To cite this version : Regnier, Jérémi and Sareni, Bruno and Roboam, Xavier and Astier, Stéphan. *Optimal design of electrical engineering systems using Pareto Genetic Algorithms.* In: 10th International Conference on Power Electronics and Applications (EPE'2003), 02-04 Sep 2003, Toulouse, France.

Optimal design of electrical engineering systems using Pareto Genetic Algorithms

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Abstract: This paper presents a Pareto Genetic Algorithm for multiobjective optimisation in electrical engineering design. Through a simple electromechanical system, based on an inverter–permanent magnet motor–reducer–load association described by analytical models, we discuss the efficiency and advantages of such methods.. We have chosen a rather simplified case study, to emphasise advantages of this approach for optimisation issues as well as a better understanding of system couplings.

Keywords: Multiobjective optimisation, Genetic Algorithms, System modelling, Coupling analysis.

I. Introduction

Strong coupling levels in electromechanical heterogeneous devices lead to study the system design as a whole. Generally speaking, a designer's purpose consists in determining the system architecture, as well as design parameters related to each sub-system, which minimise or maximise, multiple objectives (losses, costs, masses, etc.). Mathematical formulation of system approach leads to an optimisation problem characterised by an important number of discrete or / and continuous design variables. Using traditional solving techniques for this kind of problem (such as gradient search methods) is not suitable since analytical computation of objective and constraint derivatives is rather complex. Moreover, the multiobjective nature of the system approach implies several equivalent solutions (i.e. the set of best trade-offs) to the optimisation problem. The use of Genetic Algorithms (GA) is justified by the algorithm ability to explore, in the objective space, multiple solutions in parallel, to take into account multiple constraints and discrete parameters.

In this article, we introduce the application of a multiobjective GA to find the optimal design of an inverter fed permanent magnet machine association. Section II presents elementary principles of Pareto Genetic Algorithms. Section III illustrates the global optimisation problem and the analytical models we choose to describe the system. Finally, section IV deals with optimisation results and analysis we can perform thanks to these results.

II. Multiobjective optimisation with Pareto Genetic Algorithms

The purpose of multiobjective optimisation consists in minimising (or maximising) simultaneously several objectives f_i related to common design variables. The main difficulty of a multiobjective problem generally resides in the existence of conflicts between the different objectives. Therefore, there is no point in the parameters space, which leads to a simultaneous minimisation of all objectives. Consequently, the global solution of a multiobjective problem is characterised by a set of solutions expressing the best tradeoffs according to each objective. This optimal set is commonly referred to the Pareto front [1][2].

The traditional approach consists in converting the multiobjective problem into a scalar optimisation problem by aggregating or weighting the objectives in a global quality function. This technique is rather hazardous because of the difficulty to find suitable weighting coefficients and normalisation factors to homogenise the different physical criteria in the global quality function. Moreover, solving constrained problems with single objective methods leads to include penalty coefficients to the

objective function in order to traduce constraint violations. Setting these coefficients introduce additional difficulties with regard to the formulation of the problem. Furthermore, determines the complete optimal front requires to solve a set of scalar problems with different weighting factors.
Pareto GA's leads to an easier mathematical formulation of the optimisation problem. No weighting and normalisation factors are required to build objective functions and integration of constraints is done independently of objective functions.

Recently, GA's have been applied successfully to solve multiobjective optimisation problems [1][2]. GA's can explore multiple solutions in parallel thanks to s*election* and *evolution* operators [3][4].. Standard GA's can be modified to find the Pareto front of a multiobjective problem by using a specific selection method based on domination rules : if n_c objectives f_i have to be minimised, we can say that a solution A is dominating a solution B when [2] :

$$\forall i=1..n_c\ f_i(A)\leq f_i(B) \quad \text{and} \quad \exists k\in 1..n_c | f_k(A)<f_k(B) \qquad (1)$$

This domination rule is extended for constrained problems. In this case, a set of n_{const} constraints g_j are associated with each individual. An individual A is considered as non-feasible if one of its constraints is not fulfilled (i.e $\exists k\in 1...n_{const}$ such as $g_k(A)>0$). The Pareto domination rule in (1) for the selection operator is modified as follows :

- If A and B are non-feasible ⇒ Domination rules are applied in the constraint space
- If A and B are feasible ⇒ Domination rules are applied in the objective space (1)
- If A is feasible and B is non-feasible ⇒ A dominates B

