Noname manuscript No.
(will be inserted by the editor)

Monotonous Trend Estimation of Deck Displacement for Automatic Landing of Rotorcraft UAVs

Xilin Yang · Matthew Garratt · Hemanshu Pota

Received: date / Accepted: date

Abstract This paper presents a novel and practical procedure for estimating the mean deck height to assist in automatic landing operations of a Rotorcraft Unmanned Aerial Vehicle (RUAV) in harsh sea environments. A modified Prony Analysis (PA) procedure is outlined to deal with real-time observations of deck displacement, which involves developing an appropriate dynamic model to approach real deck motion with parameters identified through implementing the Forgetting Factor Recursive Least Square (FFRLS) method. The model order is specified using a proper order-selection criterion based on minimizing the summation of accumulated estimation errors. In addition, a feasible threshold criterion is proposed to separate the dominant components of deck displacement, which results in an accurate instantaneous estimation of the mean deck position. Simulation results demonstrate that the proposed recursive procedure exhibits satisfactory estimation performance when applied to real-time deck displacement measurements, making it well suited for integration into ship-RUAV approach and landing guidance systems.

Keywords Rotorcraft UAV · Automatic Landing · Prony Analysis · Recursive Least Square

Xilin Yang
School of Engineering and Information Technology, University College, University of New South Wales, Australia
Tel.: +61-02-62688518
E-mail: x.yang@adfa.edu.au

Matt Garratt
School of Engineering and Information Technology, University College, University of New South Wales, Australia
Tel.: +61-02-62688267
E-mail: m.garratt@adfa.edu.au

Hemanshu Pota
School of Engineering and Information Technology, University College, University of New South Wales, Australia
Tel.: +61-02-62688197
E-mail: h.pota@adfa.edu.au

1 Introduction

The present research is part of an effort to develop a systematic procedure for landing a RUAV on a moving platform in rough sea environments. The main challenge in fulfilling maritime landing operations results from the complicated sea environment, which consists of wave-excited deck displacement and turbulent gusts when the RUAV approaches the ship superstructure. Also, the small size and all-up weight make the RUAV more sensitive and vulnerable to external disturbances, and exacerbate the difficulty of accurate positioning and attitude control [25]. Our objective is to accurately estimate the instantaneous mean position of the landing deck. The underlying significance is that the RUAV can match its vertical position to the instantaneous mean position rather than the instantaneous deck displacement. This results in a smoother trajectory and hence will lower impact stresses on landing.

A variety of real-time dynamic systems experience significant oscillations comprising distinct sinusoidal components resulting from unknown nonlinearities, uncertainty of system dynamics, and random external disturbance. Normally, a systematic framework for theoretical analysis and applicable control strategy design is not directly available due to such complicated system dynamics. However, these nonlinear systems can be approached around a set of equilibrium points using proper linear models. Developing the form of such models depends on the specific applications under consideration. There are two mainstream approaches: the first one is to linearize the nonlinear model by expanding the nonlinear terms around the concerned equilibrium points. These equilibrium points are chosen to represent the typical working conditions the system experiences, and ignoring high-order terms would not harm system dynamics. Alternatively, curve-fitting techniques are an option to optimally fit a linear combination of terms to the measurements [27]. Since it takes tremendous efforts to build an accurate system model of deck motion due to the irrational non-minimum phase transfer function between ship motion and sea elevation [26], it is preferred to use a curve-fitting technique to analyze deck motion for real-time applications.

PA is a branch of curve-fitting techniques, which originally employs a group of exponential terms to approximate the impulse response of a dynamic system [8]. Application of PA in power systems has been subject to extensive investigations in a substantial number of papers, and significant efforts, including theoretical analysis and experimental research, have been made to deal with various practical scenarios. Hauer et al. [8],[9] present results for modal analysis and detailed model construction of power systems based on field measured data. The identification of modal content from oscillating power systems in various scenarios has been reported [7],[13],[21],[28]. Trudnowski et al. [29] extend traditional PA to allow for analyzing multiple input signals. Recently, PA was implemented to monitor power system transient harmonics, and the dominant harmonics identified were used as the harmonic reference for harmonic selective active filters in Ref. [18]. In contrast, a small number of papers address the use of PA in oscillating systems other than power systems. A recursive approach to PA estimation was employed to analyze the response of a beam to transient excitations in Ref. [4]. PA was also used for radar target identification [3, 19] and signal processing [24, 31].

The originality of this work lies in proposing and constructing a systematic recursive framework to estimate the instantaneous mean deck position. The modified PA, with model order specified based on minimizing the summed squared estimation errors and model coefficients identified using a recursive procedure, makes it well suited for analyzing the real deck motion. A dominant component selection criterion is proposed to choose the dominant components in the oscillating system. Simulation results demonstrate that the proposed methodology exhibits excellent estimation performance when applied to real-time deck motion data.

2 Research platform description

The RMAX helicopter which was originally designed for agricultural work is employed as our research platform (Fig. 1). With a maximum take-off weight of 100KG and payload capacity of 30KG, the RMAX helicopter is able to perform various operations with a flight duration of 1 hour. Presently, a considerable number of variants have been developed with similar underlying systems for different applications. e.g., agricultural spraying, airborne surveillance. Therefore, it is an idea platform for research on RUAVs. Also, a moving deck simulator has been set up by our lab to replicate deck motion dynamics. The simulator is driven by three electrical linear actuators able to deliver up to $3000N$ each. So far, dynamic motion of the ship deck has been simulated satisfactorily, and design specifications (heave of 1m and pitch of 25^o) have been achieved [5].

3 Traditional Prony Analysis

Given a sampled sequence of discrete-time system observations, numerous curve-fitting methodologies are usually available and the form of model to be chosen depends on dynamic variations revealed in the system and tractability of the estimation problem corresponding to the dynamic models [6, 14]. The real deck motion is sinusoidal, which motivates us to employ a weighted sum of several sinusoidal functions to approach real deck dynamics. The emerging PA provides a systematic means of analyzing oscillating power systems and receives extensive applications with the advancement of modern computational capacity [2, 18, 23]. This methodology can be extended to analyze dynamics of deck displacement.

