Multiobjective Optimisation by Self-adapting Pareto Genetic Algorithms for Electrical System Design

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Abstract – In this paper, Pareto Genetic Algorithms are applied to solve multiobjective optimisation problems. In particular, a recent version of the non-dominated sorting genetic algorithm (NSGA-II) is presented. A self-adaptive recombination scheme is used for crossover operators to improve the algorithm efficiency. Tests on mathematical functions of various difficulties are carried out to show the robustness of self-adaptation. Finally, the self-adaptive NSGA-II is applied to the optimal design of an electrical system based on a inverter – permanent magnet motor – reducer – load association. It allows to reduce the global losses and weight in the system and help the designer to understand couplings and interactions between design variables in relation to technological constraints and objectives.

I. INTRODUCTION

The purpose of multiobjective optimisation consists in minimising (or maximising) simultaneously several objectives f_i related to common design variables. The main difficulty of a multiobjective problem generally resides in the existence of conflicts between the different objectives. Therefore, there is no point in the design variable space, which leads to a simultaneous minimisation of all objectives. Consequently, the global solution of a multiobjective problem is characterised by a set of solutions expressing the best trade-offs according to each objective. This optimal set represents the Pareto front [HOR, 94]. The traditional approach to solve this class of problems consists in converting the multiobjective problem into a scalar optimisation problem by aggregating or weighting the objectives in a global quality function. This technique is rather hazardous because of the difficulty to find suitable weighting coefficients and normalisation factors to homogenise the different physical criteria in the global quality function. Furthermore, complete optimal front determination requires to solve a set of scalar problems with different weighting factors.

Since the mid-1990s, there has been a growing interest in solving multiobjective problems by Genetic Algorithms (GA's) [HOR, 94] [SRI, 95] [ZIT, 99] [DEB, 00]. GA's are capable of searching for multiple Pareto-optimal solutions in parallel from a single run.

Section II presents the fundamentals of Pareto Genetic Algorithms to solve multiobjective problems and refers to the second version of the non-dominated sorting genetic algorithm as example. Section III examines a self-adaptive recombination scheme to improve Pareto Genetic Algorithms efficiency. Tests on standard multiobjective problems of the literature are presented in section IV to show the interest of self-adaptation. Finally, section V illustrates the application of Pareto Genetic Algorithms to the optimal design of electrical systems.

II. PARETO GENETIC ALGORITHMS

In the past, standard GA's have been successfully used to find the global solution of single objective problems [HOL, 75][GOL, 89]. More recently, *niching methods* have been developed to minimise the effect of *genetic drift* resulting from the selection operator in the traditional GA and allow the parallel investigation of multiple solutions in the population [SAR, 98]. GA with niching can be modified to find the Pareto front of a multiobjective problem by using a specific selection operator based on a Pareto domination criterion. Assume a minimisation problem with n objectives and consider two vectors X, Y from the parameter space. Then, X is said to *dominate* Y [HOR, 94][ZIT, 99] iff:

$$\forall i = 1...n : f_i(X) \le f_i(Y) \ \ and \ \ \exists j \in 1...n \Rightarrow f_j(X) < f_j(Y)$$

Using tournaments based on this domination rule for the selection operator, GA with niching is able to approximate the solution of a multiobjective problem by distributing its population along the Pareto front. These algorithms are referred to Multi-Objective Genetic Algorithms (MOGA) or Pareto Genetic Algorithms (Pareto GA).

From a randomly initialised population, a Pareto GA evaluates the non-dominated solutions and preserves them in a specific archive (non-dominated set). At each generation, Pareto tournaments are used to select individuals from the archive to create the mating pool (*parents* of the current generation). Parents are crossed and mutated to explore new solutions (*children* of the current generation). The population of children and the archive are merged to assess the non-dominated set of the next generation. If the number of non-dominated individuals is higher than the size of the archive, a *clustering* method is used to preserve most representative solutions and eliminate others in order to keep a constant archive size. Note also that niching is used in the selection scheme when individuals involved in a tournament have the same domination rank. The structure of a Pareto GA is depicted in Fig. 1.

