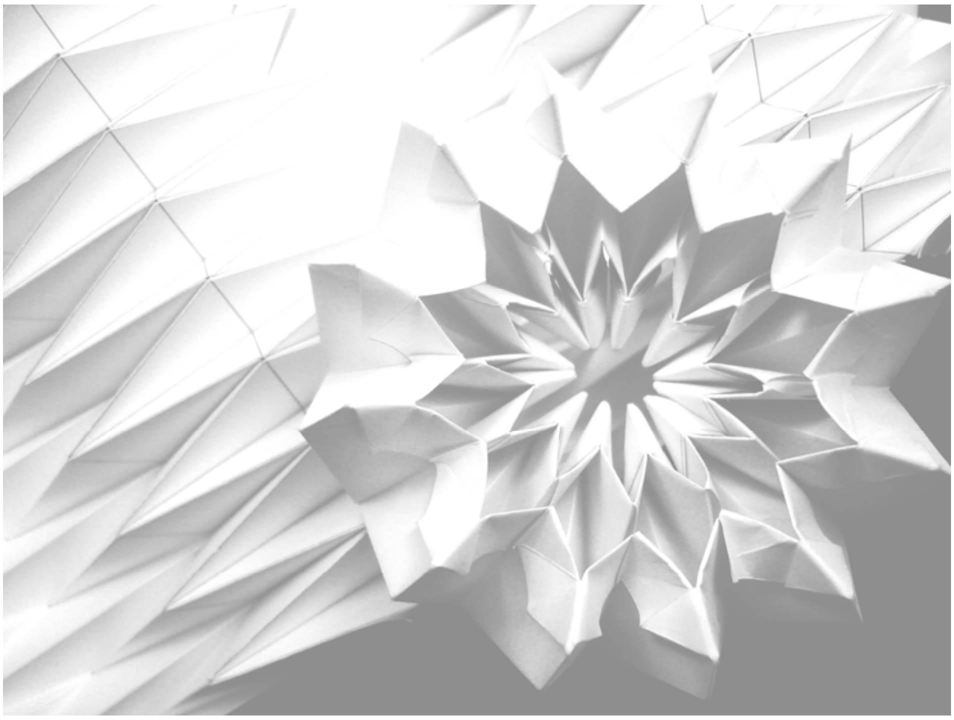

THE GRAMMAR
of
DEVELOPABLE DOUBLE CORRUGATIONS
(for FORMAL ARCHITECTURAL APPLICATIONS)

DISSERTATION REPORT

Tutors

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(figure 01 : Cover Piece : A 'Fire-Works' Origami object is juxtaposed against a 'Hill-Trough' Origami folded artifact. Source : Author)

Declaration

I, Ankon Mitra, submit this dissertation in partial fulfillment of the Degree of Master of Science (Adaptive Architecture & Computation). I also confirm that the work presented in this dissertation is my own. Where information has been derived from other sources, this has been indicated and clearly referenced in the dissertation.

The work strictly adheres to the University College London guidelines for dissertations and theses of this nature. The word count for this paper is 10, 137 (excluding - title content, abstract, page of contents, list of figures and tables and references).

Abstract

This paper investigates the geometrical basis of regular corrugations, with specific emphasis on Developable Double Corrugations (DDCs), which form a unique sub-branch of Origami Folding and Creasing Algorithms. The aim of the exercise is three fold – (1) To define and isolate a 'single smallest starting block' for a given set of distinct and divergent DDC patterns, such that this starting block becomes the generator of all DDCs when different generative rules are applied to it. (2) To delineate those generic parameters and generative rules which would apply to the starting block, such that different DDCs are created as a result (3) To use the knowledge from points (1) and (2) to create a complete family of architectural forms and shapes using DDCs. For this purpose, a matrix of 12 underlying geometry types are identified and used as archetypes. The objective is to mathematically explore DDCs for architectural form finding, using physical folding as a primary algorithmic tool. Some DDCs have more degrees of freedom than others and can fit varied geometries, while others cannot. The discussion and conclusions involve - (a) identifying why certain DDCs are ideal for certain forms and not others, when all of them are generated using the same/or similar starting block(s), (b) discussing the critical significance of flat-foldability in this specific context and (c) what we can do with this knowledge of DDCs in the field of architectural research and practice in the future.

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List of Diagrams / Illustrations / Tabulations

Note : All figures, photographs and tables, unless otherwise stated, have been created by the author, and he states his moral right to be identified as the source of these works.

Acknowledgements

Genuflection to the Ancient Chinese who invented paper and to Akira Yoshizawa, the Japanese polymath and the doyen of folding who single-handedly revived, resuscitated and reinterpreted the ancient art of Origami for the modern times. Without paper, and without Yoshizawa, this dissertation would not be possible.

I strongly believe in providence and also wish to thank serendipity, for having stumbled upon a subject as engrossing and intricate as Origami Mathematics. This is now a lifelong relationship and explorations will continue even after the MSc. Program has concluded.

I thank my tutor Sean Hanna, who suggested that I investigate the possibility of a fundamental block which could generate all Developable Double Corrugations, as also for his 'Generative Space, Form and Behaviour' lectures which opened up a new world of ideas for me.

I also express my gratefulness to Alasdair Turner, my Course Director, for his valuable feedback and assistance during the Msc. Program.

My gratitude to my parents is immeasurable.

1 Introduction

1.1 Biomimetics, The Act of Folding & the Algorithmic Process

Kostas Terzidis defines the act of folding rather well in his seminal book 'Expressive Form', and it is apt to begin this paper with his words -

> *'Folding is the process of laying one part over another. While the outcome of folding may appear as a replication of shapes, theoretically, folding is a self-referential process : no elements are added or subtracted from the folding scene. The form inverts, reverts, and entangles in multiple ways revealing repetitive patterns. Folding is an intricate process that addresses one of form's most existential qualities : the cross from one dimension into another. It is a process that involves changes that extend the geometrical properties of an object while preserving its topology. A piece of aluminium foil, for instance, when crumpled to form a ball-like shape, embodies the properties of a three dimensional solid object despite the fact that it is a two dimensional surface.'* (pg. 45, Terzidis, 2003)

Terzidis goes on to argue that the essential need for folding arises with most processes and forms in nature. The argument is that all morphogenesis and evolution in nature is in fact an attempt to constantly improve upon the act of folding and unfolding to allow for more desirable and suitable connections, efficient forms and useful motions. He gives the example of our limbs, the digits of our hands and the thumb to elucidate his argument. Thus he is essentially building an implicit case where researching folding implies Bio-mimetics. The next chapter (Two) discusses a unique example of recent research linking folding mathematics with nature.

But presently, the question arises as to how, a physical act like folding (say, of a material such as paper) becomes relevant to an MSc. program which emphasizes Adaptive Architecture, Computation and the Algorithmic Process?

Sophia Vyzoviti provides us with an insight – *'Repetitive paper folding performances evolve initial intuitive responses into primary techniques – triangulation, stress* *forming, stratification of folds, folds within folds and patterns…generative transformations are structured in [paperfold] sequences…the succession of transformations resulting to the paperfold artefact as a genetic algorithm of form. The task in this phase is to decipher the paperfold algorithm as a morphogenetic mechanism.'* (pg. 9, Vyzoviti, 2003/2006)

Is it a proposition then, that paper folding is a kind of algorithmic, computational model. But how exactly is it so?

Humaiki Huzita has described six axioms that plot points and lines, to help draw and explain folding schemes. (pg. 37-70, Huzita, 1992) Hatori has added a seventh axiom to this list (pg. 31–38, Hull, 1995). These axioms (to be explained in some detail in the next chapter) prove, that folding is an accurate, precise, and quantifiable operation. In the geometry specific to paper folding or origami, a straight line becomes a 'crease' or 'fold'. Instead of drawing straight lines, one may fold a piece of paper and flatten the crease. Although this act may seem basic and overly simplistic, its value corresponds with performing Euclidean geometrical operations using a straight edge (without markings) and a compass. Mathematicians have proved that certain geometrical problems, such as trisecting an angle and doubling a cube, for instance, are impossible with the straight edge and compass, yet possible with paper folding. Because of its accuracy, simplicity, and economy, paper-folding can contribute to the generation, analysis, and understanding of complex shapes and diagrams.

It is clear then, that the physical act of folding (as a design tool, and as a mathematical tool) solves many problems of geometry, while offering unique insights into complex forms and shapes. Paper folding as an algorithmic approach becomes clearer with Terzidis's explanation - *'…because of its step-by-step, codifiable, rational, and modular process, paper-folding may be regarded as an algorithmic mechanism for exploration of formal systems. In that sense, folding and unfolding become encoded processes through the logic of algorithmic computation.'* (pg. 49, Terzidis, 2003)

He elaborates this point further by citing the act of unfolding, as a counterpoint to folding – *'While the term unfolding may be understood as literally the reverse process of folding, its connotations extend beyond simple reversion. It suggests disclosure, revelation, elucidation, clarification and explanation. For instance, after unfolding a* *paper model until it reaches a flat configuration, a clearly defined crease pattern [and in fact folding pattern also] is revealed. This pattern is the imprinted revelation of a process. Conversely, a folded model is not a composition…but rather the encapsulation of intricate series of folding transformations.'* (pg. 51, Terzidis, 2003)

Figure 02 below shows an old Persian manuscript page copied from an earlier Arabic text where a simple folding pattern is explained algorithmically as a series of six geometric transformations, generating a square tile at the lower right hand corner of the page. Nine tiles generate a regular creasing pattern known today in origami as the 'Alligator'. (pg. 30, Engel, 1994)

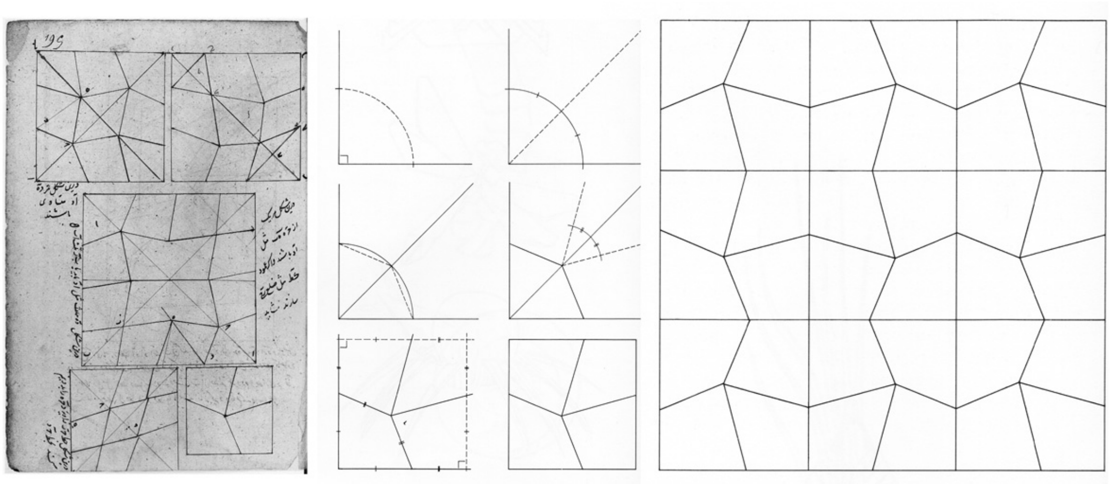


Figure 02 : An early example of a geometric algorithm for a regular tessellated folding pattern
(Source : Peter Engel, Origami from Angel Fish to Zen, pg. 30)

One of the underlying themes of this paper is to therefore clearly demonstrate (using the research of Developable Double Corrugations as a backbone), that the physical act of folding [paper], is in fact a powerful and illustrative algorithmic and computational process and should be used as a starting point and as a design tool for developing computational models for folding in/of architecture.

1.2 Folding and Kinematics

The next theme is the somewhat unformulated and ill-defined relationship between kinematics and folding. Form (especially architectural form) is usually perceived as permanent and static. This is not always true (for instance with retractable roofs or digital architecture etc.). The execution of physical folding exercises during this project have revealed fundamental relationships between folding and kinematics - studying the kinematics of folding structures reveals that each one of the units (a term called 'kernel' will be defined in due course in the paper), which springs from the intersection of crease lines in the folded arrangement, performs, as part of a mechanism, and the entire structure can be described as a single machine with many moving parts, with a series of interconnected relationships. Therefore, minor changes in the single unit or the overall creasing pattern, completely changes the relationship of the units, and therefore the design and performance of the machine. A nuanced differentiation between the act of folding and the final folded artefact is the only way to be able to comprehend this correlation. Kinematics changes the folding, which changes the folded object. So the act of folding is dynamic, the folded artefact as an end product is static, but the dynamism is embedded in it, and therefore the artefact can change when this dynamism is called into play. The best analogy from the world of physics would seem to be how kinetic energy is stored as potential energy in an object at rest.

Although kinematics (as an essential by-product of folding, and as an embedded feature of the folded artefact) is not one of the prime foci of this paper, yet, the way in which kinematics informs the static and final form of the folded object, is in itself sufficient grounds to overview this relationship in a little more detail.

'…kinematics opens up a more intricate relationship between folding and motion…rather than conceiving of folded structures as static configurations or dynamic expressions, they may instead be…transformational mechanisms. Folds…are not only mechanisms for structural, static, or dynamic support, but also means of kinematic exploration…for instance, in a symmetrical asynchronous sequential configuration, distinct steps follow a propagation effect, whereas in a synchronously deploying structure all tiles [meaning single blocks of the folded artefact] move at the same time and the motion of one affects the motion of all other tiles adjacent to it.' (pg. 51, Terzidis, 2003)

This therefore is the second underlying theme that runs through the entire paper and finds explicit expression in Chapters 4 and 5 (Observation and Discussion).

1.3 Footnote to Sections 1.1 and 1.2

An anecdotal simile from 'The Atlas of Novel Tectonics' is a somewhat eccentric but apposite footnote to sections 1.1 and 1.2 which (a) introduced the philosophy of 'folding' in architectural discourse and (b) explained the *raison d' etre* for the thematic contours of this paper. The following sections introduce DDCs, and elaborate on specific aims and objects, laying down the hypotheses.

'Architects work with matter like a chef who manages the complex 'unfoldings' of food chemistry very precisely but without necessarily knowing the science of the chemistry itself. One does not, for instance, need to know how an ovalbumen protein coagulates in order to make a superior omelet [see illustration below]. Architects, too, are in large part the managers of processes they do not, and cannot, fully comprehend.' (pg. 168, Reiser and Umemoto, 2006)

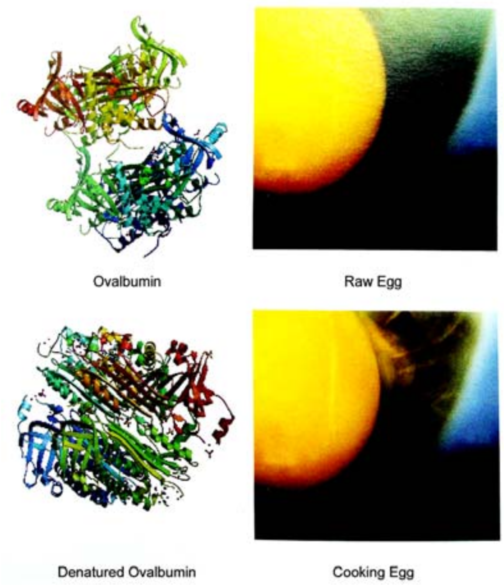


Figure 03:
How the Ovalbumin denatures during the heating process leading to the coagulation of liquid egg into solid omelette.
(Source : Reiser+Umemoto, The Atlas of Novel Tectonics, page 169)

As a corollary to this anecdote, it can be said that in the process of designing, architects juggle with geometry on a routine basis, yet, most architects are only peripherally aware of the generative algorithmic sequences that underline nearly all geometrical resolutions.

This is especially true for the act of folding – an endeavor is made in this paper to delve beyond architectural problem solving and investigate the algorithmic machinations of form-making, with folding, as a primary process. And further, within the paradigm of folding, paper folding or Origami, and specifically Developable Double Corrugations, provide an exciting meeting point for Architecture, Mathematics and the Algorithmic technique – this is the juncture at which a simple 'kernel' of triangles (defined in Section 3.3), permutes and combines with itself in scores of ways to create myriad strings of valleys and mountains, creating striking visual complexity (figure 04 below), but with a geometric regularity most sublime. This paper is foremost an attempt, at demystifying that magic.