PA was initially developed by Gaspard Riche Baron de Prony in 1795 to explain the expansion of various gases, and provides an effective way of extracting valuable information from a group of uniformly sampled data [17]. It adopts a series of damped complex exponentials to approximate system dynamics, which represents system information in terms of amplitude, frequency, phase, and damping components.

Suppose a continuous-time sequence $y(t)$ can be approximated by a weighted linear combination of exponential terms,

$$\hat{y}(t) = \sum_{i=1}^{n} B_i e^{\lambda_i t}, \tag{1}$$

where each complex residue B_i corresponds to its complex pole λ_i, where $i = 1, ..., n$ and model order is denoted by n. The proper identification of model parameters B_i, λ_i, and n enables the model to match the known measurements satisfactorily. Essentially, our objective is to determine residues B_i, poles λ_i, and the model order n, such that $\hat{y}(t)$ is the optimal approximation to the measurements $y(t)$ in the least square sense.

Practically, continuous-time data are sampled at a constant sampling period T_s. If data are sampled at $t = \tilde{k}T_s, \tilde{k} = 0, ..., N-1$, then the discrete-time form for Eq. (1) is

Fig. 1 RMAX helicopter approaching the moving deck simulator

$$\hat{y}(\tilde{k}T_s) = \sum_{i=1}^{n} B_i z_i^{\tilde{k}}, \quad (2)$$

$$z_i = e^{\lambda_i T_s}, \ \tilde{k} = 0, ..., N-1, \quad (3)$$

where the complex number z_i is termed the discrete-time system pole, and N is the number of measurements. For simplicity, let $k = \tilde{k}T_s$, then $\hat{y}(\tilde{k}T_s)$ will be replaced with $\hat{y}(k)$ in the following part of the paper.

System measurements $y(t)$ can be used to construct the Linear Prediction Model (LPM) [18]

$$y(k) = a_1 y(k-1) + \cdots + a_n y(k-n). \quad (4)$$

The traditional PA consists of three fundamental steps. The first step is to determine coefficients $a_i, \ i = 1, ..., n$ of the LPM in Eq. (4). This is paramount as the accurate estimation of residues and poles depends on the precision of these coefficients.

Normally, the length of measurements N should satisfy

$$N \geq 2n. \quad (5)$$

When $N = 2n$, coefficients a_i can be obtained by solving the LPM using the Least Square (LS) method. As there are more measurements than required when $N > 2n$, the Overdetermined Least Square (OLS) method can be employed.

In the second step, a matrix representation of sequential samples is constructed by expanding the LPM at various time instants, and coefficients a_i are acquired by inverting the matrix Q in Eq. (6)

$$D = QA, \quad (6)$$

$$D = [y(n), y(n+1), ..., y(N)]^T, \tag{7}$$

$$Q = \begin{bmatrix} y(n-1) & y(n-2) & \cdots & y(0) \\ y(n) & y(n-1) & \cdots & y(1) \\ \vdots & \vdots & \vdots & \vdots \\ y(N-1) & y(N-2) & \cdots & y(N-n) \end{bmatrix}, \tag{8}$$

$$A = [a_1, a_2, ..., a_n]^T. \tag{9}$$

The corresponding characteristic equation can be derived from coefficients a_i. From these coefficients damping factor and frequency can be acquired after zeros z_i are attained according to Eq. (10) through factorizing the following polynomial

$$z^n - a_1 z^{n-1} - \cdots - a_{n-1} z - a_n = \prod_{i=1}^{n} (1 - z \cdot z_i^{-1}). \tag{10}$$

Continuous-time pole λ_i can be accessed from discrete-time pole z_i. The zeros z_i appear only in the form of real numbers or complex conjugate pairs because a_i in Eq. (10) are real. Therefore, if z_i is completely real, then [10]

$$\lambda_i = \frac{\ln z_i}{T_s}. \tag{11}$$

Otherwise, if z_i is a complex conjugate pair,

$$\lambda_i = \alpha_i \pm j\beta_i, \tag{12}$$

$$\alpha_i = \frac{\ln |z_i|}{T_s}, \beta_i = \frac{1}{T_s} \tan^{-1}\{\frac{z_{Ii}}{z_{Ri}}\}, \tag{13}$$

where $z_i = z_{Ri} \pm j \cdot z_{Ii}$.

In the last step, the magnitude and phase are obtained through solving the following linear algebra equation

$$Y = \Psi B, \tag{14}$$

$$Y = [y(0), y(1), ..., y(N-1)]^T, \tag{15}$$

$$\Psi = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1^1 & z_2^1 & \cdots & z_n^1 \\ \vdots & \vdots & \vdots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_n^{N-1} \end{bmatrix}, \tag{16}$$

$$B = [B_1, B_2, ..., B_n]^T. \tag{17}$$

Here, the Vandermonde matrix Ψ is constructed based on the zeros z_i of characteristic equation (10), and appears as a square matrix in the traditional PA. Normally, if zeros z_i of Eq. (10) appear in conjugate pairs, the corresponding B_i in Eq. (17) will also appear in conjugate forms.

The fundamental limitation of the traditional PA lies in inverting the long matrices Q in Eq. (8) and Ψ in Eq. (16) when more measurements are available. In our case, the estimation involves dealing with a large number of instantaneous measurements, which significantly exacerbates the difficulties in the real-time implementation of the traditional PA. Also, ill-conditioned matrices may occur when inverting the above-mentioned matrices, which would cause collapse of the traditional PA. Therefore, there is a need to modify the traditional PA to deal with our application.

4 A modified Prony Analysis

4.1 The proposed recursive Prony Analysis

In the considered application, characteristics of deck dynamics and landing objective make it necessary to modify the traditional PA to suit our application. The real displacement of deck motion reflects slow-varying features, which reveals there is a close relationship between the instantaneous mean position at adjacent sampling time windows. The standard LS exclusively deals with measurements for a specified time window separately, and starts estimation without consideration of previous window information. Therefore, it is likely that estimation of instantaneous mean position is subject to significant changes when successive data windows are processed. Therefore, the inaccurate estimation would cause intricate maneuvering of the RUAV to adapt to the estimated changes. Also, there are numerical problems when standard LS is employed. The manipulation of matrix inversion suffers from singularity issues, which makes it fail in our case. To remedy its weakness, previous window information should be supplemented to improve its numerical performance. Therefore, the following factors are significant in our case:

1. How to obtain accurate and reliable model parameters when new measurements are collected?

2. How to carry forward system information for successive data windows to achieve an accurate estimation?

3. How to reduce computational burden to accomplish a rapid online estimation of instantaneous mean position to reduce hover period of the RUAV?

