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"*Entry Barriers, Limit Pricing, and Incomplete Information*"

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To my family,

Contents

Introduction

The aim of this work is to describe some of the major contributions to the theory of oligopoly with free entry, tracing the evolutionary phases and clarifying the interconnections between different models. In particular we will first analyze the starting contributions of the 1950's which gave rise to the so-called *limit pricing theory*; then we will introduce the role of capacity in deterring entry; finally we will analyze a model with incomplete information in which both established and potential firms are not fully informed about others' cost functions.

Oligopoly is a market structure characterized by the fact that industry is dominated by a small number of firms, each one possessing *market power*, i.e. the ability to alter profitably prices away from competitive levels. In such situation, the action of one firm can alter not only its own profitability, but also the one of the others and vice-versa. Therefore, competition among firms in oligopolistic market is inherently a setting of strategic interaction: each firm recognizes that its profit depends not only on its own action, but also on the action of the others. For this reason, the appropriate tool for the analysis of competition among oligopolistic firms is game theory. And indeed the models we will analyze are essentially games, each one characterized by a set of players, a structure, and one or many equilibria.

In the games we are going to analyze, competition takes place between actual (or established, or incumbent) firms and potential (or prospective entrant) ones. For the sake of simplicity we will assume the existence of just one established firm and one prospective entrant. Such simplification does not alter the basic results of the models: one might consider one established firm as collusive cooperation among oligopolistic firms; on the opposite side, n identical prospective entrant firms can be represented just as one firm, supplying n times the amount of output.

Since entry takes time, these games are characterized by many stages. At the first stage, established firms recognize that new firms want to enter the market (i.e. they recognize the *threat of entry*); hence they can try to discourage the entry of new firms by affecting their profitability perception of industry. At the second stage, prospective entrant firms make their entry decision observing the action of the first stage. Finally, if they decide to enter, firms compete under some rule.

This is the basic structure of static models, which will be subjects of our analysis; however, since the 1970's many authors examined the problem of entry in oligopolistic markets in a dynamic setting, exactly because entry is matter of time. The most famous contributions in this context are the ones of Wenders (1970), Kamien and Schwartz (1971) and Gaskins (1971).

For what concerns the rule of post-entry game, one might assume a quantity-setting oligopoly (Cournot game), a price setting oligopoly (Bertrand – Edgeworth game) or, why not, a product differentiation oligopoly (Hotelling game). Here we will deal with models in which firms compete *à la* Cournot. Hence, oligopolistic firms are supposed to manage the quantity of the good produced and sold, with the price determined by the aggregate output of industry. Another assumption is that both actual and potential firms produce homogeneous goods; that is the goods produced by firms are assumed to be *perfect substitutes*.

The effect of product differentiation on the possibilities of entry in oligopolistic markets is a matter that deserves a careful and separate analysis. Here we would like to emphasize two opposite observations: the one, according to Bain, for which product differentiation implies a *barrier* to entry in the sense that the producer of one brand cannot replicate another brand without incurring a disadvantage in either cost of sales; the other, according to Dixit, for which entry prevention is more difficult when products are poorer substitutes, since differentiation reduces the effect of output policy of one firm on others' profits; in the limiting case where products are perceived completely different by buyers the action of one firm would have no effect on the others.

According to game theory approach, the solutions of the models take the form of *Nash equilibria*. A *Nash equilibrium* is defined as a situation in which no one can do better by unilaterally changing his or her strategy; where a strategy for each player is defined as the set of action chosen at each stage (i.e. at each information set) of the game. In our context, a Nash equilibrium will be a situation where no firm can increase its profit by unilaterally changing its output policy. Moreover, as the games we are going to analyze are typically composed by many stages, for the models with complete information we will find Nash equilibria of each game using *backward induction*. However, when we will analyze the model with incomplete information, where players are not fully informed about some feature of the game, then *backward induction* will not allow us to any equilibrium. Indeed in case of uncertainty, players rely on their *beliefs* and the right solution concept to adopt will be the *perfect Bayes-Nash equilibrium*.

The notion of *entry barrier*, almost whispered above, has a precise meaning and a very special role in oligopoly theory: its determination defines the starting point for the study on the implication of entry in oligopolistic markets. In such situation, established firms can *strategically* erect entry barriers by lowering their price, with the purpose to deter entry, if they expect that the entry of a new firm will reduce substantially their profits: this is the essence of the *limit pricing theory*.

Such theory - born in the 1950's with the contributions of Bain, Sylos Labini and Modigliani - arose with the purpose to explain why monopolists or oligopolists with effective collusion on price, set prices lower than the profit-maximizing level at which marginal revenue equals marginal cost. According to Bain, Sylos Labini and Modigliani, established firms lowered their price in order to reduce the demand faced by prospective entrant firms (i.e. the *residual* demand). The assumption behind such intuition consisted in the fact that the established firms, facing the threat of entry, would have maintained fixed the level of output. Such assumption - named Sylos' postulate - was really restrictive, since in some cases an accommodating reduction of output in face of new entry could have generated greater profits than the ones resulting from the sustaining of the fixed amount of output. Such clever considerations remained without a satisfactory answer until the contribution of Dixit in the 1980's who completed and clarified the basic intuition behind limit pricing theory introducing capacity variable. Through an irreversible pre-commitment investment on capacity at the stage when entry is threatened, incumbent firms are able to give credibility to their behavior of sustaining the amount of output consistent with low prices. After Dixit's contribution the theory acquired consistency, with the consequent desertion of Sylos' postulate.

Another way for justifying the behavior of setting low prices in order to deter entry - without assuming Sylos' postulate - was provided by Milgrom and Roberts two years later Dixit's work. In a context where both actual and potential firms ignore rival's constant marginal cost, they demonstrated that established firms might set prices lower than the profit-maximizing one in order to fool the entrants, signaling a high degree of competitiveness that they really do not have. Such degree of competitiveness is enclosed in the value of firms' marginal cost, which in turn affect the amount of output and price level. Such strategy, as we will see, can constitute an equilibrium strategy for incumbent firms, in perfect harmony with limit pricing theory approach.

Milgrom and Roberts' strengthened the theory of limit pricing by which established firms can strategically erect barrier to entry by lowering their price, extending such behavior also in a context of incomplete information. The analysis of their contribution concludes this thesis.

Chapter 1

GENESIS OF ENTRY BARRIER LITERATURE

1.1 Introduction

The starting point for the study of *entry barriers* dates back to the year 1956 when two authors, Joe Bain and Paolo Sylos Labini, published "Barriers to new competition" and "Oligopolio e progresso tecnologico" respectively. Some year later (1958), Franco Modigliani formalized the basic results of Bain-Sylos' analysis in his brilliant paper "New development on the oligopoly front", which can be rightly considered a fundamental work in entry-barriers literature which was taken in consideration by future authors who faced the problem of entry in oligopolistic markets.

Precisely in Bain's work we can find the first notion of *entry barrier* in economic literature, defined as "an advantage of established sellers over potential entrant sellers, which is reflected in the extent to which established sellers can persistently raise their prices above competitive levels without attracting new firms to enter the industry" (Bain 1956, p. 3). Afterwards, with the consequent development of the research on *entry barriers*, other authors proposed different definitions. Indeed Stigler (1968, p. 67) offered an alternative definition based on cost asymmetries between incumbent and entrant firms, qualifying "entry barriers as cost that must be borne by a firm that seeks to enter an industry but is not borne by firms already in the industry". According to this approach Von Weizsacker (1980, p. 400) later emphasized welfare effects of such asymmetries, which should imply also "a distortion in the allocation of resources from the social point of view".

By those definitions, *entry barriers* arise whenever established firms possess an advantage, or, according to Modigliani, a *premium* over prospective entrant firms; such *premium*, according to Bain, allows incumbent firms to sustain a price greater than average total cost (which includes the normal remuneration from business investment) without inducing entry; on the other hand Stigler pointed out the existence of a *competitive advantage* over the entrants in terms of additional costs that potential firms have to pay if they want to come into a market characterized by *entry barriers*. Surely there is a common factor in both definitions; that is, entry is harder in markets with *entry barriers*.

The fact that entry may indeed be *free* - which means by definition that *there are no physical, monopolistic, or financial obstacles to prevent the entry of new capacity* - does not necessarily means also that it is *easy*, as Edwards (1955, p. 95) pointed out:

> "Consider first the case of Perfect Competition. Here indeed the presumption is that there is both freedom and ease of entry. The problems confronting the prospective new entrant, therefore, are of a different order as compared with the case of perfect competition. The new entrant in manufacturing industry has not only to establish his production unit; *he has also to acquire the essential connexion in the market,* which is the *condition sine qua non* of his efficient and continuing manufacture. In this sense entry, while free, is not easy."[1]

However Edwards was far away from giving a definition of *entry barriers*. His considerations should be viewed in the context of criticism in accordance with Harrod's critique of the "accepted doctrine" of imperfect competition for which conditions of imperfect competition with free entry regularly give rise to creation of *excess capacity*.

This leads us even further back in time, around the 1930's, and inside the debate of imperfect competition with free entry started with the two books of Chamberlin (1933) and Robinson (1933), afterwards developed in a critical way by Kaldor (1935).

Now, the basic result of the theory of imperfect competition with free entry is the following: under certain assumptions, long-run equilibrium is such that price equals the long-run average cost of industry. In this context there are two opposite views: in the first, supported by Kaldor, and in perfect harmony with Chamberlin and Robinson, entry in imperfect competitive markets induces inefficiencies and the arising of *excess capacity* in long-run equilibrium; the second, proposed by Harrod (1952), according to which

[1] In particular Edwards referred to the *goodwill business* acquired by established firms with their clients, meaning that buyers, especially of intermediate product, tend to look first to their customary suppliers because of the confidence gained by previous customs. As a consequence, buyers might take time to change their suppliers (even offering the product at a low cost), and in order to reduce this time-lag entrant firm may have to incur in high advertising expenditures. In this context the entrant is forced to sustain in additional costs, and this reminds to the definition of *entry barriers* given by Stigler years later.

established firms plan to charge a price yielding only normal profit in order to limit the entry of new firms, without creating *excess capacity*.

The recognition that established firms could react to potential competitors by lowering their prices was crucial in order to emphasize the relation between conditions of entry and equilibrium price of industries. The study of such relation arose in the 1950's from the contributions of Bain (1949, 1956), Harrod (1952), Edwards (1955), Lydall (1955)[2], Sylos Labini (1957)[3] and Modigliani (1958): all of them, in different ways, focused on the definition of the entry-deterring equilibrium price level.

However the works of Bain, Sylos Labini and Modigliani were a breaking point with the previous literature. Indeed Harrod, but also Edwards and Lydall, demonstrated that the level of price that did not induce the entry of new firms should have been equal to the minimum value of long-run average cost function; on the contrary Bain, Sylos Labini and Modigliani provided the possibility of long-run equilibrium where price were charged above long-run average total cost, and where established firms could earn extra-profits without inducing entry[4].

The works of Bain, Sylos Labini and Modigliani were quoted jointly by the subsequent authors who faced the problem of *entry barriers* as the Bain-Sylos Labini-Modigliani model (B-S-M model): it represented the first step towards the application of game theory and strategic behavior to oligopoly with free entry, giving rise to the *limit pricing theory*.[5]

In their critical review to Modigliani's paper, Farrar and Phillips (1959) and Fisher (1959) emphasized the analogy between B-S-M model and classical Cournot model; in particular Fisher (1959, p. 413) noted that: "the true analogy, of course, is with a Cournot "leadership" model, with all firms already in the industry playing the collective leader to the potential entrant's follower." Essentially Fisher referred to the Stackelberg (1934) oligopoly model, and he was right. In fact Osborne (1973) later demonstrated that the B-

[2] Lydall (1955) introduced the concept of "no-entry price ceiling" as "the maximum price which can be charged without provoking new entry", corresponding to the level at which, under the hypothesis of firms with identical cost curves, price "lies at the minimum point of each firm's long-run average cost (including normal profits) curve."

[3] Here we refer to the first Italian edition of Sylos Labini's work; however, a first provisional version dates back to 1956. This last version was later taken into account by Modigliani (Cfr. Rancan 2012).

[4] Recalling the definition suggested by Bain, *entry barriers* arise precisely from the "extent to which established sellers can persistently raise their prices above competitive levels without attracting new firms to enter the industry".

[5] The name derives from the concept of *limit price* introduced by Bain (1949). *Limit pricing* is a strategy that consists in setting a price low enough to make entry unprofitable (i.e. the *limit price*). For a clear explanation of *limit pricing* strategy Cfr. 1.4; for the definition of *limit price* Cfr. 1.3.2.

S-M model gives rise exactly to the Stackelberg equilibrium, at least when entry effectively occurs.

Hence, there is a double connection between the microeconomic studies of the 1930's and the emerging theory of *entry barriers* of the 1950's. First, there exists a link with the theory of imperfect competition through Harrod's principles, as Osborne (1973, p. 71) pointed out:

> "The theory of limit pricing had its genesis in a suggestion of Harrod that firms in monopolistically competitive markets might not reach the tangency solution of Chamberlin (with its implied excess capacity) but might rather set a price lower than the tangency price so as to discourage the entrance of new competitors".

Second, the B-S-M model gives rise to the equilibrium provided by Stackelberg, with incumbents acting as the *leader* firms, and the potential entrants as the *follower* firms.

In this chapter first we will describe the debate around the theory of imperfect competition; then we will show Modigliani's synthesis of Bain-Sylos Labini analysis, with an extension proposed by Bhagwati (1970) who included the *goodwill business* introduced by Edwards; finally we will expose the B-S-M model as a duopoly model, and the reaching of Stackelberg equilibrium.

1.2 Entry in the theory of Imperfect Competition

1.2.1 Equilibrium and Excess Capacity

Perfectly competitive markets are characterized by the following basic assumptions: firms produce homogeneous goods, i.e. the goods are perfect substitutes each other; firms are *price-takers* since they act as an infinitesimal element in relation to the industry; the market is *open,* without any entry or exit costs; there is perfect information in the economy. Hence, in perfect competitive markets, the demand curve facing each firm is horizontal, i.e. elasticity of every individual demand is infinite.

On the contrary, the starting point in the theory of Imperfect Competition is the proposition that firms are confronted with a downward sloping demand curve. Its development dates back to 1933 when E.H. Chamberlin and J. Robinson published respectively "The theory of Monopolistic Competition" and "The theory of Imperfect Competition", and it has been subsequently developed critically by N. Kaldor (1935).

The theory is based on four basic assumptions. First, it is assumed that there is a large number of independent firms that produce and sell only one product which is, in Kaldor's words, *slightly different* from the products of other firms; that is, firms' products are *sufficiently* different each other but at the same time similar *enough* to be considered substitutes, but not *perfect* substitutes. This implies that a firm, lowering the price of its own good, will attract *some* but not *all* others' costumers.[6] Hence each firm faces an individual demand related to its differentiated product.

Second, it is assumed that consumers' preferences are fairly evenly distributed among different varieties; and since there is a large number of firms, "any adjustment of price or of product by a single producer spreads its influence over so many of his competitors that the impact felt by any one is negligible and does not lead him to any readjustment of his own situation" (Chamberlin 1933, p. 83).

Third, the long-run cost curves of all producers are assumed to be falling up to a certain rate of output; in other words, it is assumed that up to a certain output, there are "economies of scale". Moreover the elasticities of demand curves and the cost curves of each producer are also assumed to be the same.

Finally, it is assumed that no producer possesses an *institutional monopoly* over any of the varieties produced (in forms of licenses or patents), and thus the entry of new producers into the field in general and every portion of it is free and unimpeded.

Thus, we have a market with many firms; each one is characterized by the same cost function and faces an individual demand curve, due to the possibility to produce *slightly different* goods; moreover individual demands have the same elasticities. In order to maximize its profit, each firm produces the quantity consistent with the intersection between marginal revenue and marginal cost curves; and since marginal revenue curve is lower than individual demand, each firm earns extra-profit, i.e. profit above the normal

[6] Technically this means that the cross-elasticity of demand of the goods, i.e. the elasticity of demand of one good with respect to others' price variation is not infinite.

level. Such extra-profits will attract new firms into the market. The increased supply thereby generated tends to reduce the price at which each and every firm can market a given volume of output of this kind; the particular demand curve confronting each such firm is pushed to the left and its marginal revenue curve too; this process continues until surplus profits are eliminated and with it the stimulus of new entrants. In this equilibrium position each firm, or perhaps we should say, the representative firm, makes no more than normal profit. In the positions of final equilibrium, not only marginal cost will be equal to marginal revenue, but average cost will also be equal to price, that is demand curve will be tangential to the cost curve. This process is described in figure 1.

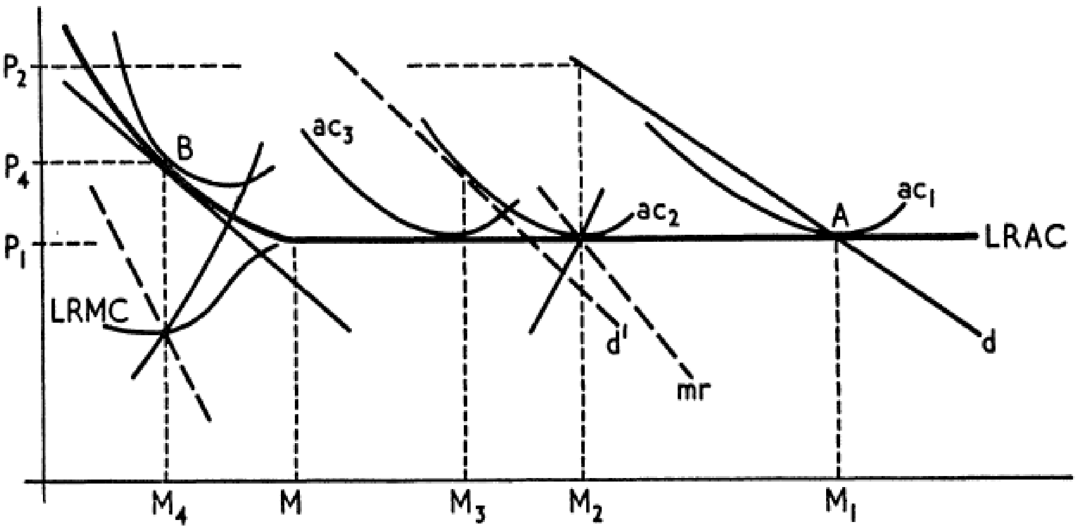

FIG 1 – The adjustment process under imperfect competition. (Source: Edwards 1955, p. 103)

Suppose that, in the initial position there is only one firm in the market, facing demand curve d. It produces the quantity M_2, at which marginal revenue equals marginal cost, and charges the price P_2, earning extra-profit. Such situation attracts new firms in the market; with the entry of new competitors, demand curve faced by established firm shifts to the left, say at d'. In such situation firms still earn extra-profits, and new firms will enter the market. Hence the equilibrium situation will be at B (the so called *tangency solution*), where price level is P_4, individual demand is tangent to long-run average cost curve, and marginal revenue equals marginal cost. During the process, firms act as short-run profit maximizers and accommodate entry of new firms. Moreover, at the final equilibrium, prices remain higher than the perfect competitive level, i.e. higher than the minimum value of long-run average cost, and production is inefficient. Indeed firms are not producing in an efficient way, since their scale of production is lower than the one consistent with minimum of long-run average cost function, that is M_1. Hence, *excess capacity* is observed in long-run equilibrium.

Now, one might first argue on the validity of the basic assumptions stated above, in particular their correspondence to the real world, as Kaldor himself did[7]. In this context his considerations on the *divisibility* assumption are of great interest. Indeed in the theory of imperfect competition, it is implicitly assumed that capital is perfectly divisible; that is, production plant can always be reduced when a reduction of output is demanded. Removing this assumption means that a certain scale of production is required in order to recover fixed cost of the *indivisible* plant. In this case, there is no reason to assume that it will stop precisely at the point where the demand and cost curves are tangent, as Kaldor (1935, p. 42) pointed out:

> "On account of the very reason of economies of scale, the potential producer cannot hope to enter the field profitably with less than a certain magnitude of output; thus the additional output may reduce demand, both to his nearest neighbors and to him, to such an extent that the demand curves will lie below the cost curves and all will be involved in losses. *The same reason therefore which prevents competition from becoming perfect, i.e. indivisible, will also prevent the complete elimination of profits*. It will secure a "monopolistic advantage" to anybody who is first in the field and merely by virtue of priority. Hence, indivisibility of capital acts as a protective shield against introducers."

[7] He argued for example that, even if it is quite reasonable to assume that the products in an industry are *slightly different*, there is no reason why the cross-elasticity of demands should be equal over the different products. This is of great importance for reaching final equilibrium, since in this case *adjustment of price or of "product" by a single producer will spread its influence evenly over all his competitors*. However Kaldor (1935, pp. 38-39) stressed the fact that

> "Pseudo-monopolists thus cannot be grouped together in a lump but can at best be placed into a series. Each product can be conceived as occupying a certain position on a scale; the scale being so constructed that those products are neighboring each other between which the consumers' elasticity of substitution is the greatest. Each producer then is faced on each side with his nearest rivals; the demand for his own product will be most sensitive with respect to the prices of these; less and less sensitive as one moves further away from him."

These considerations sound like a premonition of what Bain-Modigliani provided twenty years later, that is entry barriers emerge in industries characterized by economies of scale.[8]

1.2.2 Harrod's Critics

Given the assumptions, the theory has been considered unassailable until the contribution of R.F. Harrod, "The theory of imperfect competition revised", published in *"Economic Essays"* in 1952. He fully criticized the process according to which firms behave *naively*, maximizing their profits regardless of the possibility of entry of new firms with, as final result, the creation of *excess capacity*.

First, Harrod discussed an implicit assumption of the theory concerning fixed costs. Indeed, it was assumed that fixed equipment is an unalterable datum, having been created by each firm *once and for all in a remote past*, when the present was largely uncertain. On the contrary, Harrod pointed out that typically fixed equipment have been created step by step: at every stage, careful considerations were required in order to evaluate benefits and costs of an investment. And in this context, surely firms would consider the possibility of entry of new firms.