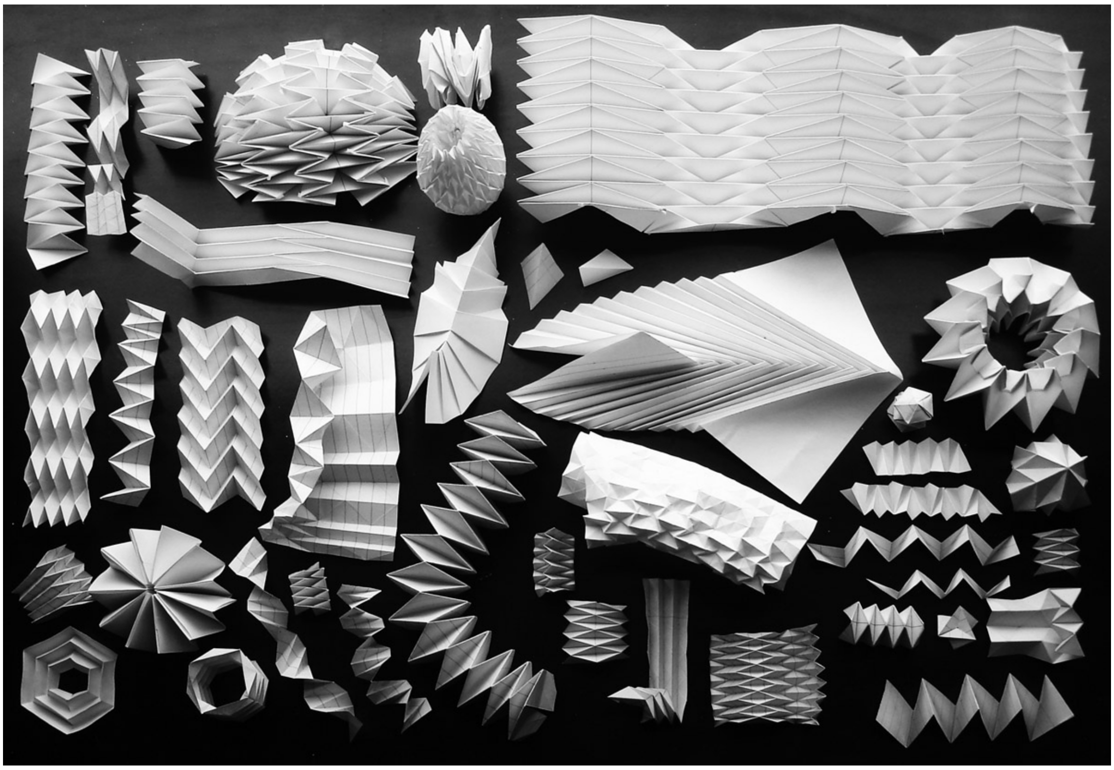


Figure 04 : Developable Double Corrugations – a grid of folded paper models

1.4 What is a DDC?

A ‘Corrugation’ is defined as a wrinkle, fold, furrow or ridge. To ‘corrugate’ means to draw or bend into folds or alternate furrows or ridges. (Source : www. Dictionary.reference.com/browse/corrugation)

In existing geometrical discourse, a Developable Double Corrugation (or DDC) is defined ‘as the repetition of a fundamental region consisting of four identical parallelograms…[and] a generalized surface which includes various shapes depending on its parameters.’ (pg. 138-139, Miura, 2002) Generically in the field of Origami, when DDCs are made by folding paper, they are referred to as *Miura-Ori*. This definition will be clarified and extended in this paper via folding experiments and analysis.

The term ‘Double’ in Developable Double Corrugations refers to the direction of the corrugations in the sheet. Crease lines in DDCs usually exhibit a variety of orientations in three dimensions, with at-least two orientations in plan (to the X and Y directions for instance). In contrast, in Developable ‘Single’ Corrugations or DSC surfaces, all corrugation lines are aligned to only one direction. The term DSC is not commonly used. Figure 05 below indicates the difference between regular DSC and DDC patterns.

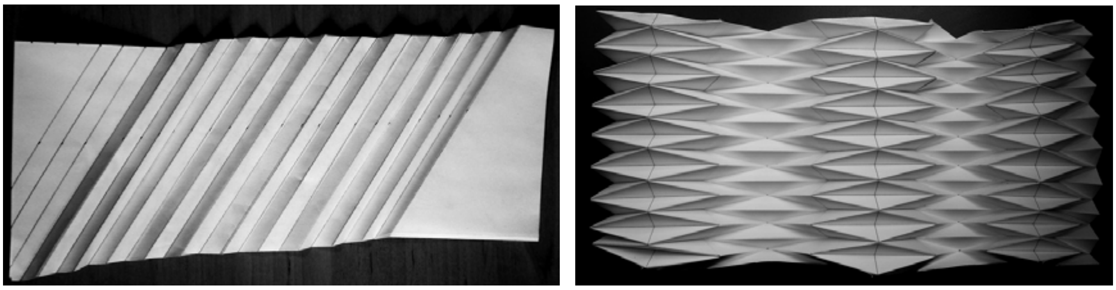


Figure 05 : Developable Single Corrugation (DSC) & Developable Double Corrugation (DDC)

The term ‘Developable’ in Developable Double Corrugations refers to a surface that has zero Gaussian curvature, such as a cylinder, a cone, a hyperbolic cylinder or most ruled surfaces.
(Weisstein, http://mathworld.wolfram.com/DevelopableSurface.html, accessed on 21.06.09)

This means that the surface does not bend or twist out of its plane. In the specific context of a Developable Double Corrugation, it means that the overall form of the DDC may or may not be approximating the curvature of a non-developable surface in space, but each constituent surface of which the corrugation is composed, is free of bending, shear or twisting, and lies in a single plane. This means that a straight line can define that constituent plane by lying on it.

A simpler explanation of this would be that a developable form can be unrolled into a flat sheet of only two dimensions. This is definitely true with reference to DDCs (figure 06 below). On the other hand an un-developable surface has compound/double curvatures, such as a hull surface or a sphere, and can never be unrolled out into a flat sheet of two dimensions accurately. It can at best be a rough approximation.

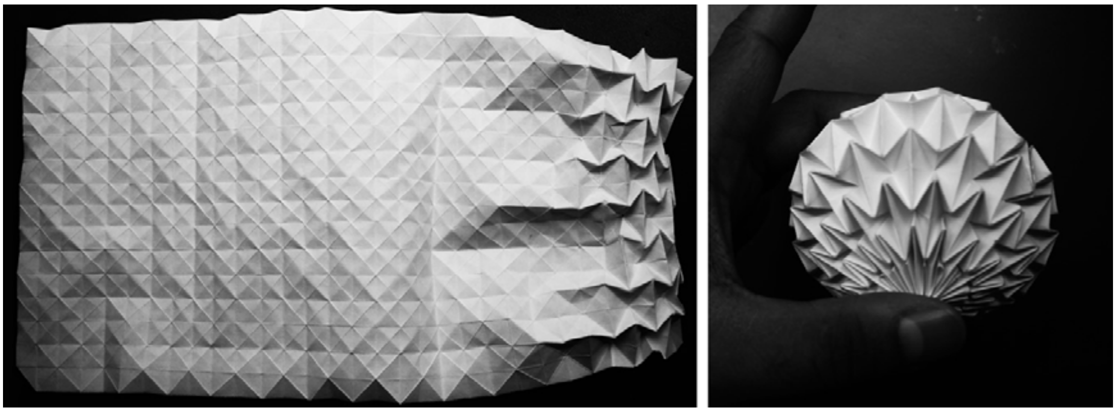


Figure 06 : A flat sheet (left), once creased using an appropriate folding pattern, can approximate a non-developable (almost) spherical form (right), using Developable Double Corrugations – the individual surfaces of the spherical form are all flat two dimensional triangular planes.

1.5 Aims and Objectives

- To critically overview the 'state of the art' in the field of Origami (paper folding)
- To explore as many known 'regular tessellation' folding paradigms in the field of Origami as possible – relevant to Developable Double Corrugations (DDCs) and Flat-foldability
- To find a 'single fundamental starting block' that is the genesis of all known DDCs
- If there is more than one 'fundamental starting block', to catalogue all the possible 'block entities' from simplest to most complicated – akin to the elements on a periodic table
- To explore how the 'fundamental starting blocks' grow into a full corrugated sheet – the mathematics of Creasing Patterns and Folding Sequences by physical folding exercises
- To document all the rules (shape grammar and iterative-algorithmic) that are involved during the generation process

- To approximate families of architectural geometries by using a large family of DDCs through experimentation (by physical folding, and by means of a visual matrix for analysis)
- To discuss all the possible applications of DDCs and flat-foldability in Architecture
- To summarise how the research undertaken in this paper can be taken to the next level of resolution through simulation and optimization

1.6 Hypotheses

One - A fundamental block or block(s) exist(s) (like the unique nucleotides in a DNA sequence AGCT - Adenine Cytosine Guanine & Thymine, for instance) which are found in all Origami developable tessellations, such as corrugations, and these fundamental blocks, by permutation and combination, helping create an entire family of tessellations, specifically all Developable Double Corrugations.

Two – It is possible to approximate and recreate all known architectural forms using the geometry of DDCs as the building sheets for the process.

2 Background and Literature Review

2.1 Origami & Mathematical and Computational Origami

Etymologically, the word Origami [or-rig-gah-mee] originates from the Japanese words *ori* or *oru* meaning 'to fold' and *kami* meaning 'paper'. The Merriam Webster Dictionary defines Origami as 'the Japanese art or process of folding squares of paper into representational shapes'. This is however a very limited definition, as we shall see during the course of this paper.

While origami was originally popularized largely by Japanese culture, its origins are believed to be pre-Japanese, roughly coinciding with the invention of paper itself. Paper, in turn, is understood to have been invented by Ts'ai Lun, a Chinese court official, in 105 A.D. (pg. 167, Demaine & O'Rourke, 2007).

The history [of Origami] within Japan is well-recorded. Time-honoured origami tradition starts with a sheet of paper (usually square). The origamist makes a succession of folds, creating a complex of creases that turns the piece of paper into a montage of polygonal facets.

The rekindling of interest in origami in the 20th century, and the proliferation of origamists throughout the world, is often attributed to the influence of the origami artist Akira Yoshizawa (1911–2005), who pioneered the origami notational system of dotted lines and arrows in his 1957 book called *Origami Dokuhon* (Kamakura Shobo,Tokyo).

The same system, slightly modified, remains in use today. Origami has an intrinsic geometry that is a natural subject of study. The oldest known reference to origami in the context of geometry is an 1840 book by Rev. Dionysius Lardner (1840), which illustrates several geometric concepts using paper folding. (pg. 168, Demaine & O'Rourke, 2007).

Origami has begun to find relevance beyond the merely ornamental (figure 07) and is continuing to expand in intricacy. In the last two decades, amazing technical and artistic advancements have been made in the field, largely due to a growing mathematical and computational understanding and analysis of the subject.

Figure 07 : 32 ways of folding an origami elephant,
Source : John Lang, Origami Design Secrets

In 1936, origami was analyzed in terms of its geometric constructions, according to a certain set of axioms, by Margherita Piazzolla Beloch. This was possibly the first contribution to 'origami mathematics'. It was followed later by Huzita's ' six axioms'. Several fundamental theorems on local crease patterns around a single flat-folded vertex were established by Jun Maekawa, Toshikazu Kawasaki, and Jacques Justin thereafter. Mathematical Origami or 'technical folding' as it began to be referred to, was christened 'Sekkei' in Japanese thereafter. Thomas Hull extended this work into the area of flat-foldability (Hull 2006).

Robert Lang developed an algorithm around 1993 and a software program thereafter for designing origami, which he called the Tree Maker, because it is based on a kind of graph theory that resembles a tree analogically. He has recently published *Origami Design Secrets* (Lang 2003) unfolding a computational approach to origami design.

Mathematical origami research generally coalesces roughly around two foci - foldability and design (pg. 170, Demaine and O'Rourke, 2007).

The first focus - origami *foldability,* generally asks which crease patterns can be folded into origami that uses exactly the given creases, no more and no other. The simplest forms occur when all creases are parallel (as the DSC described in section 1.4) or when the crease pattern has a single vertex. In these cases, we can completely characterize which CPs and FPs together would lead to a successful flat folded state.

The problem arises with crease patterns which have many vertices, all multi-pointed. This is where origami *sekkei*, or 'technical folding' begins to play a part. Through Sekkei, the four flat-foldability theorems were proved (to be discussed in section 2.4).

The second focus - origami *design* is, generally, the problem of folding a given piece of paper into an object with certain desired properties, for example, a particular shape, and specifically in the case of this paper, into architectural forms.

2.2 Crease, Crease Pattern, Folding Pattern, Mountains and Valleys

A *crease* is a line segment (or, in some rare cases, a curve) on a piece of paper. Creases may be folded in one of two ways: as a *mountain fold*, forming a protruding ridge, or as a *valley fold*, forming an indented trough.

A *Crease Pattern* (CP) maybe seen as one of two things –

(a) a collection of creases drawn on a square of paper, meeting only at common end points, which is a (usually planar straight-line) embedding of a graph.

Or alternately,

(b) a division of a square of paper into a finite set of polygonal regions by a set of straight line segments. Each polygon, which is bounded by a combination of creases and the edge of the square, may be referred to as a *facet* of the crease pattern.

Important geometrical features of crease patterns are that they exist in various types of symmetric organizations -

(i) The illustration 08(a) presents bi-axial symmetry and recurring organization of tiles. This pattern is invariable under *n*-mirroring processes.

(ii) The illustration 08(b) presents a rotational symmetry about a point. This pattern is invariable under *n*-fold rotations about the centre.

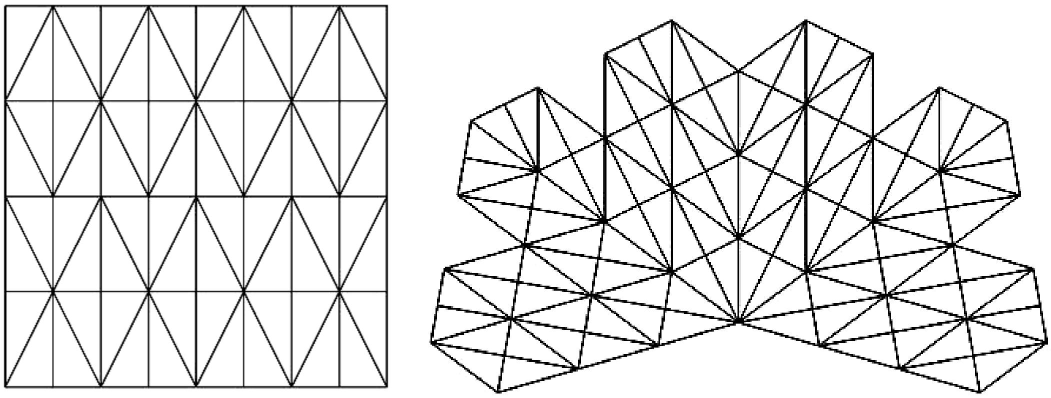


Figure 08 : Crease Patterns showing (a) bi-axial symmetry & (b) rotational symmetry
Source : drawn by Author, based on ideas from 'Expressive Form', Kostas Terzidis

A *Folding Pattern* (FP) is an identification of which creases should be folded as mountains and which as valleys. Together, a Crease Pattern (CP) and a Folding Pattern (FP) describe a Mountain–Valley Assignment (MVA). These somewhat modified definitions are derived from Demaine's original explanations in Geometric Folding Algorithms : Linkages, Origami and Polyhedra (page 169-170, Demaine and O'Rourke, 2007)

Two very important assumptions being made for a 2D surface in a 3D folded state (in Euclidean space) when referring to a CP or an FP are, that the conditions of 'Isometry' and 'Non-Crossing' be satisfied (pg. 172-173, Demaine and O'Rourke, 2007)

Isometry means that the distances between two points, measured by the shortest path on the surface of the paper, is preserved by the mapping, i.e., the mapping does not shrink or stretch the paper.

The Non-Crossing condition specifies that the paper does not cross through itself when mapped by the folded state. Portions of paper are allowed to come into geometric contact as multiple overlapping layers, yet the layers must not penetrate each other, i.e., the mapping must not tear or cut through the paper.

2.3 Huzita's Axioms

Traditionally, origami was designed by trial and error and/or heuristic techniques based on the folder's instincts. Axioms did exist surely, but they were always mathematically unproved. Humiaki Huzita finally put them down on paper by demonstrating six axioms for constructing Origami folds.

These are as follows (illustrated in figure 09) –

A1. Given two points, one can fold a crease line through them.

A2. Given two points , one can fold a crease along their perpendicular bisector, folding one point on top of the other.

A3. Given two lines, one can fold their bisector crease, folding one line on top of the other.

A4. Given a point and line, one can crease through the point perpendicular to the line, folding the line onto itself.

A5. Given two points and a line, one can fold a crease through one point that maps the other point onto the line.

A6. Given two points and two lines, one can fold a crease that simultaneously maps one point to one line and the other point to the other line.

