An Advanced Modification of Dynamic Gravitation

The adaptation of Newtonian dynamics.

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Abstract

The equations for the specification of the curvature of space-time are inherent in the general theory of relativity (GTR). However, despite its enormous success, there are a number of difficulties with GTR. Standard GTR is mathematically very complex, and it predicts the formation of black hole singularities. Here we reformulate the equations for gravitation by mathematically defining the equations for the curvature of space-time. We then translate this curvature back into equations for the force of gravity. By using the original equations for calculating the curvature of space-time used in GTR, we can translate the equations for gravitation, back into equations for a modified force of Newtonian gravity. Using worked examples, we show that such an adaptation of gravity, gives results which technically give the same results as GTR, in the mass range of the solar system. At the same time, an analysis of the data shows that with binary pulsars, the new equations can give improved results to GTR. In the case of high mass gravitational objects such as black holes, by using this advanced modified dynamic gravitation, AMDG, these equations also specifically resolve the difficulties of the formation of singularities.

Introduction:

1. Introducing Post Newtonian Dynamics

In 1915, Einstein published an equation which allowed the calculation for the advance in the perihelion of mercury (see Box 1, eq 1) [1]. In Einstein's own words this equation was a "higher degree of approximation" of GTR [2]. The difference between this higher degree of approximation and full GR, in particular with regard to the curvature of the mass that is generating the gravitational field itself, is in the order of recurring terms of $2GM/Rc^2$. So in low mass density objects the difference is very small, but in high density mass objects, this term makes a significant difference. Moreover, it is at least in part, these additional terms that make the mathematics of GTR very complex. Indeed, it was actually the equation for the "higher degree of approximation" that Einstein used to calculate the advance in the perihelion of mercury in his paper on full general relativity [2]. This general relativistic term for the perihelion advance of Mercury, can also be calculated by the equivalent of the equation 1, given by Straumann (see Box 1 eq. 2) [3]:

Box 1: Einstein's Equation used to Calculate the Effects of GTR

$$\epsilon = \frac{24\pi^3 \quad a^2}{T^2\, c^2\, (1-e^2)} \qquad (1)$$

Straumannn's Equivalent Equation

$$\Delta\varphi \approx \tan\Delta\varphi = \frac{6\pi\, m^2}{L^2} \qquad (2)$$

where $m = GM/c^2$, and $a(1-e^2) = L^2/m$, G is the gravitational constant and c is the speed of light, a is the semi-major axis and e is the eccentricity., T is the time of revolution in seconds, ϵ is the amount of the rotation of the orbital ellipse [1].

Mathematically these equivalent equations give the same answers, but differed from that of Newton. The main clue at the time that Newton's gravity may need to be modified, had been a slight advance in the perihelion of planet mercury by about 43 arc seconds per century. The total perihelion shift per century, in the case of mercury was approx. 5600′′. Of this 5557′′ could be accounted for by using Newton's formula, including some 530′′ from the effects of the other planets in the solar system. Now the remaining 43′′ advance in the perihelion in the orbit of mercury could be accounted for by this equation.

General relativity also differed from Newton in the way gravity was described. In GTR gravity was described by the curvature of space-time, and it was necessary to translate the curvature of space into an initial volume reduction. It is the Ricci curvature tensor that measures this volume change (R_{ab}). We can relate the initial volume of this sphere, as being proportional to the mass enclosed in that sphere, represented by the energy-momentum tensor and its components, given by (T_{ab}). This gave a provisional formula for general relativity (see Box2 , Eq 3). But, this effectively gave exactly the same answer as Newtonian gravity. Now that the volume in "flat-space-time" had been described, what was needed was to describe what happens when we start to additionally curve that space-time. This led to the addition of an extra term to the equation, specifically $-\frac{1}{2}R\ g_{ab}$. This means that in curved space-time the circumference and the radius will appear less than it would in flat space-time. Indeed when full general relativity was published the additional term for the equation for advanced perihelion of mercury (see Box 1), was translated into the additional term for the curvature of space-time, $-\frac{1}{2}R\ g_{ab}$ (see Box 2,Eq 4).

