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**Efficiency in the Higher Education
sector: A technical exploration**

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RESEARCH

**EFFICIENCY IN THE HIGHER EDUCATION SECTOR:
A TECHNICAL EXPLORATION**

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Contents

Contents.....	3
Executive summary.....	5
1. Introduction	7
2. Review of the literature on costs in higher education in the UK.....	9
3. Specification of the cost function: linear versus quadratic	13
3.1 Linear specification	13
3.2 Non-linear specification.....	14
4. Cost function estimation and the measurement of efficiency.....	17
5. Empirical analysis	22
5.1 Model specification.....	22
5.2 Data.....	26
5.3 Linear model over 3 time periods and for the whole time period	27
5.4 Quadratic model over 3 time periods and for the whole time period.....	32
5.5 Linear latent class model over 3 time periods and for the whole time period	33
5.6 Quadratic latent class model over 3 time periods and for the whole time period.....	38
5.7 Comparison of results with pre-defined groupings of universities.....	40
5.8 Discussion.....	44
6. Conclusions.....	45
Appendices	46
Appendix 1: Summary of results of previous studies of costs in the UK higher education sector.....	46
Appendix 2: Alternative non-linear cost function specifications	56
Appendix 3: Data definitions	57
Appendix 4: Additional results.....	59

References..... 73

Executive summary

Costs incurred by a higher education institution in producing its output vary with the levels of output that it produces. They also vary with a number of other factors such as quality, student demographics, and the nature of the real estate. Even after allowing for all relevant factors, costs are likely to vary because institutions differ in their levels of efficiency. It is important to study differences in efficiency because this offers lessons about good practice that can lead to improvements in the performance of the higher education system as a whole.

Numerous studies analysing the costs of higher education institutions have been conducted, including many in the United Kingdom. Advances in statistical methodology have allowed these simultaneously to evaluate cost structures and institutional efficiency. The most recent studies, using latent class or random parameter stochastic frontier models, do this in a context that makes allowance for differences between types of university (reflecting, for example, variation in student quality) that are not easily captured by the data.

In all exercises of this type, an important judgement call must be made concerning which of the factors affecting costs should be taken into consideration when determining institutional efficiency. Clearly allowance should be made for differences in the level of output. Arguably allowance should also be made for differences in costs that are due to, say, the historical nature of an institution's real estate, or to the role the institution plays in the widening participation agenda. In general, the more refined is the model of costs, the more efficient institutions appear to become, because the model contains more information that can be used to explain cost differences. There is no objective way of determining how much detail should be included in the analysis.

In this report, various stochastic frontier models are used to evaluate efficiency in English higher education institutions over the period 2003/04 to 2010/11, and over three sub-periods within that time frame. The stochastic frontier approach involves fitting a curve through data on costs and a variety of explanatory variables. This is not, however, a line of best fit; rather it is an envelope that defines an efficiency frontier – a curve that shows the lowest possible costs at which a given set of outputs can be produced. The position of this curve can then be used as a benchmark against which the efficiency with which each institution produces its output can be determined.

These are *estimates* of efficiency, and, like any other statistical estimates, are measured with error. It is possible for one estimate to be different from another, but not significantly so in the statistical sense. In addition, there is no consensus about what value indicates an efficient or inefficient score. Consequently, the results should be interpreted with caution.

The latent class stochastic frontier approach refines this method by separating the institutions into a number of classes based on institutional characteristics that are revealed by the data. The statistical method then estimates a cost frontier for each class, thereby

allowing each institution's efficiency to be assessed relative to other institutions of the same type.

An alternative to the latent class approach is to prescribe groups of institutions that, on a priori grounds, appear to share similarities with each other. The groups used in the recent HEFCE report on the impact of the HE reforms are used to define these clusters of institutions.

The empirical work conducted in this project involves the evaluation of a large number of models of institutions' costs, each of these models producing measures of institutions' efficiency. The models range from a simple specification in which the explanatory variables include only linear terms in the outputs produced by each institution, through models that include further variables to capture factors such as widening participation and real estate characteristics, to models that include a rich variety of interaction terms designed to capture economies of scale and scope. The models are estimated first on the assumption that the same cost structure applies to all institutions, and secondly on the assumption that different groups of institutions exist for which cost structures may differ.

A key finding of the report is that, once differences between institutions are accounted for¹, the variation in efficiency scores across institutions is greatly reduced, with a concentration of scores above 0.9 (where a score of one represents efficiency). Indeed, the relatively small number of institutions with low scores is exclusively made up of small and specialist institutions. The results do not, therefore, support the notion that substantial sector-wide gains could be made by using efficiency scores as a criterion for resource allocation.

It may be argued that the more sophisticated models are the ones most appropriate for evaluating the efficiency of institutions, since they make most allowance for the different circumstances that might influence costs. Nevertheless there are drawbacks associated with using these models as a means of understanding costs in higher education. The greater sophistication of these models comes at a price. By increasing the number of explanatory variables used in the analysis, it becomes more likely that co-movement of some of the variables reduces the precision with which the impact of any one of them on costs is estimated. Moreover, the estimation of cost models that are specific to distinct classes of institutions involves, in effect, a reduction in the sample size used to estimate the model for each class. For these reasons, the simpler models reported here have considerable merit as means of understanding cost structures.

The analyses reported in this paper provide a useful starting point in understanding differences in efficiency across higher education institutions. The data on which they are based, however, are highly aggregated, and fail to capture the detail of how and why efficiency scores vary. A more in-depth analysis of institutions that achieve scores at either end of the distribution, such as via a case study approach, would be instructive as a means of ascertaining the organisational factors underpinning efficient performance. This is likely to necessitate the collection of qualitative data of a kind that does not fit easily within the statistical approach adopted in this report.

¹ Even when only using a relatively unrefined latent class modelling procedure

1. Introduction

Under the new funding mechanism for higher education in England, many students will not pay off the whole of their debt within 30 years. The tuition fee charged by providers is not necessarily the same, therefore, as the amount paid by customers. The Resource Accounting and Budgeting (RAB) cost of the student loans scheme is estimated to amount to around 35 per cent of the value of the loan book², more than under the previous system, although there has been a simultaneous reduction in the amount of teaching grant, which, in effect, had a RAB cost of 100%. In addition, the fact that students do not make up-front payments reduces their price sensitivity. These factors have the potential to produce a market failure such that the usual competitive pressures fail to incentivise providers to become more efficient. Moreover, the government continues to subsidise both teaching and research, and has an interest in the efficient operation of all aspects of higher education. An analysis of the cost structure and efficiency of higher education institutions (HEIs) is therefore of on-going interest and importance.

Extensive work has been undertaken on evaluating efficiency in the higher education sectors of various countries. Work in the United Kingdom (UK) is of particular relevance here (see, for example, Johnes and Taylor 1990; Johnes, J 1996; Johnes et al. 2005; Johnes 2008; Thanassoulis et al. 2011). Much of the literature on efficiency measurement has emphasised the statistical evaluation of costs (Cohn et al. 1989), since efficiency concerns how a given output can be produced at as low a cost as possible. Statistical and econometric techniques have been developed which allow efficiency to be evaluated for each institution. These statistical methods do not drill down into the detail of how institutions do what they do³; rather they offer the analyst both an understanding of how costs are determined in higher education institutions as a whole, and a measure of the extent to which different institutions manage to produce their outputs efficiently. It allows an assessment to be made of the extent to which institutions differ in terms of their efficiency, and it also allows an analysis of changes in efficiency over time. At a higher level of abstraction, the method provides much the same input into benchmarking exercises as do more qualitative exercises, but it offers the advantage of a focus on the front-end activities of teaching and research. A number of studies exist which have adopted this general approach for UK higher education⁴ (Glass et al. 1995a; 1995b; Johnes, G 1996; Johnes 1997; 1998; Izadi et al. 2002; Johnes et al. 2005; Stevens 2005; Johnes et al. 2008b; Johnes and Johnes 2009; Thanassoulis et al. 2011).

² Announced by David Willetts at the Higher Education Policy Institute Spring Conference (15th May 2013) - see <https://www.gov.uk/government/speeches/david-willetts-minister-for-universities-and-science-hepi-conference-speech>.

³ Unlike, for example, the Transparent Approach to Costing (TRAC).

⁴ Note that there are also notable studies of cost structures of higher education systems of other countries such as Japan, Italy, Spain, Portugal, the USA and Germany, respectively (Hashimoto and Cohn 1997; Agasisti and Salerno 2007; Johnes and Salas Velasco 2007; Johnes et al. 2008a; Agasisti and Johnes 2009; 2010; Johnes and Schwarzenberger 2011).

The purpose of this report is to undertake an empirical study of costs and efficiency in English higher education using data from 2003/04 to 2010/11. The report is in 6 sections of which this is the first. A review of empirical studies of costs in UK higher education is presented in section 2. Section 3 examines linear and non-linear specifications of the cost function and the implications of particular choices. The methods of estimating cost functions are considered in section 4 which also looks at how estimates of efficiency can be derived from the cost function. The empirical analysis using data, in turn, from 2003/04 to 2004/05, 2005/06 to 2007/08, and 2008/09 to 2010/11, is presented in section 5. Conclusions are drawn in section 6.

2. Review of the literature on costs in higher education in the UK

There is now a considerable literature concerning the cost structure of systems of higher education, some of which also examines the efficiency levels of HEIs. The data underlying the empirical studies of costs in UK higher education span a period from the 1960s up to the early 2000s, and it is this literature which will be predominantly reviewed in this section. Additional details of the studies reviewed can be found in Appendix 1.

The first cost functions to be estimated and published for higher education in the UK relate to data from 1968 (Verry and Layard 1975) and include as outputs measures of both teaching and research. Six separate linear cost equations are estimated, one for each subject area: arts; social sciences; mathematics; physical sciences; biological sciences, and engineering. For undergraduate teaching, marginal costs are higher in the 'laboratory' science subjects than in the 'classroom' subjects such as arts, social sciences and mathematics. The marginal cost of postgraduate teaching is higher than for undergraduate teaching, and (like undergraduate teaching) is more expensive in the sciences than in the arts.

Scale economies (which arise when an expansion of scale of production leads to costs rising less than proportionately with output) are observed in all subject areas (apart from physical sciences) but are significant only in the social sciences. In fact, the linear nature of the cost function makes it inevitable that there should be scale economies so long as there exist some fixed costs of provision – and in this respect the finding for physical sciences is somewhat curious.

Though clearly dated, this study acts as a benchmark against which we can compare the results of later studies. By including measures of both teaching and research output, the Verry and Layard (1975) study is the first to recognise the impact that a multiplicity of different types of output has on costs and on the technology of production in universities. The linear cost function, however, does not allow for the possibility of synergies that arise from the joint production of the multiple outputs, and the separate estimation of cost functions by subject does not allow for synergies resulting from joint production across subject areas. This means that the methodology adopted by Verry and Layard is incapable of explaining why undergraduate education, postgraduate education and research are all carried out in the same institution; nor can it explain why different subjects are delivered within the same institution. The form of the model that is being estimated does not admit the possibility that there might be benefits (in the form of cost savings) arising from such joint production. The outputs, moreover, are limited to teaching and research only, and there is no consideration that there may be inefficiency in the sector. The omission from the sample of two universities (Oxford and Cambridge) because they are outliers raises the issue of comparability of the production units in the higher education sector – this is no doubt a growing concern as the UK higher education sector changes in its composition.

Some of these issues are addressed, though not resolved, in a companion study (Verry and Davies 1976); but the seminal work by Cohn et al. (1989) was the real trigger for more sophisticated studies of costs of higher education in the UK which gradually got to grips with the shortcomings of these early models. Four studies, which use data for the pre-1992 definition of the university sector (Glass et al. 1995a; 1995b; Johnes, G 1996; Johnes 1998), address the issue of synergies in higher education production by using complex non-linear functional forms for the cost equations and by defining teaching outputs by broad subject area. Where undergraduate teaching is split by broad subject, the studies find that average costs of teaching within the arts are lower than within the sciences (at both undergraduate and postgraduate levels). For a given subject area, however, there appears to be little difference in the average costs of undergraduate compared to postgraduate teaching (Johnes, G 1996; Johnes 1998).

With regard to ray returns to scale – that is, returns to scale that arise from a simultaneous increase in all types of output being produced – there is evidence that scale economies are significant and unexhausted for the typical university (Glass et al. 1995a; 1995b; Johnes, G 1996; Johnes 1998). But the results regarding product-specific returns to scale – the returns to scale associated with an increase in one output only – are mixed (details can be found in Appendix 1) (Glass et al. 1995a; 1995b; Johnes, G 1996; Johnes 1998).

Evidence regarding global economies of scope – the economies arising from producing all outputs together rather than separately – is also mixed. When teaching outputs are split by subject group in Johnes, G (1996) and Johnes (1998) we observe global economies of scope, but this contrasts with the finding of no significant scope economies when teaching output is aggregated across all subjects (Glass et al. 1995a; 1995b). This may be because studies where the outputs are more highly aggregated are, in effect, aggregating out the possibility of observing scope economies that exist at a finer level of analysis.

Two of these studies are the first to allow for inefficiency in the estimation of the cost function by using frontier estimation methods: stochastic frontier analysis (SFA) (Johnes, G 1996; Johnes 1998) - and data envelopment analysis (DEA) - (Johnes 1998)⁵. Efficiency of each HEI is measured on a scale of zero to 1 with the latter representing complete efficiency. The studies find that mean efficiency for the higher education sector as a whole is over 0.90 (using the DEA method). The estimates of efficiency derived from the two frontier estimation methods are positively correlated, although, at 0.133, the magnitude of the correlation coefficient is rather low, suggesting that the two frontier estimation methods provide different rankings of HEIs based on estimated efficiency. Any estimate of efficiency for an individual institution therefore needs to be treated with extreme caution.

The higher education sector in the UK saw major changes in its composition in 1992, when polytechnics were given university status, and later in 2003 when it was announced that Colleges of Higher Education would be allowed to apply for university status. Six studies have estimated cost functions from data referring to the extended higher education sector (Johnes 1997; Izadi et al. 2002; Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009; Thanassoulis et al. 2011).

⁵ See section 4 for details of the frontier methods of estimation: SFA and DEA.

For undergraduate teaching, average costs vary by subject and are highest in the sciences (or in medicine followed by other sciences where a more detailed subject split is made) and lowest in the non-sciences (Johnes 1997; Izadi et al. 2002; Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009; Thanassoulis et al. 2011). The cost of postgraduate teaching is generally higher than undergraduate teaching in the sciences and non-sciences, but lower than undergraduate teaching in medicine (Johnes 1997; Izadi et al. 2002; Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009; Thanassoulis et al. 2011), but no reasons are provided for why this might be the case.

The findings regarding economies of scale and scope, from the studies based on the extended higher education sector, differ from the findings of the earlier studies. Ray returns to scale are close to constant or decreasing for the typical university (Johnes 1997; Izadi et al. 2002; Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009) implying, in the latter case, that expanding output leads to an increase in costs. Findings on product-specific economies of scale are mixed and depend on choice of data (single-year or panel data), definition of outputs, the functional form of the cost function and the estimation method. Global diseconomies of scope are a consistent finding in these later studies (Johnes 1997; Izadi et al. 2002; Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009).

Results regarding efficiency in the extended higher education sector also vary by choice of data and estimation method. Using only a single year of data, average efficiency for the sector as a whole is estimated to be around 0.88 (Izadi et al. 2002). This result, somewhat surprisingly given the variety of HEIs included in the 2002 study, is not too dissimilar to results based on only 50 pre-1992 HEIs (Johnes 1998). There is a considerable range in efficiency, however, from under 0.40 to 0.99, and this is likely a consequence of the diversity of the HEIs in the sample. Institutions at the lower end of the distribution of efficiencies tend to have characteristics that suggest that their relatively high costs (given output) are due to idiosyncrasies that are not adequately captured by the data on outputs, and the efficiency scores attached to these institutions therefore needed to be treated with caution.

Studies which use a panel of data over a number of years find that mean efficiency is generally lower than in the single-period models. The magnitude also varies by estimation method: mean efficiency across the whole sector is 0.69 on the basis of SFA (Johnes et al. 2005; Johnes et al. 2008b), 0.863 when DEA is used (Thanassoulis et al. 2011), and 0.753 in the case of a random parameter SFA model (Johnes and Johnes 2009). A simple SFA, unlike DEA and a random parameter SFA, does not make any allowance for each HEI to have a different set of objectives or mission. Thus any efficiency results derived using SFA should be interpreted with this in mind.

There is strong evidence that efficiency varies by HEI type. Colleges of higher education appear to be least efficient and post-1992 and some pre-1992 HEIs are typically the most efficient (Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009; Thanassoulis et al. 2011). The available software has not, however, allowed evaluation of the extent to which these differences are statistically significant.

The expansion of the higher education sector calls into question the comparability of HEIs included in the sample used to estimate the cost function. We need to be sure that HEIs included in the sample are comparable in terms of their environment such as 'quality' of

students, input prices, and real estate costs. While one study finds that the proportion of students achieving first and upper second class degrees has a positive influence on both costs and on efficiency (Stevens 2005), student quality is generally not a significant determinant of costs (Verry and Davies 1976; Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009). It is possible that the random parameter specification used in some of these later studies already accounts for persistent quality differences across institutions and hence leads to the finding of insignificance of the quality variable.

Dummy variables such as a London dummy and an Oxbridge dummy, included in models to reflect, respectively, differences in input prices and costs of upkeep of ancient buildings, are not significant determinants of costs (Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009). Once again, these results may be a consequence of using a random parameter framework.⁶

Clearly much work has been undertaken on estimating the cost functions and efficiencies of UK HEIs. Many of the earlier studies restrict output to just teaching and research, are estimated on the basis of a restricted sample of HEIs, and do not allow for inefficiency in higher education production; indeed only four of all the studies reviewed include a measure of the third mission outputs of universities, use data which reflect the current composition of the English higher education sector and use a frontier estimation method (Johnes et al. 2005; Johnes et al. 2008b; Johnes and Johnes 2009; Thanassoulis et al. 2011). Thus conflicts in findings from the different studies regarding, for example, average costs and economies of scale and scope, are not surprising.

The diversity observed in the UK higher education sector raises difficulties in estimation which have not, to date, been adequately addressed. Previous studies have examined costs and efficiency amongst pre-defined mission groups and have found differences between them (Johnes et al. 2005; Johnes et al. 2008b; Thanassoulis et al. 2011). But these studies are based on preconceived notions of how costs ought to vary. In a later study (Johnes & Johnes 2009), a random parameters approach is adopted which acknowledges that each university varies in its mission and faces distinct circumstances affecting its costs, but which allows the data themselves (rather than the researchers' preconceptions) to determine the nature of each institution – and hence what each institution's cost function should look like. The random parameters frontier estimation model is an exciting development. By allowing parameters to vary across institutions, cost functions for HEIs that are clearly different from one another can be estimated in a single framework and without recourse to separate estimation for pre-determined groups of HEIs. The disadvantage is that the model can be difficult to fit; indeed such were the demands on the data in this particular study that some of the richness of the previous models was lost by amalgamating the medicine and science undergraduate teaching outputs, and by dropping the third mission output measure from the equation. The random parameters approach might also be viewed as being too permissive in that, in effect, it allows each institution to define its own mission.

⁶ Work reported later in the present paper, which does *not* use a random parameter model, finds a significant Oxbridge effect.

3. Specification of the cost function: linear versus quadratic

Costs typically increase as output increases. In many contexts, and certainly in the case of most HEIs, the output of each producer comes in a multiplicity of forms. Hence a HEI might produce graduates in a number of disciplines at a number of levels (such as bachelors, masters, doctorates), and might also produce research and engage in knowledge transfer across various fields. Each of these distinct outputs has an impact on costs. Moreover, these costs are likely to differ across institutions that have different types of student intake, and are likely to vary according to the extent to which they produce outputs of different quality.

3.1 Linear specification

The simplest way to consider the relationship between costs and output is to suppose that each unit of each type of output adds a certain (fixed) amount to total costs. This approach is appealing in that it suggests a functional form for a cost equation that is particularly simple to estimate using statistical methods. That functional form is linear, and given by an equation such as

$$C = \alpha + \beta T + \chi R \quad (1)$$

where C denotes total costs and T and R respectively denote the quantity of distinct types of output being produced, say teaching and research. The α , β and χ terms are known as parameters of the model, and they are estimated statistically.

The simplest way to do this is to conduct a least squares regression, using data for a cross-section or panel of institutions; in effect, this involves evaluating a line (or, strictly speaking, a plane) of best fit through a scatter plot of data in three dimensions – one dimension for costs, and one for each of the output variables. A more appropriate estimation technique is the stochastic frontier approach (SFA), which, rather than providing the line of best fit, evaluates the envelope of cost below which a certain combination of outputs cannot be produced, however efficient the producer. In this context, a latent class estimator can be used to estimate a separate cost equation, if desired, for each group of institutions (assuming two or more groups). More discussion of estimation methods and the implications of choice of method for efficiency estimation are provided in section 4.

