National Technical University DNIPRO POLYTECHNIC 1899

Mining of Mineral Deposits
ISSN 2415-3443 (Online) | ISSN 2415-3435 (Print)
Journal homepage http://mining.in.ua
Volume 12 (2018), Issue 2, pp. 47-57

UDC 622.271.332 https://doi.org/10.15407/mining12.02.047

EVALUATION OF OPEN PIT SLOPE STABILITY USING VARIOUS SLOPE ANGLES AND ELEMENT TYPES

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ABSTRACT

Purpose. The objective of this study is to demonstrate a method to select the optimal slope angle related to three principal factors: safety, productivity and mining costs. Also, it aims to investigate the accuracy of numerical analysis using different element types and order.

Methods. Series of two-dimensional elasto-plastic finite-element models has been constructed at various slope angles (e.g. 40°, 45°, 50°, 55°, 60°, 65°, and 70°) and different element types (e.g. 3-noded triangle (T_3), 6-noded triangle (T_6), 4-noded quadrilateral (Q_4) and 8-noded quadrilateral (Q_8).The results are presented, discussed and compared at various slope angles and element types in terms of critical strength reduction factor (CSRF) or its equivalent factor of safety (FOS), total rock slope displacement, mine production and mining costs.

Findings. The results reveal that, the mine productivity increases as slope angle increases, however, slope stability deteriorates. Alternatively, the factor of safety (FOS) decreases as slope angle becomes steeper (e.g. minimum factor of safety is obtained at highest steep angle of 70°). Despite of the increasing in computation time, the analysis shows that, the accuracy of the modelling increases when adopting high-order element types (e.g. 8-noded quadrilateral and 6-noded triangle elements).

Originality. This study provides a methodology for the application of the numerical modelling methods on open pit mine. As a result, the mine planners will be able to know ahead of time the optimal slope angle with respect to safety, production and mining costs.

Practical implications. This study sheds light on the usefulness of adopting numerical modelling analysis in the feasibility studies to determine and compare mining costs against safety and slope angle.

Keywords: *slope stability, open pit mine, critical strength reduction factor (CSRF), open pit excavation sequence, finite-element method (FEM), numerical modelling, strength criterion*

1. INTRODUCTION

Safety and productivity are the keys for the success of open pit mine extraction. The stripping ratio should be minimized to decrease the overall mining costs. Slope instability is expensive and usually has negative impacts on the mine profitability (Kumar & Parkash, 2015). Therefore, it is imperative to design slope angle that maintains open pit compete. Stability of open pit is governed by the slope geometry (e.g. height and slope angle), presence of ground water and characteristics of rock mass (e.g. quality, strength, presence of geological features such as joints, faults, etc.).

Geological features determine the pattern of slope failure and its mechanism (e.g., planar, wedge, toppling and/or circular failure) (Hoek & Bray, 1981; Bye & Bell, 2001; Hossain, 2011; Marndi, 2011; Fleurisson & Cojean, 2014; Kumar & Parkash, 2015). However, these factors are always unique to a particular site, therefore, it is difficult to generalize the slope angle for all open pit mines (e.g. due to the heterogeneity/variability of rock mass properties) (Jaeger, 1971; Goodman, 1989; Wyllie & Mah, 2004; Li, Merifield, & Lyamin, 2011).

Open pit mine may extend to hundreds of meters (e.g. South Africa Sandsloot is; located at an elevation of 1100 m, the largest open pit mining platinum in the world, about 1500 m long, 800 m wide and projected depth of 325 m). Consequently, millions of dollars may be lost if inappropriate overall slope angle is designed. Slope angle affects the stripping ratio (e.g. ratio of tonnage or volume of overburden to be removed-to-tonnage or volume of ore to be extracted) and hence, overall mine profitability (Bye & Bell, 2001; Marndi, 2011). Alternatively, the higher the stripping ratio, the more expensive

mining and the less profit is then earned. Reducing of the stripping ratio (e.g., less waste rock to be removed) requires keeping the ultimate slope angle as steep as possible (e.g., increase ore recovery). But, the steeper slope angle the more instability issues are encountered. Thus, the successful design of open pit slope has to combine stability, productivity and mining costs altogether.

Accurate information is required about geology, site characteristics (e.g. slope geometry, rock mass properties, ground water conditions and associated discontinuities, etc.). The final design of optimal slope angle of open pit is controlled not only by ore grade distribution and operational cost, but also by the overall rock mass properties. So, it is recommended that, the potential for failure should be incorporated into the ultimate open pit design. Thus, it helps, in advance, to know temporally (e.g., when) and spatially (e.g., where) actions have to be taken (Brahma, 2009).

2. METHOD OF ANALYSIS

Several methods have been existed to assess open pit slope stability such as method of limit equilibrium, analytical and numerical modelling methods. The limit equilibrium method (LEM) is widely used to estimate the FOS. Such method assumes the shape, location of failure and horizontal forces act upon the sides of the slides and their directions. It is developed and analyzed based on the actual case histories (Ching & Fredlund, 1983; Krahn, 2003; Singh, 2006; Krahn, 2007; Chatterjee & Elkadi, 2012; Berisavljević, Berisavljević, Čebašek, & Rakić, 2015). The FOS is estimated as per Equation (1):

$$FS = \frac{\tau_f}{\tau_m}, \tag{1}$$

where:

τ_f – the actual shear strength of rock mass;

τ_f – the mean shear stress on the assumed surface of failure mobilized to maintain body in equilibrium.

As introduced in Equation (1), the FOS is defined as the ratio of rock shear strength at failure to the mobilized shear stress on the surface of failure. LEM has shortcomings such as it does not consider the stress-strain behaviour of rock mass when estimating the FOS. In addition, it uses trial and errors to estimate the pattern of critical slip surface (Roosta, Sadaghiani, & Pak, 2005; Hamade, 2013; Berisavljević, Berisavljević, Čebašek, & Rakić, 2015). For more details about LEM, the reader is directed to work of Morgenstern & Price (1967) and Fredlund & Krahn (1977).

