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Synthesis of an Electro Hydrostatic Actuator from Bicausal Bond Graphs

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Keywords

Bond Graph, synthesis, bicausality, Electro Hydrostatic Actuator.

Abstract

In this paper, the structure of an Electro Hydrostatic Actuator (EHA) devoted to flight control surfaces is firstly described by its Causal Bond Graph (CBG). Then, verifying the inversion conditions, the Bicausal Bond Graph (BBG) can be obtained to synthesize the system inputs (electrical constraints) from the outputs (aerodynamic torque and position of the flight control surface) given by data stored during an actual "flight mission". The particular issues of filtering and of synthesis by model inversion are discussed and a first simulation set is provided to illustrate the capabilities of this approach by means of BBGs. Then, the same process is applied to a more complete modeling of the EHA with a BG including several non linearities inside the electro hydrostatic transmission line.

1. INTRODUCTION

Within the framework of complex electrical engineering devices, the system analysis becomes more and more complicated so that a unified formalism such as the Bond Graph (BG) [Paynter,1961], [Karnop & Al,2000] is particularly useful. This graphical method illustrates the energetic transfers in the system. By analysing the causal bond graph, it is possible to know if the association is physically (and energetically) convenient and then "numerically consistent". Homogenous modelling and system analysis methods directly applicable on bond graphs are the major interest of bond graph formalism.

Having chosen system architecture and parameter sizing, the system analysis process consists in verifying if the device fulfils the requirements: this is usually done from a system's model and its simulation by means of several iterations to verify requirements.

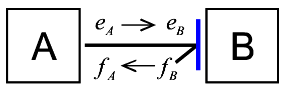
On the other hand, the synthesis process consists in choosing the system structure and its sizing, directly starting from the requirements. This "inverse process" is essential for the design of complex energetic systems and is complementary to the analysis process.

The concept of bicausality was invented and first published by P.J. Gawthrop [Gawthrop, 1995]. This proposition has opened a new research field in bond graph applications [Gawthrop & al, Mechin & al, Ngwompo & al] mainly for system inversion, state or parameter estimation.

After having presented the basic concepts of the bicausality and the model inversion procedure in the first section, our case study is proposed : a first simplified version of the Causal Bond Graph (CBG) of an EHA (Electro Hydrostatic Actuator) is described in section 2. In section 3, the inversion conditions are examined and the inverse modeling is developed by means of Bicausal Bond Graph (BBG). The results provided from the inverse model are analyzed and compared with those obtained from a classical direct model. The issue of data processing, particularly important as an inverse model is derived, is also presented in this section. The interest of this synthesis process by inversion is also emphasized. Finally, an advanced modeling of the EHA is proposed. In the corresponding CBG, several non linearities due to certain physical phenomena inside the hydrostatic transmission line are included : the applicability of the inversion process, in spite of this additional complexity, is finally analyzed.

2. BASIC CONCEPTS OF BICAUSALITY AND INVERSION CONDITIONS

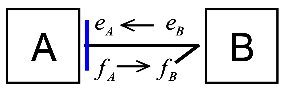
In a bicausal bond graph, the causality of each variable is separately examined. The conventional or physical causality (Figure 1.a) is a particular case of the bicausality where both causal half stokes are placed on the same side of the bond.



$e_B := e_A$

$f_A := f_B$

A imposes its effort on B whose consequence is its flow

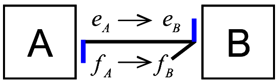


$e_A := e_B$

$f_B := f_A$

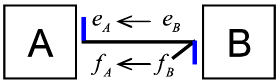
A imposes its flow on A whose consequence is its effort

(a). normal causality assignments.



$$e_B := e_A$$
$$f_B := f_A$$

A imposes its effort and flow on B



$$e_A := e_B$$
$$f_A := f_B$$

B imposes its effort and flow on A

(b). Bicausality assignments.

Figure 1: causal and bicausal assignments

In a bicausal bond graph, it is preferred to replace the notation **Sf**, **Se**, **De** and **Df** by **SS** (Source – Sensor elements) [Gawthrop & al, 1992] even if their causality is not changed (both causal half strokes are on the same side of the bond). An inverse model can only be obtained if the direct model is invertible. Therefore, it is necessary to firstly test this property on the bond graph before applying bicausality inversion process. For this purpose, several definitions are employed [Ngwompo & al].

Definition 1: two single Input/ Output (I/O) causal paths are disjoint if they do not pass through any common variable.

