University of Queensland

PAPERS

FACULTY OF ENGINEERING

Volume 1 1940 Numbers 3 and 4

3. Redundant Frames

BY

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4. The General Formulae of Moment-Distribution and their Application to Space Frames

BY

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REPRINTED
from
HE JOURNAL OF THE INSTITUTION OF ENGINEERS, AUSTRALIA,
VOL. 11., No. 12, December, 1939, pp. 421-424, and VOL. 12, No. 1, January, 1940, pp. 1-9.

FACULTY OF ENGINEERING.
VOLUME 1. 1940. NUMBER 3.

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[Reprinted from *The Journal of The Institution of Engineers, Australia,* Vol. 11, No. 12, December, 1939, pp. 421-424.]

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Redundant Frames.

BY C. N. ROSS, M.SC., B.M.E., M.I.E.AUST.*

Summary.—The author describes a modification of existing methods for finding the forces in the members of a redundant frame. By the use of trigonometrical resolution or otherwise, the forces in all members are written on the frame. Then each redundant bay in the structure is considered *by itself*, and from the strains in the six members of the bay an equation is formed which contains some of the unknown forces. Two numerical examples are used to illustrate its application.

NOTATION.

l = length of a member.
A = cross-sectional area of a member.
E = Young's Modulus for the material.
P = total force in a member caused by the load system.
μ = force in a member caused by a unit load at the point where a deflection is required and in the same direction as the required deflection.

METHOD.

Let Fig. 1 be any redundant bay forming portion of a frame.

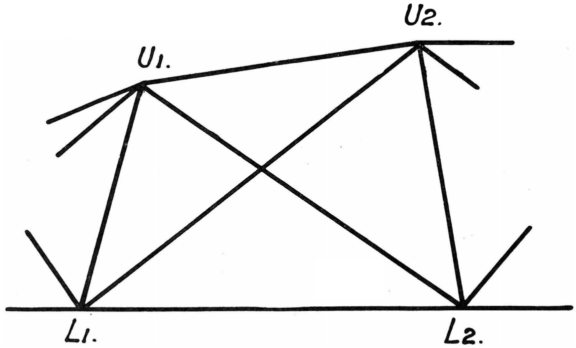

Fig. 1.

The difference of the deflections in the direction of L_1U_1, of U_1 and L_1 relative to L_2 is equal to the strain of member U_1L_1. Thus an equation is obtained connecting the six forces in the six members of the bay.

It is interesting to note that the deflections of any pair of adjacent nodes, in the direction of the line joining them, relative to either of the other nodes, can be used, thus giving eight different ways of forming the same equation.

Two more ways for obtaining this equation are to equate the relative deflections of two opposite nodes to the strain in the member joining them, e.g., the deflection of U_2, in the direction U_2L_1, relative to L_1, is equal to the strain in U_2L_1. The latter procedure will be used here.

Any suitable members are selected as the redundant ones and symbols are used for the unknown forces in these members. The forces in all the other members in the frame caused by the external load system, together with these unknown forces, are then found by trigonometrical resolution, or by some other standard method.

*This paper, No. 666, originated in the Brisbane Division of The Institution. The author is a member of the staff of the Faculty of Engineering, University of Queensland.

Each redundant bay in the frame is then treated by itself, thus giving as many equations as there are unknown forces.

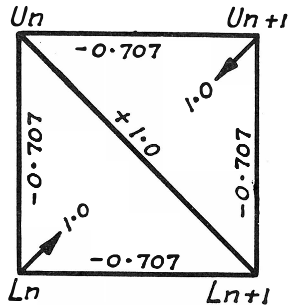

Fig. 2 (*b*).

Since E has the same value for all the members in most frames, it is assumed to be equal to unity in the following numerical examples.

Example 1. The frame shown in Fig. 2 (a) is taken from *Deflections and Statically Indeterminate Stresses* by Hudson (p. 162).

The bays are 30 ft. x 30 ft. and the loads are in units of 1,000 lb.

