# CONTINUOUS FUNCTIONS ON PRODUCTS OF COMPACT HAUSDORFF SPACES

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§1. *Introduction.* In [4], we investigated the spaces of continuous functions on countable products of compact Hausdorff spaces. Our main object here is to extend the discussion to arbitrary products of compact Hausdorff spaces. We prove the following theorems in Section 3.

THEOREM 3.1. *Let* $\{K_\gamma: \gamma \in \Gamma\}$, *with* $\Gamma$ *an arbitrary index set, be a family of compact Hausdorff spaces. Suppose that, for each finite subset* $\Phi$ *of* $\Gamma$, *the space*

$$(C(\prod \{K_\varphi: \varphi \in \Phi\}), \text{pointwise})$$

*is* $\sigma$*-fragmented. Then the space*

$$(C(\prod \{K_\gamma: \gamma \in \Gamma\}), \text{pointwise})$$

*is* $\sigma$*-fragmented.*

Here and elsewhere, a Banach space equipped with a topology, such as the weak topology or pointwise topology, is said to be "$\sigma$-fragmented" if it is $\sigma$-fragmented with respect to the norm metric.

By combining a result of Zizler [12] with one of our lemmas we also obtain a theorem.

THEOREM 3.2. *Let* $\{K_\gamma: \gamma \in \Gamma\}$, *with* $\Gamma$ *an arbitrary index set, be a family of compact Hausdorff spaces. Suppose that, for each finite subset* $\Phi$ *of* $\Gamma$, *the space*

$$C(\prod \{K_\varphi: \varphi \in \Phi\})$$

*has an equivalent locally uniformly convex norm. Then*

$$C(\prod \{K_\gamma: \gamma \in \Gamma\})$$

*has an equivalent locally uniformly convex norm.*

In the original version of the present paper submitted in 1995, we stated: "As far as we know it may be possible to prove that, if $H$ and $K$ are two compact Hausdorff spaces and ($C(H)$, pointwise) and ($C(K)$, pointwise) are both $\sigma$-fragmented, then ($C(H \times K)$, pointwise) is also $\sigma$-fragmented. If this is so, then the condition that for each finite subset $\Phi$ of $\Gamma$ the space

$$C(\prod \{K_\varphi: \varphi \in \Phi\}, \text{pointwise})$$

is $\sigma$-fragmented, can be replaced by the more elegant condition that each

[MATHEMATIKA, **46** (1999), 323–330]

($C(K_\gamma)$, pointwise) with $\gamma \in \Gamma$ is $\sigma$-fragmented, in Theorem 6.1 [Theorem 3.1 above here]."

The missing link between Theorem 3.1 and its "more elegant" version has now been supplied independently by Ribarska [**11**], Moors [**9**], and Namioka–Pol [**10**]. On the other hand, the elegant version of Theorem 3.2 still eludes us, but if the pointwise lower-semicontinuity of the norm is assumed everywhere, then it is true. For, recently Babev and Ribarska [**1**] have proved that, if $H$ and $K$ are compact Hausdorff spaces such that each of $C(H)$ and $C(K)$ can be renormed with an equivalent pointwise lower-semicontinuous locally uniformly convex norm, then $C(H \times K)$ can also be so renormed. Furthermore, Theorem 3.2 remains true if "locally uniformly convex norm" in the hypothesis and the conclusion are replaced with "pointwise lower-semicontinuous locally uniformly convex norm". The justification of this additional fact is outlined at the end of the paper.

§2. *Generalizations of two theorems and a lemma.* In order to prove one of our main theorems, we need to give simple generalizations of Theorem 6.1 of [**4**] and of Theorem 2 of [**7**]. It will also be convenient to reformulate a special case of Lemma 2 of [**12**]. In this section we give the necessary details.

