TOWARDS A RATIONAL DESIGN OF COMPOSITE-DECK SLABS SUBJECT TO FATIGUE LOAD

Jeffrey MAHACHI[1]

University of Johannesburg, Republic of South Africa

ABSTRACT

The effect of dynamic loading, in particular fatigue loading, is a major consideration in the analysis and design of various engineering structures. An experimental investigation was conducted on the effect of fatigue loading on composite deck slabs. Composite deck slabs have been used extensively under static loading conditions but with limited use under vibration conditions. In this paper, the focus is on the composite Bond-Dek slab which is formed using a cold-formed steel profile. Composite action between the steel profile and the concrete slab is provided by rolled dimples or embossments that project from the sides of the steel profile. The experimental work involved testing the slabs for ultimate static strength and then subjecting the specimens to various cyclic fatigue load levels. As a result of the study, it appears justified that under static loading, debonding of the steel/concrete interface occurs prior to ultimate load, with failure occurring as a result of shear bond rupture. Fatigue test results have indicated that the composite deck slabs can further be used in structures subjected to fatigue loading. From the experiments, it was observed that the usual S-N curve representation was not appropriate for the results since there was an abrupt transition between the failure and no-failure points, and due to the limited test results. The results are thus represented in a modified Goodman Diagram which makes the rational design approach easier to interpret and implement. The limitation of the experimental test was the type of the steel profile, the loading conditions and the frequency of the applied load, which was limited to 4 Hz.

Keywords: Fatigue load, Composite Deck Slabs, Bond-Dek, Debonding, Steel Deck Profile

1. INTRODUCTION

The trend towards longer spans and thinner sections has resulted in the use of composite steel deck slabs. These composite slabs consist of a cold-formed, profiled steel sheet which acts as formwork during construction and as tensile reinforcement during operation. Composite slabs have been used extensively world-wide under static loading conditions (Luttrel and Davison [1], Schuster [2], Harding [3], Mahachi and Dundu [4]), particularly for commercial and residential buildings.

The most commonly used steel profiles in South Africa are the Bond-Dek and Bond-Lok. For the Bond-Dek, composite action between the steel deck and the concrete slab is provided by rolled dimples or embossments that project from the sides of the profile. The embossments are capable of providing a resistance to horizontal shear and preventing vertical separation at the steel/concrete interface. A typical profile of a Bond-Dek shown in figure 1 has embossments running parallel to the web as shown in figure 2. On the other hand, the Bond-Lok profile relies mainly on its dovetail or re-entrant angle geometry and chemical adhesion to ensure composite action.

The advantages and disadvantages of using composite slabs under static load are well known and documented by Porter and Ekberg [5], Harding [3], Wright et al. [6] and many others. The major problem with composite deck slabs is the ability to achieve the structural composite action between the steel profile and concrete.

[1] Email: JMahachi@uj.ac.za

Figure 1. Bond-Dek steel profile

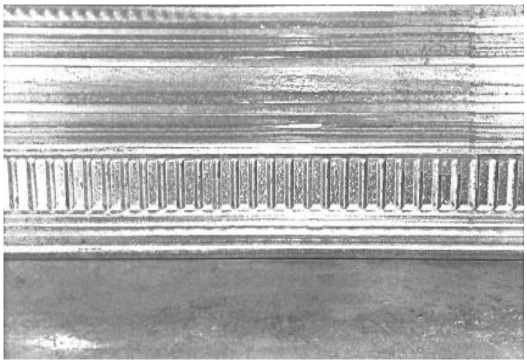

Figure 2. Details of Embossments

However, composite slabs have generally not been used extensively in structures subject to fatigue load largely because of insufficient information with regards to their behavior under such type of loading. Such loads are repetitive and severe enough to make fatigue resistance a primary design criterion. It is therefore anticipated that with proper understanding of the behavior and strength of composite slabs subjected to repeated loading, the structural use of these composite systems will be broadened. A number of other geometrical steel profiles with different orientation of embossments are in use world-wide (Murray and Abdel-Sayed [7] and Patrick and Bridge [8]). Some research conducted by Krige and Mahachi [9] and Mahachi [10] on the response of composite Bond-Lok slabs to fatigue loading indicated a favourable response. However, from the designer's point of view, little information is available for the design rules of composite slabs subjected to fatigue load.

This paper discusses the behavior of Bond-Dek composite slabs when subjected to various constant amplitude sinusoidal fatigue loads. Experimental fatigue tests were carried out on the composite Bond-Dek slabs. A rational design approach similar to that recommended for concrete by ACI Committee 215 [11] is proposed, using the modified Goodman Diagram. Due to limitations of tests that could be conducted, only two point line loading with constant shear length was considered.