A possible solution to the first question is to employ the Recursive Least Square (RLS) method which aims to estimate model coefficients such that the summation of squared errors between measured values and estimated values reaches a minimum. Apparently, all measurements are treated equally in the loss function, and the RLS averages the measurements to produce the optimal estimates [30]. In our application, where plenty of measurements are collected while the RUAV is hovering over the landing deck, the variations would be submerged when old data and new data are equally weighted. Consequently, the accumulated estimation error would increase significantly, and the estimation course is possibly subject to collapse when a substantial number of measurements are accumulated and processed.

For a dynamic system with parameters varying continuously and slowly, it is desired to introduce the conception of forgetting factor to gradually discard effect of old measurements, and highlight contributions of most recent measurements to dynamic variations [30]. Moreover, the deck displacement excited by sea-wave is not a stationary process, and it requires the employment of the Forgetting Factor Recursive Least Square (FFRLS) [16].

To implement the FFRLS, the vector of lagged input-output data

$$\varphi(t) = [y(t-1), ..., y(t-n)]^T \tag{18}$$

and coefficient vector

$$\hat{\theta}(t) = [\hat{a}_1(t), ..., \hat{a}_n(t)]^T \tag{19}$$

up to time instant t are introduced. Coefficients $\hat{a}_1(t), ..., \hat{a}_n(t)$ will be updated recursively to approach the real values $a_1, ..., a_n$. The LPM can be written in a more compact form

$$y(t) = \hat{\theta}^T(t)\varphi(t), \tag{20}$$

where $\hat{\theta}(t)$ contains the coefficients to be determined. The loss function for FFRLS is defined as [12],[30]

$$V(\hat{\theta}) = \sum_{j=1}^{t} \gamma^{t-j}[y(j) - \hat{\theta}^T(j)\varphi(j)]^2. \tag{21}$$

Here, forgetting factor is denoted by parameter γ. The principle to choose γ is to select γ such that the loss function $V(\hat{\theta})$ essentially contains those measurements mostly relevant for current properties of the dynamic system. In particular, for a system that varies gradually, forgetting factor can be set to a constant value ranging between 0.98 and 0.995. [12]

Following the definitions, model coefficients $\hat{\theta}(t)$ can be estimated using the FFRLS, and expressed as [22]

$$\hat{\theta}(t) = [\sum_{j=1}^{t} \gamma^{t-j}\varphi(j)\varphi^T(j)]^{-1}[\sum_{j=1}^{t} \gamma^{t-j}\varphi(j)y(j)]. \tag{22}$$

Alternatively, the FFRLS can be implemented recursively by

$$\hat{\theta}(t+1) = \hat{\theta}(t) + K(t+1)[y(t+1) - \varphi^T(t+1)\hat{\theta}(t)], \tag{23}$$

$$K(t+1) = P(t)\varphi(t+1)[\gamma + \varphi^T(t+1)P(t)\varphi(t+1)]^{-1}, \tag{24}$$

$$P(t+1) = [P(t) - K(t+1)\varphi^T(t+1)P(t)]/\gamma, \tag{25}$$

$$\hat{\theta}(0) = 0, \ \ P(0) = \alpha I. \tag{26}$$

Here, matrix $P(t+1)$ is referred to as error covariance matrix, matrix $K(t+1)$ denotes the updating matrix, and α is a large positive number.

Regarding the second question, the error covariance $P(t)$ and model coefficients $\hat{\theta}(t)$ are initialized once for the first data window, then the FFRLS carries them forward as the sliding window moves to the next one. This implies the model coefficient vector $\hat{\theta}(t)$ is slow-varying, and its components for adjacent data windows are closely related. Therefore, error covariance matrix $P(t)$ and estimation vector $\hat{\theta}(t)$ carry forward system information to improve estimation performance.

The third step of PA can be followed according to Eq. (14)-(17) once zeros of characteristic equation are found. Similarly, the number of measurements is more than that of the coefficients to be estimated, and RLS can be used to estimate the magnitude B_i. However, the vector of lagged input-output data $\varphi(t)$ in Eq. (18) is replaced with

$$\rho(t) = [z_1^{t-1}, ..., z_n^{t-1}]^T, \tag{27}$$

which corresponds to row components in Eq. (16). It should be noticed that the vector $\rho(t)$ at different time instants t varies significantly. Therefore, carrying forwards error covariance matrix and estimation vector is not proper in this step, as vector $\rho(t)$ for different time instants are not slow-varying. Indeed, simulation results show that the estimation performance decreases if error covariance matrix and estimation vector are carried forward owing to the fact that there are actually large discrepancies between the error covariance and model coefficients at different time instants.

4.2 Determination of model order

Regarding the third question, computational burden is significantly affected by the choice of model order n. Some available information criteria are Akaike Information Criterion (AIC) [1] and its variant Final Prediction Error (FPE) [27], Bayes Information Criterion (BIC) [20], i.e.,

$$AIC(n) = \log \sigma^2 + \frac{2n}{N}, \tag{28}$$

$$BIC(n) = \log \sigma^2 + \frac{n \log N}{N}, \tag{29}$$

$$FPE(n) = \frac{N+n+1}{N-n-1}\sigma^2, \tag{30}$$

where summed squared error (SSE)

$$\sigma^2 = \sum_{k=0}^{N-1} [y(k) - \hat{y}(k)]^2. \tag{31}$$

AIC and BIC aim to make a trade-off between estimation errors accumulated and model complexity, and the optimal order is determined when they appear in a convex trend and reach the minimum. However, in our case, AIC and BIC consistently decrease when model order becomes large, and the estimation performance does not deteriorate. Underlying this fact is that the extra exponential terms are actually trying to fit the noisy signal more accurately. Therefore, a Prony model with order much larger than the true order is still an option for estimation purpose. However, extremely large model order causes consumption of numerous computer memory allocation, and makes it difficult for real-time applications.