THEOREM 2.1. *Let $\{X_\gamma: \gamma \in \Gamma\}$ be a transfinite sequence of Banach spaces. For each $\gamma \in \Gamma$, let $\tau_\gamma$ be a topology for $X_\gamma$ with respect to which the norm of $X_\gamma$ is lower-semicontinuous. Let*

$$X = c_0\{X_\gamma: \gamma \in \Gamma\}$$

*be the $c_0$-sum of the Banach spaces and let $\tau$ be the topology on $X$ induced by the product topology of $\prod\{(X_\gamma, \tau_\gamma): \gamma \in \Gamma\}$. If each $(X_\gamma, \tau_\gamma)$ is $\sigma$-fragmented, then so is $(X, \tau)$. If each $(X_\gamma, \tau_\gamma)$ is $\sigma$-fragmented using $\tau_\gamma$-closed sets, then $(X, \tau)$ is $\sigma$-fragmented using $\tau$-closed sets.*

*Proof.* For each $\gamma \in \Gamma$ and each $r > 0$, the fact that the norm of $X_\gamma$ is lower-semicontinuous for $\tau_\gamma$ ensures that each set of the form

$$\{x \in X_\gamma: \|x\| \leq r\}$$

is $\tau_\gamma$-closed. Using this in place of the corresponding result for the weak topology, and working with the topologies $\tau_\gamma$ and $\tau$ in place of the weak topologies, the theorem follows by the proof of Theorem 6.1 of [**4**].

THEOREM 2.2. *Let $X$ be a Banach space and let $\tau$ be a topology with respect to which the norm is lower-semicontinuous. Suppose that $\{T_\gamma: \gamma \in \Gamma\}$ is a family of linear maps $T_\gamma: X \to X$ that are norm and $\tau$ continuous and satisfy the following conditions.*

(i) *For each $x \in X$, the map $\gamma \mapsto \|T_\gamma x\|$ belongs to $c_0(\Gamma)$.*
(ii) *Each $x \in X$ is in the norm closed linear span of $\{T_\gamma x: \gamma \in \Gamma\}$.*
(iii) *For each $\gamma \in \Gamma$, the norm closure $X_\gamma$ of $T_\gamma X$ in $X$, when taken with the relativization of $\tau$, is $\sigma$-fragmented.*

*Then $(X, \tau)$ is $\sigma$-fragmented.*

Note that this theorem reduces to the first part of Theorem 2 of [7] when $\tau$ is taken to be the weak topology on $X$.

*Proof.* Repeat the original proof of Theorem 2 in [7], with the weak topology replaced by $\tau$ and using Theorem 5.1 above where the original proof calls for Theorem 6.1 of [4].

LEMMA 2.1. *Let $X$ be a Banach space and suppose that, for some infinite limit ordinal $\Gamma$, there is a family $\{P_\gamma\colon 0\leqslant\gamma\leqslant\Gamma\}$ of linear maps $P_\gamma\colon X\to X$ satisfying:*

(*a*) $\|P_\gamma\|\leqslant 1$ for $0\leqslant\gamma\leqslant\Gamma$;
(*b*) *$P_\Gamma$ is the identity on $X$, and $P_0=0$;*
(*c*) $P_\alpha P_\beta=P_\beta P_\alpha=P_\alpha$ for $0\leqslant\alpha<\beta\leqslant\Gamma$;
(*d*) *for each limit ordinal $\lambda$ with $0<\lambda\leqslant\Gamma$, and for each $x\in X$,*

$$\lim_{\mu\to\lambda} P_\mu x = P_\lambda x.$$

*Then there is a family $\{T_\gamma\colon 0\leqslant\gamma<\Gamma\}$ of linear maps $T_\gamma\colon X\to X$ such that*

(i) $\|T_\gamma\|\leqslant 2, 0\leqslant\gamma<\Gamma$;
(ii) *for $x\in X$ the map $\gamma\|T_\gamma x\|$ belongs to $c_0(\Gamma)$;*
(iii) *each $x\in X$ belongs to the closed linear span of the set $\{T_\gamma x\colon 0\leqslant\gamma<\Gamma\}$;*
(iv) *for $0\leqslant\gamma<\Gamma$,*

$$T_\gamma X\subset P_\gamma+1X.$$

*Further, if $\tau$ is any topology on $X$, and the maps $P_\gamma, 0\leqslant\gamma<\Gamma$, are $\tau$-continuous, then so are the maps $T_\gamma$ for $0\leqslant\gamma<\Gamma$.*

