Inequity Aversion and Team Incentives[*]

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Abstract

We study how the optimal contract in team production is affected when employees are averse to inequity in the sense described by Fehr and Schmidt (1999). By designing a reward scheme that creates inequity off the desired equilibrium, the employer can induce employees to perform effort at a lower total wage cost than when they are not inequity averse. We also show that the optimal output choice might change when employees are inequity averse. Finally, we show that an employer can gain, and never lose, by designing a contract that accounts for inequity aversion, even if employees have standard preferences.

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"The path of the righteous man is beset on all sides by the inequities of the selfish [...] man."
Jules Wynnfield (Samuel L. Jackson) quoting The Bible before shooting two men in Pulp Fiction.

1 Introduction

In this paper, we study how managers should structure reward schemes if their employees care not only about their own direct utility (understood as the reward paid net of the effort cost of performing effort) but also about equity with respect to other employees. One of the most striking results from interview studies that economists have conducted with business leaders (Agell and Lundborg (1999), Bewley (1999), Blinder and Choi (1990), Campbell and Kamlani (1997)) is that employees report to care for the well being of co-workers and not only for several material incentives offered to workers individually. Distributional concerns are also observed in the Experimental Literature and in particular, they are one of the most accepted explanations to results in the Ultimatum Game.[1] If employees' have a preference for equity, an optimal contract offered by an employer might need to account for it. We address this idea in a theoretical framework using inequity aversion as modelled by Fehr and Schmidt (1999).[2] In prominent experimental work, F&S (2000) have argued that fairness considerations lead agents to write contracts which do not specify for all future contingencies what is going to happen and which thus implement less severe incentives than conventional theory would predict. The purpose of this paper is to investigate this claim more closely. We develop a simple model in which an employer has to design a reward scheme for two employees who dislike inequity in the way envisaged by F&S. The main message that comes out of a formal analysis of such a model is somewhat contrary to F&S's intuition. The principal can devise schemes which exploit employees' preference for equity by offering them equitable outcomes in situations where they put in the desired effort, and which threaten shirking with highly unequal outcomes. Such schemes might, for example, offer extremely unequal rewards in the case that one employee works harder than another. By constructing such schemes, the employer can implement the desired effort under a lower total wage cost than would have been possible had the employees not been inequity averse. We also show that inequity aversion might change the production level the principal wants to implement and that the principal never looses by accounting for inequity aversion in the design of contracts, even when faced with agents with standard preferences.

When comparing our model to F&S's explanation to their experimental results, one needs to keep in mind that F&S focus primarily on inequity aversion among employers and employees, whereas this paper only focuses on inequity aversion among employees. That is, in their articles, employers compare their utilities to those of employees and employees compare their utilities to those of employers. However, in our paper employees compare their direct utility with other co-workers', and not with employers', while employers only care for their material payoffs. There is no consensus about which of this directions is more relevant and there have been different attempts to study the issue. Englmaier and Wambach (2002) study the interaction between an inequity averse agent who compares himself with a selfish principal and find among other things, that linear contracts are optimal in this context. Cabrales and Calvó-Armengol (2002), use inequity aversion only among employees to justify skill segregation as employees dislike to be "close", and thus to compare themselves with, low skilled workers who are penalized by the market. We believe that in practice,

[1] See for example, Bolton and Zwick (1995), Costa-Gómez and Zauner (2001), Güth et al. (1982), Güth and Tietz (1990), Kagel et al. (1996), Ruffle (1998), Straub and Murnighan (1995), and Thaler (1988).

[2] We use F&S in the following to refer to these authors.

inequity aversion among employees is at least as plausible as inequity aversion among employers and employees. It is natural to assume that reciprocal feelings are enhanced by repeated interaction and so it is to assume that employees within the same hierarchy interact more frequently among them than with their superiors. Additionally, it could be argued that employees within the same category understand better the situation of workers within the same status, and thus utility comparisons are more meaningful among employees in the same hierarchy.[3]

When in the F&S experiments,[4] employers offer incomplete contracts that leave workers' utility above their reservation level, employees respond with higher levels of effort than the incomplete contract specifies. The conclusion that these authors reach is that it pays for principals to leave contracts incomplete and reward above reservation utilities because agents will complete those contracts by performing extra effort in their desire to please the nice principals. However, notice that the incomplete contract offered by the principals is merely cheap talk. As the contract is incomplete, there is not binding commitment from the principal to pay the agent the extra reward promised.[5] Notice that in our model, promised rewards are not cheap talk as we assume that they are enforceable by law. However, we show that even with enforceable contracts a principal who knows that agents are inequity averse might be able to exploit it by offering agents a complete contract that specifies all agents' rewards for all possible combinations of effort performed. We show that the optimal way to complete the contract is by creating inequity out of the equilibrium the principal wants to implement. This idea is in the spirit of Andreoni and Miller (2002), who claim that fairness considerations depend not only on final allocations but also on alternatives not chosen.

In this paper we do not worry about the motivations for fair behavior. We are aware that there is much debate about the reasons why we observe actions such as sharing or punishment both in experiments and in real life interactions. Rabin (1993) and Dufwenberg and Kirchsteiger (1998) stress the role of intentions as the key issue behind reciprocal behavior. For example, an agent will punish another agent who causes him some harm if he believes he did it on purpose. But others, such as Bolton et al. (1997) and Brandts and Charness (2001), emphasize the effects of distributional concerns instead of intentions. On the other hand, Binmore et al. (1995), and Postlewaite (1998), hint that behavioral rules such as sharing are observed because they might be an optimal response in the repeated Game of Life. That is, if we observe that in some cases people behave nicely to each others is not really because they care about them or about the distribution of payoffs per se, in the sense that they derive utility from others' well being, but that responding reciprocally is an evolutionary stable strategy in the Game of Life. We abstract from this debate[6] in the belief that utility functions accounting for inequity aversion can be used as a reduced form to understand short-run observed behavior and study contract design, no matter what the explanation behind observed behavior might be. We take inequity aversion as given and we focus on its consequences for the contract enforcement problem.

Finally, F&S are not the only ones proposing a method for studying inequity aversion. Bolton and Ockenfels (2000) develop an alternative utility function by which agents compare their material

[3] On this point, Dufwenberg and Kirchsteiger (2000), for example, express doubts on whether profits or the value of the firm's shares should be used for the comparison of utilities between employer and employees' utilities.

[4] See also Fehr, Klein and Schmidt (2001) and Fehr and Gächter (2002).

[5] F&S argue that it is precisely the belief on the existence of inequity aversion among employers what creates the commitment device. However, they do not notice that once employees have performed effort, employers can exploit this belief by rewarding less than expected, which might be convenient for employers even if they are truly inequity averse.

[6] For a good survey on social preferences see Sobel (2000) or Fehr and Schmidt (2000b).

payo¤ to the material average payo¤ of a reference group.[7] Charness and Rabin (2000) propose some tests to distinguish others' regarding preferences and a model in which the beliefs on the intentions of other players determine reciprocal responses. Cox (2001) proposes a di¤erent utility function together with a method of separating reciprocity and altruism and a discussion on the advantages and disadvantages of the di¤erent utility functions that have been proposed. For the purpose of this paper, we follow the F&S (1999) approach in modelling inequity aversion due to its simplicity in the binary case we study. Other models deal with the choice of the reference group with whom agents compare themselves in an unnecessary complicated way to show the main idea of this paper, which is that the presence of inequity aversion changes the optimal contract design in important ways. We believe our qualitative conclusions hold for other methods of modelling inequity aversion.

The rest of the paper is organized as follows. Section 2 describes a standard model of joint production. Section 3 solves the model under standard preferences. Section 4 solves the model under inequity aversion and discusses the possible consequences of not accounting for inequity aversion in the design of contracts. Section 5 discusses the results. Appendix A contains the proofs. Appendices B and C show two relevant examples.

2 The Model

There is a Principal and two agents named 1 and 2. The Principal pays agents $i = 1; 2$ to perform costly e¤ort e_i. Agents can either perform e¤ort, $e_i = 1$ or not, $e_i = 0$. If both agents perform e¤ort, production is normalized to 1: If only agent i performs, production is q_i: If no agent performs e¤ort, production is 0:

Output Levels

		Effort of Agent 2	
		1	**0**
Effort of Agent 1	**1**	1	q_1
	0	q_2	0

The cost for each agent $i = 1; 2$ of performing e¤ort is c_i: The cost of not performing e¤ort for each agent $i = 1; 2$ is 0. A complete contract speci…es the rewards o¤ered to the agents for all possible output levels, and not just the desired output level. In order to standardize notation, assume the principal o¤ers rewards $\{w_1; w_2\}$ to agents 1 and 2 when both agents perform, $\{w_1'; w_2'\}$

[7] For a comparison between F&S and Bolton and Ockenfels models, see Engelmann and Strobel (2000).

when agent 1 individually performs and $\{w''_1; w''_2\}$ when agent 2 individually performs. If no agent performs effort, no reward is offered to any agent:[8]

Rewards Offered

Effort of Agent 1 \ Effort of Agent 2	1	0
1	w_1, w_2	w'_1, w'_2
0	w''_1, w''_2	0 , 0

The structure of the game is as follows: the Principal proposes a wage schedule for all possible production levels, agents decide simultaneously whether to perform effort or not and, once production is realized, rewards are paid.[9] The structure of the game is common knowledge[10] and, in particular, both the Principal and the agents know output levels, rewards offered and the costs of performing effort for each agent.[11] We find the Subgame Perfect Equilibrium of this game to which in the following we refer as SPE. We also briefly discuss Equilibrium Uniqueness and other solution concepts such as Equilibria in Dominant Strategies.

We introduce the following assumptions that restrict the contracts that can be offered by the Principal.

Assumptions:

(P1) Production is always positive and increasing with the number of agents performing effort.

$$0 \le q_i \le 1 \quad \text{For } i = 1; 2$$

(C1) The sum of performing agents' costs of effort is smaller than output produced.

$$0 \le c_1 < q_1$$

[8] This is implied by assumptions (R1) and (R2) below.

[9] Notice that in this model, agents do not decide whether to accept or not the contract offered. We assume that they already work for the Principal although they can still decide not to produce at all. As we discuss in section 5, modelling the acceptance stage is not trivial in the inequity averse case and it depends crucially on how inequity aversion is assumed to affect the outside option.

[10] We here diverge from the standard moral hazard approach to Principal-Agent problems that emphasizes asymmetries of information (Holmström, 1982). The reason is that we want to stress that even if there are no informational problems, the presence of inequity aversion might change the optimal contract design.

[11] By assumption, in Section 3, the degrees of inequity aversion of each agent are also common knowledge.

$$0 \leq c_2 < q_2$$

$$c_1 + c_2 < 1$$

(R1) Limited liability: Negative rewards are not possible.

$$\begin{aligned} w_1; w_1'; w_1'' &\geq 0 \\ w_2; w_2'; w_2'' &\geq 0 \end{aligned}$$

(R2) Wages are paid from output produced.

$$\begin{aligned} w_1 + w_2 &\leq 1 \\ w_1' + w_2' &\leq q_1 \\ w_1'' + w_2'' &\leq q_2 \end{aligned}$$

(R3) Contract Commitment.

