Emergent Form from Structural Optimisation of the Voronoi Polyhedra Structure

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Abstract

In the course of the exploration of computational means in the architectural design process, in order to investigate more complex, adaptive geometries, the Voronoi diagram has recently gained some attention, being a three-dimensional space-filling structure which is modular but not repetitive. The project looks at the Voronoi diagram as a load-bearing structure, and whether it can be useful for structural optimisation. Hereby the edges of the Voronoi polyhedra are regarded as structural members of a statical system, which then is assessed by structural analysis software. Results seem to indicate that the Voronoi approach produces a very specific structural as well as spatial type of order. Through the dislocation of the Voronoi cells, the statical structure becomes more complex through emergent topology changes, and the initially simple spatial system becomes much more complex through emerging adjacencies and interconnections between spaces. The characteristics of the emerging form, however, lie rather in the complexity how shifted spaces and parts are fitted together, than in a radical overall emergent geometry. Spatially as well as a structurally, the form moves from a simple modular repetitive system towards a more complex adaptive one, with interconnected parts which cannot stand alone but rather form an organic whole.

Introduction

Alongside the introduction of computation in the design process, architects and structural engineers have been exploring the possibilities of more complex geometries and adaptive forms and structures. The Voronoi diagram has recently gained some attention in this field, being a three-dimensional space-filling structure which is modular but not repetitive, and implicitly introducing the notion of spatial relationships through adjacencies of Voronoi cells. However, the actual geometry of the Voronoi polyhedron is difficult to predict and control, as the shape of a cell is always dependent on the configuration of the entire neighbourhood. The geometry and the topology of the polyhedron – like size, proportion or the number of edges – of each cell is highly sensitive to even the slightest change of position of any point in the neighbourhood. Being precise about the geometry of space and structure, however, is what architecture is concerned with in the first place. So although the Voronoi diagram seems to work well in optimising topologies, it remains unclear in how far the difficulty to control the cell shape is a limitation for its use as a design tool in architecture.
This project explores emerging geometries of the Voronoi diagram under special regards of geometric properties of the Voronoi polyhedra. The project looks at the Voronoi diagram as a load-bearing structure, and whether it can be controlled to be useful for structural optimisation. The Voronoi structure, regarding edges of the Voronoi polyhedra as structural members, is determined statically using structural analysis tools. The system aims to optimise through systematically moving the Voronoi points. – Although the emerging geometries are assessed statically in the first place, the project aims to commence a discussion about the emerging architectonic space which develops from this.

Related work

Research has been done to investigate the potential of the Voronoi structure as a means of generating adaptive parametrised topologies, given parameters of the topology of the system [1][2]. By optimising the topology of cells the emerging geometry of the Voronoi structure is suggested to be a suitable geometrical solution for the problem, or at least to a good starting point for further optimisation.
The Kaisersrot project [1] generates layouts for housing developments, given complex input parameters like desired adjacencies, attractors and plot sizes. The process of generating the layout proceeds in two stages: At first, the topology is optimised according to the affordances of the input parameters. Having found an acceptable solution, the actual geometry is improved for example through operations like straightening out edges.
Furthermore, the Voronoi structure has been formally associated with foam-like structures such as sponges, bone structures and crystals [3]. The tradition of these formal associations reaches back to the famous work of architects like Toyo Ito, Buckminster Fuller or Frei Otto, who looked at formation principles, geometries, spatial effect and constructions in nature, using these ideas as a formal, spatial and/or constructive inspiration for architecture.

This project takes the approach to assess the geometry of the Voronoi structure rather than its topology in the first place, in order to investigate if it can be controlled sufficiently to act as a statical structure. It shall be suggested that this is possible, and, furthermore, that the emerging geometric and spatial features of the optimised structure reveal distinct characteristics which are different from former formal associations like foams, sponges or bubbles.

Setup

The software which was developed for this project consists essentially of two interacting components: a program written in Processing [4] to generate the three-dimensional Voronoi diagram and to create a statical structure from it, and the structural analysis program Oasys GSA [5] to assess the structure. GSA can be controlled remotely via a com-interface, so the process of analysing models and reimporting results can run automatically, triggered by the Processing applet.

The three-dimensional Voronoi structure is created from an initial configuration of points. Some of these Voronoi points are declared as 'structural points' which means that their Voronoi cells shall be members of the structural system, and be subject to further analysis, whilst other points are just ‘surrounding cells’. The cells of the structural points are confined by the cells of the surrounding non-structural points, and are clipped at the bottom plane. During the optimisation process, the structural Voronoi points are moved in order to seek a configuration which generates statically improved Voronoi polyhedra.

In order to translate the Voronoi polyhedra of the structural points into a statical system, the Voronoi edges are regarded as beams, interconnected through rigid nodes. The beams are assigned some material property – a circular hollow steel profile with a diameter of 0.3 m and a wall thickness of 0.02m. Beams which connect to the bottom plane are defined as fixed supports.

Several simple load cases have been tested. The structure is always considered in terms of self weight. Additionally, in some cases a moderate wind load has been applied which means horizontal force, 1 kN/m^2, and suction on roof areas.

Oasys GSA calculates the values of forces, moments and displacement of the structure. The analysis results are then reimported into the Processing applet. Now the optimisation target is to minimise the maximum displacement value of the nodes, by stepwise amending the structure through movement of the Voronoi points.