2. EXPERIMENTAL STUDY

2.1 Material Properties

Concrete mix proportions with an average 28 day cube strength of 40 MPa was used for all the composite Bond-Dek specimens used in the experimental tests. The tensile strength of the concrete was 4.2 MPa. The concrete was cast to a total depth of 150 mm, with a width of 940 mm. It should be noted that in practice a mesh reinforcement is incorporated near the top surface of the concrete. None was used in these tests since the effect of the mesh reinforcement is to control drying shrinkage and as such its absence would not affect the experimental test results. The steel deck profile had a gage thickness of 0.98 mm. The geometrical cross-sectional dimensions of the composite slab is shown in Figure 3.

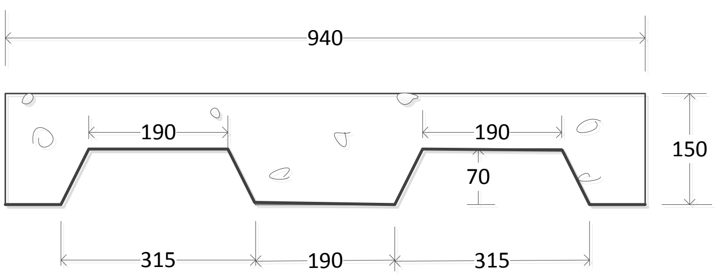

Figure 3. Cross-sectional dimensions of the Composite Bond-Dek Slab

The material used for the steel deck is ASTM Grade C galvanized to Z275. The average yield stress of the steel deck was 295 MPa with a modulus of elasticity of 202 x 10^3 MPa. The thickness of the galvanized coating was 0.015 mm, and the depth of the profile was 70 mm.

2.2 Instrumentation

Strain gauges were mounted on the top and bottom of the steel deck profiles to measure the longitudinal strains developed during both the ultimate static test and fatigue tests. Measurements were also taken of vertical displacements and relative horizontal slip between the concrete and the steel deck by means of dial gauges and Linear Variable Displacement Transducers (LVDTs). Additional measurements that were taken during the static test programme was that of damping and natural frequency. It was necessary to determine the natural frequency so as to avoid resonance with the excitation fatigue loading. Excitation frequency at or near the natural frequency would result in large displacements and stresses. This could result in premature failure of the specimen.

2.3 Test Programme

The test program was conducted in the following sequence:

i. Initial ultimate static tests were conducted on the composite slabs to establish the ultimate load (P_u), the load at which debonding of the steel-concrete interface occurs (P_d) and the initial static stiffness (k) of the slab;
ii. Fatigue tests were then carried out at various minimum and maximum stress levels; and
iii. Final ultimate static tests were conducted after either 4 million cycles or after fatigue failure.

During the tests, readings were taken of the load mid-span deflection, strains and end-slip deflection. The readings taken enabled monitoring of the deterioration of the stiffness of the slab and debonding of the steel/concrete interface.

2.4 Limitations of Experiment

Since the main object of the investigation was to study the response of the composite Bond-Dek slabs subjected to various fatigue stress levels, the following parameters were maintained constant throughout the test programme:

- A sinusoidal waveform type of loading. The effect of waveform type of loading on fatigue performance of composite slabs was not investigated. In this investigation, sinusoidal loading with a frequency of 4 Hz was chosen as it is the prevalent fatigue loading in most industrial and commercial building, due to vibratory machine loadings. The choice of 4 Hz in this investigation was considered to cover a fairly wide range of practical applications, as well as limiting the time required to complete the experiments;
- Maximum number of cycles. For this investigation, a fatigue life of 4 million cycles was considered adequate for most applications and it was necessary to limit the time spent on experimental testing;
- Concrete mix proportions. The concrete mix proportions were maintained constant for all specimens; and
- Type of loading. The type of loading was limited to four-point line loading. The shear span was kept constant. Investigations of different shear spans would require a large database, meaning that a large number of experimental tests would be required.

3. TEST PROCEDURE

3.1 Ultimate Static Tests

Static tests were first conducted to establish the ultimate static load (P_u), and the load at which debonding or slipping (P_d) occurred first. Four (4) ultimate static tests were conducted. All specimens were one way spanning, simply supported on a span of 3.5 m. A four-point line loading was applied as shown in figure 4, with a shear span of 1.2m. The load was applied across the width of the specimen by means of a servo-controlled hydraulic actuator. During the tests, readings were taken of vertical mid-span deflections and the differential movement between the steel deck and concrete. At the same time, longitudinal strains developed in the upper and lower flanges of the steel sheeting were recorded.

