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# Selection of Switching Sites in All-Optical Network Topology Design

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**Abstract**—In this paper, we consider the problem of topology design for optical networks. We investigate the problem of selecting switching sites to minimize total cost of the optical network. The cost of an optical network can be expressed as a sum of three main factors: *the site cost, the link cost, and the switch cost*. To the best of our knowledge, this problem has not been studied in its general form as investigated in this paper.

We present a mixed integer quadratic programming (MIQP) formulation of the problem to find the optimal value of the total network cost. We also present an efficient heuristic to approximate the solution in polynomial time.

The experimental results show good performance of the heuristic. The value of the total network cost computed by the heuristic varies within 2% to 21% of its optimal value in the experiments with 10 nodes. The total network cost computed by the heuristic for 51% of the experiments with 10 node network topologies varies within 8% of its optimal value. We also discuss the insight gained from our experiments.

## I. INTRODUCTION

The deployment of optical networks is becoming more favorable with the advancement of photonic and Wavelength Division Multiplexing (WDM) technologies. As a result, the size of interconnects in optical networks has increased almost exponentially over the recent past. This has led to an increase in the complexity, an increase in the number of Optical/Electronic/Optical (O/E/O) converters, and a large mismatch between the capacity and speed of the optical and electronic transmissions. This mismatch causes a critical bottleneck in electronic interconnects. Thus, eliminating the need for costly O/E/O conversions motivates the design of an all optical network where data can always be kept in the optical domain.

Fortunately, the rising costs due to complexity have been mitigated somewhat by the advancement of photonic technologies. Several cost effective designs of all-optical interconnects have been proposed [1], [2], [3], [4]. The designs of all-optical interconnects proposed in [1], [2] perform switching as a combination of wavelength and fiber crossconnect. However, complexity still dominates the cost of a design, causing *nearly quadratic* increase in the cost of interconnects with respect to the number of ports [5]. A  $1024 \times 1024$  optical switch may cost as much as \$4,000,000, whereas  $2 \times 2$  and  $8 \times 8$  optical switches may cost about \$100 and \$2,000 respectively [5].

In order to design a minimum cost topology for an optical network, we need to minimize the sum of three main factors: *the site cost, the link cost, and the switch cost*.

The cost of an optical crossconnect site (simply, site) includes the installation cost, real estate cost, and maintenance cost of the optical switch at the site. The site cost varies from one site to another and is also influenced by the geographic location. The cost of an optical link between two sites is generally proportional to the distance between them. The unit cost of a link may also vary depending on the geographic location of the link and the cost of laying or leasing optical fiber [6]. With increasing switch complexity and the increasing number of wavelengths in a fiber, the cost of an optical switch is becoming significantly important in minimizing the total network cost of an optical topology.

The total network cost should include the size of the optical switch as determined by the number of input and output lightpaths. Consider for example, a network topology shown in Figure 1(a). The solution based on maximum-leaf spanning tree method obtained for the network topology is shown in Figure 1(b). If the total network cost is only dependent on the size of the optical switches, then the network topology in Figure 1(b) has a total network cost of  $6^2 = 36$ , assuming one wavelength is available in the fiber for the network. However the minimum cost network topology shown in Figure 1(c) has a total network cost of  $2^2 + 2^2 + 2^2 + 2^2 + 2^2 = 20$ . Thus, it is important to include the size of the optical switches for minimizing the total network cost. Moreover, inclusion of the size of the optical switches in the total network cost leads to better solutions of the network cost minimization problem.

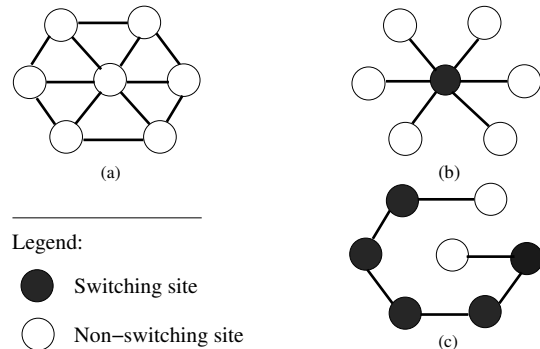


Fig. 1. (a) An optical network topology; (b) The maximum-leaf spanning tree of the network topology; and (c) A possible minimum cost network topology minimizing the size of the optical switches.