Numerical methods (e.g., finite-element methods (FEMs), finite-difference methods (FDMs) and/or discrete-element methods (DEMs)) are powerful tools, (e.g., they efficiently handle complex mine geometry), which provide an approximate solutions to the boundary value problems for partial differential equations (Hammah, Yacoub, & Corkum, 2005; Maleki, Mahyar, & Meshkabadi, 2011; Soren, Budi, & Sen 2014; Berisavljević, Berisavljević, Čebašek, & Rakić, 2015). They satisfy all requirements that have to be met for a complete solution to slope stability problems. The behaviour of the material can be modelled with various constitutive equations and numerical simulation techniques, e.g. perfect elasto-plastic analysis with Mohr-Coulomb failure criterion, creep deformation with Burger's model and jointed rock mass with interface elements (Brinkgreve & Bakker, 1991; Potts & Zdravkovic, 1999). Also, hybrid techniques have widely become acceptable in the assessment of rock slopes. These tools combine LEM and FEMs ground water flow and stress analysis (GEO-SLOPE…, 2010; Marndi, 2011; RocScience…, 2011). This study does not represent a real case study. But, it uses representative geological properties (e.g. they are obtained from Jiang & Cao, 2013) and general open pit mine geological features. Also, it is conducted to demonstrate a method to demonstrate an optimal slope angle while maintaining FOS, maximize mine productivity and minimize mining costs. For such purpose, series of models have been built using Rock-Science, RS^{2D} (RocScience…, 2016), finite-element code at various slope angles and different element types. The slope stability is evaluated using CSRF or its equivalent FOS, maximum shear strain and total displacement. The following section briefly presents the different element types and their orders that are adopted in this study.

3. ELEMENT TYPES/ORDERS

When modelling a structure/orebody, its geometry is discretized into small pieces known as "elements". Such elements could be one-, two- or three-dimensional. Alternatively, there are three types of elements namely: line, surface/area and solid/volume elements. Each element is connected to another through the nodes using shape function. The latter, shape function, defines the distribution of the displacement across the elements and it varies according to element type and order. The demonstration of the shape function for each element type and its mathematical formula is beyond the objective of this study. In this study, two-dimensional triangular and quadrilateral element types with different orders, as depicted in Figure 1, have been adopted to investigate their influence on the accuracy of the numerical results.

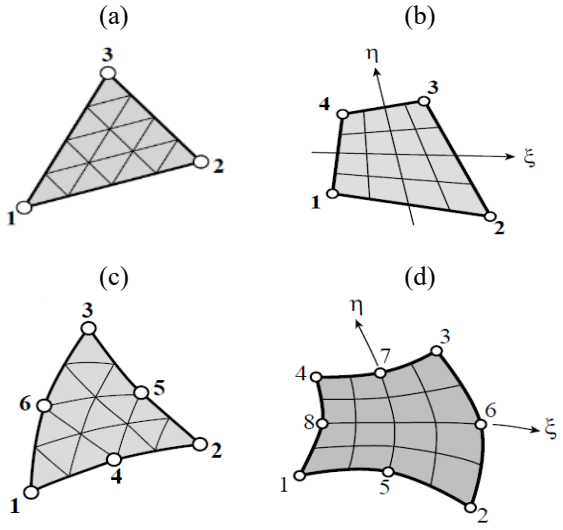


Figure 1. The geometry, type and order of the two-dimensional finite-elements employed in this analysis: (a) 3-node triangular element; (b) 4-node quadrilateral element; (c) 6-node triangular element; (d) 8-node quadrilateral element (Norton, 2006; Boeraeve, 2010; Neupane, 2014)

4. NUMERICAL MODELLING ANALYSIS

In this analysis, Mohr-Coulomb yielding-based criterion has been adopted using rock-soil, RS^{2D}, software (RocScience…, 2016). Table 1 lists the geomechanical properties of rock mass which are used as model input parameters (Jiang & Cao, 2013). Modelling set up and boundary conditions are presented in section 4.1. The results of numerical modelling analysis are introduced and discussed in section 5. The stability of rock slope is evaluated in terms of CSRF or its equivalent FOS, maximum shear strain and total displacement. Then, ore productivity and its corresponding extraction costs at various slope angles have been estimated, presented and discussed in section 5.4.

Table 1. Geomechanical properties of open pit rock mass used in the analysis (Jiang & Cao, 2013)

Rock mass property	Hanging wall (Meta-sediment)	Orebody (Copper-nickel)	Footwall (Green-stone)
Density, MN/m^3	0.0261	0.0251	0.0273
UCS, MPa	73.4	40.3	93.0
E, GPa	44.7	57.0	61.2
Poisson's ratio, υ	0.17	0.20	0.19
Cohesion, *C*, MPa	0.757	0.257	0.245
Tensile strength, σ_t, MPa	7.34	4.03	9.30
Friction angle, ϕ, deg	49	50	48
Dilation angle, Ψ, deg	0	0	0

4.1 Modelling set up

Figure 2a – d depicts the geometry and boundary conditions of the reference model at overall slope angle of 70° and several element types of different orders. The dimensions of the model are 300 × 120 m (e.g. width × height). The orebody comprises of Copper-nickel and the host rock mass consists of Greenstone (e.g. footwall) and Metasediment (e.g. hanging wall). The bottom of the model is fixed in both *X*- & *Y*-directions, while the vertical boundaries are fixed only in *X*-direction. The upper boundary (e.g., ground surface) of the model is simulated as free surface.

5. RESULTS AND DISCUSSION

As stated before, series of two-dimensional elasto-plastic finite-elements models have been developed at various ultimate slope angles (e.g. 40°, 45°, 50°, 55°, 60°, 65° and 70°) and different element types/order to examine their effect on the slope stability of open pit mine, the amounts of ore to be extracted and the corresponding mining costs. Moreover, several element types of different orders have been employed to investigate their influence on the accuracy of numerical results. The numerical modelling results of open pit at overall slope angle of 70° are presented, discussed and compared at different element types/orders in terms of CSRF or its equivalent FOS, maximum shear strain and total displacement of rock slope.

5.1 Critical strength reduction factor (CSRF)

CSRF refers to the value by which strength properties of rock mass (e.g., cohesion & friction angle) have to be reduced to reach failure. Mathematically *SRF* is expressed as follows, Equation (2):

$$SRF = \frac{\tan\varphi}{\tan\varphi_f} = \frac{C}{C_f}, \tag{2}$$

where:

SRF – strength reduction factor, which is used to define the value of rock mass strength parameters at a given stage of the analysis;

C and φ – the soil/rock mass shear strength input values/parameters;

C_f and φ_f – the soil/rock mass shear strength reduced or mobilized values used in the analysis.

