VOL. 12, NO. 11, JUNE 2017 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

CALCULATION MODEL AND METHODOLOGY FOR STIFFNESS EVALUATION IN HYDRAULICS CYLINDERS

V. Gómez Rodríguez[1], Juan José Cabello Eras[2], Noel Varela Izquierdo[2], Hernán Hernández Herrera[2] and Alexis Sagastume Gutierrez[2]

[1]Department of Commercial Education and Sciences, Bolivariano Technological Institute of Technology, Guayaquil, Ecuador
[2]Faculty of Engineering, Universidad de la Costa.Barranquilla, Colombia
E-Mail: vgomez@bolivariano.edu.ec

ABSTRACT

In the paper an analysis scheme able to considerate of all the main geometric and load factors that affect the behavior of a hydraulic cylinder in its action as a flexural-compression member. Also, experimental validations of the model are shown too. A methodology to identify the conditions leading to the instability of the cylinder is established.

Keywords: hydraulic cylinder, stability analysis, experimental validation.

1. INTRODUCTION

Hydraulic cylinders are compression members consisting of parts having different rigidity. In the simplest case there are two parts Figure-1, but in more complicated cases, such as telescopic cylinders, there are several parts [1], [2], [3]. A number of different combinations of support conditions at the ends are encountered for these members. In the simplest case there are pinned supports at both ends, while in other cases the supports may be pinned-fixed, doubly-fixed, or fixed-free. In exceptional conditions, hydraulic cylinders operate solely in the vertical condition, but more commonly they operate in an inclined position or in a completely horizontal position. In the latter position the self-weight of the cylinder and the hydraulic fluid often provide a transverse load that increases the eccentricity of axial loads. Further factors requiring consideration are the sliding joint between the rod and cylinder, and the looseness in the joints, implying elasticity in the connections or supports. The components forming the hydraulic cylinder have step-variations in their stiffness, and thus the equation governing the deflection is discontinuous over the domain, which complicates the analysis.

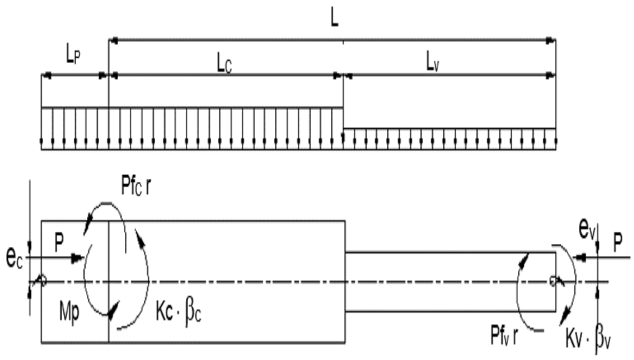
In the technical literature a number of approaches for the analysis of compression member have been presented [4]-[9], but none has completely addressed the full details that are encountered with practical hydraulic cylinders. Industrial manufacturers produce a variety of hydraulic cylinder products, and generally carry out complete designs for their products. Nevertheless, there is no available methodology, appropriate for a comprehensive analysis of hydraulic cylinders, which accounts for both transverse and axial loads. Thus a comprehensive scheme of calculation is presented herein, which addresses completely the various loading and support conditions that may arise, and which ultimately leads to a safe product having economical dimensions.

The classical method of stability analysis of a compression member is described by Timoshenko [10]; other methods exist which offer a more detailed consideration of the problem. Most of the methods cover only compression members having constant rigidity, and thus cannot be applied directly to hydraulic cylinders.

With the development of computer-aided methods and the availability of new mathematical approaches, attempts have been made to overcome difficulties posed by equations governing domains containing step-variations in properties [11]. The model proposed by [12] considered the cylinder as a beam with step variations in rigidity, subjected to perfect loading (no eccentricities), and lacking of initial curvature. In this approach the possible influence of a loose fit in the joints is not considered, nor the effect of self-weight in the case when the cylinder adopts a position at an angle to the vertical.

