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Standing Waves on the Moon

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The existence of local annular and radial structures in the environment of lunar craters, and of a global selenographic tectonic pattern, is ascribed in part to standing "moonquakes" established after impact or eruption.

Standing Waves on the Moon

Since the discovery and identification of fundamental modes of free vibration of the Earth[1] following the great Chilean (1960) and Alaskan (1964) earthquakes, the possibility of similar effects on the Moon has been considered. Such efforts have been directed at a determination of the periods of vibration for different lunar models,[2–4] in a fashion analogous to that used for the Earth.[5] No consideration has been given to the effect which free vibrations of the Moon might have, over selenological time, on surface morphology. We show in this article that both local and global vibrations have played a part in the shaping of visible selenological features and that they constitute an important lunar tectonic force.

When planetologists meet to discuss the surface of the Moon they separate into those favoring the dominance of planetesimal (meteoric) impact and those who advocate volcanic activity. It is not possible to avoid taking sides in this issue, but we point out that much of what we say applies equally well in either circumstance. The release of energy at the surface of a sphere, whether it be of endogenic (volcanic) or exogenic (impact) origin, is followed by a distribution of that energy over the entire body of the sphere; thereafter the physics of the phenomenon is to all intents and purposes identical. Only the scale of the observed effect is likely to be different.

Standing Waves on a Sphere

Standing waves on a string are characterized by point nodes. In two dimensions, as on a circular disk, nodal lines are observed—the familiar Chladni[6] figures. The observation that both concentric circles and radial lines, or a mixture of the two, are frequently seen in such patterns may be of importance in discussing the Moon, for the "rays" which surround craters on the maria lie on great circles radial to the craters themselves, while concentric circles are frequently to be seen around some of the large craters. Many lunar craters, moreover, are evidently polygonal—another characteristic of Chladni figures for a circular disk. These similarities prompted our consideration of free vibrations of the Moon. There are also distinct differences.

Determination of the detailed description of oscillations of a homogeneous elastic sphere is an old mathematical problem.[7–11] Nodal surfaces are generated which for fundamental vibrations comprise concentric cones with apices at the sphere center, intersecting the surface on small circles of latitude, and diametral planes intersecting the surface on great circles of longitude. Vibrational overtones yield concentric spheres about a common global centre. Two distinct modes of the first (C_1 or "torsional") and second ($C2$ or "spheroidal") classes may be identified, the distinction being that the former constitutes purely tangential displacements, while the latter has a radial component. Both modes are dependent on the azimuthal and polar angles through functions involving tesseral harmonics and associated Legendre functions. Table 1 summarizes the displacements u, v, w corresponding to spherical coordinates r, θ, φ for a vibrating self gravitating sphere. The zeros for each m and l of the tesseral spherical harmonics define a pattern of nodal lines on the lunar surface, the number of nodal latitude lines being $(l - m)$ and of nodal longitude lines $2m$. The value of n uniquely defines the radial character of the vibration so that $n = 0$ is the fundamental, $n = 1$ the first order overtone, and so on. For oscillations of high order m and degree l, the lunar surface is subdivided into cellular tesserae of opposite motion (Figure 1*b*).

For the Moon, it is proposed that release of sufficient energy at or near to the surface will lead to the onset of free vibrations of long period (up to of the order of 15 min). Rotation will lead to a westerly drift of the displacement pattern,[12–14] which will remove degeneracy of the associated eigenvalues, but this is expected to be of secondary importance. It is the very nature of the oscillatory forces established by the vibrations which is significant, for the forces occur on a global scale. For torsional vibrations of the general kind, at any moment of time the dominant global force is provided by the adding of forces in neighboring tesserae (Figure 1*b*) to produce huge 45° bands of unidirectional shear which can be identified on small circles running NE-SW and NW-SE (Figure 1*c*). Secondary bands of alternating shear running E-W on small circles and N-S on great circles can also be visualized. For spheroidal vibrations, a similar pattern occurs, but the radial component of displacement introduces compressive and tensile forces which are a feature of this type of oscillation (Figure 1*c*). The pattern of nodes on the surface of the globe is one of points and lines packed in a simple square fashion. This is due to a dependence of the angular components of displacement on the gradients of the tesseral harmonics.

