One Model for One Frequency Range : Comparison of Bond Graph Based Simplification Methods

G. Gandanegara[1], X. Roboam[1], B. Sareni[1], G. Dauphin Tanguy[2]

[1]LEEI, UMR INPT-ENSEEIHT/CNRS N° 5828
2 Rue Camichel BP 7122, 31071 Toulouse Cedex, France
Xavier.Roboam@leei.enseeiht.fr

[2]LAIL, Ecole Centrale de Lille, UMR CNRS N° 8021
BP 48, 59651 Villeneuve d'Ascq Cedex, France
gdt@ec-lille.fr

Abstract

Time constants of electromechanical systems are spread along a large bandwidth (from microseconds to minutes : 10^8 ratio). This issue makes difficult their analysis and increases the simulation time all the more than a system approach is considered. When only a small time interval is observed, the influence of slow modes may be neglected and vice versa. In this framework, simplifying a model with respect to the frequency range becomes really interesting. In order to simplify the model of complex devices, two model simplification methods have been applied directly on bond graphs. The Singular Perturbation Method (SPM), based on the dynamic analysis of elements, allows one to obtain several models, each one being dedicated to a class of dynamics and being simpler than the initial model. The Model Order Reduction Algorithm (MORA) is an energy-based procedure, which allows the deduction of a simplified model by suppressing the elements that are energetically inactive. Moreover, since the energy of an element depends on the frequency of the input signals, a simplified model can be obtained for the frequency range corresponding to the input signal spectrum. A railway traction system is used as our case study to compare these methods.

INTRODUCTION

A physical system can be modeled in a unified way and analyzed by using bond graph formalism [1]. When a model has time constants spread along a very large bandwidth (for example, in an electromechanical model, the time constants can be from several microseconds to several minutes, giving a ratio of 10^8 [2]), it is not obvious to analyze it. Thus, finding a model which is both simple and "just enough accurate", minimizes the simulation time while appropriate responses are maintained with respect to the goal of the run. It also contributes to clarify the analysis. This model can be obtained by simplifying the global model. Several simplification methods can be applied directly on bond graphs. Two model simplification methods will be presented and compared in this paper.

- The first method is the Singular Perturbation Method based on the dynamic analysis of elements [3][4]. This method allows one to obtain several models from a multi-time scale model, each one being dedicated to a class of dynamics where each one is simpler than the initial model.

- The second method, called MORA (Model Order Reduction Algorithm), is an energy-based procedure [5][6]. The reduced model can be obtained by suppressing the elements which are not energetically active. By using this method, we can obtain a simplified model suitable for specific input signals. Furthermore, because the energy of an element depends on the frequency of input signals, we can deduce a reduced model for the specific frequency range of input signals. For this purpose, we use several signal types such as step signals, sine waves, and sweep signals (chirp) as model inputs to determine the frequency characteristics of the initial model.

We will present these methods in the following sections. The application and comparison of these methods is illustrated on the model of a railway traction system [7]. Several simulations are carried out in order to verify our simplifications.

SINGULAR PERTURBATION METHOD

A multi-time scale bond graph can be simplified by using the singular perturbation method [3][4]. This method is based on the dynamic analysis of elements and input

signals. The dynamic elements (I and C) are classified in different categories of dynamics (fast,…slow). To determine the dynamic of elements, we examine all causal loops in the bond graph. By calculating the gain and the associated time constant (or natural period) of a causal loop, we can estimate the corresponding dynamic. Fast elements are those which have causal loops with small time constants or high harmonic pulsation. The slow ones are those with the opposite characteristics.

After determining the dynamics of all the elements, we separate the global model in several models (fast,…slow), each one being dedicated to a class of dynamics. The procedure to obtain these models is described in [3][4]. These models are all simpler than the global model.

