On the Competition of Asymmetric Agents

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Abstract

Rank-order tournaments are usually implemented in organizations to provide incentives for eliciting employees' effort and/or to identify the agent with the higher ability, e.g. in promotion tournaments. We close a gap in the literature by experimentally analyzing a *ceteris paribus* variation of the prize spread – being the major design feature of tournaments – in a symmetric and an asymmetric setting. We find that effort significantly increases with the prize spread as predicted by standard theory. However, only if the prize spread is sufficiently large weak players competing against strong players strain themselves all the more and sorting of agents is feasible.

Keywords

Tournament design; Sorting; Work incentives; Heterogeneity; Experiments

JEL Classification Codes

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1. Introduction

Tournaments have been a popular incentive mechanism in organizations for years as they may elicit higher efforts of agents even in situations where effort is not contractible (MALCOLMSON 1984, 1986). Thus, many economists have studied the behavior of agents in tournaments via theoretical modeling as well as via empirical studies (LAZEAR 1999 and PRENDERGAST 1999). In addition to using tournaments as a pure effort eliciting system rank-order tournaments can serve as a sorting device to identify the favorite among the participating agents if the talents of heterogeneous agents are unknown ex ante, e.g. in a promotion tournament: The most talented employee, who is assumed to emanate from the tournament as the winner, is promoted to a position associated with a higher hierarchical rank.[1] Hence, an important function of tournaments is the sorting of employees to jobs in an organization, e.g. on internal labor markets.

However, if the competitors know about their different talents certain behavioral incentive effects are possible. For example, one may assume that an underdog strains himself all the more when competing against a more capable player while the favorite might slack off resulting in a failure of the sorting function of tournaments. As an illustrative example for possible effects think of the famous fable of *Aesop* on a race between a hare and a tortoise. The tortoise – surprisingly for everyone – wins the race as the hare which is completely convinced that he would be the winner falls asleep while waiting for the tortoise which is using all its strengths to come closer to the finish line. Already one of the behavioral patterns, i.e. the overexertion of effort of the underdog or the retentive effort exertion of the favorite, could suffice to cause the sorting of both types of agents to fail.

In previous experimental studies, e.g. BULL, SCHOTTER and WEIGELT (1987), SCHOTTER and WEIGELT (1992) and VAN DIJK, SONNEMANS and VAN WINDEN (2001), it can be observed that the weak agent competing against a more able player oversupplies effort in the sense that she exerts more effort than theoretically predicted. Despite this oversupply of effort of the less able subjects in these studies the different types of agents can usually still be identified as the stronger agent does not slack off and wins the tournament more often.[2] However, the systematic *ceteris paribus* variation of the prize spread, i.e. the difference between winner and loser prize, which is regarded as the major design feature of tournaments, has – surprisingly – not been investigated so far in this context. Either the prize difference is not varied *ceteris paribus* (e.g. BULL, SCHOTTER and WEIGELT 1987) or participants also exert some destructive sabotage activity, a principal endogenously selects a certain tournament design and/or participants are homogenously modeled (e.g. HARBRING and IRLENBUSCH 2003, 2005, *forthcoming*, FALK and FEHR *in progress*).We close this gap by experimentally analyzing the effect of the prize spread on behavior of subjects in symmetric and asymmetric tournaments.

Using the experimental method allows for a *ceteris paribus* variation of the prize spread in tournaments with heterogeneous agents and for controlling agents' different abilities. The effort exerted by participants is quantifiable as – like in most other theoretic and experimental studies – effort exertion is abstractly modeled. Exerting an effort is done by choosing a number which is associated with a certain amount of cost. In the symmetric tournaments with identical participants the same three different prize spreads are implemented as in the asymmetric setting. In our asymmetric tournaments a weak high-cost type of player who has the same cost function as players in the symmetric setting is competing against a strong type of agent who has to bear lower costs of effort. It is important to note that both types of players receive full information on the cost of effort regarding the other player as it seems intuitive that fellow workers are aware of their heterogeneity. The employer, however, is usually not aware of the different abilities of agents as the employees are closer to each other than to the employer. In

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[1] Note that the more able contestant wins the tournament with a higher probability but the less able participant may also win sometimes (see O'KEEFFE, VISCUSI and ZECKHAUSER 1984, KOH 1992).

[2] MÜLLER and SCHOTTER (2003) conduct an experiment on the influence of prizes in contests with heterogeneous agents. They find that efforts of laboratory subjects bifurcate: While the low ability workers either drop out or exert only little effort the high ability workers oversupply effort. Their modeling of a contest, however, differs substantially from the rank-order tournaments considered in this study.

our setting agents repeatedly interact with each other. We deliberately choose this design option as this is also the case in real organizations, e.g. performance reviews are conducted by the supervisor several times a year or management panels are repeated each year.

First, our design allows us to analyze whether the effort of participants is increasing with the prize spread in symmetric as well as asymmetric tournaments.[3] This question has also been approached by several empirical studies using data from the field (e.g. EHRENBERG and BOGNANNO 1990, KNOEBER and THURMAN 1994, ERIKSSON 1999) but a comprehensive analysis with a *ceteris paribus* variation of the prize spread is difficult with real world data. Moreover, real-world data often lacks to provide an appropriate heterogeneity measure. From a theoretic perspective heterogeneity among both contestants reduces effort exertion (see e.g. LAZEAR and ROSEN 1981, MCLAUGHLIN 1988). This theoretic finding is in line with the results of SUNDE (2003) who analyzes data from tennis tournaments for professionals from the Association of Tennis Professionals (ATP). LYNCH (2005) empirically examines the incentive effects of tournament reward structures in Arabian horse racing and finds that jockeys increase their effort in the second half of races when the amount of prize money lost by dropping a place in the relative rank is greater and there is less distance between them and their closest competitors. Thus, they can confirm that effort increases with the prize spread and with an increasing effect on the winning probability. Interestingly, organizers of horse racing are aware of the inefficiency of asymmetric tournaments and introduce different types of handicaps, e.g. they assign weight burdens to very fast horses, and use sorting mechanisms to make the race more competitive.

Secondly, we can compare the behavior of the weak player in a symmetric situation with his behavior competing against a superior player in an asymmetric setting. This comparison has not been analyzed *ceteris paribus* in previous studies so far. Thus, we may approach the

[3] Note that we focus on the behavior of agents in tournaments that are exogenously imposed on agents, i.e. we do not model a principal here. If prizes are endogenously derived very high efforts can be inefficient from a theoretic perspective (AKERLOF 1976) as agents must be compensated for their investments (LAZEAR and ROSEN 1981).

question of whether an agent is exerting effort all the more when being confronted with a superior competitor and whether this behavior depends on the prize spread. The behavior of weak agents is particularly interesting as it is a typical feature of many organizations that employees are confronted with high potentials entering the hierarchy as external lateral entries at a certain level. Should the weak player – being confronted with such a superior opponent – give up and exert only minimal effort the strong player might also exert lower efforts as this suffices to win the tournament, and incentives might break down. If such a situation is anticipated an employer setting up a competition is better off when grouping subjects who are similarly talented into one tournament. Finally, the effectiveness of the sorting function of tournaments is focused, i.e. we compare the behavior of weak and strong types of players in asymmetric tournaments with different prize spreads and analyze whether the different effort levels chosen by both types allow to identify the strong type.