Forces for a Chladni figure for spherical geometry are demonstrable. By driving a balloon filled with water (an approximation to a self gravitating globe) using a signal generator and loudspeaker, it is possible to reveal the nature of surface nodes and forces. Lines of latitude and longitude can be obtained, either separately or together, packing more densely at the high frequencies. Most frequently a close-packed array of hexagons

Table 1. Coordinates of Displacement for Vibrations of the Moon

	Torsional (${}_nT_l^m$)	Spheroidal (${}_nS_l^m$)
u	0	$U(r)\ Y_l^m(\theta, \varphi)\exp(i\omega t)$
v	$\frac{W(r)}{\sin\theta}\frac{\partial}{\partial\varphi}Y_l^m(\theta, \varphi)\exp(i\omega t)$	$V(r)\frac{\partial}{\partial\theta}Y_l^m(\theta, \varphi)\exp(i\omega t)$
w	$-W(r)\frac{\partial}{\partial\theta}Y_l^m(\theta, \varphi)\exp(i\omega t)$	$\frac{V(r)}{\sin\theta}\frac{\partial}{\partial\varphi}Y_l^m(\theta, \varphi)\exp(i\omega t)$

$Y_l^m(\theta, \varphi) = P_l^m(\cos\theta)e^{im\varphi}$
P_l^m = Legendre function.

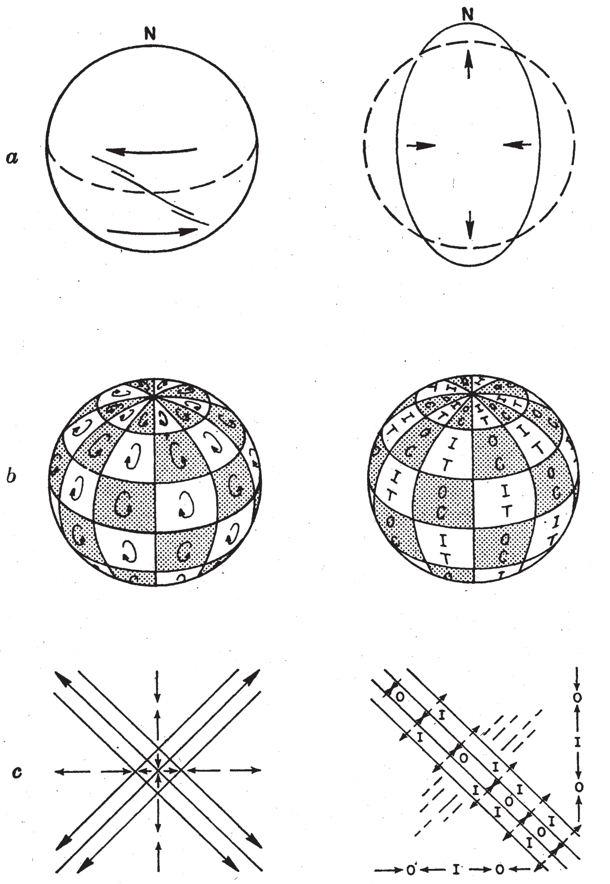


Figure 1. Schematic representation of the basic features for free torsional (left-hand side) and spheroidal (right-hand side) oscillations of a homogeneous self-gravitating sphere. ***a*,** Two typical simple oscillations are those of oscillatory twist, with the equator and the north and south poles held firm (torsional), and of alternating polar/equatorial prolation and oblation (spheroidal). Clearly the latter possesses a radial component of surface displacement. ***b*,** In more complex oscillations the surface of the globe is divided into tesserae by lines of latitude and longitude. Neighboring "cells" rotate in opposite direction in the torsional case. In the spheroidal case they are in tension (*T*) and moving in (I), or are in compression (*C*) and moving out (*O*). ***c*,** The global system of forces established by vibration is dominated by NW-SE and NE-SW systems of alternating shear (torsional) or of alternating tension and compression (spheroidal).

is observed, using a distribution of fine flour to throw the pattern into contrast (Figure 2). We do not, of course, imply that the lunar surface is a simple Chladni figure, but the basic directional features of vibration can nevertheless be shown compatible with many of the observed directional topographic structures.

