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A collision avoidance and attraction problem of a vehicle

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1 Introduction

Since automatic process in variety fields is making rapid process, the researches for collision avoidance and attraction problems are now the vogue. In particular, the fields such as moving vehicles or robot arms or agricultural instruments, etc. To solve those problems we have often used the traditional method of minimizing the introduced quadratic cost functions or the Lyapunov stability. For use of the Lyapunov stability we refer to [2,4].

In this paper, we will study the collision avoidance and attraction problem for a moving vehicle by using the Lyapunov stability. In [5, 7, 6] they studied the problems for the artificial dynamical systems by using the Lyapunov stability or Lyapunov-like stability. In this paper we will study the problems as introducing the natural dynamical system studied in [3]. That is, we will design the desirable controls to solve our problems. This paper is composed of four sections. In section 2, we review the Lyapunov stability theories. In section 3, we design the control variables. In section 4, we deduce some sufficient conditions on asymptotic stability.

2 Lyapunov stability theories

We review the tools of Lyapunov stability theory. These tools will be used to construct the controls to solve the collision avoidance and attraction problem. Let $\mathbf{R}^n$ be the n-dimensional Euclidean space with the Euclidean norm $|\cdot|$ and Ω be an open subset of $\mathbf{R}^n$ containing the origin point. By $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ denotes an element of $\mathbf{R}^n$ and $\mathbf{R}^+ = [0, \infty)$. We consider an autonomous nonlinear system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad \mathbf{x} \in \mathbf{R}^n, \tag{1}$$

where $\mathbf{f} : \Omega \to \mathbf{R}^n$. Let us assume that $\mathbf{f}$ is to be smooth enough to guarantee the existence of global solutions of the initial value problems for (1).

For the purpose of considering stability concept in the sense of Lyapunov, we assume there is $\mathbf{e} \in \mathbf{R}^n$ such that $\mathbf{f}(\mathbf{e}) = \mathbf{0}$. Then $\mathbf{x} = \mathbf{e}$ is trivially a solution of (1) and we call it an equilibrium state of (1). We omit to explain the concept of stability of $\mathbf{e}$. For it see the reference [1].

The following theorems will mainly be utilized in solving our problems. For the detailed see [1].

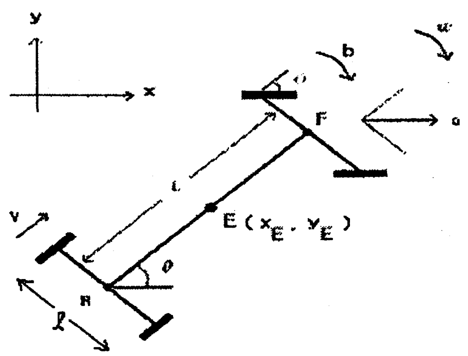
Theorem 1 (Lyapunov's stability) Let $\mathbf{e}$ be a equilibrium point of (1) and let $V : \Omega \to \mathbf{R}^+$ be a C^1 function defined on some neighborhood Ω of $\mathbf{e}$ such that (i) $V(\mathbf{e}) = 0$ and if $V(\mathbf{x}) > 0$ for $\mathbf{x} \neq \mathbf{e}$; (ii) $\dot{V}(\mathbf{x})|_{(1)} \leq 0$ in $\Omega - \{\mathbf{e}\}$. Then $\mathbf{e}$ is stable. Moreover if (iii) $\dot{V}(\mathbf{x})|_{(1)} < 0$ in $\Omega - \{\mathbf{e}\}$, then $\mathbf{e}$ is asymptotically stable.

We call V the Lyapunov function satisfying (i).

Theorem 2 (Lasalle's invariant principle) Let $V : \Omega \subset \mathbf{R}^n \to \mathbf{R}^+$, $\Omega_c = \{\mathbf{x} \in \Omega : V(\mathbf{x}) \leq c\}$, and suppose $\dot{V}|_{(1)}(\mathbf{x}) \leq 0$ on Ω_c. Let $E = \{\mathbf{x} \in \Omega_c : \dot{V}|_{(1)}(\mathbf{x}) = 0\}$. Then the trajectory tends to the largest invariant set in E as $t \to \infty$. In particular, if E contains no invariant sets other than $\{\mathbf{e}\}$, then $\mathbf{e}$ is asymptotically stable.