Chen and Tobagi have addressed the switching site selection

problem taking into account the number of switches [7] and the link cost [6]. In this paper, we present a comprehensive approach for designing minimum cost network topologies. In our formulation, the total network cost is expressed as a sum of the costs of the site, link, and switch. The problem can be formulated as a generalization of the maximum-leaf spanning tree problem, which is  $\mathcal{NP}$ -hard [8].

We present a mixed integer quadratic programming (MIQP) formulation of the problem to find the optimal total network cost. In our experiments, the MIQP formulation could not solve the problem even with 20 nodes in a reasonable amount of time. Thus, we develop an efficient heuristic to approximate the solution. The heuristic takes a finite, simple, undirected, connected graph  $G = (V, E)$  representing potential switching sites and potential links of an optical network as an input and outputs a spanning tree of minimum total network cost.

## II. RELATED WORK

A survey of models and optimization methods for designing survivable networks is presented in [9]. The identification of switching sites and the design of physical topology are considered as two independent problems in these models. The authors have studied the trade off between fibers and wavelength leading to a cost effective transport network design.

The minimum number of switching sites required in a network topology for total connectivity can be estimated using the heuristic presented in [7]. The heuristic finds a maximum-leaf spanning tree of the given network topology for minimizing the number of switching sites.

The importance of jointly optimizing the switching and fiber link costs is presented in [6]. Two heuristic algorithms are investigated that minimize the total network cost for backbone-protected and unprotected topologies. The heuristic algorithm for minimum total network cost of unprotected networks finds a minimum weight spanning tree among all the nodes identified as a switching site using the heuristic in [7]. The remaining nodes are identified as non-switching sites and are connected to one of the closest switching sites. Similarly, the heuristic for minimum total network cost of backbone-protected networks finds a simple cycle connecting all nodes identified as a switching sites using the heuristic in [7]. The heuristic also ensures that each non-switching site has at least one neighbor which is a switching site.

The network model in [6] and [7] assumes all sites in the network are equally favorable for selection as a switching site. The optical link cost is considered as the Euclidean distance between the sites satisfying the triangle inequality.

## III. PROBLEM FORMULATION

In this section, we present a mixed integer quadratic programming (MIQP) formulation of the problem.

An optical network topology is represented as a finite, simple, undirected, connected graph  $G = (V, E)$ . Thus, in the network,  $V$  represents the set of nodes (potential switching sites), and  $E$  represents the set of edges (potential links). *Site Cost*, denoted by  $\Delta_i$  is the cost of switching capability at site  $i$ ,

$i = 1$  to  $|V|$ . This cost includes the installation, real estate, and maintenance costs of an optical switch along with the cost of add/drop multiplexers at that node. *Edge Cost*, denoted by  $C_{ij}$  is the cost of the link  $i, j$  (pair of fibers one in each direction),  $i, j \in V$  and  $(i, j) \in E$ . The cost of laying and/or leasing fiber along with the cost of optical amplifiers and repeaters is also included in the *edge cost*.

As we noted earlier, the cost of an optical switch increases nearly quadratically with the increase in the size of the switch [5]. The degree of a node in the minimum cost network topology is denoted by  $d_i$ ,  $i = 1$  to  $|V|$ . A  $Wd_i \times Wd_i$  optical switch is required at a switching site  $i$ , if each fiber carries  $W$  wavelengths. The size of the optical crossconnect also increases with the increase in the number of wavelengths in a fiber. Let  $\chi$  a constant represent the rate of increase in the cost of a switch as the size of the switch increases. Thus, minimizing  $\chi \sum_{i \in V} d_i^2$  also minimizes the size of the optical switch cost. The  $\chi \sum_{i \in V} d_i^2$  term is the only quadratic term in the problem formulation requiring a MIQP formulation of the pertinent problem.