In another works [13],[14] a method is used which permits determination of stability characteristics for cylinder of any number of stages. The results obtained in that works are superior to results obtained in other works. A deficiency in previous methods stems from the fact that the self-weight of the cylinder is not considered, despite the fact that it has a substantial influence on the deformation of the rod in an articulated system. The influence is due to the bending caused by the self-weight, and to the sagging moment produced by the loose fit of the sliding joint in each stage. In a recent works [15], [16], the finite element method has been used, and empirical results have also been presented to predict the stability of hydraulic cylinders.

VOL. 12, NO. 11, JUNE 2017 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences



$\frac{P(e_v-e_c)}{L}$	$-\frac{P(e_v-e_c)}{L}$
$-\frac{P(f_v-f_c)r}{L}$	$\frac{P(f_v-f_c)r}{L}$
$\frac{K_c\beta_c-K_v\beta_v)}{L}$	$-\frac{K_c\beta_c-K_v\beta_v)}{L}$
$\frac{M_p}{L}$	$-\frac{M_p}{L}$
$\frac{1}{L}\left[W_1 I_c\left(I_v+\frac{I_c}{2}\right)+\frac{W_2 I_v^2}{2}\right]$	$-\frac{1}{L}\left[W_1 I_c\left(I_v+\frac{I_c}{2}\right)+\frac{W_2 I_v^2}{2}\right]$
$\frac{WI_v}{L}$	$\frac{WI_c}{L}$

Figure-1. Hydraulic cylinder - geometry, reactions and loads.

2. METHODOLOGY

2.1 Calculation scheme

A single stage hydraulic cylinder may be modelled as a compression member consisting of a tube (part left of C in Figure-1 and solid rod (part right of C in Figure-1). An eccentric axial load P acts on the member. As well, transverse loads arise due to gravity effects on the components and fluid (Figure-1). Following the calculation scheme presented in detail in the first article of this study (Gomez *et al*. [16]), the equation for the flexural moment in the tube is given by:

$$M_1=\frac{P(e_v-e_c)}{L}z-\frac{P(f_v-f_c)}{L}z+\frac{(k_c\beta_c-k_v\beta_v)}{L}+R_c z-M_p-P\cdot f_c-k_c\cdot\beta_c-\frac{w_1}{2}z^2+P(e_c+y_1) \quad (1)$$

Where:

- y_1- Is the deflection of the axis of the tube at the axial distance z from the origin.
- L- Is the length.
- k- Factors are rotational stiffnesses, the e factors are eccentricities.
- W_1- is a transverse load effect.

Similarly the equation for the moment in the rod is given by:

$$M_2=-\frac{P(e_v-e_c)}{L}(L-z)+\frac{P(f_v-f_c)}{L}(L-z)-\frac{[k_c\beta_c-k_v\beta_v]}{L}(L-z)+R_v(L-z)-Pf_v-K_v\beta_v-\frac{W_2}{2}(L-z)^2+P\left(e_v+y_2\right) \quad (2)$$

Where:

- y_2- Is the deflection of the axis of the rod at the axial distance z from the origin.

The governing equations for the displacement in the Euler-Bernoulli theory of beams in the tube and rod become

Where:

$$\frac{d^2y_1}{dz^2}+k_1^2y_1=k_1^2\left[-\frac{(e_v-e_c)}{L}z+\frac{(f_v-f_c)}{L}z-\frac{(k_c\beta_c-k_v\beta_v)}{PL}z-\frac{R_c}{P}z+\frac{M_p}{P}+f_c+\frac{k_c\beta_c}{L}-e_c+\frac{w_1}{2P}z^2\right] \quad (3)$$

$$\frac{d^2y_2}{dz^2}+k_2^2y_2=k_2^2\left[-\frac{(e_v-e_c)}{L}(L-z)+\frac{(f_v-f_c)}{L}(L-z)-\frac{(k_c\beta_c-k_v\beta_v)}{PL}(L-z)-\frac{R_v}{P}(L-z)+f_v+\frac{k_v\beta_v}{P}-e_v+\frac{w_2}{2P}(L-z)^2\right] \quad (4)$$