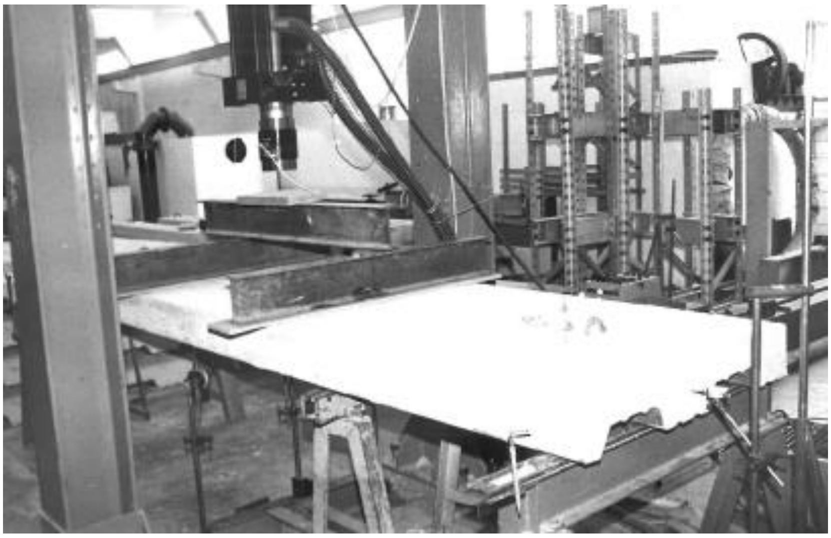

Figure 4. Four Point Testing Arrangement

3.2 Fatigue Tests

The load set-up for the fatigue tests was the same as for the static tests. Before each fatigue test commenced, the composite slab was loaded statically to a maximum load of about 90% of the debonding load (P_d) to establish the initial static stiffness of the slab. Care was taken not to exceed the load at which debonding first occurred by observing the end-slip gauge. This was then followed by an application of a sinusoidal loading at a frequency of 4 Hz. The following parameters were observed during the testing investigation:

- The stress range $(S_{max} - S_{min})$. Tests were carried out at various minimum (S_{min}) and maximum (S_{max}) stress levels. S_{min} and S_{max} are defined as the algebraic values of the minimum and maximum stresses in a cycle expressed as a percentage of the ultimate stress. Static tests were conducted after every few thousands of cycles to establish the relationship between the stiffness (k) and number of cycles. The procedure was repeated until failure occurred or until 4,000,000 cycles were reached;
- The number of cycles (N) of applied loading. Fatigue failure was deemed to have occurred when the slab specimen could not sustain the applied loading as a result of either shear bond rupture or tearing of the steel deck, with the slab subsequently not being able to carry the fatigue load. The fatigue life (N_f) was therefore defined as the number of load cycles, at a specified stress range, required to induce failure; and
- In order to reduce the number of specimens prepared, the specimens that endured 4 million cycles without noticeable deterioration in stiffness, cracking or debonding were used as specimens for another stress range, or tested statically to failure. Although there might have been some accumulated damage, specimens were

subjected to lower stress levels where the chances of failure were low.

4. TEST RESULTS

4.1 Ultimate Static Load Carrying Capacity

During the ultimate static test, failure was characterized by shear rupture of the steel/concrete interface followed by formation of a diagonal shear crack forming near one of the loading points, as shown in Figure 5. The ultimate load was approached non-linearly as the slip developed. This behavior is different to that observed for the Bond-Lok by Mahachi [10] where the composite action over the shear span length was completely lost, and accompanied by a reduced stiffness. The Bond-Lok relies only on the re-entrant geometry of the steel profile and chemical adhesion However, the dimples or embossments in the Bond-Dek profile provide a more reliable and effective mechanical interference against slip between the steel deck and concrete.

