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Optimal Resource Allocation in URLLC for Real-Time Wireless Control Systems

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Abstract—**As one of the most important communication scenarios in the coming *fifth generation* (5G) cellular networks, *ultra-reliable and low-latency communication* (URLLC) is promising to enable real-time wireless control systems. However, one of the biggest challenges is that how to integrate URLLC and control performance together to maximize the overall system performance. In this paper, we investigate the resource allocation for URLLC uplink in real-time wireless control systems. Specifically, we first discuss the relationship between communication and control performance. Based on that, we convert the hybrid co-design problem into a regular wireless resource allocation problem. Then, we propose an iteration algorithm to obtain the optimal wireless resource allocation. Simulation results indicate the performance of our method.**

Index Terms—**URLLC; real-time wireless control; co-design.**

I. INTRODUCTION

Ultra-reliable and low-latency communication (URLLC) has been identified as one of the most important communication scenarios for the coming *fifth generation* (5G) cellular networks. It is promising to enable real-time control in wireless networked control systems, where the extremely high *quality-of-service* (QoS) is required to guarantee the control performance [1]–[6]. There are huge amount of real-time wireless control scenarios, where URLLC is critical for them. For example, when the humanoid robot is carrying out a task in the fire scene, the remote control should maintain the sufficiently flexible moving, where the real-time control is needed. In summary, URLLC is very important for real-time wireless control systems.

In this paper, we intend to explore the the usage of URLLC in the real-time wireless control systems by communication-control co-design, where the communication performance and control performance are jointly considered. We notice that the communication QoS, i.e., the latency and reliability, has been sufficient discussed in wireless networked control systems [7]–[10], where most of them adopt simple assumptions. For example, the authors in [7] studied the effect of packet loss on the control cost caused by *transmission control protocol* (TCP) or *user datagram protocol* (UDP). The authors in [8] further discussed the effect of both time delay and packet loss on the control cost caused by *carrier sense multiple access/collision avoidance* (CSMA/CA). These researches indicate that the imperfect communications with time delay and packet loss result in control performance loss. However, the simple assumptions can not be used in URLLC scenarios since the resource should be guaranteed in real-time for URLLC. Furthermore, the close form expression is not obtained to show the effect of the constrained control performance on the URLLC performance. The rationale behind is that it is extremely difficult to directly evaluate the effect of the control on URLLC.

In this paper, we explore a co-design method to obtain the intrinsic relationship between the URLLC and control. Specifically, we propose optimization algorithms for communications subject to control constraints to achieve real-time control. Here, our goal is to maximize the uplink *spectral efficiency* (SE) by optimizing resource allocation while maintaining the control performance. To achieve the goal, we first study the effect of the control performance on the URLLC QoS[1] requirement. Then, we convert the hybrid communication-control co-design optimal problem into a regular optimal wireless resource allocation problem. An iteration algorithm is proposed to maximize the SE in URLLC for real-time wireless control systems, which allows us to obtain optimal resource allocation.

In the rest of this paper, we present the system model in Section II. In Section III, we formulate the optimal resource allocation problem with control performance constraint. In Section IV, we find a solution to the optimal problem. Simulations and conclusions are given in Section V and VI, respectively.

II. SYSTEM MODEL WITH LATENCY AND RELIABILITY

A. Wireless Communication Model

As shown in Fig. 1, we consider a typical centralized real-time wireless communication-control system, where we focus on the resource allocation for the uplink. Here, a remote controller is at the *base station* (BS) with coverage radius R and M plants are randomly distributed in the coverage of the BS. We furhter assume each plant is equipped with one sensor, which samples the plant state and uploads to the BS for control decision. We adopt *orthogonal frequency division multiplexing* (OFDM) system, where K subcarriers are available for the uplinks and each subcarrier occupies B_u Hz. We consider frequency domain division where subcarriers are orthogonally allocated to users. The variances of the *additive white gaussian noise* (AWGN) of the *plant-to-BS* (PB) links on each subcarrier are represented by N_0.

The transmitted data from the m-th sensor has finite length, i.e., λ_m bits payload information to be transmitted and is modulated into k_m subcarriers, where each plant is allocated to

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[1] In the rest of this paper, the QoS considered in this paper is time delay and reliability without notice.

k_m subcarriers under the criterion of selecting the best channel by the BS. Then, for the m-th plant, the received *signal-to-noise-ratio* (SNR) on the i-th subcarrier at the BS from plant m can be expressed as

$$\gamma_{m,i} = \frac{|h_{m,i}|^2 g_m P_0}{N_0 B_0}. \tag{1}$$

where $h_{m,i}$ is the small scale fading for the i-th subcarrier, g_m is the path-loss, p_0 is the allocated transmission power on each subcarrier, and B_0 is the bandwidth for each subcarrier.