It is worthy to mention that, *SRF* is set to 1.0 at the beginning of calculations (e.g., rock mass strength properties are set to their input values, cohesion & friction angle). While in case of failure the SRF, which is defined by Equation (2), corresponds to the FOS aforementioned in Equation (1). Figure 3a – d shows the CSRF for rock slope of open pit mine at overall slope angle of 70° and different element types. The CSRF varies according to element type/order for the same slope angle. For instance, the highest CSRF (e.g. CSRF = 3.40) is obtained when using 4-noded quadrilateral element (e.g. Q_4); whereas, the least CSRF (e.g. CSRF = 1.60) is obtained with 8-noded quadrilateral element (e.g. Q_8).

The CSRF or its equivalent FOS at different element types and various slope angles, is shown in Figure 4. Table 2 lists the values of CSRF or its equivalent FOS and maximum shear strain at different slope angles.

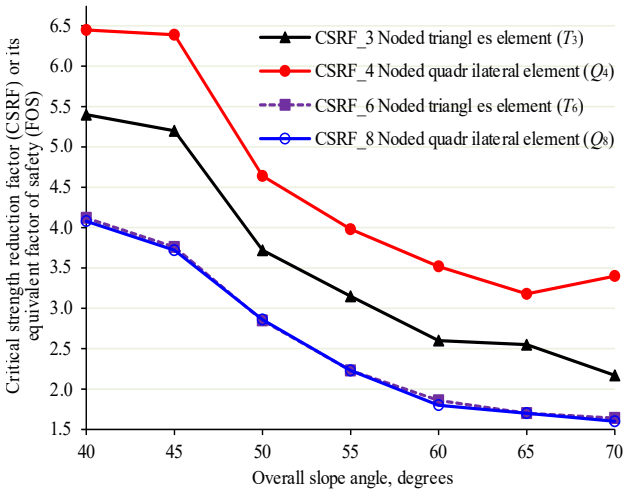
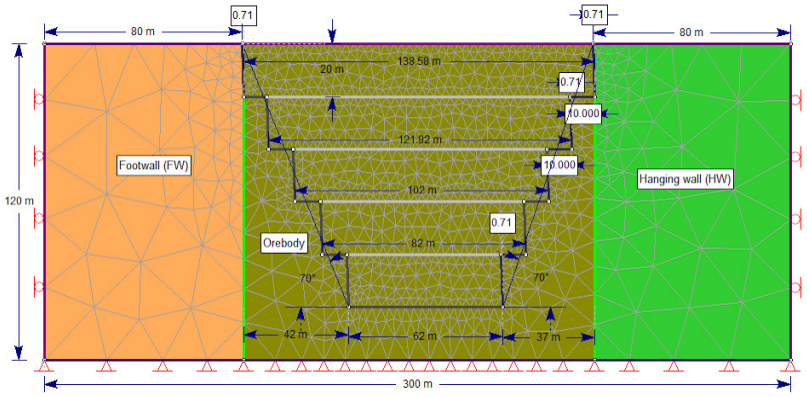


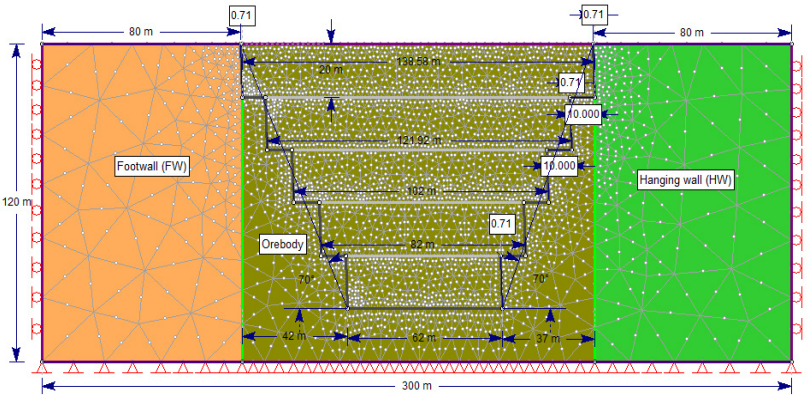
Figure 4. CSRF at various open pit slope angles and different element types

As illustrated in Figure 4, the FOS decreases as slope angle becomes steeper. Alternatively, the minimum factor of safety is obtained at highest steep angle (e.g. 70°) for all element types/order.

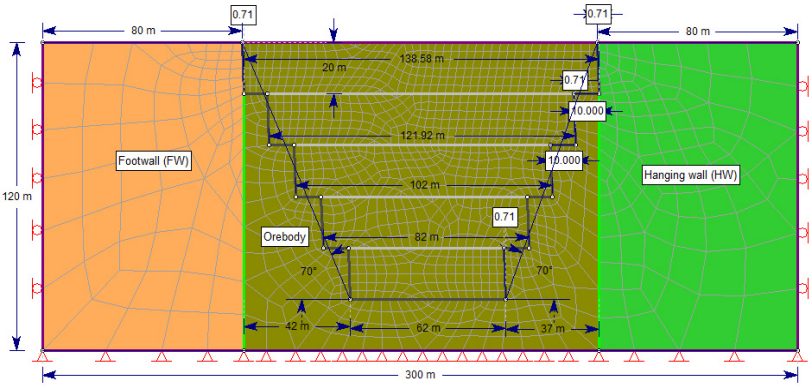
(a)



(b)



(c)



(d)

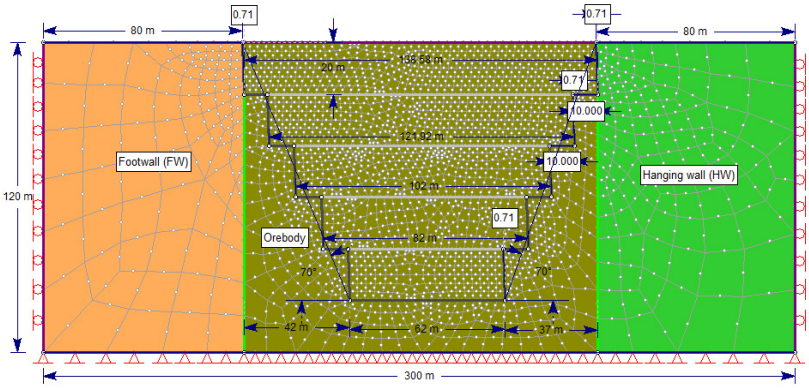


Figure 2. Boundary conditions of the reference model at slope angle of 70° and different element types/order: (a) 3-noded triangle element; (b) 6-noded triangle element; (c) 4-noded quadrilateral element; (d) 8-noded quadrilateral element