This term introduced another change from Newton in GTR, this is where what is known as the "pressures" in the material enter the equations for the curvature of space-time. In this case the pressures represent an additional volume reduction, which occurs in the volume of the planet or star that is generating the gravitational field itself. This volume reduction is represented by the same additional gravitational term, in conventional general relativity $-\frac{1}{2}R\ g_{ab}$ (Box 2, Eq 4). However, there is a difficulty in GTR with this part of the equation. If we squeeze enough mass into a small enough space, then space collapses upon itself and we seem to get a resulting mass of infinite density, known as a black hole singularity. This is also as a direct result of the term, $-\frac{1}{2}Rg_{ab}$, for the additional curvature of space-time.

To further explain this, we can demonstrate what is meant by the *additional* curvature of space-time by using a (thought) experiment. Take a piece of paper and cut the piece of paper into a circle. Now to give the paper additional curvature, affix the piece of paper on to a spherical object. You will notice two things about the dimensions of the paper circle; first if we look from above the radius of the paper will appear smaller than it was when the paper was flat. Secondly the actual circumference of the paper will decrease compared to what it was when it was flat. This is similar to the additional radius and circumference reduction we see as part of standard general relativity. It is thus possible to view the extra curvature of space-time as a diminution in the radius and in turn the circumference. In GTR this additional curvature becomes infinite in high mass density object like black holes, resulting in the formation of singularities

However, let us suppose that that the extra curvature of space-time is not seen in terms of extra curvature, but, as a straight line. In this case Nature does not "see" the diminution of the radius and in particular the circumference of a gravitating object, in terms of a curvature but in terms of a straight line, In this case, it is $\Delta\varphi$ that is the approximation (albeit a very good one), and it is $\tan\Delta\varphi$, that actually gives us the more correct answer (see Box 1, Eq 2). Moreover, in this case the difficulties with the formation of black hole singularities do not apply. Using this principle, in this paper we further develop a modification that can be applied to laws of gravity that translates the gravitational equations back into a post–Newtonian force. These modified equations, under appropriate mass density conditions are readily usable, and technically give exactly equivalent results to general relativity. † However, under high mass density conditions, such as with binary pulsars, a re-analysis of the results, shows that modified gravitation gives improved results to that of GTR. In the case of black holes, these equations also resolve the difficulties of the formation of singularities.

† For worked examples see Appendix A

Methods:

All mathematical calculations follow strict standard algebraic and standard mathematical rules. The equations presented for spherical coordinates and tensor calculus are the standard equations used in general relativity (see Box 1).

Box 2

GR, Four Dimensional Pythagoras

Provisional General Relativity

$$R_{ab} = -\frac{4\pi G}{c^4} T_{ab} \tag{3}$$

Final General Relativity

$$R_{ab} - \tfrac{1}{2}R\, g_{ab} = -\frac{8\pi G}{c^4} T_{ab} \tag{4}$$

where G the gravitational constant, R_{ab} the Ricci curvature tensor R the curvature scalar, T the energy-momentum tensor and g_{ab} the metric tensor.

The principle physics proofs are based upon standard physical formulae. The proofs offer a high degree of agreement with currently known values in GR. The paper also proposes observational experimental methods for the experimental verification of the findings, as listed in the conclusions.

Results

2. Modified General Relativity

In this paper, we address the question: what are the problems related to GTR, and can these be understood and resolved by an advanced modification of dynamic gravitation? Some authors have written on this subject but the answer remains obscure [4]. What is required, is to distil out the essential elements of Einstein's general relativity. You may have noted in the introduction, that when Einstein was developing general relativity, there was a provisional equation, which effectively gave identical results to Newtonian physics (see Box 2, Eq. 3). We explore the technique of reversing the process, with Einstein's full general relativistic equation (see Box 2, Eq. 4). Specifically to translate the equations of general relativity back from describing curved space-time into describing a force. But any modification has to describe the force from the point of view of curved space-time, with the addition of an extra term for this curvature of space-time.