Definition 2: a set S is disjoint if it consists of m disjoint I/O causal paths.

Definition 3: the order $\omega_p(u_i, y_i)$ of an I/O causal path p between an input u_i and an output y_i is defined as:

$$\omega_p(u_i, y_i) = n_I(p) - n_D(p) \qquad (1)$$

where $n_I(p)$ (respectively $n_D(p)$) is the number of dynamic elements in integral (respectively differential) causality crossed by this causal path

Definition 4: the order $\omega(S)$ of a set S of m disjoint I/O causal paths p_i, $i = 1$ to m, is:

$$\omega(S) = \sum_{i=1}^{m} \omega_{p_i} \qquad (2)$$

By using these definitions on the direct bond graph, the inversion condition for a MIMO model with m inputs and m outputs is:

- If there is no choice for the set of m disjoint I/O causal paths, the model is structurally not invertible.
- If there is only one choice for the set of m disjoint I/O causal paths, the model is structurally invertible.
- If there are several choices, we should apply the *Modified Sequential Causality Procedure for Inversion* (MSCAPI) [Ngwompo & al, 1996].
    - Determine a set S_0 whose order is the smallest among them.
    - Replace all sources and detectors associated with the control variables or inputs and outputs by **SS** elements.
    - Assign effort source – flow source causality to the **SS** output elements and propagate the causal information to the **SS** input elements. This propagation has to arrive and impose effort detector – flow detector causality on the input **SS** elements. Other elements take the causality due to the bicausality propagation and junction conventions.
    - If there is at least one causality conflict, the model is not invertible. In the opposite case, it is invertible.

When the model is invertible, the following procedure can be applied to construct the inverse model (or the synthesis model) using bicausality [Ngwompo & al, 1996] :

1. Replace all source and detector elements by **SS** elements.
2. Apply the bicausality effort source – flow source on the output elements and effort sensor – flow sensor on the input elements or on the elements whose parameters have to be synthesised, in relation to the degrees of freedom.
3. Propagate the bicausality from outputs to inputs. Other elements take the causality due to the bicausality propagation with respect to junction conventions.

The obtained bond graph is then called "*Bicausal Bond Graph"* (BBG).

Note that the specified inputs of the inverse model have to be sufficiently differentiable. In particular, if the inversion is processed along a I/O causal path with a length of n, the corresponding input of the inverse model must be n times differentiable if no symbolic manipulations are made. Filtering the specified inputs of the inverse model as proposed in the section 3 can solve this issue.

3. SIMPLIFIED BOND GRAPH OF EHA

The Electro Hydrostatic Actuator (EHA) considered here as a case study is typical of an Airbus flight control actuator. It is composed of three parts that correspond with three transformations between domains:

- The "electromechanical part" in usually constituted of a Permanent Magnet Synchronous Machine fed by a PWM controlled Voltage Source Inverter. In the following model, the motor actually used in EHAs is replaced by an energetically equivalent DC motor. This simplification is usually effected, even in manufacturer models, when only the energetic transfers or the transient analysis (network stability and quality) are focused. The equivalent DC motor is connected with a DC–DC chopper fed by an ideal voltage source.

- Regarding the mechanical – hydraulic transformation, the electrical motor drives a volumetric pump which sets the position of a hydraulic jack.
- Finally, thanks to the third domain transformation (hydraulic – mechanical), positioning of the flight control surface is possible.