Application to the Moon

The lunar surface is a conspicuous conglomerate of craters and maria.[15-17] Maria are gigantic "plains" surrounded by "mountain" ranges while craters are entities in themselves. It has been recognized that both craters [18–19] and maria are often polygonal in shape. We cite the case of the Mare Crisium which is clearly hexagonal (Figure 3). A recent study [20] separates lunar craters into two populations based on a circularity index, the author proposing formation of a subcircular population during a period of stress in the lunar crust. There is also a grid system [18, 21, 22] defined by preferential directions shown by the entire collection of linear formations of the surface revealed by crater sides, mountain chains, valleys and ridges of the lunar continents. Fractures, rilles,[23] and crater chains in the uplands and wrinkle ridges in the maria follow the same pattern. Extension of this grid system to details on a fine scale is evident in photographs taken by the Orbiter satellites [24] and in observations made from Apollo 8. Minor systems of lineaments radial or subradial to individual craters are clearly local. Major systems strike SW-NE, SE-NW, N-S and E-W, and are global. The close relation which these systems bear to the global pattern of oscillatory forces for a vibrating Moon is immediately apparent. A prima facie case for at least partial contributory cause and effect is therefore established.

If the proposal that lunar vibration is a major tectonic force is to be seriously entertained, it has to be demonstrated that other features fall into the same general pattern. This we shall do in detail elsewhere. We briefly observe, however, that polygonal craters mostly present even numbers of sides as the theory requires. Hexagons are dominant, while octagons and squares can also be identified. Triangles are never seen. The further fact that polygonal crater sides lie contiguously on or near to directions determined by the lunar grid encourages us to conclude that their shape is frequently a response to global stimuli and is not local. The existence of two populations of craters, moreover, is explicable in terms of their age. Table 2 illustrates this point by comparing the geological and selenochronological systems. It is significant that the old Pre-Imbrian craters are patently polygonal, while the younger features are undeniably round. Evidently lunar craters have been reshaped over extensive periods of selenological time. Global vibration must have played some part.

It is appropriate here to make clear the fundamental difference between the proposal that vibration is an important tectonic force and a coexistent process by means of which modifications of lunar topography, particularly height, can be made by continuous viscous adjustment [25–26] of surface irregularities. This difference reflects the time scale of the constraint. In the elastic case we seek solutions to the Laplace equation; in the viscous case of the Navier-Stokes equation.

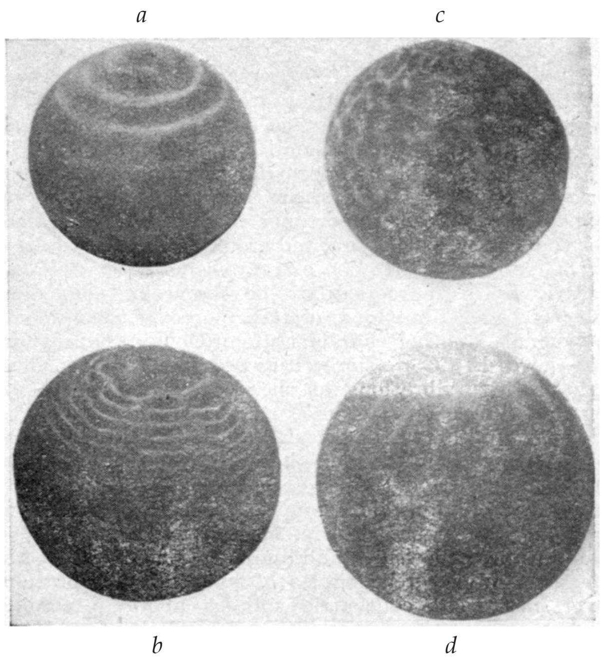


Figure 2. Chladni figures for a balloon filled with water. Annular nodal lines can be obtained (***a***); packing more densely at the higher frequencies (***b***). Most frequently a close-packed system of hexagons is obtained (***c***), though radial features are sometimes seen (***d***).