MODEL ORDER REDUCTION ALGORITHM

The Model Order Reduction Algorithm (MORA) is a simplification method based on energy [5][6]. This method uses the concept of *energy activity*. The energy activity A_i of a passive element i is calculated as follows :

$$A_i = \int_{t_0}^{t_1} |P_i(t)|\, dt = \int_{t_0}^{t_1} |e_i(t).f_i(t)|\, dt \qquad (1)$$

where e_i and f_i represent the effort and the flow in the ith element. We calculate activity of all passive elements in the model during a time window (from t_0 to t_1) fixed by the modeler. It is interesting to apply this method on a bond graph because the bond which links elements brings simultaneously the information of the effort and the flow. By normalizing the element activity with the total activity A_{total} in the model, we can obtain the *activity index AI_i* of an element i as :

$$AI_i = \frac{A_i}{A_{total}} \quad \text{with} \quad A_{total} = \sum_{i=1}^{n_k} \int_{t_0}^{t_1} |P_i(t)|\, dt \qquad (2)$$

where n_k denotes the number of passive elements in the model. The activity index of an element represents the significance of this element in the model.

We can distinguish the application of this method on linear and nonlinear systems. For linear systems, the obtained reduced model is associated to a set of input signals that gives a set of models, each one being suitable for a frequency bandwidth [5]. For nonlinear systems, we can only obtain a reduced model associated with the given inputs [6].

To reduce the initial model with respect to the frequency bandwidth, we can calculate element activities by analytical expressions or by simulating the process. For a simple model, the expressions will not be too complex. However, when the system is of high-order, it can be difficult to calculate energy activities analytically. In this case, the problem is processed by simulating the model. Sine waves that sweep all frequencies below the desired

bandwidth ω_b ($f < \omega_b$) are used as input signals. The procedure to be applied is described in [5]

To obtain a reduced model corresponding to the given inputs, the procedure consists of :
1. Simulating the initial model and calculating the activity and activity index for all passive elements during the desired duration.
2. Sorting the activity indexes in decreasing order from elements with high activity to those having low activity.
3. Summing the sorted activity indexes starting from the highest one (the most important element) until a specified *threshold* is reached. This threshold is the total activity that the modeler wants to include in the reduced model. The value of the threshold is related to the engineering specifications of the system.
4. Separating elements in 2 groups : the first one consists of elements contributing in this threshold. The other elements are classified in the second group and will be suppressed in the simplified model.
5. Constructing a reduced model by keeping the elements of the first group.

The activity depends on the frequency of input signals. Therefore, if we want to obtain a reduced model for a specific frequency range, it is necessary to use input signals which really excite the model in this desired frequency range. Several types of signals are studied in this paper. In particular, step signals, sine waves and sweep waves (chirp) can be applied to determine the frequency characteristics of our model.

APPLICATION TO A RAILWAY TRACTION DEVICE

For our case study, a model of a railway traction device is considered. This system consists of electrical parts such as LCR input filter, DC-AC voltage source inverter feeding an induction motor coupled with a mechanical transmission line. A complete description of this model can be found in [7][8]. The induction motor is modeled by using the leakage inductance related with stator. The symbols LS and LM represent the total leakage inductance located to the stator and the magnetizing inductance. The associated bond graph is displayed in Figure 1. This model is rather complex (17th order) and both the electrical and mechanical natural frequency are very close to themselves ($f_{reson} \sim 18$ Hz) creating high level interactions in the system. Therefore, simplifying the model will be very interesting to facilitate analysis of couplings.