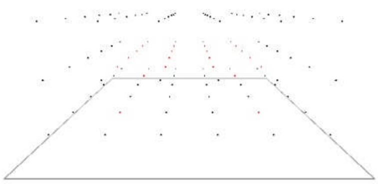


Fig 1a Configuration of points

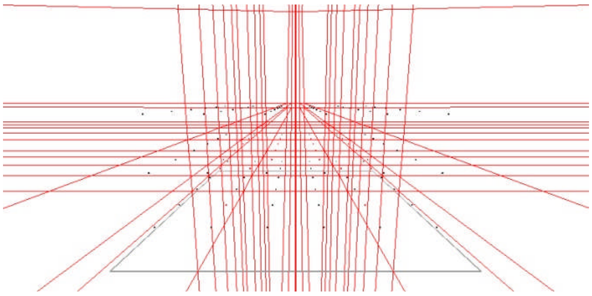


Fig 1b Voronoi polyeder

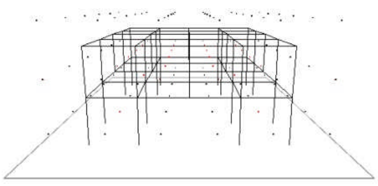


Fig 1c Polyhedra of the structural points

Fig 1: The Processing applet

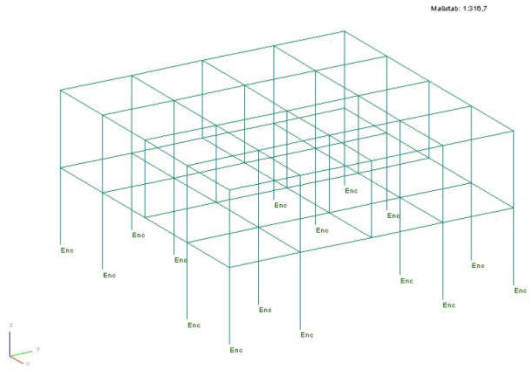


Fig 2a Beam Structure

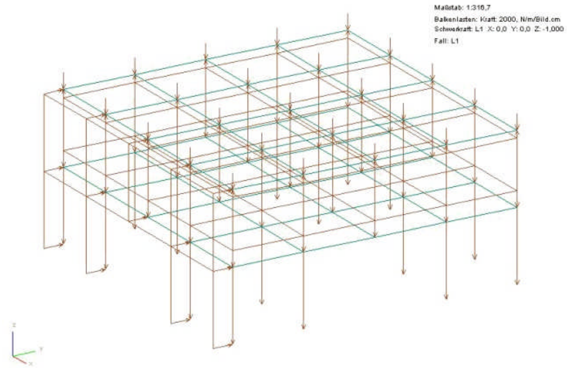


Fig 2b Loads of self-weight and wind load

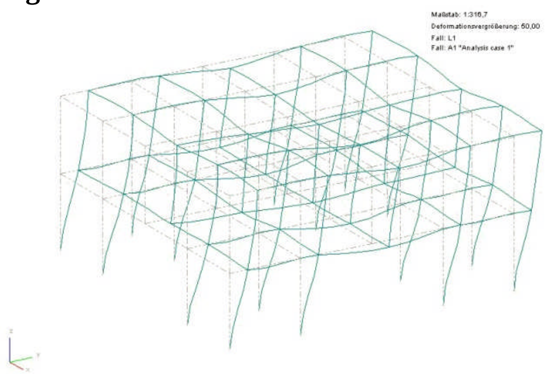
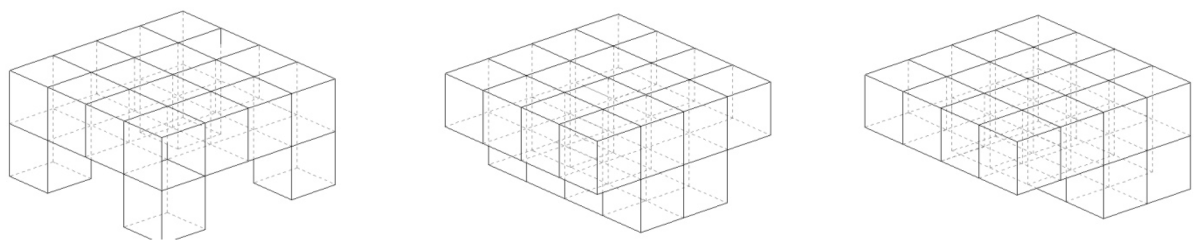


Fig 2c Displacement of nodes and beams due to loading

Fig 2: Oasys GSA

Optimisation

As initial configurations, several simple point arrangements have been tested. As all points are initially located on a grid, polyhedra are simple cubes.



type 1 **type 2** **type 3**

Fig. 3 Initial Configurations

Three optimisation techniques have been tested. The first one moves one point at a time. An array of movement vectors is created, which holds 26 normalised vectors for all directions between (-1.0,-1.0,-1.0) and (+1.0,+1.0,+1.0) One point is chosen, and the program evaluates the impact of the movement of this point, applying successively all movement vectors, then in the end the best option – if there is one - is chosen and the point is moved in this direction. Then the program moves on to the next point.
The second strategy implements a gradient descent algorithm. Hereby, any point does a trial step in any x, y and z direction. After each point has been tested in any three dimensions, the