The problem of minimizing the number of switching sites for total connectivity in an unprotected topology is  $\mathcal{NP}$ -hard [8]. It is equivalent to finding the maximum leaf spanning tree of the network topology [7]. Thus, this generalization is also  $\mathcal{NP}$ -hard.

### A. Formal Problem Statement

#### Given:

- $G = (V, E)$ : A finite, simple, undirected, connected graph representing an optical network topology.
- $C_{ij}$ : The cost the  $i, j$  link,  $\forall i, j \in V$  and  $(i, j) \in E$ .
- $\Delta_i$ : The site cost of the node  $i$ ,  $\forall i \in V$ .
- $\chi$ : The proportionality constant between the cost and size of an optical switch.

$$\text{Minimize: } \sum_{(i,j) \in E} C_{ij} X_{ij} + \sum_{i \in V} \Delta_i S_i + \chi \sum_{i \in V} d_i^2$$

$$\text{Subject to: } X_{ij} \geq X'_{ij}, \quad \forall i < j, (i,j) \in E \quad (1)$$

$$X_{ij} \geq X'_{ji}, \quad \forall i < j, (i,j) \in E \quad (2)$$

$$X'_{ij} \geq X_{ij}^{sd}, \quad \forall s, d \in V \times V, s \neq d, (i,j) \in E \quad (3)$$

$$\sum_{i:(i,j) \in E} X_{ij}^{sd} - \sum_{k:(j,k) \in E} X_{jk}^{sd} = \begin{cases} 1 & \text{if } j = d \\ -1 & \text{if } j = s \\ 0 & \text{otherwise} \end{cases} \quad \forall s, d, j \in V, s \neq d \quad (4)$$

$$d_i = \sum_{\substack{j:j < i \\ (i,j) \in E}} X_{ji} + \sum_{\substack{k:i < k \\ (i,k) \in E}} X_{ik} \quad \forall i \in V \quad (5)$$

$$S_i \geq X_{ji} + X_{ki} - 1 \quad \forall j, k \text{ such that, } \begin{matrix} (i,j), (i,k) \in E \\ j \neq k \\ j, k < i \end{matrix} \quad (6)$$

$$S_i \geq X_{ji} + X_{ik} - 1 \quad \forall j, k \text{ such that, } \begin{matrix} (i,j),(i,k) \in E \\ j \neq k \\ j < i < k \end{matrix} \quad (7)$$

$$S_i \geq X_{ij} + X_{ik} - 1 \quad \forall j, k \text{ such that, } \begin{matrix} (i,j),(i,k) \in E \\ j \neq k \\ j, k > i \end{matrix} \quad (8)$$

The first, second, and third terms of the objective function respectively minimize the link, site, and switching costs of a network topology. The minimum network cost topology is a spanning tree of the given network topology [7]. The undirected topology is converted to a directed graph in order to find the spanning tree using  $X_{ij}$  and  $X'_{ij}$  respectively.  $X_{ij}^{sd}$  is used to specify a directed path from  $s$  to  $d$  using link  $(i, j)$ .  $S_i$  identifies sites having more than one neighbors in the spanning tree, and so they must be switching sites.

The given undirected topology is first converted to a directed topology using equations (1) and (2) to find the spanning tree in polynomial time. The equations (3) and (4) intend to find a simple path between any pair of nodes, thus generating a spanning tree of minimum cost. The degree of each node in the spanning tree is calculated using the equation (5). In the spanning tree all non-terminal nodes are identified as switching sites using equations (6), (7), and (8).

When all  $\Delta_i$  are equal,  $C_{ij} = 0$ ,  $(i, j) \in E$ ,  $i, j \in V$ , and  $\chi = 0$ , the problem degenerates to the maximum-leaf spanning tree problem, which is  $\mathcal{NP}$ -hard [8]. A commercial optimizer, MOSEK could not solve MIQP formulations with 20 nodes in a reasonable amount of time. This motivates the design of a heuristic algorithm.

