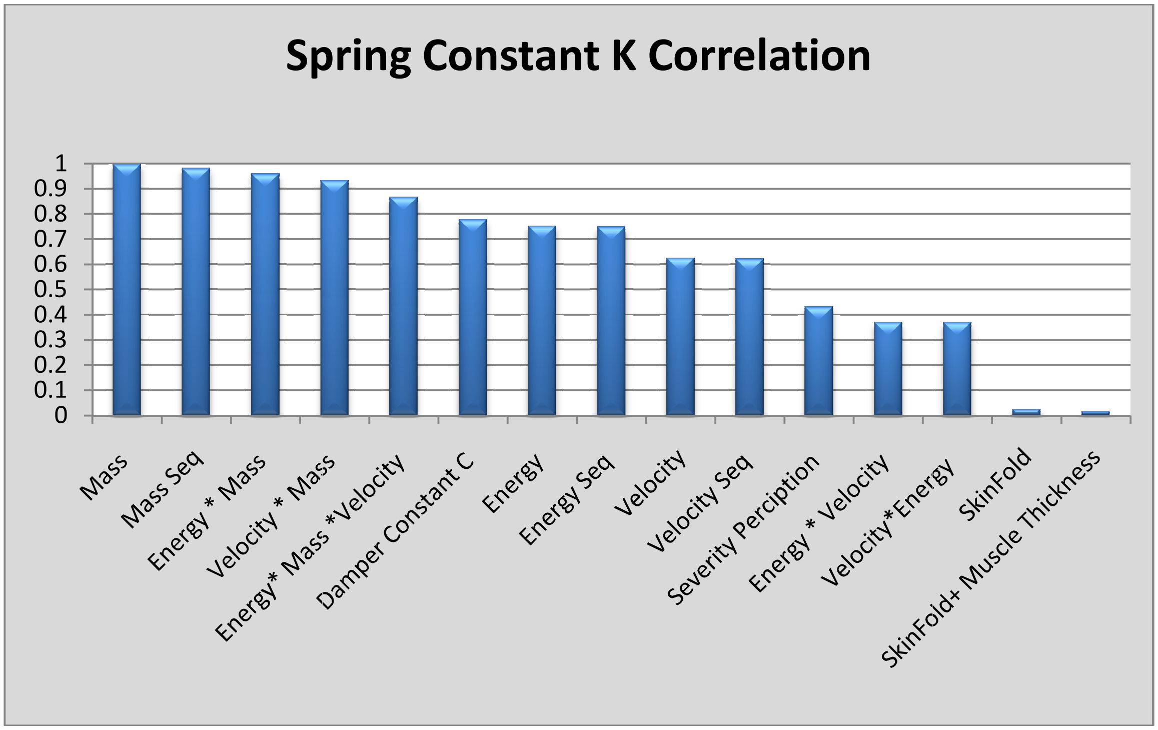

Figure 5.1: Absolute values of the Correlation values for the Spring Constants K using all of the data

Table 5.13: Correlation (Row data) with Damper Constant C

Variable	Correlation with Damper Constant C	P-Value
Mass	**0.787**	**0.000**
Mass Sq	0.785	0.000
Energy * Mass	0.751	0.000
Velocity * Mass	0.720	0.000
Energy* Mass *Velocity	0.665	0.000
Energy	0.567	0.000
Energy Seq	0.565	0.000
Velocity	- 0.500	0.000
Velocity Seq	- 0.493	0.000
Energy * Velocity	0.258	0.014
Skin Fold+ Muscle Thickness	0.079	0.458
Muscle Thickness	0.073	0.490
Skin Fold	0.072	0.496

