Recovering the initial state of dynamical systems using observers

Ghislain Haine

Institut Supérieur de l'Aéronautique et de l'Espace (ISAE)
Department of Mathematics, Computer Science, Control
Toulouse, France

CDPS 2013 – July, 1-5

Let

- X and Y be Hilbert spaces,
- $A : \mathcal{D}(A) \to X$ be a skew-adjoint operator,
- $C \in \mathcal{L}(X, Y)$ be an observation operator,
- and $\tau > 0$ be a positive real number.

Conservative systems

$$\begin{cases} \dot{z}(t) = Az(t), & \forall\, t \in [0, \infty), \\ z(0) = z_0 \in X. \end{cases}$$

Observation

We observe z *via* $y(t) = Cz(t)$ for all $t \in [0, \tau]$.

Let

- X and Y be Hilbert spaces,
- $A : \mathcal{D}(A) \to X$ be a skew-adjoint operator,
- $C \in \mathcal{L}(X, Y)$ be an observation operator,
- and $\tau > 0$ be a positive real number.

Conservative systems

$$\begin{cases} \dot{z}(t) = Az(t), & \forall\, t \in [0, \infty), \\ z(0) = z_0 \in X. \end{cases}$$

Observation

We observe z *via* $y(t) = Cz(t)$ for all $t \in [0, \tau]$.

Let

- X and Y be Hilbert spaces,
- $A : \mathcal{D}(A) \to X$ be a skew-adjoint operator,
- $C \in \mathcal{L}(X, Y)$ be an observation operator,
- and $\tau > 0$ be a positive real number.

Conservative systems

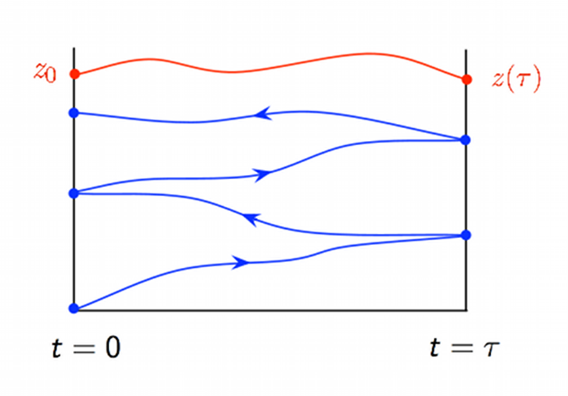
$$\begin{cases} \dot{z}(t) = Az(t), & \forall\, t \in [0, \infty), \\ z(0) = z_0 \in X. & \end{cases}$$

Observation

We observe z *via* $y(t) = Cz(t)$ for all $t \in [0, \tau]$.

K. Ramdani, M. Tucsnak, and G. Weiss, *Recovering the initial state of an infinite-dimensional system using observers*, Automatica, 46 (2010), pp. 1616–1625.

Intuitive representation



2 iterations, observation on $[0, \tau]$.

If the system is exactly observable in time τ, we can take for all $\gamma > 0$

$$\begin{cases} \dot{z}_n^+(t) = Az_n^+(t) - \gamma C^* C z_n^+(t) + \gamma C^* y(t), & \forall\ t \in [0, \tau], \\ z_0^+(0) = z_0^+ \in X, \\ z_n^+(0) = z_{n-1}^-(0), \end{cases}$$

$$\begin{cases} \dot{z}_n^-(t) = Az_n^-(t) + \gamma C^* C z_n^-(t) - \gamma C^* y(t), & \forall\ t \in [0, \tau], \\ z_n^-(\tau) = z_n^+(\tau), \end{cases}$$

and then there exists $\alpha \in (0, 1)$ such that

$$\|z_n^-(0) - z_0\| \leq \alpha^n \|z_0^+ - z_0\|.$$

In this work **we do not suppose any observability assumption**.

Then two questions arise naturally:

1. **Given arbitrary** C **and** $\tau > 0$**, does the algorithm converge ?**
2. **If it does, what is** $\lim_{n\to\infty} z_n^-(0)$**, and how is it related to** z_0 **?**

Main result

We answer these questions, and prove what the intuition suggests.

In this work **we do not suppose any observability assumption**.

Then two questions arise naturally:

1. **Given arbitrary** C **and** $\tau > 0$**, does the algorithm converge ?**
2. **If it does, what is** $\lim_{n\to\infty} z_n^-(0)$**, and how is it related to** z_0 **?**

Main result

We answer these questions, and prove what the intuition suggests.

In this work **we do not suppose any observability assumption**.

Then two questions arise naturally:

1. **Given arbitrary C and $\tau > 0$, does the algorithm converge ?**
2. **If it does, what is $\lim_{n\to\infty} z_n^-(0)$, and how is it related to z_0 ?**

Main result

We answer these questions, and prove what the intuition suggests.

Thanks for your attention !

G. Haine, *Recovering the observable part of the initial data of an infinite-dimensional linear system with skew-adjoint operator*, Mathematics of Control, Signals, and Systems (MCSS), *In Revision*.