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Non-linear Quantisation Effects in Digital Colour Systems

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Abstract

This paper shows that quantisation of colour pixels, essential to digital imagery, causes non-linear correlation in the permitted values of chromaticity coordinates in linear-intensity CIE colour spaces. It follows that colour video and image coding schemes based on these colour spaces will suffer an inherent preference for certain chromaticities.

Introduction

Colour video systems rely on the fact that any colour can be matched by an additive mixture of three primary components, red, green and blue [1]. In analysing colour, it is necessary to distinguish between our perceptual response to colour and the physical properties of the light. The radiant power or light intensity [2,3] is a physical property, defined as the integral of a spectral power distribution. When two or more light sources are mixed, their spectral power distributions add and hence their intensities add.

Grassmann's laws for colour [4,5] are based on the intensities of each of the three primary components. If C, R, G and B are intensities, C is a unit-intensity light of unknown colour, **R**, **G** and **B** are unit-intensity lights of the red, green and blue primary colours, + indicates additive mixing and $\equiv$ represents a visual colour match, the laws are:

1. for any C and C there exists numbers R,G,B such that $C(\mathrm{C}) \equiv R(\mathbf{R}) + G(\mathbf{G}) + B(\mathbf{B})$

2. if $C_1(\mathrm{C}_1) \equiv R_1(\mathbf{R}) + G_1(\mathbf{G}) + B_1(\mathbf{B})$
   and $C_2(\mathrm{C}_2) \equiv R_2(\mathbf{R}) + G_2(\mathbf{G}) + B_2(\mathbf{B})$
   then $C_1(\mathrm{C}_1) + C_2(\mathrm{C}_2) \equiv (R_1+R_2)(\mathbf{R}) + (G_1+G_2)(\mathbf{G}) + (B_1+B_2)(\mathbf{B})$

3. if $C(\mathrm{C}) \equiv R(\mathbf{R}) + G(\mathbf{G}) + B(\mathbf{B})$
   then $kC(\mathrm{C}) \equiv kR(\mathbf{R}) + kG(\mathbf{G}) + kB(\mathbf{B})$

These laws of matching, mixing and scaling apply only in colour spaces that are linear in intensity. The eye uses three types of photo-receptor, based on the red, green and blue parts of the visible spectrum, but each responds non-linearly to intensity. Our concept of lightness is roughly logarithmic to the intensity and we can distinguish lights when their intensities have a relative difference of around 1% [6].

Quantisation Model

The signal voltages used in electronic video and imaging systems are non-linear in intensity because the response of cathode ray tube displays to input voltage is proportional to that voltage raised to a power of between 2.2 and 2.8 [3]. The power index, gamma (γ), will vary according to the user's adjustment of the contrast of display [7]. As video cameras and digital scanners have in-built gamma correction to compensate for the expected gamma law, the colour values obtained electronically have a fractional power law of intensity.

A necessary feature of digital colour images is that the red, green and blue values of each pixel are quantised. One consequence is an upper limit to the number of possible colours that can be represented. . The quantisation is typically 8 bits, giving quantised levels 0 through 255 and a gamut of 16,777,216 colours. Another consequence is that any colour video or image coding scheme using these colour values will suffer image colour degradation due to the quantisation as well as the algorithm's spatial coding.

It is the electrical signal that is quantised, not the light intensity. When the electronic signal is quantised linearly, the quantisation levels follow a gamma power law in intensity. This model is shown if Figure 1.

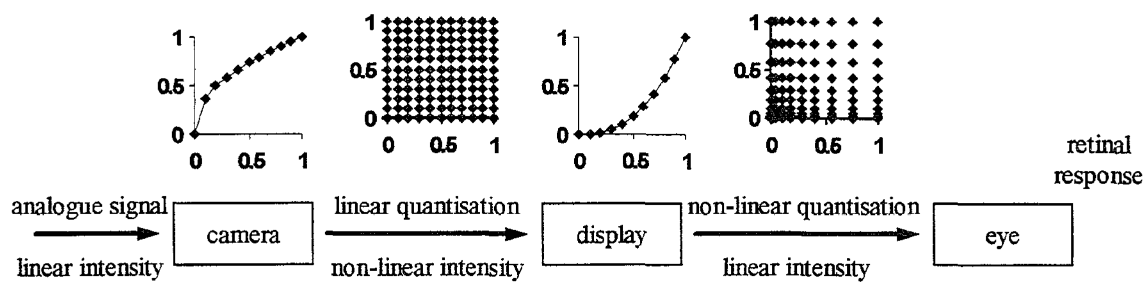


Figure 1 Model of digitisation and reproduction of each primary colour signal.