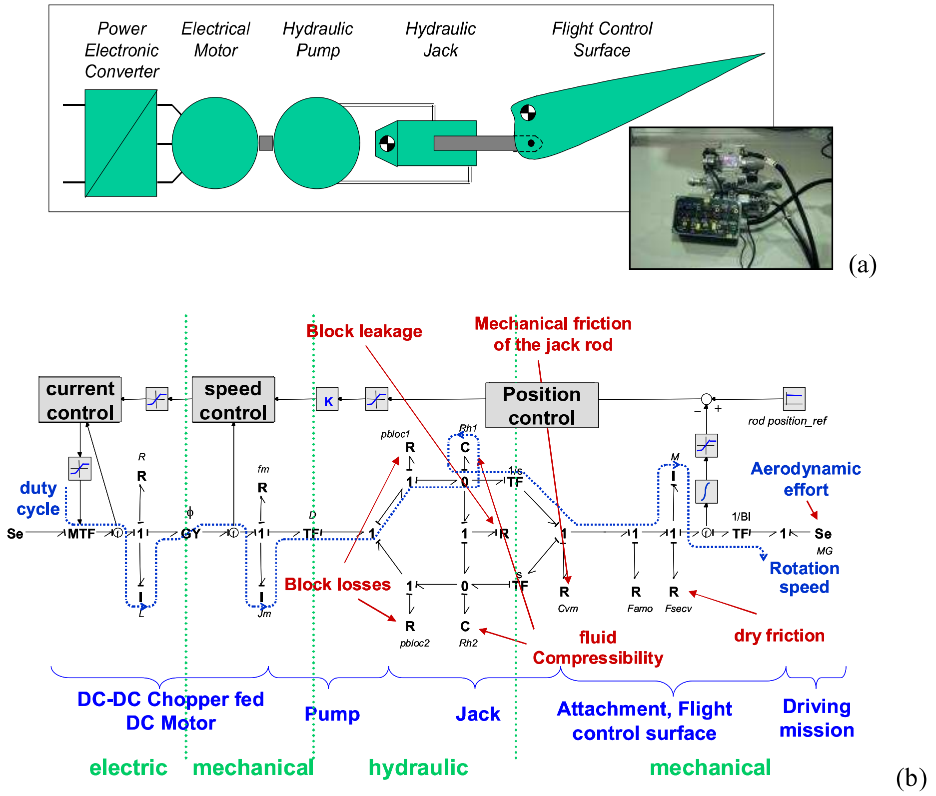


Figure 2: (a) synoptic of the EHA; (b) simplified model (CBG).

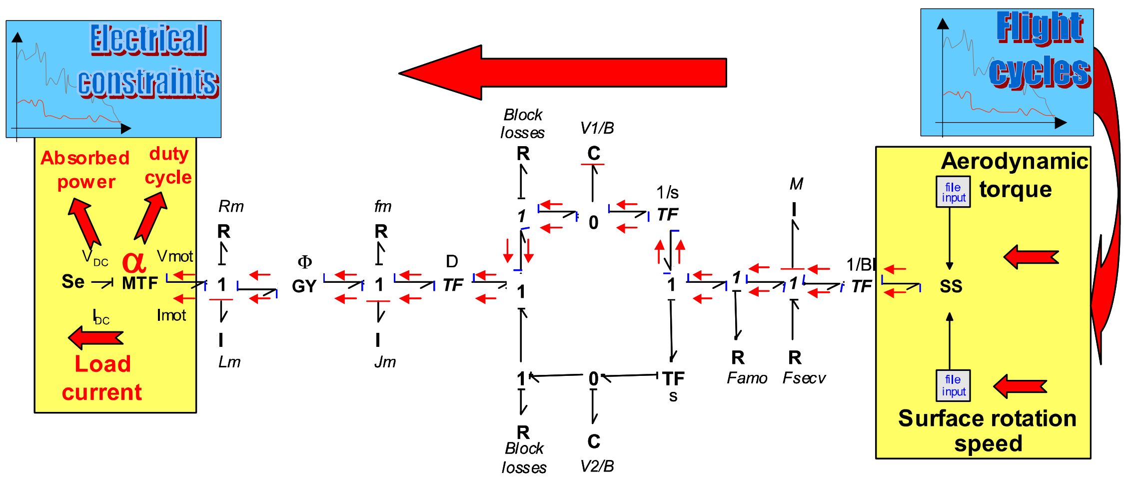


Figure 3: inverse model (BBG) of the EHA

Following the plane speed, the altitude and the position of the flight control surface, a consequent airload is applied. The synoptic (a) and a simplified vision of the CBG model (b) are given on the Figure 2. We consider here the bond graph modelling of an EHA dedicated to the position control of flight control surfaces (aileron) in the Airbus aircrafts.

4. MODEL INVERSION AND DATA PROCESSING BY FILTERING

It is possible to specify typical flight cycles (chronogram) for the "aerodynamic efforts" and for the positions of the flight control surface. A time derivative of these positions gives the values of the "flight control surface rotation speed". The issue is then to inverse the CBG model to synthesize the subsequent electrical constraints (voltage, current, electrical power) by means of the BBG.

4.1. Model inversion

In the direct CBG, the DC voltage source, the "duty cycle" of the DC/DC converter and the aerodynamic effort are fixed as inputs. On the opposite, the DC current and the control surface rotation velocity are considered as outputs. In particular, by considering the I/O causal path *L* from the control variable (i.e. the duty cycle) to the output flow variable (i.e. the control surface rotation velocity), it can be seen that there exist several choices of I/O causal paths.

