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## ***Physics*, Chapter 25: Capacitance and Dielectrics**

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# 25

## Capacitance and Dielectrics

### 25-1 Capacitance of an Isolated Sphere in Vacuum

When an isolated charged conducting sphere bears a charge  $Q$ , the potential of the sphere may be computed from the results of Section 23-6 by considering that the electric intensity outside the sphere is as though the entire charge of the sphere were concentrated at its center. The potential at a distance  $r$  from the center of the sphere is given by the formula

$$V = \frac{Q}{4\pi\epsilon_0 r}, \quad (24-6)$$

as long as  $r$  is greater than or equal to the radius of the sphere. At the surface of the sphere of radius  $a$ , the potential is

$$V = \frac{Q}{4\pi\epsilon_0 a}.$$

We may define the capacitance  $C$  of a conductor as the quotient of its charge  $Q$  divided by its potential  $V$ . That is

$$\boxed{C = \frac{Q}{V}}. \quad (25-1)$$

The capacitance is always a positive number, for if the charge on an isolated conductor is positive, its potential is positive; if the charge on an isolated conductor is negative, its potential is negative. Following the example of an isolated conductor, the capacitance for more complex cases is always given as a positive quantity.

From Equation (25-1) we find the capacitance of an isolated conducting sphere in vacuum to be

$$C = 4\pi\epsilon_0 a. \quad (25-2)$$

Thus the capacitance of a sphere is proportional to its radius. The mks unit of capacitance is called the *farad* in honor of Michael Faraday (1791–1867). The quantity  $\epsilon_0$  is often stated in units of farads per meter, as

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ farad/m,}$$

for consistency with Equation (25-2).

The farad is an impractically large unit of capacitance, being the capacitance of an enormous sphere of radius  $9 \times 10^9$  m. The distance from the planet Mercury to the sun is  $58 \times 10^9$  m. It is common practice to utilize units of microfarads (1 microfarad =  $10^{-6}$  farad), abbreviated  $\mu\text{fd}$ , or units of micromicrofarads (1 micromicrofarad =  $10^{-12}$  farad), abbreviated  $\mu\mu\text{fd}$ , to describe capacitance.

In the cgs electrostatic system of units, the potential of a charged sphere in vacuum is given by

$$V = \frac{Q}{a},$$

and the capacitance of a sphere is numerically equal to its radius in centimeters, for we find

$$C = a.$$

The cgs unit of capacitance is the capacitance of a sphere of radius one centimeter in vacuum, and is referred to as one *statfarad* (stfd). As a matter of general practice, the esu of capacitance is rarely used. From the preceding discussion

$$1 \text{ farad} = 9 \times 10^{11} \text{ stfd,}$$

so that 1  $\mu\mu\text{fd}$  is approximately equal to 1 stfd.

The capacitance of an isolated conductor of arbitrary size and shape may be bracketed between the capacitances of two spheres which just fit over the body and just fit inside the body. Thus the capacitance of an airplane is less than the capacitance of the smallest sphere which will enclose the airplane, and is greater than the capacitance of the largest sphere which will just fit inside the airplane.

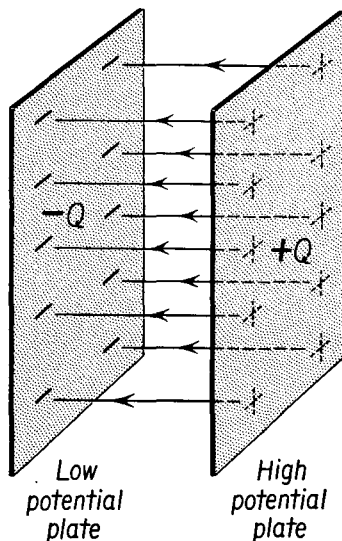
## 25-2 Capacitors

To obtain the capacitance of an isolated conducting sphere, we applied the formula for the potential of the sphere in relation to its charge, which was based upon the assumption that the lines of force emanating from the conductor terminated at infinity. The potential of the conductor was established by assigning the potential zero to a point at infinity.

In most practical problems associated with electrical apparatus, the lines of force emanating from a conductor terminate upon other conductors

in the vicinity, and one must define the capacitance between any pair of conductors in a rather complex way. In the event that we are interested in two conductors which are close together and which are relatively far from other conductors in the vicinity, the lines of force emanating from one conductor terminate upon its neighbor rather than upon any distant conductor or upon an infinitely distant charge. This means that if a charge  $+Q$  is placed on one conductor, a charge  $-Q$  must be placed on the neigh-

**Fig. 25-1** The lines of force in the parallel plate capacitor are directed from the positive (high potential) plate to the negative (low potential) plate. The electric field is in the  $-x$  direction in the figure while the potential increases in the  $+x$  direction.



boring conductor. When the two conductors are close together, Equation (25-1) must be reinterpreted so that  $V$  is the potential difference between the two conductors, and  $Q$  is the charge on either conductor without regard to sign. A *capacitor* then consists of a pair of conductors adjacent to each other. The conductors may be in vacuum, or separated by air, or, more generally, separated by an insulating material which is referred to as a *dielectric*.

Let us consider the capacitance of a parallel-plate capacitor in vacuum. To a good approximation the results obtained will also be true when the conductors are in air, but they will have to be modified when a liquid or solid dielectric fills the space between the conductors.

When a pair of plane parallel conducting plates of area  $A$  are separated by a distance  $s$ , as shown in Figure 25-1, we may compute their capacitance by assuming that one plate has a charge of  $+Q$  while the other plate has a charge of  $-Q$ . All lines of force beginning on the first plate will terminate on the second plate. We shall make the approximation that the electric field intensity is uniform between the plates. While this is strictly true

only for plates of infinite extent, it is very nearly true for plates whose dimensions are large compared to their separation.

The field outside the surface of a conductor is related to the surface density of charge on the conductor through the equation

$$E = \frac{\sigma}{\epsilon_0}. \quad (23-11)$$

The surface density of charge  $\sigma$  on the plates of the parallel-plate capacitor is given by

$$\sigma = \frac{Q}{A},$$

so that the electric intensity between the plates of the capacitor is

$$E = \frac{Q}{\epsilon_0 A}.$$

Since the field is uniform throughout the region between the capacitor plates, the work which must be done by an outside agency in moving a unit positive charge from the negative to the positive plate, through the distance  $s$ , is the potential difference  $V$  between the two plates, given by

$$V = Es = \frac{Qs}{\epsilon_0 A}.$$

From Equation (25-1) the capacitance is given by

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{s}. \quad (25-3)$$

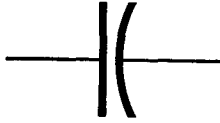
The geometric properties of the capacitor, the area and the separation between the plates, determine the capacitance of a parallel-plate capacitor, just as the capacitance of a conducting sphere is determined by its radius.

*Illustrative Example.* Determine the capacitance of a huge parallel-plate capacitor whose plates are  $1 \text{ km}^2$  in area, and which are  $1 \text{ mm}$  apart.

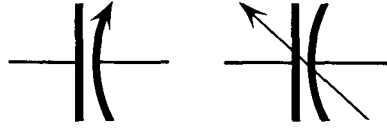
The area of each plate is  $A = 10^6 \text{ m}^2$ ; the separation of the plates is  $s = 10^{-3} \text{ m}$ ; the value of  $\epsilon_0$  is  $8.85 \times 10^{-12} \text{ farad/m}$ . Substituting these numerical values into Equation (25-3), we find

$$C = \frac{8.85 \times 10^{-12} \text{ farad/m} \times 10^6 \text{ m}^2}{10^{-3} \text{ m}} = 8.85 \times 10^{-3} \text{ farad}.$$

In schematic circuit diagrams the symbol used to represent a fixed capacitor is shown in Figure 25-2, while the symbols used to represent variable capacitors are shown in Figure 25-3.