It is interesting and very fortunate that our retinal response is roughly the inverse of the gamma law. The effect of linear quantisation of the electronic signal is barely visible on a television display because the quantised levels appear to be roughly linear in lightness [8]. The quantisation effects become apparent in a domain of linear intensity, as may be used for image coding or computer graphics requiring Grassmann's laws to model light interaction.

The effects will now be demonstrated in CIE spaces xzY and Yu'v', the basis for the CIE LUV and LAB. It will also be shown that an original colour image, without spatial coding, can produce strong patterns of artefacts in its colour scatter plot.

Effects in Linear-Intensity CIE Spaces

The CIE spaces are based on chromaticity coordinates computed from the intensities of three primaries. For tristimulus intensity values R,G,B, the CIE chromaticity coordinates are:

$$r = \frac{R}{R+G+B} \qquad g = \frac{G}{R+G+B} \qquad b = \frac{B}{R+G+B} \tag{1}$$

Quantisation of R, G and B, whether linear or not, will cause correlation in the quantised values of r, g and b due to the denominator term. The non-linear power-law quantisation of R, G and B highlights the interdependencies of the components. Figure 2 shows the discrete values of (r,g) permitted for 8-bit quantisation of R, G and B and gamma of 2.5. The pattern would be indistinguishable if all 16,777,216 points were plotted, so only those pixels with a CIE luminance [9] less than 0.005, about 1% of the pixels, have been displayed. As luminance is linearly related to r, g and b, with greatest emphasis on g and lowest on b, the densities of points are greatest at the blue and red corners.

It can be seen in Figure 2 that there are large gaps at (0,0.5), (0.5,0) and (0.5, 0.5), being the mid points of the sides, with gaps extending from each opposite corner. These points correspond to magenta, cyan and yellow respectively. It follows that the primary colours, in their purest form, are surrounded by a large number of possible chromaticity points whilst their exact opposites, at least for low luminance levels, have almost no chromaticity points and therefore lower precision.

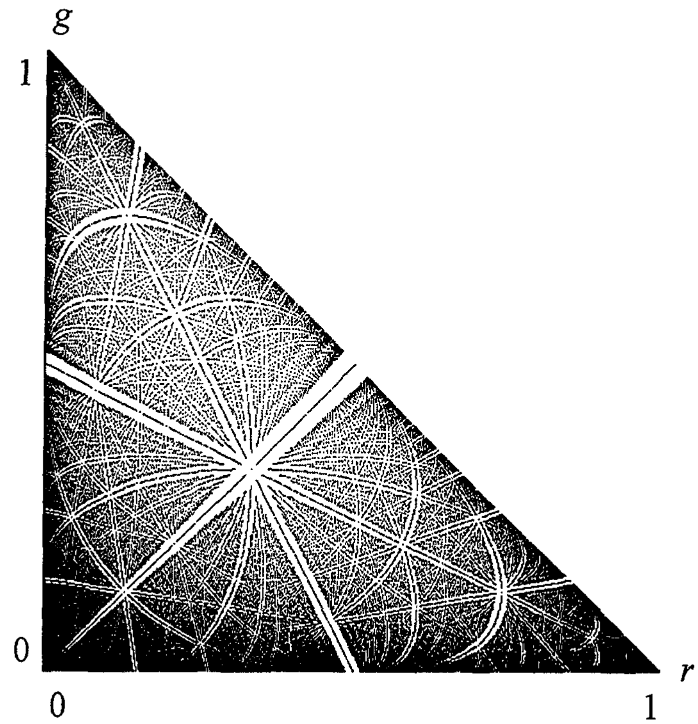
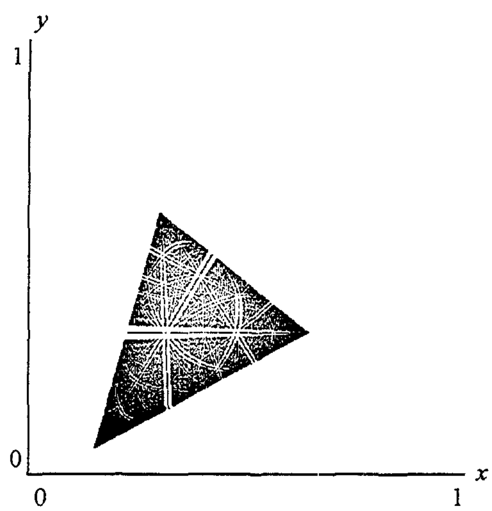


