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Abstract

This paper develops a tractable theoretical framework for analyzing the substitutability between different advertising media, the extent of marketing spillovers in the market, the allocative efficiency of advertising spending, and the sources of total advertising productivity and sales growth. Maintaining the separability assumption between sales and production technology, the proposed methodology relies on cost-function decomposition of total factor productivity and the duality between input distance and cost functions. Utilizing a flexible Translog advertising distance function, the methodology is applied to the advertising activity of meat processing firms in Greece during the period 1983-1997. Scale economies in advertising expenses turn out to be an important source of total advertising productivity changes in the Greek meat processing sector. Advertising spillovers are significant contributing to total advertising productivity observed. Our analysis also indicates that improvements in (technical and allocative) advertising efficiency are more important means of enhancing firm returns than improvements in advertising techniques.

Keywords: Advertising productivity, advertising direct distance function, media substitutability, processed meats industry, Greece.

JEL Codes: D24, L25, L66, M37.

Introduction

Advertising (persuasive or informative) is a critical element of business conduct affecting consumer valuation of advertised goods, market structure, firm profitability and social welfare. It requires significant resources as well as skills in determining the appropriate messages and the media used to communicate them. It has received considerable attention in the economic literature with the main focus being either on the market and welfare effect of different advertising strategies or on empirically estimating structural models (*i.e.*, sales or advertising cost function) aimed to characterize marketing technology and the effectiveness of marketing efforts. This strand of advertising literature, using structural models, has been based on the separability assumption between marketing and production technologies initiated by Brown (1978), Waterson (1984) and

Brensahan (1984). In these empirical models three important issues have been considered so far: *first*, the substitutability of advertising media (*e.g.*, Seldon and Jung, 1993; McCullough and Waldon, 1998; Seldon *et al.*, 2000; Silk *et al.*, 2002); *second*, the role of scale economies in advertising (*e.g.*, Seldon *et al.*, 2000); and *third*, the degree of advertising efficiency (*e.g.*, Luo and Donthu, 2001; 2005; Fare *et al.*, 2004; Vardanyan and Tremblay, 2006).

The interest in media substitutability and the role of returns to scale stems from the question regarding three separate public policy debates: *first*, the effectiveness of partial advertising media bans; *second*, the effects of merger wave among radio, television and printed media firms and; *third*, the anticompetitive effects of advertising. Firms banned from using one advertising medium could maintain a constant level of sales by increasing their use of another advertising medium if the latter is substitutable for the former. As a result, intermedia substitutability tends to limit the effectiveness of partial advertising bans but complementary relations may also be expected since firms often use more than one class of media in a given campaign (Silk *et al.*, 2002). All these are about the flexibility of advertising mix in attaining a given level of sales.[1]

On the other hand, the recent merger wave among entertainment corporations worldwide raises important concerns with regard to the market power accrue to the owners of merged broadcasting and printed media firms that sell advertising spots (Slade, 1998; Busse and Rysman, 2005). Competitive pricing of advertising outlets (*i.e.*, TV, radio, printed media) is ensured by easy of substitution between advertising media. If, however, broadcasting or printed media firms are merged, then the easiness of media substitutability across markets, will result in increased advertising spot prices and therefore in detetorating consumer's welfare. Apart of that, advertising expenses are considered as a key determinant of the barriers to entry in a market and, therefore, an important determinant of market structure. Comanor and Wilson (1969) were among the first to argue that advertising affects the height of barriers to entry through the existence of scale economies. In the presence of such economies, potential entrants face an additional (sunk) capital requirement, which can deter entry and therefore decrease competition in the market.

While the determination of effective marketing campaigns (*i.e.*, strategies that can achieve certain firm objectives) is certainly necessary for the successful performance of a firm, such a determination is not sufficient – a key issue is the identification of those effective advertising campaigns that are also efficient, *i.e.*, they can achieve the firm objectives at minimum cost. High levels of advertising inefficiency, plagues business and frustrates managers as inefficient advertising spending and misallocated resources results in lower profit margins and obstacle firm's sustainable growth. Several authors recently considered that advertising expenses might not be as efficient as it has been recognized in generating sales (*e.g.*, Smith and Park, 1992; Luo and Donthu, 2001, 2005; Färe *et al.*, 2004; Vardanyan and Tremblay, 2006).

Technical efficiency of advertising is certainly important, but it is not the sole determinant of advertising efficiency. Another but closely related issue that has not been considered previously is on whether the choice in advertising mix is in accordance with the observed prices for messages in the various media types, in firms' attempts to attain a given level of sales with the minimum possible cost. This is about allocative inefficiency in advertising and how it may affect competitiveness by increasing unnecessarily the cost of advertising compared to rival firms. With the benefits of advertising varying between different advertising media (Becker and Murphy, 1983; Tirole, 1988; Yiannaka *et al.*, 2002; Färe *et al.*, 2004),[2] the identification of the allocative efficient advertising mix can be a challenging proposition. Nerlove and Arrow (1962) and Grabowski (1970) argued that while the advertising to sales ratio varies across firms and industries, individual firms tend to allocate a nearly fixed ratio over time which, in turn, suggests that firms may be engaged in a "rule of thumb" decision making when faced with uncertainties regarding the allocation of advertising expenses. This is particularly true in highly protected sectors and oligopolistic markets where government intervention and imperfect competition create major impediments to the efficient allocation of resources (Fulginiti and Perrin, 1993).

However, media allocation is a critical dimension of advertising decision making, and it has even been suggested that it is more important than advertising spending decisions (Doyle and Saunders, 1990). Our study also attempts to complement existing literature by offering an evaluative tool for a brand's budget allocation decisions over time. As suggested by past studies, different media may exhibit different long-term effectiveness, suggesting that budget allocation plans have long-term implications. However, marketing efforts in practice are often driven by short-term profit maximization goals or by competitive reactions (Dekimpe and Hanssens, 1995; Yoo and Mandhachitara, 2003). As a result, budget allocation decisions may not be formulated based on their long-term potential. Allocative is one of the components of cost inefficiency considered by Luo and Donhu (2001, 2005), Färe *et al.*, (2004) and Vardanyan and Tremblay (2006), and it indicates how correctly firms are able to transmit prices signals into their input choices. One of the contributions of the present paper is to bring allocative inefficiency explicitly in the empirical analysis of the advertising process by examining its relative importance compared to technical inefficiency.

The second contribution is to empirically quantify the advertising spillovers among firms, that is, the effect of the marketing efforts of one firm on the marketing effectiveness of other firms within the same market (Metwally, 1975; Friedman, 1983; Sheldon *et al.*, 2000; Yiannaka *et al.*, 2002; Bagwell, 2005).[3] Empirical studies reveal that advertising spillovers are significant particularly in markets where non-cooperative behavior occurs between firms (*e.g.*, Tremblay and Polasky, 2001). The magnitude and the direction of these spillovers depend on the extent to which competing products are homogeneous with similar characteristics, the stage of market development and the kind of advertising message. On the other hand, advertising spillovers are unlikely to be significant in markets where products have very different characteristics and where consumers have a strong loyalty to a particular brand (Ackerberg, 2001).[4] In addition, advertising spillovers are not present in a monopoly market or in market where producers cooperate in their marketing efforts (Liu and Forker, 1988). However, in cases that firms are producing a homogeneous product advertising spillovers are present and affecting the demand for all brands and at the same time individual advertising expenses. The former is the *sales effect* of advertising spillovers associated with the changes in individual firm sales, whereas the latter is the *generic effect* related with the changes in individual advertising expenses. These spillovers may well be affected by the extent of advertising messages in each particular market. This *crowding-out effect* may affect both positive or negative individual marketing expenses. For instance in markets with homogeneous products, if firms are not cooperating and heavily advertise their products, the effect of spillovers may turn to be negative due to high levels of advertising expenses undertaken by rivals which increase the cost of promotion. Finally, the spillover effect may also depend on the maturity stage of the market (*maturity effect*). In later stages, when the market matured, advertising may become predatory creating negative spillovers.[5] The net spillover effect is case specific and depends on the direction of each one of these four different effects.

The third contribution of the present paper is to relate allocative inefficiency and advertising spillovers with the previously analyzed issues of scale economies and cost inefficiency in order to provide a more comprehensive picture of performance evaluation based on the notion of total advertising productivity (TAP) growth. Recently, Rust *et al.*, (2004) pinpoint concerns about the productivity of advertising and sales promotion expenditures. In particular, to rebuild confidence in advertising and marketing investment, they suggest the urgent need to “show how marketing adds to shareholder value, as the perceived lack of accountability has undermined marketing’s credibility, threatened marketing’s standing in the firm, and even threatened marketing’s existence as a distinct capability within the firm”. For the advertising process TAP refers to the increase in sales that cannot be accounted by the observed increase in the number of advertising messages in various media. It may be due to changes in advertising technology *per se*, to scale economies in advertising, to improvements in technical and allocative efficiency and, to the net advertising spillover effects. The empirical question is to estimate the relative importance of each of these sources of sales growth and then to use this information to propose appropriate measures for improving advertising effectiveness.

The objective of this paper is to address these issues developing a tractable approach for analyzing empirically the marketing process and advertising effectiveness Maintaining the separability assumption between sales and production technology, the proposed approach relies on the theoretical framework developed by Karagiannis *et al.*, (2004) which is adapted in an advertising cost function setting. Utilizing a flexible advertising distance function, the developed methodology is then applied to the advertising activity of meat processing firms in Greece during the period 1983-1997.

The rest of the paper is structured as follows. Section 2 presents the theoretical framework of analysis. Section 3 discusses the empirical model and the estimation procedure. Section 4 presents the data and the empirical results. Section 5 concludes.