Now recall figure 1 and assume that firms invest in fixed capital as Harrod described. Here the problem is the following: if the firms foresee the trend of events, i.e. that new firms will enter in the market and that the final long-run equilibrium will be at B, why not plan to have a plant that will produce output level M_4 most cheaply, rather than incur in excess capacity? But in that case (as shown in the figure by the low long-run marginal cost curve), firms will continue to earn extra-profits; then B will no more be an equilibrium, new firms will enter in the market, and a new equilibrium will be reached; the output of each firm will reduce with the creation of excess capacity (as before).

On the other side, Harrod emphasized the fact that established firms typically are not willing to sacrifice markets available to them for the sake of fleeting extra-profit, since "such sacrifice will tend to make them weaker in facing the various contingencies of an

[8] Cfr. Par. 1.3.

unforeseeable future". Such considerations acquire consistency on the assumption that firms are profit-maximizers in Robinson's sense:

> "the most valid simple generalisation is that the aim of the entrepreneur is for the firm first to survive, and secondly to growth. To this end he must pursue profit, but he must avoid action which, though profitable in the present, will damage his future position[…]". (Robinson 1933, p. 582)

If firms behave as described by Chamberlin and Kaldor, they *pursue profits, but they do not avoid action which will damage their future position*. Hence, established firms might apply an output/price policy in order to discourage entry, securing their *future position*. So long as established firms set prices higher than long-run average cost, the resulting extra-profits will attract new firms in the market. Thus, in order to limit entry, established firm must charge a price according to which they will earn only normal profits:

> "The conclusion is that, if there is free or relatively free entry, the entrepreneur […] will plan to charge a price yielding only normal profit, save to the extent that he is aware of possessing an advantage peculiar to himself, will plan to have equipment on a scale that gives the lowest cost for producing what he can sell at such a price, and, having acquired the equipment, will sell at that price, even although the short-period marginal revenue yielded by such a policy is less than the marginal cost. The equipment required for this policy will be larger and the price charged lower, and nearer, the social optimum that those entailed by "accepted doctrine". (Harrod 1952, p. 151)

Therefore, unless established firm has a cost advantage over potential competitors, equilibrium described by Harrod is point A in figure 1. In such situation, established firm earns just normal-profit, since price equal average cost, without attracting new firms in the market. Production is efficient and equilibrium price is lower than the one proposed by Chamberlin and Kaldor.

The theory of imperfect competition remains, anyway, controversial. On one hand it says essentially that free entry, which typically means more competition, induces inefficient production and high prices; on the other hand, if such entry is impeded by established firm, i.e. the industry remains highly concentrated, then price is low and production is efficient. Hence entry is supposed to have a clear positive effect in the market only by the influence to established firms that are induced to reduce the price, at least as long as they believe in *Harrod's principles*.

However, Harrod has been the first in providing an output/price policy intentionally directed to discourage entry of new firms and consistent with the objective of firms to be profit-maximizing. A strategy, this one, that can be considered the genesis of *limit pricing* strategy started with Bain-Sylos Labini-Modigliani's model.

1.3 Modigliani's Synthesis

1.3.1 Entry and Sylos' Postulate

The entry of new firms occurs when potential entrants expect to earn nonnegative profits once they will join industry competition[9]. Firms' profits are nonnegative if equilibrium price level is greater than or equal to their average total costs. Therefore potential entrant firms will enter the market if they expect that equilibrium price level *after* entry - i.e. the price generated by the increased aggregate output - will be higher than or at least equal to their average total cost. Hence established price level, i.e. the price policy actually set by established firms, has no meaning for them, since "even if the pre-entry price is above the lowest achievable cost, the additional output proposed to sell by the entrant may drive price below cost, making entry unprofitable" (Modigliani, 1958).

The price *after* entry is determined both by the entry decisions of potential firms and by the reaction of incumbents in response to the threat of entry. Thus, the effect of entry on equilibrium price level is the result of the interactions between existing firms and potential ones; and such interactions could also leave equilibrium price unchanged. Indeed consider the case of an industry with homogeneous firms, i.e. firms with the same technology, which compete on quantities facing a downward sloping aggregate demand curve. If the additional output of entrant firm reduces the price below the lowest cost achievable in the industry, then established firms may react to the threat of entry contracting the quantity

[9] Indeed if potential entrant can earn extra-profit or normal profit: in this sense costs include the normal remuneration for shareholder as an opportunity cost. Moreover it is assumed that there are no other possibilities of investment for potential entrant firm that could earn positive profits.

produced and sold: in the limiting case the aggregate quantity supplied may be the same and the price may be completely unaffected.

In order to solve this *impasse* Modigliani proposed an assumption - the so called Sylos' postulate - for which *potential entrants behave as though they expected existing firms to adopt the policy of maintaining output while reducing the price*, in order to find a well-defined solution to the problem of long-run price and output under homogeneous oligopoly with possibility of entry[10]. Under Sylos' postulate pre-entry price has a commitment value due to the fact that established firms are supposed to sustain the fixed level of production. Hence pre-entry price assumes a relevance for prospective entrant firms, since they know with certainty that equilibrium price level *after* entry will be lower than pre-entry price due to the increased aggregate output. The postulate was very powerful and it was always assumed until Dixit's work in 1980.

Modigliani's analysis can be divided into two parts: in the first one the author followed Bain's approach considering the case of an industry in which firms must produce a quantity greater than a certain amount in order to make positive profits; while, in the second part, Modigliani left such assumption providing the possibility for firms to be profitable at any quantity of output greater than zero. We will analyze the market for a homogeneous good, thus it is assumed that goods produced by both actual and potential firms are perfect substitutes; moreover Sylos' postulate applies.

1.3.2 The Model with Minimum Profitable Scale

Let us assume for simplicity that there is one established firm already operating in a market as a monopolist and one prospective entrant firm. Firms are homogeneous and produce goods that are *perfect substitutes*.[11] Moreover firms are characterized by convex long-run average cost functions, i.e. there exists a scale of output which is optimal in the sense that it is consistent with the minimum point of long-run average cost curves. Formally:

[10] "If the firm or firms that are in a position to set the price aim to prevent the entry of new firms of a certain type, they must keep the price to a level below that which ensures to such firms the minimum rate of profit" (Sylos Labini, 1956)

[11] Hence this is essentially a pure duopoly.

$$\exists \bar{x} > 0 \mid LRAC(\bar{x}) < LRAC(x), \qquad \forall x \neq \bar{x}$$

Where x is the output level and $LRAC(x)$ is long-run average cost function.
Let $X = D(P)$ be the negatively sloped aggregate demand function for the homogeneous good and $P(X) \equiv D^{-1}(X)$ the corresponding inverse aggregate demand function. Define P' as the equilibrium pre-entry price and $X' = D(P')$ the aggregate demand satisfied by monopolistic firm. By the existence of Sylos' postulate, established firm's output is assumed to be fixed for every amount of output supplied by the potential entrant; therefore entrant firm must deal with the *residual* demand curve, that is with the *segment of aggregate demand curve to the right of* P'. If it enters and supplies a certain positive quantity of output x_e, then aggregate supply becomes $X' + x_e$ and equilibrium price decreases: potential entrant will come into the market only if $P(X' + x_e) \geq LRAC(x_e)$.
Every pre-entry price that does not induce entry is said to be an *entry-preventing price*. In this context, if pre-entry price P' is such that the corresponding marginal demand curve is everywhere below entrant firm's long-run average total cost. Thus P' is an *entry-preventing price* since entry is unprofitable, as there does not exist any $x_e > 0$ such that $P(X' + x_e) \geq LRAC(x_e)$.

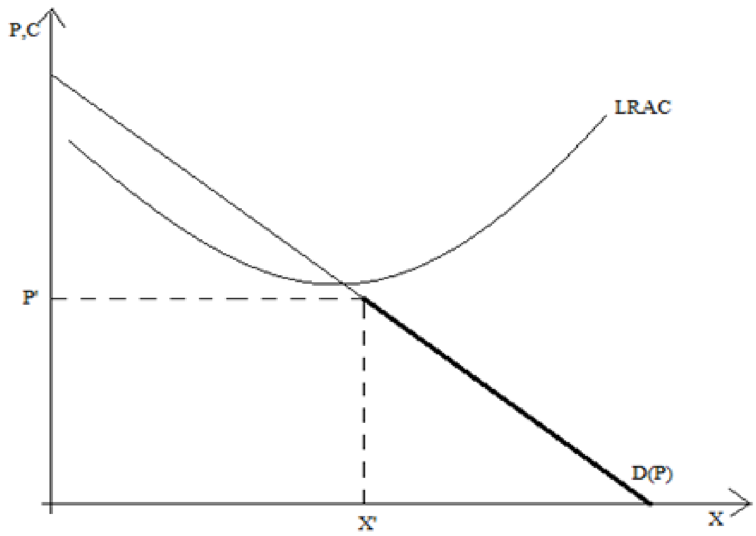

FIG 2 – An example of pre-entry price which is also an entry-preventing price

Under the definition of entry barriers proposed by Bain (1956), established firms' advantage over prospective entrants is reflected in the *extent* to which it is possible to sustain prices above competitive levels without inducing entry; hence, in order to define such *extent*, we have to focus on the *highest* entry-preventing price, i.e. the so-called *limit* *price*.[12] Let P_0 be such critical price and $X_0 = D(P_0)$ the corresponding aggregate demand (i.e. the *lowest* entry-preventing output): the heart of Modigliani's analysis consisted in finding the factors that determine P_0.

In order to explore those factors controlling P_0, denote with $\bar{x}$ the optimum scale of firms' output, that is the scale of output consistent with the lowest point of the long-run average cost, and let $\bar{c}$ be the corresponding minimum level of average cost. Denoting with P_c the perfect competitive price, in perfect competition we have $P_c = \bar{c}$ and the corresponding output $X_c = D(P_c) = D(\bar{c})$.

Modigliani defined the size of market S as the ratio between competitive output and optimum scale:

$$S = \frac{X_c}{\bar{x}} \qquad [1.1]$$

Following Bain's analysis, Modigliani considered the case where industry's technology is such that, if firms produce at scale lower than firms' optimum scale $\bar{x}$, they are not able to earn positive profits[13]; thus, entrant firm can only supply at a scale equal to or larger than $\bar{x}$. In this case entry is profitable if the aggregate output when entrant firm supplies the minimum profitable output level is no greater than perfect competitive output, i.e. if $X' + \bar{x} \leq X_c$. Hence any $\tilde{X}$ such that $\tilde{X} + \bar{x} > X_c$ is an entry-preventing output. Therefore, critical output level X_0 must be such that if a firm enters the market supplying the minimum profitable scale $\bar{x}$, then aggregate output equals perfect competitive output, i.e. $X_0 + \bar{x} = X_c$: if $X' < X_0$ then entry is attractive since $X' + \bar{x} < X_c$, while if $X' > X_0$ then entry is unattractive since $X' + \bar{x} > X_c$. Hence we get

$$X_0 = X_c - \bar{x} = X_c\left(1 - \frac{\bar{x}}{X_c}\right) = X_c\left(1 - \frac{1}{S}\right). \qquad [1.2]$$

Generally speaking, the higher is the value of X_0, the greater is the set of pre-entry output levels that make entry attractive. But X_0 depends negatively on the level of $\bar{x}$, i.e. on minimum level of profitable production scale. This means that entry is more difficult in an industry characterized by high fixed costs, in which economies of scale are incisive in

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[12] Bain (1949) first introduced the concept of *limit price*, defined as "the highest common price which the established sellers believe they can charge without inducing at least one increment to entry – presumably a significant lump increment".

[13] It can happen, for instance, in presence of sunk cost.

the sense that firms need to produce a large amount of output in order to be profitable. Thus, the incisiveness of economies of scale reduces the possibility to entry in the market. Looking at [1.2] we find the relation between critical aggregate output and size of the market. Specifically, X_0 is positively influenced by S,[14] hence the size of the market influences positively the possibility to entry in the market.
Alternatively, Modigliani introduced the following (approximated) relation between P_0 and P_c in terms of the elasticity of demand in the neighborhood of P_c:

$$P_0 \simeq P_c\left(1+\frac{1}{\eta S}\right), \qquad [1.3]$$

from which we can see the negative relation between critical price P_0 and elasticity of demand; thus elasticity of demand increases the possibility to entry in the market.[15]

1.3.3 The general case

After that, Modigliani replaced the very special cost function assumed so far with the more conventional one, falling more or less gradually up to $\bar{x}$, and providing the possibility for a firm to be profitable at a scale of production lower than $\bar{x}$. In this case, even at a level of output equal to X_0, it may be profitable for a prospective entrant firm to come into the market with a scale smaller than $\bar{x}$. This possibility and its implication can be analyzed with the following graphic apparatus proposed by the author.

[14] $X_0 = X_c\left(\frac{S-1}{S}\right)$ therefore $\frac{\partial X_0}{\partial S} = X_c\frac{1}{S^2} > 0$ thus X_0 is an increasing function of S.
[15] Quantity demanded is a negative function of price. Then if P_0decreases, X_0 raises, making entry easier.

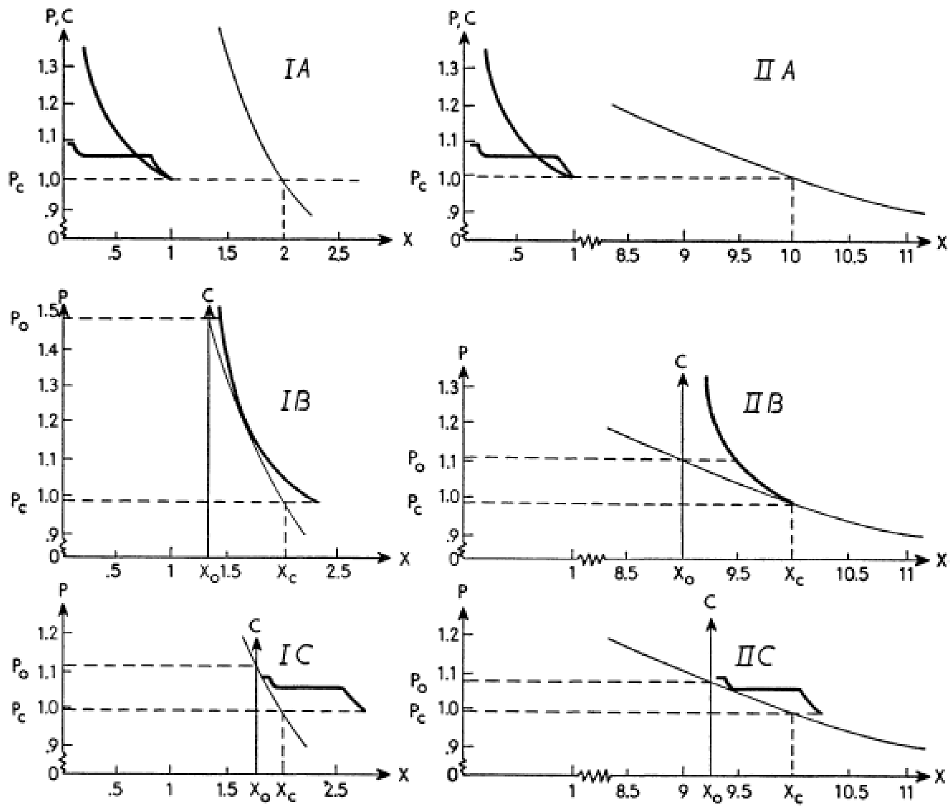

FIG 3 – Determination of P_0 with both different demand and cost functions.
(Source: Modigliani 1958, p. 219)

In panels IA and IIA, the light lines falling from the left to the right are two different market demand curves. For the sake of simplicity optimal scale output $\bar{x}$ and the corresponding minimum long-run average cost value $\bar{c}$ are respectively the unit of measurement of output X and price P. Therefore we have that $P_c = \bar{c} = 1$ and $S = X_c$ since $\bar{x} = 1$. Thus panel IA relates to an industry of size 2 while IIA to an industry of size 10.

The two heavy lines in each of the two panels represent alternative cost curves[16]. Because of the choice of unit, each curve represents long-run average cost curve in percentage of minimum level k as a function of output in percentage of optimum scale $\bar{x}$. Hence they describe the following relation:

$$\frac{LRAC}{\bar{c}} = f\left(\frac{X}{\bar{x}}\right). \quad [1.3]$$

[16] More specifically they are the branch of cost curves up to their minimum point.

The steeper of these two curves is the kind of cost function that underlies Bain's analysis and involves marked economies of scale[17]; while, the other one, involving less pronounced economies of scale, depicts the kind of cost function that underlies Sylos' numerical examples.

The critical price and output (P_0, X_0) can be found sliding the cost curve to the right parallel to itself, together with its co-ordinate axis, until no points of this curve lies inside demand curve. This step is illustrated in panels IB, IIB and IC, IIC for the two kinds of cost curves. The point at which the Y –axis so displaced cuts the demand curve represents P_0; the point at which it cuts the X –axis is X_0.[18] The portion of demand curve on the right of the axis so displaced is exactly the marginal demand curve faced by prospective entrant firm when pre-entry output is X_0.

Let us analyze the panels of the figure above. First, by comparing panel IB with IC and IIB with IIC we can see that, for a given market size, P_0 tends to be higher the steeper the cost curve, i.e. the greatest the economies of scale. Similarly, by comparing IB with IIB and IC with IIC, for a given cost curve and elasticity of demand, P_0 tends to fall with the size of the market; moreover, for a given size of the market, since a higher elasticity of demand implies a rotation counter-clockwise around competitive point, it appears that a higher elasticity acts in the same direction of market size, that is it will tend to lower P_0. Therefore, since a higher level of P_0 means a lower possibility to enter the market, also in absence of a minimum profitable scale, "there is a well-defined maximum premium that the oligopolist can command over the competitive price, and this premium tends to increase with the importance of economies of scale and to decrease with the size of the market and the elasticity of demand" (Modigliani, 1958).

[17] In the sense that the amount of output produced has a significant effect on reducing long-run average cost.

[18] Indeed imagine to slide cost curve a little bit more on the right; then, no points of long-run average cost curve lies below demand. Therefore there will be no possibilities of positive profits for new entrant.

1.3.4 An Extension

Thanks to Bhagwati (1970) we can extend the relation written above following Andrews' approach through Edwards' formal representation. As before, it is assumed that entrant firm faces a marginal demand function as described above, that there exists a minimum profitable scale $\bar{x}$ as in the first part of Modigliani's analysis, but it is added a central Andrews' assumption: costumers of the existing firms do not switch to the entrant firm's good except in pursuit of a price advantage. In this way the Bhagwati put in another variable that is the degree of confidence gained by existing firms with respect to costumers[19]. This produces an increment in *oligopolist's premium* over entrant firm.

Thus, using the same notation as before, the entry-preventing price now is given by: [20]

$$P_0 = P_c\left[1 + \frac{\bar{x}}{X_c(\eta + \epsilon)}\right], \qquad [1.4]$$

where the new variable ϵ represents the elasticity, with respect to change in price, of the transfer of current buyers to the entrant. Since the derivative of P_0 defined in [1.4] with respect to ϵ is negative, when costumers are not so willing to change their suppliers - which mean a low value of ϵ - then critical pre-entry price P_0 rises, making entry harder. Moreover in his analysis Bhagwati went further, including the case of growing demand over time. He took the arguments proposed by Modigliani and Sylos Labini for which a growing demand (over time) puts a downward pressure on the premium that can be charged by established firm (because aggregate demand is now larger). And indeed the growth in aggregate demand can be interpreted as an increase in the size of the market since it consists in a right-side shift of demand curve. As a consequence X_c raises (since $P_c = \bar{c}$ does not change) and so, recalling that $S = \frac{X_c}{\bar{x}}$, market size increases[21].

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[19] "The buyers of the product(…)will tend to look first to their customary suppliers because of the confidence gained by previous custom, and it is from these firms that they will normally buy" (Edwards 1955)

[20] Generalizing for the case of many established firms we have $P_0 = P_c\left[1 + \frac{\bar{x}}{X_c\left(\frac{\eta}{N+1}+\epsilon\right)}\right]$. The number of firms already operating in the market influences positively the entry-preventing price, making entry harder.

[21] Bhagwati proposed another approach, including in the equation of critical entry-preventing price the growth rate of aggregate demand over time:

$$P_0 = P_c\left[1 + \frac{\bar{x} - k\lambda}{X_c\left(\frac{\eta}{N+1} + \epsilon\right)}\right]$$

1.4 Entry and Stackelberg Equilibrium

1.4.1 Introduction to the Model

Entry is attractive for potential firms if expected profits are greater or equal than zero, i.e. if price after entry is greater or equal than average total cost of entrant firm. Hence, established firms may decide to apply an output/price policy in order to make entry unprofitable, creating in this way entry barriers. In this context Salop (1979) proposed a distinction between:

a) *Innocent* entry-barriers, i.e., those barriers unintentionally erected by firms as a side effect of profit maximization;
b) *Strategic* entry-barriers, i.e., those barriers erected to reduce possibility of entry.

However Bain (1956) defined, years earlier, three kinds of price/output strategies that can be used by established oligopolistic firms in face of entry:

c) Entry may be *blockaded:* this is the case in which entry is impeded simply by a price/output policy coherent with profit maximization regardless of possibility of entry;
d) Entry may be *effectively impeded*: price/output policy is designed to make entry unprofitable;
e) Entry may be *ineffectively impeded*: price/output policy is not intended to discourage potential entrants.

Indeed, points a) and c) coincides: when established firms act as simple profit maximizers they may create unawarely a barrier to entry, i.e. potential entrant firms are not able to enter the market without the existence of any change in established firms' behaviors. While, point b) is the opposite case coherent with what proposed by Bain in points d) and e): established firms apply a strategy direct to impede entry, with or without success. In this case established firms are said to apply the *limit pricing* strategy, which consists in setting a price low enough to make entry is unprofitable: the so-called *limit price*.

Where λ is the growth of aggregate demand subsequent to entry, k the proportion thereof that accrues to the entrant, and $X_c$aggregate demand at competitive price after growth. The result is the same: through the increment of X_c and the negative element in the numerator, P_0 decreases.