(pg. 37–70, Huzita, 1992)

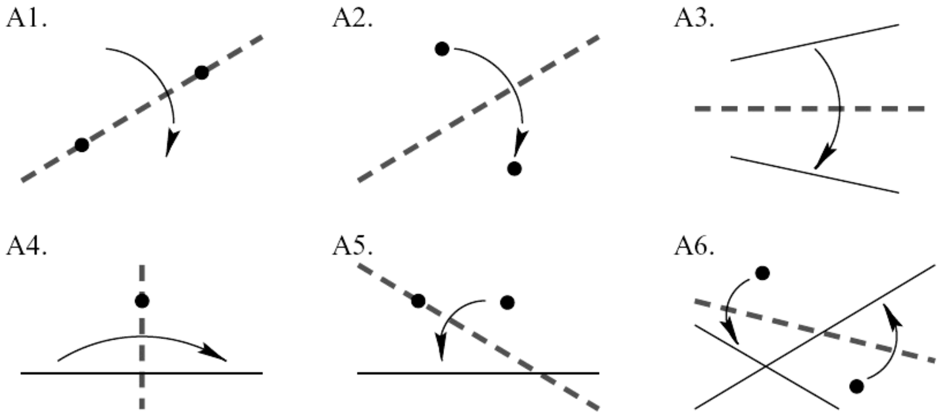


Figure 09 : Huzita's six axioms. Solid lines are existing lines; dashed lines are the new creases (Source : Humaiki Huzita, 'Understanding Geometry through Origami Axioms')

Recently Hatori suggested a seventh axiom. (pg. 31–38, Hull, 1995)

A7. Given one point and two lines, one can fold a crease perpendicular to one line so that the point maps to the other line.

Initially a sheet has only points (the corners) and no creases. The axioms create new creases from existing points and thereafter from earlier creases as well. New points are created by the crossing of previous crease lines with new ones. Huzita has proven that the axioms can construct all plane Euclidean constructions, and can also solve polynomials of degree three, such that cube doubling and angle trisection are possible. Not all kinds of folding can be solved using these axioms however. There are a class of polygons that cannot be folded with these axioms, however a large number of polygons can.

There are two important things to remember while folding -

(a) not all creases are meant to be folded, i.e., not every line on a CP will be given a mountain or valley assignment, and some will remain unused (or flat).

(b) in origami diagrams, there is an implied top surface. Valley and mountain folds are with respect to this top surface and the definition of the top surface keeps dynamically changing after every fold. Xxx Nagpal in her paper says that this is made unambiguous by defining the top surface as apical and the bottom as basal when constructing a computational algorithm (pg. 219-231, Nagpal, 2002).

2.4 Flat-Foldability Paradigms and Rules

Origami models are usually three-dimensional (and definitely all DDCs), and the challenge is to take 3D origami and collapse it into a plane without adding, undoing or damaging any creases. But if one is given just an arbitrary CP, with no ideas as to which are mountains and which are valley folds, the task of differentiating flat-foldable patterns from non-flat foldable ones turns out to be quite delicate.

Given an arbitrary CP then, would it be possible to fold the paper along the creases so that the resulting model can be collapsed into a flat plane? John Lang has proved four basic mathematical rules which apply (Schneider, 2004) : (Figures 10,11,12 & 13)

(1) Crease patterns must be two colourable – if we colour the regions of the crease pattern so that no two bordering regions have the same colour – in flat foldable origami, only two colours are sufficient to establish this pattern

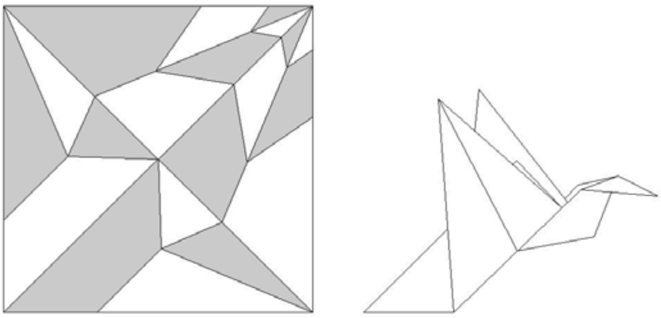


Figure 10 : 2 Colourability
(Source : John Lang – Idea + Square = Origami)

(2) At any vertex, the number of creases meeting at the vertex must be even and the sum of all the alternating angles should add up to 180 degrees; so all the odd angles would add up to a straight line, as would the even angles (Kawasaki-Justin theorem)

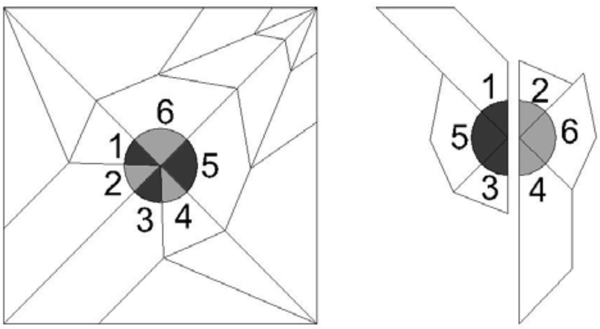


Figure 11 : Alternate angles add up to make 180 degrees
(Drawing by Author, based on original source : John Lang – Idea + Square = Origami)

(3) At any vertex, the number of valley (V) and mountain (M) folds should always differ by two either way, i.e., M – V = ±2 (Maekawa-Justin theorem)

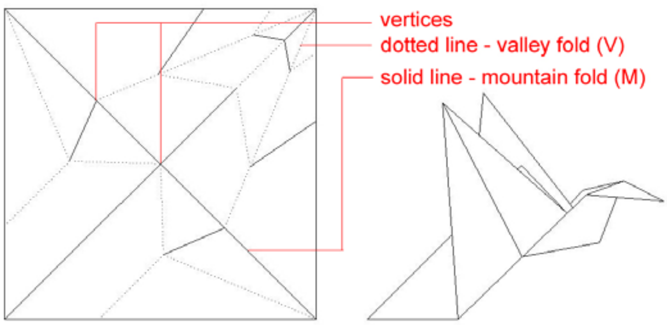


Figure 12 : M – V = ±2
(Drawing by Author, based on original source : John Lang – Idea + Square = Origami)

(4) There must be no cuts in the paper so that a sheet can never penetrate a fold OR to put it more mathematically - There must exist a superposition ordering function that does not violate the non-crossing condition.

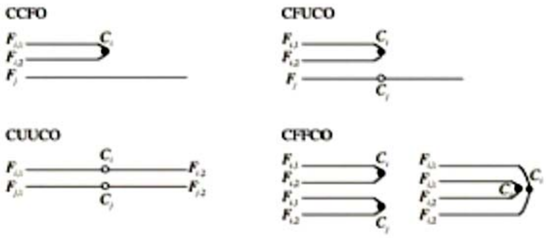


Figure 13 : No self Intersection at overlaps
(Source : John Lang – Idea + Square = Origami)

For the proofs and applications for these rules the reader is referred to Schneider's Article (see references). However, it is important to note that while creating a computational algorithm to create form through folding, if these rules are abided by, it can lead to complex forms, which at the same time are completely flat foldable. This has important implications for product design, architecture and industry. For this research project, though flat-foldability was not an essential pre-requisite, it seemed a very useful idea pertaining to Developable Double Corrugations, on the premise that that they would collapse into a 2D plane while folded, ignoring the thickness of the paper itself.

2.5 State of the Art : Origami Uses and Applications

A lot of mathematicians are studying the contributions that Origami can make to a better understanding of phenomena in the physical world.

Figure 14 : The un-creasing / unfolding of leaves in spring from closed buds

For instance a mechanism in nature, from which we can learn much, is how leaves of some plants are folded or rolled when un-blossomed inside the bud and how they unfurl thereafter during blossoming. Cedar or Beech tree leaves (figure 14) have simple and regular corrugated folding patterns.

These patterns can recommend ideas for the design of deployable forms and structures such as solar panels and light-weight antennae of satellites, or for the folding of membranes such as tents, clothes or other coverings such as large scale parasol umbrellas, which need to be tightly packed and reduced to a small size during transportation / pre-deployment and then, to expand to their full size at the site.

A number of biological folding patterns have been investigated by researchers such as Kobayashi, Kresling et al. In their paper 'The Geometry of Unfolding Tree Leaves', they describe and investigate the Miura-Ori map folding paradigm in great detail, (see references) concentrating on folding mechanisms for strengthening, deployment and other optimization criteria.

Robert Lang created a 5m wide aperture, foldable Fresnel transmissive telescope lens for the Lawrence Livermore National Laboratory which is a prototype for an eventual 100m wide aperture lens for a telescope (see figure 15). Most high-performance telescopes, like the Hubble Space Telescope, for instance, are "reflective." Their main optical element is a curved mirror - The Hubble, has a 2.4-meter-diameter mirror. But "transmissive" telescopes, like the Galileo, are tubes with lenses at each end, and these lenses need to be large to be powerful.
So designing this size of lens was a big breakthrough, because till recently, transmissive space telescope lenses were restricted in size by the diameter of the rockets that could carry them to space, or else they had to be assembled in space accompanied by a manned space vehicle. It will eventually be possible to launch extremely large telescopes with powerful foci for deep space, all made possible by advances in mathematical and computational origami. This is just one of many examples that illustrate the future potential of Origami to act as an instrument of technological problem solving.

Figure 15 : Robert Lang with the 5m Fresnel foldable lens designed using an algorithmic application of graph theory.

Source : 'Origami : Complexity in Creases', Robert Lang, 2004, Journal of Engineering and Science, No.1

3. Methodology (Forwarding the Hypothesis)

3.1 Types of DDCs

During the initial experimentation stages, 87 regular CPs were drawn up and tested for whether they folded into any forms at all, and if they did, then what was the resultant shape of the folded artifact, and finally testing the CP on the basis of the flat-foldability rules (Section 2.4 in Chapter 2), to observe if these folded artifacts collapsed into 2D without damage to the creases (somewhat like an accordion). Since the CPs were initially drawn up in a random and arbitrary process, it was surprising that around 47% folded into flat foldable symmetric geometries, 11% folded into symmetric geometries but were not flat foldable, 17% folded into asymmetric forms or arbitrary topologies that were not flat foldable, and 25% failed to fold into any forms. Although there was no selection criteria for the CPs, and the attempt was to try and randomly generate as many DDCs as possible, one point was statistically clear – of this small sample of 87, none of the asymmetric geometries were successful flat-foldable DDCs, although this in no way mathematically suggests that it is not possible for an asymmetric DDC to flat-fold. But statistically it seems unlikely. This point was investigated further and will be taken up in subsequent chapters. The exact meaning of 'symmetry' in this context will also be discussed and elaborated. The 47% successful flat-foldable DDCs (41 of 87), were categorized into 9 discernible groups based on similar CPs and FPs. No standardized or exhaustive catalog was found during the literature survey in published mathematical origami, for nomenclature and indexing of DDCs. The nine models selected for the purpose of initial enumeration and discussion in this paper, were chosen because –

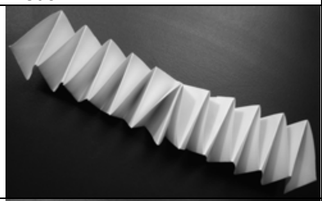
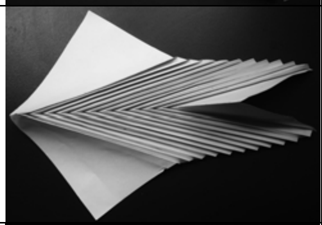
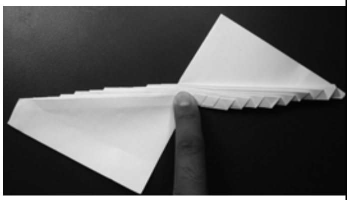
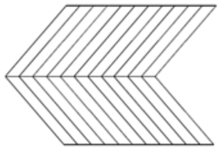
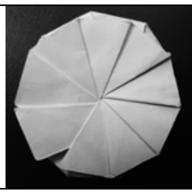
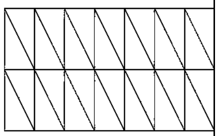
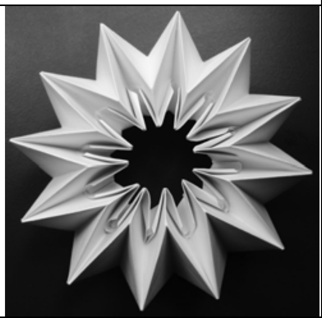
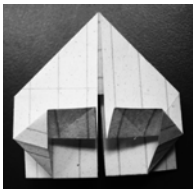
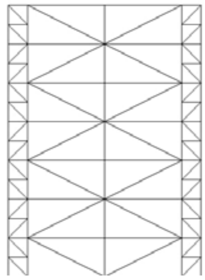
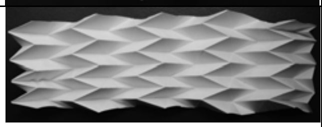
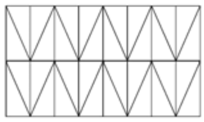
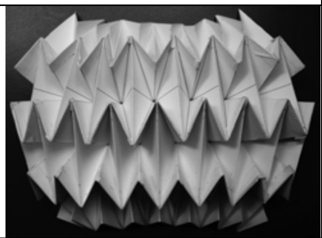
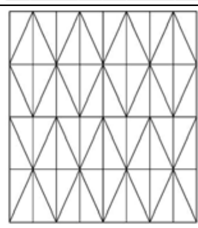
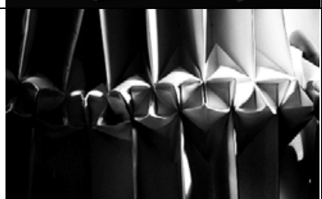
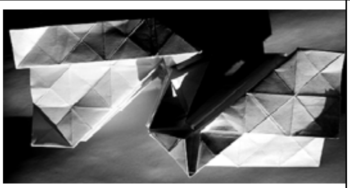
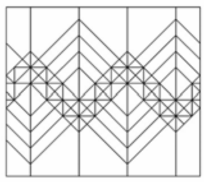
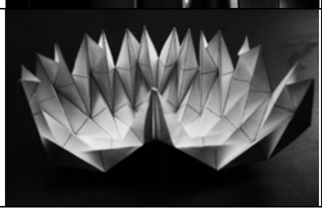
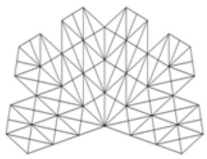
(a) All nine were flat-foldable once folded into sheet form with open ends (although some were not so in their final form, if they were closed on themselves in a circular loop).

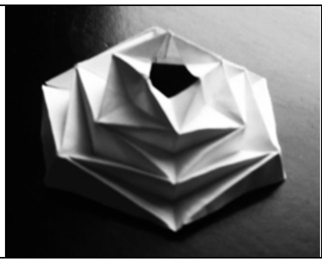
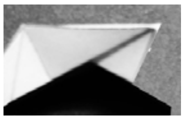
and

(b) They encapsulated a large variety of DDCs possible by being more or less representative of every possible genre of CPs and FPs encountered during the literature survey.

Because, there are no consistent nomenclature systems for DDCs, and different Origami artists and mathematicians call them by different names, for the sake of this paper, the author has chosen to follow the most commonly used names, or given a name where a DDC had none (to begin with). In the intermediate stages of the paper however, a more rigorous and logical system of nomenclature and notation will be presented. A note here - in Origami, the name of the creasing pattern (CP) becomes analogous to the name of the final folded form, so in fact a Hill-Trough CP (HT) can generate many final folded forms, based on changing FPs for the same CP. This point is also taken up and discussed in further detail, anon. Table 01 on the following page, lists the nine selected DDCs, with their Crease Patterns and flat-folded side profiles.

Table 01: Nine Initial & Generic DDC Samples (from Simplest to Complicated)

S.No	DDC Type	Visual	Flat-Folded Side Profile	Crease Pattern (CP)
1	Cater-pillar			
2	Zigzag or Tree Leaf			
3	Spring-Into-Action			
4	Fire-Works			
5	Hill - Trough			
6	Magic Ball			
7	Gothic River			
8	Lotus			

9	Pyramid			

3.2 Of Triangles : First Principles

Studying the 87 physical models (and the selected nine thereafter), it becomes clear that the most basic component involved in a DDC is a triangle; and all regular Developable Double Corrugations start with a single fold. On both sides of this single fold, there are two triangles. Sometimes, it may be that there are two quadrilaterals, but essentially hidden within those quadrilaterals are two triangles each, with the fold between them not yet manifest. It must be noted here that this is however only true for 'flat-foldable' and 'developable' double corrugations. For 'non-developable' corrugations (with non-zero Gaussian curvature surfaces) or corrugations that are not flat foldable - polygons of higher order, such as pentagons, hexagons or even non-regular geometries may be involved. Investigation of non-developable geometries is beyond the scope of this paper.

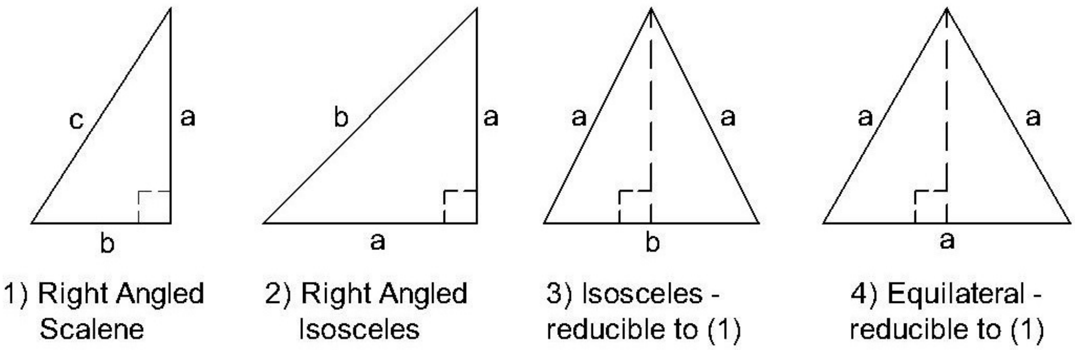


Figure 16 : Types of triangles in DDCs

It is axiomatic that all quadrilaterals are reducible to two triangles. In the case of DDCs, all quadrilaterals and triangles are reducible to right angled triangles (figure 16).