3 Control variables

In the situation which is having an autonomous vehicle with its target and an obstacle in a two-dimensional workspace, let us consider a problem of controlling a vehicle to avoid a obstacle and to reach its target, which is called a collision avoidance and attraction control problem. A designed system is called stable if the collision avoidance and attraction control problem is achieved, and is called asymptotically stable if it is stable and the vehicle is approaching its target. In this paper we will construct a feedback control to solve the collision and avoidance control problem. Furthermore we will prove the asymptotic stability. The model of an autonomous vehicle is described as follows:



We give some symbols and notations needed in the model as follows. $E = (x_E, y_E)$ is the center of vehicle on the 2-dimensional (x, y)-axis. θ is the orientation of the vehicle with respect to the axis. ϕ is the steering angle. v is the steering velocity with respect to the back wheel. b is the angular velocity of the driving wheel. ω is the angular velocity of the vehicle. a is the steering velocity for the orientation of handle. L is the distance between two wheel centers, F and R. l is the distance between back (also front) wheels. In our simplified setting of vehicle model, we suppose that $\theta = \phi$, $b = \omega$, $a = v$. By the physical laws we can produce the vehicle system of the form

$$\dot{x} = v\cos\theta - \frac{L}{2}w\sin\theta, \quad \dot{y} = v\sin\theta + \frac{L}{2}w\cos\theta, \quad \dot{\theta} = w, \tag{2}$$

where $(x, y) = (x_E, y_E)$ is the center of moving vehicle. In the system (2) the center position (x, y) can be determined by controlling the velocities v and ω, and so we want to construct such v and ω as a feedback of the state (x, y, θ) in order to solve the collision avoidance and attraction problem.

First of all we have to determine the region of the obstacle and we regard it as a circle. Hence we define the set of the obstacle, O, by $O = \{(\xi, \zeta) : (\xi - o_1)^2 + (\zeta - o_2)^2 \le r_O^2\}$, where (o_1, o_2) is the center of O and r_O is the radius of O. Similarly we regard the region of the vehicle as a circle. By neglecting the length of a wheel, the region of the vehicle, A, is defined by $A = \{(\xi, \zeta) : (\xi - x)^2 + (\zeta - y)^2 \le (L^2 + l^2)/4\}$.

Now we introduce functions for approaching and avoiding. For approaching the vehicle to the target at the desired angle of incidence we introduce a function $G(x, y, \theta) = \frac{1}{2}\{(x - \tau_1)^2 + (y - \tau_2)^2 + (\theta - \theta_3)^2\}$, which is a measure of the distance from A to T at the prescribed angle. Here by (τ_1, τ_2) denotes the center of the target, $T(\neq O)$, and by θ_3 denotes the angle of incidence at the target. For avoiding the vehicle from the fixed obstacle we introduce a function $W(x, y) = \frac{1}{2}\{(x - o_1)^2 + (y - o_2)^2 - (r_O + \sqrt{(L^2 + l^2)/4}\,)^2\}$, which is a measure of the distance from A to O.

By using the functions G and W, let us define a total Lyapunov function as follows:

$$V(x, y, \theta) = G(x, y, \theta) + \alpha\frac{G(x, y, \theta)}{W(x, y)} \tag{3}$$

for collision avoidance and attraction. Here in (3) α is a fixed positive real number which shows the tendency of avoidance. Then it is verified that the function V is defined, continuous and positive on the domain $D(V) = \{(x, y, \theta) \in \mathbf{R}^3 : W(x, y) > 0\}$. Differentiating $V(x(t), y(t), \theta(t))$ at t for the system (2), we have

$$\dot{V}(x, y, \theta)\Big|_{(2)} = (f\cos\theta + g\sin\theta)v + \frac{1}{2}L(-f\sin\theta + g\cos\theta + \frac{2}{L}h)\omega,$$

where $f = f(x, y, \theta) = \left(1 + \frac{\alpha}{W}\right)(x - \tau_1) - \frac{\alpha G}{W^2}(x - o_1)$, $g = g(x, y, \theta) = \left(1 + \frac{\alpha}{W}\right)(y - \tau_2) - \frac{\alpha G}{W^2}(y - o_2)$ and $h = h(x, y, \theta) = \left(1 + \frac{\alpha}{W}\right)(\theta - \theta_3)$. Let us take the feedback velocity controls v and ω as follows:

$$v = -k(f\cos\theta + g\sin\theta), \quad \omega = \frac{2k}{L}(f\sin\theta - g\cos\theta - \frac{2}{L}h),$$

where $k = k(x, y, \theta) > 0$ can be chosen arbitrarily. With these controls it can easily be proved that

$$\dot{V}(x, y, \theta)\Big|_{(2)} = -\frac{1}{k}(v^2 + \omega^2) \leq 0. \tag{4}$$

The function $k = k(x, y, \theta)$ plays an important part of scaling the magnitudes of v and ω. For example, if we take $k = |f| + |g| + 2|h|/L + 1$, then $|v| < 1$ and $|\omega| < \frac{2}{L}$.

