Novel Universality in Spin Transport

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(Dated: January 24, 2013)

When time-reversal symmetry is broken, quantum coherent systems with and without spin rotational symmetry exhibit the same universal behavior in their electric transport properties. We show that spin transport discriminates between these two cases. In systems with large charge conductance, spin transport is essentially insensitive to the breaking of time-reversal symmetry, while in the opposite limit of a single exit transport channel, spin currents vanish identically in the presence of time-reversal symmetry but can be turned on by breaking it with an orbital magnetic field.

PACS numbers: 72.25.Dc,73.23.-b,75.76.+j

Introduction. Fifty years ago, Dyson showed that ensembles of unitary matrices that are invariant under general symmetry groups reduce to the direct product of three irreducible ensembles [1]. These three circular ensembles are labelled by an index $\beta = 1, 2, 4$ and are respectively invariant under the transformations

$$S \rightarrow U^T S U \,, \text{ orthogonal ensemble, } \beta = 1, \tag{1a}$$
$$S \rightarrow U S V \,, \text{ unitary ensemble, } \beta = 2, \tag{1b}$$
$$S \rightarrow W^R S W \,, \text{ symplectic ensemble, } \beta = 4, \tag{1c}$$

where S is an element of the ensemble, U and V are arbitrary unitary matrices, W is a quaternion unitary matrix, U^T is the transpose of U and $W^R = \sigma^{(y)} W \sigma^{(y)}$ is the dual of W [2]. Here and below, $\sigma^{(\mu)}$, $\mu = x, y, z$ is a Pauli matrix. This classification carries over to electronic quantum transport, where the three classes are defined by the symmetry of the system [3]. Systems without time-reversal symmetry (TRS) have a scattering matrix in the $\beta = 2$ ensemble, while systems with TRS are differentiated by the presence ($\beta = 1$) or the absence ($\beta = 4$) of spin rotational symmetry (SRS). When TRS is broken, breaking SRS doubles the size of the scattering matrix, but does not generate a new ensemble.

Quantum corrections to electric transport depend on the symmetry index β, but are independent of the size N of the scattering matrix (giving the total number of transport channels from and to the scatterer) for large N [3]. Recent investigations of spin transport showed that the spin conductance

$$\mathcal{T}_{ij}^{(\mu)} = \mathrm{Tr}[S_{ij}^\dagger \sigma^{(\mu)} S_{ij}] \,, \tag{2}$$

constructed from the transmission block S_{ij} of the scattering matrix connecting terminals i and j, also exhibit a character of universality [4–6] in that var $\mathcal{T}_{ij}^{(\mu)} = 4N_i(N_i - 1)N_j/N(2N - 1)(2N - 3)$ for $\beta = 4$. Here, $N_{i,j}$ gives the number of transport channels between the system and terminals i, j, and $N = \sum_i N_i$. The spin conductance fluctuates about zero average, $\langle \mathcal{T}_{ij}^{(\mu)} \rangle = 0$ and the resulting, typically nonzero spin current is generated by the presence of the SRS breaking field. In the $\beta = 4$ ensemble one usually takes the latter field as spin-orbit interaction (SOI). In the absence of SOI, one has $\mathcal{T}_{ij}^{(\mu)} \equiv 0$. This is the case for $\beta = 1$ and, if Dyson's threefold way applies to spin transport, for $\beta = 2$. In this manuscript we demonstrate that spin transport discriminates between systems with and without SRS even when TRS is broken. Accordingly, a novel kind of universality emerges in systems with broken SRS and TRS, with charge transport properties given by those of the $\beta = 2$ ensemble, but with specific spin transport properties. The latter are similar to those of the $\beta = 4$ ensemble at large N, a finding already reported in Ref. [7] for specific four-terminal setups, but deviate from it at small N. Our finding does not invalidate Dyson's classification– the latter gives a complete classification of unitary scattering matrices and unless one introduces chiral or particle-hole symmetries [8, 9], there is no new ensemble to be found. Instead our point is that spin-dependent observables define two sub-ensembles of the $\beta = 2$ ensemble, depending on whether they commute or not with the scattering matrix. In other words, we find that while universality in charge transport is affected only by the antiunitary symmetries, universality in spin transport depends on both antiunitary and unitary symmetries.

