

# MATHEMATICAL MODELING OF VERTICAL MOVEMENT OF THE UPPER SHAFT IN CANE MILLS

<sup>1</sup>MARIO JAVIER CABELLO ULLOA. <sup>2</sup>JUAN JOSÉ CABELLO ERAS.  
<sup>3</sup>JORGE MOYA RODRIGUEZ <sup>2</sup>ALEXIS SAGASTUME GUTIERREZ.  
<sup>2</sup>HERNÁN HERNÁNDEZ HERRERA

<sup>1</sup> Centro Tecnológico Vasco. IK4-Ikerlan

<sup>2</sup> Universidad de la Costa CUC. Facultad de Ingeniería. Barranquilla. Atlántico. Colombia.

<sup>3</sup> Instituto Tecnológico Galileo. Manaus. Amazonas, Brasil.

Email: <sup>1</sup>mjaviercabello@gmail.com<sup>2</sup>jcabello2@cuc.edu.co<sup>3</sup>jorgemoyar@gmail.com<sup>4</sup>,  
 asagatu1@cuc.edu.co<sup>4</sup> hhernand16@cuc.edu.co<sup>5</sup>

## ABSTRACT

This study introduces a mathematical model to describe the floating effects of the top roll in sugarcane mills. The model is developed from experimental data measured during the operation of a sugarcane mill. The measured data is fitted using the top shaft rotation period of the mill and dividing on intervals to individually fit each interval obtaining a function defined by parts. The resulting model shows a correlation coefficient of  $R^2 \geq 0.97$ . This allow including the floating effect in the study and design of other elements of sugarcane mills.

**Keywords:** *Sugarcane Mill, Instant Floating, Dynamic Loads.*

## 1. INTRODUCTION

Sugarcane mills, used to extract the sugarcane juice in sugar factories, are essential in sugar production. The current designs of these mills dates back to the late XIX century [1] entailing three rolls and a central blade. Sugarcane juice extraction results from compressing the sugarcane as shown in Figure 1.

The bed of sugarcane bagasse resulting from the compression is of a variable height. Initially, in the first mill rolls the space where the sugarcane starts to be compressed was constant, overloading the mill and causing frequent failures. The first mill with variable wheelbase was patented in 1881 [2], with the top rolls supported on springs allowing it to float over the bagasse bed. Latter, in 1910 was patented the first mill with hydraulic systems to enable the flotation of the top rolls, which significantly increased the milling capacity.

Specialized literature reports the occurrence of rather unexpected failures in sugars mills as a distinctive feature of these technology [3-17]. The most sensitive components to frequent failures are the shaft support rolls, specially the top roll shaft [4], [14], [18], [19].

Several studies analyzed the failures in roll shafts in an effort to predict the rolls lifespan [4], [9], [10-14], [18-22], highlighting the significant influence

of the dynamic loads in the frequent failures. However, this influence is not considered in study of the roll mills as no mathematical models have been developed to this end.

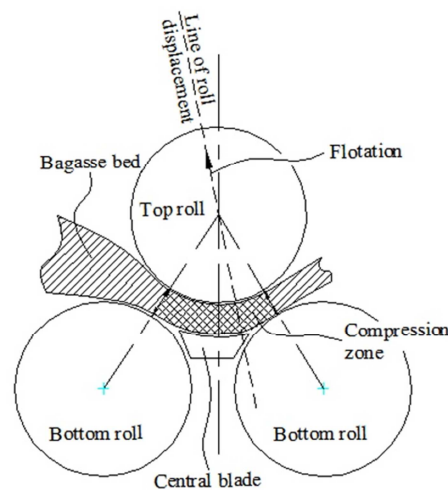


Figure 1. Sugar Cane Mills Running Principles.

Dynamic loads in sugarcane mills might be the result of:

1. The movement is transmitted to the mill through a square coupling (see Figure 2).

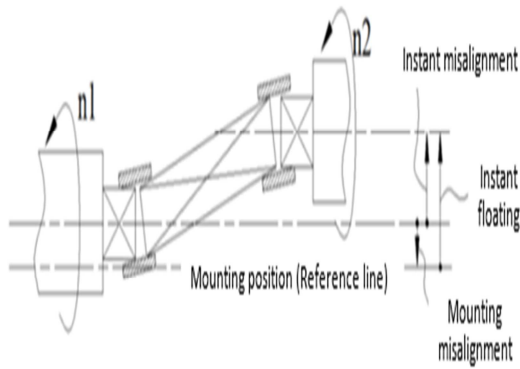


Figure 2. Square Coupling Running Principles.