Where:

$$K_1^2=\frac{P}{E_1I_1}K_2^2=\frac{P}{E_2I_2}$$

The solutions to the equations for the deflections are:

$$y_1=C_1\cos(K_1\cdot z)+D_1\sin(K_1\cdot z)-\frac{(e_v-e_c)}{L}z+\frac{(f_v-f_c)}{L}z-\frac{(k_c\beta_c-k_v\beta_v)}{PL}z-\frac{R_c}{P}z+T_1+\frac{k_c\beta_c}{P}+\frac{w_1}{2P}z^2(0\le z\le l_c) \quad (5)$$

And

$$y_2=C_2\cos(K_2\cdot z)+D_2\sin(K_2\cdot z)-\frac{(e_v-e_c)}{L}(L-z)+\frac{(f_v-f_c)}{L}(L-z)-\frac{(k_c\beta_c-k_v\beta_v)}{PL}(L-z)-\frac{R_v}{P}(L-z)+T_2+\frac{k_v\beta_v}{P}+\frac{W_2}{2P}(L-z)^2$$

$$(l_c\le z\le L \quad (6)$$

The moments for T_1 and T_2 and given by:

$$T_1=\frac{M_p}{P}+f_c-e_c-\frac{W_1}{PK_1^2} \quad (7)$$

$$T_2=f_v-e_v-\frac{W_2}{PK_2^2} \quad (8)$$

The inclinations of the elastic curve in the tube and rod are:

ARPN Journal of Engineering and Applied Sciences

$$\frac{dy_1}{dz} = -C_1K_1\sin(K_1 \cdot z) + D_1K_1\cos(K_1 \cdot z) - \frac{(e_v - e_c)}{L} + \frac{(f_v - f_c)}{L} - \frac{(k_c\beta_c - k_v\beta_v)}{PL} - \frac{R_c}{P} + \frac{w_1}{2P}z \quad (9)$$

$$(0 \le z \le l_c)$$

and

$$\frac{dy_2}{dz} = -C_2K_2\sin(K_2 \cdot z) + D_2K_2\cos(K_2 \cdot z) - \frac{(e_v - e_c)}{L} + \frac{(f_v - f_c)}{L} - \frac{(k_c\beta_c - k_v\beta_v)}{PL} + \frac{R_v}{P} - \frac{w_2}{2P}(L - z)(l_c \le z \le L) \quad (10)$$

With the application of the conditions of compatibility and of the satisfying of the various equations cited in the preceding it is possible to determine the various coefficients inherent in the flexural, slope and moment equations. Full details are provided by Gomez *et al.*[16]. Evaluation of the mechanical quantities can then be made at any point in either part of the cylinder.

To provide experimental validation for the theoretical model a hydraulic cylinder was fabricated and tested Figure-2. The properties were as follows: tube diameter = 100 mm; rod diameter = 40 mm; maximum rod extension = 200 mm; working pressure = 12 MPa; test pressure = 18 MPa; axial force = 96 kN; applied force = 79 kN.

The seals used in the cylinder permit a radial tolerance up to 2 mm. In view of the nominal dimensions and the upper and lower limits for the tolerances of these components, the maximum and minimum diametral tolerances between the body and the piston are respectively 2.22 and 0.55 mm. The experimental work was carried out with the cylinder in a horizontal position, with a hinged - hinged installation, and completely extended. Measurements were carried out for three values of pressure; 12, 18, and 22 MPa. Three sets of samples were taken at each pressure level, and the average taken. The results are presented in Table-1.