2. Theoretical Framework

Following Seldon *et al.*, (2000) and Färe *et al.*, (2004) we may distinguish between two types of processes that take place within each production unit: the pure *production* process via which conventional inputs (*i.e.*, labor, materials, capital) are converted to outputs, and the *sales promotion* process via which consumers are either informed about the existence and characteristics of a good or brand or persuaded to buy the product for real or imagined benefits mainly through advertising. Given this distinction, the firm's total cost is comprised of two separable elements, the cost of conventional factors of production and that of advertising, *i.e.*,

$$C^T\left(y,\mathbf{w^p},\mathbf{w^s}\middle|x^{sr},t\right)=C^p\left(y,\mathbf{w^p}\middle|t\right)+C^s\left(y,\mathbf{w^s}\middle|x^{sr},t\right) \quad (1)$$

where $\mathbf{w^p}\equiv\left\{w_1^p,\ \ldots,w_h^p\right\}\in R_{++}^H$ is the vector of prices of the H conventional factors of production, $\mathbf{w^s}\equiv\left\{w_1^s,\ \ldots,w_k^s\right\}\in R_{++}^K$ is the vector of K advertising media prices, y is the total firm produce, and t is an index of time. x^{sr} is the total advertising expenses of firm's rival that affect cost of sales. In addition, as suggested by Seldon and Jung (1993), firms may not plan simultaneously their production and advertising decisions. In other words, firms may want to build or deplete their inventories during the period under consideration and, therefore, the quantities in the production and sales cost function may be different, *i.e.*, $C^s\left(y^s,\mathbf{w^s}\middle|x^{sr},t\right)$ and $C^p\left(y^p,\mathbf{w^p}\middle|t\right)$ with y^s and y^p being firm sales and production volumes in period t, respectively. Therefore, we can distinguish between production and advertising cost by defining two different underlying technologies for production and sales.

Concentrating on sales, we may define an advertising technology set describing all possible media combinations, conditional on advertising expenses of the firm's rivals that are able to generate a given level of sales as:[6]

$$T^{s} \equiv \left\{ \left(y^{s}, \mathbf{x}^{\mathbf{s}} \right) \middle| \ \mathbf{x}^{\mathbf{s}} \in R_{+}^{K}, y^{s} \in \Re_{+}, x^{sr} \in \Re_{+}, \mathbf{x}^{\mathbf{s}} \ can\ produce\ y^{s} \right\} \tag{2}$$

Then for any given T^{s}, the advertising or sales function $f^{s}\left(\mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right): \Re_{+}^{K} \rightarrow \Re_{+}$ that relates the advertising expenses with firm total sales is defined by:

$$f^{s}\left(\mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right) \equiv \max_{y^{s}} \left\{ y^{s} \middle| \left(\mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right) \in T^{s} \right\} \tag{3}$$

where $\mathbf{x}^{\mathbf{s}} \equiv \left\{ x_{1}^{s}, \ \ldots, x_{k}^{s} \right\} \in R_{++}^{K}$ is the vector of advertising media utilized to promote sales, $f^{s}(\bullet)$ is a quasiconcave twice differentiable function, x^{sr} are the total advertising expenses of the firm's rivals and, t is a time index reflecting changes in advertising effectiveness in the different media due to changes in advertising techniques. Equivalently the advertising/sales technology may be represented by the following advertising requirement set:

$$L^{s}\left(y^{s} \middle| x^{sr}, t\right) \equiv \left\{ \mathbf{x}^{\mathbf{s}} \middle| y^{s} \leq f^{s}\left(\mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right) \right\} \tag{4}$$

Using the advertising media correspondence set above, the advertising technology can also be represented by the (direct) advertising distance function[7] $D^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right): \Re_{+}^{K} \times \Re_{+} \rightarrow \Re_{+} \cup +\infty \ \ \Re_{+} \cup +\infty$ (Färe and Primont, 1995, p. 19):[8]

$$D^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right) \equiv \sup \left\{ \delta > 0 \middle| \left(\mathbf{x}^{\mathbf{s}} / \delta\right) \in L^{s}\left(y^{s} \middle| x^{sr}, t\right) \ \forall y^{s} \in \Re_{+} \right\} \tag{5}$$

Ultimately, we may define the (direct) advertising cost function as the solution of the following optimization problem (Balk, 1998, p. 26):

$$\begin{aligned} C^{s}\left(y^{s}, \mathbf{w}^{\mathbf{s}} \middle| x^{sr}, t\right) &\equiv \min_{x^{s}} \left\{ \mathbf{w}^{\mathbf{s}\prime} \mathbf{x}^{\mathbf{s}} \middle| \mathbf{x}^{\mathbf{s}} \in L^{s}\left(y^{s} \middle| t, x^{sr}\right) \right\} \\ &\equiv \min_{x^{s}} \left\{ \mathbf{w}^{\mathbf{s}\prime} \mathbf{x}^{\mathbf{s}} \middle| D^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| t, x^{sr}\right) \geq 1, \mathbf{w}^{\mathbf{s}} \gg 0 \right\} \end{aligned} \tag{6}$$

Using the advertising cost function above the advertising cost efficiency can then be defined as:

$$CE^{s}\left(y^{s},\mathbf{w}^{s},\mathbf{x}^{s}\middle|x^{sr},t\right)\equiv\frac{C^{s}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)}{C^{s}} \tag{7}$$

where C^{s} is the observed total cost of media mix and $0<CE^{s}\left(y^{s},\mathbf{w}^{s},\mathbf{x}^{s}\middle|x^{sr},t\right)\leq 1$. Advertising cost efficiency is independent of media price scaling and has a clear cost interpretation with $1-CE^{s}\left(y^{s},\mathbf{w}^{s},\mathbf{x}^{s}\middle|x^{sr},t\right)$ reflecting the percentage reduction in total sales cost if cost inefficiency is eliminated (Kopp, 1981).[9] Taking the logarithm of each side of (7) and totally differentiating with respect to time yields:

$$\begin{aligned}CE^{s}\left(y^{s},\mathbf{w}^{s},\mathbf{x}^{s}\middle|x^{sr},t\right)&=\varepsilon^{y^{s}}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)\dot{y}^{s}+\mathbf{s}^{s}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)'\dot{\mathbf{w}}^{s}\\&+\varepsilon^{x^{sr}}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)\dot{x}^{sr}+C^{s_t}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)-\dot{C}^{s}\end{aligned} \tag{8}$$

where[10] $\varepsilon^{y^{s}}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)=C^{s}_{y^{s}}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)$ is the sales cost elasticity, $\varepsilon^{x^{Sr}}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)=C^{s}_{x^{sr}}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)$ is the spillover effects of rival's advertising expenses on firm's total advertising cost, $\mathbf{s}^{s}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)=\nabla_{w^{s}}C^{s}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)$ is the vector of optimum media cost shares, and $-C^{s_t}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)=C^{s}_{t}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)$ is the rate of cost diminution of advertising media. Taking the logarithms of the observed sales cost, i.e., $C^{s}=\mathbf{w}^{s\prime}\mathbf{x}^{s}$, and totally differentiating it with respect to time yields:

$$\dot{C}^{s}=\mathbf{s}^{s\prime}\dot{\mathbf{x}}^{s}+\mathbf{s}^{s\prime}\dot{\mathbf{w}}^{s} \tag{9}$$

Substituting (9) into (8) results in:

$$\begin{aligned}CE^{s}\left(y^{s},\mathbf{w}^{s},\mathbf{x}^{s}\middle|x^{sr},t\right)&=\varepsilon^{y^{s}}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)\dot{y}^{s}+\mathbf{s}^{s}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)'\dot{\mathbf{w}}^{s}\\&+\varepsilon^{x^{sr}}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)\dot{x}^{sr}+C^{s_t}\left(y^{s},\mathbf{w}^{s}\middle|x^{sr},t\right)-\mathbf{s}^{s\prime}\dot{\mathbf{x}}^{s}-\mathbf{s}^{s\prime}\dot{\mathbf{w}}^{s}\end{aligned} \tag{10}$$

Using the conventional *Divisia* index measure of total advertising productivity (*TAP*) growth of firm sales, *i.e.*, $\dot{TAP}^S = \dot{y}^s - \mathbf{s}^{\mathbf{s}\prime}\dot{\mathbf{x}}^{\mathbf{s}}$, the relation (10) can be rewritten as:

$$\begin{aligned}\dot{y}^s &= \mathbf{s}^{\mathbf{s}\prime}\dot{\mathbf{x}}^{\mathbf{s}} + \left\{1 - \varepsilon^{y^s}\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)\right\}\dot{y}^s - C^s_t\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right) \\ &\quad - \varepsilon^{x^{sr}}\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)\dot{x}^{sr} + \left\{\mathbf{s}^{\mathbf{s}} - \mathbf{s}^{\mathbf{s}}\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)\right\}'\dot{\mathbf{w}}^{\mathbf{s}} + \dot{CE}^s\left(y^s,\mathbf{w}^{\mathbf{s}},\mathbf{x}^{\mathbf{s}}\middle|x^{sr},t\right)\end{aligned} \tag{11}$$