3.3 Definitions : Hinges, Kernels & Strings

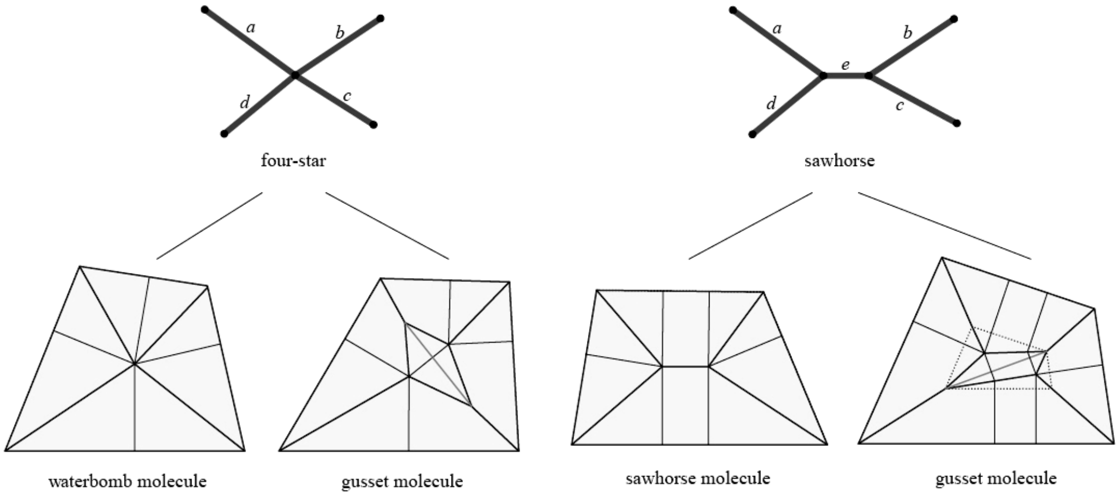


Figure 17 : A few ‘Molecules’ – water-bomb, gusset, sawhorse & gusset (type 2)
(Source : John Lang, ‘Origami : Complexity in Creases (Again)’)

Origamists define certain ‘molecules’ as the building blocks of all origami CPs and FPs. (pg.9-19, Lang, 2000&2004) Figure 17 shows some molecules. While these are fine for solving problems of folding and creasing, after closely studying physical folded artefacts, it is clear that molecules cannot solve the objectives outlined in this paper, because –

(1) There are too many ‘molecules’ in the world of Origami, and therefore it is difficult to classify them as fundamental building blocks
(2) The molecules do not share common traits and it is not possible for a CP which is defined in terms of one molecule to be re-defined in terms of another
(3) Molecules are far from simple (geometrically) and need extensive and elaborate mathematical definitions to identify them.

Thus a simpler and more rigorous system is called for to be able to classify all DDCs (except the ones with arbitrary topology).

For this purpose, three ideas are being introduced in this section, which form the backbone of this entire paper. Since these ideas were formulated as a result of folding experiments and observations made there-after, ideally these definitions should have followed the section on observations and not precede it, but explaining consequent formulations would become increasingly tedious if these definitions are not introduced at this stage. As we move into consequent sections, there will be clarity about the genesis of these concepts, and how they are very useful concepts for the problem at hand.

The first idea is that of a 'Hinge Line'. It may be defined *as the locus of points or nodes where 4 or more crease/fold lines meet*. Figure 18 illustrates this definition. The idea is christened as a 'hinge' line, because it controls the form of the DDC with its length and its shape in 3D Euclidean space. Hinge lines can be any straight, zigzagging, curvilinear, parabolic, elliptical or circular.

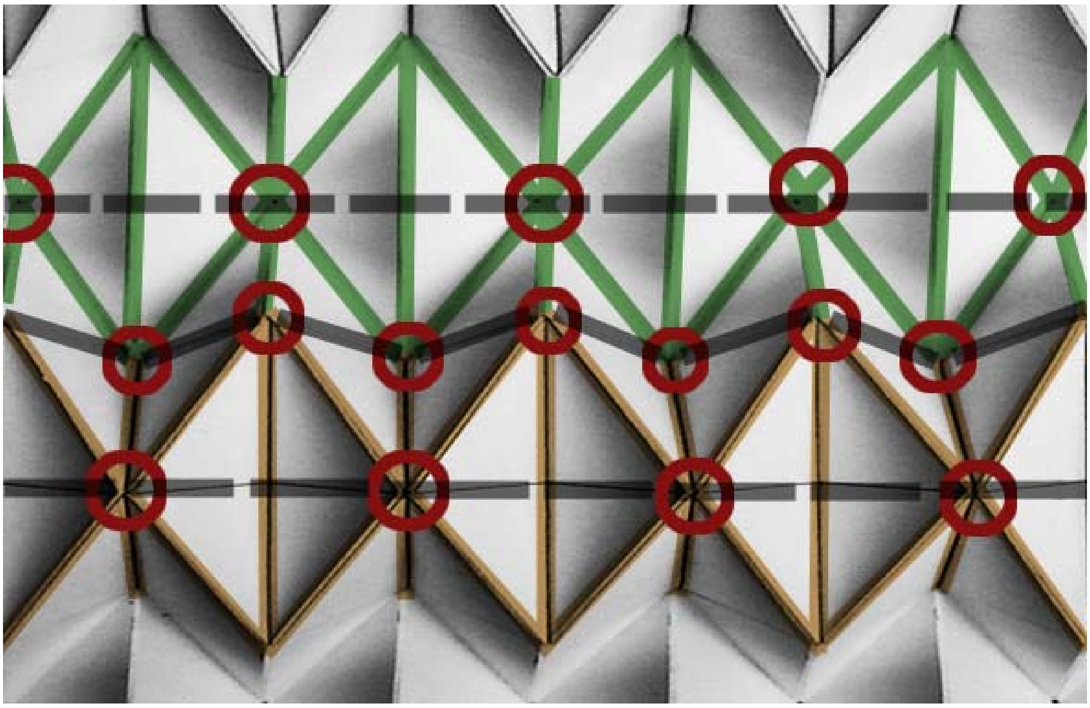


Figure 18 : Of Hinge Lines : Green and brown lines indicate the creases/fold lines. The red circles mark the nodes/points created by the creases. The grey dotted lines are the hinge lines. Note that from each node originate atleast four creases/fold-lines.

The second idea is that of a 'kernel'. A 'kernel' may be defined *as a two-triangle elementary unit (usually a rectangle or square, but always a quadrilateral) enclosed between two hinge lines on the creasing pattern of a DDC*.

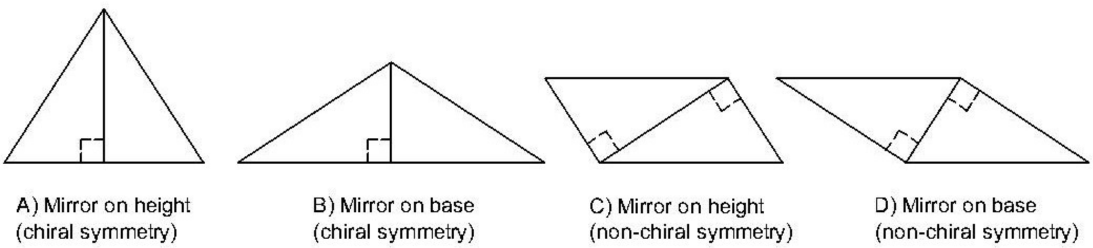


Figure 19 : All Wrong Assumptions for a Kernel

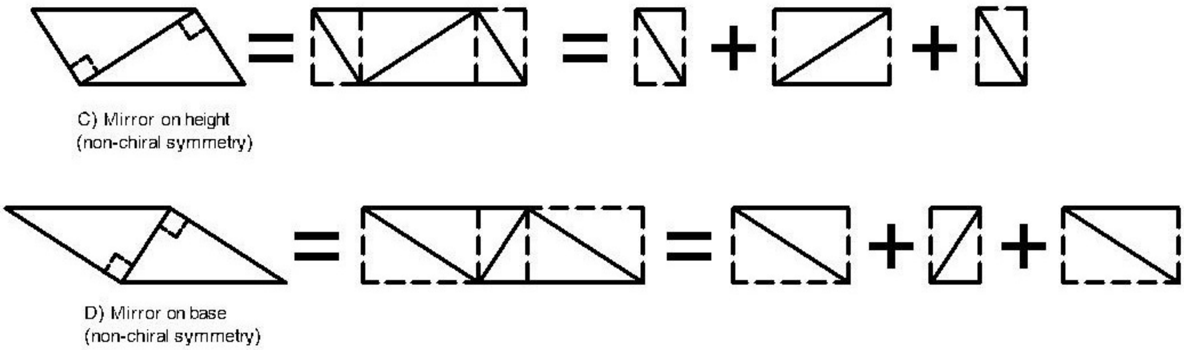


Figure 20 : Why (C) & (D) in figure 18 are not kernels

Additionally, the two triangles of the kernel are always joined to each other along their longest length (i.e., the hypotenuse). Therefore the diagrams shown in figure 19 are not kernels – (A) & (B) are not kernels because they are not quadrilaterals but bigger triangles generated by the joining of two smaller triangles, and although (C) & (D) *are* quadrilaterals, they too are not kernels, because the constituent triangles forming them do not join at their hypotenuses, but via their shorter edges. This makes it possible for us to redefine these shapes into three or lesser simpler units which then are simpler quadrilaterals than the original and therefore kernels. This is explained in figure 20. Thus, as per the definition of this paper, parallelograms and rhombuses are not kernels.

In summary, a kernel of a DDC is –

(a) a two triangle elementary unit, where the triangles join at their hypotenuses (or longest edge)
(b) always a quadrilateral
(c) the simplest quadrilateral which cannot be redefined in one simple operation as a sum of three or lesser number of kernels
(d) always enclosed between two hinge lines in a CP

Figure 21 shows all the possible types of kernels (as per the definition in this paper).

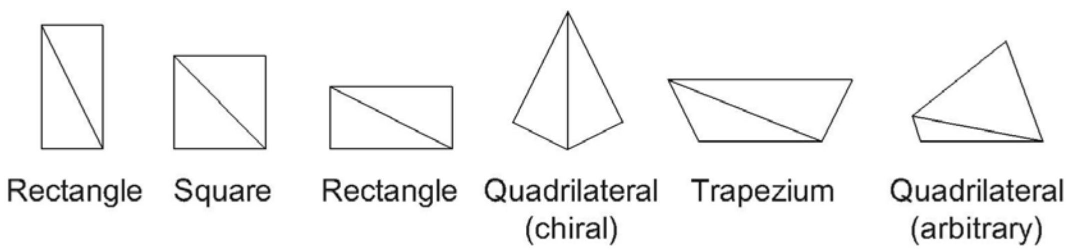


Figure 21 : Kernels

These six, most basic geometries, are 'in essence' the fundamental building blocks of all DDCs. Regarding the sixth, the 'arbitrary Quadrilateral', the same conditions are imposed as on the other five – which means, it cannot be just *any* arbitrary quadrilateral, but must fulfil all the four criteria noted above to qualify as a kernel.

Kernels can be in three and only three states –valley fold, mountain fold, or flat. Figure 22 illustrates.

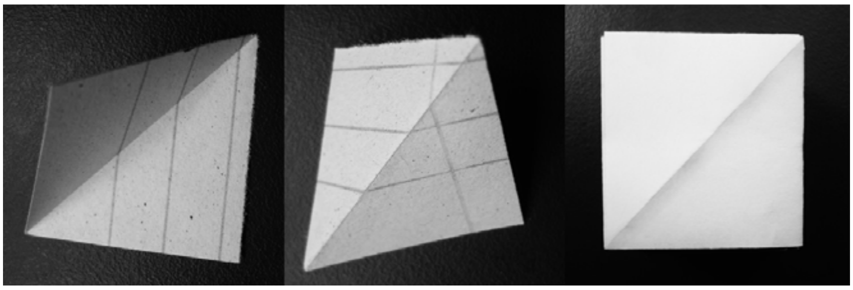


Figure 22 : Kernel states illustrated with a square kernel – valley fold, mountain fold & flat (creased but unfolded)

The third and final idea is that of a String. A 'String' is *a continuous strip of kernels connected based on accretion rules, and enclosed between two hinge lines. In a DDC creasing pattern, there can be one, and only one string between two hinge lines.* Figure 23 illustrates.

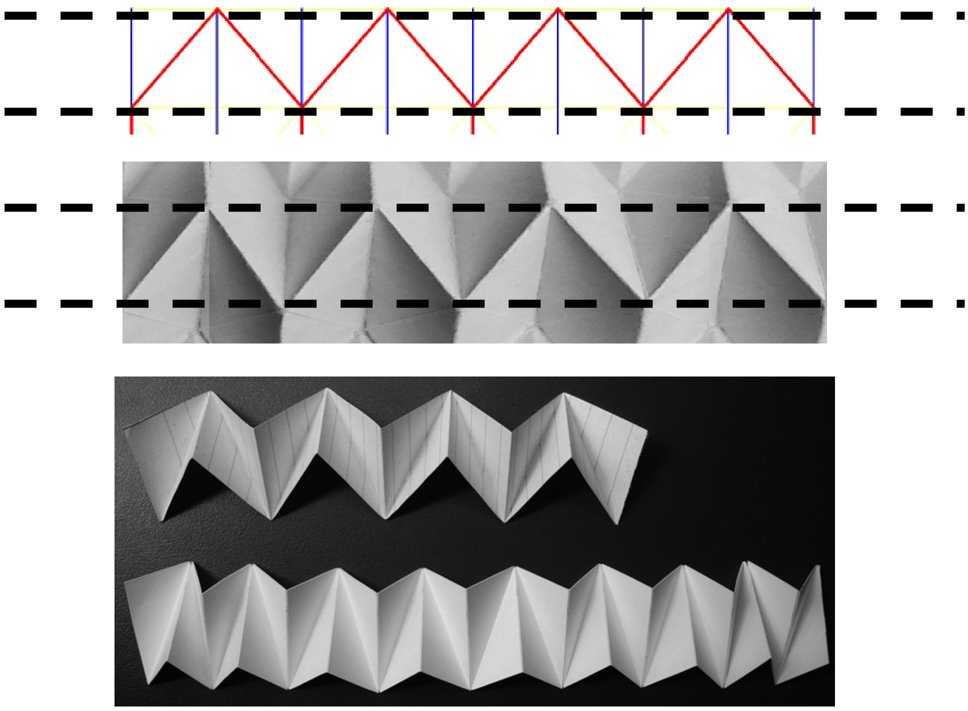


Figure 23 : A String, as seen on a CP and on a folded pattern between 2 hinge lines (black dotted); and individual strings in folded state (seen in isolation independent of a DDC surface)

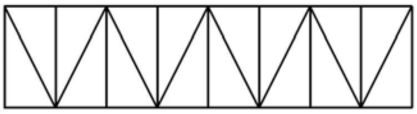
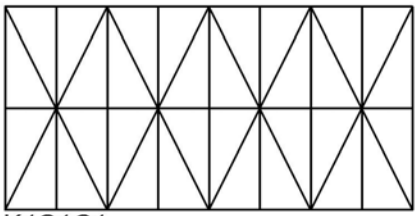
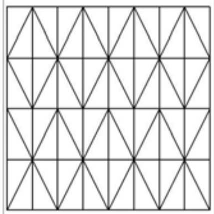
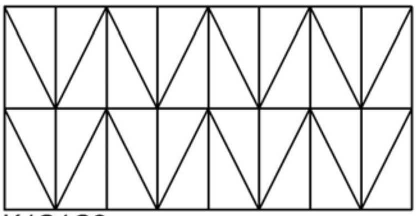
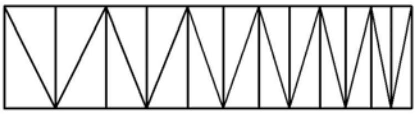
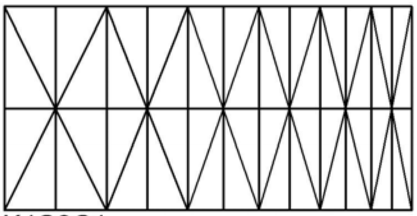
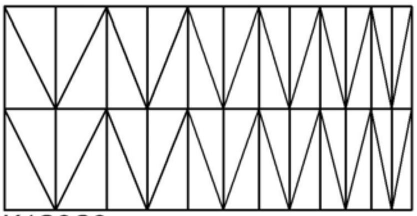
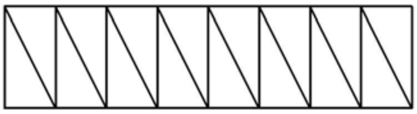
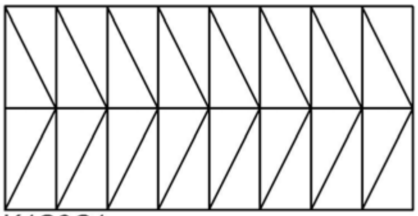
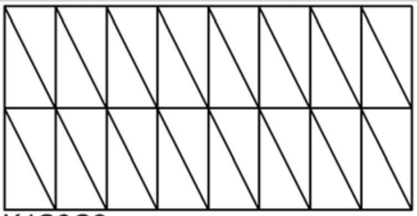
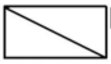
These three ideas and their definitions form the primary basis for analysis of the DDC sheets, their CPs and FPs, in this paper, and the ground on which the hypothesis is placed and proved.