In general relativity the maths seems to dovetail exactly the way it should, so that both sides of the equation match. In doing so, the actual amount additional curvature to balance the equation, for the mass producing the gravitational field, actually dropped out from first principles (see Box 2, Eq 4). Thus this bit of general relativity is a direct result of Einstein's equation. However, once we have performed the complex mathematics of general relativity we are able to get one straightforward equation, which gives answers which are very close to the original equations [5] for low masses, but differ significantly for higher mass objects (see Box 3). We can calculate this (extra) curvature of the actual gravitating body, by using direct algebra.

Box 3.

General Relativity, Standard Gravitational (mass) radius reduction (r′)

$$r' = \frac{GM}{3c^2} \tag{5}$$

where M is the mass, c is the speed of light and G the gravitational constant [12].

This algebraic equation (Box 3) [5], gives the radius reduction of the actual gravitating mass. This bit of the equation is part of that which makes relativity different from Newton, so using this as a mathematical indicator, we may be able to transform the formula for full GTR back to a formula which effectively describes the curvature of space-time in terms of a force.

Now we have come to the crucial question, is the decrease in the radius of the gravitating mass, itself related in any way to the decrease in the space-time around it? If it is then, we can reduce the complex mathematics of general relativity, specifically relating to the space-time component, which affects the mass itself and the space-time of the orbiting object, down to a direct algebraic formula. We can then progress this work relating to gravity, directly into a formula for the force of gravity using these modified equations.

A clue to this new approach to modified gravity came in 1991 and again in 2004 [3,6]. The use of supercomputers allowed the calculation of the orbit of binary pulsars, using what was the so-called DD-model (for Damour and Deruelle) , in this instance the radiation damping, is equivalent to the algebraic formula given in Box 3 [5]. This was compared to the formula for full general relativity (DDGR) [7,8]. In his most recent publication, whilst Straumann showed that there was no significant difference in these models, even at these very accurate levels of measurement, for most parameters, there was a significant difference in the parameter which described the gravitational radiation damping. This is generally measured by the secular decrease in the pulsar orbital period, given by the term $\dot{P}_b$ [3]. Straumann compared the so called DD and DDGR model for PSR B 1534+12. In the DD model the "pressures" in the material can be represented by the equation which appears in the above Box 3, which is equivalent to the tan$\Delta\varphi$ equation (see Box 1 eq, 2). The DDGR model is the full GR model equivalent to $\Delta\varphi$ (see box 1, eq 2).

Using previously published data [9], Straumann showed that the results are strikingly similar for the majority of the parameters given, apart for the results for $(\dot{P}_b)^{obs}$. This represents the observed amount of slowing in the pulsar orbital period due to gravitational radiation damping, and is given by Damour and Deruelle's equation (see Box 4) [7,8].

Box4 :

<u>Damour and Deruelle's Equation (DD Model)</u>

$$\dot{P}_b = -(192\pi/5)\,(m_1m_2G^{5/3}/c^5M^{1/3})\,(2\pi/P_b)^{5/3}f(e)$$

Where $\dot{P}_b$ is the secular decrease in the pulsar orbital period, P_b is the pulsar orbital period, G the gravitational constant , m_1m_2 the maasses of the binary pulsars, M is the total mass m_1+m_2, and the term $f(e)= (1 + 73e^2/24 +37\,e^4/96)\,(1-e^2)^{-7/2}$, where e is the eccentricity [15,16].

In the DD model the expected result is given as -0.137 x 10^{-12} sec/sec, in the full DDGR model the expected result is -0.1924 x 10^{-12} sec/sec. If we calculate the observed secular decrease in the pulsar orbital period, using the data given then $(\dot{P}_b)^{obs}$ = -0.137 x10^{-12} sec/sec, in exact agreement with the DD model [3]. This calculation for $(\dot{P}_b)^{obs}$, would tend to favor Einstein's original "higher level of approximation", as being the more correct answer, over full GTR.