What we should analyze now is when limit pricing strategy is a rational choice for established firm, i.e., when it is consistent with long-run profit maximization in an oligopolistic market of a homogeneous good.[22]

In order to do this, let us recall two basic assumptions already defined in Modigliani's synthesis: first, established firms and potential entrants seek maximum profits over the long-run; second, established firm's output is maintained fixed in face of new entry, letting the price fall and the market be ruined for all (Sylos' postulate). Both assumptions are common knowledge among firms. Moreover it is generally assumed that established firm has a first-moving advantage over any entrant firm:

> "A *pre-entry asymmetry advantage* arises from the fundamental pre-entry asymmetry between established firm and potential entrant. Before the entrant make his entry decision, the established firm has already committed resources. This prior existence gives first-move advantages is independent […] of the post-entry game that might ensue; even if the post-entry game is played according to Nash-Cournot or entrant-as-a-leader rules, the pre-entry leadership role always lies with established firm".
> (Salop, 1979, p. 335)

The presence of such asymmetries between oligopolistic firms is not new in the economic literature. Indeed H.F. von Stackelberg (1934) provided a model in which leader firms move first, enjoying such first-moving advantage, and then the follower firms move sequentially. But in Stackelberg's model, all firms are already in the market, and the problem of entry is not considered. By the way, once a first-moving advantage is recognized for established firms over prospective entrants, then Stackelberg equilibrium arises naturally in models of *entry barriers*, at least when entry is ineffectively impeded. But in this context established firms have another choice: that is to set limit price in order to deter entry. Therefore the incumbents must evaluate if the profits consistent with the limit price are greater or lower than the profits of leader firms in Stackelberg equilibrium. Another implicit assumption of the theory is that established firm recognizes the possibility of new entry and tries to determine if its profit will be grater if it deter or allow

[22] This analysis will follow Osborne's (1973) and Dixit's (1979) contributions.

entry; and if it determines that it is better to deter entry by limit pricing, then the price set at the blocking point will be kept it there presumably indefinitely.[23]

1.4.2 The model

Let us analyze entry barriers in a quantity-setting duopoly with one established firm, operating in the market as a monopolist, and one prospective entrant, both producing an homogeneous good; they face an aggregate demand function negatively sloped and they hold the same technology, i.e. they have the same cost functions. In particular, this is a two-stage game in which: at stage one (when there is the threat of entry), established firm chooses the amount of output; at stage two, potential entrant firm decides to enter or not if given established firm's output choice its own profit is non-negative. Define firm 1 the established firm and firm 2 prospective entrant firm.

Both oligopolistic firms want to maximize their long-run profits; hence they face the following maximization problem:

$$\max_{x_i} \pi_i(x_1, x_2) = P(X)x_i - C(x_i) \quad with\ i = 1,2 \qquad [1.5]$$

Where $P(X)$ is the inverse aggregate demand function with $X = x_1 + x_2$; $C(x_i)$ is the cost function assumed equal for both firms; and x_i is the output for the ith firm.

Firms' profits do not depend only on the amount produced by themselves, but also from that of the rival. Under Sylos' postulate, entrant firm takes established firm's output as a parameter on which it has no influence. But established firm knows that and, given knowledge of both entrant's cost function and market demand, it can predict potential rival's output for every level of output chosen by himself: that is, established firm realizes potential entrant's *reaction function*.

Reaction function is determined by the solution of the maximization problem written above; indeed for firm 2 we have:

[23] Someone may say, as Osborne did, that the more profitable strategy for established firm might be "to reduce the price when entry threat is imminent and raise it when the threat has subsided. This more obvious, more direct, and (if successful) more profitable way of dealing with entry threats is foreign to the point of view of the theory;[…] (firm) uses Sylos' postulate with reference to an unlimited time horizon".

$$\max_{x_2} \pi_2(x_1, x_2) = P(X)x_2 - C(x_2). \qquad [1.6]$$

By first order condition we get[24]:

$$\frac{\partial \pi_2}{\partial x_2} = P'(X)x_2 + P(X) - C'(x_2) = 0. \qquad [1.7]$$

Solving [1.7] with respect to x_2 we find the so-called *reaction function* for prospective entrant firm: it depicts how firm 2 will react given firm 1's output choice, i.e., the level of output that maximizes potential entrant firm's profit for every level of output chosen by established firm. Let us call this function $\phi_2(x_1)$; similarly for firm 1 we have its own reaction function $\phi_1(x_2)$.

Firm 2's reaction function is well-defined by established firm. Hence, under threat of entry, firm 1 must follow the following maximization problem:

$$\max_{x_1} \pi_1(x_1, \phi_2(x_1)) = P(x_1 + \phi_2(x_1))x_1 - C(x_1). \qquad [1.8]$$

The solution of this problem is the *Stackelberg point* i.e., the tangency point between firm 1's iso-profit curve and firm 2's reaction curve; this is shown in the figure below (Fig. 4).

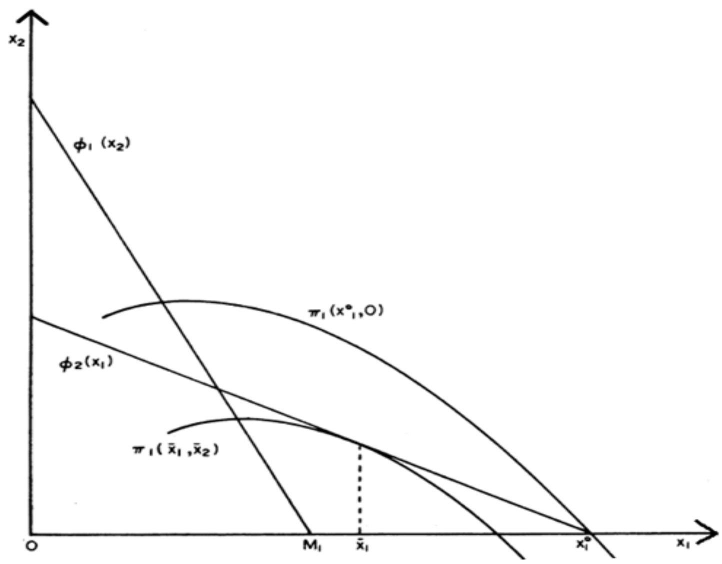

FIG 4 – Stackelberg equilibrium. (Source: Osborne, 1973)

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[24] Second order conditions are satisfied too; that is $\frac{\partial^2 \pi_2}{\partial^2 x_2} < 0$.

In the picture reaction functions are drawn for both established and potential firms[25]. Firm 1's iso-profit curves $\pi_1(x_1, x_2)$ describe the combination of x_1 and x_2 that generate the same level of profit for established firm: they are concave to x_1-axis and closer curves to the axis refers to greater level of profits. Firm 1's maximum profit occur at the point $(M_1, 0)$ where its reaction function intersects x_1-axis: this is the monopolistic behavior. But given the threat of entry and under Sylos' postulate, established firm's maximum profit occurs at the tangency point between its iso-profit curve and prospective entrant reaction function; firm 1 will produce the amount of output $\bar{x}_1$ consistent with the Stackelberg solution of the leader firm. The corresponding amount of output of firm 2 will be $\phi_2(\bar{x}_1) = \bar{x}_2$. Indeed entrant firm, given monopolist's choice, will enter the market supplying the amount of output $\bar{x}_2$ consistent with the follower's output in Stackelberg duopoly, since $\pi_2(\bar{x}_1, \bar{x}_2)$ is greater than zero. On the contrary, entry-deterring output associated with the limit price is determined by $\phi_2(x_1) = 0$ consistent with the point $(x_1^0, 0)$.

In figure 4, $\bar{x}_1 < x_1^0$; then Stackelberg output is lower than entry-deterring output. To maximize its profit firm 1 will prefer to allow entry losing its monopoly position, instead of producing the quantity x_1^0 consistent with a lower level of profit (indeed iso-profit curve $\pi_1(x_1^0, 0)$ is far away from x_1-axis). Therefore in this case established firm will not set a limit price.[26]

However $\bar{x}_1$ may be greater or equal than x_1^0, as in figure below (Fig. 5). There, the profit-maximizing output will deter entry and limit price theory will be rational. Hence, we should investigate when this condition occurs.

[25] Reaction functions are drawn linear for graphical convenience. Concavity of revenue function and convexity for cost function are assumed in order to ensure the negative slope of the curves.

[26] We neglect the possibility that $\mathrm{M}_1 = \bar{\mathrm{x}}_1$. In this case, entry would be blockaded by innocent profit maximization of monopolistic firm.

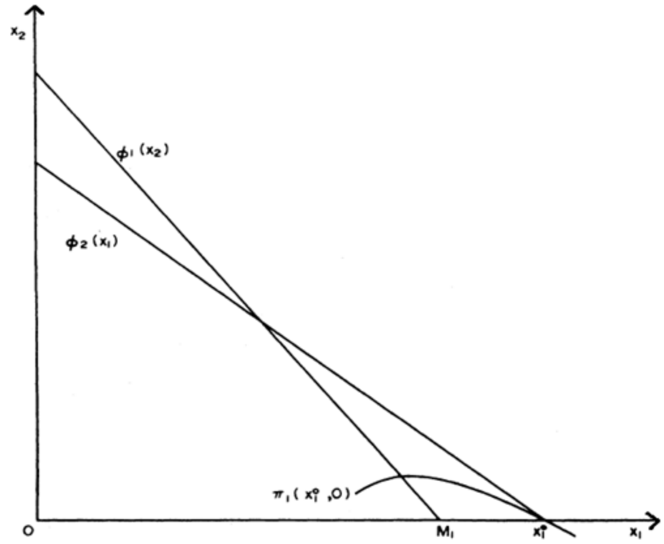

FIG 5 – Stackelberg equilibrium occurs precisely at $(x_1^0, 0)$.
(Source: Osborne, 1973)

In figure 5 at entry-deterring point entrant's reaction function is steeper than firm 1's iso-profit curve $\pi_1(x_1^0, 0)$. Hence monopolist will deter entry only if the iso-profit curve passing through entry-deterring point is no steeper than entrant's reaction function.

Let $\phi'_2(x_1^0)$ denote the slope of firm 2's reaction function, and $I'(x_1^0)$ the slope of firm 1's iso-profit curve. Then, since both slopes are negative *limit pricing consistency solution* will be:

$$I'(x_1^0) \geq \phi'_2(x_1^0). \qquad [1.9]$$

The relation can be expressed in terms of the underlying demand and cost functions. Firm 2's reaction function is such that:

$$\frac{\partial \pi_2\big(x_1, \phi_2(x_1)\big)}{\partial x_2} \equiv 0\,. \qquad [1.10]$$

Differentiating this identity and solving for $\phi_2(x_1)$ we get the slope of reaction function[27]:

$$\phi'_2(x_1) = -\frac{\partial^2 \pi_2 \big/ \partial x_2 \partial x_1}{\partial^2 \pi_2 \big/ \partial^2 x_2} = -\frac{P''(X)x_2 + P'(X)}{P''(X)x_2 + 2P'(X) - C''(x_2)}. \qquad [1.11]$$

[27]Indeed $\frac{\partial^2 \pi_2}{\partial x_2\, \partial x_1} = P''(X)x_2 + P'(X)$, and $\frac{\partial^2 \pi_2}{\partial^2 x_2} = P''(X)x_2 + 2P'(X) - C''(x_2)$.

Evaluated at entry-deterring point $(x_1^0, 0)$ we get

$$\phi'_2(x_1^0) = -\frac{P'(x_1^0)}{2P'(x_1^0) - C''(0)} < 0\,. \qquad [1.12]$$

In order to find the slope of firm 1's iso-profit curve let us first differentiate its profit function obtaining:

$$\frac{dx_2}{dx_1} = I'(x_1, x_2) = -\frac{x_1 P'(X) + P(X) - C'(x_1)}{x_1 P'(X)}. \qquad [1.13]$$

Evaluated at $(x_1^0, 0)$ is

$$I'(x_1^0, 0) = -\frac{x_1^0 P'(x_1^0) + P(x_1^0) - C'(x_1^0)}{x_1^0 P'({x_1}^0)} < 0\,. \qquad [1.14]$$

Hence $I'(x_1^0) \geq \phi'_2(x_1^0)$ is satisfied if

$$-\frac{x_1^0 P'(x_1^0) + P(x_1^0) - C'(x_1^0)}{x_1^0 P'(x_1^0)} \geq -\frac{P'(x_1^0)}{2P'(x_1^0) - C''(0)}\,. \qquad [1.15]$$

When condition [1.15] holds, Stackelberg output will deter entry and limit price $P(x_1^0)$ will be consistent with established firm's profit maximization; otherwise entry will be *ineffectively impeded*.

Summarizing, we have found two basic results: first, Sylos postulate essentially gives rise to a Stackelberg equilibrium; second, limit pricing strategy must be consistent with such equilibrium, hence it will rise only when the Stackelberg point occurs at a corner of potential entrant's reaction function.

1.4.3 The Model with Fixed Costs

In the previous model we neglected any assumption on economies of scale. Indeed we implicitly assumed that entrant firm could supply any amount of output and earn non-negative profit at least as long as price is greater or equal than average cost.

However, if potential entrant firm's technology is characterized by minimum amount of profitable scale, the story change since not all the quantities greater than zero ensure positive (or at least equal to zero) profits.

In section 1.3 we have shown, through Modigliani's results, that the existence of a minimum profitable scale constitutes a barrier to entry. Therefore, adding this hypothesis in the model described above, we should find an increase in the possibilities of deterring entry. This situation can be described by assuming that entrant firm must sustain a fixed entry cost (or *setup cost*) in order to come into the market.[28]

Let us recall the previous model, including this particular assumption. In this case firm 2's profit function will be such that:

$$\pi_2(x_1, x_2) = P(X)x_2 - C(x_2) - F, \qquad [1.16]$$

where F represents the fixed cost. Therefore firm 2 will enter the market if:

$$\pi_2(x_1) = P(X)\phi_2(x_1) - C\big(\phi_2(x_1)\big) - F \geq 0. \qquad [1.17]$$

Let $x_1^F < x_1^0$ be the level of firm 1's output such that

$$P(X)\phi_2(x_1^F) - C\big(\phi_2(x_1^F)\big) = F. \qquad [1.18]$$

Now, every $x_1^F < x_1 \leq x_1^0$ chosen by established firm will induce negative profits for prospective entrant firm. Thus if x_1 belongs to that interval, the optimal response for firm 2 is no longer given by the appropriate point on $\phi_2(x_1)$; it is better to secure zero profit by staying out of the market. Hence firm 2's reaction function is now discontinuous, as shown in the following picture.

[28] This cost can be justified in many ways: the entrant is a large firm; or the industry is characterized by an initial mandatory investment.

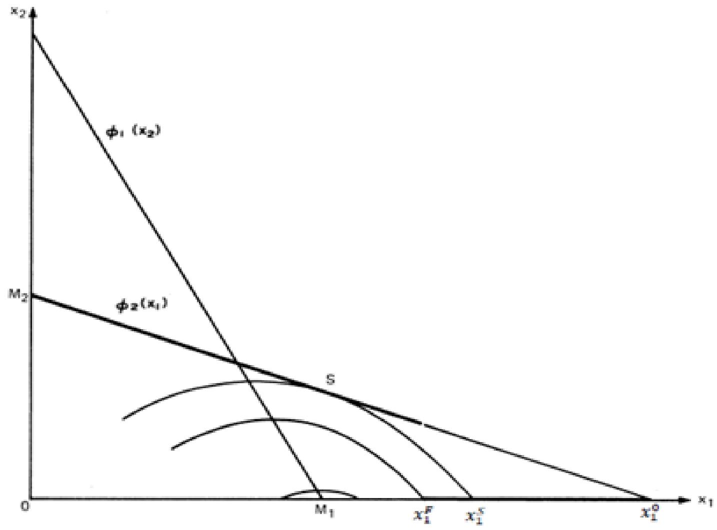

FIG 6 – Entrant's reaction curve is now discontinuous due to the existence of fixed entry cost.

The position of the discontinuity depends on the level of firm 2's fixed cost. If these were so high that firm 2 cannot even make positive profit as a monopolist, then its reaction function would simply be $0x_1^0$; this case will be ignored[29].

In figure 6, x_1^F represents the amount of firm 1's output consistent with the discontinuity point of firm 2's reaction function, while x_1^s is the amount of established firm's output consistent with Stackelberg point's profits level. If entrant's fixed cost is so small that point x_1^F of discontinuity in its reaction function lies to the right of x_1^s, the best choice for firm 1 remains Stackelberg solution, and it is optimal for established firm to allow entry. If potential entrant's fixed cost is so large that x_1^F lies to the left of M_1, the best choice for firm 1 is precisely to produce monopoly quantity M_1 blocking the entry of firm 2.

The intermediate case where $M_1 < x_1^F < x_1^S$ (the case of figure 6) needs more attention. Indeed in this last case firm 1 can do better than Stackelberg point S by setting its output somewhat below x_1^S, so that firm 2 stays out. This profit can be increased by further lowering x_1 up to any value slightly greater than x_1^F. If firm 1 sets its output exactly at x_1^F, then firm 2 is indifferent between staying out and entering the market with zero profit. But the entry would decrease substantially established firm's profit by the increase in aggregate supply and the consequent reduction in equilibrium price level. Therefore so long as there is a positive probability of entry at x_1^F, the monopolist should keep its output

[29] A discontinuity may affect firm 1's reaction function curve in presence of fixed cost for established firm too. However it is assumed that monopolist is operating in the market for a long time; hence it has already depreciated any fixed cost.

slightly greater than x_1^F - which is the *limit output* with the corresponding *limit price* $P_L \cong P(x_1^F, 0)$ - since it is the highest price that make entry unprofitable. In conclusion, in this intermediate case firm 1 finds it profitable to deter entry, and limit pricing strategy is applied by established firm.

Thus without fixed cost entry is deterred by setting limit output at x_1^0 only if condition [1.9] holds; while in presence of fixed costs there are two possibilities: if $x_1^F > M_1$ entry is effectively impeded with a limit pricing equilibrium at x_1^F; and if $x_1^F < M_1$ entry is blockaded with the pure monopoly output at M_1.

Chapter 2

CAPACITY INVESTMENT AND THE DESERTION OF SYLOS' POSTULATE

2.1 Introduction

The role of investment decision as a barrier to entry has been first developed by Spence (1977) who provided the possibility of yielding idle capacity in order to deter the entry of new firms in the market. The process is the following: at the stage when entry is threatened but not yet implemented, established firm is able to make an irrevocable investment decision on capacity; such capacity level may be higher than the level required for producing monopoly output level, but at the same time it might be necessary in order to impede the entry of a new firm, since it is assumed that the time-lag required for increasing production to the maximum capacity level is equal to the time-lag required for entering the market. Hence Spence provided the possibility for established firm to sustain a monopoly output without inducing entry, notwithstanding producing in an inefficient way and yielding *idle capacity*.

The intuition of pre-commitment capacity investment as a barrier to entry introduced by Spence (1977) was later expanded by Dixit (1980) who provided an entry barrier model that can be rightly considered a breaking point with previous literature, since the author no longer applied Sylos' postulate. Under Sylos' postulate, incumbent firms are supposed to sustain the pre-entry output level in face of entry, threating a significant reduction of price level if entry effectively takes place. As we will see, this is not a credible threat, since established firms might prefer an accommodating reduction of output. On the contrary, due to the irrevocable investment decision, Dixit provided a model where incumbent's threat is absolutely credible, i.e. it is a *best response* to the entry threat.

The connection between Spence and Dixit is strong; nevertheless they reached different results. Indeed, even if Dixit gave a central role to investment decision as an entry deterrence - as Spence did - in his model we do not observe the possibility of *idle capacity* for established firms. However, Bulow *et al.* (1985) demonstrated that, if firms face a certain type of aggregate demand, the possibility of *idle capacity* can arise also in Dixit's model. Hence, Spence's intuition for which entry threat may induce inefficient production for established firms is absolutely consistent with Dixit's approach.

In this chapter first we will introduce Spence's excess capacity model; then we will present Dixit's model, demonstrating the absence of *idle capacity*; finally, we will develop the extension proposed by Bulow *et al.*, who introduced the possibility of *idle capacity* in Dixit's model.

2.2 Excess Capacity Model

2.2.1 Capacity Constraint

So far we have implicitly assumed that both entrant and established firms were able to supply any quantity of output, from zero to infinity. However it is really hard to believe that firms can produce an infinite amount of output: typically they are characterized by a capacity constraint, i.e., by a maximum level of feasible output. The goal of this section is to explain how the problem of entry barriers changes when firms in the industry are bounded on their own capacity.

Including capacity constraint means adding another variable to our problem, which may affect the equilibria written in the previous section. For instance, established firm may not be large enough to set a limit quantity in order to deter entry; this is what suggested in the picture below, where k is the capacity measured in unit of output.

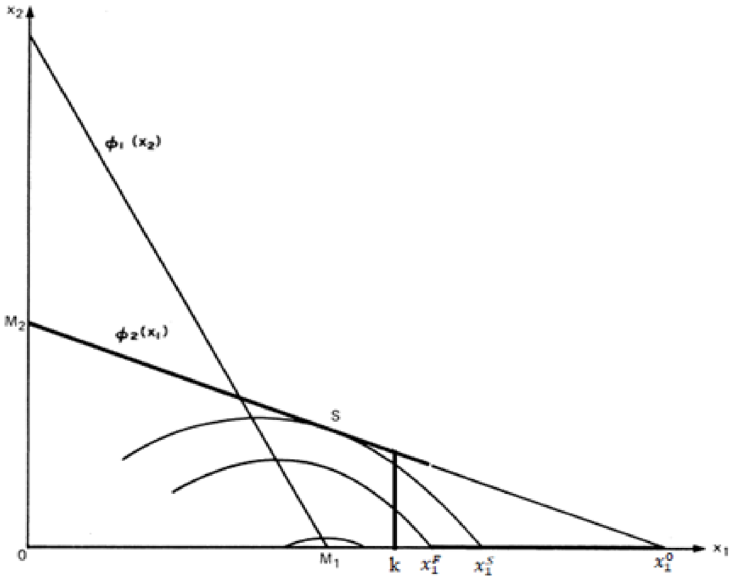

FIG 7 - Established firm cannot produce limit output quantity when $k < x_1^F$.