Given the existence of several choices as I/O causal path, we can not directly deduce if the model is invertible or not and the MSCAPI procedure has to be applied. In this case, there are two causal paths, with a minimal length (smallest order) depending whether the first (see the dotted line on Figure 2.b) or the second chamber of the hydraulic jack is crossed. The bicausality propagation does not imply any causality conflicts (see Figure 3). Therefore, the model is invertible and the bond graph obtained by the MSCAPI procedure is also the inverse bond graph.

In this inverse model, the information data related to the flight mission cycles (specified in terms of "aerodynamic torque" and of "rotation speed") are injected. In our case study, having an I/O causal path length of 4, we obtain a set of DAE (Differential Algebraic Equations) for which the output (i.e. the duty cycle) is expressed versus the input (i.e the control surface rotation speed). Without any symbolic manipulation [Van Dijk & al, 1991] of the equations, this input is differentiated 4 times in the inverse model (4 **I** and **C** elements are in derivative causality).

We use the 20 Sim software [20-Sim] with a modified library developed in our laboratory, in order to take bicausality into account. The *Backward Differentiation Formula* (BDF) is used as the integration method.

In order to validate the inversion process, we have simulated together the direct and the inverse BG models: the aerodynamic torque being imposed from the flight cycle data, the output of the direct model (i.e. the control surface rotation speed) is connected to the input of the inverse model. Then, we have superimposed the duty cycles obtained from both *direct and inverse models which are perfectly in accordance*. Note that the position/speed/current control unit is only simulated in the direct model. With such a process, we are sure that the rotation speed is differentiable at least 4 times, being obtained in the direct model through 4 dynamic elements with integral causality. In the general case, the specified input signal must be "sufficiently filtered" to inverse and differentiate the model.

4.2. Data processing : filtering of flight mission data

From a specified data file, several methods of filtering can be used with different cut off frequency. In particular, causal (low pass filter) or acausal (Centered Weighted Moving Average, CWMA) filtering methods can be applied. In the first case (causal filtering) the signal phase is delayed depending on the cut-off frequency. For this case study, a fourth order low pass filter has been considered with a bandwidth of 5Hz, the data obtained from the flight cycle being sampled at 25Hz. Note that the same data filtering has to be applied simultaneously for the aerodynamic torque as well as for the flight control position in order to obtain the same delay.

The phase delay can be avoided by means of an acausal filtering. The "Centered Weighted Moving Average" method has been selected. The filtering method supposes that the data (x_{i+j}^{data}) are *a priori* known:

$$x_i^{filtered} = \frac{\sum_{j=-k}^{k}\left(a_{i+j}.x_{i+j}^{data}\right)}{\sum_{j=-k}^{k} a_{i+j}}$$

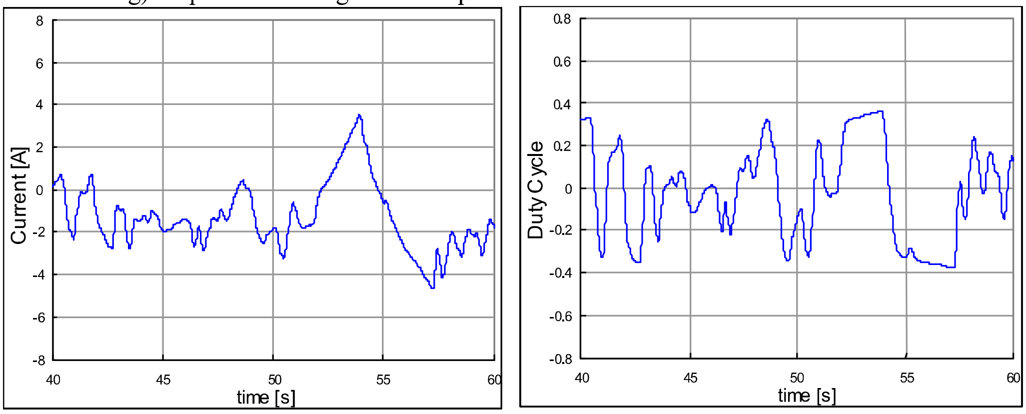
$$\dot{x}_i^{filtered} = \frac{\sum_{j=-k}^{k}\left(b_{i+j}.x_{i+j}^{data}\right)}{T_e.\left(\sum_{j=-k}^{k} j.\quad b_{i+j}\right)} \quad \ddot{x}_i^{filtered} = \frac{\sum_{j=-k}^{k}\left(c_{i+j}.x_{i+j}^{data}\right)}{T_e^2.\left(\sum_{j=-k}^{k} j^2.\quad c_{i+j}\right)} \tag{3}$$

