5th Seminar
Ordered Structures in Games and Decisions

Abstracts

November 10th, 2011

Université Paris I — Panthéon-Sorbonne

Centre d'Économie de la Sorbonne
106-112, Bd. de l'Hôpital, 75013 Paris

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PROGRAM

♦ **9:00 Welcome to participants**

❑ 9:10-10:00 : Ozer SELCUK

Average covering tree solution for directed graph games

❑ 10:00-10:50 : Miklós PINTÉR

Axioms and interpretations

♦ **10:50-11:10 Coffee break**

❑ 11:10-12:00 : Michel MINOUX

On minimizing convex Choquet integrals on polyhedra and some LP-based formulations

❑ 12:00-12:50 : Vincent IEHLÉ

Convexity and core in partition function games

♦ **12:50-14:30 Lunch**

❑ 14:30-15:20 : Jean-Luc MARICHAL

Influence and interaction indices in cooperative games: a unified least squares approach

❑ 15:20-16:10 : Kim Thang NGUYEN

Congestion Games with Capacitated Resources

♦ **16:10-16:30 Coffee break**

❑ 16:30-17:20 : Stéphane GONZALEZ

Preserving coalitional rationality for non-balanced games

1. **Ozer SELCUK (CentER, Department of Econometrics & Operations Research, Tilburg University, Tilburg, The Netherlands)**

   Average covering tree solution for directed graph games

   For a cooperative game with limited communication structure, only connected sets of players are able to cooperate. This situation is often represented by an undirected graph on the set of players, in which an (undirected) edge between two players means that both can communicate with each other. Instead of an undirected graph in which the communication is bilateral between players, we consider communication structures that are represented by a directed graph in which players could possibly communicate in only one direction. When players are able to communicate in both directions with each other, the two directed edges between them could be considered as an undirected edge. For the class of cooperative games with directed graphs as communication structure we introduce as solution concept the average covering tree solution. For games with complete communication this solution is the Shapley value and for games with undirected graphs it is the gravity center of the convex hull of marginal vectors, recently introduced by Koshevoy and Talman. We also give convexity-type of conditions under which the solution is an element of the core and therefore cannot be blocked by any coalition. In case the directed graph is a hierarchy the condition is weaker than super-additivity.

   (joint work with D. Talman and A. Khmelnitskaïa)

   Contact: ozeeer@gmail.com .

2. **Miklós PINTÉR (Department of Mathematics, Corvinus University of Budapest, 1093 Hungary, Budapest, Fővám tér 13-15)**

   Axioms and interpretations

   We consider transferable utility (TU) games, solutions of these games and axioms for, and axiomatizations of certain specific solu- tions. We focus on the "meanings" of various axioms. By "meanings" we mean interpretations, so we consider applications too. We discuss the core compatibility, the equal treatment property (symmetry), the Pareto optimality (efficiency) and various monotonicity axioms. Some steps are taken in the direction of exploring what types of solutions the combinations of axioms determine, and how these (the determinations) change when we vary the domain (sub-classes) of the considered solutions. Most importantly, we interpret monotonicity as incentive compatibility, and illustrate our argument by examples.

   Contact: miklos.pinter@uni-corvinus.hu.

3. **Michel MINOUX (Université de Paris VI, LIP6)**

   On minimizing convex Choquet integrals on polyhedra and some LP- based formulations

   We address here the problem of minimizing convex Choquet Integrals (also known as 'Lovasz Extensions') over polyhedra (extension to mixed integer solution sets is also discussed). Typical applications of such problems concern the search of compromise solutions in multicriteria optimization. We focus here on the case when the Choquet Integrals to be minimized are convex, implying that the set functions (or 'capacities') underlying the Choquet Integrals considered are submodular. We first describe an

approach based on a large scale Linear Programming (LP) formulation, and show how it can be handled via the so-called column-generation technique. We next investigate alternatives based on compact LP formulations, i.e. formulations featuring a polynomial number of variables and constraints. Results of computational experiments obtained on transportation problems and knapsack problems are provided to illustrate the practical efficiency of the proposed LP formulations.