Choose U_2L_1 and U_3L_2 as the redundant members and let the forces in them be $-x\sqrt{2}$ and $-y\sqrt{2}$ respectively as they are obviously both compressions. It is convenient to choose these forces so that their vertical components are x and y respectively.

From the given external forces and these assumed forces in the redundant members, compute the force in each member by trigonometrical resolution and write it on the frame as shown in Fig. 2 (a).

The forces in the members of a bay caused by a unit force in the direction of a diagonal member are shown in Fig. 2 (b).

The deflection of U_2 towards L_1 is equal to $\Sigma\frac{P\mu l}{A}$ of the bay $U_1U_2L_1L_2$ for all members excepting U_2L_1 of Fig. 2 (a), and the values of μ are given on Fig. 2 (b).

Member	l/A	P	μ	$\frac{P\mu l}{A}$	
U_1U_2	0.769	$-180+x$	-0.707	97.9	$-0.544x$
L_1L_2	1.364	$90+x$	-0.707	-86.75	$-0.965x$
U_1L_1	2.143	$+x$	-0.707		$-1.515x$
U_2L_2	3.333	$x+y-90$	-0.707	212.00	$-2.355x-2.355y$
U_1L_2	3.03	$90\sqrt{2}-x\sqrt{2}$	1.0	385.50	$-4.285x$

$\therefore$ deflection of U_2 towards L_1 $= 608.75 - 9.664x - 2.355y$

and this must be equal to the strain of U_2L_1,

that is to $x\sqrt{2} \times \frac{l}{A} = x\sqrt{2} \times 3.03 = 4.285x$

$\therefore\ 608.75 - 9.664x - 2.355y = 4.285x$

or $13.949x + 2.355y = 608.75$ ....(1)

The deflection of U_3 towards L_2 is equal to $\Sigma\frac{P\mu}{A}$ for all members of the bay $U_2U_3L_2L_3$ excepting U_3L_2 of Fig. 2 (a) and the values of μ are given on Fig. 2 (b).

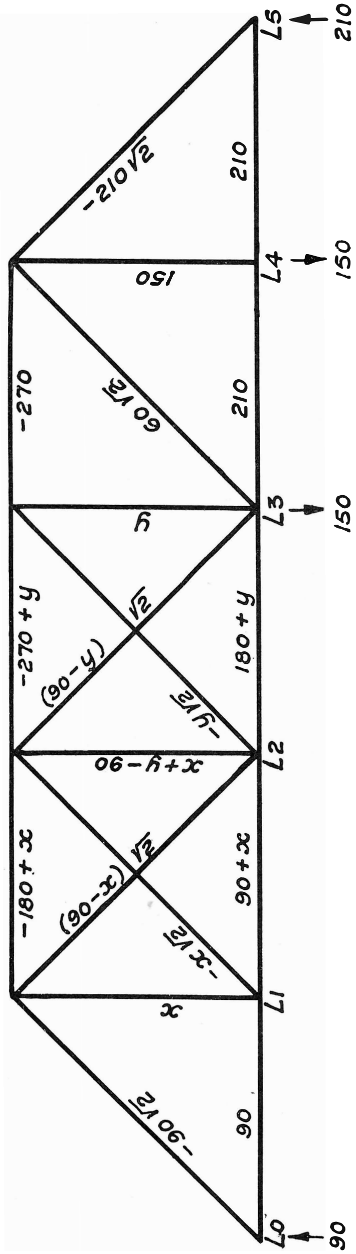
U1
U2
U3
U4
L0
L1
L2
L3
L4
L5
90
150
150
210
$-90\sqrt{2}$
90
x
$-x\sqrt{2}$
$(90-x)\sqrt{2}$
$-180+x$
$90+x$
$x+y-90$
$-y\sqrt{2}$
$(90-y)\sqrt{2}$
$-270+y$
$180+y$
y
-270
$60\sqrt{2}$
210
150
$-210\sqrt{2}$
210

Fig. 2 (*a*).