Simplifying the mechanical transmission line

In [4], we showed the application of the Singular Perturbation Method on the mechanical transmission line.
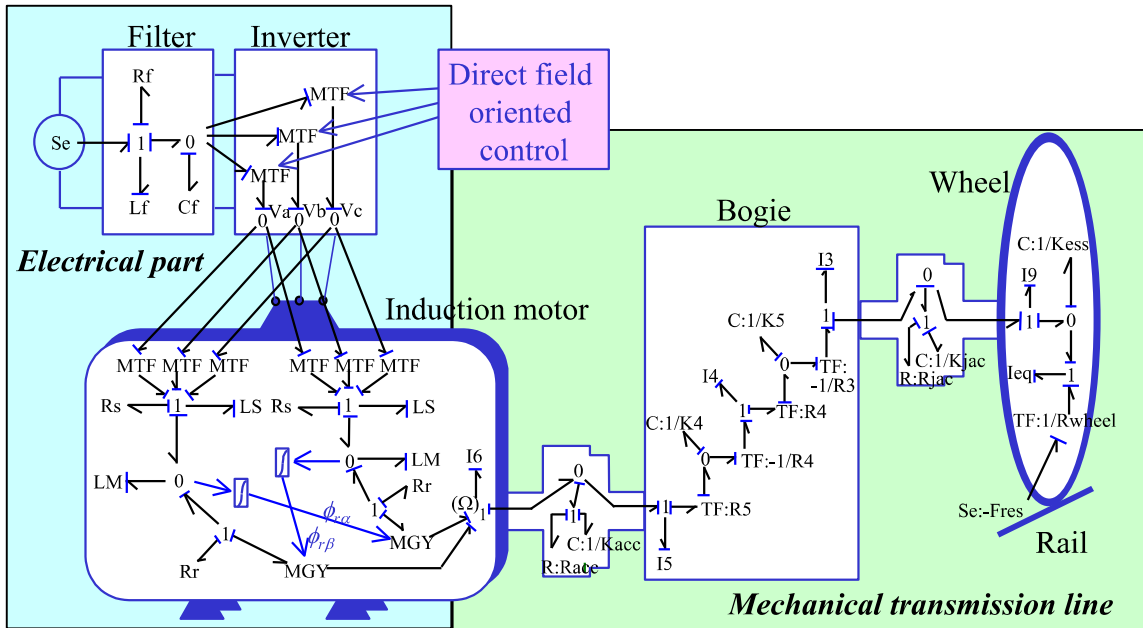
Figure 1. Associated bond graph of the railway traction system.

We have started with this reduced part of the device instead of considering the whole system because it includes the major part of dynamic elements. Furthermore, the model of this mechanical line being linear, it constitutes an important particular case study. The bond graph of this transmission line is presented in Figure 2. With this method, we reduce the model order from 11 to 3, i.e. from 17 to 9 for the global electromechanical model. The simplified model by means of SPM is described in Figure 3.

The MORA is applied to this transmission line. Two physical inputs can be considered in this case i.e. the electromagnetic torque (T_{em}) and the resistive force on the wheel (F_{res}). This model is linear, so that we can deduce a simplified model following the bandwidth (see Figure 4). The complexity of the reduced model increases with the bandwidth. Note that the reduced model strongly depends on the specified threshold. This parameter must be carefully tuned.

The reduced model is valid for all frequencies below the bandwidth. Thus, obtaining a high frequency model is impossible by means of this procedure due to the fact that low frequencies signals are always considered as inputs. Therefore, we have to use a modified approach of the MORA procedure : "simplification with respect to input signals".

To obtain a reduced model for a specific frequency range, step signals, sine waves and sweep waves (chirps) are used as input signals to excite the natural frequencies of the model. A step signal can excite a broadband frequency range which depends on the duration of this signal. Always exciting the LF (Low Frequency) elements is the main

drawback of using the step response, as HF (High Frequencies) cannot be separately excited. On the other hand, sine waves (and chirp signals) can excite a specific (range of) frequency.

On this mechanical transmission line, the following LF excitations have been applied on the electromagnetic torque, here considered as the input :

- step signal during 1 s ($T = 1$ s)
- 10 rad/s pulsation sine wave
- chirp signal whose pulsation is from 0 to 50 rad/s (the maximum value of this frequency range is below the system resonant mode : $\omega_{reson} \sim 117$ rad/s).
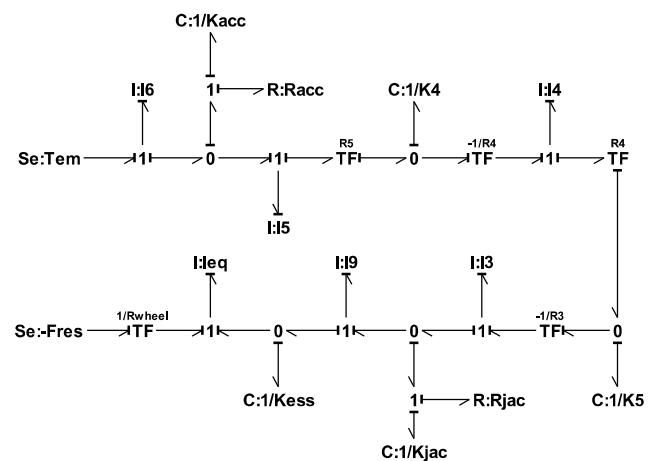

