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A Method for Static Divergence Analysis of Sting-Mounted Wind Tunnel Models

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SUMMARY

Occurrence of static divergence will result in a catastrophic failure of wind tunnel models supported on sting or other types of support. The report presents an analysis method that can be used to design and/or verify the design of a model-balance-sting system for preventing occurrence of static divergence of the system. Detailed explanations are given to provide a clear understanding of the static divergence phenomenon. A brief discussion indicating the conditions for which divergence can become a concern and various design factors that the designer can consider to meet the stipulated criteria, is also included.

In order to ensure adequate margin of safety against divergence in pitch plane the system must satisfy the following criteria: i) divergence parameter D shall not exceed a value of 0.2, and, ii) elastic deflection of the balance-sting combination should be limited to 3°.

Excellent agreement shown between calculated and test deflection data on a typical model indicates the high accuracy of the method of obtaining model deflection and angle of attack.

CONTENTS

List of Symbols

C_m : Pitching moment coefficient, pitching moment / $qS\ell_R$

C_{m_o} : Intercept of the tangent to C_m vs. α plot, drawn at α (see Fig. 3)

C_{m_α} : Local slope of C_m vs. α plot at α (see Fig. 3)

C_N : Normal force coefficient, normal force / qS

C_{N_o} : Intercept of the tangent to C_N vs. α plot, drawn at α (see Fig. 3)

C_{N_α} : Local slope of C_N vs. α plot at α (see Fig. 3)

C_o : See equation (10)

D : Static divergence parameter in pitch plane (see equation (9))

ℓ_N : Distance between front and rear normal force elements of balance

ℓ_R : Reference length for pitching moment coefficient

M : Pitching moment on model

N : Normal force on model

N1 : Force in front normal force element of balance

N2 : Force in rear normal force element of balance

q : Dynamic pressure

S : Reference area

X : Location of moment reference point w.r.t. balance centre, +ve upstream

α : Angle of attack

α_o : Initial attitude of model

$\Delta\theta$: Angular deflection of model-balance-sting system

δ_{N1} : Deflection of balance-sting combination per unit normal force applied at front normal force station of balance, deg / lb

δ_{N2} : Deflection of balance-sting combination per unit normal force applied at rear normal force station of balance, deg / lb

1. Introduction:

The sting support and the strain gauge balance employed for aerodynamic load measurements on wind tunnel models are elastic in nature. As a consequence the aerodynamic forces and moments acting on the model cause deflections of the model-balance-sting system. For a system designed with adequate factors of safety for static aerodynamic loads, these deflections are in general small and the translational deflection does not affect the flow parameters and hence ignored. However, the angular deflection of the system cannot be neglected since the angular deflection not only changes the model attitude, but it can sometimes lead to an instability phenomenon that endangers the safety of the system, as discussed below.

The change in model attitude resulting from the angular deflection alters the model angle of attack (and/or sideslip angle) giving rise to additional aerodynamic loads on the model, which in turn results in further deflection of the system, i.e. the model aerodynamics and the elasticity of the balance-sting combination interact with each other. In a properly designed system, the above interactive process stops and the model attains an equilibrium angle of attack, after some time. But, in some cases the interactive process continues without interruption and the model angle of attack (or sideslip angle) therefore increases without limit until the balance-sting combination breaks. A graphical illustration of the consequences of the occurrence of static divergence is shown in Fig.1, with the model-balance-sting system on the verge of failure. It is therefore essential to design and analyze the model-balance-sting system not only to avoid the static divergence phenomenon, but also ensure that the system possesses an adequate margin of safety against its occurrence.

Some empirical criteria have been established to ensure that the model-balance-sting combination has an adequate margin of safety against static divergence phenomenon (Refs 1 to 3). While references 1 and 2 present only statements about criteria, Reference 3 includes a brief presentation of a static divergence analysis method in addition to the criterion. However, the analysis method of Reference 3 is applicable to a system with a balance that resolves the model aerodynamic force and moment vectors into three forces (normal force, side force, axial force) and three moments (pitching moment, yawing moment and rolling moment), which is common to integral type balances used elsewhere. But, the floating-frame type balances widely used at NTAF resolve the aerodynamic force and moment vectors into five forces (front and aft normal forces, front and aft side forces and axial force) and one moment (rolling moment). Even the integral type balances used at NTAF have been wired for "five forces and one moment" measurement scheme. The method of Ref. 3 is not directly applicable to such balances. In addition, the analysis method of Reference 3 is somewhat restrictive (the method requires the moment reference point to be coincident with balance centre). A need was therefore felt of a divergence analysis method directly applicable to a system with a balance featuring "five forces and one moment" measurement scheme. The report presents an analysis method for such a system and without the restrictions of the method of Reference 3 noted above.

The method presented in this report is applicable to sting-supported models with either a floating-frame balance or an integral balance, as long as the balance resolves the force and moment vectors on the model into five forces and one moment.

2. Analysis method:

Fig. 2 shows schematically a model-balance-sting system mounted in the wind tunnel. The initial attitude of the model (i.e. in wind-off) is α_0. Under the action of the aerodynamic forces and moment that act on the model when it is exposed to flow, deflection of the system occurs. On account of the deflection of the system two effects arise: (i) the model angle of attack changes with accompanying changes in the aerodynamic forces and moments on the model. For the case shown in Fig. 2, the normal force and pitching moment on the model cause an overturning moment on the system, which in turn results in a further nose up angular deflection of the system, and, (ii) a restoring moment proportional to the angular deflection, generated by the balance-sting combination (which acts as a torsion spring due to its elasticity), acts on the system.

As these two effects oppose each other the model reaches an equilibrium position when the restoring moment equals the overturning aerodynamic moment.

