METRIC AND TOPO-GEOMETRIC PROPERTIES OF URBAN STREET NETWORKS: some convergences, divergences, and new results

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ABSTRACT

The theory of cities, which has grown out of the use of space syntax techniques in urban studies, proposes a curious mathematical duality: that urban space is locally metric but globally topo-geometric. Evidence for local metricity comes from such generic phenomena as grid intensification to reduce mean trip lengths in live centres, the fall of movement from attractors with metric distance, and the commonly observed decay of shopping with metric distance from an intersection. Evidence for global topo-geometry come from the fact that we need to utilise both the geometry and connectedness of the larger scale space network to arrive at configurational measures which optimally approximate movement patterns in the urban network. It might be conjectured that there is some threshold above which human being use some geometrical and topological representation of the urban grid rather than the sense of bodily distance to making movement decisions, but this is unknown. The discarding of metric properties in the large scale urban grid has, however, been controversial. Here we cast a new light on this duality. We show first some phenomena in which metric and topo-geometric measures of urban space converge and diverge, and in doing so clarify the relation between the metric and topo-geometric properties of urban spatial networks. We then show how metric measures can be used to create a new urban phenomenon: the partitioning of the background network of urban space into a network of semi-discrete patches by applying metric universal distance measures at different metric radii, suggesting a natural spatial area-isation of the city at all scales. On this basis we suggest a key clarification of the generic structure of cities: that metric universal distance captures exactly the formally and functionally local patchwork properties of the network, most notably the spatial differentiation of areas, while the top-geometric measures identifying the structure which overcomes locality and links the urban patchwork into a whole at different scales.

Introduction: the dual urban network

The theory of cities, which has grown out of the use of space syntax techniques in urban studies, proposes that urban street networks have a dual form: a *foreground* network of linked centres at all scales, and a *background* network of primarily residential space in which the foreground network is embedded. (Hillier 2001/2) The theory also notes a mathematical duality. On the one hand, measures which express the geometric and topological properties of the network at an extended scale, such as integration and choice measures in axial maps or segment angular maps, are needed to capture structure-function relations such as *natural movement patterns* (Hillier & Iida 2005). We can call these measures *topo-geometric*. On the other, at a more localised level, an understanding of structure-function relations often requires an account of metric properties – for example the generic, but usually local, phenomenon of grid intensification to reduce mean trip lengths in live centres (Siksna 1997, Hillier 1999), the fall of movement rates with metric distance from attractors, and the commonly observed decay of shopping with metric distance from an intersection. In terms of understanding structure-function relations. urban space seems to be globally topo-geometric but locally metric.

Here we propose to link these two dualities in a more thorough-going way. We show first that the large scale foreground network of space in cities, in spite of the claims of critics (Ratti 2004), really is *not* metric. On the contrary, the substitution of metric for topo-geometric measures in the analysis, has catastrophic effects on the ability of syntax to account for structure-function relations at this scale. At the same time, topo-geometric measures turn out to capture some interesting metric properties of the larger scale urban network. But we then show that the background network of space *really is* metric in a much more general sense than has been thought, in that metric measures at different radii can be used to partition the background network of urban space into a *patchwork* of semi-discrete areas, suggesting a natural metric *area-isation* of cities at all scales as a function of the placing, shaping and scaling of urban blocks.

On this basis we suggest a clarification of the dual structure of cities: that metric 'universal distance' (distance from all points to all others – Hillier 1996) measures can capture the spatial differentiation of the background urban network into a patchwork of local areas, while the topo-geometric measures identify the structures which overcomes locality and links the urban patchwork into a whole at different scales. The *patchwork* theory is in effect a theory of block size and shape, picking up the local distortions in urban space induced by the placing and shaping of physical structures. More generally, we can say that the local-to-global topo-geometric structure reflects the *visual*, and so *non-local* effects of placing blocks in space, while the patchwork structure reflects *metric* and so *local* effects.

The patchwork theory extends and generalises the concept of *grid intensification*, meaning the reduction of block size to reduce mean distance from all points to all others in a space network. As shown in (Hillier 2000), holding total land coverage and travellable distance in free space constant, a grid in which smaller blocks are placed at the centre and larger blocks at the edge has lower mean distance from all points to all others in the space network than a regular grid, while if larger blocks are placed at the centre and smaller blocks at the edge, then the mean distance from all points to others in the space network is higher than in a regular grid. This follows from the *partitioning theory* set out in Chapter 8 of *Space is the Machine*. (Hillier 1996)

In general in urban grids, *live centres and sub-centres* ('live' in the sense of having movement dependent uses such as retail and catering) tend to the grid intensified form to maximise the inter-accessibility of the facilities within in the centre, residential areas tend to larger block sizes, reflecting the need to restrain and structure movement in the image of a spatial culture, while the linkages between centres tend to an even larger block size again, an effect of the directional structuring of routes, so that the *network of linked centres* which dominate the spatial structure of cities tend to oscillate between a relatively large and relative small block size, with the residential background occupying the middle range. This block size pattern is explained more fully in (Hillier 2001/2).

In this paper we:

- First review the *duality* of urban space in three ways: *geometrically* to establish its empirical existence as a key dimension of urban form, *functionally* to show its implications in terms of movement and land use patterns, and *syntactically* to show the relations between the two.

- We then explore some of the suggestions that have been made about reducing the distance between syntax and more traditional metric approaches, in particular by examining the suggestion of Ratti that we should add metric weightings to the main syntax measures. We show the consequences of these suggestions for any theory which seeks to identify functionally meaningful structures in urban space

- We then suggest a general method for showing the metric effect on space of block placing and shaping, both visually and in terms of patterns in scattergrams, by showing theoretical cases

- We then apply this method to some cities and show its ability to identify if not natural spatial areas then at least a *natural periodicity* in city networks through which they tend to a natural spatial *area-isation* at all scales, reflecting the ways in which we talks about urban areas and regions at different scales.

Metric and geometric properties of the grid

First, we consider the urban duality *geometrically* by looking sections of metropolitan Tokyo and London, shown in *Figure 1*. As shown in (Hillier 2001/2) and later formalised in (Carvalho & Penn 2004), we must first remind ourselves of the *fractal* nature of urban least line networks: all are made up at all scales, from the local area to the city regions, of a small number of long lines and a large number of short lines. But there is more to be said. Longer and shorter lines form different kinds of geometric patterns. If we look for patterns in the section of Tokyo, the first thing the eye notes are line continuities. What we are seeing in effect is sequence of lines linked at their ends by *nearly straight* intersections with other lines, forming a visually dominant pattern in the network. But in general, the lines forming these nearly straight *continuities* - as Figueredo calls them (Figueredo 2003) - are longer than other nearby lines. This has the effect that if we find a locally longer line it is likely that at either end it will lead to another to which it will be connected by a nearly straight connection, and these lines will in turn be similarly connected. Probabilistically, we can say the longer the line, the more likely it is to end in a nearly straight connection, and taken together these alignments form a network of multi-directional sequences. Intuitively, the value of these in navigating urban grids is obvious, but here, following (Hillier 1999) we are making a *structural* point.

Figure 1: sections of the Tokyo and London street networks

What then of the shorter lines ? Again, in spite of a highly variable geometry, we find certain consistencies. First, shorter lines tend to form clusters, so that in the vicinity of each longer line there will be several shorter lines. These localised groups tend to form more grid-like local patterns, with lines either passing through each other, or ending on other lines, at near right angles. We can say then that the shorter the line, the more likely it is to end in a right angle or near right angle and in general to be embedded in a *nearly rectilinear* local structure. So organic grids like Tokyo and London tend to have a kind of *probabilistic geometry*, which in (Hillier 1999) we called the *hidden geometry* of deformed grids. Consulting (Hillier 2001/2) we can see that, substituting lines for near-straight alignments (or continuities), similar arguments apply to the more geometrical *interrupted grids* which also share these fractal line length properties. It should not be forgotten that these complex and consistent patterns arise only from the placing and shaping of urban blocks. This raises an interesting question. So far syntax has identified these in terms of spatial configurations. But is there also a sense in which they can be interpreted as the outcome of block patterns. The analysis of urban block patterns set out in Hillier 2001/2 suggests this could be the case. We will see below that it is the case.