Furthermore, we assume that C_m represents the Shannon capacity, which can be expressed as

$$C_m = \sum_{i=1}^{k_m} T_u B_0 \log\left(1 + \gamma_{m,i}\right). \tag{2}$$

where k_m is the number of allocated frequency band for the uplink of the m-th plant, andT_u is the allocated time resource of the uplink for the m-th plant.

Since finite block coding is used in URLLC, channel dispersion V_m is adopted to represent the capacity loss caused by the transmission error, which can be expressed as [6]

$$V_m = \sum_{i=1}^{k_m} T_u B_0 (\log e)^2 \left(1 - \frac{1}{(1+\gamma_{m,i}^2)}\right). \tag{3}$$

Then, the packet error probability can be expressed as [6]

$$\varepsilon_m = f_Q\left(\frac{C_m - \lambda_m + (\log k_m)/2}{\sqrt{V_m}}\right). \tag{4}$$

where $f_Q(\cdot)$ is the Q-function.

The above channel model consists of path-loss and small scale fading. According to [11], the path-loss g_m can be expressed as

$$g_{m_{[dB]}} = -128.1 - 37.6\,\lg(d_m), \tag{5}$$

where d_m is the distance between the m-th plant and the BS with unit km and is larger than 0.035 km. The small-scale fading $h_{m,i}$ follows Rayleigh distribution with mean zero and variance $\sigma_0^2 = 1$. In addition, small-scale fading is constant within coherence time, which is larger than the maximum *end-to-end* (E2E) time delay. Thus, we consider *quasi-static fading channel*, which is constant for each uplink subcarrier within a frame.

From (2), (3), and (4), we can obtain the SE η_m of the uplink for the m-th plant as

$$\eta_m = \frac{\lambda_m}{k_m}(1 - \varepsilon_m), \tag{6}$$

where the SE means that the successful decoding bits at the BS per subcarrier use. In this paper, we intend to obtain the uplink optimal wireless resource allocation by maximizing the SE in (6).

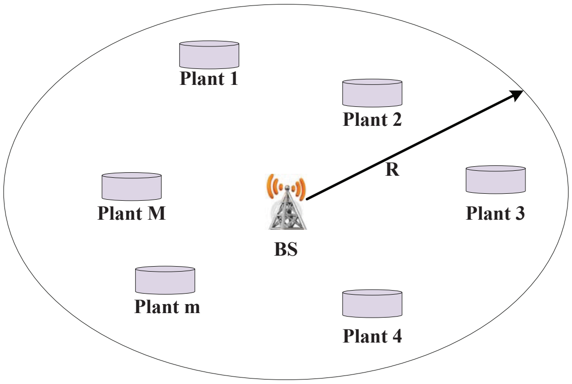

Fig. 1. Wireless communication system model.

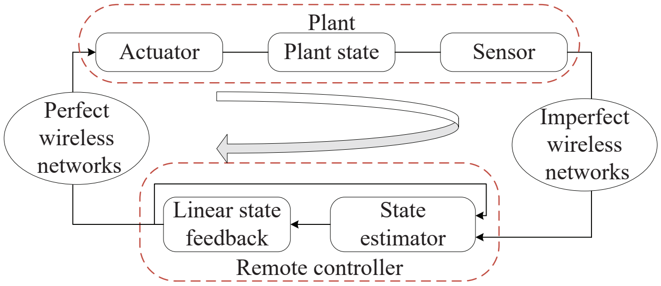

Fig. 2. Wireless control system model.

B. Wireless Control Model

In this subsection, we present the real-time wireless control model with communication time delay and reliability. As shown in Fig. 2, the control loop consists of sampling at the sensor, the current state estimation at the remote controller, linear state feedback at the controller, control input at the actuator, and the state update at the plant, periodically. Here, since we focus on the uplink design, we assume that the imperfect wireless network is adopted between the sensor and the controller, which means that the data may experience time delay and packet loss. Furthermore, perfect wireless network is adopted by the downlink from the BS to the plants.

To obtain the discrete time control model, we assume that $s_{m,n}$ represents the sample period between the last sample time $n-1$ and the current sample time n of the m-th plant at sample time index n, which consists of the wireless transmission time delay $d_{m,n}$ and an idle period $\bar{s}_{m,n}$ at sample time index n. Their relationship can be expressed as

$$s_{m,n} = \bar{s}_{m,n} + d_{m,n}, \tag{7}$$

where $n = 1, 2, \cdots, N$ represents the sampling time index in the control process.