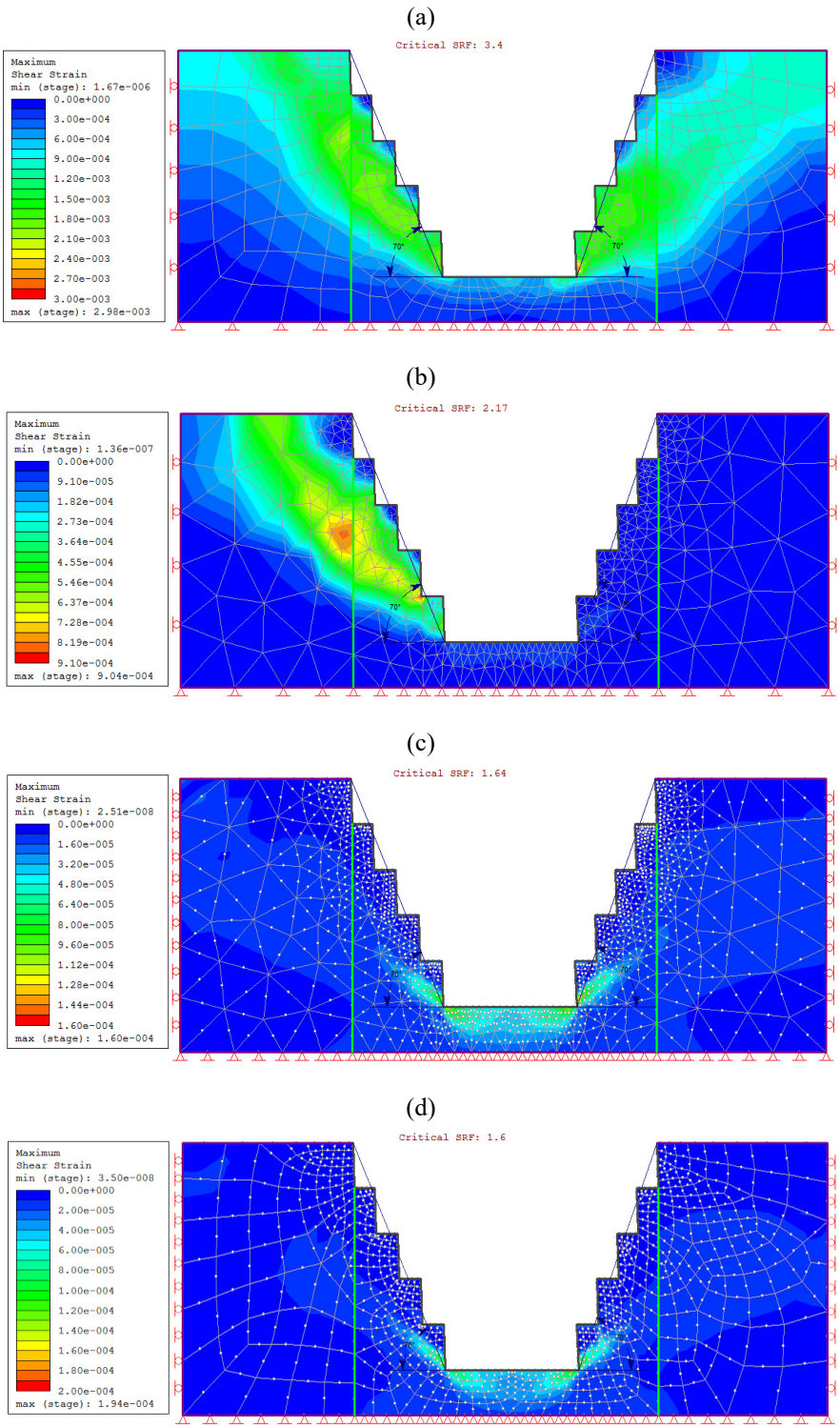


Figure 3. Maximum shear strain at slope angle of 70°: (a) 4-noded quadrilateral element (Q_4); (b) 3-noded triangle element (T_3); (c) 6-noded triangle element (T_6); (d) 8-noded quadrilateral element (Q_8)

Table 2. Strength reduction factor and factor of safety and shear strain at various slope angles and different element types

Overall slope angle, (α), deg.	CSRF or FOS				Maximum shear strain			
	T_3	T_6	Q_4	Q_8	T_3	T_6	Q_4	Q_8
40	5.40	4.12	6.45	4.08	0.000967	0.0001030	0.001310	0.0001900
45	5.20	3.76	6.39	3.72	0.000432	0.0000803	0.000891	0.0000998
50	3.72	2.85	4.64	2.86	0.000486	0.0001290	0.001270	0.0001430
55	3.15	2.23	3.98	2.23	0.001370	0.0001480	0.002550	0.0001480
60	2.60	1.86	3.52	1.80	0.001220	0.0001800	0.001610	0.0002330
65	2.55	1.70	3.18	1.70	0.001700	0.0001750	0.001450	0.0002090
70	2.17	1.64	3.40	1.60	0.000904	0.0001600	0.002980	0.0001940

5.2 Maximum shear strain

Figure 5 displays the maximum shear strain at various overall slope angles and different element types (e.g. also see Table 2). It can be shown that, maximum shear strain increases as overall slope angle increases and element order decreases. Therefore, the more stable slope is found at small overall slope angle (e.g. 40°) and high-order element types (e.g., T_6 and Q_8).

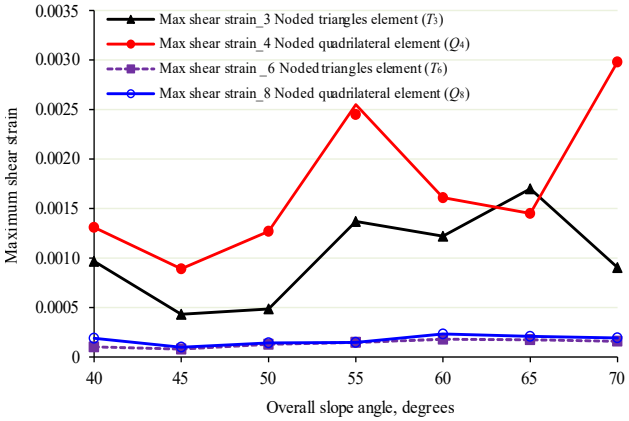


Figure 5. Maximum shear strain at various slope angles and different element types/order

5.3. Total rock slope displacement

Displacement may be adopted as an indicator to measure the movement/deformation of the rock mass. The slope stability deteriorates as rock mass deformation increases. Figure 6 illustrates the total displacement occurs at various ultimate slope angles and different element types/order.