The model. We consider a mesoscopic conductor connected to any number of external electron reservoirs. There is no ferromagnetic exchange anywhere in the system, nor is there spin accumulation in the reservoirs and we neglect spin relaxation in the terminals. The magnetoelectrically generated spin current due to the presence of SOI inside the cavity is determined by the spin-dependent transmission coefficients of Eq. (2). For instance, in the simple case of a two-terminal setup, the generated spin current in the right lead along the polarization axis $\mu = x, y, z$ is given by

$$I_{\mathrm{R}}^{(\mu)} = (e^2 V/h) \mathcal{T}_{\mathrm{RL}}^{(\mu)} \,, \tag{3}$$

with the voltage bias V applied across the sample.

Semiclassical calculation. We first calculate the average and mesoscopic fluctuations of the spin transmission coefficients using the semiclassical theory of transport [10, 11], extended to take spin transport into account [12, 13]. We write [13]

$$\mathcal{T}_{ij}^{(\mu)} = \int_i dy \int_j dy_0 \sum_{\gamma,\gamma'} A_\gamma A^*_{\gamma'} e^{i(S_\gamma - S_{\gamma'})} \mathrm{Tr}[U_\gamma \sigma^{(\mu)} U^\dagger_{\gamma'}] \,. \tag{4}$$

The sums run over all trajectories starting at y_0 on a cross-section of the injection lead j and ending at y on the exit lead i. Trajectories have a stability given by A_γ, which includes a prefactor $(2\pi i\hbar)^{-1/2}$ as well as a Maslov index, and S_γ gives the classical action accumulated on γ, in units of $\hbar$. SOI is incorporated in the matrices U_γ. The average spin conductance has been calculated semiclassically in Ref. [13]. In the absence of SOI, spins do not rotate, $U_\gamma = \sigma^{(0)}$ is the identity matrix, and one trivially obtains $\mathcal{T}_{\mathrm{ij}}^{(\mu)} \equiv 0$. The leading-order approximation is to consider $U_\gamma \in \mathrm{SU}(2)$, where SOI rotate the spin of the electron along unperturbed classical trajectories [12, 14]. In this manuscript, we will use this approximation because, even though it neglects the geometric correlations reported in Ref. [13], it is appropriate for our search of universality. At that level, the average spin conductance vanishes, $\langle \mathcal{T}_{\mathrm{ij}}^{(\mu)} \rangle_{\mathrm{semicl}} = 0$ [13], which agrees with the random matrix theory (RMT) result of Ref. [4].

Having established that the average spin conductance vanishes regardless of the presence or absence of TRS and SRS, we next calculate spin conductance fluctuations. The leading-order diagrams contributing to $\mathrm{var}[\mathcal{T}_{\mathrm{RL}}^{\mu 0}]_{\mathrm{semicl}}$ are shown in Fig. 1. They are the same as those contributing to the (charge) transmission fluctuations [substituting $\sigma^{(\mu)} \to \sigma^{(0)}$ in Eq. (2)]. In this case, Ref. [11] found that contributions c), d) and e) cancel out, furthermore, contribution b) vanishes upon breaking of TRS. This can be achieved via a magnetic flux piercing the diagram's loop. From Fig. 1, we see that contribution b) is the only one that is flux-sensitive, because the blue (dark) and the red (light) trajectories accumulate the same flux-phase. From a semiclassical point of view, this is the origin of the halving of the universal conductance fluctuations upon TRS breaking [3]. Extending this calculation to $\mathrm{var}[\mathcal{T}_{ij}^{\mu}]_{\mathrm{semicl}}$, we obtain that contributions a), b) and c) are multiplied by a spin-dependent term $\mathrm{Tr}[U^\dagger_{\gamma_1} U^\dagger_{\gamma_2} \sigma^{(\mu)} U_{\gamma_2} U_{\gamma_3}] \times \mathrm{Tr}[U^\dagger_{\gamma_3} U^\dagger_{\gamma_4} \sigma^{(\mu)} U_{\gamma_4} U_{\gamma_1}]$, while contributions d) and e) are multiplied by $|\mathrm{Tr}[U^\dagger_{\gamma_1} \sigma^{(\mu)} U_{\gamma_2}]|^2$. All these terms vanish in the absence of SOI. In the presence of SOI, we evaluate them by averaging them over a uniform distribution of all U_γ's over the SU(2) group, corresponding to totally broken SRS. Following the standard procedure of performing orbital averages and spin averages separately, we obtain that, when SRS is totally broken, contributions a), b) and c) acquire a prefactor ($\langle ... \rangle_{\mathrm{SU(2)}}$ indicates an homogeneous average over the SU(2) group)