*Proof.* We define a family $\{T_\gamma\colon 0\leqslant\gamma<\Gamma\}$ of linear maps $T_\gamma\colon X\to X$ by taking

$$T_\gamma=P_{\gamma+1}-P_\gamma \qquad \text{for } 0\leqslant\gamma<\Gamma.$$

The condition (i) follows immediately from condition (a).

To prove that condition (ii) holds, we need show that, for each $x\in X$ and for each $\varepsilon>0$, there are only finitely many $\gamma$ with $0\leqslant\gamma<\Gamma$ for which

$$\|T_\gamma x\|\geqslant\varepsilon.$$

Suppose on the contrary that there are infinitely many $\mu$ with $0\leqslant\mu<\Gamma$ and $\|T_\mu x\|\geqslant\varepsilon$. Let $\mu_1,\mu_2,\dots,\mu_n,\dots$, with $\mu_1\geqslant 1$, be an infinite increasing sequence of such ordinals. Then

$$\|(P_{\mu_i+1}x)-(P_{\mu_i}x)\|\geqslant\varepsilon \qquad \text{for } i\geqslant 1.$$

Let $\lambda$ be the supremum of the ordinals $\mu_i$, $i\geqslant 1$. Then $\lambda$ is necessarily a limit ordinal with $\lambda\leqslant\Gamma$ and

$$\lim_{i\to\infty}\mu_i=\lambda.$$

By condition (d),

$$\lim_{\mu\to\lambda} P_\mu x = \lim_{\mu\to\lambda} P_{\mu+1}x = P_\lambda x.$$

This contradicts the inequality

$$\|(P_{\mu_i+1}x)-(P_{\mu_i}x)\|\geq\varepsilon \qquad \text{for } i\geq 1.$$

Thus the map $\gamma\mapsto\|T_\gamma x\|$ belongs to $c_0(\Gamma)$ and the condition (ii) holds.

To prove (iii) we consider any point $x$ of $X$. Let $E$ be the closed linear span of $\{T_\gamma x: 0\leq\gamma<\Gamma\}$. Since, by (b) and (d),

$$x = P_\Gamma x = \lim_{\mu\to\Gamma} P_\mu x,$$

it is sufficient to show that $P_\mu x\in E$ for each $\mu$ with $0\leq\mu<\Gamma$. We do this by induction. Clearly $P_0x=0\in E$. Suppose that, for some $\lambda$ with $0<\lambda<\Gamma$, we have $P_\gamma x=E$ for each $\gamma$ with $0\leq\gamma<\lambda$. If $\lambda$ is a limit ordinal, then, by (d), $P_\lambda x=\lim_{\gamma\to\lambda}P_\gamma x\in E$. If $\lambda=\alpha+1$, then $P_\alpha x\in E$. Since $P_{\alpha+1}x-P_\alpha x=T_\alpha x$ belongs to $E$ we have $P_\lambda x=(P_{\alpha+1}x-P_\alpha x)+P_\alpha x\in E$. Hence $P_\lambda x\in E$ for $0\leq\lambda<\Gamma$ as required.

For the last condition (iv) we note that, if $0\leq\gamma<\Gamma$, then

$$T_\gamma x=(P_{\gamma+1}-P_\gamma)x=P_{\gamma+1}(x-P_\gamma x)\in P_{\gamma+1}X.$$

Thus $T_\gamma X\subset P_{\gamma+1X}$ for $0\leq\gamma<\Gamma$.

The ultimate remark about the topology $\tau$ follows immediately from the definition of the maps $T_\gamma$ in terms of the maps $P_\gamma$.

§3. *Products of compact Hausdorff spaces.* In this section we prove two of our main theorems.