(joint work with J. Lesca and P. Perny, Université de Paris VI, LIP6)

Contact: `minouxm@poleia.lip6.fr`.

4. **Vincent IEHLÉ (Université Paris-Dauphine)**

Convexity and core in partition function games

We discuss the convexity assumption in partition function form games and its relationship with the core concepts. In the standard framework of characteristic function games, convexity à la Shapley plays a key role in the analysis of stability. Convexity arises if the game displays increasing returns to scale in cooperation and is equivalent to supermodularity in the lattice of coalitions. Defining its counterpart in partition function games requires to account for the expectations of players with respect to the new coalition structure as groups merge. We introduce a definition of convexity precisely based on externality schemes, which can be related to the existing concepts of core in partition function games. We also discuss its relationship with supermodularity in the lattice of embedded coalitions.

(joint work with Giovanna Bimonte (U. Salerno, Italia))

Contact: `viehle@gmail.com`.

5. **Jean-Luc MARICHAL (University of Luxembourg, Mathematics Research Unit, 6, rue Coudenhove-Kalergi, L-1359 Luxembourg-Kirchberg)**

Influence and interaction indexes in cooperative games: a unified least squares approach

The classical Banzhaf power and interaction indexes used in cooperative game theory appear naturally as leading coefficients in the standard least squares approximation of the game under consideration by a set function of a specified degree. We observe that this still holds true if we consider approximations by set functions depending on specified variables. We show that the Banzhaf influence index can also be obtained from this new approximation problem. Considering certain weighted versions of this approximation, we also introduce a class of weighted Banzhaf influence indexes and analyze their most important properties.

(joint work with Pierre Mathonet)

Contact: `jean-luc.marichal@uni.lu`.

6. **Kim Thang NGUYEN (Laboratory IBISC, Université Évry Val d'Essonne, France)**

Congestion Games with Capacitated Resources

The players of a congestion game interact by allocating bundles of resources from a common pool. This type of games leads to well studied models for analyzing strategic situations, including networks operated by uncoordinated selfish users. Congestion games constitute a subclass of potential games, meaning that a pure Nash equilibrium emerges from a myopic process where the players iteratively react by switching to a strategy that diminishes their individual cost. With the aim of covering more applications, for instance in communication networks, we extend congestion games to the setting where every resource is endowed with a capacity which possibly limits its number of users and an order of preference over players. Though a pure Nash equilibrium is not guaranteed to exist in any case, we mainly prove that congestion games with capacities are potential games in the natural and well studied case where bundles of resources are singletons. Besides, we also investigate in efficient algorithms to compute a Nash equilibrium in case of existence.

(joint work with Laurent Gourvès, Jérôme Monnot and Stefano Moretti)

Contact: `thang@ibisc.fr`.

7. **Stéphane GONZALEZ (Université de Paris I, 106-112 Bd de l'Hôpital 75013 Paris)**

Preserving coalitional rationality for non-balanced games

A central problem in cooperative game is to define a sharing of the total worth of a game. The core is one of the most popular concept of solution because it ensures that no rational subcoalition has interest to leave the grand coalition. However, the core is often empty and other concepts of solution had to be elaborated. The k-additive core, proposed by Grabisch and Miranda, is a new concept which will prove to be an interesting tool to keep the general spirit of coalitional rationality, and which has also the advantage to be nonempty as soon as $k \geq 2$. A solution of the k-additive core gives a payment to each player and to each coalitions up to size k. Therefore, if each player accepts to pool a part of his gain with the other players, it is possible to preserve coalitional rationality. We introduce some natural solutions of the k-additive core which minimizes the amount of pooling or, equivalently, which maximizes the individual value. In a second part, we study the set of preimputations which can be obtained with this new point of view and we compare them with some classical alternatives to the core.

(joint work with Michel Grabisch)

Contact: `g1stephane@gmail.com`.