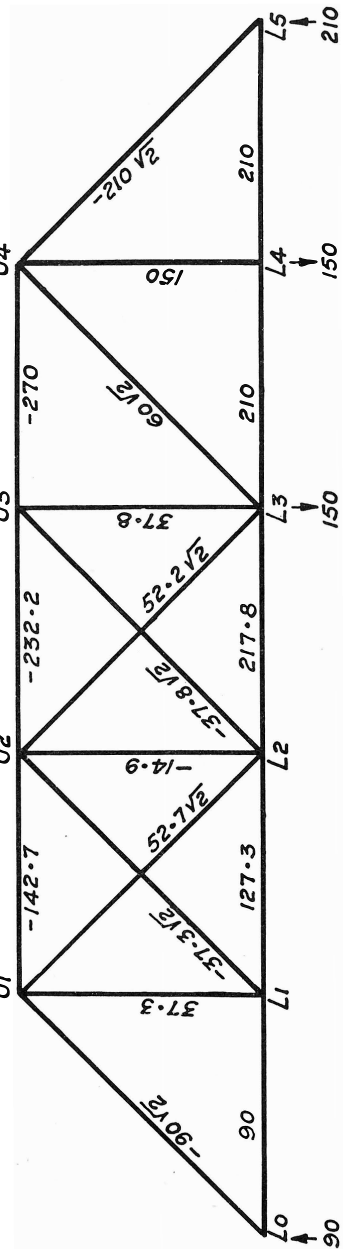
U1
U2
U3
U4
L0
L1
L2
L3
L4
L5
90
150
150
210
$-90\sqrt{2}$
90
37·3
$-37·3\sqrt{2}$
$52·7\sqrt{2}$
$-142·7$
127·3
$-14·9$
$-37·8\sqrt{2}$
$52·2\sqrt{2}$
$-232·2$
217·8
37·8
-270
$60\sqrt{2}$
210
150
$-210\sqrt{2}$
210

Fig. 2 (*c*).

Member	l/A	P	μ	$\frac{P\mu l}{A}$	
U_2U_3	0.769	$-270+y$	-0.707	146.9	$-0.544y$
L_2L_3	0.937	$180+y$	-0.707	-119.2	$-0.662y$
U_2L_2	3.333	$x+y-90$	-0.707	$212.0-2.355x$	$-2.355y$
U_3L_3	3.333	y	-0.707		$-2.355y$
U_2L_3	3.535	$90\sqrt{2}-y\sqrt{2}$	1.0	449.5	$-5.000y$

$\therefore$ deflection of U_3 towards L_2 $=$ $689.2-2.355x-10.916y$
and this must be equal to the strain of U_3L_2,

that is to $y\sqrt{2}\times\frac{l}{A}=y\sqrt{2}\times 3.535=5.00y$

$\therefore\ 689.2-2.355x-10.916y=5.00y$
or $2.355x+15.916y=689.2$ ....(2)
or $x=292.7-6.76y$
Substituting in (1)
$4080-94.2y+2.355y=608.75$
$\therefore\ y=37.8$
$\therefore\ x=292.7-6.76\times 37.8=37.3.$

Substituting these values for x and y in the forces shown on Fig. 2 (a), gives the actual forces in the members as shown on Fig. 2 (c).

Example II.—The frame shown in Fig. 3 (a), is taken from *Statically Indeterminate Stresses* by Parcel and Maney, p. 115.

Since the frame and loading are both symmetrical, only one half of the frame is shown.

The ratio of the length of a diagonal to a vertical to a horizontal member is as 1.28 : 1.0 : 0.8.

The loads are in units of 1,000 lb.

Choose U_1L_0, U_2L_1, U_3L_2, and U_4L_3 as the redundant members and let the forces in them be $-1.28w$, $-1.28x$, $-1.28y$ and $-1.28z$ respectively, as they are obviously all compressions.

From the given external forces and these assumed forces in the redundant members compute the force in each member by trigonometrical resolution and write it on the frame as shown in Fig. 3 (a).

The forces in the members of a bay caused by a unit force in the direction of a diagonal member are shown in Fig. 3 (b).