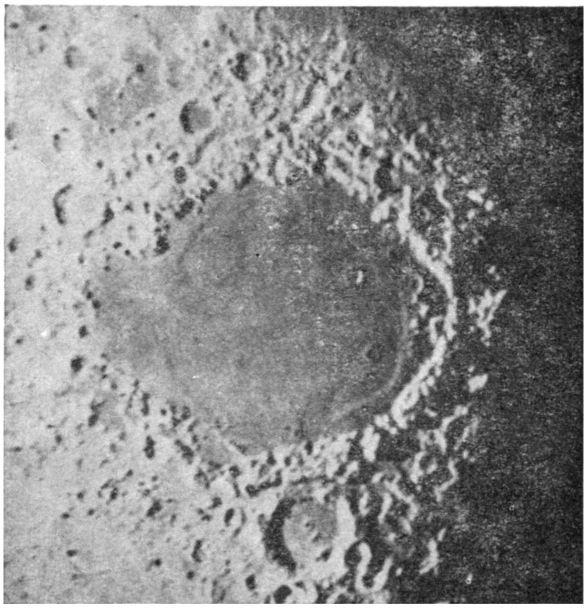


Figure 3. Terrestrial based photograph of the "old" hexagonal Mare Crisium, projected onto a sphere.

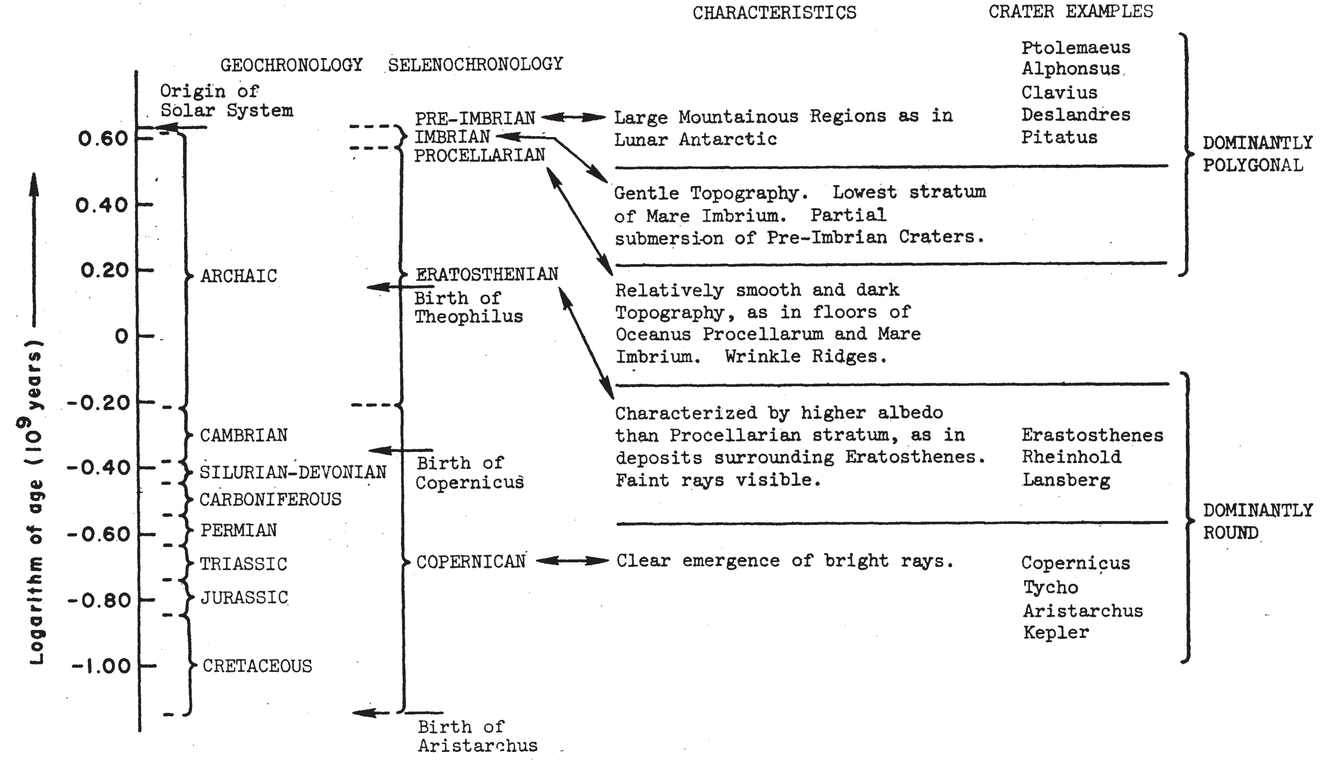
Local Vibration

Because global lineaments can be identified in part with global oscillation we might expect local lineaments, which are related to particular craters or maria, to be a consequence of local vibration. The rays which radiate from the newest craters and which are clearly visible at full Moon are a prime example of the unique identification of radial and subradial lineaments with single selenographic features. Closer investigation reveals that all such craters also present in some measure an annular system of cracks and ridges.