Following Farrell (1957), advertising cost efficiency can be decomposed as $CE^s\left(y^s,\mathbf{w}^{\mathbf{s}},\mathbf{x}^{\mathbf{s}}\middle|x^{Sr},t\right) = ITE^s\left(y^s,\mathbf{x}^{\mathbf{s}}\middle|x^{Sr},t\right)\cdot IAE^s\left(y^s,\mathbf{w}^{\mathbf{s}},\mathbf{x}^{\mathbf{s}}\middle|x^{Sr},t\right)$, with $ITE^s\left(y^s,\mathbf{x}^{\mathbf{s}}\middle|x^{Sr},t\right) = D^M\left(y^s,\mathbf{x}^{\mathbf{s}}\middle|x^{Sr},t\right)^{-1}$ and $IAE^s\left(y^s,\mathbf{w}^{\mathbf{s}},\mathbf{x}^{\mathbf{s}}\middle|x^{Sr},t\right) = \left\{D^M\left(y^s,\mathbf{x}^{\mathbf{s}}\middle|x^{Sr},t\right)C^s\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{Sr},t\right)\right\}\big/C^s$ are, respectively, the medium oriented measures of advertising technical and allocative efficiency. By definition, both measures lie within the $(0,1]$ interval, are independent of media price scaling, and have an analogous to CE^s cost-saving interpretation. Taking its time rate of change and substituting it into (11), we get the final sales growth decomposition formula as:

$$\begin{aligned}\dot{y}^s &= \underbrace{\mathbf{s}^{\mathbf{s}\prime}\dot{\mathbf{x}}^{\mathbf{s}}}_{\textit{size effect}} + \underbrace{\left\{1 - \varepsilon^{y^s}\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)\right\}\dot{y}^s}_{\textit{advertising scale effect}} + \underbrace{\left\{\mathbf{s}^{\mathbf{s}} - \mathbf{s}^{\mathbf{s}}\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)\right\}'\dot{\mathbf{w}}^{\mathbf{s}}}_{\textit{medium price adjustment effect}} \\ &\quad \underbrace{- \varepsilon^{x^{sr}}\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)\dot{x}^{sr}}_{\textit{advertising spillover effect}} \underbrace{- C^s_t\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)}_{\textit{technical change effect}} + \underbrace{\dot{ITE}^s\left(y^s,\mathbf{x}^{\mathbf{s}}\middle|x^{sr},t\right)}_{\textit{advertising TE effect}} + \underbrace{\dot{IAE}^s\left(y^s,\mathbf{w}^{\mathbf{s}},\mathbf{x}^{\mathbf{s}}\middle|x^{sr},t\right)}_{\textit{advertising AE effect}}\end{aligned} \tag{12}$$

The first term in (12) captures the contribution of aggregate advertising media growth on sales changes over time (size effect).[11] The more effective is an advertising medium, the higher its contribution to the size effect. The second term measures the relative contribution of scale economies in advertising to the growth of sales (scale effect). This term vanishes under constant returns to scale as $\varepsilon^{y^s}\left(y^s,\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right) = 1$, while it is positive (negative) under increasing (decreasing) returns to scale, as long as firm total sales increase, and *vice versa*. The third term in (12) is the medium price adjustment effect.[12] The existence of this term indicates that the aggregate measure of media-mix is biased in the presence of advertising allocative inefficiency (Bauer, 1990). Under advertising allocative efficiency, the media price adjustment effect is equal to zero as $\mathbf{s}^{s} = \mathbf{s}^{s}\left(y^{s}, \mathbf{w}^{s} \middle| x^{sr}, t\right)$. Otherwise, its magnitude is inversely related to the degree of advertising allocative inefficiency. The medium price adjustment effect is also equal to zero when advertising media prices change at the same rate.

The fourth term captures the advertising spillover effects, the impact that rival's advertising expenses may have on firm's total advertising productivity levels. It is zero in the absence of marketing spillovers and under a flexible specification of the advertising technology can be further decomposed into the four effects identified in the introductory section. The fifth term refers to the dual rate of technical change in firm's sales technology (cost diminution), which is positive (negative) under progressive (regressive) technical change. The last two terms in (12) are positive (negative) as advertising technical and allocative efficiency increase (decrease) over time. Note that there is no *a priori* reason for both types of advertising efficiency to increase or decrease simultaneously (Schmidt and Lovell, 1980) nor that their relative contribution should be of equal importance for TAP growth. More importantly, what really matters in TAP growth decomposition analysis is not the degree of advertising efficiency itself, but its improvement over time. That is, even at low levels of advertising cost efficiency, sales gains may be achieved by improving either technical or allocative efficiency, or both. However, it seems difficult to achieve substantial rates of sales growth at very high levels of technical and/or allocative advertising efficiency.

The next step concerns the recovery of all factors in (12) from a advertising-distance function, through its duality with the sales cost function. For doing so we have modified the framework developed by Karagiannis *et al.*, (2004) for the case of advertising technology. The Lagrangean multiplier and the first-order conditions of the minimization problem in (6) are given, respectively, as:

$$\Lambda\left(\mathbf{x}^{s}, \xi\right) = \mathbf{w}^{s\prime}\mathbf{x}^{s} + \xi\left\{1 - D^{M}\left(y^{s}, \mathbf{x}^{s} \middle| x^{sr}, t\right)\right\} \tag{13}$$

$$w^{s} = \xi \cdot \nabla_{x^{s}} D^{M}\left(y^{s}, \mathbf{x}^{s} \middle| x^{sr}, t\right) \tag{14}$$

and

$$1 - D^M\left(y^s, \mathbf{x}^{\mathrm{s}} \middle| x^{sr}, t\right) \tag{15}$$

where ξ is the Lagrangean multiplier. Using the first-order conditions and the linear homogeneity of the advertising-distance function in $\mathbf{x}^{\mathrm{s}}$, it can be shown (see Färe and Primont, 1995, p. 52) that $\xi\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = C^s\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right)$. In addition, by applying the envelope theorem the following relations can be obtained (Färe and Primont, 1995, p. 51):

$$C^s_{y^s}\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = -\xi\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) D^M_{y^s}\left(y^s, \mathbf{x}^{\mathrm{s}} \middle| x^{sr}, t\right) \tag{16}$$

$$C^s_{x^{sr}}\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = \xi\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) D^M_{x^{sr}}\left(y^s, \mathbf{x}^{\mathrm{s}} \middle| x^{sr}, t\right) \tag{17}$$

$$\nabla_{w^s} C^s\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = \mathbf{x}^{\mathrm{s}}\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) \tag{18}$$

and

$$C^s_t\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = -\xi\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) D^M_t\left(y^s, \mathbf{x}^{\mathrm{s}} \middle| x^{sr}, t\right) \tag{19}$$

Using (16), the first-order conditions, and $\xi\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = C^s\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right)$, Färe *et al.*, (1986) have shown that:

$$\varepsilon^{y^s}\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = D^M_{y^s}\left(y^s, \mathbf{x}^{\mathrm{s}} \middle| x^{sr}, t\right) \tag{20}$$

which provides the relationship for recovering the scale effect in (12) directly from the advertising distance function. On the other hand, using (19), the first-order conditions, and $\xi\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = C^s\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right)$, Atkinson and Cornwell (1998) have shown that:

$$-C^s_t\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = -D^M_t\left(y^s, \mathbf{x}^{\mathrm{s}} \middle| x^{sr}, t\right) \tag{21}$$

which relates the cost diminution with the primal rate of technical change in advertising techniques through the medium-distance function. Finally, using (17), the first-order conditions and $\xi\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right) = C^s\left(y^s, \mathbf{w}^{\mathrm{s}} \middle| x^{sr}, t\right)$ it can be shown that:

$$\varepsilon^{x^{sr}}\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)=D^M_{x^{sr}}\left(y^s,\mathbf{x}^s\middle|x^{sr},t\right) \tag{22}$$

Measurement of advertising allocative efficiency requires knowledge of either the minimum advertising cost $C^s\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)$ or the cost minimizing media vector $\mathbf{x}^s\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)$. Karagiannis *et al.,* (2004) have developed a procedure for deriving the latter by using the dual Shephard lemma $\breve{\mathbf{w}}^s=C^s\left(y^s,\breve{\mathbf{w}}^s\middle|x^{sr},t\right)\nabla_{x^s}D^M\left(y^s,\mathbf{x}^s\middle|x^{sr},t\right)$, where $\breve{\mathbf{w}}^s$ denotes the vector of virtual media prices and $C^s\left(y^s,\breve{\mathbf{w}}^s\middle|x^{sr},t\right)$ the minimum shadow sales cost of the observed media mix (Färe and Grosskopf, 1990).[13] Then, by definition, virtual and market media prices coincide at the (observed) advertising cost minimizing media-mix and, consequently, $C^s\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)=C^s\left(y^s,\breve{\mathbf{w}}^s\middle|x^{sr},t\right)$. This, in turn, implies that $\mathbf{w}^s=C^s\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)\ \nabla_{x^s}D^M\left(y^s,\mathbf{x}^s\middle|x^{sr},t\right)$. Karagiannis *et al.*, (2004) have shown that this system of equations can be solved for the cost minimizing medium ratios $\left(x^s_j/x^s_1\right)$ as long as the underlying technology is known and the virtual price of one advertising medium (*i.e.*, the first) coincides with its market price.[14] Using these ratios, the relation

$$\frac{C^s\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)}{x^s_1}=w^s_1+w^s_2\left(\frac{x^s_2}{x^s_1}\right)+\ldots+w^s_k\left(\frac{x^s_k}{x^s_1}\right) \tag{23}$$

can be used to compute allocative advertising efficiency as:

$$\begin{aligned}IAE^s\left(y^s,\mathbf{w}^s,\mathbf{x}^s\middle|x^{sr},t\right)&=\frac{D^M\left(y^s,\mathbf{x}^s\middle|x^{sr},t\right)C^s\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)}{\mathbf{w}^{s\prime}\mathbf{x}^s}=\\&D^M\left(y^s,\mathbf{x}^s\middle|x^{sr},t\right)\left[\frac{C^s\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)/x^s_1}{\mathbf{w}^{s\prime}\mathbf{x}^s/x^s_1}\right]\end{aligned} \tag{24}$$

where $x^s_1=x^s_1\left(y^s,\mathbf{w}^s\middle|x^{sr},t\right)$. Finally, the sales cost-minimizing media shares required to compute the third term in (12) can also be obtained from relation (23) as:

$$\mathbf{s}^{\mathbf{s}}\left(y^{s},\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)=\frac{\mathbf{w}^{\mathbf{s}\prime}\left(\mathbf{x}^{\mathbf{s}}/x_{1}^{s}\right)}{C^{s}\left(y^{s},\mathbf{w}^{\mathbf{s}}\middle|x^{sr},t\right)/x_{1}^{s}} \tag{25}$$

with the cost-minimizing share for the first advertising medium computed residually using the adding-up property.