The square coupling performance was studied by Okamura [23, 24], who proposed a number of equations to consider the additional loads introduced by these couplings, highlighting the influence of the eccentricity and its variation in such loads.

2. The transmission of movement from top to bottom rolls is through mill crown wheels, which are not involute profile gears with variable distance between crown wheel centers resulting from the flotation effect. Several studies discussed the kinematic of mill crown wheels [25-26]. Specifically [27] discusses the dynamic load resulting from the crown wheel tooth profile and the floating effect. The importance of the top rolls floating effect in the dynamic loads of wheel crowns, is stressed by several authors, although it is not considered.
3. Top rolls flotation (resulting from the varying high of the bagasse bed) causes instantaneous load changes in the top rolls.

Figure 3 shows the measurements of the torque variation during a mill operation [15], [16], [28].

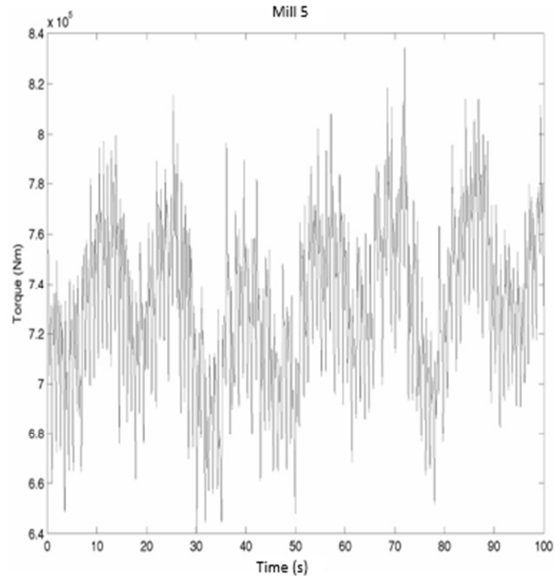


Figure 3. Variation Of Torque During The Operation Of Sugarcane.

Measurements in Figure 3 show strong variations (up to 30%) and load irregularity during the operation of sugarcane mills, mainly resulting from the floating effect of the top roll.

This paper aims to develop a mathematical model to describe the top roll flotation to study the effects of floating on the mill components.

## 2. MATERIALS AND METHODS.

In a research developed by Arzola [18] was measured the top roll floating and the sugarcane mill operation time, during the operation of a sugarcane mill in a sugar factory. The results allowed to estimate the load introduced by the square couplings in the top rolls resulting from the instant misalignment.

Moreover, another study [29] pointed to the influence of the top roll load in the shaft lifespan, concluding that the lifespan is strongly influenced by the misalignment between top roll shafts and the reduction drive output shaft.

In order to further, the influence of floating effects on the mill loads an experimental study was developed in a sugarcane mill at a sugar factory in Cienfuegos, Cuba. To this end is used the same device described in [18]. Figure 4 shows the instant floating measured in a sugarcane mill in three different periods of 180 s.

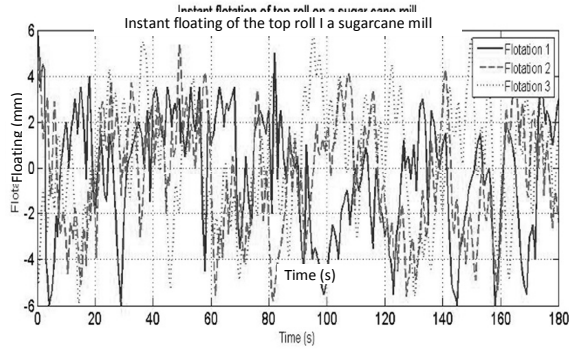


Figure 4. Top Rolls Floats Measured In July 14th Sugar Factory'S Mills.

Floating values randomly vary within a range. Therefore, the function have a cyclic behavior, so to obtain a good fit is necessary to use higher order functions, using different significant intervals, integrating them like a function defined by parts.

### 3. RESULTS AND DISCUSSION.

For modeling the floating effects is used the top shaft rotation period of the mill (1 rpm):

$$T = \frac{2 \cdot \pi}{W_1} \quad (1)$$

Where:

T: time to take for the top rolls to make one rpm (s).

$W_1$ : Mean angular velocity of the top rolls (rad/s).