If local vibration is initiated following point release of energy at the lunar surface, the system of standing waves produced will be more akin to that observed for a circular disk (Figure 4). The rays surrounding a crateriform feature might then represent fissuring of the lunar crust in response to forces established by a dominant system of radial nodal and antinodal lines (Figure 4*a*). In partial support of this hypothesis we remark that a close examination of rayed craters indicates a surprising degree of symmetry. Departures from radial to subradial patterns then simply reflect the part played by preexisting arrangements of faults associated with global tectonics. From a study of Ranger VII photographs [27] Kopal shows that if the rays of Tycho are assumed to be deposited ejectal streams consequent on impact then there is clear violation of Schroeter's rule relating the volume of the ramparts and ejecta of an impact crater to the volume of the depression. A high concentration of dimples and depressions on the rays, moreover, may in part be responsible for their high albedo. Kopal interprets these observations in terms of subsidence triggered by moonquakes. We propose that the moonquakes were established as standing waves.

The best example of a concentric ringed crater is undoubtedly that of Mare Orientale (Figure 5). Five annular rings can be identified, the diameter increasing roughly by multiples of √2. Interpretations of the mode of formation of this huge feature are as varied as for any other on the lunar surface.[28–30] One author has taken account of the relation between the ring radii. Likening the ring system to the train of water waves on a shallow layer overlying a rigid base, Van Dorn [30] shows that by suitable adjustment of the parameters of depth and time the spacings can be made compatible with the resulting dispersion curve. This proposal that Mare Orientale is a "frozen tsunami" suffers from difficulties. The first is the requirement that at a certain moment of time the traveling "fluid" wave is at all points simultaneously arrested and preserved, when the

Table 2. Comparative Geology and Selenochronology



CHARACTERISTICS	CRATER EXAMPLES	
Large Mountainous Regions as in Lunar Antarctic	Ptolemaeus Alphonsus Clavius Deslandres Pitatus	DOMINANTLY POLYGONAL
Gentle Topography. Lowest stratum of Mare Imbrium. Partial submersion of Pre-Imbrian Craters.		
Relatively smooth and dark Topography, as in floors of Oceanus Procellarum and Mare Imbrium. Wrinkle Ridges.		
Characterized by higher albedo than Procellarian stratum, as in deposits surrounding Eratosthenes. Faint rays visible.	Erastosthenes Rheinhold Lansberg	DOMINANTLY ROUND
Clear emergence of bright rays.	Copernicus Tycho Aristarchus Kepler	

pressure falls below a critical value. Second, it is possible to generate a dispersion curve in several other ways so as to fit the observed ring diameters, including that of using gravitationally affected Love and Rayleigh waves for differing solid lunar models. Finally, there would appear to be no good reason for neglecting the possibility of a sixth or possibly seventh ring with smaller amplitudes and larger radii, but not detectable in the photographs of Orbiter IV.

The annular pattern of nodes produced on a free vibrating circular disk with isotropic non-dispersive properties consists of equispaced rings (Figure 4*b*). Rings of geometrically increasing or decreasing radii can be created, however, on a disk which tapers either to the center or the edge (Figure 4*d*), in a manner analogous to the standing waves of differing wavelength on a string of linearly changing mass per unit length (as in a whip). Following impact, or eruption, the lunar surface in the immediate environment of the focal point would be subjected to a violent pressure and temperature "spike" which, except for the scale of the disturbance, would be much like that produced when a fast moving heavy charged particle enters a crystal.[31] This disturbance would spread radially, the amplitude falling in proportion to the reciprocal of the radius vector, and like Van Dorn we can conceive of a fluid-like behavior. At increasing radii, however, the medium would be progressively more viscous and finally truly solid. If the distance *l* over which this "phase change" took place was comparable with or less than the wavelength of the distance at that radius, then the standing-wave-ratio would be sufficient to permit constructive interference of stationary waves of smoothly varying length, though leakage of energy to the remaining solid globe would undoubtedly occur.

Again there is a clear difference between this proposal that standing annular moonquakes of local character can be established, and alternative suggestions involving features of collapse. Yet there does not appear to be any good reason why stationary waves immediately after the event could not layer the concentric pattern of tectonic fractures for more quiescent exploitation during longer and less violent periods of lunar time.