In the control process, we assume that $u_{m,n}^c$ represents the linear state feedback parameter and is calculated once the sample data arrives at the remote controller. We further assume that $u_{m,n}^a$ represents the control input at the actuator and is executed as soon as the state feedback arrives at the actuator. We have $u_{m,n}^a = u_{m,n}^c \triangleq u_{m,n}$ since the communication from the BS to the actuator is assumed to be perfect. Furthermore, we assume that the transmission time delay and packet loss of the wireless communication are perfectly known at the remote controller. Then, the discrete time control model with time delay $d_{m,n}$ can be obtained as [9]

$$\mathbf{x}_{m,n+1}{=}\mathbf{\Omega}_{m,n}\mathbf{x}_{m,n}{+}\mathbf{\Phi}_0^{m,n}u_{m,n}{+}\mathbf{\Phi}_1^{m,n}u_{m,n-1}{+}\mathbf{n}_{m,n}, \tag{8}$$

where the discrete time control system parameters with transmission time delay can be expressed as $\mathbf{\Omega}_{m,n} = e^{\mathbf{A}_m s_{m,n}}$, $\mathbf{\Phi}_0^{m,n} = \left(\int_0^{\bar{s}_{m,n}} e^{\mathbf{A}_m t}dt\right) \cdot \mathbf{B}_m$, and $\mathbf{\Phi}_1^{m,n} = \left(\int_{\bar{s}_{m,n}}^{s_{m,n}} e^{\mathbf{A}_m t}dt\right) \cdot \mathbf{B}_m$, respectively. In addition, $\mathbf{x}_{m,n}$ is the state of the m-th plant at time index n, $u_{m,n}$ is the control input, and $\mathbf{n}_{m,n}$ is the disturbance caused by AWGN with zero mean and variance $\mathbf{R}_n$. Furthermore, $\mathbf{A}_m$ and $\mathbf{B}_m$ represent the continuous time system parameter matrices for the m-th plant.

Assuming $\xi_{m,n} = (\mathbf{x}_{m,n}^T \;\; u_{m,n-1})^T$ is the generalized state, then the control function in (8) can be rewritten as

$$\xi_{m,n+1} = \mathbf{\Omega}_{m,d}\xi_{m,n} + \mathbf{\Phi}_{m,d}u_{m,n} + \bar{\mathbf{n}}_{m,n}, \tag{9}$$

where $\bar{\mathbf{n}}_{m,n} = (\mathbf{n}_{m,n}^T \;\; 0)^T$ and $\mathbf{\Phi}_{m,d} = \begin{pmatrix} \mathbf{\Phi}_0^{m,n} \\ \mathbf{I} \end{pmatrix}$. We assume $\mathbf{\Omega}_{m,n} = \mathbf{\Omega}_m$. Then, we have $\mathbf{\Omega}_{m,d} = \begin{pmatrix} \mathbf{\Omega}_m & \mathbf{\Phi}_1^{m,n} \\ 0 & 0 \end{pmatrix}$.

Considering the packet loss, we have $\Pr\{\alpha_{m,n} = 1\} = \Pr\{\varepsilon_{m,n} \geq \varepsilon_{th}\}$ and $\Pr\{\alpha_{m,n} = 0\} = \Pr\{\varepsilon_{m,n} < \varepsilon_{th}\}$, where ”1” means that the packet is successfully transmitted and the control is under close loop, and ”0” means that the packet is lost and the control is under open loop. In addition, we assume that the state estimator is perfect, and then a linear feedback $u_{m,n} = \Theta_m \xi_{m,n}$ is used. Then, we have the close-loop system in (9) can be rewritten as

$$\xi_{m,n+1} = \begin{cases} (\mathbf{\Omega}_{m,d} + \mathbf{\Phi}_{m,d}\Theta_m)\xi_{m,n} + \bar{\mathbf{n}}_{m,n}, & \text{if } \alpha_{m,n} = 1 \\ \mathbf{\Omega}_{m,d}\xi_{m,n} + \bar{\mathbf{n}}_{m,n}, & \text{if } \alpha_{m,n} = 0, \end{cases} \tag{10}$$