From a randomly initialised population, a Pareto GA evaluates the non-dominated solutions and preserves them in a specific archive (non-dominated set). At each generation, Pareto tournaments are used to select individuals from the archive to create the mating pool (*parents* of the current generation). Parents are crossed and mutated to explore new solutions (*children* of the current generation). Following this way, the population of children and the archive are merged to assess the non-dominated set of the next generation. If the number of non-dominated individuals is higher than the size of the archive, a *clustering* method is used to preserve most representative solutions and eliminate others in order to keep a constant archive size. Note also that niching is used in the selection scheme when individuals involved in a tournament have the same domination rank. The structure of a Pareto GA is depicted in Fig. 1.

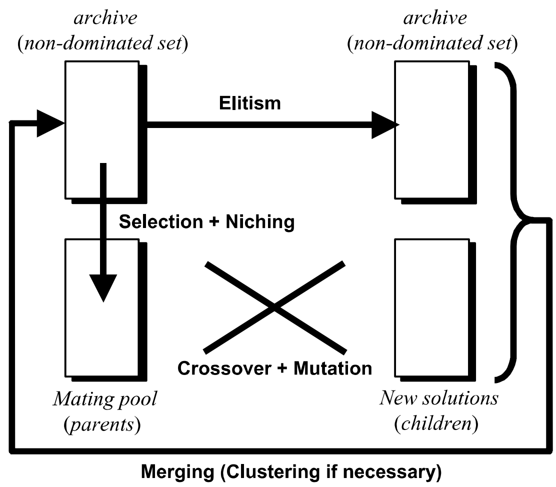


Fig.1 : Structure of a Pareto GA (one step generation)

In this article, we use a particular Pareto GA, which is the second version of the non dominated sorting genetic algorithm (NSGA-II). This algorithm is based on the principles of Pareto GA's previously exposed. In the NSGA-II, selection is performed with Pareto ranking tournaments associated with a crowded comparison operator to induce niching in the objective space. NSGA-II determines all successive fronts in the population (the best front corresponding to the non-dominated set). Moreover, a *crowding distance* is used to estimate the density of solutions surrounding each individuals on a given front. In a tournament, if individuals belong to the same front, the selected one is that with the greater crowding distance. This niching index is also used in the clustering operator to distribute uniformly the individuals on the Pareto front. All details of the algorithm can be found in [5].

III. Application to the optimal design of an electrical system

The design of complex systems composed of heterogeneous elements requires a global approach. Indeed, to guarantee the whole system efficiency, an optimisation procedure must be carried out with regard to the global behaviour of coupled sub-systems. The mathematical formulation resulting from this global approach leads to optimisation problems with continuous and discrete design variables, several constraints and multiple objectives. Efficiency of Pareto GA's in relation to optimisation considerations as been shown in [6]. Moreover, its ability to find multiple optimal trade-offs is really useful for understanding couplings. The analysis of design variables variations related to objectives and constraints along Pareto front allows to characterise and underline interactions phenomena in the system. Because of these issues, the use of Pareto GA's seems to be suitable.

We have developed a Pareto GA for the optimal design of an inverter fed permanent magnet machine – reducer association with regard to two objectives. On the one hand, the aim is to increase the whole energy efficiency by minimising system losses. On the other hand, the weight of the permanent magnet machine must be minimised. In fact, this device can be seen as a reduced part of an embedded system such as electrical vehicle for which embedded mass and consumed energy are typical optimisation criteria.

A-Modelling issues

The complete formulation of a global optimisation problem is firstly based on the use of appropriated models with regard to designer objectives. It is necessary, according to the required performances, to determine a model which provides useful information for the resolution of the problem as well as significant number of freedom degrees in order to guarantee the diversity of the solutions. These degrees freedom involve difficulties related to the range of model validity for a large variation range of design variables.
Moreover, the multi-fields, complex and global nature of the problem strongly influence the choice of the models with respect to precision and computation speed. These models must be accurate enough to allow the emergence of fundamental properties of the considered element, whatever the physical field (electric, thermal, magnetic,…). Association of different subsystems also generates constraints related to the mutual compatibility of elements. Thus, the models have to integrate required information in order to traduce association conditions.
Computing time is also an important modelling issue. It is necessary to work with "just accurate enough" models to avoid an excessive duration of the optimisation process. The management of these compromises underlines one of the main difficulties encountered in this work, namely research of models in adequacy with these requirements.