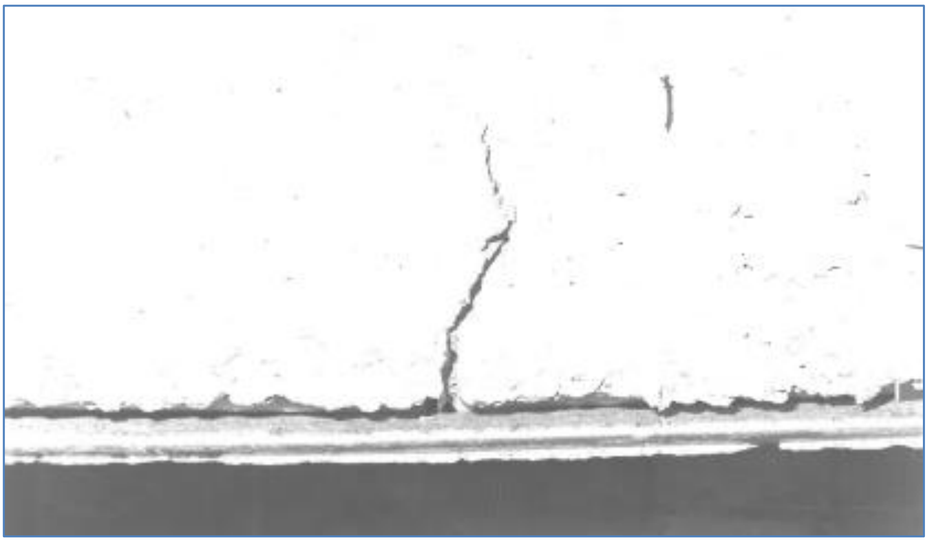

Figure 5. Shear Bond Rapture

All four static tests conducted on the Bond-Dek slabs showed a similar failure response. A summary of the static test results is shown in Table 1. A typical load vs deflection curve for a specimen subject to the ultimate static test is shown in Figure 6. The curve deviates from linear response at a load of 10.0 kN. At this load, slip between concrete and steel deck started to occur as shown by the end-slip deflection curve in the figure. A diagonal tension crack formed at a load of 11.5 kN. The ultimate load then approached rapidly during the non-linear phase as the slip developed further. Because there was no end-anchorage provided, ultimate failure occurred at a load of 18.6 kN and was accompanied by increased mid-span deflections. The maximum end-slip recorded was 2.34 mm. Tearing of the steel deck profile was not observed in any of the tests conducted.

Table 1. Summary of Static Test Results for the Bond-Dek Slabs

Specimen Identity	**Mass (kg)**	**Concrete Strength f_{cu} (MPa)**	**Debonding Load P_d (kN)**	**Ultimate Load P_u (kN)**
S6-05-10	1055	40.5	10.0	18.60
S15-10-11	1060	40.2	10.2	18.30
S5-21-09	1065	40.2	9.8	18.20
S7-05-10	1050	41.0	10.5	18.70
Average	**1058**	**40.5**	**10.1**	**18.50**

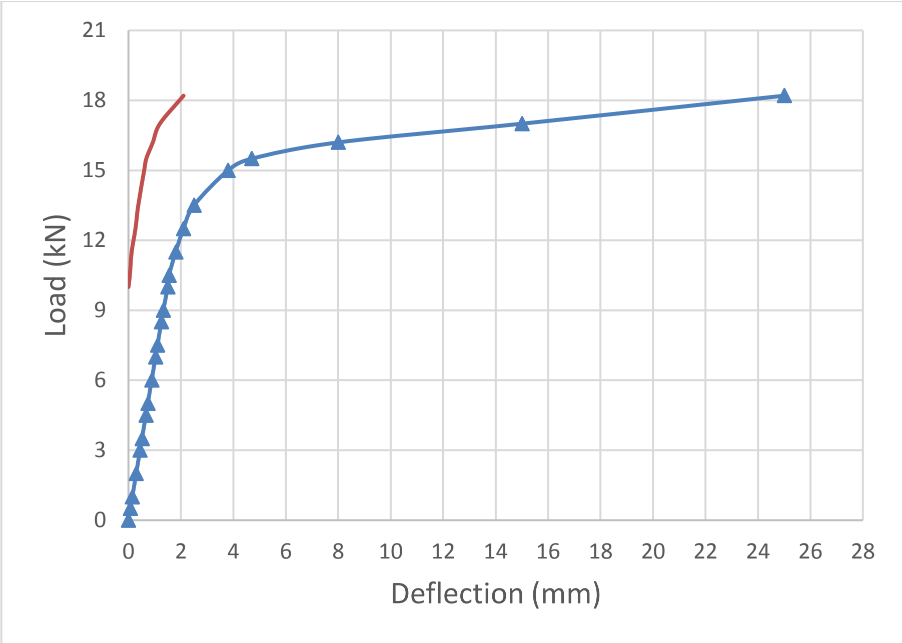

Figure 6. Load-Deflection Curve

The shear bond failure was also evidenced by the fact that no yield stresses were measured in the steel deck profile either in the top or bottom steel flanges. The maximum longitudinal stresses in the bottom flange was 190 N/mm^2 and the top flange was 122 N/mm^2. The stresses started to increase non-linearly as slip commenced at a load of about 10 kN. This was caused by the steel deck profile which now carried the load as the concrete slipped and the crack in the concrete propagated upwards.

