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Physics, Chapter 10: Momentum and Impulse

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10

Momentum and Impulse

10-1 Momentum

An extremely important concept in the development of mechanics is that of *momentum*. *The momentum of a body is defined as the product of its mass by its velocity.* We shall use the symbol \mathbf{p} to denote the momentum of a body. The momentum of a body is a vector quantity, for it is the product of mass, a scalar, by velocity, a vector. While momentum and kinetic energy are compounded of the same two ingredients, mass and velocity, they are quite different concepts, and one aspect of their difference may be seen in the fact that momentum is a vector while energy is a scalar quantity.

Newton himself recognized the importance of momentum as a mechanical concept, for a free translation of his second law of motion expressed in modern terms would read: *the rate of change of momentum is proportional to the net force and is in the direction of that force.* Expressed in the form of an equation, Newton's second law would read

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}. \quad (10-1)$$

In our discussion of Newton's second law in Chapter 5, we treated mass as a constant and obtained the result

$$\mathbf{F} = m \frac{d\mathbf{v}}{dt} = m\mathbf{a}.$$

This form of Newton's second law is true for most problems in mechanics, when the speed of the body is small in comparison with the speed of light. Newton's original formulation, as represented in Equation (10-1), remains correct even for bodies which travel at speeds approaching the speed of light, when, according to Einstein's special theory of relativity, the mass of a body may be expressed as

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad (10-2)$$

where m_0 is the mass of the body at rest, v is its speed, c is the speed of light (3×10^{10} cm/sec), and m is the mass of the moving body. For our present purposes we shall not digress further into the relativistic aspects of mechanics, but shall focus our attention upon Equation (10-1).

From Equation (10-1) we see that if an unbalanced force is applied to a body, its momentum will change at a rate determined by the force. If the unbalanced force acting on a body is zero, the change of momentum is zero; that is, the momentum of the body remains constant. Thus, in the design of space ships for interplanetary flight, consideration must be given to the provision of fuel to accelerate the ship and to decelerate it for landing at the destination, but no fuel need be provided for propelling the ship over the major portion of its path, for, acted on by no appreciable external forces, the momentum of the ship will remain substantially constant.

Our study of mechanics has thus far concentrated its attention on the motion of a particle when acted upon by a force. Suppose we consider a stream of particles of mass m , each moving with velocity v , that strike a target and come to rest in it, and inquire about the average force exerted on the target to hold it in place. From another point of view we may ask what force the particles exert on the target. Each time a particle is stopped by the target, the momentum of the particle is changed from mv to zero. The change in momentum of the particle is mv . If n particles strike the target in each second, the average rate of change of momentum, that is, the change in momentum per second, is

$$\frac{\Delta p}{\Delta t} = nmv.$$

Rewriting Equation (10-1) in incremental form, we see that the average rate of change of momentum is the average force. Thus we have

$$\bar{F} = \frac{\Delta p}{\Delta t}, \quad (10-3)$$

so that

$$\bar{F} = nmv. \quad (10-4)$$

Illustrative Example. A pitcher throws baseballs at a target mounted on a helical spring at the rate of one ball every 2 sec. The baseballs strike the target at a speed of 80 ft/sec and come to rest in it. What is the average force exerted by the baseballs against the target? A baseball weighs 5 oz.

When the baseball collides with the target, all of the momentum of the ball is absorbed by the target. Each ball has a momentum of

$$\begin{aligned} p = mv &= \frac{5}{16} \times \frac{1}{32} \text{ slug} \times 80 \frac{\text{ft}}{\text{sec}} \\ &= 0.78 \frac{\text{slug ft}}{\text{sec}}. \end{aligned}$$

Since one ball strikes the target every 2 sec, the number of balls striking the target per second is $n = \frac{1}{2}$. Substituting in Equation (10-4), we have

$$\begin{aligned}\bar{F} &= nmv \\ &= \frac{1}{2} \times 0.78 \frac{\text{slug ft}}{\text{sec}^2},\end{aligned}$$

or
$$\bar{F} = 0.39 \text{ lb.}$$

Illustrative Example. A stream of water 10 cm² in area, moving horizontally with a speed of 25 m/sec, strikes the wall of a house and splatters to the ground, losing all of its forward motion. What is the force exerted on the wall of the house by the stream of water?

