5th International Conference on
Integration of Renewable and Distributed Energy Resources
December 4-6 2012 | Berlin, Germany

Frequency Coupling in Inverter Grids

Modelling the Mutual Interference of Voltage Source Inverters in Island Grids

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Keywords: island grids, voltage source inverters, grid modelling, frequency coupling,

I. Introduction

Inverter driven island grids were investigated in [1] and a first approach of a compound model of island grids was presented in [2], where an important condition was the coupling of the inverters in one common point of connection (PoC). In [2] the grid frequency was considered as a lumped grid parameter and was defined as the arithmetic mean of all inverter frequencies. This yields each of the n connected inverters $1/n^{th}$ of the influence on the overall grid frequency. This may be right for the case of all inverters being connected in one single PoC, however, when the voltage source inverters (VSI) are distributed over the grid, interconnected with lines of different admittances, the assumption of an arithmetic mean as one single grid frequency seems fragile. In fact, the grid frequency should rather be considered as a distributed parameter in the grid, at any grid node depending on the electric distance and power of other VSI in the node's vicinity.

During transient phases, there is not one single grid frequency, but at each grid node the momentarily measurable grid frequency is slightliy different. The work in this paper proposes a formal way of modelling this distributed grid frequencies, based on the inverter power and the impedances between the inverter nodes.

II. Linear Model of Frquency Coupling

The intention is to develop a model which describes, for any node in the grid, how the frequency is determined by all VSIs, connected via resistive-inductive lines to this node, including the influence of a VSI which might be connected to this very node.

Based on graph theory, the structure of an arbitrary grid with n nodes, t branches (or twigs) and with a number of v distributed inverters can be described by the node incidence matrix $\mathcal{K}$.

Having the line admittances in the diagonal matrix $\mathcal{Y} = \mathrm{diag}(y_1, y_2, ...y_t)$, the resistance distance matrix Ω of a grid structure is calculated, containing at its elements Ω_{ij} the electric resistances between any two nodes i and j in the network according to Kirchhoff's law. The calculation of the resistance distance matrix starts with the $n \times n$ Moore-Penrose pseudo inverse $\Gamma = \mathrm{pinv}(\mathcal{K}\mathcal{Y}\mathcal{K}^T)$. The algorithm for the calculation of each element of Ω from Γ is defined as $\Omega_{ij} = \Gamma_{ii} + \Gamma_{jj} - \Gamma_{ij} - \Gamma_{ji} \qquad \forall i, j = [1 \ldots n]$.

As the mutual frequency influence is assumed to be larger with decreasing line impedance, the matrix of point to point admittances Ψ is calculated by using the inverse of each element $\psi_{ij}=1/\Omega_{ij}$ where $i \neq j$ (as the main diagonal of Ω is always zero and thus not invertible). In order to weight the VSI influence on its own PoC, an admittance weighted coupling matrix is calculated, by incorporating also the filter inductances of the connected inverters (Figure 1). This is done by introducing new virtual nodes $[A', B', C']$ (primed) in the network, separating the actual inverter as a voltage source from its PoC by the coupling filter inductance with the admittance of the purely inductive filter of $y_i=1/(j\omega L_i)$.

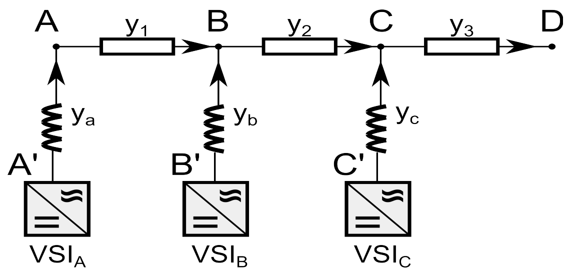


Figure 1 Sample Network Including Filter Inductances of VSIs

Calculating the by virtual nodes extended admittance distance matrix Ψ_{ext} yields a structure which is exploited to obtain the coupling matrix (see Figure 2). Ψ_{ext} is based on the node incidence matrix $\mathcal{K}_{ext}$, extended by the virtual nodes, and the admittance matrix $\mathcal{Y}_{ext}$, extended by the filter susceptances§ The green area in Figure 2 is the original, non-extended Ψ. The blue range concerns only virtual nodes and due to the sorting of $\mathcal{K}$, where VSI nodes are on top, the red area concerns only network nodes without VSIs connected.

Using the sub-matrix Λ (yellow range), weighting Λ with the respective inverter's rated power and normalizing each line to a value of 100% will yield a frequency couplding matrix M_f.

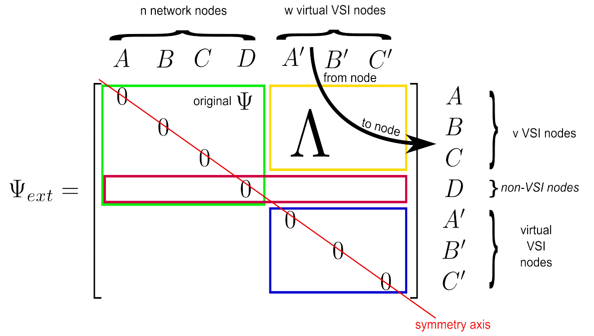


Figure 2 Structure of Node-to-Node Admittance Matrix

The advantage of a node incidence matrix based approach lies in the simple extensibility for the calculation of the extended admittanced distance matrix Ψ_{ext}. With w VSIs in the grid and n grid nodes, and with $m = n - w$, the extension of $\mathcal{K}$ with the virtual VSI nodes can be done programmatically with the $w \times w$ identity matrix $\mathcal{I}_{w \times w}$, the $w \times t$ zero matrix $\mathcal{Z}_{w \times t}$ and the $m \times w$ zero matrix $\mathcal{Z}_{m \times w}$ such that $\mathcal{K}$ becomes an extended $(n+w) \times (t+v)$ matrix of the form of Equation (1).