The time of the measurements can be divided in intervals to individually fit each interval and obtain a function defined by parts. This function describes the top roll floating of the experimental values allowing to introduce the floating effect influence in the assessment and design of sugarcane mills, especially for developing new ways to include dynamic loads.

Fitting a specific time interval ( $t_m$ ) as many subintervals as the shaft revolutions in  $t_m$  will be considered:

$$(i - 1) \cdot T \leq t_i \leq i \cdot T \quad (2)$$

Where:

$t_i$ : time of each interval (s)

i: number of interval ( $i = 1 \div \frac{t_m}{T}$ )

$t_m$ : Time interval (s).

The mean radial speed ( $N_1$ ) in the measured mill is:

$$W_1 = 4.8 \text{ rpm} = 0.5027 \frac{\text{rad}}{\text{s}} \quad (3)$$

Combining equations 1 and 3:

$$T = 12.50 \text{ s} \quad (4)$$

To model a 100 seconds interval ( $t_m=100 \text{ s}$ ) eight intervals are used.

$$i = 1 \div \frac{t_m}{T} = 1 \div 8$$

Figure 5 shows four consecutive intervals.

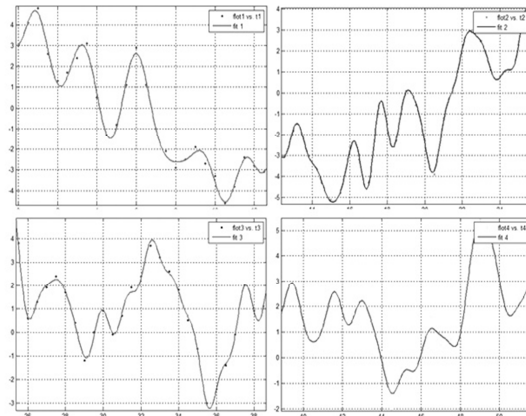


Figure 5. Four Consecutive Intervals Adjust In July 14th Sugar Factory.

The data was processed in MATLAB software. The intervals were fitted using the summatory of an eight degree sine function.

The general function used to fit the internals is:

$$\text{flot}_i = \sum_j a_{ij} \cdot \sin(b_{ij} \cdot t_i + c_{ij}) \quad (5)$$

Where:

flot - instant top rolls flotation (mm).

j: degree of the fit equation.

For floating in time segment  $t_m$ :

$$\text{flot}_i = \{\sum_j a_{ij} \cdot \sin(b_{ij} \cdot t_i + c_{ij}) \quad (i - t) \cdot T \leq t_i \leq i \cdot T\} \quad (6)$$

Where a, b and c are the fit coefficients.

$$a = [a_{ij}] \quad b = [b_{ij}] \quad c = [c_{ij}]$$

To model the operation of a top roll for 100 seconds the rotation period T is calculated from equation 4, obtaining  $i = 1 - 8$ . Following are shown the equations and coefficients for the 8 interval.

$$\text{flot}(t) = \left\{ \begin{array}{ll} \sum_{j=1}^8 a_{1j} \cdot \sin(b_{1j} \cdot t + c_{1j}) & 0 \leq t \leq 12.5 \\ \sum_{j=1}^8 a_{2j} \cdot \sin(b_{2j} \cdot t + c_{2j}) & 12.5 \leq t \leq 25 \\ \sum_{j=1}^8 a_{3j} \cdot \sin(b_{3j} \cdot t + c_{3j}) & 25 \leq t \leq 37.5 \\ \sum_{j=1}^8 a_{4j} \cdot \sin(b_{4j} \cdot t + c_{4j}) & 37.5 \leq t \leq 50 \\ \sum_{j=1}^8 a_{5j} \cdot \sin(b_{5j} \cdot t + c_{5j}) & 50 \leq t \leq 62.5 \\ \sum_{j=1}^8 a_{6j} \cdot \sin(b_{6j} \cdot t + c_{6j}) & 62.5 \leq t \leq 75 \\ \sum_{j=1}^8 a_{7j} \cdot \sin(b_{7j} \cdot t + c_{7j}) & 75 \leq t \leq 87.5 \\ \sum_{j=1}^8 a_{8j} \cdot \sin(b_{8j} \cdot t + c_{8j}) & 87.5 \leq t \leq 100 \end{array} \right.$$

