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PREDICTION OF CRACK PROPAGATION DIRECTION FOR HOLES UNDER QUASI-STATIC LOADING

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ABSTRACT

The simulation of crack propagation relies on accurate computing of stress intensity factors (SIFs) at crack tips. Numerical methods are used to calculate the crack propagation path based on the computation of the crack incremental direction and stress intensity factors K_I, K_{II} from the finite element response. This paper evaluates displacement extrapolation technique (DET) for prediction of stress intensity factors. The DET is used when the singular element is present at the crack tip and it uses the differential displacements for the adjacent nodes across the crack to compute the stress intensity factors. The crack path and its stress intensity factors are calculated in a specialized finite element program, using small crack increments. At each crack propagation step, the mesh is automatically redefined based on automatic adaptive strategy that takes into account the estimation of stress analysis in two-dimension. The crack is free to propagates without predetermine path direction. Maximum circumferential stress criterion is used as the direction criterion. Some examples are presented to show the results of the implementation.

Keywords: Finite element; crack propagation holes; stress intensity factor; adaptive mesh

INTRODUCTION

For many structures, crack propagation is an important failure mechanism requiring accurate numerical models to implement simulations essential for failure prediction. To perform numerical simulation, the computational methods must be applied to determine fracture response and reliability of cracked structure. The FEM offers solution to almost all structural analysis problems once a suitable formulation and computational model are adopted. It has been widely employed for solving linear elastic and elastic- plastic fracture problems. The evaluation of stress intensity factors in 2D geometries by FEM is a technique widely used for non-standard crack configurations. These factors define the stress field close to the crack tip and provide fundamental information on how the crack is going to propagate. Nodal relaxation is frequently used to release nodes, one by one, in order to enable the crack tip to propagate through the mesh. In contrast, methods based on near-tip field fitting procedures require finer meshes to produce a good numerical representation of crack-tip fields. The most accurate methods are those based on nodal displacements, which comprise the primary output of the finite element program (Guinea 2000).

Several methods have been proposed to numerically estimate the stress intensity factors using finite element method such as the displacement extrapolation technique, the *J*-integral and the energy domain integral. Among these methods, the displacement extrapolation technique is simplest and highly accurate (Phongthanapanich and Dechaumphai 2004).

In adaptive mesh refinement, most analysts favour either the Delaunay technique or the advancing front method over other techniques when generating meshes due to the quality of the unstructured meshes generated (El-Hamalawi 2004). The main advantage of the advancing front method is that it tends to produce nicely graded meshes and high quality triangles that are usually very close in shape to equilaterals. The boundary integrity is also preserved, since the discretisation of the domain boundary constitutes the initial front.

Rao and Rahman (2001) developed the coupled meshless-finite element method for analyzing linear-elastic cracked structures subject to Mode I and mixed-mode conditions. Bouchard et al. (2003) proposed a finite element solution based on advanced remeshing technique to modeling of two-dimensional crack propagation problems. Phongthanapanich and Dechaumphai (2004) used a finite element method, with the adaptive Delaunay triangulation as mesh generator to analyze two-dimensional crack propagation problems. They described the Delaunay triangulation procedure consisting of mesh generation, node creation, mesh smoothing, and adaptive remeshing, all with object-oriented programming. They also used the displacement extrapolation method to determine the values of stress intensity factors. Fan et al. (2004) presented an enriched partition of unity finite element method (PUFEM), which is known as one of the meshless methods to calculate the stress intensity factor in linear elastic fracture mechanics under plane stress and plane strain conditions.

The purpose of this study is to determine the effect of holes to crack propagation trajectory. The computational code is written in FORTRAN programming language for finite element analysis calculation processes, which is based on load and displacement control for linear-elastic crack propagation modeling. The mesh for the finite elements is the unstructured type; generated using the advancing front method. The global *h*-type adaptive mesh is adopted based on the norm stress error estimator. The quarter-point singular elements are uniformly generated around the crack tip in the form of a rosette. The displacement extrapolation technique used in the calculation is explained.