If Q is the volume of water that strikes the wall of the house per second, then

$$Q = Av,$$

where A is the cross-sectional area of the stream, and v is its velocity. If the water is of density ρ , the mass of water striking the house in each second is

$$m = Q\rho.$$

The water has momentum $mv = Q\rho v$ in the horizontal direction before striking the wall, and zero momentum in this direction afterward, so that the change of momentum in each second is

$$\frac{\Delta p}{\Delta t} = Q\rho v = Av^2\rho.$$

From Equation (10-3) we have, for the force in the horizontal direction,

$$\bar{F} = \frac{\Delta p}{\Delta t} = Av^2\rho. \quad (10-5)$$

Substituting numbers into Equation (10-5), we find

$$\begin{aligned}\bar{F} &= 10 \text{ cm}^2 \times (2,500)^2 \frac{\text{cm}^2}{\text{sec}^2} \times 1 \frac{\text{gm}}{\text{cm}^3} \\ &= 625 \times 10^5 \frac{\text{gm cm}}{\text{sec}^2} \\ &= 6.25 \times 10^7 \text{ dyne.}\end{aligned}$$

10-2 Impulse

In many mechanical problems the applied force is not steady, nor can the force be described in terms of simple mathematical functions. When a baseball bat strikes a ball, the force the bat exerts against the ball is zero at the initial instant of collision, then rises to some maximum value when the ball is violently deformed, and finally returns to zero when the ball leaves the bat. The behavior of materials and structures under such *impulsive*

forces is quite different from their behavior when subjected to steady forces, and we speak of materials as being brittle when they are not capable of withstanding impulsive loading. An impulsive force, such as that exerted by a baseball against a bat, might be described by the graph in Figure 10-1(a).

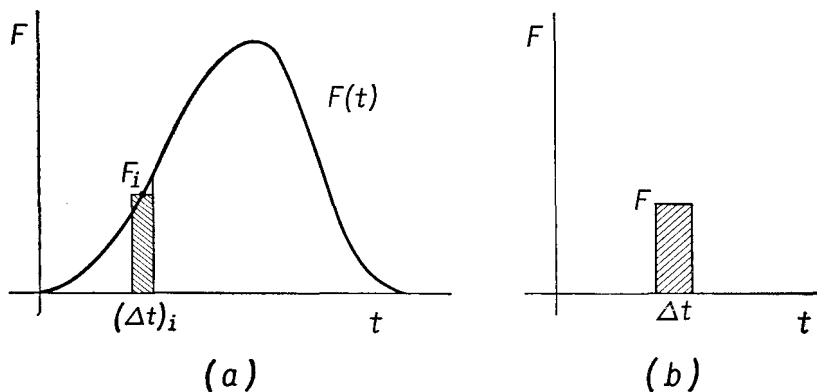


Fig. 10-1 Impulsive force.

In our earlier discussion of work and energy, we have seen the usefulness of considering the effect of a force acting through a distance. From such an analysis we derived our understanding of the concept of *work*, and we saw that the result of doing work on a particle was to change its *energy*. Another way to consider the effect of a force on a body is to study the effects produced when a force acts for a *time* interval. The product of a force by the time interval during which the force acts is called the *impulse*. When a force \mathbf{F} acts for a time interval Δt , the impulse $\Delta \mathbf{J}$ is given by the formula

$$\Delta \mathbf{J} = \mathbf{F} \Delta t. \quad (10-6)$$

We see that *impulse is a vector quantity*, for it is given by the product of force, a vector, by time, a scalar. We shall show that the effect of an impulse acting upon a particle is to produce a change in its *momentum*.

Let us consider the incremental form of Newton's second law, as given by Equation (10-3). We have

$$\bar{\mathbf{F}} = \frac{\Delta \mathbf{p}}{\Delta t}.$$

If we multiply both sides of this equation by the time interval Δt and call the force acting in this time interval F , we find

$$F \Delta t = \Delta p,$$

and, substituting from Equation (10-6), we have

$$\Delta J = \Delta p, \quad (10-7a)$$

which relates the magnitude of the impulse to the magnitude of the change in momentum. In vector form this equation becomes

$$\Delta \mathbf{J} = \Delta \mathbf{p}. \quad (10-7b)$$

Illustrative Example. A body of mass 10 gm moves along the x axis with a speed of 3 cm/sec. A force of 400 dynes is applied in the positive y direction for a time interval of 0.1 sec. Find the velocity of the particle produced by the impulse.