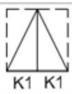
4 Applying DDCs to Form : Experiments & Observations

4.1 Analysing Kernels & Strings in DDCs

Now that we have three new concepts, it is time to apply them to actual Crease Patterns (CPs). As a result of this implementation, we get four Look-Up Tables for Kernels, Strings and Crease Patterns. These are presented next.

Table 02 : Kernels, Strings and Crease Patterns (One)

S.No	Kernel (K)	String (S)	Creasing Pattern (CP or C)	Notes
1	 K1 Rectangle (triangles with Non-chiral symmetry)	 K1S1	 K1S1C1	 Shown above is K1S1C3, a hybrid of CPs - K1S1C1 & K1S1C2.
			 K1S1C2	Many such combinations can and do exist, but all are not flat-foldable.
		 K1S2	 K1S2C1	Individually, in K1S2C1 & K1S2C2, as the width of the kernels reduce, the sheet's tendency to curl inwards increases rapidly in arithmetic progression, and soon, the sheet cannot be folded further physically, unless the pattern is abruptly changed and a hybrid of K1S2C1 & K1S2C2 is created.
			 K1S2C2	
		 K1S3	 K1S3C1	K1S3C1 CPs typically create the simplest DDCs. K1S3C2, on the other hand, shows twisting behaviour, alternating flaps fold in and *hide* inside the next flap.
			 K1S3C2	This renders it ideal for DDCs where segments of paper in the middle of the pattern are redundant and need to *go missing*.
2	 K2	K2S1, K2S2, K2S3	K2S1C1, K2S1C2, K2S2C1, K2S2C2, K2S3C1, K2S3C2	K2 & K3 CPs exhibit similar properties to those of K1; K2 folded patterns develop spiralling behaviour & form loops, physically halting the growth of the pattern
3	 K3	K3S1, K3S2, K3S3	K2S1C1, K2S1C2, K2S2C1, K2S2C2, K2S3C1, K2S3C2	

NOTES:  is not a kernel, because the hypotenuse of the constituent triangles do not join with each other.  is part of a string composed of two K1 kernels, joined together as shown 

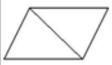
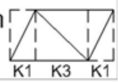
 Parallelograms (& rhombuses) are not kernels, they are reducible to more basic units.The diagram here shows how a paralleogram actually consists of a combination of two kernels of K1 & K3 type 

Table 03 : Kernels, Strings and Crease Patterns (Two)

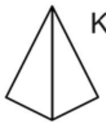
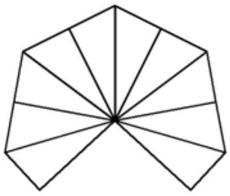
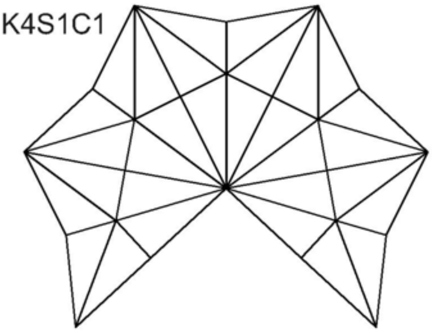
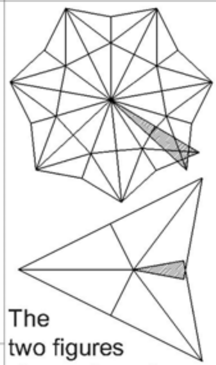
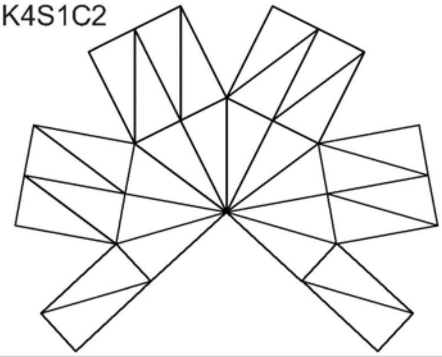
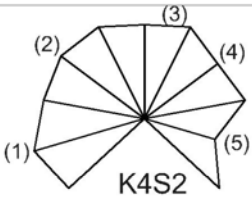
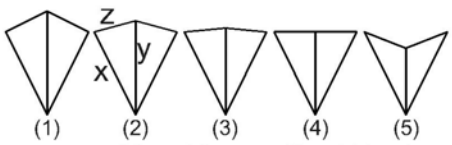
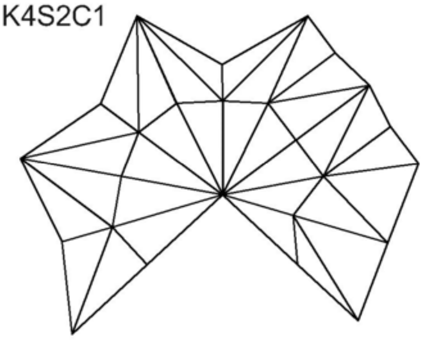
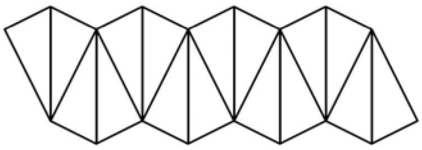
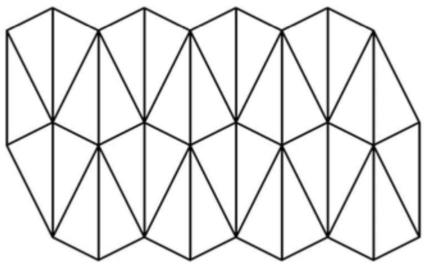
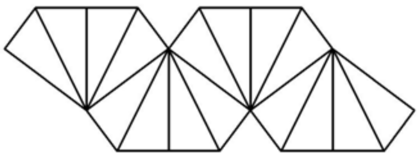
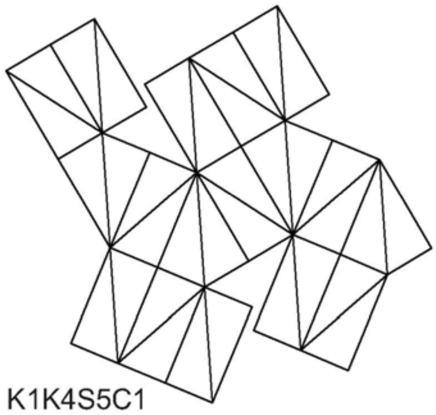
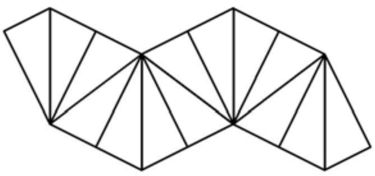
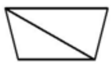
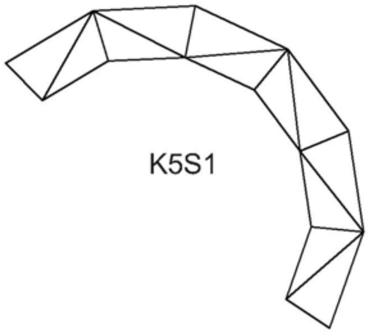
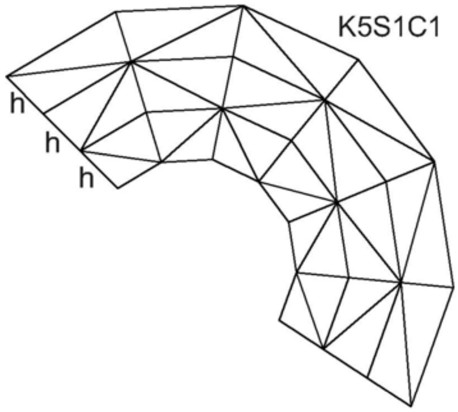
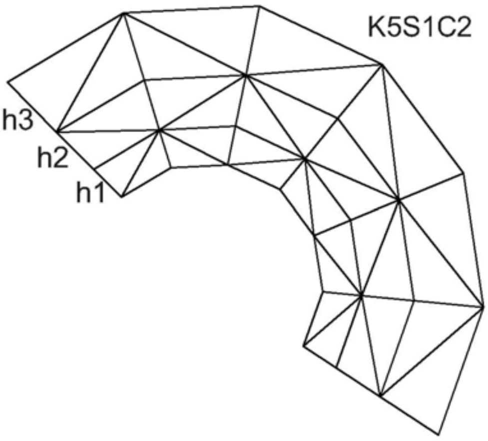
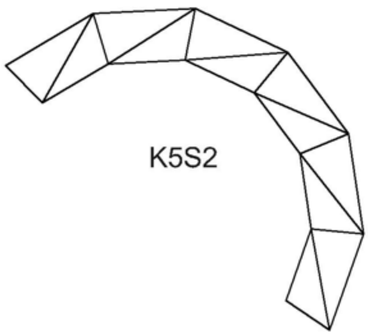
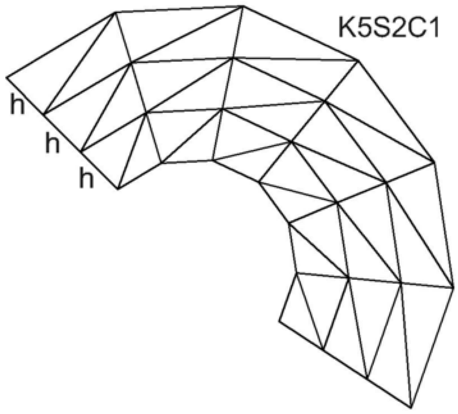
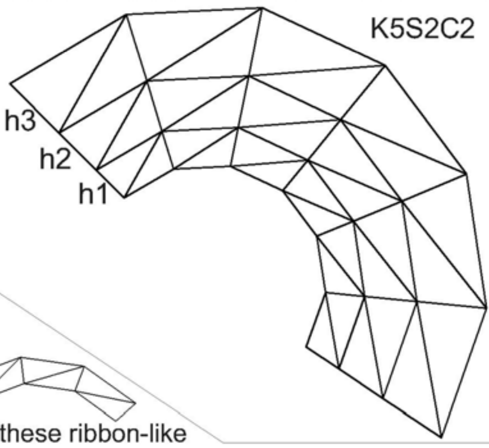
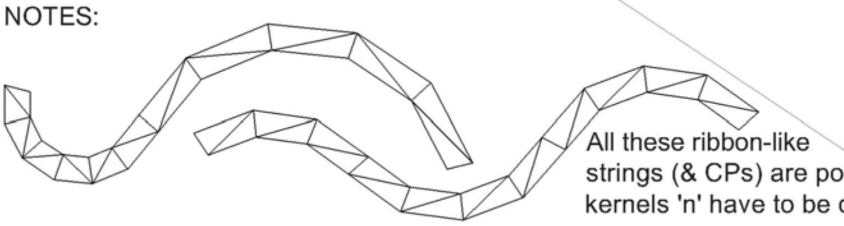
S.No	Kernel	String	Creasing Pattern (CP)	Notes
4	 K4 Quadrilateral (triangles with Chiral symmetry)	 K4S1	K4S1C1 	 The two figures above show the limiting cases of radial tesselations where the kernel eventually overlaps with another kernel and the CP is no longer printable on a single sheet of paper, although the pattern may be flat foldable.
			K4S1C2 	
		 K4S2  x,y,z are sides of the constituent triangles	K4S2C1 	As z changes, y decreases, till x>y; at that point, triangles (3), (4) & (5) are by definition, not kernels, only reducible to other kernels. Arbitrary pattern K4S2C1 is foldable, and with careful calc. of int. angles, even flat-foldable.
		 K4S3	 K4S3C1	
		 K4S4	 K1K4S5C1	(a) String K4S4 is a hybrid of two strings K4S1 & K4S2. (b) String K1K4S5 is a hybrid of two kernels, K1 & K4. Many such combinations exist, except when radiating patterns on a single sheet are restricted by the limiting case of overlapping kernels
		 K1K4S5		

Table 04 : Kernels, Strings and Crease Patterns (Three)

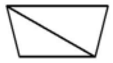
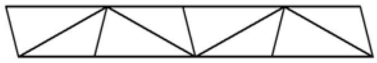
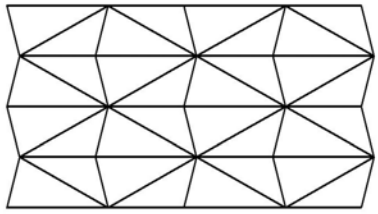
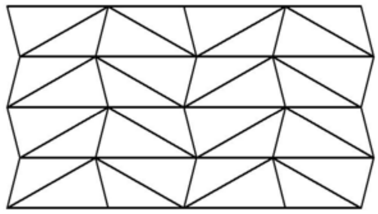
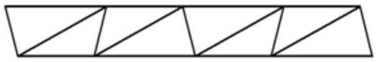
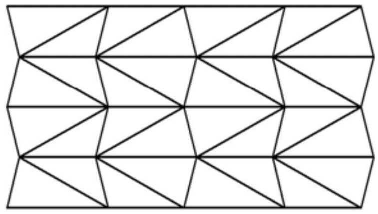
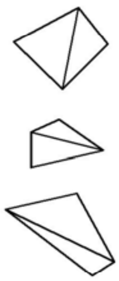
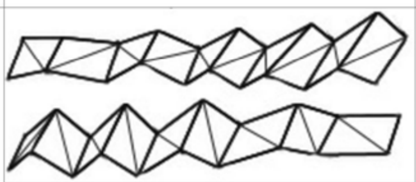
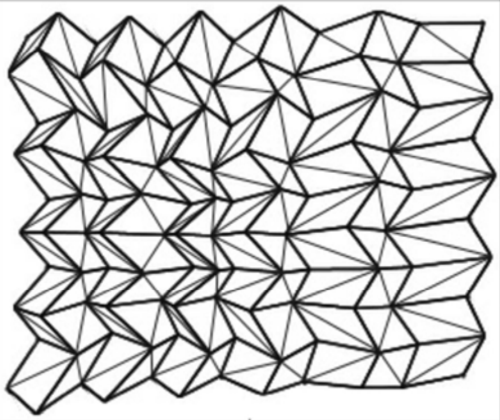
S.No	Kernel	String	Creasing Pattern (CP)	Notes
5	 Trapezium (uni-axial, non-chiral symmetry)			In K5S1C1, height h of concentric trapezium rings remains constant, and they get squarer in proportion. In K5S1C2, concentric trapezium rings are scaled replicas of each other, therefore, internal proportions of each kernel remaining the same, the hts., h1, h2, h3 and so on, keep changing in arithmetic progression. K5S1C1 & K5S1C2 both generate pyramids - of n sides when closed. K5S1C2 may generate a pyramid of golden ratio, if the h1, h2, h3 are carefully calculated.
				
				Helical & toroidal are created for CPs K5S2C1 & K5S2C2. With hybrids, sinusoidal forms are possible.
				

NOTES:



All these ribbon-like strings (& CPs) are possible with K5S1 & K5S2. The number of kernels 'n' have to be defined, & the rule for addition to the string.

Table 05 : Kernels, Strings and Crease Patterns (Four)

S.No	Kernel	String	Creasing Pattern (CP)	Notes
5	K5  Trapezium (contd.)	 K5S3	 K5S3C1	Hybrids of all three cases - K5S3C1, K5S3C2 & K5S4C1 are possible, but hybrids will usually not flat-fold (though not necessarily so), although they are certain to fold into DDC sheets. Nomenclature of hybrid sheets is discussed in detail later in the paper.
			 K5S3C2	
		 K5S4	 K5S4C1	
6	 Arbitrary quadri-laterals			Arbitrary quad-ri-laterals do generate surfaces which are flat- foldable. Shown here is a distorted Miura-Ori CP which is perfectly flat-foldable. It only has to be ensured that at each node of the crease pattern, the four laws of flat-foldablility are satisfied. Since many such surfaces can exist, based on un-countable irregular arbitrary quad-ri-laterals, this CP is not classified in this table, neither are the generator quad-ri-laterals which are all different. However, the surface generated is very much still a DDC, and for that reason is included in this table.

NOTES:

These tables are by no means an exhaustive listing, but a tabulation of kernel genres.