Figure 7 represents the same situation described in the previous chapter but now firm 1 cannot produce an amount of output greater than k; this means that established firm is not able to reach the entry-preventing output x_1^F. Therefore entry cannot be deterred, and the best for firm 1 is to set the level of output consistent with the Stackelberg solution.[30] Indeed now established firm must solve the following maximization problem:

$$\max_{x_1} \pi_1\big(x_1, \phi_2(x_1)\big) = P\big(x_1 + \phi_2(x_1)\big)x_1 - C(x_1)$$

$$s.t. \quad x_1 \leq k \qquad [2.1]$$

In [2.1] capacity is assumed to be exogenous. However, if established firm is able to make investment decisions (i.e. to choose its own capacity), then it may discourage entry of potential firms through a strategic capacity choice. This is what suggested by Spence (1977, p. 534):

> "Existing firms choose capacity in a strategic way designed to discourage entry. This strategic purpose is realized by holding excess capacity in the pre-entry period. The excess capacity permits existing firms to expand output and reduce price when entry in threatened, thereby reducing the

[30] It is just an example since k may be lower than S. In that case firm 1 will reach a corner solution, supplying an amount of output equal to k.

prospective entrant profits of the new entrant who operates in the residual demand."

2.2.2 The Model

Let us focus on established firm's behavior. When capacity is a variable that a firm can manipulate, the monopolist with no entry threat must solve the following maximization problem:

$$\max_{x,k} \pi(x,k) = R(x) - C(x) - rk$$

$$s.t.\ x \leq k \qquad [2.2]$$

where r is the annual cost of capacity (interest on debt), $R(x) = xP(X)$ is the increasing and concave revenue function and $C(x)$ is the increasing and convex variable cost function. In order to find a solution to this constrained maximization problem let us introduce the Lagrangian function:

$$L(x,k,\lambda) = R(x) - C(x) - rk + \lambda(k - x).$$

The Kuhn-Tucker conditions for this problem are:

$$\frac{\partial L}{\partial x} = 0 \ \Rightarrow\ R'(x) - C'(x) - \lambda = 0 \qquad [2.3]$$

$$\frac{\partial L}{\partial k} = 0 \Rightarrow \lambda - r = 0 \qquad [2.4]$$

$$\lambda(x - k) = 0 \qquad [2.5]$$

$$\lambda \geq 0. \qquad [2.6]$$

From [2.4] $\lambda = r$; therefore capacity constraint is binding since $\lambda > 0$. Thus, with no entry threat $x = k$, and substituting $\lambda = r$ in [2.3], at the maximum $R'(x) = C'(x) + r$

Now, suppose that existing firm sets a certain level of capacity k; then when entry occurs, it is assumed that established firm can expand the output up to k and reduce the price to $P(k)$ within the time horizon required for entry.[31]

Entrant firms face the residual demand defined as the part of aggregate demand curve on the left of the pre-entry price, as in Modigliani (1958). Suppose that pre-entry price level is $P(k)$ and prospective entrant firm supplies a positive amount of output y; then the price after-entry will be $P(k+y)$ and entry will occur if this price is greater than potential entrant's average cost. Thus entry is deterred if

$$\forall y \geq 0, \qquad P(k+y) < a(y,k).$$

Where $a(y,k)$ is the average cost function. If k increases, $P(k+y)$ falls for every y. Therefore, there exists a minimum level of k, denoted $\tilde{k}$, for which the condition above holds and entry is deterred. Hence, if established firm wants to maximize its profit and, at the same time, impede entry, it must solve the following constrained maximization:

$$\max_{x,k} \pi(x,k) = R(x) - C(x) - rk$$

$$s.t. \;\; \begin{matrix} x \leq k \\ k \geq \tilde{k} \end{matrix} \qquad [2.7]$$

The Lagrangian function for this problem is

$$L(x,k,\lambda,\mu,) = R(x) - C(x) - rk + \lambda(k-x) + \mu(k-\tilde{k})$$

And the corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial x} = 0 \;\Rightarrow\; R'(x) - C'(x) - \lambda = 0 \qquad [2.8]$$

$$\frac{\partial L}{\partial k} = 0 \Rightarrow \lambda + \mu - r = 0 \qquad [2.9]$$

$$\lambda(k-x) = 0 \qquad [2.10]$$

$$\mu\left(k-\tilde{k}\right) = 0 \qquad [2.11]$$

[31] With respect to the time horizon required for the installation of capacity, the industry is on an equal footing with the potential entrant.

$$\mu, \lambda \geq 0 \qquad [2.12]$$

We have three possible solutions to the problem:

1) $\lambda > 0, \mu = 0$. This is the profit maximization [2.2] described before by which costs are minimized, $\lambda = r$, $x = k$ and $R'(x) = C'(x) + r$. In this case entry is blockaded since innocent profit maximization establishes a level of k that make entry unprofitable.
2) $\lambda = 0, \mu > 0$. In this case the second constraint is active and $\tilde{k} = k$ i.e. capacity is set in order to deter entry. Moreover $\mu = r$ and $x < k$ is such that $R'(x) = C'(x)$. Therefore costs are not minimized given the level of output actually supplied and capacity is maintained above its efficient level in order to deter entry.
3) $\lambda > 0, \mu > 0$. Here both constraints are binding, therefore $x = k = \tilde{k}$. Unlike the second case, entry is deterred but in an efficient way since $x = \tilde{k}$ and costs are minimized. In absence of threat of entry (i.e. the second constraint is removed) capacity and output would come down together.

Let us interpret these results in limit price theory context. Case 1) describes a situation in which entry is *blockaded* since monopolist's profit maximization, regardless of potential entrant firms, is sufficient to set a level of output that make entry unprofitable. At the opposite, case 3) describes a successful *limit price strategy*: the amount of output supplied by established firm lets fall the price under prospective entrant's average costs and impedes the entry. In this case $x = \tilde{k}$ and $P(\tilde{k})$ is the corresponding price. What has been ruled out until now is the prevision of case 2). In that situation established firm carries excess capacity, given the output, and production is inefficient; moreover since $x < k = \tilde{k}$ then $P(x) > P(\tilde{k})$. Even though the price is higher than limit price, entry is deterred by excess capacity. As a result, the price will be higher than the limit one and production will be inefficient, generating *idle capacity*.

2.3 Dixit's Model

2.3.1 Problems with Sylos' Postulate

The introduction of capacity variable is the starting point in order to relax the very strong assumption so far assumed, that is Sylos' postulate. This assumption was necessary for the complicated game-theoretic aspects of entry in oligopolistic markets. Indeed, even in the simplest case of one established firm and one prospective entrant, there are many strategic interactions depending on pre-entry decisions by established firm, which will influence prospective entrant's expected profit; the monopolist will try to exploit these possibilities to its own advantage. In this context, Sylos' postulate is dubious on two opposing counts: first, faced with an irrevocable fact of entry, the established firm will usually finds it best to make an accommodating output reduction; at the opposite, it would like to threaten to respond to entry with a predatory increase in output. Its problem is *to make the latter threat credible given the prospective entrant's knowledge of the former act,* and this is not obvious. Applying Sylos' postulate means not only that established firm maintains its amount of output fixed, but also that prospective entrant strongly believes in such behavior.
Suppose the best for established firm is an accommodating reduction of output, but it wants to trick the prospective entrant threatening a predatory increase of production; is this strategy credible for the prospective entrant, i.e. will potential firm stay out of the market because of established firm's threat? The answer is no. By the basic assumptions of rationality and perfect information, there is no reason why potential firm should stay out of the market since it perfectly knows that, in face of entry, incumbent firm will reduces its output at the level that is the best for itself. Thus, the increase in output (and its sustaining) is not a credible threat. From these considerations the need arose for a new representative model of entry in oligopolistic markets that did not involve Sylos' postulate.

Dixit introduced a new approach on entry-barriers problem in 1979 ("A model of duopoly suggesting a theory of entry barriers"), afterwards expanded one year later in "The role of investment in entry-deterrence". In particular the author considered Spence's approach fpr which in the interest of entry-deterrence, the established firm may set capacity at such a high level that in the pre-entry phase it would not want to utilize it all, i.e. excess capacity would be observed. The basic point in Dixit's analysis is that "although the rules of the post-entry game are taken to be exogenous, the established firm can alter the outcome to its own advantage by changing the initial conditions. In particular, an irrevocable choice of investment allows it to alter its post-entry marginal curve, and thereby the post-entry equilibrium under any specified rule."

2.3.2 The Model with Pre-Commitment Capacity

Dixit's model is a three-stage game with one established firm (firm 1) and one potential entrant firm (firm 2). At the first stage, established firm chooses a pre-entry capacity level $\bar{k}_1$ which may subsequently be increased, but cannot be reduced. At the second stage the other firm decides to enter the market. If it enters, the two will achieve a duopoly Cournot-Nash equilibrium with quantity-setting; otherwise established firm will prevail as a monopolist. In order to find the possible equilibria for this game, we will use *backward induction*, starting the analysis from the third stage up to the investment decision of the first stage.

- ***The third stage***

Each firm is assumed to have the following cost function:

$$C_i = F_i + w_i x_i + r_i k_i \qquad i = 1,2 \qquad [2.13]$$

where F_i is the fixed set-up cost, r_i is the constant annual cost of capacity and w_i the constant average variable cost for output.[32] The revenues for both firms will be functions

$$R_i(x_1, x_2) = x_i P(X) \qquad with \quad X = x_1 + x_2 \qquad [2.14]$$

where $P(X)$ is the usual inverse aggregate demand function. Revenue function is assumed to be increasing and concave in firm's output and decreasing in the other's output.[33]

[32] Of course the two firms may have the same cost function.

[33] $\frac{\partial R_i}{\partial x_i} > 0, \frac{\partial^2 R_i}{\partial^2 x_i} < 0$ and $\frac{\partial R_i}{\partial x_j} < 0$

Now suppose that firm 1 has installed the capacity $\bar{k}_1$. If the quantity produced is within capacity level ($x_1 \leq \bar{k}_1$), then its total cost function is

$$C_1 = F_1 + r_1\bar{k}_1 + w_1 x_1. \qquad [2.15]$$

If it wishes to produce a greater amount of output, it must acquire additional capacity. Therefore if $x_1 > \bar{k}_1$

$$C_1 = F_1 + (r_1 + w_1)x_1. \qquad [2.16]$$

Firm 2 has no pre-commitment capacity, then it will acquire the efficient capacity $k_2 = x_2$; therefore

$$C_2 = F_2 + (r_2 + w_2)x_2. \qquad [2.17]$$

Let us focus on firm 1's cost function. In particular as long as its output is lower than the capacity installed, marginal cost equals w_1 while, for level of x_1 greater than $\bar{k}_1$ marginal cost is equal to $r_1 + w_1$. Therefore the choice of $\bar{k}_1$ affects the shape of firm 1's marginal cost curve, which in turn affects its reaction curve. Since the Cournot-Nash equilibrium is given by the intersection of firms' reaction curve, the choice of $\bar{k}_1$, which affects firm 1's reaction curve, influences the final duopoly equilibrium too. Indeed, write firm 1' reaction function as follows:

$$\pi'_1(x_1^C) = x_1^C P'(X) + P(X) - C'_1(x_1^C) = 0 \quad \Rightarrow$$

$$\Rightarrow x_1^C \equiv \phi_1(x_2) = -\frac{P(X) - C'_1(x_1^C)}{P'(X)} \geq 0 \qquad [2.18]$$

The increase in marginal cost produces a decrease of the x_1 consistent with reaction curve, for every value of x_2, i.e. a down-ward shift of reaction curve (Figure 8).

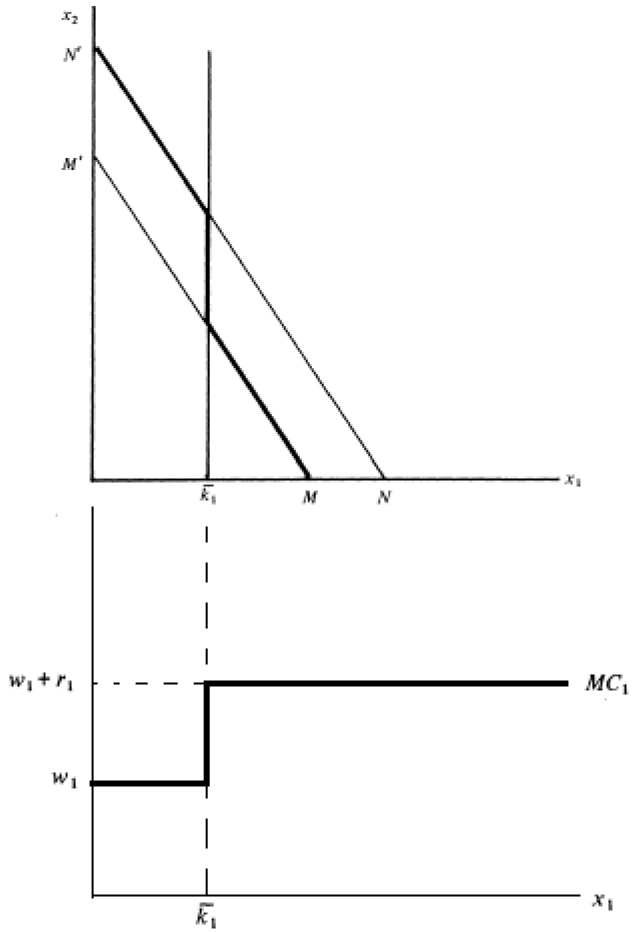

FIG 8 – The effect of marginal cost on firm 1's reaction curve.

Take a look on the first graphic of figure 8. The points M and N have the coordinates $(M_1, 0)$ and $(N_1, 0)$ respectively. The quantities M_1 and N_1 represent profit maximizing quantity choices for firm 1 when the possibility of entry is ignored (i.e. for $x_2 = 0$); but M_1 is the choice in the case in which firm 1 has not installed a pre-commitment capacity (and marginal cost equals $r_1 + w_1$), while N_1 is relevant when there is a capacity already installed. Since firm 2 has no prior capacity commitment, its reaction function RR' is straightforward as shown in figure 9.

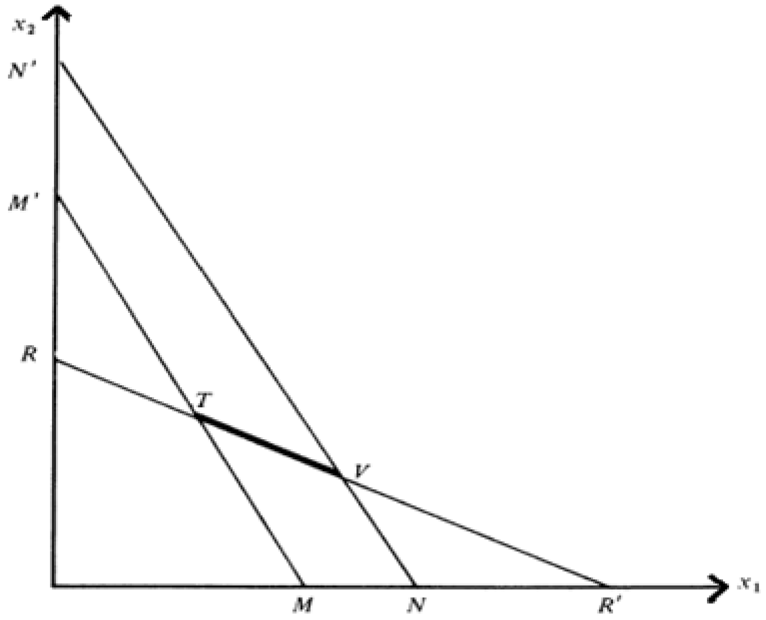

FIG 9 – The curves MM' and NN' represent firm 1's reaction curve in two opposite circumstances: MM' when capacity costs matter, while NN' when capacity has been already installed. (Source: Dixit, 1980)

For a given level of $\bar{k}_1$ we have a Cournot-Nash equilibrium at the intersection of the two reaction curves. However firm 1, choosing $\bar{k}_1$ in advance, can determine which reaction function it will present in the post-entry duopoly; surely it will choose the level of $\bar{k}_1$ that will maximize its profit given the well-defined firm 2's reaction curve. Now, let $T = (T_1, T_2)$ be the intersection point between RR' and MM' and $V = (V_1, V_2)$ the point of intersection between RR' and NN'. The points T and V are two Nash equilibria in opposite circumstances: the first equilibrium is consistence with the case in which firm 1 must acquire more capacity and this cost matters; while V is the Nash equilibrium when $x_1 \leq k_1$ and marginal cost is simply w_1.

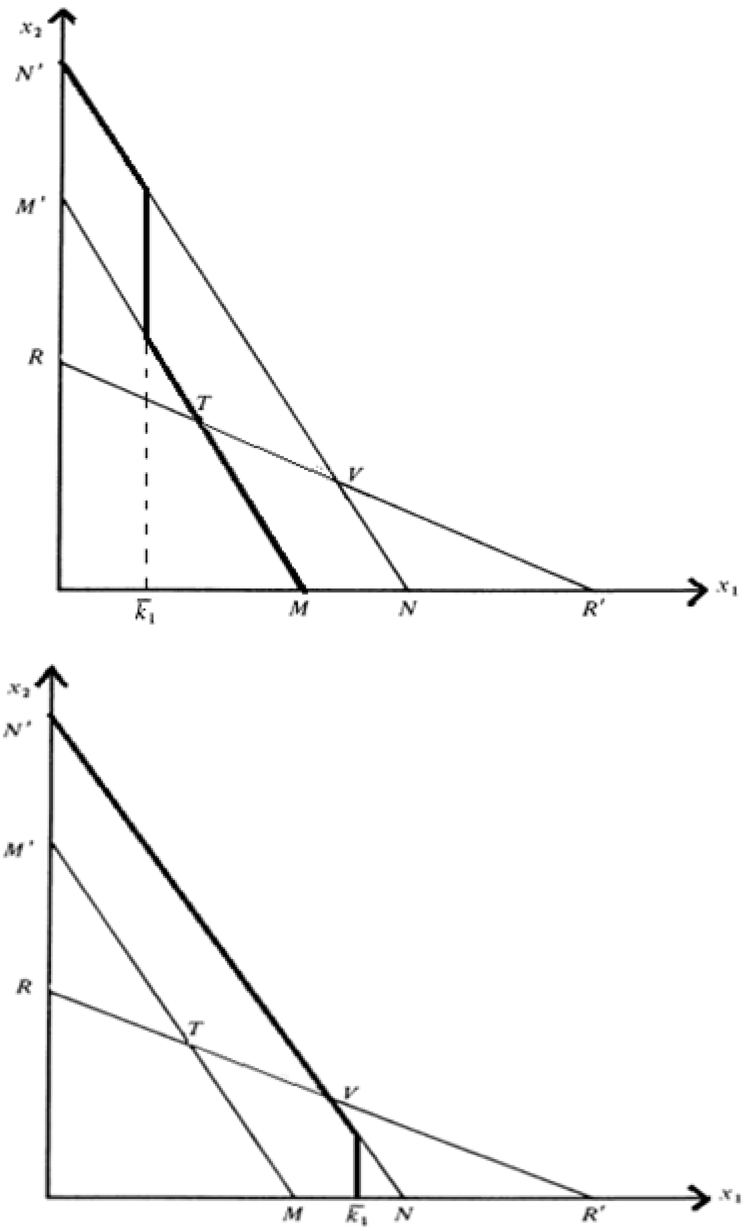

FIG 10 – In the first graphic Nash equilibrium occurs at T, while in the second at V.

For a choice of $\bar{k}_1 \leq T_1$ the post-entry Nash equilibrium will be at T while for $\bar{k}_1 \geq V_1$ it will occur at V (Figure 10). Hence, since V_1 is the maximum level of established firm's production consistent with post-entry equilibrium, any capacity level above V_1 is not a credible threat of entry deterrence.[34] Accordingly, since $N_1 > V_1$, it is evident that level of $\bar{k}_1 \geq N_1$ are not credible too.[35] Established firm will not install a pre-entry capacity

[34] "When a prospective entrant is confident of its ability to sustain a Nash equilibrium in the post-entry game, it does not fear such levels. And when the established firm knows this, it does not try out the costly and empty threat" (Dixit, 1980).

[35] In this context, Spence's excess capacity described in previous section, by which established firm set a capacity level greater than the optimal monopolist quantity in order to deter entry, will never be employed.

lower than T_1 since it needs at least T_1 in case of entry, and $M_1 > T_1$ if entry is not to occur. Hence pre-entry capacity will be chosen between these two extremis $T_1 \leq \bar{k}_1 \leq V_1$, the final equilibrium will be at the intersection point between firm 2's reaction curve and the vertical line by which $x_1 = \bar{k}_1$.

So far we have two certainties: first, established firm will choose a capacity level between T_1 and V_1; second, in order to minimize cost, it will produce the quantity $x_1 = \bar{k}_1$. Hence, post-entry equilibrium lies in the segment TV.

- ***The second stage***

The discussion concerned the third stage, i.e. the post-entry duopoly. Now we can pass on the second stage, in which firm 2 makes its decision in order to enter or not the market. As always, prospective entrant firm will come into the market if its expected profit is nonnegative,[36] that is, if

$$\pi_2(x_1, x_2) = R_2(x_1, x_2) - F_2 - (r_2 + w_2)x_2 \geq 0.$$

By the existence of set-up cost, prospective entrant's profit is non-positive for a level of established firm's output lower than R' (Figure 9)[37]. Moreover firm 2's profit decreases monotonically along its reaction function from T to V, therefore we can classify first two extreme possibilities.