Since the impulse is applied in the y direction, there is no change in the x momentum of the particle. The impulse in the y direction is

$$\begin{aligned} \Delta J_y &= 400 \text{ dynes} \times 0.1 \text{ sec} \\ &= 40 \text{ dyne sec.} \end{aligned}$$

From Equation (10-7) we have

$$\Delta p_y = 40 \text{ dyne sec.}$$

Since the momentum in the y direction was initially zero, the final momentum in the y direction is equal to 40 dyne sec, and we have

$$\begin{aligned} \Delta p_y &= mv_y = 40 \text{ dyne sec,} \\ v_y &= \frac{40 \text{ dyne sec}}{10 \text{ gm}} \\ &= 4 \frac{\text{cm}}{\text{sec}}. \end{aligned}$$

Thus the body has a velocity of 4 cm/sec in the y direction and a velocity of 3 cm/sec in the x direction. Its resultant velocity is therefore 5 cm/sec, directed at an angle of 53° with the x axis into the first quadrant.

When the impulsive force is given by a simple rectangular pulse, as in Figure 10-1(b), the evaluation of the impulse is simple and straightforward. When the impulsive force is given as an arbitrary function of time, as in Figure 10-1(a), we may follow the procedures of the integral calculus and imagine the graph of $F(t)$ to be divided into a number of rectangular pulses of different heights, in which each impulse serves to change the momentum by a small amount. The total impulse is the area under the curve. To find the over-all effect we add the changes in momentum due to each impulse and write

$$\begin{aligned} J &= \int_0^J dJ = \int_0^t F dt = \int_{p_i}^{p_f} dp, \\ J &= p_f - p_i. \end{aligned} \quad (10-8a)$$

Thus, when an arbitrary impulsive force strikes a body, the impulse is equal to the difference between the final momentum \mathbf{p}_f and the initial momentum \mathbf{p}_i . Although Equation (10-8a) was derived in scalar form, it is clear that we may consider Equation (10-8a) as an equation in one unspecified component of the more general vector equation

$$\mathbf{J} = \mathbf{p}_f - \mathbf{p}_i. \tag{10-8b}$$

In general, it is quite difficult to measure an impulse, but it is easy to observe a change in momentum. Thus the difference in momentum between a pitched baseball and a batted ball may be used to measure the impulse of the bat against the ball.

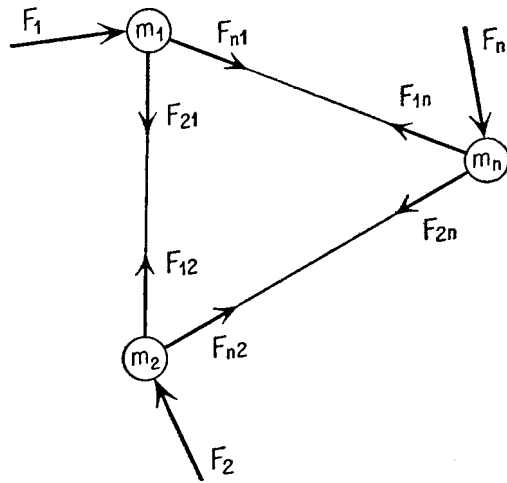


Fig. 10-2 System of particles.

10-3 Systems of Particles

Let us consider a system of particles such as the particles m_1, m_2, \dots, m_n of Figure 10-2, and let us suppose that the forces the particles of the system exert on one another are always directed along the line joining them, in the manner of gravitational forces, or of the forces which might be exerted by strings. We call the forces which one particle of the system exerts on another particle of the system *internal forces*. The forces exerted on the particles of the system from outside the collection of particles are called *external forces*.

The division of the forces on the particles of a collection into internal forces and external forces is due to the French physicist Jean le Rond d'Alembert (1717-1783). Let us call \mathbf{F}_{ab} the force exerted by *particle a on particle b*. By Newton's third law the force exerted by *a on b* must be equal in magnitude and opposite in direction to the force exerted by *b on a*

a. Forces with *double subscripts* will always denote *internal* forces, exerted by one member of the system on another member of the system, while forces with *single subscripts* will always denote the *external* force on the particle indicated by the subscript.