What then of the *functional* correlates of such patterns. We can find two kinds: *antecedent* and *consequent*. For the *antecedent* we need to understand the *origins* of the urban grid in a functionally informed process. In (Hillier & Hanson 1984, Hillier 2001/2) it was suggested that given the *basic generative process* for settlements by which dyadic cells representing buildings joined by their entrances to cells representing a piece of open space, aggregate by joining their open spaces, such dual patterns would be generated by following the rule: *don't block a longer local line when you can block a shorter one*. This in itself will create a network of longer alignments set into a background of shorter line patches. Such considerations are then *antecedent* functional correlates of the dual grid.

The *consequent* are also set out in (Hillier 2001/2). The functional patterns of cities are created by a dual process: a micro-economic process which, seeking to maximise movement and co-presence, creates the main, local-to-global structure of the grid with its longer lines and nearly straight connections; and a socio-cultural process which, seeking to modulate and structure movement and co-presence to reflect specific cultural norms, creates the background, primarily residential, patchwork of more local areas with shorter lines and more grid like connections. Thus cities tend to have a universal global form and a culturally specific local form.

The *syntactic* correlates of the dual pattern are that syntax identifies, through its measures, *configurational structures* in the network that reflect this duality. Until now, of course, syntactic measures have identified the main structure of the grid. If we can say that cities are made of two kinds of elements: local elements which play little or no role in linking the local parts into the larger scale system, and *local-to-global* elements which do play such a role, then we can say that until now syntactic measures have highlighted the *local-to-global* elements through structures such as integration cores and choice networks. Here we suggest how we can turn our attention to the local, background network. We show that although metric measures can be shown to play no part in the local to global structure, metric measures are exactly what we need to identify the patchwork of differentiated areas that make up the local background structure of the urban grid in terms of how they are formed by the block structure. We demonstrate the former first by examining the suggestion that we should improve our measures by assigning them metric weightings.

Metricising the integration measure

We begin with the integration measure. There are two ways in which we might consider metric weighting. One is simply to weight the root segment (or line) for each calculation with its metric length, and so in effect to multiply the integration value by the length of the segment. The best that can be said of this strategy is that it is harmless. The weighted and unweighted measures are barely distinguishable, and statistically the two measures correlate very closely indeed - for example .9996 for London within the north and south circular roads. This is also the case for radius restricted versions of the measure.

The second method would be to weight *each* relation in the integration measure for the metric length of the segment (or line), so in effect substituting a measure of the length of the segment or line for the topological values of 1. The effects this are dramatic, but have the effect of trivialising the measure, and on reflection this must be the case. As soon as any kind of closeness measure is metrically weighted, the only effect can be to produce a more or less smooth concentric pattern from centre to edge, reflecting the simple and obvious fact that in any system the most metrically integrated location is the centre, next the ring immediately around the centre and do on. The effect of metricising the measure is then to conceal the functionally sensitive differentiations that are shown in a normal integration map, and replace them with a trivial analysis that at best states the obvious and has at best only the broadest possible sensitivity to functional differentiation. In terms of movement prediction, the effect of metrically weighting the root is to make little difference, whereas weighting each relation destroys the ability of the measure to predict movement. This is shown in detail in (Hillier & Iida 2005) where the average correlation for metrically weighted integration in four densely observed areas of London was 'shortest path' measures was much lower than for 'fewest turns' or 'least angle change' change measures.

But there is another way of looking at this: topo-geometric measures of integration can be show to *absorb* certain non-trivial metric properties of the system. For example, whether we define the radius from a segment in metric (up to a certain distance along all streets), topological (up to so many turns away) or geometric (up to a certain amount of angular change away) terms, as we increase radius the integration measure increasingly well approximates the total length of street within that radius. For example, angular integration at angular radius 4 (up to four right angles from each root segment) for London and total street length from each segment within the radius, gives a r^2 is .97. *Figure 2* This measure closely approximates (and pre-dates) Peponis's measure of 'directional reach' (Peponis forthcoming). For metric radii, correlations are less good, but still strong, for example the r^2 i s.954 at a 5km radius in the case shown in *Figure 2* It should be noted, of course, that without restriction on radius, the total street length from each segment (or line) must be the same, since it refers to the whole system.

There are two reasons why we must expect this agreement between angular (or topological) and metric measures. The first is what we might call the *averaging effect*: that with the increasing number of segments with increasing radius, the differences in segment lengths average themselves out, so that the total segment length very closely approximates simple segment count. Simple segment count is, as Dalton shows (Dalton 2005), the strongest component of the integration measure with restricted radius. The second reason for the closeness of the two measures is what we might call the *overlapping effect*: with increasing radius the radius fields from the different root segments overlap with each other, so increasingly overlapping groups of segments are being used to calculate the measure. So in a significant sense, least angle or topological integration measures contain more useful metric information than their metrically weighted versions.

Figure 2 The correlation between segment angular integration and total street length at angular radius 4 (left) r^2 *= .970, and metric radius 5km (right)* r^2 *= .955 for London within the M25*

Metricising the choice measure

Let us now consider the effects of metricising the choice, or betweenness, measure. It was shown in (Hillier & Iida 2005) that metrically weighted - and so in effect shortest path - choice measures without radius restriction were markedly less good predictors of movement than the same measures with geometrical (least angle change) or topological (fewest turns) weightings. This study did not however consider choice measures with restricted radius, and, in view of the fact that in the 'dual' theory of urban space outlined above, at a sufficiently localised level space is expected to operate metrically, we might expect some improvement in movement prediction from metrically weighted choice with more localised radius. Here we show that we must discard this possibility. Local radius metrically weighted choice measures have far less pattern similarity with observable functional patterns (in this case land use patterns) than geometrically or topologically weighted measures. In *Figure 3*, we show in the darker colour the pattern of shops in one of the unplanned areas of Jeddah. On the right, in the context of the whole of Jeddah, we see the least angle choice measure at radius 3000 metres, with the colour spectrum adjusted to show the range of values in the area. The correspondence between the two patterns is not exact, but remarkable for a single measure.

Figure 3 On the left the pattern of shops in an unplanned area of Jeddah is shown in red, on the right the radius 3000m segment angular choice measure for the same area

We then vary the *radius* of the least angle choice measure. Reading left to right in the top row of *Figure 4*, radius 500m picks outs all the main centres, radius 1000m links the main centres together, radius 2500m shifts the focus to two main shopping streets closer to the Mecca Road in the south and radius 5000m shifts the focus to the Mecca Roads itself and its main intersector, though still maintaining a sketch of the smaller scale system of centres to the north. This is a very persuasive analysis of the relation between urban scale and the functional pattern. But if we repeat the exercise substituting metric, or shortest path choice analysis for least angle, as in the bottom row of *Figure 4*, then at all levels, the network identified is much more complex and diffused, and has very little relation to the pattern of shops. More bizarrely, with high radius the measure increasingly identifies highly complex routes through the system, with, apart from a focus on a section of the Mecca Road at high radius, absolutely no relation to the shop pattern.

Figure 4 Least angle (top) and metric (bottom) analysis at 500m, 1000m, 3000m and 5000m left to right

The difference between the two measures persists at radius-n. *Figure 5* While least angle choice continues to sketch the main functional structure of the area, metric choice identifies a network of highly complex routes with virtually no reference to the evidence provided by the shop pattern of how people actually move about the area. The dominant route - the dark diagonal - has dozens of changes of direction, and it is inconceivable that this could operate even as a main pedestrian route across the area. What might be suggested is that metric choice find the shortest paths that very highly knowledgeable movers such as taxi-drivers learn and use to *avoid* the highly uses routes in the area.

Figure 5 Least angle choice (left) and metric choice analysis (right) at radius-n

We can show why such complex routes will frequently – but arbitrarily – be identified by metric choice by a simple experiment. In *Figure 6*, we consider on the left three ways of diagonalizing a grid. In the top case, the diagonal is regular and so the length of the diagonal route is identical to that of the right side peripheral route. Bottom left, we then create an upward kink on of the line elements, with the effect of marginally increasing the length of the diagonal route compared to the peripheral route. Bottom right, we create a downward kink on one line, so marginally shortening the diagonal route compared to the peripheral route, which we show following our usual colouring convention. It follows that with the most marginal changes of this kind, shortest routes will find complex diagonals or simple peripheral routes more or less arbitrarily. This is confirmed in the right figure where we construct a system in which the two diagonals compete, and movement shifts decisively to the downward link and so the shortest path route. In real situations, then, which route is selected by the shortest path algorithm will often then depend on very minor differences in angles, and so be virtually arbitrary.