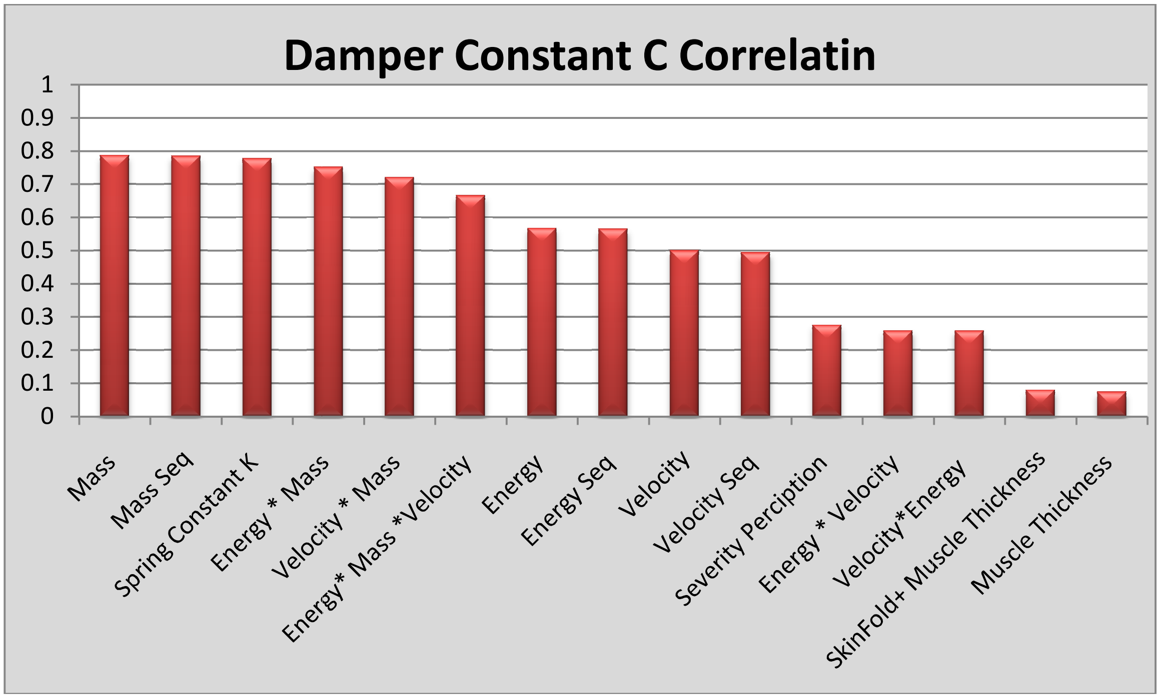

Figure 5.2: Absolute values of the Correlation values for the C using all of the data.

The correlation process was repeated for the averages of the constant estimates from Tables 5.10 and 5.11.

Table 5.14: Correlation (Average data) with Spring Constant K

Variable	Correlation with The Constant K
Mass	0.999
Mass Seq	0.983
Energy * Mass	0.962
Velocity * Mass	0.935
Energy	0.754
Energy Seq	0.750
Velocity	- 0.626
Velocity Seq	- 0.622
Energy * Velocity	0.370
Velocity*Energy	0.370
Skin Fold	- 0.191
Muscle Thickness	- 0.176
Skin Fold+ Muscle Thickness	- 0.137

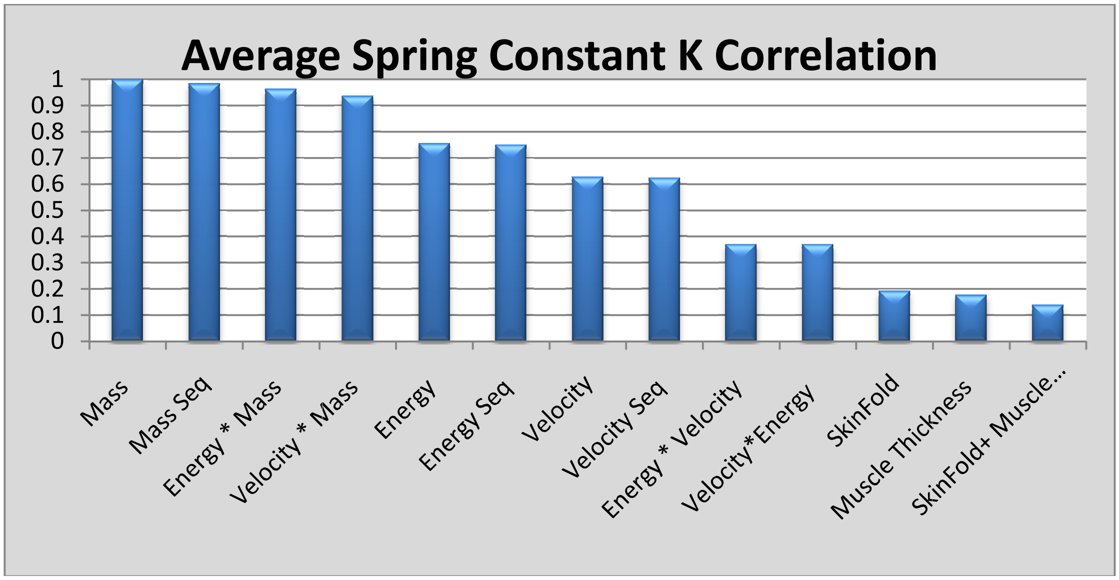

Figure 5.3: Absolute values of Correlation (Average data) with Spring Constant K

Table 5.15: Correlation (Average data) with Damper Constant C

Variable	Correlation with Damper Constant C
Mass	0.985
Mass Sq	0.983
Energy * Mass	0.940
Velocity * Mass	0.902
Energy	0.710
Energy Sq	0.708
Velocity	- 0.626
Velocity Sq	- 0.618
Skin Fold	0.456
Skin Fold+ Muscle Thickness	0.366
Energy * Velocity	0.323
Velocity*Energy	0.323
Muscle Thickness	0.283