1) $\pi_2(T) < 0$. In this case prospective entrant's profit is negative in every post-entry equilibrium. Therefore it will not try to enter the market and established firm will enjoy a pure monopoly by setting its capacity and output at M_1. *Entry is blockaded.*
2) $\pi_2(V) > 0$. Here prospective entrant will earn positive profit in every post-entry equilibrium; therefore established firm cannot deter entry. The best it can do is to compute its profits along the segment TV and choose the amount of output consistent with the higher profit. Therefore since quantity equals capacity, we can use the conventional iso-profit curve in order to find the

[36] It is imposed that established firm's profit maximum value is always positive.

[37] This has been shown in section 1.4.3. We saw that there exists a level of established firm's output denoted x_1^F where firm 2's variable profits equal fixed costs. In that point potential entrant's reaction function is discontinuous since for level of $x_1 > x_1^F, x_2 = 0$, because firm 2's profits would be negative and prospective entrant will prefer to stay out of the market.

solution. The solution will be the tangency point between firm 1's iso-profit curve and firm 2's reaction curve, i.e. the *Stackelberg solution*.[38]

However the richest set of possibilities can be found in the intermediate case in which $\pi_2(T) > 0 > \pi_2(V)$. Now there exists a point $B = (B_1, B_2)$ along the relevant segment TV such that prospective entrant's profit equals zero. If $\bar{k}_1 \geq B_1$ firm 2's expected profit will be negative and prospective entrant will not enter the market. Therefore B_1 is the entry-preventing output, and we have to distinguish among the following possibilities.

3a) $B_1 < M_1$. Here established firm's monopoly choice is sufficient to impede entry, hence *entry is blockaded.*

3b) $B_1 > M_1$. In this case established firm can deter entry only setting a capacity at a level greater than monopoly output. Therefore it should evaluate if the profit consistent with the entry-deterring quantity is greater than the profit corresponding to the entry-accommodating output. To prevent entry it needs a capacity just greater than entry-preventing output B_1. On the contrary, if firm 1 allows entry, it will reach the Stackelberg solution (tangency or corner solution as seen above): let us call the Stackelberg point $S = (S_1, S_2)$. Then, if $\pi_1(S) < \pi_1(B_1, 0)$, firm 1 will deter entry by setting a *limit output* (*limit capacity*) slightly higher than B_1, with the correspondent *limit price*. Otherwise it is better to allow entry, i.e. entry is *ineffectively impeded*, and duopoly equilibrium will occur in the Stackelberg point S.

- ***The first stage***

In the first stage firm 1 will set the capacity level consistent with post-entry Nash equilibrium in order to minimize total costs. Knowing the final result of the game, established firm has no reason for choosing a capacity level greater than equilibrium output, suffering cost for the unused capacity. Indeed, if $X^* = (x_1^*, x_2^*)$ is post-entry equilibrium output, established firm must solve the following maximization problem:

[38] If the tangency solution occurs on the right of V, there will be a corner solution at V.

$$\max_{k_1} \ \pi_1(k) = R(x_1^*, x_2^*) - F_1 - r_1 k_1 - w_1 x_1^*$$

$$s.t. \ \ k_1 \geq x_1^* \qquad [2.19]$$

The Lagrangian function for this problem is

$$L(k_1, \lambda) = R(x_1^*, x_2^*) - F_1 - r_1 k_1 - w_1 x_1^* + \lambda(k_1 - x_1^*)$$

The Kuhn-Tucker conditions for this problem are

$$\frac{\partial L}{\partial k_1} = -r_1 + \lambda = 0 \ \ \Rightarrow \ r_1 = \lambda > 0 \qquad [2.20]$$

$$\lambda(k_1 - x_1^*) = 0 \qquad [2.21]$$

$$\lambda \geq 0. \qquad [2.22]$$

Since $\lambda > 0$, [2.21] holds if and only if $k_1 = x_1^*$, and the game is solved.
In conclusion, through a pre-commitment capacity investment, established firm can alter the final outcome to its own advantage. Surely, there exists a limit; indeed, if in standard models with Sylos' postulate established firm has always the possibility to deter entry and, if it allows entry, it is because of the greater profit deriving from this last choice, in Dixit's model there is a case in which entry cannot be impeded. This happens when $\pi_2(V) > 0$. Therefore, even if established firm is able to raise barriers to entry with its behavior, in this last model this power is lower than that of an established firm in a model with Sylos' postulate.

2.3.3 Idle capacity in Dixit's model

Established firm's investment decision in pre-entry stage is the crucial element in Dixit's work. Indeed the incumbent will set a capacity level equal to the post-entry Nash equilibrium if entry cannot be impeded; otherwise, if it is able to impede entry, it must evaluate if its profit will be greater when entry is allowed or not; and if established firm finds it best to deter entry, it will install an amount of capacity consistent with the *entry-deterring output* level. Moreover, it has been shown that we do not observe Spence's excess capacity, since the monopoly output when there is enough spare capacity is greater

than the maximum level of output consistent with the intersection between the two reaction curves, i.e. greater than any post-entry Nash equilibrium.
However, as *Bulow et al.* have pointed out[39], this conclusion depends on the assumption that firm's marginal revenue (or equivalently marginal profit) is always decreasing in other's output; as a consequence, firm's reaction curve is always decreasing in rival's output. Nevertheless, this condition does not always hold.

Suppose, as before, that there is one established firm (firm 1) and one prospective entrant firm (firm 2). Recall firm 1's revenue function[40] $R_1 = P(X)x_1$, where $P(X)$ is the usual negatively sloped inverse aggregate demand function, and $X = x_1 + x_2$. Marginal revenue function for firm 1 is given by

$$MR_1 = \frac{\partial R_1}{\partial x_1} = P'(X)x_1 + P(X) > 0. \qquad [2.23]$$

If firm's marginal revenue is always decreasing in the other's output, then the derivative of firm 1's marginal revenue with respect to x_2 must be negative for every $x_2 \geq 0$. Formally

$$\frac{\partial MR_1}{\partial x_2} = P''(X)x_1 + P'(X) < 0. \qquad [2.24]$$

Since price is a negative function of aggregate output, $P'(X) < 0$. Hence, inequality [2.24] certainly holds if $P''(X)x_1 \leq 0$; and since x_1 cannot be negative as it is firm 1's output, it follows that $P''(X)$ must be non-positive, i.e., inverse aggregate demand function must be decreasing and concave in output. On the contrary, if aggregate demand is assumed to be strictly convex, i.e. $P''(X) > 0$, then there may exist an interval of values of x_2 such that ${\partial MR_1}/{\partial x_2} > 0$.
Therefore the permanent decrease in firm's marginal revenue with respect to rival's output is not obvious.[41] Indeed if we take a constant-elasticity aggregate demand curve

[39] Bulow J.I., Geanakoplos J.D. and Klemperer P.D., "Holding capacity to deter entry", 1985.
[40] Revenue function is assumed to be increasing and concave in firm's output.
[41] To ensure the concavity of revenue function in firm's output the following relation must hold:

$$\frac{\partial^2 R_1}{\partial^2 x_1} = P''(X)x_1 + 2P'(X) < 0, \qquad \forall x_1 \geq 0.$$

Therefore if aggregate demand is not concave the condition above must be checked in order to verify the existence of a maximum of revenue function.

$P = bX^{-\alpha}$ – where b is a constant parameter and $0 \leq \alpha \leq 1$ is the constant elasticity of aggregate demand – it can be shown that firm 1's marginal revenue is an increasing function of firm 2's output when $0 \leq x_2 \leq \alpha x_1$.[42]

Suppose, as in Dixit's model, firms are characterized by constant marginal cost equal to w. It has been shown in 1.4.2., that the slope of firm 1's reaction function is given by

$$\phi'_1(x_2) = -\frac{P''(X)x_1 + P'(X)}{P''(X)x_1 + 2P'(X) - C''(x_1)}. \qquad [2.25]$$

Since marginal cost is constant, $C''(x_1) = 0$. Hence, the denominator in [2.25] is essentially the second-order derivative of firm 1's revenue function with respect to x_1 (i.e. the derivative of firm 1's marginal revenue with respect to x_1), while the numerator represents the derivative of firm 1's marginal revenue with respect to x_2. So we get:

$$\phi'_1(x_2) = -\frac{\frac{\partial MR_1}{\partial x_2}}{\frac{\partial MR_1}{\partial x_1}} \lesseqgtr 0. \qquad [2.26]$$

By concavity of revenue function in firm's output, denominator in [2.26] is always negative; thus, the sign of the slope of reaction curve depends on the sign of the numerator. If we assume a constant-elasticity aggregate demand curve $P = bX^{-\alpha}$, then the slope of reaction curve will be positive until $\frac{\partial MR_1}{\partial x_2} > 0$, i.e. in the interval $0 \leq x_2 \leq \alpha x_1$, and then it will be negative. Therefore firm 1's reaction function will be initially increasing up to $x_2 = \alpha x_1$, and decreasing thereafter; the same applies for firm 2. Figure 11 should give an intuition of this case.

[42] Concavity of firm 1's revenue function with respect to x_1 is verified in case of aggregate demand with constant elasticity.

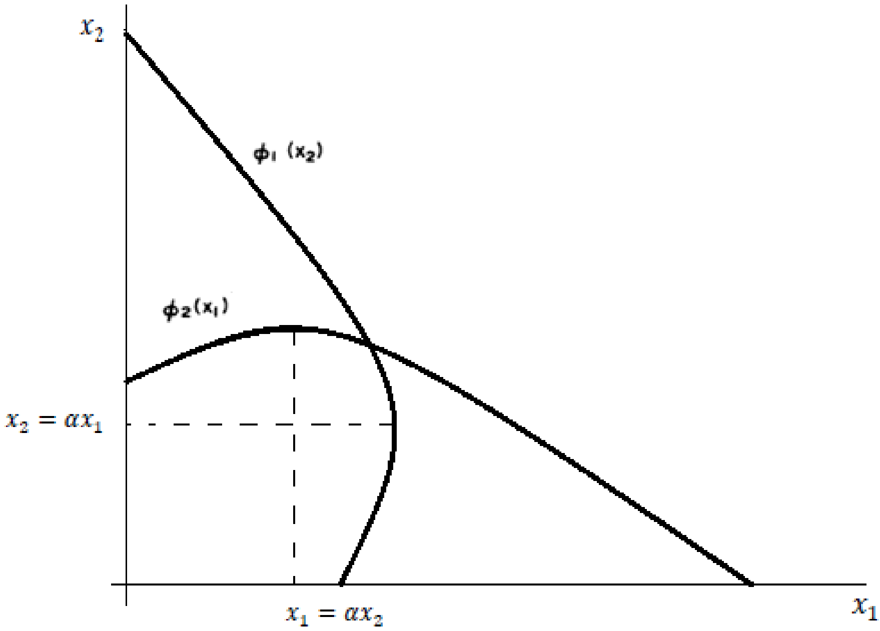

FIG 11 – The shape of reaction curves when firms face a constant elasticity aggregate demand curve.

Let us turn back to Dixit's model adopting reaction functions of this form. Figure 12 is similar to figure 9 except for the shape of reaction curves which, in this case, reflects the assumptions of this section. Indeed, as before, MM' represents firm 1's reaction function when no pre-commitment capacity has been installed (and capacity cost matters), while NN' represents firm 1's reaction function when capacity has been already installed (and marginal cost is simply w). RR' is firm 2's reaction curve.

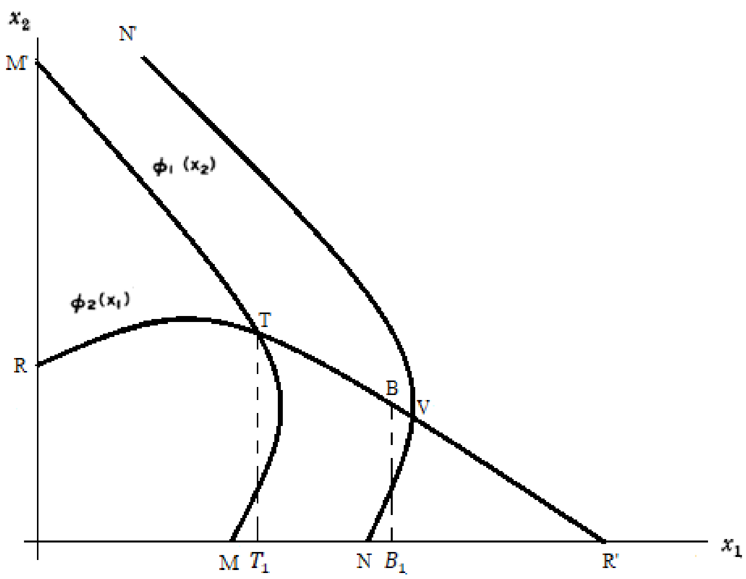

FIG 12 – Post-entry Nash equilibria with the new reaction curves.

The chord TV represents the set of possible post-entry Nash equilibria, depending, as already said, on established firm's investment decision in the first stage of the game. Moreover, by the existence of set-up costs, firm 2's profit equals zero for an amount of firm 1's output lower than R'. Let us analyze first the two extreme cases: if $\pi_2(V) > 0$, as before, entry cannot be deterred, since prospective entrant's expected profit is greater than zero in any post-entry Nash equilibrium; but if $\pi_2(T) < 0$ the story change. Now, due to the shape of reaction curves, the monopoly output (when capacity cost matters) is lower than the lowest established firm's output consistent with post-entry Nash equilibrium set ($M_1 < T_1$). Therefore, $M = (M_1, 0)$ is not a post-entry Nash equilibrium and firm 1 cannot *block* entry by setting $x_1 = M_1$.[43]

After that, focus on the intermediate case in which there exists a point $B = (B_1, B_2)$ along the relevant chord such that $\pi_2(B_1, B_2) = 0$, and suppose that this point lies as in figure 12. Moreover suppose that established firm finds it better to impede entry, i.e. that $\pi_1(B) > \pi_1(S)$, where $\pi_1(S)$ is the profit level when entry is allowed and *Stackelberg solution* occurs. Therefore in the first stage established firm will set a *limit capacity* $\bar{k}_1 \cong B_1$, but it will not produce the quantity $x_1 = \bar{k}_1$; indeed it must solve the constrained maximization problem

$$\max_{x_1} \pi_1 = R(x_1, x_2) - F_1 - r_1\bar{k}_1 - wx_1$$

$$s.t.\ x_1 \leq \bar{k}_1. \qquad [2.27]$$

The Lagrangian function for this problem is

$$L(x_1, \lambda) = R_1(x_1, x_2) - F_1 - wx_1 - r\bar{k}_1 + \lambda(\bar{k}_1 - x_1).$$

And the Kuhn-Tucker conditions for this problem are:

$$\frac{\partial L}{\partial x_1} = \frac{\partial R_1}{\partial x_1} - w - \lambda = 0 \qquad [2.28]$$

$$\lambda(\bar{k}_1 - x_1) = 0 \qquad [2.29]$$

$$\lambda \geq 0. \qquad [2.30]$$

[43] Therefore, in order to *block* entry, we must have $\pi_2(M) < 0$.

And we have two possible solutions:

1) $\lambda = 0$. In this case capacity constraint is not binding, i.e. $x_1 < \bar{k}_1$, and marginal revenue equals marginal cost. Therefore this is exactly the monopoly solution consistent with point $N = (N_1, 0)$ in figure 12.
2) $\lambda > 0$. In this case capacity constraint is binding, i.e. $x_1 = \bar{k}_1$, with $\lambda = MR_1 - w$. This is consistent with the case in which capacity is lower than monopoly output and established firm will produce the maximum feasible amount of output.

In our case $\bar{k}_1 = B_1 > N_1$ therefore we are in the first situation, where capacity constraint is not binding. Hence established firm will set *limit capacity* $\bar{k}_1$ but it will produce the amount of output $N_1 < \bar{k}_1$, i.e. we will observe *idle capacity*.

Thus, when reaction functions of both firms are not always decreasing in other's output, $B_1 > N_1$, and established firm finds it better to deter entry than to allow it, we observe Spence's excess capacity and inefficient production, contrary to what was stated by Dixit. The result depends essentially on the degree of substitutability between the goods produced by firms. Indeed if goods produced by established and prospective entrant firms are *perfect substitutes,* i.e. $\partial\pi_i / \partial x_j < 0$, then any additional quantity of rival's output reduces firm's profit and reaction functions will always be decreasing on other's output. In this case established firm will never hold *idle capacity.*
However according to *Bulow et al.*, goods are said to be *strategic substitutes*, if $\partial^2\pi_i / \partial x_j \partial x_i < 0$, i.e. any additional quantity of rival's output reduces opponent's marginal profit. In this case reaction functions may be increasing in an interval and we cannot rule out the possibility of *idle capacity* for established firm.[44]

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[44] The model has been addressed analyzing the trend of $\partial MR_1 / \partial x_2 = \partial^2\pi_i / \partial x_i \partial x_j$. However note that

$$\frac{\partial^2\pi_i}{\partial x_j \partial x_i} = \frac{\partial^2\pi_i}{\partial x_i \partial x_j}.$$

Therefore nothing changes, and the good can be rightly considered *strategic substitutes.*

Chapter 3

ENTRY UNDER INCOMPLETE INFORMATION

3.1 Introduction

The basic idea of limit pricing is that an established firm may be able to influence other's firms' perceptions of the profitability of entering the market, and that the firm may thus set its price below its monopoly profit-maximizing level in order to deter entry. Firm's rationality implies that established firm sets limit price only if its expected profit when entry is allowed is lower than that consistent with the entry-deterring price; and by complete information assumption, incumbent firm is able to make this kind of evaluation with certainty, while prospective entrant firm can clearly compute its expected profit given established firm's choice.

Moreover, it has been shown the condition of pre-entry asymmetry between the two firms. The prior existence gives a first-mover advantage to the incumbent; as a consequence, if entry is accommodated, we will observe a Stackelberg equilibrium with established firm acting as the leader firm and the prospective entrant as the follower one.

Finally, an observation about cost functions of the two firms is required. Indeed, for the sake of simplicity, we assumed that firms were symmetric, that is they had the same cost function. The only difference concerned the existence of entry-cost for prospective entrant firm which clearly influences negatively the possibility to entry, since established firm's entry-deterring output set increases. In this case entrant firm's reaction curve is discontinuous at the level of established firm's output at which entrant profit equals zero: from then on it is better to stay out of the market, and entrant's output is fixed at zero. Similarly, if potential entrant is a large firm that need a large amount of output produced and sold in order to earn non-negative profit, its possibilities of entry reduce, as shown in Modigliani's model. All of this refers, in Bain's terminology, to established firm's *absolute cost advantage* over prospective entrant firms, i.e. *a beneficial state where an incumbent firm is able to achieve and sustain lower average total costs for its products or services relative to that achievable by newer entrants.*

However, prospective entrant firm may have a cost advantage over incumbent firm due to the fact that it is new. For instance, it may possess a new technology that makes its marginal cost lower than that of established firm. In this case prospective entrant firm should enter the market relatively more easily and, in the limiting case, it may be able to push established firm out of the market. Indeed, consider Dixit's model and suppose that there exist three types of potential entrant firms as shown in figure 13. The heavy lines represent the well-known established firm's reaction curves (when capacity cost matters the lower one) while the thin ones are three possible prospective entrant's reaction curves depending on their marginal cost.

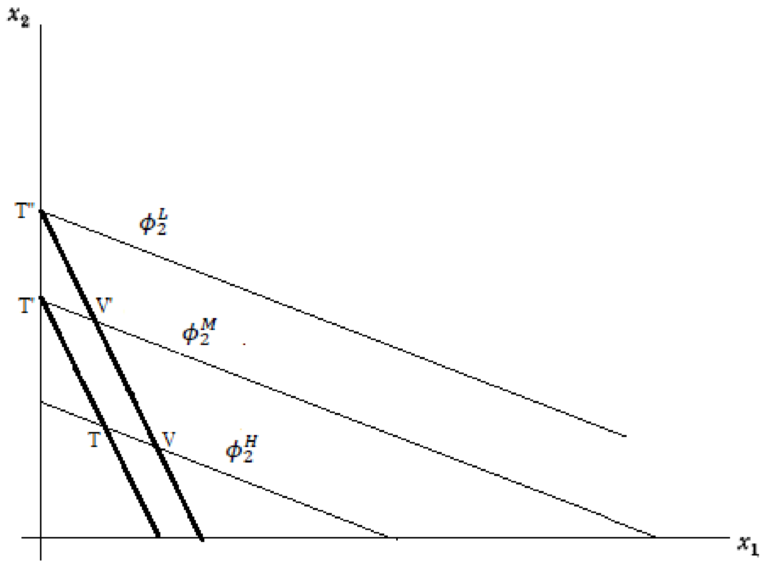


FIG 13 – The upward shift of post-entry Nash equilibria in Dixit's model due to the decrease in firm 2's marginal cost.

Recall reaction function's equation

$$\phi_i(x_j) = \frac{P(X) - C'_i}{P'(X)}; \qquad [3.1]$$

higher reaction curves refer to firms with lower marginal cost; in particular ϕ_2^H is the reaction curve of a high-cost prospective entrant, ϕ_2^M refers to medium-cost entrant, while ϕ_2^L to low-cost prospective entrant firm. As we can see in figure 13, the relevant segment of post-entry Nash equilibria TV shifts up-ward as entrant's marginal cost decreases.

Moreover, for low-cost entrant firms is much more profitable to enter the market than that for high one, since the set of post-entry Nash equilibria approaches to entrant's monopoly output. In the limiting case (the correspondent ϕ_2^L of figure 13), the only Nash equilibrium occurs at $x_1 = 0$ at which prospective entrant produces the monopoly output: in this case, the incumbent is pushed out of the market.

Now suppose established firm does not know prospective entrant's marginal cost. Clearly, it cannot make its capacity choice in the first stage of Dixit's game, since it does not know what is the relevant segment of post-entry Nash equilibria and, therefore, it cannot compute if it is in a situation of *entry-blocking*, if entry cannot be impeded (when $\pi_2(V) > 0$) or if it is better to deter entry setting a limit output/capacity. At the same time, if prospective entrant firm is ignorant about established firm's cost function, it cannot make its decision concerning on entering or not the market. Therefore, we need another approach in order to analyze oligopolistic entry-barriers in a situation of *disinformation* in which both established and prospective entrant firms are ignorant about other's cost function.