Figure 6 Different ways of diagonalising the grid, showing why minor geometrical changes can lead to near arbitrary changes in shortest paths

This arbitrary selection of complex diagonals as shortest paths will feature particularly strongly where a more regular grid system is associated with complex internal structures within grid islands. For example, in Beijing, shortest path choice analysis – right above - does not find the eight-lane boulevard between the Forbidden City and Tianamin Square, a boulevard which crosses Beijing east to west and is one of the busiest routes in Beijing. This is then a remarkable failure. It is not that the shortest path structure is not interesting, but it is quite unrealistic in terms of real flow patterns. In general, the more a grid is deformed, the more shortest path choice tends to resemble least angle change choice. However, even in a highly deformed grid such as London, shortest path choice, unlike least angle choice, does not highlight Oxford Street (the main shopping street), but a section of Aldgate east of the City of London, and then, even more strangely, Camberwell Green, a down-market inner urban centre well to the south of the central areas.

Where a *certain kind* of metric weighting does play an important role, and is often necessary, is in restricted radius choice measures. If the least angle choice measure is calculated at a low radius, then it will be very powerfully affected by small block structures, since by definition in these areas there will be very large numbers of segments acting as origins and destinations, and this will create local cluster of high local choice which would not be realistic in terms of the real numbers of buildings in those areas to act as origins and destinations. This can be eliminated by weighting each choice calculation by the product of the origin and destination segments. This is in effect a Newtonian move since what we are doing is weighting the amount of movement between the two segments by the combined 'mass' (in this case length) of the two segments. This will mean that where block sizes are small the weightings will be small, and so the choice values within the small block areas will realistically reflect the scaling of origins and destinations in these areas. Low radius choice should not be used without this Newtonian weighting, but as radius increases the need for it diminishes.

The local metric patchwork

We have shown then that metric factors play only a very limited role in the *foreground*, or local-to-global, topo-geometric structure of urban space. Theoretically, the foreground structure can be seen as arising from the impact on *visual* structure of placing and shaping objects in space (Hillier 2001/2). Because vision is not affected by distance, but, as it were, *overcomes* distance, the effects of placing objects in space is from a visual point of view *global*. In contrast the metric effects are largely *local*. If we take a set of identical urban block arranged on the on hand to allow visual connection between spaces and on the other to limit these as far as possible, then comparing the figures we see the visual structure is completely changed by the moving of the blocks, while the metric structure remains very similar.

But if we treat the metric effects as local, and analyses metric inter-segment relations at restricted radii, a wholly new type of urban pattern appears: a *patchwork* of local areas. By patchwork we mean that whole areas acquire similar values and so similar colouring, seemingly representing some natural division of the background urban network into areas. The patchwork phenomenon was first brought to light by Dalton (Dalton 2007), who took each line in a network and calculated syntactic *intelligibility* and *synergy* values for each line up to a given topological distance away from the segment. Groups of local lines often acquired similar values, giving rise to the patchwork effect when values were translated into colours, suggesting spatially defined areas based on some kind of hard-to-see discontinuities in the urban grid structure. This followed earlier work by Yang who plotted first intelligibility and synergy values with increasing radius from each line, suggesting a relation between the structure of an area and its quite remote embedding in the larger system (Yang 2005). Yang then sought to identify these discontinuities by looking at the rate of change of node count with increasing radius from each line or segment (Yang 2007). Hillier then showed that a more or less identical patchwork could be identified by simply calculating the metric mean depth from segments within a metric radius. *Figure 7* shows the 500m and 1500m patchworks, which are hard to distinguish from those identified by node count 750/250m and 2000/1000.

If we increase the radius of either measure, the scale of the patchwork increases proportionately, eventually yielding a large scale regionalisation of the urban system. What exactly is then happening ? Appendix 1 by Park show mathematically why these measures give such similar results. But theoretically it is clear that both are reflecting *discontinuities* in the urban grid at whatever radius is selected. The rate of node count change measure, as it were, *explains* the metric mean distance measure. Here we propose that what we are identifying is an extension of the *partitioning theory* set out in Chapter 8 of *Space is the Machine*, by which the local metric effects of different partitionings of the grid are predictable from a small number of simple rules. In effect, we have here a way of showing the pattern of local metric effects that come from different ways of placing and shaping urban block in space. We now explain some of the basics of this theory.

Figure 7 showing the local metric patchwork for London at radius 500m and 1500m

Theoretical foundations

Let us first look at theoretical foundations. The generative component of space syntax theory shows that as objects are placed in space, a *structure* of some kind emerges in that space. It is this spatial structure that then impacts of movement and co-presence patterns. This is a vital principle. It is not the built forms that create the pattern of co-presence, but the distortion in space created by the presence of those objects. In this sense syntax is comparable to relativity theory rather than classical physics, since there also it is the effect of objects on space that accords agency to space itself rather than to the physical structures.

The structures emerging in space from the placing and shaping of physical objects is then the key subject matter of syntactic analysis. A branch of syntax theory now deals with the laws governing the ways in which different kinds of structure emerges in space from the placing and shaping of objects (Hillier 2001/2). In a sense, all the representational techniques of space syntax are attempts to capture the structure of spatial field created by dispositions of objects. Typically, syntax has represented these patterns in two ways: as pattern of colours representing configurational values; and as plots (such as intelligibility and synergy scattergrams) of the relationships between these values.

Are there then way of capturing the effects on the metric structure of space of placing and shaping object in that space. We suggest that the answer lies in the key fact that the impact of objects on metric structure is *localised* compared with the effects of visual structure, while the metric structure of the large scale system of space is little affected by local metric variations. This we suggest guides us towards a proper assessment of the role of metric structure in urban systems: that its effects are for the most part localised, but that these localised effects constitute one of the critical dimensions of urban spatial morphology.

Consider a simple square space within a boundary. Bearing in mind that the boundary of a system is its first partitioning, is there any sense in which we can find metric structure in the space ? We propose the theory of *metric signatures.* Metric signatures are brought to light in two stages. First we analyses mean metric distance from each spatial element to all other within a series of rising radii. This produces a pattern of colours which change with increasing radius. We then plot scattergrams with the mean metric distance values at increasing radii on the y-axis and mean metric distance values at radius-n on the x-axis. The resulting pattern expresses the local metric distortion introduced into space by that partitioning against the metric pattern of the whole object. The sequence of scattergrams is then the *metric signature* of a distribution of objects in space.

If we take the simple square shape, and calculate metric mean depth , MMD, from all points to all others without radius restriction, MMD-n, with a Moore neighbourhood (8 adjacent neighbours for each cell), and colour up the results from dark for low through to light for high, we of course find (left above) a pattern in which the centres has the lowest values and the corners the highest. But if we calculate MMDr with a radius of 1 (in this case up to 3 cells away), MMD1, we find a pattern with the highest values in the centre, followed by the centre edges, and patches of low MMD1 near each of the corners, but with higher values in the corner itself. What exactly is happening ?

Figure 8 Radius-n and radius 1 (3 cells) metric means depth in a square shape

If we start with the central node, with each added level of depth with a Moore neighbourhood we find a ring of 8 additional nodes. For a corner, the added number is 2. These two rates of increase are then constant and linear. But if we multiply the node count at each level by its depth, then the total depth increases at a faster rate from a central node than a corner node because more node are being added at the deepest level. This is reflected in slightly higher mean depth in the centre compares with the corner. The difference are quite slight, and diminish from just under 6% at radius 1 to 1% at radius 24, and converge on a mean depth of 2/3 of the radius (see Appendix 1 by Park). The non-corner edges are midway in between. This is why the central area nodes are darker, and the corner nodes less dark and the edge non-corner nodes in between. Just in from the corner, however, another factor comes in. At low radius, *near-corner* node acquire shallow nodes all round, but the boundary prevents the acquisition of deeper nodes, so the differences in mean depth are greater (about 16% less than central nodes) for *near-corner* than for *at-corner* nodes. So we see that the position of nodes in relation to the boundary of the system creates the kind of *structure* in MMD at low radius that we see above right, and these effects become smaller as radius increases, eventually converging on the radius-n pattern. These patterns of MMD are then low radius, and so local effects, which vanish as the analysis becomes more globalised.