The site cost for non-switching sites is not considered in the problem formulation, since it may not have a significant impact on the topology design. However, this cost can be easily incorporated in the problem formulation by introducing an extra term in the problem statement. We assume single fiber per link formulation, but it can be easily extended to solve the multiple fibers per link using a multi-graph. For simplicity, we assume a uniform traffic model. Thus, the traffic pattern does not affect the choice of switching sites and optical links.

#### IV. PROPOSED HEURISTIC APPROACH

The idea of the proposed heuristic is similar to the heuristic in [7]. The minimum network cost topology is a spanning tree of the given network topology [7]. For total connectivity in the spanning tree, the switching sites must form a connected topology. Thus, any switching site can communicate with any other switching site in the network topology. Each non-switching site must have exactly one switching site neighbor to have connectivity among all the nodes in the network. Thus, the leaf nodes of the spanning tree are non-switching sites and the non-leaf are switching. It follows that all intermediate nodes on a path between any pair of source and destination nodes must be switching nodes.

A node is said to be **covered** if it is adjacent to a switching site. Initially, all nodes are non-switching and uncovered. The site with least site cost per uncovered neighbor among all

the nodes in the network topology is selected as the first switching site. In subsequent iterations, a new switching site is selected among all covered non-switching nodes which has the least site cost per neighbor based on previously uncovered neighbors of the selected site. This newly selected switching site is adjacent to one of the old switching sites such that sum of the link cost and the cost involved in increasing the size of the optical switch at the old switching site is minimum among all the possible choices. This greedy selection of switching sites followed by greedy selection of links between them builds a spanning tree, such that the switching sites form a connected topology. The selection of switching sites terminates when all the nodes in the network topology are covered. The leaf nodes of the spanning tree are non-switching sites and the non-leaf are switching. A link between a non-switching site and a switching site is chosen such that the sum of the link cost and the cost of increasing the size of the optical switch at the switching site is minimum among all the possible choices.

#### • Notations:

- 1)  $N(i)$ : Number of neighbors of node  $i$ .
- 2)  $N^{UC}(i)$ : Number of neighbors of node  $i$  which are uncovered.

- 1) Initialize all nodes to be non-switching and uncovered. Initialize  $d_i = 0$ ,  $\forall i = 1$  to  $|V|$ .
- 2) Select the vertex with lowest value of  $\frac{\Delta_i}{N(i)}$ ,  $i = 1$  to  $|V|$ , as the first switching site. In case of a tie, choose the node with lowest  $\Delta_i$ .
- 3) Repeat Steps 4 to 6 until all nodes are covered.
- 4) Select the node  $v$  among all covered nodes which has lowest value of  $\frac{\Delta_i}{N^{UC}(i)}$  as a switching site,  $i = 1$  to  $|V|$ ,  $i$  not a switching site, and  $N^{UC}(i) > 0$ . In case of a tie, choose the node having lowest  $\Delta_i$ .
- 5) Select a link  $(v, x)$ , such that  $x$  is a switching node and  $C_{vx} + \chi * d_x^2$  has lowest value among all choices of  $x$ . Ties can be broken arbitrarily.
- 6) Increment the values of  $d_v$  and  $d_x$  by 1.
- 7) Identify the leaf nodes of the spanning tree as non-switching sites and the non-leaf as switching.
- 8) For each non-switching site  $y$ , select a link  $(y, z)$ , such that  $z$  is a switching site, and  $C_{yz} + \chi * d_z^2$  has lowest value among all choices of  $z$ . Increment the value of  $d_z$  by 1. Ties can be broken arbitrarily.

Fig. 2. A high-level description of the proposed heuristic.

The pseudo-code of the proposed heuristic is given in Figure 2. The time complexity of the algorithm is dominated by the execution time of steps 3 to 6. The neighborhood of a vertex can be calculated in  $O(E)$  time, if the graph is represented as an adjacency list. Thus, the time complexity of the proposed heuristic algorithm is  $O(V^2E)$ .

The proposed heuristic can also be used for multi-edge network topologies by modifying the link selection for multi-graph networks. The link among several available links between two nodes with least link cost is chosen in the minimum

cost network topology. The remaining links are used as backups in the event of failure or regular maintenance. The link with second least link cost is chosen as the primary backup link. Similarly, secondary and tertiary backup paths can also be identified in the network topology, if links exist.