The purpose of this section is to show a model of entry-barrier with incomplete information.

3.2 Pre-entry price as a Signal

As already said the role of *limit pricing* is to influence entrant firms' perception about the profitability of entry in a market. In this context, established firm's price/output policy can be rightly considered as a *signal* to potential entrants that entry is not profitable. Consider for instance, a duopoly entry model in which Sylos' postulate applies and suppose established firm sets *limit price* in order to deter entry. That limit output/price policy conveys an information to potential entrant firm, i.e. that it will earn non-positive profit if it will come into the market.[45] But pre-entry price might convey also information to entrant firm about incumbent's cost function.

Imagine an established firm with a cost function that is unknown to the potential entrant and vice-versa. However, the entrant knows that the other firm's cost function must be among a certain class of cost functions. Now suppose the simplest case in which demand function is linear, i.e. $P = a - bX$ with $X = x_1 + x_2$, and cost functions are characterized by constant marginal cost; therefore both firms are uncertain about the value of rival's marginal cost. If established firm does not care about the entry threat, it will set the profit-maximizing monopoly quantity given by

$$\max_{x_1} \pi_1(x_1) = (a - bx_1)x_1 - c_1x_1. \qquad [3.2]$$

Therefore by first-order condition we get

$$\frac{\partial \pi_1}{\partial x_1} = a - 2bx_1 - c_1 = 0 \iff x_1^m = \frac{a - c_1}{2b}. \qquad [3.3]$$

Even if entrant firm does not know established firm's marginal cost, it can compute c_1 by monopolist's output policy; indeed $c_1 = a - 2bx_1^m$. Moreover x_1^m and c_1 are negatively correlated: a high value of x_1^m (and consequently low monopoly price) refers to a low-cost firm while a low value of x_1^m (high monopoly price) refers to high cost firm. Therefore if firms are ignorant about rival's marginal cost, *limit price* may be interpreted by prospective entrant firm as monopoly price of a low-cost established firm.

Now suppose the entrant can earn non-negative profit if the monopolist has a high marginal cost (with respect to its own marginal cost), but it will be forced to suffer losses if monopolists marginal cost is low. In this case prospective entrant firm can easily infer established firm's marginal cost and make its decision. But this cannot be an equilibrium situation. The incumbent would want to reduce its price (increasing output) in order to fool the entrant. But a rational entrant will perceive the possibility of such a strategy. In turn, the incumbent knows that the entrant will not be so easily fooled, and so on and so forth.

[45] We already said that this information, which is a real threat, will not be taken in consideration by prospective entrant firm since the sustaining of entry-deterring policy is not credible threat. However also capacity choice in Dixit's model can be interpreted as a signal on the profitability of entry.

Two considerations can be made at this point: first, in a context of incomplete information the role of limit price is to manipulate information contained in pre-entry price in order to derive benefits later on (*signaling game*); second, firms make their decisions under *uncertainty*, hence these decisions are sustained by *beliefs.* This means that possibility of *mistakes* is allowed, and *limit price may not limit entry*.

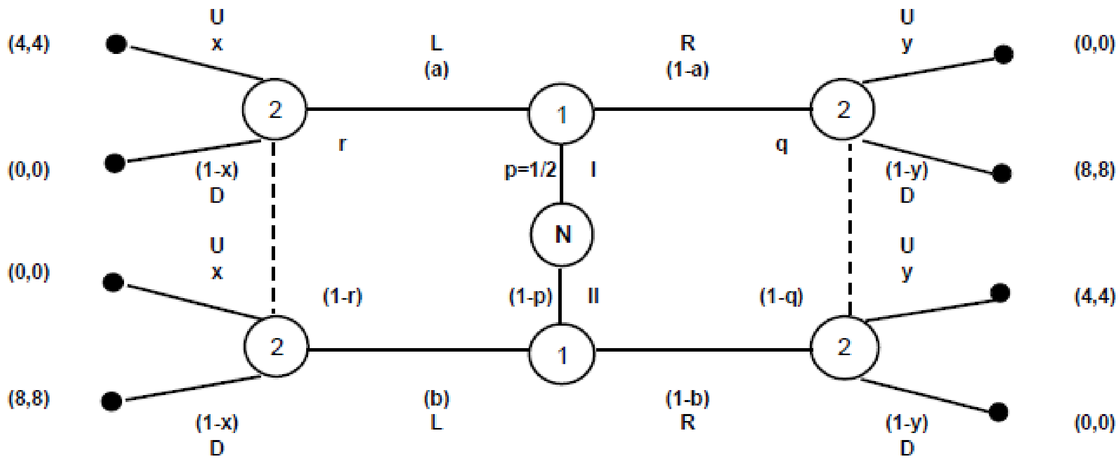
3.3 Signaling Games

3.3.1 An Intuition

Signaling games are incomplete information games with two players: a Sender (S) and a Receiver (R). The sender has a certain type t from a type space $T = \{t_1, \dots \dots \dots, t_I\}$. The specific type t_i is selected by Nature with probability $p(t_i)$. Sender observes t_i and then choose a message m_j from a message space $M = \{m_1, \dots \dots \dots, m_J\}$ to be sent to the receiver. The receiver observes the message m_j but not sender's type t_i; then he chooses an action a_k from the action space $A = \{a_1, \dots \dots \dots, a_K\}$. Therefore the payoffs for both players depend on sender's type chosen by Nature, on the message sent to the receiver, and on the subsequent action of the receiver.

In signaling games, there may exist *pooling equilibria*, *separating equilibria* or both of them. The main problem in these games is that typically there exists a *multiplicity* of equilibria. This is a great problem in economic analysis, since different equilibria have different implications in economies, especially from a welfare point of view. Multiplicity arises from the state of *uncertainty* in which players are called to act and therefore from the fact that players must rely on (rational) *expectations*, i.e. they have to maximize *expected* payoffs. Expectations take the form of *beliefs*, that is, of probability distributions over different states in which a player is called to play.

In order to understand how signaling games work consider the following extensive game.[46]



In this game the sender is player 1 and the receiver is player 2. Sender may be of type I and type II (i.e. $T = \{I, II\}$) with probability $p = {}^{1}/_{2}$, that is Nature moves first choosing sender's type according to p. Then, player 1, knowing his own type, faces the choice between Left or Right (i.e. $M = \{L, R\}$); after that, player 2 moves, choosing between Up and Down (i.e. $A = \{U, D\}$. When player 2 chooses, he does not know with certainty the real type of player 1, he just observes what player 1's message; this is depicted in the picture by the dashed lines connecting two nodes representing the *information sets* for player 2. Therefore, player 2 makes his decision under uncertainty, since he does not know which type of player 1 is facing and, as a consequence, player 1 does not know player 2's move: in this sense, backward induction is unusable in finding a solution to this game.

Indeed in this game players must rely on their *beliefs*. In this context a system of beliefs is defined as a probability distribution over the nodes at every information set. In fact, player 2 should have beliefs about the type of player 1 given the message observed. In the extensive game, these are $r = \Pr(I|L)$ and $q = \Pr(I|R)$.

Clearly this system of beliefs cannot be just anything: it must be constructed following Bayes' rule. The essence of Bayesian approach is to provide a mathematical rule

[46] The game is essentially a remake of "quiche or beer" game introduced by Cho and Kreps (1987).

explaining how one should change his/her existing beliefs in the light of new evidence. Mathematically, given an event A_1 and a subsequent event B, Bayes' rule states that

$$\Pr(A_1|B) = \frac{\Pr(B|A_1)\Pr(A_1)}{\Pr(B)}, \qquad [3.4]$$

where $\Pr(A_1|B)$ is defined as the *posterior* probability, i.e. the probability that event A_1 occurs when the subsequent event B is observed; $\Pr(B|A_1)$ is defined as the *likelihood*; $\Pr(A_1)$ is the *prior* probability that event A_1 occurs; while $\Pr(B) = \sum_i \Pr(B|A_i)\Pr(A_i)$ is a normalizing constant called *marginal likelihood*.

Hence, since the beliefs must be consistent with Bayes' rule, then in our game r and q must be such that

$$r \equiv \Pr(I|L) = \frac{\Pr(L|I)\Pr(I)}{\Pr(L|I)\Pr(I) + \Pr(L|II)\Pr(II)}, \qquad [3.5a]$$

$$q \equiv \Pr(I|R) = \frac{\Pr(R|I)\Pr(I)}{\Pr(R|I)\Pr(I) + \Pr(R|II)\Pr(II)}. \qquad [3.6a]$$

$\Pr(I)$ is exactly the probability by which Nature chooses player 1's type; thus $\Pr(I) = p$. Now, following the extensive game described above, define $\Pr(L|I) = a$ and $\Pr(L|II) = b$.[47] Hence in our game r and q are such that

$$r = \frac{ap}{ap + b(1-p)}, \qquad [3.5b]$$

$$q = \frac{(1-a)p}{(1-a)p + (1-b)(1-p)}. \qquad [3.6b]$$

Now we can introduce the concepts of both *sequential rationality* and *perfect Bayes-Nash equilibrium.*

Definition 1. *A mixed strategy s_i is sequentially rational for player i if there does not exist a deviation $s'_i \in S_i$ that strictly increases utility u_i of player $i \in N$, given a system of beliefs b_i at any information set $h_i \in H_i$ of player i; that is*

[47] Note that a represents the probability that player 1 chooses left when he is of type I. Hence a is essentially the *mixed strategy* of playing left for player 1 when he is of type I. Therefore prior beliefs are derived from player 1's mixed strategy; this fact is essential in the definition of Perfect Bayes-Nash Equilibrium of the next page.

$$u_i^{h_i}(s_i|b_i) \geq u_i^{h_i}(s'_i|b_i), \qquad for\ all\ \ s'_i \in S_i\ and\ all\ h_i \in H_i.$$

Definition 2. *A profile of mixed strategies*[48] *and a system of beliefs constitute a perfect Bayes-Nash equilibrium if and only if:*

1. *The strategy profile is sequentially rational given the system of beliefs*[49]*;*
2. *The system of beliefs is derived from the profile of mixed strategies by Bayes' rule whenever possible (i.e. at every information set that is reachable with positive probability).*

In signaling games there are two opposite types of equilibria:[50] *pooling equilibria* and *separating equilibria.* They are defined as follow.

Definition 3. *A pooling equilibrium is an equilibrium in which all types of sender send the same message.*

Definition 4. *A separating equilibrium is an equilibrium in which all types of sender send different messages.*

Let us find first separating equilibria in our game. In this case if player 1 type I chooses Left than type II will choose Right and vice-versa. Now consider this case. Player 1 chooses with *certainty* Left if he is of type I, i.e. he chooses Left when he observes type I with probability 1: that is $\Pr(L|I) = a = 1$. While, observing type II he chooses Right with probability 1, i.e. $\Pr(R|I) = b = 0$. Recalling that Nature moves with probability $p = {}^{1}/_{2}$, the player 2's system of beliefs is given by $r = 1$ and $q = 0$. This means that player 2 knows exactly in which node he is called to move (like a complete information game); that is, he recognizes the real type of the sender. Therefore player 2 will always play Up in order maximize his utility.

Now define this profile of strategies for the two players as $((L, R), (U, U))$. In order to be a Nash equilibrium, we have to provide that (L, R) is a best response for player 1 to

[48] A profile of (mixed) strategies (or strategy profile) is a set of (mixed) strategies for all players which fully specifies the actions of the game. A strategy profile must include one and only one strategy for every player.

[49] That is *sequential rationality* specified in definition 1 applies to every $i \in N$.

[50] With more than two types of senders there might also exist equilibria where some types choose the same message and some types choose to send different messages. These are called *semi-separating equilibria*.

(U, U). And indeed there is no other strategy that gives a greater expected payoff to player 1 given the player 2's strategy (U, U).
Therefore, defining N as the set of *perfect Bayes-Nash equilibria*, we have that:

$$\big((L, R), (U, U)\big) \in N. \qquad [3.7]$$

The same kind of considerations can be made in the opposite case of separating equilibrium, in which player 1 chooses Right when he is of type I and Left when he is of type II. In this case $r = 0$ and $q = 1$, and as before player 2 recognizes the real type of player 1. In order to maximize his utility he will choose strategy (D, D) and player 1's best response to player 2's strategy will be exactly (R, L); hence

$$\big((R, L), (D, D)\big) \in N. \qquad [3.8]$$

In pooling equilibria player 1 chooses Left or Right regardless of his type. Consider first the case where both types choose Left. As before, this can be described saying that both types of player 1 play Left with probability equal to 1, that is $a = 1$ and $b = 1$. Hence, in this case $r = {}^{1}\!/_{2}$ while q cannot be computed.[51] The intuition behind this result is the following: since all types signal the same, the receiver is not able to recognize real sender's type with certainty, i.e. player 2 does not know clearly in which node of the game he is called to move. This is evident if one notes that *posterior* probability r equals *prior* probability p.
Now let us compute the expected payoff for player 2 when Left is observed. He gains 2 playing Up and 4 playing Down: therefore he will play D. Now, looking at this result, one might think that $((L, L), (D, D))$ is a perfect Bayes-Nash equilibrium. But (L, L) is not a best response to (D, D) since player 1 may increase his expected payoff separating, i.e. applying strategy (R, L); hence this cannot be an equilibrium. However, (L, L) is a best response to player 2's strategy (D, U), since there is no other strategy that strictly increases player 1's payoff. Now we require that player 2 effectively chooses Up when observing Right. Clearly, player 2 will choose Up in face of Right only if his expected payoff is greater than the one if he plays Down. Therefore we require that

[51] Indeed substituting $a = b = 1$ in the equation of q we get $q = \frac{0}{0}$.

$$8q < 4(1-q) \iff q < \frac{1}{3}. \qquad [3.9]$$

Hence if $q < {}^{1}\!/_{3}$ player 2 finds it better to apply strategy (D, U). This means essentially that strategy (D, U) is *sustained by the belief* that, observing right, it is more likely that player 1 is of type II. Therefore for $r = {}^{1}\!/_{2}$ and $q < {}^{1}\!/_{3}$,

$$\big((L, L), (D, U)\big) \in N. \qquad [3.10]$$

If $q > {}^{1}\!/_{3}$ then player 2 finds it better to apply strategy (D, D) than applying (D, U) and player 1 has convenience to separate, i.e. no pooling equilibrium will be observed.

The same sort of considerations can be made when player 1 applies pooling strategy (R, R). In this case both $a = b = 0$, $q = {}^{1}\!/_{2}$ while r cannot be computed. Hence, observing Right player 2 chooses Down given belief $q = {}^{1}\!/_{2}$. However, (R, R) is a best response to player 2's strategy (U, D), then in order to have an equilibrium we require that player 2 chooses Up when observing Right. Thus, we require that

$$4r > 8(1-r) \iff r > \frac{2}{3}. \qquad [3.11]$$

When $r > {}^{2}\!/_{3}$ player 2 applies strategy (U, D). In this case, strategy (U, D) is *sustained by the belief* that, observing Left, it is more likely that player 1 is of type I. Therefore for $r > {}^{2}\!/_{3}$ and $q = {}^{1}\!/_{2}$,

$$\big((R, R), (U, D)\big) \in N. \qquad [3.12]$$

Similarly as before, if $r < {}^{2}\!/_{3}$ then player 2 finds it better to apply strategy (D, D) than applying (U, D) and player 1 has convenience to separate, i.e. no pooling equilibrium will be observed.

3.3.2 Spence's Job Market Signaling

The first application of signaling games in economics was Michael Spence's model "Job market signaling" (1973). In the model, Spence developed a game where a community of jobless people faces one employer. The community is divided into two groups based on individual productivity: in Group I there are people with *low* productivity[52] and in Group II people with *high* productivity. The employer is willing to offer a greater salary to people with *high* productivity than that to people with *low* productivity. The problem is that "in the most job markets the employer is not sure of the productive capabilities of an individual at the time he hires him.[…] The fact that these capabilities are not known beforehand makes (investment) a decision under uncertainty"(Spence, 1973 p. 356).

It is assumed that the employer cannot observe potential employees' productivity with certainty. However, there are many information that the employer can consider in order to evaluate potential employees. Spence distinguished between *unalterable attributes* defined as *indices* (sex, age, race etc.), while reserving the term *signals* for *those observable characteristics attached to the individual that are subject to manipulation by him.*

From the set of possible *signals* Spence focused on *education*, that is to a higher education should have corresponded a higher productive capabilities to work[53]. Clearly, education is costly, not only in terms of money, but mostly in terms of *opportunity costs* (time, physic fatigue, etc.); Spence defined these costs as *signaling costs*.

The critical assumption of the model is that *signaling costs are negatively correlated with productive capability*; that is, for a certain level of education people with lower capabilities have to sustain higher costs than those with higher capabilities

Hence the model describes a situation of incomplete information where the employer can only observe with certainty the level of education. Since education in negatively correlated to potential employee's capability, and at the same time higher education is a signal of higher capability (i.e. a higher wage gained), the problem for employees consists in acquiring a level of education that maximizes the difference between wage and education costs; on the other hand, employer generates a system of *beliefs* that must be

[52] Productivity is defined in terms of *capability to perform* the specific job offered by employer.
[53] Another implicit assumption is that the process of education does not improve individual capability.

Bayes' consistent, and offers wage schedules given such *beliefs*. Thus in this signaling game the employer is the Receiver while potential employees are Senders.
Under different systems of *beliefs*, Spence provided the existence of *separating* or *pooling* equilibria. In *separating equilibrium* high capability group signal a different level of education (in particular a higher level of education) than the low capability group; such equilibrium arises when the reward of being recognized with high capability is greater than low capability group payoff and, at the same time, education cost for low capability group is too high. Hence it must be the case that

$$u_H - c_H \geq u_L \geq u_H - c_L,$$

where u_H is high capability individual's payoff; c_H is education cost for high capability type; u_L is low capability individual's payoff; and c_L is education cost for low type, such that $c_H < c_L$. If such condition does not hold, then one of the two group may have an incentive not to separate, that is to declare the same as the other group. In this case we will observe a *pooling equilibrium*.

3.4 Milgrom and Roberts' Model

3.4.1 Introduction

Milgrom and Roberts (1982) designed a model of entry barriers with incomplete information where both established and potential entrant firms are ignorant about other's (constant) marginal cost, while all the other features of the market are common knowledge. It is a two-period model: in first period, which can be called the *pre-entry* period, established firm chooses its output/price policy in face of entry; in second period, i.e. the *post-entry* period, firms compete in Cournot duopoly if potential firm enters the market; otherwise incumbent enjoys pure monopoly. Established firm will choose the price/output policy that maximize the sum between *pre-entry* period profit and discounted *post-entry* period profit, while prospective entrant firm will come into the market if its expected profit is nonnegative. The main assumption of the model concerns post-entry period: if potential firm enters the market, then both firms immediately know rival's marginal cost and Cournot-Nash equilibrium is reached. Therefore *uncertainty* concerns only the first period of the model.

As already said, if the incumbent acts *naively*, i.e. it ignores the threat of entry, prospective entrant firm is able to infer incumbent's marginal cost making its decision of entry with *certainty*; knowing this, established firm will try to fool the rival, changing pre-entry price/output policy to its own advantage.

The model is essentially a signaling game and, therefore, it must be seen as one of the applications of signaling games to economic problems started with Spence's work in 1973. Here established firm acts as the Sender, and the prospective entrant firm as the Receiver. In fact established firm has a certain type, i.e. a certain marginal cost, inside the type space, i.e. the set of feasible marginal costs in industry. Nature moves first choosing a type of established firm with a given probability. Prospective entrant firm can only observe the *message,* that is pre-entry output/price policy, but it cannot observe other's marginal cost. In this model *signaling costs* consist in the loss deriving from the application of a price different from monopoly price, in the first period. That is, let $\hat{p}$ equilibrium pre-entry price and p^m monopoly price, then *signaling cost* is given by $\pi(p^m) - \pi(\hat{p})$.

The importance of cost advantage has been shown in section 3.1: firms with low marginal costs are characterized by high reaction curves, which in turn affect post-entry equilibrium in a situation much more profitable for the low cost firm. Here comes the importance of understanding the real value of incumbent's marginal cost for prospective entrant firm and vice-versa.

As in all signaling games, we will observe a (large) *multiplicity* of equilibria, both *pooling* and *separating*. This is a first limit of the model that, however, refers all models with a degree of *uncertainty.* Hence in their analysis, Milgrom and Roberts concentrate on specific families of equilibria that are relevant in their results, and provides the existence of Bayes- consistent *beliefs* that sustain such equilibria. In this context, Milgrom and Roberts' *modus operandi* is polar to that of Spence.

Another limit of the model concerns time horizon. Since established firm maximizes its profits over two period, then if entry is deterred it enjoys pure monopoly profit since no entry is allowed. This has consequences on the trend of equilibrium price with important consequences for social welfare.

Suppose established firm deters successfully entry through a first period limit pricing strategy. In this model, when entry is no more threatened, the incumbent sets the monopoly output, and the respective price emerges. Therefore, limit price is not maintained indefinitely, but it is set with the precise purpose of impeding entry, and then price rises up to the monopoly level. This approach appears incoherent with standard limit price theory so far discussed in which entry-preventing price is assumed to be maintained indefinitely. This problem has a clear implication in welfare analysis on entry barriers. Assuming that the monopoly output is the worst situation for social welfare, in complete information models entry-preventing price is typically higher than that when entry is accommodated, but is surely lower than monopoly price[54]; therefore the simple threat of entry has a positive social welfare effect *de facto*; if entry occurs, industry enjoys typically a Stackelberg equilibrium price while, if entry is deterred, the limit price lower than the monopoly one. On the contrary, in Milgrom and Roberts entry has surely a short-run effect by lowering first period price; but if entry is deterred, then price rises immediately to the initial monopoly level.[55]

However, despite its limits, Milgrom and Robert's model on entry barriers is a fundamental work on entry barriers.

