Motion control of elastic joint based on Kalman optimization with evolutionary algorithm

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Abstract—**Actual industrial ambition is to remove a maximum of sensor to improve reliability and cost. Performances are then decreasing a lot, specially for a system with variable parameters and direct drives. Moreover, a two-mass system representing numerous class of industrial problem can become unstable. Keeping stability, a simple controller and observer tuning approach and a lower time consuming are main goals of this study. A previous calculated state feedback is used as base for two Kalman filters with special a noise matrix. An evolutionary algorithm optimizes observer's degrees of freedom to keep stability all over the stiffness variation. The results show that the stability and performances are kept on an experimental test bench.**

Keywords : motion control, robust control, Kalman filtering, Stiffness variation, Evolutionary algorithm

I. INTRODUCTION

Numerous systems working in hard surroundings as crusher, rolling mill or driller have their reliability dramatically decreased by sensors. Due to large parameter variations of axis stiffness, inertia or friction depending on load (driller deepness tool, thickness of iron...), modeling such systems is possible by a well-known two-mass system. Controlling such a system is then harder due to a difference between the load and the motor speeds, then removing the load speed sensor can lead to oscillation or to an unstable system if the system is looped with motor speed or by a non adapted observer.

Several methods exist, slowing down system response by looping with the motor speed or implementing a robust observer. Previous work [2] or robust synthesis [6] prove that keeping the system performances under variations is feasible with a simple controller as an optimized state feedback. This kind of structure is highly adapted to add an observer. Different structures allow robust state reconstruction, adaptive structure [3] or extended Kalman filter [8] provide a correct control but are complex and time consuming due to the algorithm which disable faster variations of the computed parameters to avoid oscillations.

Following previous work objectives [2] (simplest and minimal time consuming control law keeping performances all over the parameter variations), two Kalman filter structures are implemented and described here, after the system presentation. In the first one, a state noise matrix is considered diagonal, the second one computes a state noise matrix representing the system dynamic matrix variation. This allows to decrease the number of parameter to tune and simplify the tuning. Nevertheless, degrees of freedom are too numerous, so, an evolutionary algorithm replaces thus human being to scan the better solution. Then the last section describes experimental results obtained with each structures.

II. SYSTEM MODELLING

A. Actuator implementation

Permanent Magnet Synchronous Motor (PMSM) is the most used drive in machine tool servos and in modern speed control applications due to its desirable features (compact structure, high air-gap flux density, high power density, high blocked torque). Moreover, the position of the motor has to be known for self-control of this kind of machine thus added known Park transformation. This transformation gives constant current values at steady state and shift problem for tracking current reference to a current regulation problem. Advantages are numerous : simpler current controllers, two controllers instead of three and most important, a lower requested controllers bandwidth. Finally in a pulse width modulation control of the inverter, the current loop dynamic is chosen slower to allow inverter linearization but as faster as necessary to neglect the influence of this one in such speed control. Furthermore, to design the current controllers, a linear PMSM model is used. This hypothesis requires to saturate current to stay into linear magnetic state.

B. Mechanical specifications

Considering that electrical parameters are constant, the torque is thus correctly applied on rotor. As explained in introduction, the stiffness can not be neglected and lead to a more complex model. Furthermore, on the considered system, the impossibility to measure the load speed for technical or financial reasons leads to implement an observer to control properly the load speed. Despite the motor speed measurement, the varying stiffness imposes to recalculate the state variable to not have an oscillating or unstable load control. To implement such automatic item, the system has to be modelled by a state space equation as shown on (1) where : T_m is the applied motor torque, T_l is the load torque, J_m, f_m and J_l, f_l are inertias and frictions of respectively the motor and load side and K_T represents the stiffness of the axis and the joint.
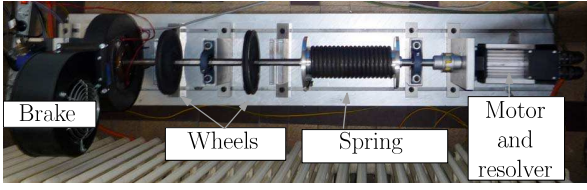
Fig. 1: The test bench

C matrix gives motor speed as output until load speed is controlled because this is the only sensor set up on test bench. Moreover, the three state variables belong to the system : the motor and the load speed (ω_m and ω_l) and the position difference ($\Delta\theta$). But to achieve convergence, it is imposed to also know the load torque (T_l) consequently a new variable representing this torque is added to the state vector. It is now a four dimension vector X_{aug}.