## V. RESULTS AND DISCUSSION

In this section, we describe the experimental setup and the experiments conducted to analyze the quality of solutions produced by the proposed heuristic.

**Experimental Setup:** We used the rectangular grid method [10] to generate random connected network topologies for simulation. In this method, nodes are assigned randomly in a rectangular grid. The probability of an edge between two nodes is a function of distance between the nodes [11]. If  $d(u, v)$  is the distance between two nodes  $u$  and  $v$ ,  $L$  is the maximum distance between any two nodes in the grid, and  $\alpha, \beta \in (0, 1]$ , then the edge probability between  $u$  and  $v$  is given by:

$$P(u, v) = \beta^{-\frac{d(u, v)}{L\alpha}}$$

The edge density of the topology increases with the increase in the value of  $\beta$ . On the other hand, short edges are more probable than long edges with smaller values of  $\alpha$  [11]. We experimented with different combinations of  $\alpha$  and  $\beta$  and generated three random connected topologies for each of 10, 20, 35, 49, and 86 nodes. We empirically selected the topologies from a set of about 20 candidate network topologies by visual inspection. The random topology generated for 10 nodes using  $\alpha = 0.6$  and  $\beta = 1.0$  is shown in Figure 3(a). Figures 3(b) and 3(c) show the random topologies generated for 49 and 86 nodes using  $\alpha = 0.40$  and  $\beta = 0.15$  and  $\alpha = 0.40$  and  $\beta = 0.047$  respectively.

In order to perform a meaningful analysis of the heuristic, we consider different values of the ratio of the site cost to the link cost and the proportionality constant  $\chi$ . The value of the proportionality constant depends on several factors. It varies not only with the technology used in the switch design, but also with the manufacturer. We believe that the following six different scenarios lead to a meaningful analysis.

- 1) site cost  $\gg$  link cost.
- 2) site cost *slightly*  $>$  link cost.
- 3) site cost  $\cong$  link cost.
- 4)  $\chi$  has a high value.
- 5)  $\chi$  has a medium value.
- 6)  $\chi$  has a low value.

The link cost is chosen as a random number between 20 to 60 units. The site cost is chosen as a random number between 200 to 250 units, 50 to 100 units, and 20 to 60 units for respective scenarios. Similarly, the high, medium, and low values of  $\chi$  are chosen as 30, 15, and 5 respectively.

For gaining insight into the impact of different network topologies on the total cost of the network, we generate 27 different network topologies for each network size of 10, 20, 35, 49, and 86 nodes. The 27 different topologies for each size

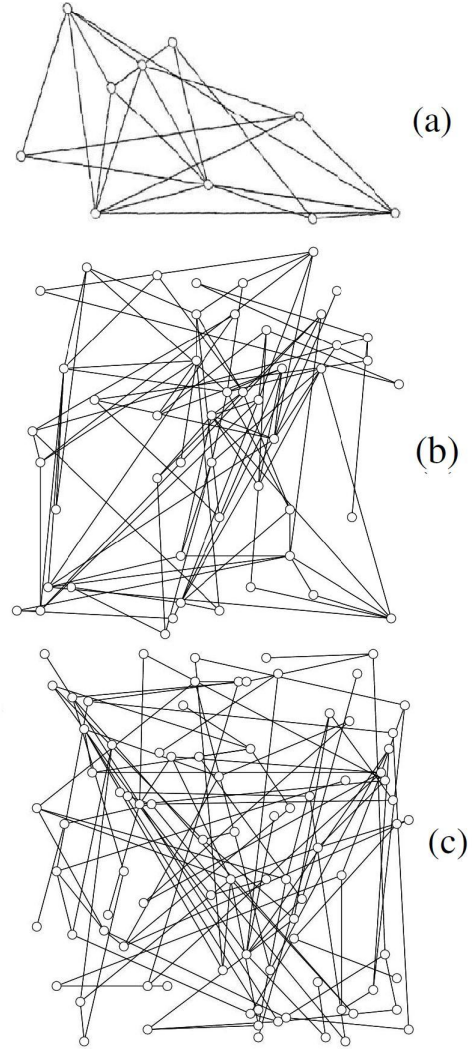


Fig. 3. Random topologies: (a) 10 nodes ( $\alpha = 0.6, \beta = 1.0$ ); (b) 49 nodes ( $\alpha = 0.40, \beta = 0.15$ ); (c) 86 nodes ( $\alpha = 0.40, \beta = 0.047$ )