Where the coefficients for each equation are:

$$a = \begin{bmatrix} 3.627 & 3.852 & 1.117 & 0.6454 & 0.566 & 0.2197 & 3.925 & 0.5642 \\ 1.3 & 3.778 & 1.023 & 0.2331 & 0.3422 & 0.3228 & 0.686 & 0.6288 \\ 1.926 & 5.816 & 0.9328 & 445.4 & 445.2 & 62.74 & 62.56 & 0.2942 \\ 10.76 & 9.164 & 0.9117 & 1.338 & 0.5698 & 0.6132 & 0.4306 & 0.4278 \\ 3.327 & 1.806 & 5.626 & 5.383 & 5.648 & 5.597 & 2.305 & 2.297 \\ 2.34 & 1.392 & 1.25 & 1.008 & 0.4788 & 0.5306 & 0.4307 & 0.3419 \\ 2.785 & 2.59 & 0.4431 & 2.668 & 2.596 & 0.5398 & 0.4413 & 0.1166 \\ 1.594 & 1.684 & 1.191 & 0.3852 & 0.327 & 4.429 & 8.079 & 3.856 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.6788 & 0.7542 & 2.489 & 2.004 & 1.671 & 3.438 & 0.1552 & 2.929 \\ 1.343 & 0.1986 & 2.062 & 2.506 & 5.552 & 4.731 & 3.395 & 4.077 \\ 0.9117 & 0.0201 & 1.273 & 2.377 & 2.377 & 4.274 & 4.275 & 5.04 \\ 0.135 & 0.2116 & 1.966 & 0.9156 & 2.428 & 3.388 & 5.064 & 3.685 \\ 0.4838 & 1.002 & 2.252 & 2.209 & 3.54 & 3.558 & 5.212 & 5.232 \\ 0.2701 & 1.152 & 1.457 & 0.7154 & 3.02 & 2.464 & 1.975 & 6.075 \\ 0.1906 & 0.5499 & 1.534 & 2.152 & 2.249 & 3.282 & 3.658 & 4.636 \\ 0.4462 & 1.023 & 1.222 & 1.957 & 2.74 & 4.803 & 4.865 & 4.93 \end{bmatrix}$$

$$c = \begin{bmatrix} -0.1481 & 2.652 & -1.123 & 1.585 & -2.467 & -0.5122 & 2.214 & -1.551 \\ -16.36 & 2.042 & -12.7 & -10.28 & 27.08 & -24.79 & -20.67 & -20.44 \\ 3.474 & 8.597 & 3.053 & 19.03 & -15.51 & -0.9796 & 20.96 & 7.421 \\ 8.895 & 14.78 & 0.8091 & 0.6684 & 1.649 & -1.014 & 11.71 & 6.749 \\ -0.516 & -0.5571 & 16.22 & -9.568 & -1.269 & 25.89 & -12.88 & 20.41 \\ -1.638 & -9.48 & 3.65 & 15.09 & -2.47 & 4.655 & 1.409 & -0.6502 \\ 7.511 & 41.43 & -0.9594 & -13.87 & 18.83 & -19.56 & -9.885 & 431.12 \\ 7.286 & -3.196 & 30.49 & 7.118 & 22.81 & -27.59 & 13.59 & 60.69 \end{bmatrix}$$

The functions fitted show correlation coefficients higher than  $R^2 > 0.97$ . Therefore, is possible to use equation 6 for modelling the top roll operation in sugarcane mills and to calculate the velocity and the acceleration of the top rolls floating variations.

The kinematic performance associated with the top rolls floating can be calculated from equation 6. From its first and second degree time derives can be

define the velocity ( $V_f = \frac{d(\text{flot})}{dt}$ ) and the acceleration ( $a_f = \frac{d^2(\text{flot})}{dt^2}$ ) associated with the floating of the top rolls. The variation of the velocity and the acceleration are shown in Figure.

6.