The deflection of U_1 towards L_0 is equal to $\Sigma\frac{P\mu l}{A}$, for all members of the bay $U_0U_1L_0L_1$ excepting U_1L_0 of Fig. 3 (a), and the values of μ are given on Fig. 3 (b).

Member	l/A	P	μ	$\frac{P\mu l}{A}$	
U_0U_1	10.4	$-280+0.8w$	-0.625	$1{,}820-5.20w$	
L_0L_1	10.4	$0.8w$	-0.625	$-5.20w$	
U_0L_0	12	$-400+w$	-0.781	$3{,}750-9.37w$	
U_1L_1	16.7	$-350+w+x$	-0.781	$4{,}565-13.04w$	$-13.04x$
U_0L_1	19.2	$448-1.28w$	1.0	$8{,}600-24.58w$	

$\therefore$ deflection of U_1 towards L_0 $=$ $18{,}735-57.39w-13.04x$
and this must be equal to the strain of U_1L_0, that is to

$1.28w\times\frac{l}{A}=1.28w\times 19.2=24.58w$

$\therefore\ 18{,}735-57.39w-13.04x=24.58w$
or $81.97w+13.04x=18{,}735$ ....(1)

The deflection of U_2 towards L_1 is equal to $\Sigma\frac{P\mu l}{A}$ for all members of the bay $U_1U_2L_1L_2$ excepting U_2L_1 of Fig. 3 (a), and the values of μ are given on Fig. 3 (b).

Member	l/A	P	μ	$\frac{P\mu l}{A}$
U_1U_2	6	$-480 + 0.8x$	-0.625	$1{,}800 - 3.00x$
L_1L_2	6	$280 + 0.8x$	-0.625	$-1{,}050 - 3.00x$
U_1L_1	16.7	$-350 + w + x$	-0.781	$4{,}565 - 13.05w - 13.05x$
U_2L_2	20	$-250 + x + y$	-0.781	$3{,}905 - 15.62x - 15.62y$
U_1L_2	24	$320 - 1.28x$	1.0	$7{,}680 - 30.72x$

$\therefore$ deflection of U_2 towards L_1 = $16{,}900 - 13.05w - 65.39x - 15.62y$

and this must be equal to the strain of U_2L_1,

that is to $1.28x\frac{l}{A} = 1.28x \times 24 = 30.72x$

$\therefore\ 16{,}900 - 13.05w - 65.39x - 15.62y = 30.72x$

or $13.05w + 96.11x + 15.62y = 16{,}900$ ....(2)

The deflection of U_3 towards L_2 is equal to $\Sigma\frac{P\mu l}{A}$ for all members of the bay $U_2U_3L_2L_3$ excepting U_3L_2 of Fig. 3 (a), and the values of μ are given on Fig. 3 (b).

Member	l/A	P	μ	$\frac{P\mu l}{A}$
U_2U_3	4.9	$-600 + 0.8y$	-0.625	$1{,}838 - 2.45y$
L_2L_3	4.9	$480 + 0.8y$	-0.625	$-1{,}471 - 2.45y$
U_2L_2	20	$-250 + x + y$	-0.781	$3{,}910 - 15.62x - 15.62y$
U_3L_3	30	$-150 + y + z$	-0.781	$3{,}516 - 23.43y - 23.43z$
U_2L_3	32	$192 - 1.28y$	1.0	$6{,}141 - 40.95y$

$\therefore$ deflection of U_3 towards L_2 = $13{,}934 - 15.62x - 84.90y - 23.43z$

and this must be equal to the strain of U_3L_2,

that is to $1.28y \times \frac{l}{A} = 1.28y \times 32 = 40.95y$

$\therefore\ 13{,}934 - 15.62x - 84.90y - 23.43z = 40.95y$

or $15.62x + 125.85y + 23.43z = 13{,}934$ ....(3)

The deflection of U_4 towards L_3 is equal to $\Sigma\frac{P\mu l}{A}$ for all members of the bay $U_3U_4L_3L_4$ excepting U_4L_3 of Fig. 3 (a), and the values of μ are given on Fig. 3 (b).