are generated by choosing 27 combinations of different values of  $\alpha$ ,  $\beta$ ,  $\chi$ , and site cost. However, due to limited space, only a reasonable sample is shown here.

The experiments were performed on an Intel Core 2 Duo 3.00 Ghz PC with 4 GB of RAM running Vista. A commercial optimization software, MOSEK Optimization Software (MOSEK) has been used to solve the MIQP formulation [12].

**Comparison of Results:** To show the effectiveness of our heuristic, we compare the results obtained by the heuristic to that obtained by the MIQP formulation of the problem solved by the optimization software, MOSEK. Comparison of the results obtained by MOSEK and the proposed heuristic for different random topologies of 10 nodes are given in Tables I, II and III. We do not compare our results with [6] because the heuristic in [6] first determines the minimum number of switching sites for total connectivity in the given network topology using the heuristic in [7]. It then finds a minimum

TABLE I  
COMPARISON OF RESULTS FOR 10 NODES( $\alpha = 0.6, \beta = 1.0$ )

Site Cost	$\chi$	Total Cost (MOSEK)	Total Cost (heuristic)	Error %age
20-60	5	616	651	5.68
20-60	15	1033	1151	11.40
20-60	30	1612	1812	4.91
50-100	5	752	767	1.99
50-100	15	1193	1247	4.52
50-100	30	1823	1967	7.89
200-250	5	1193	1234	3.40
200-250	15	1673	1714	2.45
200-250	30	2393	2434	1.71

TABLE II  
COMPARISON OF RESULTS FOR 10 NODES( $\alpha = 0.5, \beta = 0.9$ )

Site Cost	$\chi$	Total Cost (MOSEK)	Total Cost (heuristic)	Error %age
20-60	5	627	699	11.48
20-60	15	1065	1279	20.60
20-60	30	1688	1868	16.64
50-100	5	773	809	4.65
50-100	15	1229	1289	4.88
50-100	30	1859	2009	8.06
200-250	5	1057	1107	4.73
200-250	15	1637	1687	3.05
200-250	30	2400	2557	6.54

weighted spanning tree among all the switching sites and connects each non-switching site to one of the switching sites having least link cost. However, the heuristic proposed in this paper minimizes the sum of the site, link, and switch costs to obtain a spanning tree of minimum network cost of the given network topology. Therefore, comparison of our heuristic with [6] is not meaningful.

**Execution Time:** The average execution time of the heuristic was less than a second as compared to 9916 to 472785 secs needed by MOSEK to solve the MIQP formulation for 10 node topologies. The network cost calculated by the heuristic is within 21% of its optimal value in the experiments performed. Moreover, for 51% of the experiments, the network cost calculated is within 8% of its optimal value. A similar comparison for 20 or more node network topologies could not be performed, because MOSEK could not solve the MIQP formulation for 20 node network topologies even in three weeks, as compared to less than a second taken by heuristic on

TABLE III  
COMPARISON OF RESULTS FOR 10 NODES( $\alpha = 0.7, \beta = 0.8$ )

Site Cost	$\chi$	Total Cost (MOSEK)	Total Cost (heuristic)	Error %age
20-60	5	644	735	14.13
20-60	15	1078	1215	12.70
20-60	30	1686	1935	14.76
50-100	5	673	798	18.50
50-100	15	1150	1378	19.82
50-100	30	1775	1975	13.82
200-250	5	994	1103	10.96
200-250	15	1575	1683	6.80
200-250	30	2363	2553	8.04

average. Thus, for topologies with 20 or more nodes we only record the performance results of the heuristic. The average execution time of the heuristic was about two seconds for 86 node network topologies. For brevity, we only present a sample of the results in Table IV.