The choice of the optimal model order is subjective according to various scenarios [27]. Practically, the model order should be selected such that a trade-off can be achieved between estimation accuracy and computational burden. In our case, FPE is a feasible option to choose the model order. Since sliding widow length N is much larger than model order n in our case, the proper model order can be found out only by checking the SSE

$$\sigma^2 = \sum_{k=0}^{N-1} [y(k) - \sum_{i=1}^{n} B_i e^{\lambda_i k}]^2. \tag{32}$$

The estimation performance using the SSE σ^2 is close to the best available approaches which are based on maximum likelihood or on the use of eigenvector or singular value decompositions [11].

The order selection procedure consists of two steps [11]:

1. Set the predicted model order R to be larger than the maximum number of model order expected. This results in a set of R exponentials which are candidates of the Prony model;

2. Out of the R exponential functions, determine the best subset of size n^* such that the linear combination of n^* exponentials best approximates the measurements in the least square sense.

A feasible way to complete step 2 is to check the SSE for different order candidates until the rate of decrease of the SSE is small. The order candidate at which SSE shows the significant drop in rate of decrease is taken as the proper estimation of model order n^*. It is noticed that rate of change of SSE reduces slightly when order is larger than n^*. Meanwhile, as the order larger than n^* is also a possible choice, it is preferred to select small order when the curve-fitting accuracy is satisfied. By doing this, a proper trade-off can be made between estimation accuracy and computational burden.

5 Dominant components selection criterion

The proposed recursive PA procedure produces a group of poles after being applied to analyze real deck motion data. Since the coefficients of characteristic equation are real values, its zeros z_i appear as real values and conjugate pairs, which results in residues and poles also taking the form of real and/or conjugate pairs. In our project which is aimed at a successful landing, selecting the dominant residues is crucial. Here, the arithmetic mean is not preferred as it does not indicate monotonous deck trend. Practically, the dominant residues in coefficient vector B reveal the slow-varying monotonous tendency of mean deck displacement, and can be used to monitor and track variations of deck displacement.

For these λ_i with large negative real parts, the transient responses will quickly converge to zeros as the exponential amplitudes go to zeros rapidly. Therefore, the dominant residues usually correspond to a subset of poles with the negative real parts very close to imaginary axis. In the considered application, how to choose the dominant residues is a problem. Specifically, since the absolute values of real parts of the poles denote the distance between the poles and imaginary axis, then how far away from the imaginary axis can be defined as dominant components should be considered.

The proposed selection criterion is to define a box threshold to determine the dominant poles. The coefficients B_i with respect to the poles within the threshold are taken as dominant residues. It is likely that the conjugate pole pairs also appear within the threshold, and they should be considered. The box threshold is chosen according to the following criterion, as is shown in Fig. 2:

1. Choose the pole with its negative real part closest to the imaginary axis, which corresponds to the smallest horizontal distance d;

2. The threshold L_p is 5 times of the horizontal distance d.

3. The width of the rectangle W_p depends on the magnitude of rounding errors, which takes a very small value $(O(e^{-8}))$.

Poles with horizontal distance less than L_p are considered to be dominant. The closest distance d can be found out immediately following the proposed recursive PA.

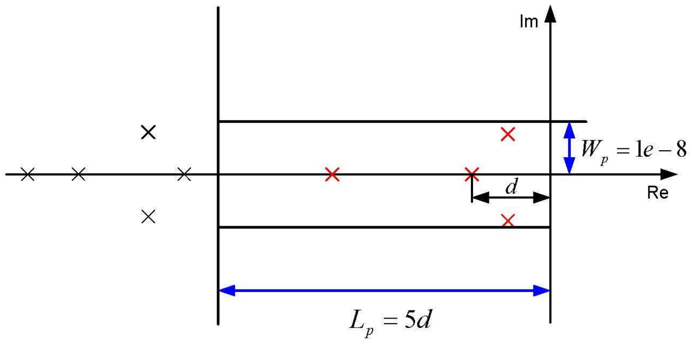

Fig. 2 Threshold to choose dominant poles

Here, the threshold L_p is chosen from the viewpoint of reducing the order of a high-order dynamic system [15]. In practice, poles of a dynamic system will be considered to determine system response when their negative real parts are within 5 times of the smallest real part. It should be noticed that when determining the dominant poles, the PA sometimes identify them with the complex parts very close to real axis (not appear in conjugate forms), this results from the rounding errors of the computer calculation. These poles should be included as dominant poles. In the considered application, $W_p = 1e^{-8}$ is a proper threshold. Once dominant poles are picked up, summation of the corresponding residues represents monotonous trend of deck motion.

6 Simulation results and analysis

6.1 Model modes identification for noise-free signal

In this section, performance of the proposed estimation procedure is investigated for the purpose of eventual applications to real scenarios. A 5^{th}-order damping system is constructed with known model modes: $\lambda_i = -1.5, \ -3 \pm j4, \ -3.5 \pm j4.5$. The data generated by the known dynamic system are employed to evaluate how well the proposed procedure is able to extract model modes.

The proposed estimation procedure is carried out for the noise-free data generated by the known model. Sliding windows are constructed for FFRLS implementations. The proper model order is sought to be identified by minimizing the SSE. As is shown in Fig. 3, SSE takes the value of $1.7647e^{-4}$ when model order $n = 4$, and SSE $= 1.2991e^{-12}$ when $n = 5$. SSE is found to be around $O(e^{-15})$ when a larger order is selected, and there is no significant decrease in SSE when model order increases. Therefore, model order $n = 5$ can be effectively identified by evaluating the SSE.

To verify the efficiency of estimating model poles using the proposed procedure, 30 groups of data are generated with 10 samples in each group. Figures 4-6 show the distributions of poles for different model modes, in which the estimated poles are very close to the real ones. Besides, the average of the estimated poles are accurate, and corresponding standard deviations are remarkably small. It is seen that the suggested PA is capable of estimating system poles with a high accuracy for noise-free data.