3. Empirical Model and Estimation Procedure

Quantitative measures of technical and allocative advertising efficiency can be obtained by econometrically estimating a advertising distance function. This is feasible with only advertising and sales quantity data, and the use of a single-equation estimation procedure since the advertising distance function is agnostic with respect to the economic motivation of decision-makers. In the present study, the translog function is used to approximate the underlying advertising distance function as a more flexible representation in the sense that it allows for substitution possibilities without restrictive assumptions about the shape of the technological relationships. Assuming that panel data are available, the translog advertising distance function with k media and exogenous factors such as technical change and rivals' advertising expenditures is given by (Lovell *et al.*, 1994; Grosskopf *et al.*, 1997; Coelli and Perelman, 1999):

$$\begin{aligned} ln\,D_{it}^{M}\left(y^{s},\mathbf{x}^{\mathbf{s}};t,x^{s_r}\right) &= \alpha_0+\zeta_t t+\frac{1}{2}\zeta_{tt}t^2+\alpha_s\,ln\,y_{it}^{s}+\frac{1}{2}\alpha_{ss}\left(ln\,y_{it}^{s}\right)^2+\sum_{k=1}^{K}\delta_{sk}\,ln\,y_{it}^{s}\,ln\,x_{kit}^{s} \\ &+\zeta_{ts}\,ln\,y_{it}^{s}+\sum_{k=1}^{K}\beta_k\,ln\,x_{kit}^{s}+\frac{1}{2}\sum_{k=1}^{K}\sum_{j=1}^{K}\beta_{kj}\,ln\,x_{kit}^{s}\,ln\,x_{jit}^{s}+\sum_{k=1}^{K}\zeta_{tk}\,ln\,x_{kit}^{s}t \\ &+\theta_o\,ln\,x_{it}^{sr}+\frac{1}{2}\theta_{oo}\left(ln\,x_{it}^{sr}\right)^2+\theta_{os}\,ln\,x_{it}^{sr}\,ln\,y_{it}^{s}+\sum_{k=1}^{K}\theta_{ok}\,ln\,x_{it}^{sr}\,ln\,x_{kit}^{s}+\theta_{ot}\,ln\,x_{it}^{sr}t \end{aligned} \tag{26}$$

where, $i=1,2,\ldots,N$ are the production units and $t=1,2,\ldots,T$ are the time periods. The required regularity conditions include homogeneity of degree one in advertising media and symmetry (of the cross terms and thus the Hessian matrix). These imply the following restrictions on the parameters of (26):

$$\sum_{k=1}^{K}\beta_k = 1,\ \sum_{k=1}^{K}\beta_{kj} = \sum_{k=1}^{K}\delta_{kj} = \sum_{k=1}^{K}\zeta_{\tau k} = \sum_{k=1}^{K}\theta_{ok} = 0 \text{ and } \beta_{kj} = \beta_{jk} \tag{27}$$

Note that the homogeneity restrictions can also be imposed by dividing the left-hand side and all advertising media quantities on the right-hand side of (26) by the quantity of any advertising medium used as *numeraire* (see Lovell *et al.*, (1994)).

Given linear homogeneity in advertising expenses, to obtain an estimable form of the advertising distance function, first rewrite (26) as $-ln\, x^s_{jit} = ln\, f\left(y^s, \mathbf{x}^s / x^s_j \middle| x^{sr}, t\right)$ $-ln\, D^M_{it}\left(y^s, \mathbf{x}^s \middle| x^{sr}, t\right)$, where j is the *numeraire* medium and $f\left(y^s, \mathbf{x}^s / x^s_j \middle| x^{sr}, t\right)$ is the right-hand side of (26) after dividing all advertising media with the j^{th} medium used as *numeraire*. Since there are no observations for $ln\, D^M_{it}$ and given that $D^M_{it}\left(y^s, \mathbf{x}^s \middle| x^{sr}, t\right) \geq 1$ and $ln\, D^M_{it}\left(y^s, \mathbf{x}^s \middle| x^{sr}, t\right) \geq 0$, the following relationship may be used to translate (26) into a stochastic frontier framework:

$$ln\, D^M_{it}\left(y^s, \mathbf{x}^s \middle| x^{sr}, t\right) = u_{it} \tag{28}$$

where u_{it} is an one-sided, non-negative error term representing the stochastic shortfall of the i^{th} firm from its sales frontier due to the existence of advertising technical inefficiency. Then, the stochastic advertising distance function model may be written as:

$$-ln\, x^s_{jit} = ln\, f\left(y^s, \mathbf{x}^s / x^s_j \middle| x^{sr}, t\right) + v_{it} - u_{it} \tag{29}$$

where v_{it} depicts a symmetric and normally distributed error term (*i.e.*, statistical noise), representing a combination of those factors that cannot be controlled by firms, omitted explanatory variables, and measurement errors in the dependent variable. It is also assumed that v_{it} and u_{it} are distributed independently of each other.

Note that the temporal pattern of technical efficiency is important in (12) as it is the changes in advertising technical efficiency over time rather than the degree of technical efficiency that matter. In this context, Battese and Coelli (1992) formulation is adopted to model the temporal pattern of medium technical inefficiency as:

$$u_{it} = \left\{ exp\left[-\eta\left(t - T \right) \right] \right\} u_i \tag{30}$$

where η captures the temporal variation of individual technical efficiency ratings, and $t \in \left[1, 2, \ldots, T \right]$.[15] If the parameter η is positive (negative), technical efficiency tends to improve (deteriorate) over time. If $\eta = 0$, advertising technical efficiency is time-invariant, and it does not contribute to advertising productivity changes.

After substituting (30) into (29) the resulting model is estimated by a single-equation estimation procedure using the *ML* method. The variance parameters of the likelihood function are estimated in terms of $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\gamma = \sigma_u^2 / \sigma$, where the γ-parameter has a value between zero and one. The closer is the estimated value of the γ-parameter to one, the higher the probability of the advertising technical inefficiency effect to be significant in the stochastic frontier model.

Firm-specific estimates of advertising technical efficiency are obtained directly from the estimated mean and variance of u_{it}. Specifically,

$$ITE_{it}^{s} = \left[exp\left(-u_{it} \right) \right]^{-1} \tag{31}$$

which is predicted using the conditional expectation of $exp\left(-u_{it} \right)$ given $\varepsilon_{it} \left(\equiv v_{it} - u_{it} \right)$.[16]

After estimating the underlying advertising distance function, the advertising scale elasticity is calculated using (20) as:

$$\varepsilon^{y^s} = -\left(\alpha_s + \alpha_{ss} \ln y_{it}^{s} + \sum_{k=1}^{K} \delta_{sk} \ln x_{kit}^{s} + \zeta_{st} t + \theta_{os} \ln x_{it}^{s_r} \right) \tag{32}$$

The hypothesis of constant returns to scale can be tested by imposing the necessary restrictions associated with the linear homogeneity of the advertising distance function with respect to sales. If this hypothesis cannot be rejected, the underlying sales technology exhibits constant returns to scale and the second term in (12) vanishes.

On the other hand, by using (21), the dual and the primal rates of technical change are related to each other as follows:

$$-C_{it}^{t} = -\left(\zeta_t + \zeta_{tt} t + \sum_{k=1}^{K} \zeta_{tk} \, ln \, x_{kit}^{s} + \zeta_{st} \, ln \, y_{it}^{s} + \theta_{ot} \, ln \, x_{it}^{s_r} \right) \tag{33}$$

The hypothesis of zero technical change can be tested by imposing the restriction that $\zeta_t = \zeta_{tt} = \zeta_{tk} = \zeta_{ts} = \theta_{to} = 0 \;\; \forall k$.[17] If the hypothesis of zero technical change cannot be rejected, the fourth term in (12) becomes equal to zero, and technical change has no effect on TAP changes.

Then using (22) the marketing spillover effect may be calculated from (26) as:

$$\varepsilon_{it}^{x^{sr}} = \theta_o + \theta_{oo} \, ln \, x_{it}^{sr} + \theta_{os} \, ln \, y_{it}^{s} + \sum_{k=1}^{K} \theta_{ok} \, ln \, x_{kit}^{s} + \theta_{ot} t \tag{34}$$

where, the first two terms in (34) $\left(i.e., \; \theta_o + \theta_{oo} \, ln \, x_{it}^{sr} \right)$ constitute the *generic effect* of marketing spillovers that is the extent that rival's advertising expenses affect firm's cost of sales, the third term $\left(i.e., \; \theta_{os} \, ln \, y_{it}^{s} \right)$ is the *sale spillover effect* capturing the extent that rival's advertising expenses affect firm's total sales, the fourth term $\left(i.e., \; \sum \theta_{ok} \, ln \, x_{kit}^{s} \right)$ is the *crowding-out effect* capturing the extent that rival's advertising expenses affect firm's media choice and the last term $\left(i.e., \; \theta_{ot} t \right)$ is the maturity effect capturing the stage of market development and it's impact on firms cost of sales. The presence of marketing spillover effects can be statistically examined by imposing the restriction $\theta_o = \theta_{oo} = \theta_{os} = \theta_{ok} = \theta_{ot} = 0 \; \forall k$ in the parameters of (26).