**Fig. 25-2** Symbol used for representing a fixed capacitor. The terminal represented by the curved line is generally nearer to ground potential.



**Fig. 25-3** Symbols used for representing variable capacitors.

### 25-3 Energy of a Charged Capacitor

When a capacitor is being charged, there is initially no charge on either plate, so that the electric field intensity between the plates is zero. To move the first increment of charge from one plate to the other requires no work. Later increments of charge transferred from one plate to the other must have work done upon them against the electric field of the charged plates. If the potential difference between the plates is  $V$ , the amount of work  $\Delta\mathcal{W}$  done in transferring charge  $\Delta q$  from the plate at low potential to the plate at higher potential will be given by

$$\Delta\mathcal{W} = V \Delta q.$$

The relationship between the potential difference and the charge  $q$  on the plates is given by Equation (25-1) as

$$q = CV.$$

Thus the work  $\Delta\mathcal{W}$  done in transferring charge  $\Delta q$  from the plate at low potential to the plate at higher potential is

$$\Delta\mathcal{W} = \frac{1}{C} q \Delta q.$$

To find the energy stored in the capacitor when it has been charged to its final potential difference  $V$  by transferring charge  $Q$  from one plate to the other, we must add all the increments in energy, and in the notation of the calculus

$$\mathcal{W} = \int d\mathcal{W} = \int_0^Q \frac{1}{C} q dq,$$

from which

$$\mathcal{W} = \frac{1}{2} \frac{Q^2}{C}. \quad (25-4a)$$

By substituting the value of  $Q$  from Equation (25-1) into Equation (25-4a), we may find alternate expressions for the energy of a charged capacitor.

We obtain

$$\mathcal{W} = \frac{1}{2}CV^2, \quad (25-4b)$$

and also

$$\mathcal{W} = \frac{1}{2}QV. \quad (25-4c)$$

From Equation (25-4c) the energy of the charged capacitor may be seen to be the energy associated with the transfer of the total charge  $Q$  through the average potential difference  $V/2$ . When mks units of coulombs, farads, and volts are substituted for the appropriate quantities in Equations (25-4), the energy  $\mathcal{W}$  will be in joules.

Equations (25-4) illustrate one application of capacitors, that is, to store electrical energy, much as a stressed spring can be used to store mechanical energy. The ability of a capacitor to store electrical energy is utilized in electronic power supplies which convert alternating to direct current, and in high-energy accelerators, such as the *betatron*, which require so much energy in a short time that power-generating stations cannot conveniently supply it.

From another point of view, the energy of a charged capacitor may be said to reside in the electric field between its plates. The energy per unit volume in an electric field can readily be calculated with the aid of the equations just derived. The energy of a parallel-plate capacitor may be expressed, from Equations (25-4b) and (25-3), as

$$\mathcal{W} = \frac{1}{2}CV^2 = \frac{1}{2} \frac{\epsilon_0 A}{s} V^2.$$

Dividing this equation by the product  $As$ , the volume of the space between the capacitor plates, we find

$$\frac{\mathcal{W}}{As} = \frac{1}{2} \epsilon_0 \frac{V^2}{s^2}.$$

The quantity  $V/s$  is the magnitude of the potential gradient, which is equal to the electric intensity  $E$  in the space between the plates of the capacitor. Let us designate the energy per unit volume in the space between the plates by  $\mathcal{W}_v$ .

Thus

$$\mathcal{W}_v = \frac{\mathcal{W}}{As},$$

and

$$E = \frac{V}{s},$$

so that

$$\mathcal{W}_v = \frac{\epsilon_0 E^2}{2}. \quad (25-5)$$

Thus the energy per unit volume in an electric field is proportional to the square of the intensity of the electric field.

*Illustrative Example.* Find the energy per unit volume in an electric field whose intensity is 100 nt/coul.

From Equation (25-5) we have

$$\begin{aligned} \mathcal{W}_v &= \frac{1}{2} \times 8.85 \times 10^{-12} \frac{\text{coul}^2}{\text{nt m}^2} \times 10^4 \frac{\text{nt}^2}{\text{coul}^2} \\ &= 4.42 \times 10^{-8} \frac{\text{nt}}{\text{m}^2} \\ &= 4.42 \times 10^{-8} \frac{\text{joule}}{\text{m}^3}. \end{aligned}$$

### 25-4 Capacitors in Series and Parallel

In many circuit applications several capacitors are electrically connected to produce the desired results. The simplest types of electrical connections are known as the *series* connection, shown in Figure 25-4(a), and the *parallel*

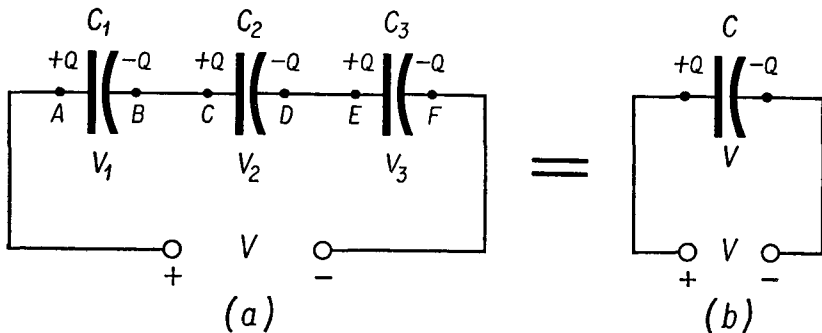


Fig. 25-4

connection, shown in Figure 25-5(a). We often wish to determine the effective capacitance of the combination; that is, we wish to determine the capacitance of that single capacitor which has the same effect in the circuit as the combination of capacitors.

Let us first consider the series connection of three capacitors  $C_1$ ,  $C_2$ ,  $C_3$ , as shown in Figure 25-4(a). When this combination of capacitors is connected to a source of electrical energy, such as a battery, electric charge flows from the source of energy to the plates of the capacitors until the plate  $A$  is at the potential of the positive terminal of the energy source, and the plate  $F$  is at the potential of the negative terminal of the energy source. The plate  $A$  bears a charge  $+Q$ , while the plate  $F$  bears a charge  $-Q$ . Lines of force emanating from plate  $A$  terminate on plate  $B$ . Thus if plate  $A$



bears a positive charge  $+Q$ , an equal negative charge  $-Q$  must appear on plate  $B$ , attracted by the electrical forces resulting from the charge on plate  $A$ .

The plate  $B$ , the plate  $C$ , and the wire  $BC$  connecting them may be considered as a single electrical conductor. This conductor was uncharged before the capacitors were connected to the source of electrical energy.

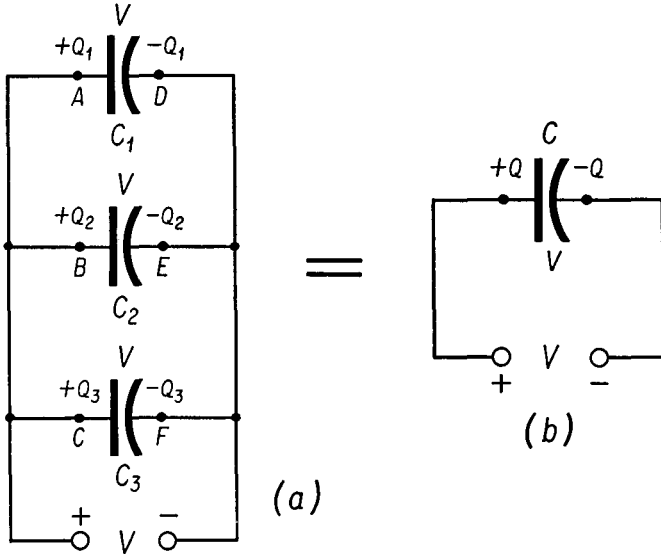


Fig. 25-5

Furthermore, this conductor is completely insulated, so that no electric charge can flow to or from the conductor. The only way a charge  $-Q$  can appear on plate  $B$  is for an equal and opposite charge  $+Q$  to appear on plate  $C$ . Continuing this argument for each of the capacitors of the series combination, we see that the charge on each capacitor is the same and is equal to  $Q$ . If  $Q_1$  is the charge on capacitor  $C_1$ ,  $Q_2$  the charge on  $C_2$ , and  $Q_3$  is the charge on  $C_3$ , we have