The parameter *k* deals with the sampling number considered to average the data, *Te* being the sampling period. The signal is more and more filtered as *k* is increased. More than the absence of a phase delay, the main advantage of this filtering process is the direct analytical derivation of the first and second derivatives obtained from the data as mentioned in (3). The weighting coefficients (a_i, b_i, c_i) are constants depending on *k*. The Table 1 gives the example for *k*=2.

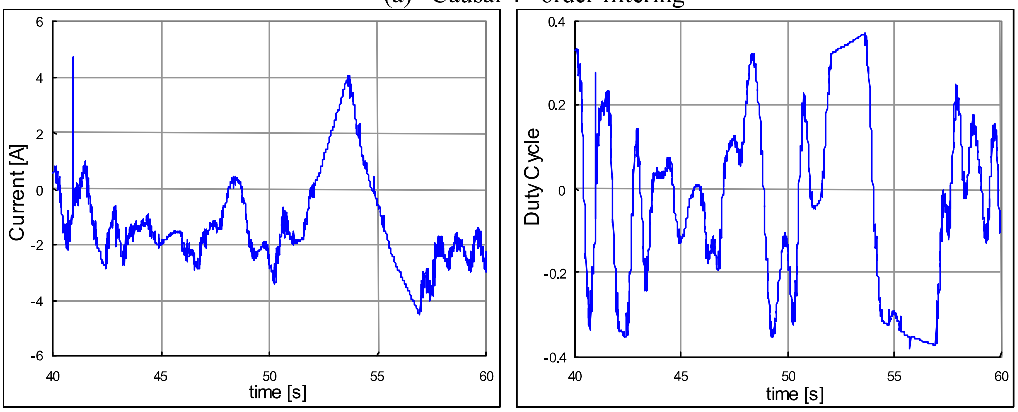
Table 1: weighting coefficients for k=2

	i–2	*i*–1	*i*	*i*+1	*i*+2
Filtered signal : a_{i+j}	-3	12	17	12	-3
1st derivative: b_{i+j}	-2	-1	0	1	2
2nd derivative : c_{i+j}	2	-1	-2	-1	2

Finally, as presented on the Figure 4, the inverse model (BBG) obtained from the filtered data allows showing up the electrical constraints (i.e. the DC side current and the electrical power) and the control constraints (i.e the duty cycle). If the phase delay is not problematic, the causal filtering is efficient in terms of waveform quality. The quality obtained with the acausal CWMA filtering is lower. However, no delay are provoked and the first derivative of the surface position (i.e. the control surface rotation speed) is directly (analytically) obtained. Note that this latter process (acausal filtering) requires knowing data samples before the simulation process.



(a) Causal 4th order filtering



(b) acausal CWMA filtering with k=6

Figure 4: simulation of the .inverse model (BBG) of the EHA : synthesis of electrical and control constraints

4.3. How exploiting the bicausal synthesis of the EHA?

Several opportunities to exploit the inverse modeling by means of the BBG can be addressed. Among them, one can think about :

Synthesizing the "cost" of the control dynamic on the electrical constraints: an example of this issue is illustrated on the synoptic of the Figure 5 : in this case, the position reference (Θ_{CS}^{ref}) provided by the "cruise control" is transformed through a closed loop transfer function (G_{cl}) fixed from the control performance requirements. After having filtered the position (Θ_{CS}) of the desired surface control, the inverse modeling (BBG) allows synthesizing a constraint vector (C_{elec}) composed (for example) of the electrical constraints (current, power). From this approach, *given the sizing of the actuator,* it is possible to *evaluate "the cost" of the desired dynamic performance in terms of electrical constraints.*