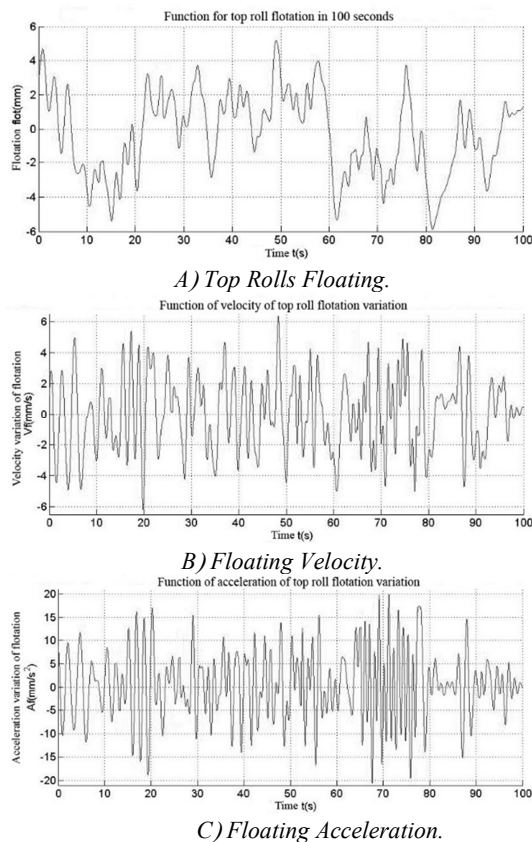


Figure 6. Kinematic Performance Of The Top Rolls Floating.

Figure 6b shows that for the studied sugarcane mills the floating velocity in the top rolls varies from  $-6$  mm/s to  $6$  mm/s. Moreover, fig. 6c shows that the acceleration varies between  $-20$  mm/s<sup>2</sup> to  $20$  mm/s<sup>2</sup>. These are relatively low values (in the order of mm), yet considering that the top rolls and crown wheels of sugarcane mills weights in the order of a few tons per component, these small variations introduce significant dynamic loads, strongly influencing the mill resistance and reliability [4, 11, 15, 16, 29]

#### 4. CONCLUSIONS

The proposed equations for modeling the floating of top rolls in sugarcane mills validate the research hypothesis. The model is based on the correlation of experimental data, measured during the operation of sugarcane mills in a sugar factory, resulting in correlations ( $R^2$ ) higher than 0.97. Consequently, the equations can be implemented in the calculation of dynamic loads in sugar cane mills with good accuracy, also allowing to calculate the velocity and acceleration of the floating movement in the upper shaft, one of the main contributors to this load.

This work represents an important step toward the introduction of the dynamics loads of sugarcane mill components in the design methodologies, to reduce the occurrence of unexpected failures.

#### REFERENCES:

- [1] A. BERNARD, Societe dite: Compagnie de Fives-Lille, assignee. Sugar Cane Mill. Patent 2612101. Paris, France. February 22. 1946.
- [2] D.WILDE, Daniel Wilde, of Washington, Iowa, assignee. *Cane-Mill*. Washington, United States patent 255228. 1881 December 2, 1881.
- [3] N. ARZOLA, R. GOYTISOLO, A. FERNANDEZ, "Determination of an Optimal Assembly Misalignment in Sugar Cane Mills". ASME Conference Proceedings, 2005, IMECE2005-80435, pp. 555-559. ISBN 0-7918-4223-1.
- [4] C. ADAM, J. G. LOUGHRAN, "Finite element prediction of the performance of sugarcane rolling mills". International Sugar Journal, 2007, 109(1301), pp. 272-284.
- [5] C. ADAM, J. G. LOUGHRAN, "The effect of blanket thickness on extraction energy in sugarcane rolling mills: a finite element investigation". Biosystems Engineering, 2005, 92(2), pp. 255-263.
- [6] A. KANNAPIRAN, "Computational and experimental modelling of the crushing of prepared sugar cane". PhD thesis: James Cook University of North Queensland, 2003, pp. 237.
- [7] G.A. KENT, "Increasing the capacity of Australian raw sugar factory milling units", PhD thesis, James Cook University of North Queensland; 2003, pp. 248
- [8] W. MORA, J. RIVAS, J. CORONADO, "Design of two bearings with sealing system for the top roll shaft of sugar cane mill". Rev. Téc. Ing. Univ. Zulia, Vol.28. No 3, 2005, pp. 219-232.
- [9] J. RIVAS, S. RODRÍGUEZ, J. CORONADO, "Análisis de la confiabilidad de los ejes de molino de caña de azúcar". Tecnura, Vol. 8 No 15, 2004 pp. 45-54.
- [10] S. RODRÍGUEZ, J. CORONADO, N. ARZOLA, "Predicción de vida remanente en ejes de masa superior de molino de caña". Ingeniería e Investigación, Vol. 26, No 1, 2006, pp.84-91.
- [11] S. RODRÍGUEZ, J. CORONADO, N. ARZOLA, "Life prediction for the top roller