First because they provided an analysis where oligopolistic firms are not *open boxes* in which anyone can take a look realizing with certainty how they work. This is of great relevance if one considers that firms typically do everything possible to ensure that no one outside firms can understand their functioning. And if the competition is between established and potential entrant firms, then it is really hard to believe that incumbent is able to see the mechanism of a firm which is just *potential*, unless it already operates in another (observable) market.

Second, the authors clarify the reason why established firms can discourage entry by charging a low pre-entry price, that is how low prices could deter entry.

[54] In this context we are referring to the case of *entry effectively impeded*, and not to entry blockaded. Indeed in the last case the simple monopoly policy is sufficient to deter entry, and monopoly price is maintained.

[55] This approach is coherent with what suggested by Osborne (1973) for which the more profitable strategy for established firm might be "*to reduce the price when entry threat is imminent and raise it when the threat has subsided.*"

One possibility is that the price of established firm has a commitment value, i.e. entrants expect that pre-entry price will prevail after entry. In Modigliani (1958) this assumption is enclosed in Sylos' postulate, and it has been already shown that is not a credible threat. Moreover, as Tirole (1988) has pointed out, *entry into markets is a decision that covers a period of many months or years, whereas a price can often be changed within few days or weeks. Consequently, any loss that a potential entrant may suffer from a low pre-entry price is negligible.* Another possibility, provided by Dixit (1980), is that low pre-entry price is related to high pre-entry capacity. But in that case, the source of entry barrier is incumbent's capacity rather than its price.

Milgrom and Roberts intervened in the debate clarifying the role of low pre-entry price. In fact in their model, pre-price incorporates information about market profitability for potential entrants. In particular, by the positive relationship between pre-entry price and established firm's cost function, low price conveys bad news to potential entrants, in the sense that it signals the high degree of competitiveness of established firm. In this context Milgrom and Roberts' contribution can be considered as another way in order to overcome Sylos' postulate, since the two authors demonstrated the existence of equilibrium strategies in which established firms set prices lower than the profit-maximizing ones without assuming that potential entrants take the output of incumbents as given.

Finally, Milgrom and Roberts' model shows that limit pricing strategy occurs in a situation of incomplete information too, but at the same time it may not limit entry. Indeed, by the existence of many equilibria, in some of them limit prices will emerge without, however, deterring entry. In this sense, limit price is not always successful, with all the consequences from the standpoint of a welfare analysis on entry barriers.

3.4.2 A Simplified Model

Let us give a simplified version of Milgrom and Roberts' model that contains many of the relevant features.[56] It is a two-period game with one established firm (firm 1) and one

[56] This analysis follows Fundeberg and Tirole's (1986) contribution.

prospective entrant (firm2): in first period, the monopolist chooses a price[57] p_1; in second period firm 2 observes p_1 and decides whether to enter the market or not. All the relevant parameters of industry (such as aggregate demand and cost functions) are common knowledge but firms' marginal costs. In particular cost functions of both firms are characterized by constant marginal cost, i.e. $C_i(x_i) = c_i x_i$ with $i = 1,2$. Moreover, if firm 2 comes into the market, both firms immediately recognize other's marginal cost.

First, let us focus on prospective entrant firm's decision in second stage. In this simplified model, established firm's marginal cost c_1 can take two values $\overline{c}_1$ and $\underline{c}_1$, with $\overline{c}_1 > \underline{c}_1$, each with probability $1/2$.[58] Firm 2's profit, net of entry cost, is $\pi_2^d(\overline{c}_1)$ or $\pi_2^d(\underline{c}_1)$, and zero if it does not enter.[59] To have a non-trivial problem assume that

$$\pi_2^d(\underline{c}_1) < 0 < \pi_2^d(\overline{c}_1), \qquad [3.13]$$

i.e. if entrant firm has complete information, it would enter only if the incumbent has high cost. Moreover, assume that without any market information, entrant firm would want to enter. Assuming risk neutrality, this implies that

$$\frac{1}{2}\left(\pi_2^d(\underline{c}_1) + \pi_2^d(\overline{c}_1)\right) > 0. \qquad [3.14]$$

Finally, let $\overline{\pi}_1(p_1)$ and $\underline{\pi}_1(p_1)$ denote respectively the high cost and the low cost incumbent's first period profit function[60]; and let $\delta\overline{\pi}_1{}^m$ and $\delta\overline{\pi}_1{}^d$ denote the high cost incumbent's discounted second period profit when it is in a situation of monopoly or duopoly (the same for low cost firm) such that $\overline{\pi}_1{}^m > \overline{\pi}_1{}^d$.

Now assume that established firm behaves naively, i.e. it ignores the threat of entry; it chooses the monopoly price in the first period $\underline{p}_1^m$ or $\overline{p}_1^m$ depending if it has low costs or high costs, and such that $\overline{p}_1^m > \underline{p}_1^m$. In this case prospective entrant firm can infer firm

[57] Therefore we concentrate on price and not on quantity. However firms compete in the second stage on quantities, thus firms cannot choose directly the price, which is rather the result of output choices.
[58] Indeed, the game of *incomplete information* is replaced by a game of *complete* but *imperfect information* through Harsanyi's transformation, in which Nature moves first selecting $\overline{c}_1$ or $\underline{c}_1$ according to the given probability.
[59] Common knowledge variables have been omitted in the definition of firm 2' profit.
[60] It is increasing and concave in firm's output.

1's marginal cost; then it will enter if $p_1 = \overline{p}_1^m$. High cost established firm will be willing to fool prospective entrant firm (and sacrifice first period monopoly profit) only if

$$\overline{\pi}_1{}^m + \delta\overline{\pi}_1{}^d < \overline{\pi}_1(\underline{p}_1{}^m) + \delta\overline{\pi}_1{}^m; \qquad [3.15]$$

i.e. only if the sum of first period monopoly profit and second-period discounted duopoly profit is lower than the sum of first period profit when firm 1 pretends to be a low cost firm and second-period discounted monopoly profit.

Now consider rational firms. The monopolist chooses a first period price that depends on its cost: $p_1(c_1)$. Then the entrant observes p_1 and forms *beliefs* about c_1. Let $\underline{q}(p_1)$ be the posterior probability that established firm has low cost associated to firm 2's beliefs (and $\overline{q}(p_1) = 1 - \underline{q}(p_1)$ the probability that established firm has high cost). We require such probability distribution be Bayes-consistent with monopolist strategy on p_1. Then prospective entrant firm will compare

$$\underline{q}(p_1)\pi_2^d(\underline{c}_1) + \overline{q}(p_1)\pi_2^d(\overline{c}_1) \lesseqgtr 0 \qquad [3.16]$$

in order to decide whether to enter or not. Therefore established firm can influence firm 2' entry choice through p_1. We then require p_1 to be intertemporally optimal for the incumbent given its cost and prospective entrant's reaction to p_1.
Thus, this is basically a *signaling game* and there can be two regimes of equilibria:

a) *Pooling equilibria*: monopoly's price in unrelated to c_1 and every firm declares the same;
b) *Separating equilibria*: different types of monopolists charge different prices in order to be recognized.

To find a perfect Bayesian equilibrium we consider a particular regime, compute entrant firm's belief associated with chosen regime and check that different types of monopolists want to behave consistently with this regime.

a) *Pooling equilibrium*: in this case monopoly's price does not depend on c_1 and prospective entrant firm has no information about monopolist's marginal cost function. Therefore firm 2's posterior probability distribution after observing p_1 is $\underline{q}(p_1) = \overline{q}(p_1) = {}^1\!/_2$ and prospective entrant comes into the market.

But each type of monopolist would then prefer (strictly prefer for at least one type) playing its first period monopoly price since it cannot impede entry and at the same time, it is not maximizing its first period profit. Hence, firms set different prices; therefore there cannot exist a pooling equilibrium.

b) *Separating equilibrium*: in this case p_1 is related to c_1. Here there are two necessary conditions: first, each type of established firm wants to be distinguished by entrant firm, therefore low-cost type does not want to pick high-cost type's equilibrium price and vice-versa; second that the separating equilibrium price correspond to a *perfect Bayesian Nash equilibrium.*

Let $\overline{p}_1$ and $\underline{p}_1$ denote high and low cost incumbent's first period equilibrium price. The entrant comes into the market if it observes $\overline{p}_1$. If established firm is a high-cost type, it will set its monopoly price $\overline{p}_1^m$ since this price maximizes high-cost incumbent's first period profit and cannot induce more entry than $\overline{p}_1$ does. Hence firm 2 enters when observing high cost pre-entry price $\overline{p}_1 = \overline{p}_1^m$ and firm 1 payoff will be $\overline{\pi}_1{}^m + \delta\overline{\pi}_1{}^d$. This strategy is an equilibrium for high-cost type if this payoff is greater than that when entry is deterred and equals $\overline{\pi}_1(\underline{p}_1) + \delta\overline{\pi}_1{}^m$. Hence,

$$\overline{\pi}_1{}^m + \delta\overline{\pi}_1{}^d \geq \overline{\pi}_1(\underline{p}_1) + \delta\overline{\pi}_1{}^m \Leftrightarrow \qquad [3.17]$$

$$\Leftrightarrow \overline{\pi}_1{}^m - \overline{\pi}_1(\underline{p}_1) \geq \delta(\overline{\pi}_1{}^m - \overline{\pi}_1{}^d).$$

Similarly, the low-cost type must be maximizing its profit by choosing $\underline{p}_1$. In this case, if established firm charges its monopoly price it will get at worst $\underline{\pi}_1^m + \delta\underline{\pi}_1^d$ (i.e. at worst it induce entry). Hence, in order to be an equilibrium, price for low-cost incumbent $\underline{p}_1$ must be such that

$$\underline{\pi}_1^m + \delta\underline{\pi}_1^d \leq \underline{\pi}_1(\underline{p}_1) + \delta\underline{\pi}_1^m \;\Leftrightarrow \qquad [3.18]$$

$$\Leftrightarrow \; \underline{\pi}_1^m - \underline{\pi}_1(\underline{p}_1) \leq \delta(\underline{\pi}_1^m - \underline{\pi}_1^d).$$

These two conditions define and interval of equilibrium values for $\underline{p}_1$: denote this set of equilibrium prices for low-cost established firm as E; hence $\underline{p}_1 \in E$.

However, in order to specify this set, we need additional assumptions on second-period duopoly outcomes, which in turn are affected by the assumptions on demand and cost function. Let us skip for the moment this issue, and demonstrate that such price really constitutes an equilibrium.

In order to do this, suppose that high-cost incumbent does not choose $\overline{p}_1^m$ and that low-cost one does not choose $\underline{p_1} \in E$. When a price that differs from these two is observed, prospective entrant firm is facing an unexpected event and Bayes' rule does not pin down firm 2's posterior belief. In order to demonstrate that equilibrium prices are best response for firm 1, we choose the worst situation for the incumbent firm and verify that it will not deviate from equilibrium prices.

Therefore, concentrate on the most aggressive situation in which entrant firm strongly believes that incumbent is a high cost firm, i.e. $\underline{q}(p_1) = 0$; doing so, prospective entrant comes into the market with certainty. Now, let us check that no incumbent type wants to deviate from equilibrium prices. Indeed, high cost firm is not willing to lose its first period monopoly price choosing another price that induces entry anyhow. Similarly for low-cost established firm, since there is no possibility to deter entry it will not deviate from its equilibrium price set since for every $\underline{p_1} \notin E$, $\underline{\pi}_1^m > \underline{\pi}_1(\underline{p_1})$.[61]

Now we can analyze the set of equilibrium prices for low-cost established firm. Indeed $\underline{p_1}$ must satisfy both [3.17] and [3.18]: let $\tilde{p}$ denote the level of $\underline{p_1}$ that satisfies [3.17] with equality, and $\tilde{\tilde{p}}$ the one that satisfies [3.18] with equality. At this point, let us define the two functions

$$y = \pi_1^m - \pi_1(\underline{p_1}) \qquad [3.19]$$

$$z = \delta(\pi_1^m - \pi_1{}^d) \qquad [3.20]$$

in the space $(y, \underline{p_1})$; the following picture should give a suggestion of these two functions.

[61] Recall that $\underline{\pi}_1^m - \underline{\pi}_1(\underline{p_1}) \leq \delta(\underline{\pi}_1^m - \underline{\pi}_1^d)$ if $\underline{p_1} \in E$. Since entry cannot be deterred, the right-hand side of the inequality is equal to zero and $\underline{\pi}_1^m = \underline{\pi}_1(\underline{p_1})$. On the contrary, if $\underline{p_1} \notin E$ then $\underline{\pi}_1^m - \underline{\pi}_1(\underline{p_1}) > 0$; therefore $\underline{\pi}_1^m > \underline{\pi}_1(\underline{p_1})$.

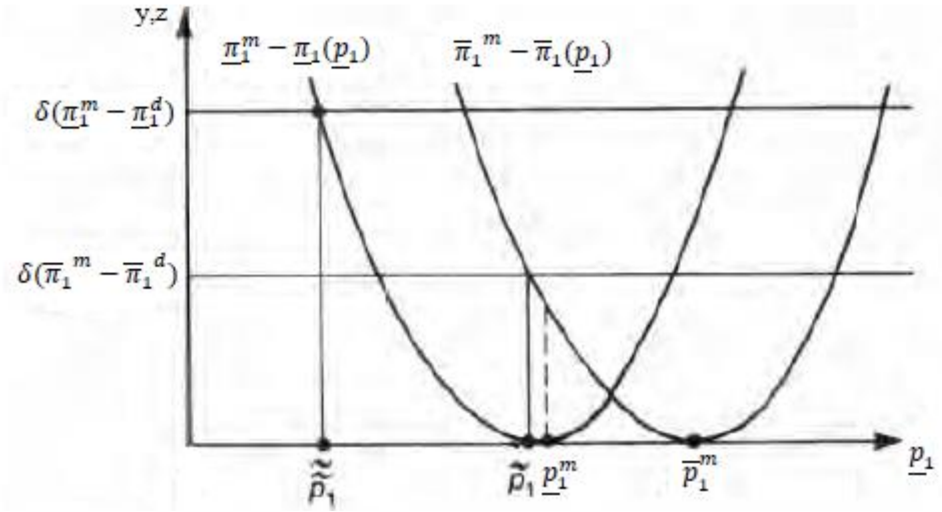


FIG 14 – The set of equilibrium prices in separating equilibrium. (Source: Tirole, 1988)

Figure 14 represents both functions y and z for high and low cost firm 1. The two function y are decreasing up to monopoly price and increasing thereafter. But let us focus on the two types of function z. In particular we should understand if $\left(\overline{\pi}_1{}^m - \overline{\pi}_1{}^d\right) \lesseqgtr \left(\underline{\pi}_1^m - \underline{\pi}_1^d\right)$; i.e., we should understand the trend of z with respect to c_1; that is

$$\frac{\partial z}{\partial\, c_1} \equiv \frac{\partial(\pi_1^m - \pi_1{}^d)}{\partial\, c_1} = \frac{\partial \pi_1^m}{\partial c_1} - \frac{\partial \pi_1^m}{\partial c_1} \lesseqgtr 0. \qquad [3.21]$$

Let us find the sign of this derivative using envelope theorem. Indeed π_1^m is given by the solution of monopolist's maximization problem, and can be defined as $\pi_1^m = f(x_1^m(\, c_1), c_1)$ where x_1^m is the *argmax* of π_1^m; while $\pi_1{}^d$ is the profit given by the maximization in duopoly, which can be define as $\pi_1{}^d = g(x_1^d(c_1, c_2), x_2^d(c_1, c_2), c_1)$ where x_1^d maximizes $\pi_1{}^d$ given x_2^d. By the envelope theorem we have that

$$\frac{\partial \pi_1^m}{\partial c_1} = \frac{\partial f}{\partial c_1}, \qquad [3.22a]$$

$$\frac{\partial\, \pi_1{}^d}{\partial c_1} = \frac{\partial g}{\partial c_1} + \frac{\partial g}{\partial x_2^d}\,\frac{\partial x_2^d}{\partial c_1}. \qquad [3.23a]$$

Since marginal cost is assumed to be constant, it follows that

$$\frac{\partial \pi_1^m}{\partial c_1} = -x_1^m, \qquad [3.22b]$$

$$\frac{\partial \pi_1{}^d}{\partial c_1} = -x_1^d + \frac{\partial g}{\partial x_2^d}\frac{\partial x_2^d}{\partial c_1}. \qquad [3.23b]$$

Where x_1^m is the monopoly output while x_1^d is firm 1's duopoly output. Moreover the term $\frac{\partial g}{\partial x_2^d}\frac{\partial x_2^d}{\partial c_1}$ is negative.[62] Finally we get

$$\frac{\partial(\pi_1^m - \pi_1{}^d)}{\partial\, c_1} = -x_1^m - \left(\frac{\partial g}{\partial x_2^d}\frac{\partial x_2^d}{\partial c_1} - x_1^d\right) < 0 \;\Leftrightarrow\; x_1^m > x_1^d + \frac{\partial g}{\partial x_2^d}\frac{\partial x_2^d}{\partial c_1}. \qquad [3.24]$$

If the inequality holds, function z is decreasing in marginal cost[63]; therefore, as in figure 14, $\left(\overline{\pi}_1{}^m - \overline{\pi}_1{}^d\right) < \left(\underline{\pi}_1^m - \underline{\pi}_1^d\right)$.

Suppose to be in this case as shown in figure 14. The intersection between curves y and z for each level of cost, represents the boundaries of equilibrium price levels for low cost incumbent firm. Indeed for every $\underline{p}_1 > \tilde{\tilde{p}} \Rightarrow \underline{\pi}_1^m - \underline{\pi}_1(\underline{p}_1) < \delta(\underline{\pi}_1^m - \underline{\pi}_1^d)$, while for every $\underline{p}_1 < \tilde{p} \Rightarrow \overline{\pi}_1{}^m - \overline{\pi}_1(\underline{p}_1) > \delta(\overline{\pi}_1{}^m - \overline{\pi}_1{}^d)$; thus $E = [\tilde{\tilde{p}}, \tilde{p}]$ and $\underline{p}_1 \in [\tilde{\tilde{p}}, \tilde{p}]$.

Now some consideration about $E = [\tilde{\tilde{p}}, \tilde{p}]$ is required. Indeed, even if every $\underline{p}_1 \in [\tilde{\tilde{p}}, \tilde{p}]$ is in principle an equilibrium price for low-cost established firm, surely firm 1 will choose the equilibrium price closest to its monopoly price in order to maximize its profit in the first period. This is true if we assume, as we have always done so far, that firms' profit function is increasing and concave, with maximum at the monopoly price/output.

In this context, low-cost firm's monopoly price may or may not belong to the set of equilibrium prices. This depends on the gap between the cost of high and low-type firm, which in turn affects the gap of the two monopoly prices. However if the two types are taken *sufficiently different* each other, $\underline{p}_1{}^m$ does not belong to E as in figure 14; therefore low-cost firm 1 will choose $\tilde{p}$, i.e. *low cost firm will charge the highest $\underline{p}_1$ such that the high-cost type's first period loss of charging $\underline{p}_1$ exceeds its gain from deterring entry.*

[62] Indeed under regular conditions, $\frac{\partial g}{\partial x_2^d} < 0,$ since firm 1's profit decreases if firm 2's equilibrium output increases, while $\frac{\partial x_2^d}{\partial c_1} > 0,$ since firm 2's equilibrium output usually increases with respect to firm 1's marginal cost.

[63] The sign depends essentially on the assumption we made about demand function, and on the amount of marginal costs.

Hence in order to separate and be recognized as a low-cost type, established firm will set a *limit price*.

3.4.3 The model with an interval of continuous values of c_1

Let us concentrate on the model of entry-barrier with complete information developed by Milgrom and Roberts (1982) in which, instead of being only two possible values of marginal cost, both for established and prospective entrant firms there exists an interval $[\overline{c}_i, \underline{c_i}]$ with $i = 1,2$ of possible values of marginal costs. Each firm knows the interval in which its rival's marginal cost lies but ignores the exact value. Firms produce identical products whose inverse demand function is supposed to be linear, i.e. $P = a - bX$.
Let us recall the main features of the model. It is a two-period game in which: in the first period, established firm chooses its output/price policy; prospective entrant firm, after observing firm's one choice, decides whether to enter the market or not. If it enters, both firms recognize other's marginal cost and compete on quantities, reaching a Cournot equilibrium. Knowing that both firms have cost functions characterized by constant marginal cost, discounted profit functions can be constructed, starting at the second period. If firm 2 does not enter, then firm 1 is a simple monopolist facing no entry threat in the second period, with profit equal to

$$\pi_1^m = \frac{(a - c_1)^2}{4b} \quad with \;\; x_1^m = \frac{a - c_1}{2b}.$$

If firm 2 enters, the two firms choose Cournot equilibrium output levels with certainty, since they know other's marginal cost. Thus, their second-period profits are

$$\pi_i^c(c_i, c_j) = \frac{\left(a - 2c_i + c_j\right)^2}{9b}.$$

Therefore, once entry decision is made, the rest of the play is determined with certainty. Since firm 2 observes first period output of firm 1 before making its entry decision, firm 2's choice can be written as $v(c_2, x_1)$ where only two values of v are possible: $v(c_2, x_1) = 1$ if firm 2 enters, and $v(c_2, x_1) = 0$ if it does not enter.

Recalling that when firm 1 chooses its first period output it is ignorant of c_2, and when firm 2 decides whether or not to enter the market it is ignorant of c_1, let $H_2(c_2)$ be the probability distribution that firm 1 supposes for c_2, and let $H_1(c_1)$ be the distribution that firm 2 supposes for c_1.[64] Moreover, letting $\eta(c_1)$ be firm 1's first period output choice as a function of c_1, expected established firm's discounted profit is given by

$$\Pi_1\big(c_1, E(c_2)\big) = \pi_1(\eta(c_1), c_1) +$$
$$+\delta \int_{\underline{c}_2}^{\overline{c}_2} \{\pi_1^c(c_1, c_2) v\big(c_2, \eta(c_1)\big) + \pi_1^m(c_1)[1 - v(c_2, \eta(c_1))]\}\, dH_2(c_2) \qquad [3.25]$$

where δ is, as before, the discounted factor. On the other side, prospective entrant firm's discounted profit is given by

$$\Pi_2\big(c_2, E(c_1)\big) = \int_{\underline{c}_1}^{\overline{c}_1} [\delta \pi_2^c(c_1, c_2) - F] v(c_2, \eta(c_1))\, dH_1(c_1) \qquad [3.26]$$

where F is the entry fixed cost.