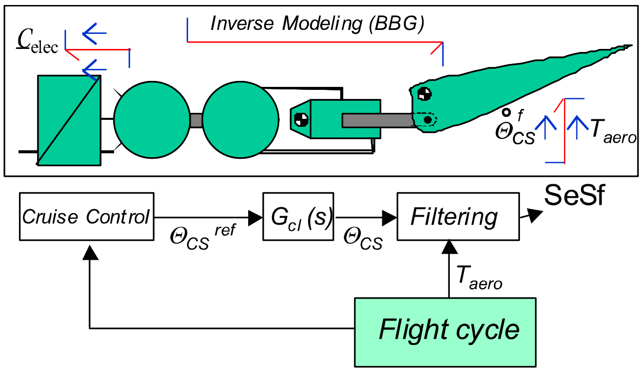


Figure 5: synthesis of electrical constraints (C_{elec}) for a given control dynamic (G_{cl})

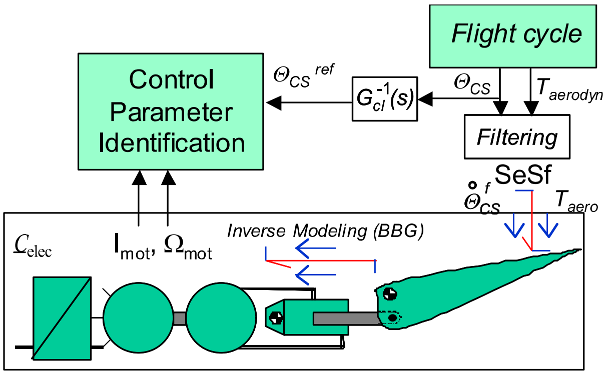


Figure 6: identification of the control parameters by means of the bicausal synthesis process

Synthesizing the control parameters by means of an identification coupled with the BBG : this second issue is related to the identification of the control parameters (for example the state feedback control parameters) by means of the bicausal synthesis process. Indeed, thanks to the BBG, information such as the reference position (Θ_{CS}^{ref}), the motor rotation speed or/and the load current can be exploited to identify to control parameter as illustrated on the Figure 6.

In the example of the Figure 6, the reference trajectory of the position (Θ_{CS}^{ref}) can be synthesized from the actual position of the actuator (Θ_{CS}), by inversing the closed loop transfer function.

5. SECOND BOND GRAPH OF THE EHA: A MORE COMPLETE VERSION

A second Bond Graph of the EHA has been proposed to take into account the main nonlinearities of the hydrostatic transmission line. With reference to the first CBG of the Figure 2, the physical phenomena added in this second version are:

- The mechanical transmission stiffness (r_t) between the jack rod and the control surface;
- The mechanical attachment stiffness (r_a) between the jack body and the airframe;
- The hydraulic accumulator with the re-feeding valves: two valves are used together with an accumulator to ensure a minimal pressure inside each hydraulic line (see Figure 7.b). These valves are modeled with nonlinear *R* elements;
- The leakages inside the hydraulic pump: leakages between each pump chamber and drain (*R1,R2*) as well as those between drain and re-feeding accumulator (*R3*) are considered. Note that the causal loops linking these *R* elements involve algebraic loops which slow down the computation;

The hydraulic capacitance (C_{ch}) of each jack chamber which is non linear due to the volume variations when the piston surface moves: .$C_{ch} = V_{ch}/\beta$, where β is the compressibility module and V_{ch} the volume of a jack chamber which varies with the position of the piston surface.

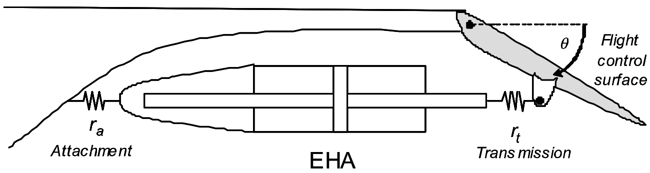
More information on this bond graph model is detailed in [Langlois & al., 2005]. The accuracy of this model has been analysed by comparison with actual tests. Furthermore, simulation tests are in accordance with a physical model developed on the AMESim solver [AMESim].

Several nonlinearities are included in this bond graph as for the re-feeding valves or the hydraulic capacitances of the jack. The bond graph model inversion is possible by applying the MSCAPI procedure as in the previous section. In spite of the nonlinear elements, de BBG can be simulated on 20Sim (with backward Euler algorithm). The structural singularities due to the causal loops between the *R* elements representing the pump leakages slow down the simulation and provoke signal chattering. The Figure 8 shows up the comparison between inverse and direct models with (a) and without (b) the pump leakages. For each case a causal filtering (5Hz) of the flight control surface position and the aerodynamic torque is applied: the inverse model is directly synthesized from the filtered position $\theta^{filtered}$ and the corresponding aerodynamic torque. For the direct model, the control input is the reference position. This latter can be obtained by estimating the closed loop control dynamic.