$$Q = Q_1 = Q_2 = Q_3.$$

The potential difference between the plates of each of the capacitors of the assembly can be found from Equation (25-1). Thus

$$V_1 = \frac{Q_1}{C_1} = \frac{Q}{C_1},$$

$$V_2 = \frac{Q}{C_2},$$

and 
$$V_3 = \frac{Q}{C_3}.$$

The potential difference between the points  $A$  and  $F$  is equal to the work done in transporting a unit positive test charge from terminal  $A$  to terminal  $F$  against the electric forces. In carrying a test charge  $q$  from  $A$  to  $B$ , a quantity of work  $V_1q$  is done on the charge. Since there is electrostatic equilibrium, no work is done in moving the test charge along the wire  $BC$ , for this wire is a conductor and is an equipotential region. In transporting the test charge from plate  $C$  to plate  $D$ , the work done is  $V_2q$ , and in moving the charge from  $E$  to  $F$  the work done is  $V_3q$ . The total work done in moving the test charge from  $A$  to  $F$  may be represented by  $Vq$ , where  $V$  is the potential difference between  $A$  and  $F$ . Thus we have

$$Vq = V_1q + V_2q + V_3q,$$

or 
$$V = V_1 + V_2 + V_3.$$

If we are to replace the series combination by a single capacitor of capacitance  $C$  having the same effect in the circuit as the series combination, as shown in Figure 25-4(b), a charge  $Q$  must flow to the plates of that capacitor when the potential between its plates is  $V$ . From Equation (25-1)

$$\begin{aligned} \frac{1}{C} &= \frac{V}{Q} \\ &= \frac{V_1 + V_2 + V_3}{Q} \\ &= \frac{Q/C_1 + Q/C_2 + Q/C_3}{Q}. \end{aligned}$$

Thus we find

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \quad (25-6)$$

When any number of capacitors is connected in series, the reciprocal of the effective capacitance is equal to the sum of the reciprocals of the individual capacitances. The charge on the plates of each capacitor of the combination is the same, but different potential differences appear between the plates of the individual capacitors. Since practical capacitors are rated for maximum potential difference as well as for capacitance, it is necessary to exercise caution when connecting capacitors in series so as not to exceed the rated value of the potential difference of any one of the capacitors.

When capacitors are connected in parallel, as shown in Figure 25-5(a), one terminal of each capacitor is connected to one terminal of the source

of electrical energy. The other terminal of each capacitor is connected to the other terminal of the source of electrical energy. Thus the plates  $A$ ,  $B$ , and  $C$  are all at one potential, while the plates  $D$ ,  $E$ , and  $F$  are all at a different common potential. The potential difference between the plates of each capacitor is the same and is equal to the potential difference between the terminals of the source of electrical energy. Thus we have

$$V = V_1 = V_2 = V_3.$$

Unlike the series case, the charge on the plates of the individual capacitors is different, and we have

$$Q_1 = C_1 V_1 = C_1 V,$$

$$Q_2 = C_2 V,$$

and

$$Q_3 = C_3 V.$$

If we wish to replace the parallel combination of capacitors by a single capacitor of capacitance  $C$ , whose effect in the circuit is the same as the parallel combination, as shown in Figure 25-5(b), the plates of that capacitor should bear a charge equal to the charge which flows from the source of electrical energy to the parallel combination when the potential difference between the plates is  $V$ . Thus the charge on the equivalent capacitor of Figure 25-5(b) must be  $Q$  where

$$Q = Q_1 + Q_2 + Q_3.$$

Thus we have

$$CV = C_1 V + C_2 V + C_3 V,$$

so that

$$C = C_1 + C_2 + C_3. \quad (25-7)$$

The effective capacitance of a combination of several capacitors connected in parallel is equal to the sum of the individual capacitances. The potential difference between the plates of each of the capacitors is the same, but different quantities of charge appear on the plates of the individual capacitors.

*Illustrative Example.* (a) Find the effective capacitance of the series-parallel combination of capacitors shown in Figure 25-6(a). (b) Find the charge on the  $5\text{-}\mu\text{fd}$  capacitor when the potential difference between the points  $a$  and  $b$  is 300 volts.

(a) As a first step, let us find the equivalent capacitance of the parallel combination of the  $3\text{-}\mu\text{fd}$  and  $5\text{-}\mu\text{fd}$  capacitors. From Equation (25-7) we find that the effective capacitance of the combination is  $8\mu\text{fd}$ . Let us imagine that the parallel combination is replaced by a single  $8\text{-}\mu\text{fd}$  capacitor, as shown in Figure 25-6(b). The equivalent capacitance of the series combination of Figure 25-6(b) is given by Equation (25-6) as

$$\frac{1}{C} = \frac{1}{4\ \mu\text{fd}} + \frac{1}{8\ \mu\text{fd}},$$

or

$$C = \frac{8}{3}\ \mu\text{fd}.$$

Thus the entire series-parallel combination may be replaced by a single capacitor of  $\frac{8}{3} \mu\text{fd}$  capacitance, as shown in Figure 25-6(c).

(b) The charge  $Q$  which appears on the single capacitor of Figure 25-6(c), when a potential difference of 300 volts exists between points  $a$  and  $b$ , is

$$Q = CV = \frac{8}{3} \mu\text{fd} \times 300 \text{ volts};$$

$$Q = 800 \mu\text{coul.}$$

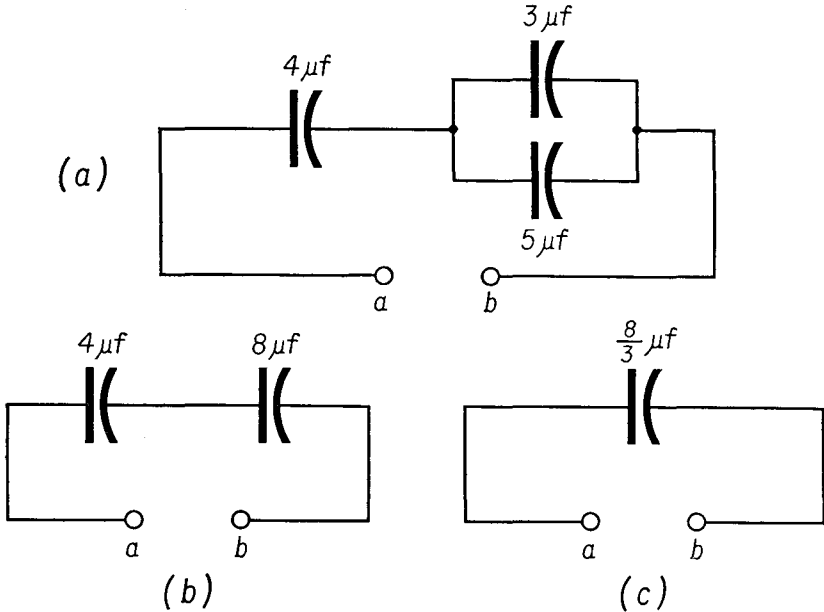


Fig. 25-6

The circuit of Figure 25-6(c) may now be replaced by the circuit of Figure 25-6(b), where a charge of  $800 \mu\text{coul}$  must appear on the plates of each of the two capacitors in series. The potential difference between the plates of the  $8\text{-}\mu\text{fd}$  capacitor may be evaluated as

$$V_1 = \frac{800 \mu\text{coul}}{8 \mu\text{fd}} = 100 \text{ volts.}$$

The circuit of Figure 25-6(b) may now be replaced by the original circuit of Figure 25-6(a). A potential difference of 100 volts appears between the plates of each of the capacitors of the parallel combination. The potential difference between the plates of the  $5\text{-}\mu\text{fd}$  capacitor is 100 volts, so that the charge on the plates of the  $5\text{-}\mu\text{fd}$  capacitor is

$$Q_1 = C_1 V_1 = 5 \mu\text{fd} \times 100 \text{ volts};$$

so that

$$Q_1 = 500 \mu\text{coul.}$$

It is now easily seen that the charge on the  $3\text{-}\mu\text{fd}$  capacitor is  $300 \mu\text{coul}$ , and that the potential difference across the  $4\text{-}\mu\text{fd}$  capacitor is 200 volts.