$$X_{aug} = \begin{bmatrix} \omega_m & \omega_l & \Delta\theta & T_l \end{bmatrix}^T$$

$$\begin{cases} \dot{X} = \underbrace{\begin{pmatrix} -\frac{f_m}{J_m} & 0 & -\frac{K_T}{J_m} & 0 \\ 0 & -\frac{f_l}{J_l} & \frac{K_T}{J_l} & -\frac{1}{J_l} \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{A} X + \underbrace{\begin{pmatrix} \frac{1}{J_m} \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{B} T_m \\ y = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}}_{C} X \end{cases} \quad (1)$$

All the test bench parameters and so the simulated system parameters have been chosen to describe an industrial case of a such problem. The test bench id presented on Fig. 1. the three phases motor is powered by an inverter and produces torque. The varying stiffness is created by replacing spring. Two different stiffness are available to have from a flexible to a rigid system $[K_{Tmin}, K_{Tmax}]$. Following sections describe the control laws implemented on system and their performance upon the stiffness variation.

III. CONTROL LAW

Previous work [2] focused on how to design a constant gain controller to minimize effect of parameter variations using a linear quadratic criterion. This controller uses all state variables and is convenient to be coupled with an observer. Nevertheless as explained in [2], the current control is neglected in the speed loop. To achieve this assumption, the dynamic of each loop is carefully chosen to avoid them to impact on the other loop. In fact, inner dynamic loop is 5 to 10 times larger than outer dynamic loop.

To meet the previous system requirements, an observer is implemented and the control law scheme become as drawn on Fig. 2. The load parameter variations imply a modification of matrix A and remove possibility of using a Luenberger observer. In fact, a Luenberger observer is sensitive to system evolutions and that can drive this system to unstability. However, other techniques exists to solve such problem. Kalman filtering [5] is a quite interesting method by considering the parameter variations as a state variable noise. The two following sections present two methods of Kalman filtering gain computation and out tuning algorithm.

figure

IV. KALMAN FILTER

A Kalman filter is an optimal observer in noises rejection point of view. From the motor speed ω_m measurement, the provided torque T_m and the dynamic sampled model based on mechanical relation (2) discretized and noised from (1), state may be rebuild.

$$\begin{cases} X_{k+1} = AX_k + BT_{mk} + w_k \\ \omega_{mk} = CX_k + v_k \end{cases} \quad (2)$$

X is the state vector, T_m the input, ω_m the output and w and v are respectively the state and output noise. However, to assure filter convergence, critical assumption must stay effective, noise have to be white Gaussian noise with a null mean.

Then, implementing the filter on a calculator follows the three sequential steps :

Init : Initial value of state vector $X_{0|0}$ and the error covariance matrix $P_{0|0}$ are given to the observer.

1 : The algorithm's loop is beginning at the correction step by computing the filter gain (3). Then state variables (4) and the covariance matrix (5) are updated with new measurement.

2 : The last step is predicting the new state used by controller (6) and covariance (7).

This last step allows to compare the prediction with the measurement by looping at previous step and step by step having the correct state estimation.

$$\begin{align} K_{k+1} &= P_{k+1|k}C^T(CP_{k+1|k}C^T + R)^{-1} \quad &(3) \\ \hat{X}_{k+1|k+1} &= \hat{X}_{k+1|k} + K_{k+1}((\omega_m)_{k+1} \quad &(4) \\ & \quad -C\hat{X}_{k+1|k}) \\ P_{k+1|k+1} &= (I - K_{k+1}C)P_{k+1|k} \quad &(5) \\ \hat{X}_{k+1|k} &= A\hat{X}_{k|k} + BU_k \quad &(6) \\ P_{k+1|k} &= AP_{k|k}A^T + Q \quad &(7) \end{align}$$

The main challenge of this control structure is to tune the Q and R matrices defining the output shape. Two methods are now proposed, one commonly used and one other using model variation. Note that a normalisation is made to avoid numerical quantification problem between state variable composed with heterogenous quantity. T_4 matrix in Fig 2 is diagonal and bounds state variables between -1 and 1.