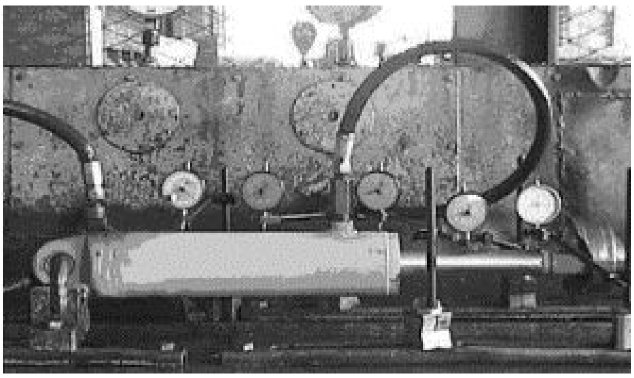


Figure-2. Experimental apparatus for cylinder test.

In the validation of the model, it was assumed that there was no eccentricity in the axial load. The effect of self-weight of the cylinder and hydraulic fluid were considered. Following recommendations given in the literature three values were assumed for the initial angle of inclination β; 0, 0.05, and 0.1. In Table-1 the average difference between the experimental and theoretical values is 9.21%.

Several factors were not considered in the validation that could have reduced the size of the average difference. With the assumption of ideal conditions, β = 0, i.e. zero initial slope, the average error was 10.6%. On the other hand for assumptions of initial angle of inclination of β = 0.05, 0.01, the average differences were 9.8% and 7.3%.

Table-1. Comparison of theoretical and experimental displacements (P is the assumed angle of initial slope).

Pressure = 12MPa					
	Distance from support to measurement (mm)				
	115	215	300	395	505
β=0	20.6	62.0	81.1	83.8	35.5
β =0.05	21.2	63.3	84.5	90.8	30.0
β =0.1	24.0	64.1	87.0	79.0	28.5
Expt.	23.5	67.5	86.0	72.5	36.0

Pressure = 18 MPa					
	Distance from support to measurement (mm)				
	115	215	300	395	505
β =0	34.1	83.6	92.1	82.0	39.2
β =0.05	45.2	85.2	93.7	76.0	49.7
β =0.1	36.5	85.9	94.2	79.0	53.9
Expt.	40.7	89.0	101	85.0	50.3

Pressure = 22 MPa					
	Distance from support to measurement (mm)				
	115	215	300	395	505
β =0	59.1	101.9	130	109	69.0
β =0.05	68.5	106	139	111	65.0
β =0.1	63.7	118	125	112	75.0
Expt.	71.0	111	130	117	70.8

2.2. Initiation of loss of stability

Using the expression (2) for the flexural moment in the rod, one can pose the condition for the initiation of loss of stability of the hydraulic cylinder. This condition is characterized by the appearance of plastic deformations at the positions of maximum normal stress at the critical section of the rod, which are at the extreme fibres. The stresses there are then given by:

$$\sigma_{maxv} = \sigma_{maxcv} + \sigma_{maxfv} = \sigma_f \quad (11)$$

Where

σ_{maxcv} = compressive stress in the rod due to the axial force P.

σ_{maxfv} = flexural stress in rod due to flexural load.

ARPN Journal of Engineering and Applied Sciences

There is a deviation from the condition that the limit in compression is equal to the limit in tension for the materials used for the rod, and the loss of stability in the hydraulic cylinder occurs in the rod part.

The maximum stress in the rod (for the case of a massive rod) can be calculated using the expression:

$$\sigma_{maxv} = \frac{P}{A} + \frac{M_{fmaxv}}{W_x} = \sigma_f \quad (12)$$

Where A= $\pi d_v^2/4$ (cross-sectional area of rod).
W_x = = $\pi d_v^3/32$ (flexural modulus of the rod).

Then the maximum stress becomes:

$$\sigma_{maxv} = \frac{4 \cdot P}{\pi \cdot d_v^2} + \frac{32 \cdot M_{fmaxv}}{\pi \cdot d_v^3} = \sigma_f \quad (13)$$

Where
d_v = cross-sectional diameter of the rod,
P = axial load on the cylinder.
σ_f = plastic flow stress for the rod material.