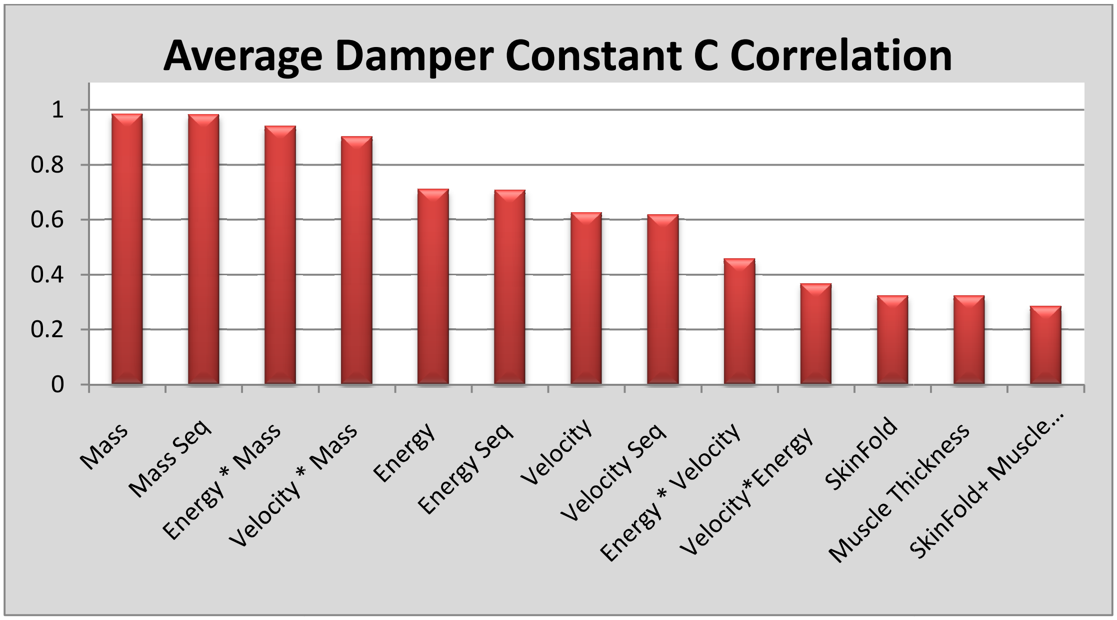

Figure 5.4: Absolute values of Correlation (Average data) with Damper Constant C

Correlation analysis indicated that the estimate of both the spring constant (K) and Damper constant have strong relationship with mass of the hitting object. This strong was directed in figure 5.5 and 5.6.