TABLE IV  
COMPARISON OF RESULTS

# of Nodes	Site Cost	$(\alpha, \beta)$	$\chi$	Total Cost
20	20-60	(0.6, 0.35)	5	1624
20	200-250	(0.45, 0.45)	15	3779
20	50-100	(0.6, 0.5)	30	4125
35	20-60	(0.35, 0.2)	5	2918
35	50-100	(0.4, 0.15)	15	5147
35	200-250	(0.5, 0.25)	30	8974
49	20-60	(0.4, 0.15)	5	4725
49	50-100	(0.25, 0.2)	15	7339
49	200-250	(0.35, 0.2)	30	15789
86	20-60	(0.3, 0.039)	5	10047
86	50-100	(0.4, 0.041)	15	15978
86	200-250	(0.4, 0.047)	30	28713

**Performance Analysis:** The proposed heuristic performs better when the site cost is higher than the optical link cost. In such scenarios, the site cost dominates the total network cost of the optical network. Thus, the greedy approach ensures proper selection of switching sites in each iteration which dominates the total network cost. As a result the total network cost is closer to its optimal value.

For the scenarios where the site and link costs are comparable, the heuristic does not perform as well. This can be explained by the nature of the heuristic. Since the site cost does not dominate the link cost, the link cost is equal to or better than the site cost in these scenarios. But the heuristic makes a greedy choice of a switching site first, followed by a greedy selection of an optical link. Thus, the performance of the heuristic can simply be explained by the greedy strategy. However, the strategy can be easily modified to obtain better solutions for scenarios where the link cost dominates the site cost.

The greedy choice of switching sites followed by the greedy selection of links results in close to optimal solution for the topologies with higher site cost than link cost in Tables I and II. However, some particular setting of the parameters leads to the worst error rate of 20.60% in Table II.

The performance of the heuristic is also quite good for high density topologies. With the increase in the density of the topology, the number of possible choices for selection of switching sites and optical links increases. The number of switching sites required for total connectivity is also smaller in dense topologies as compared with sparse topologies with equal number of nodes. For most instances of the topologies shown in Table III, the low density of the network topology can be shown as the main reason for not so good performance of the heuristic. Low density results in reduced number of choices in switching sites and links in each iteration leading to an increase in the number of switching sites required for total connectivity as compared to the optimal value.

**Nature of Solution:** Our experimental results show that the proposed heuristic is very effective and is not biased toward any specific kind of solutions. For example, consider the simple example shown in Figure 1. It appears that our heuristic will find a solution like the one shown in Figure 1(c), and would favor a high-congestion (a bus) network with long paths over designs with high-degree switches, Figure 1(b). However, from extensive simulations on network topologies of different sizes, we observe that the proposed heuristic is not biased toward any particular type of networks, it just finds near minimum cost networks.

**Total Cost vs. Topology Design:** We obtained interesting insight about the impact of different network topologies on the total cost of the network from our experiments. We discover that dense random optical network topologies with longer optical links have lower total network cost. Moreover, dense networks with fewer constraints are probably better than sparse networks with more constraints.

The number of possible choices of switching sites and links increases with the increase in density of the network topology. The number of switching sites required for total network connectivity for dense networks is also less than that required for sparse network topology. Hence, high density topologies have lower total network cost. The effect of longer links on the total network cost becomes significant in network topologies with a large number of nodes. The number of switching sites required for total connectivity in network topology decreases with the presence of longer links. Thus, network topologies with longer links have lower total network cost. However, this insight about long links is essentially just a small-world-graph phenomenon [13].

The proposed heuristic does not attempt to differentiate the building cost of the network from operational and maintenance costs. Such issues can be addressed by appropriate amortization schemes.

## VI. CONCLUSION

In this paper, we investigate the problem of selection of switching sites for minimizing the total network cost. In our model, the cost of a network can be expressed as a sum of three main factors: the site cost, the optical link cost, and the optical switch cost.