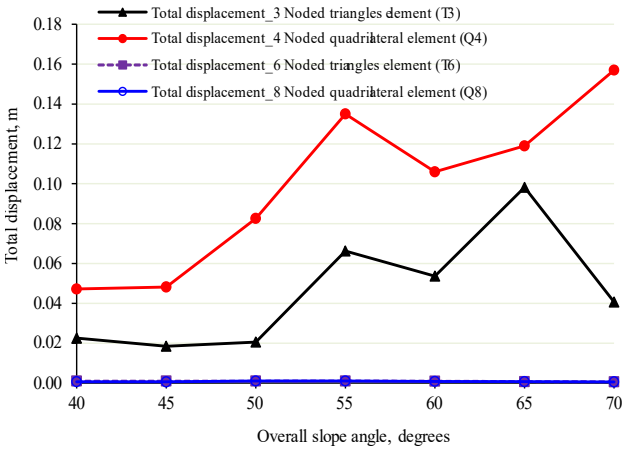


Figure 6. Total displacement occurs at various slope angles and different element types

The results show that, the rock slope displacement increases as slope angle increases and element types/order decreases. The total displacements, at overall slope angle of 70°, are 0.175, 0.0448, 0.0067 and 0.0006 m when adopting element types/order 4-noded, 3-noded, 6-noded and 8-noded respectively. Table 3 lists the total displacements obtained at various slope angles and different element types/order. Figure 6 depicts the total displacements against the ultimate slope angles for each element type.

Table 3. Total displacement at different slope angles and various element types

Overall slope angle, (α), deg.	Total displacement, m			
	T_3	T_6	Q_4	Q_8
40	0.0226	0.001000	0.0472	0.000700
45	0.0185	0.001010	0.0482	0.000605
50	0.0206	0.001010	0.0826	0.001020
55	0.0663	0.001080	0.1350	0.001090
60	0.0537	0.000955	0.1060	0.000850
65	0.0982	0.000736	0.1190	0.000727
70	0.0407	0.000664	0.1570	0.000594

5.4. Tonnage of extracted ore and mining costs

The tonnage of ore to be extracted and its associated mining cost are estimated at different slope angles. It is noteworthy to mention that, the mining costs should include both cost of overburden removal and cost of ore extraction. Generally, the cost of overburden removal decreases as slope angle increases and vice versa. However, when slope angle becomes steep, the slope stability deteriorates (e.g. factor of safety decreases).

Alternatively, low slope angles result in higher factor of safety, but larger costs than steep slope angles. In this study, for simplicity, only the cost of ore extraction is considered (e.g. cost of removal of overburden is not taken into account in addition to costs of machines, transportation, mine operators, blasting and explosive, drilling, milling, concentration, storage, etc.). Figure 7a – g gives the dimensions of expected ore to be extracted at various slope angles.

The analysis shows that, the expected amount of ore to be mined out is directly proportional to slope angle. The highest tonnage of ore will be obtained at slope angle of 70° (e.g. about 0.02560451 million tons and its extraction cost is about M$ 0.05863). The calculation methodology dimensions and boundaries of amounts ore to be extracted at different slope angles and their corresponding operating costs are given at the end of this manuscript (Table 4, Appendix A). Also, Figure 8 depicts the estimated total tonnages of excavated ore and their corresponding costs at different rock slope angles. It can be shown that, the ore productivity and its mining cost increase as slope angle increases.

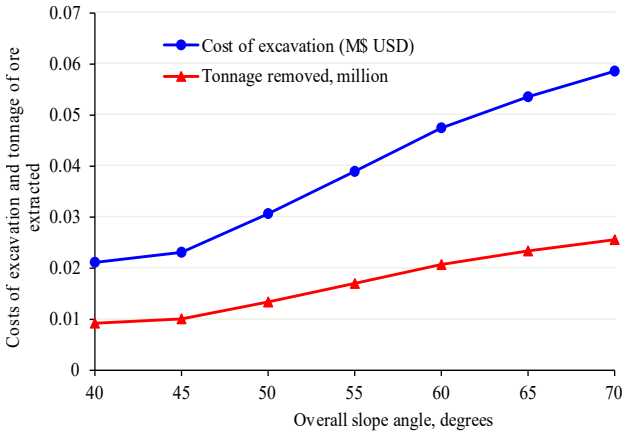
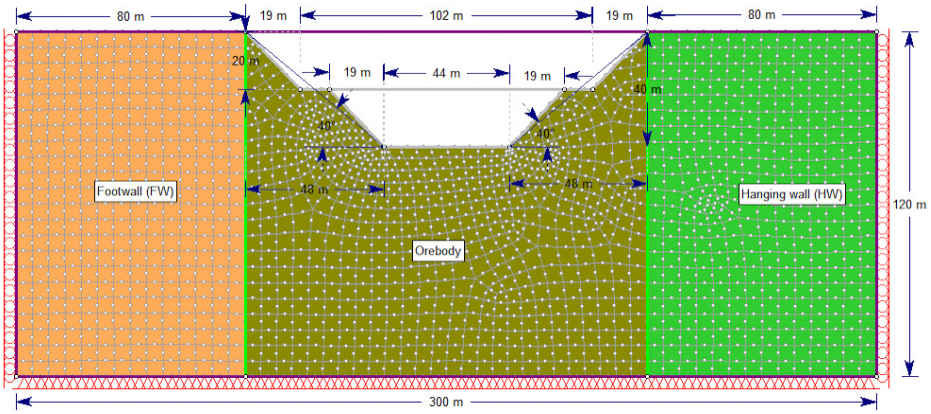
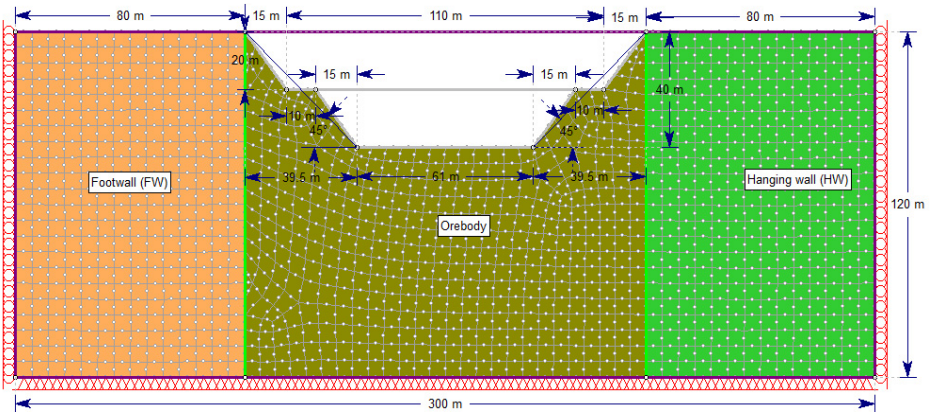