'best move' for each point is guessed from the results of the trial steps, by multiplying the amount of success from each trial with the respective coordinate, creating a movement vector for each point. Finally, all points are moved simultaneously according to their movement vector.
However, both optimisation strategies do not seem to be able to effectively optimise the structure. As the cell geometry is extremely sensitive to even the slightest movements of the points, the relocation of a single point often leads to abrupt changes in the topology of the structure. For strategy one, this causes the optimisation process constantly being trapped in local optima. For example, if a point already is in line with its neighbours, and the edges forming continuous elements, any movement of a single point out of this line means a decline of stability through the loss of connectivity to the other principal elements (Fig 5). The optimised results of strategy one seem quite random, with large - and critical - areas remaining unchanged as no better solution could be found for them.
The second strategy is also constricted by a lack of topology control: As Voronoi points are moved, one point and one dimension at a time, and a guess for the best move is made from this isolated movement, it often occurs that none of the topological features which would be of advantage actually happen in the end, when all points are moved simultaneously. - The resulting structure also seems random, and does not improve steadily, but oscillates between extremes of better and worse results
Considering the condition of the Voronoi diagram that a particular edge of a polyhedron is always dependent on the simultaneous influence of neighbouring points, a third strategy has been adopted, which groups similar and adjacent points, and moves them in respect to each other in any possible direction, to evaluate the best combinational move. This technique allows the structure to change gradually whilst maintaining continuously linked members between cells where necessary. (Fig. 6)

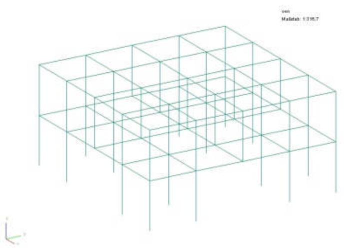


Fig 4a Initial configuration

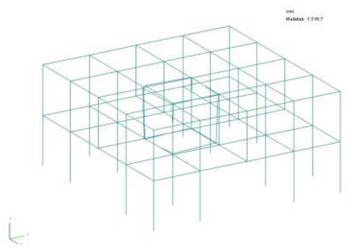


Fig 5a Moving one point

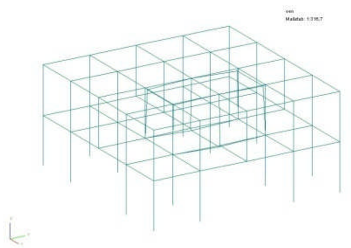


Fig 6a Moving two points

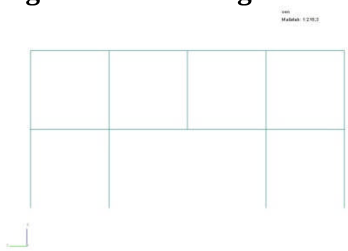


Fig 4b Elevation

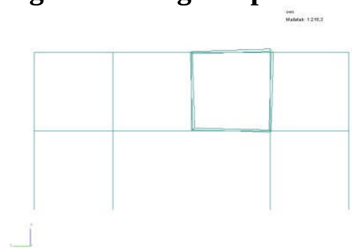


Fig 5b Elevation

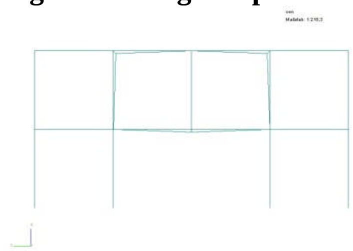


Fig 6b Elevation

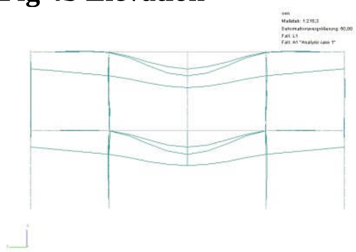


Fig 4c Deformed Elevation: max. Displacement = 89.62mm

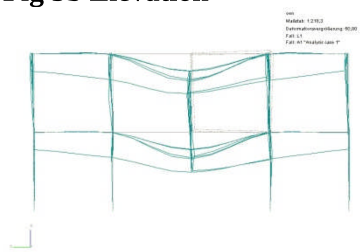


Fig 5c Deformed Elevation max. Displacement = 102.1mm

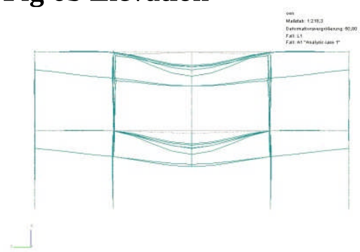


Fig 6c Deformed Elevation max. Displacement = 82.44mm

Test cases

Each of the three configuration types have been tested under self-weight conditions as well as with additional wind loading (Fig 5).