Evaluating the expression (13) one can find the value of the axial load P at which the plastic deformations begin in the rod, and thus one can find the load for instability initiation.

3. RESULTS AND DISCUSSIONS

The fact that plastic deformations commence in the rod does not signify at all a complete loss of stability in the hydraulic cylinder. The normal stresses due to flexure are distributed in the transverse section of the rod, in a manner proportional to the the distance from the neutral axis, and the start of the plastic deformations signifies solely that the maximum stresses in the section reach, locally, the capacity of the material Figure-3 a.

From this loading level onward, owing to the increase of the flexural moment, the plastic stresses expand within the section, proceeding from the extreme fibres towards the neutral axis Figure-3b. There is complete loss of stability when the plastic stresses reach completely across the cross-section Figure-3c. Subsequently, the deformations of the rod continue to increase without increase in load.

When the critical load is reached a plastic hinge is formed at the critical cross-section, which can carry only a flexural moment equal to the flexural capacity of the cross-section. This moment capacity is given by:

$$M_{lim} = \sigma_f \cdot W_{pl} \quad (14)$$

Where M_{lim} is the plastic moment capacity of the section. For a circular section one has:

$$W_{pi} = 1.7 \cdot W_x = \frac{1.7 \cdot i \cdot d_v^3}{32} = 0.167 \cdot d_v^3 \quad (15)$$

At the complete loss of stability the stress is given by

$$\sigma_{max} = \frac{P}{A} + \frac{M_{fmaxv}}{W_{Pl}} \quad (16)$$

From this expression is obtained:

$$\sigma_{max} = \frac{4 \cdot P}{\pi \cdot d_v^2} + \frac{6 \cdot M_{fmaxv}}{d_v^3} \quad (17)$$

This latter expression is used to obtain the value of the axial load P at which there is complete loss of stability.

In the proposed method of analysis an initial trial axial load P is considered to be applied to the hydraulic cylinder, and the equation of the deflected curve is determined. The slopes and the flexural moment in this solution are determined, and the analysis is continued up to the determination of the condition for the initial, and then the complete loss of stability.

If the condition (17) is not satisfied by the given load P, an increase P is made to the load, and the entire analysis is repeated at this higher load level. The process is continued until the failure state is reached.

In the stability calculations, it is often assumed that the hydraulic cylinder acts as a column axially loaded, with a diameter equal to the diameter of the rod, and with a length (L_c) equal to the total length of a hydraulic cylinder in the most extended condition Figure-4a. Alternatively, it may be assumed that the hydraulic cylinder acts as a column, fixed at one end, and hinged at the other Figure-4b. The diameter is taken equal to the diameter of the rod, and the length equal to the length of the completely extended rod ($L_c/2$). In some industries the first scheme is used in the calculations, verifying the rigidity of the hydraulic cylinder at the design stage.