### 25-5 Dielectric Constant

Capacitors are commonly built with a solid or liquid insulating material, called a *dielectric*, placed between their plates. To understand the effect of a dielectric, suppose that the plates of a parallel-plate capacitor, with a vacuum between the plates, are charged to a potential difference  $V_{\text{vac}}$  and then disconnected from the source of charge; the charge on each plate will be  $Q$ . If a sheet of dielectric material is inserted between the plates, the potential difference will be found to decrease to some value  $V_{\text{die}}$ . Since the energy of a charge capacitor is  $\frac{1}{2}QV$ , the energy of the capacitor is decreased when a dielectric material is put between the plates of a charged capacitor. When the dielectric is removed, the potential difference returns to its initial value. This implies that there is an attractive force on the dielectric which draws it into the region of more intense electric field. Work is done by the electric field on the dielectric. When work is done by an external agency in removing the dielectric, the electrical energy of the capacitor is restored to its initial value.

From Equation (25-1) we see that the capacitance of a vacuum capacitor is less than the capacitance of the same pair of plates when a dielectric is inserted between them. If the capacitance in vacuum is  $C_{\text{vac}}$ , and the capacitance with the dielectric inserted is  $C_{\text{die}}$ , we have

$$C_{\text{vac}} = \frac{Q}{V_{\text{vac}}},$$

and

$$C_{\text{die}} = \frac{Q}{V_{\text{die}}}.$$

Let us suppose that the voltage  $V_{\text{vac}}$  is some number  $\kappa_e$  times the voltage of the capacitor with the dielectric between its plates. We may divide the second of these equations by the first to find

$$\frac{C_{\text{die}}}{C_{\text{vac}}} = \frac{V_{\text{vac}}}{V_{\text{die}}} = \kappa_e. \quad (25-8)$$

The ratio of  $C_{\text{die}}/C_{\text{vac}}$ ,  $\kappa_e$ , is called the *dielectric constant*. It is also sometimes called the *dielectric coefficient*, the *relative permittivity*, or the *specific inductive capacity*. Experiment shows that  $\kappa_e$  is not truly a constant but varies somewhat with temperature, with the state of internal stress in the dielectric, and with other factors as well. By definition, the value of the dielectric constant is unity for vacuum. For air and most gases, the value deviates but slightly from unity, being 1.0006 for air at standard conditions of temperature and pressure. For most solids and liquids the dielectric constant ranges in value from about 1 to 100, although crystalline materials have been found with dielectric constants greater than 1,000.

We must be careful to distinguish between the *dielectric constant* and the *dielectric strength*. The dielectric constant affects the capacitance of a capacitor. The dielectric strength is the property which determines the maximum potential difference which can be imposed upon the plates of a capacitor without destroying its insulating properties. The dielectric constant and dielectric strength of several insulating materials are given in Table 25-1.

TABLE 25-1 PROPERTIES OF SOME DIELECTRIC MATERIALS

Material	Dielectric Constant (Relative Permittivity)	Dielectric Strength (kv/cm)
Vacuum	1	$\infty$
Air (1 atm)	1.0006	30
Air (100 atm)	1.055	
Polystyrene	2.6	200-1,400
Polyethylene	2.3	160-240
Teflon	2.0	170-190
Paraffin wax	2.0-2.5	100
Porcelain	6.0-8.0	16-1,600
Glass	5-10	200-400
Rubber	3-6	160-480
Transformer oil	2.2	50-150
Water	55-88	
Barium titanate	$\sim 10,000$	

The capacitance of a parallel-plate capacitor whose plates are separated by a medium of dielectric constant  $\kappa_e$  is given by the equation

$$C = \frac{\kappa_e \epsilon_0 A}{s} = \frac{\epsilon A}{s}. \quad (25-9)$$

In the mks system of units, the product  $\kappa_e \epsilon_0$  is called the *permittivity* of the medium and is represented by  $\epsilon$ ; thus

$$\epsilon = \kappa_e \epsilon_0. \quad (25-10)$$

$\epsilon_0$  is called the permittivity of free space, and  $\kappa_e$  is referred to as the relative permittivity.

*Illustrative Example.* A parallel-plate capacitor for use in a high-voltage circuit is made of sheets of tin foil plated on a large glass sheet whose dimensions are  $2 \text{ m} \times 1.8 \text{ m} \times 0.02 \text{ m}$ . Assuming the relative permittivity of the glass to be 8 and the dielectric strength of the glass to be 300 kv/cm, determine (a) the capacitance of this capacitor and (b) the maximum potential difference which may be applied to its plates.

(a) From Equation (25-9) we have

$$C = \frac{\kappa_e \epsilon_0 A}{s} = \frac{8 \times 8.85 \times 10^{-12} \text{ farad/m} \times 3.6 \text{ m}^2}{0.02 \text{ m}},$$

$$C = 127 \times 10^{-10} \text{ farad.}$$

(b) The dielectric strength of the glass is 300 kv/cm or  $3 \times 10^7$  volts/m. The thickness of the glass is 0.02 m. The maximum potential difference which can be applied to the plates of the capacitor without destroying the insulating properties of the glass is given by

$$\frac{V}{s} = 3 \times 10^7 \frac{\text{volts}}{\text{m}},$$

so that

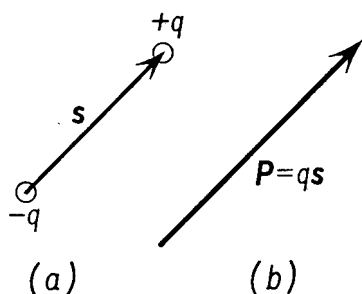
$$V = 3 \times 10^7 \frac{\text{volts}}{\text{m}} \times 0.02 \text{ m},$$

$$V = 6 \times 10^5 \text{ volts.}$$

### 25-6 The Material Medium; Polarization

The chemist describes the molecules of a substance by the term *polar* if the center of positive electricity within the molecule does not coincide with the center of negative electricity. If these

two centers coincide, the molecule is called *nonpolar*. An ionic crystal such as sodium chloride is composed of positive sodium ions and negative chlorine ions and is highly polar. There are points within the substance where there is a high concentration of positive charge, while at other points there is a high concentration of negative charge. Molecular crystals, in which nonpolar molecules form the basic building blocks of the crystal, are generally only slightly polar or nonpolar.



**Fig. 25-7** (a) Electric dipole, and (b) dipole moment vector  $P$ .

The polarity of a molecule may be described by its *dipole moment*. If the displacement vector directed from the center of negative charge  $-q$  to the center of positive charge  $+q$  is  $s$ , as shown in Figure 25-7 the dipole moment  $p$  is given by the product of the charge by the displacement. Thus

$$p = qs.$$

(25-11)

The dipole moment is a vector quantity whose magnitude is the product

of the magnitude of the charge  $q$  by the distance  $s$  between the two charges, while the direction of the dipole-moment vector is from the negative to the positive charge.