Applying Newton's second law to each of the particles of the collection in turn, we find

$$\begin{aligned}\mathbf{F}_1 + \mathbf{F}_{21} + \cdots + \mathbf{F}_{n1} &= \frac{d\mathbf{p}_1}{dt}, \\ \mathbf{F}_2 + \mathbf{F}_{12} + \cdots + \mathbf{F}_{n2} &= \frac{d\mathbf{p}_2}{dt}, \\ \cdots & \\ \mathbf{F}_n + \mathbf{F}_{1n} + \mathbf{F}_{2n} + \cdots + \mathbf{F}_{(n-1)n} &= \frac{d\mathbf{p}_n}{dt}.\end{aligned}$$

Adding these equations, and remembering that $\mathbf{F}_{12} = -\mathbf{F}_{21}$, and so on, we find that

$$\mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n = \frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_n).$$

Thus *the vector sum of all the external forces acting on the system of particles is equal to the rate of change of the total momentum of the system of particles.* If \mathbf{p} is the sum of the momenta $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \cdots + \mathbf{p}_n$, and \mathbf{F} is the sum of the external forces acting on each of these particles $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_n$, we may write

$$\boxed{\mathbf{F} = \frac{d\mathbf{p}}{dt}}. \quad (10-9)$$

In the special case in which $\mathbf{F} = 0$, Equation (10-9) shows that \mathbf{p} is constant.

Thus *if a system of particles is acted on by no external force, the total momentum of the system of particles is constant.* This statement is known as the *principle of conservation of momentum*; it is of very great importance in the analysis of mechanical systems. The principle of conservation of momentum, like the principle of conservation of energy, is valid in all realms of physics, from subatomic to astronomical.

A theorem which is extremely useful in understanding the behavior of many mechanical systems, and one which is very easily proved, states that *the momentum of a system of particles is equal to the product of the mass of the entire system by the velocity of its center of mass.* To prove the theorem

we recall that the coordinates of the center of gravity of a system of particles are given by Equations (4-4) as

$$x_0 = \frac{w_1x_1 + w_2x_2 + \cdots + w_nx_n}{w_1 + w_2 + \cdots + w_n}, \quad (4-4a)$$

and similarly for the y coordinate of the center of gravity y_0 and the z coordinate z_0 . If we substitute $w = mg$ into the above equation and clear it of fractions, we find

$$(m_1 + m_2 + \cdots + m_n)x_0 = m_1x_1 + m_2x_2 + \cdots + m_nx_n.$$

Thus x_0 is the *center of mass* of the system of particles; it coincides with the center of gravity in any region where g is constant. Let us differentiate both sides of the equation with respect to time to obtain

$$(m_1 + m_2 + \cdots + m_n) \frac{dx_0}{dt} = m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt} + \cdots + m_n \frac{dx_n}{dt},$$

which proves the theorem. The left-hand side of the equation is the mass of the system of particles times the x component of the velocity of the center of mass, while the right-hand side of the equation is the sum of the x components of the momenta of the particles of the system. Since analogous equations may be obtained immediately for the other two components of the momentum, we may combine these results into a single vector equation:

$$M\mathbf{v}_0 = \mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_n, \quad (10-10)$$

where M is the total mass of the system of particles and \mathbf{v}_0 is the velocity of its center of mass.

Let us examine the implications of Equation (10-10). If the sum of the external forces acting on a system of particles is zero, we see from Equation (10-9) that the momentum of the system must remain constant. From Equation (10-10), therefore, the velocity of the center of mass of the system must remain constant. If a shrapnel shell explodes in mid-air, the trajectory followed by the center of mass will be the same as it would have been if the shell had not burst, and the total momentum which the shrapnel fragments acquire in a direction normal to the path of the unexploded shell must be zero, as it was before the explosion, for the forces involved in the explosion are all internal forces. When a gun fires a shell, we observe that the center of mass of the system consisting of gun, shell, and explosive gases was at rest before the firing; hence the center of mass of this system must be at rest after the firing. If the projectile has acquired forward momentum, the gun must acquire an equal backward momentum; it must recoil.

10-4 Conservation of Momentum

The impact between two isolated bodies in space may be most easily understood in terms of the principle of conservation of momentum. In addition, many problems of propulsion may be most easily understood in terms of momentum conservation.