In order to analyze (2) for hidden meaning let us replace v and ω into (2) and simplify it. Then we have the designed feedback system

$$\begin{aligned} \dot{x} &= -k\left(f - \frac{2h}{L}\sin\theta\right), \\ \dot{y} &= -k\left(g + \frac{2h}{L}\cos\theta\right), \\ \dot{\theta} &= \frac{2}{L}k\left(f\sin\theta - g\cos\theta - \frac{2}{L}h\right). \end{aligned} \tag{5}$$

In our setting of approaching to the target the angle θ should approach to θ_3 when the vehicle approaches to the target. It is easily verified that the point $\mathbf{e} = (\tau_1, \tau_2, \theta_3)$ is the equilibrium point of (5).

Summarizing these we have the following theorem.

Theorem 3 The equilibrium point $\mathbf{e} = (\tau_1, \tau_2, \theta_3)$ of (5) is stable.

Proof: Since $G(\mathbf{e}) = 0$, $f(\mathbf{e}) = g(\mathbf{e}) = h(\mathbf{e}) = 0$. Hence $\mathbf{e}$ is the equilibrium of (5). Then it is clear that $V(\mathbf{e}) = 0$ and $V(x, y, \theta) > 0$ for all $(x, y, \theta) \in D(V) - \{\mathbf{e}\}$. By (4) we get $\dot{V}(x(t), y(t), \theta(t))|_{(5)} \leq 0$ for all $(x, y, \theta) \in D(V)$. Therefore this theorem is proved.

4 Sufficient conditions on asymptotic stability

Unfortunately we could observe that the designed controls were not suitable for solving our problems, because it was not natural to control the three variables v, ω and θ, simultaneously. Hence we consider the following specific feedback control law:

$$v = -k_0(f_0\cos\theta + g_0\sin\theta), \quad \omega = \frac{2k_0}{L}(f_0\sin\theta - g_0\cos\theta),$$

where $k_0 = k_0(x, y) > 0$, $G_0(x, y) = \frac{1}{2}\{(x - \tau_1)^2 + (y - \tau_2)^2\}$ and $f_0 = f_0(x, y) = \left(1 + \frac{\alpha}{W}\right)(x - \tau_1) - \frac{\alpha G_0}{W^2}(x - o_1)$, $g_0 = g_0(x, y) = \left(1 + \frac{\alpha}{W}\right)(y - \tau_2) - \frac{\alpha G_0}{W^2}(y - o_2)$, which are independent of θ. In this case the feedback system is given by

$$\dot{x} = -k_0 f_0, \quad \dot{y} = -k_0 g_0, \quad \dot{\theta} = \frac{2}{L}k_0(f_0\sin\theta - g_0\cos\theta). \tag{6}$$

The first two equations in (6) are independent of θ, i.e., the position (x, y) is determined by f_0 and g_0 only. Sequently we separate the system (6) from θ and we study the system of

$$\dot{x} = -k_0(x, y)f_0(x, y), \quad \dot{y} = -k_0(x, y)g_0(x, y). \tag{7}$$

Of course, the angle θ is determined by the position (x, y). Then we have the following stability result on the subsystem (7).

Theorem 4 The equilibrium point $\mathbf{e} = (\tau_1, \tau_2)$ of (7) is stable.

Proof: Similar to Theorem 3 this theorem is easily proved if we consider the Lyapunov function V defined by $V(x, y) = G_0(x, y) + \alpha\frac{G_0(x,y)}{W(x,y)}$ on the domain $D(V) = \{(x, y) \in \mathbf{R}^2 : W(x, y) > 0\}$.

Unfortunately we can see that $\mathbf{e}$ is not asymptotically stable. We denote by l the straight line jointing $\mathbf{e} = (\tau_1, \tau_2)$ with $\mathbf{o} = (o_1, o_2)$. Here we can make the solutions of (7) move on l. That is, with an initial condition (x_0, y_0) on l, we can prove that the form of functions $x = \eta(t), y = \zeta(t) = \tau_2 + m(x - \tau_1), m = (o_2 - \tau_2)/(o_1 - \tau_1)$ with $\eta(0) = x_0,\ \zeta(0) = y_0$ satisfies (7). This means that $\mathbf{e}$ is not asymptotically stable and there are another equilibrium other than $\mathbf{e}$. It is easily shown that $E = \{(x, y) : \dot{V}(x(t), y(t)) = 0\} \subset l$. Hence by Lasalle's invariant principle all trajectories must converge to some points on the straight line l. Let $\mathbf{e}^* \neq \mathbf{e}$ be another equilibrium point on l and let us assume that $\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{e}^*$, which is possible due to $\dot{V}|_{(7)} = 0$.