- shafts of sugar mills". Journal of the Mechanical Behavior of Materials, Vol.17. No 5, 2006, pp. 327-336.
- [12] N. ARZOLA, R. GOYTISOLO, L. SUAREZ, "Efficiency Increase in the Extraction of Sugar Cane Juice in the Sugar Cane Mills by Means of the Regulation of Hydraulic Pressures". ASME Conference Proceedings, 2005, (42193), pp. 637-641.
- [13] J. CORONADO, J. RIVAS, A. LEÓN, "Estudio tribológico en chumaceras y ejes de molino de caña de azúcar". Dyna (Medellin), No 071, 2004, pp. 1-8.
- [14] J. CORONADO, "Fracture mechanics approach of repaired top roll shafts in cane mill". Journal of the Mechanical Behavior of Materials, Vol. 16. No 6, 2005, pp. 419-429.
- [15] G. MUÑOZ, J. LEWINSKY, "Analysis of the mechanical performance of a sugar cane mill. International Sugar Journal", 98(1175), 1996, pp. 574-578.
- [16] E. ROSERO, J. RAMIREZ, "Modelado y control de molinos de caña de azúcar usando accionamientos eléctricos". Revista Iberoamericana de Automática e Informática Industrial (RIAI), Vol. 6. No 3, 2009, pp. 44-53.
- [17] A. MENDOZA, M. CABELLO, J. CABELLO, R. GOYTIZOLO, J. MOYA, "Modelación del contacto entre dos pares de dientes en las coronas de molinos". Ingeniería Mecánica, Vol. 16. No 3. 2014, pp 178-185.
- [18] N. ARZOLA, "Esquema de análisis para los árboles de los molinos de caña de azúcar y aplicación de la Mecánica de la Fractura en la evaluación de la falla por fatiga". Tesis de Doctorado. Universidad Central de las Villas Santa Clara. Cuba. 2003. pp. 145.
- [19] N. ARZOLA, R. GOYTISOLO, R. PEREZ,; et al. "Prediction of the Sugar Mill Shaft Failure Using a Fracture Mechanics Method". ASME Conference Proceedings, 2005, 2005(42150), pp. 749-755.
- [20] N. ARZOLA, R. GOYTISOLO, R. PEREZ, et al. "Determinación de la vida remanente de los árboles de los molinos de caña de azúcar con grieta semielíptica superficial". Ingeniería Mecánica, Vol. 6. No 2. 2003, pp. 43-52.
- [21] N. ARZOLA, R. GOYTISOLO, J. CABELLO, "Utilización del factor de densidad de energía de deformación en el modelo de crecimiento de la grieta en árboles de molinos de caña de azúcar". Ingeniería Mecánica, Vol.8. No 2, 2005, pp. 7-14.
- [22] J. ADAM, J. LOUGHRAN, "Multivariate analysis of frictional interaction between grooved rollers and prepared sugarcane. International". Journal of the American Society of Agricultural and Biological Engineers, Vol. 47 No 5, 2004, pp. 1611-1618.
- [23] H. OKAMURA, H. TANAKA, M. TERAQ, "Square box couplings in cane mill Drives-1". International Sugar Journal, 74(886), 1972, pp. 291-293.
- [24] H. OKAMURA, H.; TANAKA, M. TERAQ, "Square box couplings in cane mill Drives-2". International Sugar Journal, 74(887), 1972, pp. 323-327.
- [25] J. CABELLO, "Cinemática, transmisión de la carga, lubricación y resistencia superficial de las coronas de molinos de caña de azúcar". Tesis de Doctorado. Santa Clara: Universidad Central de las Villas, Cuba. 1999.
- [26] J. MOYA, "Diseño de coronas de molinos de caña de azúcar". Tesis de Doctorado. Santa Clara: Universidad Central de Las Villas; Cuba 1994.
- [27] L. NEGRÍN, R. FRANCO, "Estudio de las curvas epicicloide y evolvente para formar el perfil de los engranajes que operan con distancia entre centros variable". Ingeniería Mecánica, Vol. 10 No 3, pp. 71-76.
- [28] M. CABELLO "Modelo matemático para el análisis cinemático y dinámico de las coronas de molinos de caña de azúcar". Tesis de Maestría. Universidad de Cienfuegos: Cienfuegos. 2010.
- [29] A. ARTANEN, R. BITMEAD, "The application of an iterative identification and controller design to a sugar cane crushing mill. Automática", Vol. 31. No 11, 1995, pp. 1547-1563.