However, under certain circumstances the rate of increase of the overturning moment due to aerodynamic loads will be greater than the rate of increase of the restoring moment provided by the structure, consequently the deflection of the system and hence the model angle of attack increase without limit, resulting in structural failure of the sting-balance. The conditions under which such divergence phenomenon which is primarily a static instability of the system, occurs can be predicted. An analysis method for predicting this instability phenomenon is presented below.

Assume that the model-balance-sting system characteristics are such that the restoring moment provided by the elasticity of the sting-balance combination equals the overturning aerodynamic moment and consequently the model reaches an equilibrium position. Let the aerodynamic loads acting on the model in this position be represented by a normal force N and a pitching moment M acting at the moment reference point B, and the deflection of sting-balance combination from its initial position be $\Delta\theta$.

The deflection of sting-balance combination depends on the flexibility of the sting-balance combination and the magnitude of forces and moments acting on the balance. While the sting has infinite degrees of freedom, the balance can be considered as having six degrees of freedom. As stated earlier, only angular deflections need to be considered for analysis of static divergence. In addition, coupling between the three angular deflections i.e., pitch, yaw and roll can be ignored without loss of accuracy for the purpose of divergence analysis. With this assumption, analysis can be made separately for pitch, yaw and roll cases, considering the system flexibilities and the model force and moment appropriate to the relevant degree of freedom.

We are considering here the analysis of static divergence in pitch plane, with a pitch angle deflection $\Delta\theta$ caused by the model normal force N and pitching moment M. The system features a balance that resolves the model aerodynamic force and moment vectors into five forces and one moment. Flexibility of such a balance is expressed in terms of angular deflections per unit normal force at the front and aft normal force element stations.

Let δ_{N1} and δ_{N2} be the angular deflections of the upstream end of the balance-sting combination due to a unit normal force applied respectively at the front and aft normal force stations (Fig.2). Since the model is mounted on the balance, the quantities δ_{N1} and δ_{N2} (usually called as 'deflection constants') also represent the model deflections per unit force at front and aft normal force elements. Hence the total angular deflection $\Delta\theta$ of the model can be obtained as,

$$\Delta\theta = \delta_{N1}.N1 + \delta_{N2}.N2 \qquad \dots (1)$$

where N1 and N2 are the forces induced respectively in the front and aft normal force elements of the balance, due to normal force N and pitching moment M acting at point B on the model. These element forces can be calculated by taking moments about the aft and front normal force element stations,

These are given by,

$$N1 = \frac{1}{\ell_N}\left[M + N\left(X + \frac{\ell_N}{2}\right)\right] \qquad \dots (2)$$

$$N2 = \frac{-1}{\ell_N}\left[M + N\left(X - \frac{\ell_N}{2}\right)\right] \qquad \dots (3)$$

Substituting for N1 and N2 in equation (1), the angular deflection $\Delta\theta$ of the model under the action of the aerodynamic force N and moment M is given by,

$$\Delta\theta = \frac{\delta_{N1}}{\ell_N}\left[M + N\left(X + \frac{\ell_N}{2}\right)\right] - \frac{\delta_{N2}}{\ell_N}\left[M + N\left(X - \frac{\ell_N}{2}\right)\right]$$

After simplification and rearrangement the above equation can be written as,

$$\Delta\theta = M\,\frac{(\delta_{N1} - \delta_{N2})}{\ell_N} + \frac{N}{\ell_N}\left[X(\delta_{N1} - \delta_{N2}) + \frac{\ell_N}{2}(\delta_{N1} + \delta_{N2})\right] \qquad \dots (4)$$

The normal force N and pitching moment M, assumed to be linear over a small range of angle of attack, can be expressed in terms of corresponding nondimensional aerodynamic derivatives and coefficients, free-stream dynamic pressure, and reference area and length as,

$$\left.\begin{aligned} N &= qS\,(C_{N_o} + \alpha C_{N_\alpha}), \\ M &= qS\ell_R\,(C_{m_o} + \alpha C_{m_\alpha}) \end{aligned}\right\} \qquad \ldots (5)$$

where the terms C_{N_o} and C_{m_o} are the intercepts made by the tangents drawn respectively to the C_N vs. α and C_m vs. α plots and C_{N_α} and C_{m_α} are the local slopes at α, as shown in Fig. 3.

Substituting equations (5) in equation (4) and rearranging the terms, the final equation for angular deflection of the model can be written as,

$$\Delta\theta = \alpha qS\left[\left(\ell_R\, C_{m_\alpha} + X C_{N_\alpha}\right)\frac{(\delta_{N1} - \delta_{N2})}{\ell_N} + C_{N_\alpha}\frac{(\delta_{N1} + \delta_{N2})}{2}\right]$$

$$+\, qS\left[\left(\ell_R\, C_{m_o} + X C_{N_o}\right)\frac{(\delta_{N1} - \delta_{N2})}{\ell_N} + C_{N_o}\frac{(\delta_{N1} + \delta_{N2})}{2}\right] \qquad \ldots (6)$$

The angle of attack of the model taking into account the above deflection is therefore,

$$\alpha = \alpha_o + \Delta\theta \qquad \ldots (7)$$

Substituting for $\Delta\theta$ from equation (6) and rearranging the terms, the equation for the model angle of attack can be written as,

$$\alpha\left[1 - qS\left(\ell_R\, C_{m_\alpha} + X\, C_{N_\alpha}\right)\left(\frac{\delta_{N1} - \delta_{N2}}{\ell_N}\right) + C_{N_\alpha}\left(\frac{\delta_{N1} + \delta_{N2}}{2}\right)\right]$$

$$= \alpha_o + qS\left[\left(\ell_R\, C_{m_o} + X\, C_{N_o}\right)\left(\frac{\delta_{N1} - \delta_{N2}}{\ell_N}\right) + C_{N_o}\left(\frac{\delta_{N1} + \delta_{N2}}{2}\right)\right] \qquad \ldots (8)$$