By manipulating the dual Shephard's lemma for the translog advertising distance function, evaluating them for the sales cost minimizing medium vector, and taking ratios results in:

$$\left(\frac{w_{kit}^{s}}{w_{1it}^{s}} \right) \left(\frac{x_{kit}^{s}}{x_{1it}^{s}} \right)^{0} = \frac{\beta_k + \sum_{j=1}^{K} \beta_{kj} \, ln \left(\frac{x_{jit}^{s}}{x_{1it}^{s}} \right) + \delta_{ks} \, ln \, y_{it}^{s} + \zeta_{kt} t + \theta_{ko} \, ln \, x_{it}^{s_r}}{\beta_1 + \sum_{j=2}^{K} \beta_{1j} \, ln \left(\frac{x_{jit}^{s}}{x_{1it}^{s}} \right) + \delta_{1s} \, ln \, y_{it}^{s} + \zeta_{1t} t + \theta_{1o} \, ln \, x_{it}^{s_r}} \tag{35}$$

After few manipulations (35) may be written as:

$$\left(\frac{w_{kit}^{s}}{w_{1it}^{s}}\right) exp\left(\lambda_{jit}^{s}\right)\left[\beta_{k}+\sum_{j=2}^{K}\beta_{kj}\lambda_{jit}^{s}+\delta_{ks}\, ln\, y_{it}^{s}+\zeta_{kt}t+\theta_{ko}\, ln\, x_{it}^{s_r}\right]= \\ =\beta_{1}+\sum_{j=2}^{K}\beta_{1j}\lambda_{jit}^{s}+\delta_{1s}\, ln\, y_{it}^{s}+\zeta_{1t}t+\theta_{1o}\, ln\, x_{it}^{s_r} \tag{36}$$

where $\lambda_{jit}^{s}=ln\left(x_{jit}^{s}/x_{1it}^{s}\right)$. This is a system of *n-1* non-linear equations because of the exponential terms on the left hand-side of (36) and it has no closed form solution. In principle it can be solved to obtain $\left(x_{jit}^{s}/x_{1it}^{s}\right)$ for *j=2,…,K* in terms of the observed media prices, the observed sales, the index of technical change, and the estimated advertising technology parameters. Karagiannis *et al.*, (2005) have provided an approximate based on $\Lambda_{jit}^{s}=exp\left(z_{jit}^{s}\right)\cong 1+z_{jit}^{s}$. In this case, (36) may be rewritten as:

$$\left(\frac{w_{kit}^{s}}{w_{1it}^{s}}\right)\left[\beta_{k}+\sum_{j=2}^{K}\beta_{kj}\Lambda_{jit}^{s}+\delta_{ks}\, ln\, y_{it}^{s}+\zeta_{kt}t+\theta_{ko}\, ln\, x_{it}^{s_r}\right]= \\ =\beta_{1}+\sum_{j=2}^{K}\beta_{1j}\Lambda_{jit}^{s}+\delta_{1s}\, ln\, y_{it}^{s}+\zeta_{1t}t+\theta_{1o}\, ln\, x_{it}^{s_r} \tag{37}$$

The advantage of this approximation is that the system of equations becomes linear and can be solved using standard techniques.[18] Then, these cost minimizing advertising media ratios are substituted into (23) and then into (24) to derive firm-specific estimates of advertising allocative efficiency. Using the dual Shephard lemma and the obtained virtual prices we can evaluate the extent of over- or under- utilization of advertising expenses by comparing virtual with observed media shares as:

$$MU_{k}=\frac{s_{k}^{s}\left(y^{s},\breve{w}^{s}\middle|x^{sr},t\right)}{s_{k}^{s}}\Rightarrow ln\, MU_{k}=ln\, s_{k}^{s}\left(y^{s},\breve{w}^{s}\middle|x^{sr},t\right)-ln\, s_{k}^{s} \tag{38}$$

Values greater than unity (zero) indicate under-utilization of the k^{th} advertising medium, whereas values less than one (zero) over-utilization. Further, using virtual prices we can evaluate the slope of the isoquant at the observed media mix *i.e.*, the marginal rate of technical substitution, as follows:

$$MRTS_{kj}^{s} = \frac{D_{k}^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right)}{D_{j}^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right)} \tag{39}$$

where the subscripts refer to partial derivatives of the advertising distance function. If the advertising technology satisfies all regularity conditions, the $MRTS_{kj}^{s}$ will increase as the ratio of advertising media falls because increased use of one medium alone occurs at higher opportunity cost. This requires the normalization of (39) by the observed media mix (Grosskopf *et al.*, 1995b), *i.e.*,

$$Sub_{kj}^{s} = \frac{MRTS_{kj}^{s}}{x_{j}^{s} / x_{k}^{s}} \tag{40}$$

for all $k \neq j$ which is the ratio of marginal rate of technical substitution (or relative opportunity cost) to relative media mix, *i.e.*, normalized $MRTS_{kj}^{s}$. Values of Sub_{kj}^{s} greater (less) than unity reflect relative difficulty (ease) in substitution between media *k* and *j*.

Following Blackorby and Russell (1989), Grosskopf *et al.*, (1995b) and Sharma (2002), the *Morishima* indirect advertising elasticities of substitution can be computed from the advertising distance function as:

$$\sigma_{kj}^{M_s} = x_{k}^{s} \frac{D_{kj}^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right)}{D_{j}^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right)} - x_{k}^{s} \frac{D_{kk}^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right)}{D_{k}^{M}\left(y^{s}, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right)} \tag{41}$$

Media *j* and *k* are *Morishima* substitutes if $\sigma_{kj}^{M_s} > 0$ and *Morishima* complements if $\sigma_{kj}^{M_s} < 0$. Following Blackorby and Russell (1989), the percentage change in the relative shares of media *k* and *j* induced by a given percentage change in their relative prices due to a change in the price of medium *j* is equal to $\left(1 - \sigma_{kj}^{M_s}\right)$. Thus, for a given increase in the price of the j^{th} medium, the relative share of medium *j* decreases if $\sigma_{kj}^{M_s} > 1$ and increases if $\sigma_{kj}^{M_s} < 1$. In addition, the *Allen* elasticities of advertising substitution can be obtained in terms of the advertising distance function as:

$$\sigma_{kj}^{A_s} = \frac{D^M\left(y^s, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right) D_{kj}^M\left(y^s, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right)}{D_k^M\left(y^s, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right) D_j^M\left(y^s, \mathbf{x}^{\mathbf{s}} \middle| x^{sr}, t\right)} \tag{42}$$

which, in contrast with *Morishima*, are symmetric.[19]

4. Data and Empirical Results

The data used in this paper are firm level data on sales and disaggregated advertising expenditures for all firms that operated systematically in the Greek meat processing sector during the period 1983-1997. Systematically operating firms are defined as those whose balance sheets are published annually by ICAP Hellas SA that provides individual annual balance sheet data for all manufacturing sectors in Greece. Sample participants account for most of the industry production (96.5 per cent in 1997)[20] and are the ones who advertised their products during the study period.

The panel data set used in the analysis is unbalanced due to late entries and early exits from the market. The number of systematically operating firms varied between 22 in 1983 and 34 in the period 1989 to 1991. Data on total firm sales were obtained from ICAP Hellas SA (1983-1997), while those for advertising expenses from Nielsen Hellas (1983-1997). The data on advertising indicates that total advertising expenses almost tripled between 1983 and 1997 in real 1983 prices. On average, approximately 80 percent of the industry's advertising was carried out by seven of the biggest firms of the sector. Variation in the ranking of the top four firms in terms of advertising budgets indicates competition in advertising among firms of the sector.

Media used in processed meats advertising included television, radio, magazines and newspapers with the television being the favored medium for firms of the industry. Total television advertising expenditure was greater than the aggregate of the three other media for the whole study period. A decrease in television's share during the last years benefited other media, especially radio, which was a remarkably well-utilized medium for the leading advertisers of the sector. The high utilization of both magazines and newspapers by the systematic advertisers in the sector should also be noted. The seven firms that advertised consistently accounted, on average, for 90 percent of print media expenditures, and about 70 percent of total magazine and newspaper advertising was carried out each year by the four firms with the largest advertising budgets. The latter observation may be consistent with the argument that for high-quality firms with the incentive to reveal their products' superior quality, print media and especially newspapers are the most appropriate and, therefore, favorite advertising media (Tirole, 1988). A brief summary statistics of the variables used in the analysis is presented in Table 1.

Television advertising expenses were used as the *numeraire* medium in imposing the linear homogeneity in media restriction. To avoid problems associated with units of measurement, all variables included in the stochastic medium-distance function were converted into indices. The basis for normalization was the firm with the smallest deviation of its sales and advertising expenses levels from the sample means.

The estimated parameters of the translog advertising distance function are presented in Table 2. According to the estimated parameters, the translog advertising distance function is found, at the point of approximation, to be non-increasing in sales and non-decreasing in advertising media. Also, at the point of approximation, the Hessian matrix of the first- and second-order partial derivatives with respect to advertising media is found to be negative definite and that with respect to sales to be positive definite. These indicate the concavity and convexity of the underlying advertising distance function with respect to media and sales, respectively. The logarithm of the likelihood function indicates a satisfactory fit for the particular functional form. The ratio parameter, γ, is significant at the 1 per cent level implying that firm-specific advertising technical inefficiency is important in explaining the total variability of realized sales. Obviously, the remained unexplained portion is due to measurement errors and misspecification problems from factors that have not been included in the stochastic advertising distance function. The statistical significance of modeling advertising technology is examined using the likelihood ratio test.[21] Several hypotheses are considered for different model specifications and the results are presented in Table 3.

It is evident that the "average" advertising distance function does not adequately represent the structure of Greek meat processing firms in the sample. The null hypothesis that $\gamma = \mu = \eta = 0$ is rejected at the 5 per cent level of significance indicating that the advertising technical inefficiency is in fact present. Moreover, Schmidt and Lin's (1984) test for the skewness of the composed error term also confirms the existence of advertising technical inefficiency.[22] This indicates that the majority of Greek meat processing firms in the sample operate below their stochastic advertising frontier. Thus, a significant part of sales variability among firms is explained by the existing differences in their degree of advertising technical inefficiency. In addition, the hypothesis that $\mu = \eta = 0$ is also rejected at the 5 per cent level of significance indicating that the stochastic frontier cannot be reduced to the Aigner *et al.*, (1977) model, where advertising technical efficiency scores is time-invariant following a half-normal distribution. This is also true for time-varying advertising technical efficiencies as the hypothesis that $\mu = 0$ is also rejected. Finally, the hypothesis that advertising technical inefficiency is time-invariant, $\eta = 0$, is also rejected at the same level of significance.