Once the estimation has been conducted, it is straightforward to interpret α as fixed costs (the costs that would be incurred even if production of teaching and research were zero), β as the (marginal) cost associated with each unit of teaching, and χ as the (marginal) cost associated with each unit of research. In the case of the latent class model, the parameters α , β and χ will be different for each of the classes – indicating that one group of institutions has different fixed costs to the other, and that the (marginal) costs associated with teaching and research also differ across the groups.

This specification of the cost equation has the considerable merit of simplicity. The model has few parameters, and this reduces the likelihood with which statistical problems will be met that hamper the estimation. This is a particularly important consideration if the models being estimated are rich in terms of the number of explanatory variables being considered as potential determinants of costs. Moreover, the estimated parameters from a linear specification lend themselves to straightforward interpretation.

The linear specification of the cost function allows a limited consideration of returns to scale. If α is equal to zero, then a given percentage increase in both outputs, T and R, leads to the same percentage increase in C, thus implying constant returns to scale. If, on the other hand, α is greater than zero, a given percentage increase in both outputs leads to a smaller percentage increase in costs. This is because, with the increase in T and R, fixed costs can now be spread over a higher number of units of output. In this case we observe increasing returns to scale. It is possible also to observe diseconomies of scale (where α is less than zero), though this would be somewhat counterintuitive in the case of higher education since it would imply that system-wide costs are minimised by organising provision through a large number of very small providers.

While the linear specification of costs can provide some information about returns to scale, however, the specification itself is clearly highly restrictive in this regard. If returns to scale are increasing for some level of output, then they must be increasing for every level of output. Likewise, if they are decreasing (or constant) for some level, they must be decreasing (or constant) at every level. This does not correspond to the conventional thinking that returns to scale are initially increasing but subsequently constant or decreasing as output rises. Neither does it correspond to the stylised facts: while there are relatively few very small institutions in existence, the higher education system is not dominated by a single very large institution, sweeping up all the available economies of scale. It seems more reasonable to consider a specification of the cost equation that is capable of accommodating both increasing and decreasing returns to scale at different levels of output.

3.2 Non-linear specification

The preceding points suggest that a nonlinear specification of the cost function might have merit, and this in turn raises the question of what type of nonlinear representation of the cost equation might be appropriate. One specification that has been particularly popular in the literature is the quadratic cost function⁷. Using the same two outputs as above, this has the form

$$C = \delta + \phi T + \gamma R + \eta T^2 + \kappa R^2 + \lambda TR \quad (2)$$

The squared terms in T and R, and the interaction term (where T and R are multiplied) give this equation its quadratic (nonlinear) character. The equation has several appealing

⁷ Appendix 2 presents some alternative non-linear cost function specifications. The quadratic specification is commonly used in the literature, largely because it can be regarded as a linear equation in variables that are nonlinear combinations of outputs. This makes estimating the equation considerably more straightforward than estimating other non-linear functions.

properties. First, returns to scale can be different at different levels of output. Depending on the parameter values, it is possible for there to be economies of scale at low levels of output, and diseconomies of scale at very high levels of output, thus providing a rationale for the existence of institutions that are neither very small nor very large. Secondly, the interaction term allows for the possibility that the joint production of the distinct types of output under consideration can yield economies. These are known as economies of scope, or synergies. In an important sense, the (potential) existence of economies of scope explains the existence of organisations as we know them. In the case of HEIs, such economies might explain why research is conducted within the same organisation as teaching, why postgraduates receive their training in the same organisations as undergraduates, or why tuition and research in the arts are delivered in the same organisations as tuition and research in the sciences.

The quadratic function thus provides a much richer framework within which to analyse costs than does the linear function. Both are parametric, and therefore impose restrictions on the shape that the cost curve can take. But, since the quadratic nests the linear as a special case⁸, the quadratic function is clearly more general. It is important to note, however, that increasing the number of output types has very different implications for the two types of specification. In the linear case, increasing the number of outputs by one leads to an increase of one in the number of terms on the right hand side of the equation. In the quadratic case, increasing the number of outputs by one results in an increase of $2+x$ in the number of parameters, where x is the number of output types. This means that, even when considering only a modest number of outputs, the specification of the quadratic cost function involves many terms on the right hand side of the equation. This can result in statistical problems owing to over-parameterisation and multicollinearity⁹. For this reason, the quadratic model is most suitable when a fairly parsimonious model is under consideration.

A further characteristic of the quadratic model is that each output appears more than once in the set of explanatory variables, because it appears in squared and interaction terms, not just as a linear term. This makes the interpretation of parameters more difficult than in the linear model. It is, however, possible to extract information from the model about measures that are of policy interest, such as the (marginal and average) cost associated with the provision of each type of output.

Once the quadratic cost function has been estimated – using the same types of statistical methods as are used to evaluate the parameters of a linear function – it is straightforward to calculate the marginal costs associated with each type of output. This measure indicates how much an extra unit of output adds to total costs¹⁰. In contrast to the linear case, however, where the cost function is quadratic the marginal cost will not be a

⁸ If in the special case that the coefficients η , κ , λ in the quadratic model (equation (2)) are all zero, then equation (2) simply becomes the linear model of equation (1) with $\delta=\alpha$, $\phi=\beta$, and $\gamma=\chi$. Thus equation (2) is said to 'nest' equation (1).

⁹ Multicollinearity occurs when two or more of the variables on the right hand side of the equation are highly correlated. It can lead to imprecise estimates of the coefficients of the model, with small changes in model specification often leading to large changes in the estimated impact of each variable on left hand side variable – costs in this case.

¹⁰ Formally, the marginal cost is found by differentiating costs with respect to the output type of interest.

constant. It will depend on the level of output. Owing to the existence of the squared and interaction terms, the marginal cost associated with each type of output will vary with the amount of that output type and with the amount of other output types produced. Hence the quadratic form of the cost function allows the returns to both scale and scope to vary, depending on the output profile of an institution.

The average incremental cost (AIC) associated with the production of a particular output type can be calculated as the difference between total costs at the outturn level of output and the estimate of what total costs would be if none of the output type of interest were produced (all other outputs remaining equal), expressed as a proportion of the outturn level of that output type¹¹.

¹¹ See Baumol *et al.* (1982).

4. Cost function estimation and the measurement of efficiency

The previous section made allusions to various statistical methods for estimating cost functions (such as least squares regression, stochastic frontier analysis, and latent class models). The choice of estimation method implies assumptions regarding the efficiency of the group of organisations whose cost function is being estimated, and hence has implications for the measurement of efficiency. This section examines the issues of cost function estimation and the measurement of efficiency in more detail.

Organisations of various types have a variety of motivations that lead them to seek to be efficient. In sectors characterised by intense competition, efficiency is a prerequisite of survival. Elsewhere, efficiency is needed in order to ensure that the objectives of the organisation can be maximised. Where organisations are funded, at least in part, by the public purse, government has a responsibility to the taxpayer to ensure that resources are used efficiently. Yet the evaluation of efficiency is not straightforward.

Efficiency refers to the process whereby inputs are converted to outputs. The ratio of the value of outputs to the value of inputs provides one means whereby efficiency can be measured, and requires knowledge of costs, outputs and the estimation of the cost function. Early cost function studies used simple least squares regression to estimate a line of best fit through the data. This approach calculates, over a number of organisations, the average value of total cost associated with producing a given level of output, and does not tell us how cheaply it is possible to produce that output. We know, because we can see it happening, that it is possible to produce the output more cheaply than the average. In analysing costs and efficiency, therefore, it is important that we should be able to identify an envelope below which it is technically impossible for costs to go. Rather than a line of best fit (which may be estimated by regression), we need to identify the position of a cost *frontier*, a cost curve that would typically lie below the best fit line.

In practice, the method used to estimate the parameters of the cost frontier involves a modification of the basic least squares regression method. In the least squares method, the residuals (the gap between observed values for each data point and the line of best fit) are required to follow a normal distribution with a zero mean. This means that the observed values are as likely to lie below the estimated cost curve as above it, and that the sum of deviations below the line is as great as the sum of deviations above the line. This clearly violates the requirement that the line should represent a frontier. For a stochastic frontier model¹², the requirement that the residuals follow a normal distribution is replaced by a specification in which the residuals are made up of two components: one is normal, with zero mean, and is designed to capture measurement error; the second

¹² The stochastic frontier model was introduced by Aigner *et al.* (1977).

component is non-normal (usually a half-normal distribution is assumed¹³), and is designed to capture differences in efficiency across the observed units¹⁴. The effect of introducing this latter component of the residual is to shift the line so that it becomes a frontier rather than a line of best fit. The parameters of the model may be estimated using maximum likelihood techniques¹⁵. An advantage of this method is that it allows the magnitude of the non-normal residual associated with each observation to be calculated; this may then be interpreted as a measure of efficiency. More commonly, the ratio of predicted costs to the sum of predicted costs and the non-normal residual is used to define an efficiency score¹⁶.

These measures of efficiency are interesting and are likely to be instructive at least inasmuch as they provoke questions. It should be remembered, however, that they are obtained from a statistical exercise in which a line (or plane) is fitted through data that do not allow a perfect fit. They are *estimates* of efficiency, and, like any other statistical estimates, are measured with error. The efficiency estimate for each data point (in our case, each HEI) may be different from the estimate for other data points, but not (in the statistical sense) significantly so. Moreover, there is no consensus about the precise value an efficiency score should have in order for the unit to be deemed efficient or inefficient. As a result, caution is needed in their interpretation.

The stochastic frontier estimation approach (like earlier estimation methods) has the underlying assumption that all production units under examination are directly comparable. It may be the case, however, that the cost associated with a given level of production may be higher in one organisation than in another for reasons that may reflect differences in the cost and production structures of different organisations rather than differences in efficiency. For example, an organisation that enjoys the use of new, purpose-designed buildings may enjoy lower costs than one that uses antiquated accommodation. This may mean that the former is more efficient than the latter; but equally it may mean that the latter produces, as a by-product, intangible outputs (such as architectural heritage) which it is obliged to preserve. Likewise it is possible that different institutions differ in terms of the quality of students they can attract, and in the quality of graduates that they produce. It is important therefore to recognise the danger that the measurement of efficiency may be conflated with the issue of legitimate differences between organisations in cost and

¹³ Alternative distributions for this error component include the truncated normal, the exponential and the gamma distributions. The half-normal and exponential distributions have a mode at zero while the gamma and truncated normal models have a much wider range of distribution shapes. The parameters of the gamma and truncated normal, however, are much more difficult to estimate than for the half-normal and exponential distributions. While the values of efficiency scores can be sensitive to the precise distribution that they are assumed to follow, when observations are ranked on the basis of efficiency the rankings are usually not sensitive to choice of distribution (Coelli *et al.* 2005).

¹⁴ It is possible to test whether the half-normal distribution of efficiencies (across all units) is significantly different from zero using a statistic λ which is calculated from the variance of the random error component and the variance of the inefficiency component (Coelli *et al.* 2005). If λ is not significantly different from zero, then there is no significant inefficiency component.

¹⁵ Maximum likelihood estimation involves using an iterative procedure to find the parameters of a given statistical model that maximize the likelihood of observing the particular set of data.

¹⁶ The efficiency score typically lies between zero and one, with one representing efficiency. It is possible for the score to be outside that range under some circumstances. This can happen if predicted costs are less than zero.

production structures. ***What constitutes legitimacy in this context inevitably involves a judgement call. It is possible, of course, to compare like with like, but someone has to make a judgement about how alike the members of each cluster of organisations have to be in order to be deemed comparable.***

These difficulties have been acknowledged by researchers for some time, and efforts have been made to develop methods that allow efficiency to be evaluated while ensuring that the organisations that are being compared with each other are indeed comparable. An important contribution to this research effort is the development of the *latent class stochastic frontier model*¹⁷. This is a statistical model that allows the analyst simultaneously to estimate the parameters of the cost structures of two or more groups of organisations and to evaluate the efficiency of each organisation in each group, while also determining which organisations comprise the membership of each group.

At the same time, the position and shape of this cost frontier needs to be evaluated separately for each of a number of groups of organisations. This is in recognition of the fact that different groups of organisations face different challenges and have different missions. The structure of costs is not expected to be the same across all organisations, simply because the characteristics of these organisations vary widely. There may be good reasons to suppose that there are (say) two distinct groups – or what we might call *latent classes* – of organisations included within the data set. The analyst can then set up a problem that can be solved to provide estimates of the position and shape of two cost curves – one for each group. Part of the solution of this model involves establishing which organisations belong in which latent class. The problem is solved using maximum likelihood estimation methods (see footnote 15). To be clear, the estimation methodology simultaneously provides information about what organisations comprise which group *and* provides estimates of the parameters of the cost equation for each group.

To summarize, it is possible to combine the stochastic frontier and latent class approaches so that (i) cost frontiers (or envelopes) are estimated (ii) yielding measures of the efficiency of each organisation in the data set and (iii) establishing which organisations belong in each of the latent classes or groups. This is illustrated in Figure 1. This shows a scatter plot of points, each of which describes the costs and output levels of a single observation. Each observation might represent a decision-making unit or organisation – for example a HEI. Where panel data are used, each observation might represent a particular organisation in a particular time period. A straightforward latent class analysis of these data might involve the analyst in specifying that there are two¹⁸ different types of organisation in the data set. The latent class model therefore fits two lines to the data. These are shown by the two dashed lines. In fitting these two lines, the model also determines which observations belong to which of the two latent classes – thus the model classifies some of the cost-output pairings into class X and some into class Y. These letters are shown as the data points on the diagram, but it should be emphasised that the observations are placed in these classes by the maximum likelihood algorithm used in the

¹⁷ While the latent class model was introduced by Lazarsfeld and Henry (1968), the frontier version of the latent class model was much later (Orea and Kumbhakar 2004; Greene 2005).

¹⁸ It is, of course, possible to develop latent class models with more than two classes. More discussion on this is provided in section 5.

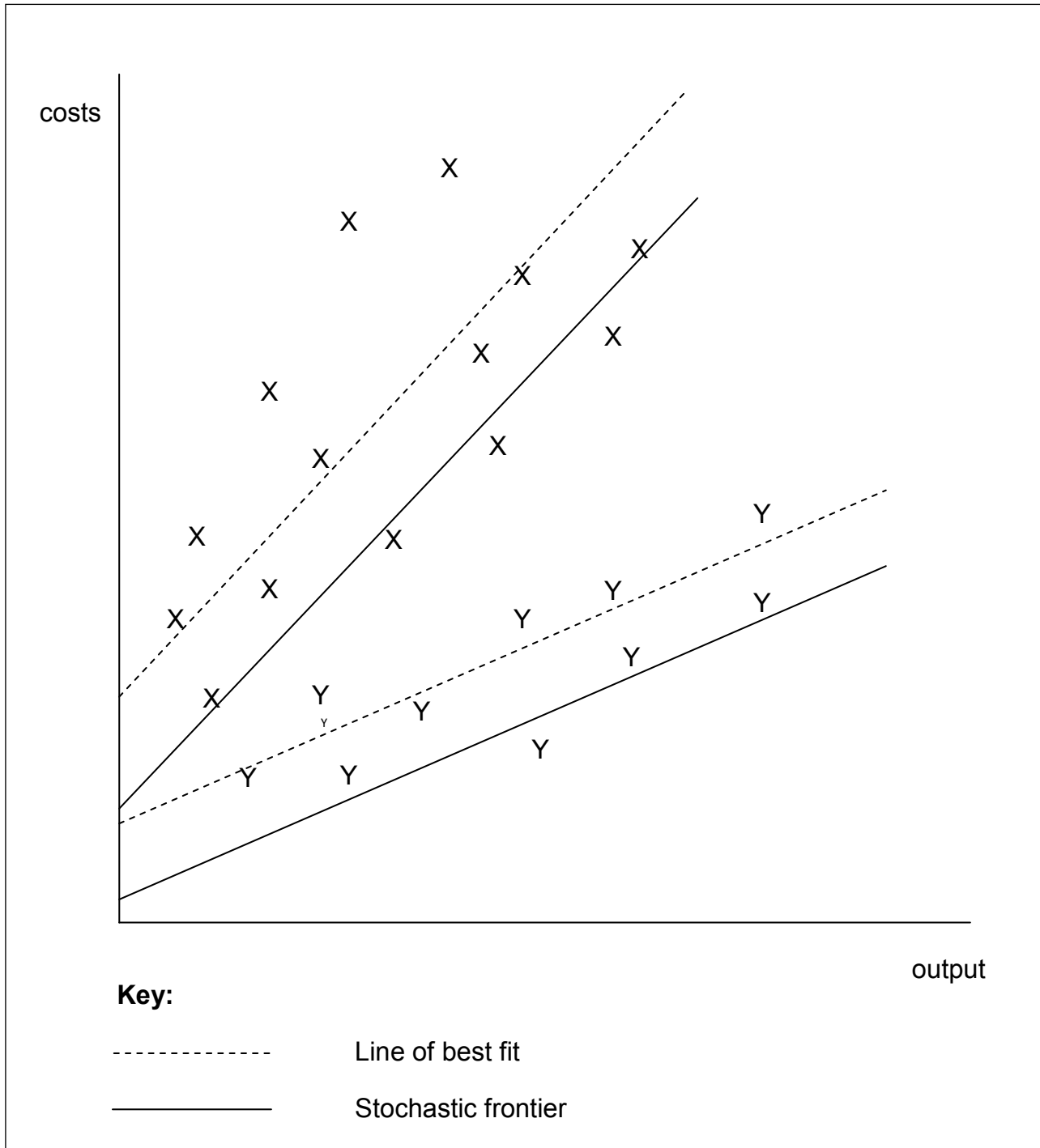
latent class estimation itself; the observations are not placed within one class or the other by the analyst.

The two dashed lines represent the best fit that is associated with the observations (given that there are two latent classes), but they do not represent the cost envelope faced by organisations within each of these two classes. To find these cost envelopes, the latent class method must be used alongside a stochastic frontier model. Doing this moves the lines down (and this is not necessarily a parallel shift). The resultant cost envelopes are represented by the solid lines. Note that, within each latent class, some observations lie below the cost frontier (because of the stochastic error component), but most lie above. The preponderance of observations above the frontiers represents inefficiency. The technique allows the efficiency of each observation to be evaluated by reference to its position *relative to the frontier for the latent class to which the observation belongs*¹⁹. Caution should therefore be exercised when interpreting the results from a latent class model. In particular, ***while it is valid to compare HEIs within a group (because they are all being evaluated relative to the same frontier) it is not appropriate to make comparisons across groups, since the estimated frontier may be different for each group***²⁰.

¹⁹ This is done using a method developed by Jondrow *et al.* (1982).

²⁰ In any case, the point of a latent class model is that HEIs are different and therefore comparisons should not be made across HEIs in different classes.

Figure 1: Illustration of the latent class approach



5. Empirical analysis

Several different specifications of the model of costs are reported in the tables that follow. These include both linear and nonlinear models; the former benefit from simplicity, but the latter have the advantage of allowing more sophisticated analysis of economies of scale and of scope. The simplest estimates reported below are based on an assumption that all institutions belong to a single class – that, while institutions might differ vastly in both scale and in the mix of outputs produced, the underlying technology is common, so that costs are determined in the same way in all institutions. The more sophisticated models assume that there are two or more latent classes, so that cost structures differ across these classes.

5.1 Model specification

The explanatory variables in all models include a set of outputs and a number of controls. The outputs are: full-time equivalent (FTE) student numbers in each of four categories – undergraduates in medicine (UGMED), in other science (UGSCI), and in other subjects (which, for conciseness, are referred to as ‘arts’, though this set of subjects also includes humanities and social sciences - UGARTS) and the total number of FTE postgraduates (PG); research income (RESEARCH); and a measure of income from intellectual property (IPINCOME). This last variable is intended to proxy the output of third mission work undertaken by institutions. The control variables, used in some models, are the number of students at the institution that come from neighbourhoods with low levels of participation in higher education (LOWPNO) and the area of the institution’s estate that has listed building status (LISTED). A binary variable is also included to identify the ancient institutions (Oxford and Cambridge – OXBRIDGE), and, since the data used in the analysis are in the form of a panel of institutions over several time periods, year dummies are used to capture sector-wide changes over time. A complete list of variables with their precise definitions is provided in Appendix 3. All variables measured in monetary units are deflated to 2011 values by the Office for Budgetary Responsibility’s GDP deflator.

It is worth making a number of observations about the choice of explanatory variables used in the models. First, while undergraduates are disaggregated by broad subject area, the same is not done for postgraduates. Considerable efforts were made to evaluate models in which postgraduates are disaggregated into subject groups, but these proved to be unsuccessful, yielding results that were suggestive of statistical problems. Institutions that are major providers of postgraduate education in one area of their activity tend also to be highly active in training postgraduates in other areas. Hence several variables in the model were highly correlated with one another, thus making it impossible accurately to determine the effect on costs of each variable. This problem is known as multicollinearity, and it leads to imprecise estimates of the coefficients of the model, with small changes in model specification often leading to large changes in the estimated impact of each variable on costs. Aggregating across subjects at postgraduate level appears to mitigate this problem and results in a considerably more robust model specification.