Defining $D = qS\left[\left(\ell_R\, C_{m_\alpha} + X\, C_{N_\alpha}\right)\left(\frac{\delta_{N1} - \delta_{N2}}{\ell_N}\right) + C_{N_\alpha}\left(\frac{\delta_{N1} + \delta_{N2}}{2}\right)\right]$ ...(9)

and $C_o = qS\left[\left(\ell_R\, C_{m_o} + X\, C_{N_o}\right)\left(\frac{\delta_{N1} - \delta_{N2}}{\ell_N}\right) + C_{N_o}\left(\frac{\delta_{N1} + \delta_{N2}}{2}\right)\right]$ ... (10)

equations (6) and (8) can be written in shorter forms as

$$\Delta\theta = \alpha\, D + C_o\,,$$

and, $\alpha\,(1 - D) = \alpha_o + C_o\,,$

On simplification, equations for model deflection and angle of attack are obtained as,

$$\Delta\theta = \frac{(D\,\alpha_o + C_o)}{(1 - D)} \qquad \text{... (11)}$$

and $\alpha = \frac{(\alpha_o + C_o)}{(1 - D)}$ ... (12)

where D and C_o are given by equations (9) and (10) respectively.

The term (1 – D) appearing on the right hand side of equations (11) and (12), can be interpreted as a measure of the overall or combined stiffness of the model-balance-sting system, with contributions from structural stiffness of the balance-sting combination and the aerodynamic stiffness determined by the aerodynamic characteristics of the model. In fact, the term (1 – D) can be shown to be equal to the ratio, (structural stiffness + aerodynamic stiffness) / (structural stiffness), in the case of a system with the model aerodynamic centre coinciding with the balance centre. While the structural stiffness is always positive producing a restoring moment, the aerodynamic stiffness which is determined by the aerodynamic characteristics of model can be positive, negative or even zero. Hence the combined stiffness of the model-balance-sting combination can, in general, be positive, negative or zero, depending on the relative magnitudes of structural and aerodynamic stiffness terms.

The sign of the term (1 – D) which, as noted earlier, is a measure of the combined stiffness of the system, can likewise be positive or negative or its value can even become zero. Static stability of the system, which is determined by the combined stiffness of the system, is therefore governed by the sign and magnitude of the term (1 – D).

The qualitative behaviour of the system can be explained by considering different ranges of the values of the parameter D. When $0 < D < 1$ the term (1 –D) is positive, which indicates that the combined stiffness of the system is positive and the system is therefore statically stable. Consequently the model set initially at an angle α_o reaches an equilibrium angle of attack α, which, in this case will be higher than α_o depending on the magnitude of (1 – D).

When D = 1 the term (1 – D) = 0, which means that the combined stiffness of the system is zero and thus, when the model deflects under the action of the aerodynamic force and moment, no restoring moment acts on the model. Consequently, the overturning moment due to aerodynamic loads tends to increase the model angle of attack infinitely i.e. the system diverges. When D > 1 the term (1 – D) becomes negative, which implies that the overall system stiffness is negative, consequently the angle of attack α again increases without limit. To summarize, the system becomes unstable and the angle of attack α increases without limit i.e. the system diverges when the parameter D is ≥ 1. The parameter D given by equation (9) is called the static divergence parameter.

When the parameter D is negative the term (1 – D) is positive and its magnitude is > 1, which means that the overall stiffness of the system is not only positive but its magnitude is higher than that of the structural stiffness. Consequently, the system is statically stable and the angle of attack reaches an equilibrium value which will be lower than α_o. (see equation (12)). If D = 0 (which implies that the negative aerodynamic stiffness is equal to the positive structural stiffness) the system has neutral stability.

The above analysis deals with divergence of the system in the pitch plane of the model. The set of equations derived above are also valid for the case of divergence of the system in the yaw plane of model, if quantities appropriate to the yaw plane are used, i.e., yaw deflection $\Delta\Psi$ (equal to $-\Delta\beta$) instead of $\Delta\theta$, lateral aerodynamic derivatives C_{S_β} and C_{n_β} instead of longitudinal derivatives C_{N_α} and C_{m_α}, and side force deflection constants δ_{S1} and δ_{S2} in place of δ_{N1} and δ_{N2}.

It is however noted that, the longitudinal aerodynamic derivatives are much larger in magnitude than the corresponding lateral derivatives for aircraft configurations, while the longitudinal and lateral derivatives are equal for the configurations featuring axial symmetry (such as missile configurations), Consequently critical case for static divergence studies is generally the model pitch plane involving longitudinal aerodynamic derivatives, except in special cases featuring a balance + sting combination with a very low stiffness in the yaw plane or about its roll axis (such as due to a non-circular sting cross section with the short side lying in the yaw plane and/or highly flexible balance side force or rolling moment elements).

3. Static divergence criteria

As discussed in section 2, to prevent occurrence of static divergence phenomenon the system characteristics should be such that the value of the static divergence parameter D is less than 1. In addition to preventing the occurrence of static divergence it is also necessary to have an adequate margin of safety with regard to this phenomenon. To ensure adequate margin of safety against static divergence, empirical criteria involving certain characteristics of the model-balance-sting system have been established by some wind tunnel facilities.

The criteria established by the Propulsion Wind Tunnel facility at AEDC, states that "the ratio of model air load increase caused by a change in angle of attack $(\Delta N/\Delta\alpha)$ must not exceed four-tenths of the calculated support system restoring force generated by such an angle change $(\Delta F/\Delta\theta)$, i.e., $\Delta N/\Delta\alpha \leq 0.4\ \Delta F/\Delta\theta$", (Ref.1). Note that the above AEDC criterion expressed in the terminology and symbols of the present report is equivalent to a statement that the second term of the parameter D (see equation (9)) must not exceed 0.4. A similar criterion established by NASA Langley Research Center limits the value of the above ratio to 0.5 instead of 0.4 (Ref. 2).