which can be rewritten in a general way as

$$\xi_{m,n+1} = \begin{cases} \mathbf{\Omega}_{e_1}\xi_{m,n} + \bar{\mathbf{n}}_{m,n}, & \text{if } \alpha_{m,n} = 1 \\ \mathbf{\Omega}_{e_0}\xi_{m,n} + \bar{\mathbf{n}}_{m,n}, & \text{if } \alpha_{m,n} = 0, \end{cases} \tag{11}$$

where $\mathbf{\Omega}_{e_1} = \mathbf{\Omega}_{m,d} + \mathbf{\Phi}_{m,d}\Theta_m$ is the parameter of the control system with time delay when the packet is successfully transmitted, and $\mathbf{\Omega}_{e_0} = \mathbf{\Omega}_{m,d}$ is the parameter of the control system with time delay when the packet transmission is failed. Furthermore, the expression in (11) indicates that the proposed method in this paper can be extended into general control functions without considering the transmission time delay.

III. COMMUNICATION-CONTROL CO-DESIGN

Our goal is to maximize the communication SE and maintain the control performance by designing the communication coefficients. Thus, in the following of this section, we first discuss the constraints from the perspectives of communication and control, respectively. Then, we formulate the co-design problem based on the constraints.

A. Constraints

1) Communication Constraint: To guarantee the successful decoding at the BS, the received SNR at the BS should be maintained within a high regime, i.e.,

$$\gamma_{m,i} \geq \gamma_{th}. \tag{12}$$

In addition, the successful transmission probability can be expressed as

$$\Pr\{\alpha_{m,n} = 1\}{=}\Pr\{\gamma_{m,i} \geq \gamma_{th}\}{=}\, 1{-}\,\varepsilon_m \geq 1{-}\,\varepsilon_{th}, \tag{13}$$

and the failed transmission probability can be expressed as

$$\Pr\{\alpha_{m,n} = 0\} = \Pr\{\gamma_{m,i} < \gamma_{th}\} = \varepsilon_m \leq \varepsilon_{th}, \tag{14}$$

where ε_{th} is the packet error probability bounded by the URLLC QoS requirement. Furthermore, the communication time delay should also be bounded by the URLLC QoS requirement. Then, we have

$$T_u \leq T_{th}. \tag{15}$$

2) Control Constraint: To obtain the constraint from the control aspect, we consider Lyapunov-like function for each plant, which can be expressed as [12]

$$\Delta_m(\xi_m) = \xi_m^T \mathbf{Q}_m \xi_m, \tag{16}$$

where $\mathbf{Q}_m$ is a given positive definite matrix. The requirement for the Lyapunov-like function is that these functions should decrease at given rates $\rho_m < 1$ for the close loop during the control process. Note that small ρ_m means that the plant state updates smoothly, which can guarantee good control performance [12]. For any possible value of the current plant states $\xi_{m,n}$, the Lyapunov-like functions needs to satisfy

$$\mathbb{E}[V_m(\xi_{m,n+1})|\xi_{m,n}] \leq \rho_m \Delta_m(\xi_{m,n}) + Tr(\mathbf{Q}_m W_m) \tag{17}$$

where $\mathbb{E}[\cdot]$ represents the expectation operator.

B. Problem Formulation

In this subsection, we formulate the communication-control co-design problem. Here, we intend to maximize the communication SE with constraints on URLLC QoS requirements, channel coefficients, and control performance. The problem can be expressed as

$$\max_{k_m, p_0} \quad \eta = \sum_{m=1}^{M}\sum_{n=1}^{N} \eta_{m,n} \tag{18a}$$

s.t.

$$\varepsilon_{m,n} \approx f_Q\left(\frac{C_{m,n} - \lambda + \log(k_{m,n}T_u B_0)/2}{(\log e)\sqrt{k_{m,n}T_u B_0}}\right) \leq \varepsilon_{th}, \tag{18b}$$

$$\gamma_{m,n,i} \geq \gamma_{th}, \tag{18c}$$

$$T_u \leq T_{th}, \tag{18d}$$

$$\mathbb{E}[\Delta_{m,n}(\xi_{m,n+1})|\xi_{m,n}] \leq \rho_m \Delta_m(\xi_{m,n}) + Tr(\mathbf{Q}_m W_m). \tag{18e}$$

where we use the fact that the term $1/(1 + \frac{|h_{m_i}|^2 g_m P_0}{N_0 B_0})^2$ in (3) approximately equals to 0 when SNR is large enough.