(U1) The Principal maximizes production minus rewards paid.

Assumption (P1) implies that an extra agent performing effort always increases production. Assumption (C1) implies that there always exists a surplus above the cost of effort performed. Assumption (R1) is a limited liability constraint restricting agents' possible direct punishment for not performing effort.[12] Assumption (R2) is a budget constraint for the Principal, implying that all rewards must be made from output produced. (R3) implies that offered rewards must be paid ex-post by the Principal. This assumption is imposed in order to avoid the problem of cheap talk that would make our model uninteresting. Assumption (U1) is the simplest functional form imposing the Principal is not inequity averse.

3 Solution of the model without inequity aversion

>From here on, we name the utility functions of agents who are not inequity averse, "standard utility functions". Standard agents derive utility only from their own rewards and disutility from the cost of effort performed.

Assumption:

(U2) Standard Agents' utility is equal to rewards minus the cost of effort performed.

According to (U1), the Principal maximizes production minus rewards paid to the agents. To do so, the Principal chooses the minimum rewards in equilibrium such that agents do not deviate from the output level the Principal wants to implement. This solution is a Subgame Perfect Equilibrium. Notice that to find this solution we need to answer the following two questions:

[12] We prove that when agents are inequity averse, it is possible to create inequity by redistributing rewards among agents. We show that this redistribution produces disutility for the agents, and thus can be interpreted as an indirect way of punishing them.

1. Which is the optimal reward design if the Principal is to implement each production level?

2. Given the optimal reward design for each case and the productivity parameters, which production does the Principal optimally implement?

We answer these questions below.

3.1 Optimal reward design under standard preferences

Given the assumptions above, the utility of standard agents for di¤erent levels of e¤ort performed is:

Agents' Utility

Effort of Agent 2

Effort of Agent 1	**1**	**0**
1	$w_1 - c_1, w_2 - c_2$	$w'_1 - c_1, w'_2$
0	$w''_1, w''_2 - c_2$	0 ,0

Notice that the optimal reward design requires to …nd the optimal values for six parameters ($w_1; w_2; w'_1; w'_2; w''_1$ and w''_2) under the di¤erent output levels the Principal might want to implement: Lemma 1 shows a general principle on how rewards should be designed that applies for all possible cases.

> Lemma 1 Under standard preferences, the optimal reward design implies paying a wage in equilibrium that exactly compensates for the cost of e¤ort of each agent performing and not rewarding non-performing agents.

Intuitively, when agent i does not perform e¤ort, the Principal should pay agent i the lowest possible wage in order to avoid extra reward costs. Due to assumptions (R1) and (R2), the minimum an agent can be paid is 0 and thus, his direct utility is 0. To obtain a SPE in which agent i performs e¤ort, such agent must obtain positive utility when performing. As the cost of e¤ort is c_i; any wage higher than c_i leaves agent i with positive utility. By paying exactly c_i when the agent performs e¤ort and 0 when he does not, a SPE in which agent i performs e¤ort can be implemented at the minimum possible wage cost.

Notice that for the standard case, agent j's utility does not enter in agent i's utility, and thus, this lack of interdependencies allows to apply Lemma 1 both if the Principal implements joint or individual production in equilibrium, with no need of specifying some of the rewards offered out of equilibrium. However we have emphasized in the introduction that when we move to the inequity aversion case, rewards offered out of equilibrium are crucial. Notice also, that although promising zero rewards to both agents out of the desired equilibrium under the standard case is the most straightforward solution, several other out of equilibrium promised rewards implement the same SPE. In particular, any out of equilibrium reward that at most compensates for the cost of effort of the agent performing out of equilibrium implements the same SPE with no extra cost for the Principal. Finally, notice that the proof for Lemma 1 includes a discussion on Uniqueness of Equilibria and Dominant Strategies Implementation.[13] In particular, in the proof we introduce a negligible payment of ε that by assumption does not increase the reward cost for the Principal, to obtain uniqueness of equilibria under the standard case. This negligible payment ε is used when summarizing the results of this section.

3.2 Optimal implementation of effort under standard preferences

Once we know how the optimal reward matrix is designed in the standard case, we turn to the question of what is the optimal production the Principal wants to implement depending on optimal rewards and productivity. As expected, the higher the marginal productivity (q_i) of an agent and the lower his cost of effort, the more the Principal wants that agent to perform effort in equilibrium. Given that the minimum cost of inducing each agent to perform under standard preferences is each agents' cost of effort and not performing agents are paid 0, the Principal implements the equilibrium in which output minus costs of effort of performing agents are higher. Therefore, the Principal compares:

Utility of the Principal if joint production: $1 - w_1 - w_2$.

Utility of the Principal if agent 1 individually performs: $q_1 - w_1'$.

Utility of the Principal if agent 2 individually performs: $q_2 - w_2''$;

where, from Lemma 1, the optimal values for the equilibrium wages are:

$$
\begin{aligned}
w_1 &= c_1 \\
w_2 &= c_2 \\
w_1' &= c_1 \\
w_2'' &= c_2.
\end{aligned}
$$

Substituting it is straightforward to see that:

If the net product of agent 1 individually performing is bigger than the net product of agent 2 individually performing, i.e., $(q_1 - c_1 \geq q_2 - c_2)$; the Principal implements joint production if $1 - c_1 - c_2 \geq q_1 - c_1$; which simplifies to $1 - q_1 \geq c_2$, i.e., if the marginal product of agent 2 performing

[13]Notice that for this case, an Unique SPE and an Equilibrium in Dominant Strategies are implemented exactly in the same way.

$(1 - q_1)$ is bigger than agent's 2 cost of effort (c_2): If $1 - q_1 < c_2$; the Principal implements agent 1 performing individual production.

If the net product of agent 1 individually performing is smaller than the net product of agent 2 individually performing, i.e., $(q_1 - c_1 < q_2 - c_2)$; the Principal implements joint production if $1 - c_1 - c_2 \geq q_2 - c_2$; which simplifies to $1 - q_2 \geq c_1$; i.e., if the marginal product of agent 1 performing $(1 - q_2)$ is bigger agent's 1 cost of effort (c_1) : If $1 - q_2 < c_1$; the Principal implements agent 2 performing individual production.

These conditions are not trivial. Intuitively, it would appear that if agents' efforts are complements, i.e., if the marginal productivity of one agent increases when the other agents is performing $(1 - q_1 \geq q_2)$, the Principal always wants to implement joint production. This intuition is right. However, as the costs of effort also play a role, it is possible that for costs of effort sufficiently different, the Principal optimally implements joint production even if efforts are substitutes $(1 - q_1 < q_2)$. Therefore, complementarity of agents' effort is not the only condition under which joint production is optimally implemented and both productivities and costs of effort need to be taken into account.

3.3 Summary of the solution under standard preferences

We can summarize the most natural solution of the standard case that creates a Unique SPE as:

1. If conditions for the principal to implement joint production hold, in equilibrium the Principal compensates both performing agents for their cost of effort plus a negligible positive premium. Out of equilibrium, the Principal compensates the performing agent for his cost of effort plus a negligible positive premium and pays zero to the agents who do not perform.
2. If conditions for joint production do not satisfy, in equilibrium the Principal compensates the cost of effort of the more productive agent (the one for whom $q_i - c_i$ is higher) plus a negligible premium. The Principal offers no reward to the more productive agent out of equilibrium. The less productive agent is paid 0 both if he performs and if he does not.

In the next section, we study how the solutions to this standard problem change when agents are inequity averse. In particular, we emphasize how the total cost of implementing effort changes and how the conditions for the Principal to implement joint or individual production are affected by inequity aversion. However, notice that when inequity aversion exists, it is not only that the total cost of implementing production varies but that the whole optimal contract (including rewards offered off the equilibrium) can change.

4 Solution of the model with inequity aversion

As explained in the introduction, we follow F&S (1999) in their modelling of inequity aversion. However, we need to adapt their utility function to our specific problem. The "transformed utility function" of inequity averse agents in this context is U_i^F where:

$$U_i^F = U_i - \alpha \max[U_j - U_i; 0] - \beta \max[U_i - U_j; 0] \quad \text{for } i; j = 1; 2; \quad i \neq j$$

where, as before U_i for $i = 1; 2$ is equal to rewards offered minus the cost of effort performed. As in the previous section, we call U_i "direct utility".

Assumptions:

(U3) Agents dislike inequity:

$$\alpha \geq 0$$
$$\beta \geq 0.$$

(U4) Agents care more for their own direct utility than for inequity:

$$\alpha, \beta \in [0; 1).$$

Assumption (U3) imposes inequity aversion. Although it is natural to assume that agents are negatively inequity averse, i.e., experience disutility when they are worse off than other agents ($\alpha \geq 0$), it is not so natural to assume that agents are positively inequity averse, i.e., they dislike being better off than others ($\beta \geq 0$).[14] In fact, it has been experimentally observed that agents derive, under some circumstances, utility from being better off than others, which we could call pride. However, it has also been observed that in some cases, experimental subjects are willing to incur monetary losses to reestablish equity even when they are better off than other subjects, which we could interpret such as they obtain disutility form unequal distributions because of altruism.[15] As what we are interested in is inequity aversion, we therefore stick to both $\alpha \geq 0$ and $\beta \geq 0.$[16] Assumption (U4) implies that agents derive more utility from their own direct utilities than from the comparison with other agents' direct utilities.[17] Finally, notice that for simplicity, we assume that different agents have the same α and β, not allowing for differences in the degrees of inequity aversion among agents.[18]

Figure 1 shows the transformed utility function U_i^F accounting for inequity aversion as a function of the original utility functions U_i and U_j for $i, j = 1, 2$ and $i \neq j$. Notice that the transformed utility function U_i^F changes slope depending on whether agent i is obtaining more or less direct utility U_i than his peer j. When agent i is worse off than agent j, the transformed utility function of agent i, U_i^F is driven by agent's i own direct utility and by the envy of being worse off than j, and thus the slope is $\frac{\partial U_i^F}{\partial U_i} = 1 + \alpha$. When agent i is better off than agent j, the transformed utility function of agent i, U_i^F, is driven by his own direct utility U_i and by the disutility of altruism of being better off than agent j, and thus the slope is $\frac{\partial U_i^F}{\partial U_i} = 1 - \beta$, always smaller than when agent i is worse off than agent j.

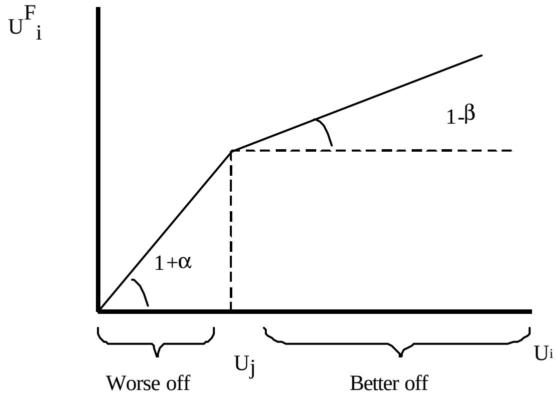
[14] To clarify the exposition, in the following we loosely refer to negative inequity aversion as envy, while we loosely refer to positive inequity aversion as altruism. However, we are aware that there is no consensus in the Literature (neither in Economics nor in Philosophy) on the formal definitions of Altruism and Envy.