The Vought wind tunnels at LTV Aerospace and Defense Company used the static divergence criteria established by AEDC (noted above) up to the year 1983. But, they revised this criterion subsequently and established a new criterion according to which the maximum permissible value of static divergence factor is limited to 0.2 (Ref. 3). Higher values may be allowed if some special procedures are used. Reasons given for adopting this criterion, which is more stringent than those of Refs. 1 and 2, are (i) the dynamic pressure in the tunnel could exceed the nominal or programmed value by a factor of 3.3 in the event of a tunnel pressure control system failure, and (ii) the estimates of local values of aerodynamic derivatives could be larger than preliminary estimates by a factor of 1.5.

Although not explicitly stated, it appears that the criteria of Refs. 1 and 2 are applicable to model-balance-sting systems meant for use in continuous wind tunnels in which the possibility of a large increase in dynamic pressure following tunnel control system failure, is unlikely. On the other hand the criterion of Ref. 3 and the explanations given in establishing this criteria suggest that the criterion is primarily applicable for blowdown tunnels. Since the wind tunnels at NTAF are of blowdown type the criterion of Ref. 3 are more relevant than those of Refs 1 and 2 and hence the criteria of Ref. 3 are adopted at NTAF. These criteria are: (i) the divergence parameter D shall not exceed a value of 0.2, and (ii) higher values of the predicted divergence parameter can be permitted if special precautions are taken, such as use of accurate experimental aerodynamic data in calculating D, and/or with fail-safe tunnel pressure control system. However, as noted earlier, since the analysis method of Ref. 3 is not directly applicable to a system with a balance involving a measurement scheme with five forces and one moment, the method presented here should be used for static divergence analysis of model-balance-sting systems used at NTAF.

It can be shown that equivalent criterion in case of yaw plane divergence is : divergence parameter shall be ≥ -0.2.

In addition to the limit on maximum permissible value of static divergence parameter D, it has also been found necessary to limit the maximum deflection of the model-balance-sting system to 3°. The latter limit is imposed from two considerations, viz., (i) the usual methods of stress analysis adopted for sizing the sting, and also the static divergence analysis method presented here is valid for "small" elastic deflections, and (ii) to prevent inadvertent overloading of the balance, since the test program specification of the preset pitch angle range does not take account of the elastic deflections of the system. This ceiling on maximum deflection may lead to imposition of pitch angle vs. Mach number test envelope restrictions in some cases, especially for large aircraft models.

4. Accuracy of the method

Accuracy of the above method for predicting the static divergence of a model-balance-sting system depends on the accuracy of input data used in equations (9) and (10), i.e. the accuracy of aerodynamic derivatives and coefficients and the deflection constants of the balance-sting combination. Estimated aerodynamic data from handbook and/or other simple methods will be adequate for design purposes, provided these are conservative. However, the angular deflection of the model needs to be determined accurately since, as discussed earlier, the deflection alters the angle of attack (and/or sideslip), which should be measured as accurately as possible.

The most common method of accounting for the angular deflection involves the use of experimentally determined deflection constants along with measured values of balance element loads in equation (1), which is then added algebraically to the initial angle to obtain the true angle of attack, equation (7). A similar procedure is adopted to obtain the true sideslip angle. These procedures (of computing the deflections which are utilized to obtain the true angle of attack and sideslip) are incorporated into the tunnel data reduction program. The model deflection data obtained from the wind tunnel data reduction program can therefore be regarded as experimental data. Availability of such experimental data provides a means of validating some of the equations derived here and also to obtain an idea of the accuracy of model deflection calculations from the present method by comparing such calculations with test data.

Fig. 4 shows plots of C_N vs. α and C_m vs. α data obtained form wind tunnel tests at a Mach number of 0.95 on an aircraft model. These plots were utilized to obtain the aerodynamic derivatives and coefficients needed to calculate the model deflection using equation (6). While the aerodynamic derivatives C_{N_α} and C_{m_α} were obtained as the local slopes of the respective plots at two selected values of α (α=8.08° and 13.24°), the coefficients C_{N_0} and C_{m_0} were obtained as the intercepts made by the tangents drawn to the respective plots at the selected values of α, as indicated in Fig. 3 (Fig. 4 shows the tangents drawn at α = 8.08°). The above aerodynamic data along with experimental values of deflection constants were utilized in equations (9) to (12) to compute the model deflection and angle of attack. The calculated data are presented in Table 1, along with data obtained from wind tunnel tests. The calculated values of model angle of attack are seen to be in excellent agreement with the test data (the small differences seen are mainly due to the errors in deriving the aerodynamic data from the plots), which indicates the high accuracy of the method for obtaining the deflection and the true angle of attack.

5. Model-sting design factors influencing static divergence

A brief discussion on various conditions for which static divergence can become a concern and some of the remedial steps that the designer of model and support system can adopt, is presented below.

The above analysis shows that occurrence of static divergence phenomenon can become a possibility when (i) values of q and S are large, (ii) slopes C_{N_α} and C_{m_α} are positive in sign and are large in magnitude, (iii) the balance center location is far downstream of center of pressure location and (iv) the sting support is long and slender.

The aerodynamic derivatives C_{N_α} and C_{m_α} which have a major influence on divergence are model configuration dependent, and the designer has no choice over these. The designer should, however, be aware of the type of configurations that feature high values of the above aerodynamic derivatives. Among the various types of flight vehicle configurations, aircraft configurations with high aspect ratio unswept wings feature highest values of C_{N_α} per unit area and the static divergence can become a critical issue when large size models of such configurations are to be tested especially at transonic speeds and high dynamic pressures.