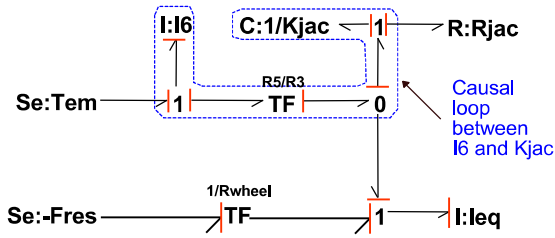**Figure 2.** Bond graph of mechanical transmission line.

Figure 3. Mechanical transmission line : simplified model obtained by the SPM

Elements in the reduced model :

2 : I_{eq} and I_6

3 : I_{eq}, I_6, K_{jac}

4 : I_{eq}, I_6, K_{jac}, R_{jac}

5 : I_{eq}, I_6, K_{jac}, R_{jac}, K_5

6 : I_{eq}, I_6, K_{jac}, R_{jac}, K_4, I_3 (o) or K_5 (×)

7 : I_{eq}, I_6, K_{jac}, R_{jac}, K_4, K_5, I_3

8 : I_{eq}, I_6, K_{jac}, R_{jac}, K_4, K_5, I_3, I_4

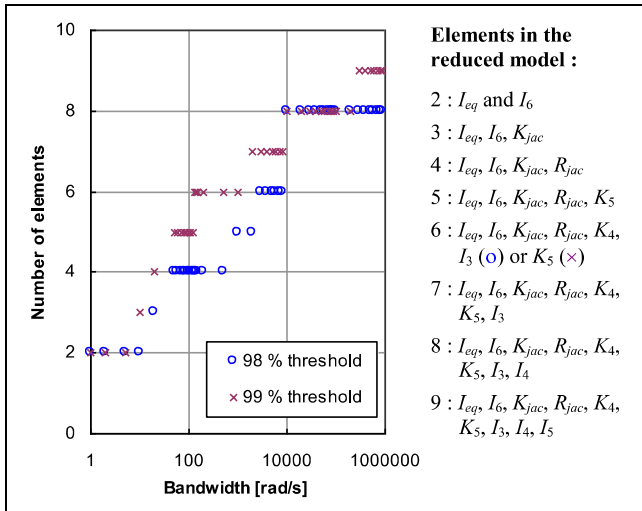9 : I_{eq}, I_6, K_{jac}, R_{jac}, K_4, K_5, I_3, I_4, I_5

Figure 4. Important elements in function of bandwidth.

The element activity indexes are presented in Figure 5.(a). The reduced model for this type of excitation has to consist of I_6, I_{eq}, K_{jac} and R_{jac}. This simplified model is the same as the one obtained by the Singular Perturbation Method (see Figure 3).

Coupling Analysis : in this model, we can find a causal loop between I_6 and K_{jac} which is linked to the natural pulsation corresponding to the resonance frequency of the mechanical transmission line (see [4] for details). Let notice that this causal loop did not exist in the complete model. Thus, we can underline the advantage of simplification methods : reducing the model order helps to reveal the elements responsible for system perturbations such as resonance.

HF inputs characterized by a chirp signal (from 1000 to 2000 rad/s) and a sine wave (1000 rad/s) are applied to obtain the HF model. All elements are important, except I_9, K_{acc}, K_{ess} and R_{acc}.(see Figure 5.(b)) The reduced model is presented in Figure 6.

Several simulations have been carried out in order to validate the simplified models (see Figure 7). For a HF excitation, the response of the HF model obtained by MORA is similar to the response of the full model. For a LF excitation, the behaviors of the initial and reduced model are also identical.

