Employment, Hours of Work and the Optimal Taxation of Low Income Families

Richard Blundell
Andrew Shephard

THE INSTITUTE FOR FISCAL STUDIES
WP08/01

Employment, Hours of Work and the Optimal Taxation of Low Income Families

Richard Blundell and Andrew Shephard
University College London and Institute for Fiscal Studies

October 24, 2009

Abstract

This paper examines the tax schedule for low income families with children. We take an optimal tax approach based on a structural labour supply model which incorporates unobserved heterogeneity, fixed costs of work, childcare costs and the detailed non-convexities of the tax and transfer system. The motivation is the British earned income tax credit reform (WFTC) and its interaction with the tax and transfer system for lone parents. Our analysis also examines the case for the use of hours-contingent payments. The results point to a tax schedule which depends on the age of children, with tax credits only optimal for low earners with school age children. The results also suggest a welfare improving role for hours-contingent payments although this is mitigated when hours cannot be monitored or recorded accurately by the tax authorities.

Acknowledgements: This study is part of the research program of the ESRC Centre for the Microeconomic Analysis of Public Policy at the IFS. We thank Stuart Adam, Mike Brewer, Guy Laroque, Ian Preston, Emmanuel Saez, seminar participants at MIT, Jerusalem, NIESR, participants in the Mirrlees Review and the referees for helpful comments. We are responsible for all errors and interpretations.

1 Introduction

The empirical analysis of labour supply behaviour has strong implications for the design of earnings taxation. Our aim here is to use a microeconometric labour supply model to assess the design of tax rate reforms for the low paid. In particular, to examine policies that aim at reducing the effective tax rates on work for low income families, as in the significant expansions of earned income tax credits in the UK and the US.[1]

Tax credit reforms have been evaluated extensively in the UK and elsewhere. The evidence that tax credit policies encourage work is compelling and the positive impact on employment has been found to be particularly strong for single mothers, see for example Eissa and Liebman (1996) and Blundell et al. (2000). These and other studies tell us about the labour supply impact of tax credit reforms. Given that such labour supply responses also help us to learn about preferences, it is possible to move beyond the evaluation of particular reforms, and consider problems related to the optimal design of the tax and transfer system. In the spirit of Mirrlees (1971), we shall ask: how should the government best allocate a fixed amount of revenue to the design of earnings taxation?

The analysis draws on the microeconometric and the optimal taxation literature. In the microeconometric literature certain common and robust features of estimated labour supply responses of the low paid have emerged. Specifically, the importance of distinguishing between the intensive margin of hours of work and the extensive margin where the work decision is made. Labour supply elasticities appear to be much larger at the extensive margin, at least for certain household demographic types, see Blundell and Macurdy (1999).

The optimal taxation literature explores consequences for design. In parallel with the empirical regularities, the literature on the design of tax and transfer systems has increasingly focussed on the extensive margin and the use of work conditions, see for

[1] See Blundell and Hoynes (2004), for example.

example Beaudry et al. (2008), Besley and Coate (1992), Choné and Laroque (2005), Laroque (2005), Moffitt (2006), Phelps (1994) and Saez (2001, 2002). Our approach is closest to that by Saez (2002) who, building on earlier work by Diamond (1980), examines the optimality of tax credit designs within a Mirrlees framework but one which acknowledges the distinction between the extensive margin and intensive margin of labour supply. Indeed, Saez (2002) derives approximate optimal tax formula in terms of representative labour supply elasticities at the extensive and intensive margin. Recently, Immervoll et al. (2007) implement this approach and suggest that for reasonable welfare weights, tax credits would be an optimal policy across a wide set of economies. As part of the Mirrlees Review, Brewer et al. (2009) use this approach to explore the taxation of families in the UK.

The contribution of this paper is threefold. First, we take the structural model of employment and hours of work seriously in designing the structure of taxes and transfers, allowing the distribution of earnings, fixed costs of work and demographic differences to influence the design of tax policy. Second, we consider the case where hours of work are partially observable to the tax authorities and consider the case for hours contingent reforms. Third, we assess the role of conditioning on the age of children in the rate schedule for earnings taxation.

Our exploration of hours contingent reforms is motivated by the common use of hours based eligibility in the tax credit systems of countries like the UK, Ireland and New Zealand. Hours information is also used in the design of work conditioned earnings supplements, for example in the Canadian Self-Sufficiency Project (Card and Robins, 1998) and in the TANF programme of welfare payment in the US (Moffitt, 2003). It has also been proposed as a mechanism for improving tax design, see Keane (1995), although not within an optimal tax framework. Given the likely difficulties in recording and monitoring hours of work, our analysis also considers scenarios where hours are subject to measurement error, or where individuals may directly misreporting their hours of work to the tax authorities.

The microeconometric analysis we follow is based on a stochastic discrete choice labour supply model (Hoynes, 1996; Keane and Moffitt, 1998; Blundell et al., 2000; van Soest et al., 2002). This model allows for discrete choices over non-linear budget constraints and fixed costs of work to re-examine the optimal design problem. The optimal tax model is then derived directly from the labour supply model together with the estimated distribution of earnings, fixed costs of work, childcare costs, demographic differences and unobserved heterogeneity.[2]

The analysis is set in a static environment with fixed costs of work and stigma costs of accessing welfare benefits. We are therefore ignoring dynamic effects in both labour supply choices and in the design of the tax structure. Our focus is on the design of the tax schedule for low earners and the role of tax credits. Although an experience pay-off in earnings would change the optimal structure, we think our approach captures the most important aspects of design for this group. The evidence points to relatively low or negligible experience effects for low earnings single parents, see Card and Hyslop (2005) and Gladden and Taber (2000). A more subtle dynamic effect may act through fertility decisions. Keane and Wolpin (2007) note that fertility effects may largely counteract the direct impact on labour supply. However, the effect of tax reform on fertility behaviour is generally found to be significant but small, see Hoynes (2009). A further key dynamic aspect of tax design is the interaction with savings taxation and the taxation of lifetime income. In certain circumstances, the taxation of saving can be used to relax the incentive compatibility constraint on earnings taxation (see Banks and Diamond, 2009). However, with fixed costs of work, credit constraints and earnings uncertainty there is likely to remain a strong role for nonlinear earnings tax design of the type described here.

The results of our analysis point to marginal tax rates that are broadly increasing in earnings, and that are lower than under the current UK system. Moreover, we show that heterogeneity is important. In particular, we present a case for pure tax credits at low earnings but only for mothers with school aged children. It is also found that

[2] An alternative model which incorporates constraints on labour supply choices in an optimal design problem is developed in Aaberge and Colombino (2008).

hours contingent payments can improve design. Indeed, if hours can be accurately observed, we present an empirical case for using a full-time work rule rather than the part-time rule currently in place for parents in the UK. While this is found to be a more effective instrument, the welfare gains remain modest in size for all but parents with older children. These welfare gains are also shown to reduce significantly with moderate amounts of misreporting or measurement error.

The paper proceeds as follows. In the next section we develop the analytical framework for optimal design within a stochastic structural labour supply model. In section 3 we outline the WFTC reform in the UK and its impact on work incentives. Section 4 outlines the structural microeconometric model, while in section 5 we describe the data and model estimates. Section 6 uses these model estimates to derive optimal tax schedules. We provide evidence for lowering the marginal rates at lower incomes and also document the importance of allowing the tax schedule to depend on the age of children. We also discuss how introducing hours rules affects tax credit design, and how important these are likely to be in terms of social welfare. Finally, section 7 concludes.

2 The Optimal Design Problem

The policy analysis here concerns the choice of a tax schedule in which the government is attempting to allocate a fixed amount of revenue R to a specific demographic group – single mothers – in a way which will maximise the social welfare for this group. Such a schedule balances redistributive objectives with efficiency considerations. Redistributive preferences are represented through the social welfare function defined as the sum of transformed individual utilities, where the choice of transformation reflects the desire for equality.

In this section we develop an analytical framework for the design of tax and transfer policy that allows for two scenarios. In the first only earnings are observable by the tax authority, in the second we allow for partial observability of hours of work. Rather than assuming that individuals are unconstrained in their choice of hours, we suppose that only a finite number of hours choices are available, with hours of work h chosen from the finite set $\mathcal{H} = \{h_0, \ldots h_J\}$. The formulation of the optimal tax design problem will depend upon what information is observable to the tax authorities. We always assume that the government can observe earnings wh and worker characteristics X, and we shall also allow for the possibility of observing some hours of work information. In much of our analysis we will assume that rather than necessarily observing the actual hours h that are chosen, the tax authorities is assumed to only be able to observe that they belong to some closed interval $\mathbf{h} = [\underline{h}, \overline{h}] \in \mathcal{H}$ with $\underline{h} \leq h \leq \overline{h}$. For example, the tax authorities may be able to observe whether individuals are working at least h_B hours per week, but conditional on this, not how many. Depending on the size of the interval, this framework nests two important special cases; (i) when hours are perfectly observable $h = \underline{h} = \overline{h}$ for all $h \in \mathcal{H}$; (ii) only earnings information is observed $\mathbf{h} = \mathcal{H}_{++}$ for all $h > 0$. In general this is viewed as a problem of partial observability since actual hours h are always contained in the interval $\mathbf{h}$. In our later analysis in section 6.3 we will explore the effect that both random hours measurement error, and possible direct hours misreporting have upon the optimal design problem.

Work decisions by individuals are determined by their preferences over consumption c and labour hours h, as well as possible childcare requirements, fixed costs of work, and the tax and transfer system. Preferences are indexed by observable characteristics X, including the number and age of her children, and vectors of unobservable (to the econometrician) characteristics ϵ and ε; the distinction between these vectors will be made in section 4. We let $U(c, h; X, \epsilon, \varepsilon)$ represent the utility of a single mother who consumes c and works h hours. We will assume that she consumes her net income which comprises the product of hours of work h and the gross hourly wage w plus non-labour income and transfer payments, less taxes paid, childcare expenditure, and fixed costs of work. In what follows we let F denote the distribution of state specific errors ε, and G denote the joint distribution of (X, ϵ).[3]

In our later empirical analysis individual utilities $U(c, h; X, \epsilon, \varepsilon)$ will be described by a parametric utility function and a parametric distribution of unobserved heterogeneity (ϵ, ε). Similarly, a parametric form will be assumed for the stochastic process determining fixed costs of work and childcare expenditure. To maintain focus on the optimal design problem, we delay this discussion regarding the econometric modelling until section 4; for now it suffices to write consumption c at hours h as $c(h; T, X, \epsilon)$,[4] where $T(wh, \mathbf{h}; X)$ represents the tax and transfer system. Non-labour income, such as child maintenance payments, enter the tax and transfer schedule T through the set of demographics X, and for notational simplicity we abstract from the potential dependence of the tax and transfer system on childcare expenditure. Taking the schedule T as given, each single mother is assumed to choose her hours of work $h^* \in \mathcal{H}$ to maximize her utility. That is:

$$h^* = \arg\max_{h \in \mathcal{H}} U(c(h; T, X, \epsilon), h; X, \epsilon, \varepsilon). \tag{1}$$

We assume that the government chooses the tax schedule T to maximize a social welfare function W that is represented by the sum of transformed utilities:

$$W(T) = \int_{X,\epsilon} \int_{\varepsilon} \Upsilon(U(c(h^*; T, X, \epsilon), h^*; X, \epsilon, \varepsilon)) dF(\varepsilon) dG(X, \epsilon) \tag{2}$$

where for a given cardinal representation of U, the utility transformation function Υ determines the governments relative preference for the equality of utilities.[5] This maximization is subject to the incentive compatibility constraint which states that lone mothers choose their hours of work optimally given T (equation 1) and the government resource constraint:

$$\int_{X,\epsilon} \int_{\varepsilon} T(wh^*, \mathbf{h}^*; X) dF(\varepsilon) dG(X, \epsilon) \geq \overline{T}(\equiv -R). \tag{3}$$

[3] Throughout our analysis we assume that ε is independent of both ϵ and X.

[4] The assumptions that we later make regarding the error term ε ensure that consumption will not depend on ε for given work hours h.

[5] Given the presence of preference heterogeneity, a more general formulation would allow the utility transformation function Υ to vary with individual characteristics.

In our empirical application we will restrict T to belong to a particular parametric class of tax functions. This is discussed in section 6 when we examine the optimal design of the tax and transfer schedule.