IV. OPTIMAL RESOURCE ALLOCATION FOR THE PROPOSED CO-DESIGN

To solve the optimal problem in (18), it is critical to deal with the control performance constraint in (18e). Thus, in the following of this section, we first explore the relationship between the control and communication to find a method converting the control constraint (18e) into the communication performance constraint. Then, we discuss the solution for the problem (18) based on the relationship.

A. Relationship Between Control and Communication

From (11), we can obtain the relationship between the Lyapunov-like constraint and the communication reliability as [12]

$$\begin{aligned}\mathbb{E}[V_m(\xi_{m,n+1})|\xi_{m,n}] = \Pr\{\alpha_{m,n}=1\}\xi_{m,n}^T\mathbf{\Omega}_{e_1}^T\mathbf{Q}_m\mathbf{\Omega}_{e_1}\xi_{m,n}\\ + \Pr\{\alpha_{m,n}=0\}\xi_{m,n}^T\mathbf{\Omega}_{e_0}^T\mathbf{Q}_m\mathbf{\Omega}_{e_0}\xi_{m,n}\\ + Tr(\mathbf{Q}_m W_m).\end{aligned} \tag{19}$$

Then, for $\xi_{m,n} \neq 0$, submitting (19) into (18e), we can obtain

$$\Pr\{\alpha_{m,n}=1\} \geq \frac{\xi_{m,n}^T(\mathbf{\Omega}_{e_0}^T\mathbf{Q}_m\mathbf{\Omega}_{e_0} - \rho_m\mathbf{Q}_m)\xi_{m,n}}{\xi_{m,n}^T(\mathbf{\Omega}_{e_0}^T\mathbf{Q}_m\mathbf{\Omega}_{e_0} - \mathbf{\Omega}_{e_1}^T\mathbf{Q}_m\mathbf{\Omega}_{e_1})\xi_{m,n}}, \tag{20}$$

which means that the lower bound of the successful transmission probability can be obtained from the control performance. In other words, the upper bound of the control performance is bounded by the successful transmission probability.

Let

$$c_m = \sup_{y\neq 0}\frac{y^T(\mathbf{\Omega}_{e_0}^T\mathbf{Q}_m\mathbf{\Omega}_{e_0} - \rho_m\mathbf{Q}_m)y}{y^T(\mathbf{\Omega}_{e_0}^T\mathbf{Q}_m\mathbf{\Omega}_{e_0} - \mathbf{\Omega}_{e_1}^T\mathbf{Q}_m\mathbf{\Omega}_{e_1})y} \tag{21}$$

represent the supremum of the left-hand term in (20). The supremum c_m^* in (21) can be easily obtained by the method in [12]–[14].

Based on the above discussion, we can obtain the following theorem about the relationship between control and communication.

Theorem 1. *The communication reliability is effected by both the communication coefficients and the control performance. On the one hand, the URLLC transmission reliability should be no more than $1-\varepsilon_{th}$ according to the URLLC QoS. On the other hand, the reliability is constrained by the control performance c_m^*. In summary, the communication reliability can be obtained by*

$$\Pr\{\alpha_{m,k}=1\} \geq \max\{1-\varepsilon_{th}, c_m^*\}, \tag{22}$$

where $\max\{\cdot\}$ *means taking the maximum value operation.*

B. Optimal Resource Allocation

In this subsection, we solve the optimal problem in (18). First, we convert the optimal problem into a solvable problem. Then, we give an algorithm to obtain the solution for optimal resource allocation.

1) Problem Conversion: Since the optimal resource allocation is independent over time and is independent among M plants, we can drop the time indices k and decompose Problem (18) into M subproblems. In addition, the constraints in (18b) and Theorem 1 are both about the communication reliability, which can be jointly considered. Thus, (18) can be rewritten as

$$\max_{k_m,p_0} \quad \eta_m = \frac{\lambda_m}{k_m}(1-\varepsilon_m) \tag{23a}$$

s.t.

$$\varepsilon_m \leq \min\{\varepsilon_{th}, 1-c_m^*\}, \tag{23b}$$

$$\gamma_{m,i} \geq \gamma_{th}, \tag{23c}$$

$$T_u \leq T_{th}. \tag{23d}$$

Our goal is to maximize the wireless SE by optimal resource allocation, and meanwhile consume less resource. To achieve this goal in solve the problem in (23), the time delay should be long enough, i.e.,

$$T_u = T_{th}, \tag{24}$$

which is because large time domain resource can reduce the other resource consumption. In addition, once the optimal subcarrier allocation is obtained, the optimal power allocation can be obtain by

$$\begin{aligned}\varepsilon_m' &= f_Q\left(\frac{C_m - \lambda + \log(k_m T_{th} B_0)/2}{(\log e)\sqrt{k_m T_{th} B_0}}\right)\\ &= \min\{\varepsilon_{th}, 1-c_m^*\}\end{aligned} \tag{25}$$

to reduce the power consumption since ε_m' increases monotonously with p_0, where p_0 satisfing $\gamma_{m,i} = \frac{h_{m_i}^2 g_m P_0}{N_0 B_0} \geq \gamma_{th}$. Then, (23) can be rewritten as

$$\max_{k_m,p_0} \quad \frac{\lambda_m}{k_m p_0}(1-\varepsilon_m) \tag{26a}$$

s.t.