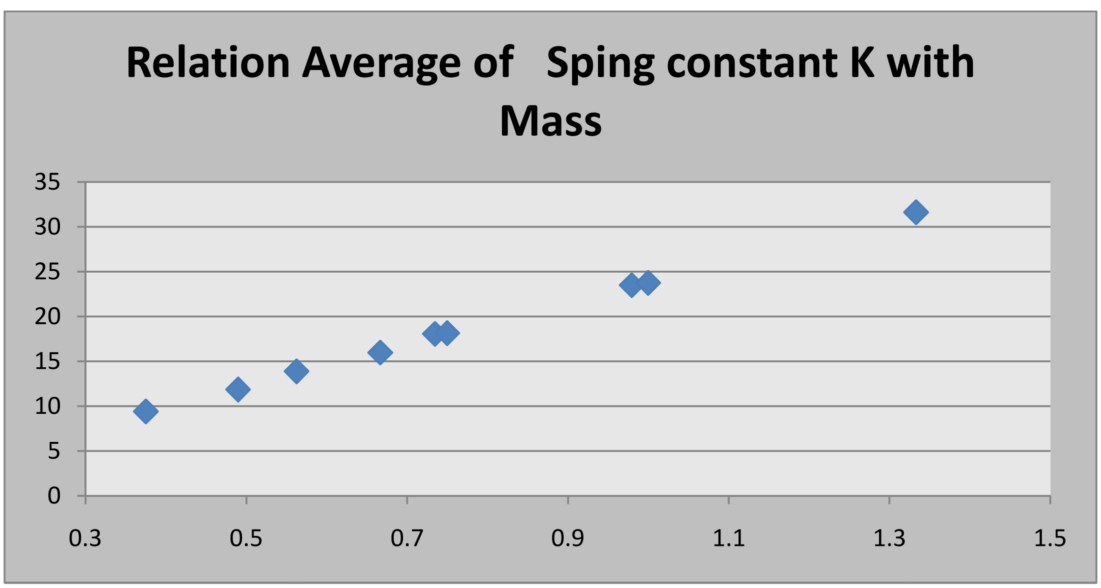

Figure 5.5: Depiction the relation between K and Mass

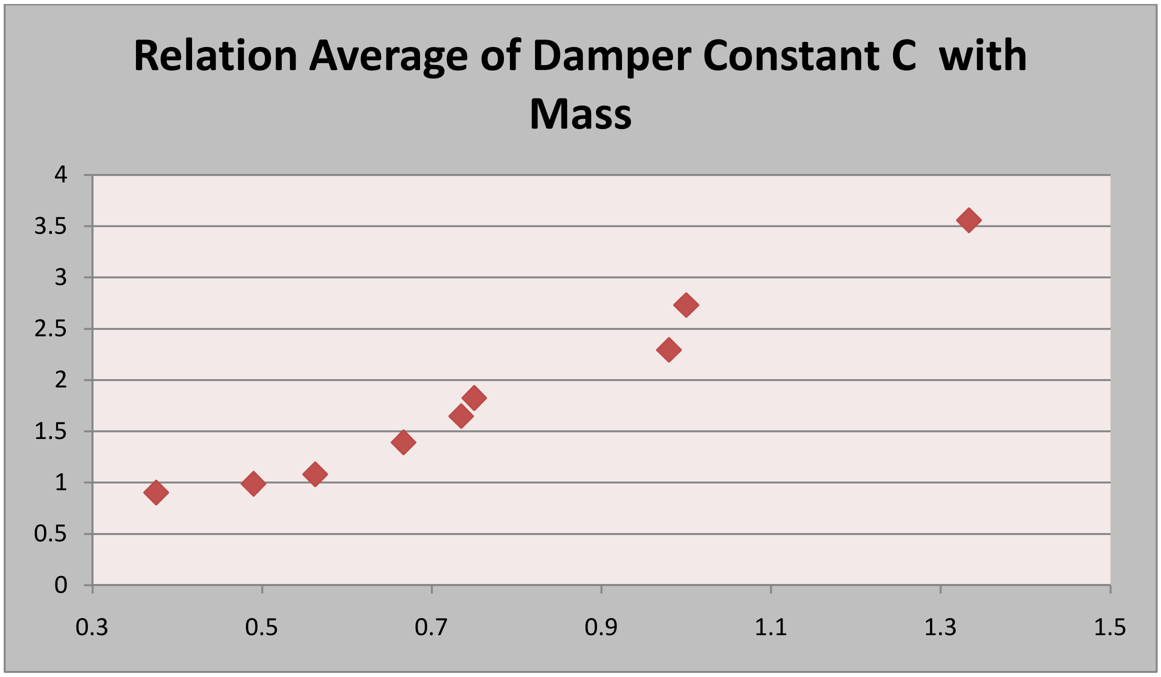

Figure 5.6: Depiction the relation between C and Mass

5.4 Stepwise regression

In addition to correlation analysis, a step-wise analysis has been conducted. Table 5.16 contains the variables available for inclusion in the stepwise analyses. Variables that were significant in both spring constant and Damper constant are contained in Table 5.17. Table 5.18 presents the variables included with the model R square values.

Table 5.16: A list of variables available for inclusion in the stepwise regression procedure.

Variables Available
Energy
Velocity
Mass
Skin Fold Thickness
Muscle Thickness
Skin Fold + Muscle Thickness
Velocity Square
Mass Square
Energy * Velocity
Energy * Mass
Velocity * Mass
Energy Square

Table 5.17: Stepwise regression results of spring and Damper Constants.

Model	**Regression of the Constant K**		**Regression of the Constant C**	
	Variables Entered	**Variables Not Entered**	**Variables Entered**	**Variables Not Entered**
1	Mass	Mass Sq	Mass	Mass Sq
2	Skin Fold Thickness	Energy * Mass		Energy * Mass
		Velocity * Mass		Velocity * Mass
		Energy		Energy
		Energy Sq		Energy Sq
		Velocity		Velocity
		Velocity Sq		Velocity Sq
		Energy * Velocity		Energy * Velocity
		Velocity*Energy		Velocity*Energy
		Muscle Thickness		Skin Fold Thickness
		Skin Fold+ Muscle Thickness		Muscle Thickness

Table 5.18: Variables included with the model R square values

Model Summary			
Spring Constant K		**Damper Constant C**	
Model	**R Square**	**Model**	**R Square**
1 Mass	0.994	1 Mass	0.619
2 Skin Fold Thick	0.991		

Twelve variables were available for inclusion in the step-wise analysis. Mass was significant in both spring and damper constant. Additionally, skin fold thickness was statistically significant in spring constant, K, which can be justified by the elasticity human skin has Daly (1982). As a result, when skin thickness increases it is expected to see an increase in spring constant. Step-wise analysis confirmed that Mass found to be statistically significant in both spring constant and damper constant.