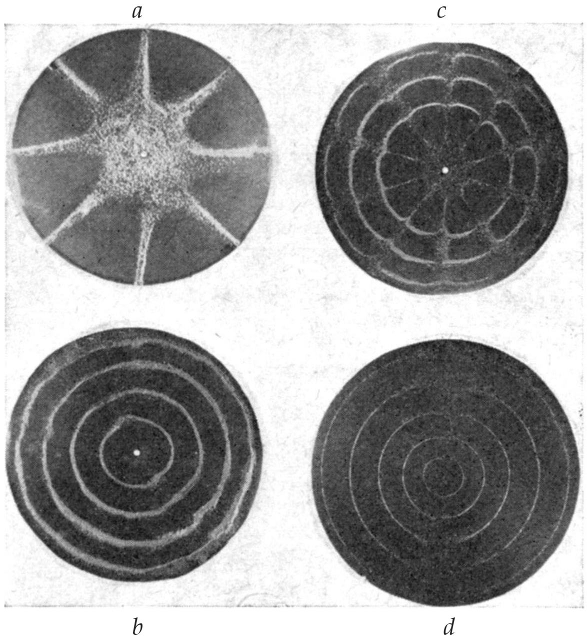


Figure 4. Chladni figures for a uniformly thick disk consisting of rays (*a*), concentric rings (*b*) or a mixture of the two (*c*). For a disk which tapers toward the center (*d*), the rings have progressively increasing radii (compare Figure 5).

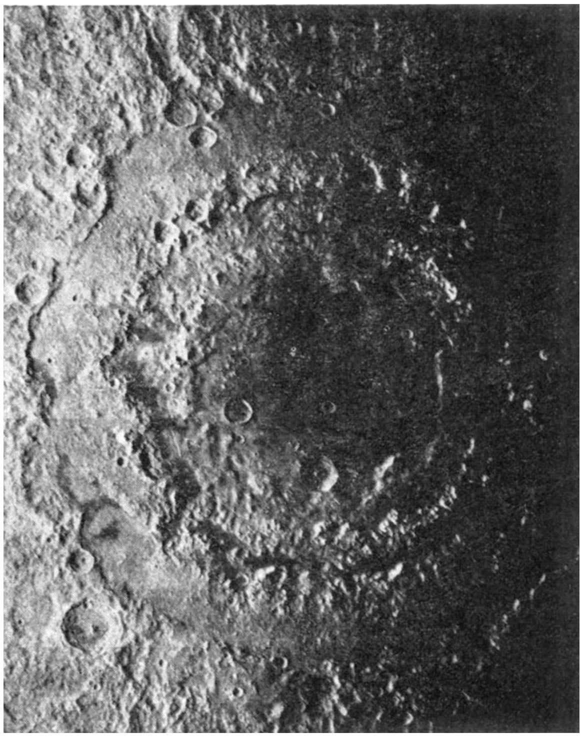


Figure 5. Orbiter IV photographs of the great Mare Orientale near the terminator shows prominent concentric rings.

We suggest that the selenomorphology of many of the observed features on the surface of our satellite is due to standing moonquakes established after either planetesimal impact or violent eruption. In local terms, this "ringing" of the Moon gives rise to radial and concentric patterns in the immediate environment of a crater. For total global oscillation the reshaping of older craters and the production of tectonic features lying on a lunar grid take place over extensive periods of selenological time. These processes do not replace others which occur simultaneously, such as convections,[32] tidal flow, and crustal collapse, but should be accounted for in any effort to understand the long and complex nature of lunar history.

In a separate communication [33] we consider sources of energy and discuss the origin of the symmetry of global vibration due to impact, relative to the distorted shape of a nearby Moon, to behavior of the bound Earth-Moon system in a gravitational dipole and to "nesting" of areal crater density contours.[34]

We thank the staff of the North American Rockwell Science Center library for their assistance in making a full literature survey of the physics of the Moon.

Note added in proof.