$$\varepsilon_m \leq \min\{\varepsilon_{th}, 1-c_m^*\}, \tag{26b}$$

$$\varepsilon_m' = \min\{\varepsilon_{th}, 1-c_m^*\}, \tag{26c}$$

$$\gamma_{m,i} \geq \gamma_{th}, \tag{26d}$$

This is the final expression of the problem formulation. Next, we focus on the solution to this problem.

2) Problem Solution: Here, we intend to solve the problem (26) by iteration method. From (18b), we can obtain that ε_m decreases monotonously with the transmission power p_0. Then, if we can select a optimal k_m^*, the corresponding p_0^* is unique.

Given p_0, we can obtain that $\lim\limits_{k_m\to 0^+}(\frac{\lambda_m}{k_m}(1-\varepsilon_m)) = 0$ and $\lim\limits_{k_m\to +\infty}(\frac{\lambda_m}{k_m}(1-\varepsilon_m)) = 0$. In addition, $(\frac{\lambda_m}{k_m}(\varepsilon_m - 1)) > 0$ when $k_m \in (0,+\infty)$. Thus, there is an optimal k_m^* for p_0, which can be obtained by heuristic algorithm [15]. Finally, we propose an iterative method to find the optimal m^* and p_0^* to maximize the SE, which is summarized in the following Algorithm 1.