$$\langle \mathrm{Tr}[U^\dagger_{\gamma_1} U^\dagger_{\gamma_2} \sigma^{(\mu)} U_{\gamma_2} U_{\gamma_3}] \, \mathrm{Tr}[U^\dagger_{\gamma_3} U^\dagger_{\gamma_4} \sigma^{(\mu)} U_{\gamma_4} U_{\gamma_1}] \rangle_{\mathrm{SU(2)}} = 0 \,, \tag{5}$$

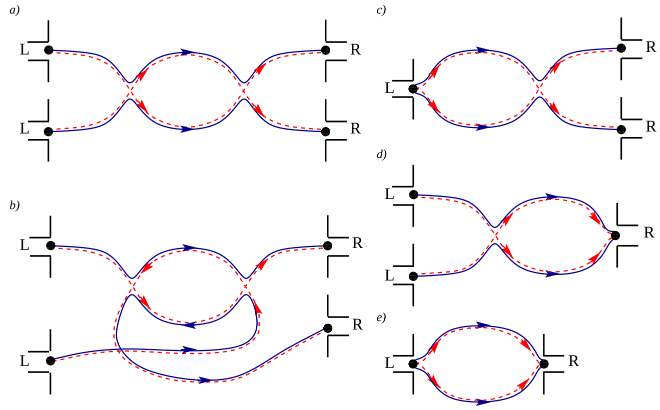

Figure 1: (Color online) Semiclassical diagrams determining the conductance and spin conductance fluctuations to leading order in the number $N \gg 1$ of transport channels. Blue (dark) and red (light) trajectories travel in opposite direction in diagram b), which consequently vanishes in the presence of a large magnetic flux piercing the loop. All other diagrams are insensitive to the breaking of time-reversal symmetry.

and thus vanish identically, while contributions d) and e) are multiplied by

$$\langle |\mathrm{Tr}[U^\dagger_{\gamma_1} \sigma^{(\mu)} U_{\gamma_2}]|^2 \rangle_{\mathrm{SU(2)}} = 1 \,. \tag{6}$$

We conclude that the semiclassical contributions to the spin conductance fluctuations are those with a correlated encounter at the exit terminal, which in particular has the consequence that contribution b) vanishes.

We obtain the variance of the spin conductance coefficients as the sum of contributions d) and e), i.e.

$$\mathrm{var}[\mathcal{T}_{ij}^{(\mu)}]_{\mathrm{semicl}} = N_i N_j N - N_i N_j^2 / N^3 \,. \tag{7}$$

The key point is that this result holds both in the absence and in the presence of TRS, because both relevant contributions d) and e) are sensitive neither to magnetic fluxes piercing their loops, nor to orbital magnetic field effects that do not alter the ergodicity of the classical trajectories. Thus, Eq. (6) gives the leading-order semiclassical expression for $\mathrm{var}\, G_\mu$ for systems without SRS (with SOI) in both cases of conserved or broken TRS, as well as in the intermediate regime of partially broken TRS. Therefore, to leading order in the number $N \gg 1$ of transport chanels, spin conductance fluctuations are insensitive to the breaking of TRS. In the next section, this result is confirmed using RMT.