In case of models of long slender configurations, although the normal force slope per unit area is much smaller compared to the aircraft configurations, the balance center is often located far downstream of the center of pressure (primarily to provide adequate clearance between the sting and the model base). In addition, the slender configurations exhibit pronounced non-linearity in their normal force characteristics. As a result, the normal force slope of such configurations at higher alpha can become much larger than its linear range value, and this is often accompanied by large upstream shift in the center of pressure location. As both these factors tend to promote occurrence of static divergence, it is prudent to check the design to ensure adequate safety margins against static divergence, also for such slender configurations.

It is emphasized that in all the above cases, static divergence analysis should be carried out using the highest values of local slopes C_{N_α} and C_{m_α} rather than linearized values along with corresponding center of pressure location, in the Mach number – alpha test envelope.

Since the reference area S depends on both the model configuration and the model scale, the designer can exercise some control over this quantity by choosing the model scale to meet the divergence criteria. It is however to be noted that other considerations such as provision of adequate clearance between sting and model base, proper simulation of model geometry, aerodynamic load measurement accuracy etc. are also important factors in deciding the model scale. The location of balance in the model is primarily the designer's choice, which is normally decided to optimize balance element loads (such as equal sharing of normal force between the two normal force elements and prevention of balance overloading due to starting loads, if test Mach number > 2) But, if it becomes necessary, the designer can choose the balance center location primarily to meet static divergence criteria although this location may be non-optimum from balance element load considerations noted above.

In general, occurrence of static divergence can be minimized by locating the balance center close to the center of pressure. In fact, a balance center location as far ahead of the center of pressure as possible, will be helpful in satisfying the divergence criteria, since such a location tends to inhibit static divergence.

Flexibility of the (balance + sting) combination is an important parameter influencing static divergence. While both the balance and sting are elastic, major contribution to the flexibility of the combination (expressed in term of δ_{N1} and δ_{N2}, as noted in section 2) arises from the sting, primarily because of its long length. Consequently, if it becomes necessary to provide a stiffer (balance + sting) combination to meet static divergence criteria, the designer should opt to use or design a stiffer sting rather than a stiffer balance. Stiffness of the sting can in general, be increased by reducing its length and increasing its diameter. But, both these factors tend to increase aerodynamic interference (known as support interference). Thus the requirements of the sting to provide high stiffness and low aerodynamic interference conflict with each other, and a compromise will be necessary to obtain a satisfactory solution.

6. Summary of static divergence criteria and equations

For convenience and quick reference the static divergence criteria and the important equations derived in the report for the case of pitch plane are summarized below:

6.1. Static divergence criteria:

- Static divergence parameter, $D \leq 0.2$

  Higher values of D can be permitted if special precautions are taken.
- Maximum deflection, $\Delta\theta \leq 3$ deg

6.2. Summary of equations:

- Static divergence parameter D is given by,

$$D = qS\left[\left(\ell_R C_{m_\alpha} + X C_{N_\alpha}\right)\left(\frac{\delta_{N1} - \delta_{N2}}{\ell_N}\right) + C_{N_\alpha}\left(\frac{\delta_{N1} + \delta_{N2}}{2}\right)\right]$$

where the aerodynamic derivatives C_{m_α} and C_{N_α} are the local slopes of C_m vs. α and C_N vs. α plots, at the value of α giving highest value of C_{N_α} (see Fig. 3).

- Deflection of the model-balance-sting system is given by,

$$\Delta\theta = \delta_{N1}.N1 + \delta_{n2}.N2$$

$$= \frac{(D.\alpha_o + C_o)}{(1 - D)}$$

where C_o is given by

$$C_o = qS\left[\left(\ell_R C_{m_o} + X C_{N_o}\right)\left(\frac{\delta_{N1} - \delta_{N2}}{\ell_N}\right) + C_{N_o}\left(\frac{\delta_{N1} + \delta_{N2}}{2}\right)\right]$$

and the aerodynamic coefficients C_{m_o} and C_{N_o} are the intercepts made by the tangents drawn respectively to C_m vs. α and C_N vs. α plots at α corresponding to maximum local slope, C_{N_α} (see Fig.3).

- Model angle of attack α is given by,

$$\alpha = \frac{(\alpha_o + C_o)}{(1 - D)}$$

where C_o and D are given by the equations noted above.

The above set of equations can also be used for yaw plane divergence analysis with appropriate changes noted in section 2 along with the relevant criteria noted in section 3. But, for the reasons noted in section 2, critical case for divergence analysis occurs normally for the model pitch plane, and a system that satisfies divergence criteria in pitch plane will in general, also be satisfactory in the yaw plane, and about the roll axis, except in special cases.

7. Concluding remarks:

Occurrence of static divergence will result in a catastrophic failure of wind tunnel models supported on sting or other types of support. The report presents an analysis method that can be used to design and/or check the design of a model-balance-sting system for preventing occurrence of static divergence of the system. In order to ensure adequate margin of safety against divergence in pitch plane the system must satisfy the following criteria:

i) divergence parameter D shall not exceed a value of 0.2. Higher values of the parameter can however, be permitted if special precautions are taken,

ii) elastic deflection of the balance-sting combination should be limited to 3°.

In general, a system that satisfies divergence criteria in pitch plane will also be satisfactory in the yaw plane and about the roll axis, except in special cases.

Excellent agreement shown between calculated and test deflection data on a typical model indicates the high accuracy of the method of obtaining model deflection and angle of attack.