Lemma 1 The equilibrium $\mathbf{e}^*$ is not asymptotically stable.

Proof. Let us prove this lemma by using the fact that $\dot{V}(x(t), y(t)) \leq 0$ and the definition of $V(x, y)$. By C_G and C_W denote the circles at centers $\mathbf{e}$ and $\mathbf{o}$ with radii $|\mathbf{e} - \mathbf{e}^*|$ and $|\mathbf{e}^* - \mathbf{o}|$, respectively. This picture shows that $V(\mathbf{e}^*) > V(x_1, y_1)$ for all $(x_1, y_1) \neq \mathbf{e}^*$ on the circle C_W. Since $\dot{V}(x(t), y(t)) \leq 0$, it is not possible that $\lim_{t\to\infty}(x(t), y(t)) \notin l$, where $(x(t), y(t))$ is the solution with an initial point on C_W.

We consider the linearlized system of (7):

$$\dot{\tilde{\mathbf{x}}} = A\tilde{\mathbf{x}}, \quad A = \partial\mathbf{f}(\mathbf{e}^*), \quad \tilde{\mathbf{x}} = \mathbf{x} - \mathbf{e}^*. \tag{8}$$

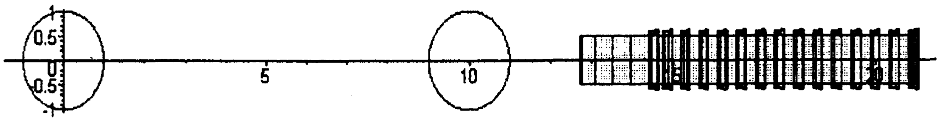
Since $x = x(t), y = y(t) = \tau_2 + m(x - \tau_1), m = (o_2 - \tau_2)/(o_1 - \tau_1)$ is the solution of (7) with an initial condition on l, at least one of the eigenvalues of A must have negative real part. But all two eigenvalues of A can't have negative real parts owing to Lemma 1. Therefore the eigenvalues of A are two real roots with opposite sines. There is one-dimensional manifold S containing $\mathbf{e}^*$ such that the solutions $(x(t), y(t))$ of (7) with the initial points on the manifold

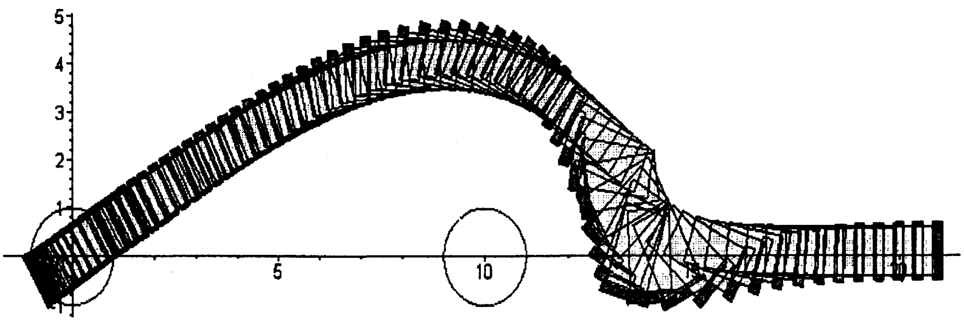
S satisfies $(x(t), y(t)) \to \mathbf{e}^*$ as $t \to \infty$. Here $S = l$. Therefore all the solutions $(x(t), y(t))$ with the initial conditions not on l tends to (τ_1, τ_2) as $t \to \infty$.

Therefore we obtain the main theorem as follows.

Theorem 5 The equilibrium point (τ_1, τ_2) of (7) is asymptotically stable if the initial conditions are not on l.

Example 1 When $x(0) = 20, y(0) = 0, \theta(0) = 0$, the vehicle stopped before the obstacle. With change of $x(0) = 20.1$ the vehicle went to the target. Note that we took $k = |f| + |g| + 1$, $\alpha = 1$, $(\tau_1, \tau_2) = (0, 0)$ and $(o_1, o_2) = (10, 0)$.





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