Consider the problem of an airplane moving through the air. We may think of the system consisting of the airplane and a volume of air around it as constituting a region isolated in space and acted on by no external forces. If the airplane's velocity in the forward direction is to be changed, its momentum in the forward direction must be changed. But the system, having no external forces acting on it, is not permitted to change its total momentum. If the momentum of the airplane is increased in the forward direction, the air must acquire an equal and opposite momentum in the backward direction. The function of the propellers is not to "screw the airplane through the air" but rather to deliver this backward component of momentum to the air. The jet engine of an airplane takes in a quantity of air, and, as a result of combustion processes, that air is expelled to the rear with greater velocity, in the form of exhaust gases. In this sense there is only a superficial mechanical difference between a propeller-driven airplane and a jet airplane, for both the propeller and the jet serve the same function—to give the air a backward component of momentum.

In exactly the same way the oars of a rowboat or the propellers of an ocean-going vessel propel the craft by delivering a backward momentum component to the water. The propellers of a ship are considerably smaller, in comparison to its mass, than the propellers of an airplane, because a smaller volume of water can carry off the required momentum, the water having far greater mass per unit of volume.

The problem of propelling a rocket is similar to that of propelling a jet airplane, but there is the additional complication that the rocket must carry along all the mass which must be ejected as momentum, while the airplane has a readily available supply of mass, in the form of the surrounding air, which it acquires at one speed and expels with greater speed as needed.

10-5 Elastic and Inelastic Impact

While in every collision between bodies the total momentum remains the same before and after the impact when the system is acted upon by no external forces, the mechanical energy of the system does not necessarily remain constant. Collisions in which the total kinetic energy remains constant are called *elastic*, while collisions in which the total kinetic energy is less after the collision than it was before the collision are called *inelastic*.

In an inelastic collision the kinetic energy lost in the impact is transformed to sound energy, to heat energy, and to the energy required to deform or fracture a body.

Consider the central collision between two spheres of masses M and m , moving with velocity \mathbf{U} and \mathbf{u} before collision and with velocity \mathbf{V} and \mathbf{v}

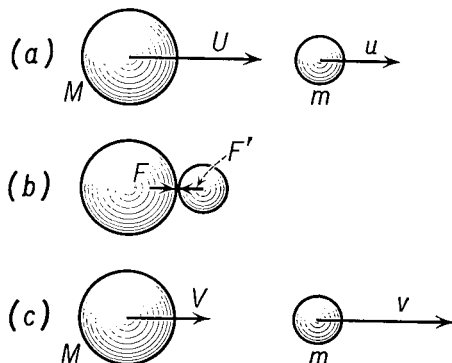


Fig. 10-3 Central collision of two spheres: (a) velocities before collision; (b) forces acting during collision, $F = -F'$; (c) velocities after collision.

after collision, respectively, as illustrated in Figure 10-3. Since the system, composed of the two spheres, is acted upon by no external forces, the total momentum of the system before collision will be equal to the total momentum of the system after collision. If, in addition, the collision is elastic, the total kinetic energy before collision will be equal to the total kinetic energy after collision. Writing the energy equation first, we have

$$\frac{1}{2}MU^2 + \frac{1}{2}mu^2 = \frac{1}{2}MV^2 + \frac{1}{2}mv^2.$$

The equation representing conservation of momentum is

$$MU + mu = MV + mv.$$

Transposing quantities for the sphere M to the left-hand side and quantities for the sphere m to the right-hand side of both equations, and simplifying, we get

$$M(U^2 - V^2) = m(v^2 - u^2),$$

and,

$$M(U - V) = m(v - u);$$

and, dividing the first equation by the second, we have

$$U + V = v + u,$$

or

$$U - u = v - V; \quad (10-11)$$

that is, the velocity with which the two spheres approach each other is numerically equal to the velocity with which they leave each other.

In general, one cannot say much about an arbitrary inelastic collision.

If the collision is completely *inelastic*, a solution may be obtained by noting that the two bodies stick together after the collision; that is, they have a common final velocity.

In some cases it is possible to characterize inelastic collisions by making use of a quantity called the *coefficient of restitution* e , defined by the equation

$$e = \frac{v - V}{U - u}. \quad (10-12)$$

The coefficient of restitution is a positive number whose value is given by the ratio of the velocity at which the two particles leave each other in a central collision to the velocity with which the two particles approach each other before the collision. From Equation (10-11) we see that $e = 1$ for an elastic collision. In a perfectly inelastic collision the particles stick together after the impact, so that their separation velocity is zero. Thus $e = 0$ in a perfectly inelastic collision.