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Table 1 : Comparison of calculated angle of attack with test data

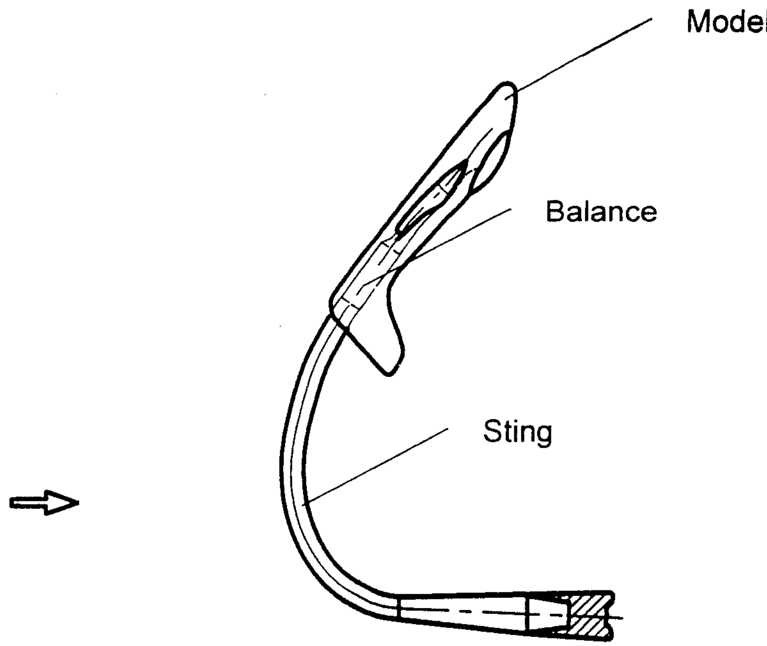
α_0	6.82 deg	11.01 deg
q	8.845 psi	8.834 psi
$C_{N\alpha}$	0.0662/deg	0.0583/deg
C_{N0}	-0.234	-0.165
$C_{m\alpha}$	-0.00537/deg	-0.00817/deg
C_{m0}	0.069	0.116
D (eqn. (9))	0.1907	0.1394
C_0 (eqn. (10))	-0.2591 deg	0.3765 deg
$\Delta\theta$ (eqn. (11))	1.28 deg	2.22 deg
α (eqn. (12))	**8.105 deg**	**13.227deg**
α (test data)	**8.08 deg**	**13.24 deg**

Note: (1) Aerodynamic data derived from Fig.4

(2) Other data used in eqns. (9) to (12) are:

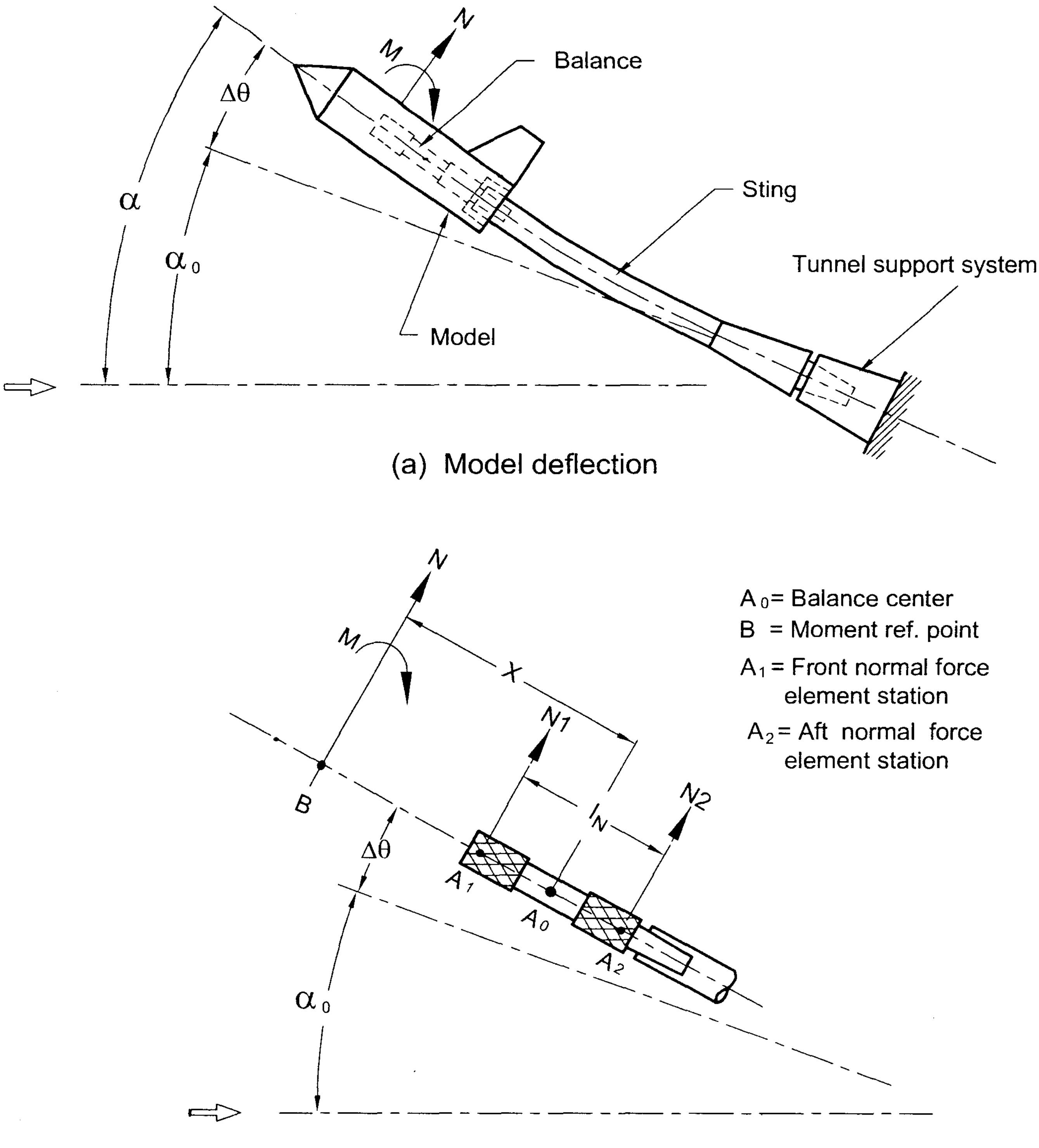
$S = 264.59\ in^2$, $\ell_R = 14.7''$, $X = 0.355''$