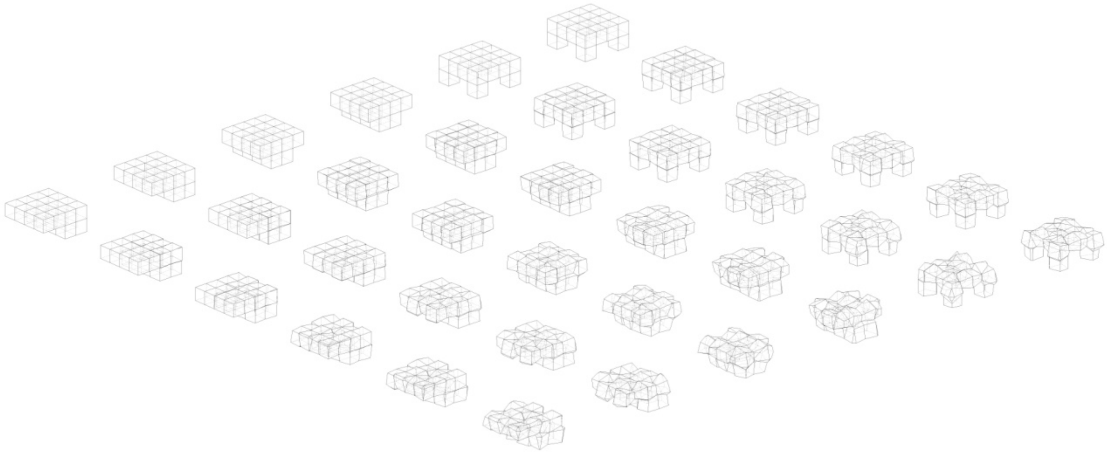


Fig 5a Test cases

					1.01.001					
				1.02.001		1.01.013				
			2.01.001		1.02.013		1.01.025			
		2.02.001		2.01.013		1.02.025		1.01.061		
	3.01.001		2.02.013		2.01.025		1.02.061		1.01.097	
3.02.001		3.01.013		2.02.025		2.01.061		1.02.097		1.01.193
	3.02.013		3.01.025		2.02.061		2.01.097		1.02.193	
		3.02.025		3.01.061		2.02.097		2.01.193		
			3.02.061		3.01.097		2.02.193			
				3.02.097		3.01.193				
					3.02.193					

Fig 5b captions of test cases

The optimisation process starts by slightly contorting and twisting the cubes against each other, and then moving on to developing more strongly distorted and deformed polyhedrons. Under self-weight conditions, displacement of the nodes is predominantly in the vertical direction, leading to tilts of the horizontal members. Considering more complex load cases including wind load, the contortion is more complex and includes shifts of the vertical members more frequently.
Fig 5a shows the six test cases as they develop over time. The captions should be read

1.	*01.*	*001*
Type	*load case* *01 = self weight* *02 = self weight + wind load*	*time step*

During ongoing transformation, the structure seems to pass through several typical formal stages (Fig 6, 7 and 8). The first stage is characterised through slightly twisted elements. In the next stage, the polyhedra dislocate more strongly, whereby more complex adjacencies between formerly not connected spaces appear. Finally, the spaces transform into complex polyhedra with strongly tilted planes, with little similarity to the original shapes.