A polar substance is made up of many dipoles. The substance is electrically neutral, for any macroscopic volume element, containing of the order of a few hundred atoms or molecules, contains an equal amount of positive and negative charge. When a polar substance is placed in an electric field, the positive charge experiences a force in the direction of the field, while the negative charge experiences a force in the opposite direction. The resultant torque tends to align the dipole parallel to the field, as shown in Figure 25-8. At room temperature the thermal energy of the molecules tends to disorient the dipole, while the elastic binding forces tend to restrain this rotation. Nevertheless, in an electric field a partial alignment does occur for many substances. In these the component of the dipole moment in the direction of the field, averaged over many molecules, is proportional to the electric field intensity.

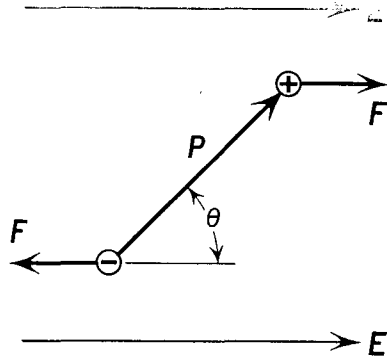


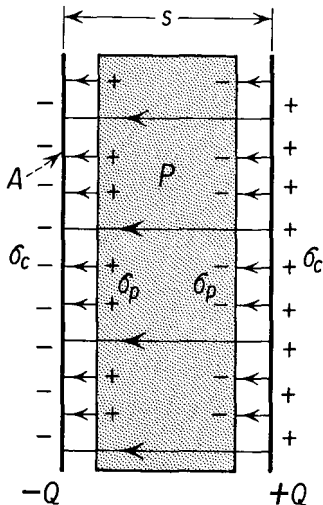
Fig. 25-8

Nonpolar substances do not possess a dipole moment in the absence of electric fields, but in the presence of an electric field the positive charges of a molecule experience forces in the direction of the field, while the negative electrons experience forces in the opposite direction so that a dipole moment may be induced. The *induced dipole moment* is parallel to the applied field. In many substances the magnitude of the induced dipole moment is proportional to the applied electric field. Thus in both polar and nonpolar substances the effect of an applied electric field is to increase the component of the polarization vector in the direction of the electric field. Rather than speaking of individual atoms and molecules, it is convenient to think of the aggregate effect produced by the electric field. We may speak of the total dipole moment per unit volume of the material. This is the vector sum of all the moments of all the elementary dipoles in a unit volume. The *dipole moment per unit volume* is called the *polarization*, designated by the symbol  $\mathbf{P}$ . In the absence of an electric field, the dipoles of a substance are randomly oriented, so that the polarization is zero, but when the field is applied, the polarization is proportional to the electric intensity within the dielectric. The constant of proportionality relating the polarization  $\mathbf{P}$  to the electric intensity  $\mathbf{E}$  is called the *electric susceptibility*  $\chi_e$  (chi sub  $e$ ). In the form of an equation,

$$\mathbf{P} = \chi_e \mathbf{E}. \quad (25-12)$$



Let us analyze, in terms of the concept of polarization, the effect of placing a dielectric between the plates of a parallel-plate capacitor. Let us suppose that the plates of a parallel-plate capacitor have been charged in vacuum to a charge  $Q$ , as shown in Figure 25-9. When a dielectric is inserted between the plates of the capacitor, the polarization induced in the dielectric is parallel to the field. The polarization vector is directed from



**Fig. 25-9** The effect of placing a dielectric between the charged plates of a capacitor, thus polarizing the dielectric, may be most easily understood by imagining the dielectric to be replaced by two sheets of polarization charge located at the surfaces of the dielectric.

the negative to the positive charge of the dipoles induced in the dielectric. Since the positive and negative charges of a dipole are displaced through a comparatively small distance, the net effect is as though only the surfaces of the dielectric are charged with charges of opposite sign. It will therefore be convenient to imagine the dielectric to be replaced by two oppositely charged sheets of charge located at the surface of the dielectric. The imaginary sheet of negative charge is adjacent to the positively charged plate of the capacitor, while the imaginary sheet of positive charge is adjacent to the negatively charged plate of the capacitor. Let us call these imaginary sheets of charge the *polarization charge*.

Some of the lines of force emanating from the positively charged plate of the capacitor terminate on the adjacent sheet of negative polarization charge. The number of lines of force passing from the positive to the negative plate of the capacitor is therefore smaller in the presence of the dielectric than in vacuum. Since the number of lines of force per unit area is a measure of the electric field intensity, the electric field intensity within the dielectric is smaller than the electric field intensity in vacuum. The potential difference between the plates is therefore diminished, while the charge on the plates has remained the same. Thus the capacitance of the

parallel-plate structure has been increased by the insertion of the dielectric between the capacitor plates.

Let us represent the surface density of the imaginary polarization charge by the symbol  $\sigma_p$ , while the surface density of the free charge or conduction charge on the plates is represented by  $\sigma_c$ . The area of the capacitor plates is  $A$ , and their separation is  $s$ . The total polarization charge in one sheet is the product of the surface density of the polarization charge  $\sigma_p$  by the area of the plate  $A$ . The two sheets of polarization charge are separated by the distance  $s$ , so that the dipole moment due to the polarization charge is  $\sigma_p As$ . Let us now replace the polarization generated by the sheets of polarization charge by the induced polarization in the dielectric. The total dipole moment of the dielectric in which the dipole moment per unit volume is  $P$  is simply  $PAs$ ; equating this to the dipole moment of the imaginary sheets of polarization charge, we get

$$PAs = \sigma_p As.$$

This yields  $P = \sigma_p$ . (25-13)

Hence the magnitude of the polarization or dipole moment per unit volume turns out to be equal to the magnitude of the surface density of the imaginary polarization charge. Substituting this value of  $P$  into Equation (25-12), we find

$$\sigma_p = \chi_e E. \quad (25-14)$$

To compute the electric intensity between the plates of the capacitor when there is a dielectric material between the plates, let us construct a Gaussian pillbox of unit surface area and of sufficient depth to penetrate the dielectric, as shown in Figure 25-10. The electric field intensity is given by Equation (23-11) as

$$E = \frac{\sigma}{\epsilon_0}.$$

The total charge now consists of the conduction charge and the polarization charge. At the positive plate of the capacitor  $\sigma_c$  is positive, but the polarization charge  $\sigma_p$  is negative; hence

$$\sigma = \sigma_c - \sigma_p = \sigma_c - \chi_e E,$$

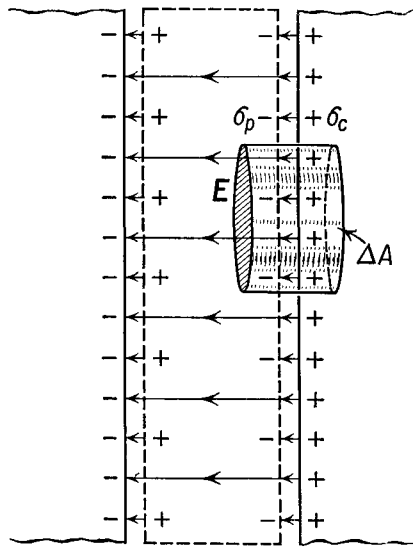


Fig. 25-10

so that 
$$E = \frac{(\sigma_c - \chi_e E)}{\epsilon_0}.$$

When this equation is solved for  $E$ , we obtain

$$E = \frac{1}{\epsilon_0} \frac{\sigma_c}{(1 + \chi_e/\epsilon_0)}. \quad (25-15)$$

Equation (25-15) is the expression for the electric field intensity between the plates of the capacitor containing a dielectric; hence it is the electric field intensity within the dielectric. Let us designate it by the symbol  $E_{\text{die}}$  and rewrite the equation as

$$E_{\text{die}} = \frac{\sigma_c/\epsilon_0}{1 + \chi_e/\epsilon_0}.$$

Let us compare this value with the electric field intensity when there is a vacuum between the plates. In the latter case,  $\sigma_p = 0$ , so that  $\sigma = \sigma_c$ ; calling the electric field intensity now  $E_{\text{vac}}$ , we get

$$E_{\text{vac}} = \frac{\sigma_c}{\epsilon_0}.$$

Hence 
$$\frac{E_{\text{die}}}{E_{\text{vac}}} = \frac{1}{1 + \chi_e/\epsilon_0}. \quad (25-16a)$$