THEOREM 3.1. *Let $\{K_\gamma: \gamma\in\Gamma\}$, with $\Gamma$ an arbitrary index set, be a family of compact Hausdorff spaces. Suppose that, for each finite subset $\Phi$ of $\Gamma$, the space*

$$(C(\prod\{K_\varphi: \varphi\in\Phi\}), \text{pointwise})$$

*is $\sigma$-fragmented. Then the space*

$$(C(\prod\{K_\gamma: \gamma\in\Gamma\}), \text{pointwise})$$

*is $\sigma$-fragmented.*

THEOREM 3.2. *Let $\{K_\gamma: \gamma\in\Gamma\}$, with $\Gamma$ an arbitrary index set, be a family of compact Hausdorff spaces. Suppose that, for each finite subset $\Phi$ of $\Gamma$, the space*

$$C(\prod\{K_\varphi: \varphi\in\Phi\})$$

*has an equivalent locally uniformly convex norm. Then*

$$C(\prod \{K_\gamma: \gamma \in \Gamma\})$$

*has an equivalent locally uniformly convex norm.*

We first prove two lemmas.

LEMMA 3.1. *Let $\{K_\gamma: 1 \leq \gamma < \Gamma\}$, with $\Gamma$ an infinite limit ordinal, be a family of compact Hausdorff spaces. Write*

$$K = \prod \{K_\gamma: 1 \leq \gamma < \Gamma\},$$

$$K^{(\alpha)} = \prod \{K_\gamma: 1 \leq \gamma < \alpha\},$$

*for $1 < \alpha \leq \Gamma$ and $K^{(1)} = \{\varnothing\}$. Then there is a family $\{T_\gamma: 0 \leq \gamma < \Gamma\}$ of norm and pointwise continuous linear maps $T_\gamma$: $C(K) \to C(K)$ with the following properties.*

(i) $\|T_\gamma\| \leq 2$ *for* $0 \leq \gamma < \Gamma$.

(ii) *For $f \in C(K)$ the map $\gamma \mapsto \|T_\gamma f\|$ belongs to $c_0(\Gamma)$.*

(iii) *Each $f$ in $C(K)$ belongs to the closed linear span of the set $\{T_\gamma f: 0 \leq \gamma < \Gamma\}$.*

(iv) *For $0 \leq \gamma < \Gamma$, the norm closure of the set $T_\gamma C(K)$ in $C(K)$ is linearly isometric and linearly pointwise homeomorphic to a closed linear subspace of $C(K^{(\gamma+1)})$.*

*Proof.* For each $\gamma$ with $1 \leq \gamma < \Gamma$, we select a base point $k_\gamma$ in $K_\gamma$. We use

$$x = \{x_\gamma: 1 \leq \gamma < \Gamma\},$$

with $x_\gamma \in \mathrm{K}_\gamma$ for $1 \leq \gamma < \Gamma$, to denote the typical point in $K$. For $x$ in $K$ and $1 \leq \alpha \leq \Gamma$ we write

$$x^{(\alpha)} = \{x_\gamma: 1 \leq \gamma < \alpha\}$$

for a point in $K^{(\alpha)}$ corresponding to $x$, and we use

$$(x^{(\alpha)}, k^{[\alpha]})$$

for the point $y$ in $K$ with

$$y_\gamma = x_\gamma, \qquad 1 \leq \gamma < \alpha,$$

$$y_\gamma = k_\gamma, \qquad \alpha \leq \gamma < \Gamma.$$

The map $x \mapsto (x^{(\alpha)}, k^{[\alpha]})$ is clearly a continuous retraction of $K$ onto a compact subset homeomorphic to $K^{(\alpha)}$. Let $P_\alpha$ be the corresponding map from $C(K)$ to itself carrying the function $f$ of $C(K)$ to the function $g$ of $C(K)$ defined by

$$g(x) = f(x^{(\alpha)}, k^{[\alpha]}),$$

and let $P_0 = 0$. Note that $P_\alpha$ is a bounded linear projection from $C(K)$ to $C(K)$ that is continuous from $(C(K)$, pointwise) to itself.