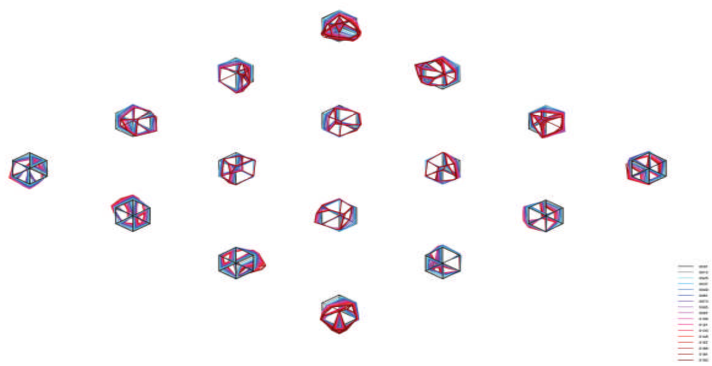


Fig 6 Type 1.02: Overlaid shapes of cells during optimisation

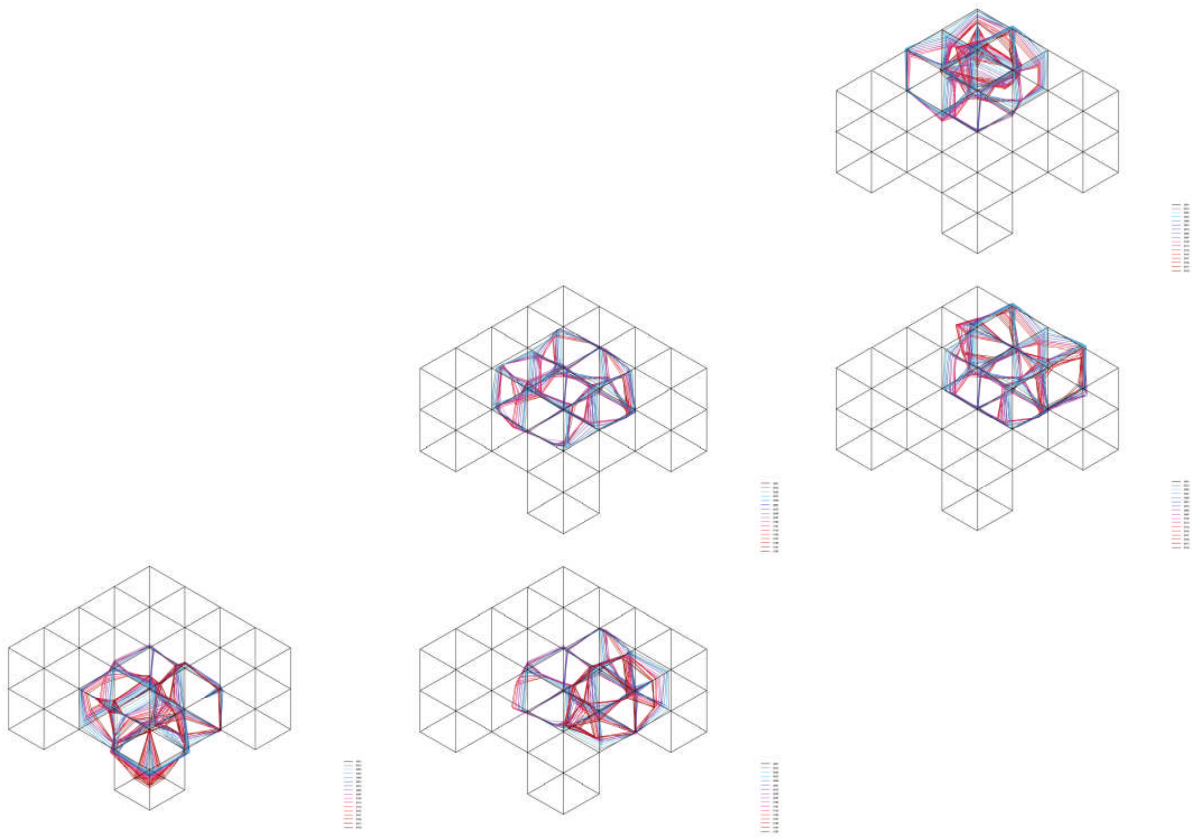


Fig 7 Type 1.02: Overlaid shapes of respectively four adjacent cells during optimisation