The second version of the non-dominated sorting genetic algorithm (NSGA-II) is based on the principles of Pareto GA's previously exposed. In the NSGA-II, selection is performed with Pareto ranking tournaments associated with a crowded comparison operator to induce niching in the objective space. NSGA-II determines all successive fronts in the population (the best front corresponding to the non-dominated set). Moreover, a *crowding distance* is used to estimate the density of solutions surrounding each individuals on a given front. In a tournament, if individuals belong to the same front, the selected one is that with the greater crowding distance. This niching index is also used in the clustering operator to distribute uniformly the individuals on the Pareto front. All details of the algorithm can be found in [DEB, 00]

III. Self-Adaptive Recombination Scheme

Most of researches in the field of Pareto GA's have been concentrated on selection, elitism and niching operators. Only few works have been done on recombination procedures. In this paper, we examine the efficiency of three different crossovers for real encoded GA's i.e. the simulated binary crossover (SBX), the blend crossover (BLX) [ESH, 93] and the crossover used in the Breeder Genetic Algorithm (BGAX) [SCH, 94]. Finally, a self-adapting scheme is proposed to help the GA to use the most suitable crossover operator in relation to the characteristics of objective functions during the search.

A. The blend crossover (BLX-α)

From two parent solutions p_1^i and p_2^i, the BLX-α crossover creates one child c^i as follows :

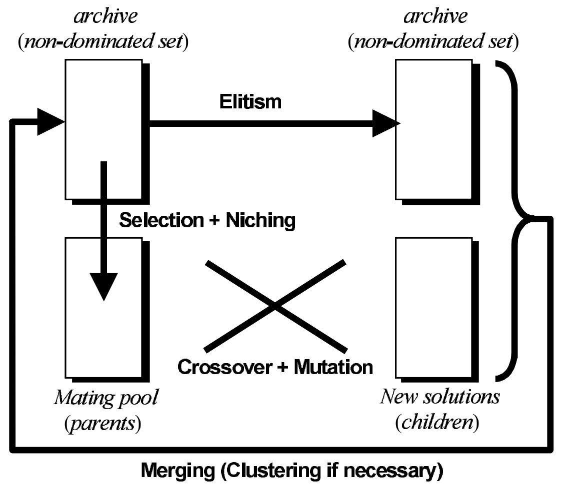

Fig. 1. Structure of a Pareto GA (one step generation)

$$c^i = p_1^i + \beta(p_2^i - p_1^i) \quad (1)$$

where β is a random variable in the interval $[-\alpha, 1+\alpha]$, i denoting the index related to parameters of the child and parents solutions. If α is set to zero, this crossover creates a random solution inside the range defined by the parents similarly to the arithmetical crossover [MIC, 92]. Eshelman and Schaffer have reported that BLX-0.5 (with $\alpha = 0.5$) performs better than BLX with any other α value in a number of test problems [ESH, 93].

B. The simulated binary crossover (SBX)

SBX operator simulates the working principle of the single point crossover operator on binary strings. From two parent solutions p_1^i and p_2^i, it creates two children c_1^i and c_2^i as follows :

$$\begin{cases} c_1^i = 0.5[(1+\beta)p_1^i + (1-\beta)p_2^i] \\ c_2^i = 0.5[(1-\beta)p_1^i + (1+\beta)p_2^i] \end{cases} \quad (2)$$

with a spread factor β defined by (3),

$$\beta = \begin{cases} (2u)^{\frac{1}{\eta+1}} & \text{if } u < 0.5 \\ \left(\dfrac{1}{2(1-u)}\right)^{\frac{1}{\eta+1}} & \text{otherwise} \end{cases} \quad (3)$$

where u is a random variable in the interval [0,1] and η is a nonnegative real number that characterises the distribution of the children in relation to their parents. A large value of η gives a higher probability for creating children near parents. Acting alone and without any mutation scheme, SBX presents interesting properties of self-adaptation similarly to Evolution Strategies [DEB, 99].

C. The Breeder GA crossover (BGAX)

From two parent solutions p_1^i and p_2^i, the BGAX crossover creates one child c^i as follows :

$$c^i = p_1^i \pm \frac{(p_2^i - p_1^i)}{\left\| p_2^i - p_1^i \right\|} \Delta_i \delta \quad (4)$$

where Δ_i is normally set to 0.5 time the domain definition of the parameter i and the metric denotes the Euclidean distance in the parameter space. δ is computed from a distribution that favours small values :

$$\delta = 2^{-ku} \quad (5)$$

where u is a random variable in the interval [0,1], the precision constant k being typically set to 16. Note that in [SCH, 94], the child was placed more often in the direction to the best parent, p_1^i being the parent with the better fitness and the minus sign in (4) was chosen with probability 0.9. In our work, we decide to not favour any parent (the choice of p_1^i and the sign in (4) are made with a probability 0.5).