Secondly, research income is used as a measure of research activity. This is standard in the literature, but is nonetheless worth commenting upon. The measurement of research undertaken by a university raises questions about how the quantity and the quality (or impact, perhaps) of research should be weighted. By using research income as a measure, these questions can be finessed. Income provides a measure of the valuation that is put on research by clients, and hence implicitly provides the appropriate weights on quantity and quality. It is recognised that the clients in this case are not necessarily all operating in competitive markets, but nonetheless the use of this measure offers (implicit) weights that are not arbitrary.

Alternative measures of research activity are available and have been considered for use in this study. Data on numbers of publications (PUBLICATIONS), and on the number of times work from each institution has been cited (CITATIONS), are available from the Web of Science. The correlation between research income, publications and citations measures of research activity is high, as is demonstrated in Table 1. Early experimentation with models similar to those reported below, using the publications and citations variables rather than research income, suggested that results are robust with respect to the choice of variable used to measure research activity. This being the case, and to be consistent with the received literature, the results reported below use the income measure of research.

Table 1: Correlation between various possible measures of research output (2008/09 to 2010/11)

Variable	PUBLICATIONS	CITATIONS
RESEARCH	0.973	0.776
PUBLICATIONS	-	0.788

Thirdly, alternative measures of third mission activity are available from the Higher Education Business and Community Interaction Survey, and were considered for use in the present analysis. The number of people attending (ATEVENT), or the number of staff days involved in (EVENTS), events such as concerts, exhibitions, public lectures etc. at institutions are examples of such measures. Examination of the data on events reveals that the quality of the data is poor. Some institutions that are known to have arts centres report no attendance at events, for example. Moreover the data for single institutions often vary considerably, implausibly so, from year to year. This suggests that the interpretation of these variables differs both across time and across institutions. We experimented with inclusion of the variables (both individually and together) in early estimations, but the results confirmed that they were unfit for use in the statistical analysis. Therefore, this study only reports results where IPINCOME is the measure of third mission activity.

Fourthly, the control variables used in the models reported below require some justification. The character of an institution's real estate is likely to be a major influence on maintenance costs. Two measures of the nature of the estate were considered as candidate control variables in the current exercise. The first is the institutions' self-reported figure for the estimated cost of upgrading their real estate to newly refurbished condition (UPGRADE). While superficially attractive, these data suffer a major drawback in the present context. Institutions which, over the period of analysis, engage in major refurbishment works see a fall in the value of this variable while simultaneously increasing their expenditures. This causes ambiguity in the impact that the upgrade measure is

expected to have on costs. For this reason, the second variable – namely the area of the institution’s estate that is accounted for by listed buildings (LISTED) – is preferred. The variable UPGRADE was included in some early estimations but the results were unsatisfactory (in that the coefficient was not significant and had a sign which did not accord with intuition), and hence our reported results only consider the impact of LISTED on costs.

Fifthly, the inclusion of a dummy variable for the ancient universities is worthy of discussion. It is not surprising to find that the cost structures of these universities are different from those of other HEIs. The nature of their estate, their organisational structures, the balance of their activities (with a relatively heavy concentration on postgraduate and research activities), and their positions in international rankings of universities all distinguish these universities from others in the country. One option would be to exclude them from the analysis, but this resulted in implausible values for some coefficients when estimation was based on all other observations, and in a latent class model which failed to converge. It appears appropriate therefore to employ an alternative approach of including a dummy variable to identify the Oxbridge institutions²¹.

Some further variables were considered for inclusion in the model, but do not appear in the preferred specifications reported below. Data from Unistats on graduate earnings, based on the Destinations of Leavers of Higher Education (DLHE) survey, provide a market based measure of the quality of institutions’ output (NMEAN) which is assumed to reflect inter-institution variations in quality of teaching output. There are various problems with including this variable in the cost equation. The data are based on survey data with different response rates for each institution. In addition, the data refer to graduates’ success in the labour market 6 months after graduation, and it is debateable whether this provides an adequate reflection of graduate quality. Finally, there is a problem with using average graduate earnings as a control variable in an equation which has total costs as the dependent variable. This would suggest that institutions’ fixed costs vary with quality (as measured by graduate earnings) but that variable costs do not. This is clearly implausible. Nevertheless, graduate earnings were used as a control variable in some early specifications of the cost equation (not reported here), and consistently proved to be insignificant as a determinant of costs.

A further measure of quality that was considered for use in the analysis was institutions’ performance in the National Student Survey – specifically the percentage positive response to the question ‘overall, I am satisfied with the quality of the course’ (NSS). In common with the graduate earnings variable, this measure of quality suffers the drawback that its inclusion in a cost equation would imply that it can affect fixed but not variable costs. Its use as an explanatory variable in some specifications of the cost function

²¹ It would be possible to allow for this in another way. A random parameter model would allow each institution, including Oxford and Cambridge, to have a distinct cost structure. Such a model might, however, be considered to be too permissive in the sense that it would, in effect, allow a distinct cost function to be estimated for each institution, thus allowing each institution to claim that differences in costs are due to differences in structure rather than differences in efficiency. The latent class model goes some way towards the random parameter specification without allowing quite so much flexibility – it does this by identifying two or more separate classes of institution, but, on the evidence of the data used here, this mechanism is not refined enough to allocate Oxford and Cambridge to their own class.

produced unsatisfactory results, typically leading to an estimated equation with efficiencies which have the wrong skew²².

Other factors which might be considered of interest, a priori, have not been included in the analysis because it has not been possible to obtain satisfactory measures. This is the case with, for example, the quality of student intake. We note, however, that the latent class approach adopted in some of the work which follows is designed precisely to allow for differences between HEIs which are not otherwise observed in the data.

We end this section by highlighting the points of original contribution of the empirical work reported below:

- We estimate cost equations for English higher education over a period of 8 years of data, and also for sub-periods of 2 and 3 years within that period. This is a considerably longer period than any used in previous literature: typically, analysis has been based on one year of data, or, in the case of panel data studies, on a maximum of 3 years of data.
- We include a measure of third mission output. Attempts have been made to measure third mission in some previous work, but the variable used here is more appropriate than any previously used. In addition, we consider the interaction of third mission and research output in some models reported below, and this is new to the literature.
- Undergraduate teaching is broken down into 3 subject areas (medicine, other sciences and non-sciences). Previous studies have typically divided teaching only into two subject areas; where three groups have been used in previous studies the approach has not been combined with a third mission measure and its interaction with research. Separation of undergraduate teaching into this many groups poses challenges in estimation which will be discussed in the context of the results presented below.
- We investigate for the first time in the literature the effect of other possible determinants of costs, in particular, the effect of recruiting students from traditionally low participation neighbourhoods (LOWPNO), and the effect of having buildings with potentially costly upkeep (LISTED).
- We investigate the possibility that the cost function varies for distinct groups of universities, as revealed by the data, using stochastic frontier latent class estimation. This is the first time this approach has been used in the context of English higher education.

²² The method used to evaluate efficiency, stochastic frontier analysis, requires that variation between institutions' costs that are unexplained by the output and control variables should follow a distribution that comprises a normal and a 'one-sided' element. The latter should all be positive values (in a cost function context), reflecting the extent of inefficiency observed in each institution. To obtain such a one-sided component of the residual, the total residual has to be skewed in a certain direction. If the skew goes the wrong way, one cannot interpret the one-sided element as a measure of inefficiency. In this case, the ordinary least squares estimator provides the best estimate of the cost function, and the data indicate that all institutions are efficient.

- We compare the cost functions and efficiency derived from the latent class approach with the cost functions and efficiency estimated using pre-defined classes of 'similar' institutions. These pre-defined classes are the same as those used by the Higher Education Funding Council for England (HEFCE 2013).

5.2 Data

Summary statistics for the variables used in the models for each of the time periods are displayed in Table 2. It should be noted that values of n (number of observations on which the mean is calculated) vary because of missing data for some variables.

Table 2: Summary statistics for the data

a) 2008/09 to 2010/11

	Mean	Standard deviation	Minimum	Maximum	n
COST	156,943.20	178,620.10	5,424.49	1,249,909.00	387
UGMED	1,333.43	1,457.47	0.00	6,839.48	387
UGSCI	2,684.83	2,344.37	0.00	9,249.96	387
UGARTS	5,101.03	3,703.83	0.00	16,162.84	387
PG	2,229.55	1,890.64	0.00	9,457.09	387
RESEARCH	38,928.08	81,725.65	0.00	490,105.10	387
IPINCOME	19,958.34	28,859.26	0.00	163,684.00	384
LOWPNO	220.47	214.79	0.00	1,090.00	375
LISTED	16,624.64	28,023.97	0.00	182,536.00	367

b) 2005/06 to 2007/08

	Mean	Standard deviation	Minimum	Maximum	n
COST	136,959.20	156,131.80	3,987.54	1,120,624.00	394
UGMED	1,339.37	1,510.61	0.00	6,851.22	394
UGSCI	2,474.75	2,560.99	0.00	22,110.14	394
UGARTS	4,760.30	4,153.28	0.00	39,186.91	394
PG	1,972.08	1,688.05	0.00	8,668.88	394
RESEARCH	33,316.15	68,630.43	0.00	412,845.00	392
IPINCOME	17,310.54	22,930.70	0.00	122,171.90	387
LOWPNO	205.64	197.31	0.00	1,100.00	369
LISTED	16,005.44	29,582.77	0.00	197,000.00	349

c) 2003/04 to 2004/05

	Mean	Standard deviation	Minimum	Maximum	n
COST	118,689.00	128,321.60	4,698.33	790,109.10	263
UGMED	1,305.93	1,523.77	0.00	6,890.56	263
UGSCI	2,487.89	2,713.90	0.00	22,322.49	263
UGARTS	4,546.28	4,407.67	0.00	40,237.30	263
PG	1,954.64	1,631.95	0.00	7,927.64	263
RESEARCH	29,874.27	59,394.44	0.00	311,924.80	259
IPINCOME	14,752.30	20,184.71	0.00	100,684.80	258
LOWPNO	219.59	199.40	0.00	975.00	243
LISTED	16,762.90	30,890.83	0.00	197,000.00	206

d) 2003/04 to 2010/11

	Mean	Standard deviation	Minimum	Maximum	n
COST	140,016.70	159,277.10	3,987.54	1,249,909.00	1042
UGMED	1,330.20	1,493.97	0.00	6,890.56	1042
UGSCI	2,560.84	2,522.88	0.00	22,322.49	1042
UGARTS	4,841.46	4,061.73	0.00	40,237.30	1042
PG	2,067.03	1,754.97	0.00	9,457.09	1042
RESEARCH	34,582.19	71,811.05	0.00	490,105.10	1037
IPINCOME	17,657.22	24,773.57	0.00	163,684.00	1029
LOWPNO	214.91	204.55	0.00	1,100.00	986
LISTED	16,421.15	29,245.00	0.00	197,000.00	922

5.3 Linear model over 3 time periods and for the whole time period

Table 3 reports the coefficient estimates obtained in a stochastic frontier regression of costs against linear terms in the various outputs and a set of control variables – including year dummies, an Oxbridge indicator, area of real estate comprising listed buildings, and the number of students originating from traditionally low participation neighbourhoods. The specifications of the model (and the models that follow) allow efficiency to vary across time for each institution in the data set. Costs are measured in thousands of pounds, so the coefficients on the student number variables each represent the sum (in thousands of pounds) that the marginal student adds to total costs. Hence, for example, one extra science undergraduate costs a typical university an extra £7,775 per year during the latest time period (measured at 2011 prices). It is readily observed that, within each of the (two or three year) time periods under investigation, the undergraduates that impose the highest costs on institutions are those studying medicine, followed by those studying other sciences, followed by those studying other subjects. The higher costs of these subjects are recognised by the support given by HEFCE for band A and band B disciplines. Postgraduate provision is generally more costly than undergraduate provision (much

postgraduate provision involves one-to-one supervision), except undergraduate provision in medicine.

The value of the constant in each equation is worthy of further discussion. A negative constant term in a cost equation suggests that fixed costs are negative. In some cases, the estimated coefficient is not significantly different from zero. Where a negative constant term is significant it becomes somewhat difficult to interpret. It should be noted, however, that the cost functions estimated in exercises of this type are based on data for institutions whose output exceeds zero. The equations may provide an imprecise guide to what costs would be in the hypothetical case of an institution that produced nothing. While they provide a good fit to the data over the range of data that is actually observed, the fit outside this range may be less satisfactory. A negative constant term does not therefore mean that institutions would make a profit by producing nothing; it means, rather, that, within the range of data observed, some diseconomies of scale may exist.

Table 3: Linear model with a complete set of controls

AICs	2008/09 to 2010/11	2005/06 to 2007/08	2003/04 to 2004/05	2003/04 to 2010/11
UGMED	13.48440	13.86610	9.74789	13.92710
UGSCI	7.77511	7.04032	5.60877	7.11761
UGARTS	4.57408	6.65664	3.95087	7.13533
PG	13.95320	9.40896	9.81821	12.21420
RESEARCH	0.81612	0.83696	1.18236	0.89197
IPINCOME	0.81920	1.15411	0.35017	0.81935
CONTROLS				
2003/04			-2,485.99	-17,257.20
2004/05				-14,638.20
2005/06				-10,858.80
2006/07		3,640.36		-6,838.74
2007/08		10,429.90		-182.20
2008/09				3760.20
2009/10	-2,813.51			1437.09
2010/11	-4,070.18			
OXBRIDGE	322,696.00	184,217.00	111,966.00	205,326.00
LISTED	0.31	0.25	0.22	0.12
LOWPNO	-37.29	-33.06	-3.41	-28.63
CONSTANT	-8,426.55	-39,195.80	-3,237.03	-42,488.50
Is λ significantly different from zero at the 5% significance level? ²³	YES	YES	YES	YES

Notes:

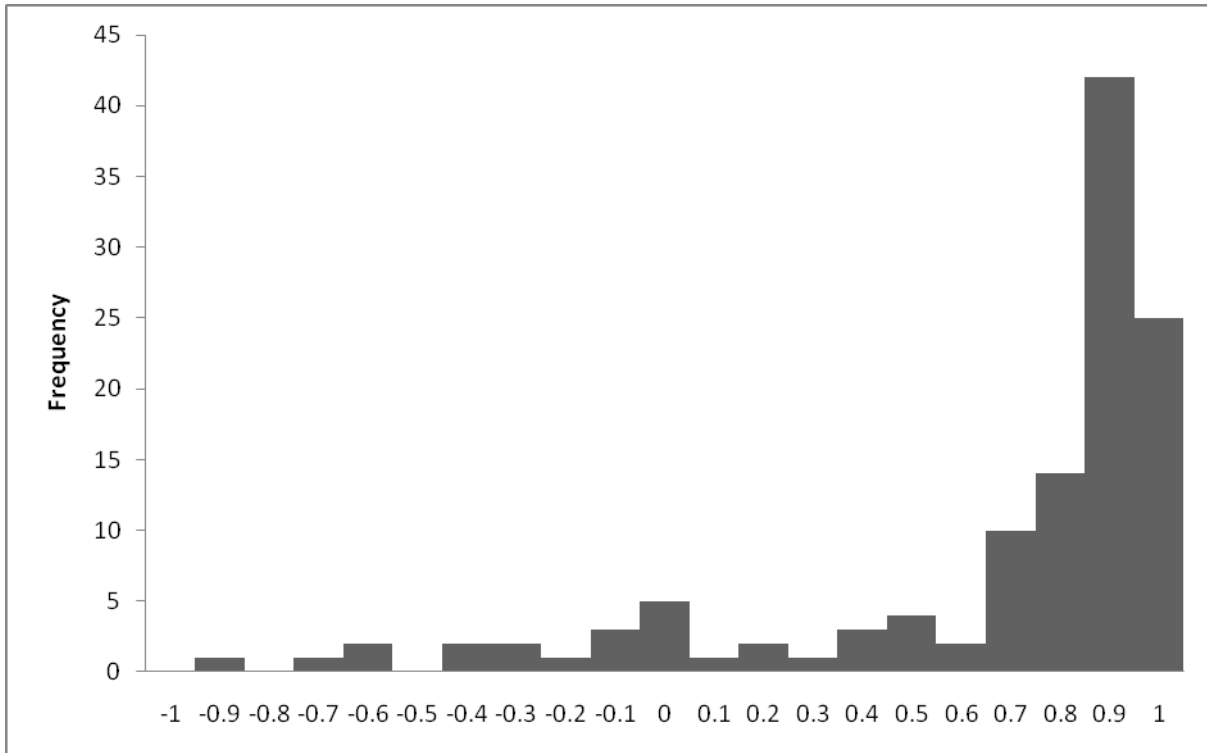
1. Controls: LISTED; LOWPNO; OXBRIDGE; YEAR dummies.
2. Coefficients in bold are statistically significantly different from zero at the 5% significance level.

The universities of Oxford and Cambridge are distinctive owing to their antiquity, organisational structures and academic orientation, and this is reflected in higher costs. Those institutions whose real estate includes a higher area covered by listed buildings typically have higher costs than others. Finally, those institutions that admit relatively high numbers of students from low participation neighbourhoods tend to have lower costs. This is, at first sight, a somewhat surprising result. The direction of causality, however, is open to debate: it may be that students from low participation neighbourhoods are attracted to institutions that have relatively low costs, possibly because of the type of subjects provided or because they undertake less research. The correlations between LOWPNO and, respectively, UGMED, UGSCI, UGARTS and RESEARCH are 0.51, 0.67, 0.78 and -0.08 and are therefore consistent with this hypothesis. We do not investigate this further as it is not the main issue of interest.

²³ This row in the table indicates whether or not the one-sided inefficiency term is statistically significantly different from zero across all observations. Here they are but in later tables (particularly ones relating to latent class models) they are not. These provide us with a check on the confidence with which we can interpret the efficiency scores.

Estimation of the equations that comprise our models of costs also allows computation of efficiency scores for each institution. These are obtained from the one-sided residual in the stochastic frontier estimator, and may be expressed as the predicted value of costs divided by the predicted value plus the one-sided residual. A score of one thus implies efficiency, while lower scores imply the existence of some inefficiency. The distribution of efficiency scores suggests that some institutions are more efficient than others. To illustrate, the distribution associated with the 2008/09 to 2010/11 model reported in Table 3 is shown in Figure 2 (efficiency distributions associated with the other models in Table 3 are reported in Appendix 4.1). This indicates that the majority of HEIs have efficiency scores above 0.8, but that there is a noticeable tail of institutions which, *on this measure*, appear to be less efficient. Some institutions have an efficiency score of less than zero. This is possible where the output levels of the institution are very low²⁴, and indicates that the model of costs does not satisfactorily explain the relationship between costs and outputs for such small institutions. In Figure 2, the single observation at the bottom end of the efficiency distribution is the Rose Bruford College of Theatre and Performance which is a small specialist institution. As we shall see later, more refined models of costs tend to produce distributions of efficiencies that look rather different from those reported here.

Figure 2: Histogram of efficiency scores – final year of 2008/09 to 2010/11 (linear model)



In Table 4 we investigate the effect of excluding the control variables from the equation. The results are broadly similar, though some observations are warranted. First, there is a noticeable shift in the coefficient values between the two time periods reported in this table – and, indeed, for the 2005/06 to 2007/08 period, between the results obtained in this table

²⁴ The efficiency score is defined as predicted costs divided by the sum of predicted costs and the one-sided residual. An efficiency score can be less than zero if predicted costs are negative.

and those reported earlier. While, in the latest time period, it costs more to produce a marginal science undergraduate than an undergraduate in non-science subjects, the reverse is true in the earlier time period. The counterintuitive result obtained here for the 2005/06 to 2007/08 time period serves as a warning that some of the statistical results are not robust to minor changes in specification or modelling strategy.

Table 4: Linear model with a limited set of controls

AICs	2008/09 to 2010/11	2005/06 to 2007/08	2003/04 to 2010/11
UGMED	13.71790	14.17640	14.42160
UGSCI	7.34657	3.17296	3.89932
UGARTS	3.01348	6.04901	5.68941
PG	18.41520	13.46680	16.13060
RESEARCH	0.93983	0.93658	0.92238
IPINCOME	0.53013	1.11874	0.87065
CONTROLS			
2003/04			-15,018.50
2004/05			-12,543.60
2005/06			-9,984.87
2006/07		5,220.86	-4,594.99
2007/08		10,904.60	1,229.65
2008/09			5,046.32
2009/10	-3,838.74		1,583.07
2010/11	-5,372.17		
OXBRIDGE	309,483.00	186,112.00	205,629.00
CONSTANT	-18,258.00	-44,053.60	-41,383.20
Is λ significantly different from zero at the 5% significance level?	YES	YES	YES

Notes:

1. Controls: OXBRIDGE; YEAR dummies.
2. Coefficients in bold are statistically significantly different from zero at the 5% significance level.
3. The 2003/04 to 2004/05 model does not converge.