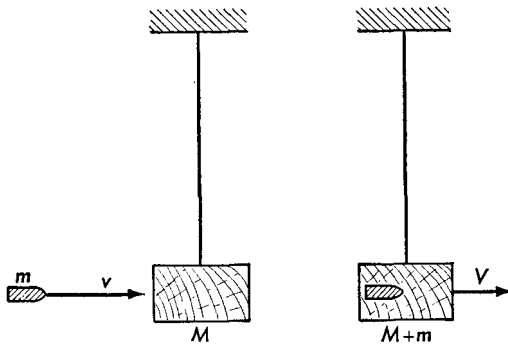


Fig. 10-4 (a) Initial momentum of system as bullet approaches target is mv ; (b) final momentum of system just after bullet has penetrated target is $(M + m)V$.

Illustrative Example. A block of balsa wood whose mass is 600 gm is hung from a cord of negligible weight. A bullet whose mass is 2 gm and which has a muzzle velocity of 16,000 cm/sec is fired into this block at close range and becomes embedded in it. Determine the velocity with which the balsa wood is set in motion. What is the energy of the bullet before the collision? What is the energy of the system after the collision?

If we consider the bullet and the balsa-wood block as a single system, there is no external force acting on it, hence the momentum of the system is conserved in the collision process and is the same before the collision as after. The collision is a completely inelastic one, for, after the impact the bullet and the block of wood move as one, with common velocity V , and the final momentum of the system is $(M + m)V$, where M is the mass of the block and m is the mass of the bullet. The total momentum of the system before collision is mv , where v is the velocity of the bullet before the collision, as shown in Figure 10-4. From the principle of conservation of momentum, we write

$$mv = (M + m)V,$$

from which

$$V = \frac{m}{(M + m)} v,$$

and, substituting numerical values, we get

$$V = \frac{2 \text{ gm}}{602 \text{ gm}} \times 16,000 \frac{\text{cm}}{\text{sec}},$$

or
$$V = 52.2 \frac{\text{cm}}{\text{sec}}.$$

The kinetic energy of the system before collision is \mathcal{E}_i , where

$$\begin{aligned} \mathcal{E}_i &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} \times 2 \text{ gm} \times 256 \times 10^6 \frac{\text{cm}^2}{\text{sec}^2}, \end{aligned}$$

so that
$$\mathcal{E}_i = 256 \times 10^6 \text{ ergs},$$

and this is the total kinetic energy of the system before the collision, for the balsa block is at rest. The final kinetic energy of the system is \mathcal{E}_f , where

$$\begin{aligned} \mathcal{E}_f &= \frac{1}{2} (M + m) V^2 \\ &= \frac{1}{2} \times 602 \text{ gm} \times \left(52.2 \frac{\text{cm}}{\text{sec}} \right)^2, \end{aligned}$$

so that
$$\mathcal{E}_f = 0.84 \times 10^6 \text{ ergs}.$$

Note that mechanical energy has been lost in the collision process, a sign of an inelastic collision. From the principle of conservation of energy, we know that the energy has not vanished but has been transformed into heat, sound, deformation, and chemical change of the bullet and the balsa block.

Collision problems are of great importance in all of physics, from the realm of everyday experience to microscopic or atomic phenomena and to astronomical phenomena. Let us consider the collision of two perfectly elastic objects of the same mass, such as between two billiard balls, or between two nuclear particles such as two protons or two neutrons, or between a proton and a neutron (although the proton and neutron have slightly different masses). Let us suppose that before the collision one of these objects is at rest, and that the direction of the initial velocity U is along the line joining the centers of the two bodies. This will be called a central collision. Following the notation of the preceding discussion, we will suppose that $M = m$, and that u , the initial velocity of m , is equal to zero; that is, the sphere of mass m is at rest. From the momentum conservation equation, we find

$$MU = MV + mv,$$

but since

$$M = m,$$

we may write

$$U = V + v.$$

But from Equation (10-11), remembering that $u = 0$, we have

$$U = v - V.$$

Adding these two equations, we find that

$$U = v,$$

and that

$$V = 0.$$

Thus when two objects of equal mass collide in a central collision, all of the velocity of the moving object is transferred to the object which was formerly at rest. A similar analysis shows that a very light object bounces off a very heavy one with little transfer of energy, while, for objects of equal mass, all of the energy is transferred to the second object.