In Fig. 2, the probability density function per child for the three investigated crossover operators is depicted. The corresponding parents p_1 and p_2 are marked with a full circle. Note that BGAX with k=16 essentially reinforces local exploration since it tends to create children in the neighbourhood of their parents.

D. *Towards a self-adapting recombination scheme*

As it is not possible to *a priori* know which crossover operator will be the most efficient on a specific problem, we propose a self-adaptive scheme similar to that of Spears for binary encoded GA's [SPE, 95]. It consists in associating in the chromosome of individuals an additional gene (X-*gene*) that codes the type of crossover to apply during the recombination. When recombining two parents, the type of crossover to operate is chosen randomly from the *X-gene* of the parents. Using this procedure, the GA will favour the crossover that produces the best children through the selection operator. To avoid premature convergence to a particular type of crossover, the X-*gene* also undergoes mutation.

IV. Mathematical Tests

A. *Test functions*

We consider three multiobjective problems of the literature [DEB, 00][ZIT, 98] displayed in Table I.

EC4 is a multimodal continuous problem which contains 21^9 local Pareto fronts. The global Pareto front is obtained with g=1 and is convex. EC6 has a non-uniformly distributed search space with solutions non-uniformly distributed along the Pareto front (the front is biased for solutions for which $f_1(x_1)$ is close to one). The Pareto front is obtained with g=1 and is non-convex. SCH is a generalisation of the Schaffer's problem. It is characterised by a large variable space domain and a convex Pareto front.

B. *Performance criteria*

To assess the efficiency on the previous test problems of the NSGA-II with the crossover operators presented in section III, we propose different performance criteria :

- Average deviation to the Pareto-optimal front

The average distance of the non-dominated set to the Pareto-optimal front $\bar{\varepsilon}$ is computed as follows [ZIT, 99]:

$$\bar{\varepsilon} = \frac{1}{|F|}\sum_{a \in F} \min\left\{ \| a - a^* \| \quad a^* \in F^* \right\} \qquad (6)$$

where F (respectively F*) denotes the non-dominated set in the final population (respectively the theoretical Pareto-optimal front), a and a* belonging to each subset. The metric in (6) is the Euclidean distance computed in the objective space.

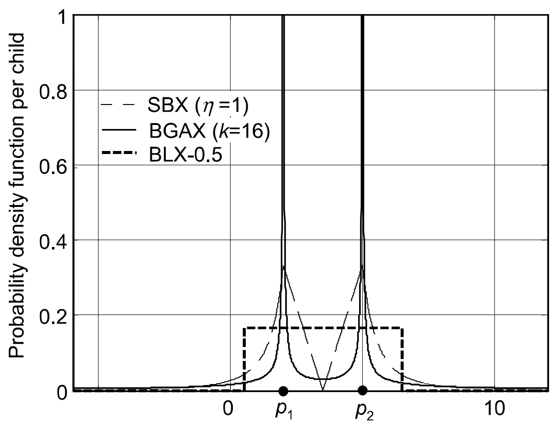

Fig. 2. Probability distribution of children solutions

TABLE I : MULTIOBJECTIVE TEST PROBLEMS

Problem	Characteristics
EC4	$f_1(x) = x_1 \quad 0 \le x_1 \le 1$ $f_2(x) = g\left(1 - \sqrt{\frac{x_1}{g}}\right) \quad -5 \le x_i \le 5 \quad i = 2,...,10$ where $g = 91 + \sum_{i=2}^{10} (x_i^2 - 10\cos(4\pi x_i))$
EC6	$f_1(x) = 1 - \exp(-4x_1)\sin^6(6\pi x_1)$ $f_2(x) = g(1-(f_1/g)^2) \quad 0 \le x_i \le 1 \qquad i = 1,...,10$ where $g = 1 + 9\left(\sum_{i=2}^{10} x_i / 9\right)^{0.25}$
SCH	$f_1(x) = \frac{1}{40}\sum_{i=1}^{10} x_i^2 \quad -1000 \le x_i \le 1000 \quad i = 1,...,10$ $f_2(x) = \frac{1}{40}\sum_{i=1}^{10} (x_i - 2)^2$

- Spread

We define the spread $\bar{\varepsilon}_{\min}$ as the average minimum distance of the non-dominated set to the Pareto-optimal solutions that minimises each objective independently :

$$\bar{\varepsilon}_{\min} = \frac{1}{n}\sum_{i=1}^{n} \min\left\{ \| a - a^*_{i\min} \| \quad a \in F \right\} \qquad (7)$$

where n is the number of objectives and $a^*_{i\min}$ represents the theoretical solution of the Pareto-optimal front that minimises the ith objective.