No equation is reported in Table 4 for the 2003/04 to 2004/05 time period. This is because the algorithm used to obtain the maximum likelihood results for these periods failed to converge. This is likely to be because different solutions to the model (different sets of coefficients and different parameterisations of the structure of the residuals) yield similar likelihoods. This simply means that the data in these cases are not suitable for estimating a model of this kind. Efficiency distributions associated with the models displayed in Table 4 can be found in Appendix 4.2.

5.4 Quadratic model over 3 time periods and for the whole time period

We now turn to consider the quadratic stochastic frontier model. The model includes as explanatory variables:

- linear terms in all variables
- squared terms in each of the student number variables, research, and income from intellectual property
- a full set of interaction terms between the student number variables, and between each of these and research
- an interaction term between research and income from intellectual property.

The model also includes a full set of controls. The estimated coefficients on the control variables are similar to those obtained in earlier models, and are not discussed further here.

Rather than report the estimated parameters of the full quadratic model, which are difficult to interpret, Table 5 reports the average incremental costs (AICs) associated with each of the outputs. We use the definition of average incremental costs given in section 3.2 and evaluate at mean values of each of the explanatory variables. It is readily observed that these follow a similar pattern to that observed in the linear models described earlier – of undergraduates, students in medicine are the most costly, followed by those in other sciences. With the exception of the low estimate for non-science undergraduates in the 2003/04 to 2004/05 period, the estimates of average incremental costs look broadly plausible. The costs associated with postgraduates are lower than in the estimates provided by the linear model, and those associated with research are higher. It is likely that collinearity between these two variables reduced the precision of the estimates. The negative estimate of average incremental costs associated with postgraduate provision in the final column of the table is suggestive of statistical problems, and should be treated with scepticism. The relatively high values associated with undergraduate provision and, especially, third mission activity in this column indicates that multicollinearity could be adversely affecting the precision of these estimates.

Table 5: Quadratic model with a complete set of controls

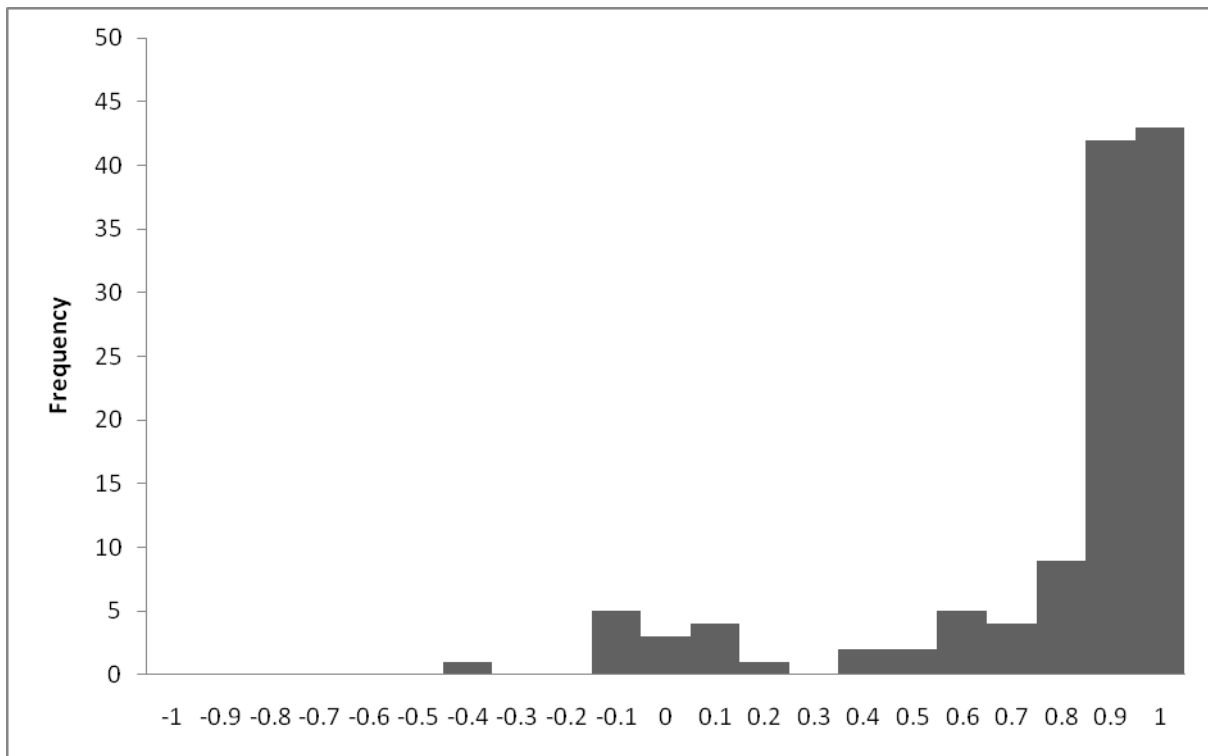
AICs	2008/09 to 2010/11	2005/06 to 2007/08	2003/04 to 2004/05	2003/04 to 2010/11
UGMED	16.03379	15.00020	9.19486	17.43360
UGSCI	7.85770	9.44366	4.59139	7.21945
UGARTS	5.45938	4.58650	0.32875	5.12836
PG	5.27499	2.60062	7.07272	-0.80377
RESEARCH	1.27087	1.32466	1.34063	1.13070
IPINCOME	1.00667	1.74035	0.89848	1.75663
Is λ significantly different from zero at the 5% significance level?	YES	YES	YES	YES

Note:

1. Controls: LISTED; LOWPNO; OXBRIDGE; YEAR dummies.

Efficiencies are estimated for HEIs using the 2008/09 to 2010/11 model, and these are plotted for the final year only in Figure 3. The distribution of efficiencies indicates that, compared with the linear model, the efficiency scores are, in general, higher when the quadratic specification of the cost equation is used. This is not surprising – a richer model allows more of the differences between institutions to be explained, thus leaving less to be accounted for by a residual which is (misleadingly, perhaps) labelled inefficiency. The observation at the lower extreme of the distribution is Heythrop College, which is a small institution specialising in philosophy and theology.

Figure 3: Histogram of efficiency scores – final year of 2008/09 to 2010/11 (quadratic model)



The effect of excluding the control variables from the quadratic equation is examined; AICs are reported in Appendix 4.3 and histograms of efficiencies are reported in Appendix 4.4.

5.5 Linear latent class model over 3 time periods and for the whole time period

The above results all impose a single structure on costs for all institutions. It is clear, however, that institutions of higher education in England are not homogenous in character. To allow for this, we turn to the evaluation of latent class variants of the stochastic frontier model. In these, we suppose that institutions are all one of two types²⁵. We do not constrain any institution to be in the same latent class in all years. Table 6 shows the results obtained when a latent class stochastic frontier is applied to a linear model of costs

²⁵ We do in fact try estimating models with different numbers of classes, in particular 3- and 4-class models. In each case, estimated AICs for one class are implausible, presumably because of the small number of observations on which the estimates are based. More details are provided at the end of this section.

with a full set of controls. There are several striking features that emerge from the results. First, in all time periods, the parameter associated with (non-medical) science undergraduates is relatively low in the first latent class, but relatively high in the second. This may reflect differences in the precise mix of science subjects provided in institutions within each class. Secondly, two parameters are striking by virtue of being surprisingly low – those on postgraduates in the first latent class in the 2008/09 to 2010/11 period, and on undergraduates in medicine in the first latent class in the 2003/04 to 2004/05 period. It may be the case that the institutions in these classes do not produce sufficient medical students and postgraduates respectively to provide reliable estimates of these parameters. We shall examine the membership of each latent class later. Thirdly, the negative coefficient on postgraduates in the first latent class in the 2003/04-2010/11 period is curious and should be treated with scepticism²⁶.

²⁶ Note also that the inefficiency component for this model is not significantly different from zero at the 5% significance level (as indicated by the value of λ) and this provides further reason to be cautious about this model.

Table 6: Linear latent class model with a complete set of controls (2 classes)

AICs	2008/09 to 2010/11		2005/06 to 2007/08		2003/04 to 2004/05		2003/04 to 2010/11	
	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2
UGMED	10.86470	7.77427	9.73153	6.62318	2.40610	9.44584	6.01711	10.04800
UGSCI	1.93079	8.47168	1.74760	8.64085	2.53791	7.05533	2.99762	5.61400
UGARTS	9.35289	2.75710	8.16584	4.65941	6.50197	4.42665	12.14230	6.07852
PG	0.24627	18.69390	10.45850	5.75412	13.43170	8.61365	-17.48280	9.70639
RESEARCH	1.50834	0.97330	1.65369	0.96609	1.15465	1.45594	1.43784	1.27254
IPINCOME	1.07664	0.62224	-0.07846	1.50001	0.82643	-0.11413	2.93942	0.52597
CONTROLS								
2003/04					-1,114.38	-1,735.85	-206,226.00	-11,577.00
2004/05							-182,882.00	-9,695.82
2005/06							-34,486.50	-8,308.94
2006/07			7,911.85	-171.29			-42,794.40	-4,722.14
2007/08			18,511.40	2,281.25			-20,563.60	581.69
2008/09							-33,363.10	3,819.46
2009/10	-1,139.03	-465.19					-13,166.80	1,547.01
2010/11	1,095.46	-4,842.08						
OXBRIDGE	431,387.00	104,301.00	379,554.00	104,248.00	77,371.10	43,327.90	302,035.00	52,040.60
LISTED	-0.10	0.15	-0.18	-0.09	0.74	-0.03	-0.18	0.01
LOWPNO	-31.10	-4.97	-0.56	-9.63	-36.32	2.81	-44.11	-12.87
CONSTANT	-519.58	-557.62	-19,537.50	-924.37	-965.39	-2,224.99	36,444.30	-1,164.20
Number in each class	121	234	111	216	60	136	38	840
Is λ significantly different from zero at the 5% significance level?	YES	YES	YES	YES	NO	YES	NO	YES

Notes:

1. Controls: LISTED; LOWPNO; OXBRIDGE; YEAR dummies.
2. Coefficients in bold are statistically significantly different from zero at the 5% significance level.
3. The number in each class is the number of observations over the estimation period, not the number of HEIs. Given that HEIs can be in different classes in different years, dividing by the number of years on which the analysis is based does not give the number of HEIs.

As is the case for the simpler models estimated above, it is possible to report the distributions of efficiency scores obtained by institutions when applying the latent class model. In this instance, however, the efficiency of each institution is evaluated by comparing the institution only to others in its own class. Since this is more akin to comparing like with like, the average efficiency score is higher than was observed in the simpler models. The distributions of efficiency scores for the two latent classes, using the model for 2008/09 to 2010/11 are shown in Figure 4.

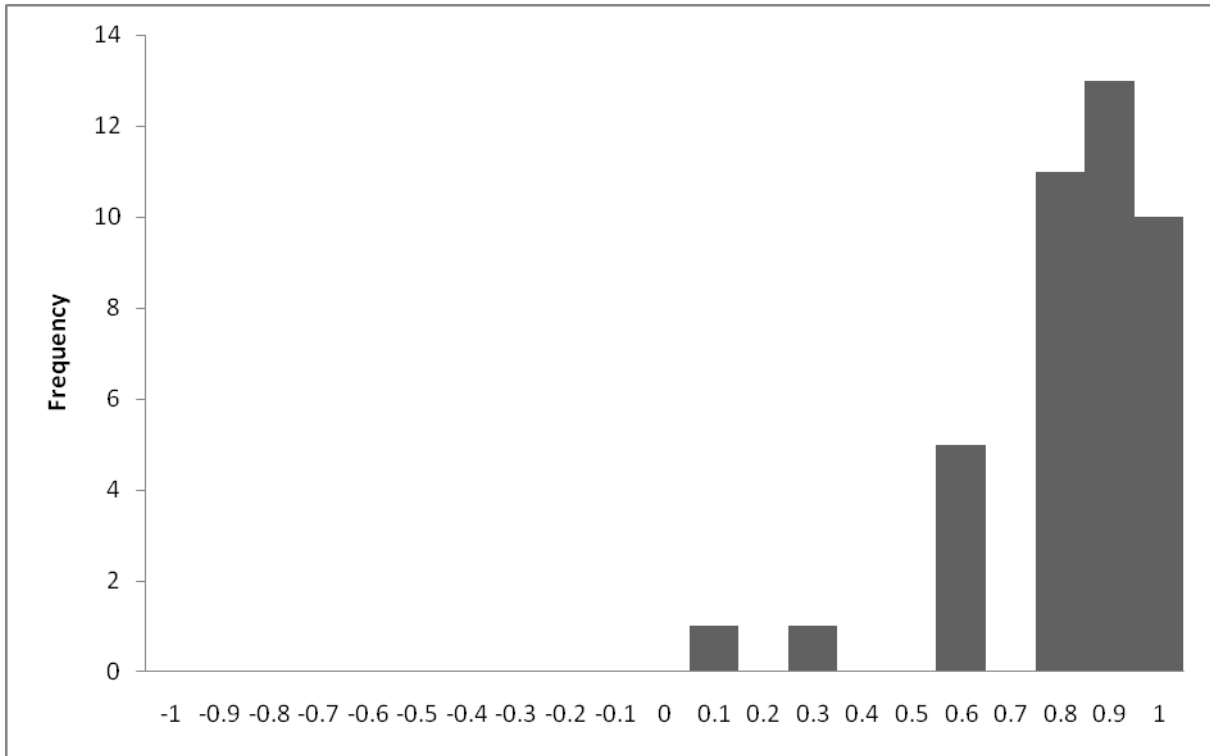
For the second latent class, most HEIs have efficiency scores which are above 0.6. For the first class, just two HEIs have efficiency scores less than 0.6: the London University (Institutes and Activities) and Trinity Laban Conservatoire of Music and Dance. The former is unusual in that it comprises a small and highly specialised collection of research centres, while the latter specialises in music and dance.

We do not constrain HEIs to be in the same latent class across all years within any period of analysis. In Appendix 4.5 we present a table showing the membership of each of the two latent classes in each year of the 2008/09 to 2010/11 period. It is readily observed that individual institutions switch between the classes quite frequently. Moreover, it is difficult to provide a clear rationale for the membership of the latent classes in any given year, beyond noting that the classes are defined by the statistical method. Since the latent classes are not easily explained by appeal to intuition, we present an alternative means of disaggregating the data by institution type later in this report (see section 5.7).

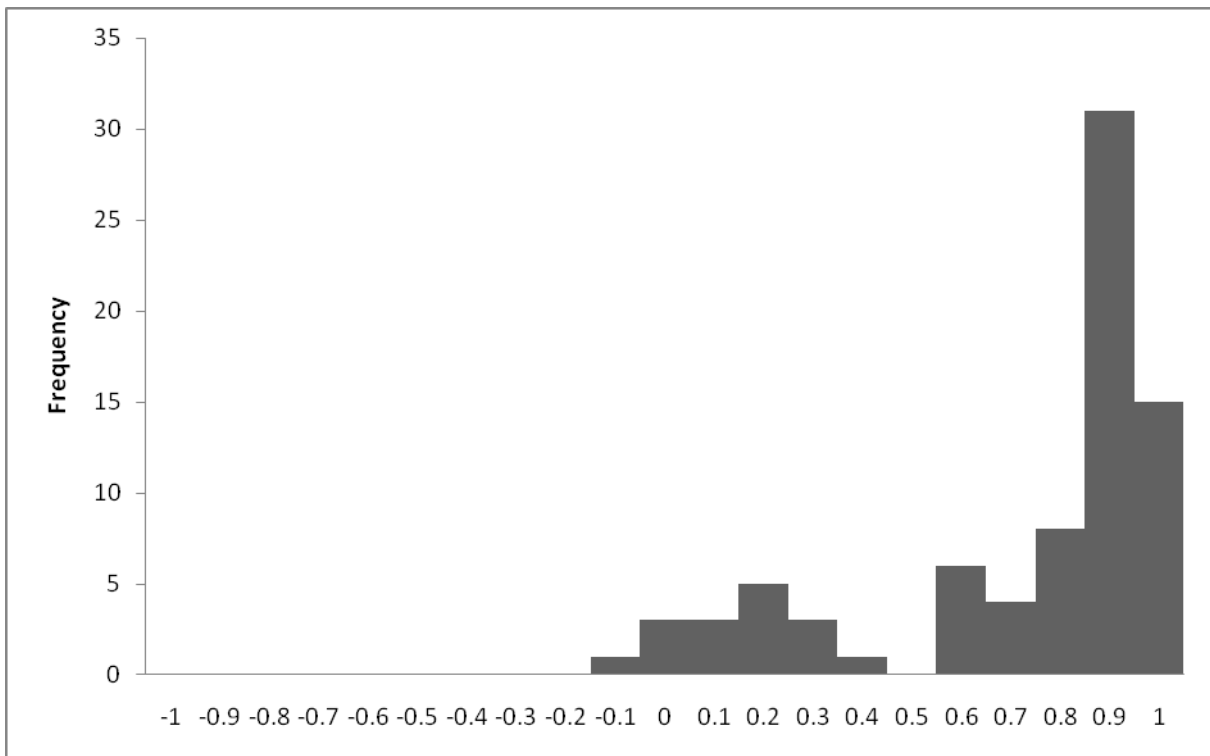
Results obtained by estimating linear models with 3 and 4 latent classes (respectively) are reported in Appendix 4.6. In each case, one of the latent classes is small and yields implausible estimates of average incremental costs. We do not, therefore, report the distributions of efficiency scores associated with these analyses.

Figure 4: Histogram of efficiency scores – final year of 2008/09 to 2010/11 (linear latent class model)

a) Latent class 1



b) Latent class 2



Note: These histograms relate to the equations in Table 6 (2008/09 to 2010/11 model), and are drawn for the final year of the estimation period.

5.6 Quadratic latent class model over 3 time periods and for the whole time period

Now consider the latent class stochastic frontier model with a quadratic specification. As earlier, rather than report the coefficients of the full model, which are difficult to interpret, we tabulate the average incremental costs, evaluated at mean values of the explanatory variables within each class. These appear in Table 7. We note some caveats that should attach to the results reported here, particularly for the 2003/04-2004/05 period. In this case, the model has many explanatory variables and the sample used to estimate the parameters in this two year period is small. Indeed, owing to missing values for the control variables, this is an issue that applies, to a greater or lesser extent, to all estimates obtained for this earliest period. The negative estimates obtained for some average incremental costs in the 2005/06-2007/08 period suggest that the results for this period, too, should be treated with caution²⁷. Our confidence in these results has to be conditioned by the observation that we are drilling down to subgroups of institutions, with data collected over short periods, and using a complex specification with numerous explanatory variables; the numbers of observations available are insufficient to allow precise estimates of the parameters.

The distributions of efficiency scores associated with the model in Table 7 (2008/09 to 2010/11 model) are reported in Figure 5 for the final year of the estimation period. The majority of HEIs (across both classes) have efficiency scores above 0.9, but a number of small specialist institutions once again achieve much lower scores. This poses questions about the validity of including these in the analysis.

²⁷ It should also be noted that for each of the 2005/06 to 2007/08 and the 2003/04 and 2004/05 periods, the inefficiency component is not significantly different from zero (at the 5% significance level) for either of the latent classes as indicated by λ .

