An Experimental Test of Rubinstein's Bargaining Model:

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Abstract: This paper offers an experimental test of a version of Rubinstein's bargaining model in which the players' discount factors are unequal. We find that learning, rationality, and fairness are all significant in determining the outcome. In particular, we find that a model of myopic optimization over time predicts the sign of deviations in the opening proposal from the final undiscounted agreement in the previous period rather well. To explain the amplitude of the deviations, we then successfully fit a perturbed version of the model of myopic adjustment to the data that allows for a bias toward refusing inequitable offers.

An Experimental Test of Rubinstein's Bargaining Model

by Ken Binmore, Chris Tomlinson, and Joseph Swierzbinski[1]

1 Introduction

Twenty-five years ago, economists regarded the bargaining problem as indeterminate. Psychologists and philosophers might have something to say about bargaining, but the orthodoxy was that the subject lay beyond the reach of economic theory. Nash's [17, 18] arguments to the contrary were largely ignored, and it wasn't until Ståhl [27] and Rubinstein [22] employed Selten's [26] newly minted notion of a subgame-perfect equilibrium to obtain unique predictions in bargaining games with alternating offers that bargaining was generally recognized as an area to which economists could sensibly contribute.

Empirical research on bargaining models has lagged behind the theory. One reason is that Ariel Rubinstein—as in the final paragraphs of this paper—argues against using his model for predictive purposes. We agree that the pure form of his model is highly idealized, but nothing prevents our considering perturbed versions that take into account some of the peculiarities of human psychology.

Another reason is that subgame-perfection has been discredited, both theoretically and empirically. Numerous experiments show that subjects don't employ backward induction, even in bargaining games much simpler than the Rubinstein bargaining model (Camerer *et al* [11, 12]). But Rubinstein's results don't require the full force of subgame-perfection to survive (Binmore *et al* [4, 6]). In particular, the simple model of myopic adjustment used in this paper to help to make sense of the data works mainly because Rubinstein's prediction is a Nash equilibrium with stationary expectations (sections 2 and 3).

A third reason is that fairness is widely thought to trump strategic considerations. It is certainly true that fairness considerations are important in determining bargaining behavior in many experiments, although no single theory of

[1]We gratefully acknowledge funding from the UK Economic and Social Research Council through the Centre for Economic Learning and Social Evolution at University College London. We owe an additional debt to the UK Environmental Agency, which also contributed funding through its support of Joseph Swierzbinski.

how fairness effects the behavior of subjects in laboratories commands consensus support. However, Camerer [11, p.173] draws attention to the fact that both fairness and strategy seem relevant to laboratory behavior in alternating-offers bargaining games with only two stages, on which there is a great deal of data.[2]

The empirical work on alternating-offers games with an infinite horizon has been assessed by Camerer [11, Chapter 4] and by Weg and Zwick [28, Chapter 11]. Weg and Zwick [28, p.288] summarize the behavior of subjects in the work they survey, much of which is their own, by saying:

> People are reasonable within their cognitive limitations and moral constraints. This does not mean that they behave rationally according to point specifications. No one expects them to. Rather, people respond to changing bargaining conditions, in general, in the right directions.

However, the fact that both players discount time at the same rate in much of this work finesses some fairness issues, since the predicted outcome then approximates an equal split of the surplus, unless someone has a larger outside option.

The current paper considers the case when the players' discount factors are unequal. We find qualified support for the Rubinstein model, in that the subjects' behavior clearly exhibits a strong tendency to exploit the first-mover advantage enjoyed by the player with the opportunity to make the next offer. But the final outcomes are shifted away from the Rubinstein prediction by a countervailing tendency that favors fair outcomes.

We find it possible to make sense of the data by modeling the tendency to favor fairness as a perturbation of the basic model used to justify the Rubinstein prediction. We briefly review the possibility that some agents are slower than others in abandoning fairness norms that are not adapted to the game played in the laboratory, However, our chief effort is devoted to exploring the extent to which our data can be explained by perturbing our model of myopic adjustment by assuming that proposers modify their offers because they believe that a more unfair proposal is less likely to be accepted.

2 Outline of the Experiment

The design of the experiment is partly motivated by the success of a previous experiment on the Nash Demand Game (Binmore *et al* [8, 3]). This experiment

[2] Relevant data is summarized in figures 3.2 and 4.1 of Binmore [3].

began with a conditioning phase in which the subjects knowingly played against robots. In different treatments, the robots were programmed to converge on one of a number of different focal points. When the subjects later played each other, they began by playing as they had been conditioned, but then gradually adjusted their behavior until they were playing one of the exact Nash equilibria of the game.

We suspect that the events observed in this abstract game replicate in miniature what happens in many laboratory experiments. The conditioning phase mimics the socializing processes of real life through which we learn to operate social norms. When faced with an unfamiliar laboratory game, subjects then begin by simply operating whatever social norm is triggered by the way the game is framed. As Henrich *et al* [15] say: "Experimental play often reflects patterns of interaction found in everyday life." But as subjects gain experience through repeated play with different partners, their play gradually adapts to the laboratory game they are actually playing.

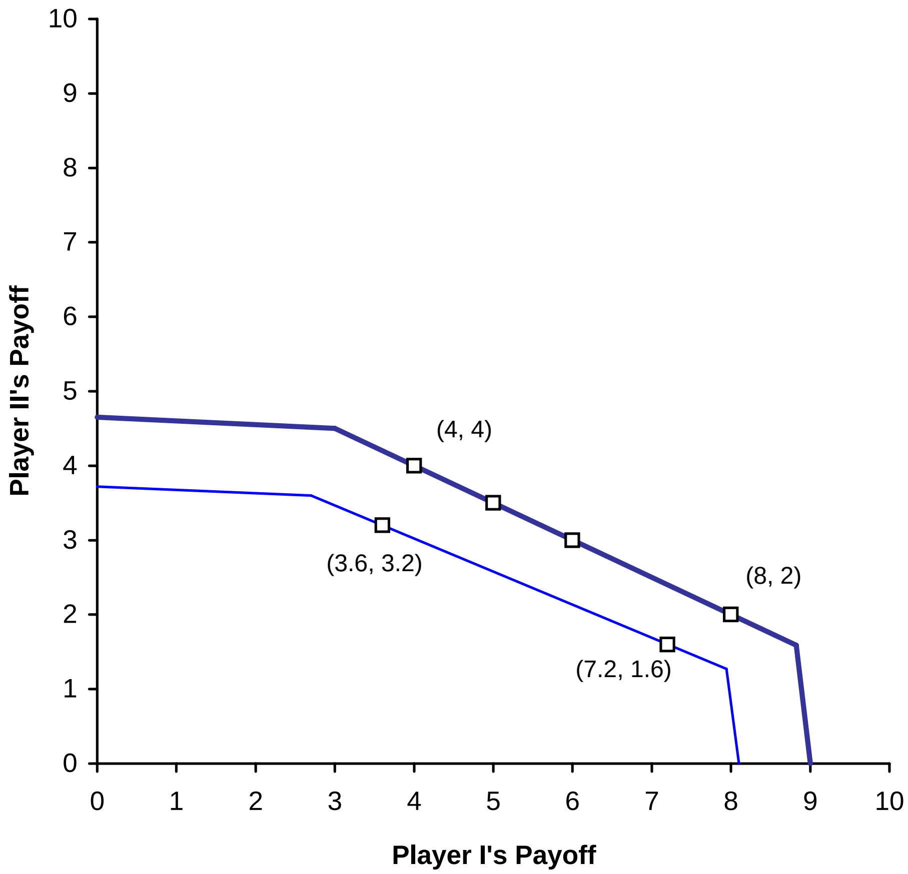
We attach considerable importance to the learning phase, in the absence of which we think it unreasonable to expect game theory to predict in the laboratory. In real life, we commonly receive a great deal of feedback from all kinds of sources when learning how to behave in a new economic environment. For example, rooky stockbrokers learn the ropes from their more experienced colleagues; shoppers tell each other where the best bargains are to be found. In the experiments reported in this and other papers, the quality of the feedback provided is therefore rich when compared with related experiments like that of Ochs and Roth [20]. It is therefore not surprising that we find more evidence of the subjects learning to respond to the strategic realities of the games they play.

The feasible set in the bargaining problem faced by the subjects in the experiment reported in this paper is shown in figure 1. The points $(4, 4)$ and $(8, 2)$ are significant. We refer to $(4, 4)$ as the egalitarian or fair outcome. We refer to $(8, 2)$ as the utilitarian outcome, although it is only an approximation to the true utilitarian outcome.

In our plan for the experiment, subjects were first conditioned on one or other of the egalitarian or utilitarian outcomes in a preliminary "practice" period, in which the subjects knowingly played against the computer. As it turned out, our attempt to condition subjects to begin bargaining near the utilitarian outcome was only partially successful.

After the conditioning phase, the subjects then began a learning phase in which they repeatedly played a version of the Rubinstein bargaining game against

Figure 1: Feasible Agreements, Stages 1 and 2



a new opponent each time. When player I's discount factor is $\delta_1 = 0.9$ and player II's is $\delta_2 = 0.8$, the Rubinstein solution is the utilitarian outcome $(8, 2)$. When the players' discount factors are exchanged, the Rubinstein solution is the egalitarian outcome $(4, 4)$. Four treatments need to be distinguished:

Treatment 1 Subjects conditioned on $(8, 2)$. Rubinstein solution $(4, 4)$.

Treatment 2 Subjects conditioned on $(4, 4)$. Rubinstein solution $(8, 2)$.

Treatment 3 Subjects conditioned on $(8, 2)$. Rubinstein solution $(8, 2)$.

Treatment 4 Subjects conditioned on $(4, 4)$. Rubinstein solution $(4, 4)$.

The questions to be asked were simple. Would the subjects move away from their conditioned behavior in treatments 1 and 2 towards the Rubinstein solution? Would the subjects continue with their conditioned behavior in treatments 3 and 4?

3 Theoretical Considerations

In Rubinstein's [22] Alternating Offers Game, two players alternate in proposing how to split a shrinking cake. We model the cake at time 1 as the set

$$X_1 = \{x \in I\!R^2 : x_2 \le g(x_1)\},$$

where $g : I\!R \to I\!R$ is strictly decreasing and concave. Its inverse function is denoted by $h : I\!R \to I\!R$. The feasible set in our experiment can be treated as the special case when the boundary of X_1 is $x_1 + 2x_2 = 12$, since the chunk cut away from this set in figure 1 is irrelevant to any calculations.[3]

We attribute the limited success that the Rubinstein prediction has enjoyed in past laboratory experiments to the fact that the prediction is consistent with the use of a Nash equilibrium with stationary expectations. In order that the players need only look one period into the future when forming expectations (rather than two), our experiment modified the rules of Rubinstein's [22] original paper. We used instead a format in which, from the point of view of the subjects, the next proposer is always randomly chosen from the two players with equal probability.

[3]Although not to focal point considerations, as the midpoint $(6, 3)$ on the hypotenuse would clearly have strong focal properties if the cutaway chunk were present.

Random proposers. The model with random proposers assumes that at each stage $s = 1, 2, \ldots$ of the game before an agreement, an independent chance move chooses player I or II with equal probability to act as proposer or responder. The proposer then makes a demand that the responder can accept or refuse. If the demand is accepted, the proposer receives his demand, and the responder is assigned whatever remains of the cake.

The shrinkage of the cake is modeled by assigning discount factors δ_1 and δ_2 to the two players. After a refusal at time s, the cake shrinks from X_s to

$$X_{s+1} = \{(x_1\delta_1, x_2\delta_2) \in I\!R^2 : x \in X_s\}.$$

Since we assume $0 < \delta_i < 1$, the cake shrinks to zero if all proposals are refused.

The game has a unique subgame-perfect equilibrium (Binmore [2]). In equilibrium, the expected payoffs to the two players in our experiment are

$$r_1 = \frac{12(1-\delta_2)}{2-\delta_1-\delta_2}; \qquad r_2 = \frac{6(1-\delta_1)}{2-\delta_1-\delta_2}. \tag{1}$$

In equilibrium, the opening proposal is always accepted, but previous experimental work shows that this feature of the prediction fails (Binmore *et al* [7, 5]). The best one can hope for is that the theory predicts the payoffs eventually agreed when these are discounted back to the first stage of the game.

The case when the interval between successive proposals becomes vanishingly small is of particular interest. Replacing δ_i by $e^{-\rho_i\tau}$ and allowing $\tau \to 0$, we are led to the asymmetric Nash bargaining solution of the bargaining problem $(X_1, 0)$ with bargaining powers $1/\rho_1$ and $1/\rho_2$. Applying the same procedure in the case of Rubinstein's original alternating-offers model yields exactly the same result.

Stationary expectations. We don't defend the Rubinstein prediction (1) on the grounds that it is the unique subgame-perfect equilibrium of the game with random proposers. We think it worth testing the Rubinstein prediction because it is the unique Nash equilibrium with stationary expectations. In such an equilibrium, a proposer will always make an offer (either $\delta_1 r_1$ or $\delta_2 r_2$) that leaves the responder indifferent between accepting and refusing. To arrive at the Rubinstein prediction (1) on this basis merely requires solving the equations

$$\begin{aligned} 2r_1 &= h(\delta_2 r_2) + \delta_1 r_1, \\ 2r_2 &= g(\delta_1 r_1) + \delta_2 r_2. \end{aligned}$$

Myopic adjustment. The same result can be obtained from a model of myopic adjustment.The subjects in our experiment participated in 16 bargaining sessions against a potentially new opponent each time. Let $r_i(t)$ denote the undiscounted mean payoff obtained by subjects occupying the role of player i in session t. (The undiscounted payoffs corresponding to a deal are those that would apply if the deal had been made at the opening proposal of a game). Since we only allow efficient deals, all the undiscounted payoffs lie on the Pareto frontier of X_1 so that $r_1(t) = h(r_2(t))$ and $r_2(t) = g(r_1(t))$.

Our model of myopic adjustment assumes that, at each stage s in session t, player I makes an optimal choice assuming that he or she will receive the (discounted) payoff $\delta_1^s r_1(t-1)$ if the game proceeds to the next stage. Similarly, we suppose that player II expects to receive the (discounted) payoff $\delta_2^s r_2(t-1)$ if play continues to stage $s+1$ and makes an optimal choice given this expectation.