The regression equations are:

$$K = + 0.695 + 23.21\ \text{Mass} + 0.0028\ \text{SFT}, \quad R^2 = 0.99$$

$$C = - 0.444 + 2.962\ \text{Mass}, \quad R^2 = 0.97$$

CHAPTER 6 DISCUSSION

The discussion chapter examines the estimates obtained, the variances of those estimates, the ranges of those estimates, the resulting correlation values, and the resulting regression relationships. This is followed by an examination of the results concerning what was expected and what was not expected with an effort to explain the possible reasons for the unexpected very high correlation values and subsequent regression analyses associated the estimated values of K and C and the mass of the pendulum.

6.1 Spring Constant – K

The estimates of K had an average of 18.470 with a variance of 42.872 and a range of 23.339 for all 90 estimates obtained. The values ranged from 8.833 to 32.172. This represents a ratio of 3.64 or a greater than threefold difference in estimates that are supposed to be estimating the same parameter, K, for the same material.

When the estimated values for K were averaged for each trial those averages had a range of 22.218. These averages ranged from 9.414 to 31.632. This represents a ratio of 3.36 which is still greater than a threefold difference and does not represent much difference from the values obtained for the individual values. The mass for the individual trials were all different so trial and mass in this analysis are representing the same thing.

The very wide range of the 90 estimates and the very wide range of the trial averages of these estimates are a concern. Supposedly, these estimates should be estimating the same parameter. The relationships between the estimates and the physical variables involved in the experimentation were examined. Mass was the variable with the highest correlation with K (0.997) and all of the five highest correlation values had mass as a component. When a stepwise regression was conducted, mass was the only variable included in the final model. Therefore, for this experimentation, the estimates were highly dependent on the mass of the impacting pendulum. This is problematic and the possible reasons for its occurrence are examined in more detail later in this chapter.

6.2 Damper Constant – C

The estimates of C had an average of 1.824 with a variance of 1.120 and a range of 4.504 for all 90 estimates obtained. The values ranged from .0122 to 4.626. This represents a ratio of 37.90 or a greater than thirty fold difference in estimates that are supposed to be estimating the same parameter, C, for the same material.

When the estimated values for C were averaged for each trial those averages had a range of 2.653. These averages ranged from 0.904 to 3.557. This represents a ratio of 3.936 which is still greater than a threefold difference but not the extreme of 37.91.

The very wide range of the 90 estimates and the very wide range of the trial averages of these estimates are of concern. Once again, supposedly, these estimates should be estimating the same parameter. The relationships between the estimates and the physical variables involved in the experimentation were examined. Mass was the variable with the highest correlation with C (0.787) and all of the five highest correlation values had mass as a component. When a stepwise regression was conducted, mass was the only variable included in the final model. Therefore, for this experimentation, the estimates were highly dependent on the mass of the impacting pendulum. As was the case with K, this is problematic and the possible reasons for its occurrence are examined in more detail later in this chapter.

The estimates of K and C, which represent the elastic resistance of the tissue, were found to be in agreement of the literature (Maurel et al., 1998). Maurel studied the mechanical properties of human tissues and the relationship between stress and strain is depicted in Figure 6.1. This curve can be divided into three stages. In the first stage (I), at low strain, collagen fiber response can be neglected and the elastin fibers are responsible for the skin stretching and the relation between stress-strain is approximately linear and the angle is very low. This implies that the elastic resistance is low when the stress applied is low.

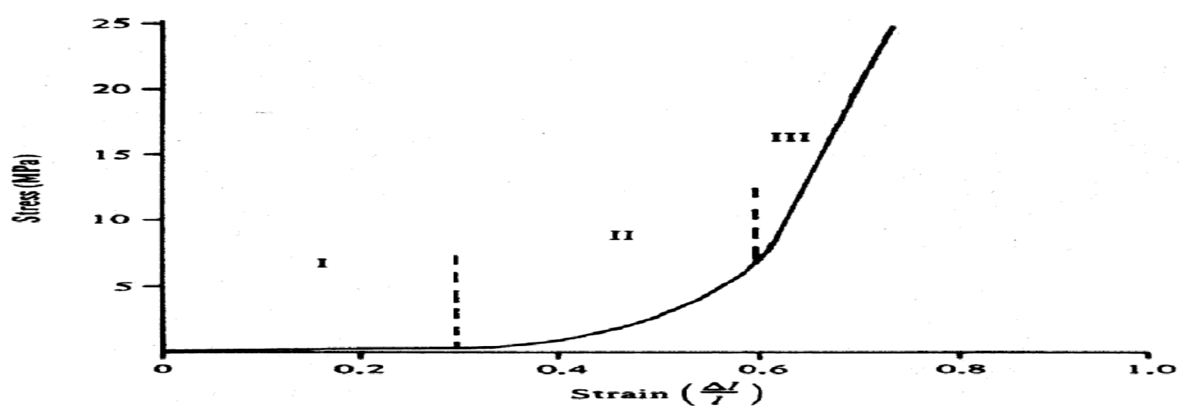

Figure 6.1 Stress-strain diagram for skin showing the different stages (Maurel et al., 1998)