Table 7: Quadratic latent class model with complete set of controls (2 classes)

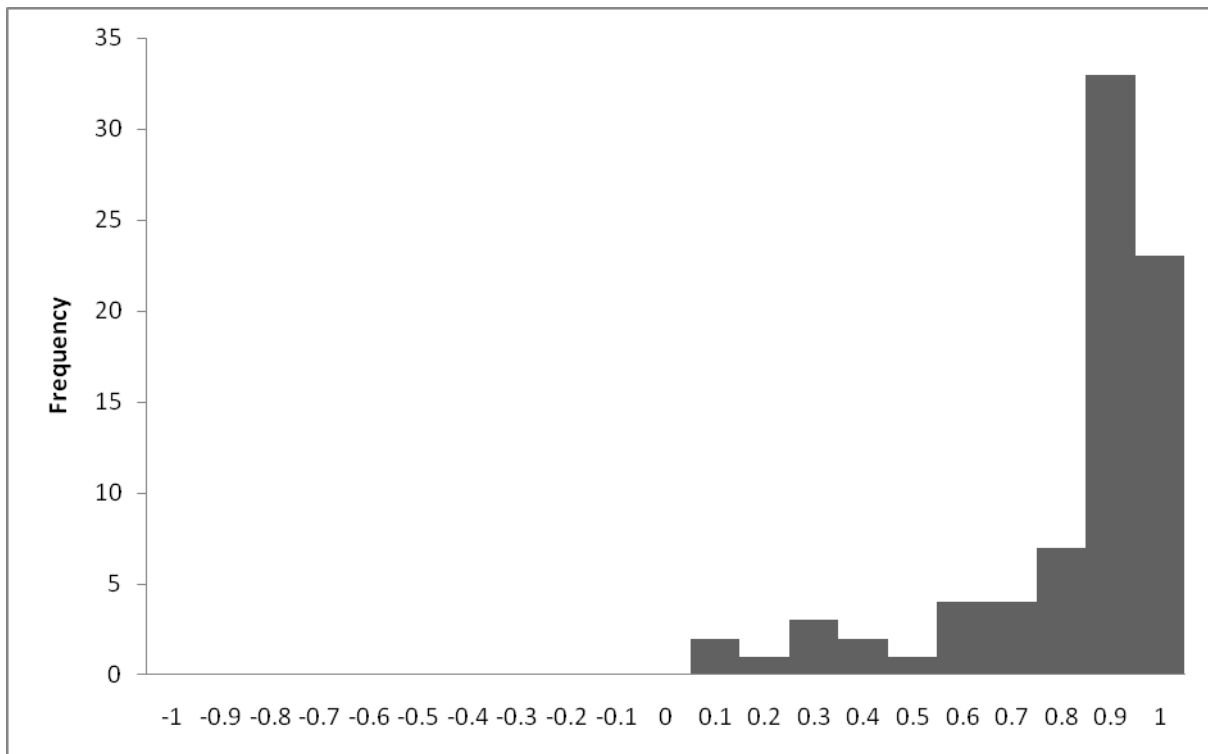
AICs	2008/09 to 2010/11		2005/06 to 2007/08		2003/04 to 2004/05	
	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2
UGMED	8.72000	19.59518	8.35074	8.93250	3.95822	4.96233
UGSCI	5.25962	7.18468	7.70809	11.10917	0.86012	8.75251
UGARTS	5.88274	2.17572	-2.35441	6.14633	0.76439	6.57626
PG	7.83897	1.24167	-10.07146	0.30639	-4.89484	0.37550
RESEARCH	1.12587	1.14127	0.89196	1.32144	1.64641	1.30870
IPINCOME	1.02964	0.75173	2.79682	0.26046	1.73449	0.20759
Number in each class	236	119	132	195	100	96
Is λ significantly different from zero at the 5% significance level?	YES	YES	NO	NO	NO	NO

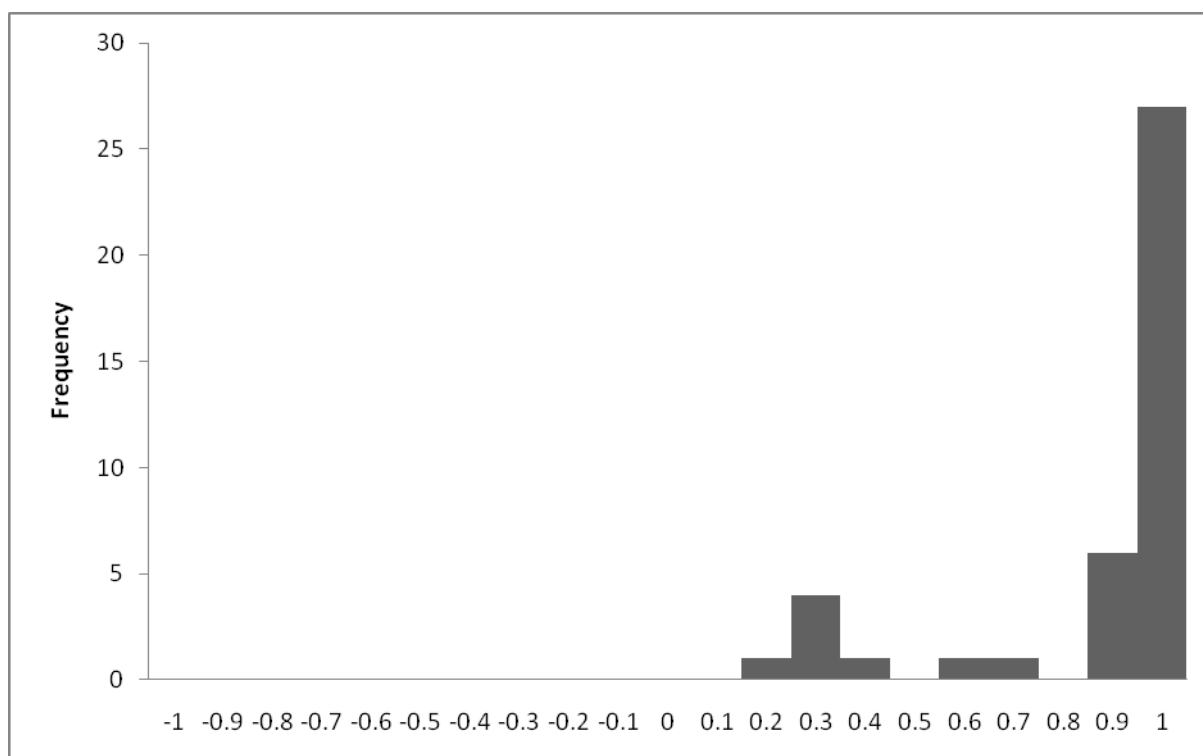
Notes:

1. Controls: LISTED; LOWPNO; OXBRIDGE; YEARS.
2. The 2003/04 to 2010/11 model does not converge.
3. The number in each class is the number of observations over the estimation period, not the number of HEIs. Given that HEIs can be in different classes in different years, dividing by the number of years on which the analysis is based does not give the number of HEIs.

Figure 5: Histogram of efficiencies – final year of 2008/09 to 2010/11 (quadratic latent class model)

a) Latent class 1



b) Latent class 2**Note:**

These histograms relate to the equations in Table 6 (2008/09 to 2010/11 model), and are drawn for the final year of the estimation period.

5.7 Comparison of results with pre-defined groupings of universities

An obvious alternative to latent class modelling involves prescribing groups within which institutions are deemed to share certain characteristics. One such categorisation is the HEFCE impact report groups – a categorisation that divides institutions into four types: (1) specialist (2) high tariff (3) medium tariff and (4) low tariff. Results obtained by estimating linear stochastic frontier models (with only a limited set of controls) for each of these groups are reported in Table 8.

We do not report results for a quadratic model because numbers of observations within each group are too low. Also, note that, in contrast to the tables reported earlier, and owing to the small sample size represented within each group, the specifications of the models reported here do not include control variables (area of estate covered by listed buildings, and numbers of students from low participation neighbourhoods). The results are instructive. Most of the specialist institutions are specialising either in medicine or in the applied arts. The low parameter on undergraduate provision in other sciences that is obtained in all years for this group is therefore unsurprising – few students in these institutions fall into this subject category. The high cost associated with non-science undergraduates in high tariff institutions (type 2) in the later two time periods should be

treated with caution; with only 28 universities in this group, it is the smallest of the HEFCE groupings and many coefficients are imprecisely estimated²⁸.

The efficiency scores that result from the analysis of HEFCE impact report groups are reported, for the final year of the 2008/09 to 2010/11 period, in Appendix 4.7. For groups 3 and 4 (medium and low tariff groups) the efficiency scores are generally clustered around a very high level of efficiency, suggesting that there is little scope to differentiate between institutions within each of these groups on the basis of efficiency score. For groups 1 and 2 (specialist and high tariff groups) the distribution of efficiencies is wide, but it should be borne in mind that these are derived from poorly estimated equations where many coefficients are not statistically significantly different from zero.

²⁸ It is also the case that the one-sided inefficiency component is not significantly different from zero as signalled by the value of λ .

Table 8: Linear model with a limited set of controls – pre-defined classes

AICs	2008/09 to 2010/11				2005/06 to 2007/08			
	Type 1	Type 2	Type 3	Type 4	Type 1	Type 2	Type 3	Type 4
UGMED	12.17810	8.26523	8.41394	8.83894	12.09190	8.14358	7.10110	8.79205
UGSCI	2.08030	9.82652	8.08465	5.02403	2.06171	4.17238	4.40898	4.69115
UGARTS	12.26320	14.85030	3.22720	6.92543	11.02950	12.58070	4.84906	7.23211
PG	6.41110	11.35770	14.60920	11.08710	8.04269	17.63310	11.47020	6.33013
RESEARCH	1.25448	1.70680	1.30524	1.00174	1.06033	1.55152	1.02540	1.51193
IPINCOME	1.85920	-0.25084	0.69636	0.51220	1.56877	0.26268	1.31932	0.70375
CONTROLS								
2003/04								
2004/05								
2005/06								
2006/07					520.84	11,005.40	3,766.26	5,581.02
2007/08					2,602.96	20,886.30	9,953.88	8,545.21
2008/09								
2009/10	-150.01	-5,397.57	-3,048.97	-5,298.47				
2010/11	-2,489.73	-3,924.66	-5,275.52	-6,620.22				
CONSTANT	-35,205.60	-152,147.00	4,518.53	458.43	-32,350.50	-141,177.00	-10,162.20	-9,191.27
Number in class	111	84	96	87	106	83	109	82
Is λ significantly different from zero at the 5% significance level?	YES	NO	NO	YES	NO	NO	YES	YES

Table 8: continued

AICs	2003/04 to 2004/05				2003/04 to 2010/11			
	Type 1	Type 2	Type 3	Type 4	Type 1	Type 2	Type 3	Type 4
UGMED	15.17180	3.41735	6.54379	6.91507	22.90240	7.88866	7.42526	8.56164
UGSCI	4.55565	5.46450	3.35649	7.61685	4.47340	8.22235	5.98942	5.10155
UGARTS	10.50840	4.51969	5.51970	5.79197	11.97400	12.32610	4.11670	6.80409
PG	0.21740	19.26820	10.62440	11.25630	3.09518	16.18410	11.87630	9.64011
RESEARCH	1.36032	1.61664	1.12153	0.79771	1.06451	1.61885	1.12084	1.18349
IPINCOME	1.69211	-0.22979	0.68564	0.24722	2.08913	-0.24532	0.89513	0.52735
CONTROLS								
2003/04	-533.96	-6,326.86	-1,705.88	-2,463.74	-4,017.72	-20,621.80	-14,264.10	-10,991.80
2004/05					-3,836.98	-15,402.60	-12,569.10	-8,805.18
2005/06					-2,996.30	-19,980.70	-10,042.10	-6,951.10
2006/07					-2,488.56	-7,621.67	-5,622.40	-906.64
2007/08					-460.13	2,289.81	534.88	2,514.16
2008/09					2,228.48	6,937.04	4,736.52	6,696.75
2009/10					2,062.71	-156.11	2,826.78	1,518.68
2010/11								
CONSTANT	-21,623.30	-49,047.60	-8,504.08	-9,021.09	-31,205.70	-126,630.00	113.46	-4,549.38
Number in class	62	55	88	50	279	222	293	219
Is λ significantly different from zero at the 5% significance level?	NO	NO	YES	YES	NO	YES	YES	YES

Notes:

1. Controls: YEAR dummies
2. Coefficients in bold are statistically significantly different from zero at the 5% significance level.
3. The number in each class is the number of observations over the estimation period, not the number of HEIs. Given that HEIs can be in different classes in different years, dividing by the number of years on which the analysis is based does not give the number of HEIs.

5.8 Discussion

Numerous models have been presented in this report, and it is useful at this stage to present some evaluation of their relative merits. The simplest specification is presented in Table 4 – this is the linear model with a limited set of controls, where a single set of parameters attaches to all institutions in the sample. The estimated parameters are broadly plausible, though those on miscellaneous science and non-science undergraduates appear to be quite volatile from sub-period to sub-period. Moreover this model could not, for statistical reasons, be estimated for the 2003/04-2004/05 period. Adding control variables to this model produces the results in Table 3, where the pattern of costs associated with undergraduate provision is seen to be stable across subject areas, with medicine being the most costly and non-science the least costly subjects. The cost attached to postgraduate provision (across all subjects) is broadly on a par with those associated with undergraduate medicine.

The parsimonious specification of the model – where all institutions are assumed to have the same structure of costs and where no accommodation is made for the possible existence of scale and scope economies – works well as a description of how costs are determined in higher education. But it leaves out a lot of information that may be relevant to the evaluation of efficiency. So the more refined models, where institutions are divided into classes (either by the latent class method or by a priori assignment of institutions into groups) may be more informative about efficiency. The distributions of efficiencies that emerge from a quadratic specification of a latent class model with only two latent classes (in Figure 5) are quite compressed, with a concentration of institutions having efficiency scores above 0.9. There is nevertheless a (fairly small) number of institutions with low scores. These are all unusual by virtue of being small, specialist institutions.

The latent class specifications of the model – whether linear or quadratic – also have the advantage of allowing some correction for unobserved heterogeneity across institutions. Albeit only by allowing two classes of institutions (in the results reported here), this makes some allowance for differences in (amongst other things) quality of provision that might influence costs. A more refined approach, using random parameters so that the distinctiveness of each institution is accommodated, would more fully allow for these qualitative differences, but might be viewed as being too permissive, and this approach has not, therefore, been employed here.

6. Conclusions

This report shows how statistical models of the determination of costs faced by HEIs can be constructed. These can help to explain the relationship between costs and both outputs and other characteristics of institutions. They also allow the evaluation of an efficiency score for each institution.

It is important to note that efficiency is a slippery concept. A user of the results of a statistical analysis may deem some characteristics of institutions, but not others, to be legitimate explanations of cost variations. This issue is further complicated by the fact that some of the characteristics that influence costs can be measured whereas others cannot – though, using panel data, both observable and unobservable characteristics can be allowed for in the calculation of an efficiency score.

A key finding of the report is that, once differences between institutions are accounted for, even by a relatively unrefined latent class modelling procedure, the variation in efficiency scores across institutions is greatly reduced. Indeed, the relatively small number of institutions with low scores is exclusively made up of small and specialist institutions. The results do not, therefore, support the notion that substantial sector-wide gains could be made by using efficiency scores as a criterion for resource allocation.

That said, the fact that frontier models can be estimated without difficulty in this context confirms that, in general, the residuals have the right skew. In other words, there does exist a distribution of efficiencies across institutions. It is not possible, from the analysis reported above, to explain these differences in measured efficiency – the nature of the analysis is that efficiency is calculated as an unexplained residual. This suggests that there may be advantage in conducting an analysis aimed at comparing institutions with higher and lower efficiency scores. Such an analysis would need to gather qualitative information of a kind that does not fit easily within the statistical approach adopted in the current report. Case studies would allow evaluation of the extent to which organisational structures, management styles and other more qualitative characteristics affect organisational efficiency. We leave this to future research.

Appendices

Appendix 1: Summary of results of previous studies of costs in the UK higher education sector

	Verry & Layard (1975)	Verry & Davies (1976)	Cohn et al (1989)	Johnes (1990)	Glass et al (1995a)
Period covered	1968/69	1968/69 (approx)		1985, 1986, 1987	1989/90
Coverage of HEIs	UK universities except Oxford and Cambridge	UK universities except Oxford and Cambridge UK polytechnics	1887 US universities	45 UK universities	61 UK universities
Frontier estimation	No	No	No	No	No
Functional form	Linear	Linear Quadratic Multiplicative	Quadratic	Linear	Hybrid translog
Definition of costs	Estimation by 6 departments: Departmental current costs	Estimation by 6 departments: Departmental current costs Also: Recurrent central university costs	Total cost	Total general expenditure on academic departments divided by FTE students	Total cost
Definition of outputs	<ul style="list-style-type: none"> •No. of UG students in the given year •No. of PG students in the given year (split by coursework and research in some runs) •Annual quality weighted hours spent on research 	<ul style="list-style-type: none"> •No. of UG students in the given year •No. of PG students in the given year (split by coursework and research in some runs) •Annual quality weighted hours spent on research (sometimes replaced by number of articles and books) 	<ul style="list-style-type: none"> •UG FTE enrolment •PG FTE enrolment •Research grants 		<ul style="list-style-type: none"> •No. FTE Ugs •No. FTE PGs •1989 RAE aggregate research score
Third mission	No	No	No	No	No
Input prices	No	No	Average faculty salaries	No	Price of capital Price of labour

	Verry & Layard (1975)	Verry & Davies (1976)	Cohn et al (1989)	Johnes (1990)	Glass et al (1995a)
Quality	No	Undergraduate quality is considered by using a value added measure based on A level results, degree results, & salaries.	No	No Subject mix found to be an important determinant of unit costs	No
Geographical location	No	London and Scotland considered	No	No	No
Mission group		<ul style="list-style-type: none"> •Redbricks •New •Medical schools •ExCats •Polytechnics 	Separate estimation for public and private universities	No	Research groups: <ul style="list-style-type: none"> •Top-ranking •Middle-ranking •Bottom-ranking
Economies of scale	<ul style="list-style-type: none"> •Arts: 1.069 •Social science: 1.197 •Maths: 1.012 •Physical science: 0.986 •Biological science: 1.045 •Engineering: 1.060 	Departmental costs <ul style="list-style-type: none"> •Arts: 1.069 •Social science: 1.197 •Maths: 1.012 •Physical science: 0.986 •Biological science: 1.045 •Engineering: 1.060 Recurrent central university costs: Ray: 1.141	Public institutions: <ul style="list-style-type: none"> •Ray: 1.045 •UG: 0.944 •PG: 1.685 •Research: 1.273 Private institutions: <ul style="list-style-type: none"> •Ray: 1.213 •UG: 0.954 •PG: 0.674 •Research: 0.694 		<ul style="list-style-type: none"> •Ray: 1.131 •Research: 1.695 •PG: 1.263 •UG: 2.578
Economies of scope	Assumed zero	The multiplicative models find the interaction terms are generally insignificant	Public institutions: <ul style="list-style-type: none"> •Ray: -0.064 Private institutions: <ul style="list-style-type: none"> •Ray: 0.179 		<ul style="list-style-type: none"> •Global economies of scope not significantly different from 0. •Product specific for R and PG are not significantly different from 0. •UG has significant diseconomies of scope.

	Verry & Layard (1975)	Verry & Davies (1976)	Cohn et al (1989)	Johnes (1990)	Glass et al (1995a)
Average or marginal costs	<p>MC Arts:</p> <ul style="list-style-type: none"> •UG = £310 •PG = £710 <p>MC Social Science:</p> <ul style="list-style-type: none"> •UG = £310 •PG = £860 <p>MC Maths:</p> <ul style="list-style-type: none"> •UG = £350 •PG = £1470 <p>MC Physical Science:</p> <ul style="list-style-type: none"> •UG = £480 •PG = £2,100 <p>MC Biological Science:</p> <ul style="list-style-type: none"> •UG = £550 •PG = £1,580 <p>MC Engineering:</p> <ul style="list-style-type: none"> •UG = £680 •PG = £1,610 	<p>Departmental costs</p> <p>MC Arts:</p> <ul style="list-style-type: none"> •UG = £134 •PG = £468 <p>MC Social Science:</p> <ul style="list-style-type: none"> •UG = £133 •PG = £620 <p>MC Maths:</p> <ul style="list-style-type: none"> •UG = £118 •PG = £902 <p>MC Physical Science:</p> <ul style="list-style-type: none"> •UG = £243 •PG = £1,533 <p>MC Biological Science:</p> <ul style="list-style-type: none"> •UG = £310 •PG = £1,012 <p>MC Engineering:</p> <ul style="list-style-type: none"> •UG = £441 PG = £1,049 <p>Central costs</p> <p>MC Arts:</p> <ul style="list-style-type: none"> •UG = £171 •PG = £242 <p>MC Science:</p> <ul style="list-style-type: none"> •UG = £235 PG = £564 			<p>MCs:</p> <ul style="list-style-type: none"> •PG = 0.074 •UG = 0.350
Efficiencies	N/A	N/A	N/A	N/A	N/A

	Glass et al (1995b)	Johnes (1996)	Johnes (1998)	Johnes (1997)	Izadi et al (2002)
Period covered	1992	1989/90	1989/90	1994/95	1994/95
Coverage of HEIs	53 traditional UK universities plus 8 London colleges	UK universities	50 UK universities	99 UK universities	99 UK HEIs
Frontier estimation	No	•OLS •SFA	•SFA •DEA	No	SFA
Functional form	Hybrid translog	Quadratic	Quadratic	CES	CES
Definition of costs	Total cost	Total recurrent expenditure	Total recurrent expenditure	Total expenditure	Total expenditure
Definition of outputs	<ul style="list-style-type: none"> •No. FTE UGs •No. FTE PGs •1992 RAE aggregate research score 	<ul style="list-style-type: none"> •FTE UGs in Arts •FTE UGs in Science •FTE PGs in Arts •FTE PGs in Science •Research income in Arts •Research income in Science 	<ul style="list-style-type: none"> •FTE UGs in Arts •FTE UGs in Science •FTE PGs in Arts •FTE PGs in Science •External research grants in Arts •External research grants in Science 	<ul style="list-style-type: none"> •UG load in Arts •UG load in Science •PG load •Research grants and contracts <p>Note that each part-time student is assumed to be 0.5 times a full-time student to measure student load.</p>	<ul style="list-style-type: none"> •UG load in Arts •UG load in Science •PG load •Research grants and contracts <p>Note that each part-time student is assumed to be 0.5 times a full-time student to measure student load.</p>
Third mission	No	No	No	No	No
Input prices	Price of capital Price of labour	No	No	No	No
Quality	No	No	No	No	No
Geographical location	No	No	No	No	No
Mission group	Research groups: <ul style="list-style-type: none"> •Top-ranking •Middle-ranking •Bottom-ranking 	No	No	<ul style="list-style-type: none"> •Pre-1992 HEIs •Post-1992 HEIs <p>Results run for each of 2 groups and no significant difference in cost function found.</p>	<ul style="list-style-type: none"> •Arts-biased •Science-biased