We have already shown that

$$\frac{V_{\text{die}}}{V_{\text{vac}}} = \frac{1}{\kappa_e}. \quad (25-8)$$

Since  $V = Es$  for a parallel-plate capacitor, we can write

$$\frac{E_{\text{die}}}{E_{\text{vac}}} = \frac{V_{\text{die}}}{V_{\text{vac}}} = \frac{1}{\kappa_e}. \quad (25-16b)$$

Equating the denominators of Equations (25-16a) and (25-16b) we get

$$\kappa_e = 1 + \frac{\chi_e}{\epsilon_0}. \quad (25-17)$$

Multiplying Equation (25-17) by  $\epsilon_0$ , we find

$$\kappa_e \epsilon_0 = \epsilon_0 + \chi_e,$$

and from Equation (25-10)

$$\epsilon = \epsilon_0 + \chi_e. \quad (25-18)$$

The permittivity of the dielectric is made up of the sum of  $\epsilon_0$ , the permittivity of free space, and  $\chi_e$ , the susceptibility of the medium.

There must be atoms or molecules present if the dielectric constant is to differ from unity, or if  $\epsilon$  is to differ from  $\epsilon_0$ . If matter is in the gaseous phase between the plates of a capacitor, the number of molecules per unit volume is quite small, and even though the induced dipole moment of each molecule is comparatively large, the total dipole moment per unit volume, the polarization, must be small. Thus the dielectric constant of a gas must be close to 1. If the pressure of a gas is increased, or if the gas is liquefied, the number of molecules per unit volume increases by a large factor, perhaps even a factor of 1,000. The polarization is then increased, and the dielectric constant may now substantially differ from unity.

It is possible to manufacture artificial dielectrics by distributing a large number of small conducting spheres in an insulator, as shown in Figure 25-11. A dipole is induced in each sphere by an electric field. If there are

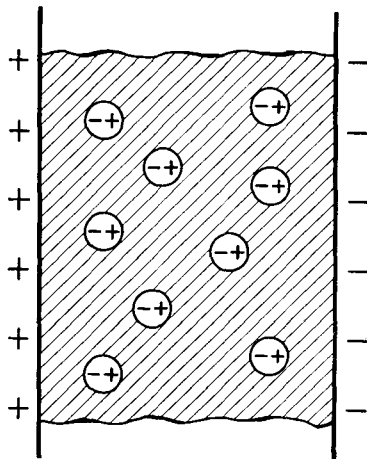


Fig. 25-11

$N$  such spheres in a unit volume, each sphere of radius  $a$ , it may be shown that the susceptibility of such a distribution is  $Na^3$ . Dispersions of conducting spheres in an insulating material have been used to produce artificial dielectrics for microwave lenses employed in high-frequency radio communication.

In the event that the space between a pair of capacitor plates is filled with a conducting substance, the electric field is reduced to zero. From Equations (25-15) and (25-17) we see that the electric field intensity between the plates of a capacitor is inversely proportional to the dielectric constant. Thus the dielectric constant of a perfect conductor is infinite.

**25-7 Electric Field of a Point Charge in an Infinite Dielectric**

When a point charge is placed in an infinite homogeneous dielectric, we can expect the electric field produced to be radial, from considerations of symmetry. To compute the effect of the dielectric on the field, let us imagine that the dielectric has a small spherical hole of radius  $a$ , centered on the point charge, as shown in Figure 25-12. Just as in the case of the parallel-plate capacitor, the electric field generated by the point charge will induce polarization in the dielectric whose effect can be calculated by imagining the dielectric to be replaced by a layer of polarization charge on

the surface of the spherical hole of charge density  $\sigma_p$  given by

$$\sigma_p = \chi_e E. \tag{25-14}$$

The total polarization charge  $q_p$  induced on the surface of the spherical hole is the product of the charge density and the surface area of the sphere and is negative in the neighborhood of a positive point charge, as given by

$$q_p = -4\pi a^2 \sigma_p = -4\pi a^2 \chi_e E_a. \tag{25-19}$$

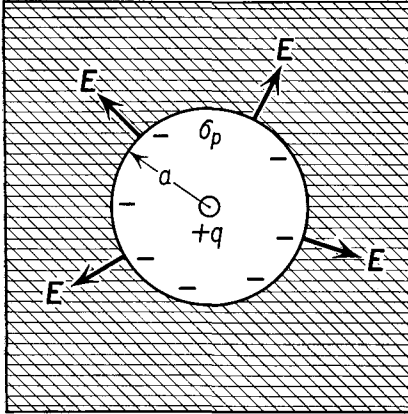


Fig. 25-12

Just as in the case of the parallel-plate capacitor, we may compute the electric field within the dielectric by considering that this electric field is generated by the original point charge and the uniformly charged shell of polarization charge. From Gauss's theorem we know that the field generated by a uniformly charged shell may be calculated as though the entire charge of the shell were

concentrated at its center, for field points on or outside the shell. The electric field at the surface of the shell  $E_a$  is made up of the field generated by the original point conduction charge  $q_c$  and the field of the polarization charge  $q_p$ . Thus we have

$$E_a = \frac{q_c + q_p}{4\pi\epsilon_0 a^2}. \tag{25-20}$$

Substituting from Equation (25-20) into Equation (25-19), we find

$$q_p = -4\pi a^2 \chi_e \frac{q_c + q_p}{4\pi\epsilon_0 a^2},$$

so that

$$q_p = -q_c \frac{\chi_e/\epsilon_0}{1 + \chi_e/\epsilon_0}. \tag{25-21}$$

In Equation (25-21) we see that the value of the polarization charge  $q_p$  does not depend upon the radius of the spherical hole. We may therefore shrink the cavity down to infinitesimal radius and imagine the polarization charge to be located at the position of the point charge.

The results of the preceding discussion are such as to indicate that the electric field within the dielectric may be found by replacing the dielectric by a point polarization charge  $q_p$ , of magnitude given by Equation (25-21),

located at the position of the free charge  $q_c$ . The electric field intensity within the dielectric is therefore

$$E = \frac{q_c + q_p}{4\pi\epsilon_0 r^2},$$

$$E = \frac{q_c}{4\pi\epsilon_0 r^2} \left( 1 - \frac{\chi_e/\epsilon_0}{1 + \chi_e/\epsilon_0} \right),$$

$$E = \frac{q_c}{4\pi\epsilon_0(1 + \chi_e/\epsilon_0)r^2}.$$

We recall from Equation (25-17) that the quantity in the parentheses in the above equation is equal to the dielectric constant  $\kappa_e$ , so that the electric field of a point charge embedded in an infinite dielectric may be expressed as

$$E = \frac{q_c}{4\pi\epsilon r^2}. \quad (25-22)$$

Since  $q_c$  is simply the original charge placed in the dielectric, we may drop the subscript  $c$  from  $q_c$ , for there is no longer any need to distinguish it from the polarization charge. Rewriting Equation (25-22) in vector form, recalling that the field generated by a point charge is radial, we obtain

$$\mathbf{E} = \frac{q}{4\pi\epsilon r^2} \mathbf{1}_r. \quad (25-22a)$$

If two point charges are embedded in an infinite dielectric, the force exerted by the charge  $q_1$  on the charge  $q_2$  may be obtained from Equation (25-22a) as

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\epsilon r^2} \mathbf{1}_r. \quad (25-23)$$

This is the form of Coulomb's law applicable to charges placed in an infinite dielectric medium of permittivity  $\epsilon$ .