In the second stage, (II), a gradual straightening of undulated collagen fibers causes an increase in skin tissue stiffness. Collagen fibers are the main components of the skin and they are strong and stiff. There is an intimate connection between the various skin layers. Collagen fibers are the major components of the dermis (77% of the fat-free dry weight) and form an irregular network of wavy coiled fibers which run parallel with the human skin surface (Finlay, 1969).

Collagen fibers have high strength (tensile strength of 1.5-3.5 Mega Pascal), low extensibility (rupture at strains in the order of 5-6%), and high stiffness (Young's modulus approximately 0.1 Giga Pascal (Manschot, 1985) to 1 Giga Pascal in the linear region (Maurel et al., 1998).

In the third stage (III), collagen fibers are straight and are at high levels of strain. The stress-strain relationship becomes linear again but with an angle is very steep. This means that elastic resistance of the skin is very high when the applied stress is high. In this investigation of both K and C, it was found that these estimates have a strong positive correlation with mass. It was found that the correlation between mass and k was 0.992 with P-value approaching zero and 0.787 with P-value is almost zero with C. Because these estimates represent the elastic properties of the tissue it means that when mass increases the elastic resistance increases. When the findings of Maurel et.al. are examined, stress can be broken up into its initial component which is (Force /Area). Area is a fixed variable in our estimate and the force can be represented by mass (F= m*a, Newton's law). As a result, it can be said that when mass increases, the elastic resistance increases and this was what the analysis of the estimate confirmed.

In phase (I) as shown in Figure 6.1, skin demonstrates elastic behavior, while in phase (II) and (III) skin shows visco-elastic behavior. It seems that when the mass is relatively small the tissue demonstrates elastic behavior in which the spring constant takes most of the responsibility protecting the inner tissue. And with larger masses it appears that the tissue shows visco-elastic behavior which may mean that a combination of the spring and the damper is resisting the impact. It seems reasonable that the estimates of K and C are explaining logically the finding found by Maurel et.al.

6.3 Analyses

The methods of analysis used to analyze the data obtained were – correlation analysis and stepwise regression. It was found that mass's relationships with both estimated values of spring and damper constants are very linear as is depicted in Figures 5.5 and 5.6. As mass increased, the spring constant increased. The same was true for the damper constant. The graphs show that Mass and the spring constant have an almost a perfect linear relationship (R= 0.999). This is also true for the damper constants (R= 0.985). This might imply that the tissue is increasing its elastic resistance to counter the stimuli caused by the increases in the mass of the impacting object. It is possible that the system of the underlying tissues might change their characteristics in order to absorb as much energy as it can to prevent more penetration into the tissue to avoid damage.

Correlation analysis showed that both the spring constant, K, and the Damper constant, C, were highly correlated with Mass. Stepwise regression showed that Mass was found to be a statistically significant predictor of both spring constant and damper constant. These results imply that mass is the most important variable affecting damping characteristics.

From Tables 5.12, 5.13 and 5.14 it is apparent that no personal factors (Skin- Fold, Muscle, and the Skin-Fold plus Muscle) were statistically significant for the correlation analyses, and the stepwise regression analyses. It was expected that these characteristics would affect the estimates. Therefore, these results might imply that the human tissues in the area tested might have similar damping characteristics regardless of the thickness of the skin or the muscle layers.

6.4 Rationale

It is apparent that for this data the mass of the impacting object affects damping characteristics of the tissue.

Mathematically, mass, energy, and velocity are components that represent the kinetic energy of the impacting object such that once two of the three are set, the other is determined. The experimental data used to estimate the constants had different levels of energy, velocity and mass. The mathematical relationship is that the kinetic energy of a moving object is equal to the mass times the velocity squared. For each trial mass was adjusted to achieve specified levels of energy and velocity. In the estimating process mass was the dominant factor. Energy and Velocity were highly correlated with the estimates of C and K but were secondary to Mass such that the models arrived at using stepwise regression, only Mass was included. Mass was utilized directly in the modeling process. Energy and Velocity were included indirectly as they would influence the characteristics of the acceleration curve and therefore the deformation curve. Increasing the mass may cause the tissues to alter their damping characteristics. From the estimates obtained, it appears that the limits of this change of dampening characteristics was not reached.