The paper proceeds as follows: In section 2, the model with the theoretic prediction as well as the experimental design is described. In section 3, we derive our behavioral hypotheses based on previous empirical results. Section 4 summarizes the experimental results, followed by a discussion and conclusion of the results in section 5.

2. Model and Experimental Design

In this section, we describe the tournament model which is based on the seminal paper by LAZEAR and ROSEN (1981) as well as the experimental design first introduced by BULL, SCHOTTER and WEIGELT (1987) and derive the theoretic prediction for agents in symmetric and asymmetric two-person tournaments. After the game theoretic analysis of the model the experimental design is described.

For studies on tournaments with an endogenous principal who selects certain design alternatives see HARBRING and IRLENBUSCH (2003, 2005) and the work in progress by FALK and FEHR.

2.1 Model

Our experiment is based on a non-cooperative game with two players. Throughout our theoretic model we assume that agents are risk neutral and aim to maximize their monetary payoffs. The agents i (i = 1, 2) simultaneously choose an effort level e_i out of the interval [0, ..., 100] which represent the agents' pure strategies. The output y_i of agent i is determined by the following production function: $y_i = e_i + \varepsilon_i$ while ε_i is a random variable which is uniformly distributed over the interval $[-\bar{\varepsilon}, +\bar{\varepsilon}]$ and assumed to be i.i.d. for both agents. The random component can be thought of as a measurement error or a true randomness in the technology.

Agents compete for the winner prize M and receive a loser prize m with $M > m$, if they perform relatively worse than the opponent in the tournament, i.e. the agent with the lower output receives the loser prize m. If the two agents have the same output level such that a winner cannot unambiguously be determined, a fair random move ascertains the winner. We denote the prize spread $(M - m)$ by Δ.

If an agent exerts effort he has to bear costs which are given by the convex function $C_i(e_i)$ with $C_i(0) = 0$, $C_i'(e_i) > 0$ and $C_i''(e_i) > 0$. The expected payoff for agent i with his effort choice e_i and the effort choice of the other agent e_j is given by

$$Ez_i(e_i, e_j) = \pi_i(e_i, e_j)M + (1 - \pi_i(e_i, e_j))m - C_i(e_i) = m + \pi_i(e_i, e_j)\Delta - C_i(e_i)$$

with $\pi_i(e_i, e_j)$ denoting the probability for agent i to win the tournament and receive the winner prize. Every agent collects the loser prize for sure, additionally with his probability of winning he obtains the prize spread, but has to bear the cost of effort resulting from his own effort choice.

In our experiment, we consider two types of players who differ in their ability, which is modeled through different cost functions. There are strong players who have to bear costs given by $C_s(e) = e^2/c$ and weak players whose cost function is equal to $C_w(e) = \alpha e^2/c$ with $\alpha > 1$ and c > 0. Note that the strong players have lower costs than the weak players for each effort level. In symmetric tournaments, two weak players compete against each other whereas in the asymmetric case a weak and a strong player participate in the tournament.

Assuming existence of an interior equilibrium in pure strategies[4] the following first-order conditions for asymmetric tournaments must be fulfilled (with i denoting the strong and j the weak player): $\frac{\partial \pi_i(e_i, e_j)}{\partial e_i}\Delta = \frac{2e_i}{c}$ and $\frac{\partial \pi_j(e_i, e_j)}{\partial e_j}\Delta = \frac{2\alpha e_j}{c}$

In symmetric tournaments we have $\frac{\partial \pi_l(e_i, e_j)}{\partial e_l}\Delta = \frac{2\alpha e_l}{c}$ with $l = i, j$ since both players have the same cost function. Given the assumptions one can show that in a tournament with heterogeneous agents the marginal probabilities of winning are

$$\frac{\partial \pi_l(e_i, e_j)}{\partial e_l} = \frac{1}{2\bar{\varepsilon}} - \frac{e_i - e_j}{4\bar{\varepsilon}^2} \text{ with } l = i,j \text{ if } e_i > e_j$$

$$\frac{\partial \pi_l(e_i, e_j)}{\partial e_l} = \frac{1}{2\bar{\varepsilon}} - \frac{e_j - e_i}{4\bar{\varepsilon}^2} \text{ with } l = i,j \text{ if } e_i < e_j$$

while in a symmetric tournament they reduce to $\frac{\partial \pi_l(e_i, e_j)}{\partial e_l} = \frac{1}{2\bar{\varepsilon}}$ with $l = i,j$.[5]

In the asymmetric tournament, the marginal probabilities of winning depend on the size of the interval from which the random shock in the production function is drawn as well as the difference of the efforts chosen by the two agents. The absolute effort levels do not have an influence on the winning probability, but their difference. In the symmetric tournament, only the realization of the random components determines who is going to obtain the winner prize as effort choices in equilibrium are symmetric.

[4] If there are equilibria in pure strategies it is usually assumed that they guide behavior rather than mixed-strategy equilibria which imply complicated computation and often seem quite unintuitive. Therefore, we only consider equilibria in pure strategies here.

[5] Further details are provided in the mathematical appendix.

The marginal probabilities of winning lead to the following effort levels played in the asymmetric equilibrium: $e_j^* = \frac{\frac{c\,\Delta}{4\bar{\varepsilon}\alpha}}{1+\frac{(\alpha-1)}{4\bar{\varepsilon}^2}\frac{c\,\Delta}{2\alpha}}$ and $e_i^* = \alpha\, e_j$.

The advantaged low-cost type chooses an effort level that is α times higher than the effort level of the disadvantaged player. The effort levels in the symmetric equilibrium of the tournament with homogeneous players $(\alpha = 2)$ are equal for both players: $e_i^* = e_j^* = \frac{c\,\Delta}{4\alpha\bar{\varepsilon}}$.

Assuming that an interior solution exists agents must have no incentive not to exert any effort at all, i.e. cost of effort in equilibrium may not exceed the expected gain.[6]

2.2 Experimental Design

The computerized[7] experiment was conducted in the *Laboratorium für experimentelle Wirtschaftsforschung* at the University of Bonn. 108 students of different disciplines participated in the experiment, 18 in each treatment. Each subject participated only once. Sessions lasted about 90 minutes including instruction time. Subjects were paid according to their performance. During the experiment the payoffs were given in the fictitious currency "taler" which were changed into Euro by an exchange rate of 250 talers per Euro after the experiment. Payment was anonymous. The average payoff was 13.75 € across all treatments.

The experiment consisted of six treatments: Three symmetric tournaments among homogeneous players with prize spreads of 20 talers, 60 talers and 100 talers (*Hom20*, *Hom60* and *Hom100*) and three asymmetric tournaments among heterogeneous players with the same prize differences (*Het20*, *Het60* and *Het100*). The loser prize *m* was always 100 talers. During the experiment the subjects chose an integer effort level $e \in \{0,...,100\}$. The random component ε_i was uniformly distributed over the integer interval {-60, +60}. The cost function for weak

[6] The following condition must be fulfilled for each player: $\frac{1}{2}\Delta \geq c(e^*)$. Furthermore, in the experiment the range of effort participants may choose a number from is chosen such that e^* is feasible and not at the corner of the feasible effort choices.

players was given by $C_w(e) = e^2/100$ and for strong players by $C_s(e) = e^2/200$, so that for a given effort level the cost of the disadvantaged player was twice as high as for the advantaged player $(\alpha = 2, c = 200)$. The design of the experiment is summarized in Table 1 which also depicts the equilibrium efforts for each treatment.