ARPN Journal of Engineering and Applied Sciences

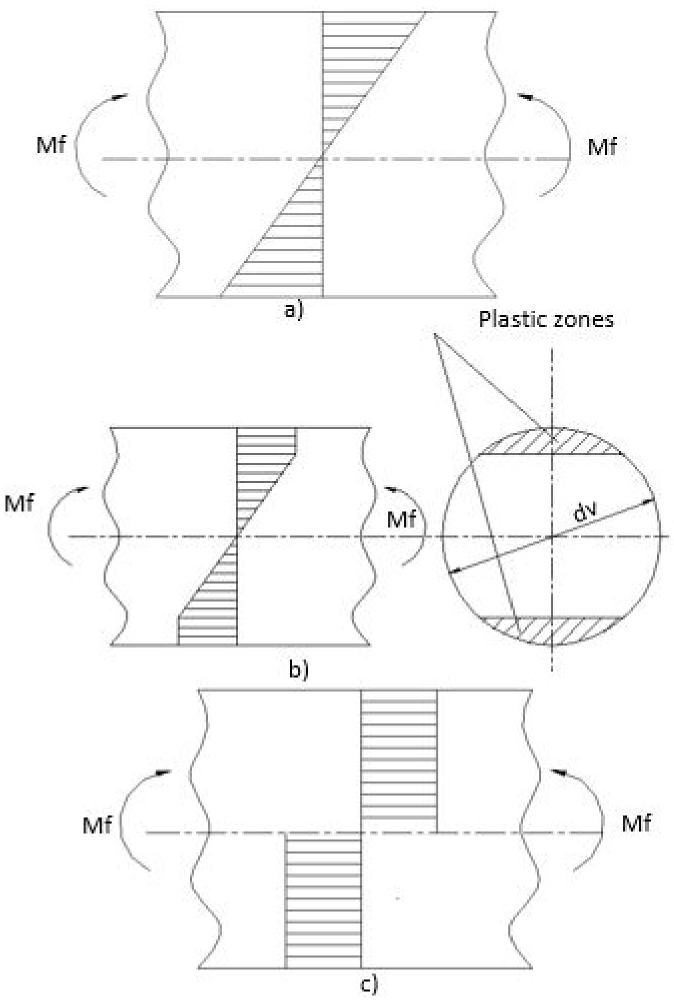


Figure-3. Growth of plastic zone in rod cross-section.

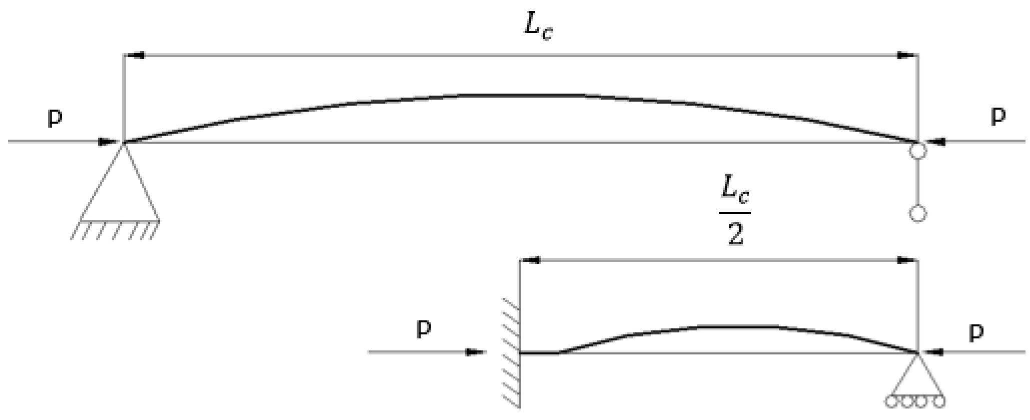


Figure-4. Idealization of columns; top: hinged - hinged, bottom: fixed - hinged.

For a cylinder with a total length Lc = 2000 mm, and a diameter dv = 25 mm, with the hinged - hinged idealization of Figure-4a, one obtains a critical load using the classical Euler formula as:

$$P_{crít} = \frac{\pi^2 \cdot E \cdot I_{min}}{L^2} = 963{,}82\ kgf \quad (18)$$

Using the fixed - hinge idealization of Figure 4b the value of the critical load is

P_{crit} = 5782.97 kgf

Using the expression (17) and the other equations of the method for the displacements and moments, the critical load at the initiation of the loss of equilibrium, assuming a rod material with σ_f = 3600 kgf/cm^2, isPcrit= 1947 kgf From the preceding analysis it is concluded that the values for the limit forces, recommended by the industrial manufacturers, are below the limit values obtained by the proposed method. The actual value of the supported load by this cylinder is about 1.5 times (i.e. above) the value presently recommended. On the other hand, those industrial manufacturers that use, in the determination of the critical load the fixed-hinged scheme, propose values equal to double those obtained by the method proposed in the current study. Thus, for those cylinders the loss of total stability will occur at a value far below the value of the limit load presently recommended.