Figure 9 Colouration and scattergrams for metric mean distance at radius 2, MMDr2, for a square shape, then the shape with a central and then corner object

We can picture the distortions in space that appear with restricted radius MMD by scattergrams plotting the MMD pattern at that radius on the y-axis against MMD at radius-n on the x-axis. *Figure 9* In this case we use a much denser analysis, and in this case in fact we use DepthMap segment analysis rather point analysis. The coloration in the scattergram is vertical and so shows low radius-n MMD in dark through to high in light. The vertical fluctuation show the fall and rise of MMD at radius 2 in this case, meaning a radius of a quarter of the diameter of the system. The points high on the left are the central segments, the central, slightly lower peak, points are the edges and the lower third peak on the right are the corner segments. The falling curve represents the four near corner low MMD peaks. The scattergram thus shows the metric distortion in the pattern of space created by the simple fact of the boundary. These effects would disappear if the bounded shape were rendered unbounded by rolling the shape up into a torus in the manner discussed in Chapter 8 of *Space is the Machine*. *Figure 9* shows the scattergrams for a simple square shape, and then the same shape with a central and then corner square object.

With a little practice, we can learn to interpret the scatters in terms of the shadings (or colours) and what they mean. However, we shade the scattergram left right for the MMDn pattern to establish a convention in which the shades show the radius-n pattern and the rise and fall the restricted radius, MMDr, pattern. This means that the colours in the scattergram are the opposite of the colours in the colouration, but it seems better to make the colours and shapes show the different dimensions of the scatter. The pattern in space that we see in the first case is the effect of the boundary, which we should see as the first partitioning of the system. With increasing radius, the scatter will of course converges on the radius-n pattern.

We can explore increasing radius first by experimenting with boundary shapes in this case using the two smallest 'soundalike drums' as in *Figure 10* and showing their *metric signature* as the seqeunce of scattergrams.

Figure 10 The 'metric signatures' of the two smallest 'soundalike drums': radius 1 top left, radius 2 right, radius 3 bottom left, radius 4 right. A radius is ¼ of the object radius

We can then use the technique to explore the metric distortions of space brought about by placing multiple objects in space by looking at their pattern of shading, or coloration, at different radii. We have already shown that the patterns that come to light are brought about by the *discontinuities* in the space established by local variations in the block structure. The scattergrams show the metric *shape* of space of the patchwork through the metric signatures at different radii and the pattern of peaks and troughs that are found in the scattergram. In *Figure 11*, for example, we show the colouration pattern and metric signatures for 10 objects placed randomly in a square.

Figure 11 The metric signatures of ten randomly place objects

Metric signatures of urban spatial networks

We can now use this technique to clarify and explore the patchworks that appear in urban systems under restricted radius mean metric distance analysis. In *Figure 12* we show the patchworks at radii of 500m, 1500m and 3500m for part of Central London, with MMDn on the horizontal axis, and MMDr on the vertical. The darker patches, which are metrically integrated zones, show initially as thin peaks, and these become broader with increasing radius, and at higher radii yield a large scale regional picture of the city.

Figure 12 The metric signature of part of Central London at radii .5km, 1.5km and 3.5km

But do these represent real patterns ? The test of structure is function, so we must ask if the patchwork corresponds in any sense to functional differentiations ? Intuitively, this does seem to be the case. In *Figure 13* left we take the radius .5 kilometre anlaysis of part of central London. 1 is the immediate area of Marylebone High Street, and the blue colour indicates local grid intensification. 2 is the adjacent residential and non-live business area of north Marylebone. 3 is the live area of Goodge Street, and 4 the live Coptic Street area south of the British Museum, and the adjacent live Seven Dials area. 5 is the South Bank Cultural Centre with its two level grid, 6 is the very different adjacent Coin Street area while 7 is the area beyond which has been subject to a quite successful urban regeneration.8 is then the Roupell Street residential area of small terraced houses, 9 is the live St Andrews's Hill local area in the City of London, while 10 is the upmarket but non-live area of St James. At an intuitive level then there seems quite a strong agreement between the patchwork and functional variation, even at this small scale.

Figure 13 Central London patchworks at .5km and 2km with patches numbered

On the right of *Figure 13*, with MMD set at radius 2km, much larger areas are of course identified. 1 corresponds to the City of London (UK's financial centre) plus its northern business extensions, 2 is the area around Borough Market, a highly active regenerated area, 3 is Pimlico, 4 is the very upmarket area bounded by Knightsbridge and the Kings Road, 5 corresponds to the part of London north of Trafalgar Square most heavily populated with tourists including Leicester Square, Picadilly Circus, Soho and Covent Garden, 6 is the gentrifying area of Clerkenwell, and 7 is a popular and active area around The Cut on the South side of the river. Again the larger patches seem to be broadly reflected in functional differentiation.

But of course, we can also use the scattergrams to explore areal differentiations, either by selecting peaks or troughs in the scattergram and seeing where they are in the maps, or vice versa. If we take the intermediate level of analysis of central London, as in *Figure 14*, the small leftmost peak in the scattergram corresponds very closely with the main tourist area of London, as we saw in *Figure 13*. The second, higher peak is the City of London, the financial centre and historic core of London. This peak turns out to conceal another. If instead of selecting the peak in the scattergram we select an area from the map, in this case the very active residential, tourist and shopping area between the Kings Road and the Fulham Road in Chelsea, we find it take the form of a peak hidden by the higher peak of the City of London. The higher peaks to the right should be treated with caution, since their location may subject them to the edge effect by which the system boundary cuts deeper nodes from peripheral locations.

Figure 14 Selecting from the scattergrams to show peaks are areas in Central London

What about more regular grids, say Manhattan island ? *Figure 15* shows the patchwork for Manhattan at a radius of 2 kilometres, and scattergram which shows a series of peaks.

Figure 15 Manhattan patches at a 2km radius

Starting with the scattergrams in *Figure 16* the small leftmost peak, is Greenwich Village, East Village, and the second, much higher peak, is the financial district. This peak seems to conceal another, slightly to the right and just visible in the original scatter.

Figure 16 Peaks and patches in Manhattan

We can find this by reversing the selection process, and instead of selecting a region in the scattergram, we select an area of the map. We find then that the peak concealed by the financial district peak is the Upper West Side area and behind that is another which is Upper East Side. The smaller peak to the right is an area to the west of the north end of Central Park, the sub peak on the right is East Harlem, and the main peak is the main Harlem area north of the Park. Again there seems to be a strong relation between the peaks and patches and the functional differentiation of areas.

In general, low radius peaks tend to identify the historic centres of cities. In Barcelona (left in *Figure 17*), the first main peak identifies the old part of the old city, which show a clear differentiation into three different areas. At higher radius, this differentiation disappears and the old city reads a single system. The minor peak to the left identifies the four dark regions to the north of the diagonale. In Atlanta, the first peak at radius 1 is again the offset grid of the original centre.

Figure 17 The centres of Barcelona (left) and Atlanta (right)

As with Barcelona, the first radius in Amsterdam creates a patchwork of differentiated areas within the old central area, but with higher radius the area as a whole becomes more like a patch. But compared with Barcelona, the transition happens more quickly. In Hamedan the first peak identifies the centre at radius 2. In Konya, all low radii identify the centre. In Jeddah. the first peak is the historic centre. The second peak is the unplanned University area that we looked at earlier on. This is just a selection of cases that have been examined so far.

The technique can plausibly be used archaeologically. In *Figure 18* we show the reconstructed plan of Teotihuacan with a radius 3 MMD analysis and scattergram. The first peak is the area around the Pyramid of the Sun, the biggest structure in Teotihuacan, and the large peak is in fact conjectural original settlement which existed prior to the building of the huge ceremonial centre which makes up most of the city. It is clear that this area is morphologically quite distinct form the rest of the settlement. The space complex leading to the Pyramid of the Moon is the first peak on the radius 2 map.

Figure 18 The central peak is the original settlement at Teotihuacan

Discussion

These results suggest not that there are, spatially speaking, natural areas in cities, but something more interesting and perhaps more lifelike: that at each scale there is a *natural area-isation* of the city into a patchwork of spatially distinguishable zones. This is after all how we talk about cities. We do not mentally regionalise them at one level only. But they do suggest that the area structure of the city is a *dependent variable of the grid*, and it must be among the objects of a theoretical model of the city to identify these.