Recent results of R. A. Wells (***Geophys. J. Roy. Astron. Soc.*, 17,** 209; 1969) show that the canals of Mars fall on a global grid system essentially similar to that of the Moon. Coupled with Mariner observations of Martian polygonal craters, this strongly suggests a similar origin for both sets of surface features. We support the hypothesis of N. A. Barricelli and R. Metcalfe (*Icarus,* **10**, 144; 1969) that the Moon, slowly increasing its distance from Earth, has cleaned up the skies at all distances between its original and present orbits. It is a well known feature of a major satellite system (for example, Jupiter and Saturn) that only a portion of the moons, namely those closest to the mother planet, rotate in the equatorial plane, while external moons show very great inclinations and may even be retrograde. This would lead one to expect that the most recent satellite impacts are often those located at the greatest distance from the Moon's equator. Such impacts would preferentially establish the global modes of vibration we have described, and would be in keeping with the observed distribution of numbers of craters and with their distribution in size.

Mars therefore takes a position in the solar system comparable with that of the Moon and its relation with Earth. The proximity of the asteroid belt is also clearly of importance.

References

1. Bolt, B. A., *Phys. Chem. Earth,* **5**, 55 (1963).
2. Bolt, B. A., *Nature*, **188**, 1176 (1960).
3. Carr, R. E., and Kovach, B. L., *Icarus*, **1**, 76 (1962).
4. Takeuchi, H, Saito, M., and Mobayashi. N., *J. Geophys. Res.*, **66**, 3689 (1961).
5. Alterman, Z., Jarosch, H., and Pekeris. C. L., *Proc. Roy. Soc., A,* **252**, 80 (1959).
6. Chladni, E. F. F., *Die Akustik* (Leipzig, 1830).
7. Jeans, J. H., *Phil. Trans. Roy. Soc., A,* **201**, 157 (1903).
8. Lamb, H., *London Math. Soc. Proc.,* **13**, 51 (1882).
9. Lamb, H., *London Math. Soc. Proc.,* **13**, 189 (1882).
10. Lamb, H., *London Math. Soc. Proc.*, **14**, 50 (1883).
11. Baker, W. E., *J. Acoust. Soc. Amer.*, **88**, 1749 (1961).
12. Backus, G., and Gilbert, F., *Proc. US Nat. Acad. Sci.*, **47**, 362 (1961).
13. Pekeris, C. L., Alterman, Z., and Jarosch. H., *Phys. Rev.,* **122**, 1692 (1961).
14. Pekeris, C. L., Alterman, Z., and Jarosch, H., *J. Geophys. Res.,* **68**, 2887 (1963).
15. Kopal, Z., *An Introduction to the Study of the Moon* (Gordon and Breach, New York, 1966).
16. Baldwin. R. B., *The Nature of the Moon* (University of Chicago Press, 1963).
17. Kopal, Z., *Photographic Atlas of the Moon* (Academic Press, New York, 1965).
18. Fielder, G., *Lunar Geology* (Defour, Pennsylvania, 1965).
19. Davydov, V. D., *Sov. Physics-Dokl.,* 13 ,77 (1968).
20. Ronca, L. B., and Salisbury, J. W., *Icarus*, **5**, 130 (1966).
21. Strom, R. G., *Commun. Lunar Planet. Lab.,* **2**, 205 (1964).
22. Whitaker, E. A., *The Nature of the Lunar Surface,* 79 (Johns Hopkins, Baltimore, 1966).
23. Peale, S. J., Schubert, G., and Lingonfelter. R. E., *Nature*, **220**, 1222 (1968).
24. Fulmer, C. V., and Roberts, W. A., *Icarus*, **7**, 394 (1967).
25. Takeuchi, H., and Hasegawa, Y., *Geophys. J.,* **9**, 503 (1965).
26. Shimazu, Y., *Icarus*, **5**, 455 (1966).
27. Kopal, Z., *Icarus*, **5**, 207 (1966).
28. Miyamota, S., *Icarus*, **9**, 373 (1968).
29. Hartmann, W. K., and ,Yale, F. G., *Commun. Lunar Planet. Lab.,* **117** (1968).
30. Van Dorn, W. G., *Nature*, 220, 1102 (1968).
31. Chadderton, L. T., *Radiation Damage in Crystals* (Methuen, London, 1965).
32. Runcorn, S. K., *Nature*, **195**, 1150 (1962).
33. Chadderton, L. T., Krajenbrink, F. G., Katz, R., and Poveda, A. (to be published 1969).
34. Ronca, L. B., and Green, R. R., *Nature*, **218**, 1147 (1968).