In ordinary experience these considerations help to determine the weight of a hammer used to drive a spike. If the hammer is too heavy, very little of the momentum and energy of the hammer will be transferred to the spike; the energy expended in driving the hammer will be inefficiently used, and, in addition, the work will be badly damaged by penetration of the hammer head itself. If the hammer is too light, it will bounce off the spike. In the ideal situation all of the energy and momentum of the hammer will be absorbed by the work, and the hammer will stop dead after the impact. Precisely the same considerations apply to the matching of the mass of the head of a croquet mallet to the mass of the ball and to countless other situations.

In one form of nuclear reactor, it is desired to use slow neutrons, although fast neutrons are emitted from uranium atoms which have undergone fission (see Chapter 46). If these neutrons are to be utilized efficiently in the reactor, they must be slowed down as rapidly as possible to very low speeds. This is accomplished by incorporating a *moderator* into the construction of the reactor. The moderator must contain nuclei whose mass is close to the mass of the neutron. Protons, having almost the same mass as neutrons, may be used to slow down the neutrons by elastic collision processes. Ordinary water is a convenient source of protons and can be used as a moderator; the kinetic energy and momentum of the neutrons are transferred to the protons in water in collision processes.

In atomic processes as well as in macroscopic processes, we may speak of inelastic collisions. In a collision between a rapidly moving proton and a molecule, the kinetic energy of the proton may dissociate the molecule, thus doing work against the forces which hold the molecule together. In the collision between an electron and an atom, the kinetic energy of the electron may be absorbed by the atom and converted into internal energy;

some of the internal energy may subsequently be emitted as light, as in the neon tube. In atomic and nuclear physics the principles of the conservation of energy and momentum and the concepts of elastic and inelastic collisions are extremely important in interpreting the data gathered from high-energy accelerators, from gaseous discharge tubes, from nuclear fission, and from many other atomic processes.

Problems

10-1. The hammer of a pile driver weighs 500 lb. The hammer is dropped from a height of 10 ft onto a pile, and drives the pile 3 in. (a) What is the momentum of the hammer before the impact? (b) What is the impulse delivered by the hammer to the pile? (c) Assuming that the pile exerts a steady force against the hammer, what is the magnitude of that force? (d) For how long a time interval does the hammer exert a force on the pile?

10-2. A 5-gm bullet is fired from a gun whose barrel is 60 cm long. The bullet leaves the gun with a muzzle velocity of 25,000 cm/sec. (a) What was the average force acting on the bullet? (b) What is the momentum of the bullet when it leaves the gun? (c) What impulse was delivered to the bullet? (d) How long was the bullet in the gun barrel?

10-3. A man driving a golf ball gives it a speed of 3,000 cm/sec. (a) If the mass of the golf ball is 50 gm, what impulse was imparted to it by the driver? (b) If the head of the driver has a mass of 75 gm, what fraction of the energy of the driver was delivered to the ball? Assume an elastic collision.

10-4. An ivory ball of mass 100 gm, moving with a velocity of 80 cm/sec, strikes a stationary ivory ball of equal mass. The velocity is parallel to the line joining the centers of the two balls. Assume a perfectly elastic collision. Determine the velocity of the two balls after collision.

10-5. An ivory ball of mass 400 gm, moving with a velocity of 90 cm/sec, strikes a stationary ivory ball having a mass of 100 gm. (a) Assuming perfect elasticity and a central collision, determine the velocity of each ball after collision. (b) What fraction of the energy of the first ball was transferred to the second ball?

10-6. An ivory ball of mass 200 gm, moving with a velocity of 100 cm/sec in the positive x direction, strikes a second ivory ball of mass 300 gm a glancing blow, so that the first ball is deflected by an angle of 30° from its initial direction and has a speed of 75 cm/sec after the collision. Find the speed and direction of motion of the second ball after the collision.

10-7. A man fires an automatic rifle, shooting a clip of 16 shells in 4 sec. Each bullet weighs 2 oz and has a muzzle velocity of 2,500 ft/sec. The man stands on a perfectly smooth floor. If the man and rifle weigh 160 lb, find the velocity with which the man is sliding backward after the 4-sec period.