The hypotheses of no technical change (*i.e.*, $\zeta_t = \zeta_{tt} = \zeta_{tj} = 0 \ \forall j$) and Hicks-neutral technical change (*i.e.*, $\zeta_{tj} = 0 \ \forall j$) are rejected at the 5 per cent significance level indicating that technical change has been a significant source of total advertising productivity growth in the Greek meat processing industry and it should be taken into account in equation (12). The neutral component of technical change is found to be progressive at a constant rate as the estimates for the parameters ζ_t and ζ_{tt} are both negative and statistically significant at the 5 per cent significance level (see Table 2). Regarding technological biases, technical change is found to be print media-using and television-saving, but radio-neutral (as the relevant estimated parameter is not statistically different from zero). On the other hand, the null hypothesis of a linearly homogeneous production technology is also rejected at the 5% significance level, indicating non-constant returns to scale. Thus, the advertising scale effect is a significant source of productivity growth and should be taken into account in equation (12).

According to our empirical results, production was characterised by increasing returns to advertising scale, with average scale parameter of 1.267 during the period 1983-97. An implication of this result is that large-scale advertising undertaken by meat processors has been a barrier to entry into the Greek processed meats market. This result is in line with previous evidence reported in the literature (Peles, 1971; Brown, 1978) although recently Seldon *et al.*, (2000) found contradictory evidence in the US brewery sector. Increasing returns to advertising scale also imply that advertising expenditures have not grown beyond the potential capabilities of the sales technology. Even though the average advertising expenditures were less than the advertising expenditures that maximize the ray average productivity, the continued increase in average advertising expenditures resulted in returns to scale following a declining trend over time. The relevant point estimate of returns to media scale fell from 1.423 in 1983 to 1.109 in 1997.

The marketing spillover effects are important in modelling advertising technology of Greek processed meat firms. Statistical testing rejects the hypothesis of zero marketing spillovers at the 5% significance level (see Table 3). Further, examining the individual statistical significance of the relevant parameters reported in Table 2, it reveals that all four spillover effects identified herein are present affecting the advertising cost function of Greek processed meat firms. Using these parameter estimates the separate marketing spillover effects calculated using (34) were found to be -0.3898 for the generic effect, 0.3227 for crowding-out effect, 0.1424 for sales effect and 0.0270 for maturity effect. These figures mean first, that advertising expenses in the market are affecting negatively the individual cost of sales implying a low degree of product differentiation in the market. Given the nature of the sector which heavily advertised newly developed processed meat products during the year of survey, implies that generic advertising effects resulted in the increase for the demand of these newly developed products that affected negatively the individual advertising expenses. This is also evident from the sales effect which is also positive. However, these positive spillovers are lessened from the extensive advertising campaigns from all firms in the market that forces increased individual advertising expenses in order to maintain market share. Finally, the maturity effect was found to be positive (*i.e.*, increasing advertising cost) as market expands confirming previous findings reported in the literature. These marketing spillovers once are present are important factors explaining total advertising productivity and should be taken into account in (12).

Using the parameter estimates of the advertising distance function and the relations (39) to (42), we have calculated normalized and non-normalized marginal rates of technical substitution as well as the *Morishima* and *Allen* elasticities of substitution of all media pairs.[23] Summary statistics of those calculations for the whole period are presented in Table 4. The highest *MRTS* are those between television and the other two advertising media. The same pattern remains when the *MRTS* are normalized by the observed media-mix (see *Sub* in Table 3). In general, there is an ease of substitution between TV and radio or print media. Looking at the individual point estimates, large firms with high market share exhibit higher *MRTS* values implying that larger firms are better able to adjust their media-mix in response to changes in their respective shadow prices. Turning to the *Morishima* and *Allen* elasticities of substitution (also presented in Table 4), we find some interesting differences across media types. Recall that negative values of the *Allen* elasticities imply that the two media are substitutes, whereas positive values imply the two media are complements.[24] Hence, according to the relevant point estimates, television advertising messages are complementary to both print and radio messages, while radio and print messages appear to be substitutes.

Although the *Morishima* elasticities of substitution do not always have the same sign with the *Allen*, the results reported in Table 4 are rather similar.[25] Table 4 reveals that the estimated *Morishima* elasticities of substitution between television and radio messages and between radio and television messages are positive and statistically significant at the 5 per cent level. Hence, television and radio messages are *Morishima* substitutes irrespective of whether the price of radio or the price of television messages rises. Moreover, it holds that $\sigma^{M}_{TVR} < \sigma^{M}_{RTV}$ as the average estimates over firms and time are 0.3018 and 0.9389, respectively. This implies that when the price of television messages rises there is an increase in the radio to television messages ratio which is greater than the increase in the television to radio messages that occurs when the price of radio messages rises. For example, an one per cent increase in the price of television messages causes the radio over television messages ratio to change by 0.9389 per cent while a one per cent increase in the price of radio messages causes the television over radio messages ratio to rise by 0.3018 per cent. On the other hand, television and print messages as well as print and television messages are *Morishima* complements as the corresponding point estimates are negative in both cases. Finally, radio and print messages exhibit negative elasticity values implying *Morishima* complementarily. However, in all cases substitutability is weak among television and the other two media types implying that, if the price of TV broadcasting messages increases, there is a relatively low opportunity for substitution.

The estimated advertising technical efficiency using relation (31) was found to be 66.14 per cent during the period 1983-97 (see Table 5). Thus, on the average, a 33.86 per cent decrease in total advertising costs could have been achieved during this period, without altering the total sales, sales technology, and media use. The vast majority of the firms in the sample (76.5 per cent) have consistently achieved scores of technical efficiency greater than 70 per cent. On the other hand, there are 8 firms in the Greek meat processing industry that are faced with significant advertising technical inefficiency problems as their respective scores are below 60 per cent. However, the portion of firms with low advertising technical efficiency problems has been decreased over the period analyzed. In general, during the 1983-97 period, advertising technical efficiency tended to increase as the estimated parameter η_t was positive and statistical significant. Specifically, advertising technical efficiency increased from 60.09 per cent in 1983 to 72.34 per cent in 1997, implying that its contribution to sales growth was positive during the same period. The average annual rate of increase in advertising technical efficiency was calculated to be 0.34 per cent.

Mean advertising allocative efficiency was found to be higher than the corresponding technical efficiency during the period analyzed (see Table 5). This indicates that Greek meat processing firms were more successful in allocating their advertising expenses to the different media than in achieving the maximum attainable sales for given advertising expenses. Specifically, mean advertising allocative efficiency was found to be 77.28 per cent ranging from a minimum of 48.17 per cent to a maximum of 96.81 per cent. This average value implies that, in light of the prevailing prices for advertising media, a 22.72 per cent reduction in cost of sales would be possible by means of further re-allocation of media-mix for any given level of firm sales. The majority of Greek meat processing firms (21 firms) have consistently achieved scores of advertising allocative efficiency above 80 per cent. In addition, this portion tended to increase since 1983. Generally speaking, advertising allocative efficiency increased at a higher rate than advertising technical efficiency during the 1983-97 period. In particular, advertising allocative efficiency has been increasing at an average annual rate of 0.76 per cent and has contributed positively to both total advertising productivity (TAP) and sales growth.

Since advertising expenses are undertaken before sales are realized, there is a considerable uncertainty regarding the benefits from an advertising campaign in a particular medium. Hence, it is quite likely that firms will not always choose the optimal media-mix of advertising ex post. Apart from the riskiness of advertising campaigns, it seems that firms spent either too much or too little on one advertising medium versus another. Using relation (38) the difference between virtual and observed media mix has been calculated and the average values are reported to Figure 1. These values indicate that Greek meat processing firms consistently spent too much on television advertising and too little on print and radio advertising. However, it seems that individual firm perception have been improved since 1983 as the difference between observed and virtual media mix has been decreasing over time. Greek meat processing firms have had increasing success in choosing the most efficient media mix as their respective indices are getting closer to zero over time. However, there are still systematic errors indicating the existence of unexploited benefits from redistributing advertising budgets among media types. Specifically, firms in the sample would benefit from redistributing their advertising spending from television to the other two media (*i.e.*, print media and radio).

Finally, mean advertising cost efficiency was found to be 52.18 per cent for the firms in the sample. This value is calculated as the ratio of minimum to actual advertising cost and implies that significant cost savings can be realized (47.82 per cent) by eliminating both technical and allocative advertising inefficiency. Only a small portion of firms (23.5 per cent) in the sample achieved efficiency scores greater than 80 per cent. In general, there is a considerable variability in media-mix inefficiency among Greek meat processing firms. This seems reasonable, as the expected benefits of advertising are uncertain and vary considerably between the different media. Specifically, individual advertising cost inefficiency ranges from a minimum of 12.28 per cent to a maximum of 88.32 per cent. Advertising cost inefficiency increased over the period analyzed from 48.76 to 59.23 per cent. Nevertheless, its annual rate of increase was greater than that of technical efficiency as allocative efficiency tended to increase at a higher rate. Regarding the potential relationship between firm success (as measured by individual market shares) and sales efficiency, the Spearman correlation coefficient was 0.737 and statistically significant at the 5 per cent level, indicating a positive and strong relationship between these two variables.

The decomposition of sales growth for the Greek meat processing industry during the 1983-97 period is given in Table 6. During that period, average annual sales growth for the firms in the sample was 3.14 per cent. A greater share of the observed sales growth (63.0 per cent) was due to total advertising productivity growth and a smaller share (53.0 per cent) due to the size effect (*i.e.*, aggregate media-mix use). Specifically, average annual rate of TAP growth was estimated at 1.97 per cent, whereas aggregate media-mix use increased at an average annual rate of 1.66 per cent. Most of the aggregate media use growth is associated with increase in television (32.5 per cent) and radio (11.5 per cent) advertising expenses, whereas expenses in print media advertising have accounted for only 9.0 per cent of the total media expenses growth. However, substantial growth in firm sales can still be achieved by improving total advertising productivity.