Using Straumann's earlier observations [6] the late Professor Paul Marmet was later able to reformulate the equation and theoretically develop this relation further in terms of a modification of Newton [10]. Importantly, the calculation of the change in the circumference and in turn the radius of the orbit of mercury, using the reformulated equation (see Box 5), was very revealing because we find that it related in some way to the change in the radius of the actual gravitating mass (see Box 3). With this new result (see Box 5) and some relatively straightforward calculations we can work out the change in the circumference of the orbit of mercury and in turn the change in the perihelion of Mercury.

Box 5

<u>Straumann's Advance Perihelion of the orbit of Mercury</u>

$$\Delta\varphi = \frac{6\pi\, GM_S}{c^2\, a(1-e^2)} \tag{6}$$

where $\Delta\varphi$ is the advance in the perihelion of mercury in radians, M_s is the mass of the Sun, c is the speed of light and G the gravitational constant. $a(1-e^2) = \ell$, the semi latus rectum, where a is the semi major axis and e the eccentricity [3].

First we divide the formula in Box 5, by 2π to change radians to the circumference change. For a circular orbit the ratio of the change of the radius is exactly that of circumference.† Now, very interestingly it turns out that this change in orbital space-time radius *does* relate directly to our relativistic change in radius of the actual gravitating mass (see Box 3).

† For a elliptical orbit, to be precise, the change in the circumference, should be corrected by the formula for the ratio of the average radius of an ellipse compared to the radius of a circle, by the standard term $(1-e^2)$.

The clue to the realisation of modified gravity is that matter, as general relativity (and string theory) describes it, has effectively nine space components. This is perhaps not unexpected, because the standard tensor field is a rank 2, 3x3 tensor, (although in GR, the resultant 4x4 tensor is used for *space-time* together, there are only ten components).The answer is then all that is required, is to multiply the formula for the standard radius reduction of matter in general relativity (see Box 3), by a factor of 9. Interestingly, we get our algebraic conversion for the reduction in the orbital radius of the space itself (see Box 6). Importantly, this gives exactly the same as the change in radius as calculated by Straumann [3].

Box 6

Relativistic Orbital (Space) Radius Reduction (R′)

$$R' = \frac{3GM}{c^2} \qquad (8)$$

where M is the mass, c is the speed of light and G the gravitational constant, R′ is the relativistic space-time radius reduction.

So in the range, where general relativity applies, then the answers are the same. This is the radius reduction equivalent of general relativity for the radius reduction of *the surrounding space* itself and gives answers that in very close agreement to general relativity for Mercury †. Indeed we can also do the same calculation for other planets and moons in the solar system. Importantly, this mathematical agreement with general relativity is not just a coincidence it is a constant relationship. We can by the same means also calculate, using worked examples, that the (average) radius of the orbit of the moon around the Earth is reduced by 1.323 cm compared to Newton's

† For worked examples Appendix A.

gravity, which gives the same result as general relativity.† For further proof, we can do the same calculation, for the change in the perihelion of the Earth around the Sun and we again, get exactly the same answer as general relativity, 3.84 arc sec per century.† Recent evidence confirms that this is the same as the experimentally determined advance in the perihelion of Earth, 3.84′′ ± 0.1 arc sec/cy. [11]. A similar calculation may be performed for any gravitational body in this mass density range. Recent experiments have been able to estimate the advance in the perihelion of Mars, and we again get a result which agrees with general relativity and the experimental advance in the perihelion of Mars, 1.35 ± 0.1 arc sec/cy [11]. To calculate this with GR, would normally take an in depth knowledge of tensor calculus and reams of calculations. We can readily demonstrate worked examples using a few lines of algebra, which technically gives exactly the same answer as general relativity. †

Now we can go on to develop an equation for the change in the *force* of gravitation by taking into account the extra curvature of space-time. Technically, this again gives answers that are no different than the equations for general relativity, technically giving exactly the same answer as general relativity does for bodies like the planet Mercury (see Box 5). The equation has just been derived from the translation of describing the curvature of space-time back into describing the effective *force* of gravity. Indeed, in this range of mass densities, we reproduce the effect that general relativity has on the curvature of space-time.