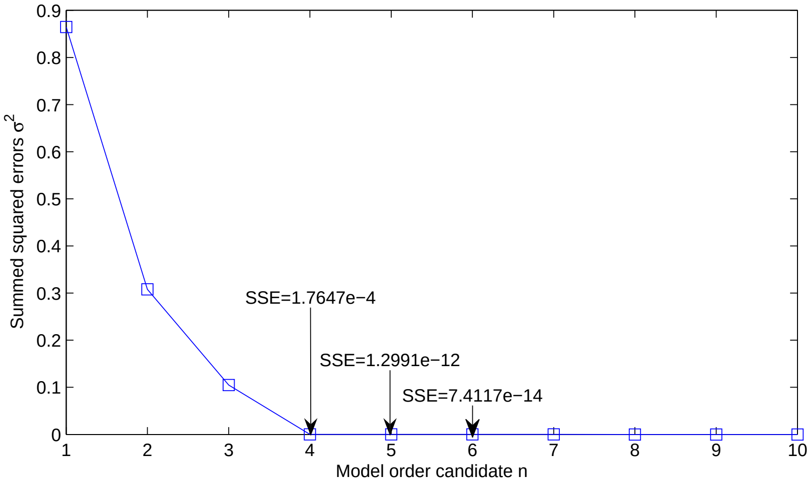

Fig. 3 Summed squared errors versus model order candidates

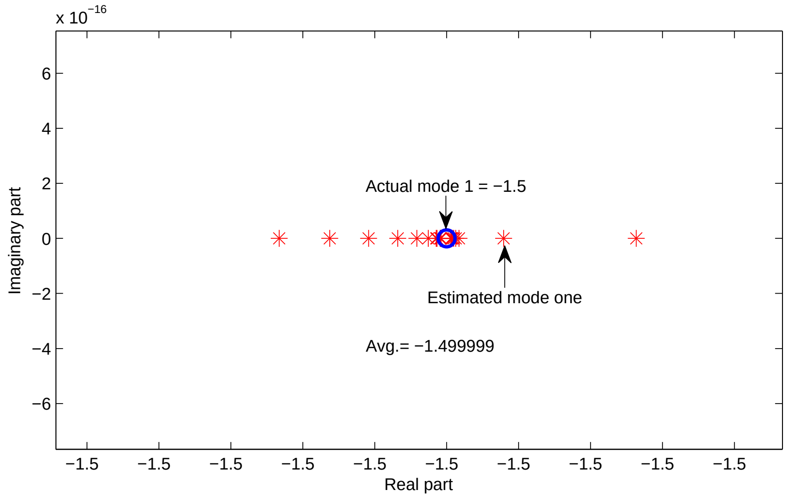

Fig. 4 Estimation of the first mode

Measurement noise is an inevitable factor significantly affecting estimation performance of PA in the real applications [27]. In our case, a Kalman filter (KF) has been designed to give a smooth estimation of deck position. The KF attenuates the noise effect greatly, and makes the proposed PA work well for deck motion contaminated by sensor noise.

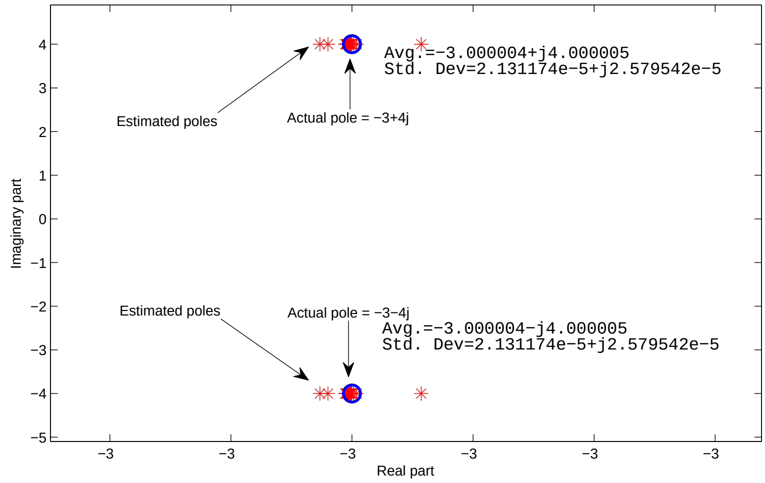

Fig. 5 Estimation of the second mode

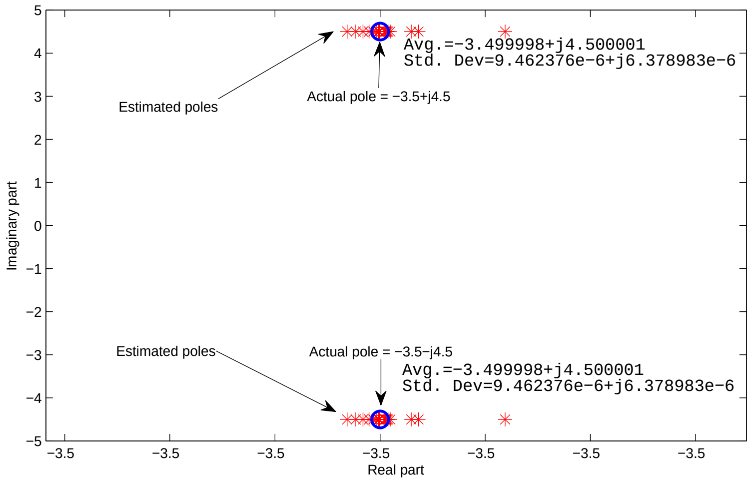

Fig. 6 Estimation of the third mode

6.2 Extracting instantaneous mean position of real deck displacement

In this section, we aim to extract the mean position of real deck displacement for which the model dynamics are unknown. The real data of deck displacement motion were collected by the onboard inertial measurement unit for ANZAC warship operating in a harsh sea environment. The ANZAC ship is able to embark a multi-role Sikorsky S-70B-2 Seahawk helicopter. Therefore, ship motion data collected from ANZAC are