$\delta_{N1} = 0.00216$ deg/lb, $\delta_{N2} = 0.00071$ deg/lb



Occurence of divergence phenomenon causes the deflection of the model - balance -sting system to increase without limit. Figure shows the system at an intermediate position with large deflection and prior to failure

Fig. 1 Schematic illustration static divergence



(a) Model deflection

(b) Balance element loads due to model aerodynamic loads

Fig.2 Static deflection of model due to balance-sting flexibility

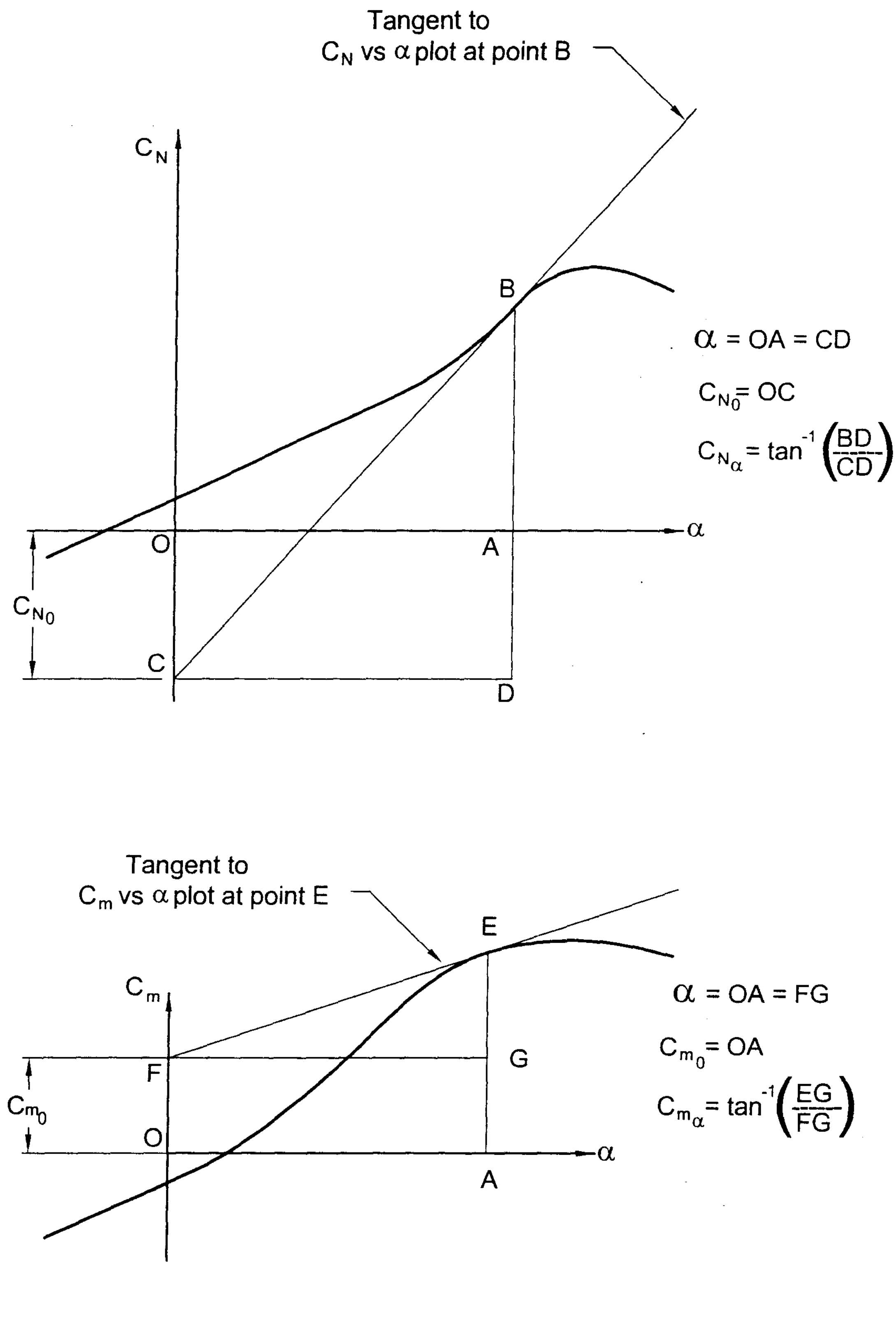
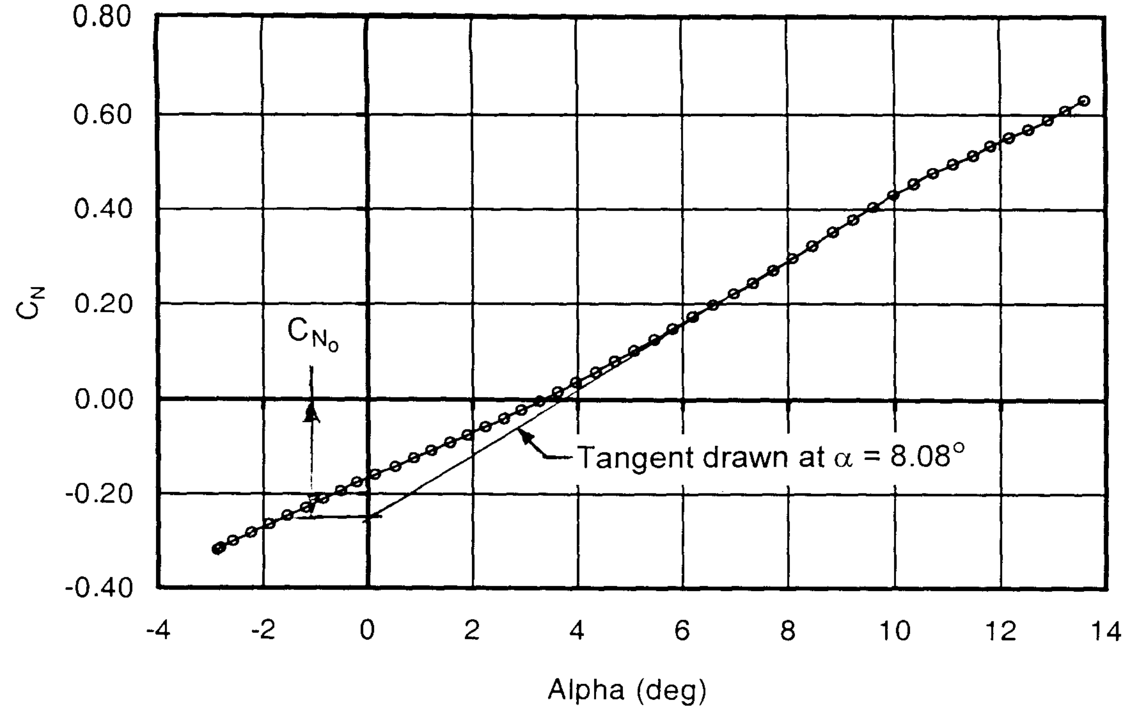
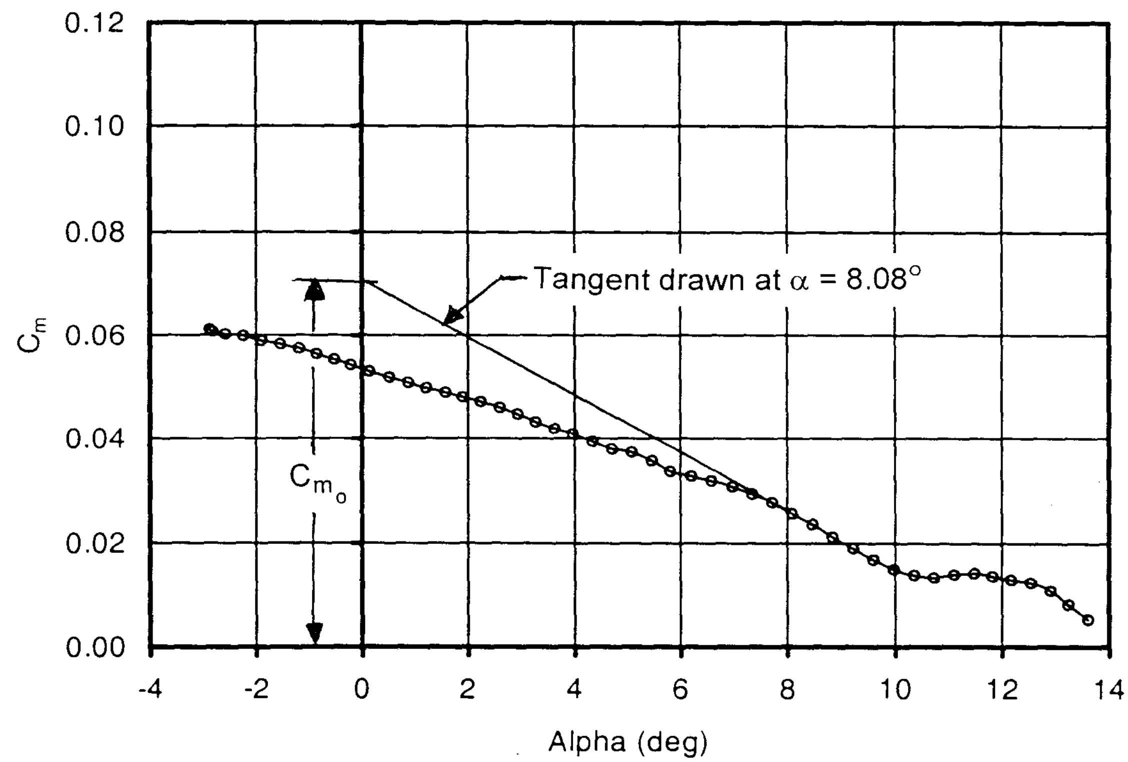


Fig. 3 Definition of intercepts and local slopes of

C_N vs α and C_m vs α plots



(a) C_N vs Alpha



(b) C_m vs Alpha

Fig. 4 Plots of normal force and pitching moment data on a sting mounted aircraft model

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Abstract

Occurrence of static divergence will result in a catastrophic failure of wind tunnel models supported on sting or other types of support. The report presents an analysis method that can be used to design and/or verify the design of a model-balance-sting system for preventing occurrence of static divergence of the system. Detailed explanations are given to provide a clear understanding of the static divergence phenomenon. A brief discussion indicating the conditions for which divergence can become a concern and various design factors that the designer can consider to meet the stipulated criteria, is also included.

In order to ensure adequate margin of safety against divergence in pitch plane the system must satisfy the following criteria: i) divergence parameter D shall not exceed a value of 0.2, and, ii) elastic deflection of the balance-sting combination should be limited to 3°.

Excellent agreement shown between calculated and test deflection data on a typical model indicates the high accuracy of the method of obtaining model deflection and angle of attack.

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