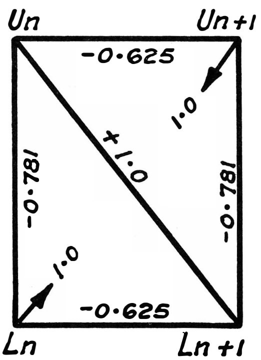

Fig. 3 (*b*).

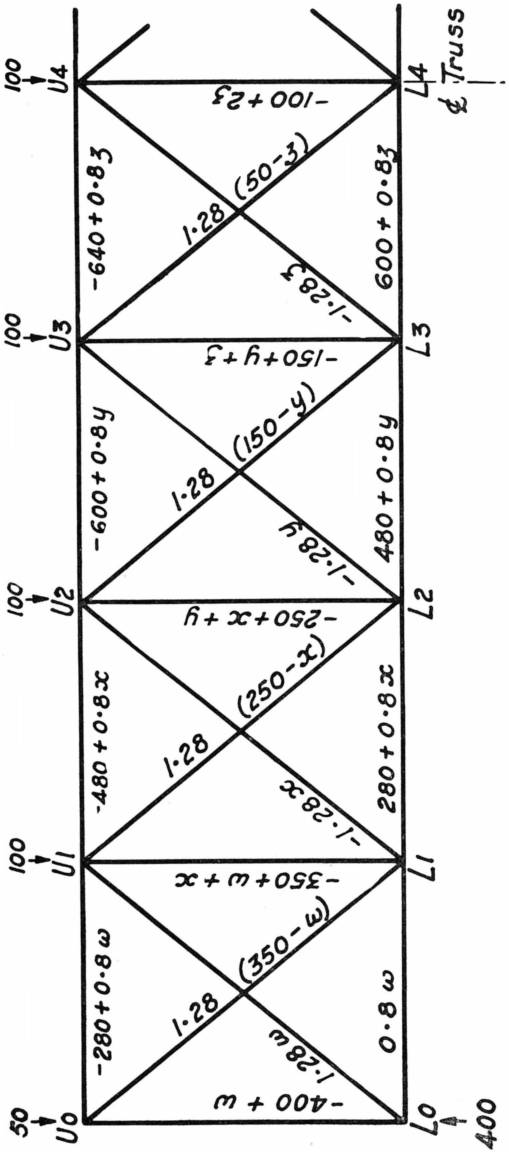
50
100
100
100
100
U0
U1
U2
U3
U4
L0
L1
L2
L3
L4
400
℄ Truss
-280+0·8ω
-480+0·8x
-600+0·8y
-640+0·83
0·8ω
280+0·8x
480+0·8y
600+0·83
-400+ω
-350+ω+x
-250+x+y
-150+y+3
-100+23
1·28
(350-ω)
-1·28ω
(250-x)
-1·28x
(150-y)
-1·28y
(50-3)
-1·283

Fig. 3 (a).