4. CONCLUSIONS

A proposal for a new scheme for the evaluation of the stability condition for a single stage hydraulic cylinder has been presented. The proposed scheme is more comprehensive than those currently in use. Experimental results obtained demonstrate the validity of the proposed approach, with theoretical and experimental results differing by an average value less than 10%.

REFERENCES

[1] G. Seebacher, H. Winkler. Jul. 2014. Evaluating flexibility in discrete manufacturing based on performance and efficiency. Int. J. Production Economics. 153: 340-351.

[2] A. Trąbka. Nov. 2014.Dynamics of telescopic cranes with flexible structural components. International Journal of Mechanical Sciences. 88: 162-174.

[3] B, Posiadala, D. Cekus. Oct. 2011. Vibration model and analysis of three-member telescopic boom with hydraulic cylinder for its radius change. International Journal of Bifurcation and Chaos. 21: 2883-2892.

[4] H.G. Puhl. 1975. Static analysis and experimentation of elastic hydraulic cylinders. Forschungsheft. 34: 237-244.

[5] W.P. Dawkins, S.K.R. Iyengar. October 1975. Stress Analysis of Hydraulic Cylinders. National Conference on Fluid Power, Chicago, NCFP Proc.Vol. 29.

[6] C. H. Yoo, C. R. Siegel. Feb 2003. Column loadings on telescopic power cylinders. Computers & structures. 22(3): 245-251.

[7] S. Baragetti, A. Terranova. May. 2001. Bending behaviour of double-acting hydraulic actuator’s. Proc. of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science. 215(5): 607-619.

[8] T. Nguyen, K. H. Tan. Oct. 2014. Thermal-induced restraint forces in reinforced concrete columns subjected to eccentric loads. Fire Safety Journal. 69: 136-146.

[9] M. R. Sheidaii, R. Bahraminejad. Mar. 2012. Evaluation of compression member buckling and post-buckling behavior using artificial neural network. Journal of Constructional Steel Research. 70: 71-77.

[10] S.P. Timoshenko. 1983. History of Strength of Materials, Dover, New York, USA.

[11] O. Navarro. Jul 2001. Stability of Telescopic Hydraulic Cylinders. M. Eng. Thesis, University of Cienfuegos.

[12] M. Torres del Toro. May 1999. Application of Differential Equations to the Analysis of the Stability of Telescopic Hydraulic Cylinders. M. Eng. Thesis, University of Cienfuegos.

[13] H. Hernandez Herrera. Jul 2000. Stability Analysis of Hydraulic Telescopic Cylinders of Several Stages in a Vertical Position. M. Eng. Thesis, University of Cienfuegos.

[14] V. Gómez Rodríguez, J.J. Cabello Eras, H. Hernández Herrera, R. Goytisolo Espinosa, Dec 2016-Jan 2017. Static Analysis of a Single Stage Hydraulic Cylinder. International Journal of Engineering and Technology (IJET). 8(6): 2443-2451.

[15] E. Salazar, M. Khamashtha, E. Codina, E., Viedma. 2006. Study of Hydraulic Cylinders Load Capacity. Proc. 3rd Int. Conf. on Advances in Mechanical Engineering and Mechanics (ICAMEM 2006), Hammamet, Tunisia. p. 7.

[16] L.Zhua, S. Shuang, Xi.Chenga, Y.Qia, J.RuGea, Cheng. Long Yina, T. Chien, November. 2016. Combined finite element and multi-body dynamics analysis of effects of hydraulic cylinder movement on ploughshare of Horizontally Reversible Plough. 163: 168-175.

[17] V. Gomez. J.J. Cabello, R. Goytisolo, O. Redekop. 2006. Static Analysis of a Single Stage Hydraulic Cylinder. Proc. 3rd Int. Conf. on Advances in Mechanical Engineering and Mechanics, (ICAMEM 2006), Hammamet, Tunisia. p. 6.