It should be underlined that no DC component has been superposed on the excitations, contrary to the next section where the perturbation signals will be added to the normal operating point of the traction system.
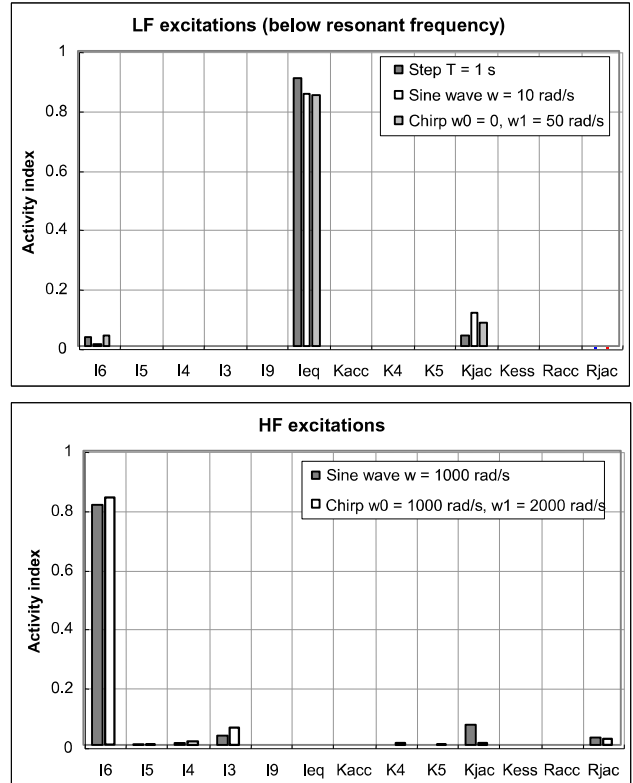

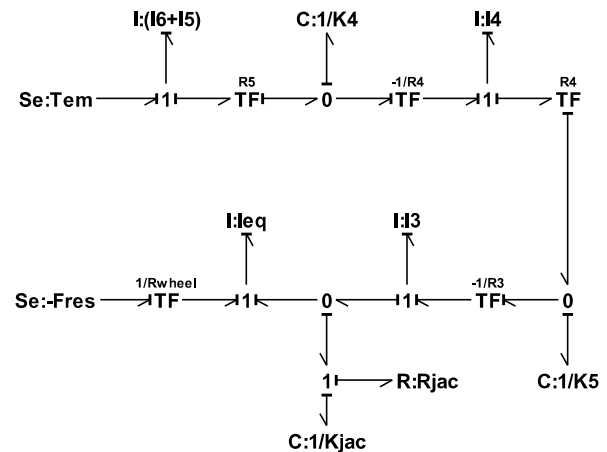**Figure 5.** Mechanical transmission : element activity index for (a) LF excitations and (b) HF excitations.

Figure 6. Reduced model of mechanical transmission line for HF excitations.
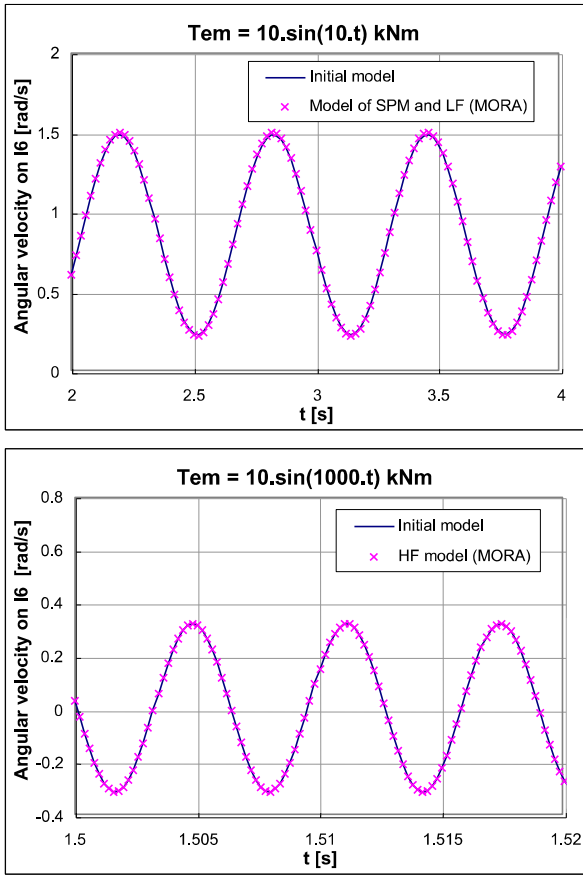
Figure 7. Mechanical transmission line : comparison between initial and reduced model for (a) LF and (b) HF input signal at $Se : T_{em}$.