[15] See Huck, Müller and Norman (2001).

[16] We have conducted similar calculations for $\alpha \geq 0$ and $\beta \leq 0$ and our main result holds: the Principal can still exploit this inequity averse preferences to implement the desired production level with a smaller total wage cost than under standard preferences, although the optimal reward design is much more complicated.

[17] Fehr and Schmidt's (2000) original formulation allows for $\alpha \geq 1$, and thus, agents might care more for the comparison of being worse off than their peers than for their direct utility of their rewards. We assume $\alpha \leq 1$ to show that even if inequity aversion is not dominant, its effects on the optimal contract design can still be substantial.

[18] Fehr and Schmidt (2000) allow for different values of α and β among agents. Differences in these values might have important behavioral effects, as for example, agents obtaining relatively higher direct utility might be able to afford being inequity averse, and thus give up some direct utility to reestablish equity. However, we believe that in this context, allowing for different degrees of inequity aversion would only complicate the exposition of an effect that is clearer under symmetry.



Inequity Averse Preferences of Agent i

Once we have understood how inequity averse utility functions di¤er from the standard ones, we proceed analogously to the standard case and we study how contract design is a¤ected by inequity aversion. We study this question in the following two subsections.

4.1 Optimal reward design under inequity aversion

Notice than when agents are inequity averse, agents' utility does not only depend on their rewards and their e¤ort costs, but also on the rewards and the costs of e¤ort of agents to whom they compare. Following the notation in Section 2, the transformed utility of each agent in each case depending on rewards o¤ered and costs of e¤ort is:

		Effort of Agent 2	
		1	0
Effort of Agent 1	1	$w_1-c_1-a\max[w_2-c_2-w_1+c_1,0]-\text{ß}\max[w_1-c_1-w_2+c_2,0]$, $w_2-c_2-a\max[w_1-c_1-w_2+c_2,0]-\text{ß}\max[w_2-c_2-w_1+c_1,0]$	$w_1'-c_1-a\max[w_2'-w_1'+c_1,0]-\text{ß}\max[w_1'-c_1+w_2',0]$, $w_2'-a\max[w_1'-c_1-w_2',0]-\text{ß}\max[w_2'-w_1'+c_1,0]$
	0	$w_1''-a\max[w_2''-c_2-w_1'',0]-\text{ß}\max[w_1''-w_2''+c_2,0]$, $w_2''-c_2-a\max[w_1''-w_2''+c_2,0]-\text{ß}\max[w_2''-c_2+w_1'',0]$	0, 0

Therefore, the no deviation conditions for each agent now depend on more parameters than under the standard case and the design of the reward matrix is more complicated. The main idea of constructing the optimal reward matrix is that once the Principal knows which situation to implement (joint production or individual production), he needs to carefully design the whole reward matrix, and not only the rewards that entered in the agents' no deviation conditions without inequity aversion. The reason being that inequity aversion creates more interdependencies among agents' utilities and a careful account of these interdependencies can be beneficial for the Principal. The optimal reward design is carried out in such a way that it exploits agents' inequity aversion. Because now agents' transformed utilities depend also on the equity of the distribution of direct utilities, agents might trade own rewards with equity to allow a more equitable distribution of direct utilities in equilibrium. Thus, by creating extra inequity out of the equilibrium, the Principal might be able to implement the desired equilibrium at a lower wage cost than under standard preferences.

Notice that, for simplicity, we develop here general results that apply to all possible implementations of output that the Principal might want to enforce in equilibrium. Proofs in Appendix A show how to construct the optimal reward matrix for each possible output decision.

> **Lemma 2** The minimum reward needed to implement individual production as a SPE under inequity aversion is the cost of effort of the agent individually performing in equilibrium.

Intuitively, when the Principal implements one of the agents individually performing effort as a SPE, the agent performing effort has to prefer to individually perform than not to perform when the other agent is not performing. If no agent performs, both agents obtain the same transformed utility (0), as costs of effort are 0 and due to assumption (R2), rewards are also 0. Thus, when no agent performs, equity is maximized. Therefore, the only way to use inequity aversion to implement an equilibrium in which only one agent performs is by not creating additional inequity in equilibrium. To maximize equity in this situation under the lowest possible wage cost, in equilibrium it is optimal to exactly compensate the agent performing for his cost of effort, leaving the performing agent with zero direct utility, and paying 0 to the agent that does not perform, leaving the not performing agent with zero direct utility. Thus, direct utilities for both agents in the implemented SPE with individual production are the same and equal to 0 and, as equity is maximized, transformed utilities take the same value as direct utilities.

Notice that Lemma 2 only refers to optimal rewards in equilibrium when individual production is implemented. However, we have argued that with inequity aversion it is optimal to offer complete contracts, i.e., to also specify the rewards offered out of the equilibrium implemented. The proof for Lemma 2 specifies these rewards and also discusses the optimal rewards out of equilibrium that do not enter into the agents' no deviation conditions. Notice that some of the out of equilibrium rewards are not relevant to make individual production a SPE but they do play a role if the equilibrium is to be implemented in Dominant Strategies or the SPE is to be unique.

In general, notice that the optimal design of the off equilibrium rewards that enter into the agents' no deviation conditions implies offering very extreme rewards to the agents out of equilibrium, so as to maximize the effect of inequity aversion. The way to implement a SPE at he minimum equilibrium cost for the Principal is by maximizing the disutility of the agents out of equilibrium. To do so, the agent who performs in equilibrium must not be offered any reward off equilibrium, i.e., when not performing. But with inequity aversion, the Principal can even do better when designing the other agent's rewards that enter into the performing agent's no deviation conditions. By offering extreme rewards to the other agent (either no reward or all the output produced) the disutility caused by envy or altruism is maximized. To maximize envy, all output produced must be offered to the other agent. To maximize altruism, no reward must be offered to the other agent. The choice between offering no reward or all the output depends on whether the maximum potential effect of envy or altruism is bigger out of equilibrium for the agent who performs effort in equilibrium.

Finally, notice that in the case in which the Principal implements individual production, inequity aversion cannot be exploited by the Principal to his benefit because the minimum total cost of implementing individual production with inequity aversion is the same as without it.[19] However, as we see below, when joint production is implemented, there is room for inequity aversion to be exploited. In the following two lemmas, we show the optimal rewards offered out of equilibrium when the Principal implements joint production in equilibrium..

Lemma 3 If joint production is to be implemented in equilibrium then it is always optimal to offer zero rewards to an agent who does not perform effort out of the equilibrium.

Intuitively, if the Principal is to implement joint production, in equilibrium both agents must prefer to perform effort than not, given that the other agent is performing. Therefore, the Principal designs the reward matrix such that both agents obtain the highest possible disutility out of the equilibrium, i.e., when individually not performing effort but the other agent performs. Given that there is a limited liability constraint by which negative rewards are not possible (assumption (R1)) and that agents care more for their direct utility than for the comparison with the other agent's direct utility (assumption (U4)) the disutility of the agent not performing effort is maximized when he is not rewarded at all.[20]

Once we know what the optimal rewards for the agent who does not perform out of equilibrium when joint production is implemented are, we complement Lemma 3 with Lemma 4 which shows optimal wages to the agent who performs effort out of equilibrium.

Lemma 4 To implement joint production in equilibrium, it is optimal to offer extreme rewards to the agent who performs effort off the equilibrium (agent i). If the potential of envy is relatively high ($\alpha(q_i - c_i) \geq \beta c_i$), the agent who performs out of equilibrium should be rewarded with all the output produced (q_i). If, in contrast, the potential altruism is relatively high ($\alpha(q_i - c_i) < \beta c_i$), the agent who performs out of the equilibrium must not be offered any reward.

Extreme rewards are used to maximize the effect of inequity aversion out of equilibrium. The reward offered to the agent who performs effort out of equilibrium only appears in the no deviation condition of the agent who does not perform effort out of equilibrium. Thus, this reward must be chosen such as it maximizes the disutility of the agent who does not perform out of equilibrium. The non-performing agent obtains disutility from both envy and altruism, but not both at the same time. If the potential to exploit envy is higher than the potential to exploit altruism, ($\alpha(q_i - c_i) \geq \beta c_i$), then the offered reward must be the one that maximizes envy. By offering all the output available when only agent i performs (q_i) to the agent who performs out of equilibrium the effect of envy is maximized. Maximizing the negative effect of altruism requires offering no reward (0) to the agent that performs out of equilibrium.

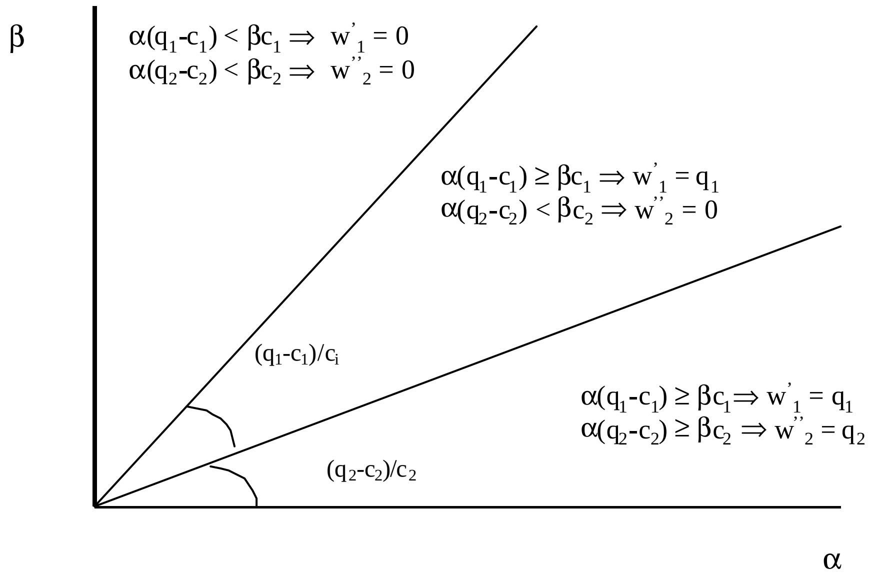
Notice that in the conditions that determine whether envy or altruism have more potential to harm the agent not performing off joint production equilibrium, do not only enter the inequity aversion parameters (α and β), but also the costs of effort relatively to productivity. Thus, it is easy to reinterpret these conditions in terms of the costs of effort. Intuitively, if the cost of effort

[19] Although, as explained in the proof, in some cases, inequity aversion can be used to help select an Unique Nash Equilibrium. additionally, we prove below that it is possible that the optimal production level changes to joint production when facing inequity averse agents.