The problem is formulated as a generalization of the maximum-leaf spanning tree problem, which is  $\mathcal{NP}$ -hard. We present a mixed integer quadratic programming (MIQP) formulation of the problem to find the optimal total network cost. The MIQP formulation did not solve problems even for network topologies with 20 nodes in a reasonable amount of time. We also present an efficient heuristic to approximate the solution in polynomial time.

The total network cost for random topologies of 10 nodes calculated by the proposed heuristic is compared with its optimal value obtained from the MIQP formulation of the problem. In our experiments with 10 node topologies, the CPU execution time of the MIQP formulation varied within 9916 to 472785 secs, whereas the proposed heuristic takes

less than a second on average. The results of the proposed heuristic vary within 2% to 21% of its optimal value in the experiments performed. The total network cost for 51% of the experiments with 10 node network topologies varies within 8% of its optimal value.

The heuristic proposed in this paper is intended to be used by network designers as a tool for making decisions on the selection of the switching sites, selection of the size of an optical switch for the switching site, and selection of optical fiber links for communication between the nodes in the topology.

It would be interesting to see if the performance of the heuristic can be improved by applying meta-heuristic approaches to the base heuristic proposed in this paper. A comprehensive study of parameter space for better design of optical network topologies with minimum total network cost is another interesting research problem. The proposed network model does not consider having any extra paths open in the event of failure or regular maintenance. Hence, the design of a similar heuristics for fault tolerant optical networks is also an interesting research problem. It would also be interesting to formulate the problem where there is no planned switching capability at the nodes. This would result in using unused links for communication in the event of failure or regular maintenance and establishing a new switching site in the event of unfortunate node failure.

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## REFERENCES

- [1] H. S. Hamza and J. S. Deogun, "WDM Optical Interconnects: A Balanced Design Approach," *IEEE/ACM Trans. Networking*, vol. 15, no. 6, pp. 1565-1578, Dec. 2007.
- [2] H. S. Hamza and J. S. Deogun, "Wavelength exchanging cross-connect (WEX) - A new class of photonic cross-connect architectures," *Journal of Lightw. Technol.*, vol. 24, no. 3, pp. 1101-1111, Mar. 2006.
- [3] Y. Yang and J. Wang, "Designing WDM Optical Interconnects with Full Connectivity by Using Limited Wavelength Conversion," *IEEE Trans. Computers*, Vol. 53, no. 12, pp. 1547-1556, Dec. 2006.
- [4] X. Qin and Y. Yang, "Nonblocking WDM Switching Networks with Full and Limited Wavelength Conversion," *IEEE Trans. Comm.*, Vol. 50, no. 12, pp. 2032-2041, Dec. 2002.
- [5] Y. Leung, "Lightpath Concentrators for All-Optical Networks," *Journal of Lightw. Technol.*, Vol. 24, no. 9, pp. 3259-3267, Sept. 2006.
- [6] B. K. Chen and F. A. Tobagi, "Network Topology Design to Optimize Link and Switching Costs", *IEEE International Conference on Communications (ICC)*, pp.2450-2456, June 2007.
- [7] B. Chen and F. Tobagi, "Optical network design to minimize switching and transceiver equipment costs", *BROADNETS*, pp.1-10, 2006.
- [8] M. R. Garey and D. S. Johnson, "Computers and Intractability: A Guide to the Theory of  $\mathcal{NP}$ -Completeness", W. H. Freeman, 1979.
- [9] H. Kerivin and A. R. Mahjoub, "Design of survivable networks: A survey", *Networks*, vol. 46, no. 1, pp. 1-21, June 2005.
- [10] B. M. Waxman, "Routing of multipoint connections", *IEEE J. Sel. Areas Comm.*, vol. 6, pp. 1617-1622, Dec. 1988.
- [11] Wan-Yi Lin and S. Tjoa, "Network Topology Generators", *ENEE698A Communications Seminar*, University of Maryland, May 2006.
- [12] MOSEK Optimization Software, <http://www.mosek.com/>
- [13] M. Buchanan, "Nexus: Small Worlds and the Groundbreaking Theory of Networks", *W. W. Norton & Company*, 2002.