It is important to note that we model a continuous strategy space which cannot be implemented in an experiment. Thus, participants could only choose integer numbers during the experiment while the equilibrium efforts are primarily real numbers. Although this procedure is often applied in experimental economics (see e.g. BULL, SCHOTTER and WEIGELT 1987) it is essential to check which of the discrete choices constitute equilibria. In our setting, the equilibrium in discrete numbers is given by the integer values next to the real numbers depicted in Table 1, either rounded up or down.[8]

Table 1: Design of the experiment

	treatments					
	Hom20	*Hom60*	*Hom100*	*Het20*	*Het60*	*Het100*
Design						
# rounds	30	30	30	30	30	30
# participants	18	18	18	18	18	18
thereof weak / strong player type	18 / 0	18 / 0	18 / 0	9 / 9	9 / 9	9 / 9
# independent observations	9	9	9	9	9	9
prize spread Δ	20	60	100	20	60	100
winner prize M	120	160	200	120	160	200
cost function of weak player	$e^2/100$	$e^2/100$	$e^2/100$	$e^2/100$	$e^2/100$	$e^2/100$
cost function of strong player	-	-	-	$e^2/200$	$e^2/200$	$e^2/200$
Nash equilibrium of stage game						
weak type of player	8.33	25.00	41.67	7.79	20.69	30.93
strong type of player	-	-	-	15.58	41.38	61.86
average of both types of players	8.33	25.00	41.67	11.69	31.03	46.39

[7] The experiment was programmed and conducted with the software z-Tree (FISCHBACHER *forthcoming*).

[8] Numerical computation which is available from the authors upon request shows that the equilibrium efforts are the following if participants may only choose integer numbers: 8 in *Hom20*, 41 in *Hom100*, 8 for weak and 16 for strong players in *Het20*, 21 for weak and 41 for strong players in *Het60*, 31 for weak and 62 for strong players in *Het100*.

Upon arrival each subject received instructions[9] as well as a cost table which showed the cost for each possible number (effort). In the treatment with heterogeneous agents participants received two cost tables, one for each type of agent. After the instruction had been read, three examples were explained to illustrate the procedure in each round.[10] Hereafter, subjects were randomly allocated to computer terminals so that they could not influence who the other subject in the group was. Additionally, in sessions with heterogeneous players the player types were randomly assigned to the participants when they entered their cubicles. The participants knew that they were paired with a player of the other type. As they were supplied with both cost tables they were informed about the cost of effort associated with each decision. During the whole session the language was kept neutral, avoiding words like "tournament", "prize" and "effort". Instead, in each round the subjects chose a "number" and received a "payment" which was either high or low. The group composition remained unchanged and was kept anonymous during the experiment. Subjects were not allowed to communicate with each other. One session consisted of 30 rounds with identical tournaments.[11]

In every round, subjects entered their chosen number (effort) on the computer screen. The computer program determined the winner and loser in each group based on the chosen efforts and the individual random numbers. After each round, the following feedback was given to each subject: own effort choice, achieved payment (winner or loser prize), cost of chosen number and payoff in this particular round. In addition, subjects were able to see their accumulated profits during the whole session. No information was given on the behavior of the other subject in the group as well as the two individual random numbers.

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[9] The instructions are available from the authors upon request.

[10] Before the introduction started we asked participants to choose random numbers from 0 to 100, which were used for the examples. This procedure was used to keep possible suggestive influences as small as possible.

[11] We implemented a restart procedure after 15 rounds to analyze behavioral changes due to restarting the experiment which are known from public good experiments (e.g. ISAAC and WALKER 1988, COOKSON 2000). As we find no systematic behavioral changes due to restarting the sessions in this experiment we refrain from reporting further details.

3. Hypotheses

Like in most other experimental studies on tournaments (e.g. BULL, SCHOTTER and WEIGELT 1987, ORRISON, SCHOTTER and WEIGELT 2004 etc.) we theoretically analyze a static stage game while we repeat the game with the same participants in the experiment. We opt for a repetition of the game in a partner setting as this is the realistic procedure in organizations. Participants in real-world tournaments have the opportunity to react towards their competitors' behavior either because the tournament is repeated with the same participants or because they may gather some information while competing. The analysis of the stage game does not reflect any learning processes or strategic considerations, e.g. participants might base their belief towards the other participant's decision on the outcome in the previous round. Based on the *learning direction theory* (SELTEN 1998) participants might not increase (reduce) their effort in the subsequent round after having received the winner (loser) prize (see HARBRING and IRLENBUSCH 2004 for an empirical analysis of such a learning process in tournaments).[12] Or, participants might try to build up a reputation as a cooperator and choose low efforts in the beginning (see e.g. KREPS et al. 1982 for a theoretic analysis of cooperation in a finitely repeated prisoner's dilemma). A theoretic elaboration on such dynamic aspects of behavior in tournaments should definitely be the focus of future research on tournament models.

With the study at hand, however, we close a gap in the empirical literature while we are aware of the fact that the theoretic model does not capture the dynamic aspects of our experimental setting. In contrast to previous studies we are, therefore, careful in comparing actual behavior to the theoretic prediction and use it as a first benchmark. We are convinced, however, that the explanation of the theoretic model in subsection 2.1 is essential for understanding the strategic setting we are analyzing here and enables us to integrate our study in the literature where the same theoretic framework is used. Thus, our behavioral hypotheses which are described in the following are mainly based on results of previous empirical studies using field and laboratory

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[12] For an overview on learning models see CAMERER (2003).

data and are related to comparisons between treatments and comparisons between different types of players within a treatment. We refer to our theoretic benchmark as additional evidence.

Many empirical studies using data from the field – mostly from sports (e.g. EHRENBERG and BOGNANNO 1990) – support the theoretic prediction of standard tournament theory on the basis of LAZEAR and ROSEN (1981) that effort increases with the prize spread. Experimental studies indicate that this result can be confirmed, but so far the prize spread has not been varied *ceteris paribus* in a setting as it is modeled here. Based on the empirical observations made so far we hypothesize that effort in symmetric as well as asymmetric tournaments increases with the prize spread which is summarized by hypothesis 1:

Hypothesis 1 – Incentive effect of prize spread:

a. Effort increases with the prize spread in symmetric tournaments.

b. Effort increases with the prize spread in asymmetric tournaments.