[20] This principle also applies to the out equilibrium rewards when individual production is implemented.

of the agent performing out of the equilibrium is low ($c_i \to 0$), the potential to harm the agent who does not perform effort due to altruism is low, because for the agent performing effort it is not very costly to perform. Thus, by rewarding effort as high as possible (limited by the amount of total output produced) the Principal optimally exploits envy. In contrast, if the cost of effort is high ($c_i \to q_i$), the potential for the Principal to exploit altruism by offering no reward to the agent who performs effort is high, and thus it is optimal not to reward the agent who performs effort out of the equilibrium.

Figure 2 uses this intuition to show the agent's performing out of equilibrium optimal rewards, w_1' and w_2'', as a function of the degrees of envy (α) and altruism (β), given the productivity parameters $(q_1; q_2)$ and the costs of effort $(c_1; c_2)$, when joint production is implemented.



Optimal rewards to the agent individually performing off Joint Production Equilibrium

As seen in Lemma 4, the optimal rewards out of equilibrium are more complicated under inequity aversion than in the standard case because of the possible combinations of parameter values, and so does happen with the optimal rewards in equilibrium. As we are interested in studying the effect of inequity aversion when introduced in a standard setting, instead of calculating the optimal reward design for all the possible parameter values, we now state a general result that compares the total cost of implementing joint production with and without inequity aversion. However, the proof of Proposition 1 illustrates how the optimal reward matrix is designed for all possible parameter values.

Proposition 1 The cost of implementing joint production as a SPE is never higher with inequity aversion than without it. By creating inequity off the equilibrium, it can be lower.

Intuitively, the Principal can always implement a SPE in which both agents perform by exactly compensating agents for their cost of effort in equilibrium and not rewarding agents out of equilibrium. The reason is that in equilibrium, both agents obtain the same transformed utilities and therefore, as equity is maximized, agents are not worse o¤ with inequity aversion than with standard preferences. However, the Principal can do better than exactly compensate the costs of e¤ort in equilibrium. By creating extra inequity o¤ the equilibrium, agents might obtain extra disutility out of equilibrium. Thus, rewarding the agents with less than their cost of e¤ort but maintaining more equity in equilibrium than out of equilibrium, joint production can be implemented at a lower total cost for the Principal than the sums of the costs of e¤ort. Notice that this does not mean that equity is now maximized in joint production equilibrium, but that there is less inequity with joint production than with individual production.[21] The proof for Proposition 1 in Appendix A contains an example which shows how the Principal optimally designs the reward matrix depending on parameter values.

Finally, notice Lemma 5 regarding the implementation of Unique SPE or Equilibria in Dominant Strategies.

Lemma 5 A di¤erent contract might be needed to implement joint production as a Unique SPE (or an Equilibrium in Dominant Strategies) than to implement joint production as a SPE.

Intuitively, a contract that implements joint production as a SPE might induce no production as another SPE, specially if the agent who individually performs o¤ the equilibrium is not compensated o¤ equilibrium for his cost of e¤ort. To eliminate the no production equilibrium it is required to leave the agent who does not perform e¤ort out of equilibrium with transformed utility above the one when there is no production. As these rewards are o¤ered out of equilibrium, they do not imply an extra cost for the Principal. However, these rewards o¤ered out of equilibrium to each agent when individually producing do enter in the no deviation conditions needed to implement joint production as a SPE, and thus, the optimal rewards o¤ered in equilibrium might increase from the ones calculated in the Proof for Proposition 1. The proof for this Lemma 5 shows however, that the optimal rewards in joint production in equilibrium with inequity aversion are still below the ones in the standard case as there is still more inequity o¤ the equilibrium than in the Unique SPE.

Once we know how the optimal reward matrix is designed, we turn in the next two subsections to the other question that interests us, which is how inequity aversion changes the optimal output choice the Principal wants to implement.

4.2 Optimal implementation of e¤ort under inequity aversion

Under inequity aversion the objective of the Principal is the same as in the standard case: maximize production minus rewards paid to the agents. However, due to the interdependencies on rewards that inequity aversion creates, it is important to study if the conditions for the implementation of output being optimal with standard preferences change when agents are inequity averse. We study this question in the following lemma.

Proposition 2 The Principal might implement joint production under inequity aversion even if individual production is implemented without inequity aversion.

Intuitively, Lemma 2 shows that under inequity aversion, the minimum cost of implementing individual production in equilibrium is equal to the cost of e¤ort of the agent who performs. However,

[21] Naturally, with no production equity is still maximized.

Proposition 1 tells us that under inequity aversion it is possible to implement joint production by rewarding agents with less than their costs of effort. Therefore, when the Principal optimally exploits inequity aversion, he might save by paying agents less than agents' cost of effort to implement joint production in equilibrium, and thus, for sufficiently high differences between joint production levels with respect to individual production levels, it is optimal to change the production decision from individual production in the standard case to joint production in the inequity aversion case. However, the opposite change, from joint production without inequity aversion to individual production with it, is not possible. The reason is that the minimum rewards paid to induce agents to perform in both cases are the same, their cost of effort, and production is always bigger when both agents perform than when only one performs. Finally, as the costs of implementing individual performance of effort are the same with and without inequity aversion, the change from one agent performing effort in the standard case to the other agent performing under inequity aversion is never optimal.

The proof is straightforward, given the results in the previous Lemmas 1 to 4 and Proposition 1. Appendix B shows a numerical example which proves that, for given parameter values, it can be optimal to change from individual production without inequity aversion to joint production with inequity aversion.

In this subsection we have seen possible changes of equilibria when optimally accounting for inequity aversion. However, another interesting issue is what happens to production if the Principal does not design the reward matrix optimally. We deal with this issue in the next subsection.

4.3 Non-optimal implementation of effort under inequity aversion

Standard contract theory does not account for inequity aversion. However, the fact that contract design has not studied until recently inequity aversion, does not mean that employees might not behave in real life as if they were inequity averse neither that real life employers are not accounting for inequity aversion and other non-standard preferences in the design of real life contracts. An interesting way of proving the theoretical relevance of our results is to check what would be the effect of offering "standard" contracts to agents motivated by inequity aversion. We use two different approaches to deal with this issue. In the first one, we study whether inequity averse agents would deviate from the effort decision that a Principal tries to implement with a standard contract resulting in a different SPE than the desired by the Principal. In the second one, we calculate the possible loss (or gain) for the Principal of offering standard contracts to inequity averse agents even if the SPE does not change.

4.3.1 Change of the implemented equilibrium when not accounting for inequity aversion

An employer not aware that his employees are inequity averse, would offer a contract such as the one described in section 3. Therefore, in equilibrium, the Principal designs the reward matrix such as it exactly compensates performing agents for their costs of effort and pays 0 to not performing agents. In the case agents have standard preferences, the Principal does not need to worry about the rewards offered out of the desired equilibrium, as agents' effort decisions do not depend on the rewards offered to other agents. With standard agents, the Principal only needs to make sure that each agent obtains more direct utility in the desired equilibrium than out of it and so, he does not need to worry about equity in the distribution of utilities out of equilibrium. However, if rewards o¤ered out of equilibrium are not carefully designed, it is possible that the distribution of utilities out of equilibrium is more equitable than the one in the SPE the Principal has tried to implement. Thus, it is possible that inequity averse agents might deviate to this new equilibrium in search of more equity. In this sense, we can say that when inequity aversion exists, optimal contracts are more "complete" as they must be completed by carefully specifying rewards o¤ered out of the desired equilibrium.

Notice that this issue is di¤erent to what we studied in Proposition 2. Here we show that if the Principal does not behave optimally, and thus, he does not realize that agents might be inequity averse, he might o¤er a contract that implements a di¤erent equilibrium than the one that would be optimal.

The following Proposition 3, shows the change of equilibrium that can occur when the contract o¤ered is not optimally designed.

> **Proposition 3** A contract designed to implement individual production as a unique SPE under standard preferences might implement joint production as the unique SPE if agents are inequity averse.

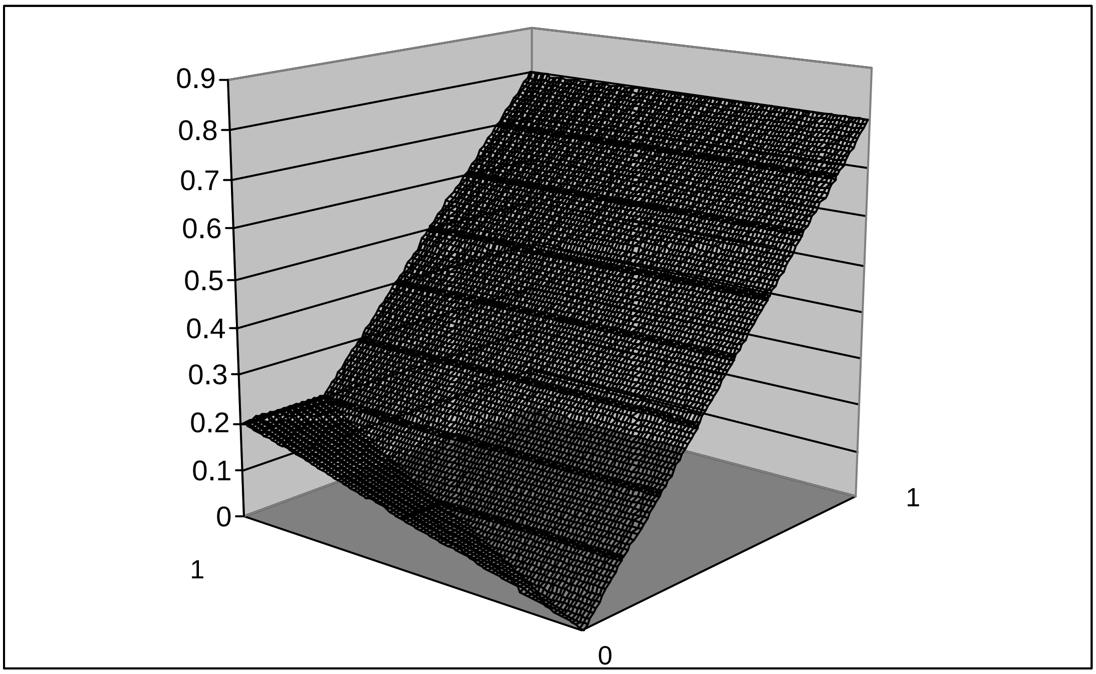
Intuitively, a contract that implements individual production in equilibrium under standard preferences, creates inequity in the SPE. The reason is that by Lemma 1, the agent who does not perform when individual production is not rewarded at all, while the performing agent is rewarded above his cost of e¤ort, and thus the non-rewarded agent will feel envy. However, if out of equilibrium, when both agents performs, both agents are o¤ered rewards that exactly compensate their costs of e¤ort, equity is maximized in joint production and both agents prefer to perform e¤ort than not perform, and thus agents deviate to a new SPE, di¤erent than the individual production, desired by the Principal. The proof in Appendix A also contains and explanation of why a contract that implements joint production under standard preferences, implements the same equilibrium under inequity aversion. The main reason being that to implement joint production, both with and without inequity aversion, equity is maximized in equilibrium, and so it is not possible to create extra equity in equilibrium to obtain the SPE at a lower cost.