B-Permanent magnet motor analytical model

In this part, we describe the principles of the analytical model of the permanent magnet machine. In the design procedure, this model established a link between the motor geometry (radius, length, air gap dimension, yoke thickness,…) , magnetic (induction, flux,…) and electrical circuit parameters (inductance, resistance,…). In order to avoid excessive number of motor parameters relatively to the other sub-systems, we choose to associate three design variables to the motor : the number of pole pairs p , the slot current density J_S and the radius / length ratio R_{rl} . Characterisation of permanent magnet machine requires two steps. From the motor steady state torque C_b and design variables p ,

J_S and R_{rl}, we determine geometric dimensions of the motor (see Fig. 2). The bore radius r_s is related to B_{1g}, fundamental value of air gap induction, R_{dr}, slot depth / bore radius ratio (typical values $0.2 < R_{dr} < 0.4$) :

$$r_s = \left(C_b R_{rl} \frac{1}{J_S K_r B_{1g} R_{dr} \pi} \right)^{\frac{1}{4}} \tag{2}$$

K_r is the slot filling coefficient (typical values $0.4 < K_r < 0.6$)
B_{1g} is deduced from magnet properties (relative permeability $\mu_R = 1.05$, remanent induction $B_R = 1.1$ T and electrical half pole width $\alpha = 75°$) by :

$$B_{1g} = \frac{4}{\pi} B_R \frac{l_m/g'}{\mu_R + (l_m/g')} \sin(\alpha) \tag{3}$$

l_m/g' is the ratio between the magnet thickness and the air gap corrected by Carter coefficient (typical values $3.5 < l_m/g' < 5$). Air gap motor g depend on r_s and l_r by the empiric relation :

$$g = 0.001 + 0.003\sqrt{r_S l_R} \tag{3}$$

Magnet thickness is computed thanks to :

$$l_m = g' \frac{\mu_r}{\frac{B_r}{\hat{B}_g} - 1} \tag{5}$$

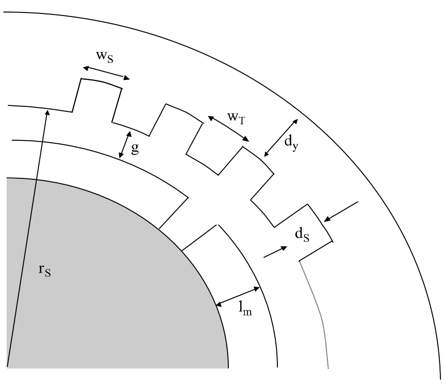
Yoke thickness is designed to limit the yoke induction $\hat{B}_y$ to 1.6 T :

$$d_y = \frac{r_s}{p} \alpha \frac{\hat{B}_g}{\hat{B}_y} \tag{6}$$

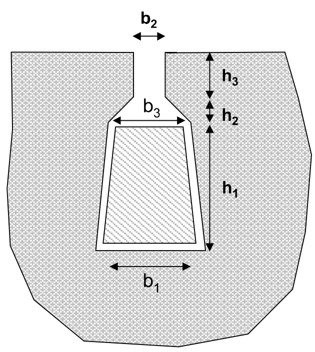
All details of the analytical model can be found in [7]. The geometrical dimensions allows the computation of the flux Φ and of the main inductance of one motor phase as follows :

$$L_p = \frac{6 \mu_0 r_S l_R}{\pi (K_c g + l_m)} N_{epp}^2 K_{1b}^2 N_{ce}^2 \tag{7}$$

$$\Phi_s = 2 K_{1b} N_{epp} B_{1g} r_S l_R N_{ce} \tag{8}$$



(a) Geometrical characteristics of the motor



(b) Geometrical characteristics of a slot

Fig. 2 : Permanent magnet machine dimension

Leakage inductances are computed considering a trapezoidal slot, as shown in Fig. 2(b), by using analytical formulation of [8]. Dimensions $h_1, h_2, h_3, b_1, b_2, b_3$ are related to d_s and w_s via empiric relations. Therefore, the global leakage inductance of one phase is :

$$L_f = 2\mu_0 l_R p N_{epp} \lambda_{enc} N_{ce}^2 \qquad \text{with } \lambda_{enc} = \frac{2h_1}{3(b_1+b_3)} + \frac{2h_2}{b_2+b_3} + \frac{h_3}{b_2} \tag{9}$$

In equations (1), (2) and (3), only one parameter N_{ce}, i.e. the number of conductors in one slot, is unknown. N_{ce} depends on the supply voltage level and the motor rotation frequency. Permanent magnet machine must be able to provide the torque $C = C_b$ under supply voltage $V = V_b$ at electrical pulsation $\omega = \omega_b$.