The estimation methodology used in this research functions under a number of assumptions. The first is that there is a valid deformation curve. The deformation curve is used to extract the variables of spring and damper constants. In order to get a valid deformation curve a further set of assumptions was made. The first was that the impacted material is homogenous and the reaction duo to the impact is the same throughout its depth. Moreover, the impacted material must be stationary. Also, it is assumed that the impacted tissue is healthy and the subject has the same health condition in all trails. The second assumption was that the impacted material have the same consistency for each trial. The third assumption was that the impacted material was of sufficient thickness that the penetration of the pendulum is constrained only by that material. This is assumption is aimed to eliminate the possibility of a wall effect. Additionally, the modeling methodology functions under the assumptions that the model used which consists of a parallel set of one damper and one spring is appropriate and a strategy of converting the discreet model to a continuous model using equal covariance is valid.

The estimates obtained have a wide range that is highly associated with mass. This may indicate that one or more of the assumptions given above are not met. Likely reasons for this are:

- Experimental method
    - Subject anticipation
    - Involuntary muscle contraction
- Modeling
    - Infinite material/Wall effect
    - Equal covariance
    - Appropriate model

6.4.1 Subject Anticipation. It is possible that the subjects anticipated the impact differently for different pendulum masses. The experimental methodology was such that the subjects observed the mass changes for each trial. As a result, the subjects may have anticipated a stronger impact when the mass was increased and therefore might have allowed them to unconsciously tense their muscle in the impact area. It is reasonable to expect higher resistance which translates to higher estimates of the spring constant and the damper constant for a more flexed or rigid muscle. Therefore, when the mass of the impacting object increases the subject's anticipation of the impact might unconsciously increase the stiffness of the muscle in the impacted area resulting in less deformation and therefore less estimated displacement.

6.4.2 Involuntary Muscle Contraction. The only active action a muscle can perform is to contract and shorten its length. Muscle fibers are enervated by motor nerves emanating from the spinal cord. Muscles can be enervated at a conscious level or on a reflex level. Reflexes require a minimum of two neurons, sensory neuron (input) and a motor neuron (output). Sensory neuron are sensitive to a wide variety of stimuli. Once stimulated, they send a signal toward the central nerves system (Robert. 2003). The sensory neuron synapses with a motor neuron which innervates the effecter tissue which could contract the muscle fibers. (Sanders and McCormick, 1993 , Jensen and Murray,2003).

Spinal reflexes happen very quickly. This is because they involve a few number of neurons and because the electrical signal is not required to travel to the brain and back. Spinal reflexes only go to the spinal cord and back. This is a much shorter distance and faster than traveling to the brain (Jensen and Murray,2003 , Winter, Sharon,2003). Additionally there is no brain processing time.

Therefore, it is possible that the larger the mass of the impacting pendulum the more reflexive action of the muscle to the impact. As a result, the relationship between the mass of the pendulum and the estimated constants might be attributed to this muscle reflex property which causes the muscle contract unconsciously and be more rigid resulting in an increase its resistance and therefore the estimates of the spring constant and damper constant.

6.4.3 Wall Effect. The experimentation used to estimate the spring and dampening constants used live human subjects. The thickness of the skin and muscle of these subjects varied from 0.5 to 1.4 mm and 2.2 to 4.9 mm respectively. This was supported by bone – a fairly rigid material. The estimating process assumes a homogenous material of sufficient depth without obstruction.(Fujii, Y. (2005 and Luciana,2006). Therefore, the amount of material impacted and the rigid backing may have affected the resulting estimates. Increasing the mass might make it more likely that the rigid bone backing would affect the estimates in such a way that the values increased as mass increased. The reasoning on this is that the limited depth of the elastic material and the presence of bone as a support material would cause the pendulum to penetrate less than would be expected with an increase in mass. It is not that the penetration was less, it is that the penetration was less than would be expected with an increase in mass. It can be that when the pendulum is confronted by a solid bone, the kinetic energy would be transferred to the bone, or even the whole body of the subject.

6.4.4 Equal Auto Covariance. A basic assumption of the modeling process used here is that the discreet second order autoregressive model and the continuous second order autoregressive which represents mass-spring-damper system have equal covariance**.** It can be that converting the discreet second order model into a continuous second order model of the displacement data based on the assumption of equal covariance could be a reason for having a wide range of estimates with high correlations with mass. It was stated in the literature concerning the use of this model that the estimates obtained should be used with caution and should be used to conceive trends and behaviors (Pandit and Wu 2001).

6.4.5 Appropriate Modeling. It was assumed that human tissue is similar to the system of one damper and one spring in parallel. It could be that the system hypothesized is not appropriate. It could be just a spring, just a damper, or a combination of a set of dampers and springs. Also, a complex combination of springs and dampers might represent the tissue better than the system used. Using a complicated model would require the development of a methodology different from the one used in this research. It might require a tissue to be subjected to more complicated testes. Therefore, it is possible that the model of a single spring and a single damper is not appropriate.