When player I makes the first proposal in session t, player I will offer player II $\delta_2 r_2(t-1)$, which player II will accept. In games where player II makes the first proposal, player II will offer player I $\delta_1 r_1(t-1)$, which player I will accept. The x–coordinates of the optimal proposals made by players I and II at the first stage of session t are then $h(\delta_2 r_2(t-1))$ and $\delta_1\, r_1(t-1)$.

If proposers are drawn at random from an infinite population of subjects, then $r_1(t) = \frac{1}{2}(h(\delta_2 r_2(t-1)) + \delta_1 r_1(t-1))$ $(t \geq 1)$, and so

$$r_1(t) = 6(1-\delta_2) + \tfrac{1}{2}(\delta_1+\delta_2)\, r_1(t-1)\,.$$

The solution of this linear, first-order difference equation converges monotonically to its steady state value, which is the Rubinstein prediction for r_1 of equation (1).

We do not have an infinite number of subjects in our experiment. Nor were the proposers independently chosen each time. The choice of proposer at each stage of all games in each session was determined once-and-for-all in advance of the experiment so that each data point would be comparable. The subjects were told only that it would be hard to predict who would be the next proposer. Our simulation of the evolution of the mean proposal using the myopic adjustment dynamics described in the previous paragraphs and our actual experimental set-up, shows that the mean proposal still converges toward a neighborhood of the Rubinstein prediction. However, the convergence is more ragged than in the case when equal numbers of each type of player make a first proposal in each session.

4 Experimental Design

The experiments were conducted at the experimental laboratory of the ESRC Center for Economic Learning and Social Evolution (ELSE) at University College London. Each experiment used 12 subjects who sat at networked microcomputers. Screens blocked each subject's view of other subjects' computer screens or actions.

A subject in an experiment played the same bargaining game 24 times, first in 8 "practice" games with the computer and then in 16 "real games". Subjects were matched anonymously at the start of each real game with another human subject.

Before participating in any bargaining game, subjects participated in an interactive demonstration designed to familiarize them with the details of the game, and how their payoffs could be converted into money.

Lottery tickets. Subjects bargained over lottery tickets. After each set of 8 bargaining games, every subject independently participated in a lottery in which the prizes were either £10 (about $16) or nothing. If the subject had accumulated N lottery tickets in the previous 8 bargaining games, then the subject's probability of winning the lottery was $1.5N\%$. So each lottery ticket increased the subject's chance of winning £10 by 1.5% . Subjects were paid at the same rate in both the practice and the real games.

Feasible agreements. Figure 1 shows the set of feasible agreements in stages 1 and 2 of the bargaining game for one combination of discount factors. Throughout a given bargaining game, a subject was always either player I, with a payoff from an agreement that was indicated on the horizontal axis of a graph like figure 1, or player II, with a payoff indicated on the vertical axis. Subjects were assigned the role of player I or player II in different games. Rather than simply alternating roles in successive games, subjects were assigned the role of player I or player II in a pattern that was deterministic but not easily predictable by the subjects. By the end of the experiment, every player had the opportunity to bargain in each role an equal number of times.

The darker set of line segments that form the upper right-hand boundary of figure 1 indicate the set of feasible agreements that could be reached in the first stage of the bargaining game studied in our experiments. In the actual graph shown to subjects in the experiment, the region under the set of feasible agreements in the first stage was shown in deep blue with the set of feasible agreements indicated by thin, white line segments following the upper boundary of the blue region.

Choice of proposer. At each stage of the game, one of the two players was chosen to make a proposal. If player I was chosen, then player I registered a proposal by moving a cursor along the horizontal axis. As the cursor moved, the proposal was indicated by an orange rectangle framed by the axes and with an upper right vertex that moved along the set of feasible agreements. The length of the rectangle indicated the number of lottery tickets proposed for player I and the height of the rectangle indicated the number of lottery tickets proposed for player II. When player II was chosen to make a proposal, the procedure was similar except that player II registered a proposal by moving a cursor along the vertical axis.

After the proposer confirmed a proposal, the proposal was communicated to the other player, who could decide whether to accept or reject it. If the proposal was accepted the game ended, and each player received the designated number of lottery tickets. If the proposal was rejected, the game proceeded to the next stage.

In Rubinstein's original model, the two players were assumed to alternate in making proposals. In our experiment, the selection of which player makes a proposal at each stage was also deterministic but arranged in a way that was intended to be difficult to predict by the subjects. So, for example, in our game player I or player II might have the opportunity to make two proposals in succession. The unpredictable assignment of the the next proposer was designed to simulate a version of Rubinstein's model in which the proposer at each stage is chosen randomly. The control afforded by keeping the assignment of the right to propose deterministic facilitates the presentation and analysis of the results.

Discounting. The role of discounting in Rubinstein's model is implemented in our experiment by shrinking the set of feasible agreements each time a proposal is rejected. The inner set of line segments in figure 1 shows how the set of feasible agreements shrinks from stage 1 to stage 2 when the discount factor for player I is $\delta_1 = 0.9$ and the discount factor for player II is $\delta_2 = 0.8$. For example, discounting the egalitarian agreement $(4, 4)$ by one period yields $(3.6, 3.2)$, which is one of the feasible agreements at stage 2 shown in figure 1. Similarly, discounting the utilitarian agreement $(8, 2)$ by one period yields $(7.2, 1.6)$, which is one of the feasible agreements at stage 2 shown in figure 1.

In the graph presented to the subjects, the shrinking of the set of feasible agreements was highlighted by showing the shrunken region in a different color. During the demonstration that preceeded the bargaining games, subjects had several opportunities to observe how shrinking the set of agreements from stage to stage eroded the value of an agreement.[4]

Forced termination. In Rubinstein's original model, bargaining could continue indefinitely, but this isn't feasible in an experiment. During the demonstration, subjects were informed: "To speed things up, the computer will interrupt games in which all proposals keep being refused, but it will be hard to guess exactly when this will happen." The number of proposals that needed to be rejected before the computer terminated the game was set as a parameter for each game and varied from 3 to 7. In practice games, the subject received 0 when a game was interrupted by the computer. In real games, subjects whose game was interrupted by the computer received the appropriately discounted median payoff from the uninterrupted agreements of the other pairs of subjects in that game.[5] The maximum number of rejected offers allowed before termination is reported for each real game in Table A.1. The fraction of the individual bargaining games terminated by the computer is also reported in Table A.1 for each real game and each treatment. As can be observed, this fraction is typically quite low. Less than 5 % of the individual bargaining games played during the real games of our experiments were terminated by the computer.

Feedback. The graphical interface in the experiment was also used to provide information about the recent behavior of subjects in order to facilitate learning. Six small red boxes were shown whose locations indicated the payoffs obtained in recent agreements and the stages at which the agreements were reached.

[4] Much preliminary effort was devoted to trying to present the shrinking of the cake by rescaling the axes so as to emphasize the stationary features of the bargaining problem. But subjects then largely ignored the discounting altogether.

[5] Subjects whose games were terminated by the computer were awarded the median uninterrupted payoff in order to make the anticipated continuation payoff from an interrupted game similar to what could be expected from an uninterrupted game.

After the first real game, the six boxes showed all the agreements reached in the previous real game. Since there were twelve separate agreements in a practice game (one for each subject), a subset of recent agreements was randomly chosen for display in the practice games and the first real game.

The six boxes were displayed at the beginning of each stage of bargaining. Once the boxes were displayed, subjects pressed a button that discounted the location of each box so as to show the payoffs that subjects would obtain if they had to wait for the agreement represented by the box until the next stage—the stage that would occur if the current proposal were rejected. Figure 1 shows six boxes similar to those used in the experiment (although the numerical values of the payoffs corresponding to a box weren't displayed to the subjects).

Conditioning. In addition to providing experience with the bargaining protocol, the practice rounds were used to try and condition subjects to begin bargaining in the real rounds close to one of two focal points shown in figure 1. The egalitarian point, $(4, 4)$, was one focal point, and the utilitarian point $(8, 2)$ was the other. (The precise utilitarian maximum lies at the "corner" slightly to the right of $(8, 2)$ in figure 1.)

When the computer was called upon to make a proposal in the practice rounds, it always made proposals close to the focal point on which we were trying condition the subjects. When the computer was called upon to accept or reject a proposal, the probability that the computer accepted was related to the distance between the proposal and the focal point.

Treatments. Four different versions of the experiment were used in our study. These four treatments differed both in the focal point which was used by the computer in the practice rounds and in the choice of the discount factors for player I and player II. The same set of discount factors was used throughout an experiment.

In treatment 1, we attempted to condition subjects in the practice rounds to begin bargaining at the focal point $(8, 2)$. The discount factors used in treatment 1 were $\delta_1 = 0.8$ for player I and $\delta_2 = 0.9$ for player II.

The Rubinstein prediction for our model depends on which player is chosen to make the first proposal. For the discount factors used in treatment 1, Rubinstein's solution predicts that the agreement $(4.8, 3.6)$ would be reached in stage 1 when player I is the first proposer while the agreement $(3.2, 4.4)$ would be reached in those games where player II is selected as the first proposer. When it is equally likely that player I and player II will be chosen as the first proposer, the average payoff obtained when Rubinstein's solution is played will therefore be $(4, 4)$. In assessing Rubinstein's model for treatment 1, we wish to consider the extent to which the actual agreements in real games move from the initial conditioning point $(8, 2)$ toward the "average" Rubinstein solution $(4, 4)$.

Treatment 2 considers the reverse case where subjects were conditioned to begin bargaining close to the focal point $(4, 4)$ and the discount factors were $\delta_1 = 0.9$ and $\delta_2 = 0.8$. For this set of discount factors, the mean payoffs for player I and player II produced by the Rubinstein solution are $(8, 2)$.

Treatments 3 and 4 were "control" treatments. In treatment 3, the conditioning point in the practice rounds was $(8, 2)$ and the average Rubinstein solution was also $(8, 2)$. In treatment 4, the conditioning point in the practice rounds was $(4, 4)$ and the average Rubinstein solution was $(4, 4)$.

5 Description of the Results

This paper reports the results of 60 experiments, each involving 12 subjects. Each subject participated in 24 individual bargaining games, including 8 "practice" games playing against the computer and 16 "real" games bargaining with other subjects.

Primary and subsidiary data sets. A *primary* data set consisted of 10 experiments numbered 1 to 10 for each of the 4 treatments described in section 4. A *subsidiary* data set consisted of 5 additional experiments, numbered 11 to 15, for each treatment. In the subsidiary data set, the type of the first proposer in each real game was reversed. In part, the subsidiary data set represented a chance to try out our best fitting model for each treatment in an "out-of-sample" test.

Graphical summaries of mean data. Figures 2a through 2h summarize our results graphically for each of the four treatments with both the primary and the subsidiary data sets. Each of the eight figures shows four graphs. Two graphs describe how the mean actual behavior of the subjects represented in each data set evolves over time from the first real game (game 1) to the last real