In order to determine N_{ce}, we need to compute the stator resistance R_1 and current I_1 of the motor for one conductor per slot ($N_{ce} = 1$).

$$R_1 = \rho_{\text{cuivre}} \frac{2pN_{\text{epp}}}{S_{\text{enc}}} l_{\text{cond}} \tag{10}$$

$$I_1 = \frac{J_s d_s K_r \pi r_s}{6pN_{\text{epp}}} \tag{11}$$

The values of equivalent electrical model parameters of motor can be expressed, in relation to N_{ce}, by the following relations :

$$L_p = N_{ce}^2 L_{p1}, \; L_f = N_{ce}^2 L_{f1}$$
$$R_s = N_{ce}^2 R_1, \; \Phi_s = N_{ce}\Phi_1, \; I_s = \frac{I_1}{N_{ce}} \tag{12}$$

The electrical diagram of the machine operating at point (C_b, ω_b) is displayed in Fig. 3 :

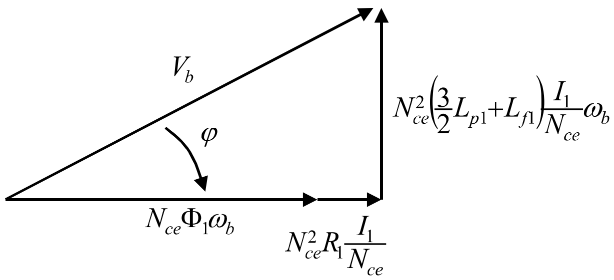


Fig. 3: Electrical diagram of the permanent magnet machine

The number of conductors in a slot is expressed as follows :

$$N_{ce} = \frac{V_b}{\sqrt{(\Phi_1\omega_b + R_1 I_1)^2 + ((\frac{3}{2}L_{p1} + L_{f1})\, I_1 \omega_b)^2}} \tag{13}$$

Motor characteristics are thus completely known and the criteria of losses chosen for optimisation can be evaluated, as well as the mass of the various motor parts.

- Motor weight is the sum of magnet weight M_{mag}, stator weight M_{stator}, rotor weight M_{rotor} and copper weight M_{copper}
- Iron losses P_{iron} are expressed by $P_{iron} = 0.0025\left(\frac{\omega_b}{2\pi}\right)^{1.45}(B_y)^{1.91} M_{stator}$ [8]
- Joules losses P_j are expressed by $P_j = 3R_s I_s^2$

C-Inverter modelling

Inverter losses are evaluated by means of an analytical model which takes into account switching and conduction losses [10]. Assessment of switching losses is carried out using technical data, by interpolating switching energy data supplied by IGBT manufacturer :

$$P_{sw}=6\frac{f_{sw}}{2\pi}\int_{\varphi}^{\varphi+\pi}\left[W_{on}(\theta)+W_{off}(\theta)\right]\mathrm{d}\theta$$
$$P_{sw}=6f_{sw}\left\{\frac{a_{on}+a_{off}}{2}+\left(b_{on}+b_{off}\right)\frac{I_{\max}}{\pi}+\left(c_{on}+c_{off}\right)\frac{I_{\max}^2}{4}\right\} \quad \text{with} \begin{cases} W_{on}=a_{on}+b_{on}I_c+c_{on}I_c^2 \\ W_{off}=a_{off}+b_{off}I_c+c_{off}I_c^2 \end{cases} \tag{14}$$

Conduction losses are computed by using characteristics of diodes ($V_{D(on)}, R_{D(on)}$) and IGBT's ($V_{CE(on)}, R_{IGBT(on)}$) such as :

$$P_{cond}= 6\left(V_{D(on)}\,I_{moy}+R_{D(on)}\,I_{D\_rms}^2\right) +6\left(V_{CE(on)}\,I_{moy}+R_{IGBT(on)}\,I_{IGBT\_rms}^2\right) \quad \text{with} \begin{cases} I_{moy}=\frac{I_{\max}}{2\pi}\left(1+\frac{\pi}{4}m_a\cos\varphi\right) \\ I_{D\_rms}^2=\frac{I_{\max}{}^2}{8}\left(1-\frac{8}{3\pi}m_a\cos\varphi\right) \\ I_{IGBT\_rms}^2=\frac{I_{\max}{}^2}{8}\left(1+\frac{8}{3\pi}m_a\cos\varphi\right) \end{cases} \tag{15}$$

In these relations, $I_{\max}$ denotes the maximum peak value of motor current and $m_a=2\sqrt{2}V_b/E$ represents the modulation factor .