Previous experimental studies (BULL, SCHOTTER and WEIGELT 1987, SCHOTTER and WEIGELT 1992) compare the agents' behavior with the equilibrium prediction and find that subjects being disadvantaged exert a much higher effort than theoretically predicted while the observed behavior in symmetric tournaments is roughly in line with theory.[13] BULL, SCHOTTER and WEIGELT (1987) find that an agent competing against a strong player seems to exert a higher effort than in a symmetric tournament. However, the weak agent they model does not have exactly the same cost function in the symmetric and the asymmetric setting and also other parameters are varied across treatments. VAN DIJK, SONNEMANS and VAN WINDEN (2001) find in a real effort experiment that subjects with a lower ability continuously try to win a tournament against a stronger competitor although they are losing in most rounds and although they could also earn money under a piece-rate scheme. Our design enables us to compare an

[13] KNOEBER and THURMAN (1994) analyze data from the field on the performance of broiler producers. They use the performance measure as a proxy for grower quality and find that less able producers adopt riskier strategies. They do not investigate the impact of different abilities on performance.

agent of the same ability in a symmetric and an asymmetric tournament *ceteris paribus*. Based on earlier empirical findings we propose – as a behavioral hypothesis – that weak players in the asymmetric tournament exert higher efforts than in the symmetric tournament:

Hypothesis 2 – Behavior of weak player:

The effort of the weak player in the asymmetric tournament is higher than the average effort of players in the symmetric tournament.

In previous experimental studies sorting was still possible although the disadvantaged players oversupplied effort (BULL, SCHOTTER and WEIGELT 1987) as the advantaged players did not slack off and also slightly oversupplied effort compared to the equilibrium prediction. In VAN DIJK, SONNEMANS and VAN WINDEN (2001) both the weak and the strong agent increase their efforts in a repeated tournament setting compared to a previous piece-rate scheme. Based on these results we hypothesize the following:

Hypothesis 3 – Sorting function:

The effort of strong players is higher than the effort of weak players in asymmetric tournaments.

4. Experimental Results

This section describes the statistical analysis[14] of our experimental findings regarding our hypotheses. Additionally, we analyze the change of behavior over rounds.

4.1 Incentive Effect of Prize Spread

In Table 2 the average effort exerted is depicted. By applying the Jonckheere-Terpstra test we can confirm our first hypothesis:

[14] Regarding the statistical results supplied in this section please note the following: If not explicitly mentioned otherwise, e.g. in section 4.4, we always aggregate the values over all rounds. For the comparison of different treatments either the average of each group or each type of player in the group is treated as a statistically independent variable. If behavior within one treatment is compared, e.g. the comparison of the strong and weak player, the two players' average values over all rounds in one group are treated as dependent pairs.

Observation on Hypothesis 1:

a. Effort significantly increases with the prize spread in symmetric tournaments.

b. Effort significantly increases with the prize spread in asymmetric tournaments.

The null hypothesis that there is no ordering of the medians of the three treatments can be rejected in favor of the alternative hypothesis that there is an ordering of the medians of the treatments according to the amount of the prize spread (Jonckheere-Terpstra test, p=0.000 for symmetric as well asymmetric tournaments, two-tailed). Effort increases with the prize spread in both settings. Note that also the average efforts of strong and weak players in the asymmetric tournament – when analyzed separately – are significantly increasing with the prize spread.[15]

Table 2: Results regarding variation of prize spread
(Standard deviation over rounds is given in brackets.)

	symmetric tournaments			
	Hom20	*Hom60*	*Hom100*	Jonckheere-Terpstra test[16]
Average effort	16.31	26.87	38.66	p = 0.000
	(10.66)	(13.03)	(19.01)	p = 0.001
Effort in equilibrium of stage game	8.33	25.00	41.67	

	asymmetric tournaments			
	Het20	*Het60*	*Het100*	Jonckheere-Terpstra test
Average effort	16.77	38.03	58.55+	p = 0.000
	(15.80)	(14.65)	(18.42)	p = 0.255
Average effort in equilibrium of stage game	11.69	31.03	46.39	

By using the Binomial test[17] (one-tailed) we state the level of significance at which the null hypothesis can be rejected in favor of the alternative hypothesis that the average values are more often above (below) the equilibrium level than below (above):

\+ *significantly above the equilibrium level:* $0.05 \leq p \leq 0.1$

[15] Given that participants repeatedly interact one may assume – based on previous experiments – that some groups manage to collude and exert only minimal efforts. A tendency to collude can best be observed in *Hom20* and also in *Het20*. In both treatments the incentive to deviate from minimal efforts is smallest. After some difficulties in the beginning some participants make it to exert minimal or very low efforts in most of the rounds and sometimes even stick to it in the last two rounds. In the other treatments, however, the tendency to collude is rather rare. Quite stable collusion evolves in tournaments of two participants if they may observe each other after each round and if no random noise distorts effort (see HARBRING 2006). Figures on individual behavior are available from the authors upon request.

[16] The Jonckheere-Terpstra test is a non-parametric test for ordered differences among classes. The alternative hypothesis assumes a certain ordering of the medians of k statistically independent samples. All average efforts – each of a statistically independent observation from a treatment with the same prize spread – are assigned to one class. The given values are results of a two-tailed test. For further details on non-parametric tests see SIEGEL and CASTELLAN (1988).

[17] The Binomial test is a non-parametric test which compares the observed frequencies of the two categories of a dichotomous variable to the frequencies expected under a binomial distribution with a specified probability of 0.5 in our procedures. In our setting, we check whether the average values over all rounds for each group or each type in the group lie above or below the equilibrium level.

In earlier experiments a high variability of behavior in tournaments – particularly in contrast to other incentive systems (see e.g. BULL, SCHOTTER and WEIGELT 1987, NALBANTIAN and SCHOTTER 1997) – has been observed. The variability of behavior in our setting resembles the one in previous studies. Moreover, we find that the average standard deviation of behavior over rounds increases with the prize spread in symmetric tournaments (Table 2). Finally, we supply some additional evidence on the comparison of actual average effort and the theoretic prediction. By applying the Binomial test we find that effort is significantly higher than predicted only in *Het100* (indicated by "+" in the table).

4.2 Behavior of Weak Player

In Table 2 only effort choices aggregated over different types of players are presented. We now compare the weak player's behavior competing with a player of the same type with the weak player's choices who is challenged by a strong player in the asymmetric setting.

Table 3: Comparison of weak players' behavior
(Standard deviation over rounds is given in brackets.)

	Hom20	*Het20* (weak player)	*Hom60*	*Het60* (weak player)	*Hom100*	*Het100* (weak player)
Average effort	16.31 (10.66)	17.46 (16.92)	26.87 (13.03)	36.02 (16.56)	38.66 (19.01)	45.46+ (22.56)
Mann-Whitney-U-test (one-tailed)	not significant		not significant		$p = 0.05$	
Effort in equilibrium of stage game	8.33	7.79	25.00	20.69	41.67	30.93
Loss of payoff due to deviation from equilibrium (in % of equilibr. payoff)[18]	7.31	11.36	0.18	10.63	0.37	9.94

By using the Binomial test (one-tailed) we state the level of significance at which the null hypothesis can be rejected in favor of the alternative hypothesis that the average values are more often above (below) the equilibrium level than below (above):
\+ *significantly above the equilibrium level:* $0.05 \leq p \leq 0.1$

[18] We compute the payoff if one player chooses the actual average effort and the other player the equilibrium effort. Table 3 supplies the difference between this payoff and the payoff in equilibrium as a percentage of equilibrium payoff. The loser prize is not included in the payoffs as it is constant across participants and treatments.