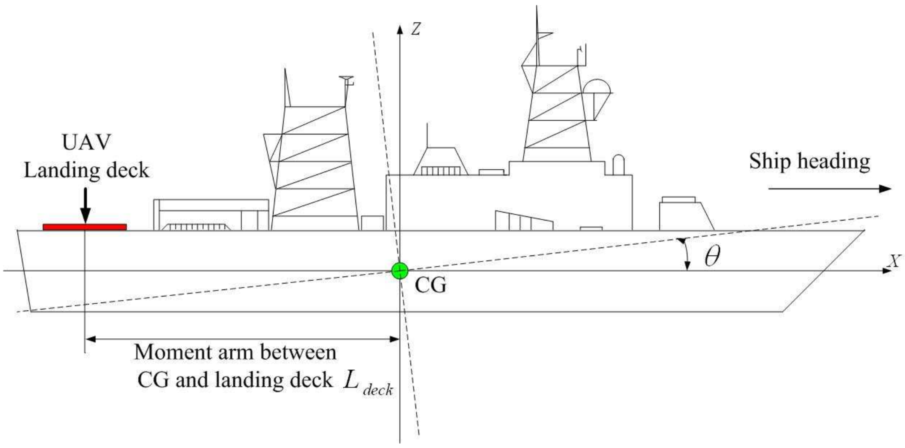

Fig. 7 Geometry configuration of the landing deck

representative and provide insight into displacement motion of the landing deck. Pitch motion η at the Center of Gravity (CG) of the ship, collected every 0.1s, is multiplied by the moment arm $L_{deck} = 67.7m$ to produce the local deck motion, as depicted in Fig. 7. Here, the deck displacement is expressed as

$$Z_{deck} = Z_{CG} + R_{3\times3} \begin{bmatrix} 0 \\ 0 \\ L_{deck} \end{bmatrix}, \tag{33}$$

with rotation matrix

$$R = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\psi + c_\phi s_\theta c_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix}, \tag{34}$$

where $c_{(\cdot)} = \cos(\cdot)$ and $s_{(\cdot)} = \sin(\cdot)$, Z_{CG} is heave at the CG. Here, pitch motion is denoted by θ, yaw motion ψ, and roll motion ϕ. It can be noted that small ship displacement at the CG will result in significant deck displacement.

For landing operations, our objective is to estimate the instantaneous mean position accurately and rapidly. Therefore, adequate length of deck displacement data should be collected firstly. To identify the model order more reliably, the RUAV is supposed to hover for some time, then displacement data collected during this period are used to identify model order. Two groups of displacement data with length of 2000 samples are tested. It is seen from Fig. 8 that there is a significant drop in SSE when model order is 13, and the SSE would decrease only slightly when model order is larger. Therefore, model order is chosen to be 13 such that not only the Prony model can match measurements accurately, but also the recursive procedure is easy to be implemented with reduced computational burden. Generally speaking, it is found out that model order $n = 13$ is suitable for most of the ship motion data.

Once the optimal order is determined, poles and residues are to be estimated using the proposed procedure, then the dominant residues will be sought according to the criterion described before. The instantaneous means are given in Fig. 9 and Fig. 11 for

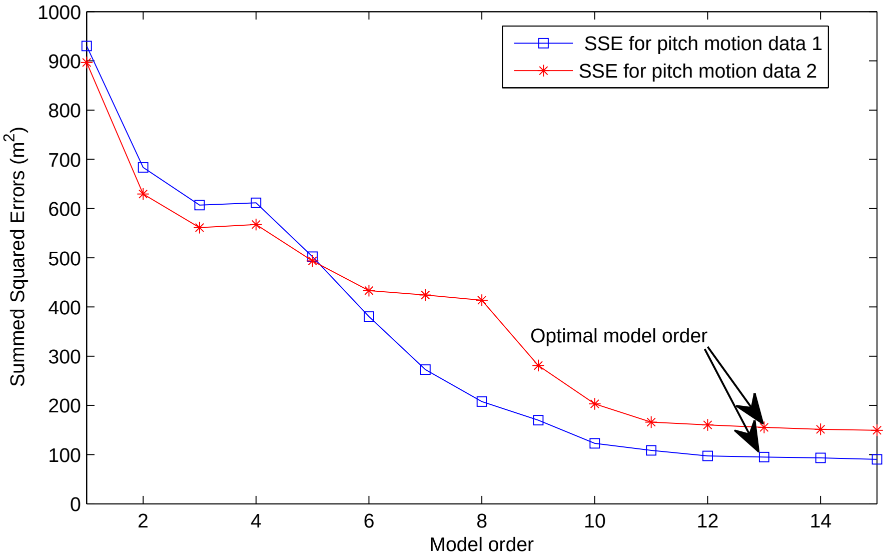

Fig. 8 Summed squared errors for different model orders

two different groups of real deck motion data (cyan curves). Also, the estimated deck displacement using the proposed PA is shown on the same graphs (blue curves), it is seen that data produced by the Prony model match the measurements well. The standard deviations are 2.95cm and 3.51cm for Fig. 9 and Fig. 11. To determine the best time to start PA, estimation results using different data length are compared. As shown in Fig. 10 and Fig. 12, the green curves are the estimated means when measurements are collected until $30s$ (300 samples), cyan curves correspond to the estimated instantaneous means when measurements are collected until $60s$ (600 samples), and pink curves $90s$ (900 samples). Practically, the RUAV is supposed to hover 30s-2min before landing operation is triggered. It is seen that there are oscillations evident when starting estimation from 30 seconds, and estimation performance can be improved noticeably after the RUAV hovers above the deck for 1 minute. However, there is no significant enhancement for a longer hover period (90s). Therefore, it takes 60 seconds to obtain the accurate estimation of instantaneous means in the considered applications.

Of particular interest here is the fact that the proposed procedure performs satisfactorily for slow-varying dynamic systems due to the advantage of carrying forward the error covariance which enables transmission of system information.