Algorithm 1 The proposed algorithm to obtain optimal resource allocation to maximize SE.

```
Input: $\varepsilon_{th}$, $c_m^*$, $B_0$, $T_{th}$, $\gamma_{th}$, $\lambda_m$, the initial transmission
    power $p_{m,0} = \arg\max_i \left( \frac{\gamma_{th} N_0 B_0}{|h_{m,i}|^2 g_m} \right)$
1: Using the heuristic algorithm in [14] to obtain $k_m^*$ for
   given $p_0$
2: while $\varepsilon_m \geq \min\{\varepsilon_{th}, 1 - c_m^*\}$ or $\frac{|h_{m,i}|^2 g_m P_{m,0}}{N_0 B_0} < \gamma_{th}$ do
3:     Obtaining new $p_0$ by $\varepsilon_m' = \min\{\varepsilon_{th}, 1 - c_m^*\}$
4:     Repeating Step 1
5: end while
Output: $m^*$ and $p_0^*$.
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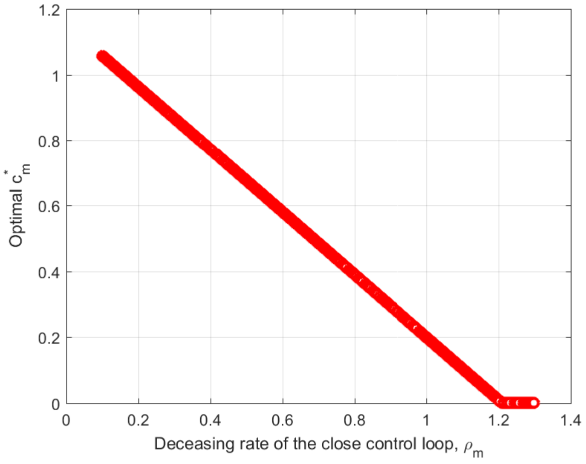

Fig. 3. The optimal c_m^* in (21) with different decreasing rate ρ_m.

V. NUMERICAL SIMULATIONS

In this section, we provide simulation results to demonstrate the performance of the proposed method, where the communication system model is the same as shown in Fig. 1 and the control system model is the same as shown in Fig. 2. From the perspective of wireless communication, we assume that the bandwidth of each subcarrier is 1 kHz, the single-sided noise spectral density is -174 dBm/Hz. For the URLLC, the maximum packet transmission error probability is 10^{-5}, and the maximum transmission time delay is $T_{th} = 0.5$ ms for the uplink from the plant to the BS. From the perspective of wireless control, since M plants are independent in communication-control co-design, we assume that $M = 2$ is considered in the simulations. For simplicity, we assume that both plants have the identical dynamics parameters. Here, we adopt $\Omega_{e_0} = 1.1$ and $\Omega_{e_1} = 0.4$.

Fig. 3 shows the relationship between the optimal lower bound of the effect of the control performance constraint on the communication reliability[2] c_m^* in (21) when the control decreasing rate ρ_m is different and the payload information is $\lambda_m = 100$ bits in each packet. From the figure, the curve decreases monotonously with ρ_m until $c_m^* = 0$. This

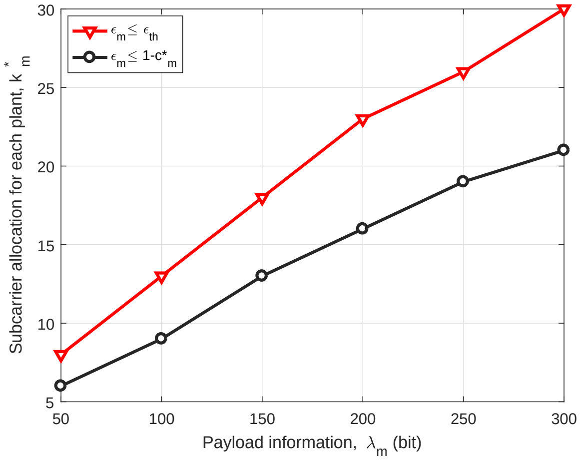

Fig. 4. The optimal subcarrier allocation with different payload information λ_m.

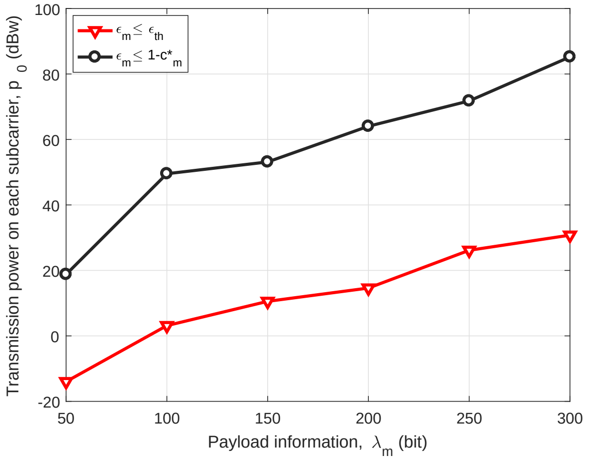

Fig. 5. The optimal power allocation with different payload information λ_m.

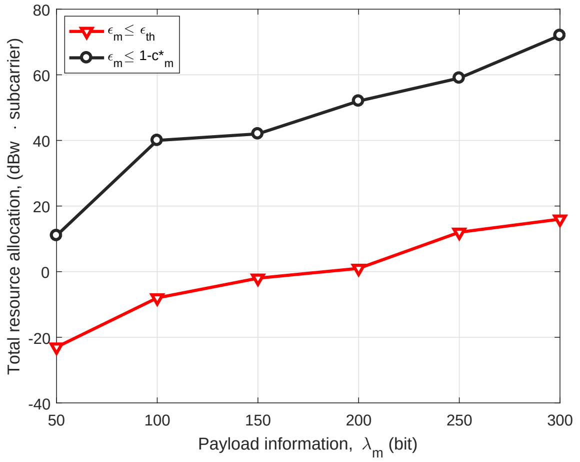

Fig. 6. The total resource allocation with different payload information λ_m.

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[2] Here, it is also the upper bound of the effect of the communication reliability constraint on the control performance.

means that from the control perspective, small decreasing rate ρ_m requires high reliability bound c_m^*. In other words, good control performance needs small decreasing rate ρ_m, which further leads to high reliability bound c_m^*. In summary, high control performance requires high communication reliability. We further assume that $\varepsilon_{th} = 1 - c_{th}^*(\rho_{th})$. Then, the effect of the control performance constraint on the optimal wireless resource allocation can be divided into two parts, i.e., $\varepsilon_m \le \varepsilon_{th}$ when $\rho_m \ge \rho_{th}$ and $\varepsilon_m \le 1 - c_m^*$ when $\rho_m < \rho_{th}$. Thus, in the following simulations, two ρ_m values are adopted. We assume that $\rho_{m,1} \ge \rho_{th}$ leading to that $\varepsilon_m \le \varepsilon_{th}$, and $\rho_{m,2} < \rho_{th}$ leading to that $\varepsilon_m \le 1 - c_m^*$, where we let $(1 - c_m^*) = 10^{-9}$.