B-Constraints of optimisation problem

As mentioned in section III-A, the large variation range of design variables imposes the presence of constraints to guarantee the feasibility of obtained solutions during the optimisation process. In our problem, we define six constraints :

- Steady state current must be lower than demagnetisation current of magnet
- Slot opening must be higher than minimum winding section
- Number of conductor per slot must be higher than 1
- Minimum switching frequency must be higher than 20 times motor electrical frequency
- Number of stator slot must be lower than 72
- Raise of winding temperature T_{copper} must be lower than 150°C (thermal limit of winding insulation)

The increase of winding temperature is obtained from a thermal model which take into account effects of iron and joules losses on the global heating. This model includes conductive transfers in the machine materials as well as convective transfer between yoke and ambient air.
This model is presented in Fig. 4. R_1, R_2, R_3 and R_4 respectively represent the thermal resistance of the slot copper, the bottom slot insulation, the stator iron (conductive phenomena) and the thermal resistance related to the exchange between external motor surface S_{ext} and ambient air (convective phenomenon). The external thermal resistance R_4 is a main parameter of the thermal model because it depends on the cooling system associated to the permanent magnet machine. R_4 is expressed as follows :

$$R_4 = \frac{1}{hS_{ext}} \tag{16}$$

h is the heat transfer coefficient $5 < h < 20$ for a self cooled motor.

The cooper temperature is expressed by the following expression :

$$T_{copper} = \Delta T_{copper} + T_{ambient} \tag{17}$$

All details for this model can be found in [9].

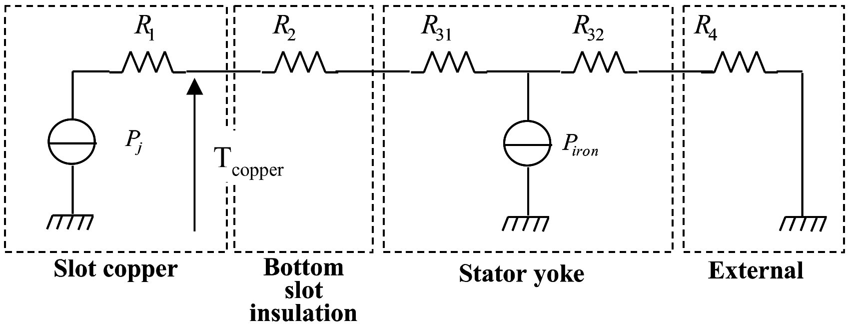
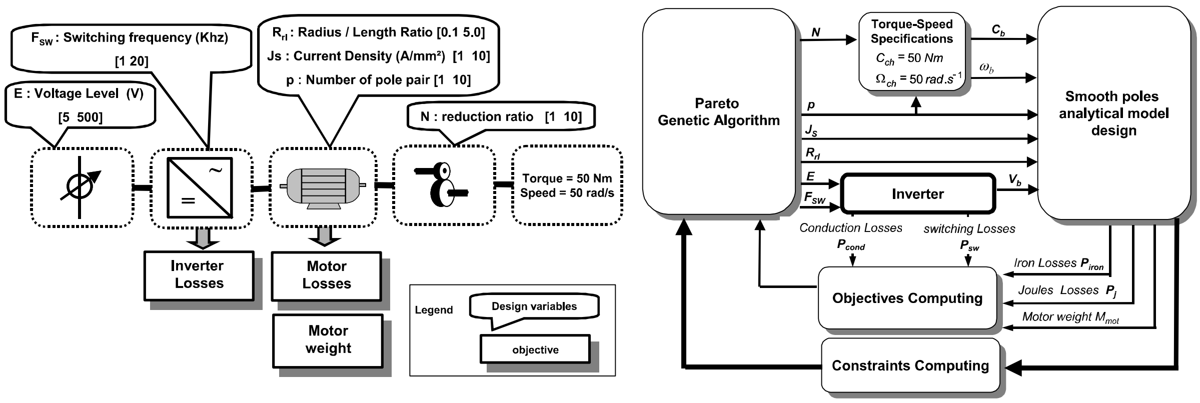