Theoretical calculations based on the steel deck failing in tension show that the ultimate load carrying capacity would be 55 kN as opposed to the ultimate failure load of 18.6 kN. This clearly indicates that the experimental failure mode was a shear bond rupture.

4.2 Response to Fatigue Load

After completing the static tests, fatigue tests were carried out using four point line loading as in the static tests for various stress levels S_{min} and S_{max}. Before each test, the natural frequency of each specimen was determined. The average natural frequency of 18.1 Hz was well above the excitation frequency of 4 Hz. Thus there was no chance of resonance occurring.

A summary of fatigue test results is presented in Table 2. The slabs were sub-divided into Groups (I to VI) depending on the minimum stress level (S_{min}) applied as shown in the table.

Specimen Group I: $S_{min} = 30\%$

For Group I specimens with $S_{min} = 30\%$, the response of the composite slabs under repeated high stress range (S) was inferior to that observed for the low stress range. Slabs subjected to $S_{max} \leq 50\%$ were not affected by fatigue load. These slabs managed to sustain 4 million cycles. For a typical slab subjected to $S_{min} = 30\%$ and $S_{max} = 50\%$, Figure 7 shows that the initial static stiffness was reduced by only 13% from 7,760 kN/m to 6,764 kN/m after 4 million cycles. The slight increase in stiffness after 1,250,000 cycles could have been due to "bedding-in" of the supports. Figure 8 shows the corresponding load mid-span deflection curves of the specimen after a number of cycles. After 4 million cycles, it was decided to subject the specimen to an ultimate static test. From the figure, it is apparent that the ultimate load carrying capacity for the slab could still be developed after 4 million cycles. Ultimate failure occurred at a load of 17.8 kN and was accompanied by an end slip of 1.3 mm.

For a slab subjected to $S_{max} = 54\%$, a shear crack developed at one of the load points after only 660,000 cycles. The stiffness of the slab was reduced from 7,850 kN/m to 5,850 kN/m, a reduction of almost 25% after 660,000 cycles. However, ultimate static tests after failure show that the ultimate load carrying capacity was unaffected by fatigue load. The ability to carry the ultimate load was due to the mechanical interlock of the embossments of the steel deck which provided the required shear resistance. Thus for this Group with $S_{min} = 30\%$, it was deduced that the maximum stress level (S_{max}) that can be carried in order to endure 4 million cycles is below 54% and somewhere between 50% and 54%.

Table 2. Fatigue Test Results

Group Identity	**Load Range (%)**		**R (S_{min}/S_{max})**	**ΔS (%) (S_{max}-S_{min})**	**Cycles to Failure (N_f)**	**Stiffness at Failure[1] kN/m**
	S_{min}	**S_{max}**				
I	30	44	0.682	14	*	7,800
I	30	46	0.652	16	*	7,760
I	30	50	0.600	20	*	6,764
I	30	54	0.556	24	660,000	5,850
II	36	51	0.706	15	*	8,400
II	36	54	0.667	18	*	8,250
II	36	56	0.643	20	2,600	4,854
III	39	46	0.848	7	*	8,105
III	39	54	0.722	15	*	8,600
III	39	58	0.672	19	2,340	5,584
IV	46	54	0.852	8	*	8,557
IV	46	56	0.821	10	1,100,000	5,460
IV	46	59	0.780	13	271,000	6,876
V	53	54	0.981	1	*	7,825
V	53	59	0.898	6	3,900,000	5,620
VI	54	61	0.885	7	697	6,455

* No failure after 4 million cycles
[1] Stiffness at failure or after 4 million cycles