A. Common Q and R matrices (KS4)

The first method consists choosing matrices as white uncorrelated noise with a null mean, so matrices are diagonals as detailed in (8). Matrices are normally first set up with estimated noise values and tuned by trial and error. Matrices
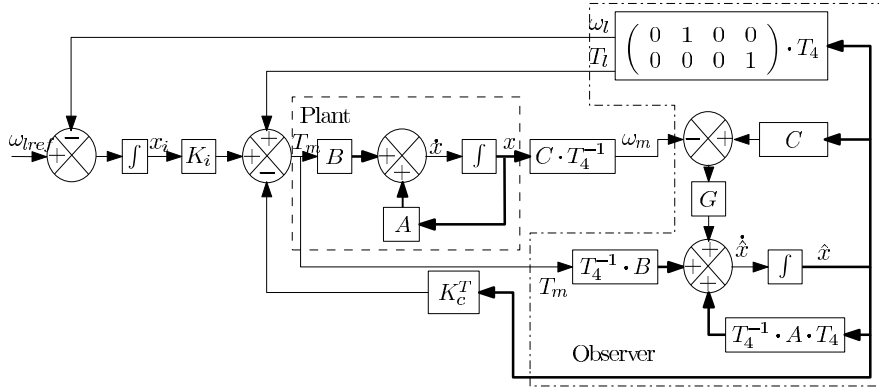
Fig. 2: Implemented sensorless speed control

Q and R with a four order state vector let five parameters to tune (one on input and four on state) by the algorithm.

$$E[w_k] = 0 \quad E[w_k w_i^T] = \begin{cases} Q_k & i = k \\ 0 & i \neq k \\ R_k & i = k \\ 0 & i \neq k \end{cases} \quad (8)$$

$$E[v_k] = 0 \quad E[v_k v_i^T] = \begin{cases} R_k & i = k \\ 0 & i \neq k \end{cases}$$

Parameter variations imply that matrices do not match with classical noises. They have to describe state variables shape modifications which are not gradable as a white gaussian noise. Then to optimally solve the parameters, an evolutionary algorithm presented in section V tunes the matrices.

B. Model variation tuning (KR4)

The second method is extracted and theoretically proved in [7]. A dynamic matrix additive variation $(A + \Delta A)$ is taken into account by modifying the state system as shown in (9) to meet a standard robust representation.

$$\begin{cases} X_{k+1} &= AX_k + B_{1b}w_k + BT_{mk} \\ z_k &= KX_k \\ \omega_{mk} &= CX_k + v_k \\ w_k &= \Gamma z_k \\ B_{1b} &= e^{A(K_{Tmax})T_{em}} - e^{A(K_{Tmin})T_{em}} \end{cases} \quad (9)$$

So B_{1b} represents the available additive variation range (9) here, stiffness variation and Γ the rate of variation at step k. This uncertainty is extracted from the system in another gain block Γ and the system is then linearized. If Γ infinity norm is lower than 1 then system can be rebuild by Kalman filter unless noises are correlated. The open loop system is stable consequently, the calculation of $H\infty$ norm of transfert function $\frac{Z(k)}{W(k)}$ proves that system observes small gain theorem condition. This allows to compute a stable observer. However, the implementation of the covariance error prediction (7) changes to become (10).

$$P_{k+1|k} = AP_{k|k}A^T + Q + B_1 B_1^T \quad (10)$$

$$B_1 = \frac{B_{1b}}{t} \quad (11)$$

B_1 (11) represents the variation rated by t of the dynamic sampled matrix. It is multiplied by its transpose to have a semi-definite positive matrix. Some state variables are not affected by model variation as the load torque. Into the Kalman filter theory, only variables with a non nul variance evolves. The Q matrix allows this kind of variable to evolve by giving them a non nul variance as in standard Kalman filter definition and converge to a correct value. Finally this method has three unknown to be tuned by algorithm : the rating of B_1 matrix, the input noise and the load torque noise. Then, to have best solution for comparison with previous design shape, parameters are tune by the same algorithm to have comparable results. Following section describes the implemented evolutionary algorithm.

V. EVOLUTIONARY ALGORITHM

Given the human abilities, the both presented structures require to deals with a great number of variables. To overcome this difficulty and to scan faster a large space of possible solution, an evolutionary algorithm has been implemented. Extracted from [1], and first presented in [4] by Fogel, this algorithm searches the best set of unknown by evolving item parameters as made in nature. It means that only a combination and a mutation are allowed between input parameters weighted by a normal distribution. These parameters are the noise variances necessary to tune the filter (3 for kr4 and 5 for ks4). The final value is the best solution found.