STRESS INTENSITY FACTOR, CRACK GROWTH AND CRACK DIRECTION CRITERIA

The stress intensity factor K_I is a measure of the intensity of the stress field near the crack tip under the opening mode (mode I) deformation. A crack growth criterion is used to determine when the crack starts. Then, the crack-growth processes are simulated with an incremental crack-extension analysis. For each increment of the crack extension, a stress analysis is carried out and the stress intensity factors are evaluated. The crack path, predicted on an incremental basis, is computed by a criterion defined in terms of the stress intensity factors. The stress intensity factors are determined from

$$K_I = \frac{E}{3(1+\nu)(1+\kappa)}\sqrt{\frac{2\pi}{L}}\left(4(v_b - v_d) - \frac{(v_c - v_e)}{2}\right) \quad (1)$$

$$K_{II} = \frac{E}{3(1+\nu)(1+\kappa)}\sqrt{\frac{2\pi}{L}}\left(4(u_b - u_d) - \frac{(u_c - u_e)}{2}\right) \quad (2)$$

where E is the modulus of elasticity, υ is the Poisson's ratio, κ is the elastic parameter defined by $(3\text{-}4\upsilon)$ for plane strain and $(3\text{-}\upsilon)/(1+\upsilon)$ for a plane stress problem and L is the element. The u and v are the displacement components in the x and y directions, respectively; the subscripts indicate their position as shown in Figure 1.

The isoparametric six-node triangular elements with mid side nodes displaced from their nominal position to quarter points at the crack tip were employed to form up a circular zone surrounding the tip in order to better capture the stress field.

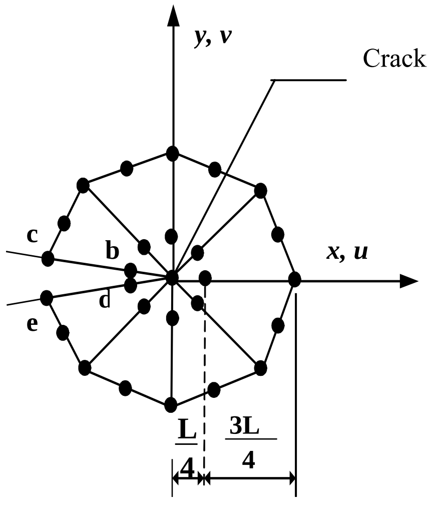

FIGURE 1 Quarter-point triangular elements around the crack tip

There are several methods use to predict the direction of crack growths such as the maximum circumferential stress, the minimum strain energy density, maximum stain energy release rate and maximum principle stress. In the maximum circumferential stress theory, which is used in the present paper, the direction of the crack propagation may be computed from

$$K_I \sin\theta + K_{II}\left(3\cos\theta - 1\right) = 0 \tag{3}$$

Analyzing Eq. (3) for the two pure modes, it is found for pure mode I that $K_{II} = 0$, $K_I \sin\theta = 0$ and $\theta = 0^\circ$, and for pure mode II that $K_I = 0$, and $\theta = \pm 70.5^\circ$. This values of θ are the extreme values of the crack propagation angles. The intermediary values are found solving Eq. (3) for θ considering the mode I, resulting in:

$$\theta = \pm\cos^{-1}\left\{\frac{3K_{II}^2 + K_I\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right\} \tag{4}$$

In order to ensure that the opening stress associated with the crack direction of the crack extension is maximum, the sign of θ_0 should be opposite to the sign of K_{II} (Shen and Lee 1982). The two possibilities are illustrated in Figure 2.

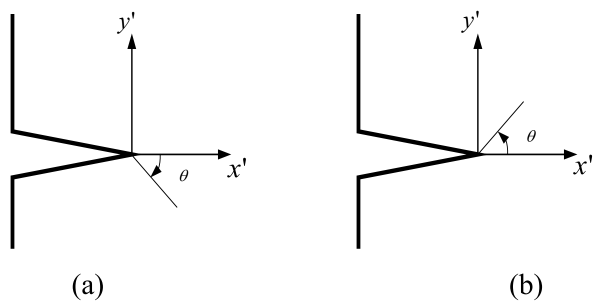

FIGURE 2 Sign of the propagation angle (a) positive and (b) negative

The criterion for crack to propagate from crack tip is based on the material toughness, K_C. If the calculated stress intensity factor, $K_I \geq K_C$ then the crack will propagate to the direction θ expressed by Equation 4.

RESULTS AND DISCUSSION

In order to carry out a comprehensive evaluation of the crack propagation trajectory approximated by the developed program, two well-known plate geometries, double edge cracked with two holes and single edge cracked with three holes are being considered.

Double edge cracked plate with two holes

The geometry and the final adaptive mesh of this specimen are shown in Figure 3. The plate was simply fixed at the bottom edge and subjected to a concentrated load at the upper edge.