The fact is that not only do the new advanced modified dynamic gravitation, AMDG, equations give the correct answer, but they do equate to general relativity - at the appropriate mass densities. The effect on the curvature of space-time in

† For worked examples see Appendix A

general relativity has just been directly translated back into an equation for the expression of the acceleration due to gravity. From that it is a straightforward matter to calculate the force due to gravity. The equivalence of AMDG and GTR, at this mass density, is based on the same principle that Einstein's provisional formula for curvature equated to Newton's formula (Box 1, Eq. 3).

The beauty of this approach is that we can also solve these gravitational problems without resorting to highly complex mathematics. We can also get rid of the concept of singularities (except perhaps for the one that led to the big Bang). This can be achieved using the same principal equation (see Box 7).

Box 7:

Modified General Relativity Force Equations (F_q)

$$F_q = \frac{GMm}{R^2}[1 + 3GM/Rc^2]^2 \tag{9}$$

For elliptical orbits

$$F_q = \frac{GMm}{R^2}[1 + 3GM/\ell c^2]^2 \tag{10}$$

where M is the larger mass, m is the smaller mass, c is the speed of light and G the gravitational constant, R is the distance, (normally taken as the radius) and $\ell = a(1-e^2)$, where *a* is the semi major axis and e is the eccentricity.

Of course it is still possible to use the tensor calculus involved in GR, however, it is now possible to use the modified equivalent, but without the mathematical difficulties, and importantly this technically gives exactly the same answer as general relativity, where GTR applies. † Specifically, with low and medium density masses, where GR has been thoroughly tested, then the answers agree very closely with GR.

† For worked examples see Appendix A.

A re-analysis, of the recent data for the binary pulsar data, PSR B 1534 + 12, tends to favour the DD model, which is equivalent to the tan$\Delta\varphi$ equation, and in turn the modified equations for the force of gravity presented in this paper (see Box 1 eq, 2).

But accuracy and ease of use, is not the only criteria. Just as Einstein was able to resolve an astronomical anomaly using his formula, so should the modified formula be able to resolve a gravitational anomaly. The equivalence of modified dynamic gravity and GTR is based on the same principle that Einstein's provisional formula for curvature effectively equated to Newton's formula (Box 2, Eq. 3). So the principle difference between modified gravity and standard GTR, paradoxically enters the equations not on the small scale but on the large scale. That scale starts to be important in our treatment of objects with the mass of binary pulsars (as previously demonstrated) and in particular that of black holes. Firstly a black hole becomes an infinitely dense singularity in GTR. In GTR there is effectively an infinite force at the event horizon [12]. In modified general relativity, the force required would only be the normal force of gravity, multiplied by 6.25 (see Box 8).

Box : 8

Quintessence Force of Gravity at the Event Horizon

$$R_s = 2GM/c^2 \tag{12}$$

and

$$F_q = \frac{GMm}{R_s^{\,2}}[1 + 3GM/ R_s c^2]^2 \tag{9}$$

$$F_q = \frac{GMm}{R_s^{\,2}}[1 + 3/2]^2 \quad = \quad F_q = \frac{GMm}{R_s^{\,2}} x\ 6.25 \tag{13}$$

where M is the larger mass, m is the smaller mass, c is the speed of light and G the gravitational constant, R is the distance, (normally taken as the radius)

In general relativity it is impossible to say what is beyond the event horizon. Using modified dynamic gravity, we can reasonably estimate the forces exerted at the event horizon and inside a black hole. Additionally the event horizon now describes the radius for the escape velocity of light. Importantly, because it is now generally accepted that the speed of *space-time itself* is allowed to exceed the speed of light [19], then the presumed singularities that appeared in general relativity do not appear in modified dynamic gravity.

More importantly using modified gravity we should be able to go further than GTR and Newton's laws of gravity. The addition of an extra mathematical term, which takes into account the "pressures" in the gravitating mass then can be translated to make the correction for the radius of orbital objects (see Box 4). This in turn gives the equivalent formula to GTR (see Box 7) with low and medium density bodies.† With high mass density bodies, such as with binary pulsars, these equations show greater accuracy. Equally well these equations show how the force of gravity at the event horizon can be calculated (see Box 8).