- Spacing

Spacing Δ is a measure based on consecutive distances among the solutions of the non-dominated set. It assesses the ability of the GA to distribute its population uniformly along the Pareto-optimal front :

$$\Delta = \frac{1}{|F|-1}\sum_{i=1}^{|F|-1} \left| d_i - \bar{d} \right| \qquad (8)$$

where d_i is the Euclidean distance between two consecutive solutions of the non-dominated set, $\bar{d}$ being the average of these distances. A value of zero for this metric indicates all the non-dominated solutions found are equidistantly spaced. Unlike the definition of Δ in [DEB, 00], we do not include in the non-dominated set the boundary solutions of the theoretical Pareto-optimal front to take into account the spread (spread is independently evaluated by (7)).

C. *Tests results*

All tests are made with the same number of objective function evaluations. The NSGA-II is run for 200 generations with a population size of 100. The archive size is also set to 100 and the crossover probability is 1. NSGA-II uses the BGA mutation operator [SCH, 94] with a mutation rate of $1/m$ (where m is the number of variables). The X-*gene* undergoes mutation with a probability of 5%. For all investigated tests, 100 runs are made with random populations to take into account the stochastic nature of the GA. An average statistic is taken from the final population for the performance criteria.

We present in Tables II–IV the values of the performance criteria on the investigated problems for the NSGA-II in relation to each crossover operator. Best values are indicated in bold types and margin errors with 95% confidence are given in brackets. From these results, we propose a sort of the crossover operators (ranking efficiency) on each problem.

Note that *best results are always obtained by a simple crossover acting alone*. As it can be seen from Table II, NSGA-II with BLX-0.5 clearly outperforms other operators on EC4. On the other hand, NSGA-II with BGAX gives the best results on EC6 but performs extremely poorly on SCH (convergence was not achieved after 200 generations ; only one non-dominated individual was found in the final population in all runs). Except with SBX crossover, NSGA-II fails to spread correctly its population on SCH. Therefore, SBX is ranked at the top of the sort for this problem despite a slightly lowest quality for $\bar{\varepsilon}$ and Δ.

These results indicate that the sensitivity to the crossover operator can not be neglected. Therefore, *using multiple crossover operators through a self-adaptive scheme tends to improve the robustness of the Pareto GA*. We verify this property in our tests since the self-adaptive scheme performs extremely well whatever the type of problem (the ranking efficiency always equals 2).

V. Optimal Design of Electrical Systems

The design of complex systems composed of heterogeneous elements requires a global approach which takes into account couplings and interactions between the different sub-systems The mathematical formulation resulting from this global approach leads to optimisation problems with continuous and discrete design variables, several constraints and multiple objectives. Because of these issues, the use of Pareto GA seems to be suitable. In this section, we illustrate the application of the NSGA-II with the previous self-adaptative recombination scheme to the optimal design of an electrical system based on an inverter fed permanent magnet machine – reducer – load association. The optimisation procedure consists in finding optimal configurations with regard to two objectives. On the one hand, the aim is to increase the whole energy efficiency by reducing system losses. On the other hand, the weight of the permanent magnet machine has to be minimised. The system structure, associated objectives and design variables are depicted in Fig 3.