CHAPTER 7 SUMMARY AND CONCLUSIONS

7.1 Summary

This research has been conducted to study and estimate the damping characteristics of human soft tissue. Reviewing the available literature revealed that there is a little knowledge regarding damping characteristics of human tissue.

This research utilized data collected where human deltoid muscles were impacted with different energies and velocities. Different energy and velocity combinations were in part achieved by varying the mass of the pendulum. A continuous second order system that represents Mass-spring-damper systems by fitting a discrete model ARMA(2,1) was applied to the data obtained from experiments in which the a discrete data of the deceleration of a pendulum was recorded as it impacted a human shoulder. The data of the deceleration recorded was integrated twice to obtain the data estimates of the displacement. The integration was done until the maximum displacement occurred (velocity = 0). The discreet ARMA (2, 1) model was fitted using the displacement data and using the DDS (Data-Dependent-System) technique. It was then converted to a continuous second order autoregressive model A(2) using the concept of Green's Function and the auto covariance. The A(2) model was used to obtain the physical characteristics of data extracted from the experiments. (Pandit 1985).

The continuous second order model obtained considered as mass-spring-damper system to represent the resistance the tissue makes when subjected to an impact. (Pandit 1985).

The external pendulum force can be represented by Z(t) which causes the displacement obtained X(t) by integrating the experimental deceleration twice. The elastic resistance of human tissue is represented by the term $\omega_n^2 x(t)$. And the damping characteristics can be represented by the term $2\zeta\omega_n \frac{dx(t)}{dt}$.

Utilizing the principles of a mass-spring-damper system enabled the estimation of the spring constant (K) and damper constant (C) for each trail of the experiment. Correlation and stepwise regression analysis were used to study the trends of the numerical estimates obtained of damping characteristics K and C.

7.2 Conclusions

Estimated constants for C and K varied from 0.903 to 3.557 and 9.413 to 31.631 respectively.

No past estimates for C and K were found in the literature so it is impossible to determine if these estimates are reasonable or not.

The estimates varied very widely which is concerning. The possible reasons for these large variations were explored in the preceding section. Any of them individually, or in combination, may be responsible for this wide variation.

The mass of the impacting object was the dominant variable in determining the estimates of the spring and dampening constants.

The methodology utilized to estimate the constants did not consider either the size or shape of the impacting object. This may be a serious shortcoming of the methodology.

Estimating the dampening constants of human tissue is problematic. Humans are not homogenous – they differ in composition. They are alive and react in ways that might affect the results. In this experimentation, the subjects might have moved, anticipated the impact, or had a reflex action that changed the resulting data collected. Any of these could affect the resulting estimates. Furthermore, the depth of the tissue and the fact that it is backed by rigid material, bone, would affect the estimates and would affect those situations involving the most deformation or penetration. Finally, the tissues impacted were composed of two layers that were not consistent in thickness from subject to subject. This research essentially assumed homogenous material.

CHAPTER 8 LIMITATIONS AND FUTURE STUDY

8.1 LIMITATIONS

It was of the importance to study the effect of the size of the impacting head but due to the use of only one ball size, size was not taken into account. Also, the size of the impacting object was not included in the model and the size can be studied by repeating the analysis using the same model. This is one of the limitations in this research. Also, the levels of energies and velocities used were restricted to a narrow range. Moreover, the impacting spot was fixed in one area. There could be different results since the tissue might be different. Additionally, due to the use of live subjects this research encountered problems that might have affected the results. For instance, subjects must have anticipated the impact, had unconscious reflex, and moved. For this reason, these difficulties had made the research limited to human reaction which might have interfered in the results. Finally, it was of the interest to study the effect of the shape of the impacting object but it was not addressed in this research.

8.2 FUTURE STUDY

Future studies ought to address the limitations explained above. However, relationships between damping characteristics and clinical needs should be studied. In addition, it is highly recommended a future study using different set of spring and damper. For instance, a series set of spring and damper or a combination of more than one spring and/or one damper.

Furthermore, it is recommended the effect of the size of the impacting object be investigated. One way to do this is by repeating the method of analyses using data derived from different ball size. Also, it is recommended to address the effect of the shape of the impacting object.

In addition, it is recommended to study these estimate before the point of zero velocity. This would be lucrative and gives more understanding to what affect these estimates.

Finally, more studies, better guidelines and safety precautions can add important knowledge to have a safer work place and more comfortable work environment for workers. And also additional studies would be key of ergonomists and scientists developments and to better understand of damping characteristics of human tissue.

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