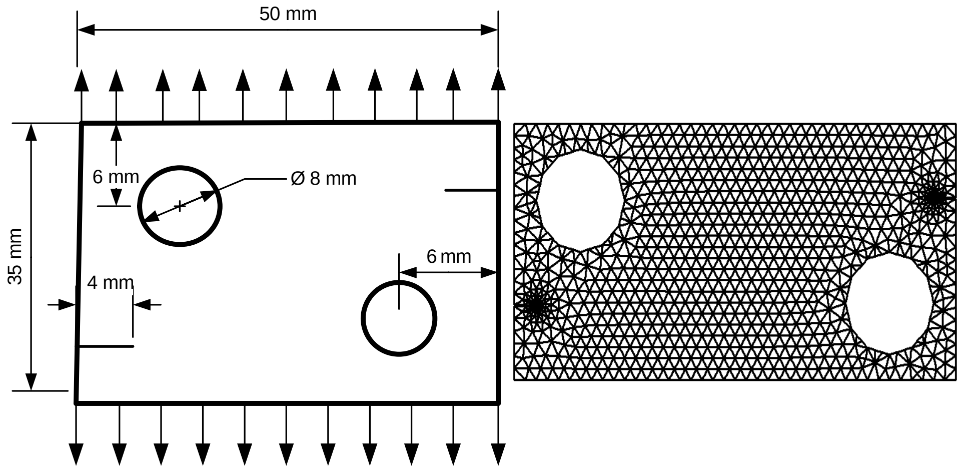

FIGURE 3 Problem statement and the final mesh of the initial crack for the double edge cracked plate with two holes

The material properties are Young modulus, E = 98 GPa and Poisson's ratio ν = 0.3. Each crack was found to grow towards the nearest hole as seen in, Figure 4(a). Then, the crack reoriented horizontally since the cracks have modified the stress distribution at each other's tip as seen in, Figure 4(c). Eventually, the cracks were attracted again by the opposite holes and curved towards the holes, (Figure 4(d)). The result illustrated that the crack propagated with the same trajectory.

This proofed Bouchard et al.'s (2003) statement which said that, two cracks can propagate with the same length if they are symmetric. However, for multiple cracks, some authors (Wang 1994) suggested that the cracks propagate one after the other according to the values of the stress intensity factors. As can be seen in Figure 4(d), a slight convergence of the crack paths could be detected at the areas close to the holes. The present work simulated the crack propagation for the double edge crack plate simultaneously. Each crack tip had its own refinement with its own concentric mesh that did not depend on each other.

The stress distribution is also presented in terms of the Von Mises equivalent stress field, as in Figure 5. The figure clearly shows the effects of each other's stresses. Comparison between the present results and those obtained by Bouchard et al. (2003) shows that there are good agreement for crack trajectory and stress distribution.

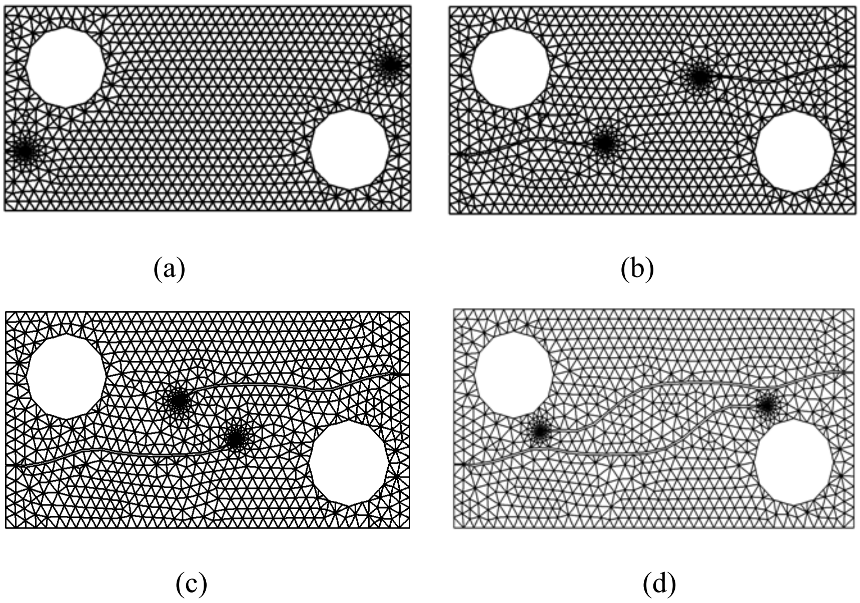

FIGURE 4 Crack growth trajectories for the double edge cracked plate with two holes