Fig. 4 : Thermal model of the permanent magnet machine

III. Optimisation results

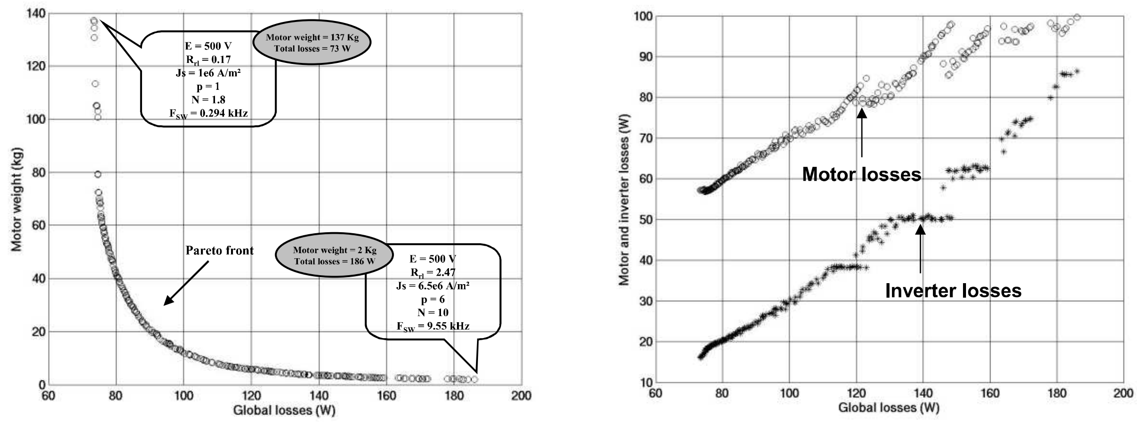
The whole optimisation problem is depicted in Fig. 5(a). The global optimisation strategy is shown in Fig. 5(b). The optimisation of the electrical system is carried out using the NSGA-II for 100 generations. 10 runs are made to take into account the stochastic nature of the GA. The global Pareto-optimal front resulting from these runs is displayed in Fig. 6(a). Boundary configurations of the Pareto-optimal front are shown. Motor weight presents large variations for solutions having the lowest global losses (from 73 to 80 W). In this region of the optimal front, solutions have approximately the same global losses but motor weight is rapidly decreasing. For global losses variations from 120 to 180 W, the motor weight is slightly improved in comparison to other region of the optimal front. With regard to the chosen objectives, the best trade-off region can be located from 80 to 120 W for global losses, corresponding to a motor weight variations from 6 to 40 kg. Note that the final choice between optimal solutions can be done by considering additional criteria (for example financial, or technological issues.