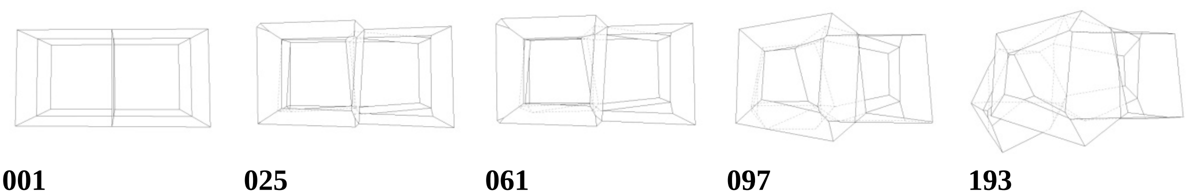


Fig 8 Type 1.02: Perspective of four adjacent cells during optimisation

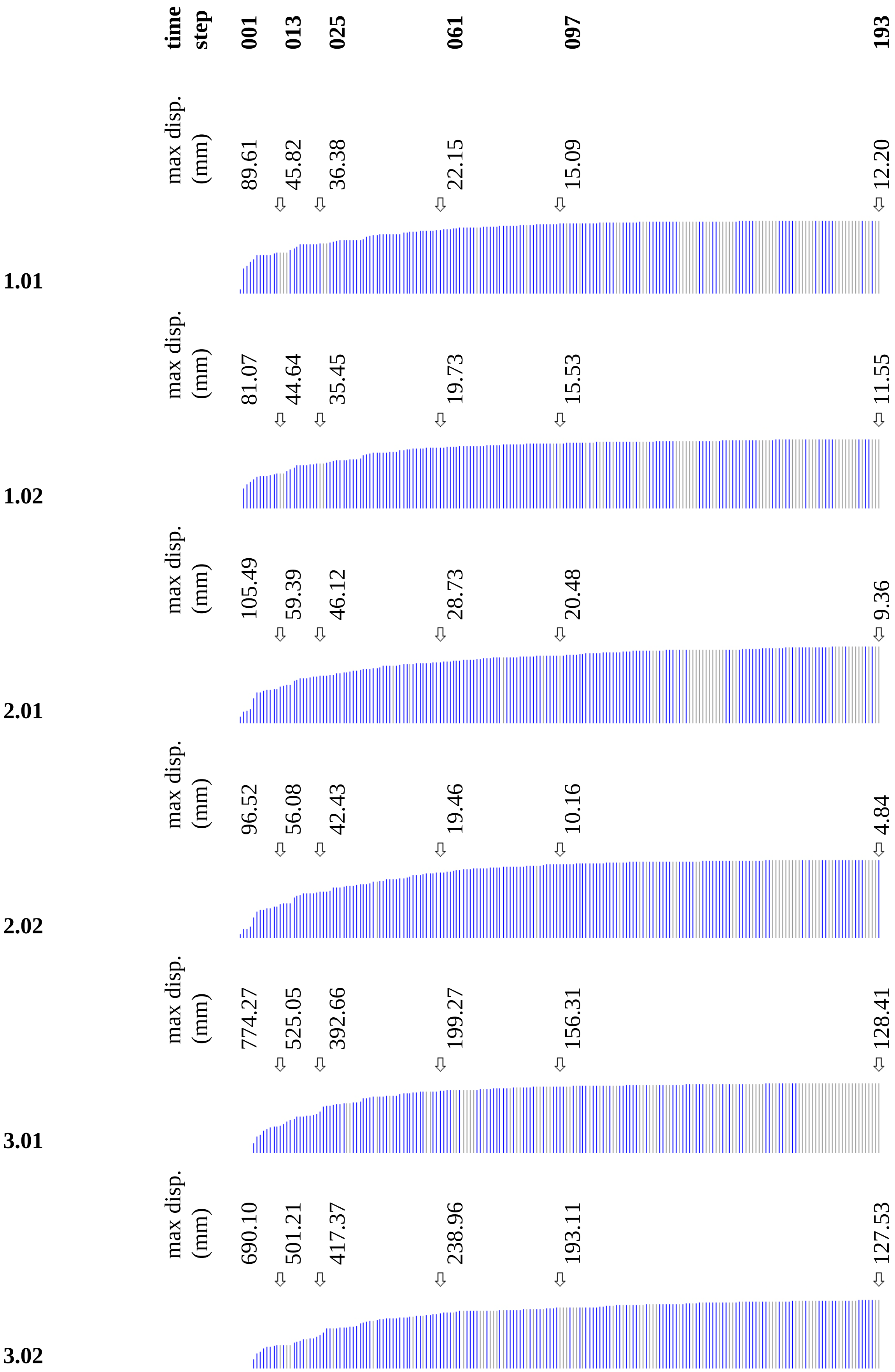
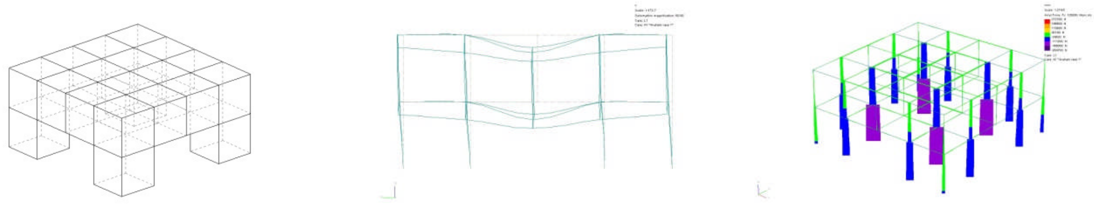
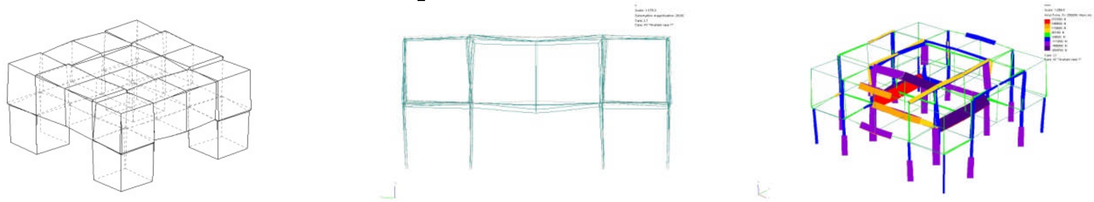


Fig 9 Optimisation statistics

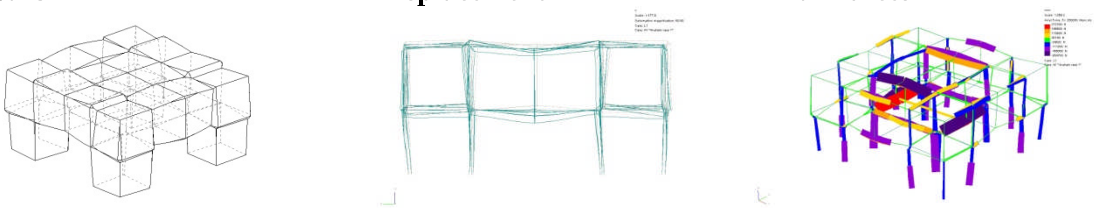
The optimisation statistics (Fig 9) reveal that, in all cases, there is a significant improvement even in early stages, as optimisation success usually happens to follow a logarithm - shaped curve. Although in later stages the structure changes strongly – the twisted cubes turn into more bubble-shaped complex polyhedra, the increase in fitness is relatively lower than in the first stage.



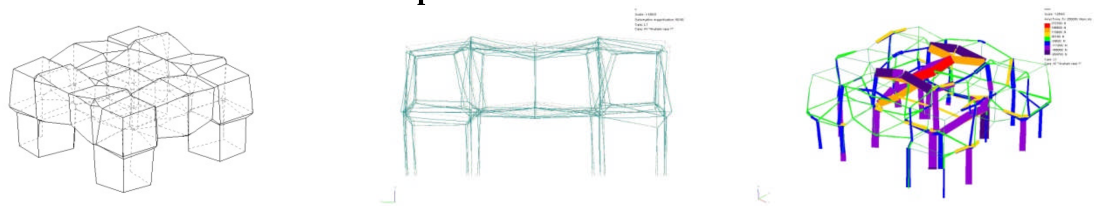
1.02.001 **Displacement** **Axial Forces**