3 Tax Credit Reform

The increasing reliance on tax-credit policies during the 1980s and 1990s, especially in the UK and the US, reflected the secular decline in the relative wages of low skilled workers with low labour market attachment together with the growth in single-parent households (see Blundell, 2002, and references therein). The specific policy context for this paper is the Working Families' Tax Credit (WFTC) reform which took place in the UK at the end of 1999. A novel feature of the British tax credit system is that it makes use of hours conditions in addition to an earnings condition. Specifically WFTC eligibility required a working parent to record at least 16 hours of work per week. Moreover there was a further hours contingent bonus for working 30 hours or more.

As in the US, the UK has a long history of in-work benefits, starting with the introduction Family Income Support (FIS) in 1971. Over the years, these programmes became more generous, and in October 1999, Working Families' Tax Credit was introduced, replacing a similar, but less generous, tax credit programme called Family Credit (see Blundell et al., 2008, for example). As noted above, an important feature of British programmes of in-work support since their inception – and in contrast with programmes such as the US Earned Income Tax Credit – is that awards depend not only on earned and unearned income and family characteristics, but also on a minimum weekly hours of work requirement. In April 1992, the minimum hours requirement fell from 24 to 16 hours a week. The impact of this reform on single parents' labour supply is ambiguous: those working more than 16 hours a week had an incentive to reduce their hours to (no less than) 16, while those previously working fewer than 16 hours had an incentive to increase their labour supply to (at least) the new cut-off. Figure 1 shows that the pattern of observed hours of work over this period strongly reflects these incentives. Single women without children were ineligible.[6]

The tax design problem we discuss here relates directly to the features of the WFTC. Indeed we assess the reliability of our labour supply model in terms of its ability to explain behaviour before and after the reform. There were essentially five ways in which WFTC increased the level of in-work support relative to the previous FC system: (i) it offered higher credits, especially for families with younger children; (ii) the increase in the threshold meant that families could earn more before it was phased out; (iii) the tax credit withdrawal rate was reduced from 70% to 55%; (iv) it provided more support for formal childcare costs through a new childcare credit; (v) all child maintenance payments were disregarded from income when calculating tax credit entitlement. The main parameters of FC and WFTC are presented in Table 1.

The WFTC reform increased the attractiveness of working 16 or more hours a week compared to working fewer hours, and the largest potential beneficiaries of WFTC were those families who were just at the end of the FC benefit withdrawal taper. Conditional on working 16 or more hours, the theoretical impact of WFTC is as follows: (i) people receiving the maximum FC award will face an income effect away from work, but not below 16 hours a week; (ii) people working more than 16 hours and not on maximum FC will face an income effect away from work (but not below 16 hours a week), and a substitution effect towards work; (iii) people working more than 16 hours and earning too much to be entitled to FC but not WFTC will face income and substitution effects away from work if they claim WFTC (see Blundell and Hoynes, 2004).

When analyzing the effect of the WFTC programme it is necessary to take an integrated view of the tax system. This is because tax credit awards are counted as income when calculating entitlements to other benefits, such as Housing Benefit and Council Tax Benefit. Families in receipt of such benefits would gain less from the

[6]In 1995, there was another reform to Family Credit, in the form of an additional (smaller) credit for those adults working full time (defined as 30 or more hours a week).

WFTC reform than otherwise equivalent families not receiving these benefits; Figure 2 illustrates how the various policies impact on the budget constraint for a low wage lone parent. Moreover, there were other important changes to the tax system affecting families with children that coincided with the expansion of tax credits, and which make the potential labour supply responses considerably more complex. In particular, there were increases in the generosity of Child Benefit (a cash benefit available to all families with children regardless of income), as well as notable increases in the child additions in Income Support (a welfare benefit for low income families working less than 16 hours a week).[7]

4 A Structural Labour Supply Model

The labour supply specification develops from earlier studies of structural labour supply that use discrete choice techniques and incorporate non-participation in transfer programmes, specifically Hoynes (1996) and Keane and Moffitt (1998). Our aim is to construct a credible model of labour supply behaviour that adequately allows for individual heterogeneity in preferences and can well describe observed labour market outcomes. As initially discussed in section 2, lone mothers have preferences defined over consumption c and hours of work h. Hours of work h are chosen from some finite set $\mathcal{H}$, which in our empirical application will correspond to the discrete weekly hours points $\mathcal{H} = \{0, 10, 19, 26, 33, 40\}$.[8] We augment the model discussed in section 2 to allow the take-up of tax-credits to have a direct impact on preferences through the presence of some stigma or hassle cost (discussed further below), and we use P (equal to one if tax credits are received, zero otherwise) to denote the endogenous

[7]For many families with children, these increases in out-of-work income meant that, despite the increased generosity of in-work tax credits, replacement rates remained relatively stable. There were also changes to the tax system that affected families both with and without dependent children during the lifetime of WFTC: a new 10% starting rate of income tax was introduced; the basic rate of income tax was reduced from 23% to 22%; there was a real rise in the point at which National Insurance (payroll tax) becomes payable.

[8]These hours points correspond to the empirical hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively.

programme participation decision.[9] These preferences may vary with observable demographic characteristics X (such as age, region, the number and age of children), and vectors of unobservable (to the econometrician) characteristics ϵ and ε. Here ε is used specifically to denote the additive state specific errors which are attached to each discrete hours point and are assumed to follow a standard Type-I extreme value distribution so that:

$$U(c, h, P; X, \epsilon, \varepsilon) = u(c, h, P; X, \epsilon) + \varepsilon_h.$$

While we will later consider alternative preference specifications, our results will largely assume Box-Cox preferences of the form:

$$\begin{aligned} u(c, h, P; X, \epsilon) = \alpha_y(X, \epsilon)\frac{c^{\theta_y} - 1}{\theta_y} + \alpha_l(X, \epsilon)\frac{(1 - h/H)^{\theta_l} - 1}{\theta_l} \\ + \alpha_{yl}(X)\frac{c^{\theta_y} - 1}{\theta_y}\frac{(1 - h/H)^{\theta_l} - 1}{\theta_l} - P\eta(X, \epsilon) \quad (4) \end{aligned}$$

where $H = 168$ denotes the total weekly time endowment, and where the set of functions $\alpha_y(X, \epsilon)$, $\alpha_l(X, \epsilon)$, $\alpha_{yl}(X)$ and $\eta(X, \epsilon)$ capture observed and unobserved preference heterogeneity. The function $\eta(X, \epsilon)$ is included to reflect the possible disutility associated with claiming in-work tax credits ($P = 1$), and its presence allows us to rationalize less then complete take-up of tax credit programmes. In each case we allow observed and unobserved heterogeneity to influence the preference shifter functions through appropriate index restrictions. We assume that $\alpha_{yl}(X) = X'_{yl}\beta_{yl}$, $\log \alpha_y(X, \epsilon) = X'_y\beta_y + \epsilon_y$ and $\log \alpha_l(X, \epsilon) = X'_l\beta_l + \epsilon_l$, with programme participation costs also assumed to be linear in parameters, $\eta(X, \epsilon) = X'_\eta\beta_\eta + \epsilon_\eta$. We do not impose concavity on the utility function.

The choice of hours of work h affects consumption c through two main channels: firstly, through its direct effect on labour market earnings and its interactions with the tax and transfer system; secondly, working mothers may be required to purchase childcare for their children which varies with maternal hours of employment. Given the rather limited information that our data contains on the types of childcare use,

[9] All other transfer programmes are assumed to have complete take-up.

we take a similarly limited approach to modelling, whereby hours of childcare use h_c is essentially viewed as a constraint: working mothers are required to purchase a minimum level of childcare $h_c \geq \alpha_c(h, X, \epsilon)$ which varies stochastically with hours of work and demographic characteristics. Since we observe a mass of working mothers across the hours of work distribution who do not use any childcare, a linear relationship (as in Blundell et al., 2000) is unlikely to be appropriate. Instead, we assume the presence of some underlying latent variable that governs both the selection mechanism and the value of required childcare itself. More specifically, we assume that the total childcare hours constraint is given by:

$$\alpha_c(h, X, \epsilon) = \mathbf{1}(h > 0) \times \mathbf{1}(\epsilon_{c_X} > -\beta_{c_X} h - \gamma_{c_X}) \times (\gamma_{c_X} + \beta_{c_X} h + \epsilon_{c_X}) \qquad (5)$$

where $\mathbf{1}(\cdot)$ is the indicator function, and where the explicit conditioning of the parameters and the unobservables on demographic characteristics X reflects the specification we adopt in our estimation, where we allow the parameters of this stochastic relationship to vary with a subset of observable characteristics X_c (specifically, the number and age composition of children). Total weekly childcare expenditure is then given by $p_c h_c$ with p_c denoting the hourly price of childcare. Empirically, we observe a large amount of dispersion in childcare prices, with this distribution varying systematically with the age composition of children. This is modelled by assuming that p_c follows some distribution $p_c \sim F_c(\cdot; X_c)$ which again varies with demographic characteristics. We approximate this distribution by discretizing the empirical childcare price distribution including zero price and conditional on X_c.

Individuals are assumed to face a budget constraint, determined by a fixed gross hourly wage rate (assumed to be generated by a log-linear relationship of the form $\log w = X_w' \beta_w + \epsilon_w$) and the tax and transfer system. We arrive at our measure of consumption by subtracting both childcare expenditure $p_c h_c$ (which also interacts with the tax and transfer system) and fixed work-related costs from net-income. These fixed work-related costs help provide a potentially important wedge that separates the intensive and extensive margin. They reflect the actual and psychological costs that an individual has to pay to get to work. We model work-related costs as a fixed, one-off, weekly cost subtracted from net income at positive values of working time: $f = \alpha_f(h; X, \epsilon) = \mathbf{1}(h > 0) \times (X_f' \beta_f + \epsilon_f)$. It then follows that consumption at a given hours and programme participation choice is given by:

$$c(h, P; T, X, \epsilon) = wh - T(wh, \mathbf{h}, P; X) - p_c h_c - f \tag{6}$$

where non-labour income, such as child maintenance payments, enter the tax and transfer schedule T through the set of demographic characteristics X, and with the explicit conditioning of T on childcare expenditure suppressed for notational simplicity.

In order to fully describe the utility maximization problem of lone mothers, we denote $P^*(h) \in \{0, E(h; X, \epsilon)\}$ as the optimal choice of programme participation for given hours of work h, where $E(h; X, \epsilon) = 1$ if the individual is eligible to receive tax credits at hours h, and zero otherwise. Assuming eligibility, it then follows that $P^*(h) = 1$ if and only if the following condition holds:

$$u(c(h, P = 1; T, X, \epsilon), h, P = 1; X, \epsilon) \geq u(c(h, P = 0; T, X, \epsilon), h, P = 0; X, \epsilon) \tag{7}$$

where $c(h, P; X, \epsilon)$ is as defined in equation 6. It then follows that the optimal choice of hours $h^* \in \mathcal{H}$ maximizes $U(c(h, P^*(h); T, X, \epsilon), h, P^*(h); X, \epsilon, \varepsilon)$ subject to the constraints as detailed above.

5 Data and Estimation

5.1 Data

We use six repeated cross-sections from the Family Resources Survey (FRS), from the financial year 1997/8 through to 2002/3, which covers the introduction and subsequent expansion of WFTC. The FRS is a cross-section household-based survey drawn from postcode records across Great Britain: around 30,000 families with and without children each year are asked detailed questions about earnings, other forms of income and receipt of state benefits. Our sample is restricted to lone mothers who are aged between 18 and 45 at the interview date, not residing in a multiple tax unit household, and not in receipt of any disability related benefits. Dropping families with missing observations of crucial variables, and those observed during the WFTC phase-in period of October 1999 to March 2000 inclusive, restricts our estimation sample to 7,110 lone mothers.

5.2 Estimation

The full model (preferences, wages, and childcare) is estimated simultaneously by simulated maximum likelihood; the likelihood function is presented in Appendix A.[10] We incorporate highly detailed representations of the tax and transfer system using FORTAX (Shephard, 2009). The budget constraints vary accurately with individual circumstances, and reflect the complex interactions between the many components of the tax and transfer system. To facilitate the estimation procedure, the actual tax and transfer schedules are modified slightly to ensure that there are no discontinuities in net-income as either the gross wage or child care expenditure vary for given hours of work. We do not attempt to describe the full UK system here, but the interested reader may consult Adam and Browne (2009) and O'Dea et al. (2007) for recent surveys; see Shephard (2009) for a discussion of the implementation of the UK system in FORTAX.