Figure 2a. Treatment 1, cp = 8,2, rs = 4,4

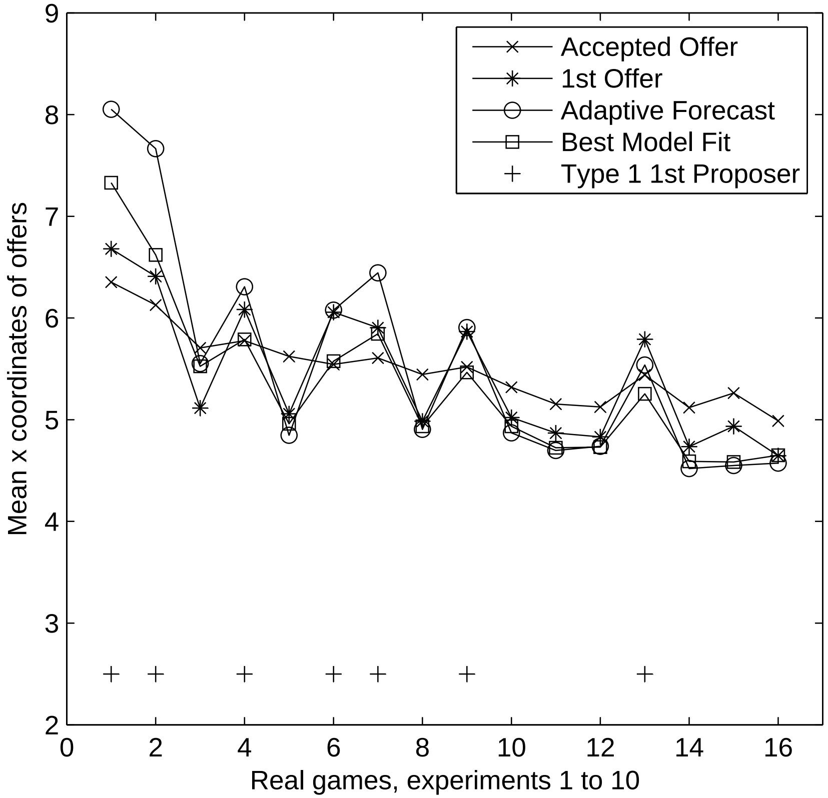


Figure 2c. Treatment 3, cp = 8,2, rs = 8,2

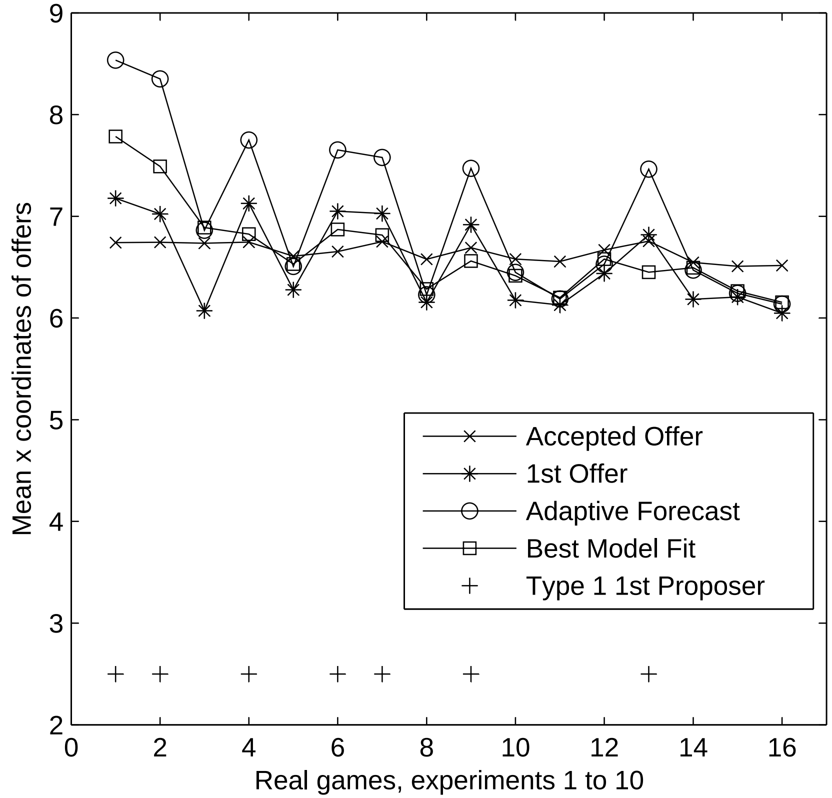


Figure 2b. Treatment 2, cp = 4,4, rs = 8,2

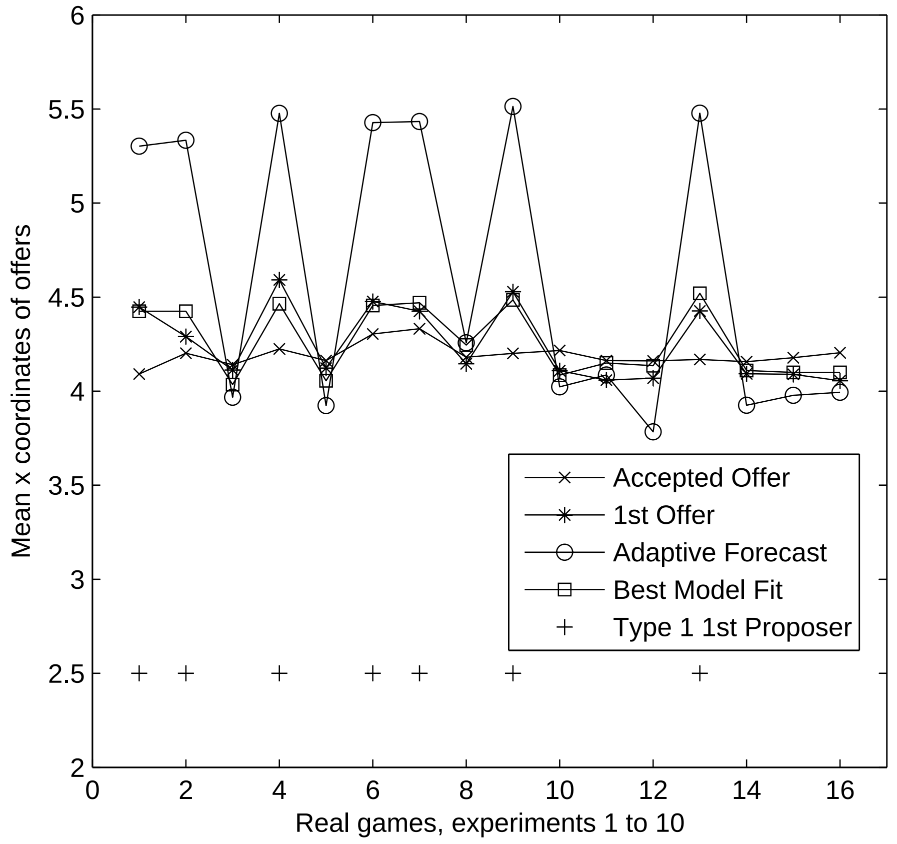
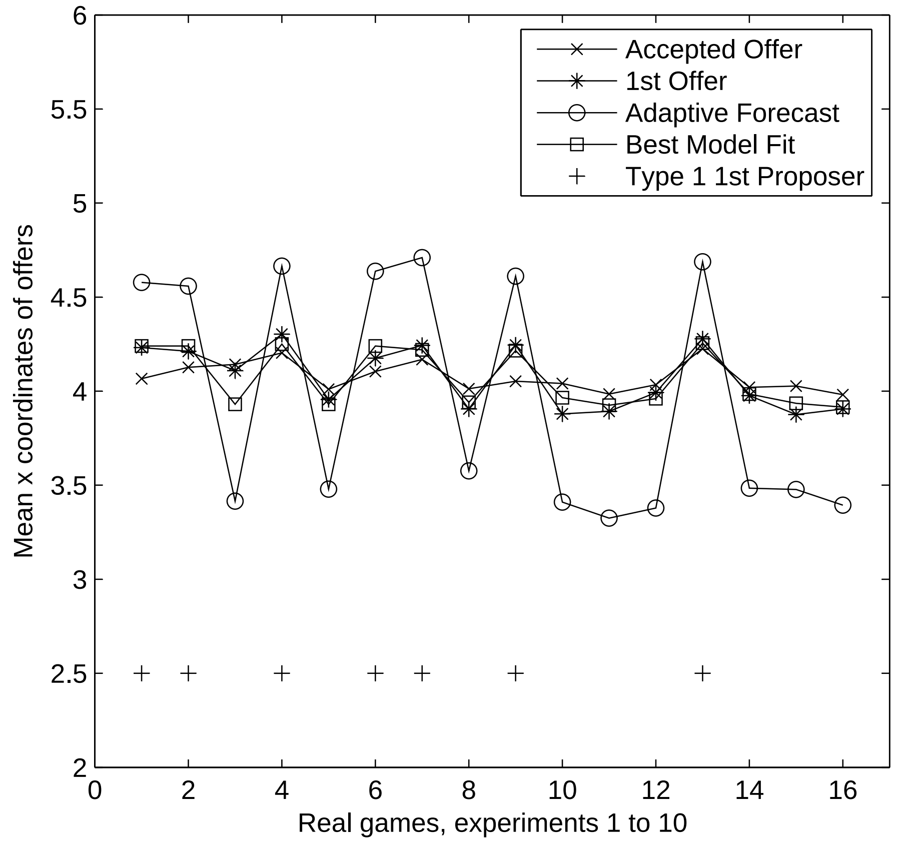


Figure 2d. Treatment 4, cp = 4,4, rs = 4,4



game (game 16). The other two graphs show the predictions of models that use versions of adaptive optimal strategies to forecast subjects' behavior (section 6).

Table A.1 in the appendix provides the data used to draw figures 2a through 2h as well as further supporting information for each data set. Table 1 provides additional information to support the interpretation of the data presented in figures 2a through 2h. Figure 3 shows some features of the statistical model used to analyze the data.

As an illustration, consider figure 2a, which describes the results from the first 10 experiments of treatment 1. The points marked with an "x" in figure 2a indicate the mean of the payoffs to player I (the x–coordinates in figure 1) from the accepted offers in each of the 16 real games for experiments 1 to 10 of treatment 1. Since six pairs of subjects bargained in each real game of each experiment, each data point shown in figure 2a is the average of 60 actual payoffs. In order to make the payoffs from agreements reached at different stages comparable, the payoffs shown are inflated where necessary to indicate the "undiscounted" payoffs that would have been obtained if the agreement in question had been reached in the first stage of bargaining. Column 4 of table A.1a in the appendix reports the numerical values of each of these data points. Tables A.1b through A.1h report similar numerical information for each of the other data sets, while the graphs marked with x's in figures 2b through 2h show the evolution of the mean accepted offers graphically for each of the other data sets.

Influence of discount factors. In treatments 1 and 4, the discount factors for players I and II were chosen so that the mean payoff at Rubinstein's solution was (4,4). For treatment 1, figures 2a and 2e show a gradual evolution in the x–coordinate (player I's payoff) of the mean accepted proposal toward the Rubinstein solution. For the primary data set consisting of experiments 1 to 10, the x–coordinate of the mean acceptance decreased from 6.353 in the first real game to 4.989 in the last real game. Similarly, in the second data set the x–coordinate of the mean acceptance decreased from 6.514 in the first real game to 5.713 in the last. In treatment 4, subjects were conditioned to start bargaining in the real games close to the Rubinstein solution. Figures 2d and 2h show that, for treatment 4, the mean accepted offers remained close to (4,4) from the first to the last real games.