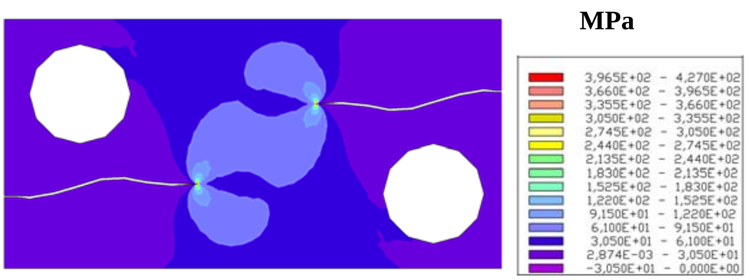

FIGURE 5 Stress field distribution at 8 step propagation

Single edge cracked with three holes

Figure 6 shows the geometry of the single edge cracked plate with three holes and its final adaptive mesh of the initial crack for cases I and II. The material is polymethylmethacrylate (PMMA). The experiments were conducted by Ingraffea and Grigoriu (1990) from Cornell University. The plate was simply supported near the lower corners and subjected to a concentrated load at the center of the upper edge. The crack trajectory was predicted for two different initial notch configurations which were case I and case II. This configuration was chosen because two different crack growth trajectories had been predicted by the Finite

Element modeling of the holed specimens, depending on the initial notch geometry (Ala and Jin 2003).

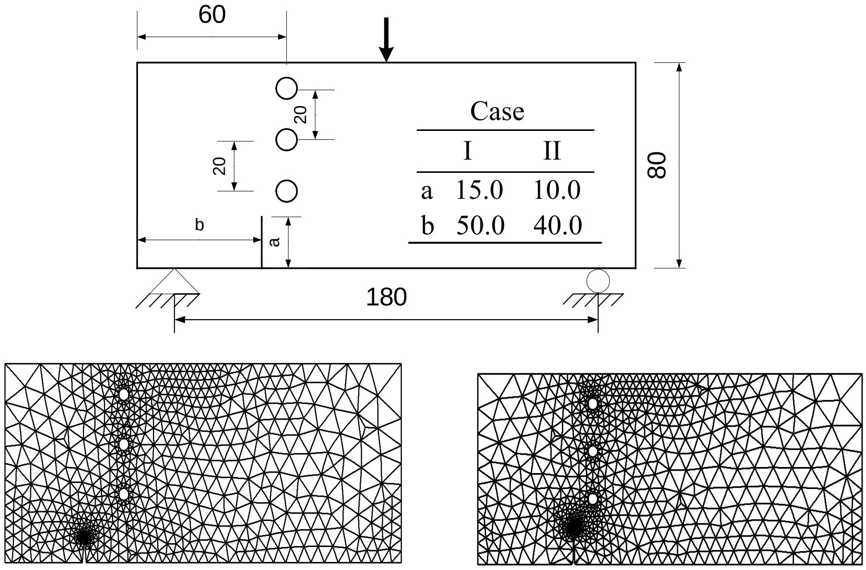

FIGURE 6 Problem statement and the final mesh of the initial crack for the single edge cracked plate with three holes for cases I and II

Figure 7 and 8 shows the crack trajectories obtained by the numerical simulation using the present method. The predictions indicated that the crack was always attracted by the hole. For case I (as shown in Figure 7(c)), the crack growth trajectory passed just above the lower hole and ended at the middle hole (Figure 7(d)). The numerical results are obtained by considering Δa =3.0 mm and 12 steps.

Figure 8(d) demonstrates that the crack propagated towards the middle hole. The numerical results were obtained by considering Δa =2.0 mm and 11 steps. As expected, there was significant difference in crack trajectories between Figure 7 and 8. The results proofed that the crack trajectory was dependent on the initial crack location. An initial location close to the hole could cause the crack path to go towards the hole, while the crack path is far from initial locations remote from the hole. As can seen in Figure 8, the crack trajectory still moved towards the hole although the initial crack was far from the hole compared to that in Figure 7. However for both cases, the predicted crack growth trajectories gave a good agreement with the results obtained by Phongthanapanich and Dechaumpai (2004). Figure 9(a) and 10(a) shows the Von Mises plot for the stress distribution in the final step propagation. The results showed a slightly different stress distribution for each specimen. Note that, for case I, the stress on the left-hand side was less than that on the right-hand side. For case II, the stress on the left-hand side was higher than on the right-hand side. As a consequence, the crack propagated inwards starting from the boundary, towards the region where the stresses were much higher. The crack trajectories in the present study for both cases are very similar to those represented in Figure 9(b) and 10(b).