Figure 8. Expected amounts of ore to be extracted and their extraction costs at various slope angles

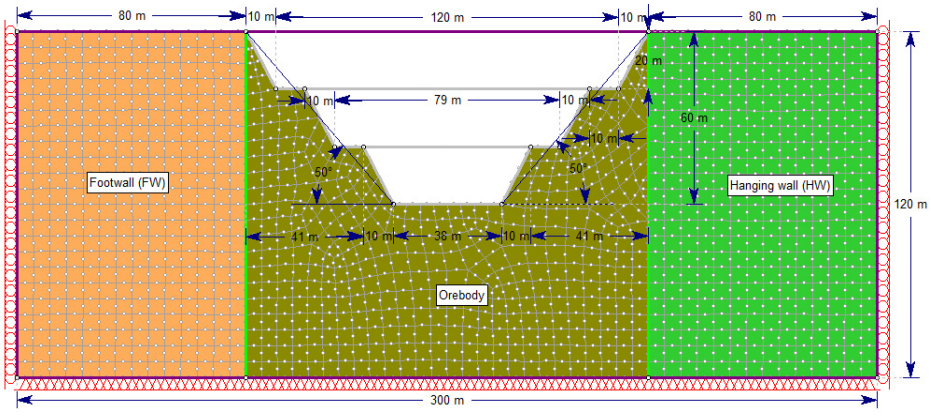
(a)


80 m
19 m
102 m
19 m
80 m
20 m
19 m
44 m
19 m
40 m
40°
40°
Footwall (FW)
18 m
18 m
Hanging wall (HW)
Orebody
120 m
300 m

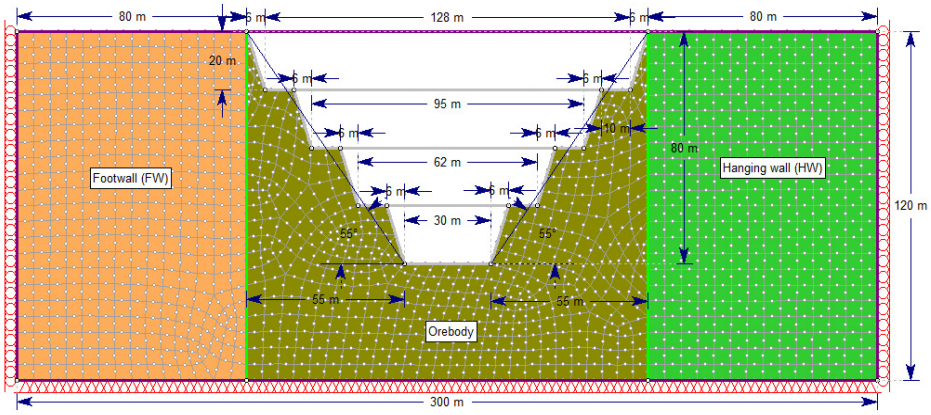
(b)


80 m
15 m
110 m
15 m
80 m
20 m
15 m
15 m
40 m
10 m
10 m
45°
45°
Footwall (FW)
39.5 m
61 m
39.5 m
Hanging wall (HW)
Orebody
120 m
300 m

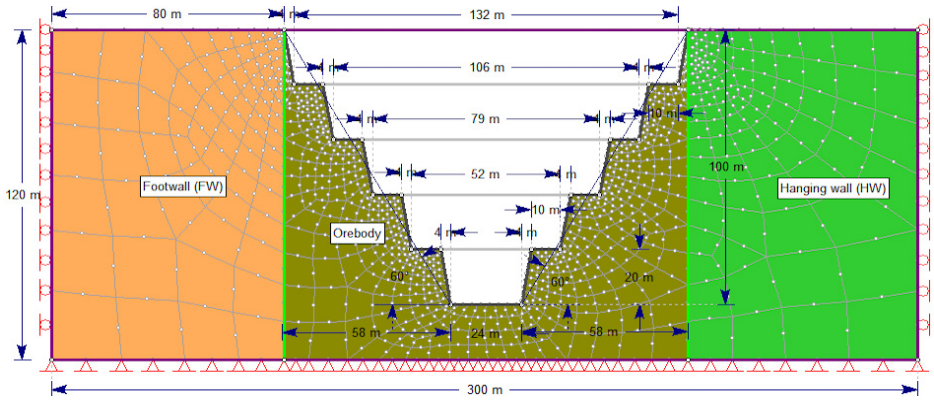
(c)


80 m
10 m
120 m
10 m
80 m
20 m
10 m
79 m
10 m
60 m
10 m
50°
50°
Footwall (FW)
Hanging wall (HW)
41 m
10 m
36 m
10 m
41 m
Orebody
120 m
300 m

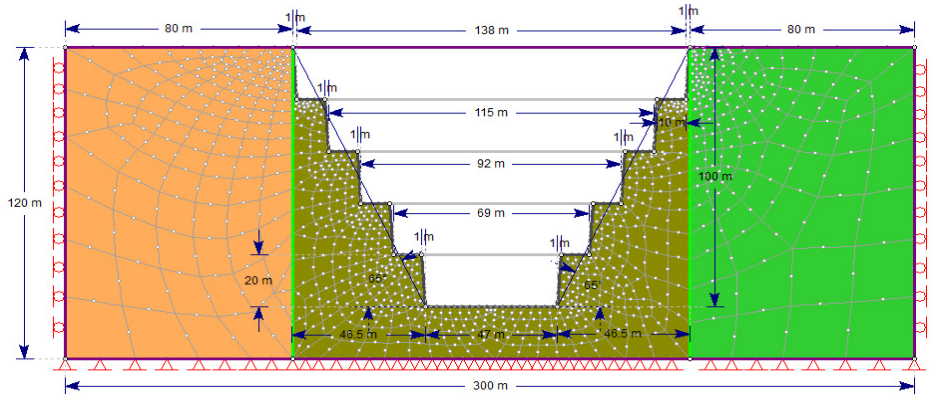
(d)