For the purpose of modelling childcare, we define six groups by the age of youngest child (0–4, 5–10, and 11–18) and by the number of children (1 and 2 or more). The stochastic relationship determining hours of required childcare $\alpha_c(h, X, \epsilon)$ varies within each of these groups, as does the child care price distribution $F_c(\cdot; X_c)$. Using

[10] This simultaneous estimation procedure contrasts with existing UK-centric labour supply studies that have used discrete choice techniques. Perhaps largely owing to the complexity of the UK transfer system, these existing studies (such as Blundell et al., 2000) typically pre-estimate wages which allows net-incomes to be computed prior to the main preference estimation. In addition to the usual efficiency arguments, the simultaneous estimation here imposes internal coherency with regards to the various selection mechanisms.

data from the entire sample period, the childcare price distribution is discretized into either four price points (if the youngest child is aged 0–4 or 5–10) or 2 points (if the youngest child is aged 11-18). In each case, the zero price point is included, and the probability that lone mothers face each of these discrete price points is estimated.

The unobserved wage component ϵ_w and the random preference heterogeneity terms $(\epsilon_y, \epsilon_l, \epsilon_f, \epsilon_\eta, \epsilon_{c_X})$ are assumed to be normally distributed. Given the difficulty in identifying flexible correlation structures from observed outcomes (see Keane, 1992), we allow ϵ_y to be correlated with ϵ_w, but otherwise assume that the errors are independent. In the later results presented we additionally restrict the standard deviation of both ϵ_l and ϵ_f to be zero as we found them to be both very small in magnitude and imprecisely estimated. The integrals over ϵ in the log-likelihood function are approximated using simulation methods (see Train, 2003); we use 400 quasi-random draws generated using Neiderreiter's method. The model is estimated using a sequential quadratic programming method.

5.3 Specification and Structural Parameter Estimates

The estimates of the parameters of our structural model are presented in Table 12. The age of the youngest child has a significant impact on the estimated fixed costs of work α_f; fixed work related costs are higher by around £15 per week if the youngest child is of pre-school age. The presence of young children also has a highly significant effect on the interacted leisure-consumption parameter α_{yl}, but does not have any quantitatively large or significant effect on the linear preference terms α_y and α_l. Whilst the age of the youngest child is important, the actual number of children does not have a significant effect upon the preference parameters.

Lone mothers who are older are estimated to have a lower preference for both consumption and leisure, but higher costs of claiming in-work support. Meanwhile, the main impact of education comes primarily on the preference for leisure α_l; mothers who have completed compulsory schooling have a lower preference for leisure. Ethnicity enters the model through both fixed costs of work and programme participation costs η; we find that programme participation costs are significantly higher for non-white lone mothers. Programme participation costs are found to fall significantly following the introduction of WFTC, although the reduction in the first year is small (as captured by the inclusion of a variable equal to one in the first year of WFTC).

Both the intercept γ_c and the slope coefficient β_c in the child care equation are lower for those with older children. This reflects the fact that lone mothers with older children use child care less, and that the total childcare required varies less with maternal hours of work. To rationalize the observed distributions, we require that the standard deviation σ_c is also larger for those with older children. The price distribution of childcare for each group was discretized in such a way that amongst those mothers using paid childcare, there are equal numbers in each discrete price group. Our estimates attach greater probability on the relatively high childcare prices (and less on zero price) than in our raw data. Individuals who do not work are therefore more likely to face relatively expensive childcare were they to work.

The hourly log-wage equation includes years of education completed (which enters positively), and both age and age squared (potential wages are increasing in age, but at a diminishing rate). Lone mothers who reside in the Greater London area have significantly higher wages, and the inclusion of time dummies track the general increase in real wages over time. Unsurprisingly, there is considerable dispersion in the unobserved component of log-wages.

The within sample fit of the model is presented in Tables 2 and 3. We match the observed employment states and the take-up rate over the entire sample period very well (see the first column of Table 2). We slightly under predict the number of lone mothers working 19 hours per week, and slightly over predict the number working either 26 or 33 hours per week, but the difference is not quantitatively large. Similarly, we obtain very good fit by age of youngest child. The fit to the employment rate is particularly good, and the difference between predicted and empirical hours frequencies never differs by more than around two percentage points.

The fit of the model over time is presented in Table 3. Fitting the model over time is more challenging given that time only enters our specification in a very limited manner - through the wage equation and via the change in the stigma costs of the accessing the tax credit. Despite this we are able to replicate the 9 percentage point increase in employment between 1997/98 and 2002/03 reasonably well with our model, although we do slightly under predict the growth in part-time employment over this period.

To understand what our parameter estimates mean for labour supply behaviour we simulate labour supply elasticities under the actual 2002 tax system across a range of earnings and household types. The results of this exercise are presented in Table 5. Participation elasticities are lowest for single mothers whose youngest child is under 4 (an elasticity of 0.57), while they are significantly higher for mothers with school aged children (0.82 if youngest child is aged 5-10; 0.72 if the youngest child is aged 11-18). Across all child age groups, extensive elasticities are higher than intensive elasticities at low earnings, but at higher earnings levels the intensive elasticities dominate.[11] Intensive elasticities are typically higher for lone mothers with older children, as are the extensive elasticities except at low earnings levels; extensive elasticities are very similar for lone mothers whose youngest child is aged 5-10 or aged 11-18. The individual behaviour that these summary elasticity measures reflect will have implications for the optimal design of the tax and transfer system (see section 6).

5.4 Simulating the WFTC Reform

Before we proceed to consider optimal design problems using our structural model, we first provide an evaluation of the impact of the WFTC reform discussed in section 3 above on single mothers. This exercise considers the impact of replacing the actual 2002 tax systems with the April 1997 tax system on the 2002 population. This exercise is slightly different to simply examining the change in predicted states over this time period as it removes the influence of changing demographic characteristics.

[11] See the note accompanying Table 5 for a precise definition of these elasticities.

The results of this policy reform simulation are presented in Table 4. Overall we predict that employment increased by 4 percentage points as a result of these reforms, with the increase due to movements into both part-time and full-time employment. Comparing with Table 3 we find the reform explains a little under half of the rise in employment over this period. The predicted increase in take-up of tax credits is also substantial, with this increase driven both by the changing entitlement and the estimated reduction in programme participation costs.

6 The Optimal Design of the Tax and Transfer Schedule

In this section we use our structural model to examine the design of the tax and transfer schedule. We show the importance of allowing the schedule to depend on the age of children. One of the key results is that marginal rates should be lower for low earnings families with older children. Given the use of a minimum hours condition for eligibility in the British tax credit system, we also consider the design in the case of a minimum hours rule. We show that if hours of work are partially (but otherwise accurately) observable, then there can be non-trivial welfare gains from introducing an hours rule for lone mothers with older children. However, accurately observing hours of work is crucial for this result. Our results suggest that if hours of work are subject to measurement error – whether this be random or due to direct misreporting – then the welfare gains that can be realised may be much reduced. Our analysis here therefore supports the informal discussion regarding the inclusion of hours in the tax base in Banks and Diamond (2009). Before detailing these results, we first turn to the choice of social welfare transformation and the parameterisation of the tax and transfer schedule.

6.1 Optimal Tax Specification

We have shown that using parameter estimates from a structural model of labour supply, the behaviour of individuals can be simulated as the tax and transfer system is varied. With these heterogeneous labour supply responses allowed for, the structural model provides all the necessary information to maximise an arbitrary social welfare function, subject to a government budget constraint. Note that our analysis here integrates that tax and transfer system.

To implement the optimal design analysis we approximate the underlying non-parametric optimal schedule by a piecewise linear tax schedule that is characterized by a level of out-of-work income (income support), and seven different marginal tax rates. These marginal tax rates, which are restricted to lie between -100% and 100%, apply to weekly earnings from £0 to £300 in increments of £50, and then all weekly earnings above £300. We do not tax any non-labour sources of income, and do not allow childcare usage to interact with tax and transfer schedule unless explicitly stated. When we later allow for partial observability of hours we introduce additional payments that are received only if the individual fulfills the relevant hours criteria.

The optimal tax schedule is solved separately for three different groups on the basis of the age of youngest child: under 4, aged 5 to 10 and 11 to 18. For these illustrations, we have also conditioned upon the presence of a single child. For each of these groups we set the value of government expenditure equal to the predicted expenditure on this group within our sample.[12] Conditioning upon this level of expenditure we calculate the tax and transfer schedule that maximizes social welfare in each of these groups. We adopt the following utility transformation in the social welfare function:

$$\Upsilon(U;\theta) = \frac{(\exp U)^{\theta} - 1}{\theta} \tag{8}$$

which controls the preference for equality by the one dimensional parameter θ and also permits negative utilities which is important in our analysis given that the state

[12] To date we have made no attempt to calculate what the optimal division of overall expenditure is between these three groups. This therefore makes an implicit assumption regarding the value that the government attaches on the welfare of these groups.

specific errors ε can span the entire real line. When θ is negative, the function (8) favours the equality of utilities; when θ is positive the reverse is true. By L'Hôpital's rule $\theta = 0$ corresponds to the linear case. We solve the schedule for a set of parameter values $\theta = \{-0.4, -0.2, 0.0\}$ and then derive the social weights that characterise these redistributive preferences. We do not consider cases where $\theta > 0$. The presence of state specific Type-I extreme value errors, together with our above choice of utility transformation has some particularly convenient properties, as the follow Proposition now demonstrates.

Proposition 1. *Suppose that the utility transformation function is as specified in equation 8. If $\theta = 0$ then conditional on X and ϵ the integral over (Type-I extreme value) state specific errors ε in equation 2 is given by:*

$$\log\left(\sum_{h\in\mathcal{H}} \exp(u(c(h;T,X,\epsilon),h;X,\epsilon))\right) + \gamma$$

where $\gamma \approx 0.57721$ is the Euler-Mascheroni constant. If $\theta < 0$ then conditional on X and ϵ the integral over state specific errors is given by:

$$\frac{1}{\theta}\left[\Gamma(1-\theta) \times \left(\sum_{h\in\mathcal{H}} \exp(u(c(h;T,X,\epsilon),h;X,\epsilon))\right)^{\theta} - 1\right]$$

where Γ is the gamma function.

Proof. The result for $\theta = 0$ follows directly from an application of L'Hôpital's rule, and the well known result for expected utility in the presence of Type-I extreme value errors (see McFadden, 1978). See Appendix B for a proof in the case where $\theta < 0$. □

This proposition, which essentially generalizes the result of McFadden (1978), facilitates the numerical analysis as the integral over state specific errors does not require simulating. Moreover, the relationship between the utilities in each state, and the contribution to social welfare for given (X, ϵ) is made explicit and transparent.

6.2 Implications for the Tax Schedule

The underlying properties from the labour supply model, together with the choice of social welfare weights, are the key ingredients in the empirical design problem. We have seen from Table 5 that the intensive and extensive labour supply responses differ substantially. They also vary with the age of the youngest child. As expected this is reflected in the optimal tax results. For the choice of utility transformation function in equation 8 we examine the impact of alternative θ values. In Table 7 we present the underlying social welfare weights evaluated at the optimal schedule (discussed below) across the different child age groups according to these alternative θ values. For all three values of θ considered here the weights are broadly downward sloping. For the most part we focus our discussion here on the -0.2 value, although we do provide a sensitivity of our results to the choice of θ and find the broad conclusions are robust to this choice.

In Table 6 we present the optimal tax and transfer schedules across the alternative θ values and for all child age groups (also see Figure 3(a)–(c) for $\theta = -0.2$). In all the simulations performed here, the structure of marginal tax rates is broadly progressive with lower rates at lower earnings levels. In particular, marginal rates are typically much lower in the first tax bracket (earnings up to £50 per-week) and for lone mothers with a child aged between 11 and 18 we obtain pure tax credits (negative marginal tax rates) in this bracket. Marginal tax rates are typically much higher in the second bracket (weekly earnings between £50 and £100), but then fall before proceeding to generally increase with labour earnings. As we increase the value of θ (corresponding to less redistributive concern), we obtain reductions in the value of out-of-work income. This is accompanied by broad decreases in marginal tax rates, except in the first tax bracket where marginal tax rates increase. The social welfare weights presented in Table 7 reflect these changes.