The principle of this algorithm is presented on Fig. 3. Firstly, classifying items from the most competitive to the less interesting one is a priority, then an evaluation criterion should be calculated. In fact, this optimization ensures that the system has the minimal output deviation for all system variations. The criterion is the difference between the time response of the system with sensor with maximal stiffness $\Theta_{ref} = \begin{pmatrix} J_l, & f_l, & K_{Tmax} \end{pmatrix}$ and the load speed measured for all variations taken into account. The optimization is obtained when the global deviation is minimized. This system has the requested response which the observer and the controller try to maintain whatever parameters are. Then a set of parameter variations Θ is defined. Θ represents all the parametric varia-
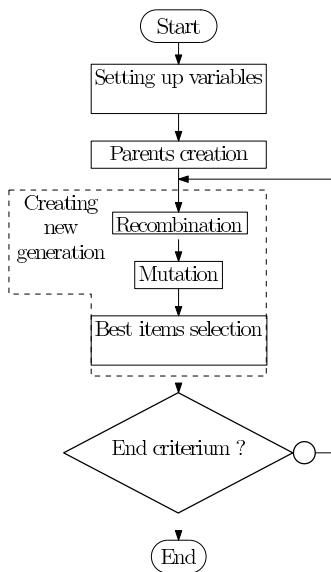
Fig. 3: Evolutionary algorithm

2 : The children's birth is operated with two calculations, a recombination is the first. Two parents are randomly chosen and each parameter is modified by a weighted mean of both parents parameters. The weighting is randomly defined by an uniform distribution.

The second operation is a mutation. The parameters used as standard deviation mutate following a normal distribution with a constant standard deviation.

The observer parameters mutate with a normal distribution with previous mutated standard deviation which are included in the set of parameters. This ambiguous method is used to achieve a quick overview of the allowed space.

3 : Finally, the algorithm is ended when the criterion value is quite similar between two consecutive iteration after a minimal number of steps.

All methods and constants to be defined are explained into the standard reference [1]. Tuning the filter's gains is now possible in two or three hours with a Pentium 4 2.4 Ghz. The last section will show and compare the experimental results of the proposed filters.

VI. RESULTS

The test bench showed in Fig. 1 has mechanical characteristics printed into Table. I. Axis is driven by a 2.5 kW PMSM powered through a PWM inverter switching at 15 kHz. The overall system is controlled with a DSpace 1104 controller. Current ADC have a ten bit accuracy. Motor resolver has 4096 points per turn accuracy and load position sensor has a 3000 points per turn accuracy. As explained, this sensor is only used to save speed shape for offline comparison.

First of all, the two figures showed are the system response optimized for a load torque step and for both stiffness bounds (250 and 2000$Nm.rd^{-1}$). This kind of trial is relevant for this system because the speed regulations have most of time no varying reference and have to avoid speed deviation under load variation. Tests show first a response to a 30 $rd.s^{-1}$ speed step and at time 0.5s a load torque variation from 0 to 5 Nm.

In the rigid version Fig. 4(a), the speed step response is a little faster for kr4 with an overshoot equal to 5%, ks4 observer does not provide overshoot then, the time response is a little slower. Contrarily, the load torque variation response is quite slower for kr4. At the minimal stiffness Fig. 4(b), kr4 is always a little faster for the load speed step time response but slower for the load torque variation. The shaping response is a dealing between increasing the effect of parameter variations or the load torque variations. The maximal stiffness response is the fastest of the both and torque oscillations for the load torque variation shows that system is reacting as quick as

tion bounds to be taken into account. In this case, Θ has two sets of parameter which are the upper and lower bounds of stiffness variation. All Θ time responses of the system driven with the chosen observer are computed. After all experiments, the criterion (12) is computed to evaluate the performance of the structure. The criterion is computed for each variations of the set Θ and all effects are summed. Hence, all variations have to be as closed as possible to reference without any rating between bounds. The purchased purpose is to have an overall optimization and not one optimized variation.

$$
\begin{aligned}
f \;=\; & \textstyle\sum_{i=0}^{n} (\int_{0}^{T_{end}} (\omega_l(t,\Theta_{ref}) - \\
& \omega_l(t,\Theta_i))^2 dt + \int_{0}^{T_{end}} abs((T_m(t,\Theta_{ref}) \\
& -T_m(t,\Theta_i))) \cdot t \cdot dt)
\end{aligned}
\tag{12}
$$

A close regards on criterion reveals two summed items. The first item is the integral quadratic error between the tested response and the reference, this error is relevant when the difference is high thus here for the transient time. Nevertheless, the algorithm takes high coefficients to minimize the transient deviation and this increases the noise's effects which create small speed variations avoided by speed regulation. Consequently, the second item is the integral time absolute error which amplifies the error for steady state because the error is multiplied by time (ITAE).