It is hard to judge how useful this will all be in the long run. The relations we have shown between the patchwork at various radii are suggestive but no more. It will take some time to develop functional measures which show unequivocally that the ways in which the block shapes and sizes create the patchwork is a significant force in shaping the functional patterns of the city. What is clear is that the measures *does* bring to light the metric imprint on urban space of the pattern of large and small discontinuities that result from the block pattern. The fact that the Yang's measure of node count change of (NC*r+r/*2)/(NC*r-r/*2) produces a very similar result to MMD*r* means that one measure explains the other. One interpretation of Park's Appendix to this paper would be that MMD*r* shows the effect of discontinuities, while node count change shows where they are.

Even so, at first sight, the *periodicity* exhibited by the MMDr measures is at first sight unnerving. The first thought must be that it is an artefact of some kind. But if so, it is far from clear what kind of an artefact it could possibly be. The only artificial aspect of our procedure has been to use the shading (or colour) spectrum to highlight local differences, and the legitimacy of this would seem to be confirmed by the strongly differentiated patterns shown in the scattergrams. The second thought then is that it must be real. But how could such a periodicity have arisen ? A natural answer would seem to lie in the *generative* process we have described. The *periodicity* of the network is could plausibly be established by the generative process of block placing that establishes the fractal line structure through the rule: *don't block a longer local alignment if a shorter one can be blocked*. This necessarily gives rise to the network of longer lines connected a nearly straight angles that constitute the foreground structure of the network, and it is in the nature of things that this line network is the means by which the local parts formed by the shorter line complexes are linked into a whole system. The choice measure finds this network. The converse of this is that the clusters of shorter line do not do this but tend to form more localised patterns. One way in which this might be evidenced is the tendency of the lines making up the foreground network to separate the two sides of the line by not having short lines which pass through the main line to the other side. This is why main alignments in cities seem so often both to be centres of integration for neighbouring areas, but also to separate them from each other and so give rise to areas with different functional and spatial characteristics on either side.

So these preliminary results suggest the technique is an interesting one, but as yet no more than that. Considerable work will be needed to find unequivocal functional tests for the kinds of patterns that have been brought to light. But in the meantime, great care must be exercised with this measure as there are a number of health warnings:
- as we showed with the theoretical examples, a patch can appear integrated because local boundaries allow locally shallow but not deep nodes. This means that that small isolated clusters of lines can appear metrically integrated, which of course they are , but only because they are small. This is a particular problem near the edge of the system, or where large holes exist in the urban fabric. |But commonsense can avoid this problem.

- where a number of lines intersect a local metrically integrated patch is likely to show. This is realistic, but of course the patch is not one defined by groups of buildings, but by a complex intersection. We can call these *trivial patches*. Again we can avoid this problem by commonsense.
- the scaling of patches must reflect the regional scale of the urban grid – for example MMD at a low radius produce a good patchwork in the central areas but may need a higher radius to get a good patchwork in suburban areas with their generally greater block size.
- careful adjustment of the colour spectrum is usually needed to show in the images the patterns that are clearly present in the scattergram. Typically, at low radius – 1 or 2 is the default radius is being used rather than a real metric radius – three clicks are need to bring in the blue spectrum and one click on in the red
- it must be made clear that although MMDr does reflect block size and shape, it is not in itself enough to account for live centres. It is a only where MMDr works alongside measures of the foreground structure – as in Marylebone High Street for example - that we find centres forming. Global – or at least local-to-global – factors are normally conjoined with local factors in centres formation, and of course grid intensification often increases as a consequence of centre formation.

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APPENDIX 1

This appendix aims to clarify the relationship between metric mean depth and the rate of change of node count. We assume that metric depth is defined in real number and node count is differentiable accordingly with respect to metric depth.

(I) *Local metric mean depth*, $\mu(v,r)$, of node v for radius r is:

$$\mu(v,r) = \frac{1}{N(v,r)} \int_0^r x \frac{\partial N(v,x)}{\partial x} dx \tag{1}$$

where $x = x(v,u)$ is metric depth of node u from the reference node v and $N(v,r)$ is the number of nodes with $x(v,u) \le r$, that is:

$$N(v,r) = \int_0^r \frac{\partial N(v,x)}{\partial x} dx \tag{2}$$

(II) *The rate of change of node count*, $\theta(v,x)$, of v at metric depth x has been defined as (Yang 2007):

$$\theta(v,x) = \frac{\partial \log N(v,x)}{\partial \log x} = \frac{x}{N(v,x)} \cdot \frac{\partial N(v,x)}{\partial x} \tag{3}$$

Hence,

$$\frac{\partial N(v,x)}{\partial x} = \frac{N(v,x)\theta(v,x)}{x} \tag{4}$$

Substituting (4) into (1), we have:

$$\mu(v,r)N(v,r) = \int_0^r N(v,x)\theta(v,x)dx \tag{5}$$

where $\mu(v,r)N(v,r)$ is simply total metric depth.

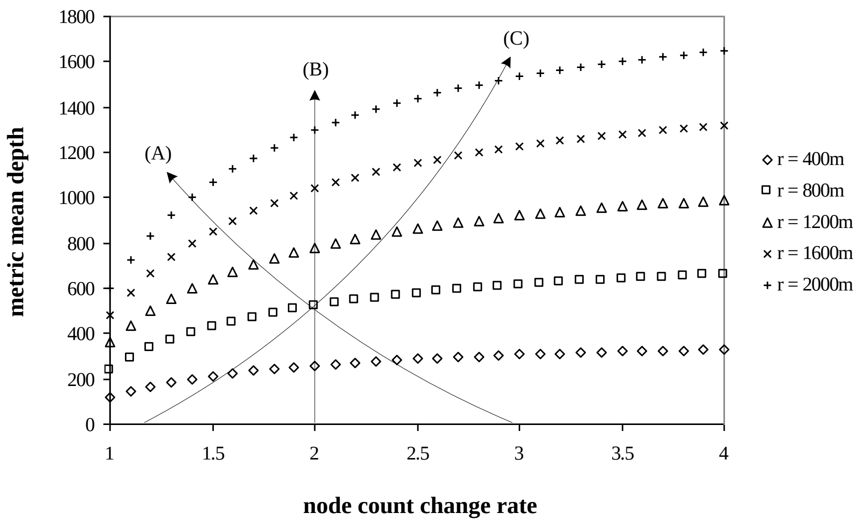
Differentiating both sides of (5) with respect to r will yield:

$$\begin{aligned}
&\frac{\partial \mu(v,r)}{\partial r} N(v,r) + \mu(v,r)\frac{\partial N(v,r)}{\partial r} = N(v,r)\theta(v,r) \\
&\Rightarrow \frac{\partial \mu(v,r)}{\partial r} N(v,r) + \mu(v,r)\frac{N(v,r)\theta(v,r)}{r} = N(v,r)\theta(v,r) \\
&\Rightarrow \frac{\partial \mu(v,r)}{\partial r} + \frac{\theta(v,r)}{r}\mu(v,r) = \theta(v,r)
\end{aligned} \tag{6}$$

Since metric mean depth and radius have the same linear dimension, the first derivative term in (6) must be independent of r, that is, $\partial\mu(v,r)/\partial r = c(v) < 1$. Consequently, we can simplify (6) into:

$$\mu(v,r) = r\left(1 - \frac{c(v)}{\theta(v,r)}\right) \quad (7)$$

The figure below is drawn according to (7) to show the relationship between node count change rate and metric mean depth for a single node having, for instance, $c(v) = 0.7$. On the one hand, for any fixed radius, we should always be able to expect a positive correlation between the two. On the other, if radius increases, we may consider three possible monotonous routes of development: (A) $\mu(v,r)$ increases while $\theta(v,r)$ decreases; (B) $\mu(v,r)$ increases while $\theta(v,r)$ remains stationary; (C) both $\mu(v,r)$ and $\theta(v,r)$ increase. Note how these different routes would induce an 'inversion of centrality': nodes that are more central (i.e. lower mean depth) at smaller radii tend to be decentralised at larger radii. Which route of development a node will take depends entirely on a scaling between radius and its node count.



(III) It is often observed that node count scales with radius following the *power-law*, such that:

$$N(v,r) = kr^{\alpha(v)} \quad (8)$$

where $\alpha(v)$ is the scaling (or fractal) dimension starting from v and k is some constant of proportionality.