Since the hypotheses of constant returns to scale and zero technical change have been rejected at the 5 per cent level of significance (see Table 3), the effect of advertising scale economies and technical change should also be taken into account when analyzing advertising productivity. Parameter estimates of the advertising distance function indicate an average 0.73 per cent technological progress in advertising during the 1983-97 period. The neutral component dominated the non-neutral one, with the latter being statistically significant (the hypothesis of neutral technical change has also been rejected at the 5 per cent level of significance). Specifically, the neutral component accounted for 17.8 per cent of total sales growth and the non-neutral one for 5.6 per cent. It is important to note that, unlike most previous studies analyzing productivity growth, technical change has not been the main source of TAP growth accounting for 39.1 per cent of TAP growth and 23.4 per cent of total sales growth.

On the other hand, given that the hypothesis of linear homogeneous sales technology has been rejected at the 5 per cent level, the scale effect in relation (12) is present, contributing to advertising productivity changes and sales growth. Since the firms in the sample exhibited increasing returns to advertising scale and the aggregate sales index increased over time, the scale effect in TAP growth is positive. On the average, the degree of advertising scale economies was estimated at 1.267 during the period 1983-97. As a result, economies of scale enhanced annual sales growth by an average annual rate of 0.66 per cent (see Table 6). In relative terms, the scale effect was the third largest factor influencing TAP and sales growth, after allocative efficiency and changes in advertising technology. In particular, media scale effect accounted for the 21.1 per cent of total sales growth and for the 35.3 per cent of total advertising productivity growth. Marketing spillover effects are also present contributing for the 3.3 per cent of total sales growth during the analyzed period. The highest positive impact arises from generic spillovers which account on the average for the 1.14 per cent increase in firm's sales, whereas crowding-out, sales and maturity spillover effects are all negative accounting on the average for the -0.58, -0.47 and -0.08 per cent of total sales growth, respectively. In total marketing spillovers due to the significant generic spillovers enhanced annual sales growth by an average annual rate of 0.10 per cent.

Both advertising technical and allocative efficiency have affected TAP and sales growth in the same manner (*i.e.*, positively). Specifically, the effect of both changes is positive as the pattern of technical inefficiency indicated movements towards the sales frontier over time, whereas that of allocative inefficiency showed moves towards an optimal utilization of media mix in light of the prevailing media prices. The relative contribution of each index depends on their rate of change over time, rather than their absolute magnitudes. As shown in Table 6, the relative contribution of allocative efficiency on sales growth was higher than that of technical efficiency. In particular, changes in allocative inefficiency enhanced sales growth by an average annual rate of 0.76 per cent whereas that of technical inefficiency by 0.34 per cent. Overall, advertising cost inefficiency accounted for 35.3 and 94.2 per cent of average annual productivity and sales growth, respectively. Finally, the price adjustment effect was found to have a relatively significant impact on both TAP and sales growth. On average, the price adjustment effect accounted for 20.1 per cent of sales slowdowns (0.63 per cent average annual decrease). However, given that allocative inefficiencies are indeed present, their impact cannot be neglected when attempting to accurately measure the rate of TAP growth. After accounting for all theoretically proposed sources of TAP growth and for the media effect, a -15.9 per cent of observed sales growth remained unexplained.

5. Concluding Remarks

This paper developed a theoretical framework for analyzing the substitutability between different advertising media, the allocative efficiency of advertising spending, the extent of marketing spillovers and the sources of total advertising productivity growth. Maintaining the separability assumption between sales and production technologies, the paper proposed methods of decomposing advertising productivity growth into scale economies, changes in advertising techniques, marketing spillovers and advertising technical and allocative efficiency as well as methods of estimating these effects.

The proposed methodology was applied to an unbalanced panel data set on sales and media advertising of the Greek meat processing industry over the period 1983-97. Our analysis indicates the existence of significant economies of scale in the Greek meat processing sector implying that large scale advertising undertaken by firms in our sample acts as a barrier to entry in this market. The estimated Morishima elasticities of substitution indicate that television and radio messages are substitutes irrespective of whether the price of radio or the price of television messages rises. This substitutability is weak, however, indicating a low opportunity for substitution of TV messages. On the other hand, television and print messages as well as radio and print advertisements are found to be complements.

The estimated advertising technical efficiency was found to be 66.14 per cent on the average suggesting that total sales can be significantly increased without altering the sales technology and media use. On the other hand, measures of advertising allocative efficiency indicate that Greek firms were more successful in allocating advertising media-mix than in achieving the maximum potential sales from given advertising expenses. Nevertheless, our analysis suggests that there is still room for important cost reductions from a further re-allocation of media expenses.

Finally, the productivity decomposition analysis revealed that Greek firms have experienced an average annual sales growth of 3.14 per cent. The greater part of this growth is attributed to increased advertising productivity and, to a lesser extent, to increases in aggregate media-mix. Overall sales cost efficiency accounted for the 35.3 per cent of total advertising productivity indicating that Greek meat processing firms exhibited movements towards their sales frontier over time as well as an optimal utilization of media-mix in the light of prevailing media prices.

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Table 1. Descriptive Statistics of the Variables (average values for the 1983-97 period).

Variable	Mean	Min	Max	StDev
Sales	967	16	11,009	1,435
Advertising Expenses:				
Television	3,389	229	70,833	10,571
Radio	1,225	125	95,434	6,355
Print Media	401	124	20,882	1,661
Rivals Adv Expenses	3,938	754	11,516	2,784
Advertising Mix:				
TV/Radio	8,117	25	70,331	6,837
TV/Print media	24,823	6	64,709	15,399
Radio/Print media	8,975	6	51,219	2,888

Note: All variables are expressed in euros except of total sales and rivals advertising expenses which are in ths euros.

Table 2. Parameter Estimates of the Stochastic Translog Medium-Distance Function.

Parameter	Estimate	StdError	Parameter	Estimate	StdError
β_0	3.1628	(0.1179)*	δ_{SV}	-0.0315	(0.0143)*
β_R	0.1641	(0.0519)*	ζ_T	-0.0706	(0.0220)*
β_P	0.6017	(0.0461)*	ζ_{TT}	-0.0761	(0.0144)*
β_V	0.2342	(0.0123)*	ζ_{TR}	-0.0969	(0.1059)
β_{RP}	0.0658	(0.0249)*	ζ_{TP}	-0.2451	(0.0542)*
β_{RV}	-0.0258	(0.1213)**	ζ_{TV}	0.3421	(0.1209)*
β_{RR}	-0.0400	(0.0154)*	ζ_{TS}	-0.0605	(0.0763)
β_{PV}	-0.0415	(0.0234)**	θ_O	-0.4135	(0.0144)*
β_{PP}	-0.0244	(0.0082)*	θ_{OV}	-0.3380	(0.1192)*
β_{VV}	0.0673	(0.0546)	θ_{OR}	0.1302	(0.0684)**
α_S	-0.6432	(0.1145)*	θ_{OP}	0.2079	(0.0551)*
α_{SS}	-0.1562	(0.0320)*	θ_{OO}	-0.1266	(0.1597)
δ_{SR}	-0.0085	(0.0591)	θ_{OS}	0.1434	(0.0784)**
δ_{SP}	0.2400	(0.0399)*	θ_{OT}	-0.1527	(0.3055)*
σ^2	3.2113	(0.9769)*	μ	-3.2422	(0.8284)*
γ	0.8184	(0.0568)*	η	0.0708	(0.0153)*
Ln(θ)	-527.378				

Note: *R* refer to radio advertisement, *P* to print media, *V* to television, *S* to total sales, O for rivals advertisement expenditures, and *T* for time. *(**) indicate that the parameter is significant at the 1(5) per cent level. The standard errors of the parameters obtained using the homogeneity restrictions in (27) are computed using delta method.

Table 3. Model Specification Tests.

Hypothesis	LR-statistic	Critical Value (α=0.05)
$\gamma = \mu = \eta = 0$	91.36	$\chi_3^2 = 7.81$
$\mu = \eta = 0$	55.84	$\chi_2^2 = 5.99$
$\mu = 0$	15.63	$\chi_1^2 = 3.84$
$\eta = 0$	15.27	$\chi_1^2 = 3.84$
CRTS	92.14	$\chi_6^2 = 12.6$
Absence of Marketing Spillovers	57.64	$\chi_7^2 = 14.1$
Zero Technical Change	43.69	$\chi_7^2 = 14.1$
Hicks Neutral Technical Change	34.18	$\chi_5^2 = 11.1$

Table 4. MRTS and Elasticities of Substitution of Advertising Expenditures (average values for the 1983-97 period).

Media	MRTS	Sub	Allen	Morishima
TV x Radio	0.6206	6.0545	-0.1909	0.3018
	(0.1091)	(2.3421)	(0.0893)	(0.1342)
TV x Print	0.9871	6.1901	-0.2645	-0.1655
	(0.1234)	(2.6743)	(0.0938)	(0.0653)
Radio x Print	0.4615	0.6954	0.3858	-0.5278
	(0.1192)	(0.3442)	(0.1982)	(0.1832)
Radio x TV	-	-	-	0.9389
				(0.2342)
Print x TV	-	-	-	-0.0030
				(0.0127)
Print x Radio	-	-	-	-0.1679
				(0.0464)

Note: Standards errors computed using delta method are in parentheses.