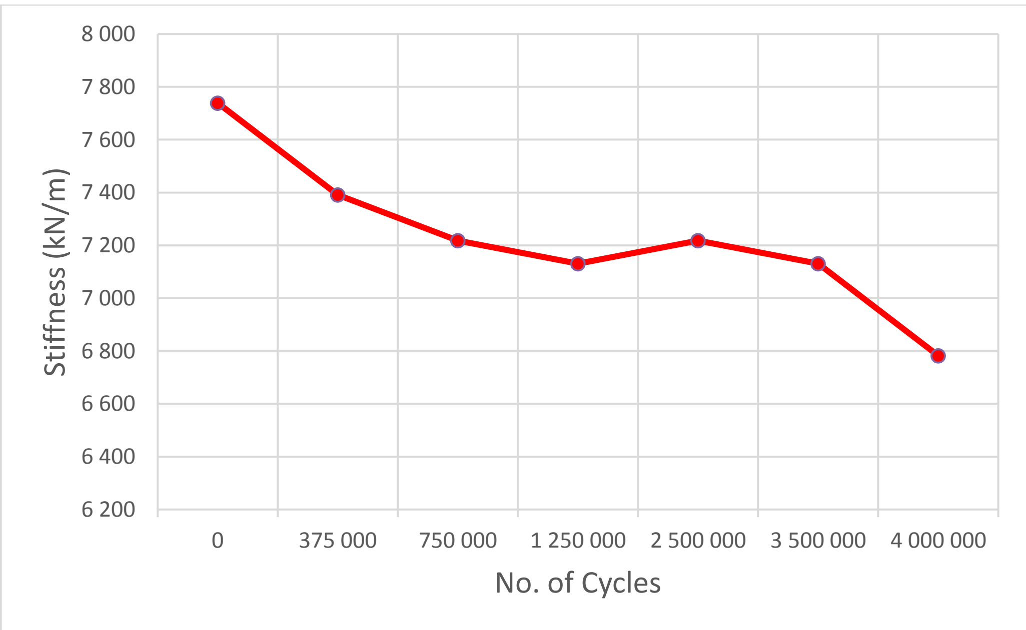

Figure7. K-N Curve

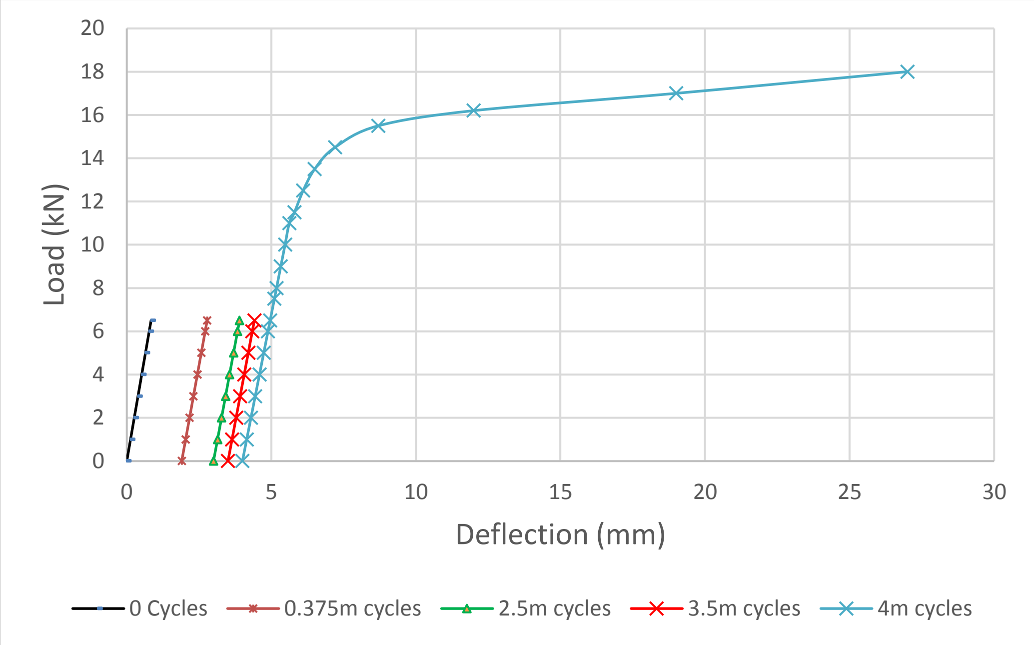

Figure 8. Load-Deflection Curves

Specimen Group II: $S_{min} = 36\%$

Behaviour of this Group was similar to that of Group I, with the lower stress ranges being able to sustain 4 million cycles. However, for a higher stress range of 20%, the slab could not sustain more than 2,600 cycles of repeated load. Failure was also characterized by a shear bond rupture and a shear crack forming at one of the load points similar to that shown in Figure 5. The maximum stress level that can be sustained for 4 million cycles is therefore between 54% and 56%.

Specimen Group III: $S_{min} = 39\%$

For $S_{max} = 46\%$ and 54%, there were no observed deterioration in shear bond or stiffness of the slabs. The slabs managed to sustain 4 million cycles. However, for $S_{max} = 58\%$, the slab could only sustain 2,340 cycles. The mode of failure was also shear. Thus for this Group S_{max} is between 54% and 58%.