From (3), the rate of change of node count under the power-law scaling becomes:

$$\theta(v,r) = \frac{r}{kr^{\alpha(v)}} \cdot \alpha(v)kr^{\alpha(v)-1} = \alpha(v) \quad (9)$$

from which it is clear that the rate of change of node count under the power-law is equivalent to the scaling dimension and must remain stationary, independently of radius. This means that it is the route (B) that will be realised under the power-law.

Now by solving the differential equation (6) with the integrating factor $e^{\int \alpha(v)/r dr} = r^{\alpha(v)}$, we have:

$$
\begin{aligned}
& r^{\alpha(v)} \frac{\partial \mu(v,r)}{\partial r} + r^{\alpha(v)} \frac{\alpha(v)}{r} \mu(v,r) = \alpha(v) r^{\alpha(v)} \\
& \Rightarrow \frac{\partial}{\partial r}\left(r^{\alpha(v)} \mu(v,r)\right) = \alpha(v) r^{\alpha(v)} \\
& \Rightarrow r^{\alpha(v)} \mu(v,r) = \frac{\alpha(v)}{\alpha(v)+1} r^{\alpha(v)+1}
\end{aligned}
\qquad (10)
$$

Therefore:

$$
\mu(v,r) = \frac{\alpha(v)}{\alpha(v)+1} r \qquad (11)
$$

Empirically, it can be shown that $\alpha(v)$ has a typical value of 2, which implies that the network in question is close to a 2-dimensional entity. In this case, metric mean depth will be in average just 2/3 of the radius applied.
HTP □

Figure 1: sections of the Tokyo and London street networks

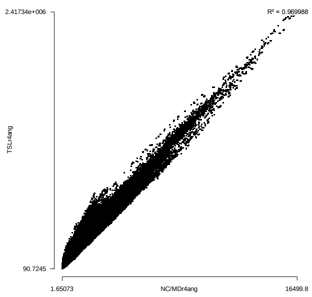


Figure 2 The correlation between segment angular integration and total street length at angular radius 4 (left) r^2= .97 for London within the M25.

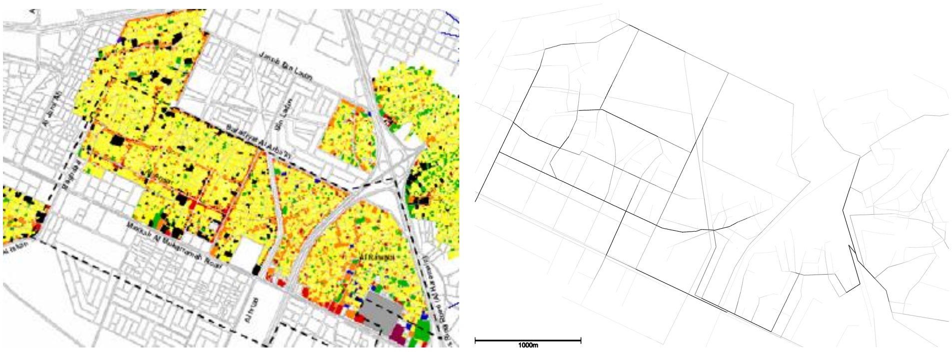


Figure 3 On the left the pattern of shops in an unplanned area of Jeddah is shown in red, on the right the radius 3000m segment angular choice measure for the same area

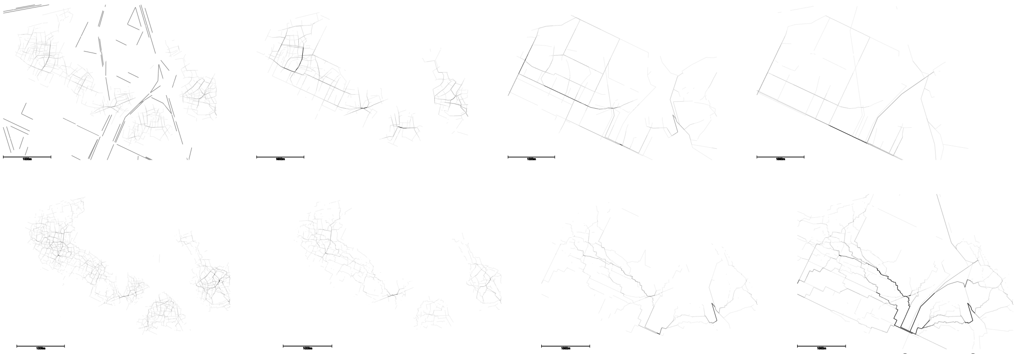


Figure 4 Least angle (top) and metric (bottom) analysis at 500m, 1000m, 3000m and 5000m from left to right

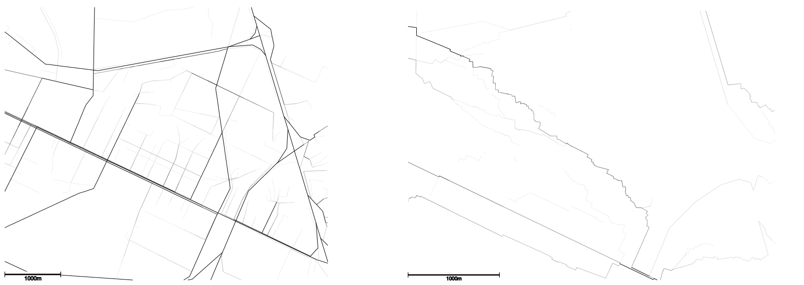


Figure 5 Least angle choice (left) and metric choice analysis (right) at radius-n

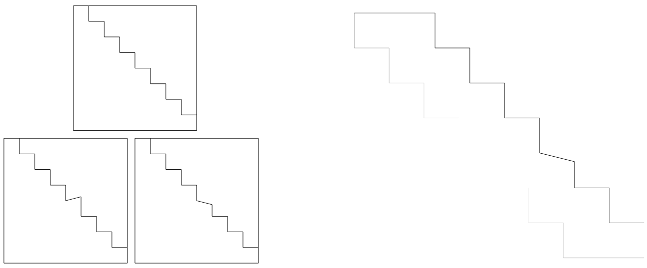


Figure 6 Different ways of diagonalising the grid, showing why minor geometrical changes can lead to near arbitrary changes in shortest paths