It follows immediately from the definitions that conditions (a), (b), and (c) of Lemma 2.1 are satisfied with $X = C(K)$.

We now prove two facts.

(d) For each limit ordinal $\beta$ with $0<\beta\leqslant\Gamma$, and for each $f$ in $C(K)$,

$$\lim_{\alpha\to\beta} P_\alpha f = P_\beta f$$

in the norm topology.

(e) For $0\leqslant\alpha<\Gamma$, the range of $P_\alpha$ in $C(K)$ is a closed linear subset of $C(K)$ that is linearly isometric and linearly pointwise homeomorphic to $C(K_{(\alpha)})$.

We prove (d). Let $f$ in $C(K)$ and $\varepsilon>0$ be given. Just as in Theorem 4.4 of **[4]**, there is a finite subset $\Phi\subset\Gamma$ such that, whenever $x, y\in K$ and

$$x_\varphi = y_\varphi \qquad \text{for } \varphi\in\Phi,$$

then

$$|f(x)-f(y)|<\varepsilon.$$

Since $\beta$ is a limit ordinal, there is an $\alpha_0<\beta$ with $[\alpha_0, \beta)\cap\Phi=\varnothing$. Now, whenever $\alpha\in[\alpha_0, \beta)$ and $x\in K$, we have

$$(x^{(\alpha)}, k^{[\alpha]})_\varphi = (x^{(\beta)}, k^{[\beta]})_\varphi \qquad \text{for } \varphi\in\Phi,$$

so that

$$|f(x^{(\alpha)}, k^{[\alpha]}) - f(x^{(\beta)}, k^{[\beta]})|<\varepsilon.$$

Thus

$$\|P_\alpha f - P_\beta f\|<\varepsilon.$$

Since $\gamma>0$ is arbitrary, this ensures that

$$\lim_{\alpha\uparrow\beta} P_\alpha f = P_\beta f,$$

as required.

To prove (e) we introduce, for $1\leqslant\alpha<\Gamma$, maps $\varphi_\alpha$ from $C(K^{(\alpha)})$ to $C(K)$ and $\psi_\alpha$ from $C(K)$ to $C(K^{(\alpha)})$. For $g$ in $C(K^{(\alpha)})$ we take $\varphi_\alpha(g)$ to be the function in $C(K)$ given by

$$\varphi_\alpha(g)(x) = g(x^{(\alpha)}).$$

For $f$ in $C(K)$ we take $\psi_\alpha(f)$ *to be the function in* $C(K^{(\alpha)})$ given by

$$\psi_\alpha(f)(x^{(\alpha)}) = f(x^{(\alpha)}, k^{[\alpha]}).$$

Clearly

$$P_\alpha = \varphi_\alpha\circ\psi_\alpha$$

and

$$\psi_\alpha\circ\varphi_\alpha$$

is the identity on $C(K^{(\alpha)})$.

Furthermore, $\psi_\alpha$ maps $C(K)$ onto $C(K^{(\alpha)})$ and $\varphi_\alpha$ is an isometric embedding of $C(K^{(\alpha)})$ into $C(K)$. It can also be seen directly that $\varphi_\alpha$ embeds

($C(K^{(\alpha)})$, pointwise) homeomorphically into ($C(K)$, pointwise), It follows that

$$\text{range } P_\alpha = \text{range}\,(\varphi_\alpha \circ \psi_\alpha) = \text{range } \varphi_\alpha,$$

so that range $P_\alpha$ is isometrically isomorphic and pointwise homeomorphic to $C(K^{(\alpha)})$. This proves (e).

The conditions (a) to (d) of Lemma 2.1 are now satisfied with $X = C(K)$, and further, for $0 \leqslant \alpha < \Gamma$, the space $P_{\alpha+1}C(K)$ is isometrically isomorphic and pointwise homeomorphic to $C(K^{(\alpha+1)})$. Thus, by Lemma 2.1, there is a family $\{T_\gamma : 0 \leqslant \gamma < \Gamma\}$ of norm and pointwise continuous linear maps $T_\gamma$: $C(K) \to C(K)$ satisfying the conditions (i) to (iv).