In treatments 2 and 3, the discount factors were chosen so that the mean

Figure 2e. Treatment 1, cp = 8,2, rs = 4,4

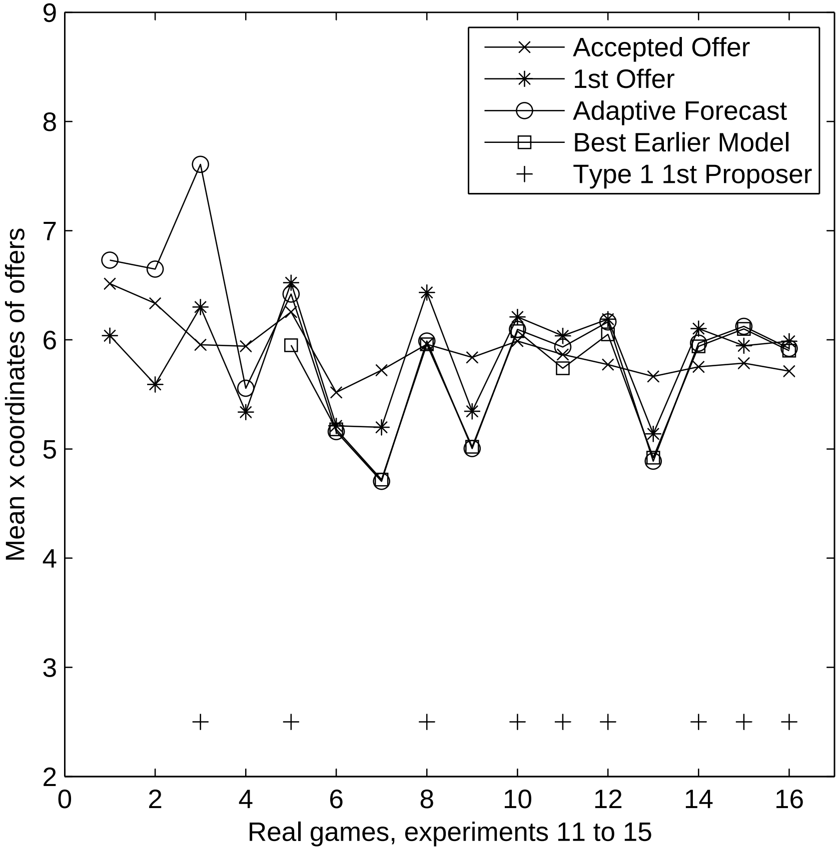


Figure 2g. Treatment 3, cp = 8,2, rs = 8,2

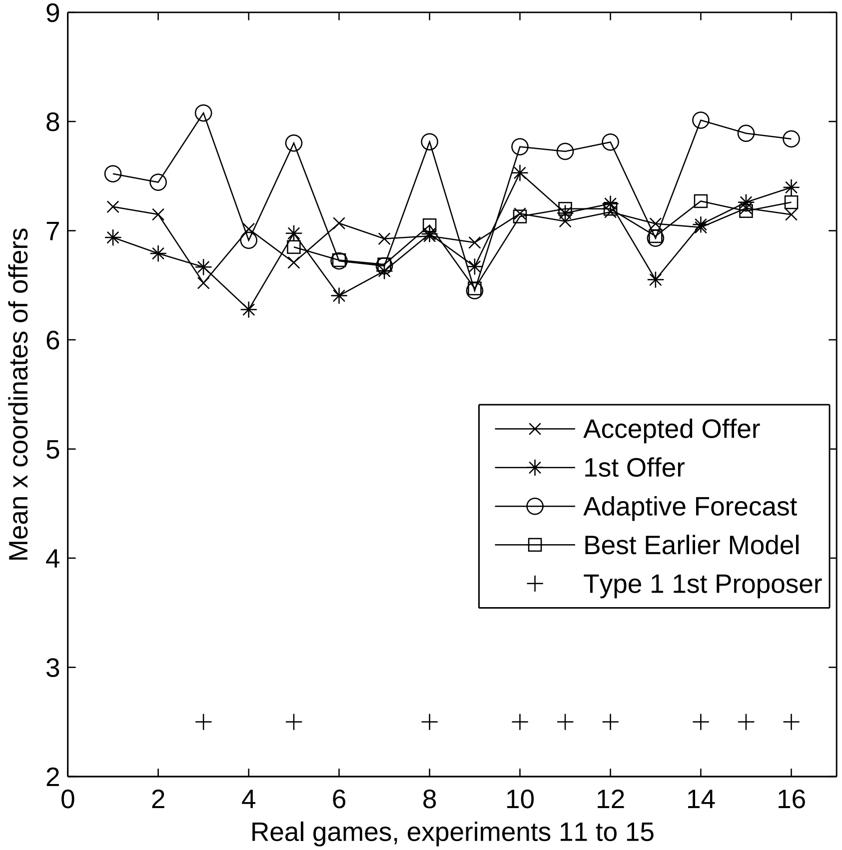


Figure 2f. Treatment 2, cp = 4,4, rs = 8,2

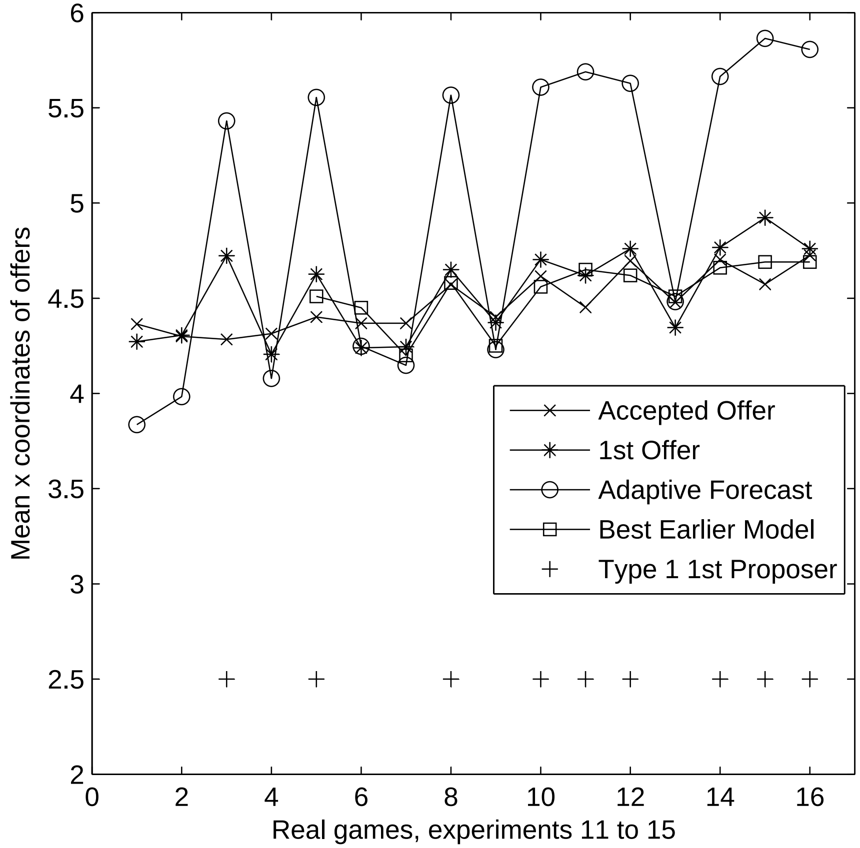
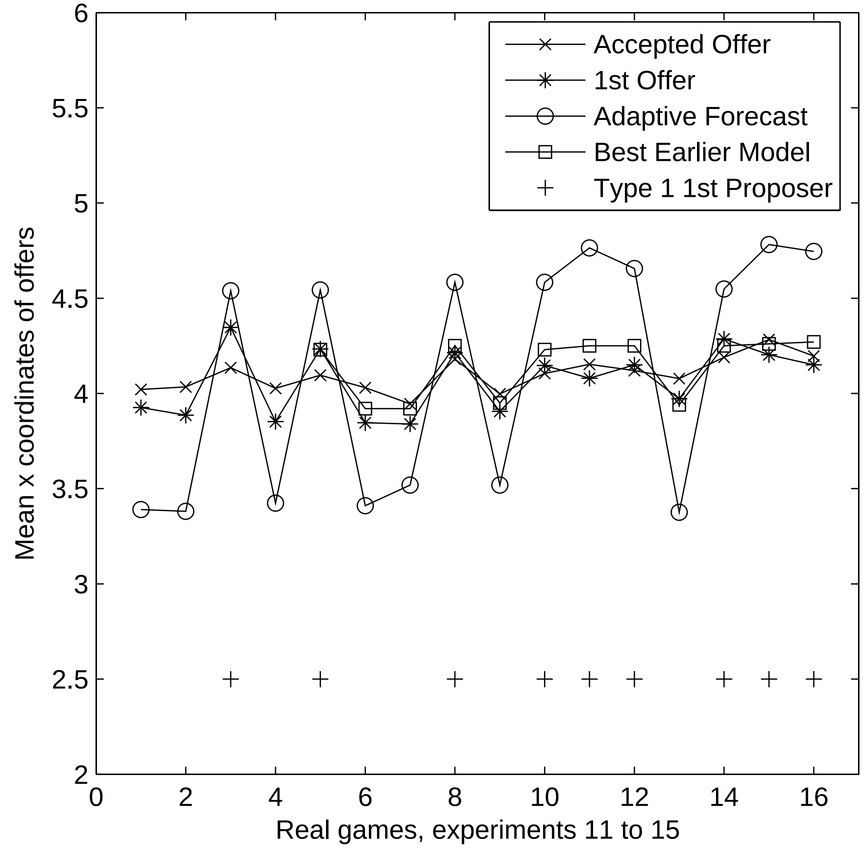


Figure 2h. Treatment 4, cp = 4,4, rs = 4,4



payoff at Rubinstein's solution was (8,2). Figures 2b and 2f show that in treatment 2, when subjects began the real games with bargains close to (4,4), the mean accepted offers tended to remain close to (4,4) throughout the 16 real games. However, figures 2c and 2g show that in treatment 3, where the mean x–coordinates of the accepted offers in the first real game were much closer to the (8,2) mean Rubinstein offer, the mean accepted offers also tended to remain close to the mean accepted offer in the first real game.

The observations in the previous two paragraphs don't support the hypothesis that subjects immediately find their way to bargains that are close to the Rubinstein solution. On the other hand, as described below, comparisons of the mean accepted offers across treatments show that the different combinations of discount factors used in our experiments influenced the accepted offers in the direction predicted by Rubinstein's theory.

We can control for the effect of our attempts to condition subjects by comparing the mean accepted offers in the final real games of the main treatments, 1 and 2, with the mean acceptances in the corresponding control treatments, 3 and 4. As in the previous paragraphs, such comparisons could be made using the graphs in figures 2a through 2h. The means and standard deviations reported in table A.1 can also be used to construct two-sample t-tests to make such comparisons.[6] It should be noted that in constructing the t-tests mentioned below (and the corresponding standard deviations reported in table A.1) we assume that the mean offers accepted by subjects within a given real game and a given experiment constitute a single observation. Hence, we make the conservative assumption that each experiment provides only one data point for each real game.[7]

Treatment 1 (main) and treatment 3 (control) differed only in the discount factors that governed the shrinking of the set of feasible bargains. For treatment 1, the discount factors were chosen so that the mean payoffs in the Rubinstein solution were $(4, 4)$ while, for treatment 3, the mean Rubinstein payoffs were $(8, 2)$. Two-sample, 1-sided t-tests using either the primary data set (experiments 1 to 10) or the subsidiary data set (experiments 11 to 15) reject at the 1%

[6] The graphical comparisons discussed in the previous paragraphs could also be supplemented by comparisons using t-tests constructed with the information in table A.1. For example, t-tests could be used to compare the mean accepted offers in the last real games to the Rubinstein solution or to compare the mean accepted offers in the first and last real games.

[7] If the offers accepted by each individual subject-pair could be regarded as independent observations, then each experiment would provide six data points for each real game.

level of significance the hypothesis that the mean x–coordinates of the accepted offers in real game 16 (the last real game) are equal for the two treatments in favor of the hypothesis that the mean x–coordinate is greater for treatment 3 than for treatment 1. Similarly, treatment 2 (main) and treatment 4 (control) differed only in that the mean Rubinstein payoffs were $(8, 2)$ for treatment 2 and $(4, 4)$ for treatment 3. The mean acceptances in the last real games were much closer for treatments 2 and 4. Nevertheless, for the primary data set, a two-sample t-test rejects the hypothesis that the mean acceptances were equal in the last real game of each treatment at the 10%, but not the 5%, level of significance. For the subsidiary data set, this hypothesis would be rejected at the 1% level of significance. In each case, the alternate hypothesis is that the mean x–coordinate is greater for treatment 2 than for treatment 4 as predicted by Rubinstein's theory.[8]

Influence of conditioning. In treatments 1 and 3, we attempted to condition subjects in the practice rounds played against the computer to begin bargaining with real subjects near the agreement (8,2). However, we were only partially successful in this task. As can be seen in figure 2a, as well as figures 2c, 2e, 2g, and table A.1, the x–coordinates of the mean offers accepted by subjects in the first real game, when subjects began to bargain with each other, ranged from 6.353 to 7.219 for the data sets involving treatments 1 and 3. In treatments 2 and 4, we attempted to condition subjects to begin bargaining for real near (4,4). For the data sets involving treatments 1 and 4, the x–coordinates of the mean accepted offers in the first real game ranged from 4.021 to 4.364.

As with the influence of the discount factors, comparisons between the mean offers in the first real games of treatments 1 and 4 and treatments 2 and 3 can be used to demonstrate the effect of conditioning in the practice rounds while controlling for the different combinations of discount factors used in the various treatments. For each data set, two-sample t-tests confirm the clear impression from the graphs in figure 2 that the conditioning always had an important effect

[8] The two-sample t-statistics used to compare the mean x–coordinates of the accepted offers in the last real games of treatments 3 and 1 were $tp = 6.959$ for the primary data set and $ts = 3.881$ for the second data set. The number of observations in each sample is 10 for the primary data set (one for each of 10 experiments) and 5 for the second data set. The corresponding two-sample t-statistics used to compare the sample means for treatments 2 and 4 were $tp = 1.554$ for the primary data set and $ts = 3.189$ for the subsidiary data set.

on where subjects began bargaining in the real games.[9]

6 Fitting an Adaptive Model

To examine the extent to which the subjects' behavior is consistent with an adaptive version of Rubinstein's theory, we turn to a consideration of the first proposals made in our experiment.

The points marked with an asterisk in figure 2a indicate the mean of the payoffs to player I from the proposals made at the first stage of each of the 16 real games for experiments 1 to 10 of treatment 1. Column 5 of table A.1a in the appendix reports the numerical values of each of these data points. As with the mean accepted offer, tables A.1b through A.1h report similar numerical information for each of the other data sets, while the graphs marked with asterisks in figures 2b through 2h show the evolution of the mean first proposals graphically for each of the other data sets.

By comparing the graphs in figures 2a through 2h that show the x–coordinates of the mean (undiscounted) accepted offer and the mean first proposal made in each real game, we can observe how subjects responded to the opportunity to make a first proposal. Rubinstein's prediction that agreements will always be reached in the first stage of the bargaining game will be seen to fail. However, many of the individual bargaining games in our experiments concluded after only a few stages of bargaining. As can be seen in column 2 of table A.1, the mean number of offers made in each bargaining game were all between 1 and 2.1. We nevertheless focus on the first proposals, because this is where we have most data.

Recall from section 4 that the type of the proposer at each bargaining stage was specified as a parameter in our experiments, although the type of the proposer (player I or player II) varied from session to session and from stage to stage in a way that was intended to be difficult for the subjects to predict. The type of the subject making the first proposal in a given real game was the same for experiments 1 through 10 of each treatment. The types making the first proposal in each real game were reversed for experiments 11 through 15. In the graphs of

[9] The two-sample t-statistics for comparing the mean of the accepted offers in real game 1 of treatment 1 with that of treatment 4 are $tp = 14.348$ for the primary data set and $ts = 16.148$ for the subsidiary data set. The corresponding two-sample t-statistics for comparing treatments 2 and 3 are $tp = 13.051$ and $ts = 9.672$.

figure 2, games marked with a "+" indicate games where player I made the first proposal. Column three of table A.1 also reports the type of the first proposer in each game.

Because the type of the players making first proposals in a given real game is the same for all the experiments in each data set shown in figures 2a through 2h, these figures make it easy to review the behavior discussed in the previous section.

Consider, for example, the data shown for the real games in figure 2a. If subjects behave in an opportunistic way, we would expect that, in a real game where player I made the first proposal, the mean x–coordinate of the first proposal would generally be greater than the x–coordinate of the mean acceptance in the previous game as the type I subjects attempt to push deals in a direction favorable to themselves. Similarly, we would expect the mean x–coordinate of first proposals by player II to be smaller than the mean x–coordinate of the accepted offers in the previous period. Bargaining in later stages might then be expected to moderate somewhat the effect of the first proposals and move the mean accepted offer in the current period back toward the mean acceptance in the previous period. One result is that the mean accepted offer typically evolves more smoothly from game to game than does the mean first proposal.

The give-and-take pattern of opportunistic behavior described in the previous paragraph can be observed in each of the figures 2a through 2h.

The numerical data reported in table A.1 confirms that the patterns observed in figures 2a through 2h are not a trick of the eye. In particular, for real games 2 through 16 of each data set, one can calculate the difference between the x–coordinate of the mean first proposal reported in column 5 and the x–coordinate of the mean accepted offer in the previous game reported in column 4 of the previous row.[10] If subjects behave opportunistically, this difference should generally be positive when the first proposals are made by player I and generally negative when the first proposals are made by player II. Eight data sets and fifteen real games provide a total of 120 observations. Of these, 112 observations or 93.3 % are in the direction predicted by the hypothesis of opportunistic behavior.[11]

[10] For the first real game in each data set, the accepted offers in the previous game involve bargaining with the computer. Hence, we omit the first real game in each data set from the analysis discussed in this paragraph.

[11] The eight exceptions are game 12 of treatment 4, experiments 1 to 10, game 3 of treatment 1, experiments 11 to 15, games 3, 5, and 14 of treatment 3, experiments 11 to 15, and games 11, 12, and 16, of treatment 4, experiments 11 to 15.

In order to choose a first offer, opportunistic subjects must weigh the gain from an offer that is more advantageous for themselves against the risk that such an offer will be rejected, and the cost of such a rejection. The idea of a myopically optimal strategy discussed in section 3 is well suited to modeling such a tradeoff. As noted in section 4, the graphical interface used in our experiments provided subjects in each game with information about the proposals agreed by the subjects in previous games. In subsections 6.1 and 6.2 we use versions of a myopically optimal strategy to forecast how subjects used the information we provided to make their first proposals.

6.1 An Adaptive Forecast of the First Proposal

We now propose a model of adaptive optimal play to obtain putative forecasts of the first proposals made by our subjects. We then consider how well these adaptive forecasts explain the actual proposals made by the subjects.

Calculating myopically optimal first proposals. Recall from section 4 that the subjects were given information about previous agreements in the form of six red boxes. In any given real game in a particular experiment, all subjects observed the same set of red boxes. For real games 2 through 16, these boxes indicated offers agreed in the previous bargaining session. At each bargaining stage, the location of each box was shifted to indicate the payoffs that subjects would obtain if the agreement represented by the box were to be reached in the stage following the present stage.

For a given real game within a given experiment, let $i = 1, 2, \ldots, 6$ index the agreements indicated by the red boxes. Let $\xi = (\xi_1, \xi_2, \ldots, \xi_6)$, where ξ_i is the payoff to player I if the agreement specified by box i were to be reached in the first stage of the current game. Suppose that x denotes the x–coordinate of some feasible agreement in stage 1 of the bargaining game. Let $y(x)$ $(0 \leq x \leq 9)$ denote the monotonic decreasing function that specifies the set of feasible stage 1 bargains indicated by the set of heavy line segments drawn in figure 1. Using this notation, $y(\xi_i)$ indicates the payoff to player II when the agreement represented by box i is accepted at stage 1.

Our calculation of an adaptive forecast assumes that subjects use the locations of the red boxes to forecast both the probability that a proposal will be rejected, and the payoff they would receive when such a rejection occurs. When player I contemplates making a first proposal $(x, y(x))$, we suppose that he or she acts as though the offer will be rejected with probability

$$R_{II}(x, \xi_i) = \begin{cases} 1\,, & \text{if} \quad y(x) < \delta_2 y(\xi_i)\,, \\ 0\,, & \text{if} \quad y(x) \geq \delta_2 y(\xi_i)\,, \end{cases}$$

when bargaining with a player II whose agreement in the previous game is indicated by box i. Subjects who reject the current offer and so precipitate a further bargaining stage are treated as though they will receive the (appropriately discounted) payoff to which they agreed in the previous game.