4.3.2 Possible loss for the Principal when not accounting for inequity aversion

In the previous subsection, we saw that the implemented SPE can change if the Principal designs a contract without accounting for inequity aversion. In this subsection, we take a di¤erent perspective. We here show an example in which the optimal SPE implemented is the same with and without inequity aversion, and we measure the possible extra costs for the Principal when using a standard contract and thus, not exploiting inequity aversion optimally.

The example is constructed for a symmetric case in which the conditions to implement joint production both with and without inequity aversion hold. We assume $q_1 = q_2 = 0{:}5$ and $c_1 = c_2 = 0{:}4$; and we calculate the possible loss for the Principal for those values. The loss is de…ned as the di¤erence in the Principal's utility (production minus rewards paid) when o¤ering a standard contract to inequity averse agents as a proportion of the total output implemented in joint production (equal to 1), depending on the possible values of envy (®) and (¯). The calculations are explained in Appendix C. The loss for the Principal of o¤ering a standard contract is drawn in …gure 3.

Notice that it is particularly interesting that the Principal always loses by o¤ering a standard contract to inequity averse agents, he never gains. The reason for this result is that under standard preferences, the minimum cost of implementing an agent to perform e¤ort is the one that exactly compensates the agent for his cost of effort. However, we have shown in Proposition 1 that, under inequity aversion, it is possible for the Principal to exert effort at a smaller cost than the one that compensates effort. In particular, the Principal can implement joint production at a smaller cost with inequity aversion than without it. Finally, by rewarding both agents exactly for their cost of effort, joint production is also implemented as a SPE under inequity aversion.



Loss for the Principal when offering a standard contract to Inequity Averse Agents

Figure 3 shows that the loss from not taking into account inequity aversion can be extremely high. For the parameter values assumed, if the degrees of envy and altruism are high enough, the loss can be up to an 80% of the total output produced. The Experimental Literature[22] agrees that fairness concerns do not disappear under high stakes and thus, the real loss for an employer of not accounting for inequity aversion in the design of his contracts can be far from negligible, specially since the employer can never lose by designing the out of equilibrium rewards.[23]

5 Discussion

We have proved that the existence of inequity aversion among employees might change the optimal output decision taken by an employer. Additionally, it is possible that the employer can exploit inequity aversion and thus implement the desired effort levels at a lower total wage cost. The employer just needs to create inequity out of the equilibrium and redistribute rewards in equilibrium in a more equitable way. Finally, we have shown that when employees are inequity averse but the

[22] Cameron (1999) and Fehr, Fischbacher and Tougareva (2001) review these results.

[23] A different issue would be if there were a cost of designing more complete contracts, which we do not study.

employer does not account for it in the design of the reward scheme offered, the Principal always loses, never gains. The reason is that it is possible that undesired levels of effort or non-optimal total wage costs appear in equilibrium.

However, our model is very stylized and only pretends to add some theoretical analysis to an effect we believe is already being taken into account by firms' Human Resources Departments in real contracts design. One particular restriction is that we assume that the enforcement situation occurs only once. However, work relationships usually last more than one period and issues such as reciprocity, modelled as the reaction to another agent's decision, will be crucial. Additionally, it could be argued that inequity aversion might be enhanced by repeated interaction and thus, inequity aversion could increase over periods in repeated games.

A second restriction of our model is that it focuses on only two agents and a principal. Generalizing the model to N-agents would not be straightforward as we would face the problem of whom agents compare with that differentiates the models of F&S and Bolton and Ockenfels. We do not claim that this step is not important but that more research is needed on how agents care about fairness when the reference groups are N-dimensional, before modeling applications to the multi-agents case.

A potential problem comes from the fact that when inequity aversion is optimally exploited, employees could be better off not working for the firm at all. This depends on how the outside option is modelled. We believe our model adjusts well for jobs where joint production is a requirement. [24] If this is the case, an agent who does not accept a contract because his inequity aversion is exploited, has only two options: either accept a contract in a different firm in which there would be others workers for which the agents will feel equally inequity averse and so he will be equally exploited, or not work for any firm and thus obtain even less utility than when accepting the contract.[25] What we want to emphasize here is that preferences are given at one point in time. Either agents have a preference for equity or they do not. They cannot decide whether they want to have a taste for equity or not. Thus, if agents are inequity averse, the moment they are put in a situation in which there is interaction with other people, the moment they start to care about equity. The only way to avoid feeling inequity aversion would be to live totally isolated, but that could be quite a worse life than being partially exploited at work.

A limitation of our model is that effort is discrete. Either agents perform effort or they do not, but they cannot decide to trade a bit less of effort for some extra equity. However, in our model rewards can be marginally adjusted by the Principal. It could be argued that it might be relevant to provide agents with the choice to marginally adapt their effort choice to account for inequity aversion if precisely the agents are the ones assumed to be inequity averse. We believe that our main result still holds if effort is a continuous variable and thus, still an egoist Principal is able to exploit inequity aversion to his benefit in such a model. The reason is that no matter how much choice discretion agents have, still rewards can create more inequity out of the desired equilibrium than in equilibrium. In a different paper with a co-author,[26] we study a genuine team problem in which there is no principal and output is split among co-workers. In this model, inequity averse agents are allowed to continuously adapt their effort choice and we look at the optimallity of sequential effort choice versus simultaneous choice. However, we still observe that with continuous effort choice the

[24] We have seen that when the Principal implements individual production, inequity aversion has no effect neither on output decisions nor in costs of implementation.

[25] The normalization to zero utility when not performing effort and not being rewarded is just a normalization. Utility of not having a job can be assumed to be even lower.

[26] Huck and Rey Biel (2002).

e¤ects of inequity aversion on e¤ort are important and interesting.

We have not discussed in this paper the possibility of collusion among agents. This issue is particularly relevant for the case in which joint production is optimally implemented because joint production could not be the unique SPE. We observe that no production might also be a SPE and it could be argued that agents would coordinate on this equilibrium because it yields higher utilities for both of them. However, we have argued that although the optimal contract might change, it is still possible to implement joint production as a unique equilibrium, which weakens the incentives for collusion. In real …rms, other forms of collusion would be possible, as it seems intuitive that employees can agree not to make noticeable that they do care about welfare comparisons among them to the employer. But, at the same time, it is also true that it is precisely when the employer creates inequities when this collusive behavior is threaten and thus, it is not so uncommon to observe manifestations of envy or altruism among employees working together.

We should not forget that the motivation for our analysis comes from experimental work. Once we have provided a simple model to study some of the e¤ects of inequity aversion on contract design, a natural step would be to carry out experiments in which to test this model. We intend to do this on future research.

In any case, our model tries to provide some insights on how managers use non-standard contract theory to organize their …rms. Just as in the quote from the …lm Pulp Fiction (Quentin Tarantino, 1994) that opens this article, fair (or righteous) agents might be exploited by sel…sh principals by creating inequities. Optimally exploiting inequity aversion would imply designing work structures in a way that maximizes inequity when company demands are not met. But this approach could be extended beyond our story about paying di¤erent wages to di¤erent agents out of equilibrium. In particular, an employer might be able to create inequity among employees in several other ways such as in the assignment of holidays periods, working conditions or maternity leaves. What our model hinges, is that to be able to use these inequities in the bene…t of the employer it is a good idea to make information about these issues easily available to employees, such as they use it to compare themselves. Thus, our model might provide a rationale to such company policies such as making wages publicly known within the …rm, or whether the workplace should be designed such as co-workers' e¤orts are easily observed. We intend to study this issue further in future work.

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7 Appendix A

Proof of Lemma 1

We study two cases, depending on whether the Principal implements individual production (a)) or joint production (b)).

a) Optimal reward design if the Principal implements individual production.

Assume the Principal wants agent 1 to perform individual effort. Then, agent 1 must obtain more utility when performing individual effort than when not performing, given that agent 2 does not perform. Thus, given that by assumption (R2) rewards are 0 when there is no production, agent's 1 no deviation condition is:

$$w_1' - c_1 \geq 0.$$

If agent 1 performing effort individually is to be a SPE, agent 2 must obtain more utility when not performing effort than when performing, given that agent 1 individually performs. Thus, agent's 2 no deviation condition is:

$$w_2' \geq w_2 - c_2.$$

In order to make agent 1 individually performing effort a SPE at the cheapest possible cost for the Principal, we only need to find the minimum $w_1'; w_2'$ in equilibrium and a w_2 out of equilibrium such as these two no deviation conditions hold. The most natural solution is:

$$\begin{aligned} w_1' &= c_1 \\ w_2' &= 0 \\ w_2 &= 0. \end{aligned}$$

However, this is not the only possible solution. As w_2 is a reward offered out of the desired equilibrium, any $w_2 \in [0; c_2]$ still allows agent 1 performing effort individually to be a SPE with no extra cost for the Principal. Additionally, if we want to find the SPE in Dominant Strategies, it requires that $w_1^{'} = c_1 + \varepsilon$ where $\varepsilon \to 0$.[27] Notice that we cannot claim this to be the only SPE of this game, as we do not calculate all the rewards offered out of equilibrium, and some of the unspecified rewards might create other equilibria. However, if we require agent 1 individually performing effort to be the Unique SPE of this game, we need to specify rewards for all possible production levels. Thus, the optimal reward design is:

Rewards Offered

Effort of Agent 2

Effort Of Agent 1	1	0
1	$[0,c_1)$, $[0,c_2]$	$c_1+\varepsilon$, 0
0	0, $[0,c_2)$	0 , 0

The solution is symmetric if agent 2 is to individually perform effort.

b) Optimal reward design if the Principal implements joint production.

If agent 1 is to perform when agent 2 is also performing, agent's 1 no deviation condition is:

$$w_1 - c_1 \geq w_1^{''}.$$

If agent 2 is to perform when agent 1 is also performing, agent's 2 no deviation condition is:

$$w_2 - c_2 \geq w_2^{'}.$$

In order to make joint production a SPE at the cheapest possible cost for the Principal, we need to find the minimum $w_1; w_2$; and some $w_1^{''}$ and $w_2^{'}$ out of equilibrium such as these two no deviation conditions hold jointly, as all the other rewards are outside the implementation of this equilibrium. The most natural solution is:

$$\begin{aligned} w_1 &= c_1 \\ w_2 &= c_2 \\ w_1^{''} &= 0 \\ w_2^{'} &= 0. \end{aligned}$$

[27] As ε is marginally small, the increase in the wage cost for the Principal is negligible.