	Glass et al (1995b)	Johnes (1996)	Johnes (1998)	Johnes (1997)	Izadi et al (2002)
Economies of scale	<ul style="list-style-type: none"> •Ray: 1.16 •PG: 7.057 •UG: 2.789 	SFA <ul style="list-style-type: none"> •Ray: 1.05 •PG: 5.06 •Research: 2.41 •All others unity 	<ul style="list-style-type: none"> •Ray: 1.07 •Arts UG: 1.32 •Science UG: 0.93 •PG: 1.81 •Research: 1.44 	SFA <ul style="list-style-type: none"> •Ray: 1.61 •PG Science: 5.03 •Unit for other outputs 	<ul style="list-style-type: none"> •Ray: 1.01 •UG Arts: 1.20 •UG Science: 1.03 •PG: 3.34 •Research: 1.39
Economies of scope	Not calculated	Global: 0.18	Global: -0.08	Global: 0.17	Global: -0.63
Average or marginal costs	<ul style="list-style-type: none"> •PG = 0.118 •UG = 0.328 	Average incremental costs <ul style="list-style-type: none"> • UG Arts = £6,239 • UG Science = £8,261 • PG Arts = £4,599 • PG Science = £8,322 • Research Arts = £4.95 • Research Science = £2.19 	Average incremental costs <ul style="list-style-type: none"> • UG Arts = £3,920 • UG Science = £6,090 • PG = £11,120 	Average incremental costs <ul style="list-style-type: none"> • UG Arts = £6,239 • UG Science = £8,275 • PG Arts = £4,598 • PG Science = £8,327 	Average incremental costs <ul style="list-style-type: none"> • UG Arts = £3,241 • UG Science = £6,714 • PG = £22,789 • Research = £3.20
Efficiencies	N/A	Not reported	N/A	Mean DEA efficiency = 0.91 Rank correlation between SFA and DEA efficiencies = 0.133	Mean efficiency = 0.78 Minimum = 0.374 Maximum = 0.991

	Stevens (2005)	Johnes et al (2005)	Johnes et al (2008)	Johnes & Johnes (2009)	Thanassoulis et al (2011)
Period covered	1995/96 to 1998/99	2000/01 to 2002/03	2000/01 to 2002/03	2000/01 to 2002/03	2000/01 to 2002/03
Coverage of HEIs	80 HEIs in England and Wales	121 HEIs in England	121 HEIs in England	121 HEIs in England	121 HEIs in England
Frontier estimation	SFA	<ul style="list-style-type: none"> •RE •SFA •DEA 	<ul style="list-style-type: none"> •RE •SFA 	<ul style="list-style-type: none"> •RE •Random Parameter RE •SFA •Random Parameter SFA NB: Only UG Science coefficient is random	<ul style="list-style-type: none"> •DEA •Malmquist
Functional form	Translog	Quadratic for parametric models	Quadratic	Quadratic	N/A
Definition of costs	Total expenditure	Total operating costs (excluding catering and student accommodation)	Total operating costs (excluding catering and student accommodation)	Total operating costs (excluding catering and student accommodation)	Total operating costs (excluding catering and student accommodation)
Definition of outputs	<ul style="list-style-type: none"> •UG Science •UG Arts •PG •Research income 	<ul style="list-style-type: none"> •FTE UGs in Medicine •FTE UGs in Science •FTE UGs in Non-science •FTE PGs •Quality related research funding and research grants •Income from other services rendered 	<ul style="list-style-type: none"> •FTE UGs in Medicine •FTE UGs in Science •FTE UGs in Non-science •FTE PGs •Quality related research funding and research grants •Income from other services rendered 	<ul style="list-style-type: none"> •FTE UGs in Science (inc medicine) •FTE UGs in Non-science •FTE PGs •Quality related research funding and research grants 	<ul style="list-style-type: none"> •FTE UGs in Medicine •FTE UGs in Science •FTE UGs in Non-science •FTE PGs •Quality related research funding and research grants •Income from other services rendered
Third mission	No	Yes	Yes	No	Yes
Input prices	Average staff costs	No	No	No	No

	Stevens (2005)	Johnes et al (2005)	Johnes et al (2008)	Johnes & Johnes (2009)	Thanassoulis et al (2011)
Quality	<ul style="list-style-type: none"> •Average A level score •Proportion of firsts and upper seconds 	A value added variable is added to equation but is not significant	A value added variable is added to equation but is not significant	No	No
Geographical location	No	London considered but barely significant	London considered but not significant	No	No
Mission group	No	<ul style="list-style-type: none"> •Colleges of higher education •Pre-1992 HEIs •Pre-1992 HEIs with medical schools •Post-1992 HEIs 	<ul style="list-style-type: none"> •Colleges of higher education •Pre-1992 HEIs •Pre-1992 HEIs with medical schools •Post-1992 HEIs 	<ul style="list-style-type: none"> •Top 5 •Civics •ExCATs and Greenfields •Other pre-1992 HEIs •Post-1992 HEIs •Colleges of higher education 	<ul style="list-style-type: none"> •Colleges of higher education •Pre-1992 HEIs •Pre-1992 HEIs with medical schools •Post-1992 HEIs
Economies of scale	Not calculated	<p>Random Effects</p> <ul style="list-style-type: none"> •Ray economies: 1.13 •UG Medicine: 0.98 •UG Science: 0.89 •UG Non-science: 0.86 •PG: 0.99 •Research :1.05 	<p>Random Effects</p> <ul style="list-style-type: none"> •Ray economies: 1.09 •UG Medicine: 1.01 •UG Science: 0.99 •UG Non-science: 0.95 •PG: 1.00 •Research: 1.07 <p>SFA</p> <ul style="list-style-type: none"> •Ray economies: 0.96 •UG Medicine: 0.98 •UG Science: 1.01 •UG Non-science: 1.02 •PG: 0.87 •Research: 1.07 	<p>Random parameters random effects</p> <ul style="list-style-type: none"> •Ray economies: 1.10 •UG Science: 0.87 •UG Non-science: 0.96 •PG: 1.22 •Research: 1.04 <p>Random parameters SFA</p> <ul style="list-style-type: none"> •Ray economies: 0.98 •UG Science: 0.98 •UG Non-science: 0.79 •PG: 1.30 •Research: 1.08 	Not calculated

	Stevens (2005)	Johnes et al (2005)	Johnes et al (2008)	Johnes & Johnes (2009)	Thanassoulis et al (2011)
Economies of scope	Not calculated	<p>Random effects</p> <ul style="list-style-type: none"> • Global economies: 0.58 • UG Medicine: 0.10 • UG Science: 0.23 • UG Non-science: 0.14 • PG: 0.07 • Research: 0.08 • Third mission: 0.11 	<p>Random effects</p> <ul style="list-style-type: none"> • Global economies: 0.38 • UG Medicine: 0.06 • UG Science: 0.17 • UG Non-science: 0.07 • PG: 0.03 • Research: 0.04 • Third mission: 0.08 <p>SFA</p> <ul style="list-style-type: none"> • Global economies: -0.18 • UG Medicine: -0.05 • UG Science: 0.07 • UG Non-science: -0.04 • PG: -0.08 • Research: -0.09 • Third mission: -0.04 	<p>Random parameters random effects</p> <p>Global economies: 0.30</p> <p>Random parameters SFA</p> <p>Global economies: -0.17</p>	Not calculated

	Stevens (2005)	Johnes et al (2005)	Johnes et al (2008)	Johnes & Johnes (2009)	Thanassoulis et al (2011)
Average or marginal costs	Not calculated	Average incremental costs Random Effects <ul style="list-style-type: none"> •UG Medicine = £17,769 •UG Science = £5,079 •UG Non-science = £3,217 •PG = £9,569 SFA <ul style="list-style-type: none"> •UG Medicine = £15,973 •UG Science = £5,506 •UG Non-science = £3,665 •PG = £6,980 	Average incremental costs Random Effects <ul style="list-style-type: none"> •UG Medicine = £21,220 •UG Science = £6,196 •UG Non-science = £3,308 •PG = £10,664 SFA <ul style="list-style-type: none"> •UG Medicine = £17,603 •UG Science = £6,368 •UG Non-science = £3,925 •PG = £7,574 	Average incremental costs Random Parameter Random Effects <ul style="list-style-type: none"> •UG Science = £5,516 •UG Non-science = £2,869 •PG = £16,215 Random Parameter SFA <ul style="list-style-type: none"> •UG Science = £6,452 •UG Non-science = £3,126 •PG = £10,527 	Average incremental costs DEA <ul style="list-style-type: none"> •UG Medicine = £13,121 •UG Science = £5,627 •UG Non-science = £4,638 •PG = £3,828 SFA <ul style="list-style-type: none"> •UG Medicine = £15,973 •UG Science = £5,506 •UG Non-science = £3,665 •PG = £6,979

	Stevens (2005)	Johnes et al (2005)	Johnes et al (2008)	Johnes & Johnes (2009)	Thanassoulis et al (2011)
Efficiencies	<p>Mean Efficiency</p> <ul style="list-style-type: none"> • Model 1: 0.789 • Model 2: 0.782 • Model 3: 0.777 <p>Second stage equation</p> <ul style="list-style-type: none"> • Staff variables: Higher proportions of staff aged 50 or more leads to lower efficiency. • Higher proportions of staff who professors, SLs, research active lead to higher efficiency. • Student variables: Higher proportions of students with first and upper seconds, mature students, students of low socio-economic class lead to higher efficiency. 	<p>Final year of study</p> <ul style="list-style-type: none"> • Mean efficiency = 0.75 • Post-1992 HEIs = 0.85 • Pre-1992 HEIs = 0.82 • Colleges of higher education = 0.56 	<ul style="list-style-type: none"> • Mean efficiency = 0.69 • Post-1992 HEIs = 0.84 • Pre-1992 HEIs = 0.80 • Colleges of higher education = 0.43 	<ul style="list-style-type: none"> • Mean efficiency = 0.753 • Top 5 = 0.942 • Civics = 0.919 • ExCATs and Greenfields = 0.844 • Other pre-1992 HEIs = 0.712 • Post-1992 HEIs = 0.859 • Colleges of higher education = 0.499 	<ul style="list-style-type: none"> • Mean DEA efficiency = 0.863 • Mean DEA efficiency year 3 = 0.854 • Mean SFA efficiency year 3 = 0.837

Appendix 2: Alternative non-linear cost function specifications

In addition to the quadratic specification, two other functional forms can be used to provide a non-linear representation of costs.

- i) The constant elasticity of substitution (CES) function

Using the two outputs of teaching (T) and research (R) as in the text, the CES function is specified as follows:

$$C = \alpha + [T^\phi + R^\gamma]^\beta$$

- ii) The hybrid translog function

$$\ln C = \alpha + \frac{\phi(T^\theta - 1)}{\theta} + \frac{\gamma(R^\theta - 1)}{\theta} + \delta \left[\frac{(T^\theta - 1)}{\theta} \right] \left[\frac{(R^\theta - 1)}{\theta} \right] + \frac{1}{2} \beta \left[\frac{(T^\theta - 1)}{\theta} \right]^2 + \frac{1}{2} \beta \left[\frac{(R^\theta - 1)}{\theta} \right]^2$$

All the non-linear specifications considered here and in the text satisfy, at least for certain parameter vectors, the three desiderata identified by (Baumol et al. 1982), namely that the cost function should:

- be a 'proper' cost function that is consistent with cost minimization given input and output cost. In particular it should be a non-negative and non-decreasing function;
- sensibly predict costs where outputs of some products are zero. This is essential if average incremental costs and measures of economies of scale and scope are being calculated;
- not either preclude or enforce the existence of economics (or diseconomies) of scale or scope.

It is easily seen that these alternative specifications are highly nonlinear and more complicated than the quadratic specification used in this report. Indeed, it is not possible to estimate frontier variants of these models using standard software without undertaking considerable work to evaluate the likelihood functions and undertake the programming needed to maximise these likelihoods. For this reason, the analysis of non-linear models in this report is limited to the quadratic specification.

Appendix 3: Data definitions

Variable name	Definition	Units
Dependent variable		
COST	Total expenditure minus expenditure on residences and catering operations	£000s in 2011 prices
Undergraduate teaching		
UGMED	Undergraduate students (first degree and other) in medicine	FTEs
UGSCI	Undergraduate students (first degree and other) in sciences other than medicine	FTEs
UGARTS	Undergraduate students in all other subjects	FTEs
Postgraduate teaching		
PG	Postgraduate students in all subjects	FTEs
Research		
RESEARCH	HEFCE R plus income from research grants and contracts	£ in 2011 prices
PUBLICATIONS	Publications per HEI from Web of Science	Number
CITATIONS	Citations per HEI from Web of Science	Number
Third mission		
EVENTS	Staff time devoted to public events (free and chargeable) Events include: public lectures; performance arts; exhibitions; museum education and other.	Days
ATEVENT	Number of attendees at public events (free and chargeable) Events include: public lectures; performance arts; exhibitions; museum education and other.	Number of people
IPINCOME	Income from third mission activity i.e. the sum of: <ul style="list-style-type: none"> Total income from collaborative research involving both public funding and funding from business (a5) Total value of contract research (excluding any already returned in a5) (b8) Total value of consultancy contracts (c8) Total value of contracts for facilities and equipment related services - organisations involved and income (d8) Total revenue for courses for business and the community - CPD courses and CE (excluding those funded by the NHS or TDA) (e5) Total income from regeneration and development programmes (f7) Total revenue from: IP income from SMEs plus IP income from other (non-SME) commercial businesses plus IP income from other non-commercial organisations plus sale of shares in spin-offs (m7) 	£000s in 2011 prices

Additional factors		
LOWPNO	Number of young full-time undergraduate entrants from low participation neighbourhoods	
LISTED	Area of a HEI's buildings which are designated as listed	Metres squared
UPGRADE	Cost of upgrade to upgrade all non-residential space in condition D to condition B (D20bC13UPDB) plus the cost of upgrade to upgrade all space in condition C to condition B (D")bC13UPCB) where condition B is defined as: Sound, operationally safe and exhibiting only minor deterioration.	£ in 2011 prices
NMEAN	Mean salary of graduates (from DLHE)	£ in 2011 prices
NSS	The percentage positive response to the question from the National Student Survey: 'Overall, I am satisfied with the quality of the course'	Percentage
Dummy variables		
2003/04	1 if year is 2003/04	Dummy variable
2004/05	1 if year is 2004/05	Dummy variable
2005/06	1 if year is 2005/06	Dummy variable
2006/07	1 if year is 2006/07	Dummy variable
2007/08	1 if year is 2007/08	Dummy variable
2008/09	1 if year is 2008/09	Dummy variable
2009/10	1 if year is 2009/10	Dummy variable
2010/11	1 if year is 2010/11	Dummy variable
OXBRIDGE	1 if HEI is Oxford or Cambridge	Dummy variable

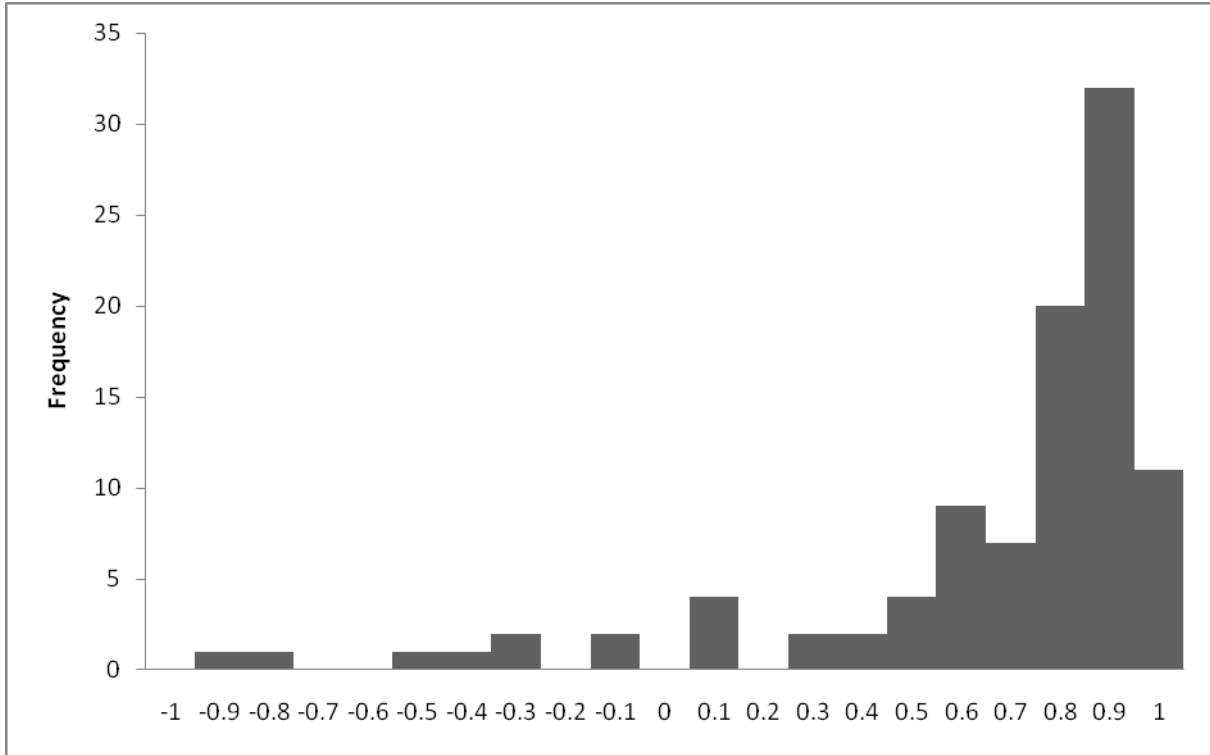
Note: Subject definitions are as follows

- **Medicine:** Medicine & dentistry; Subjects allied to medicine;
- **Other science:** Biological sciences; Veterinary science; Agriculture and related subjects; Physical sciences; Mathematical sciences; Computer science; Engineering and technology; Architecture, building and planning;
- **Non-science:** Social studies; Law; Business and administrative studies; Mass communications and documentation; Languages; Historical and philosophical studies; Creative arts and design; Education.
- There are a small number of students in combined subjects. These have been divided and allocated according to the proportion of students observed in each subject.