10-8. A bullet whose mass is 2 gm is fired from a rifle with a muzzle velocity of 30,000 cm/sec into a piece of balsa wood mounted on a car with frictionless wheels. The total mass of the balsa wood and car is 1,500 gm. Determine (a) the initial momentum of the system, (b) the velocity of the balsa wood and car after the bullet was embedded in the wood, and (c) the mechanical energy lost in the impact.

10-9. An automobile weighing 2,400 lb, driving east at a speed of 50 mi/hr, collides with a heavily loaded truck driving west at a speed of 60 mi/hr. The loaded truck weighs 10,000 lb. If the collision is inelastic, find (a) the impulse received by the truck and (b) the impulse received by the car.

10-10. Four croquet balls are lined up in a frictionless trough. A fifth croquet ball, moving with speed v , strikes the end ball. Prove that only one ball will leave the other end with speed v in the same direction and of the same magnitude as the incident ball. Assume elastic collisions.

10-11. Show that a ball striking a wall in a perfectly elastic collision will bounce off the wall, making an equal angle with the wall. Is momentum conserved in this collision?

10-12. A fireman holds a fire hose which expels a stream of water from a 2-in.-diameter nozzle at a speed of 40 ft/sec. Find the force the fireman must exert to keep the nozzle stationary.

10-13. A ball is dropped onto an anvil from a height of 10 ft. If the coefficient of restitution is 0.1, to what height will the ball rise (a) after the first impact? (b) After the second impact?

10-14. If a ball of mass m is dropped from height h onto an anvil, and if the collision is perfectly elastic, find the time between successive impacts.

10-15. Repeat Problem 10-13 if the coefficient of restitution is e . Is the time between the first and second impact the same as the time between the second and third impact?

10-16. A ball A whose mass is 50 gm is moving to the right with a velocity of 80 cm/sec, and another ball B with a mass of 75 gm is moving to the left with a velocity of 120 cm/sec along the x axis. Determine (a) the initial momentum of the system, (b) the velocity of the center of mass of the system, and (c) the velocity of each ball after collision, assuming the collision to be elastic.

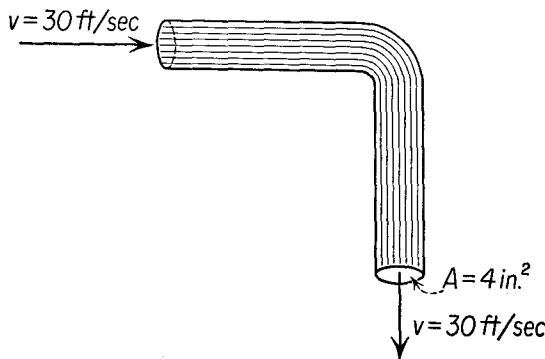


Fig. 10-5

10-17. A stream of water is moving through a horizontal pipe of uniform cross section of 4 in.² with a speed of 30 ft/sec. The pipe has a right-angle bend in it, as shown in Figure 10-5. Determine the force that has to be exerted at the bend to hold the pipe in equilibrium.

10-18. A steel ball weighing 1 lb drops from a height of 16 ft, strikes a steel anvil, and rebounds to the same height. (a) Calculate the impulse on the ball. (b) If the time of contact between the ball and anvil is 0.002 sec, determine the impulsive force on the anvil.

10-19. The truck and car of Problem 10-9 are driven by men who weigh 160 lb. The drivers are fastened to their respective vehicles by seat belts. Find the impulse experienced by (a) the driver of the car, and (b) the driver of the truck in the collision.

10-20. A baseball of mass 250 gm leaves the bat at an angle of 30° above the horizontal with a velocity of 150 m/sec. Assume that the pitch was traveling horizontally with a velocity of 80 m/sec. If the bat was in contact with the ball for 0.05 sec, what was (a) the average horizontal component and (b) the average vertical component of the force exerted by the bat on the ball?

10-21. A ball of putty with mass 2 kg and velocity 25 m/sec strikes a wall and sticks. The collision lasts for 0.10 sec. (a) What is the magnitude of the average force exerted on the wall during the collision? (b) How much mechanical energy is lost during the collision? (c) What is the magnitude of the impulse transmitted to the wall?