TABLE II : PERFORMANCE CRITERIA ON PROBLEM EC4

Crossover scheme	Deviation $\bar{\varepsilon}$	Spread $\bar{\varepsilon}_{\min}$	Spacing Δ	Ranking efficiency
BGAX (k=16)	2.231 [0.185]	1.769 [0.148]	0.112 [0.011]	4 poor
SBX (η=1)	1.961 [0.170]	1.656 [0.138]	0.033 [0.009]	3 good
BLX-0.5	**1.483** [0.151]	**1.275** [0.122]	**0.019** [0.006]	**1** excellent
Self-adaptive	1.688 [0.164]	1.437 [0.136]	0.021 [0.008]	2 very good

TABLE III : PERFORMANCE CRITERIA ON PROBLEM EC6

Crossover scheme	Deviation $\bar{\varepsilon}$	Spread $\bar{\varepsilon}_{\min}$	Spacing Δ	Ranking efficiency
BGAX (k=16)	**0.000** [0.000]	**0.000** [0.000]	**0.006** [0.000]	**1** excellent
SBX (η=1)	0.185 [0.018]	0.001 [0.000]	0.202 [0.022]	3 good
BLX-0.5	3.167 [0.097]	2.399 [0.078]	0.415 [0.036]	4 poor
Self-adaptive	0.014 [0.006]	0.000 [0.000]	0.024 [0.010]]	2 very good

TABLE IV : PERFORMANCE CRITERIA ON PROBLEM SH

Crossover scheme	Deviation $\bar{\varepsilon}$	Spread $\bar{\varepsilon}_{\min}$	Spacing Δ	Ranking efficiency
BGAX (k=16)	no convergence			4 extremely poor
SBX (η=1)	0.007 [0.000]	**0.086** [0.006]	0.006 [0.000]	**1** excellent
BLX-0.5	**0.004** [0.000]	0.209 [0.008]	**0.004** [0.000]	3 good
Self-adaptive	**0.004** [0.000]	0.118 [0.009]	0.005 [0.000]	2 very good

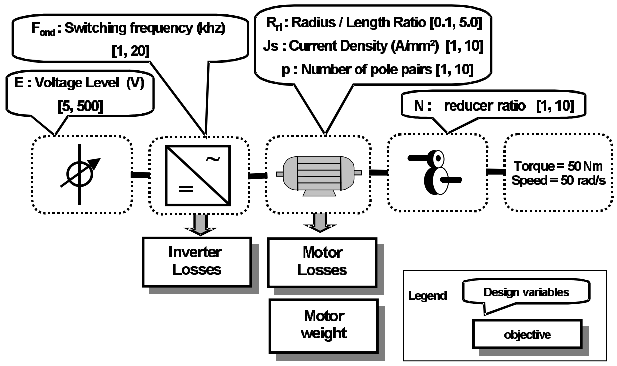

Fig. 3. System structure – objectives and design variables

A. *Optimisation procedure*

Finding optimal configurations for the system needs the use of appropriate models. The choice of these models must be done with accuracy and speed computation considerations. Including these requirements, the permanent magnet machine appears like the most complex sub-system. The use of finite elements is banned with regard to the speed computation but we must be able to evaluate electrical parameters related to geometric variations. Consequently, we choose an analytical model for the motor design and losses (iron and joule) computation [SLE, 92]. Inverter conduction and switching losses are also based on analytical models [TUR, 01]. Note that the system design is characterised by a vector of six parameters $X = \{E, J_s, R_{rl}, p, N, F_{ond}\}$. The number of pole pairs p and the reducer ratio N are discrete variables. Four additional constraints are related to association restrictions between sub-systems and technological limits :

- temperature of motor winding lower than 150°C
- permanent current lower than demagnetisation current
- slot opening higher than minimum winding section
- inverter switching frequency higher than 20 times electrical frequency of the permanent magnet machine

The structure of the optimisation procedure is shown in Fig 4.