However, this is not the only possible result. As w_1'' and w_2' are rewards offered out of the desired equilibrium, any $w_1'' \in [0; c_1]$ and $w_2' \in [0; c_2]$ still allows joint production to be a SPE with no extra cost for the Principal. Additionally, if we want to find the SPE in Dominant Strategies it requires that $w_1 = c_1 + \varepsilon$ and $w_2 = c_2 + \varepsilon$ where $\varepsilon \to 0$: Notice that we do not claim this to be the only SPE of this game. The reason is that to make both agents performing effort a SPE of this game, we do not need to calculate some of the optimal rewards offered out of the equilibrium, and some of the unspecified rewards might create other equilibria. However, if we require joint production to be the Unique SPE of this game, the optimal design of the reward matrix is:

Rewards Offered

		Effort of Agent 2	
		1	**0**
Effort of Agent 1	**1**	$c_1+\varepsilon$, $c_2+\varepsilon$	$c_1+\varepsilon$, 0
	0	0, $c_2+\varepsilon$	0 , 0

Proof of Lemma 2

We prove it for the case in which the Principal wants agent 2 to perform individual effort.

Step 1:

Under inequity aversion, for agent 2 performing individual production to be a SPE, the two following no deviation conditions need to be satisfied:

For agent 1 :

$$w_1'' - \alpha \max\left[w_2'' - c_2 - w_1''; 0\right] - \beta \max\left[w_1'' - w_2'' + c_2; 0\right] \geq \qquad (1)$$

$$w_1 - c_1 - \alpha \max[w_2 - c_2 - w_1 + c_1; 0] - \beta \max[w_1 - c_1 - w_2 + c_2; 0]:$$

For agent 2 :

$$w_2'' - c_2 - \alpha \max\left[w_1'' - w_2'' + c_2; 0\right] - \beta \max\left[w_2'' - c_2 - w_1''; 0\right] \geq 0: \qquad (2)$$

We then need to find the smallest possible values of w_1'' and w_2'', such that conditions (1) and (2) hold. However, because of interdependencies in utilities, we also need to find the optimal values for the rewards offered out of equilibrium, w_1 and w_2:

Step 2: Optimal Choice of w_1:

This reward (w_1), only appears on the right-hand side (RHS onwards) of condition (1), namely, the utility of agent 1 when both agents perform (joint production). Let us denote this utility as U_1^{JP}:

U_1^{JP} should be the smallest possible so as to make condition (1) hold at the cheapest possible cost for the Principal. The optimal choice is $w_1 = 0$.

Notice that inequity aversion acts in such a way such as an agent obtains disutility either from being better off or worse off than the other agent, but not from both at the same time. Therefore, only one of the two terms in brackets on the RHS of condition (1) is different from zero.

a) If agent 1 is worse off than agent 2, envy dominates and $w_2 - c_2 - w_1 + c_1 \geq 0$: Thus, to make agent 1 worse off out of equilibrium, $w_1 = 0$, as $\frac{\partial U_1^{JP}}{\partial w_1} = 1 + \alpha > 0$, by assumption (U3):

b) If agent 1 is better off than agent 2, altruism dominates and $w_1 - c_1 - w_2 + c_2 \geq 0$: Thus, to make agent 1 worse off out of equilibrium, $w_1 = 0$, as $\frac{\partial U_1^{JP}}{\partial w_1} = 1 - \beta > 0$, by assumption (U4).

Step 3: Optimal Choice of w_2:

This reward (w_2); only appears on the RHS of condition (1), which we have denoted as U_1^{JP}: Again, U_1^{JP} should be the smallest possible so as to make condition (1) hold at the cheapest possible cost for the Principal.

a) To maximize the effect of envy, it is optimal to reward agent 2 as much as possible. Due to assumption (R2) the maximum the Principal can reward agent 2 when both agents perform is $w_2 = 1$:

b) To maximize the effect of altruism, it is optimal to reward agent 2 as little as possible. Due to assumption (R2) the minimum the Principal can reward agent 2 when both agents perform is $w_2 = 0$:

Again, because of the way we have modelled inequity aversion, only one of the two terms in brackets on the RHS of condition (1) is different from zero. Thus, the optimal choice of w_2 depends on whether the maximized effect of envy or altruism is bigger than the other.

The optimal payment for agent 2 when both agents perform is:

$$w_2 = 1 \text{ if } \alpha[1 - c_2 + c_1] \geq \beta[c_2 - c_1]$$

and

$$w_2 = 0 \text{ if } \alpha[1 - c_2 + c_1] < \beta[c_2 - c_1]:$$

Step 4: Optimal choice of w_1'' and w_2'':

Both rewards (w_1'' and w_2'') appear simultaneously in both conditions (1) and (2): Thus, we find the optimal values of w_1'' and w_2'' using both conditions at the same time. We need to check two cases, depending on the optimal values found for w_1 and w_2 in step 3:

1. Assume $\alpha[1 - c_2 + c_1] \geq \beta[c_2 - c_1]$. Thus, the Principal wants to maximize the effect of envy by setting $w_1 = 0$ and $w_2 = 1$.

Conditions (1) and (2) are then:

$$w_1'' - \alpha \max\left[w_2'' - c_2 - w_1''; 0\right] - \beta \max\left[w_1'' - w_2'' + c_2; 0\right] \geq -c_1 - \alpha[1 - c_2 + c_1]$$
$$w_2'' - c_2 - \alpha \max\left[w_1'' - w_2'' + c_2; 0\right] - \beta \max\left[w_2'' - c_2 - w_1''; 0\right] \geq 0.$$

a) Assume $w_1'' - w_2'' + c_2 \geq 0$, that is, agent 1 is better off than agent 2.

Thus, the conditions are:

$$w_1'' - \beta\left[w_1'' - w_2'' + c_2\right] \geq -c_1 - \alpha[1 - c_2 + c_1]$$
$$w_2'' - c_2 - \alpha\left[w_1'' - w_2'' + c_2\right] \geq 0.$$

As we assume $w_1'' - w_2'' + c_2 \geq 0$, the second condition is more restrictive. The reason is that w_2'' in the second condition needs to be bigger or equal than a strictly positive number, while w_1'' only needs to be bigger than 0 (by assumption (R1)). As we are looking for the smallest possible values of these parameters, we impose $w_1'' - w_2'' + c_2 = 0$, which leads to

$$\begin{aligned} w_1'' &= 0 \\ w_2'' &= c_2. \end{aligned}$$

Notice than under these values, both conditions for agent 1 and agent 2 satisfy and the assumption $w_1'' - w_2'' + c_2 \geq 0$ holds.

b) Assume $w_2'' - c_2 - w_1'' \geq 0$, agent 2 is better off than agent 1.

Thus the conditions are:

$$w_1'' - \alpha\left[w_2'' - c_2 - w_1''\right] \geq -c_1 - \alpha[1 - c_2 + c_1]$$
$$w_2'' - c_2 - \beta\left[w_2'' - c_2 - w_1''\right] \geq 0.$$

As we assume $w_2'' - c_2 - w_1'' \geq 0$, the second condition is more restrictive. The reason is that w_2'' in the second condition needs to be bigger or equal than a strictly positive number, while w_1'' only need to be bigger than 0 (assumption (R1)). As we are looking for the smallest possible values of these parameters, we impose $w_2'' - c_2 - w_1'' = 0$, which leads to

$$\begin{aligned} w_1'' &= 0 \\ w_2'' &= c_2. \end{aligned}$$

Notice than under these values, both conditions for agent 1 and agent 2 satisfy and the assumption $w_2'' - c_2 - w_1'' \geq 0$ holds.

2. Assume $\alpha[1 - c_2 + c_1] < \beta[c_2 - c_1]$. Thus the Principal wants to maximize the effect of altruism by setting $w_1 = 0$ and $w_2 = 0$.

Conditions (1) and (2) are then:

$$w_1' - \alpha \max\left[w_2'' - c_2 - w_1''; 0\right] - \beta \max\left[w_1'' - w_2'' + c_2; 0\right] \geq 0$$

$$w_2'' - c_2 - \alpha \max\left[w_1'' - w_2'' + c_2; 0\right] - \beta \max\left[w_2'' - c_2 - w_1''; 0\right] \geq 0.$$

Using the same procedure as in 1., and given assumptions (R1) and (R2), it is straightforward to see that the smallest values such as these two conditions hold jointly are:

$$\begin{aligned} w_1'' &= 0 \\ w_2'' &= c_2. \end{aligned}$$

Step 5: Optimal choice of w_1' and w_2'.

Notice that neither w_1' nor w_2' enter into any of the agents' no deviation conditions. Therefore, their optimal values are only relevant for the issues of Equilibrium Uniqueness and the implementation of the SPE in Dominant Strategies.

With respect to Equilibrium Uniqueness, if the Principal chooses the smallest possible values for these rewards, $w_1' = 0$ and $w_2' = 0$, avoids making individual production by the other agent to be a SPE. The issue is then that both individual production by the desired agent and not production are SPE. To obtain equilibrium uniqueness, the Principal can proceed as in the standard case and pay in equilibrium a negligible extra reward of ε to the performing agent to. Notice that this same procedure allows individual production to be a SPE in Dominant Strategies.

The optimal reward matrix would the be:

Rewards Offered

Effort of Agent 2

Effort Of Agent 1	**1**	**0**
1	$0, w_2$	$0, 0$
0	$0, c_2+\varepsilon$	$0, 0$

where

$$w_2 = 1 \text{ if } \alpha[1 - c_2 + c_1] \geq \beta[c_2 - c_1]$$

and

$$w_2 = 0 \text{ if } \alpha[1 - c_2 + c_1] < \beta[c_2 - c_1].$$

A symmetric reasoning holds for the case in which the Principal wants agent 1 to perform individual effort.

Proof of Lemma 3

Step1

If no agent performs effort, there is no production and thus, by assumption (R2), both agents are not rewarded.

Step 2

Assume agent 2 individually performs effort out of equilibrium. The Principal's objective is to maximize the disutility of agent 1 out of equilibrium such as agent 1 does not deviate from the desired equilibrium. We calculate the optimal reward for agent 1, w_1'', when agent 2 individually performs and is paid w_2''.

The utility of agent 1 when agent 2 individually performs effort is:

$$U_1'' = w_1'' - \alpha \max\left[w_2'' - c_2 - w_1''; 0\right] - \beta \max\left[w_1'' - w_2'' + c_2; 0\right].$$

Notice that inequity aversion imposes that an agent obtains disutility either from being better off or worse off than the other agent, but not from both at the same time.

a) If agent 1 is worse off than agent 2, the effect of envy dominates and $w_2'' - c_2 - w_1'' \geq 0$. Thus, to make agent 1 worse off out of equilibrium, $w_1'' = 0$, as $\frac{\partial U_1''}{\partial w_1''} = 1 + \alpha > 0$, by assumption (U3).

b) If agent 1 is better off than agent 2, the effect of altruism dominates and $w_1 - w_2 + c_2 \geq 0$. Thus, to make agent 1 worse off out of equilibrium, $w_1'' = 0$, as $\frac{\partial U_1''}{\partial w_1''} = 1 - \beta > 0$, by assumption (U4).

A symmetric argument holds for w_2' if it is agent 1 who performs individual effort out of equilibrium.

Proof of Lemma 4

Assume agent 2 individually performs effort out of the desired equilibrium (joint production).