In Figure 1 the average effort choices of both types of players are depicted separately. Furthermore, Table 3 supplies results on the behavior of weak players in symmetric as well as asymmetric tournaments including the test statistics regarding the weak player's behavior.

The null hypothesis that the average effort in symmetric tournaments is equal to the weak player's average effort in asymmetric tournaments can be rejected in favor of the alternative behavioral hypothesis that the effort in asymmetric tournaments is higher in *Het100* (Mann-Whitney-U-test[19], p=0.05, one-tailed). In all other treatments, the effort level of the weak player does not significantly vary between the symmetric and the asymmetric setting. Therefore, we can only confirm our hypothesis 2 for the treatment with the large prize spread:

Observation on Hypothesis 2:

The effort of the weak player in the asymmetric tournament (*Het100*) is significantly higher than the average effort of weak players in the symmetric tournament (*Hom100*) only if the prize spread is large.

Additional results are provided by a comparison with theory. According to our theoretic prediction the effort of the weak player is slightly lower in the asymmetric tournament compared to his effort choice in the symmetric setting which is – obviously – in contrast to actual behavior if the prize spread is large. Moreover, note that only the weak player's effort in *Het100* is significantly higher than theoretically predicted although particularly the actual efforts in all asymmetric treatments seem considerably higher than in equilibrium.[20] To supply an impression of the real impact of the participants' deviation from equilibrium we depict the loss of payoff due to the deviation. Note that in the asymmetric treatments an agent deviating from the equilibrium as depicted receives an approx. 10% lower payoff in each round given

[19] The Mann-Whitney-U-test checks whether two independent samples come from the same population. It is the most popular of the two-independent-samples non-parametric tests.

[20] Although the average effort in the asymmetric treatments is always well above the predicted effort level, this is not significant in *Het20* and *Het60*. This is due to the fact that only some groups considerably oversupply effort. However, the Binomial test to be significant requires that most of the group averages lie above the prediction.

that the other player chooses the equilibrium strategy compared to the payoff she would receive if she chose the equilibrium effort.

4.3 Sorting Function

Finally, we analyze whether the tournament device can serve to identify the good type of player, i.e. the strong low-cost type. Thus, in this subsection we compare the behavior of weak and strong players in the asymmetric tournaments. In Figures 2-4 the average effort of each type of player over rounds for all treatments is depicted, always for one prize spread. One can easily see that only in the treatment with the large prize spread of 100 effort levels of weak and strong players obviously drift apart. Our results are stated in Table 4 which also shows the absolute average effort levels of strong and weak players for each prize spread.

Only in *Het100* the null hypothesis that the average effort of strong players is equal to the average effort of weak players can be rejected in favor of the alternative hypothesis that the strong player's effort is higher (Wilcoxon-Signed-Rank test, p=0.005, two-tailed). In the other treatments with lower prize spreads no significant difference can be found with regard to effort levels. Thus, we can confirm hypothesis 3 only for *Het100*, i.e. if the prize spread is large:

> **Observation on Hypothesis 3:**
>
> The effort of strong players is higher than the effort of weak players only in the asymmetric tournament with the large prize spread (*Het100*).

Effort levels are not observable by the employer and the different effort levels exerted by agents result in the winning probabilities of each type of player: The tournament is significantly more often won by the strong player when the prize spread is large (see also Table 4). This result is also supported by the finding that the strong player wins more often than the weak player in each statistically independent group. Interestingly, in our setting theory predicts that the sorting of asymmetric agents is facilitated if the prize spread is increased, i.e. in equilibrium the difference of effort levels and thus, the winning probability of the strong agent, increases with the prize spread which helps to differentiate between both agents.[21]

Table 4: Strong and weak players in asymmetric treatments
(Standard deviation over rounds is given in brackets.)

	Het20		*Het60*		*Het100*	
	weak player	strong player	weak player	strong player	weak player	strong player
Average effort	17.46 (16.92)	16.08 (14.67)	36.02 (16.56)	40.04 (12.73)	45.46$^{+/*}$ (22.56)	71.63 (14.28)
Wilcoxon-Signed-Rank test (two-tailed)[22]	not significant		not significant		p = 0.005	
Effort in equilibrium of stage game	7.79	15.58	20.69	41.38	30.93	61.86
Best reply to other player's actually given effort	7.76	16.76	21.04	45.89	25.73	67.81
Average winning probability	0.51^{+}	0.49 $^{\theta}$	0.45	0.55	0.33^{+}	0.67 $^{\theta}$
Wilcoxon-Signed-Rank test (two-tailed)	not significant		not significant		p = 0.001	
Winning probability in equilibrium of stage game[23]	0.44	0.56	0.34	0.66	0.28	0.72

By using the Binomial test (one-tailed) we state the level of significance at which the null hypothesis can be rejected in favor of the alternative hypothesis that the average values are more often above (below) the equilibrium level than below (above):

θ *significantly below the equilibrium level:* $0.05 \leq p \leq 0.1$

+ *significantly above the equilibrium level:* $0.05 \leq p \leq 0.1$

By using the Binomial test (one-tailed) we state the level of significance at which the null hypothesis can be rejected in favor of the alternative hypothesis that the average values are more often above the best reply to the other player's given effort than below:

* *significantly above the best reply to the other player's given effort* $0.05 \leq p \leq 0.1$

Table 4 also supplies the best replies of agents to the other agent's actually given effort as additional information. This gives an impression on how far agents are away from playing the best response given the other player's decisions. Though, participants are only informed about their payoff after each round and thus, cannot directly react to the other agent's choice. We find no systematic behavioral over- or undersupply of effort compared to these best replies.

[21] The detailed analysis is given in the mathematical appendix.
[22] The Wilcoxon-Signed-Rank test is a non-parametric test comparing the distributions of two related variables.
[23] In equilibrium the realization of the random move can still result in either of both types of players to be the winner. For the winning probability see the appendix.

4.4 Change of Behavior over Rounds

In order to examine whether behavior changes over rounds we analyze the first and last five rounds separately and shortly sketch the results in this section. In the first five rounds hypothesis 1 can be confirmed for heterogeneous agents only (Jonckheere-Terpstra test, p=0.01, one-tailed). Moreover, hypothesis 2 can be confirmed for tournaments with a large prize spread as the average effort of the weak player is weakly significantly higher in the asymmetric than in symmetric treatment (Mann-Whitney-U-test, p=0.1, one-tailed). Finally, we find that the strong agent's average effort is significantly higher than the weak agent's only in *Het100* validating hypothesis 3 for tournaments with a large prize spread (Wilcoxon-Signed-Rank text, p=0.05, two-tailed). In the last five rounds average effort increases with the wage spread in symmetric as well asymmetric tournaments (Jonckheere-Terpstra test, both: p=0.000, one-tailed). Hypothesis 2 cannot be confirmed for the last five rounds.[24] And again, the strong agent exerts significantly more effort than the weak agent only in *Het100* (Wilcoxon-Signed-Rank test, p=0.05, two-tailed) partly confirming hypothesis 3.

5. Conclusion

We experimentally investigate the influence of different prize spreads in tournaments with homogenous and heterogeneous players. In the asymmetric treatments, a strong type of player competes against a weak type of player who is confronted with cost of effort twice as high. In the symmetric tournaments, two weak players compete for obtaining the winner prize. The same three prize spreads are implemented in the symmetric and the asymmetric setting.