7 Conclusion and future work

In this paper we concentrate on building a proper procedure for estimating the instantaneous mean position of deck displacement. A modified version of PA was proposed with model order identified by minimizing squared estimation errors and model coefficients determined using the FFRLS. Also, the dominant residues were found out

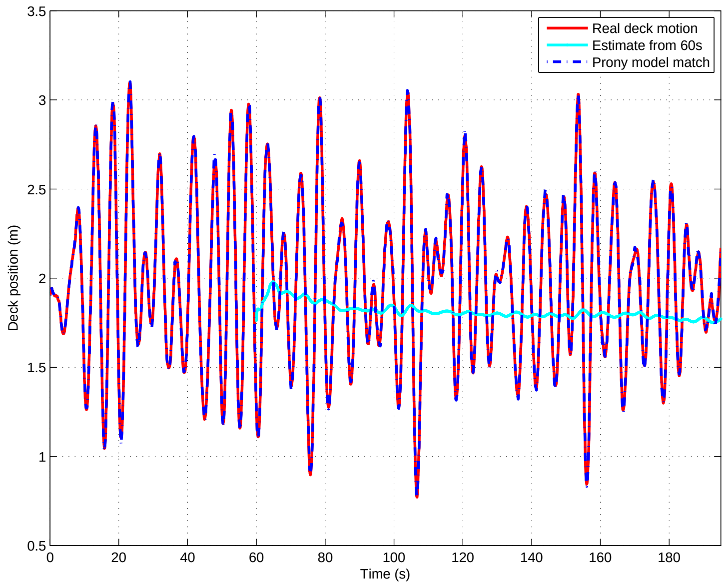

Fig. 9 Extracting monotonous trend of real deck displacement (group 1)

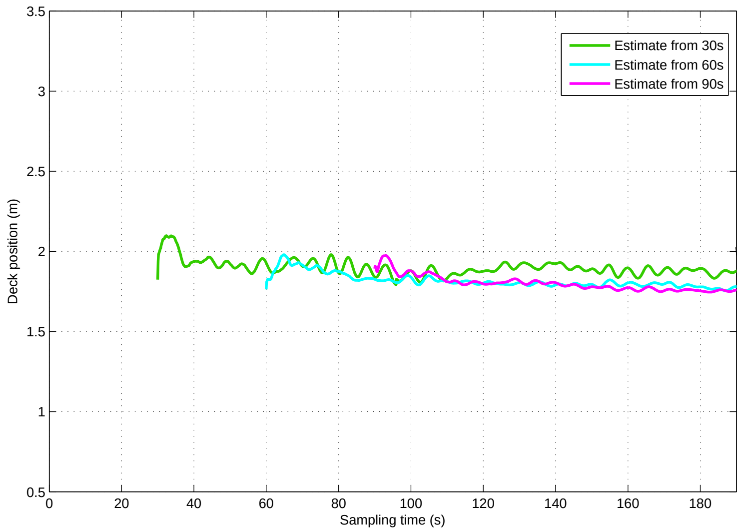

Fig. 10 Comparison of estimation results using different data length (group 1)

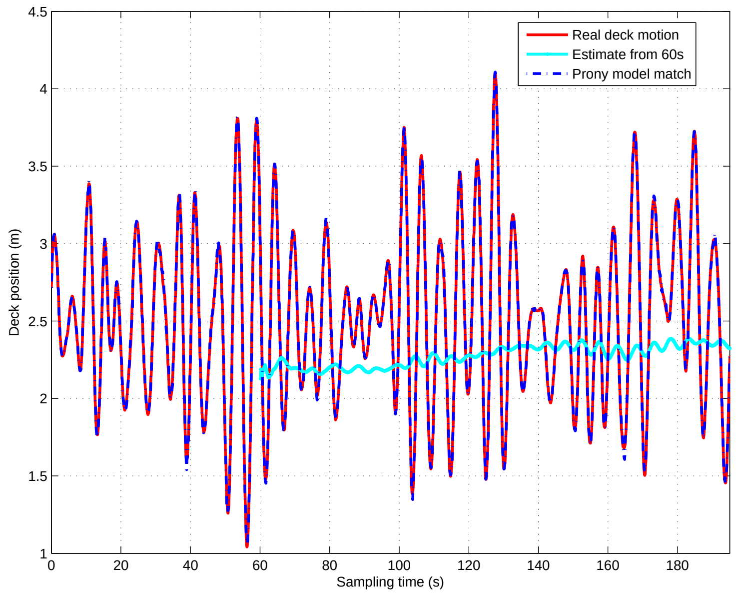

Fig. 11 Extracting monotonous trend of real deck displacement (group 2)

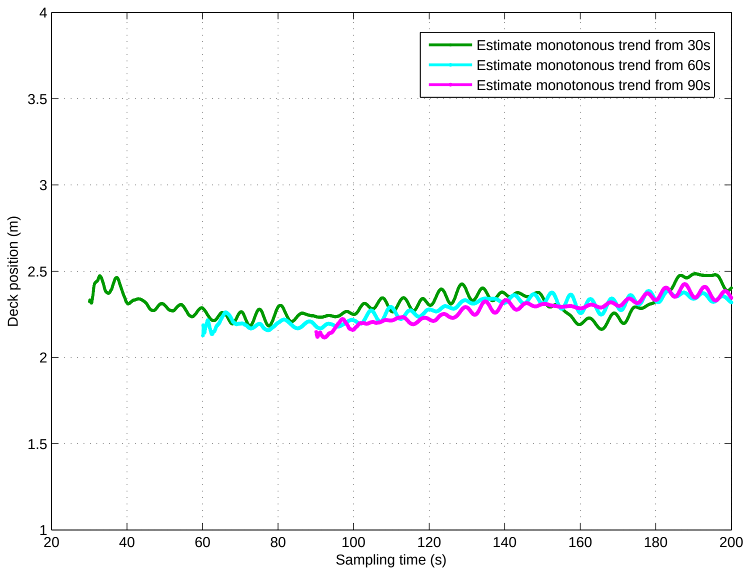

Fig. 12 Comparison of estimation results using different data length (group 2)

based on a threshold selection criterion. Simulation results justify the suitability of our procedure for analyzing real ship motion data. The proposed estimation procedure has been tested using real deck displacement data in the simulation. Work is in progress to apply our methodology to extract the tendency of the moving deck simulator.