Our optimal tax simulations reveal some important differences by the age of children. In particular, marginal tax rates tend to be higher at low earnings for lone mothers with younger children, but lower at high earnings. There are two important observations to make here. Firstly, there are far fewer lone mothers with young children who obtain high earnings under the respective optimal tax and transfer systems: only around 25% of lone mothers whose child is aged 0–4 have earnings that exceed £100 per week; in contrast, around 70% of lone mothers with children in the oldest age group have earnings exceeding this amount. Secondly, the childcare requirements of mothers with young children are considerably higher (see Table 12). As such, the marginal rates presented in Table 6 understate the effective marginal tax rates that mothers with young children face. If we explicitly allow the tax system to subsidize childcare expenditure (we consider a 70% subsidy, which corresponds to the formal childcare subsidy rate under WFTC), then the level of out-of-work income remains effectively unchanged (since non-working mothers do not require childcare in our structural model), while marginal tax rates increase across the entire distribution of earnings for mothers with very young children. There are small increases for mothers with children aged 5–10, and effectively no change for mothers with children aged 11–18. Full results are available upon request.

In the simulation results in Table 6 we also present standard errors for the parameters of the optimal tax schedule. We obtain these by sampling 500 times from the distribution of parameter estimates and re-solving for the optimal schedule conditional on the sample distribution of covariates. The standard errors that we obtain are typically quite small, but this does raise some concern that our results may be sensitive to our particular specification of the utility function. Before proceeding further, we consider the robustness of our main results to the utility function parameterization by estimating our labour supply model with different preference representations, and then exploring the implications for design under each of these. We consider two alternative representations: (i) modify the utility function presented in equation 4 by adding squared Box-Cox transformations of consumption and leisure (henceforth referred to as *utility 2*); (ii) preferences that are quadratic in leisure and consumption[13]

[13]That is: $u(c, h, P; X, \epsilon) = \alpha_y c^2 + \alpha_l l^2 + \alpha_{ly} cl + \beta_y c + \beta_l l - P\eta$, with observable heterogeneity

as in Blundell et al. (2000) (referred to as *utility 3*). The results of this robustness exercise are presented in Table 8 in the case when $\theta = -0.2$. Across all the different age groups, we find that the schedules are very similar to those arrived at using our original utility representation (referred to as *utility 1* in the table). This therefore suggests that the results we present are not too dependent upon our choice of utility function.

6.3 Introducing an Hours Rule

For several decades the UK's tax credits and welfare benefits have made use of rules related to weekly hours of work. As discussed in section 3, individuals must work at least 16 hours a week to be eligible for in-work tax credits, and receive a further smaller credit when working 30 or more hours. While many theoretical models rule out the observability of any hours information, this design feature motivates us to explore the optimal structure of the tax and transfer system when hours can be partially observed as set out in section 2. We begin by assuming that the tax authority is able to observe whether individuals are working 19 hours or more, which roughly corresponds to the placement of the main 16 hours condition in the British tax-credit system, and for now we do not allow for any form of measurement error. In this case the tax authority is able to condition an additional payment on individuals working such hours. When the tax authority is only able to observe earnings, it is unable to infer whether an individual with a given level of earnings is low wage-high hours, or high wage-low hours. Since the government may value redistribution more highly in the former case, it may be able to better achieve its goals by introducing an hours rule into the system.

The results of this exercise are presented in Figure 3(a)–(c) with $\theta = -0.2$ and assuming an hourly wage rate of £6 for all child age groups. The figures show that the size of the hours bonus exhibits a very pronounced age gradient; we obtain a

X influencing the coefficients through linear index restrictions, and with unobserved preference heterogeneity ϵ entering the model similarly.

weekly hours bonus equal to £23, £38 and £45 for lone mothers with children aged 0–4, 5–10 and 11–18 respectively.[14] It therefore appears that there is a much smaller requirement for a part-time hours bonus for families with children aged below 5. But as the children age the optimal schedule changes quite dramatically with a strong move towards an hours bonus.

Relative to the optimal system when such a rule is not implementable, the hours bonus increases marginal rates in the part of the earnings distribution where this hours rule would roughly come into effect (particularly in the £50 to £100 earnings bracket) while marginal rates further up the distribution, as well as the level of out-of-work support, are essentially unchanged. As a result of this, some non-workers with low potential wages may be induced to work part-time, while some low hours individuals will either not work or increase their hours. Similarly, some high earnings individuals will reduce their hours to that required for the bonus. The hours bonus is sufficiently large for lone mothers with school aged children, that it implies a negative participation tax rate at 19 hours when earning the minimum wage rate.

Although there are some notable changes in the structure of the constraint when hours information is partially observable (particularly for lone mothers with older children), it does not follow that it necessarily leads to a large improvement in social welfare. Indeed, in the absence of the hours conditioning, there are only few individuals working less than 19 hours (see Figure 4(a)–(c)) so the potential that it offers to improve social welfare appears limited. We now attempt to provide some guidance concerning the size of the welfare gain from introducing hours rules. The exact experiment we perform is as follows: we calculate the level of social welfare under the optimal schedule with hours contingent payments, and then determine the increase in expenditure per-person that is required to obtain the same level of social welfare in the absence of such hours conditioning. In conducting this experiment we

[14] We also explore the impact that varying the redistributive taste parameter θ has on the size of the hours bonus at 19 hours and on the overall structure of the budget constraint: when $\theta = -0.4$ there is little change in the size of the bonus; when $\theta = 0.0$ the optimal bonus is approximately halved for all child age groups.

allow all the parameters of the (earnings) tax schedule to vary so this is obtained at least cost.

The results of this analysis are presented in Table 9. Unsurprisingly, when children are aged less than 5 the increased expenditure required to achieve the level of social welfare obtained under the 19 hour rule is negligible. However, even when children are of school age, the required increased expenditure is found to be small (and is clearly negligible when the less redistributive preferences are considered). Even without allowing for any form of measurement error, it follows that unless the costs of partial hours observability is sufficiently low, it would appear difficult to advocate the use of a 19 hour rule based upon this analysis. This has very important policy implications given that the UK tax credit system makes heavy use of very similar hours conditions.[15]

6.3.1 An Optimal Hours Rule?

The social welfare gains from introducing a 19 hours rule appear to be only very modest in size at best. In this section we explore whether there are potentially larger gains by allowing the choice of the point at which the hours rule becomes effective to be part of the optimal design problem. The optimal schedules with $\theta = -0.2$ are also shown in Figure 3(a)–(c). In all cases, we get an optimal hours rule at the fifth (out of six) discrete hours point, which corresponds to 33 hours per week.[16] We also note that the size of the optimally placed hours bonus always exceeds that calculated

[15]This finding contrasts with Keane and Moffitt (1998) which considered introducing a work subsidy in a model with three employment states (non-workers, part-time and full-time work) and multiple benefit take-up. Even small subsidies were found to increase labour supply and to reduce dependence on welfare benefits. In contrast to our application (where we are moving from a base with marginal rates well below 100% at low earnings), their simulations considered introducing the subsidy in an environment where many workers faced marginal effective tax rates which often exceeded 100%.

[16]As was the case with the 19 hours rule, we find that with $\theta = -0.4$ there is essentially no change in either the size or placement of the hours bonus. However, when $\theta = 0.0$ we find that the size of the optimal bonus is approximately halved for all child age groups, whilst the optimal placement shifts to 40 hours per-week.

when the hours rule became effective at 19 hours per week. The age gradient that we observed previously is still preserved. Introducing an hours rule further up the hours distribution allows the government to become more effective in distinguishing between high wage/low effort and high effort/low wage individuals than at 19 hours to the extent that few higher wage individuals would choose to work very few hours. Relative to the schedule when the hours rule is set at around 19 hours, this alternative placement tends to make people with low and high earnings better off, while people in the middle range lose. While we again find that very little happens to the level of out-of-work income, there are much more pronounced changes to the overall structure of marginal rates. In particular, there are large reductions in the marginal tax rate in the first tax bracket for all groups (there is now a tax credit of −0.20 for lone mothers with children aged 11–18, and −0.08 for lone mothers with children aged 5–10), while marginal rates now become higher at higher earnings (especially in the presence of older children). Figure 4(a)–(c) show the resulting impact on the hours distribution.

As before, we attempt to quantify the benefits from allowing for hours conditioning. Performing the same experiment as we conducted under the 19 hours rule we find that the required increase in expenditure is considerably larger than that obtained previously (again, see Table 9). For lone parents with children aged 11–18, an 8.5% increase in expenditure would be required to achieve the same level of social welfare when $\theta = -0.2$. We believe that if hours can be accurately observed (as this analysis so far assumes), then this represents a non-trivial welfare gain. For lone mothers with younger children, the welfare gains are far more modest. In any case, if the government wishes to maintain the use of hours conditional eligibility, the analysis here suggests that it may be able to improve design by shifting towards a system that primarily rewards full-time rather than part-time work.[17]

[17] We also considered alternative social welfare functions where the government places an explicit weight on employment. In these simulations we obtained lower out-of-work income, together with lower marginal tax rates at low earnings. However, such considerations did not have a large impact on either the size or placement of the optimal hours bonus.

6.4 Measurement error and hours misreporting

The results presented so far have not allowed for any form of measurement error. While earnings may not always be perfectly measured, it seems likely that there is more scope for mismeasurement of hours as they are conceivably harder to monitor and verify. Indeed, the presence of hours rules in the tax and transfer system presents individuals with an incentive to not truthfully declare whether they satisfy the relevant hours criteria. Relative to when hours are always accurately reported, this would seem to weaken the case for introducing a measure of hours in the tax base. In this section we quantify the importance of such measurement error by considering two alternative scenarios: firstly, we consider the case where hours are imperfectly observed due to random measurement error; secondly, we allow individuals to directly misreport their hours of work to the tax authorities.

In performing this analysis it is necessary to modify our analytical framework from section 2 to distinguish between actual hours of work h, and reported hours of work h_R. While actual hours continue to determine both leisure and earnings, reported hours of work directly affect consumption through the tax schedule, with $T = T(wh, \mathbf{h_R}; X)$. They will also have a direct impact on utility when we allow for individual hours misreporting (discussed below).

6.4.1 Measurement error

We allow for random measurement error by adding an independent and normally distributed error term ν to work hours h to form a pseudo reported hours measure, $\tilde{h}_R = h + \nu$. Actual reported hours h_R are then given by the nearest discrete hours point in the set of hours $\mathcal{H}_{++}$. We assume that ν has zero mean, and in Table 10 we show how the size of the hours bonus and the associated welfare gain, vary as the standard deviation of the measurement error term σ_ν increases in value. A clear pattern emerges. Across all groups, the optimal size of the hours bonus declines as reported hours become less informative. Furthermore, the placement of the optimal hours rule is reduced from 33 to 26 hours for relatively high values of σ_ν. In the simulations where the standard deviation of the error term is equal to 8 (so that a single standard deviation results in reported hours differing from actual hours by a single category), the welfare gain from using hours information is more than halved relative to no measurement error. The presence of random measurement error clearly reduces the desirability of conditioning upon hours, and if it is modest or large in size, then the welfare gains that are achievable are only small, even amongst lone mothers with older children.

6.4.2 Hours misreporting

We have shown that random measurement error reduces the extent to which the government may wish to condition upon hours of work, and it also diminishes the welfare gains that are achievable. In the case of hours conditioning, it is plausible that the form of misreporting is likely to be more systematic than random measurement error. Here we modify our setup to allow individuals to directly misreport their reported hours of work. We let h_B be the required hours of work to receive a bonus (received if $h \geq h_B$), and we continue to let h_R denote reported hours of work. Misreporting is only possible if $h > 0$, so that the tax authorities can always accurately observe employment status. If individuals misreport their hours of work then they must incur a utility cost, which is assumed to depend upon the distance $h_R - h$. Since misreporting hours is costly, it is only necessary to consider the cases when hours are truthfully revealed $h_R = h$, or when $h_R = h_B > h$.