† For a worked examples see Appendix A.

3: Conclusions and Discussion

The principle findings in this paper are, that using an *advanced modified dynamic gravitation,* we can formulate the equations for gravity and proceed to develop these equations in a way, which very closely agrees, with general relativity (where general relativity is applicable), yet greatly simplifies the calculations involved. By such means we can readily calculate the advance in the perihelion of mercury, to a very good degree of accuracy.[†] We can also by the same means, using a worked example, calculate that for instance, the radius of the orbit of the moon around the Earth is reduced by 1.323 cm, in keeping with standard GTR.[†] For further proof, we can do the same calculation, for the advance of the perihelion of the Earth around the Sun and we get the same answer as general relativity, 3.84 arc sec per century. † Indeed recent experimental evidence for the advance perihelion of Earth agrees with these findings, 3.84 ± 0.1 arc sec [11]. These results suggest a straight line correlation between modified and standard GR, for low and medium mass density gravitational bodies.

A similar calculation may be performed for any gravitational body in this mass density range. For instance, with the advance in the perihelion of Venus, the results are again in general agreement with GR, although the inaccuracy in the experimental data cannot be used to confirm the result. Additionally, recent experiments have been able to estimate the advance in the perihelion of Mars, and we again get a result which agrees very closely with general relativity 1.35 ± arc sec/cy,[†] and in particular this agrees with the experimental advance in the perihelion of Mars, 1.35 ± 0.1 arc sec/cy [11]. These results further confirm a straight line correlation between modified gravity and standard GTR, at these mass densities.

† For worked example see Appendix A

Where advanced modified dynamic gravity, AMDG, may improve upon standard GTR, is at higher mass densities, such as in binary pulsars. This is relevant particularly in the observation of the secular decrease in the pulsar orbital period of binary pulsar PSR B 1534 +12, given by the term $\dot{P}_b$ [3]. In the So called DD model (equivalent to the modified gravity presented here) the expected decreased result is given as: -0.137 x 10^{-12} sec/sec, in the DDGR model (equivalent to full GTR) the expected result is: -0.1924 x 10^{-12} sec/sec. However, if we calculate the observed result from the presented data, $(\dot{P}_b)^{obs}$ = -0.137 x10^{-12} sec/sec; it is in exact agreement with the DD model [3]. Indeed, the results of full GTR, *overestimate* this parameter by the factor predicted in this paper, specifically, $2GM/Rc^2$. Moreover, even if a putative correction to the observed result of $(\dot{P}_b)^{GR}$ of 0.037 x10^{-12} is applied, which is subtracted to give: -0.174 x 10^{-12} sec/sec, this brings the result closer to the DDGR model, but still well short of the value of: -0.1924 x 10^{-12} sec/sec, predicted by DDGR[3]. In the DD model (equivalent to the modified gravity presented here) the expected result is exactly given as: $(\dot{P}_b)^{obs}$ = -0.137 x 10^{-12} sec/sec, and this very accurately agrees with the observed result without any correction factors. This experimental data for $(\dot{P}_b)^{obs}$, does tend to favor Einstein's original "higher level of approximation" as a more accurate representation of the experimental results, in particular with regards the gravitating mass itself.

The corollary is that the difference between this "higher degree of approximation" and full GTR means that the formation of singularities, at the mass density of black holes, does not necessarily occur. It is also important to note that whilst advanced modified dynamic gravity AMDG, agrees very closely with GTR in the low and medium mass densities, it does not break down in high density gravitational objects. In high mass density objects, like black holes, the major difficulty that arises is that infinite densities or "singularities" appear from the equations for GTR. One reason for the production of the singularities in general relativity is that in the equation for full GTR an extra term, which is in the order of recurring terms of $2GM/Rc^2$ is required. So in low mass density objects, the difference is very small, but in high density mass objects when the Schwarzschild radius is reached (see Box 9), this difference mathematically results in the formation of black hole singularities. In the model for modified gravity presented here, the Schwarzschild radius (Box 9) describes the event horizon, the horizon for the escape velocity of light, but it is not necessarily the "limit" for the formation of a infinitely dense singularity.