The function R_{II} and a similarly defined R_I incorporate the idea of stationary expectations within the same bargaining session, and adaptive expectations across different bargaining sessions. In particular, they specify the optimal behavior of a myopic but opportunistic subject in receipt of an offer $(x, y(x))$.

When proposing, we assume that players I and II act as though it is equally likely that they are bargaining with an opponent whose experience in the previous game is summarized by any of the boxes in the vector ξ. They then (myopically) choose $(x, y(x))$ to maximize the respective expected payoffs:

$$V_I(x, \xi) \;=\; \frac{1}{6}\sum_{i=1}^{6} \left[(1 - R_{II}(x, \xi_i))\, x \;+\; R_{II}(x, \xi_i)\, \delta_1 \xi_i\right] \;. \qquad (2)$$

$$V_{II}(x, \xi) \;=\; \frac{1}{6}\sum_{i=1}^{6} \left[(1 - R_I(x, \xi_i))\, y(x) \;+\; R_I(x, \xi_i)\, \delta_2 y(\xi_i)\right] \;. \qquad (3)$$

The adaptive forecasts of first proposals obtained in this way vary with the experiment as well as the real game. However, the adaptive forecast is the same for all subjects in a particular game, since they all then observe the same set of boxes.

Comparing the mean forecasts with the data. We calculated separate adaptive forecasts of the first proposal for each experiment and each real game. However, as with the x–coordinates of the accepted offers and actual first proposals, the points labeled with a circle in figure 2a show the mean of the adaptive forecasts for each designated real game in experiments 1 to 10 of treatment 1. Column 6 of table A.1a reports the numerical values of these average forecasts. The values in parentheses beneath the means in column 6 report the standard deviations of the adaptive forecasts for each real game, where the adaptive forecast for a particular experiment constitutes one observation. Figures 2b through

2h and tables A.1b through A.1h report similar information for the other seven data sets.

A comparison of the graphs of the adaptive forecasts and actual first proposals in figures 2a through 2h suggests that the adaptive forecasts track the direction of movement of the first proposals from game to game rather well. In some cases, for example in figure 2a, the adaptive forecasts also appear to perform reasonably well in predicting the magnitudes of the first proposals. In other cases, such as figure 2b, the adaptive forecasts substantially overshoot the actual first proposals made in some games. We return to the question of overshooting in subsection 6.2.

We measure the observed direction of movement in the mean first proposal by the difference between the x–coordinate of the mean actual first proposal in a given real game and the x–coordinate of the mean accepted offer in the previous game. As in figures 2a through 2h and the corresponding entries in table A.1, the first proposals and accepted offers are averaged over all individual bargaining games in the experiments that comprise each data set. We similarly measure the predicted direction of movement in the first proposals by the difference between the mean adaptive forecast for a given real game and the mean accepted offer in the previous game. As in figures 2a through 2h, the adaptive forecasts are also averaged over the experiments in each data set.

The data in table A.1 can be used to construct 120 observed directions of movement for the mean first proposals—a direction for real games 2 through 16 for each of 8 data sets. Of these 120 observations, the directions predicted by the adaptive forecasts agree for 106 observations (88.3 % of the total). Eight of the fourteen exceptions are cases noted earlier where the observed direction of movement in the first proposals isn't what might be expected if the proposers behaved in an opportunistic way.[12] Of the remaining six cases, five refer to early real games in treatments where we hadn't fully succeeded in persuading subjects in the practice rounds to begin bargaining near (8,2). In these five cases, the mean first proposals were even more aggressively favorable for the proposers than the means of the offers deemed optimal for player II by our model. Some player IIs in these early games were possibly trying to take advantage of the apparent willingness of player Is to accept low offers.

[12]The remaining six cases, where the observed and predicted directions fail to agree, are game 3 of treatment 3 (primary data set), games 2 and 11 of treatment 1, and games 2, 4, and 6, of treatment 3 (subsidiary data sets).

6.2 Adaptive Forecasts with a Preference for Equity

The adaptive forecast developed in the previous subsection substantially overestimates the magnitude of the actual first proposals made in some real games. This is particularly noticeable in the graphs of figures 2b and 2f, which show data from treatment 2, and figures 2d and 2h, which show data from treatment 4, although it also occurs in some of the other data. In treatments 2 and 4, we conditioned subjects to begin bargaining close to the egalitarian outcome (4,4), and the agreements tended to stay close to (4,4) throughout all the real games in these treatments.

Inequity aversion? One possible explanation for both the apparent opportunistic behavior of proposers and the proposers' apparent caution in making proposals is that proposers were concerned that the subjects receiving proposals were likely to reject proposals that deviated too greatly from the egalitarian outcome (4,4). Such a preference for equity on the part of responders might arise because the egalitarian payoffs (4,4) evoked a fairness norm which subjects brought with them to the laboratory. As we will observe later, we may also have inadvertently encouraged the perception that proposals that deviated too greatly from (4,4) were risky by the way that we conditioned subjects in the practice rounds of treatments 2 and 4 to begin bargaining close to (4,4).

In this subsection, we modify the model of adaptive forecasts described in subsection 6.1 to include a preference for equity that increases the risk that proposals deviating from (4,4) will be rejected. The modified model (of adaptive forecasts with a preference for equity) appears to provide one reasonable explanation for both the observed directions and the observed magnitudes of the average first proposals made by our subjects.

Our model differs from the models of inequity aversion of Bolton and Ockenfels [9, 10] and Fehr and Schmidt [13, 14] in numerous ways. In particular, the postulated level of equity preference in our model depends on the preliminary conditioning of the subjects. Unsurprisingly, we find that a greater level of equity preference needs to be postulated after the subjects have been conditioned to be equitable.

Modeling a preference for equity. Suppose that player I is in receipt of a proposed agreement, $(x, y(x))$. Let $z_1(x) = \max\,[0, (4-x)/b\,]$, where b is a positive parameter. The quantity $z_1(x)$ is a measure of the standardized distance between the payoff offered to player I and the egalitarian payoff. The distance measure is asymmetric since we assume that player I wouldn't object to a more favorable offer than the egalitarian payoff. Similarly, suppose that player II receives a proposal $(x, y(x))$. Let $z_2(x) = \max[\,0, (4 - y(x))/b\,]$, where b is the same parameter that was used in the definition of z_1.

The quantity b is a scale factor that translates payoff differences into standardized distances. Smaller values of b increase the distance measures $z_1(x)$ and $z_2(x)$.[13] One reason for introducing the scale factor b is that it enables us to allow for different degrees of equity preference in the different treatments by adjusting the value of b.

Suppose that $S(z_1(x))$ denotes the probability that player I rejects the offer $(x, y(x))$ because of a preference for equity. We suppose that $S(z)$ is a smoothly increasing function with $S(0) = 0$ and that $S(z)$ approaches 1 as z becomes large. Similarly, let $S(z_2(x))$ denote the probability that player II rejects the offer $(x, y(x))$ because of a preference for equity. For concreteness, we use one of the family of gamma cumulative distribution functions to represent $S(z)$ (Mood *et al* [16]). Specifically, we assume that

$$S(z) \;=\; \frac{1}{\Gamma(3)} \int_0^z x^2 \exp(-x)\, dx \;=\; 1 - \exp(-z)\,(1 + z + z^2/2)\;. \qquad (4)$$

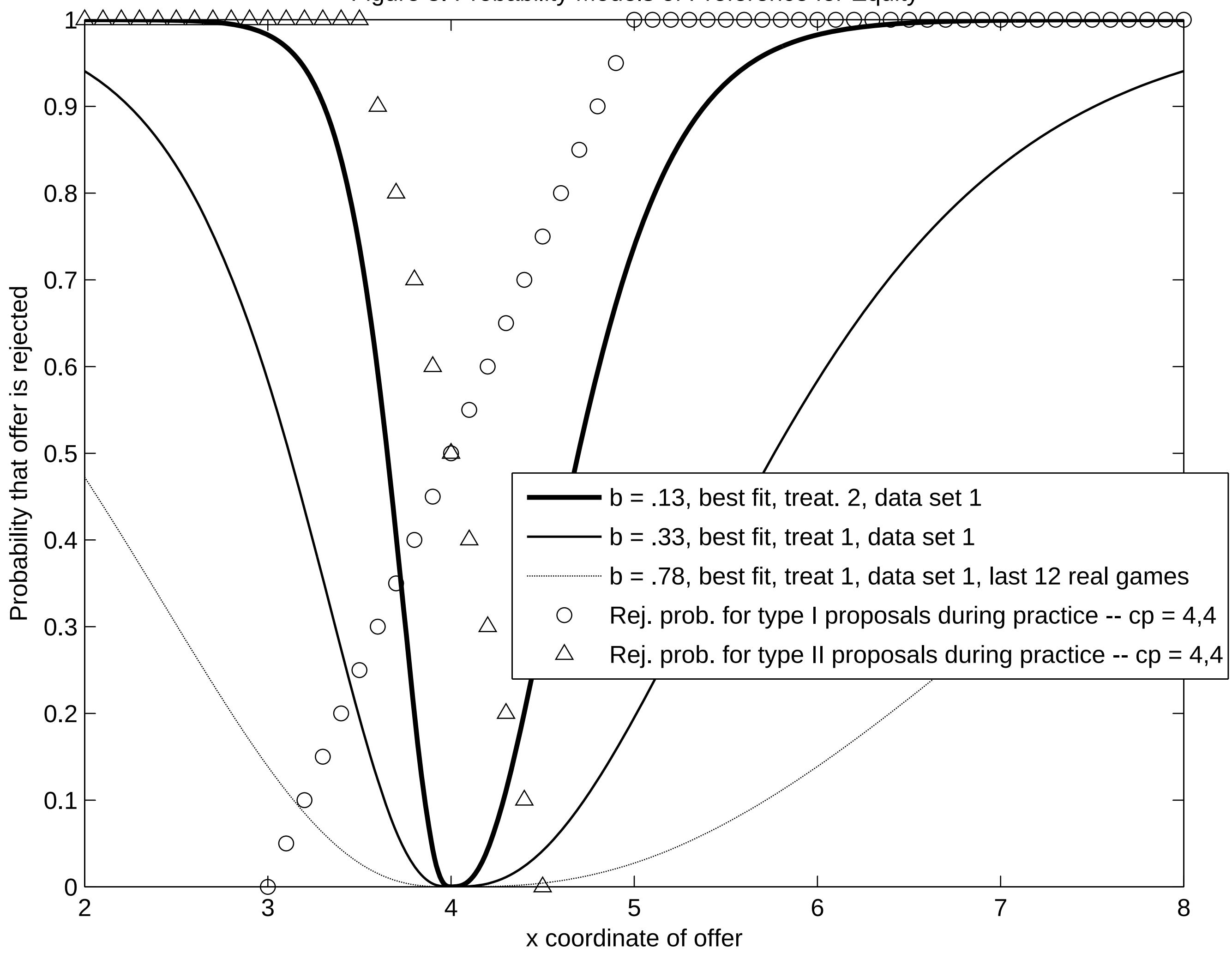
Figure 3 shows the rejection probabilities, $S(z_1(x))$ and $S(z_2(x))$, for selected values of the scale factor b. The horizontal axis in figure 3 shows the x–coordinate of a proposal made in stage 1. For values of x greater than 4 and, hence, values of $y(x)$ less than 4, the curves show $S(z_2(x))$, the probability that a type II player is assumed to reject such a proposal because of a preference for equity. Similarly, for values of the x–coordinate less than 4, the curves show $S(z_1(x))$, the probability that a type I player is assumed to reject the offer x because of a preference for equity. Note that, for a given value of x, the rejection probabilities are higher for smaller values of the scale factor b.[14]

In the practice rounds of treatments 2 and 4, subjects were encouraged to begin bargaining for real near the offer (4,4), both by observing the proposals made

[13] We considered introducing separate scale factors b_1 and b_2 for players I and II. However, it seemed reasonable to start by assuming that both types of subjects responded to payoff differences in the same way.

[14] The curves in figure 3 are flatter to the right of $x = 4$ than to the left, because the horizontal axis in figure 3 shows the value x of the offer to player I, while the rejection probabilities are assumed to depend on the payoffs of the player receiving the offer.

Figure 3. Probability Models of Preference for Equity



by the computer and by observing which proposals were accepted or rejected by the computer. For reference, the curves indicated by the circles and triangles in figure 3 show the probabilities with which the computer rejected particular offers during the practice rounds of treatments 2 and 4. The curve consisting of circles indicates the probability that the computer, when acting as player II, rejected proposals made by player I. Similarly, the curve consisting of triangles indicates the probability with which the computer rejected offers by player II.[15]

To construct an adaptive forecast that includes a preference for equity, we replace the objective functions V_I and V_{II} of equations (2) and (3) by W_I and W_{II}, where

$$W_I(x,\xi)=\frac{1}{6}\sum_{i=1}^{6}\left\{\overline{R}_{II}(x,\xi_i)\overline{S}(z_2(x))x+\left[1-\overline{R}_{II}(x,\xi_i)\overline{S}(z_2(x))\right]\delta_1\xi_i\right\} \quad (5)$$

$$W_{II}(x,\xi)=\frac{1}{6}\sum_{i=1}^{6}\left\{\overline{R}_{I}(x,\xi_i)\overline{S}(z_1(x))y(x)+\left[1-\overline{R}_{I}(x,\xi_i)\overline{S}(z_1(x))\right]\delta_2 y(\xi_i)\right\} \quad (6)$$

The term $\overline{R}_{II}(x,\xi_i)=1-R_{II}(x,\xi_i)$ in equation (5) is the probability (either 0 or 1) that a player II whose agreement in the previous game is summarized by box i will accept the offer $(x,y(x))$ on "strategic" grounds. The term $\overline{S}(z_2(x))=1-S(z_2(x))$ is the probability that the player will accept the offer "in spite of" a preference for equity. The product $\overline{R}_{II}(x,\xi_i)\overline{S}(z_2(x))$ is the overall probability that the offer $(x,y(x))$ will be accepted. The complementary probability $1-\overline{R}_{II}(x,\xi_i)\overline{S}(z_2(x))$ is the probability that the offer is rejected.