The reward offered to agent 2 when agent 2 individually performs, w_2'', only appears in the no deviation condition of agent 1. The objective of the Principal is to maximize the disutility of agent 1 out of the equilibrium.

By Lemma 2, we know that he optimal payment to agent 1 when agent 2 individually performs is $w_1'' = 0$.

The utility of agent 1 when agent 2 individually performs is thus:

$$-\alpha \max\left[w_2'' - c_2; 0\right] - \beta \max\left[-w_2'' + c_2; 0\right]$$

where by (R2),

$$w_2'' \in [0; q_2];$$

and by (C1),

$$0 \leq c_2 \leq q_2:$$

Thus, minimizing the utility of agent 1 implies:

$$w_2'' = q_2 \quad \text{if} \quad \alpha(q_2 - c_2) \geq \beta c_2$$

and

$$w_2'' = 0 \quad \text{if} \quad \alpha(q_2 - c_2) < \beta c_2$$

A symmetric argument holds for w_1' if it is agent 1 who individually performs effort out of the desired equilibrium.

Proof of Proposition 1

We prove it in two steps. First we show that, under inequity aversion, the maximum needed total wage cost to implement joint production in equilibrium is the sum of the costs of exactly compensating both agents for their costs of effort. By Lemma 1, this is the same as the cost of implementing joint production with standard agents. We then show an example of how the total cost of implementing joint production can be smaller that the sum of the costs of effort.

Step 1

Under inequity aversion, it is always possible to exactly compensate the agents for their cost of effort in equilibrium and implement joint production.

To implement joint production, both agents must prefer to perform effort than not performing when the other agent is performing. Thus, the objective of the Principal is to maximize agents' disutility out of the equilibrium, i.e., in the situation when one agent individually performs.

Assume agent 2 individually performs off the equilibrium. The transformed utility of agent 1 is:

$$w_1'' - \alpha \max\left[w_2'' - c_2 - w_1''; 0\right] - \beta \max\left[w_1'' - w_2'' + c_2; 0\right]:$$

By rewarding $w_1'' = 0$ and $w_2'' \in [0; c_2)$ the utility of agent 1 out of the equilibrium is always negative.

Assume agent 1 individually performs off the equilibrium. The transformed utility of agent 2 is:

$$w_2' - \alpha \max\left[w_1' - c_1 - w_2'; 0\right] - \beta \max\left[w_2' - w_1' + c_1; 0\right]:$$

By rewarding $w_2' = 0$ and $w_1' \in [0; c_1)$ the utility of agent 2 out of the equilibrium is always negative.

Therefore, when comparing the transformed utility of each agent in joint production:

For agent 1 :

$$w_1 - c_1 - \alpha \max[w_2 - c_2 - w_1 + c_1; 0] - \beta \max[w_1 - c_1 - w_2 + c_2; 0]$$

and for agent 2 :

$$w_2 - c_2 - \alpha \max[w_1 - c_1 - w_2 + c_2; 0] - \beta \max[w_2 - c_2 - w_1 + c_1; 0]:$$

Each one needs to be bigger than a negative value.

However, by paying $w_1 = c_1$ and $w_2 = c_2$ the transformed utility in joint production equilibrium of each agent is zero, both no deviation conditions hold, and the total wage cost, $w_1 + w_2 = c_1 + c_2$; is exactly the same as in the standard case.

Step 2

An example on how to design the reward matrix in such a way that the total wage cost of implementing joint production under inequity aversion is smaller than in the standard case.

Lets first use the preceding Lemmas to find the optimal rewards out of equilibrium.

By Lemma 3 it is always optimal to pay 0 the agent who does not perform out of equilibrium:

$$w_1'' = 0$$

$$w_2' = 0:$$

By Lemma 4 the optimal rewards to the agent who performs out of equilibrium depend on the potential effect of envy and altruism. Therefore:

$$\text{If } \alpha(q_1 - c_1) \geq \beta c_1 \text{ then } w_1' = q_1$$

$$\text{If } \alpha(q_2 - c_2) \geq \beta c_2 \text{ then } w_2'' = q_2$$

$$\text{If } \alpha(q_1 - c_1) < \beta c_1 \text{ then } w_1' = 0$$

$$\text{If } \alpha(q_2 - c_2) < \beta c_2 \text{ then } w_2'' = 0$$

There are therefore, four possible optimal combinations of rewards depending on parameter values. For the purpose of this example we focus on one of them. The reasoning for the remaining cases is analogous.

Assume $\alpha(q_1 - c_1) \geq \beta c_1$ and $\alpha(q_2 - c_2) \geq \beta c_2$.

Thus, by Lemma 4 the optimal payments for the agents performing effort out of equilibrium (joint production) are:

$$w_1' = q_1$$

$$w_2'' = q_2:$$

The no deviation conditions for the agents in joint production are thus:

$$w_1 - c_1 - \alpha \max[w_2 - c_2 - w_1 + c_1; 0] - \beta \max[w_1 - c_1 - w_2 + c_2; 0] \geq -\alpha(q_2 - c_2)$$

$$w_2 - c_2 - \alpha \max[w_1 - c_1 - w_2 + c_2; 0] - \beta \max[w_2 - c_2 - w_1 + c_1; 0] \geq -\alpha(q_1 - c_1):$$

Assume $q_1 - c_1 \geq q_2 - c_2$:

a) Conjecture that the minimum w_1 and w_2 satisfy $w_1 - c_1 \geq w_2 - c_2$. Then:

$$w_1 - c_1 - \beta[w_1 - c_1 - w_2 + c_2] \geq -\alpha(q_2 - c_2)$$

$$w_2 - c_2 - \alpha[w_1 - c_1 - w_2 + c_2] \geq -\alpha(q_1 - c_1).$$

Solving this system of inequalities for the minimum possible values of w_1 and w_2 :

$$w_1 = \frac{c_1(1 + \alpha - \beta - \alpha\beta) + \alpha\beta q_1 + \alpha q_2(-1 - \alpha) + \alpha c_2(1 + \alpha)}{1 + \alpha - \beta}$$

and

$$w_2 = \frac{\alpha c_1(1 - \beta) + \alpha q_1(\beta - 1) - \alpha^2 q_2 + c_2(\alpha^2 + \alpha + 1 - \beta)}{1 + \alpha - \beta}$$

which satisfies $w_1 - c_1 \geq w_2 - c_2$.

b) Conjecture, on the contrary, that the minimum w_1 and w_2 satisfy $w_1 - c_1 < w_2 - c_2$. Then:

$$w_1 - c_1 - \alpha[w_2 - c_2 - w_1 + c_1] \geq -\alpha(q_2 - c_2)$$

$$w_2 - c_2 - \beta[w_2 - c_2 - w_1 + c_1] \geq -\alpha(q_1 - c_1).$$

Solving this system of inequalities for the minimum possible values of w_1 and w_2 :

$$w_1 = \frac{c_1(1 + \alpha(1 + \alpha) - \beta) + \alpha c_2(1 - \beta) + \alpha q_1(\beta - 1) + \alpha^2 q_2}{1 + \alpha - \beta}$$

and

$$w_2 = \frac{\alpha c_1(1 + \alpha) + c_2(1 - \beta(1 + \alpha) + \alpha) + \alpha\beta q_1 - \alpha q_2(1 + \alpha)}{1 + \alpha - \beta}$$

which satisfies $w_1 - c_1 \geq w_2 - c_2$ only as long as $q_1 - c_1 < q_2 - c_2$, which contradicts the assumption.

Therefore the minimum total wage bill with inequity aversion (TWB^{IA}) is the sum of the rewards ($w_1 + w_2$) from case a):

$$TWB^{IA} = \frac{c_1(1 - \beta + 2\alpha - 2\alpha\beta) + c_2(1 - \beta + 2\alpha^2 + 2\alpha) + \alpha q_1(2\beta - 1) + \alpha q_2(-2\alpha - 1)}{1 + \alpha - \beta}$$

which we can compare with the total wage bill under standard preferences ($TWB^S = c_1 + c_2$) :

a) If $\beta \leq \frac{1}{2}$ then $TWB^{IA} \leq TWB^S$.

b) If $\beta > \frac{1}{2}$ then:

b1) If $(q_1 - c_1)(2\beta - 1) \leq (c_2 - q_2)(-2\alpha - 1)$ then $TWB^{IA} \leq TWB^S$.

b2) If $(q_1 - c_1)(2\beta - 1) > (c_2 - q_2)(-2\alpha - 1)$ then $TWB^{IA} > TWB^S$. However, by the first step of this proof, the Principal can always reward $w_1 + w_2 = c_1 + c_2$ in equilibrium and implement joint production with the same cost as in the standard case.

The reasoning is the same for $q_1 - c_1 < q_2 - c_2$; conjecturing that the minimum w_1 and w_2 satisfies $w_2 - c_2 - w_1 + c_1 \geq 0$.

Proof of Lemma 5

Step 1

Notice that when the conditions for at least one of the agents who individually performs effort off equilibrium to be rewarded with all input produced (either $w_1' = q_1; w_2' = q_2$ or both) hold, joint production as implemented in Step 2 of the Proof of Proposition 1 is a Unique SPE (and a Unique Equilibrium in Dominant Strategies).

Step 2

Notice that

$$\text{If } \alpha(q_1 - c_1) < \beta c_1$$

and

$$\text{If } \alpha(q_2 - c_2) < \beta c_2;$$

we saw that the optimal rewards to implement joint production as a SPE were:

$$w_1' = 0$$

and

$$w_2'' = 0.$$

However, these rewards make the agent who individually performs effort out of equilibrium worse off when individually performing than when no agent performs at all and thus making no production a SPE.

What is needed is to reward the agent who individually performs effort off the equilibrium above his cost of effort:

$$w_1' > c_1$$

and

$$w_2'' > c_2.$$

By doing so, given that by Lemma 3 it is optimally not to reward the agent who does not perform off the equilibrium, the transformed utilities of the agent who does not perform off the equilibrium when the other agent is individually performing are:

For agent 1:

$$-\alpha(w_2'' - c_2);$$

and for agent 2:

$$-\alpha(w_1' - c_1);$$

which are always negative given that $w_1' > c_1$ and $w_2'' > c_2$ in order to make the other agent to prefer to individually perform.

In order to maximize the disutility of the agent who does not perform when the other agent is individually performing, it is now optimal to choose:

$$w_1' = q_1$$

and

$$w_2'' = q_2;$$

which by assumption (C1) are bigger than the costs of effort.

Rewards Offered

Effort of Agent 1		Effort of Agent 2: 1	Effort of Agent 2: 0
	1	w_1, w_2	q_1 , 0
	0	0, q_2	0 , 0

It is straightforward to see that joint production rewards w_1 and w_2 as calculated in Step 2 of the Proof of Proposition 1 are the Unique SPE and that optimal rewards in equilibrium with inequity aversion are smaller or equal than under standard preferences.

Proof of Proposition 3

Below we study the consequences of o¤ering standard contracts to inequity averse agents both if the desired equilibrium is joint or individual production.