### 25-8 The Electric Displacement

In dealing with problems associated with electric fields in dielectrics, it is convenient to introduce an auxiliary vector called the *electric displacement*, represented by the symbol  $\mathbf{D}$ , a concept introduced into the study of electricity by James Clerk Maxwell (1831–1879). We may define  $\mathbf{D}$  by the equation

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}. \quad (25-24)$$

The concept of lines of force was very useful for describing the electric field in vacuum. From this concept we were able to develop Gauss's theorem and to obtain considerable insight into the structure of electric fields. When we came to face the problem of the dielectric medium, the calculation of electric fields became much more difficult, and we were forced to invent the construct of a polarization charge.

The displacement vector is used in electrostatics for the purpose of recovering the convenience of lines of force when dielectric media are involved. Instead of lines of force, it is possible to describe the electric field by *lines of electric displacement*.

If we take account of the polarization charge  $q_p$  on the surface of a dielectric which is enclosed within a Gaussian surface, Gauss's law becomes

$$\int \mathbf{E} \cdot d\mathbf{A} = \frac{q_p}{\epsilon_0} + \frac{q}{\epsilon_0},$$

where  $q$  is the original conduction charge enclosed within the Gaussian surface. The total polarization charge  $q_p$  on the surface of the dielectric may be represented as

$$q_p = - \int \mathbf{P} \cdot d\mathbf{A},$$

for the polarization  $\mathbf{P}$  is a vector quantity. Substituting into the above equation, transposing, and multiplying through by  $\epsilon_0$ , we obtain

$$\int (\epsilon_0 \mathbf{E} + \mathbf{P}) \cdot d\mathbf{A} = q.$$

From the definition of  $\mathbf{D}$  given in Equation (25-24), Gauss's law takes the form

$$\boxed{\int \mathbf{D} \cdot d\mathbf{A} = q.} \quad (25-25)$$

Thus in the mks system of units we may assert that one  $D$  line emerges from each real, positive, conduction charge of 1 coul. In this system of units,  $D$  is expressed in coulombs per square meter, for consistency with Equation (25-25).

In the case of the parallel-plate capacitor with a dielectric we saw how some of the lines of force, or  $E$  lines, originating on the free or conduction charges, terminated on polarization charges on the surface of the dielectric. If we add to the  $E$  lines within the dielectric additional lines associated with the polarization, as indicated by Equation (25-24), the resultant lines are lines of electric displacement, or  $D$  lines. We find that the  $D$  lines are continuous from one plate of the capacitor, through the dielectric, to the

other plate. Thus the  $D$  lines originate on positive conduction charges and terminate on negative conduction charges, and are continuous on passing through a dielectric. They do not terminate abruptly on passing into a dielectric.

Having obtained a solution to the distribution of  $D$  lines in space for a particular charge distribution, we must still transcribe that solution so that it is stated in terms of the electric field intensity  $E$ . The transcription from  $D$  to  $E$  is especially simple for *homogeneous isotropic dielectrics*, such as we have been discussing. If we substitute from Equation (25-12) into Equation (25-24), we find

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = (\epsilon_0 + \chi_e) \mathbf{E}.$$

From Equation (25-18)

$$\epsilon_0 + \chi_e = \epsilon,$$

so that

$$\boxed{\mathbf{D} = \epsilon \mathbf{E}.} \tag{25-26}$$

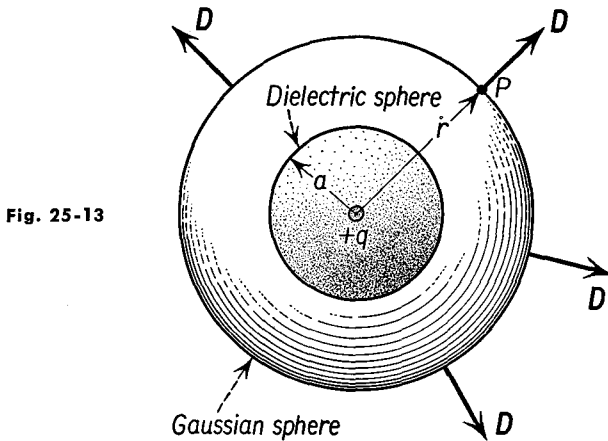


Fig. 25-13

*Illustrative Example.* Find the electric intensity at a distance  $r$  from a point charge  $q$  embedded at the center of a dielectric sphere of radius  $a$ , as shown in Figure 25-13.

From symmetry, the  $D$  lines are radial and continuous. If we draw a Gaussian sphere of radius  $r$  whose center is at  $q$ , we may apply Gauss's theorem in the form of Equation (25-25) to find

$$4\pi r^2 D = q,$$

so that

$$D = \frac{q}{4\pi r^2}.$$



TABLE 25-2 PRINCIPAL EQUATIONS IN MKS AND CGS UNITS

Equation	MKS	CGS	
(25-1)	$C = Q/V$	Same as mks	Capacitance
(25-2)	$C = 4\pi\epsilon_0 a$	$C = a$	Sphere
(25-3)	$C = \frac{\epsilon_0 A}{s}$	$C = \frac{A}{4\pi s}$	Parallel-plate capacitor in vacuum
(25-4a)	$\mathcal{W} = \frac{1}{2} \frac{Q^2}{C}$	Same as mks	Energy of a charged capacitor
(25-5)	$\mathcal{W}_v = \frac{\epsilon_0 E^2}{2}$	$\mathcal{W}_v = \frac{E^2}{8\pi}$	Energy density of electric field in vacuum
(25-6)	$\frac{1}{C} = \sum \frac{1}{C_i}$	Same as mks	Series capacitors
(25-7)	$C = \sum C_i$	Same as mks	Parallel capacitors
(25-9)	$C = \frac{\epsilon A}{s}$	$C = \frac{\kappa_e A}{4\pi s}$	Parallel-plate capacitor with dielectric
(25-11)	$\mathbf{p} = q\mathbf{s}$	Same as mks	Dipole moment
(25-12)	$\mathbf{P} = \chi_e \mathbf{E}$	Same as mks	Susceptibility
(25-17)	$\kappa_e = 1 + \chi_e/\epsilon_0$	$\kappa_e = 1 + 4\pi\chi_e$	Dielectric constant
(25-18)	$\epsilon = \epsilon_0 + \chi_e$	No analogous equation	
(25-22a)	$\mathbf{E} = \frac{q}{4\pi\epsilon r^2} \mathbf{1}_r$	$\mathbf{E} = \frac{q}{\kappa_e r^2} \mathbf{1}_r$	Point charge in an infinite dielectric
(25-24)	$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$	$\mathbf{D} = \mathbf{E} + 4\pi \mathbf{P}$	Displacement
(25-25)	$\int \mathbf{D} \cdot d\mathbf{A} = q$	$\int \mathbf{D} \cdot d\mathbf{A} = 4\pi q$	Gauss's theorem
(25-26)	$\mathbf{D} = \epsilon \mathbf{E}$	$\mathbf{D} = \kappa_e \mathbf{E}$	Displacement

When  $r$  is greater than  $a$ , the field point is in vacuum, so that

$$D = \epsilon_0 E,$$

and

$$E = \frac{q}{4\pi\epsilon_0 r^2}.$$

When  $r$  is less than  $a$ , the field point is within the dielectric, so that

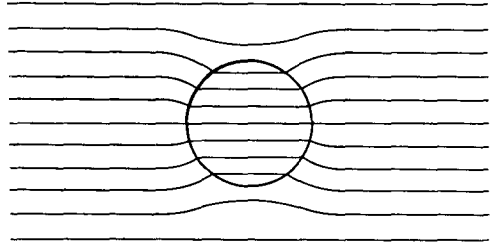
$$D = \epsilon E,$$

and

$$E = \frac{1}{4\pi\epsilon} \frac{q}{r^2}.$$

The distribution of lines of electric displacement for the case of a

dielectric sphere placed in a uniform electric field is shown in Figure 25-14. Problems of this sort may be solved by mathematical methods beyond the



**Fig. 25-14** Lines of electric displacement  $D$  for a dielectric sphere in a uniform electric field.

scope of this book through the application of the basic concepts developed here. The electric displacement is an important concept for describing the electric field in any case involving the use of insulating materials, and therefore in all practical engineering design. We shall see in subsequent chapters that the concepts developed here for the treatment of the electric field in the material medium will be of value in our study of the magnetic field.