Since the optimization is only over a single variable x, it is straightforward to calculate numerically the values of x that maximize the objective functions in equations (5) and (6) for given values of the scale factor b and the vector ξ of box coordinates. These optimal values of x determine the new adaptive forecast for a given value of the parameter b. In particular, the optimal value of x for the actual type of the proposer in a designated real game and experiment is the new adaptive forecast conditional on the parameter b. As with the original adaptive forecast defined in section 6.1, the new adaptive forecast is the same for all the subjects in a particular real game of a particular experiment, so it cannot explain the variation in the x–coordinates of the first proposals of individual subjects within a given game and experiment.

[15]For offers from both players I and II, the computer's rejection probabilities were specified by a uniform distribution defined on a payoff interval centered on the payoff 4 and extending for .5 units on either side of this value (figure 3).

Choosing the scale factor b. In order to choose appropriate values of the scale factor b for each treatment, we need a measure of the "goodness-of-fit" of our forecasts. The measures we use are unitless and so resemble the usual R^2 statistic used in linear regression. We always calculate the goodness-of-fit measures for a single data set at a time.

Let N_D denote the number of experiments included in each of the eight data sets. The primary data set for each treatment consisted of 10 experiments. For these data sets, $N_D = 10$ and we index the experiments by $k = 1 \ldots 10$, with the index corresponding to the number of the experiment. The subsidiary data set for each treatment consisted of 5 additional experiments labeled 11 to 15. For these data sets, $N_D = 5$ and we index the sessions by $k = 1 \ldots 5$, with $k = 1$ corresponding to experiment 11, $k = 2$ corresponding to experiment 12 and so on.

We index the real games included in the goodness-of-fit measure by an index j that runs from j_0 to 16. There were a total of 16 real games in each experiment which, for example, are labeled 1 to 16 in figures 2a through 2h. We often wish to include data from all the real games in our measures of goodness-of-fit, in which case $j_0 = 1$. Sometimes, we will wish to consider the goodness-of-fit for the last 12 real games, in which case $j_0 = 5$. For a given value of j_0, the total number of real games being considered is $17 - j_0$.

For a particular real game j of experiment k, let $a_{jk}(b)$ denote the adaptive forecast of the x–coordinates of the first proposals, given the scale factor b. Let m_{jk} denote the mean of the x–coordinates of the first proposals actually observed in game j of experiment k. Let MFP denote the mean of the observed first proposals taken over all the real games and all the experiments under consideration. In this case, MFP satisfies the equation

$$\mathrm{MFP} \;=\; \frac{1}{(17 - j_0)N_D} \sum_{j=j_0}^{16} \sum_{k=1}^{N_D} m_{jk} \; .$$

For a given data set and a given set of real games, the experiment-by-experiment R^2 is generalized to EGR^2 by the following equation:

$$\mathrm{EGR}^2 \;=\; 1 \;-\; \frac{\sum_{j=j_0}^{16} \sum_{k=1}^{N_D} \left(m_{jk} - a_{jk}(b)\right)^2}{\sum_{j=j_0}^{16} \sum_{k=1}^{N_D} \left(m_{jk} - \mathrm{MFP}\right)^2} \; . \tag{7}$$

The quantity EGR^2 so defined can be given the following interpretation. Suppose that the total variability of the observed mean first proposals, m_{jk}, about some forecast is measured by the mean square error. Then EGR^2 measures the fraction of the total variability around the overall mean, MFP, that is explained by the adaptive forecasts, $a_{jk}(b)$. Since it involves the ratio of two mean square errors, EGR^2 is invariant to changes in the units with which the payoffs are measured. However, unlike the standard R^2 statistic, EGR^2 can be negative, since there is typically no value of b for which the adaptive forecasts $a_{jk}(b)$ are all equal to MFP.

To forecast the first proposals for each data set, we chose values of b that maximized the appropriate version of EGR^2. As with the adaptive forecasts themselves, the values of EGR^2 were calculated numerically for various values of b and compared to find the value of b that maximized EGR^2.

Although we used EGR^2 to choose the best fitting values of the scale factor b, most of the discussion up to now has involved quantities that have been averaged over experiments to reduce noise. We define a second but related measure, the generalized R^2, to assess how well the adaptive forecasts averaged over experiments fit the observed first proposals when these are also averaged over experiments.

For each real game j, let m_j denote the mean first proposal averaged over the experiments in the data set under consideration. Similarly, let $a_j(b)$ denote the average of the adaptive forecasts over relevant experiments for a given real game j and a given value of the scale factor b. These quantities satisfy the equations

$$m_j \;=\; \frac{1}{N_D}\sum_{k=1}^{N_D} m_{jk} \quad \text{and} \quad a_j(b) \;=\; \frac{1}{N_D}\sum_{k=1}^{N_D} a_{jk}(b) \quad .$$

For a given data set and a given set of real games, we define GR^2 to be the generalized R^2 given by

$$\mathrm{GR}^2 \;=\; 1 \;-\; \frac{\sum_{j=j_0}^{16} \left(m_j - a_j(b)\right)^2}{\sum_{j=j_0}^{16} \left(m_j - \mathrm{MFP}\right)^2} \quad . \tag{8}$$

When the observed first proposals and the adaptive forecasts are averaged over experiments, GR^2 specifies the fraction of the total variability of the averaged first proposals across real games explained by the average adaptive forecasts.

Table 1, figures 2a through 2h, and column 7 of table A.1 report data concerning the adaptive forecasts with a preference for equity.

Experiments 1 to 10 constitute the primary data set for each of our treatments, and, for each treatment, column 1 of table 1 reports information concerning the best-fitting adaptive forecast when all the real games in experiments 1 to 10 are considered. Row 1 of column 1 lists the values of the scale factor b that maximize EGR^2 calculated using all the real games of experiments 1 to 10. Row 3 of column 1 in table 1 reports the maximized values of EGR^2. The fourth row reports GR^2, which measures how well the adaptive forecasts averaged over experiments 1 through 10 predict the mean observed first proposals in each real game when these proposals are also averaged over experiments.

Understanding how a change in the scale factor b translates into a change in the probabilities that various offers are rejected may not be intuitively obvious. To aid intuition, the second rows of table 1 state for each value of b reported in the first rows, the corresponding values of $S(z_1(3.5))$, which is the probability that player I would reject an offer of 3.5 because of a preference for equity. Note that this probability is small when the subjects have been conditioned on the utilitarian outcome $(8, 2)$ but large when they have been conditioned on the egalitarian outcome $(4, 4)$. Curves showing all rejection probabilities appear in figure 3 for the values of b reported in columns 1 and 2 of table 1 for treatments 1 and 2.

Figures 2a through 2d report the results from the primary data sets for each of the treatments in our experiments. The points labeled with small boxes in these four figures show, for each treatment, adaptive forecasts that include a preference for equity, whose intensity is described by the values of b reported in column 1 of table 1. Each of the small boxes shows, for each real game j, the average of the adaptive forecasts for experiments 1 to 10 of the designated treatment. Column 7 of tables A.1a through A.1d report the numerical values of these average, best-fitting adaptive forecasts.

A comparison of the circles and boxes in figures 2a through 2d shows how the addition of a preference for equity moderates the swings in the adaptive forecasts from game to game. A comparison of the asterisks and boxes in these figures suggests that the modified adaptive forecasts perform reasonably well in describing both the direction and the magnitude of change in the first proposals. The generalized R^2, GR^2, for the main treatments, 1 and 2, is greater than 0.75 for each treatment.

It might be expected that subjects take a little time to adjust to bargaining with real subjects. Column 2 of table 1 reports information on the best-fitting adaptive forecasts when the data under consideration are the last 12 of the 16

Treatment 1

Conditioning point = (8,2)   Rubinstein solution = (4,4)

	Best fit Primary data set all real games	Best fit Primary data set last 12 games	Out-of-sample test Subsidiary data set last 12 games
b	0.33	0.78	0.78
$S(z_1(3.5))$	0.1950	0.0274	0.0274
EGR^2	0.5099	0.4959	0.2878
GR^2	0.7540	0.8745	0.5945

Treatment 2

Conditioning point = (4,4)   Rubinstein solution = (8,2)

	Best fit Primary data set all real games	Best fit Primary data set last 12 games	Out-of-sample test Subsidiary data set last 12 games
b	0.13	0.13	0.13
$S(z_1(3.5))$	0.7385	0.7385	0.7385
EGR^2	0.2866	0.2982	0.4265
GR^2	0.8533	0.8873	0.6142

Treatment 3

Conditioning point = (8,2)   Rubinstein solution = (8,2)

	Best fit Primary data set all real games	Best fit Primary data set last 12 games	Out-of-sample test Subsidiary data set last 12 games
b	0.22	0.28	0.28
$S(z_1(3.5))$	0.3967	0.2656	0.2656
EGR^2	0.0986	0.2013	0.4493
GR^2	0.3291	0.7934	0.5946

Treatment 4

Conditioning point = (4,4)   Rubinstein solution = (4,4)

	Best fit Primary data set all real games	Best fit Primary data set last 12 games	Out-of-sample test Subidiary data set last 12 games
b	0.11	0.11	0.11
$S(z_1(3.5))$	0.8315	0.8315	0.8315
EGR^2	0.3389	0.3687	0.4276
GR^2	0.8592	0.9244	0.6969

Table 1. Scale factors and goodness-of-fit measures for each treatment: For each treatment and each of the data sets indicated, the table reports values of the scale factor b that are used to construct adaptive forecasts. The value of $S(z_1(3.5))$, the probability that player I would reject an offer of 3.5 because of a preference for equity is also reported for each b value. The goodness-of-fit measures EGR^2 and GR^2 are also reported for each data set. The text explains how the quantities b, $S(z_1(3.5))$, EGR^2 and GR^2 were chosen for each data set.

real games in experiments 1 to 10. A comparison of GR^2 reported in columns 1 and 2 of table 1 indicates that the goodness-of-fit measure improves somewhat if the first 4 real games are excluded.

For brevity, the adaptive forecasts calculated using the best-fitting values of b reported in column 2 of table 1 are not reported in figures 2a through 2d and tables A.1a through A.1d. However, we used these values of the scale factors to conduct an out-of-sample test of our model of adaptive forecasts with a preference for equity using our subsidiary data set.

Experiments 11 to 15 for each treatment constitute a subsidiary data set that wasn't used in choosing the best-fitting scale factors reported in table 1. These experiments were not simply a replication of the sessions conducted in experiments 1 to 10. The conditions facing subjects in these experiments were varied by reversing the types of the subjects who made the first proposals. For example, as shown in figures 2a and 2e, player I made the first proposal in real game 1 of experiments 1 to 10, while player II made the first proposal in real game 1 of experiments 11 to 15. The same reversal of types occurs for each real game of each treatment.

For each treatment, the best-fitting scale factors for the last 12 real games of experiments 1 to 10 were used to construct adaptive forecasts of the first proposals made in the last 12 real games of experiments 11 to 15.[16] Column 3 of table 1, figures 2e through 2h, and column 7 of tables A.1e through A.1h report the results.

The values of b used to construct the adaptive forecasts for experiments 11 to 15 are those reported in column 2 of table 1. Hence, rows 1 and 2 of column 3 are always the same as rows 1 and 2 of column 2. Row 3 of column 3 reports EGR^2 for each treatment, and row 4 reports GR^2. As might be expected, this goodness-of-fit measure decreases from column 2 to column 3. However, the average out-of-sample adaptive forecasts still explain a considerable fraction of the variability of the average first proposals from game to game.[17]

The points labeled with small boxes in figures 2e through 2h show the adap-

[16] The out-of-sample adaptive forecasts for experiments 11 to 15 used the information from the boxes shown in experiments 11 to 15. Only the value of b was determined using information from experiments 1 to 10.

[17] It may seem odd that EGR^2 increases from column 2 to column 3 of table 1 for some treatments. However, the subsidiary data sets consist of only 5 experiments (instead of 10), so that the level of experiment-by-experiment variability to be explained by the adaptive forecasts is likely to be greater for the primary data sets.

tive forecasts calculated using the values of b reported in column 3 of table 1. Specifically, the average of the adaptive forecasts over experiments 11 to 15 is shown for each of the last 12 real games. Column 7 of tables A.1e through A.1h reports the corresponding numerical values for these average forecasts.

In treatments 2 and 4 subjects were conditioned to begin bargaining for real near the agreement (4,4). Table 1 shows that the best fitting values of the scale factor b are similar for these two treatments and that they result in substantially greater probabilities that an offer is rejected on equity grounds than for the other two treatments. One explanation is that the early experience with offers near an equitable outcome like (4,4) evokes a fairness norm in a way that a history involving other offers does not. It is, however, intriguing to compare the best-fitting curve of rejection probabilities shown in figure 3 for treatment 2 with the curves showing the probabilities with which the computer rejected offers in the practice rounds of treatments 2 and 4. Perhaps the high rejection probabilities used in the practice games inadvertently led subjects to infer that proposals which deviate significantly from (4,4) were just very risky rather than "unfair."

7 Robustness of the Rubinstein Model

Like many overly idealized models in game theory, the predictions of Rubinstein's bargaining model are not robust to small perturbations of various kinds.[18] For example, our model of adaptive optimization converges on the Rubinstein solution, but when perturbed by assuming that the subjects have a small preference for equity as in treatment 3, the model tracks the experimental results quite well although these show no sign of converging on the Rubinstein solution.

We are not sold on equity preference as the most important perturbation of the Rubinstein model that might be introduced. Other perturbations can also modify the Rubinstein solution significantly. In this section, we briefly explain how assuming that some subjects are slower than others to throw off any conditioning can totally alter the results of an equilibrium analysis of Rubinstein's model.

Strategists or robots? We modify the version of the Rubinstein model with random proposers so that each player may be either a strategist or a robot.

[18]Other examples are the Ultimatum Game, Public Goods Games with Punishment, and Gift Exchange Games (Binmore and Swierzbinski [3, Chapter 8].

Instead of a single chance move that decides whether a player will be a robot or a strategist at the start of the game (as in Abreu and Gul [1]), we introduce independent chance moves immediately following each refusal that permanently transform a player who has been a robot hitherto into a strategist from now on with probability $1-\theta$.

We take a robot to be a player who has been conditioned to believe that the correct proposal is some efficient point f of X_1. A robot in the role of player I therefore always demands f_1 when proposing, and accepts f_2 or better when responding. A robot in the role of player II always demands f_2 when proposing, and accepts f_1 or better when responding. To keep things simple we assume that the initial probability that a player is a robot is $\theta > 0$. If we also assume that a newly created strategist has the same beliefs that any other strategist would have on reaching the same point in the game, we thereby create a game with a stationary structure.