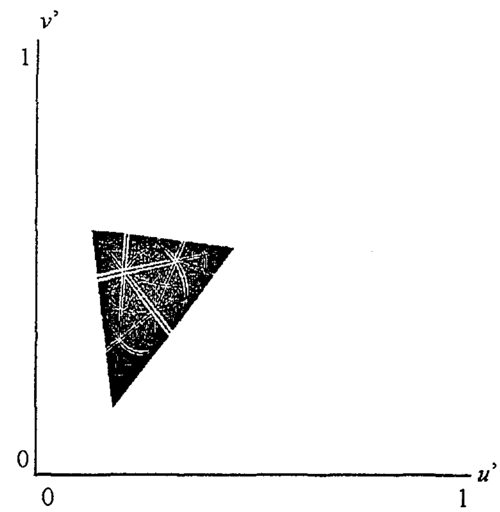
Figure 2

Loci of possible chromaticity coordinates (r,g) for 24-bit quantised colour pixels with gamma (γ) of 2.5.

For picture clarity, only pixels with CIE luminance less than 0.005 have been displayed.



CIE 1931 (x,y) chromaticity coordinates



CIE 1976 (u',v') chromaticity coordinates

Figure 3 Loci of possible chromaticity coordinates (x,y) and (u',v') for 24-bit quantised colour pixels with gamma (γ) of 2.5. For picture clarity, only pixels with CIE luminance less than 0.002 have been shown.

A transformation from any set of RGB primaries to the CIE XYZ primaries and (x,y) chromaticity coordinates will preserve the topology of the quantisation pattern because the transformation is based on linear multiplication by a 3x3 matrix [4,9]. This paper uses the transformation from ITU Rec. 709 primaries [9] with daylight illuminant D65. The chromaticity coordinates (x,y) are obtained by division by $X+Y+Z$ in a similar way to equation 1.

The chromaticity coordinates (u',v') are the basis for the LUV uniform colour space defined by the CIE in 1976 [6] and they are obtained from (x,y) by:

$$u' = \frac{2x}{6y - x + 1.5} \qquad v' = \frac{4.5y}{6y - x + 1.5} \tag{2}$$

The patterns for (x,y) and (u',v') are shown in Figure 3. The pattern for (x,y) has a similar topology to (r,g) from Figure 2 as expected. The progression to (u',v') involves a sequence of two divisions, via equations 1 and 2, so the pattern of the loci of (u',v') is permuted further than that for (x,y).

Effects on Digital Images

The quantisation effect is clearly visible in the CIE chromaticity coordinates of pixels from digital images. Figure 4 is an image of a group of colourful aircraft parked in a grass field in Perth. This image was digitised to 24 bits per pixel, 8 bits per primary. The index of gamma correction ($1/\gamma$) used by the digital scanner is unknown but estimated at 0.45. The scatter plot of (r,g) chromaticity coordinates for this image is shown in Figure 5. As the image contains 746×575 pixels, the scatter plot is dense and the exact pattern of quantisation cannot be seen easily. Thus Figure 6 shows only those pixels in the image with luminance less than 0.026 and Figure 7 shows their chromaticity coordinates. The resulting scatter plot exhibits both the underlying pattern of Figure 2 and also a secondary effect, possibly due to quantisation by the scanner's analogue to digital conversion prior to gamma correction, but this remains unverified.