Random matrix theory calculation. We next use the method of Ref. [15] to calculate the RMT average and fluctuations of the spin conductance. We write [4]

$$\mathcal{T}_{ij}^{(\mu)} = \mathrm{Tr}\,[Q_{\mathrm{i}}^{(\mu)} S Q_{\mathrm{j}}^{(0)} S^\dagger], \tag{8a}$$

$$[Q_{\mathrm{i}}^{(\mu)}]_{m\eta,n\nu} = \begin{cases} \delta_{mn}\ \sigma^{(\mu)}_{\eta\nu}, & m \in i \\ 0, & \text{otherwise}\,, \end{cases} \tag{8b}$$

$$[Q_{\mathrm{j}}^{(\mu)}]_{m\eta,n\nu} = \begin{cases} \delta_{mn}\ \sigma^{(\mu)}_{\eta\nu}, & m \in j, \\ 0, & \text{otherwise}\,, \end{cases} \tag{8c}$$

where m and n are channel indices, η and ν are spin indices and $\sigma^{(0)}$ is the 2×2 identity matrix. The trace in Eq. (8a) is taken over both sets of indices. We find that the average of the spin transmission vanishes in all cases,

$$\langle \mathcal{T}_{ij}^{(\mu)} \rangle_{\rm RMT} = 0 \,. \tag{9}$$

For the $\beta = 4$ ensemble, this result was first obtained in Ref. [4]. We further obtain

$$\begin{aligned} \mathrm{var}[\mathcal{T}_{ij}^{(\mu)}]_{\beta=2;\mathrm{SRS}} &= 0 \,, && (10a) \\ \mathrm{var}[\mathcal{T}_{ij}^{(\mu)}]_{\beta=2;\not{\mathrm{SRS}}} &= 4\frac{N_i N_j N - N_i N_j^2}{N(4N^2-1)} \,, && (10b) \\ \mathrm{var}[\mathcal{T}_{ij}^{(\mu)}]_{\beta=4} &= 4\frac{N_i N_j (N-1) - N_i N_j^2}{N(2N-1)(2N-3)} \,. && (10c) \end{aligned}$$

Eq. (10c) first appeared in Ref. [4]. We see that Eqs. (7), (10b) and (10c) all agree in the limit $N_{i,j} \gg 1$, however, while the semiclassical expression Eq. (7) is valid only in that limit, Eqs. (10) are exact for any number of channels. Most interestingly, for a two-terminal setup with $N_i = 1$, Eq. (10c) gives $\mathrm{var}[\mathcal{T}_{ij}^{(\mu)}]_{\beta=4} = 0$ and thus $G_\mu = 0$, in agreement with Ref. [16]. This restriction no longer applies once TRS is broken, as reflected in Eq. (10b) – breaking TRS can turn spin currents on.

Numerical simulations. We numerically confirm our findings using the spin kicked rotator model [17]. It is represented by a $2M \times 2M$ Floquet matrix [18]

$$\mathcal{F}_{ll'} = (\Pi U X U^\dagger \Pi)_{ll'} \,, \quad l, l' = 0, 1, \ldots, M-1, \tag{11a}$$

$$\Pi_{ll'} = \delta_{ll'} e^{-i\pi(l+l_0)^2/M} \sigma_0, \tag{11b}$$

$$U_{ll'} = M^{-1/2} e^{-i2\pi l l'/M} \sigma_0, \tag{11c}$$

$$X_{ll'} = \delta_{ll'} e^{-i(M/4\pi)V(2\pi l/M)} \,. \tag{11d}$$

The matrix Π represents free ballistic motion, periodically interrupted by spin-independent and spin-dependent kicks given by the matrix X, and corresponding to scattering at the boundaries of the quantum dot, as well as SOI. We choose

$$V(p) = K \cos(p + \theta)\, \sigma_0 + K_{\rm so}(\sigma_x \sin 2p + \sigma_z \sin p) \,. \tag{12}$$