We describe three types of stationary expectations equilibria in which strategists always accept proposals made in equilibrium by strategists. Any refusal therefore signals to a strategist that the opponent is currently a robot, who will remain a robot only with probability θ in the next round. A strategic proposer then sometimes has two possibly optimal demands to compare: a larger demand that makes a strategic responder indifferent between accepting and refusing, and a possibly smaller demand that will also be accepted by a robot responder. In the following three different types of equilibrium, different choices of these possibilities turn out to be optimal:

Rubinstein equilibria: A strategist always makes a demand that renders another strategist indifferent between accepting and refusing. In equilibrium, strategists always accept.

Fair equilibria: A strategist always makes the fair demand. In equilibrium, strategists always accept.

Hybrid equilibria: A strategist plays as in a Rubinstein equilibrium or as in a fair equilibrium, depending on whether assigned the role of player I or player II. In equilibrium, strategists always accept.

The existence of these equilibria is proved in Binmore [3, Section 8.5]. The point of mentioning them here is that their existence—particularly the existence of a fair equilibrium—demonstrates the fragility of the Rubinstein solution to psychologically plausible perturbations other than equity preference.

8 Conclusions

This paper describes an experiment on Rubinstein's bargaining model with unequal discount factors. We find that a model of myopic optimization over time predicts the sign of deviations in the opening proposal from the final undiscounted agreement rather well. To explain the amplitude of the deviations, we then successfully fit a perturbed version of the model of myopic adjustment to the data that allows for a bias toward refusing inequitable offers. However, we are aware than other perturbations could be introduced that would have the same effect.

Our conclusion is that the underlying structure of Rubinstein's solution to the bargaining problem holds up unexpectedly well. It would therefore be a mistake to abandon the Rubinstein model on the grounds that subgame-perfect equilibrium has been shown to predict badly in alternating-offer bargaining games with a finite horizon. However, the precise form of the Rubinstein solution is fragile. Introducing psychological perturbations can alter the predicted outcome substantially when players adjust their behavior over time. Future research therefore needs to focus on the nature of the psychological quirks that perturb Rubinstein's basically sound model in real bargaining situations.

9 Ariel Rubinstein's Comments

(1) In general, economists are very conservative about the structure of their papers. The implicit requirement that a paper must have one agreed-upon conclusion discourages the disagreements that provide the oxygen for original and critical thinking and encourages the view that economics papers are meant to expose some truth. For a long time now, I have disagreed with this approach (see Osborne and Rubinstein [21] which explicitly includes the disagreements between the authors and Rubinstein [24] which includes a critique by Herbert Simon). Perhaps I should be pleased with the results presented in this paper, but I also feel obligated to express my skepticism regarding the interpretation of experimental results such as these, whether or not they support any particular model. Therefore, I was pleased with the invitation of the authors to ask me to add comments to their paper.

(2) I was attending a conference in Oberwolfach, Germany in the late eighties when Hugo Sonnenschein approached me with a draft of what was to become

Neelin, Sonnenschein and Spiegel [19]. The paper looked like a refutation of the "prediction" of the alternating offers model. I recall being quite shocked — not by the fact that an experiment had been designed whose results were not compatible with the unique subgame perfect equilibrium of the alternating offers model but by the fact that a distinguished group of researchers were of the opinion that the model is meant to be predictive.

I have never thought that the alternating offers model (or any other model in economic theory for that matter) is meant to have any predictive power. I always viewed economic models as fables that are linked to reality in a more subtle way, like the way in which a fairy tale is related to the real world (see Rubinstein [25]). My favorite interpretation of the alternating offers model (see Rubinstein [23]) is based on the notion of an acceptable agreement, which has also been used to interpret the Nash bargaining solution. The basic idea is that for an agreement x to be acceptable it has to be immune to an objection raised by player A in which he demands an agreement y that is more worthwhile for him (even taking into account the delay due to bargaining). The objection y should be rejected by the other player, B, due to his anticipation of some agreement z which is (i) acceptable in the circumstances that exist after the objection y has been raised and rejected and (ii) is more worthwhile for B than accepting y, even taking into account the delay involved. Apparently, the set of acceptable agreements is very close to the outcome of the alternating offers model.

(3) The current paper is based on the idea that game theoretical "predictions" are relevant to real world situations only when those situations are often repeated and after the players gained plenty of experience. If we insist that confirming a game prediction can be done only if the same game is repeated, and the players operate in laboratory-like circumstances, then game theory becomes very narrow in scope.

To have any relevance to real life, we need to test a prediction on individuals imagining a realistic scenario like the following: "An amount of $10,000 in compensation is to be paid to two neighbors (call them A and B), who live in the same apartment building, by a third neighbor in the building who is enlarging his apartment.

Each neighbor believes that he is entitled to a larger share of the money. Both have a sizable overdraft and need the money quickly. However, A pays 20% interest on his overdraft whereas B pays only 10%."

If the players time preferences are driven from the interest rates, then bargainer A should get around 1/3 of the sum. In any case, A is more impatient than player B and thus the standard game theoretic prediction would be that A gets less than $5000.

Students in game theory courses were assigned this problem by their teachers through my website: http://gametheory.tau.ac.il and were asked to give their prediction of the outcome of the bargaining. Admittedly, the text describing the scenario needed improvement. I have recently modified it in order to emphasize that the players have to negotiate an agreement and that neither of them will get a penny before they strike a deal. Nevertheless, the results, presented here for the first time, deserve closer analysis and are relevant to the paper's message.

More than 2000 students in 76 courses from 21 countries responded to the question. The average sum that player A would receive according to their responses was 4996, which is remarkably close to half of the stake! About 40% of the subjects said that A would get more than $5000 while only 28% said that he would get less. In all of the 9 courses that had more than 50 students, more than one half of the students thought that A would get more than $5000. In fact, 65% of the subjects chose one of the following 9 responses:
0 (1%) 3333-4 (2%) 4000 (8%) 5000 (31%) 6000 (12%) 6666-7 (6%) 10000 (5%).

One might suspect that many of the subjects did not devote enough attention to the question. However, among the half of the population whose response time was above the median, the picture is even more dramatic: 44% thought that A would get more than $5000 as opposed to 30% who thought he would get less.

This finding is puzzling. The subjects should have had enough bargaining experience to realize that the more impatient bargainer gets less than the more patient one. Yet, their responses were the opposite to what the alternating offers model predicts.

(4) Game theory is, in my opinion, no more than an analysis of the considerations that are used in strategic situations. Various considerations can appear simultaneously in the mind of a single player in a given situation and of course they can appear in the minds of different players in the same situation. Game theory has almost nothing to say about the circumstances in which various considerations appear in our minds and how we choose between conflicting considerations. Thus, game theory can contribute little to predicting the way in which games are played in the real world.

My approach is that experimental game theory can do no more than identify the appearance of certain considerations in some contexts. This does not conflict with the authors' view. They are careful in interpreting the results of their experiment as no more than an indication of the presence of adaptive dynamics, which converge to an outcome that closely resembles the equilibrium agreement of the alternating offers model.

Obviously, I don't dispute this interesting finding but I think that it has to be interpreted with caution. I suspect that many economists would consider results like those presented here as confirmation of their model but I cant accept it. I stick to my view that fitting experimental and empirical data to theoretical models in economics is just like attaching real life events to fables.

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Appendix: Summary Data

This appendix contains tables A.1a through A.1h with the summary data for all treatments. For each real game (1 to 16), columns four through seven in each table report the undiscounted x–coordinates of respectively (col. 4) accepted proposals, (col. 5) first proposals, (col. 6) adaptive forecasts, and (col. 7) best-fitting adaptive forecasts with a preference for equity. In each case, the x–coordinates are averaged over the data from all the subject-pairs in the indicated sets of experiments. (Standard deviations of the undiscounted x–coordinates are reported in parentheses. For a given game, these standard deviations take the mean x–coordinate of subject-pairs within an experiment as the unit of observation.) The type of the first proposer in each game is reported in column 3. Column 2 lists the mean number of offers made in each game up to and including an agreement. (The maximum number of rejected offers allowed by the computer in each game and the fraction of games terminated by the computer are listed in parentheses in column 2.)

Real Game	Mean Number of Offers	Type 1st P	Mean Undiscounted X-coordinates of Proposals: Accepted	First	Adaptive Forecast	Best Fit
1	1.333	1	6.353	6.680	8.051	7.330
	(6, 0.000)		(0.481)	(0.424)	(0.091)	(0.071)
2	1.450	1	6.129	6.410	7.666	6.620
	(4, 0.017)		(0.427)	(0.502)	(0.601)	(0.375)
3	1.650	2	5.706	5.115	5.552	5.525
	(3, 0.133)		(0.493)	(0.490)	(0.657)	(0.611)
4	1.600	1	5.778	6.083	6.307	5.790
	(4, 0.017)		(0.654)	(0.501)	(0.605)	(0.646)
5	1.900	2	5.623	5.059	4.848	4.960
	(7, 0.033)		(0.459)	(0.653)	(0.577)	(0.677)
6	1.600	1	5.543	6.057	6.077	5.575
	(3, 0.100)		(0.548)	(0.489)	(0.783)	(0.545)
7	1.517	1	5.607	5.905	6.444	5.845
	(5, 0.033)		(0.464)	(0.539)	(0.699)	(0.420)
8	1.783	2	5.444	4.985	4.905	4.935
	(4, 0.083)		(0.549)	(0.694)	(0.605)	(0.615)
9	1.717	1	5.518	5.867	5.905	5.465
	(5, 0.100)		(0.493)	(0.571)	(0.742)	(0.614)
10	1.600	2	5.318	5.022	4.871	4.935
	(3, 0.083)		(0.445)	(0.571)	(0.599)	(0.584)
11	1.850	2	5.153	4.869	4.698	4.725
	(7, 0.033)		(0.393)	(0.399)	(0.556)	(0.565)
12	1.500	2	5.126	4.832	4.738	4.730
	(3, 0.083)		(0.644)	(0.669)	(0.342)	(0.357)
13	1.433	1	5.441	5.792	5.538	5.255
	(5, 0.033)		(0.557)	(0.503)	(0.591)	(0.427)
14	1.617	2	5.119	4.735	4.519	4.590
	(4, 0.017)		(0.573)	(0.531)	(0.623)	(0.549)
15	1.367	2	5.263	4.935	4.550	4.585
	(6, 0.000)		(0.852)	(0.958)	(0.431)	(0.436)
16	1.450	2	4.989	4.645	4.574	4.650
	(7, 0.000)		(0.528)	(0.666)	(0.619)	(0.538)

Table A.1a: Treatment 1, Experiments 1 to 10.
Conditioning point–$(8, 2)$, Rubinstein solution–$(4, 4)$

Real Game	Mean Number of Offers	Type 1st P	Mean Undiscounted X-coordinates of Proposals: Accepted	First	Adaptive Forecast	Best Fit
1	1.583	1	4.091	4.447	5.302	4.425
	(6, 0.050)		(0.143)	(0.352)	(0.096)	(0.026)
2	1.267	1	4.202	4.290	5.334	4.425
	(4, 0.050)		(0.176)	(0.194)	(0.117)	(0.054)
3	1.300	2	4.141	4.112	3.967	4.035
	(3, 0.033)		(0.123)	(0.149)	(0.218)	(0.176)
4	1.633	1	4.225	4.592	5.477	4.465
	(4, 0.067)		(0.227)	(0.353)	(0.199)	(0.078)
5	1.250	2	4.163	4.122	3.923	4.055
	(7, 0.017)		(0.206)	(0.166)	(0.184)	(0.119)
6	1.283	1	4.304	4.477	5.428	4.455
	(3, 0.050)		(0.224)	(0.328)	(0.111)	(0.090)
7	1.350	1	4.332	4.425	5.433	4.470
	(5, 0.033)		(0.270)	(0.249)	(0.178)	(0.048)
8	1.600	2	4.180	4.146	4.257	4.245
	(4, 0.100)		(0.124)	(0.118)	(0.268)	(0.267)
9	1.350	1	4.201	4.528	5.514	4.485
	(5, 0.017)		(0.191)	(0.405)	(0.154)	(0.058)
10	1.283	2	4.216	4.109	4.023	4.085
	(3, 0.050)		(0.248)	(0.155)	(0.204)	(0.153)
11	1.867	2	4.162	4.059	4.086	4.150
	(7, 0.050)		(0.407)	(0.221)	(0.285)	(0.224)
12	1.400	2	4.161	4.069	3.783	4.135
	(3, 0.067)		(0.209)	(0.168)	(0.429)	(0.192)
13	1.517	1	4.168	4.427	5.478	4.520
	(5, 0.033)		(0.194)	(0.193)	(0.191)	(0.136)
14	1.383	2	4.156	4.092	3.925	4.110
	(4, 0.067)		(0.281)	(0.216)	(0.320)	(0.228)
15	1.250	2	4.178	4.089	3.978	4.100
	(6, 0.017)		(0.134)	(0.133)	(0.393)	(0.190)
16	1.883	2	4.204	4.056	3.994	4.100
	(7, 0.067)		(0.435)	(0.154)	(0.276)	(0.172)

Table A.1b: Treatment 2, Experiments 1 to 10.
Conditioning point–$(4, 4)$, Rubinstein solution–$(8, 2)$