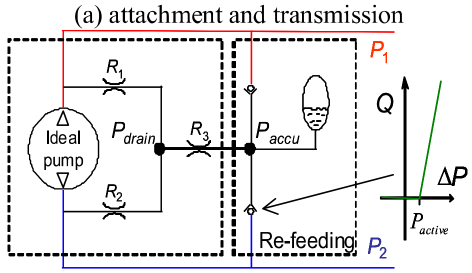
Considering a second order dynamic, the reference is obtained as follows :

$$\theta^{ref}(t) = \theta^{filtered}(t) + 2\frac{\zeta}{\omega_n}\dot{\theta}^{filtered}(t) + \frac{1}{\omega_n^{\ 2}}\ddot{\theta}^{filtered}(t) \qquad (4)$$

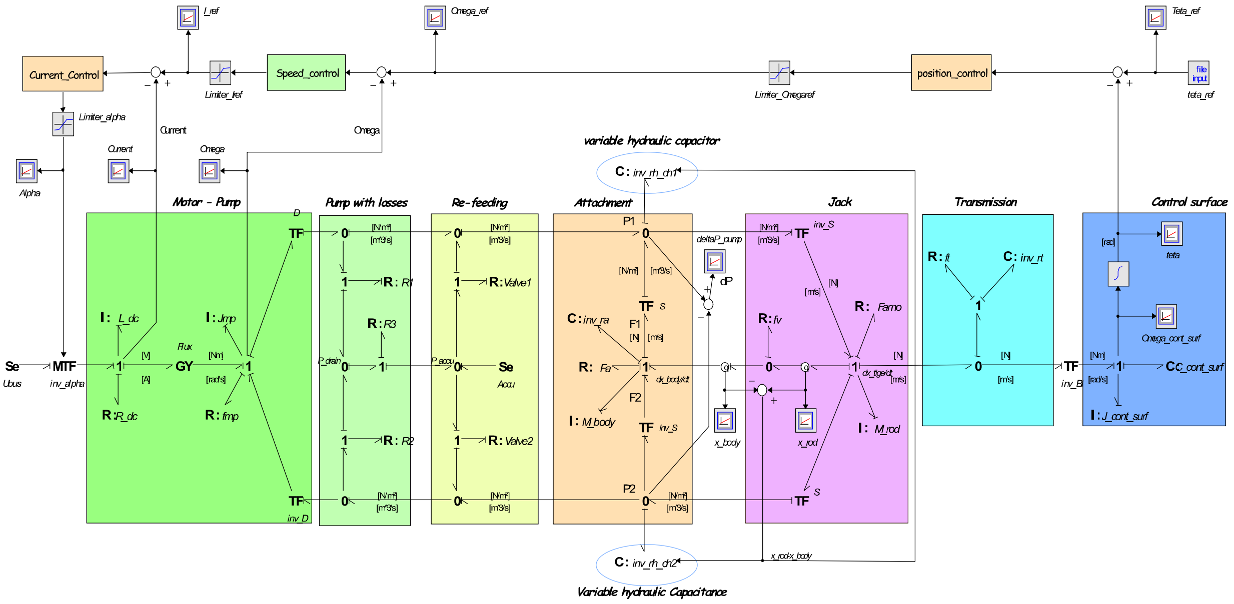
Some slight differences appear between direct and inverse responses mainly due to the estimation of the closed loop control dynamic (Eq. 4). Anyway, this test shows that the inversion process is applicable even with this highly non linear BG.



(a) attachment and transmission



(b) actual pump with leakages and accumulator



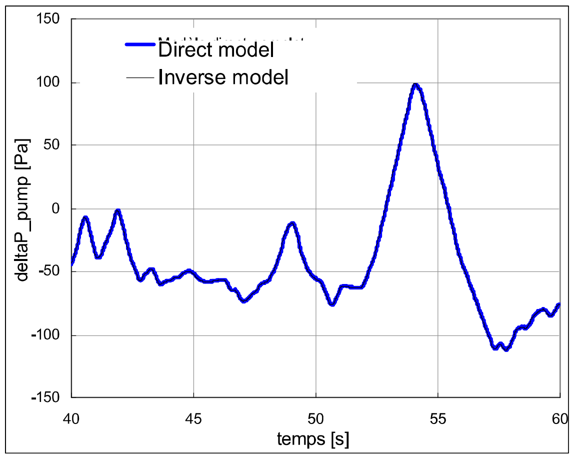
(c) "advanced CBG"