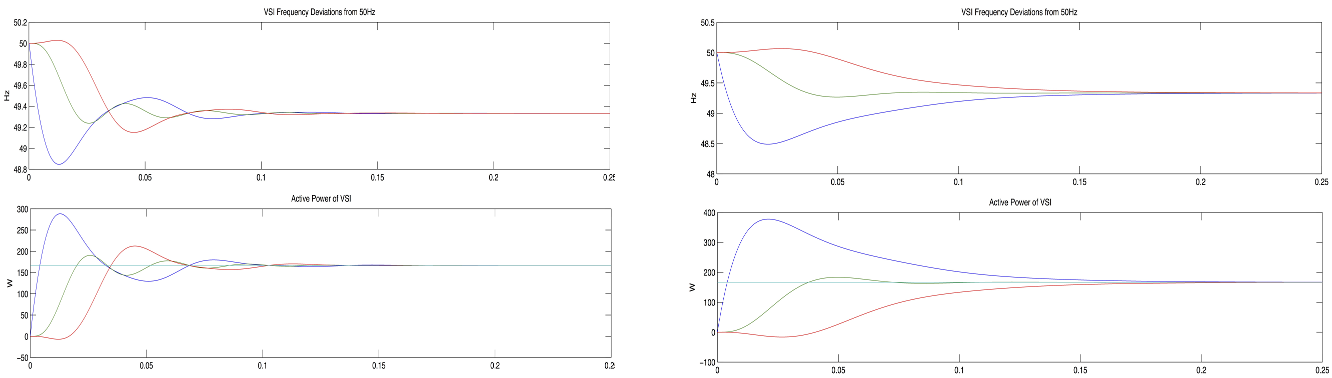
$$M_f = \begin{array}{c} \begin{matrix} A' & B' & C' \end{matrix} \\ \begin{bmatrix} 0.858 & 0.093 & 0.049 \\ 0.089 & 0.822 & 0.089 \\ 0.049 & 0.093 & 0.858 \end{bmatrix} \end{array} \begin{matrix} A \\ B \\ C \end{matrix} \quad (1)$$

Example: For a sample low voltage grid of Figure 1, assuming 400m underground cable connections (Z'=0.642+j0.083 Ω/km) and equal inverter powers, the frequency coupling matrix $\mathcal{M}_f$ can be calculated to (2).

$$K_{ext} = \begin{bmatrix} K & \mathcal{I}_{w \times w} \\ & \mathcal{Z}_{m \times w} \\ \mathcal{Z}_{w \times t} & -\mathcal{I}_{v \times w} \end{bmatrix} \quad (2)$$

The example indicates that VSI_A (first row of M_f) is measuring via its PLL a frequency value which is determined by 85.8% through itself, but by 9.3% disturbed through the adjacent VSI_B and by 4.9% disturbed by VSI_C which is located at a higher distance and thus has a higher impedance to VSI_A.

Simulating two scenarios with a load step at node D in the grid of Figure 1, one scenario based on the presented disturbance model with the matrix M_f and one based on the assumption of a single lumped grid frequency. Both yield coherent results with equal steady state values (49.3Hz, 166W) but different transient phases (see Figure 3).



a) lumped grid frequency

b) impedance dependent frequency disturbance

Figure 3 Simulation of Load Step in Sample Network with Three VSIs (top: injection frequency, bottom: injected power)

The way of modelling the frequency coupling between inverters has an obvious effect on the transient phases. As the frequencies of voltage source inverters are not physically coupled like those of rotating generators, where energy is exchanged over the power grid and stored as kinetic energy within the generators, a plausible model for purely inverter-driven grids is needed. The proposed model assumes the line impedances as a mainly influencing factor when calculating the inverter's voltage angle drift with respect to the grid frequency, as the drift depends both on the VSI frequency and the frequency at the point of connection.

The proposed model is currently verified in a laboratory setup in the Netpower Lab of the Interdisciplinary Center for Security, Reliability and Trust, SnT, at the University of Luxembourg, affiliated with the European Network of Excellence DERlab e.V. First results indicate a confirmation of the proposed model in the sense, that a frequency variation of one VSI is registered by adjacent VSIs with an amplitude variance linearly depending on line impedance.

This work is supported by the National Research Fund of Luxembourg FNR.

Fonds National de la Recherche Luxembourg

[1] *A. Engler. Regelung von Batteriestromrichtern in modularen und erweiterbaren Inselnetzen. PhD thesis, Universität Kassel, May 2001.*

[2] *O. Osika. Stability of MicroGrids with High Share of Inverter-dominated and Decentralised Sources. Phd thesis, University of Kassel, 2005.*