Simplifying the global model

We presented in the previous section the application of the MORA compared with SPM procedures on the linear model of the mechanical transmission line. On the opposite of the latter case study, the whole model of the locomotive is a nonlinear system. The nonlinear model is due to the controller that commands the DC-AC inverter (the control signals modulate transformers for each inverter leg) and also to the gyrators which represent the multiplication of the rotor currents and the rotor fluxes for the electromechanical transformation. In such a case, we can only obtain reduced models for given inputs. This system is normally excited by three inputs : U_{cont}, T_{ref}, F_{res}.

We present in Figure 8 the activity index of each element for a normal operating mode. A step signal is given as torque reference (T_{ref}) after having obtained the desired flux value. We can see that a brief step signal ($T = 0.1$ s) excites the mechanical elements underlined in the previous section (I_6, I_{eq}, K_{jac} and R_{jac}). Their activity indexes decrease (except for I_{eq}) for longer step duration.

In the next simulation results, only the DC source U_{cont} is considered as the model input. Using this input (U_{cont}) requires to superpose two different signal components :
- a continuous component ($U_{cont} = 2750$ V) which corresponds to the normal operating mode of the system (feeding of the voltage source inverter).
- a perturbation input (sine wave or chirp signal) that corresponds to the excitation signal for the MORA.

Both LF and HF perturbations are used to excite the system. LF perturbations are inside a frequency range nearby the resonance frequency of the filter and the mechanical transmission ($f_{reson} = 18.7$ Hz, associated with a pulsation of 117 rad/s). The perturbations have an amplitude of 3 % of the continuous value. By applying a threshold of 99.8 %, we observed the importance of the electrical elements. As a result, no electrical elements can be neglected. On the other hand, all elements in the mechanical transmission line "are supposed" to be negligible with this excitation input, if we except I_6 and I_{eq} which are included in the reduced model for a perturbation at the resonance frequency of the transmission line.

Based on these latter results, major conclusions and restrictions are required to process the model reduction from the MORA method :

- When the excitation signals are superimposed on the normal operating of the system, "masking effects" can sometimes be observed. This can be due to the small amplitude of perturbations with respect to the continuous value. Therefore, mechanical elements that are very important when excitations have no continuous value (see Figure 5) have a small influence when a normal operating is added. Note that in this particular example, chirp or sine signals with higher amplitudes (or without a continuous component) cannot be directly applied on the global model at the continuous input voltage U_{cont} since the control system could become unstable.

- The mechanical elements are situated far from the U_{cont} excitation, whose influence is here attenuated successively by the input filter, the induction machine (inductance) and finally by the motor inertia (I_6). More generally, when a multi input system is considered, every input must be excited to implement the MORA and to deduce the reduced model.