For the next lemma, we retain the notation used in the statement of Lemma 3.1. In particular, $\Gamma$ is an infinite limit ordinal.

LEMMA 3.2. (1) *If, for each $\alpha$ with $1 \leqslant \alpha < \Gamma$,* ($C(K^{(\alpha)})$, pointwise) *is $\sigma$-fragmented, then* ($C(K)$, pointwise) *is also $\sigma$-fragmented.*

(2) *If, for each $\alpha$ with $1 \leqslant \alpha < \Gamma$, $C(K^{(\alpha)})$ has an equivalent locally uniformly convex norm, then $C(K)$ also has an equivalent locally uniformly convex norm.*

*Proof.* By Lemma 3.1 there is a family $\{T_\alpha : 1 \leqslant \alpha < \Gamma\}$ of linear maps $T_\alpha$: $C(K) \to C(K)$ satisfying the conditions (i) to (iv) of that lemma. Condition (iv) ensures that ($T_\alpha C(K)$, pointwise) is $\sigma$-fragmented in case (1) and that the norm closure of $T_\alpha C(K)$ in $C(K)$ has an equivalent locally uniformly convex norm in case (2).

The result (1) follows by Theorem 2.2 above. The result (2) follows by use of Theorem 1 of Zizler **[12]**.

We can now give the proofs of Theorems 3.1 and 3.2.

*Proof of Theorem* 3.1. We use induction on the cardinality of $\Gamma$. There is nothing to prove if $\Gamma$ is finite. So we suppose that, for some infinite cardinal $\aleph_\delta$, the theorem holds for all cardinals less than $\aleph_\delta$. After well-ordering, we may suppose that $\Gamma$ is the least ordinal with cardinal $\aleph_\delta$. Then the hypotheses of Lemma 3.2(1) are satisfied, and we conclude that ($C(K)$, pointwise) is $\sigma$-fragmented when $|\Gamma| = \aleph_\delta$. The result follows by transfinite induction.

*Proof of Theorem* 3.2. The result follows as in the proof of Theorem 3.1 using Lemma 3.2(2) rather than Lemma 3.2(1).

§4. *Final remarks.* (1) As mentioned in the Introduction, this paper is a revision of an earlier one of 1995. Many results in the earlier version have been made obsolete by the subsequent development, and they are omitted in the present paper. However the following result seems new and worth mentioning here. The proof is rather long. *Let $K_1$, $K_2, \ldots,$ be totally ordered spaces that are compact in their order topology. Write $K = \prod_{i=1}^{\infty} K_i$. Then* ($C(K)$, pointwise) *has a countable cover by sets of small local norm diameter.*

(2) In the Introduction we have remarked that Theorem 3.2 remains valid if each "locally uniformly convex (luc) norm" in the statement is replaced with "pointwise lower-semicontinuous (lsc) luc norm". We see this as follows: In Lemma 3.1, assume that, for each $\gamma$ with $0 \leqslant \gamma < \Gamma$, $C(K^{\gamma+1})$ admits an equivalent pointwise lsc luc norm. Then, for each $\gamma$, the map $T_\gamma$ is pointwise continuous and $T_\gamma(C(K))$ admits an equivalent pointwise lsc luc norm. Also the supremum norm of $C(K)$ is clearly pointwise lsc. Now given these facts, the luc norm constructed from the family $\{T_\gamma: 0 \leqslant \gamma < \Gamma\}$ using Theorem 1 of Zizler [**12**] is easily seen to be pointwise lsc. (The same kind of argument has been used by Deville and Godefroy in [**2**].) It follows that Lemma 3.2(2), and hence Theorem 3.2, remains valid when pointwise lower-semicontinuity is added to the assumption and the conclusion.

For a discussion of compact totally ordered spaces, see [**8**].

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