In this framework, a *non-cooperative equilibrium* is defined as a pair of strategies, $\eta(c_1)$ for firm 1 and $v(c_2, \eta(c_1))$ for firm 2 such that neither firm can increase its profit by unilaterally altering its strategy (taking H_1 and H_2 as fixed). Thus, $\{\eta^*(c_1), v^*(c_2, \eta^*(c_1))\}$ is an equilibrium pair of strategies if: (a) for any $c_1 \in [\underline{c}_1, \overline{c}_1]$ and any $\eta(c_1)$ such that $[\underline{c}_1, \overline{c}_1] \longrightarrow \mathbb{R}_+$,

$$\pi_1(\eta^*(c_1), c_1) +$$
$$+\delta \int_{\underline{c}_2}^{\overline{c}_2} \{\pi_1^c(c_1, c_2) v^*\big(c_2, \eta^*(c_1)\big) + \pi_1^m(c_1)\big[1 - v^*\big(c_2, \eta^*(c_1)\big)\big]\}\, dH_2(c_2) \geq$$
$$\geq \pi_1(\eta(c_1), c_1) +$$
$$+\delta \int_{\underline{c}_2}^{\overline{c}_2} \{\pi_1^c(c_1, c_2) v^*(c_2, \eta(c_1)) + \pi_1^m(c_1)[1 - v^*(c_2, \eta(c_1))]\}\, dH_2(c_2) \qquad [3.27]$$

[64] H_1 and H_2, which are essentially firms' *beliefs* of others' marginal costs, incorporate the probability distribution of the different states of Nature.

and (b) for any $c_2 \in [\underline{c}_2, \overline{c}_2]$ and any $v(c_2, \eta^*(c_1))$ with domain $[\underline{c}_2, \overline{c}_2] \times \mathbb{R}_+$ and range $\{0,1\}$,

$$\int_{\underline{c}_1}^{\overline{c}_1} [\delta \pi_2^c(c_1, c_2) - F] v^*(c_2, \eta^*(c_1))\, dH_1(c_1) \geq$$

$$\geq \int_{\underline{c}_1}^{\overline{c}_1} [\delta \pi_2^c(c_1, c_2) - F] v(c_2, \eta^*(c_1))\, dH_1(c_1). \quad [3.28]$$

- ***A numerical example***

First, a numerical example used by Milgrom and Roberts (1982) will clarify the working of the model. Let $a = 10$, $b = 1$, $\underline{c}_1 = 0.5$, $\overline{c}_1 = 2$, $\underline{c}_2 = 1.5$, $\overline{c}_2 = 2$, $\delta = 1$ and $F = 7$. Moreover, assume that the only possible values for c_1 and c_2 are $\{\underline{c}_1, \overline{c}_1\}$ and $\{\underline{c}_2, \overline{c}_2\}$ respectively. Finally, let h_2 the probability that $c_2 = \overline{c}_2$ and let h_1 be the probability that $c_1 = \overline{c}_1$. Then the following can be readily calculated:

$$\pi_1^c(\underline{c}_1, \underline{c}_2) = 12.25 \qquad \pi_2^c(\underline{c}_1, \underline{c}_2) - F = -0.75 \quad [3.29]$$

$$\pi_1^c(\underline{c}_1, \overline{c}_2) = 13.44 \qquad \pi_2^c(\underline{c}_1, \overline{c}_2) - F = -2.31 \quad [3.30]$$

$$\pi_1^c(\overline{c}_1, \underline{c}_2) = 6.25 \qquad \pi_2^c(\overline{c}_1, \underline{c}_2) - F = 2 \quad [3.31]$$

$$\pi_1^c(\overline{c}_1, \overline{c}_2) = 7.11 \qquad \pi_2^c(\overline{c}_1, \overline{c}_2) - F = 0.11 \quad [3.32]$$

$$x_1^m(\underline{c}_1) = 4.75 \qquad \pi_1^m(\underline{c}_1) = 22.56 \quad [3.33]$$

$$x_1^m(\overline{c}_1) = 4 \qquad \pi_1^m(\overline{c}_1) = 16 \quad [3.34]$$

It is immediately evident from equations [3.29] through [3.32] that entry is unprofitable for entrant firm when the incumbent has a low marginal cost and profitable when firm 1 has the high marginal cost.[65]

[65] Therefore this case is similar to the one developed in the previous paragraph: the main difference lies on prospective entrant firm. In the simplified model, it is assumed, for the sake of simplicity, that the entrant has not specific type. The only assumption we made concerned its *propensity to enter*: indeed we assumed that $\frac{1}{2}\left(\pi_2^d(\underline{c}_1) + \pi_2^d(\overline{c}_1)\right) > 0$. As a consequence, it entered the market surely when posterior probability equals prior probability; therefore no pooling equilibrium exists since entry cannot be deterred and at least one type of established firm strictly prefers to set monopoly output. This is exactly equilibrium strategy for low-cost firm 2. However, since in this model firm 2 can be high-cost firm, and in that case it would not

From this example, two different sorts of equilibria exist: a *pooling equilibrium,* at which $\eta^*(\underline{c}_1) = \eta^*(\overline{c}_1)$, and a *separating equilibrium*, at which $\eta^*(\underline{c}_1) \neq \eta^*(\overline{c}_1)$. As already seen, at a separating equilibrium firm 2 can infer the correct value of c_1 by observing established firm's first period output choice; however this is impossible at a pooling equilibrium.

At this point the analysis is focused in showing the existence of specific equilibria inside the set of possible equilibria. As described above, an equilibrium a mutual best response for both players given equilibrium strategy of the other, given a system of beliefs.

Thus, let us demonstrate that the existence of a pooling equilibrium given by the following profile of pure strategies:

$$\eta^*(\underline{c}_1) = \eta^*(\overline{c}_1) = x_1^m(\underline{c}_1) = 4.75$$

$$v^*(\underline{c}_2, \eta^*(\overline{c}_1)) = 1$$

$$v^*(\overline{c}_2, x_1) = \begin{cases} 0 & if\ x_1 \geq 4.75 \\ 1 & otherwise \end{cases}. \qquad [3.35]$$

First note that at the pooling equilibrium, firm 2 decides to enter the market if it has the low marginal cost; however, if it has the high cost, if enters only if $x_1 < 4.75$. In order to demonstrate that such strategies constitute an equilibrium, we have to find a system of beliefs that sustains such equilibrium, i.e. we have to find value for h_1 and h_2 for which η^* a best response to v^* and vice-versa. Therefore expected profits for both firms are[66]

$$\Pi_1 = \begin{cases} \pi_1^m(\underline{c}_1) + \delta\left[\pi_1^m(\underline{c}_1)h_2 + \pi_1^c(\underline{c}_1, \underline{c}_2)(1-h_2)\right] & if\ \ c_1 = \underline{c}_1 \qquad [3.36a] \\ \\ \pi_1^m(x_1^m(\underline{c}_1)) + \delta\left\{\pi_1^m(\overline{c}_1)h_2 + \pi_1^c(\overline{c}_1, \underline{c}_2)(1-h_2)\right\} & if\ c_1 = \overline{c}_1 \qquad [3.37a] \end{cases}$$

$$\Pi_2 = \begin{cases} \left[\pi_2^c(\overline{c}_1, \underline{c}_2) - F\right]h_1 + \left[\pi_2^c(\underline{c}_1, \underline{c}_2) - F\right](1-h_1) & if\ c_2 = \underline{c}_2 \qquad [3.38a] \\ \\ 0 & if\ \ c_2 = \overline{c}_2 \qquad [3.39a] \end{cases}$$

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enter in the market, then a possibility of deterring entry acquire consistency, and a pooling equilibrium can be sustained by a system of beliefs.

[66] $\pi_1^m(x_1^m(\underline{c}_1))$ is high-cost type firm 1 profit when it set low-cost type monopoly output. Therefore, it is not as optimal condition for high-cost monopolist.

and substituting the real values we get for firm 1

$$\Pi_1 = \begin{cases} 22.56(1+h_2) + 12.25(1-h_2) \;\; if \quad c_1 = \underline{c}_1 & [3.36b] \\ \\ 15.44(1+h_2) + 6.25(1-h_2) \;\; if \quad c_1 = \overline{c}_1 & [3.37b] \end{cases}$$

while for firm 2

$$\Pi_2 = \begin{cases} 2h_2 - 0.75(1-h_1) \qquad if \quad c_2 = \underline{c}_2 & [3.38b] \\ \\ 0 \qquad\qquad\qquad\qquad if \;\; c_2 = \overline{c}_2 & [3.39b] \end{cases}$$

Thus, pooling strategies form an equilibrium if $h_2 > 0.67$ and $0.273 < h_1 < 0.953$. Indeed, if firm 1 is a low-cost type equation $[3.36b]$ is optimal for every $0 \leq h_2 \leq 1$, since there does not exist another first period quantity choice that gives a larger payoff; but if it has the high cost, equation $[3.37b]$ is best only if $h_2 > 0.67$, since it must be the case that

$$\pi_1^m\big(x_1^m(\underline{c}_1)\big) + \delta\big\{\pi_1^m(\overline{c}_1)h_2 + \pi_1^c\big(\overline{c}_1, \underline{c}_2\big)(1-h_2)\big\} >$$
$$> \pi_1^m(\overline{c}_1) + \delta\big[\pi_1^c(\overline{c}_1, \overline{c}_2)h_2 + \pi_1^c\big(\overline{c}_1, \underline{c}_2\big)(1-h_2)\big].$$

Similarly for firm 2, if it is a low-cost type $[3.38b]$ is best if $h_1 > 0.273$, since it is such that $2h_1 - 0.75(1-h_1) > 0$, while if it is a high cost type $[3.39b]$ is the best if $h_1 < 0.953$, since it is such that $[\pi_1^c(\overline{c}_1, \overline{c}_2) - F]h_1 + \big[\pi_2^c(\underline{c}_1, \overline{c}_2) - F\big](1-h_1) < 0$.
By choosing $x_1 = 4.75$ in first period, high-cost established firm engages in limit pricing when it has the high marginal cost. Indeed in that case, $x_1 > x_1^m$ and therefore $p(x_1) < p(x_1^m)$. However, if firm 2 has the low marginal cost, the limit pricing has no effect on entry, but if its marginal cost is high, limit pricing prevents entry. Thus, from the standpoint of firm 1, limit pricing is effective with probability h_1, and from the standpoint of firm 2, it is being subjected to limit pricing with probability h_2.

Let us now demonstrate the existence of a separating equilibrium given by the following profile of pure strategies[67]:

[67] Separating equilibrium quantity for low-cost firm is found as in the simplified model. Indeed, in order to separate, $\eta^*(\underline{c}_1)$ must be such that $\overline{\pi}_1{}^m - \overline{\pi}_1(\underline{p}_1) \geq \delta(\overline{\pi}_1{}^m - \overline{\pi}_1{}^d)$ and $\underline{\pi}_1^m - \underline{\pi}_1(\underline{p}_1) \leq \delta(\underline{\pi}_1^m - \underline{\pi}_1^d)$.

$$\eta^*(\underline{c}_1) = 7.2, \qquad \eta^*(\overline{c}_1) = x_1^m(\overline{c}_1) = 4,$$

$$v^*(c_2, x_1) = \begin{cases} 1 \ \ if \ x_1 < 7.2 \\ 0 \ \ otherwise \end{cases}. \qquad [3.40]$$

First note that since $\eta^*(\underline{c}_1)$ is greater than low-cost monopoly output; hence η^* is a limit pricing strategy. In separating equilibrium, observing the equilibrium choice of established firm allows a precise and accurate inference to be made about firm's characteristic. Hence v^* is clearly a best response to firm 1's equilibrium strategy: observing $\eta^*(\underline{c}_1)$ it recognizes the low value of firm 1's marginal cost and therefore it does not enter; on the other side, observing $\eta^*(\overline{c}_1)$ it recognize that established firm is a high-cost type and therefore enters the market. Thus, we need to check that η^* is effectively optimal given v^*. Note first that unless high-cost firm 1 produces $\eta^*(\underline{c}_1)$ it cannot deter entry. But $\eta^*(\underline{c}_1)$ is high enough that $\overline{\pi}_1{}^m + \delta\overline{\pi}_1{}^d \geq \overline{\pi}_1(\underline{p}_1) + \delta\overline{\pi}_1{}^m$, hence established firm finds it better to allow entry. On the other hand, if it produces a quantity lower than $\eta^*(\underline{c}_1)$, it is sure to face entry, and thus its best choice is to produce monopoly output; then $\eta^*(\overline{c}_1) = x_1^m(\overline{c}_1)$ is a optimal strategy for high-cost incumbent. If firm 1 is low-cost type, it has no reason to produce more than $\eta^*(\underline{c}_1)$. And if it would produce less, it surely face entry, and it will produce its monopoly quantity. But $\eta^*(\underline{c}_1)$ is such that $\underline{\pi}_1^m + \delta\underline{\pi}_1^d \leq \underline{\pi}_1(\underline{p}_1) + \delta\underline{\pi}_1^m$, then it prefers to set $\eta^*(\underline{c}_1) = 7.2$; hence, this is the optimal strategy for low-cost established firm.

Thus, in separating equilibrium expected profit for firm 1 is $\Pi_1 = \pi_1^m(\eta^*(\underline{c}_1)) + \delta\pi_1^m(\underline{c}_1) = 39.12$ if it is a low-cost firm, while if $c_1 = \overline{c}_1$,

$$\Pi_1 = \pi_1^m(\overline{c}_1) + \delta\left[\pi_1^c(\overline{c}_1, \overline{c}_2)h_2 + \pi_1^c(\overline{c}_1, \underline{c}_2)(1 - h_2)\right] = 22.25 + 7.11h_2.$$

Firm 2 enters only if first period firm 1's output is lower than 7.2, i.e. only if established firm has high cost, and it will get expected profit equal to $2h_1$ for $c_2 = \overline{c}_2$, and equal to $0.11h_1$ for $c_2 = \underline{c}_2$.

Here too, firm 1 sets limit pricing output part of the time: when it has a low cost. It does not fool prospective entrant firm. On the contrary, it signals its cost to firm 2, and firm 2

From the set of separating quantities, established firm will choose the amount closest to monopoly output, i.e. $\eta^*(\underline{c}_1) = 7.2$.

stays out knowing that entry would be unprofitable. At the same time, the strategy followed by entrant firm makes cost-revealing strategy of firm 1 optimal for firm 1.
A useful way to think about these results is to consider limit pricing as the outcome of competition between the types of established firm, with high cost types attempting to mimic low cost ones and low cost firms attempting to distinguish themselves from the high cost ones. Then, whether a pooling or a separating equilibrium is established, is a matter of whether it is a high or a low cost.

- ***Limit pricing and separating equilibrium in continuous interval***

The numerical example described above shows situations in which the best for established firm is to accommodate entry, setting in the first period the simple profit maximizing output. Therefore this kind of strategy could arise with other specification too, and we are not able to identify the role of limit pricing behavior in this framework. However it can be shown that if there are a continuum of types (i.e. of cost levels) possible for established firms, and if firm 1's equilibrium strategy is a strictly decreasing function in c_1, then it is always better to set a limit pricing in face of a threat of entry.
Suppose that the distribution of c_i is given by a continuous probability density function $h_i(c_i)$ which is positive on $[\underline{c}_i, \overline{c}_i]$ and let us concentrate first on separating equilibria. Assume that entrant firm believes that established firm will play a certain strategy $x_1 = \tilde{\eta}(c_1)$. Since we are concentrating on separating equilibria, firm 2 can infer firm 1's marginal cost through the output choice; indeed $c_1 = \tilde{\eta}^{-1}(x_1)$. Therefore prospective entrant firm will come into the market if and only if its expected profit $\delta\pi_1^c(c_1, c_2) - F$, with $c_1 = \tilde{\eta}^{-1}(x_1)$, is positive. Let $\bar{\bar{c}}_2$ be highest value of c_2 that make entry profitable when $c_1 = \tilde{\eta}^{-1}(x_1)$; therefore it must be the case that $c_2 \leq \bar{\bar{c}}_2(x_1) \equiv g(\tilde{\eta}^{-1}(x_1))$.[68] If $\tilde{\eta}(c_1)$ is monotone decresing, as well as $\tilde{\eta}^{-1}(x_1)$, then $\bar{\bar{c}}_2$ is a singleton. Hence, in this case firm 2's best response is given by

$$v(c_2, x_1) = \begin{cases} 1 & if \ \ c_2 \leq \bar{\bar{c}}_2(x_1) \\ 0 & otherwise \end{cases} \quad . \qquad [3.41]$$

[68] A necessary assumption in order to have a non-trivial problem is that $\bar{\bar{c}}_2 \in [\underline{c}_2, \overline{c}_2]$. Indeed if $\bar{\bar{c}}_2 < \underline{c}_2$ entry is always unprofitable, while if $\bar{\bar{c}}_2 > \overline{c}_2$ entry cannot be deterred.

Now suppose that established firm believes that v is of that general form, so that entry will be deterred if c_2 exceeds some value $\hat{c}_2(x_1)$. Then firm 1's expected profit will be

$$\Pi_1 = \pi_1^0(c_1, x_1) +$$

$$+\delta\left[\int_{\underline{c}_2}^{\hat{c}_2(x_1)} \pi_1^c(c_1, c_2)h_2(c_2)\, dc_2 + \int_{\hat{c}_2(x_1)}^{\overline{c}_2} \pi_1^m(c_1, c_2)h_2(c_2)\, dc_2\right] \qquad [3.42]$$

Where π_1^0 refers to first-period profit for established firm. Maximizing with respect to x_1 we get

$$\frac{\partial \pi_1^0}{\partial x_1} - \delta h_2(\hat{c}_2(x_1))\hat{c}'_2(x_1)\{\pi_1^m(c_1, \hat{c}_2(x_1)) - \pi_1^c(c_1, \hat{c}_2(x_1))\} = 0. \qquad [3.43]$$

Let $R(c_1, c_2) = \pi_1^m(c_1, c_2) - \pi_1^c(c_1, c_2) > 0$ be firm 1's reward from deterring entry. Then

$$\frac{\partial \pi_1^0}{\partial x_1} - \delta h_2\big(\hat{c}_2(x_1)\big)\hat{c}'_2(x_1)R(c_1, \hat{c}_2(x_1)) = 0 \qquad [3.44]$$

must hold. But in equilibrium the conjectures must be correct, i.e. $x_1 = \tilde{\eta} = \eta^*$ and $\hat{c}_2(x_1) = \bar{\bar{c}}_2(x_1) = g(\eta^{*-1}(x_1))$, hence the following equality must hold

$$\frac{\partial \pi_1^0(c_1, \eta^*(c_1))}{\partial x_1} - \frac{\delta R\big(c_1, g(c_1)\big)h_2(g(c_1))g'(c_1)}{\partial \eta^* / \partial\, c_1} = 0. \qquad [3.45]$$

Note that so long as η^* is differentiable in c_1,

$$\frac{\delta R\big(c_1, g(c_1)\big)h_2(g(c_1))g'(c_1)}{\partial \eta^* / \partial\, c_1} < 0, \qquad [3.46]$$

since $\frac{\partial \eta^*}{\partial c_1} < 0$ is negative by hypothesis, while all the other elements are positive; therefore the equality holds if and only if $\frac{\partial \pi_1^0}{\partial x_1} < 0$. Thus, the simple monopoly solution x_1^m which is defined by $\frac{\partial \pi_1^0}{\partial x_1} = 0$ cannot arise in equilibrium.[69] Indeed, if the entrant were to believe that $\tilde{\eta} = x_1^m(c_1)$ responding optimally, then by a little increase in output

[69] Note that the level of $x_1 : \frac{\partial \pi_1^0}{\partial x_1} < 0$ is greater than $x_1 : \frac{\partial \pi_1^0}{\partial x_1} = 0$ by the assumption that profit function is increasing and concave in output. Hence, firm 1 engages in limit pricing output.

$x_1^m(c_1) + \epsilon = \tilde{\eta}(c'_1)$ established firm can eliminate the threat of entry from firms in the interval of $c_2 \in (g(c'_1), g(c_1)]$. Indeed, with a continuum of possible types, is always possible to find an $\epsilon > 0$ such that

$$\pi_1(x_1^m(c_1) + \epsilon) + \delta\pi_1^m > \pi_1^m + \delta\pi_1^c.$$

This means that the increase in first period output has a negative but negligible effect on first period profit, but a non-negligible positive effect on the total amount on expected profit due to the reward on deterring entry.

Therefore in this model so long as (i) it is more profitable to be a monopolist than sharing the market, (ii) beliefs are given with positive probability, and (iii) higher costs for established firm encourage entry, firm must be limit pricing in a separating equilibrium.

Finally, although we have concentrated on separating equilibria, pooling equilibria are conceptually possible in the continuum of types framework. In any pooling equilibrium, all established firm types are better off producing the equilibrium output η^* than they are if they deviate from equilibrium and facing a different probability of entry related to the new output. This happens tipically when the deviation from η^* induce an *almost sure entry*; then if high-cost type firms (with respect to the type related to η^*) are willing to produce η^* a pooling equilibrium will be maintained.

In conclusion, by modeling the decision making of both established firm and prospective entrant firm through a game of incomplete information, it has been shown two important results: first, it is in the interest of established firm to reduce price, i.e. to apply a limit pricing strategy, in order to deter entry; second, that *equilibrium limit pricing does not necessarily limit entry.*

Indeed, *in equilibrium potential entrants cannot be consistently fooled, or more precisely, their beliefs cannot be systematically biased.* In this framework, limit pricing has an impact on the possibility of entry, but it is not always successful: the comparison is between the entry occurring in equilibrium limit pricing (and relative expected profit) and that which would occur if no limit pricing were to take place and prospective entrant firm were informed of this.

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