Appendix 4: Additional results

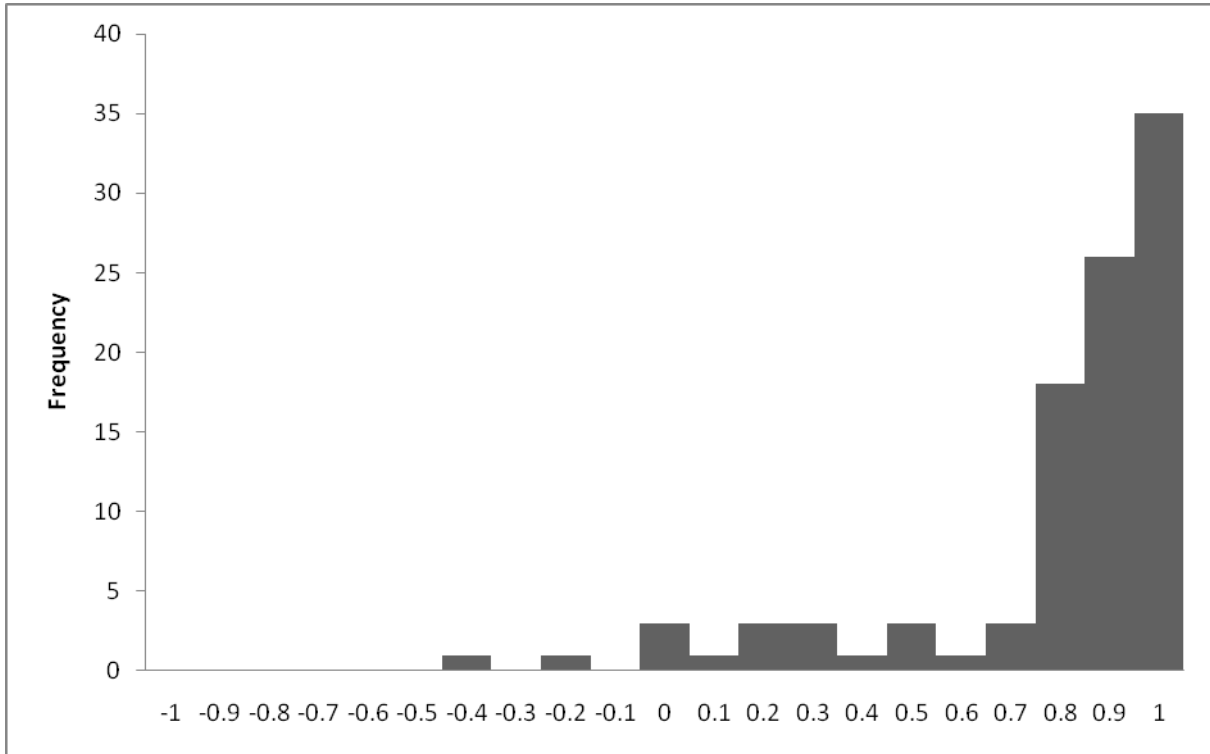
A4.1 Efficiency distributions associated with linear models with a full set of controls presented in Table 3

a) Final year of 2005/06 to 2007/08 model

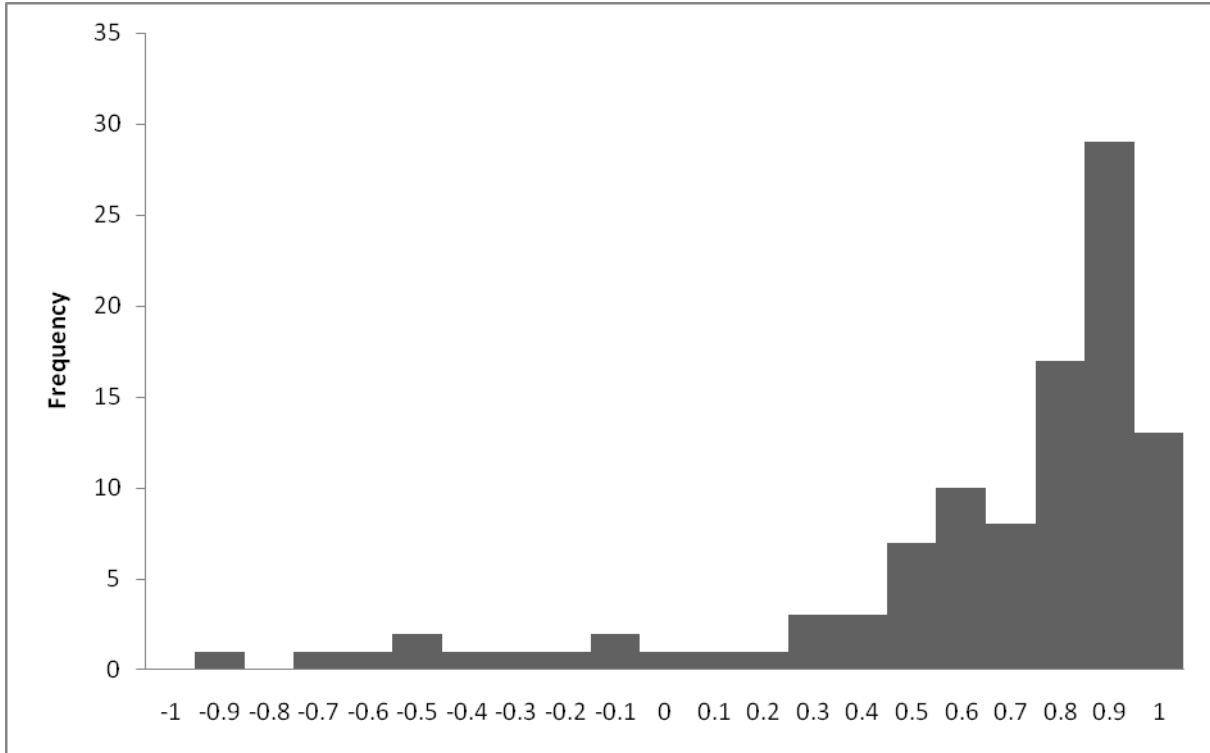


Note: Efficiencies for 14 HEIs lie outside the range from -1 to +1.

b) Final year of 2003/04 to 2004/05 model



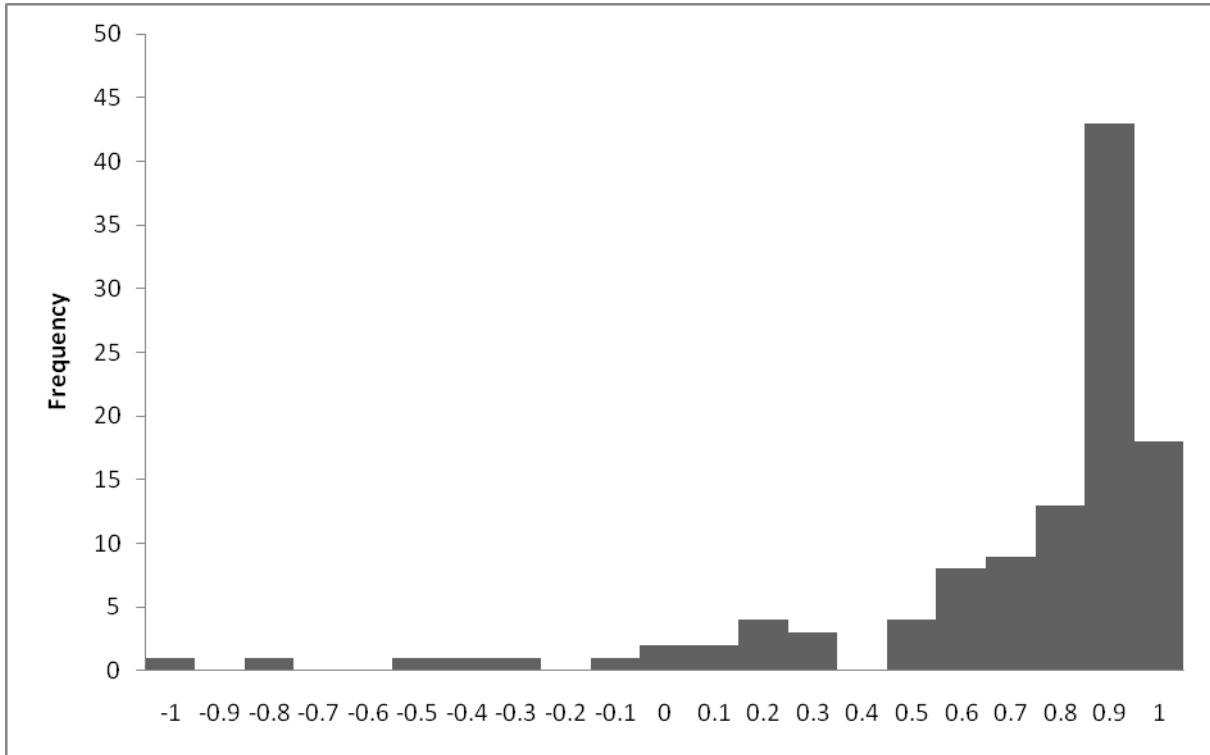
c) Final year of 2003/04 to 2010/11 model



Note: Efficiencies for 18 HEIs lie outside the range from -1 to +1.

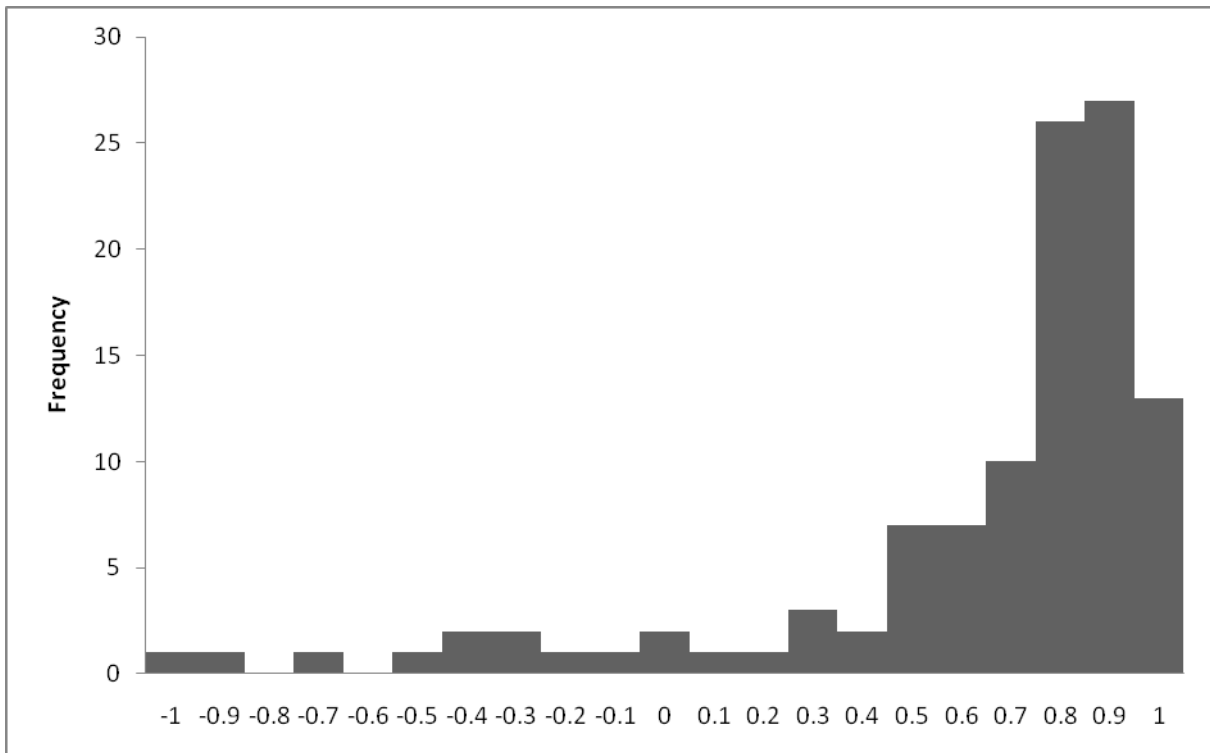
A4.2 Efficiency distributions associated with linear models with a limited set of controls presented in Table 4

a) Final year of 2008/09 to 2010/11 model



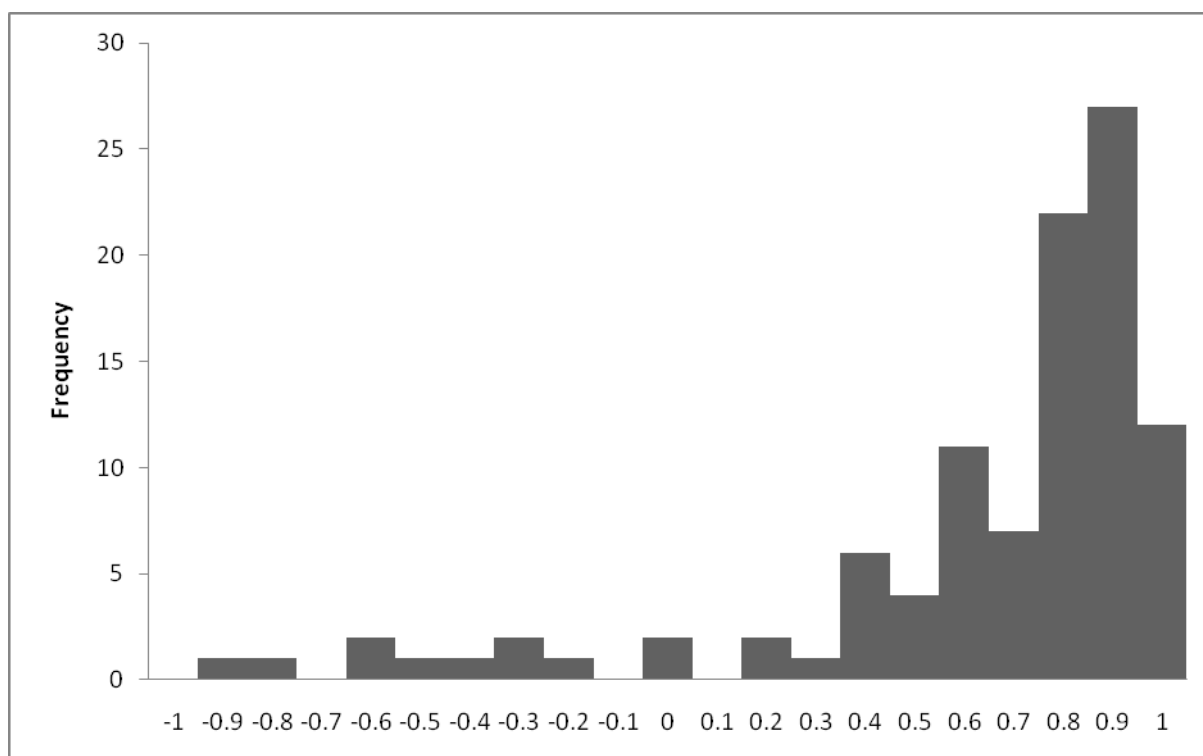
Note: Efficiencies for 17 HEIs lie outside the range from -1 to +1.

b) Final year of 2005/06 to 2007/08 model



Note: Efficiencies for 20 HEIs lie outside the range from -1 to +1.

c) Final year of 2003/04 to 2010/11 model



Note: Efficiencies for 18 HEIs lie outside the range from -1 to +1.

A4.3 AICs for the quadratic model with a limited set of controls

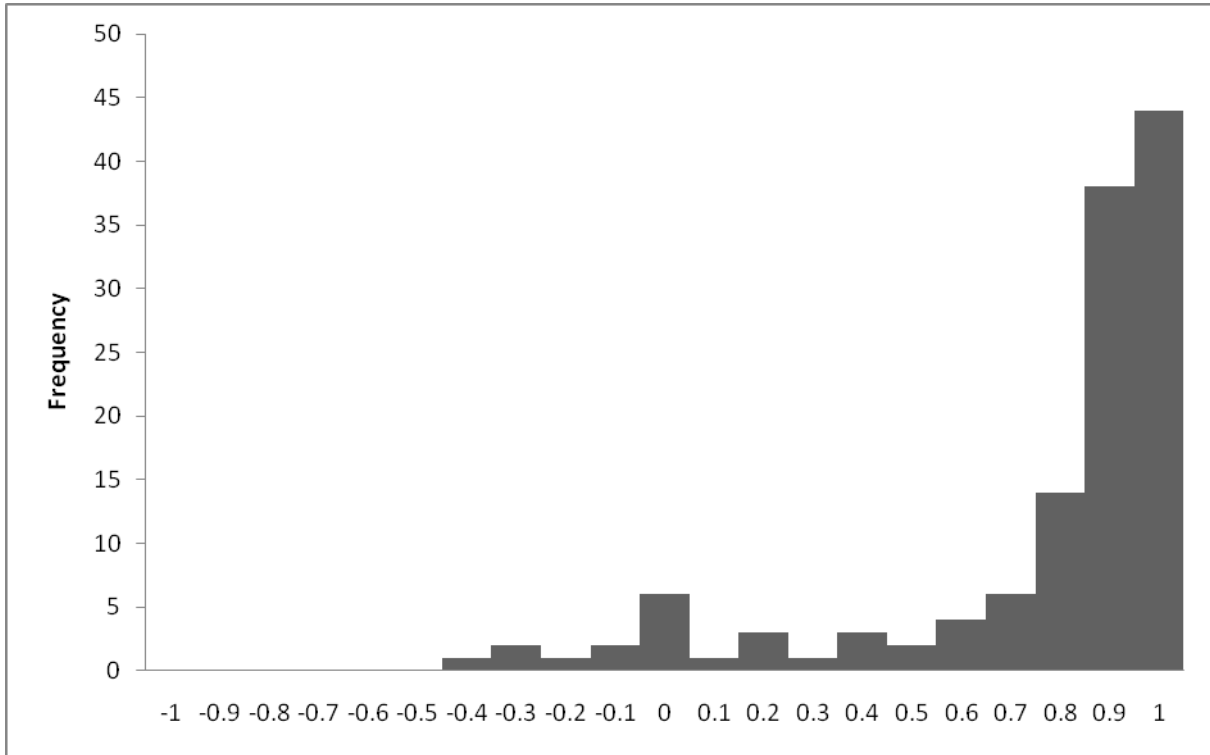
AICs	2008/09 to 2010/11	2003/04 to 2004/05	2003/4 to 2010/11
UGMED	17.88119	9.50562	12.58065
UGSCI	6.25241	3.31588	4.80015
UGARTS	4.44784	3.89313	4.87373
PG	13.69599	8.54893	0.28524
RESEARCH	1.22823	1.19398	1.14478
IPINCOME	1.13426	0.59380	0.92564

Notes:

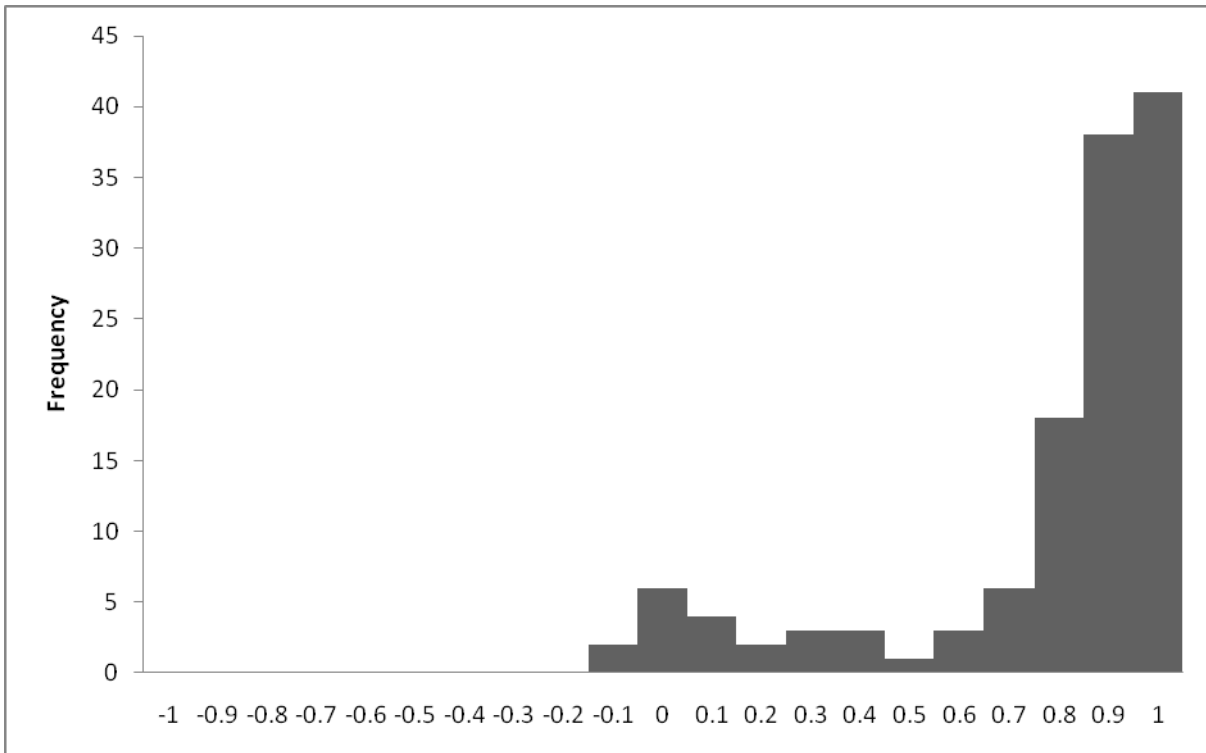
1. Controls: OXBRIDGE; YEAR dummies
2. The 2005/06 to 2007/08 model does not converge

A4.4 Efficiency distributions associated with quadratic models with a limited set of controls presented in Appendix 4.3 above

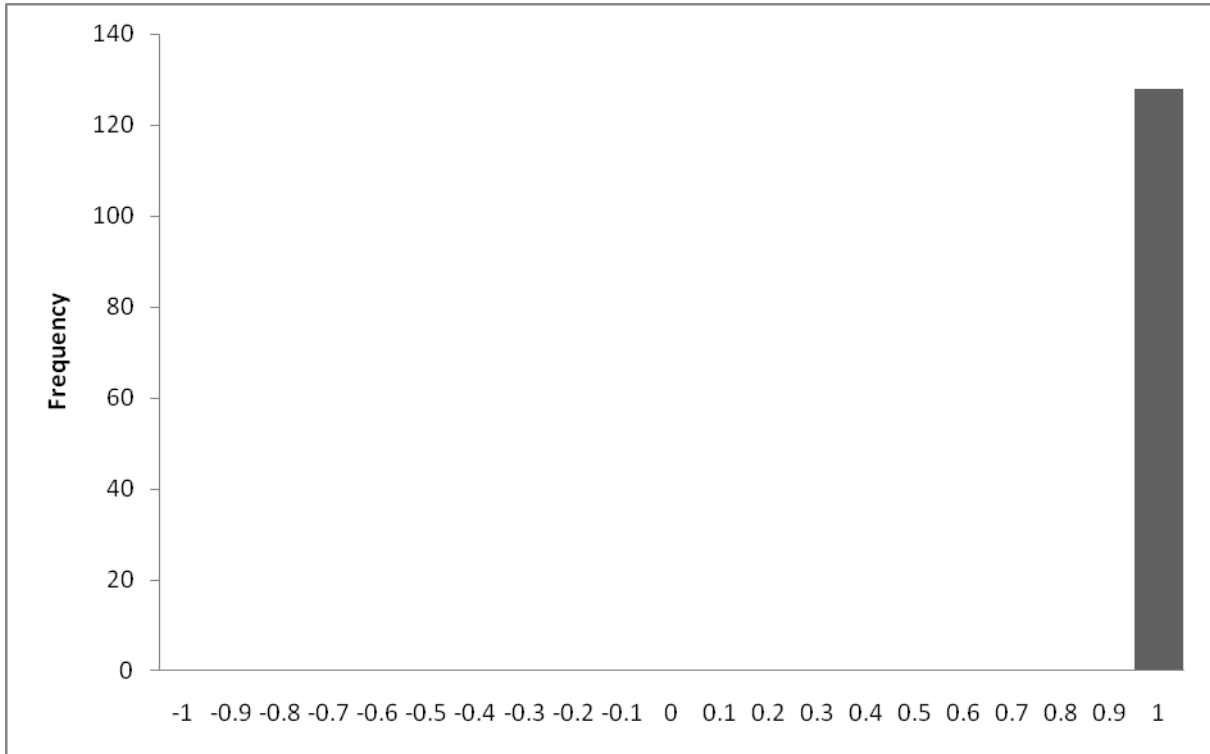
a) Final year of 2008/09 to 2010/11 model



b) Final year of 2003/04 to 2004/05 model



c) Final year of 2003/04 to 2010/11 model



Note: The quadratic model for 2005/06 through 2007/08 fails to converge. Once the data for this period are included alongside data for the 2003/04 through 2004/05 and 2008/09 through 2010/11 periods, there is little skew in the residuals, and so all efficiency scores are estimated to be close to one.