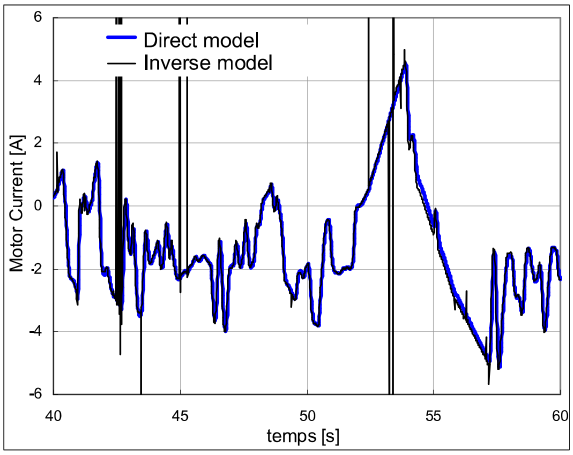
Figure 7: advanced modeling of the EHA

6. CONCLUSION

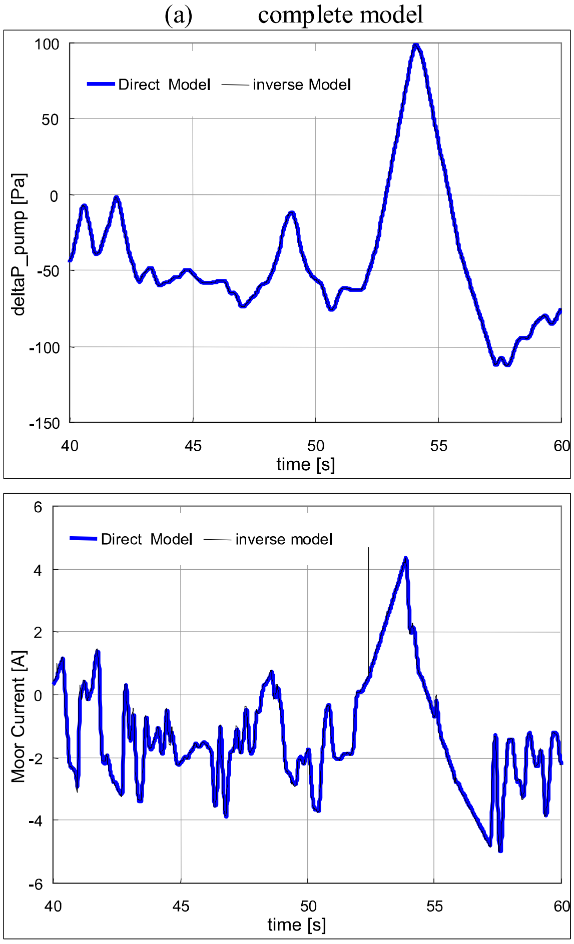
Several issues have been processed in this paper with their related conclusions:

- The BG model inversion procedure has been described and applied to the case study of an EHA which constitutes a typical example of multi domain system. The inversion conditions have been verified by applying the MSCAPI procedure. The obtained Bicausal Bond Graph (BBG) has been validated by comparison with a direct classical approach: the comparative results are perfectly in accordance.
- The output data have to be sufficiently differentiable to inverse the model depending on the number of dynamic elements in derivative causality: a data processing by filtering is then necessary. The efficiency of several data processing approaches has been discussed: a causal filtering is preferable in terms of wave form quality if the issue of phase delay is not critical. On the opposite, an acausal filtering is preferable if the data are a priori known (given from flight mission). It also offers the advantage of an analytical expression for the first and the second derivatives of the filtered data.
- Some examples have put forward the capabilities of the system synthesis procedure by model inversion: a first example, is the opportunity to evaluate "the cost" of the control dynamic in terms of electrical constraints and given the sizing of the actuator. Another example, is related to the identification of the control parameters by means of the bicausal synthesis process.
- Finally, an advanced modeling of the EHA has been developed, involving several nonlinear elements. In spite of these nonlinearities, the system synthesis procedure by model inversion has been applied and has shown its efficiency.





(a) complete model



(b) model without the pump leakages (R1,R2,R3)

Figure 8: comparison between direct (CBG) and inverse (BBG) bond graphs

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