(a) Design variables and objectives (b) Optimisation procedure

Fig. 5 : Characteristics of optimisation problem

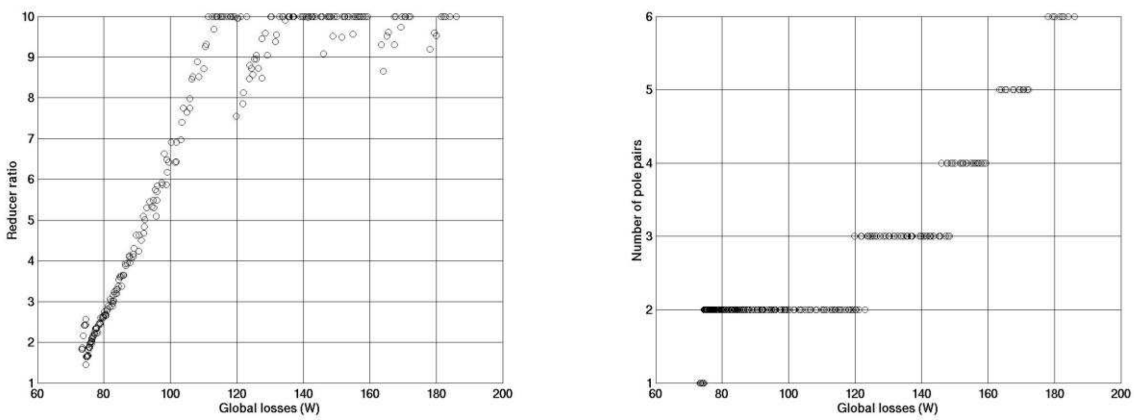
Moreover, in order to help the designer in the optimisation results analysis, variations of design variables, constraints and sub-criteria along the front are studied. Fig 6(b) shows the evolution of motor and inverter losses as a function of global losses. Inverter losses essentially depend on the steady state current I_s of the motor and switching frequency F_{SW}. Optimisation results show that supply voltage E is always near from its maximum value of 500 V. Since the load power $P_{ch}=2.5kW \approx EI_s$ is constant, this high level voltage allows the minimisation of steady state current, and consequently the minimisation of inverter conduction losses. Moreover, design variables N, p and F_{SW} are directly linked through the constraint defined by $F_{SW} \geq 20pN\Omega_{ch}$. The variations of N and p in Fig. 7(a)(b) explain the increase of the inverter switching frequency F_{SW} (Fig. 8(a)) which leads to inverter losses growth. Iron and Joule losses evolution is shown in Fig. 8(b) and 9(a). The minimisation of iron losses is linked to the minimisation of the stator weight and electrical frequency of the motor. In order to balance iron losses increase related to the stator weight, we see that the heaviest machines are characterised by a low pole number and a low reducer ratio value. When the motor weight is decreasing, p and N can increase without damaging iron losses.



(a) Optimal front of the problem

(b) Variations of motor and inverter losses

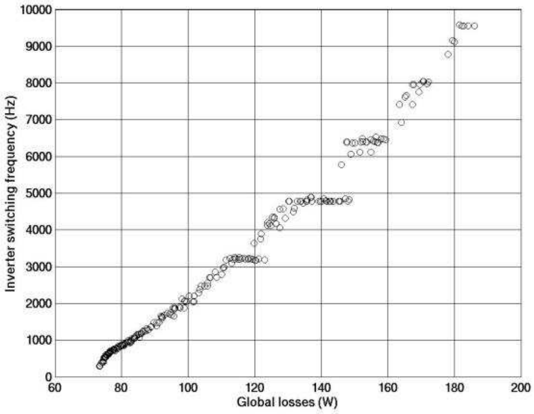
Fig. 6 : Optimisation results and variations of sub-criteria along the Pareto front



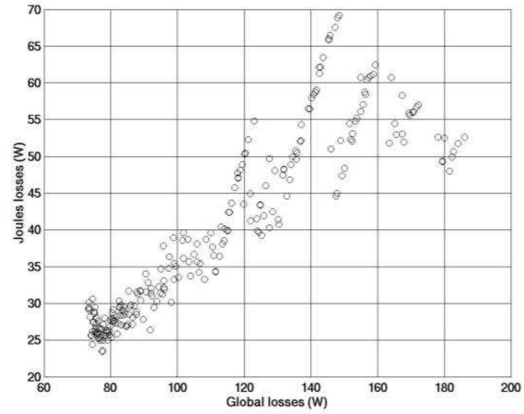
(a) Variations of N

(b) Variations of p

Fig. 7 : Variations of the reducer ratio and number of pole pairs along the Pareto front



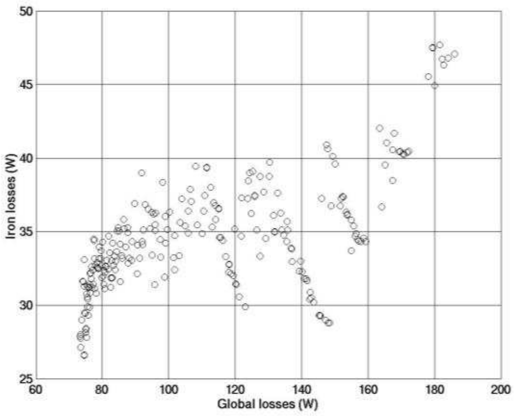
(a) Variations of F_{SW}



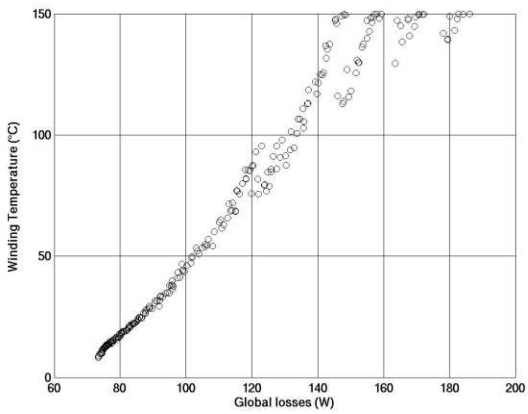
(b) Variations of P_j

Fig. 8 : Variations of the switching frequency and joule losses along the Pareto front