80 m
5 m
128 m
5 m
80 m
20 m
5 m
5 m
95 m
5 m
5 m
10 m
80 m
62 m
Footwall (FW)
Hanging wall (HW)
5 m
5 m
30 m
65°
65°
55 m
55 m
Orebody
120 m
300 m

(e)



(f)



(g)

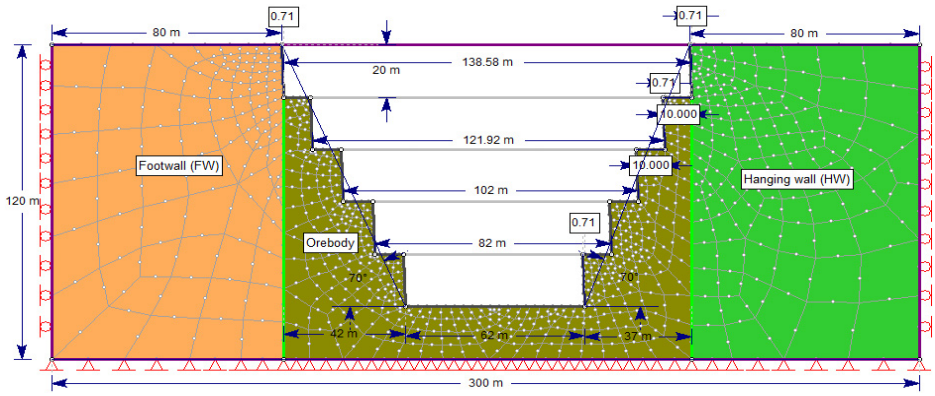


Figure 7. Dimensions of expected ore to be extracted at various slope angles: (a) 40°; (b) 45°; (c) 50°; (d) 55°; (e) 60°; (f) 65°; (g) 70°

6. CONCLUSIONS

Stability of slope is the major concern in open pit mines. Otherwise, slope instability leads to hazards to personnel, machinery and loss of mine profitability. The Overall slope angle has crucial impact on the stability of open pit mine and therefore, its productivity. To maximize the productivity of open pit, the ultimate slope angle should be designed as steep as possible. However, the FOS decreases as slope angle becomes steeper.

In this study, numerical modelling analysis has been conducted to propose a methodology to select the optimal slope angle based on three major factors namely: safety, productivity and costs. Also, the study aims to investigate the effect of different types/order on the accuracy of the modelling results. The results reveal that, as slope angle increases the stability of open pit deteriorates (e.g. factor of safety decreases). Therefore, the better stability is obtained at low slope angle. In addition to, the expected total amount of ore to be excavated is increased as slope angle becomes steep. Furthermore, the analysis shows that, the mining cost increases as slope angle increases. However, this cost does not include the cost of removal of overburden, mine machinery, operators, drilling, blasting, transportation, oil, milling and crushing, etc. Generally, the cost of removal of overburden decreases as slope angle increases. Despite of the computation time, the results show that, the accuracy of the numerical analysis increases when employing high order element type (e.g. 8-noded quadrilateral and 6-noded triangle elements).

APPENDIX A

Table 4. Tonnage of ore to be excavated and its assumed corresponding mining costs*

Overall slope angle, α	Area of excavated blocks				
	Block 1	Block 2	Block 3	Block 4	Block 5
40	2 × (0.5 × 19 × 20) + + (102 × 20) = 2420	2 × (0.5 × 19 × 20) + + (44 × 20) = 1260	—	—	—
45	2 × (0.5 × 15 × 20) + + (110 × 20) = 2500	2 × (0.5 × 15 × 20) + + (61 × 20) = 1520	—	—	—
50	2 × (0.5 × 10 × 20) + + (120 × 20) = 2600	2 × (0.5 × 10 × 20) + + (79 × 20) = 1780	2 × (0.5 × 10 × 20) + + (38 × 20) = 960	—	—
55	2 × (0.5 × 6 × 20) + + (128 × 20) = 2680	2 × (0.5 × 6 × 20) + + (95 × 20) = 2020	2 × (0.5 × 6 × 20) + + (62 × 20) = 1360	2 × (0.5 × 6 × 20) + + (30 × 20) = 720	—
60	2 × (0.5 × 4 × 20) + + (132 × 20) = 2720	2 × (0.5 × 4 × 20) + + (106 × 20) = 2200	2 × (0.5 × 4 × 20) + + (79 × 20) = 1660	2 × (0.5 × 4 × 20) + + (52 × 20) = 1120	2 × (0.5 × 4 × 20) + + (24 × 20) = 560
65	2 × (0.5 × 1 × 20) + + (138 × 20) = 2780	2 × (0.5 × 1 × 20) + + (115 × 20) = 2320	2 × (0.5 × 1 × 20) + + (92 × 20) = 1860	2 × (0.5 × 1 × 20) + + (69 × 20) = 1400	2 × (0.5 × 1 × 20) + + (47 × 20) = 960
70	2 × (0.5 × 0.71 × 20) + + (138.58 × 20) = 2785.8	2 × (0.5 × 0.71 × 20) + + (121.92 × 20) = 2452.6	2 × (0.5 × 0.71 × 20) + + (102 × 20) = 2054.20	2 × (0.5 × 0.71 × 20) + + (82 × 20) = 1654.20	2 × (0.5 × 0.71 × 20) + + (62 × 20) = 1254.20

Continuation of Table 4

Overall slope angle, α	Total area, m^2	Excavated depth, m	Volume, m^3	Excavated tonnage area × γ, t	Cost M$ USD ($ 2.29/ton)
40	3680	40	3680	3680 × 2.51 = 9236.8	$9236.8 \times 2.29 \times 10^{-6}$ = = M$ 0.02115 USD
45	4020	40	4020	4020 × 2.51 = 10090.2	$10090.2 \times 2.29 \times 10^{-6}$ = = M$ 0.02311 USD
50	5340	60	5340	5340 × 2.51 = 13403.4	$13403.4 \times 2.29 \times 10^{-6}$ = = M$ 0.0307 USD
55	6780	80	6780	6780 × 2.51 = 17017.8	$17017.8 \times 2.29 \times 10^{-6}$ = = M$ 0.03897 USD
60	8260	100	8260	8260 × 2.51 = 20732.6	$20732.6 \times 2.29 \times 10^{-6}$ = = M$ 0.0475 USD
65	9320	100	9320	9320 × 2.51 = 23393.2	$23393.2 \times 2.29 \times 10^{-6}$ = = M$ 0.0536 USD
70	10201	100	10201	10201 × 2.51 = 25604.51	$25604.51 \times 2.29 \times 10^{-6}$ = = M$ 0.05863 USD