Discussion

Colour video and image coding schemes usually transform RGB pixel colours into modified colour spaces such as CIE LUV or CCIR Y'CbCr prior to applying spatial coding algorithms. It has been shown that the quantisation of RGB causes intricate patterns of non-linear correlation in the permitted values of the chromaticity coordinates. Whilst the effect on colour image coding schemes has not yet been analysed fully, it can be expected that the chromaticity patterns will be preserved by spatial coding and data compression because they are independent of pixel location. Algorithms based on Y'CbCr, such as JPEG File Interchange Format [10], may suffer reduced quantisation effects because the transformation from the digitised *R*, *G* and *B* to Y', Cb and Cr signals is linear. The Y'CbCr space is linear in quantisation but non-linear in intensity [9].

Figure 4 Image of coloured aircraft, digitised to 24 bits per pixel with $1/\gamma$ estimated as 0.45.

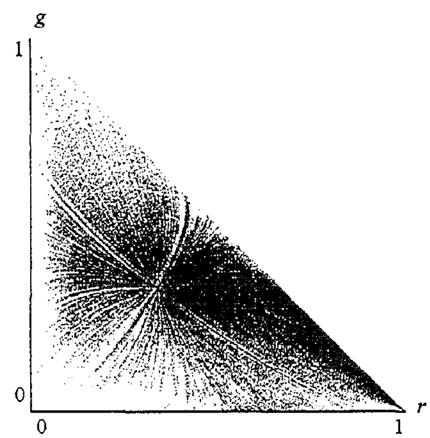


Figure 5 Scatter plot of (r,g) chromaticity coordinates of pixels from Figure 4.

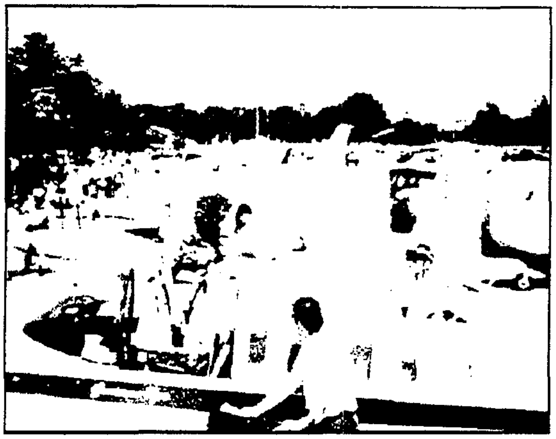


Figure 6 The pixels in the image of Figure 4 having CIE luminance less than 0.026.

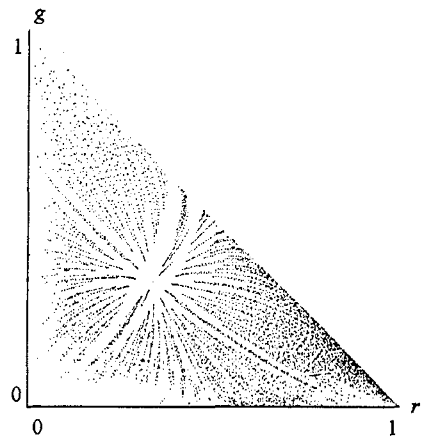


Figure 7 Scatter plot of (r,g) chromaticity coordinates of those pixels in Figure 6.

Conclusion

Quantised RGB pixels from digital colour images produce intricate patterns of preferred chromaticites in linear-intensity CIE spaces such as xzY and Yu'v'. Patterns in the (r,g) chromaticity coordinates arise from division of non-linearly quantised intensities R and G by the sum $R+G+B$. CIE chromaticity coordinates (x,y) are derived by a linear transformation from primaries RGB to XYZ and exhibit the same quantisation pattern as (r,g). The pattern in the chromaticity coordinates (u',v') of the CIE uniform colour space LUV is complicated by another division involving x and y.

The quantisation effect has been demonstrated on an original digitised image, without coding or data compression. Any coding algorithm applied to the image will further encode the internal pattern. The decoded image will then exhibit this encoded pattern whilst simultaneously conforming to the same quantisation grid pattern as the original and any other digital image. Further investigation is required.

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