However, when the reduction of the motor weight is not enough to balance the increase of iron losses, the increase of *p* leads to higher iron losses. Moreover, it can be seen that the reducer ratio is rapidly growing to its maximum value in order to guarantee minimum motor weight. Minimising the motor weight as the reducer ratio equals 10 can be done by increasing the current density and the number of pole pairs. Fig 9(b) shows that thermal constraint on winding machine temperature is closed to the limit of 150°C when global losses are higher than 140W. While iron losses are growing with the increase of *p*, joule losses and current density are limited by the thermal constraint (see Fig.7(b), 8(b) and 10(a)). These observations explain the evolution of the motor shape in Fig. 10(b). Optimisation shows that minimising the machine weight, with respect to the maximum temperature, leads to a change of the motor shape, expressed by the variations of the motor radius/length ratio R_{rl}. Thermal behaviour of winding is linked to the ability of the motor to exchange calories with ambient air. Therefore, R_{rl}. is increasing to guarantee the maximisation of the motor exchange surface S_{ext} (so the minimisation of parameter R_4 of the thermal model in section III-C) with ambient air in order to limit heating effects.

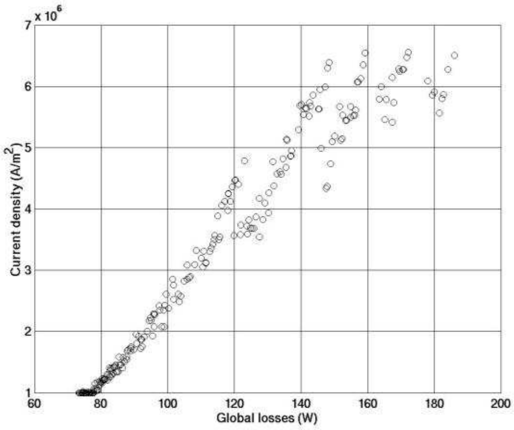


(a) Variations of P_{iron}

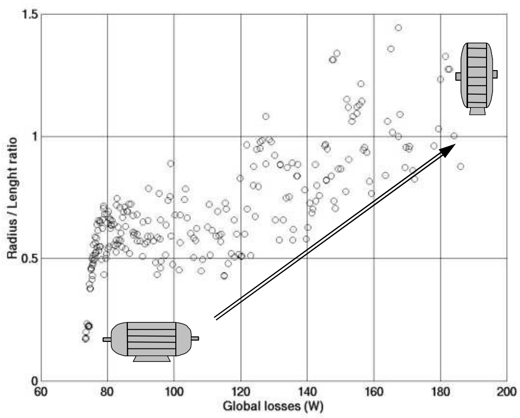


(b) Variation of T_{copper}

Fig. 9 : Variations of iron losses and winding temperature along the Pareto front



(a) Variations of J_S



(b) Variations of R_{rl}

Fig. 10 : Variations of the current density and radius / length ratio of the permanent magnet machine

IV. Conclusion

In this paper, a Pareto GA is applied to solve multiobjective optimisation problem. The interest of this algorithm is not only justified by its ability to minimise multiple objectives simultaneously by approximating the Pareto front of the problem. Applied to the optimal design of an electrical system based on an inverter fed permanent magnet machine – reducer – load association, the Pareto GA allows the system designer to compare and study characteristics and particularities of various optimal configurations. The final choice between all Pareto-optimal configurations can be *a posteriori* done in relation to other considerations : total harmonic distortion, cogging torque, economical costs, … Modelling sub-systems with regard to the requirements of the global system optimisation needs the use of an appropriate approach in order to adapt characteristics of models to the formulation of objectives. Many investigations are required in this field to improve the application range of this approach. Finally, we show that the exploitation of design variable, constraint and objective variations along the optimal front help to understand coupling phenomena in the whole system. Understanding these correlations thanks to a preliminary analysis is rather complex and Pareto GA appears like an efficient tool for system design. Future research on more complex systems, such as electrical vehicle design, will emphasis interest of this method.

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