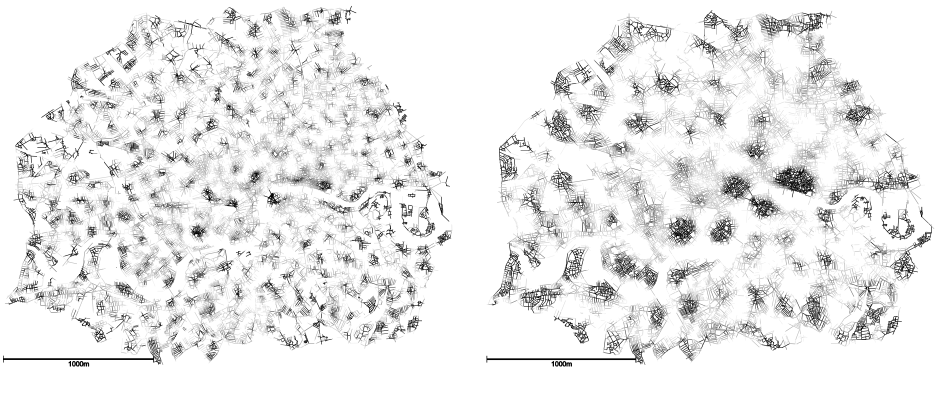


Figure 7 showing the local metric patchwork for London at radius 500m and 1500m

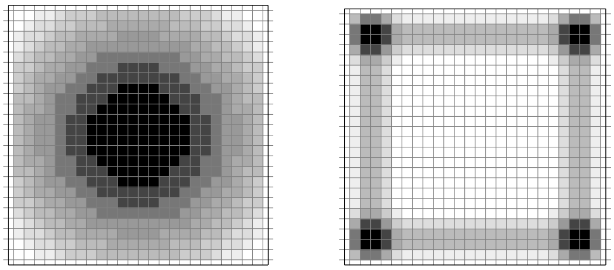


Figure 8 Radius-n and radius 1 (3 cells) metric means depth in a square shape

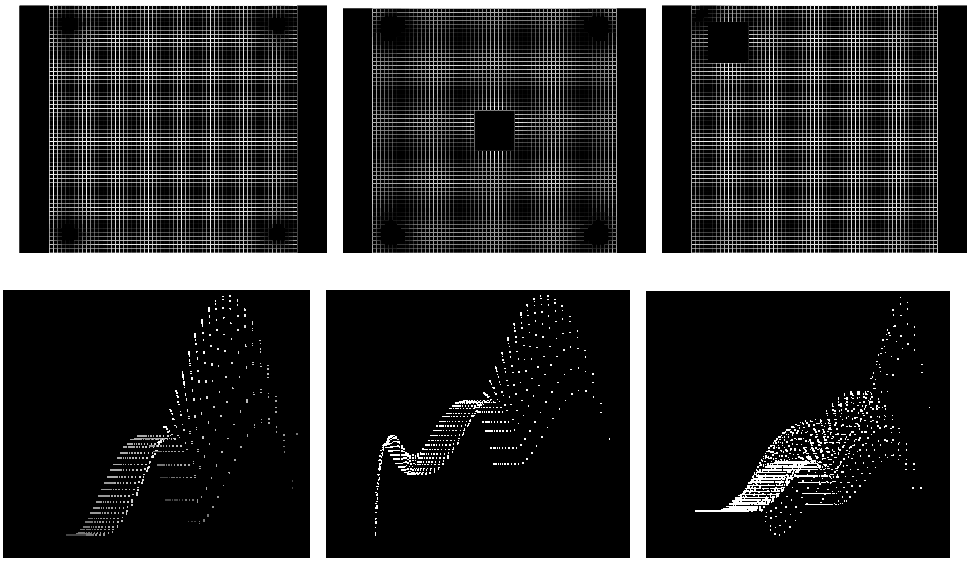
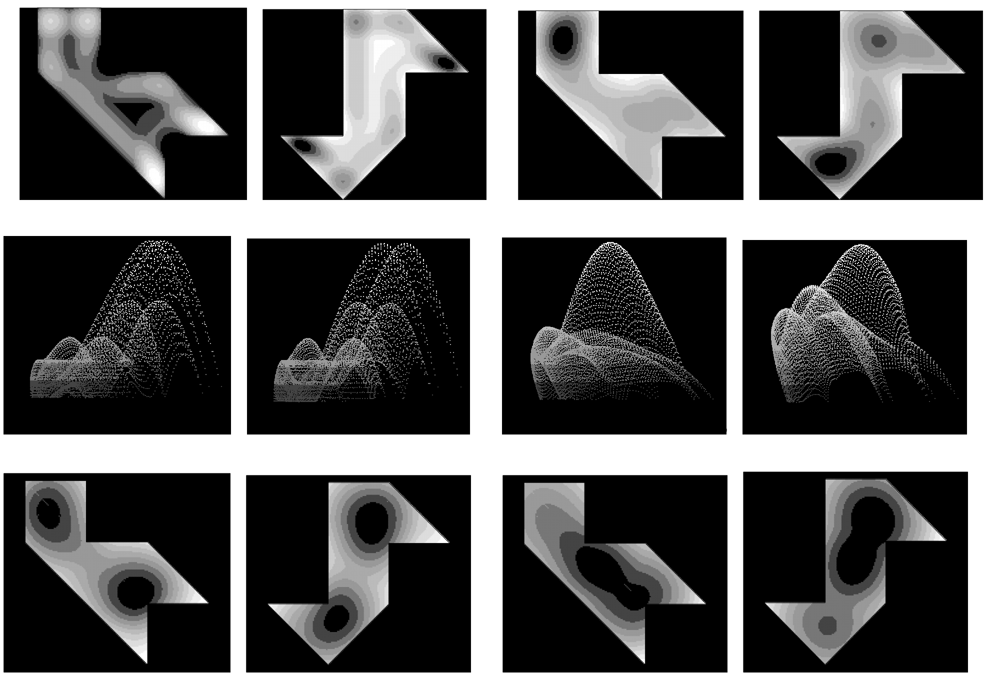


Figure 9 Colouration and scattergrams for metric mean distance at radius 2, MMDr2 for a square shape, then the shape with a central and then corner object



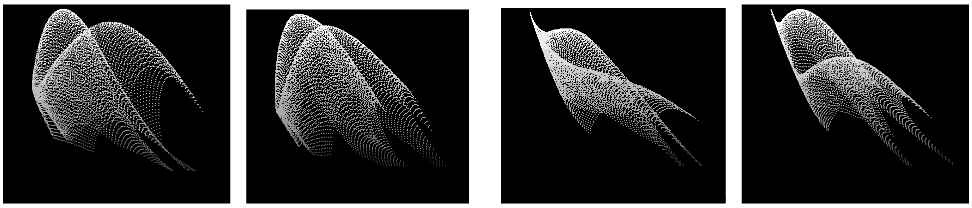


Figure 10 The ‘metric signatures’ of the two smallest ‘soundalike drums’: radius 1 top left, radius 2 right, radius 3 bottom left, radius 4 right. A radius is ¼ of the object radius