Specimen Group IV, V & VI

As the minimum load level (S_{min}) increased above 46%, it was observed that the maximum load level (S_{max}) could not increase to more than 61%. From the ultimate static tests carried earlier on, it had been observed that cracking of the slabs occurred at a load of about 11.5 kN (63% of the ultimate load) For S_{min} greater than 54%, the composite slabs could not sustain any fatigue load. It was also noted that although the stiffness was reduced, ultimate static loads could still be achieved.

4.3 Summary of Observations

The following are some of the general observations made:

- Ultimate static tests revealed that the Bond-Dek composite slabs have a high carrying capability even after the first slip due to the effective mechanical shear bond interlocking system. As such, deflections at ultimate load were of the order of 30 mm;
- Failure due to static load was caused by shear bond rupture of the steel/concrete interface accompanied by a diagonal tension crack forming at one of the loading points. Tearing of the steel deck was not observed in any of the tests conducted;
- When subjected to cyclic fatigue, there was a slight increase in stiffness during the

first few thousands of cycles followed by a reduction in stiffness. The initial increase in stiffness was of the order of 3-5% and was attributed to the initial "bedding-in" of the specimens at the supports;

- Residual deflections of the order of 2.5 mm were observed after a few thousands of cycles. These deflections were still within the permissible deflection of 14 mm (Span/250) according to South African National Standard SANS 1010162 [12]. However, for specimens that failed, the residual deflections were of the order of 15 mm;
- A static loading test after 4 million cycles revealed that the ultimate load carrying capacity could still be attained;
- Where fatigue failure occurred, it was a result of shear bond rupture accompanied by a shear crack at one of the loading points. No steel fracture or yielding of the steel deck profile was observed; and
- A general trend is that, as the minimum load level (S_{min}) increases, the stress range (ΔS) under which the slab can withstand 4 million cycles decreases. This trend suggests that a good representation of this fatigue study is a modified Goodman [13] diagram, similar to that recommended by ACI Committee 215 [11].

5. DESIGN FORMULATION AND PROPOSAL

5.1 Fatigue Design Formulation

Experimental fatigue tests that were carried out on the Bond-Dek composite slabs enable mathematical models to be formulated so as to evaluate the fatigue strength of these slabs when subject to fatigue load. Based on the fatigue strength results, a guide for the design of composite slabs can now be formulated in the form of a "modified Goodman Diagram" and algebraic expressions that can be utilized for design purposes. In developing the mathematical model, the main focus involved the identification of the following:

- The form of the model to be used. From the fatigue test results, the maximum stress level, S_{max}, that could be sustained for 4 million cycles increased with increasing minimum load level, S_{min}, although the stress range decreased. Such characteristics would fit quite well with the modified Goodman diagram [13]. In a Goodman diagram, the effect of stress range and maximum stress is depicted by assuming a linear decrease of the stress range as the maximum stress increases. It was also observed from the tests that low cyclic fatigue failure occurred after only a few hundred cycles implying that the usual S-N curve representation was not appropriate for the composite deck slabs;
- The proper choice of the dependent variable and the calculation of constants for the model. In this mathematical model, the maximum stress level (S_{max}) was taken as the dependent variable, and the minimum stress level (S_{min}) as the independent variable. This was in line with the experimental observation;
- How well the model describes the test data. As discussed above, the modified Goodman diagram was chosen as it incorporated all the test results and stress ranges, for a fatigue life of 4 million cycles; and
- How reliably the model can be used in predictions, including extrapolations beyond the range of data. The model developed had some characteristics which for design purposes can be adapted to predict the fatigue life for specimens of different spans and load distributions.

5.2 Rational Design Proposal

The most valuable portion of fatigue investigations involve the graphical representation of data and the formulation of mathematical models for the test results. A graph of the test results presented in Table 2 is plotted as shown in Figure 9. The points denoted as "failure" indicate a point where the slab failed (S_{max}) for a particular minimum stress level (S_{min}). Likewise the point denoted as “no failure” corresponds to a point where the slab experienced no failure for a given (S_{min}).

An initial assumption to formulate a design line equation was to take the average of the upper and lower bounds of the “no failure” and “failure” points for a given minimum stress level, S_{min}. For example, for $S_{min} =$ 30%, the average failure point is taken as mid-way of 46% (no failure point) and 54% (failure point), i.e. 50%.These points are designated as “Design Points” in the figure.