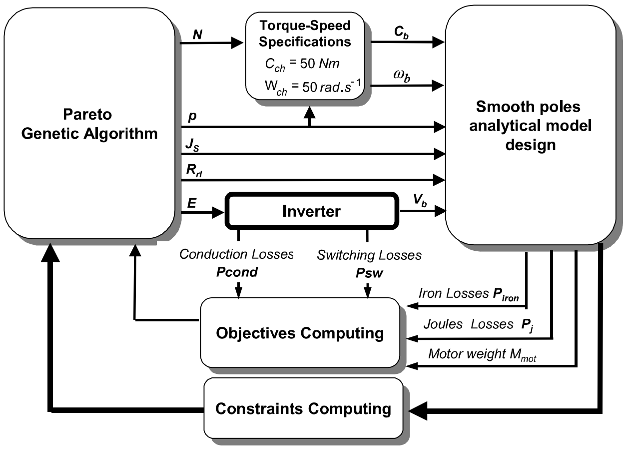

Fig. 4. Optimisation procedure

B. *Optimisation results and analysis*

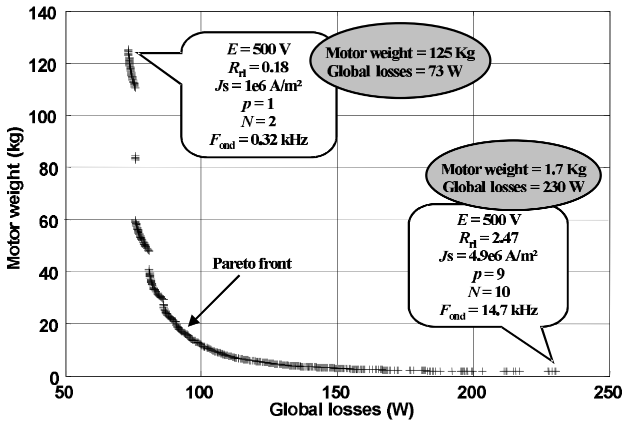

Fig. 5. Pareto-optimal front of the electrical system design problem

The optimisation of the electrical system is carried out using the NSGA-II with the characteristics defined in section IV-C for 100 generations. 10 runs are made to take into account the stochastic nature of the GA. The global Pareto-optimal front resulting from these runs is displayed in Fig. 5. Boundary configurations of the Pareto-optimal front are shown. Note that the discontinuity of the front is related to the discrete nature of p and N. We also indicate in Fig. 6 the evolution of the rate of children created with each crossover operators in the self-adaptive recombination scheme during generations. In the first generations, each crossover is approximately applied with the same rate. Around the 30th generation, NSGA-II converges to the Pareto optimal configurations of the system. Therefore, finding better solutions becomes harder and harder. Only local search can improve solutions by increasing accuracy. This explains why NSGA-II tends to favour the BGAX comparatively to SBX and BLX-0.5 which are more suitable for global search.

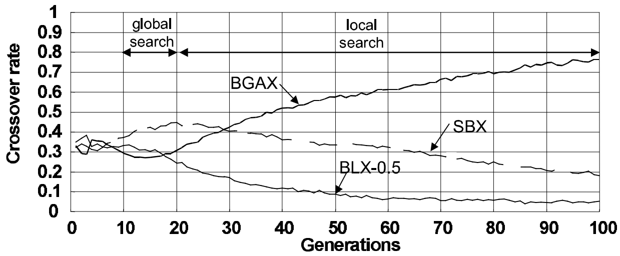

Fig. 6. Crossover rate per generation (average of 100 runs)

In order to help the designer in the optimisation results analysis, variations of design variables, constraints and sub-criteria along the front are studied. Fig 7 shows the evolution of motor and inverter losses as a function of global losses.

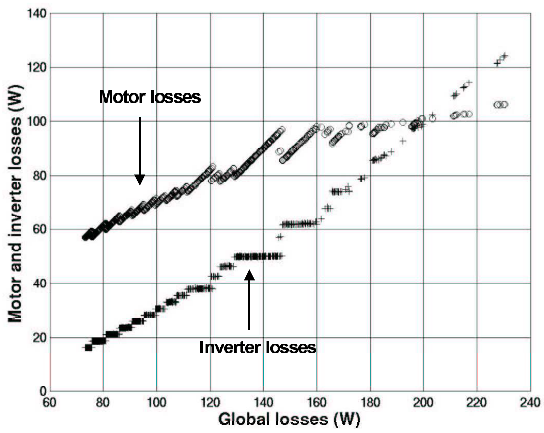

Fig. 7. Variations of motor and inverter losses

Inverter losses essentially depend on the steady state current I_M of the motor and switching frequency F_{ond}. Optimisation results show that supply voltage E is always near from its maximum value of 500 V. Since the load power $P_{ch} = 2.5kW \approx EI_M$ is constant, this high level voltage allows the minimisation of steady state current, and consequently the minimisation of inverter conduction losses. Moreover, design variables N, p and F_{ond} are directly linked through the constraint defined by $F_{ond} \geq 20pN\Omega_{ch}$. The variations of p and N in Fig 8-9 explain the increase of the inverter switching frequency (Fig. 10) which leads to inverter losses growth.