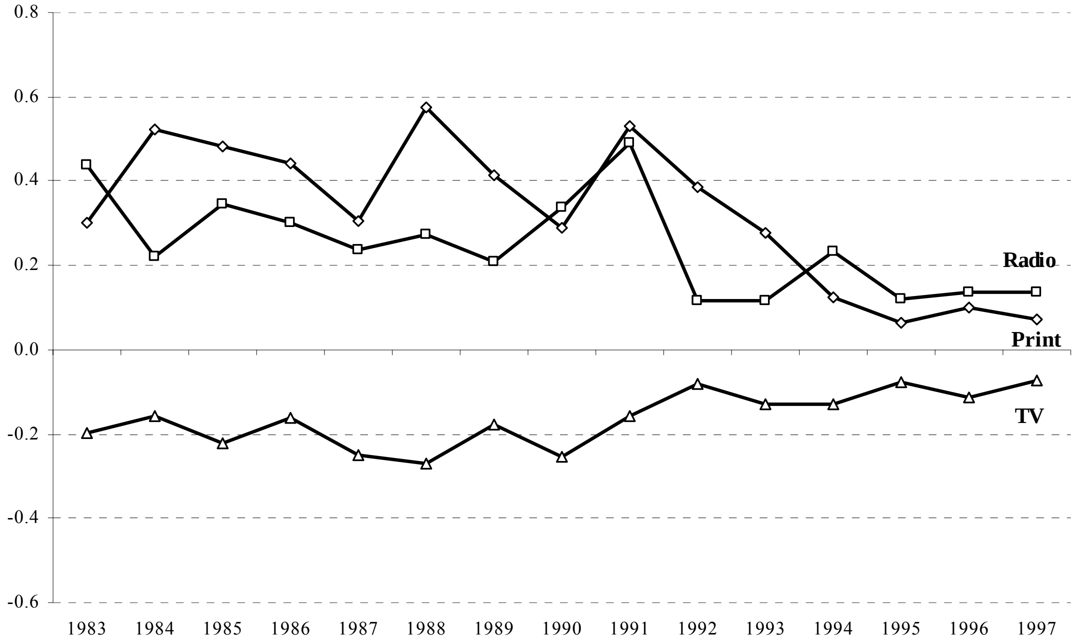
Table 5. Frequency Distribution of Medium Technical, Allocative and Cost Efficiency of Advertising Expenses (average values for the 1983-97 period).

Efficiency (%)	ITE^s	IAE^s	CE^s
<10	0	0	0
10-20	0	0	0
20-30	0	0	0
30-40	2	0	2
40-50	1	1	4
50-60	2	4	8
60-70	3	2	9
70-80	16	6	5
80-90	8	13	6
90-100	2	8	0
N	34	34	34
Mean	66.53	76.33	58.44
Min	37.44	49.09	32.78
Max	92.22	97.66	87.43

Table 6. Decomposition of Total Sales Growth for Greek Processed Meat Industry (average values for the 1983-97 period).

	Average Annual Rate of Change	Percentage
Total Sales Growth	3.14	(100)
Aggregate Media Growth	1.66	(53.0)
Television	1.02	(32.5)
Radio	0.36	(11.5)
Print Media	0.28	(9.0)
TAP Growth	1.97	(63.0)
Technical Change	0.73	(23.4)
Neutral	0.56	(17.8)
Biased	0.17	(5.6)
Total Spillover Effect	0.10	(3.3)
Generic Effect	1.14	(36.2)
Crowding-Out Effect	-0.58	(-21.8)
Sale Effect	-0.37	(-15.0)
Time Effect	-0.08	(-2.6)
Scale Effect	0.66	(21.1)
Change in TE	0.34	(10.9)
Change in AE	0.76	(24.4)
Price Adjustment Effect	-0.63	(-20.1)
Unexplained Residuals	-0.50	(-15.8)

Figure 1. Advertising Media-Mix Utilization, 1983-97 (average values over firms).



Endnotes

[1] For instance the partial ban of advertising alcoholic beverages on TV and radio in both US and EU may not be such effective in reducing alcohol consumption, as suggested by Saffer (1991) and Tremblay and Tremblay (1995), if advertising media are close substitutes. In that instances firms can maintained their marketed output regardless the restrictions imposed by the relevant legislation.

[2] The choice of the appropriate medium for implementing an advertising campaign is dictated by the available budget, the target audience, and the nature of the message, since the particular medium has to be well suited to communicating the desired message. The advantages that large firms enjoy in advertising are thought to be greater in television advertising. The latter has lower informational content since most TV commercials provide no information beyond existence. Becker and Murphy (1993) considering the welfare effects of advertising discriminate between media on the basis of the information provided. They show that television and radio advertising tend to lower the utility to consumers while highly informative messages, which are presumed to be communicated mainly through magazines and newspapers, are welfare increasing.

[3] Advertising spillovers may well exists also within multi-brand firms. The advertising of one product may reduce (expand) the sales of firm's other products if these are substitutes (complements) within a particular market. However, in our subsequent empirical analysis we are focused only to intra-firm advertising spillovers.

[4] Persuasive advertising (*e.g.*, cosmetics) that create subjective product differentiation and increase brand loyalty, advertising spillovers are dampened (Tremblay and Polasky, 2001).

[5] Farr *et al.*, (2001), in contrast with the early study by Tennant (1950), found that cigarette advertising became in the last decade predatory, producing negative spillovers.

[6] We assume that the advertising technology satisfies all axioms described by Färe and Primont (1995, p. 27)

[7] The direct input distance function was introduced by Shephard (1953) and independently in the context of consumer theory by Malmquist (1953).

[8] The advertising distance function defined in (5) is linearly homogeneous in $\mathbf{x}^{\mathbf{s}}$, *i.e.*, $D^M\left(\lambda\mathbf{x}^{\mathbf{s}}, y^s \middle| x^{sr}, t\right) = \lambda D^M\left(\mathbf{x}^{\mathbf{s}}, y^s \middle| x^{sr}, t\right) \;\; \forall\lambda > 0$, non-increasing in y^s and non-decreasing in $\mathbf{x}^{\mathbf{s}}$, *i.e.*, $D^M\left(\overline{\mathbf{x}}^{\mathbf{s}}, y^s \middle| x^{sr}, t\right) \geq D^M\left(\mathbf{x}^{\mathbf{s}}, y^s \middle| x^{sr}, t\right)$ and, $D^M\left(\mathbf{x}^{\mathbf{s}}, \overline{y}^s \middle| x^{sr}, t\right) \leq D^M\left(\mathbf{x}^{\mathbf{s}}, y^s \middle| x^{sr}, t\right)$ $\forall\ \overline{\mathbf{x}}^{\mathbf{s}} \geq \mathbf{x}^{\mathbf{s}},\ \overline{y}^s \geq y^s$. The direction of change with respect to x^{sr} depends on the advertising spillovers which in turn are affecting from the structure of the market, the type of product and the kind of advertising messages.

[9] That is, scaling all media prices equally or each medium price individually will have no effect on the input-oriented measure of advertising cost inefficiency. This property of input-oriented measures is due to their radial nature and it will be proved important in panel data studies where there are no price data for individual firms. Apparently, it allows the use of regional, or even national, price data to be used in estimating efficiency measures, without altering the final outcome.

[10] The dot over a variable or function indicates its time rate of change, while the subscripts denote the logarithmic partial derivatives.

[11] Aggregate advertising medium growth is measured as a *Divisia* index. The use of observed media cost shares as weights for individual media growth rise to the third term in (12).

[12] The existence of the medium price adjustment effect is closely related to the definition of *TAP*, which is based on observed media and sales quantities.

[13] Virtual prices consist of the vector of media prices that make the (observed) input technically inefficient media-mix input allocatively efficient. That is, virtual prices may be interpreted as the marginal products of advertising at the observed media-mix (Grosskopf *et al.*, 1995a).

[14] In fact, the number of unknowns in this system is greater than the number of equations. For this reason, each equation is divided by let's say the first equation to eliminate $C^s\left(y^s, \mathbf{w}^{\mathbf{s}} \middle| x^{sr}, t\right)$ and then solve the resulting system of *n-1* equations for the *n-1* cost minimizing advertising ratios by assuming that the virtual and market price of the first advertising medium coincide.

[15] In the case of unbalanced panels, t includes a subset of integers representing the periods for which observations on individual producers are obtained.

[16] Battese and Coelli (1988) have developed a consistent predictor for u_{it} based on that developed by Jondrow *et al.*, (1982).

[17] Analogously, the hypothesis that technical change is Hicks neutral can be tested by imposing the following restrictions in (16): $\zeta_{tk} = \zeta_{ts} = \theta_{to} = 0 \ \forall \ k$.

[18] From (37) it is clear that firm-specific estimates of the cost minimizing media ratios can be obtained even if there are no firm-specific media price data, as is the case in the present study.

[19] Morishima elasticities of substitution are symmetric only in the case of a CES production function. Blackorby and Russell (1989) discuss why elasticities of substitution should be asymmetric. In particular, elasticities of substitution measures the curvature along the isoquant when the factor price ratio w_j / w_k changes due to change in w_k.. However, the direction of change is different if the factor price ratio changes due to change in w_j and therefore the revealed curvature of the isoquant is different.

[20] The remaining firms are mainly small producers serving local markets that are not using advertisement campaigns to promote their products.

[21] The generalized likelihood-ratio test statistic is $\lambda = -2\{ln L(H_0) - ln L(H_1)\}$ where $L(H_0)$ and $L(H_1)$ denote the values of the likelihood function under the null (H_0) and the alternative (H_1) hypothesis, respectively. It follows approximately a χ^2 distribution, except in the case where the null hypothesis also involves γ=0. Then, its asymptotic distribution is a mixed *chi-squared* and the appropriate critical values are obtained from Kodde and Palm (1986, Table 1).

[22] The test-statistic computed as $\sqrt{b_1} = m_3 / m_2^{3/2}$ (with m_3 and m_2 being the third and second moments of the residuals and b_1 the coefficient of skewness) is 1.907, well above the corresponding critical value at the 5% level of significance (0.298).

[23] Although Morishima elasticities of substitution are more meaningful preserving the salient characteristics of the original Hicksian concept of substitution than their Allen counterparts (Blackorby and Russell, 1989), we report both estimates here.

[24] Since the *Morishima* elasticities are computed also from $\sigma_{kj}^{M} = e_{kj} - e_{jj}$, where the *e's* are the cross and own demand elasticities, it may take on values greater than, less than or equal to zero depending on the signs and the magnitudes of the two terms.

[25] Note that σ_{kj}^{M} represents the percentage change in the k^{th} to j^{th} quantity ratio $\left(i.e., \; x_k^s / x_j^s\right)$, when the relative prices w_j^s / w_k^s are changed by changing w_j^s and holding w_k^s constant.