The map is classically chaotic for kicking strength $K \gtrsim 7.5$, and $K_{\rm so}$ is related to the SO coupling time $\tau_{\rm so}$ (in units of the stroboscopic period) through $\tau_{\rm so} = 32\pi^2/K_{\rm so}^2 M^2$ [17]. From (11), we construct the quasienergy-dependent scattering matrix as

$$S(\varepsilon) = P[e^{-i\varepsilon} - \mathcal{F}(1 - P^T P)]^{-1} \mathcal{F} P^T, \tag{13}$$

with P a $2N \times 2M$ projection matrix

$$P_{k\alpha, k'\beta} = \begin{cases} \delta_{\alpha\beta} & \text{if } k' = l^{(k)}, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The $l^{(k)}$ ($k = 1, 2, \ldots, 2N$, labels the modes) give the position in phase space of the attached leads. The mean dwell time $\tau_{\rm D}$ is given by $\tau_{\rm D} = M/N$. The parameter $K_{\rm so}$ breaks SRS over a scale $K_{\rm soc} = 4\pi\sqrt{2}/M\tau_{\rm D}^{1/2}$ and θ breaks time-reversal symmetry over a scale $\theta_c = 4\pi/KM\tau_{\rm D}^{1/2}$ when l_0 is finite. In our numerics we fix $l_0 = 0.14$. When $K \gg 1$ and $\theta/\theta_c \gg 1$, the charge conductance properties are those of the $\beta = 2$ ensemble, while for $\theta = 0$ and $K_{\rm so}/K_{\rm soc} \gg 1$ they are those of the $\beta = 4$ ensemble [17]. In our numerics, we fix $K_{\rm so}/K_{\rm soc} = 120$ and vary θ to gradually break TRS, starting from $\theta = 0$. For simplicity, we specify to two-terminal setups and accordingly calculate the dimensionless spin conductance defined by Eq. (3) as $G_\mu = \mathcal{T}_{\rm RL}^{(\mu)}$ for $\mu = z$. We checked, but do not show, that numerical results remain the same if instead we consider $\mu = x, y$.

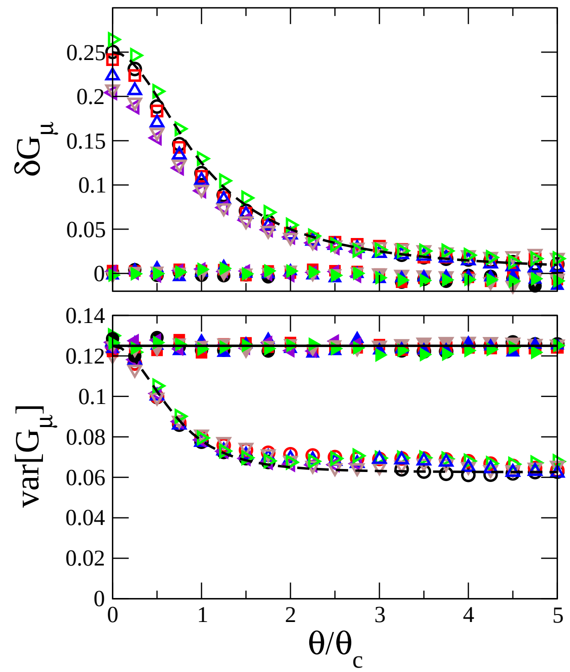

Figure 2: (Color online) Weak localization corrections to (top), and variance of (bottom) the charge (empty symbols) and spin (full symbols) conductance for the two-terminal kicked rotator of Eqs. (11). Parameters are $\tau_{\rm D} = 10, 20$, $K = 40, 60, 80, 90$, $K_{\rm so} = 120\, K_{\rm soc}$ and $M = 128, 256, 512$. The dashed lines indicate the RMT predicted crossover from $\beta = 4$ to $\beta = 2$ [17]. Our semiclassical prediction of Eq. (7) is illustrated by the straight black line in the bottom panel. For all data, $N > 10$.

Fig. 2 first shows data for quantum corrections to the charge and spin conductance, as TRS is gradually broken. The top panel shows that weak localization corrections to the charge conductance are damped by a Lorentzian $\sim [1 + (\theta/\theta_c)^2]^{-1}$ as predicted by RMT [3] and semiclassics [10]. There is no weak localization correction to the average spin conductance, both with and without TRS, in agreement with Ref. [4]. The bottom panel shows that charge conductance fluctuations are halved upon TRS, breaking and their behavior once again agrees