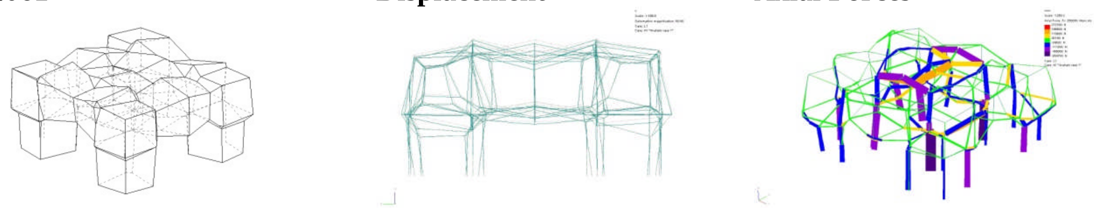
1.02.013 **Displacement** **Axial Forces**



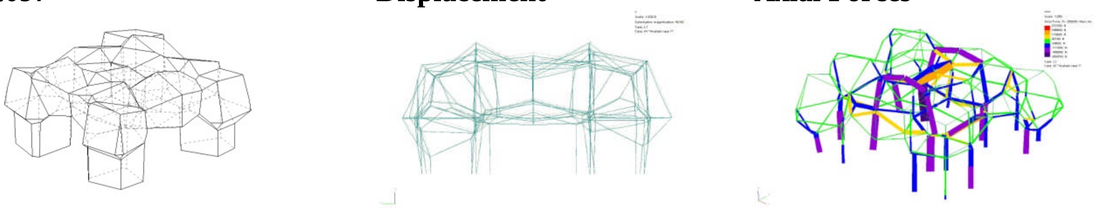
1.02.025 **Displacement** **Axial Forces**



1.02.061 **Displacement** **Axial Forces**



1.02.097 **Displacement** **Axial Forces**



1.02.193 **Displacement** **Axial Forces**

Fig 10 Type 1.02: Geometry, displacement and axial forces during optimisation

What happens in the first stage is that, although the cubes do not change radically in shape and proportion, so does the topology of the beams (Fig 10). The beams tend to double up when the cells move out of the grid, and the members themselves contort against each other making the overall structure more stable. – The axial forces diagram reveals that, in opposition to the original structure, which is predominantly stressed by pressure forces, the doubled edges have a pressure and a tension stressed member (colours red to yellow represent tension, green to lilac indicate pressure). This effect diminishes in later stages of the optimisation.

Conclusion

These findings seem to indicate that a considerable improvement in structural performance can already be achieved through slight contortments and topological local changes of the structure. The usage of the Voronoi diagram as underlying system hereby plays a crucial role, as these emergent topologies are a key property of the Voronoi diagram itself. It shall be suggested this property of the 'instability of the topology', which has initially be considered as a threat, has turned out to provide rather interesting system conditions, which can be exploited for optimisation in a very distinct manner, and produces rather unique structural systems.

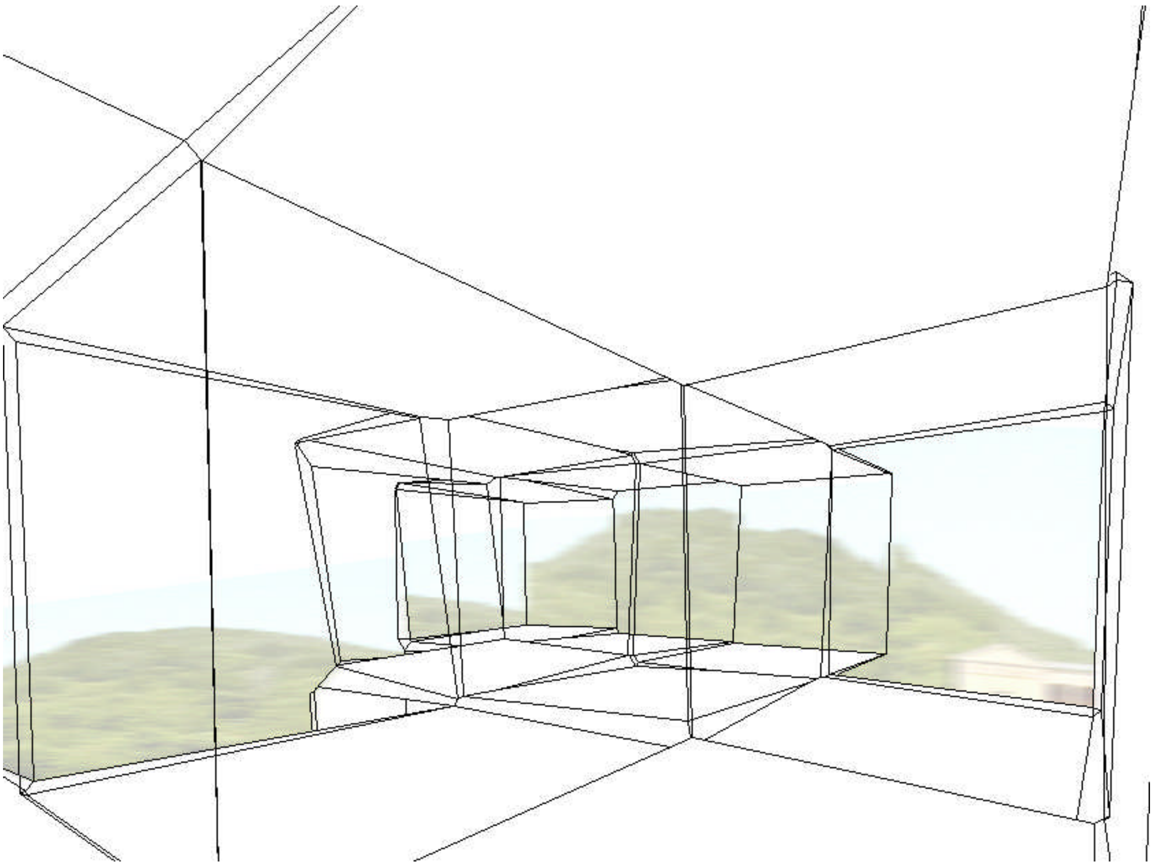


Fig 11 Interior perspective of an optimised structure in the early stage

Furthermore, it shall be suggested that the emerging geometries might provide an interesting research field in terms of their spatiality. In the course of optimisation, space undergoes