We therefore modify the individual utility function by including $h_R - h$ as an explicit argument, so that $U = u(c, h, h_R - h; X, \epsilon) + \varepsilon_h$. This modified utility function is as in equation 4 but now with the additional cost term $b \times (h_R - h)$ subtracted from u whenever $h_R > h$.[18] If misreporting is not possible, then this is equivalent to $b = \infty$. We do not allow individuals to manipulate their earnings wh. At a given actual hours of work $h < h_B$ individuals will report their hours as $h_R = h_B$ if and

[18]In practice misreporting costs are likely to vary with both observed and unobserved worker characteristics. While it is sufficient to model this as a single cost for the purpose of our discussion and simulations here, our framework can easily be extended to incorporate such heterogeneity.

only if the utility gain exceeds the cost. That is:

$$u(c(h, T(wh, \mathbf{h_B}; X), X, \epsilon), h, h_B - h; X, \epsilon) > u(c(h, T(wh, \mathbf{h}; X), X, \epsilon), h, 0; X, \epsilon).$$

We refer to the parameter b as the misreporting cost, and in the results presented in Table 11 this is measured relative to the standard deviation of the state specific error ε. With an hours bonus payable at 33 hours per week (for example), a value of $b = 0.16$ would mean that the utility cost of reporting 33 hours when actual hours are 26 is equivalent to a $0.16 \times (33 - 26) = 1.12$ standard deviation change in the realisation of the state specific error. The table illustrates that as the utility cost of misreporting becomes very low, the welfare gain from using reported hours of work effectively disappears (but the optimal placement remains at 33 hours for all values considered). Again, this suggests that the welfare gains from using hours of work information may be small unless the scope for misreporting hours of work is limited.

7 Conclusions

The aim of this paper has been to examine the optimal schedule of marginal tax rates and design of earned income tax credits. The context for this design problem has been the tax and transfer schedule for lone parents in Britain. To address this tax design problem we developed a structural labour supply model which incorporated unobserved heterogeneity and the non-convexities of the tax and welfare system as well as allowing for childcare costs and fixed costs of work. We also explicitly allow for different labour supply responses at the intensive and extensive margins.

To mirror the hours contingent nature of the British tax credit system we developed an analytical framework that explicitly allowed for the tax authorities to have partial observability of hours of work. We contrasted this to the standard case in which only earnings (and employment) are revealed to the tax authority.

The structural labour supply model appeared reliable and the estimated model suggested that lone parents with very young children are much less responsive to changes in financial work incentives than are lone parents with children of school age.

This has implications for tax design. For those with very young children – where the marginal value of leisure is high – the optimal policy design suggests it is better to offer high levels of income support together with higher marginal tax rates when in work. In contrast, for those with school age children, where leisure is valued less highly, the results suggest a move to a lower level of income support but also lower marginal tax rates, increasing the incentives to work.

Our results highlight a role for conditioning effective tax rates on the age of children. Tax credits being found to be most important for low earning families with school age children. Hours contingent payments, as feature in the British tax credit system, are also found to lead to improvements in the tax design at least for those parents with school age children. If the tax authorities are able to choose the lower limit on working hours that trigger eligibility for such families, then we find an empirical case for using a *full-time* work rule rather than the part-time rule currently in place for parents in the UK. While this is found to be a more effective instrument, the welfare gains remain modest in size for all but parents with older children. These welfare gains are also shown to reduce significantly with moderate amounts of misreporting or measurement error.

Appendix

A Likelihood function

In what follows let $\mathcal{P}_j(X, p_{c_k}, \epsilon) \equiv \Pr(h = h_j | X, p_{c_k}, \epsilon)$ denote the probability of choosing hours $h_j \in \mathcal{H}$ conditional on demographics X, the childcare price p_{c_k}, and the vector of unobserved preference heterogeneity $\epsilon = (\epsilon_w, \epsilon_{c_X}, \epsilon_y, \epsilon_l, \epsilon_f, \epsilon_\eta)$. Given the presence of state specific Type-I extreme value errors, this choice probability takes the familiar conditional logit form. We also use $\pi_k(X) \equiv \Pr(p_c = p_{c_k} | X)$ to denote the probability of the lone mother with characteristics X facing childcare price p_{c_k}. In the case of non-workers ($h = h_0$), neither wages nor childcare are observed so that the likelihood contribution is simply given by:

$$\sum_k \pi_k(X) \int_\epsilon \mathcal{P}_0(X, p_{c_k}, \epsilon) dG(\epsilon).$$

Now consider the case for workers when both wages and childcare information is observed so that h_c is not censored at zero. Using $E_h \equiv E(h; X, p_c, \epsilon)$ to denote eligibility for in-work support we define the indicator $\mathcal{D}(e, p) = \mathbf{1}(E_h = e, P = p)$. We also let $\Delta u(h_j | p_{c_k}, X, \epsilon_{|\epsilon_\eta = 0})$ denote the (possibly negative) utility gain from claiming in-work support at hours h_j, conditional on demographics X, the childcare price p_{c_k}, and the vector of unobserved preference heterogeneity ϵ with $\epsilon_\eta = 0$. Suppressing the explicit conditioning for notational simplicity, the likelihood contribution is given by:

$$\prod_k \pi_k(X)^{\mathbf{1}(p_c = p_{c_k})} \iiint_{\epsilon_y, \epsilon_l, \epsilon_f} \Bigg\{ \mathcal{D}(1,1) \int_{\epsilon_\eta < \Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h = h_j)}$$
$$+ \mathcal{D}(1,0) \int_{\epsilon_\eta > \Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h = h_j)} + \mathcal{D}(0,0) \int_{\epsilon_\eta} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h = h_j)} \Bigg\}$$
$$dG(\epsilon | \epsilon_w = \log w - X'_w \beta_w, \epsilon_c = h_c - \gamma_{c_X} - \beta_{c_X} h)$$
$$g_{w,c}(\log w - X'_w \beta_w, h_c - \gamma_{c_X} - \beta_{c_X} h).$$

If working mothers are not observed using childcare, then h_c is censored at zero and the childcare price also unobserved. If $\overline{\epsilon}_c = -\gamma_{c_X} - \beta_{c_X} h$, then the likelihood contribution is given by:

$$\sum_k \pi_k(X) \iiiint_{\epsilon_c < \overline{\epsilon}_c, \epsilon_y, \epsilon_l, \epsilon_f} \Bigg\{ \mathcal{D}(1,1) \int_{\epsilon_\eta < \Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)}$$
$$+ \mathcal{D}(1,0) \int_{\epsilon_\eta > \Delta u} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} + \mathcal{D}(0,0) \int_{\epsilon_\eta} \prod_j \mathcal{P}_j(X, p_{c_k}, \epsilon)^{\mathbf{1}(h=h_j)} \Bigg\}$$
$$dG(\epsilon | \epsilon_w = \log w - X_w' \beta_w) g_w(\log w - X_w' \beta_w).$$

Our estimation also allows for workers with missing wages. This takes a similar form to the above, except that it is now necessary to also integrate over the unobserved component of wages ϵ_w.

B Proof of Proposition

For notational simplicity we abstract from the explicit conditioning of utility on observed and unobserved preference heterogeneity and let $u(h) \equiv u(c(h), h; X, \epsilon)$. We then define V as the integral of transformed utility over state specific errors conditional on (X, ϵ):

$$V \equiv \int_\varepsilon \Upsilon \left(\max_{h \in \mathcal{H}} [u(h) + \varepsilon_h] \right) dF(\varepsilon) \quad \text{(A-1)}$$

To prove this result we first differentiate V with respect to $u(h)$:

$$\begin{aligned} \frac{\partial V}{\partial u(h)} &= \int_\varepsilon \left(\frac{\partial \Upsilon \left(\max_{h \in \mathcal{H}} [u(h) + \varepsilon_h] \right)}{\partial u(h)} \right) dF(\varepsilon) \\ &= \int_\varepsilon \Upsilon' \left(u(h) + \varepsilon_h \right) \times \mathbf{1} \left(h = \arg\max_{h' \in \mathcal{H}} [u(h') + \varepsilon_{h'}] \right) dF(\varepsilon) \end{aligned}$$

Given our choice of utility transformation function in X and our distributional assumptions concerning ε the above becomes:

$$\begin{aligned}\frac{\partial V}{\partial u(h)} &= \int_{\varepsilon_h=-\infty}^{\infty} \left\{e^{(u(h)+\varepsilon_h)}\right\}^{\theta} \left(\prod_{h'\neq h} e^{-e^{-\{\varepsilon_h+u(h)-u(h')\}}}\right) \times e^{-\varepsilon_h} e^{-e^{-\varepsilon_h}} d\varepsilon_h \\ &= \left\{e^{u(h)}\right\}^{\theta} \int_{\varepsilon_h=-\infty}^{\infty} \left\{e^{\varepsilon_h}\right\}^{\theta} \times \exp\left(-e^{-\varepsilon_h} \sum_{h'\in\mathcal{H}} e^{-(u(h)-u(h'))}\right) e^{-\varepsilon_h} d\varepsilon_h\end{aligned}$$

We proceed by using the change of variable $t = \exp(-\varepsilon_h)$ so that the above partial derivative becomes:

$$\frac{\partial V}{\partial u(h)} = \left\{e^{u(h)}\right\}^{\theta} \int_{t=0}^{\infty} t^{-\theta} \times \exp\left(-t \sum_{h'\in\mathcal{H}} e^{-(u(h)-u(h'))}\right) dt$$

By defining $z \equiv t \times \sum_{h'\in\mathcal{H}} e^{-(u(h)-u(h'))}$ we can once again perform a simple change of variable and express the above as:

$$\begin{aligned}\frac{\partial V}{\partial u(h)} &= \left\{e^{u(h)}\right\}^{\theta} \left\{\sum_{h'\in\mathcal{H}} e^{-(u(h)-u(h'))}\right\}^{\theta-1} \int_{z=0}^{\infty} z^{-\theta} e^{-z} dz \\ &= e^{u(h)} \left\{\sum_{h'\in\mathcal{H}} e^{u(h')}\right\}^{\theta-1} \int_{z=0}^{\infty} z^{-\theta} e^{-z} dz \\ &= e^{u(h)} \left\{\sum_{h'\in\mathcal{H}} e^{u(h')}\right\}^{\theta-1} \Gamma(1-\theta) \qquad \text{(A-2)}\end{aligned}$$

where the third equality follows directly from the definition of the Gamma function $\Gamma(\cdot)$. Note that this integral will always converge given that we are considering cases where $\theta < 0$. Integrating equation A-2 we obtain:

$$V = \frac{1}{\theta}\left[\Gamma(1-\theta) \times \left(\sum_{h'\in\mathcal{H}} \exp\left\{u(h')\right\}\right)^{\theta} - 1\right] \qquad \text{(A-3)}$$

where the constant of integration is easily obtained by considering the case of a degenerate choice set and directly integrating A-1. This completes our proof of the Proposition.

References

Aaberge, R. and U. Colombino (2008): "Designing Optimal Taxes with a Microeconometric Model of Household Labour Supply," CHILD Working Paper 06/08. 4

Adam, S. and J. Browne (2009): "A survey of the UK tax system," Briefing Note 9, Institute for Fiscal Studies, London. 14

Banks, J. and P. Diamond (2009): "The base for direct taxation," in *Dimensions of Tax Design: The Mirrlees Review*, ed. by J. Mirrlees, S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Grammie, P. Johnson, G. Myles, and J. Poterba, Oxford University Press for Institute for Fiscal Studies. 4, 18

Beaudry, P., C. Blackorby, and D. Szalay (2008): "Taxes and Employment Subsidies in Optimal Redistribution Programs," *American Economic Review*, Forthcoming. 3

Besley, T. and S. Coate (1992): "Workfare versus Welfare: Incentive Arguments for Work Requirements in Poverty-Alleviation Programs," *The American Economic Review*, 82, 249–261. 3

Blundell, R. (2002): "Welfare-to-work: Which policies work and why?" in *Proceedings of the British Academy*, vol. 117, 477–524. 8

Blundell, R., M. Brewer, and M. Francesconi (2008): "Job Changes and Hours Changes: Understanding the Path of Labor Supply Adjustment," *Journal of Labor Economics*, 26, 421–453. 8

Blundell, R., A. Duncan, J. McCrae, and C. Meghir (2000): "The Labour Market Impact of the Working Families' Tax Credit," *Fiscal Studies*, 21, 75–104. 2, 4, 12, 14, 23

Blundell, R. and H. W. Hoynes (2004): "Has "In-Work" Benefit Reform Helped the Labor Market?" in *Seeking a Premier Economy: The Economic Effects of British Economic Reforms, 1980-2000*, ed. by R. Blundell, D. Card, and R. B. Freeman, Chicago: University Of Chicago Press, 411–459. 2, 9

Blundell, R. and T. Macurdy (1999): "Labor supply: A review of alternative approaches," Elsevier, vol. 3, Part 1 of *Handbook of Labor Economics*, 1559–1695. 2

Brewer, M., E. Saez, and A. Shephard (2009): "Means testing and tax rates on earnings," in *Dimensions of Tax Design: The Mirrlees Review*, ed. by J. Mirrlees, S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Grammie, P. Johnson, G. Myles, and J. Poterba, Oxford University Press for Institute for Fiscal Studies. 3