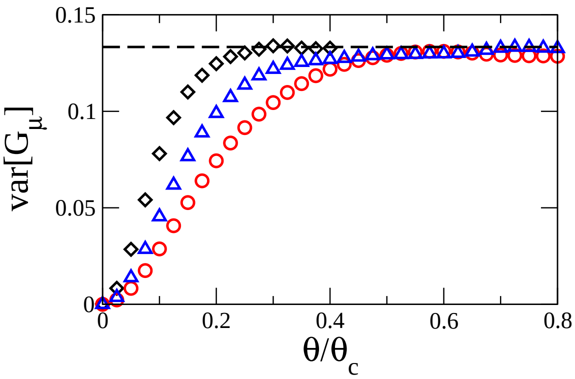

Figure 3: Spin conductance fluctuations for the kicked rotator with SOI defined in Eq. (11) vs. the rescaled TRS breaking parameter θ/θ_c for $N_{\rm R} = N_{\rm L} = 1$. For $\theta = 0$, one is in the $\beta = 4$ ensemble and TRS forces the spin conductance to vanish [16]. Breaking TRS results in a finite variance of the spin conductance. Dashed line: RMT prediction $\mathrm{var}[G_\mu] = 4/30$ for $N_{\rm R} = N_{\rm L} = 1$ [see Eq. (10b)]. Data correspond to $K = 45$, $K_{\rm so} = 384(\pi/M)^2$ with $M = 128$ (red circles), 256 (blue triangles) and 512 (black diamonds). The curves do not lie on top of one another, because the rescaling of the horizontal axis with θ_c assumes $N_{\rm R,L} \gg 1$ [17].

well with theoretical predictions. The situation is entirely different, however, for the spin conductance fluctuations, which are essentially insensitive to the breaking of TRS. This is in agreement with our predictions, Eqs. (7) and (10) for the large number of channels $N > 10$ considered in all data in Fig. 2. The new universal behavior corresponding to broken SRS and TRS emerges at larger θ, where the charge conductance corresponds to the $\beta = 2$ Dyson ensemble, while the spin conductance is very close to those of the $\beta = 4$ ensemble.

Fig. 3 best illustrates the new universal behavior. When the exit lead carries a single transport channel, TRS requires that the spin conductance vanishes [16], regardless of the presence or absence of SRS. Fig. 3 shows that, when SRS is broken, breaking TRS turns spin currents on, whose variance is given by Eq (10b) once TRS is totally broken. Note that the magnitude of the field necessary to break TRS for $N_{\rm R,L} = 1$ becomes smaller and smaller in the semiclassical limit, $M \to \infty$ as the dwell time grows in that limit, $\tau_{\rm D} \sim M$.

Conclusions. By direct calculation we have shown that the spin conductance is an observable that is sensitive to the presence or absence of SRS even when TRS is broken. Breaking of SRS is necessary to magnetoelectrically generate a spin current, thus to acquire a finite spin conductance, but the latter is affected by TRS only when there are very few transport channels. Accordingly, we conclude that the $\beta = 2$ universality class splits into two different subsets for spin transport. In both cases, charge transport properties correspond to the $\beta = 2$ class, however, the spin conductance vanishes identically when SRS is preserved, but exhibits a universal behavior when RS is broken, see Eq. (10b). Spin and charge transport universality classes are related to TRS and SRS in Table. I. Examples of systems with broken SRS and TRS include spin-orbit coupled systems under external magnetic fields that we discussed, but also systems with spin textures and even spin valves with non-aligned magnetizations.

TRS	SRS	Charge transport	Spin transport
Yes	Yes	$\beta = 1$	$\beta = 1$; $G_\mu \equiv 0$
Yes	No	$\beta = 4$	$\beta = 4$; Eqs. (9) and (10c)
No	Yes	$\beta = 2$	$G_\mu \equiv 0$
No	No	$\beta = 2$	Eqs. (9) and (10b)

Table I: Universality behavior of charge and spin transport properties in the four possible cases of broken or unbroken SRS and TRS. When both symmetries are broken, the spin transport properties correspond to those of the $\beta = 4$ Dyson ensemble in the limit $N_{\rm R}, N_{\rm L} \gg 1$. Deviations from $\beta = 4$ are given in Eq. (10) for the spin conductance variance. They are largest for small number of channels.

We thank M. Büttiker for several interesting discussions at various stages of this project.

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