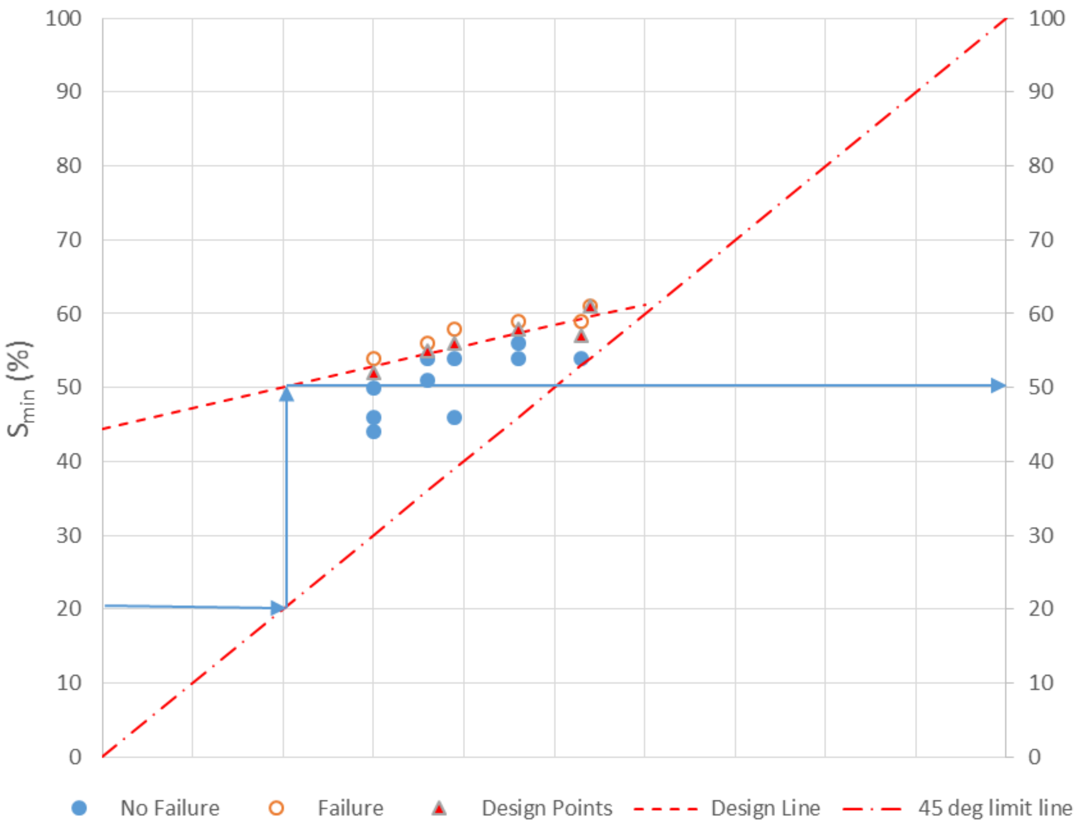

Figure 9. Modified Goodman Diagram

A linear regression analysis on the design points gives the “Design Line” which can be expressed mathematically as:

$$S_{max} = \frac{7}{25} \cdot S_{min} + 44.5 \; for \; S_{min} < 60\% \qquad (1)$$

For $S_{min} \geq 60\%$, fatigue failure will inevitably occur and $S_{max} = S_{min}$.

For a safe design, it may be necessary to introduce a margin of safety to Equation (1). The Design Equation therefore gives the maximum load level (S_{max}) that can be applied for a given (S_{min}) if the slab is to sustain 4 million cycles, or alternatively the modified Goodman Diagram as presented in Figure 9 can be used. For example, for $S_{min} =$ 20%, $S_{max} = 50\%$.

6. CONCLUSIONS

Experimental tests were carried out on composite Bond-Dek slabs under static and fatigue loading. After establishing the performance of the composite deck slabs under static load, tests were then carried under fatigue loading. The set-up of the fatigue experiment was similar to that carried out for the static tests. Specimens were subjected to sinusoidal cyclic loading at a constant frequency of 4 Hz. Test results have indicated that the slabs can be used in industrial buildings and warehouses where they are subjected to both static and fatigue loading. Although the tests performed had some experimental limitations, it has been demonstrated that the composite deck slabs can resist repeated loading to a certain degree. From the experimental observations carried out, it was observed that the usual S-N curve representation was not appropriate for the composite deck slabs since there was an abrupt transition between "no fatigue failure" and low cyclic fatigue failure. A better presentation of the results thus suggested was the modified Goodman diagram. Within the experimental constraints highlighted, a rational design approach was then proposed. Since the research was only limited to a specific type of steel deck, further research can be conducted on other decking profiles, with different designs of embossments. Embossments play a big role in the performance of steel decks both under static and fatigue loading. Effects of different shape of steel profile also need to be investigated.

7. ACKNOWLEDGEMENT

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