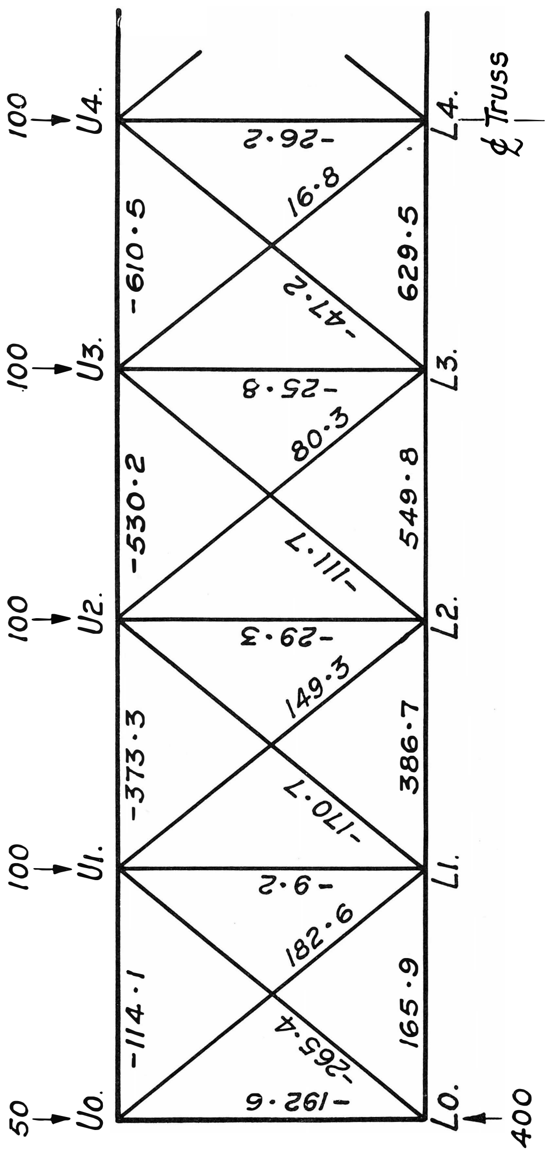
50
100
100
100
100
U0.
U1.
U2.
U3.
U4.
L0.
L1.
L2.
L3.
L4.
400
℄ Truss
-114·1
-373·3
-530·2
-610·5
165·9
386·7
549·8
629·5
-192·6
-9·2
-29·3
-25·8
-26·2
-265·4
182·6
-170·7
149·3
-111·7
80·3
-47·2
16·8

Fig. 3 (c).

Member	l/A	P	μ			
U_3U_4	4.5	$-640+0.8z$	-0.625	1,800		$-2.25z$
L_3L_4	4.5	$600+0.8z$	-0.625	1,688		$-2.25z$
U_3L_3	30	$-150+y+z$	-0.781	3,516	$-23.43y$	$-23.43z$
U_4L_4	30	$-100+2z$	-0.781	2,343		$-46.86z$
U_3L_4	38.4	$64-1.28z$	1.0	2,458		$-49.15z$

$\therefore$ deflection of U_4 towards L_3 = $8{,}429 - 23.43y - 123.94z$
and this must be equal to the strain of U_4L_3

that is to $1.28z \times \frac{l}{A} = 1.28z \times 38.4 = 49.15z$

$\therefore\ 8{,}429 - 23.43y - 123.94z = 49.15z$

or $23.43y + 173.09z = 8{,}429$ ....(4)

$15.62x + 125.85y + 23.43z = 13{,}934$ ....(3)

$13.05w + 96.11x + 15.62y = 16{,}900$ ....(2)

$81.97w + 13.04x = 18{,}735$ ....(1)

From (1) $w = 228.6 - 0.159x$
substituting in (2) $2{,}998 - 2.07x + 96.11x + 15.62y = 16{,}900$
or $94.04x + 15.62y = 13{,}902$
or $x = 147.9 - 0.166y$.
Substituting in (3) $2{,}310 - 2.59y + 125.85y + 23.43z = 13{,}934$
or $123.26y + 23.43z = 11{,}624$
or $y = 94.3 - 0.19z$.
Substituting in (4) $2{,}210 - 4.45z + 173.09z = 8{,}429$
or $168.64z = 6{,}219$ or $z = 36.9$
$\therefore\ y = 94.3 - 0.19 \times 36.9 = 87.3$
$\therefore\ x = 147.9 - 0.166 \times 87.3 = 133.4$
$\therefore\ w = 228.6 - 0.159 \times 133.4 = 207.4$.

Substituting these values for w, x, y, and z in the forces shown on Fig. 3 (a) gives the actual forces in the members as shown on Fig. 3 (c).

Conclusion.

This method considers the forces caused by the load system and by the redundant members together, and is therefore shorter than the usual method.

All forces are shown on the members of the frame so that from the first step the procedure can be visualised. The sets of quantities from which the equations are formed have similar signs, thus eliminating the possibility of an error in sign. Obviously these signs change beyond the node where the shearing force changes sign.