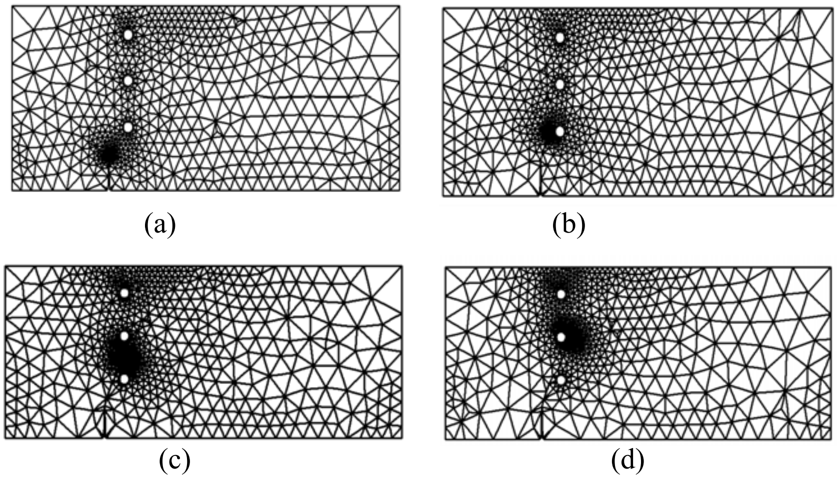

FIGURE 7 Crack growth trajectories for the single edge cracked plate with three holes for case I

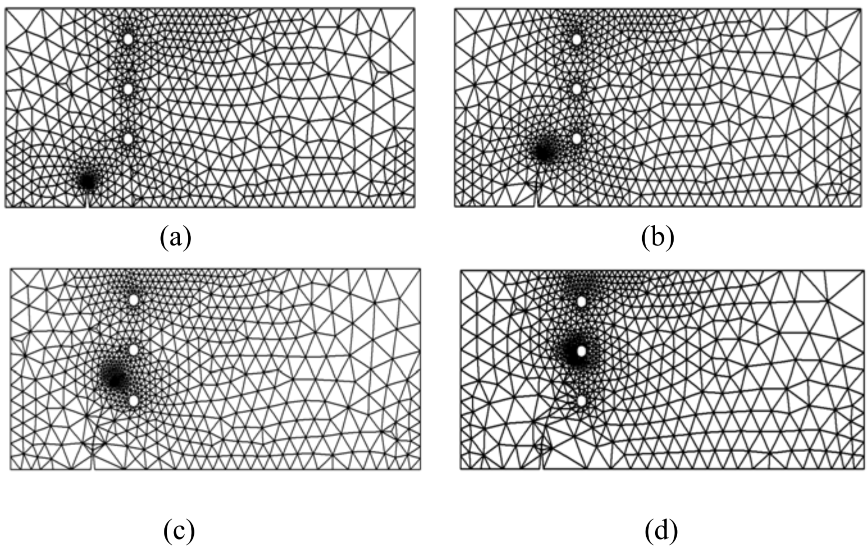

FIGURE 8 Crack growth trajectories for single edge cracked plate with three holes for case II

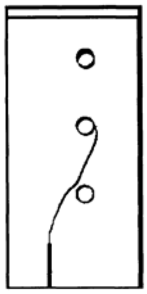

FIGURE 9 (a) Simulated crack trajectory for the single edge cracked plate with three holes stress field distribution for case I (b) experimental trajectory (Ingraffea and Grigoriu 1990)

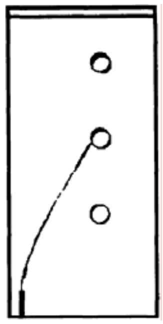

FIGURE 10 (a) Simulated crack trajectory for the single edge cracked plate with three holes stress field distribution for case II (b) experimental trajectory (Ingraffea and Grigoriu 1990)

CONCLUSION

The adaptive finite element program using the advancing front method has been developed to simulate linear elastic crack propagation using Fortran language. The norm stress error was taken as a posterior estimator for the *h*-type adaptive refinement. The strategy has been used successfully to simulate the propagation of cracks in plate specimens with holes. The presence of holes in the plates disturbed the stress and strain fields providing interesting crack trajectories. The crack simulations for mode I and mixed mode cases showed the acceptable crack path predictions. The results of the assessments strongly indicated that the finite element simulation for two-dimensional linear elastic fracture mechanics problems has been successfully employed.

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