**TABLE 25-3 CONVERSION FACTORS RELATING MKS AND CGS UNITS**

Quantity	Symbol	MKS Unit	CGS Unit
Capacitance	$C$	1 farad	$= 9 \times 10^{11}$ stfd (esu)
Displacement	$D$	1 coul/m <sup>2</sup>	$= 3 \times 10^5$ stcoul/cm <sup>2</sup> (esu)
Dipole moment	$p$	1 coul m	$= 3 \times 10^{11}$ stcoul cm (esu)
Polarization	$P$	1 coul/m <sup>2</sup>	$= 3 \times 10^5$ stcoul/cm <sup>2</sup> (esu)
Susceptibility	$\chi_e$	$1 \frac{\text{coul}^2}{\text{joule m}}$	$= 9 \times 10^9 \left( \frac{\text{stcoul}^2}{\text{erg cm}} \right)^*$ (esu)
Potential	$V$	1 volt	$= 300$ statvolts (esu)
Charge	$Q$	1 coul	$= 3 \times 10^9$ stcoul (esu)
Electric intensity	$E$	1 volt/m	$= \frac{1}{3 \times 10^4}$ statvolt/m (esu)

$$\text{Permittivity of free space: } \epsilon_0 = 8.85 \times 10^{-12} \frac{\text{farad}}{\text{m}} = 8.85 \times 10^{-12} \frac{\text{coul}^2}{\text{joule m}}$$

\*The electric susceptibility is dimensionless in the cgs electrostatic system of units. The dimensions in the parentheses cancel; they are included in the table to facilitate conversion between the two systems of units.

**Problems**

25-1. What is the capacitance in microfarads of a sphere 150 cm in radius (a) in air? (b) Immersed in an infinite bath of oil of dielectric constant 1.5?

25-2. What are the upper and lower limits for the capacitance of a charges conducting cube 1 m on an edge, in vacuum?

25-3. Show that the units of  $\epsilon_0$  in farads per meter are equivalent to unitd of coul<sup>2</sup>/nt m<sup>2</sup>.

25-4. A parallel-plate capacitor whose plates are 10 cm<sup>2</sup> in area has a charge of  $10^{-9}$  coul. The potential difference between the plates is 100 volts. What is the separation between the plates of the capacitor?

25-5. A capacitor of 2  $\mu$ fd capacitance is charged until the difference of potential between its plates is 120 volts. (a) Determine the charge on the capacitor. (b) Determine the amount of work done in charging it.

25-6. Two parallel-plate capacitors of identical dimensions differ only in that one has air between the plates and the other has oil of dielectric constant 2 between the plates. The capacitance of the air capacitor is 175  $\mu$ fd. (a) What is the capacitance of the oil capacitor? (b) Each capacitor is charged to a potential difference of 30 volts. Determine the charge on each capacitor. (c) Determine the energy of each capacitor.

25-7. Each of the two plates of a parallel-plate capacitor has an area of 400 cm<sup>2</sup>. The plates are 2 mm apart in vacuum. (a) Determine the capacitance in statfarads. (b) If a potential difference of 125 statvolts is maintained between the plates, determine the charge on each plate. (c) Determine the energy of this charged capacitor.

25-8. The plates of a parallel-plate capacitor are arranged so that the distance between them can be varied. When the distance between them is  $d$ , the capacitor is charged until the difference of potential is  $V$ . The plates are then separated until the distance between them is  $2d$ . Assuming that the charge  $Q$  on the plates is unchanged, determine (a) the difference of potential between the plates of this capacitor, (b) the change in energy due to the increase in the distance between the plates, and (c) the work done in separating the plates.

25-9. Derive a formula for the capacitance between a pair of concentric conducting spheres of radii  $r_1$  and  $r_2$ , in vacuum.

25-10. Derive a formula for the capacitance per unit of length of a pair of coaxial cylinders in vacuum. (See Problem 23-14.)

25-11. Two capacitors, one of 3  $\mu$ fd capacitance and the other of 5  $\mu$ fd capacitance, are connected in parallel and charged until the potential difference is 100 volts. Determine (a) the charge on each capacitor, (b) the equivalent capacitance of the system, and (c) the energy of this system.

25-12. Two capacitors, one of 4  $\mu$ fd capacitance and the other of 6  $\mu$ fd capacitance, are connected in series and charged to a difference of potential of 120 volts. Determine (a) the equivalent capacitance of the combination, (b) the charge on each capacitor, (c) the potential difference across each capacitor, and (d) the energy of the system.

25-13. A capacitor of 4  $\mu$ fd capacitance has a charge of 40  $\mu$ coul, and a capacitor of 3  $\mu$ fd capacitance has a charge of 10  $\mu$ coul. The negative plate of each one is connected to the positive plate of the other. Determine (a) the charge on each capacitor and (b) the potential difference across each one.

25-14. If the charged capacitors of Problem 25-13 are connected so that plates of like charge are connected together, determine (a) the initial potential

difference across each capacitor, (b) the final potential difference across each capacitor, (c) the initial energy of each capacitor, (d) the final energy of the combination, and (e) the energy lost in connecting them together.

25-15. Two capacitors of 3 and 4  $\mu\text{fd}$  are connected in parallel, and the combination is connected in series to a third capacitor of 5  $\mu\text{fd}$ . Determine (a) the effective capacitance of the combination, (b) the potential difference across the 3- $\mu\text{fd}$  capacitor when 100 volts are applied across the entire combination, and (c) the charge on the 4- $\mu\text{fd}$  capacitor under these circumstances.

25-16. It is desired to construct a parallel-plate capacitor of aluminum foil and polystyrene sheeting 5 mils thick (1 mil = 0.001 in.). The dielectric strength of polystyrene is 2,500 volts/mil, and the dielectric constant of this material is 2.5. The capacitor is to be able to withstand a maximum voltage of 100,000 volts. How large an area of plate will be required to yield a capacitance of 10  $\mu\text{fd}$ ?

25-17. What is the dipole moment of a pair of charges of opposite sign of 5 stcoul separated by a distance of 2 cm? State the units as well as the magnitude.

25-18. A parallel-plate capacitor, whose plates are 0.5 m<sup>2</sup> in area and are separated by 0.01 m, has a potential difference of 100 volts across its plates. The plates are separated by a dielectric whose dielectric constant is 3. Determine (a) the susceptibility of this dielectric, (b) the polarization of the dielectric, and (c) the polarization charge density at the surface of the dielectric.

25-19. Two point charges of 5 stcoul and  $-10$  stcoul lie along the  $x$  axis separated by a distance of 5 cm. The charges are immersed in insulating oil of dielectric constant 2.5. (a) Find the attractive force between them. (b) Find the electric field intensity at a point 3 cm from the 5-stcoul charge and 4 cm from the  $-10$ -stcoul charge.

25-20. A parallel-plate capacitor whose plates are separated by a distance  $s$  has a sheet of dielectric constant 2.5 and thickness  $0.95s$  inserted between its plates, the remainder of the space between the plates being filled by air of dielectric constant 1. A potential difference  $V$  is applied to the plates. (a) What is the electric field intensity in the air? (b) What is the electric field intensity in the dielectric? (c) How does this compare with the electric field intensity in an identical capacitor in which the space between the plates is completely filled by air or by dielectric? (Solve by use of the displacement  $\mathbf{D}$ .)

25-21. By use of a parallel-plate capacitor filled with a dielectric, find the energy per unit volume of the electric field in a medium of dielectric constant  $\kappa_e$ .