Fig. 4, Fig. 5 and Fig. 6 demonstrate the resource allocation when payload information λ_m is different. From Fig. 4 and Fig. 5, both the allocated subcarriers and allocated transmission power increase monotonously with λ_m, which means that more payload information needs more resources for transmission. Furthermore, from Fig. 4, when $\varepsilon_m \le 1 - c_m^*$, the allocated subcarriers is more than that when $\varepsilon_m \le \varepsilon_{th}$, which means that more frequency resource is needed for higher control performance. However, from Fig. 5, when $\varepsilon_m \le 1 - c_m^*$, the allocated transmission power on each subcarrier is less than that when $\varepsilon_m \le \varepsilon_{th}$. This is reasonable since more frequency resource leads to less power consumption on each subcarrier. Fig. 6 shows the total resource allocation. From the figure, the total resource allocation when $\varepsilon_m \le 1 - c_m^*$ is more than that when $\varepsilon_m \le \varepsilon_{th}$. In summary, the higher control performance needs more communication resources.

VI. CONCLUSIONS

In this paper, we proposed an optimal resource allocation of URLLC when the control performance is constrained in real-time wireless control systems. In our method, the control performance constraint was converted into a constraint on the wireless communication reliability. Then, the difficult hybrid optimal problem can be replaced by a regular wireless resource allocation problem. To solve the problem, we developed an iteration algorithm based on heuristic method, which allows us to obtain the optimal resource allocation. The simulation results showed that the proposed method achieves the maximum spectral efficiency while maintaining the control performance.

REFERENCES

[1] B. Chang, “QoS-constrained area spectral efficiency in ultra-reliable and low-latency industrial wireless networks,” *IEEE Asia-Pacific Conf. Commu. (APCC)*, Dec. 2017, pp. 1-4.

[2] C. She, Ch. Yang, and T. Q. S. Quek, “Uplink transmission design with massive machine type devices in tactile internet,” *IEEE Globecom Workshops (GC Wkshps)*, Dec. 2016, pp. 1-6.

[3] C. She, C. Yang, and T. Quek, “Cross-layer optimization for ultra-reliable and low-latency radio access networks,” *IEEE Trans. Wireless Commun.*, vol. 17, no. 1, pp. 127-141, Jan. 2018.

[4] C. She, C. Yang, and T. Quek, “Radio resource management for ultra-reliable and low-latency communications,” *IEEE Commun. Mag.*, vol. 55, no. 6, pp. 72-78, Jun. 2017.

[5] G. Durisi, T. Koch, and P. Popovski,“Toward massive, ultrareliable, and low-latency wireless communication with short packets,” *IEEE Proc.*, vol.104, no.9, pp. 1711-1726, Aug. 2016.

[6] Y. Polyanskiy, H. V. Poor, and S. Verdu, “Channel coding rate in the finite blocklength regime,” *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2307-2359, May 2010.

[7] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poola, and S. Sastry, “Foundations of control and estimation over lossy networks,” *IEEE Proc.*, vol. 95, no. 1, pp. 163-187, Jan. 2007.

[8] P. Park, J. Araújo, and K. H. Johansson, “Wireless networked control system co-design,” *IEEE Inter. Conf. Networking, Sensing and Control (ICNSC)*, 2011, pp.486-491.

[9] P. Park, S. Ergen, C. Fischione, C. Lu, and K. Johansson, “Wireless network design for control systems: a survey,” *IEEE Commun. Surveys & Tutorials*, pp. 1-1, Dec. 2017.

[10] A. Cetinkaya, H. Ishii, and T. Hayakawa, “Networked control under random and malicious packet losses,” *IEEE Trans. Automatic Control*, vol. 62, no. 5, pp. 2434-2449, Sep. 2017.

[11] 3GPP, *Study on Scenarios and Requirements for Next Generation Access Technologies*. Technical Specification Group Radio Access Network, Technical Report 38.913, Release 14, Oct. 2016.

[12] K. Gatsis, M. Pajic, A. Ribeiro, and G. J. Pappas, “Opportunistic control over shared wireless channels,” *IEEE Trans. Automatic Control*, vol. 60, no. 12, pp. 3140-3155, Dec. 2015.

[13] S.Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[14] S. Ruscheweyh and T. Small, *Comment. Math. Helv.*, 1973.

[15] E. Chong and S. Żak, *An Introduction to Optimization (4th Edition)*, New York: Wiley, 2001.