Although the notation system seems unwieldy and impractical to use, it serves two very important purposes -

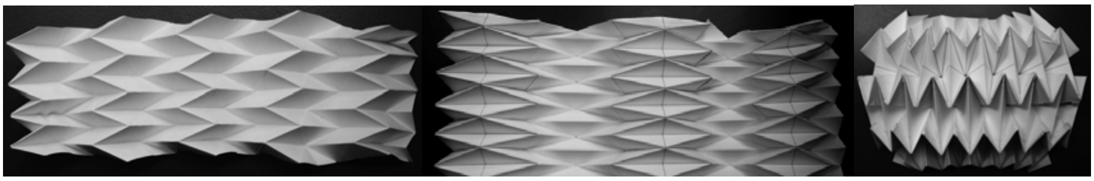
(a) each name illuminates in a single line of alphabets and numbers, the complete genetic code of the crease pattern (i.e., the rules of replication and generation are embedded in the name of the pattern itself).
(b) it is possible to add newer elements at each level - for instance, after K5, kernels K6 and K7 may be added; after K1S4, strings K1S5 and K1S6 may be added; and all these insertions can be made at any level without disturbing the order of the table, due to the strict bottom up hierarchical nature of the nomencalture system.

However, this system of defining the final folded patterns is as yet incomplete, because the effect of FPs has not yet been factored in. The next section deals with this paradigm.

4.2 Making Sense of Folding Patterns (FPs)

From the tables 02-05 we now have a clear classification system for Crease Patterns (CPs). We can also recognize (a) kernels, (b) strings they generate, (c) possible configurations of CP sheets that these strings then join together to forge and (d) hybrids. Further we have a notation system which allows us to recognize and categorize a CP using a look-up table as reference.

But we still cannot fold a CP into a DDC. For that we need an FP. Let us look closely at the FPs of a few DDCs in conjunction with their CPs. Of the nine DDC types identified in section 3.1, let us choose two types of 'Hill-Trough Creasing' and one 'Magic Ball Creasing', for their generic linear regularity and ease of translation to a comparative matrix (figure 24) .



Hill-Trough One (HT1) Hill-Trough Two (HT2) Magic Ball (MB)

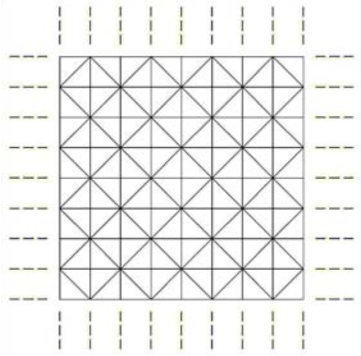
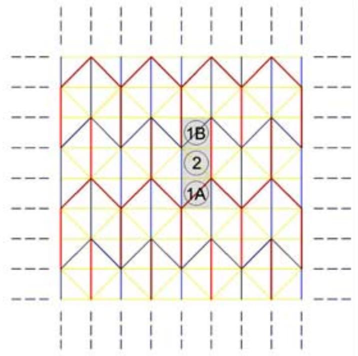
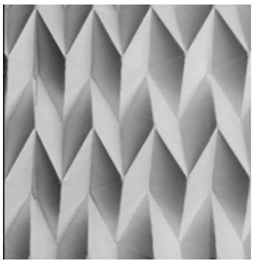
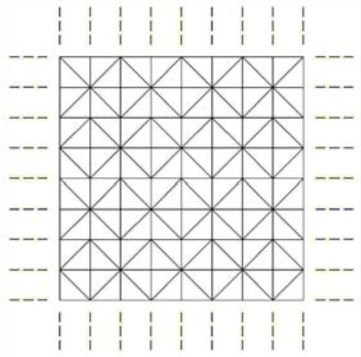
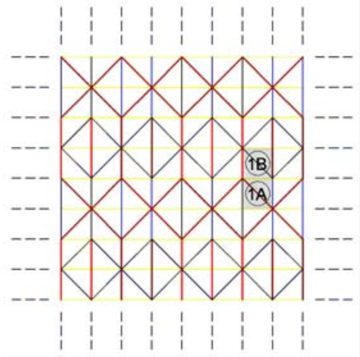
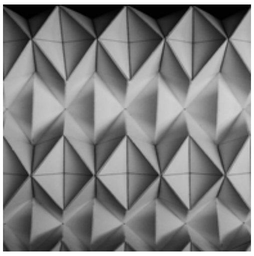
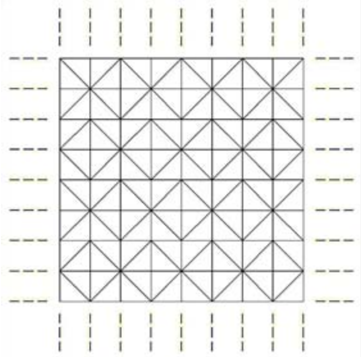
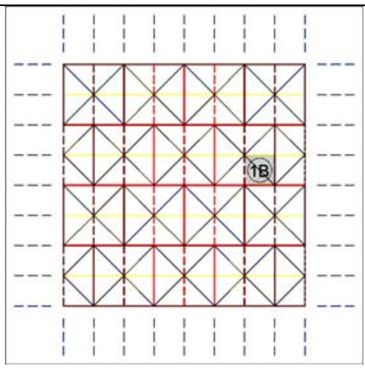
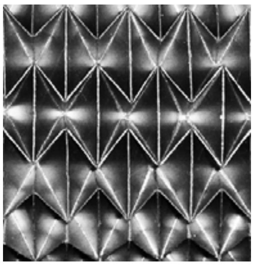
Figure 24 : Three DDCs as samples for comparative analysis of CPs and FPs.

Table 06 illustrates the creasing pattern (CP), folding pattern (FP) and final corrugation of three DDCs – two of them of the HT type, and one of the MB type. In the column of folding patterns (FPs), the red lines indicate crest lines / mountain folds, and the blue lines indicate trough lines / valley folds. The faint yellow lines indicate flat folds (unfolded creases). The dotted blue-grey lines are the hinge lines.

We already know that a kernel can be in three states – mountain fold, valley fold or flat. A kernel can connect to two other kernels in a string each of which can be in any one of these three states. Each combination of string thus formed can combine with two other strings, each of which can be in any configuration. This allows for a confounding number of possible folding patterns for a single crease pattern. In reality only about a dozen or so FPs are practically possible for a given CP, generating 12 different kinds of corrugations with a single crease pattern.

In the tabulation below, HT2 and MB share an identical CP, but very different FPs, and this reflects in the difference of the final corrugations that emerge.

Table 06 : CP, FP & Corrugation : Comparative Matrix

	Crease Pattern (CP)	Folding Pattern (FP)	Final Corrugation
Hill-Trough Creasing Type One (HT1)			
Hill-Trough Creasing Type Two (HT2)			
Magic Ball Creasing (MB)			

	Crest Line / Mountain Crease (Continuous)
	Crest Line / Mountain Crease (Discontinuous)
	Trough Line / Valley Crease
	Unfolded Line / Flat Crease
	Hinge Line
1B	Kernel

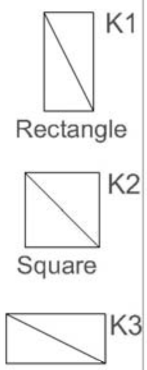
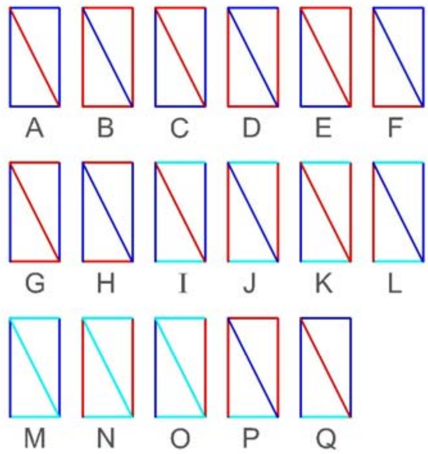
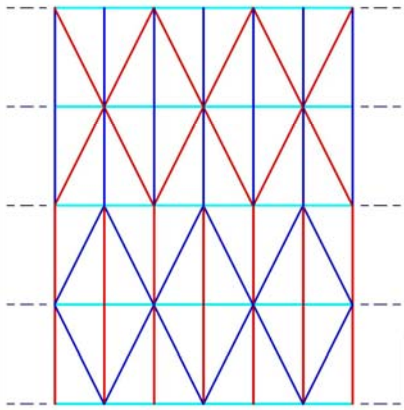
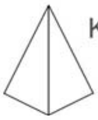
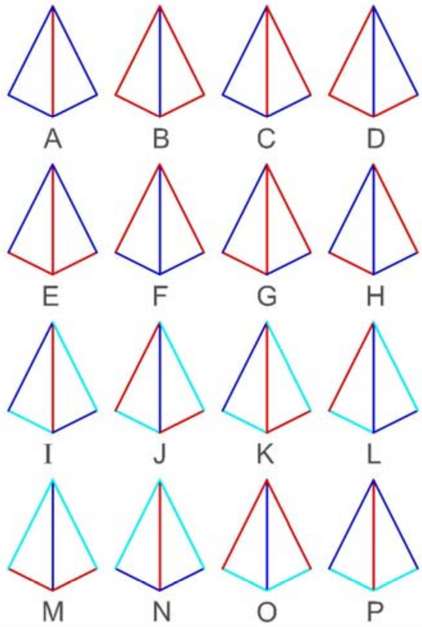
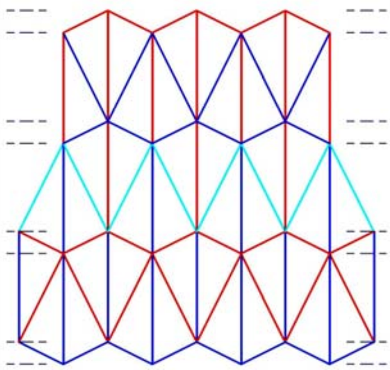
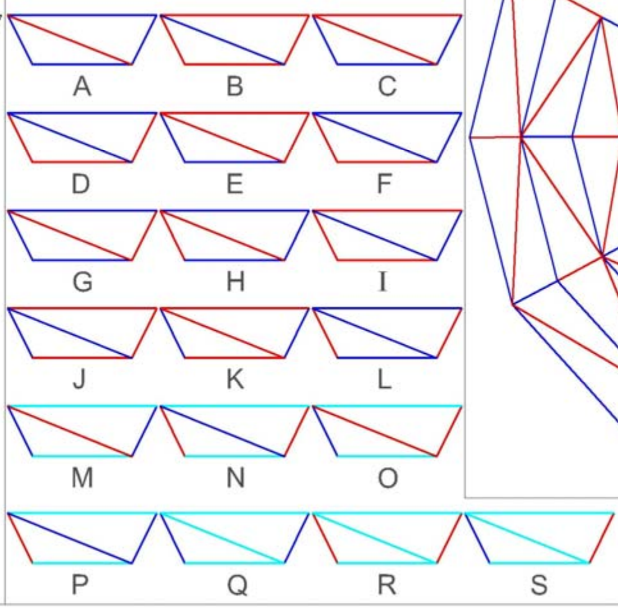
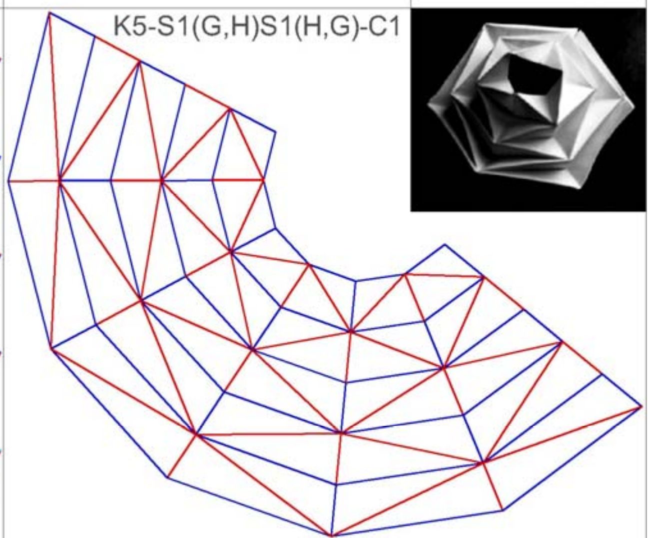
We conclude, that we need to factor in the FP into the notation system, to be able to completely define and categorize a DDC. The notation system described and elucidated in tables 02-05, although very useful, is still incomplete. And here the role of the next table 07 comes into play.

It completely describes all the possible states a kernel can be in, vis-à-vis fold conditions, and each kernel state is given an alphabet. In the notation system, K1-S1(I)S1(J)-C3 (see table 07) means that the DDC is composed of the kernel K1, the string is of the S1 type, in the first string, the state of the kernel is I, and in the next string, the state of the kernel is J, and strings alternating thus continuously, such that the pattern formed is of the C3 type, specific to the kernel K1. Thus a single line of alphabets and numerals completely describes a DDC, and all that is needed are two look-up tables – one for 'kernels, strings and CPs', and another for all possible kernel states.

A slightly complex configuration from the table - K4-S3(E,A)S3(M,N)S3(F,B)-C1 is explained thus. Composed of the kernel K4, the DDC is of the pattern C1 type. The first string contains K1 kernels of the states E and A in alternating order. The next string contains K1 kernels of the states M and N in alternating order and the third string contains K1 kernels of the states F and B in alternating order. Thereafter the first string repeats and so on.

Theoretically, we now have a complete definition of all DDCs with only simple kernels as a basis, using the look-up tables 02-05 and 07.

Table 07 : Kernels of all possible FP States

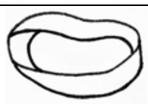
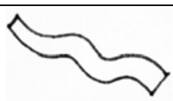
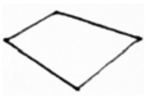
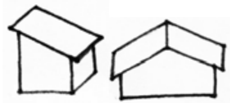
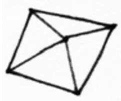
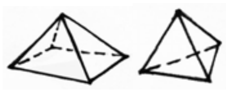
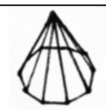
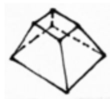
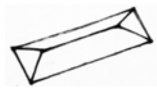
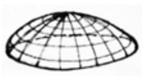
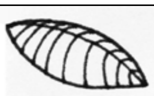
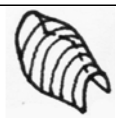
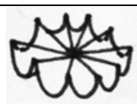
S.No	Kernel (K)	Possible State Permutations	Example FP with full notation	Corrugation
1	 K1 Rectangle K2 Square K3	 A B C D E F G H I J K L M N O P Q	 K1-S1(I)S1(J)-C3	
2	 K4 Quadrilateral (chiral)	 A B C D E F G H I J K L M N O P	 K4-S3(E,A)S3(M,N)S3(F,B)-C1	
3	K5  Trapezium	 A B C D E F G H I J K L M N O P Q R S	K5-S1(G,H)S1(H,G)-C1 	

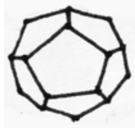
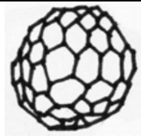
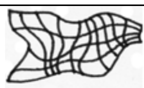
KEY TO LINE COLOURS :
- Mountain Fold / Crest Creasing
- Valley Fold / Trough Creasing
- No Fold / Flat Crease

4.3 Architectural Geometries

With this section, we initiate the process of understanding and interpreting DDCs in the role of Architectural Geometries. The table below outlines commonly employed families of Architectural Geometries.

Table 08 : Family of Architectural Geometries

S. No	Architectural Geometry Genre	A	B	C	D	E
1	Ribbons	 Mobius	 Wave			
2	Flat Panels	 Flat	 Lean-To and Pitched	 Prismatic	 Folded Plate	
3	Pyramidal	 Shallow	 Deep : Square or Triangular Base	 Prismatic	 Truncated or Mansard	 Hipped
4	Conical	 Shallow	 Deep	 Truncated		
5	Spherical	 Shallow	 Deep	 Pinched		
6	Ellipsoidal / Oval	 Truncated	 Inverted Boat			
7	Cylindrical	 Barrel Vault	 Varying section	 Varying Ht.		
8	Groined Vaults	 True Arch based	 Pointed Arch based	 Multiple Rib Vault		

9	Toroidal	 Or part of a Torus				
10	Polyhedral	 Dodecahedron & other Platonic Solids	 Great Stellated Dodeca-hedron & other Kepler-Poinsot Solids	 Cuboctahedron & other Archimedean Solids	 Geodesic (Buckyball)	 Poly-polyhedra
11	Hyperbolic Paraboloid	 Saddle/Pringle	 Hypar Sections	 Tensile/ Reverse Catenaries		
12	Arbitrary Topology					

Note : (1) Some forms such as Ellipsoidal and Spherical etc., are mathematically not developable into flat panels with zero Gaussian curvature (see definition of ‘developable surface’ in the Introduction), however, architecturally, their forms can be imagined as faceted approximations of flat panels when resolved.
(2) Since this is a ‘family’ of geometries, there are overlaps in definitions – for instance 8C shares traits with 11B, though it is not identical. Similarly, 5C is a revolved surface not unlike conical surfaces, but it is a hybrid, in that it sits more properly geometrically within the spherical family.

The next section 4.4 begins to deliberate on how DDCs can be adapted for architectural form making.