A4.5 Membership of latent classes

Institution	2008-09	2009-10	2010-11
Anglia Ruskin University	2	1	2
Arts University College at Bournemouth	1	2	1
Aston University	2	2	2
Bath Spa University	2	1	2
Birkbeck College	1	1	1
Birmingham City University	2	1	2
Bishop Grosseteste University College Lincoln	2	2	2
Bournemouth University	2	2	2
Brunel University	1	2	1
Buckinghamshire New University	1	1	2
Canterbury Christ Church University	2	1	1
Central School of Speech and Drama	2	2	2
City University	2	2	2
Courtauld Institute of Art	2	2	2
Coventry University	1	2	2
Cranfield University	1		
De Montfort University	1	1	1
Edge Hill University	1	1	1
Goldsmiths College	2	1	2
Guildhall School of Music and Drama	2	2	2
Harper Adams University College	2	2	2
Heythrop College	2	2	2
Imperial College of Science, Technology and Medicine			2
Institute of Cancer Research			2
Institute of Education	1	1	1
King's College London	2	2	1
Kingston University	2	2	1
Leeds College of Music		2	2
Leeds Metropolitan University	2	2	2
Leeds Trinity University College	2	2	2
Liverpool Hope University	2	2	2
Liverpool Institute for Performing Arts	2	2	2
Liverpool John Moores University	2	2	2
London Metropolitan University	2	2	2
London School of Economics and Political Science	2	2	2
London School of Hygiene and Tropical Medicine		2	
London South Bank University	1	1	2
Loughborough University	1	1	1
Manchester Metropolitan University	2	1	1
Middlesex University	2	2	1
Newman University College	2	2	2
Norwich University College of the Arts	2	2	2
Nottingham Trent University	2	2	2
Oxford Brookes University	2	2	2
Queen Mary and Westfield College	1	1	1

Institution	2008-09	2009-10	2010-11
Ravensbourne College of Design and Communication	2	2	2
Roehampton University	2	2	2
Rose Bruford College	2	2	2
Royal Academy of Music	2	2	2
Royal Agricultural College	2	2	2
Royal College of Art			2
Royal College of Music	2	2	2
Royal Holloway and Bedford New College	2	2	2
Royal Northern College of Music	2	2	2
Royal Veterinary College	1	1	1
School of Oriental and African Studies	2	2	2
School of Pharmacy	2	2	
Sheffield Hallam University	2	2	2
Southampton Solent University	1	1	1
St George's Hospital Medical School	1	1	1
St Mary's University College, Twickenham	2	2	2
Staffordshire University	2	2	2
Thames Valley University	1	1	2
Trinity Laban Conservatoire of Music and Dance	2	2	1
University College Birmingham	2	2	1
University College Falmouth	1	1	1
University College Plymouth St Mark and St John	1	1	2
University for the Creative Arts	1	1	1
University of Bath	1	1	1
University of Bedfordshire	1	2	1
University of Birmingham	2	2	2
University of Bolton	2	2	2
University of Bradford	2		2
University of Brighton	2	2	2
University of Bristol	1	1	1
University of Cambridge	1	1	1
University of Central Lancashire	2	2	1
University of Chester	2		2
University of Chichester	2	2	2
University of Cumbria	1	1	2
University of Derby	1	1	1
University of Durham	2	2	1
University of East Anglia	2	2	2
University of East London	2	2	2
University of Essex	2	2	2
University of Exeter	2	2	2
University of Gloucestershire	2	2	2
University of Greenwich	1	2	1
University of Hertfordshire	1	2	2
University of Huddersfield	1	2	1
University of Hull	2	2	2
University of Keele	1	1	1

Institution	2008-09	2009-10	2010-11
University of Kent	2	1	1
University of Lancaster	1	1	1
University of Leeds	2	2	1
University of Leicester	1	2	2
University of Lincoln	2	1	2
University of Liverpool	1	1	1
University of London (Institutes and activities)			1
University of Manchester	1	1	1
University of Newcastle-upon-Tyne	2	2	2
University of Northampton	2	1	2
University of Northumbria at Newcastle	2	2	1
University of Nottingham	1	1	2
University of Oxford	2	2	2
University of Plymouth	2	2	2
University of Portsmouth	2	2	1
University of Reading	1	1	1
University of Salford	2	2	2
University of Sheffield	2	2	
University of Southampton	1	1	1
University of Sunderland	1	1	2
University of Surrey	2		2
University of Sussex	2	2	2
University of Teesside	2	2	2
University of the Arts, London	1	1	1
University of the West of England, Bristol	1	2	2
University of Warwick	1	1	1
University of Westminster	2	2	2
University of Winchester		2	
University of Wolverhampton	2	2	2
University of Worcester	2	2	2
University of York	2	2	2
Writtle College	2	2	2
York St John University	2	2	2

Note:

1. The information in this table allows a sense check to be conducted on the analysis, in that it shows the extent to which institutions that are known to be of broadly similar type tend to be clustered together in latent classes.
2. If an institution has missing data for a particular variable for all three years, they were omitted from this analysis. This affected the Conservatoire for Dance and Drama, London Business School, University College London and the University of Buckingham.

A4.6 Results from 3- and 4-class latent class models: linear specification with only a limited set of controls

We estimate latent class models with 3 and with 4 classes, using a linear model with no controls in the latest run of 3 years. In each case, latent class 1 is small, and in each case this class produces implausible coefficients, while the coefficients for the other latent classes look broadly sensible.

a) 3-class linear latent class model

2008/09 to 2010/11

AICs	Class 1	Class 2	Class 3
UGMED	-14.61370	4.43692	13.23580
UGSCI	-1.12589	7.98182	4.40658
UGARTS	5.85165	3.77974	6.67897
PG	-16.22300	15.41280	9.11164
RESEARCH	1.88377	1.09513	1.22724
IPINCOME	1.29048	1.19655	0.46838
Controls			
2009/10	8,689.96	-3,019.64	-1,255.80
2010/11	12,905.60	-1,380.47	-4,880.82
OXBRIDGE	315,210.00	61,296.50	79,624.30
CONSTANT	104,531.00	-162.52	7,270.85
Number in class	18	254	112
Is λ significantly different from zero at the 5% significance level?	NO	NO	NO

Notes:

1. Controls: OXBRIDGE; YEAR dummies
2. Coefficients in bold are statistically significantly different from zero at the 5% significance level.
3. The number in each class is the number of observations over the estimation period, not the number of HEIs. Given that HEIs can be in different classes in different years, dividing by the number of years on which the analysis is based does not give the number of HEIs.

b) 4-class linear latent class model**2008/09 to 2010/11**

AICs	Class 1	Class 2	Class 3	Class 4
UGMED	-11.38990	3.33611	13.90920	14.66870
UGSCI	3.48366	9.76754	2.81820	7.69351
UGARTS	7.74338	3.87148	4.74366	3.75975
PG	-23.71430	12.90010	18.58490	3.94800
RESEARCH	2.23325	1.10102	1.11549	1.08442
IPINCOME	0.25371	1.26308	0.30541	0.97486
Controls				
2009/10	11,443.20	-1,442.06	-2,210.09	-982.70
2010/11	20,637.40	-1,692.57	-1,627.01	-982.17
OXBRIDGE	316,556.00	62,387.10	106,659.00	97,948.70
CONSTANT	103,672.00	378.39	-142.69	8,289.91
Number in each group	20	231	102	31
Is λ significantly different from zero at the 5% significance level?	NO	NO	NO	NO

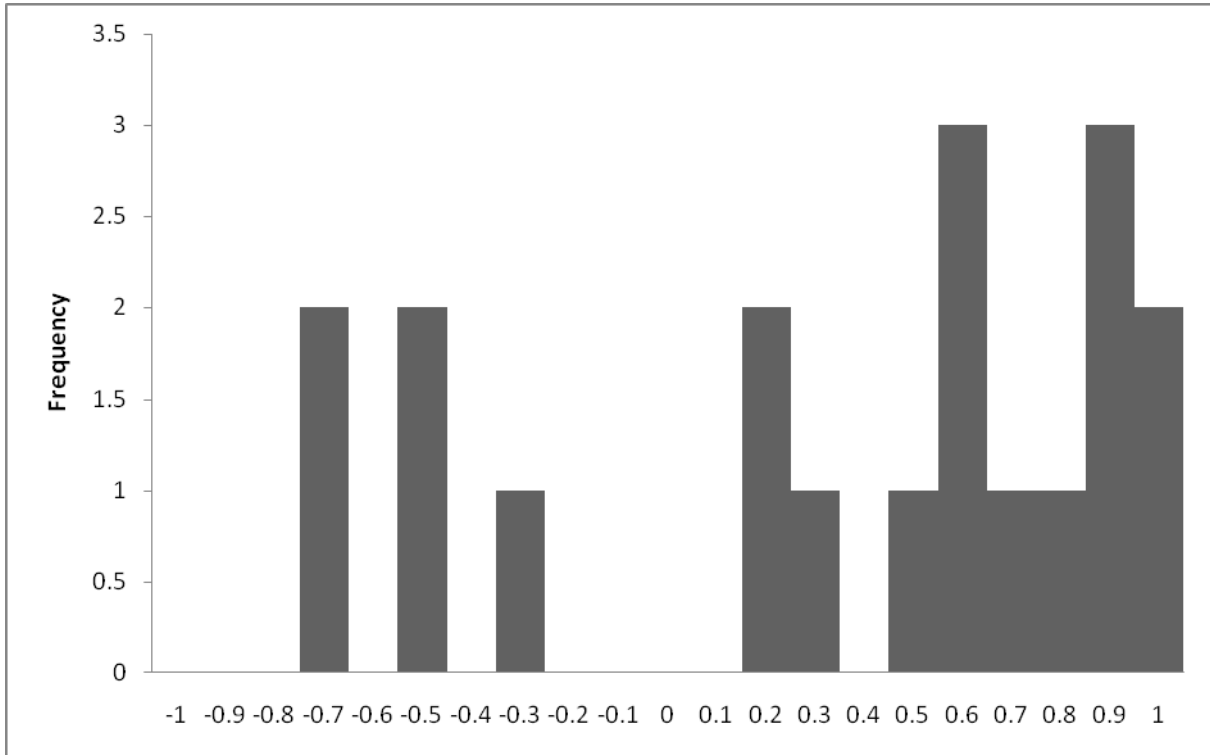
Notes:

1. Controls: OXBRIDGE; YEAR dummies.
2. Coefficients in bold are statistically significantly different from zero at the 5% significance level.
3. The number in each class is the number of observations over the estimation period, not the number of HEIs. Given that HEIs can be in different classes in different years, dividing by the number of years on which the analysis is based does not give the number of HEIs.

A4.7 Distributions of efficiency scores obtained from the linear model in which institutions are classified by HEFCE impact report groups – final year of 2008/09 to 2010/11 model

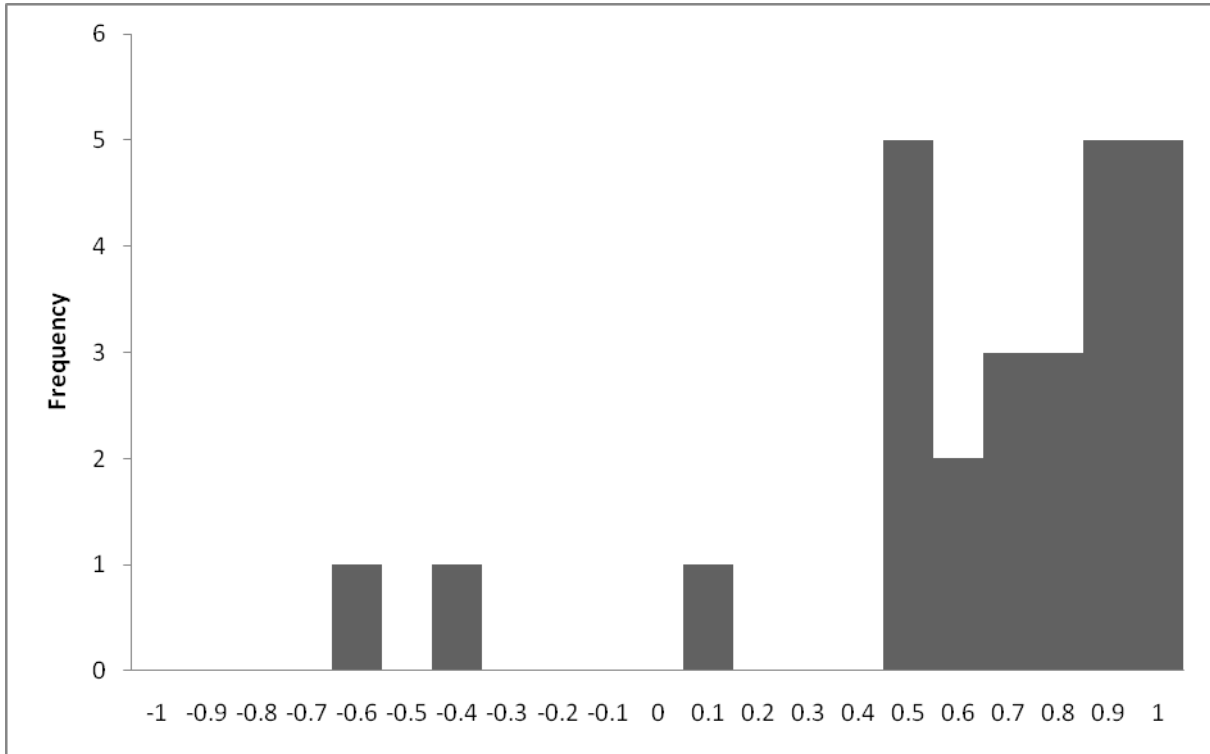
These histograms relate to the equations in Table 8 (2008/09 to 2010/11 models), and are drawn for the final year of the estimation period.

a) Group 1 - specialist

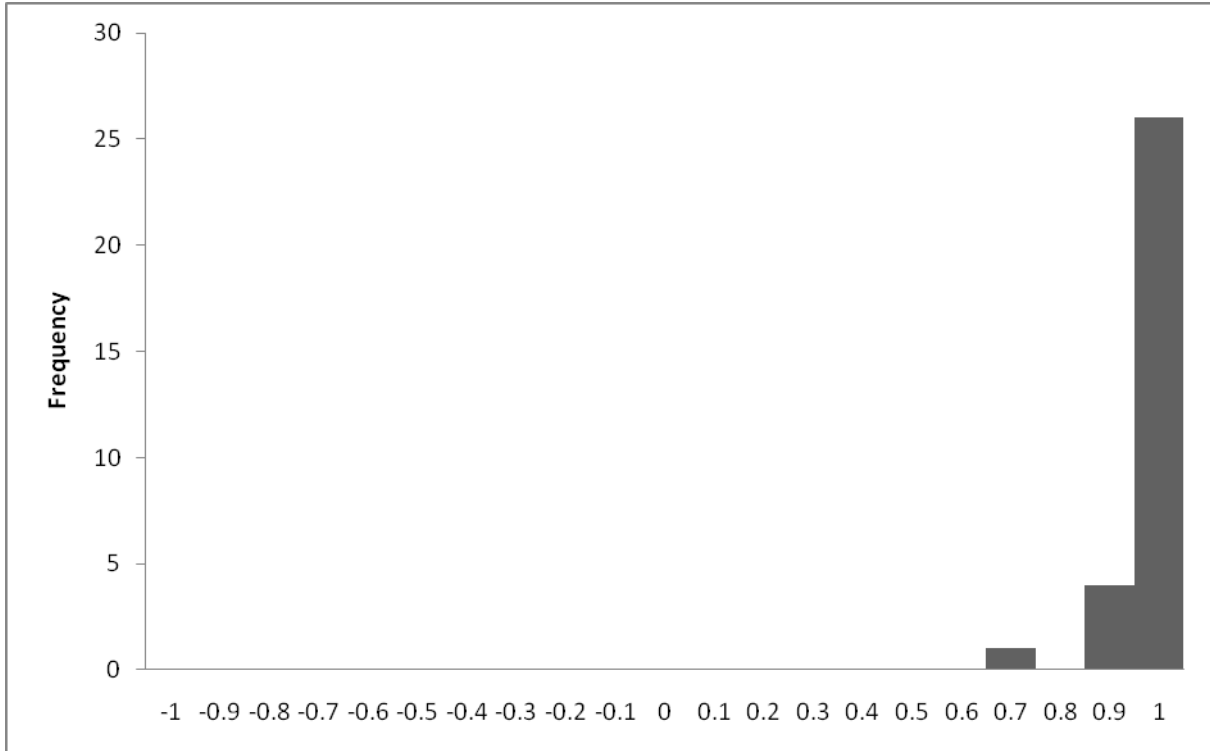


Note: Efficiencies for 16 HEIs lie outside the range from -1 to +1.

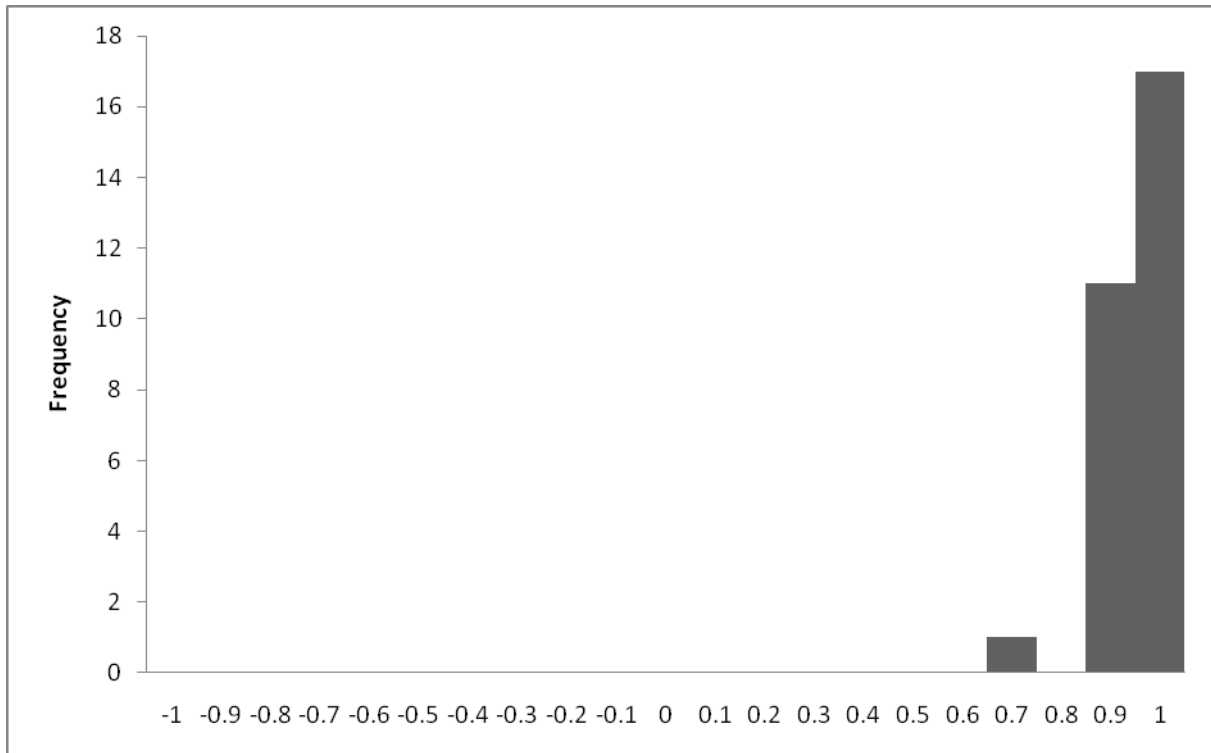
b) Group 2 – high tariff



c) Group 3 – medium tariff



d) Group 4 – low tariff



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