Card, D. and R. Hyslop (2005): "Estimating the Effects of a Time-Limited Earnings Subsidy for Welfare-Leavers," *Econometrica*, 73, 1723–1770. 4

Card, D. and P. Robins (1998): "Do financial incentives encourage welfare recipients to work? Evidence from a randomized evaluation of the Self-Sufficiency Project," *Research in Labor Economics*, 17, 1–56. 3

Choné, P. and G. Laroque (2005): "Optimal incentives for labor force participation," *Journal of Public Economics*, 89, 395–425. 3

Diamond, P. (1980): "Income taxation with fixed hours of work," *Journal of Public Economics*, 13, 101–110. 3

Eissa, N. and J. B. Liebman (1996): "Labor Supply Response to the Earned Income Tax Credit," *The Quarterly Journal of Economics*, 111, 605–637. 2

Gladden, T. and C. Taber (2000): "Wage Progression Among Low Skilled Workers," in *Finding Jobs: Work and Welfare Reform*, ed. by D. Card and R. M. Blank, Russell Sage Foundation. 4

Hoynes, H. W. (1996): "Welfare Transfers in Two-Parent Families: Labor Supply and Welfare Participation Under AFDC-UP," *Econometrica*, 64, 295–332. 4, 10

——— (2009): "Commentary on Chapter 2, Means Testing and Tax Rates on Earnings," in *Dimensions of Tax Design: The Mirrlees Review*, ed. by J. Mirrlees, S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Grammie, P. Johnson, G. Myles, and J. Poterba, Oxford University Press for Institute for Fiscal Studies. 4

Immervoll, H., H. J. Kleven, C. T. Kreiner, and E. Saez (2007): "Welfare reform in European countries: a microsimulation analysis," *The Economic Journal*, 117, 1–44. 3

Keane, M. (1995): "A New Idea for Welfare Reform," *Federal Reserve Bank of Minneapolis Quarterly Review*, 19, 2–28. 3

Keane, M. and R. Moffitt (1998): "A Structural Model of Multiple Welfare Program Participation and Labor Supply," *International Economic Review*, 39, 553–589. 4, 10, 25

Keane, M. P. (1992): "A Note on Identification in the Multinomial Probit Model," *Journal of Business & Economic Statistics*, 10, 193–200. 15

Keane, M. P. and K. I. Wolpin (2007): "Exploring the Usefulness of a Nonrandom Holdout Sample for Model Validation: Welfare Effects on Female Behavior," *International Economic Review*, 48, 1351–1378. 4

Laroque, G. (2005): "Income Maintenance and Labor Force Participation," *Econometrica*, 73, 341–376. 3

McFadden, D. (1978): "Modeling the Choice of Residential Location," in *Spatial Interaction Theory and Planning Models*, ed. by A. Karlqvist, L. Lundqvist, F. Snickars, and J. Weibull, North Holland, 75–96. 20

Mirrlees, J. A. (1971): "An Exploration in the Theory of Optimum Income Taxation," *The Review of Economic Studies*, 38, 175–208. 2

Moffitt, R. (2003): "The Temporary Assistance for Needy Families Program," in *Means-Tested Transfer Programmes in the US*, ed. by R. Moffitt, University of Chicago Press. 3

——— (2006): "Welfare work Requirements with Paternalistic Government Preferences," *The Economic Journal*, 116, F441–F458. 3

O'Dea, C., D. Phillips, and A. Vink (2007): "A survey of the UK benefit system," Briefing Note 13, Institute for Fiscal Studies, London. 14

Phelps, E. S. (1994): "Low-Wage Employment Subsidies versus the Welfare State," *The American Economic Review*, 84, 54–58. 3

Saez, E. (2001): "Using Elasticities to Derive Optimal Income Tax Rates," *The Review of Economic Studies*, 68, 205–229. 3

——— (2002): "Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses," *The Quarterly Journal of Economics*, 117, 1039–1073. 3

Shephard, A. (2009): "FORTAX: Reference Manual," University College London, Manuscript. 14

Train, K. (2003): *Discrete Choice Methods with Simulation*, Discrete Choice Methods with Simulation, Cambridge University Press. 15

van Soest, A., M. Das, and X. Gong (2002): "A structural labour supply model with flexible preferences," *Journal of Econometrics*, 107, 345–374. 4

Table 1: Parameters of FC/WFTC

	April 1999 (FC)	October 1999 (WFTC)	June 2000 (WFTC)	June 2002 (WFTC)
Basic Credit	49.80	52.30	53.15	62.50
Child Credit				
under 11	15.15	19.85	25.60	26.45
11 to 16	20.90	20.90	25.60	26.45
over 16	25.95	25.95	26.35	27.20
30 hour credit	11.05	11.05	11.25	11.65
Threshold	80.65	90.00	91.45	94.50
Taper rate	70% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance
Childcare	Expenses up to £60 (£100) for 1 (more than 1) child under 12 disregarded when calculating income	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £135 (£200) for 1 (more than 1) child under 15

Notes: All monetary amounts are in pounds per week and expressed in nominal terms. Minimum FC/WFTC award is 50p per week in all years above.

Table 2: Predicted and empirical frequencies, age of youngest child

	All		0-4		5-10		11-18	
	Predicted	Empirical	Predicted	Empirical	Predicted	Empirical	Predicted	Empirical
0 hours	0.551	0.550	0.709	0.708	0.491	0.488	0.319	0.320
	(0.005)	(0.006)	(0.007)	(0.008)	(0.008)	(0.010)	(0.012)	(0.013)
10 hours	0.069	0.068	0.053	0.050	0.080	0.082	0.080	0.081
	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.005)	(0.006)	(0.007)
19 hours	0.101	0.121	0.085	0.099	0.114	0.139	0.114	0.130
	(0.003)	(0.004)	(0.003)	(0.005)	(0.003)	(0.007)	(0.004)	(0.009)
26 hours	0.081	0.070	0.056	0.044	0.093	0.084	0.113	0.098
	(0.002)	(0.003)	(0.002)	(0.004)	(0.002)	(0.005)	(0.003)	(0.008)
33 hours	0.092	0.077	0.051	0.042	0.105	0.087	0.157	0.136
	(0.002)	(0.003)	(0.002)	(0.004)	(0.003)	(0.005)	(0.004)	(0.009)
40 hours	0.106	0.115	0.046	0.058	0.117	0.120	0.217	0.235
	(0.003)	(0.004)	(0.003)	(0.004)	(0.005)	(0.006)	(0.009)	(0.012)
Take-up	0.766	0.765	0.822	0.788	0.767	0.783	0.709	0.715
rate	(0.007)	(0.005)	(0.010)	(0.007)	(0.009)	(0.008)	(0.011)	(0.012)

Notes: Empirical frequencies calculated using FRS data with sample selection as detailed in section 5. The discrete points 0, 10, 19, 26, 33 and 40 correspond to the hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively. Empirical take-up rates calculated using reported receipt of FC/WFTC with entitlement simulated using FORTAX. Predicted frequencies are calculated using FRS data and the maximum likelihood estimates from table 12. Standard errors are in parentheses, and calculated for the predicted frequencies by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table 3: Predicted and empirical frequencies, 1997-2002

	1997		2002	
	Predicted	Empirical	Predicted	Empirical
0 hours	0.592	0.600	0.507	0.508
	(0.007)	(0.014)	(0.007)	(0.013)
10 hours	0.071	0.080	0.069	0.062
	(0.003)	(0.008)	(0.003)	(0.006)
19 hours	0.092	0.100	0.114	0.140
	(0.003)	(0.009)	(0.003)	(0.009)
26 hours	0.072	0.052	0.091	0.079
	(0.002)	(0.006)	(0.002)	(0.007)
33 hours	0.080	0.064	0.103	0.093
	(0.002)	(0.007)	(0.002)	(0.008)
40 hours	0.094	0.104	0.115	0.120
	(0.003)	(0.009)	(0.003)	(0.009)
Take-up rate	0.716	0.688	0.817	0.838
	(0.011)	(0.013)	(0.008)	(0.010)

Notes: Empirical frequencies calculated using FRS data with sample selection as detailed in section 5. The discrete points 0, 10, 19, 26, 33 and 40 correspond to the hours ranges 0, 1–15, 16–22, 23–29, 30–36 and 37+ respectively. Empirical take-up rates calculated using reported receipt of FC/WFTC with entitlement simulated using FORTAX. Predicted frequencies are calculated using FRS data and the maximum likelihood estimates from table 12. Standard errors are in parentheses, and calculated for the predicted frequencies by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table 4: Impact of reforms, 1997-2002

	2002 system	1997 system	change
0 hours	0.507	0.547	-0.039
	(0.006)	(0.007)	(0.003)
10 hours	0.069	0.072	-0.002
	(0.003)	(0.003)	(0.001)
19 hours	0.114	0.098	0.015
	(0.003)	(0.004)	(0.002)
26 hours	0.091	0.078	0.013
	(0.002)	(0.002)	(0.001)
33 hours	0.103	0.089	0.014
	(0.002)	(0.002)	(0.001)
40 hours	0.115	0.117	-0.001
	(0.004)	(0.004)	(0.001)
Take-up	0.817	0.683	0.134
rate	(0.008)	(0.019)	(0.015)

Notes: impact of tax and transfer system reforms on hours of work and take-up simulated using FRS 2002 data by replacing actual 2002 tax systems with the April 1997 tax system. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table 5: Simulated elasticities, age of youngest child

Weekly	0-4		5-10		11-18	
Earnings	Extensive	Intensive	Extensive	Intensive	Extensive	Intensive
50	0.168	0.025	0.205	0.085	0.144	0.130
	(0.017)	(0.003)	(0.020)	(0.009)	(0.018)	(0.016)
100	0.128	0.055	0.178	0.177	0.151	0.269
	(0.012)	(0.008)	(0.012)	(0.018)	(0.011)	(0.030)
150	0.100	0.077	0.155	0.239	0.153	0.387
	(0.010)	(0.012)	(0.009)	(0.025)	(0.008)	(0.042)
200	0.067	0.076	0.112	0.231	0.116	0.394
	(0.006)	(0.012)	(0.005)	(0.024)	(0.005)	(0.041)
250	0.043	0.066	0.074	0.194	0.077	0.340
	(0.004)	(0.010)	(0.004)	(0.020)	(0.004)	(0.035)
300	0.027	0.051	0.046	0.147	0.045	0.252
	(0.003)	(0.007)	(0.002)	(0.014)	(0.003)	(0.024)
350	0.016	0.035	0.028	0.102	0.025	0.170
	(0.002)	(0.005)	(0.002)	(0.009)	(0.002)	(0.015)
400	0.024	0.034	0.039	0.094	0.028	0.140
	(0.002)	(0.004)	(0.003)	(0.008)	(0.002)	(0.011)
Participation	0.566		0.820		0.720	
elasticity	(0.047)		(0.042)		(0.036)	

Notes: All elasticities simulated under actual 2002 tax systems with complete take-up of WFTC. Earnings are in pounds per week and are expressed in April 2002 prices. Participation elasticities simulated by increasing consumption at all positive hours choices by 1%. Extensive and intensive earnings elasticities simulated by increasing consumption at the hours point closest to the respective earnings point. Extensive elasticities measure the increase in the employment rate following a 1% increase in consumption at the respective level of earnings. Intensive elasticities measure the increase in the proportion of individuals at each earnings point from any positive hours point following a 1% increase in consumption at the respective level of earnings. Standard errors are in parentheses, and calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table 6: Optimal marginal tax schedules, age of youngest child