4.4 Mapping DDCs onto Geometries using Hinge Lines

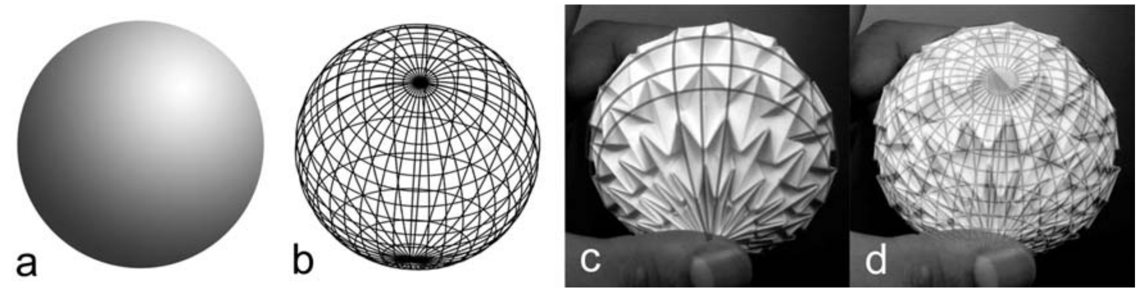


Figure 25 : Mapping DDCs onto Geometries : Illustration using a folded Magic Ball as example

Figure 25 shows (a) a sphere, and (b) a sphere as represented by a grid of circular lines. All geometries can be approximated in this manner as grids of lines. In fact this is essentially what all CAD packages do to approximate surfaces and forms.

Now it is also true for DDC surfaces that they can essentially be represented by a grid of hinge lines that govern them. Each DDC has its own unique set of hinge lines. Hinge lines represent the degrees of freedom that a DDC surface enjoys geometrically, and therefore using hinge lines as a basis, it is possible to define at any given time the state of existence of a DDC. Figure 25 (c) shows a few hinge lines overlaid on a Magic Ball when it is in its spherical form. (d) shows the complete grid of hinge lines mapped onto the Magic Ball. Given that (b) and (d) are identical – if the number of lines on the sphere and the number of hinge lines on the DDC are deliberately designed to be the same – then, a one to one correspondence of lines would allow a natural adaptation of the DDC to the relevant architectural geometry.

Thus, we can state that if we know the capabilities of the hinge lines of a DDC, then we can recognize and match a DDC to its natural formal geometry.

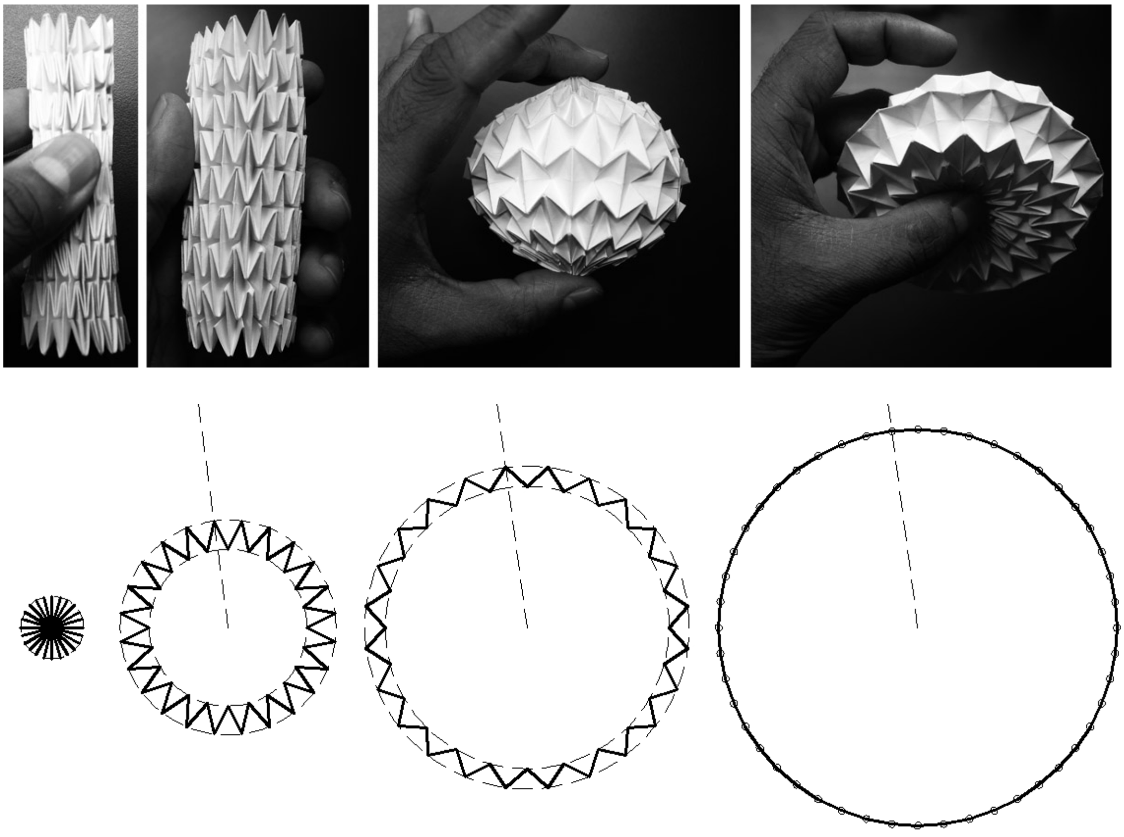


Figure 26 : Planimetric cross-sections right through the middle of the Magic Ball model revealing the 'state' of the kernels and the diameters of the inner and outer hinge line circles.

There is a second important property of hinge lines. There are usually two hinge lines at any given cross-section of a folded DDC, an inner and an outer, and enclosed between them is the maximum possible depth of the DDC. How this works in form manipulation is explained using the diagram in figure 26. The figure shows the various formal states of the Magic Ball and corresponding diagrammatic cross-sections through the middle of the form. The dotted circles represent the hinge-lines, the black lines the folded pleats of the corrugation and the straight dotted line, the central axis of the form. When the corrugation is packed at its tightest into a cylinder, the inner hinge line vanishes to a point (extreme left diagram). As the structure expands, the internal depth of the structure decreases and the inner and outer hinge lines get closer and closer, till the inner and outer hinges overlap completely. At that point the DDC has expanded to its maximum capacity, and in this case to a 48-gon which closely approximates a circle (extreme right diagram). Since the number of nodes on a hinge line are fixed, how much it can expand or contract becomes a function of whether the inner and outer hinge lines can completely overlap or not, and/or whether any one of the hinge lines can theoretically vanish to a point. Since DDCs are developable and can only 'approximate' non-developable surfaces, this property of hinge lines is an important evaluation criteria for form determination. This is further illustrated with Erik Demaine's diagrams (figure 27), where curves have been reduced to triangular faceted DDCs. Although Demaine does not apply the concept of hinge lines to his diagrams, on doing so, it becomes apparent that they play a critical role in describing the transformation from a non-developable surface to a developable double corrugation.

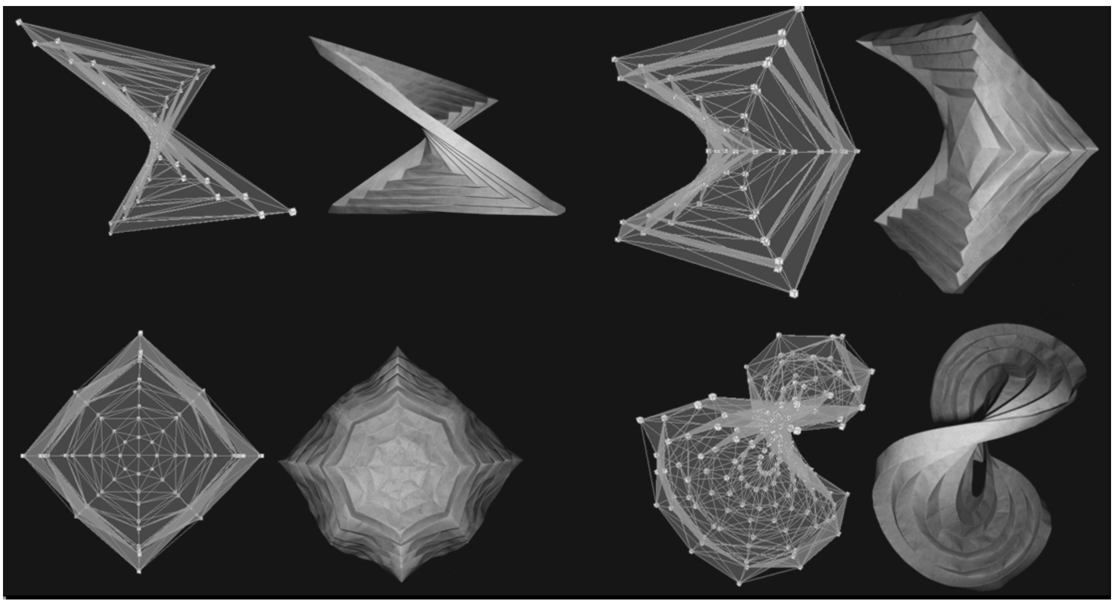


Figure 27 : DDCs which approximate non-developable forms/surfaces (source : Erik Demaine)

4.5 From Algorithm to Form

What is an Algorithm? *'An algorithm is a computational procedure for addressing a problem in a finite number of steps. It involves deduction, induction, abstraction, generalization, and structured logic. It is the systematic extraction of logical principles and the development of a generic solution plan. Algorithmic strategies utilize the search for repetitive patterns, universal principles, interchangeable modules, and inductive links. The intellectual power of an algorithm lies in its ability to infer new knowledge and to extend certain limits of the human intellect.'* (pg. 65, Terzidis, 2003)

This definition above sums up well what this paper aims to achieve with DDCs. The phrases *'…search for repetitive patterns, universal principles, interchangeable modules, and inductive links'*, specially apply to DDCs, vis-à-vis the kernels and the strings.

Based on the results and analysis, we are now in a position to theoretically formulate an algorithm for folding architectural forms using DDCs. This would work as follows –

(1) Select architectural form from table 08

(2) Describe architectural form in terms of a mesh or grid of lines (determine intensity of mesh, based on resolution of form desired)

(3) Identify and classify the constituent lines of the mesh as straight, segmented, curvilinear, circular, parabolic, elliptic or arbitrary

(4) Randomly select kernel from tables 02-05

(5) Run kernel through a sequential process of String and Creasing Pattern (CP) generation iterations, based on tables 02-05 for all possible CPs, corresponding to that particular kernel in the look-up table. (Important to parametrically link the kernels during the generation process of the CPs).

(6) Randomly pick one of the CPs created and generate all possible folding patterns (FPs) for the same. (Hundreds of patterns will be generated, of which in reality only 5-6% will fold).

(7) Attempt to fold all the FPs into DDCs using the notation system as an embedded code which specifies not only the kernel and how the kernels join together to create a string, but also the sequence of folding operations (such as translation and rotation) via the structure and order of the notation itself.

(8) For the FPs that successfully fold into DDCs, generate hinge lines as per the location of the strings on the folded up DDCs. Parametrically link the hinge lines to the DDCs.

(9) Manipulate the hinge lines (length, shape, Degrees of Freedom – to be discussed in detail in section 5.2)) etc. This will also change the DDC surfaces as they are parametrically linked to the hinge lines.

(10) Compare the hinge lines of each of the DDCs to the grid lines of the desired architectural form.

(11) If a shape match is found, jump to step 14, else go back to step 6 and pick another crease pattern for the same kernel.

(12) Repeat steps 7 to 11, until a matching set of hinge lines (and consequently matching DDC) is found corresponding to the desired architectural geometry. If not, go back to step 4 and pick a new kernel.

(13) Repeat steps 5 through to 12 until a match is found.

(14) Initiate form optimization sub-routine.

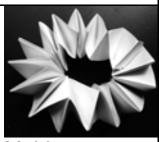
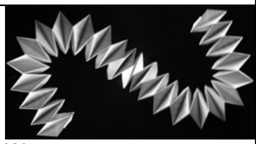
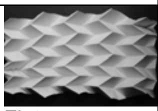
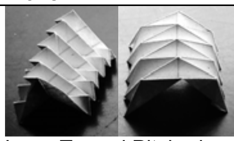
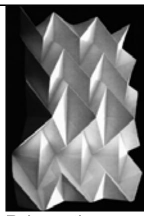
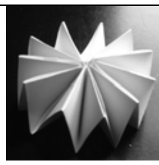
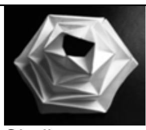
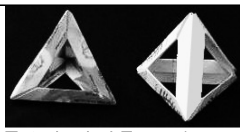
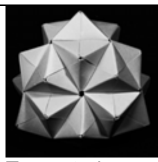
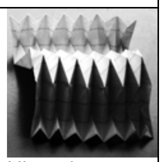
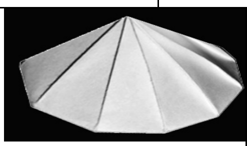
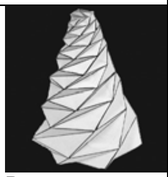
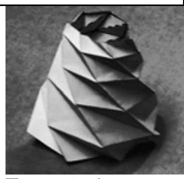
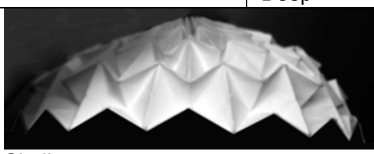
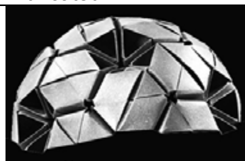
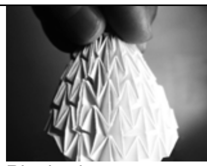
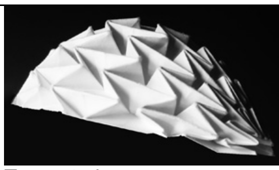
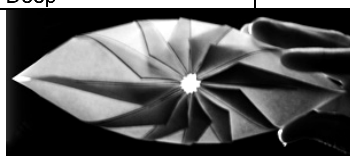
(15) Finally test the CP of the optimized corrugation (DDC) for flat-foldability using the 4 flat-foldability rules. A successful test indicates that not only is the DDC an optimized architectural form but also a flat foldable one.

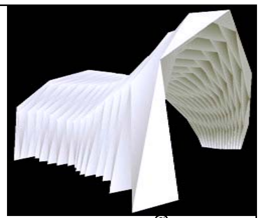
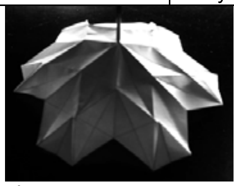
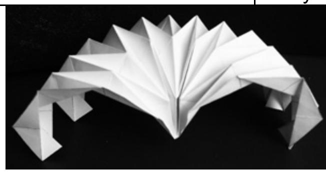
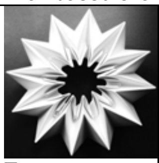
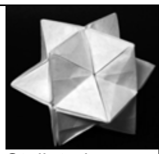
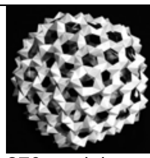
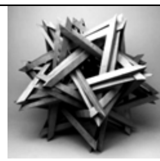
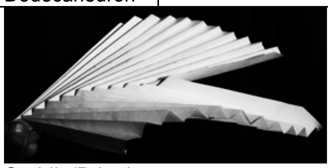
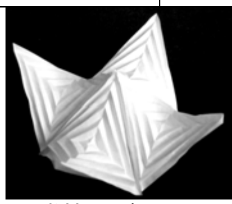
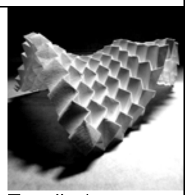
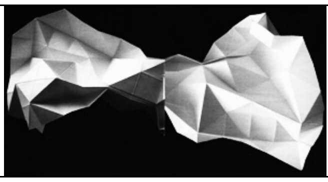
5 Results & Discussion

5.1 DDCs as Architectural Geometries

In light of the formulations and understanding gained in Chapter 4, we are now in a position to generate a full DDC version tabulation of the Architectural Geometries as below.

Table 09 : Revised Table showing Architectural Geometries using DDCs : All the physical models were generated during the research project unless otherwise stated.

S. No	Architectural Geometry Genre	A	B	C	D	E
1	Ribbons	 Mobius	 Wave			
2	Flat Panels	 Flat	 Lean-To and Pitched	 Prismatic	 Folded Plate	
3	Pyramidal	 Shallow	 Tetrahedral Frame*	 Prismatic	 Truncated Tetrahedron*	 Hipped
4	Conical	 Shallow	 Deep	 Truncated		
5	Spherical	 Shallow		 Deep %	 Pinched	
6	Ellipsoidal / Oval	 Truncated		 Inverted Boat		

7	Cylindrical	 Barrel Vault  Varying section @  Varying Height @
8	Groined Vaults	  Arch based examples  Multiple Rib Vault
9	Toroidal	 Torus
10	Polyhedral	 Modular - Origami Dodecahedron*  Section through a Kepler-Poinsot Solid  Stellated Rhombic Dodecahedron  270 module Geodesic (Buckyball)*  K2 -20x1x3 Poly-polyhedra *
11	Hyperbolic Paraboloid	 Saddle/Pringle  4-unit Hypar ^  Tensile / Distorted Miura Ori #
12	Arbitrary Topology	

* indicates a work of modular origami - not constructed from a single sheet, but a single origami module used many times
\# physical model, created by Tactom – web reference (http://www.flickr.com/photos/tactom/3113852024/)
^ created by Tomohiro Tachi
@ virtual model, computer generated by Tactom – web reference (http://www.flickr.com/photos/tactom/346434097/in/set-72157594370508788/)
% based on Ron Resch's tessellation pattern

This table shows only one DDC type adapted to each given Architectural Geometry, whereas, for some of the Geometry genres, it was apparent that more than one DDC type could be easily adapted to exhibit that geometry. Since all models shown here were physically folded, it is suggested that more DDC adaptations should be attempted through virtual simulation.