Box 9

Schwarzschild Radius R_s

$$R_s = \frac{2GM}{c^2} \tag{14}$$

where M is the mass, c is the speed of light and G the gravitational constant.

In modified gravitation, the problem of the singularity does not arise (see Box 8), and it is possible to begin to model the physics of what might be occurring inside the event horizon.

In AMDG, we can calculate the effects of gravity at a particular radius and the same force law applies, whatever radius we choose. Equally well this force law applies as much for an observer, for instance, at the event horizon, as it does for a distant observer - both observers will determine the same force at any particular radius, so the laws of gravity are maintained for all observers. From the point of view of observers at the event horizon and observers distant to the event horizon, they will both measure a total force that is 6.25 times greater than the standard

Newtonian force at the event horizon. This means that the laws of gravity remain equal for all observers.

Additionally, the other benefit of modified gravity is that it is relatively straight forward to use. On the other hand GTR, is mathematically very difficult to use, indeed for many years only a few solutions were known explicitly because of the difficulty with the calculations. Notably, one of those was the Schwarzschild radius, the radius of a black hole (see Box 9). Only more recently with the advent of supercomputers has it been possible to derive more explicit answers. Even some of these answers give more theoretical than physical results. A lot of solutions have no real relevance to everyday physics. With modified gravity it is entirely possible to resolve these problems using equations (see Boxes 7 & 8) that greatly ease the mathematics involved (see worked examples, Appendix A).

But ease of use, is not the only criteria. Just as GTR was able to resolve an astronomical anomaly, so should the advanced dynamic modified gravitational formula be able to resolve a gravitational anomaly. Indeed there are a number of anomalies in gravitational physics, which remain unsolved. Modified gravity predicts, as does standard GTR, an increase in the gravitational field (above that of Newton) around gravitating bodies, and this principle can be applied to these gravitational anomalies. One anomaly, which remains unresolved relates to the Pioneer 10 & 11, Galileo and Ulysses missions. In each case there appears to be an acceleration *towards* the Sun acting upon these probes of $\sim 8 \times 10^{-10}$ m/sec^2. At least some, if not all of this effect on gravity *can* be accounted for by dynamic modified gravity, from the known mass of the Sun and from the increased gravitational effects of this (and the close flybys of planets such as Jupiter and Saturn) on the trajectory of these probes. In GTR one can calculate these increased effects at a particular set of coordinates. In modified dynamic gravity we can calculate the *cumulative* effects of this increased force of gravity, which allows us a good estimate of this effect. The results of more precise telemetry would be needed for accurate confirmation.

Equally this advanced modification of gravity may also help explain the apparent missing mass of the galaxy. This missing mass may be due to the presence of dark matter, but a percentage may also be due to the gravitational effects described by modified gravity. Evidence suggests that MOND, by an increased effect of gravity, may explain the apparent behaviour of galaxies and galaxy clusters without invoking cold dark matter [13-16]. In addition to these possibilities some have suggested that the extra mass may be derived from remnant primordial black holes, but in that model an additional force of gravity, may be required to provide sufficient gravitational effects [17]. The effects of modified dynamic gravity in terms of interstellar distances is sufficient, in the presence of primordial black holes and/or relativistic neutrinos, to explain the missing mass of the galaxy.

The additional importance of this work, is that we can obviate the very difficult mathematics of general relativity. We can then begin to explain such anomalies as the additional gravitational pull that Pioneer and Voyager are experiencing as they leave the solar system. AMDG also resolves the problems related to the formation of singularities. If we go back to using the force of gravity we no longer encounter this difficulty (see Box 8).

Additionally, as regards the recently discovered dark energy [25, 26], by using the same principles of modified gravity, it may be possible to begin to define the properties, and the very nature of the field equations of dark energy. Indeed, some of the most fundamental aspects of standard quantum physics might also be explained, by similarly using the concepts described here.