Weekly	0-4			5-10			11-18		
Earnings	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$
0–50	0.107	0.150	0.241	0.020	0.043	0.120	-0.045	-0.028	0.060
	(0.026)	(0.021)	(0.029)	(0.034)	(0.023)	(0.020)	(0.056)	(0.040)	(0.028)
50–100	0.618	0.486	0.205	0.631	0.470	0.154	0.552	0.369	0.101
	(0.045)	(0.046)	(0.057)	(0.036)	(0.037)	(0.040)	(0.044)	(0.042)	(0.034)
100–150	0.239	0.177	-0.024	0.325	0.259	0.043	0.407	0.322	0.080
	(0.023)	(0.025)	(0.037)	(0.024)	(0.023)	(0.027)	(0.025)	(0.024)	(0.028)
150–200	0.424	0.367	0.144	0.513	0.437	0.127	0.565	0.468	0.098
	(0.014)	(0.016)	(0.030)	(0.012)	(0.016)	(0.034)	(0.015)	(0.019)	(0.040)
200–250	0.444	0.407	0.136	0.523	0.476	0.202	0.582	0.522	0.219
	(0.010)	(0.012)	(0.029)	(0.009)	(0.012)	(0.035)	(0.011)	(0.013)	(0.041)
250–300	0.384	0.338	0.118	0.517	0.461	0.096	0.580	0.507	0.094
	(0.015)	(0.016)	(0.038)	(0.011)	(0.015)	(0.038)	(0.014)	(0.018)	(0.044)
300+	0.559	0.542	0.343	0.602	0.575	0.298	0.663	0.631	0.335
	(0.010)	(0.010)	(0.032)	(0.008)	(0.009)	(0.040)	(0.008)	(0.009)	(0.044)
Out-of-work	142.545	141.401	133.762	135.548	131.041	108.591	123.733	114.296	79.458
income	(1.273)	(1.188)	(1.270)	(1.833)	(1.752)	(3.200)	(3.579)	(3.451)	(4.651)

Notes: Table presents optimal structure of marginal tax rates and out-of-work income by age of child and under range of distributional taste parameters θ. All incomes are in pounds per week and are expressed in April 2002 prices. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table 7: Social welfare weights under optimal system, age of youngest child

Weekly	0-4			5-10			11-18		
Earnings	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$
0	1.226	1.208	1.143	1.493	1.418	1.228	1.701	1.539	1.238
0–50	1.034	0.966	0.856	1.381	1.282	1.076	1.680	1.497	1.174
50–100	0.838	0.837	0.784	1.103	1.092	0.968	1.352	1.284	1.047
100–150	0.643	0.714	0.802	0.886	0.950	0.952	1.119	1.140	1.016
150–200	0.524	0.647	0.851	0.704	0.828	0.969	0.883	0.980	1.015
200–250	0.423	0.563	0.842	0.562	0.707	0.929	0.705	0.834	0.971
250–300	0.335	0.483	0.883	0.440	0.595	0.912	0.549	0.702	0.948
300+	0.202	0.331	0.775	0.253	0.397	0.860	0.323	0.479	0.905

Notes: Table presents social welfare weights under optimal structure of marginal tax rates and out-of-work income by age of child and under range of distributional taste parameters θ as presented in Table 6. All incomes are in pounds per week and are expressed in April 2002 prices. Social weights are normalized so that the sum of weights multiplied by earnings density under optimal system is equal to unity.

Table 8: Optimal marginal tax schedules, robustness exercise

Weekly	0-4			5-10			11-18		
Earnings	Utility 1	Utility 2	Utility 3	Utility 1	Utility 2	Utility 3	Utility 1	Utility 2	Utility 3
0–50	0.150	0.181	0.125	0.043	0.019	0.015	-0.028	0.006	0.014
	(0.021)	(0.029)	(0.032)	(0.023)	(0.025)	(0.030)	(0.040)	(0.063)	(0.060)
50–100	0.486	0.596	0.335	0.470	0.439	0.257	0.369	0.327	0.247
	(0.046)	(0.062)	(0.044)	(0.037)	(0.042)	(0.042)	(0.042)	(0.093)	(0.063)
100–150	0.177	0.170	0.261	0.259	0.220	0.271	0.322	0.298	0.309
	(0.025)	(0.055)	(0.015)	(0.023)	(0.025)	(0.021)	(0.024)	(0.064)	(0.027)
150–200	0.367	0.362	0.361	0.437	0.413	0.374	0.468	0.453	0.432
	(0.016)	(0.027)	(0.022)	(0.016)	(0.020)	(0.019)	(0.019)	(0.069)	(0.023)
200–250	0.407	0.411	0.410	0.476	0.461	0.452	0.522	0.510	0.512
	(0.012)	(0.016)	(0.019)	(0.012)	(0.015)	(0.016)	(0.013)	(0.070)	(0.020)
250–300	0.338	0.353	0.353	0.461	0.447	0.416	0.507	0.495	0.477
	(0.016)	(0.022)	(0.021)	(0.015)	(0.019)	(0.016)	(0.018)	(0.026)	(0.020)
300+	0.542	0.564	0.557	0.575	0.570	0.583	0.631	0.622	0.646
	(0.010)	(0.014)	(0.025)	(0.009)	(0.012)	(0.016)	(0.009)	(0.013)	(0.020)
Out-of-work	141.401	141.276	140.637	131.041	129.398	125.817	114.296	113.329	111.085
income	(1.188)	(1.407)	(1.217)	(1.752)	(1.954)	(2.292)	(3.451)	(6.336)	(4.966)

Notes: Table presents optimal structure of marginal tax rates and out-of-work income by age of child and range of utility function specifications (*utility 1*, *utility 2*, and *utility 3* – see section 6 for details) with $\theta = -0.2$. All incomes are in pounds per week and are expressed in April 2002 prices. Standard errors are in parentheses and are calculated by sampling 500 times from the distribution of parameter estimates and conditional on the sample distribution of observables.

Table 9: Quantifying the welfare gain of hours rules

	19 hours			optimal hours		
	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$	$\theta = -0.4$	$\theta = -0.2$	$\theta = 0.0$
0–4	0.250	0.213	0.05	0.782	0.854	0.956
	(0.2%)	(0.2%)	(0.0%)	(0.7%)	(0.7%)	(0.8%)
5–10	1.118	0.884	0.130	2.760	2.711	1.476
	(1.3%)	(1.0%)	(0.2%)	(3.2%)	(3.2%)	(1.7%)
11–18	1.592	1.083	0.08	5.016	4.471	1.720
	(3.0%)	(2.1%)	(0.2%)	(9.5%)	(8.5%)	(3.3%)

Notes: Table shows the additional expenditure requirement per person by age of child and under range of distributional taste parameters θ that is necessary to achieve the same level of social welfare as under the respective hours rules with a schedule that varies only with earnings. All incomes are in pounds per week and are expressed in April 2002 prices. Figures in parentheses correspond to the proportional increase in required expenditure.

Table 10: The effect of random measurement error on the optimal hours bonus

Standard	0–4			5–10			11–18		
Deviation	bonus	hours	welfare	bonus	hours	welfare	bonus	hours	welfare
0	39.54	33	0.7%	51.02	33	3.2%	60.42	33	8.5%
2	37.90	33	0.7%	49.42	33	3.0%	58.87	33	8.2%
4	33.87	33	0.6%	43.38	33	2.5%	52.07	33	6.9%
6	29.13	33	0.5%	36.99	33	2.0%	43.52	33	5.4%
8	23.88	33	0.3%	29.91	33	1.4%	33.42	33	3.7%
10	19.24	33	0.3%	23.83	33	1.1%	30.44	26	2.8%
12	15.06	33	0.2%	20.13	26	0.8%	24.26	26	2.1%
14	13.07	33	0.1%	17.49	26	0.6%	20.76	26	1.7%
16	11.70	26	0.1%	15.73	26	0.6%	18.24	26	1.4%

Notes: Table shows how the optimal placement and size of hours contingent payments varies with random hours measurement error by age of youngest child and with $\theta = -0.2$. Standard Deviation refers to the standard deviation of the additive independent normally distributed hours measurement error term. The columns "welfare" refer to the percentage increase in required expenditure to achieve the same level of social welfare compared to when no hours conditioning is performed. All incomes are in pounds per week and are expressed in April 2002 prices.

Table 11: The effect of hours misreporting on the optimal hours bonus

Misreporting	0–4			5–10			11–18		
Cost	bonus	hours	welfare	bonus	hours	welfare	bonus	hours	welfare
∞	39.54	33	0.7%	51.02	33	3.2%	60.42	33	8.5%
0.64	39.54	33	0.7%	51.01	33	3.2%	60.41	33	8.5%
0.32	38.54	33	0.7%	49.03	33	3.1%	57.92	33	8.4%
0.16	29.85	33	0.6%	34.12	33	2.6%	41.38	33	7.0%
0.08	17.35	33	0.4%	19.50	33	1.7%	23.44	33	4.6%
0.04	8.58	33	0.2%	11.04	33	1.0%	12.14	33	2.6%
0.02	5.30	33	0.1%	6.16	33	0.6%	6.73	33	1.5%
0.01	2.75	33	0.1%	3.22	33	0.3%	3.77	33	0.8%

Notes: Table shows how the optimal placement and size of hours contingent payments varies with the utility cost of hours misreporting by age of youngest child and with $\theta = -0.2$. "Misreporting Cost" refers to the additive utility cost associated with misreporting, and is measured per-hour overstated and relative to standard deviation of the state specific error ε. The columns "welfare" refer to the percentage increase in required expenditure to achieve the same level of social welfare compared to when no hours conditioning is performed. All incomes are in pounds per week and are expressed in April 2002 prices.

Table 12: Simulated maximum likelihood estimation results

Preference parameters

	constant	youngest child 0-4	youngest child 5-10	number of children-1	age	compuslory schooling	non-white	London	WFTC period	year 2000
α_y	1.566	-0.104	-0.029	-0.010	-0.010	-0.027	–	–	–	–
	(0.131)	(0.119)	(0.108)	(0.031)	(0.005)	(0.083)	–	–	–	–
α_l	2.781	0.030	0.024	0.057	-0.047	-(0.407)	–	–	–	–
	(0.187)	(0.168)	(0.157)	(0.044)	(0.007)	(0.085)	–	–	–	–
α_{yl}	4.112	7.578	3.587	–	–	–	–	–	–	–
	(1.630)	(2.065)	(1.849)	–	–	–	–	–	–	–
θ_y	0.302	–	–	–	–	–	–	–	–	–
	(0.111)	–	–	–	–	–	–	–	–	–
θ_l	2.813	–	–	–	–	–	–	–	–	–
	(0.816)	–	–	–	–	–	–	–	–	–
α_f	0.284	0.151	0.043	0.044	0.006	0.081	-0.035	0.228	–	–
	(0.083)	(0.084)	(0.068)	(0.032)	(0.005)	(0.063)	(0.053)	(0.046)	–	–
η	0.760	–	–	–	0.028	-0.058	0.328	–	-0.475	0.394
	(0.177)	–	–	–	(0.008)	(0.146)	(0.153)	–	(0.102)	(0.114)

Continued . . .

Table 12: (continued)

Childcare parameters

	1 child youngest age 0-4	1 child youngest age 5-10	1 child youngest age 11-18	2 children youngest age 0-4	2 children youngest age 5-10	2 children youngest age 11-18
γ_c	4.481	-7.767	-27.833	5.035	-25.872	-58.522
	(2.041)	(1.494)	(5.354)	(3.646)	(3.319)	(11.016)
β_c	0.701	0.672	0.309	1.163	1.308	0.639
	(0.066)	(0.049)	(0.157)	(0.133)	(0.115)	(0.323)
σ_c	13.171	11.783	24.814	26.944	27.420	42.667
	(0.466)	(0.312)	(2.274)	(0.905)	(0.868)	(3.757)
$\Pr(p_{cc}^1)$	0.181	0.172	0.153	0.159	0.133	0.178
	(0.019)	(0.018)	(0.036)	(0.019)	(0.016)	(0.049)
$\Pr(p_{cc}^2)$	0.205	0.179	–	0.194	0.146	–
	(0.021)	(0.019)	–	(0.023)	(0.018)	–
$\Pr(p_{cc}^3)$	0.240	0.194	–	0.267	0.164	–
	(0.023)	(0.020)	–	(0.028)	(0.020)	–
p_{cc}^0	0.000	0.000	0.000	0.000	0.000	0.000
p_{cc}^1	0.972	0.810	1.820	0.541	0.570	1.658
p_{cc}^2	2.172	1.594	–	1.555	1.474	–
p_{cc}^3	3.436	2.576	–	2.942	2.474	–

Wage equation

constant	education	age	age squared	London	non-white	1998	1999	2000	2001	2002	σ_w
0.250	0.081	0.052	-0.054	0.191	-0.030	-0.013	0.028	0.130	0.138	0.146	0.406
0.043	(0.002)	(0.012)	(0.017)	(0.024)	(0.027)	(0.025)	(0.029)	(0.025)	(0.023)	(0.024)	(0.005)

Notes: All parameters estimated simultaneously by simulated maximum likelihood, using FRS data and with sample selection as detailed in section 5. Incomes are expressed in hundreds of pounds per week in April 2002 prices. Age and age squared are defined in terms of deviations from the median value; age squared is divided by one hundred. Compulsory schooling is equal to 1 if the individual completed school at age 16 or above. Education measures years of education completed. London is equal to one if resident in the Greater London area. WFTC period is equal to one if individual is interviewed post-October 1999. Standard errors are presented in parentheses.