First, we can confirm one of the main findings of standard tournament theory, i.e. effort increases with the prize spread. This holds for symmetric as well as asymmetric tournaments after participants have gained some experience with the situation. Our second result is novel and complements existing research: We *ceteris paribus* compare players in a symmetric and an

[24] The weak agents exert higher efforts in asymmetric than in symmetric tournaments with a large prize spread particularly in the first 10 rounds (Mann-Whitney-U-test, p=0.05, one-tailed).

asymmetric tournament competing against a superior participant which has not yet been analyzed in other experimental studies. Moreover, the influence of the prize spread has not been investigated in this context so far. We find that particularly inexperienced agents exert higher efforts in asymmetric tournaments compared to the symmetric setting, but only if the prize spread is large. Other experimental studies (BULL, SCHOTTER and WEIGELT 1987, SCHOTTER and WEIGELT 1992 and VAN DIJK, SONNEMANS and VAN WINDEN 2001) indicate that weak agents oversupply effort when competing with a superior competitor compared to the theoretical benchmark. These studies, however, only analyze very large prize spreads that amount to at least 100% of the loser prize. Thus, those settings can only be compared with our treatment *Het100* implementing the large prize spread. Our result adds to the existing literature by indicating that obviously the weak player's exertion of very high efforts is also dependent on the size of the prize spread. This is quite interesting for practitioners as winner prizes implemented as some kind of variable pay component only seldomly amount to 100% of fixed pay. One possible interpretation for this second finding is provided by KRÄKEL (2004) who integrates emotions into the standard tournament model.[25] The assumptions underlying his model are quite intuitive: Weak agents feel pride if they make it to achieve the winner prize while competing against a superior player. This pride leads to additional incentives by enlarging the subjectively perceived winner prize. He concludes that pride may induce higher efforts than standard theory predicts. Combining our finding with the explanation of KRÄKEL we may conclude that the extent of pride seems to vary with the prize spread.[26]

Also, our final result complements previous studies and seems vital for practitioners: We find that the sorting of both types of agents is only feasible if the prize spread is large, i.e. the effort of the strong player (and the resulting winning probability) is higher than that of the weak player in asymmetric tournaments. This is qualitatively in line with tournament theory and

[25] Other models deviating from standard tournament theory are provided by GRUND and SLIWKA (2005) integrating inequity aversion and KRÄKEL (2000) integrating relative deprivation into a tournament model.

could be due to the considerable variability of behavior invoked by such tournament structures that has already been stated in other experiments (see e.g. NALBANTIAN and SCHOTTER 1997). This result is of considerable practical relevance as it emphasizes that the prize spread needs to be sufficiently large to achieve significant differentiation of different types of agents. Tournaments set up as a sorting device in organizations should be implemented with high prize spreads only, e.g. as job promotion tournaments offering an attractive job on a higher rank. However, as pointed out in HARBRING and IRLENBUSCH (2004, *forthcoming*) increasing the prize spread in tournaments might result in higher destructive activities, i.e. sabotage, among the participating agents and might not be bolstered by an equivalent increase of productive efforts.

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[26] KRÄKEL (2004) also assumes that the strong player feels anger if she does not receive the winner prize which leads to a reduction of the subjectively perceived loser prize and, thus, an increased prize spread. However, our results do not point to a significant oversupply of the strong player's effort compared to theory.

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Figures

Figure 1: Average effort for each type of player

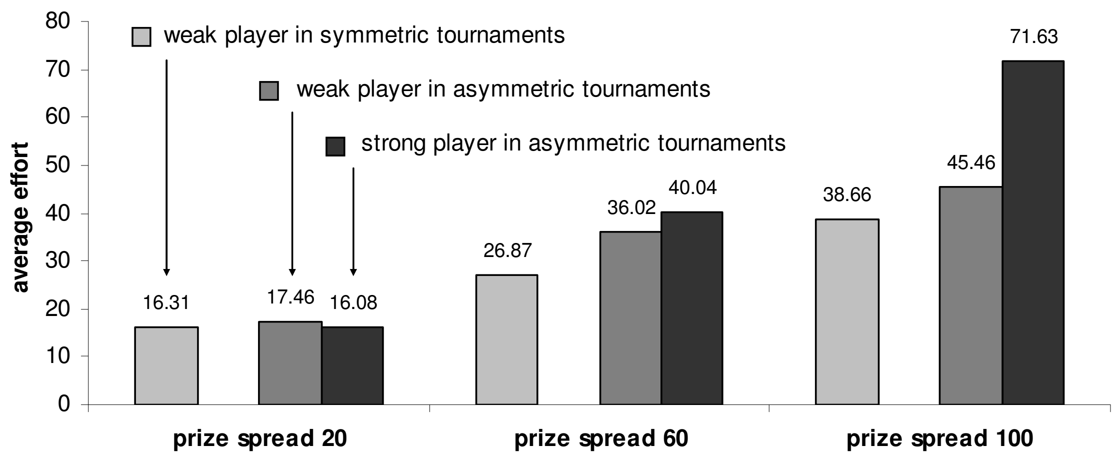


Figure 2: Average effort for each type of player over all rounds in *Het20*
(The restart of sessions is presented by the gap between round 15 and 16.)