certain distinct 'phases': from a cube-like additive space to a stage where spaces are contorted, still ‘Cartesian’ but more interwoven, until in the end orthodox geometry gets lost and gives way to more 'organic' bubble-shaped forms. It shall be suggested that it might be the earlier stages of the process which might be of special interest, structurally as well as spatially.
Here it shall be referred to some work of the Swiss Architect Valerio Olgiati, who, in his built and theoretical work, has been deliberately developing the distortion of simple geometries as an architectural as well as a structural means.

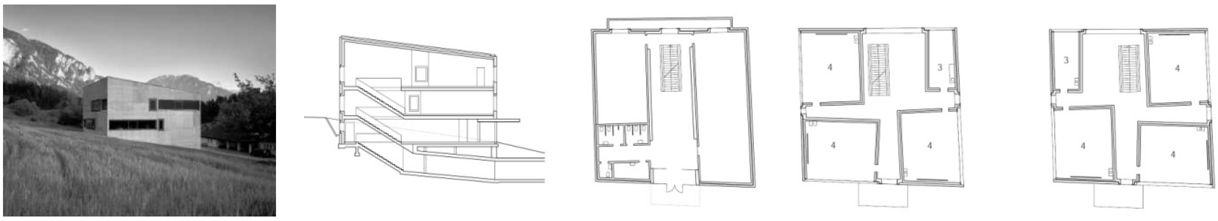


Fig 12   V. Olgiati, School in Paspels, 1999 [6]

For example, Olgiati’s school in Paspels seems to be a simple-shaped building at first sight. However, the seemingly simple geometry is distorted slightly on the verge of degree what is perceivable, following only a limited number of shifts and contortments "which might seem imperceptible but produce a variety of chain reactions ... this spatiality provide the chance to take up many viewpoints, all different, impossible to locate within a system of orthogonal axes, providing a variety of perspective views...." [7]. From these spatial operations, Olgiati develops a very unique architectural expression of complexity which acts as a self-contained frame of reference for this very unique building. Olgiati is interested in the contrast which is created through the shift which cause the building to step away from being ‘Cartesian’ and modular, but rather being an 'organic whole' [8] (without being 'organically' shaped in a ostensible formal manner).

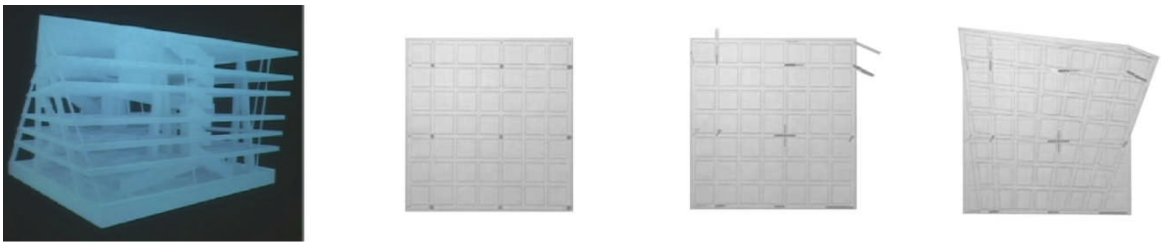


Fig 13   V. Olgiati, University in Lucerne, 2003 [9]

The design for the University of Lucerne was a winning contribution to a competition tendered in 2003. The statical structure, distorted and seemingly coincidental, is in fact precisely derived from static and functional preconditions. The slight contortion of the building is exploited to stabilise the structure in any three directions, with as few pillars as necessary. There are two types of pillars, the main load-bearing ones which are mainly stressed by pressure and which push up, accompanied by additional thin pillars which pull down at certain points where the horizontal beams cantilever and tend to bend upwards. This interplay of supporting and tearing elements 'makes the structure thinner and more efficient' [10], 'The building is a skeleton building, but on the other hand it is also an organic building, that is not modular anymore, even though it is based on the typology of a piloty system.' [11]

It shall be suggested that maybe an approach as outlined above can provide a field of research to explore optimised statical systems on the one hand, which improve through topological changes and contortments, and on the other hand, to explore a certain type of spatiality, which brings about a complex ‘organic’ adaptive space, without being ostensibly ‘organic-shaped, an adaptive spatiality different from known metaphors and analogies of ‘organic architecture’.

Credits

Many thanks to Tristan Simmonds for his introduction to Oasys GSA, his advice on the analysis setup, and the very helpful reviews of the analysis results. Many thanks also to Daniel Glaessl for architectural and conceptual discussions!

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