*Some assumptions have been made:
– cost of extraction of 1-ton of ore is assumed to equal $2.29 USD;
– as 2D-analysis has been conducted, thus, the projected length of open pit in the third dimension is taken as unity;
– other mining costs such as hauling, hoisting, crushing, labour, supporting and upgrading are not considered here;
The costs may be changed according to market prices and ore type and therefore, net profit of ore can be practically estimated.

ACKNOWLEDGEMENTS

The authors acknowledge the support of Rocscience Inc. for getting a free two-week full-version of RS^{2D} (Rock-Soil two-dimensional finite-element analysis program). The authors are grateful for their support.

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ОЦІНКА СТІЙКОСТІ БОРТУ КАР'ЄРА ПРИ РІЗНИХ КУТАХ ЙОГО НАХИЛУ З ВИКОРИСТАННЯМ ЕЛЕМЕНТІВ АНАЛІЗУ РІЗНОГО ТИПУ

В.Р. Абделлах, М.М. Беблаві, М.Т. Мохамед

Мета. Розробка методики для підбору оптимального кута нахилу борта кар'єру з видобутку мідно-нікелевих руд з урахуванням трьох головних чинників: безпека, продуктивність та витрати, а також перевірка точності чисельного аналізу при використанні елементів різного типу і порядку.

Методика. Побудовано серії двовимірних пружно-пластичних кінцево-елементних моделей (КЕМ) для різних кутів нахилу борта кар'єру (наприклад, 40°, 45°, 50°, 55°, 60°, 65° і 70°) та з елементами різного типу (3-вузловий трикутник (T_3), 6-вузловий трикутник (T_6), 4-вузловий чотирикутник (Q_4) і 8-вузловий чотирикутник (Q_8)). Чисельне моделювання виконано у програмному продукті Rock and Soil 2-Dimensional Analysis Program. В якості критерію руйнування прийнято критерій міцності Кулона-Мора.

Результати. Дослідження показали, що продуктивність шахти зростає зі збільшенням кута нахилу борта кар'єру; однак при цьому зменшується його стійкість, і навпаки, чим крутіше кут нахилу борта, тим менше коефіцієнт безпеки. Так, мінімальному значенню коефіцієнта безпеки відповідає найбільший кут нахилу 70°. Незважаючи на більш тривалі обчислення, аналіз показав, що точність моделювання зростає при використанні елементів високого порядку (8-вузлового чотирикутника і 6-вузлового трикутника).

Наукова новизна. Розроблено новий методичний підхід для застосування чисельного моделювання для оцінки стійкості бортів кар'єрів з точки зору граничного коефіцієнта зниження міцності або його еквівалентного коефіцієнта безпеки, загального зсуву схилів, продуктивності та витрат на видобуток.

Практична значимість. Дослідження доводять ефективність застосування чисельного моделювання для визначення доцільності витрат при різних кутах нахилу борта для забезпечення безпеки робіт. В результаті його застосування проектувальники кар'єрів зможуть заздалегідь спланувати оптимальний кут нахилу борта з урахуванням безпеки, продуктивності і витрат.

Ключові слова: *стійкість борту, кар'єр, граничний коефіцієнт зниження міцності (ГКЗМ), метод скінченних елементів (МСЕ), чисельне моделювання, критерій міцності*

ОЦЕНКА УСТОЙЧИВОСТИ БОРТА КАРЬЕРА ПРИ РАЗЛИЧНЫХ УГЛАХ ЕГО НАКЛОНА С ИСПОЛЬЗОВАНИЕМ ЭЛЕМЕНТОВ АНАЛИЗА РАЗНОГО ТИПА

В.Р. Абделлах, М.М. Беблави, М.Т. Мохамед

Цель. Разработка методики для подбора оптимального угла наклона борта карьера по добыче медно-никелевых руд с учетом трех главных факторов: безопасность, производительность и затраты, а также проверка точности численного анализа при использовании элементов различного типа и порядка.

Методика. Построены серии двумерных упругопластических конечно-элементных моделей (КЭМ) для разных углов наклона борта карьера (например, 40°, 45°, 50°, 55°, 60°, 65° и 70°) и с элементами разного типа (3-узловой треугольник (T_3), 6-узловой треугольник (T_6), 4-узловой четырехугольник (Q_4) и 8-узловой четырехугольник (Q_8)). Численное моделирование выполнено в программном продукте Rock and Soil 2-Dimensional Analysis Program. В качестве критерия разрушения принят критерий прочности Кулона-Мора.

Результаты. Исследования показали, что производительность шахты растет с увеличением угла наклона борта карьера, однако при этом уменьшается его устойчивость, и наоборот, чем круче угол наклона борта, тем меньше коэффициент безопасности. Так, минимальному значению коэффициента безопасности соответствует самый большой угол наклона 70°. Несмотря на более длительные вычисления, анализ показал, что точность моделирования возрастает при использовании элементов высокого порядка (8-узлового четырехугольника и 6-узлового треугольника).

Научная новизна. Разработан новый методический подход для применения численного моделирования для оценки устойчивости бортов карьеров с точки зрения предельного коэффициента снижения прочности или его эквивалентного коэффициента безопасности, общего смещения склонов, производительности и затрат на добычу.

Практическая значимость. Исследования доказывают эффективность применения численного моделирования для определения целесообразности затрат при различных углах наклона борта для обеспечения безопасности работ. В результате его применения проектировщики шахт смогут заранее спланировать оптимальный угол наклона борта с учетом безопасности, производительности и затрат.

Ключевые слова: *устойчивость борта, карьер, предельный коэффициент снижения прочности (ПКСП), метод конечных элементов (МКЭ), численное моделирование, критерий прочности*

ARTICLE INFO

Received: 23 January 2018
Accepted: 16 March 2018
Available online: 26 March 2018

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