CONCLUSIONS

In this paper, several simplification methods have been presented and applied on a complex railway traction system. The SPM based on dynamic analysis of elements does not need any simulation and gives a direct indication of elements that can be neglected in the model. Generally, the MORA excited with specific input signals such as chirp, step or sine waves allows the validation of the hypothesis of the SPM and improves model simplification
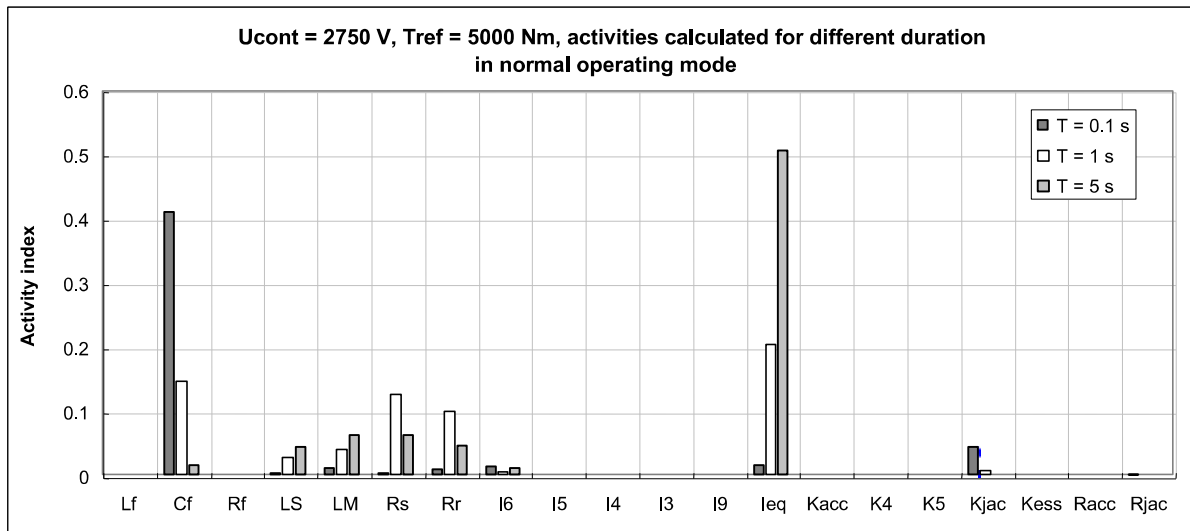
Figure 8. Element activity indexes for different duration of the torque reference in a normal operating mode.

in a specific frequency range. Nevertheless, some particular aspects of the MORA must be underlined. On the one hand, the threshold that specifies the elements to be suppressed must be carefully tuned since it considerably influences the topology of the reduced model. On the other hand, results strongly depend on the excitation of the system (type of excitation signals and amplitude of excitations, choice of inputs...) and some "masking effects" could appear in some cases due to the normal system operation. Every input of MIMO systems should be excited and considered to determine the reduced model.

In this particular application (i.e. a railway traction system), only the mechanical model can be reduced. However, it has been shown that reducing this model allows simplifying the coupling analysis. Indeed, it has become possible to point out the elements responsible of the main mechanical resonance.

Obviously, this simplification procedure is rather long and complex to handle, but it is certainly "one cost to be paid" to process the design problem at a "system level", the device being considered and analyzed as a whole.

REFERENCES

[1]. Karnopp, D.; D. Margolis; R. Rosenberg. 2000. *System Dynamics : Modeling and Simulation of Mechatronic Systems*. John Wiley & son. 3rd ed.

[2]. Roboam, X.; C. Lochot; B. de Fornel. 1999. "Contribution à la Conception Global d'une Chaîne de Traction Ferroviaire." *La Revue 3EI*, No. 19, 35-45.

[3]. Sueur C. and G. Dauphin-Tanguy. 1991. "Bond Graph Approach to Multi-time Scale Systems Analysis." *Journal of the Franklin Institute*, vol. 328, 1005-1026, Pergamon press.

[4]. Gandanegara G.; X. Roboam; B. Sareni; G. Dauphin-Tanguy. 2001. "Modeling and Multi-time Scale Analysis of Railway Traction Systems Using Bond Graphs." In *Proceedings of International Conference on Bond Graph Modeling and Simulation. ICBGM'01*. Phoenix, Arizona.

[5]. Louca L.S. and J.L. Stein. 1999. "Energy-based Model Reduction of Linear Systems." In *Proceedings of International Conference on Bond Graph Modeling and Simulation. ICBGM'99*. San Francisco, CA.

[6]. Louca L.S.; J.L. Stein; G.M. Hulbert. 1998. "A Physical-based Model Reduction Metric with an Application to Vehicle Dynamics." In *Proceedings of 4th IFAC Nonlinear Control Systems Symposium*. Enschede, The Netherlands.

[7]. Lochot C. 1999. "Modélisation et Caractérisation des Phénomènes Couplés dans Une Chaîne de Traction Ferroviaire Asynchrone." Thèse INP Toulouse, France.

[8]. Lochot C.; X. Roboam; B. de Fornel; F. Moll. 1997. "High Speed Railway Traction System Modelling for Simulating Electromechanical Interactions." *WCRR'97*. Firenze, Italia.