The estimation efficiency of the proposed PA can be enhanced if smoother measurements are available. In real-time applications, noisy deck motion measurements can be filtered using the KF before being processed by the proposed PA.

8 ACKNOWLEDGEMENTS

We would like to thank the Australian Defence Science and Technology Organization for providing the ship motion data used for testing our methodology.

References

1. Akaike, H.: A new look at the statistical model identification. IEEE Transactions on Automatic Control **AC-19**(6), 716–723 (1974)
2. Bracale, A., Caramia, P., Carpinelli, G.: Adaptive prony method for waveform distortion detection in power systems. International Journal of Electrical Power and Energy Systems **29**(5), 371–379 (2007)
3. Carriere, R., Moses, R.L.: High resolution radar target modeling using a modified prony estimator. IEEE Transactions on Antennas and Propagation **40**(1), 13–18 (1992)
4. Davies, P.: A recursive approach to prony parameter estimation. Journal of Sound and Vibration **89**(4), 571–583 (1993)
5. Garratt, M.A., Pota, H.R., Lambert, A., Maslin, S.E.: Systems for automated launch and recovery of an unmanned aerial vehicle from ships at sea. Proceedings of the 22nd International UAV Systems Conference pp. 1401–1415 (2007)
6. Goshtasby, A.A.: Fitting parametric curves to dense and noisy points. In: 4th International Conference on Curves and Surfaces. St. Malo, France (1999)
7. Hauer, J.: The use of prony analysis to determine modal content and equivalent models for measured power system response. In: IEEE Symposium on Eigenanalysis and Frequency Domain Methods for System Dynamic Performance, pp. 105–115. IEEE Publication 90 TH0292-3-PWR (1989)
8. Hauer, J.F.: Application of prony analysis to the determination of modal content and equivalent models for measured system response. IEEE Transactions on Power Systems **6**, 1062–1068 (1991)
9. Hauer, J.F., Demeure, C.J., Scharf, L.L.: Initial results in prony analysis of power system response signals. IEEE Transactions on Power Systems **5**(1), 80–89 (1990)
10. Johnson, M.A., Zarafonitis, I.P., Calligaris, M.: Prony analysis anad power system stability some recent theoretical and applications research. In: Power Engineering Society Summer Meeting, 2000. IEEE, vol. 3, pp. 1918–1923 (2000)
11. Kumaresan, R., Tufts, D.W., Scharf, L.L.: A prony method for noisy data: Choosing the signal components and selecting the order in exponential signal models. In: Proceedings of the IEEE, pp. 230–233 (1984)
12. Ljung, L.: System Identification: Theory for the User. Prentice-Hall (1987)
13. Lorsaiswami, R., Liu, W.: Real-time estimation of the parameters of power system small signal oscillations. Wireless Personal Communications **8**(1), 74–83 (93)
14. Moussa, M.A.A., Cheema, M.Y.: Non-parametric regression in curve fitting. Journal of the Royal Statistical Society. Series D (The Statistician) **41**(2), 209–225 (1992)
15. Ogata, K.: Modern Control Engineering, 3 edn. Prentice Hall (1997)
16. Park, D.J., Jun, B.E., Kim, J.H.: Fast tracking rls algorithm using novel variable forgetting factor with unity zone. Electronics Letters **23**(23) (1991)
17. Prony, G.R.B.: Essai experimental et analytique des vibrations libres amorties. J. de l'Ecole Polytech **1**, 24–76 (1795)

18. Qi, L., Quan, L., Woodruff, S., Cartes, D.: Prony analysis for power system transient harmonics. EURASIP Journal of Advances in Signal Processing **2007**, 1–12 (2007)
19. Sacchini, J.J., Steedly, W.M., Moses, R.L.: Full polarization two-dimensional prony modeling with application to radar target identification. In: Automatic Object Recognition III, vol. Vol. 1960, pp. 125–139. SPIE (1993)
20. Schwarz, G.: Estimating the dimension of a model. Annals of Statistics (6), 461–464 (1978)
21. Smith, J., Fatehi, F., Woods, S., Hauer, J., Trudnowski, D.: Transfer function identification in power system application. IEEE Transactions on Power Systems **8**(3) (1993)
22. Soderstrom, T., Stoica, P.: System Identification. Prentice-Hall (1989)
23. Tawfik, M.M., Morcos, M.M.: Prony application for locating faults on loop systems. IEEE Power Engineering Review pp. 41–43 (2001)
24. Therrien, C.W., Velasco, C.H.: An iterative prony method for arma signal modeling. IEEE Transactions on Signal Processing **43**(1), 358–361 (1995)
25. Toffoletto, R., Reddy, R., Lewis, J.: Effect of ship frontal shape variation on flow field in flight deck region. In: Proceedings of the 2nd International Conference on Computational Fluid Dynamics, ICCFD. Sydney, Australia (2002)
26. Triantafyllou, M.S., Bodson, M., Athans, M.: Real time estimation of ship motions using kalman filtering techniques. IEEE Journal of Oceanic Engineering **OE-8**(1), 9–20 (1983)
27. Trudnowski, D.J.: Characteristics of identifying linear dynamic models from impulse response data using prony analysis. Tech. rep., Pacific Northwest Laboratory (1992)
28. Trudnowski, D.J., Donnelly, M.K., Hauer, J.F.: Estimating damping effectiveness of **BPA**'s thyristor controlled series capacitor by applying time and frequency domain methods to measured response. IEEE Transactions on Power Systems **11**(2), 761–766 (1996)
29. Trudnowski, D.J., Johnson, J.M., Hauer, J.F.: Making prony analysis more accurate using multiple signals. IEEE Transactions on Power Systems **14**(1), 226–231 (1999)
30. Vahidi, A., Stefanopoulou, A., Peng, H.: Recursive least squares with forgetting for online estimation of vehicle mass and road grade: Theory and experiments. Vehicle System Dynamics **43**(1), 31–55 (2005)
31. Viti, V., Petrucci, C., Barone, P.: Prony methods in nmr spectroscopy. International Journal of Imaging Systems and Technology **8**(6), 565–571 (1998)