In this paper, overall it has been shown that modified gravity can explain the physical phenomena of gravity in a way which very closely agrees with general relativity (where GTR is applicable). Modified gravity predicts, as does standard GR, an increase in the gravitational field around gravitational bodies. As a result modified dynamic gravity using a cumulative model, can account for the anomalous gravity, which affects the Pioneer, Galileo and Ulysses space probes. Under high mass density conditions, such as with binary pulsars, a re-analysis of the results, shows that modified gravitation gives improved results to that of GTR. Modified gravity can additionally resolve the difficulties associated with the formation of singularities. Where GTR is applicable, not only does *advanced modified general relativity AMDG,* technically give exactly the same answers as general relativity [†], but it does have that same inherent symmetry.

[†] For worked examples see Appendix A.

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Appendix

1). Advance in the Perihelion of Mercury (worked example).

$$\Delta_{circ} = \frac{3GM_S}{c^2\, a(1-e^2)} \qquad = 7.987 \times 10^{-8}$$

multiplied by the no. of orbits in a century,

$$= \quad 3.316 \times 10^{-5}$$

the ratio of circumference to arc second

$$= 360 \times 3600 = 1.296 \times 10^{6}$$

calculated advance in the perihelion of Mercury per century

$3.316 \times 10^{-5} \times 1.296 \times 10^{6}$ = <u>42.98 arc sec</u>.

Equivalent *general relativistic* value per century

= 42.98 arc sec

Experimentally estimated advance in the perihelion of Earth per century [18].
= 43 ± 0.1 arc sec

where Δ_{circ} is the change in circumference of the orbit of Mercury, G is the gravitational constant, Ms the mass of the Sun, c the speed of light, a is the semi major axis of Mercury 's orbit (in meters), e is the eccentricity.

2). *Reduction in the radius of the orbit of the Moon (worked example)*

$$R'_M = \frac{3GM_E}{c^2} \qquad = \quad 1.323 \text{ cm}$$

Equivalent *general relativistic* value.

= 1.323 cm

where R'_M is the change in the radius of the orbit of Moon, G is the gravitational constant, M_E the mass of the Earth, and c the speed of light.

3). *Advance in the Perihelion of Earth (worked example).*

$$\Delta_{circ} = \frac{3GM_s}{c^2\, a(1-e^2)} = 2.961 \times 10^{-8}$$

in a century $= 2.961 \times 10^{-6}$

the ratio of circumference to arc second

$$= 360 \times 3600 = 1.296 \times 10^{6}$$

calculated advance in the perihelion of Earth per century

$$2.961 \times 10^{-6} \times 1.296 \times 10^{6} = \underline{3.84 \text{ arc sec}}.$$

Equivalent *general relativistic* value per century

$$= 3.84 \text{ arc sec}$$

Experimentally estimated advance in the perihelion of Earth per century [18].

$$= 3.84 \pm 0.1 \text{ arc sec}$$

where Δ_{circ} is the change in circumference of the orbit of Earth, G is the gravitational constant, Ms the mass of the Sun, c the speed of light, a is the semi major axis of Earth 's orbit (in meters), e is the eccentricity.

4). *Advance in the Perihelion of Mars (worked example).*

$$\Delta_{circ} = \frac{3GM_S}{c^2\, a(1-e^2)} = 1.9595 \times 10^{-8}$$

multiplied by the no. of orbits in a century,

$$= 1.0416 \times 10^{-6}$$

the ratio of circumference to arc second

$$= 360 \times 3600 = 1.296 \times 10^{6}$$

calculated advance in the perihelion of Mars per century

$$1.0416 \times 10^{-6} \times 1.296 \times 10^{6} = \underline{1.35 \text{ arc sec}}.$$

Equivalent *general relativistic* value per century

$$= 1.35 \text{ arc sec}$$

Experimentally estimated advance in the perihelion of Mars per century [18].

$$= 1.35 \pm 0.1 \text{ arc sec}$$

where Δ_{circ} is the change in circumference of the orbit of Mars, G is the gravitational constant, Ms the mass of the Sun, c the speed of light, a is the semi major axis of Mars orbit (in meters), e is the estimated eccentricity (0.0041988) - *excluding the proportion of eccentricity that Jupiter produces on the Mars orbit.*

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