Also, ideally all the DDC models generated (and shown in the table above) as a solution set, should have the corresponding full notations stated alongside, as per the notation system developed in this paper. However this annotating is an incomplete task for the future due to constraints of time.

Many of the folding patterns used to create these models have been known from very long ago but rarely thought of in the context of architectural form; some have been created by mathematicians, folders and origamists in the last 15 years or so – and wherever these FPs can be sourced to an individual, the name has been cited below the table. A few of the FPs may have 'emerged' during the research for this paper, but that assertion cannot be made with certainty. It may merely be that such patterns have been in use, but may perhaps not been catalogued earlier or else, their source is either obscure or has remained undocumented.

5.2 Degrees of Freedom (of DDCs)

We now finally come to a discussion of the concept of Kinematics in DDCs, initially described in the introductory chapters. 'In Mechanics, Degrees of Freedom (DF) are the set of independent displacements and/or rotations that specify completely the displaced or deformed position and orientation of the body or system. This is a fundamental concept relating to systems of moving bodies in mechanical engineering, aeronautical engineering, robotics, structural-engineering,etc.' (http://en.wikipedia.org/wiki/Degrees_of_freedom_(mechanics), accessed on 25/08/09))

'A particle that moves in three dimensional space has three translational displacement components as DFs, while a rigid body has at most six DFs including three rotations. Translation is the ability to move without rotating, while rotation is angular-motion about some axis.' (http://en.wikipedia.org/wiki/Degrees_of_freedom_(mechanics), accessed on 25/08/09)

A DDC is in essence a system of rigid surfaces which are hinged to each other. Each surface can rotate about its hinged connection, but only upto a certain extent, because it is also connected to other surfaces which prevent displacement.

Such a system with several rigid surfaces working in concert would have a combined DF that is the sum of the DFs of the individual surfaces (bodies), minus the internal constraints that exist for relative motion. A DDC like surface may often have many more degrees of freedom than a single rigid surface of the same dimensions.

In three dimensions, the six DOFs of a rigid body are sometimes described using these nautical names:

1. Moving up and down (heaving)
2. Moving left and right (swaying)
3. Moving forward and backward (surging)
4. Tilting forward and backward (pitching)
5. Turning left and right (yawing)
6. Tilting side to side (rolling)

(http://en.wikipedia.org/wiki/Degrees_of_freedom_(mechanics), accessed on 01/09/09)

The angles by which these motions occur may properly be referred to as Euler Angles, placed within the framework of a rotation matrix. To give an object a specific orientation it may be subjected to a sequence of three rotations described by the Euler angles. This means that a rotation matrix is being described as a product of three elemental rotations.

However our main interest in DF is not in how much the DDC surface will move or sway, or even in how far the terminal element of the corrugation will go, but in the shapes a given DDC surface can assume, as a result of the Degrees of Freedom allowed to it by its own inherent geometry. Here the term DF is specifically being used to describe the number of parameters needed to specify the spatial positions of a loci of linkages. Thus we can say that we want to calculate mechanism topology. Mobility criteria based on mechanism topology allows us to compute the mobility depending solely on the number of links, joints and joints type.

To calculate the DF of a DDC, the Grubler formula for planar mechanisms is -

$$F = 3(l-1) - 2j_1 - j_2$$

Where - - F :degrees-of-freedom of the mechanism (D.F.);
- f_i : degrees-of-freedom of the ith kinematic pair;

- *l* : number of links (frame included);
- *j* : number of kinematic pairs;
- *ji* : number of kinematic pairs with *i* degrees-of-freedom;
- λ: (mobility number) degree-of-freedom of space within which the mechanism operates *e.g.* (=3 for planar and spherical space), (=6 spatial space);

(pg.4, Pennestri et al., 2005)

Calculation of DFs for DDC types enumerated in this paper is not in the scope of this discussion, but from the formula above, four fundamental aspects of DFs as applicable to DDCs can be formulated –

(1) Sheets with more links have more scope for maneuverability. This means that for the same folding pattern, if a sheet has 50 kernels in a row, and another has 100, the one with the larger number of kernels will have more maneuverability, although this does not mean that it can roll into a fundamentally different shape, but within a family of forms (for instance spheroidal dispositions), it will display more flexibility.
(2) Sheets with more internal constraints (such as axial rigidity in one dimension due to their form) will have less degrees of freedom than 'flat sheets'. Figure 28 shows the possible formal variations possible with sheet K1-S1(I)S1(J)-C3. On the other hand, radially folding sheet K4-S1(P)S1(K1-I,K4-P)S1(K1-I,K1-I,K4-P)C1 seen in figure 29 is already restricted by the central axis or pivot about which it unfurls and therefore shows lesser degrees of freedom. In general, flat and linear DDC patterns have more degrees of freedom than radial patterns.

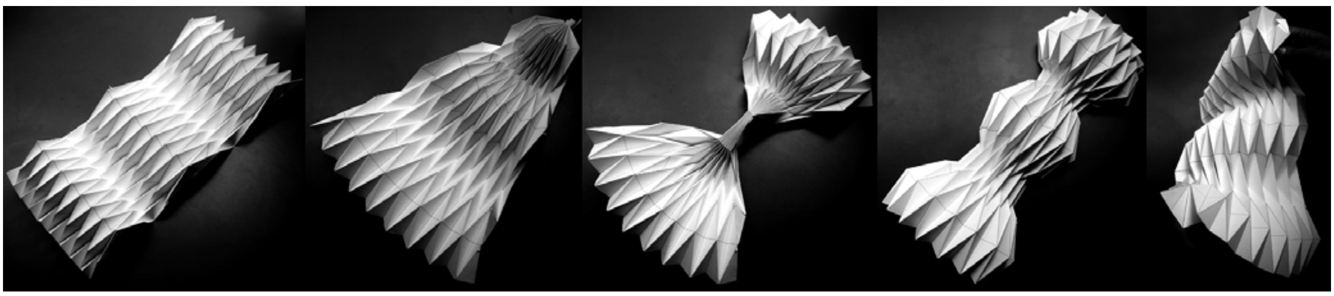


Figure 28 : Degrees of Freedom of a 'flat sheet' K1-S1(I)S1(J)-C3

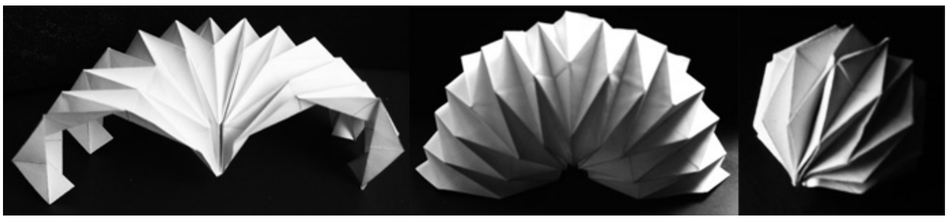


Figure 29 : Degrees of Freedom of 'radially-folding sheet' K4-S1(P)S1(K1-I,K4-P)S1(K1-I,K1-I,K4-P)C1

(3) The angles described in the making of the CP also have an important bearing on the DF of the final generated corrugation. More acute angles indicate rigidity and inflexibility. However, this must be read strictly in conjunction with the FP. Consecutive mountain folds or valley folds (of one type only) along sequential hinge lines indicates that the form will fold inwards in one direction, creating axial rigidity, whereas, alternating mountain and valley folds along sequential hinge lines nullifies that effect and creates more flattened linear sheets.

(4) Squarer grids in the CP indicate more flexibility and degrees of freedom in the final folded DDC. K1S1(P)C3 (christened as per this paper) with a grid of perfect squares shows remarkable degrees of freedom and can generate a cylinder, a disc or a sphere. For this reason Origamists fondly refer to it as the Magic Ball (figure 30). In general, corrugations folded from CPs with squarer grids enjoy more DF than those folded from CPs with rectangular grids, simply because a square is directionless, whereas a rectangle already has an in-built orientation.

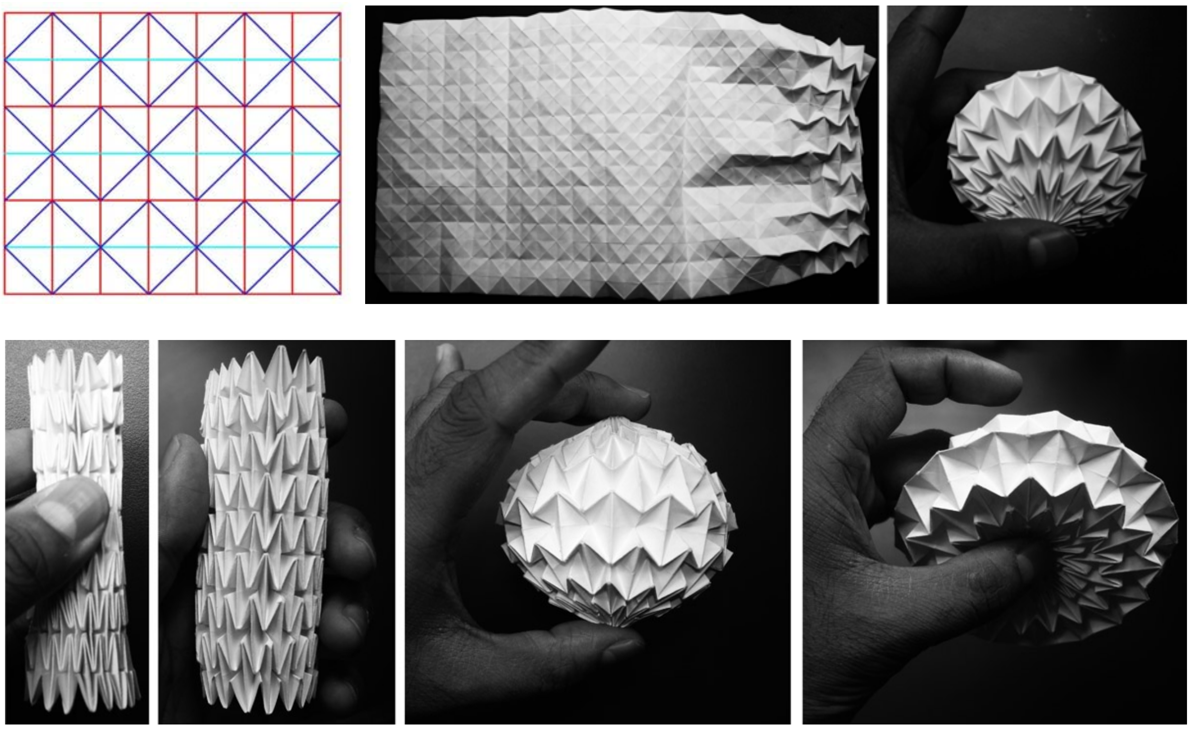


Figure 30 : An FP of K1S1(P)C3 of square grid CP folds up into a Magic Ball and also showing various other formal possibilities

5.3 The significance of Flat Foldability

Flat foldability has important implications for architectural applications –

(1) it allows for off-site manufacture of elements and prefabrication, saving significant transportation costs as well as at-site on demand in-situ technical complications
(2) development of standard kit of parts makes it possible for modular systems to be unassembled and re-assembled on demand – this is especially useful for mobile and transient architecture (such as pavilions and temporary exhibition structures)
(3) large unwieldy elements of design can be re-invented as smaller flat-foldable elements, saving logistical and construction costs

In all this DDCs have a significant role to play, and therefore if a Developable Double Corrugation solution is also flat foldable, its utility and functionality increases many fold. Using the flat-foldability rules outlined in section 2.4, all DDCs can checked for validity with ease, and preference given to those patterns (during form generation), which are flat foldable.

6 Conclusions & Further Work

6.1 The Fundamentals of this Research

The paper delineates a language of folding for developable double corrugations – a shape grammar for origami as it were, but specific to regular corrugations, identifying which folding patterns can generate architectural forms and how those forms could be generated. It is as though, an instruction manual were being written for a robot - a robot that is being trained in the art and science of folding – a robot that folds, in other words, a *'Fold-o-Bot'*. The term sums up well the process of creating a system of recognizing and distinguishing between folding patterns, understanding what goes into creating that pattern, and then using that knowledge and applying it to a form creation paradigm.

The attempt was also to collapse the myriad and often confusing varieties of regular and irregular origami tessellations into families, a cataloguing of sorts at one level, and trace the lineage of these families back to an original gene pool of a few basic starting blocks – the 'kernels' as defined in this paper; to that extent, the research was able to identify and clearly establish a 'gene pool', as well as various 'reproduction and growth strategies'.

6.2 Applications

The results of this research could be used for –

(a) creating roofing and walling systems (in an architectural context) for a wide variety of shapes and profiles, for medium to large scale column free spaces

(b) designing structural frameworks for roofing systems, where the roof itself is not corrugated, but the supporting structure underneath is a system of lines in space not unlike the crease lines of corrugated folding surfaces of DDCs

(c) generating retractable roofing designs for large span spaces, using flat-foldability as a fundamental constraint. These ideas would not be restricted to only retractable roofing but be extended to other varieties of transformative architecture (for instance Masashi Tanaka's XOR transformations) or kinetic design (xxx design feature) – see related reading for links to both these examples.

(d) reducing architectural forms with surfaces having Gaussian curvature to approximate DDC forms with triangular facets which have zero Gaussian curvature (this is significant for construction, where it is economical and beneficial to use flat panels and sheets rather than parts of spheres for generating surfaces) – figure 31 shows the Assembly Hall at the University of Illinois designed by Max Abramovitz. It is a non-developable spherical section. Using a DDC, namely K4S1(P)S1(K1-I,K4-P)S1(K1-I,K1-I,K4-P)C1, an attempt was made to approximate the form using the same proportions as the original. The result is shown in figure 32.

(e) extending this study to also include cases where surfaces do not exhibit zero Gaussian curvature

Figure 31 : Assembly Hall, University of Illinois, Architect : Max Abramovitz
Source : University of Illinois, Library Archives

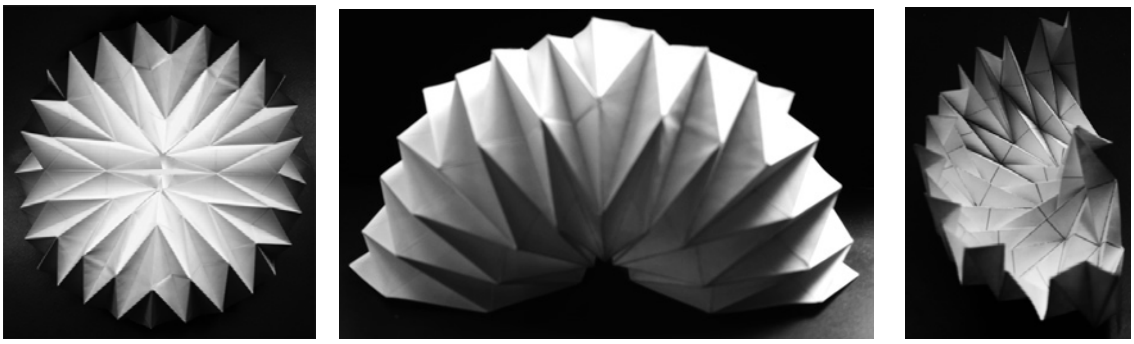


Figure 32 : An attempt to adapt the non-developable Assembly Hall roof form of the University of Illinois to a developable double corrugation namely, K4S1(P)S1(K1-I,K4-P)S1(K1-I,K1-I,K4-P)C1

6.3 Optimization & Applying Genetic Algorithms (or perhaps Neural Networks)

(a) of the possible next steps, the first would involve running simulations based on the fundamental kernels defined, the hinge lines, the generator folding patterns and the sequence rules. The simulations would not only validate the generative process outlined here-in, but also allow for many more permutations and combinations of folding patterns to be generated, the sheer variety of which was not possible physically folding by hand.

(b) the second step would involve optimizations –

(i) for geometry (to fit the profile of the prescribed architectural form as closely as possible).

(ii) for functional efficiency (as an architectural form) - given a flat sheet of paper of fixed dimensions, to begin with).

(iii) for minimum energy (minimum folds and consequently minimum number of kernels used to create a form. This optimization would directly conflict with the first optimization of the geometry, as large kernels which are lesser in number would not create as accurate an architectural form as smaller kernels, more in number. Some sort of negotiated solution would need to be achieved between the two optimizations.

(iv) for stress and strain (and other structural optimizations).

(v) testing the generated forms in the context of real construction materials with thickness and limiting properties – wood, steel sheets, aluminum panels etc.

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