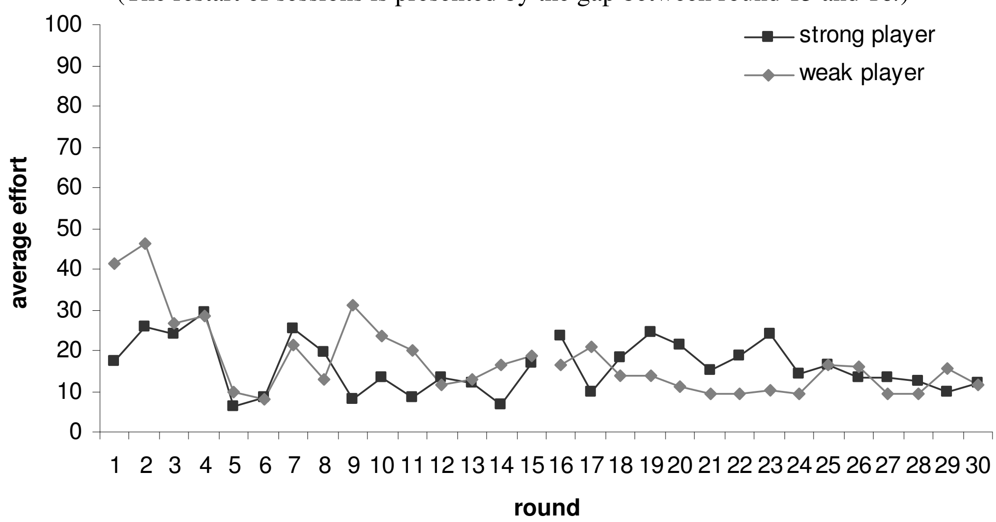


Figure 3: Average effort for each type of player over all rounds in *Het60*

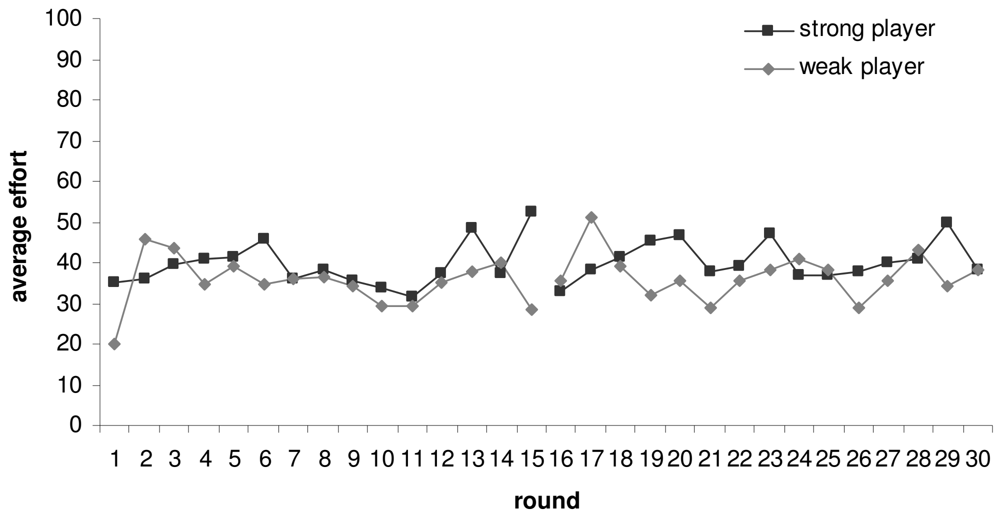
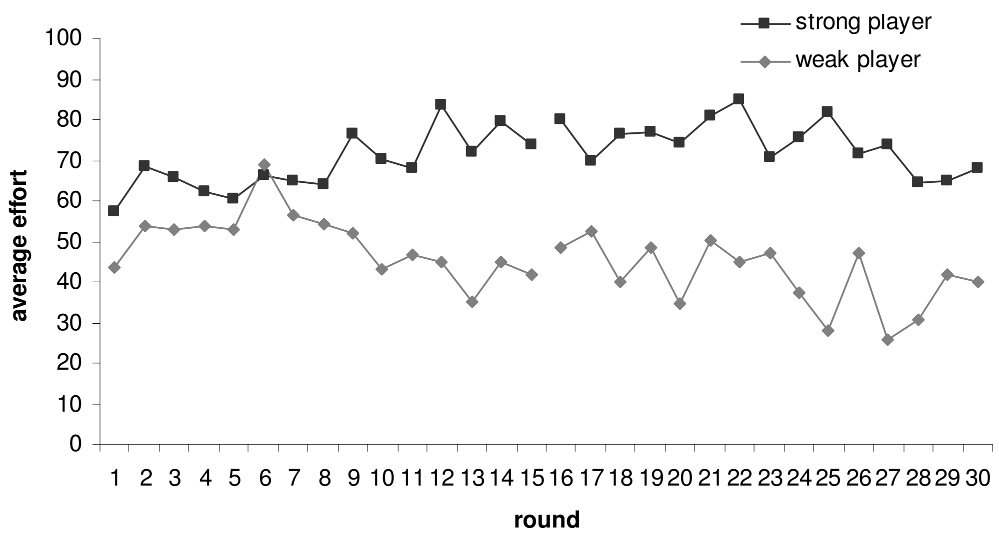


Figure 4: Average effort for each type of player over all rounds in *Het100*



Mathematical Appendix

Marginal probabilities of winning

A tournament with two agents (*i, j*) is won by agent *i* if $y_i > y_j \Leftrightarrow e_i + \varepsilon_i > e_j + \varepsilon_j$.

Now we assume (without loss of generality) that agent *i* is the one who provides a higher effort level: $e_i > e_j$. Given agent *i*'s random realization ε_i agent *i* wins if $e_i + \varepsilon_i > e_j + \varepsilon_j \Leftrightarrow \varepsilon_j < e_i - e_j + \varepsilon_i$. That the realization of ε_j is so small and the inequality is fulfilled is given by the following conditional probability of winning:

$$P(\text{agent } i \text{ wins} \mid \varepsilon_\text{i}) = \int_{-\bar{\varepsilon}}^{e_i - e_j + \varepsilon_i} \frac{1}{2\bar{\varepsilon}} d\varepsilon_j = \frac{\bar{\varepsilon} + e_i - e_j + \varepsilon_i}{2\bar{\varepsilon}} = \frac{A_j + \varepsilon_i}{2\bar{\varepsilon}} = \frac{1}{2} + \frac{e_i - e_j}{2\bar{\varepsilon}} + \frac{\varepsilon_i}{2\bar{\varepsilon}} \quad \text{with}$$

$A_j = \bar{\varepsilon} + e_i - e_j$. It is equal to $\frac{1}{2}$ plus a term which is proportional to the difference in the effort levels of both agents, plus another term which is proportional to ε_i. The factor of proportionality is $\frac{1}{2\bar{\varepsilon}}$.[27] To find the unconditional probability of winning one has to integrate over all possible realizations of ε_i. The realizations lie in the interval from $-\bar{\varepsilon}$ till $+\bar{\varepsilon}$. Agent *i* wins the tournament for sure if $\varepsilon_i > e_j - e_i + \bar{\varepsilon} = B_i$. This subinterval $\lfloor e_j - e_i + \bar{\varepsilon}, \bar{\varepsilon} \rfloor$ can be denoted as "Sure Win Interval": $P(\text{agent } i \text{ wins in this subinterval}) = \int_{e_j - e_i + \bar{\varepsilon}}^{\bar{\varepsilon}} \frac{1}{2\bar{\varepsilon}} d\varepsilon_i$

Only with certain realizations of the random component agent *i* can win in the remaining subinterval $\lfloor -\bar{\varepsilon}, e_j - e_i + \bar{\varepsilon} \rfloor$:

$$P(\text{agent } i \text{ wins in this subinterval}) = \int_{-\bar{\varepsilon}}^{e_j - e_i + \bar{\varepsilon}} \left(\frac{A_j + \varepsilon_i}{2\bar{\varepsilon}} \right) \frac{1}{2\bar{\varepsilon}} d\varepsilon_i = \int_{-\bar{\varepsilon}}^{e_j - e_i + \bar{\varepsilon}} \left(\frac{\bar{\varepsilon} + e_i - e_j + \varepsilon_i}{4\bar{\varepsilon}^2} \right) d\varepsilon_i$$

The unconditional winning probability of agent *i* results out of those two parts:

$$\pi_i(e_i, e_j) = \int_{e_j - e_i + \bar{\varepsilon}}^{\bar{\varepsilon}} \frac{1}{2\bar{\varepsilon}} d\varepsilon_i + \int_{-\bar{\varepsilon}}^{e_j - e_i + \bar{\varepsilon}} \frac{\bar{\varepsilon} + e_i - e_j + \varepsilon_i}{4\bar{\varepsilon}^2} d\varepsilon_i = \left[\frac{\varepsilon_i}{2\bar{\varepsilon}} \right]_{e_j - e_i + \bar{\varepsilon}}^{\bar{\varepsilon}}$$

$$+ \left[\frac{\bar{\varepsilon}\varepsilon_i + e_i\varepsilon_i - e_j\varepsilon_i + \frac{1}{2}\varepsilon_i^2}{4\bar{\varepsilon}^2} \right]_{-\bar{\varepsilon}}^{e_j - e_i + \bar{\varepsilon}} = \frac{1}{2} + \frac{e_i - e_j}{2\bar{\varepsilon}} - \frac{(e_i - e_j)^2}{8\bar{\varepsilon}^2}$$

Consequently, the marginal winning probability of agent *i* is: $\frac{\partial \pi_i(e_i, e_j)}{\partial e_i} = \frac{1}{2\bar{\varepsilon}} - \frac{e_i - e_j}{4\bar{\varepsilon}^2}$

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[27] For the derivation of the density function see ORRISON, SCHOTTER and WEIGELT (2004) as well as HARBRING and IRLENBUSCH (2005).