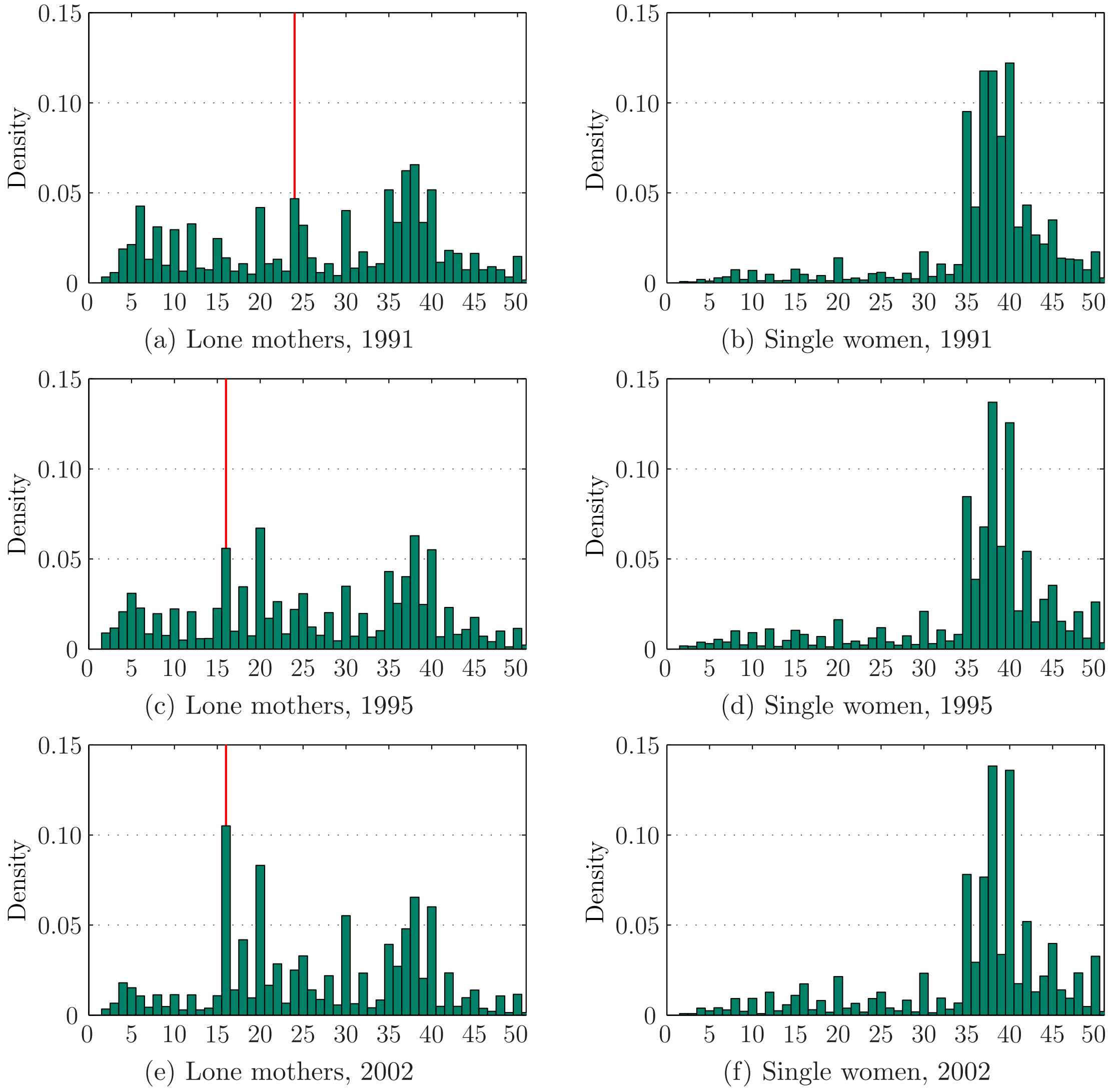


Figure 1: Female hours of work by survey year. Figure shows the distribution of usual hours of work for women by year and presence of children. Sample is restricted to women aged 18–45. Calculated using UK Labour Force Survey data (for 1991) and UK Quarterly Labour Force Survey data (1995 and 2002). Horizontal axes measure weekly hours of work; the vertical line indicates the minimum hours eligibility.

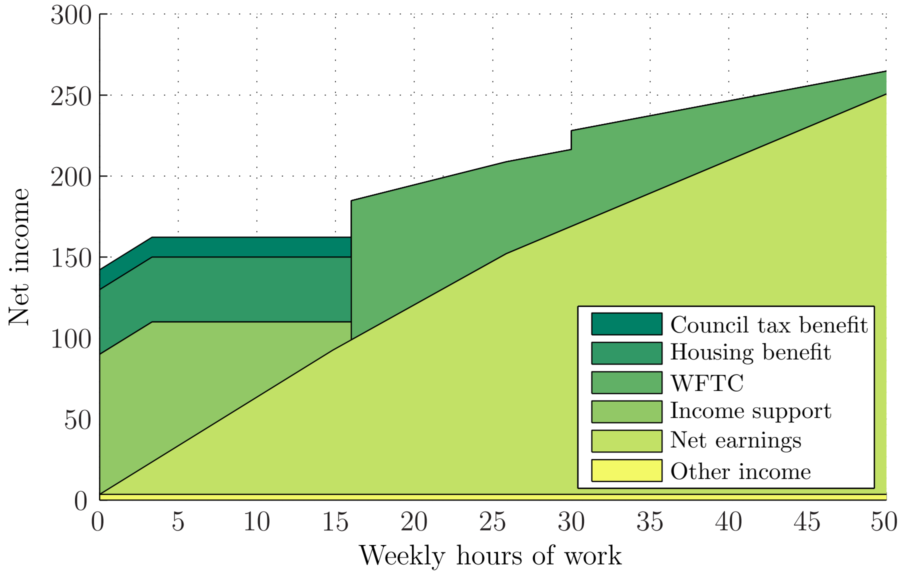
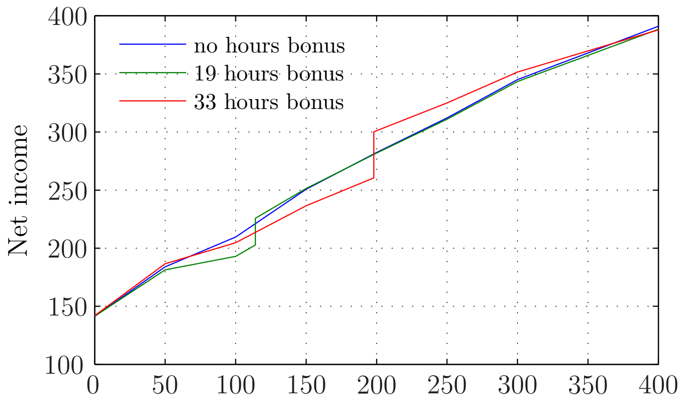
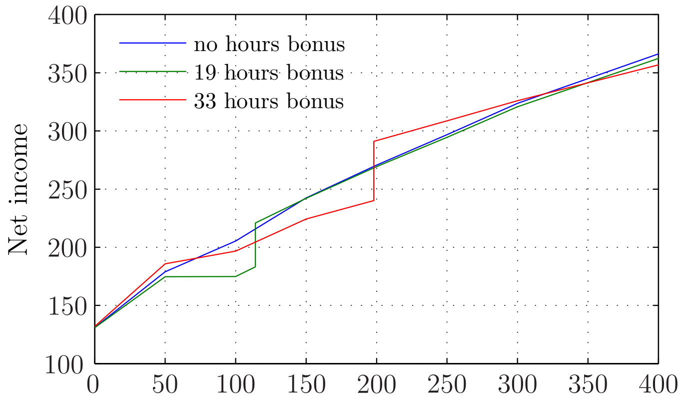


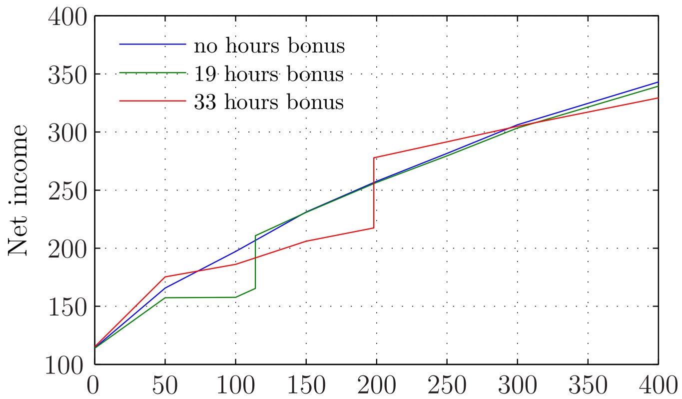
Figure 2: Tax and transfer system interactions. Figure shows interaction of tax and transfer system under April 2002 system for a lone parent with a single child aged 5, average band C council tax, £40 per week housing costs, and no childcare costs. All incomes expressed in April 2002 prices. Calculated using FORTAX.



(a) Youngest child aged 0–4

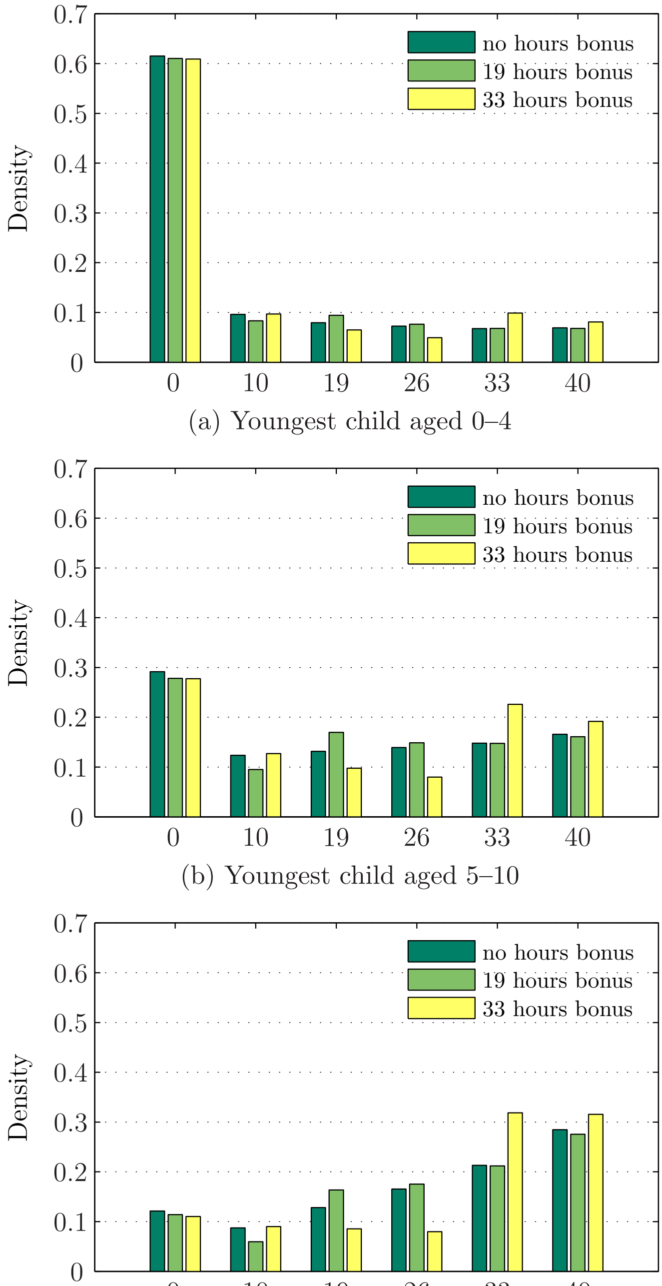


(b) Youngest child aged 5–10



(c) Youngest child aged 11–18

Figure 3: Optimal tax schedules with hours bonuses. All schedules are calculated with $\theta = -0.2$ and assuming an hourly wage of £6. All incomes are measured in April 2002 prices. Horizontal axis measures earnings in pounds per week.



(a) Youngest child aged 0–4

(b) Youngest child aged 5–10

(c) Youngest child aged 11–18

Figure 4: Hours distributions under optimal schedules. Hours distributions are calculated under the respective optimal tax systems with $\theta = -0.2$. Horizontal axis measures hours of work per week.