If the Principal implements joint production under the standard case, the optimal reward matrix is:

Rewards Offered

Effort of Agent 1		Effort of Agent 2: 1	Effort of Agent 2: 0
	1	$c_1+\varepsilon$, $c_2+\varepsilon$	$c_1+\varepsilon$, 0
	0	0, $c_2+\varepsilon$	0 , 0

We already discussed in section 3 that under standard preferences, this contract implements a Unique SPE. This does not change under inequity aversion as the no deviation conditions still only satisfy for joint production. We can be certain that under inequity aversion individual performance is not a SPE because the agent not performing out of the joint production equilibrium always loses by not performing. The reason is that the agent is not rewarded at all and the reward offered out of equilibrium to the agent who individually performs, creates now disutility to agent not performing agent because of inequity in the distribution of rewards. A more subtle argument exists to disregard no production (neither agent performing effort) as a SPE of this game. It seems that the total equity of the distribution (both agents obtain the same utility when not performing, zero) makes no-production a candidate to be a SPE. However, notice that each agent would be willing to deviate from this possible equilibrium and perform because they would see their effort cost compensated (plus a term ε). Although this payment creates inequity in the distribution of rewards, and thus disutility to the agent performing, it is important to keep in mind that we assume $\alpha; \beta \in [0; 1)$; and thus, the effect of this inequity (which in any case is motivated by an ε inequity in the distribution of rewards) is always dominated by the agent's performing effort own direct rewards. Therefore, the Principal can be sure that the standard contract implements the same joint production SPE if the agents are inequity averse.

However, things can change when the Principal implements individual production in the standard case. We discuss it assuming the Principal implements agent 1 performing as the unique SPE of the standard case without loss of generality. The discussion for the implementation of agent 2 performing is symmetric. In section 3 we see that if the Principal implements agent 1 performing effort in the standard case, the optimal reward matrix is:

Rewards Offered

Effort of Agent 2

Effort Of Agent 1		**1**	**0**
	1	$[0,c_1), [0,c_2]$	$c_1+\varepsilon, 0$
	0	$0, [0,c_2)$	$0, 0$

Notice that in this matrix some rewards are not totally specified and can take different values. However, if some of these values are not carefully chosen, they might implement a different equilibrium under inequity aversion. Following the discussion above, it is easy to see that this contract creates inequity in the equilibrium where agent 1 individually performs effort. Therefore, if off the equilibrium the Principal offers rewards to both agents such as they have the same transformed utilities (for example, by exactly compensating for their costs of effort when both agents perform, $w_1 = c_1$ and $w_2 = c_2$), agent 2 will deviate and will perform. If additionally, there is some inequity in the utilities when agent 2 performs and agent 1 does not, for example, $w_1'' = 0; w_2'' = c_2 - \varepsilon$, agent 1 will also prefer to perform when agent 2 is performing and thus the only SPE of this game with inequity averse agents is joint production. Therefore, in this case, there can be a change from a SPE under standard preferences (individual production) to under inequity averse preferences (joint production).

8 Appendix B

Numerical example showing the result in Proposition 2 is possible.

Assume $\alpha = 0.9; \beta = 0.1; q_1 = 0.7; c_1 = 0.5; q_2 = 0.5$ and $c_2 = 0.4.$

Agent 1's individually performing no deviation condition without inequity aversion is satisfied as

$$1 - c_2 \le q_1 \quad \text{if} \quad (q_1 - c_1) > (q_2 - c_2).$$

substituting;

$$1 - 0.4 \le 0.7 \quad \text{with} \quad (0.7 - 0.5) > (0.5 - 0.4);$$

as

$$0.6 < 0.7 \text{ with } 0.2 > 0.1.$$

Therefore, by Lemma 1, in equilibrium with standard preferences, agent 1 is paid his cost of effort for individually performing ($w_1' = 0.5$) and agent 2 is not rewarded at all ($w_2' = 0$) and individual production is implemented.

However, we now show that for the given parameter values, the Principal is better off implementing joint production when agents are inequity averse.

Implementation of Individual Production with Inequity Aversion

By Lemma 2, the minimum reward needed to implement individual production as a SPE under inequity aversion is the cost of effort of the agent individually performing in equilibrium.

By Lemma 3, the agent who does not perform when the other agent individually performs is not rewarded at all.

Therefore, if agent 1 is to individually perform under inequity aversion, $w_1' = 0.5$ and $w_2' = 0.$

Additionally, if agent 2 is to individually perform under inequity aversion, $w_1'' = 0$ and $w_2'' = 0.4.$

Implementation of Joint Production with Inequity Aversion.

We now use Lemma 4 to show the optimal reward matrix under inequity aversion to implement joint production which appears below. The values for w_1 and w_2 still need to be determined.

Rewards Offered

Effort of Agent 2

Effort of Agent 1		**1**	**0**
	1	w_1, w_2	0.7 , 0
	0	0 , 0.5	0 , 0

The no deviation conditions for joint production to be a SPE under inequity aversion are:

$w_1 - 0.5 - 0.9\max[w_2 - 0.4 - w_1 + 0.5; 0] - 0.1\max[w_1 - 0.5 - w_2 + 0.4; 0] \geq -0.9\max[0.5 - 0.4; 0]$

$w_2 - 0.4 - 0.9\max[w_1 - 0.5 - w_2 + 0.4; 0] - 0.1\max[w_2 - 04 - w_1 + 0.5; 0] \geq -0.9\max[0.7 - 0.5; 0];$

which simplify to:

$w_1 - 0.9\max[w_2 - w_1 + 0.1; 0] - 0.1\max[w_1 - w_2 - 0.1; 0] \geq 0.41$

$w_2 - 0.9\max[w_1 - w_2 - 0.1; 0] - 0.1\max[w_2 - w_1 + 0.1; 0] \geq 0.22.$

Solving these two inequalities for the lowest possible values of w_1 and w_2 yields:

$$w_1 = 0.365$$

$$w_2 = 0.215.$$

Notice that it is then optimal for the Principal to implement joint production when there is inequity aversion.

Utility for the Principal if joint production is implemented:

$$1 - w_1 - w_2 = 1 - 0.365 - 0.215 = 0.42.$$

Utility for the Principal if agent 1 individually performs:

$$q_1 - w_1^{0} = 0.7 - 0.5 = 0.2.$$

Utility for the Principal if agent 2 individually performs:

$$q_2 - w_2^{00} = 0.5 - 0.4 = 0.1.$$

Utility of the Principal if no agent performs:

$$0.$$

As $0.42 > 0.2 > 0.1 > 0$, the Principal implements joint production when there is inequity aversion.

9 Appendix C

Numerical example showing the possible loss of not accounting for inequity aversion.

Assume the following values for the parameters:

$$q_1 = q_2 = 0.5$$

$$c_1 = c_2 = 0.4.$$

Therefore the conditions for the Principal to implement joint production are satisfied in the standard case:

$$1 - q_1 \geq c_2 \quad \text{if} \quad (q_1 - c_1) \geq (q_2 - c_2)$$

as

$$1 - 0.5 \geq 0.4 \quad \text{if} \quad (0.5 - 0.4) \geq (0.5 - 0.4).$$

Under the standard case, the total cost of implementing joint production (TWB^S) is the sum of the costs of effort of both agents:

$$TWB^S = w_1 + w_2 = c_1 + c_2 = 0.8$$

The condition for implementing joint production under inequity aversion,

$$1 - w_1 - w_2 \geq q_1 - c_1 ,$$

is satisfied if

$$1 - w_1 - w_2 \geq 0.5 - 0.4 ,$$

thus if

$$w_1 + w_2 \leq 0.9.$$

Under inequity aversion, the agent who individually performs effort out of equilibrium is compensated for its cost of effort if:

$$\alpha(q_j - c_j) \geq \beta c_j ,$$

substituting,

$$\alpha(0.5 - 0.4) \geq \beta(0.4)$$

thus if,

$$\alpha \geq 4\beta.$$

Alternatively, if $\alpha < 4\beta$, the agent who individually performs effort off the equilibrium is paid 0.

a) Assume $\alpha \geq 4\beta$. The no deviation conditions for each agent to perform effort when the other agent is performing are:

$w_1 - 0.4 - \alpha \max[w_2 - 0.4 - w_1 + 0.4, 0] - \beta \max[w_1 - 0.4 - w_2 + 0.4, 0] \geq -\alpha \max[0.5 - 0.4, 0] - \beta \max[-0.5 + 0.4, 0]$

$w_2 - 0.4 - \alpha \max[w_1 - 0.4 - w_2 + 0.4, 0] - \beta \max[w_2 - 0.4 - w_1 + 0.4, 0] \geq -\alpha \max[0.5 - 0.4, 0] - \beta \max[-0.5 + 0.4, 0]$

which simplify to:

$$w_1 - 0.4 - \alpha \max[w_2 - w_1, 0] - \beta \max[w_1 - w_2, 0] \geq -0.1\alpha$$
$$w_2 - 0.4 - \alpha \max[w_1 - w_2, 0] - \beta \max[w_2 - w_1, 0] \geq -0.1\alpha.$$

Thus, the minimum possible values of w_1 and w_2 such as these two conditions hold are:

$$w_1 = w_2 = 0.4 - 0.1\alpha.$$

b) Assume $\alpha < 4\beta$. The no deviation conditions for each agent to perform effort when the other agent is performing are:

$$w_1 - 0.4 - \alpha \max[w_2 - 0.4 - w_1 + 0.4, 0] - \beta \max[w_1 - 0.4 - w_2 + 0.4, 0] \geq -\alpha \max[-0.4, 0] - \beta \max[0.4, 0]$$
$$w_2 - 0.4 - \alpha \max[w_1 - 0.4 - w_2 + 0.4, 0] - \beta \max[w_2 - 0.4 - w_1 + 0.4, 0] \geq -\alpha \max[-0.4, 0] - \beta \max[0.4, 0].$$

which simplify to:

$$w_1 - 0.4 - \alpha \max[w_2 - w_1, 0] - \beta \max[w_1 - w_2, 0] \geq -0.4\beta$$
$$w_2 - 0.4 - \alpha \max[w_1 - w_2, 0] - \beta \max[w_2 - w_1, 0] \geq -0.4\beta.$$

Thus, the minimum possible values of w_1 and w_2 such as these two conditions hold are:

$$w_1 = w_2 = 0.4(1 - \beta).$$

Therefore, the condition to implement joint production under inequity aversion ($w_1 + w_2 < 0.9$) is satisfied for both cases as $\alpha, \beta \in [0, 1)$.

We calculate the Principal's possible loss as the difference between the Principal's utility (production minus rewards) with and without inequity aversion. As the production when both agents perform effort is standardized to 1, this loss is expressed in terms of the total production exerted.

Thus, the loss function is

$$\begin{array}{ll} [1 - 2(0.4 - 0.1\alpha)] - [1 - 0.8] & \text{if } \alpha \geq 4\beta \\ [1 - 2(0.4)(1 - \beta)] - [1 - 0.8] & \text{if } \alpha < 4\beta. \end{array}$$

Figure 3 in section 5 draws this loss function for all the possible values of α and β.