Real Game	Mean Number of Offers	Type 1st P	Mean Undiscounted X-coordinates of Proposals: Accepted	First	Adaptive Forecast	Best Fit
1	1.383	1	6.741	7.177	8.536	7.785
	(6, 0.000)		(0.626)	(0.394)	(0.102)	(0.058)
2	1.567	1	6.744	7.023	8.350	7.490
	(4, 0.067)		(0.555)	(0.417)	(0.263)	(0.266)
3	1.650	2	6.734	6.071	6.862	6.890
	(3, 0.100)		(0.480)	(0.566)	(0.698)	(0.698)
4	1.467	1	6.747	7.127	7.750	6.825
	(4, 0.050)		(0.516)	(0.335)	(0.422)	(0.408)
5	1.683	2	6.607	6.278	6.507	6.530
	(7, 0.017)		(0.427)	(0.416)	(0.811)	(0.811)
6	1.850	1	6.654	7.050	7.652	6.870
	(3, 0.217)		(0.455)	(0.283)	(0.546)	(0.488)
7	2.083	1	6.751	7.028	7.579	6.815
	(5, 0.150)		(0.429)	(0.426)	(0.403)	(0.456)
8	1.683	2	6.578	6.155	6.229	6.285
	(4, 0.067)		(0.341)	(0.612)	(1.019)	(0.930)
9	1.483	1	6.691	6.918	7.471	6.560
	(5, 0.033)		(0.546)	(0.576)	(0.543)	(0.228)
10	1.567	2	6.579	6.175	6.449	6.415
	(3, 0.100)		(0.551)	(0.678)	(0.478)	(0.594)
11	1.683	2	6.556	6.128	6.188	6.200
	(7, 0.017)		(0.433)	(0.661)	(0.573)	(0.576)
12	1.500	2	6.669	6.438	6.529	6.580
	(3, 0.100)		(0.564)	(0.705)	(0.445)	(0.467)
13	1.217	1	6.756	6.818	7.464	6.450
	(5, 0.033)		(0.405)	(0.379)	(0.410)	(0.610)
14	1.767	2	6.547	6.184	6.471	6.495
	(4, 0.100)		(0.495)	(0.536)	(0.570)	(0.576)
15	1.617	2	6.509	6.206	6.243	6.265
	(6, 0.067)		(0.542)	(0.609)	(0.412)	(0.410)
16	1.750	2	6.517	6.048	6.138	6.155
	(7, 0.033)		(0.451)	(0.663)	(0.554)	(0.553)

Table A.1c: Treatment 3, Experiments 1 to 10.
Conditioning point–$(8, 2)$, Rubinstein solution–$(8, 2)$

Real Game	Mean Number of Offers	Type 1st P	Mean Undiscounted X-coordinates of Proposals: Accepted	First	Adaptive Forecast	Best Fit
1	1.450	1	4.067	4.232	4.578	4.240
	(6, 0.033)		(0.150)	(0.273)	(0.165)	(0.032)
2	1.450	1	4.127	4.212	4.559	4.240
	(4, 0.000)		(0.143)	(0.206)	(0.165)	(0.021)
3	1.183	2	4.142	4.109	3.415	3.930
	(3, 0.033)		(0.294)	(0.262)	(0.181)	(0.026)
4	1.400	1	4.204	4.303	4.665	4.250
	(4, 0.017)		(0.201)	(0.271)	(0.209)	(0.024)
5	1.117	2	4.010	3.956	3.479	3.930
	(7, 0.000)		(0.127)	(0.103)	(0.209)	(0.035)
6	1.233	1	4.105	4.175	4.638	4.240
	(3, 0.017)		(0.081)	(0.110)	(0.133)	(0.021)
7	1.233	1	4.169	4.243	4.710	4.220
	(5, 0.000)		(0.154)	(0.239)	(0.203)	(0.079)
8	1.317	2	4.012	3.906	3.576	3.940
	(4, 0.000)		(0.158)	(0.070)	(0.241)	(0.021)
9	1.567	1	4.053	4.245	4.611	4.215
	(5, 0.050)		(0.140)	(0.229)	(0.182)	(0.113)
10	1.267	2	4.041	3.879	3.411	3.965
	(3, 0.033)		(0.110)	(0.152)	(0.183)	(0.071)
11	1.317	2	3.985	3.892	3.325	3.925
	(7, 0.017)		(0.133)	(0.098)	(0.151)	(0.035)
12	1.167	2	4.034	3.992	3.379	3.960
	(3, 0.033)		(0.286)	(0.292)	(0.177)	(0.057)
13	1.317	1	4.226	4.278	4.688	4.255
	(5, 0.000)		(0.231)	(0.241)	(0.276)	(0.083)
14	1.117	2	4.021	3.976	3.484	3.985
	(4, 0.017)		(0.298)	(0.319)	(0.276)	(0.129)
15	1.250	2	4.027	3.876	3.477	3.935
	(6, 0.000)		(0.180)	(0.113)	(0.283)	(0.034)
16	1.133	2	3.981	3.906	3.394	3.915
	(7, 0.000)		(0.129)	(0.141)	(0.196)	(0.024)

Table A.1d: Treatment 4, Experiments 1 to 10.
Conditioning point–$(4, 4)$, Rubinstein solution–$(4, 4)$

Real Game	Mean Number of Offers	Type 1st P	Mean Undiscounted X-coordinates of Proposals: Accepted	First	Adaptive Forecast	Best Model (from earlier data)
1	1.433 (6, 0.000)	2	6.514 (0.321)	6.038 (0.621)	6.730 (0.076)	— (—)
2	1.667 (4, 0.033)	2	6.334 (0.498)	5.592 (0.549)	6.649 (0.188)	— (—)
3	1.500 (3, 0.067)	1	5.955 (0.700)	6.300 (0.529)	7.608 (0.394)	— (—)
4	1.633 (4, 0.067)	2	5.942 (0.711)	5.338 (0.628)	5.556 (0.841)	— (—)
5	1.700 (7, 0.000)	1	6.255 (0.543)	6.523 (0.284)	6.420 (0.555)	5.950 (0.793)
6	1.433 (3, 0.067)	2	5.519 (0.947)	5.212 (1.123)	5.160 (0.819)	5.180 (0.810)
7	1.467 (5, 0.033)	2	5.721 (0.621)	5.198 (0.731)	4.704 (0.908)	4.720 (0.922)
8	1.600 (4, 0.000)	1	5.960 (0.426)	6.433 (0.325)	5.988 (0.433)	5.960 (0.428)
9	1.567 (5, 0.000)	2	5.838 (0.636)	5.345 (0.714)	5.004 (0.675)	5.020 (0.682)
10	1.567 (3, 0.033)	1	5.989 (0.578)	6.210 (0.451)	6.096 (0.369)	6.080 (0.356)
11	1.933 (7, 0.067)	1	5.865 (0.589)	6.037 (0.657)	5.934 (0.390)	5.740 (0.349)
12	1.733 (3, 0.100)	1	5.773 (0.631)	6.190 (0.484)	6.164 (0.591)	6.050 (0.624)
13	2.000 (5, 0.100)	2	5.665 (0.846)	5.139 (0.905)	4.888 (0.565)	4.920 (0.556)
14	1.633 (4, 0.033)	1	5.753 (0.615)	6.103 (0.778)	5.969 (0.578)	5.940 (0.586)
15	1.400 (6, 0.000)	1	5.786 (0.664)	5.947 (0.699)	6.123 (0.567)	6.100 (0.569)
16	1.733 (7, 0.033)	1	5.713 (0.418)	5.987 (0.593)	5.921 (0.692)	5.900 (0.675)

Table A.1e: Treatment 1, Experiments 11 to 15.
Conditioning point–$(8, 2)$, Rubinstein solution–$(4, 4)$

Real Game	Mean Number of Offers	Type 1st P	Mean Undiscounted X-coordinates of Proposals: Accepted	First	Adaptive Forecast	Best Model (from earlier data)
1	1.533	2	4.364	4.272	3.837	—
	(6, 0.000)		(0.180)	(0.164)	(0.165)	(—)
2	1.467	2	4.300	4.306	3.983	—
	(4, 0.033)		(0.084)	(0.109)	(0.137)	(—)
3	1.800	1	4.284	4.723	5.431	—
	(3, 0.167)		(0.218)	(0.475)	(0.122)	(—)
4	1.733	2	4.314	4.206	4.079	—
	(4, 0.100)		(0.224)	(0.293)	(0.340)	(—)
5	1.667	1	4.401	4.627	5.555	4.510
	(7, 0.067)		(0.066)	(0.336)	(0.102)	(0.089)
6	1.700	2	4.369	4.239	4.248	4.450
	(3, 0.167)		(0.268)	(0.234)	(0.302)	(0.221)
7	1.800	2	4.369	4.246	4.148	4.200
	(5, 0.133)		(0.185)	(0.126)	(0.304)	(0.265)
8	1.467	1	4.571	4.650	5.566	4.580
	(4, 0.067)		(0.295)	(0.273)	(0.257)	(0.104)
9	1.350	2	4.401	4.372	4.230	4.250
	(5, 0.033)		(0.247)	(0.266)	(0.142)	(0.158)
10	1.600	1	4.616	4.703	5.608	4.560
	(3, 0.133)		(0.285)	(0.209)	(0.237)	(0.139)
11	1.800	1	4.452	4.620	5.688	4.650
	(7, 0.067)		(0.110)	(0.347)	(0.253)	(0.146)
12	1.300	1	4.696	4.760	5.628	4.620
	(3, 0.067)		(0.240)	(0.278)	(0.284)	(0.125)
13	1.733	2	4.478	4.346	4.482	4.510
	(5, 0.067)		(0.263)	(0.264)	(0.195)	(0.192)
14	1.100	1	4.703	4.767	5.664	4.660
	(4, 0.000)		(0.229)	(0.305)	(0.401)	(0.204)
15	1.400	1	4.573	4.923	5.864	4.690
	(6, 0.000)		(0.219)	(0.367)	(0.364)	(0.152)
16	1.167	1	4.725	4.760	5.806	4.690
	(7, 0.000)		(0.275)	(0.316)	(0.330)	(0.129)

Table A.1f: Treatment 2, Experiments 11 to 15.
Conditioning point–$(4, 4)$, Rubinstein solution–$(8, 2)$

Real Game	Mean Number of Offers	Type 1st P	Mean Undiscounted X-coordinates of Proposals: Accepted	First	Adaptive Forecast	Best Model (from earlier data)
1	1.300	2	7.219	6.937	7.521	—
	(6, 0.000)		(0.635)	(0.554)	(0.132)	(—)
2	1.300	2	7.149	6.792	7.444	—
	(4, 0.000)		(0.357)	(0.279)	(0.093)	(—)
3	1.233	1	6.520	6.667	8.077	—
	(3, 0.000)		(0.644)	(0.546)	(0.542)	(—)
4	1.900	2	7.015	6.278	6.912	—
	(4, 0.000)		(0.596)	(1.052)	(0.389)	(—)
5	1.700	1	6.709	6.977	7.803	6.850
	(7, 0.033)		(0.414)	(0.613)	(0.647)	(0.756)
6	1.633	2	7.068	6.404	6.722	6.730
	(3, 0.067)		(0.508)	(0.708)	(0.515)	(0.518)
7	1.767	2	6.926	6.631	6.677	6.690
	(5, 0.067)		(0.545)	(0.584)	(0.600)	(0.607)
8	1.767	1	6.951	6.970	7.814	7.050
	(4, 0.133)		(0.792)	(0.946)	(0.500)	(0.432)
9	1.500	2	6.892	6.671	6.450	6.470
	(5, 0.033)		(0.447)	(0.450)	(0.743)	(0.734)
10	1.533	1	7.156	7.530	7.768	7.130
	(3, 0.033)		(0.417)	(0.348)	(0.434)	(0.448)
11	1.533	1	7.085	7.160	7.726	7.200
	(7, 0.067)		(0.529)	(0.542)	(0.574)	(0.366)
12	1.233	1	7.171	7.247	7.812	7.200
	(3, 0.067)		(0.375)	(0.427)	(0.523)	(0.596)
13	1.433	2	7.064	6.551	6.930	6.950
	(5, 0.033)		(0.645)	(0.661)	(0.505)	(0.504)
14	1.033	1	7.030	7.057	8.012	7.270
	(4, 0.000)		(0.548)	(0.521)	(0.514)	(0.382)
15	1.067	1	7.206	7.260	7.892	7.180
	(6, 0.000)		(0.489)	(0.539)	(0.291)	(0.315)
16	1.300	1	7.147	7.397	7.840	7.260
	(7, 0.000)		(0.538)	(0.500)	(0.697)	(0.493)

Table A.1g: Treatment 3, Experiments 11 to 15.
Conditioning point–$(8, 2)$, Rubinstein solution–$(8, 2)$

Real Game	Mean Number of Offers	Type 1st P	Mean Undiscounted X-coordinates of Proposals: Accepted	First	Adaptive Forecast	Best Model (from earlier data)
1	1.233	2	4.021	3.926	3.390	—
	(6, 0.000)		(0.127)	(0.123)	(0.125)	(—)
2	1.333	2	4.035	3.886	3.381	—
	(4, 0.033)		(0.116)	(0.117)	(0.130)	(—)
3	1.533	1	4.136	4.347	4.539	—
	(3, 0.033)		(0.132)	(0.529)	(0.112)	(—)
4	1.367	2	4.027	3.853	3.424	—
	(4, 0.033)		(0.094)	(0.093)	(0.070)	(—)
5	1.567	1	4.096	4.233	4.544	4.230
	(7, 0.033)		(0.089)	(0.190)	(0.095)	(0.027)
6	1.500	2	4.030	3.846	3.410	3.920
	(3, 0.100)		(0.121)	(0.096)	(0.153)	(0.027)
7	1.167	2	3.945	3.839	3.519	3.920
	(5, 0.000)		(0.070)	(0.068)	(0.187)	(0.027)
8	1.300	1	4.181	4.213	4.584	4.250
	(4, 0.033)		(0.126)	(0.171)	(0.151)	(0.000)
9	1.233	2	3.997	3.906	3.519	3.950
	(5, 0.000)		(0.069)	(0.076)	(0.165)	(0.000)
10	1.267	1	4.105	4.147	4.584	4.230
	(3, 0.000)		(0.064)	(0.088)	(0.080)	(0.027)
11	1.533	1	4.153	4.080	4.764	4.250
	(7, 0.033)		(0.094)	(0.043)	(0.103)	(0.000)
12	1.333	1	4.120	4.150	4.656	4.250
	(3, 0.067)		(0.086)	(0.103)	(0.151)	(0.000)
13	1.200	2	4.078	3.972	3.376	3.940
	(5, 0.000)		(0.239)	(0.295)	(0.143)	(0.022)
14	1.267	1	4.190	4.287	4.548	4.250
	(4, 0.033)		(0.194)	(0.234)	(0.161)	(0.061)
15	1.767	1	4.283	4.203	4.782	4.260
	(6, 0.033)		(0.109)	(0.084)	(0.040)	(0.022)
16	1.367	1	4.196	4.150	4.746	4.270
	(7, 0.000)		(0.249)	(0.120)	(0.080)	(0.027)

Table A.1h: Treatment 4, Experiments 11 to 15.
Conditioning point–$(4, 4)$, Rubinstein solution–$(4, 4)$