Analogously the following unconditional winning probability of agent j can be derived:

$$\pi_j(e_i,e_j)=\int_{-(e_j-e_i+\bar{\varepsilon})}^{\bar{\varepsilon}}\left(\frac{\bar{\varepsilon}+e_j-e_i+\varepsilon_j}{2\bar{\varepsilon}}\right)\frac{1}{2\bar{\varepsilon}}d\varepsilon_j$$

$$=1-\left(\frac{1}{2}+\frac{e_i-e_j}{2\bar{\varepsilon}}-\frac{(e_i-e_j)^2}{8\bar{\varepsilon}^2}\right)=\frac{1}{2}-\frac{e_i-e_j}{2\bar{\varepsilon}}+\frac{(e_i-e_j)^2}{8\bar{\varepsilon}^2}$$

The marginal winning probability of agent j is:

$$\frac{\partial\pi_j(e_i,e_j)}{\partial e_j}=\frac{1}{2\bar{\varepsilon}}+\frac{2(e_i-e_j)(-1)}{8\bar{\varepsilon}^2}=\frac{1}{2\bar{\varepsilon}}-\frac{e_i-e_j}{4\bar{\varepsilon}^2}$$

Note that both agents have the same marginal probability of winning.

Equilibrium effort in asymmetric tournaments:

The first-order conditions of the expected payoff of the two agents who differ in their abilities are: $\frac{\partial Ez_i}{\partial e_i}=\frac{\partial\pi_i(e_i^*,e_j^*)}{\partial e_i}\Delta-\frac{2e_i^*}{c}=0$ *and* $\frac{\partial Ez_j}{\partial e_j}=\frac{\partial\pi_j(e_i^*,e_j^*)}{\partial e_j}\Delta-\frac{2\alpha\, e_j^*}{c}=0$

Since the marginal probabilities of winning are equal for both agents, one can deduce from $\frac{\partial Ez_i}{\partial e_i}=\frac{\partial Ez_j}{\partial e_j}$ the following relation: $e_i^*=\alpha\, e_j^*$. If $\alpha>1$, agent i has a higher equilibrium effort, so that inserting the marginal probabilities of winning into the first-order conditions leads to: $\frac{\partial Ez_i}{\partial e_i}=\left(\frac{1}{2\bar{\varepsilon}}-\frac{e_i^*-e_j^*}{4\bar{\varepsilon}^2}\right)\Delta-\frac{2e_i^*}{c}=0$ and $\frac{\partial Ez_j}{\partial e_j}=\left(\frac{1}{2\bar{\varepsilon}}-\frac{e_i^*-e_j^*}{4\bar{\varepsilon}^2}\right)\Delta-\frac{2\alpha\, e_j^*}{c}=0$

With $e_i^*=\alpha\, e_j^*$ the following equilibrium effort for agent j can be derived:

$$\frac{\partial Ez_j}{\partial e_j}=\left(\frac{1}{2\bar{\varepsilon}}-\frac{\alpha\, e_j^*-e_j^*}{4\bar{\varepsilon}^2}\right)\Delta-\frac{2\alpha\, e_j^*}{c}=0\Leftrightarrow\frac{\Delta}{2\bar{\varepsilon}}=e_j^*\left(\frac{2\alpha}{c}+\frac{(\alpha-1)\Delta}{4\bar{\varepsilon}^2}\right)\Leftrightarrow e_j^*=\frac{\frac{\Delta}{2\bar{\varepsilon}}}{\frac{2\alpha}{c}+\frac{(\alpha-1)\Delta}{4\bar{\varepsilon}^2}}$$

The fraction is expanded with $\left(\frac{c}{2\alpha}\right)/\left(\frac{c}{2\alpha}\right)$ and leads to $e_j^*=\dfrac{\frac{c\Delta}{4\bar{\varepsilon}\alpha}}{1+\frac{(\alpha-1)}{4\bar{\varepsilon}^2}\frac{c\Delta}{2\alpha}}$

According to this result the equilibrium effort of agent i is: $e_i^*=\dfrac{\frac{c\Delta}{4\bar{\varepsilon}}}{1+\frac{(\alpha-1)}{4\bar{\varepsilon}^2}\frac{c\Delta}{2\alpha}}$

Equilibrium effort in symmetric tournaments:

In symmetric tournaments, both agents have the same cost function which means that they do not differ in their abilities $(\alpha = 2)$. The expected payoff of both agents is given by $Ez_l(e_i, e_j) = m + \pi_l(e_i, e_j)\Delta - \frac{\alpha e_l^2}{c}$ with $l = i, j$. The necessary condition to maximize the expected payoff for both agents demands that the first-order condition in equilibrium is equal to zero: $\frac{\partial Ez_l}{\partial e_l} = \frac{\partial \pi_l(e_i^*, e_j^*)}{\partial e_l}\Delta - \frac{2\alpha e_l^*}{c} = 0$. Due to symmetry we know that $e_i^* = e_j^*$. The marginal probability of winning reduces therefore to: $\frac{\partial \pi_l(e_i, e_j)}{\partial e_l} = \frac{1}{2\bar{\varepsilon}}$. This leads to the following equilibrium efforts: $\frac{1}{2\bar{\varepsilon}}\Delta = \frac{2\alpha e_l^*}{c} \Leftrightarrow e_l^* = \frac{c\Delta}{4\alpha\bar{\varepsilon}}$ with $l = i, j$.

Quality of sorting

The unconditional winning probability depends on the effort difference of the two agents (*i, j*): $\pi_i(e_i, e_j) = \frac{1}{2} + \frac{e_i - e_j}{2\bar{\varepsilon}} - \frac{(e_i - e_j)^2}{8\bar{\varepsilon}^2}$. Assuming that $e_i > e_j$ and $\bar{\varepsilon} > \frac{1}{2}(e_i - e_j)$ the winning probability of agent *i* increases with the effort difference: $\frac{\partial \pi_i(e_i, e_j)}{\partial (e_i - e_j)} = \frac{1}{2\bar{\varepsilon}} - \frac{e_i - e_j}{4\bar{\varepsilon}^2} > 0$. Moreover, as $\frac{\partial^2 \pi_i(e_i, e_j)}{\partial^2 (e_i - e_j)} = -\frac{1}{4\bar{\varepsilon}^2} < 0$ the winning probability is a concave function of the effort difference.