Secondly, each item has a vector of parameters : the three or five coefficients listed in section IV that observers need (the output of the algorithm), plus one standard deviation per observer's parameters to achieve the mutation.

Thirdly, the optimization needs following steps to be effective :

1 : The parent's creation allows the algorithm to start. The observer parameters are randomly chosen inside the allowed bounded space by an uniform distribution.

Parameter	motor	load
Torque (Nm)	16	7 (brake)
Inertia ($kg \cdot m^2$)	$2.6 \cdot 10^{-3}$	$50.5 \cdot 10^{-3}$
Friction ($Nm \cdot s \cdot rd^{-1}$)	$5.4 \cdot 10^{-3}$	$11.4 \cdot 10^{-3}$
Stiffness ($Nm \cdot rd^{-1}$)	[500; 2000] (springs)	

TABLE I: Test bench parameters

(a) maximal stiffness $(2000Nm.rd^{-1})$

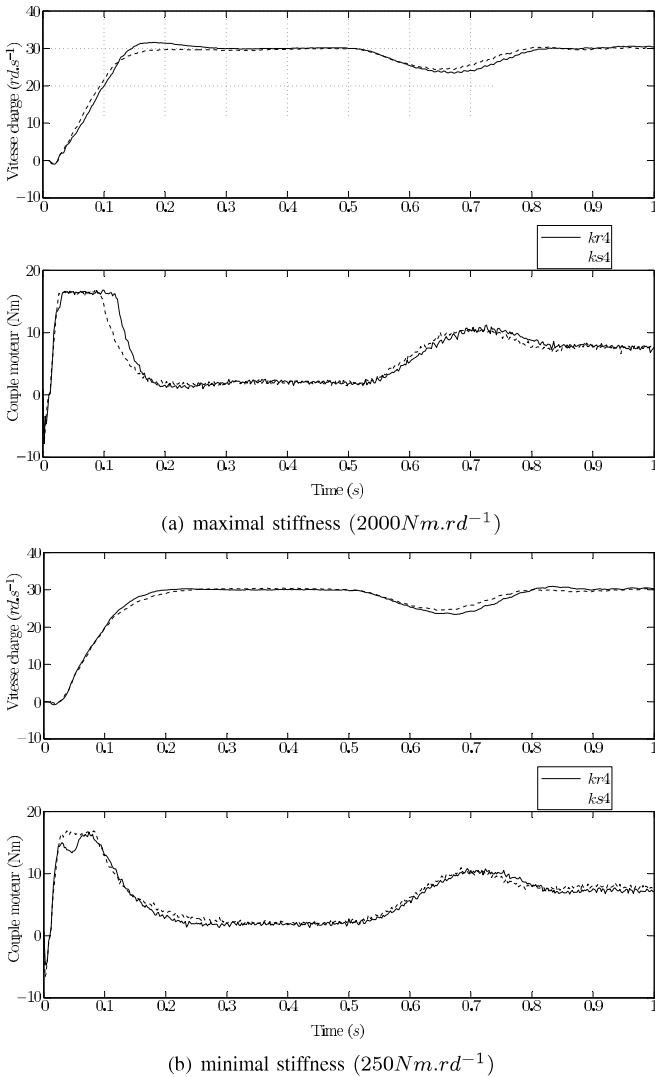(b) minimal stiffness $(250Nm.rd^{-1})$

Fig. 4: Response to a load seep step and a load torque disturbance

possible before becoming unstable. ks4 has a better load torque response because of its uncorrelated parameter allowing a more precise design. But the load speed step response is faster for kr4 (Fig. 4(a)) because each parameters of the state noise matrix is defined to be proportionally valued for the system variation. Additionally, kr4 has the lower number of parameter to tune than ks4 and state noise matrix is chosen with the system variation and not by trial and error. This allows the algorithm to save a lot of time, nearly 25% faster by decreasing the space to scan.

VII. CONCLUSION

Two methods of tuning a Kalman filter are experimented for parameter variant two-mass system. Only stiffness variations are experimented. Both of them have quite similar response after tuning of the five noise variances of ks4 and two noise variances and rating of the variation of the dynamic matrix

with an evolutionary algorithm. ks4 manages to regulate a load torque variation a slighter better but contrarily occurs for load speed variation. The main difference is due to the number of parameter to tune. ks4 with its highest number of uncorrelated parameters happens to manage a slighter better regulation. Nevertheless, kr4 design by using directly state matrix into noise definition has a smaller space to scan and more intuitive design which allow to gain on design time.

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