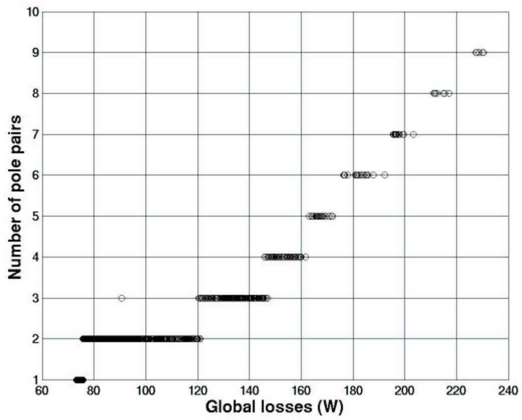

Fig. 8. Variation of the number of pole pairs along the Pareto front

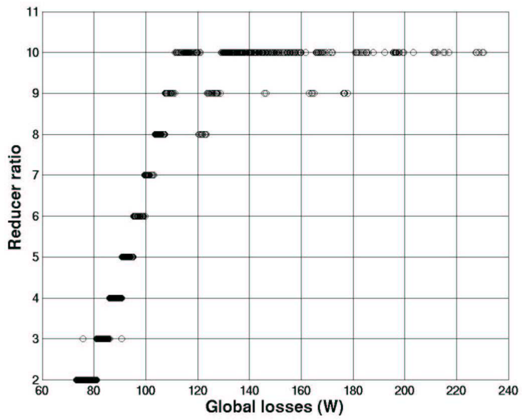

Fig. 9. Variation of the reducer ratio along the Pareto Front

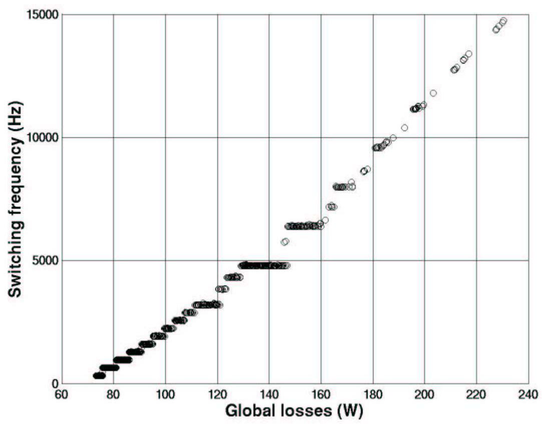

Fig. 10. Variation of the switching frequency along the Pareto front

Iron and Joule losses evolution is shown in Fig. 11-12. The minimisation of iron losses is linked to the minimisation of the stator weight and electrical frequency of the motor. In order to balance iron losses increase related to the stator weight, we see that the heaviest machines are characterised by a low pole number and a low reducer ratio value. When the motor weight is decreasing, p and N can increase without damaging iron losses. However, when the reduction of the motor weight is not enough to balance the increase of iron losses, the increase of p leads to higher iron losses. Moreover, it can be seen that the reducer ratio is rapidly growing to its maximum value in order to guaranty minimum motor weight. Minimising the motor weight as the reducer ratio equals 10 can be done by increasing the current density and the number of pole pairs. Fig 13 shows that thermal constraint on winding machine temperature is near from the limit of 150°C when global losses are higher than 160W. While iron losses are growing with the increase of p, joule losses and current density are limited by the thermal constraint (see Fig.11-14). These observations explain the evolution of the motor shape in Fig. 15. Optimisation shows that minimising the machine weight, with respect of the maximum temperature, leads to a change of the motor shape, expressed by the variations of the motor radius/length ratio R_{rl}. Thermal behaviour of winding is linked to the ability of the motor to exchange

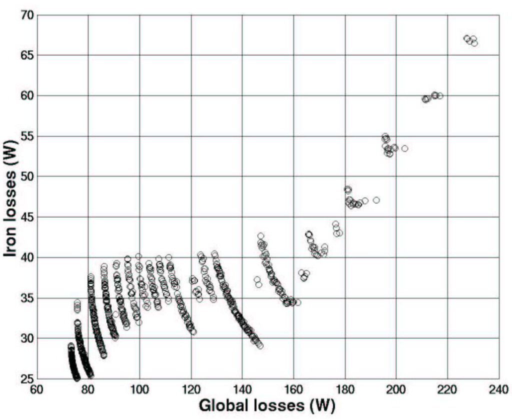

Fig 11. Variations of iron losses along the Pareto front

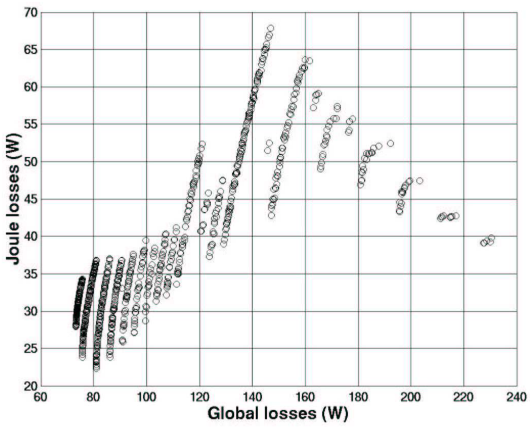

Fig 12. Variations of joule losses along the Pareto front

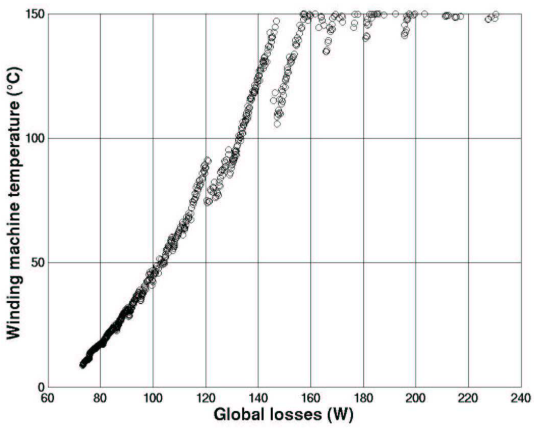

Fig 13. Variations of the motor temperature along the Pareto front

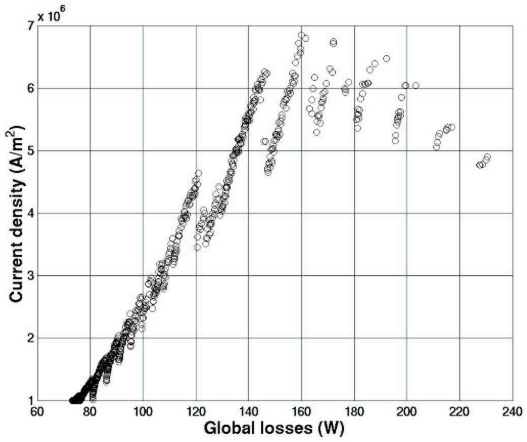

Fig 14. Variations of the current density along the Pareto front

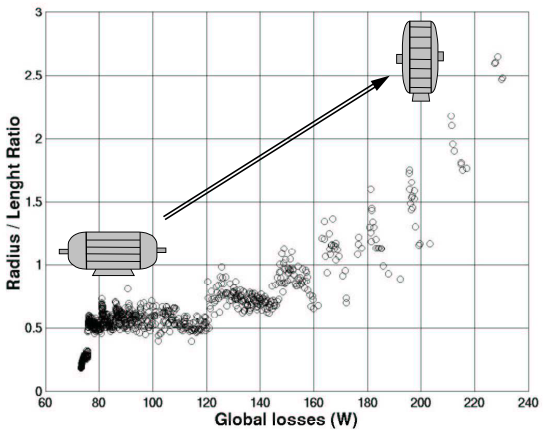

Fig 15. Variations of the motor radius/length ratio along the Pareto front

calories with ambient air. Therefore, R_{rl}. is increasing to guaranty the maximisation of the motor exchange surface with ambient air in order to limit heating effects.

IV. Conclusion

In this paper, a self-adaptive Pareto GA is applied to solve multiobjective optimisation problems. The robustness of the proposed self-adaptive recombination scheme is shown on mathematical test functions of various difficulties. The interest of this algorithm is not only justified by its ability to minimise multiple objectives simultaneously by approximating the Pareto front of the problem. Applied to the optimal design of an electrical system based on an inverter fed permanent magnet machine – reducer – load association, the Pareto GA allows the system designer to compare and study characteristics and particularities of various optimal configurations. Exploiting design variable, constraint and objective variations along the optimal front help to understand coupling phenomena in the whole system. The final choice between all Pareto-optimal configurations can be *a posteriori* done in relation to other considerations : total harmonic distortion, cogging torque, economical costs, …

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