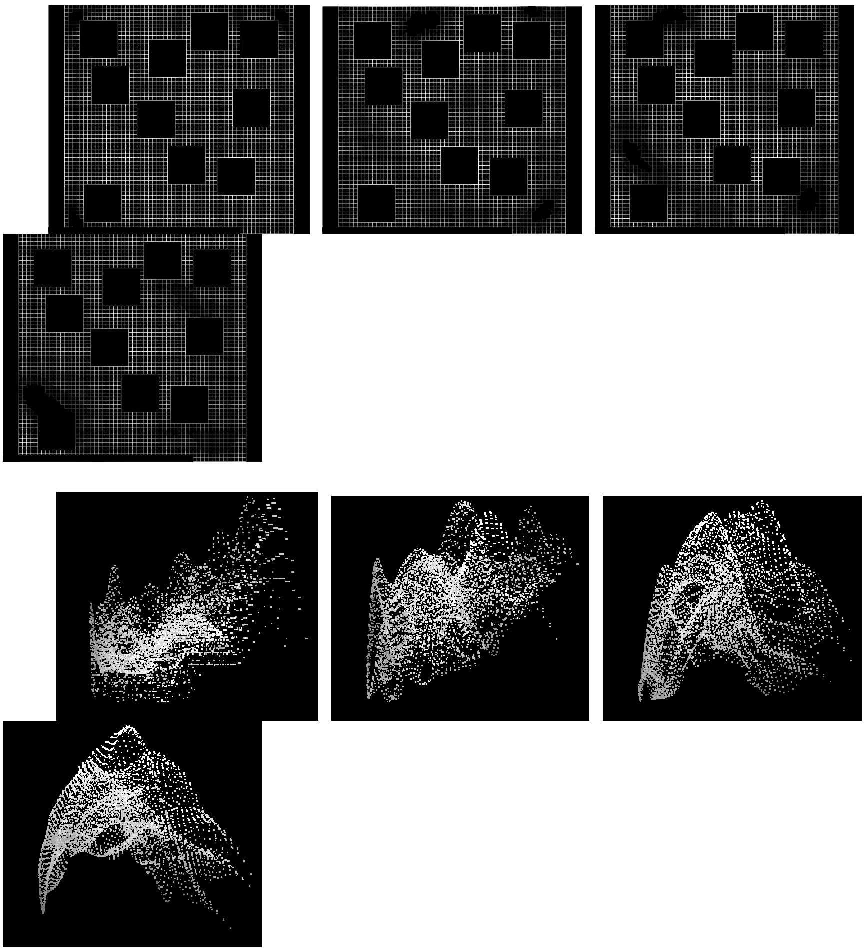


Figure 11 The metric signatures of ten randomly placed objects

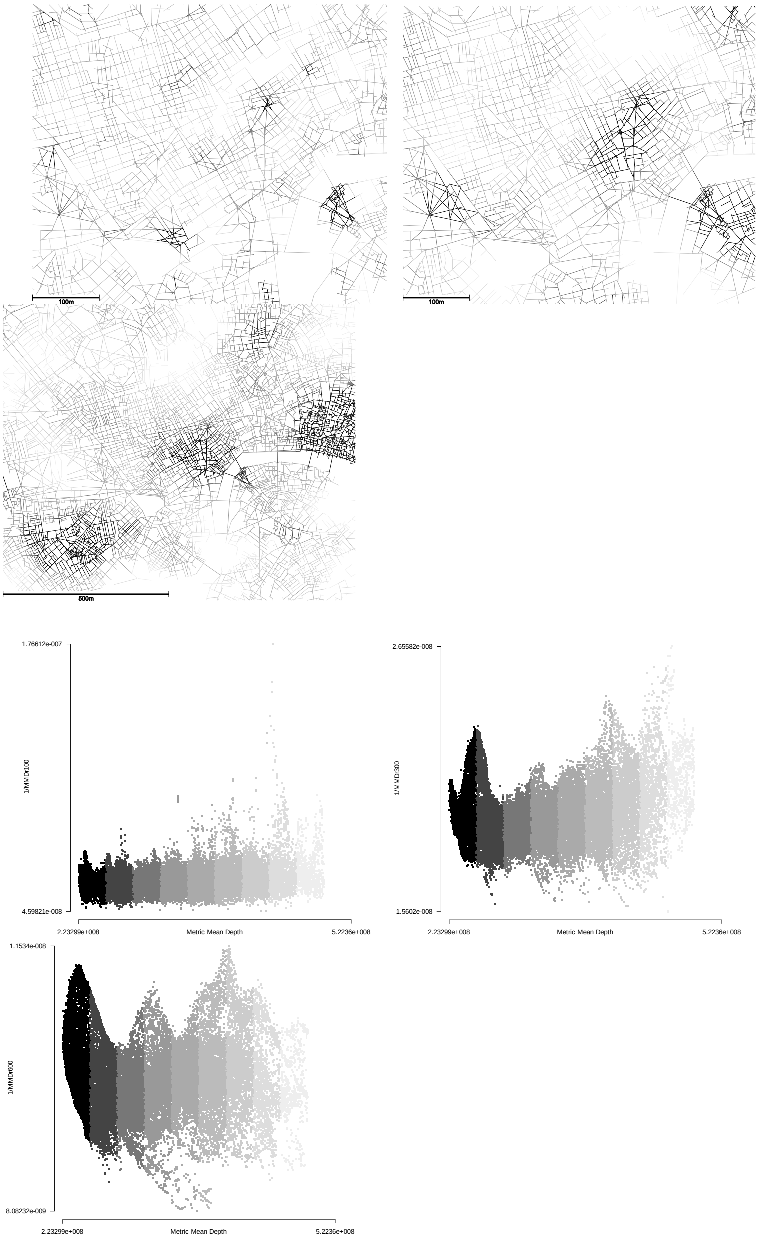


Figure 12 The metric signature of part of Central London at radii .5km, 1.5km and 3.5km

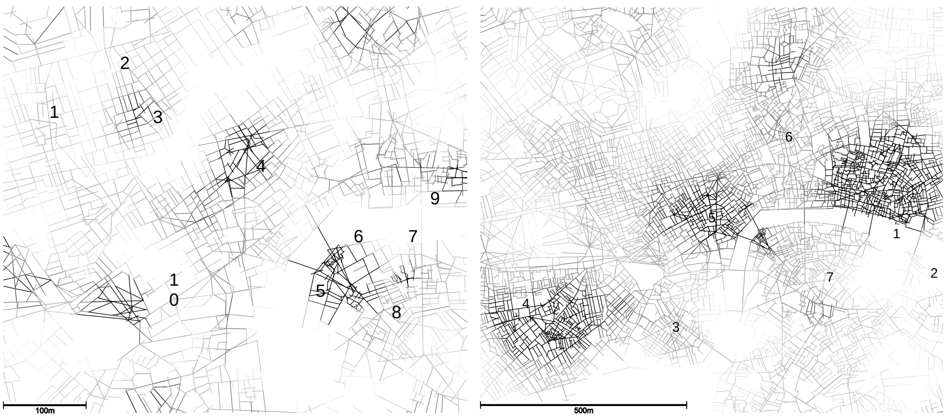


Figure 13 Central London patchworks at .5km and 2km with patches numbered

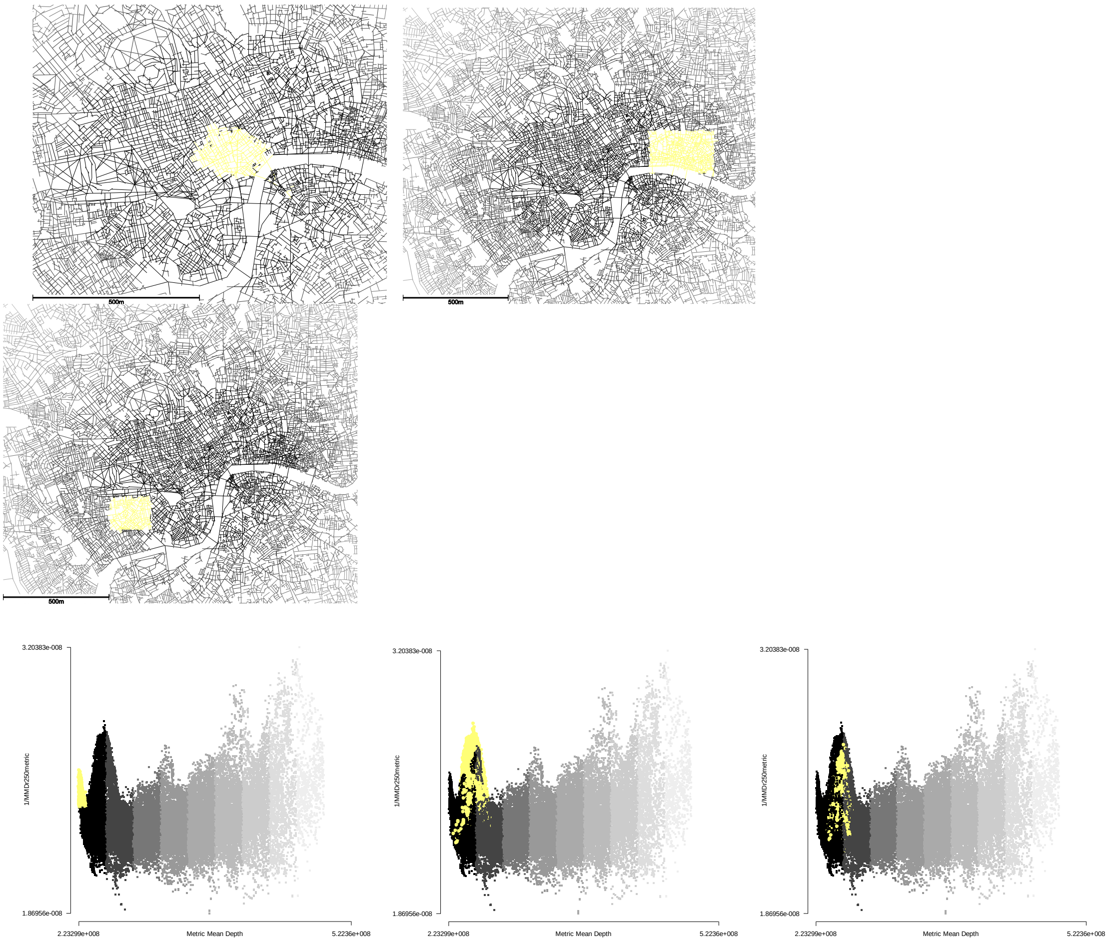


Figure 14 Selecting from the scattergrams to show peaks are areas in Central London

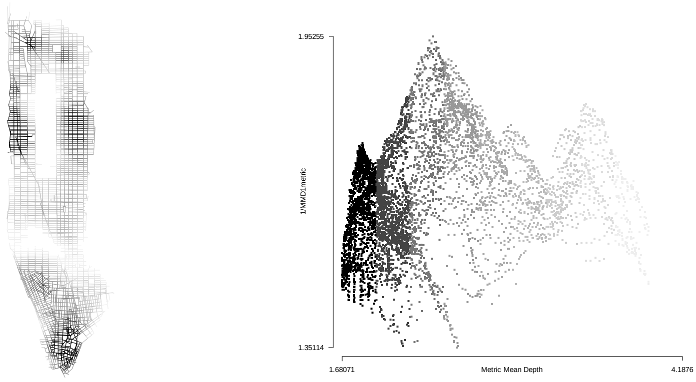


Figure 15 Manhattan patches at a 2km radius

Figure 16 Peaks and patches in Manhattan

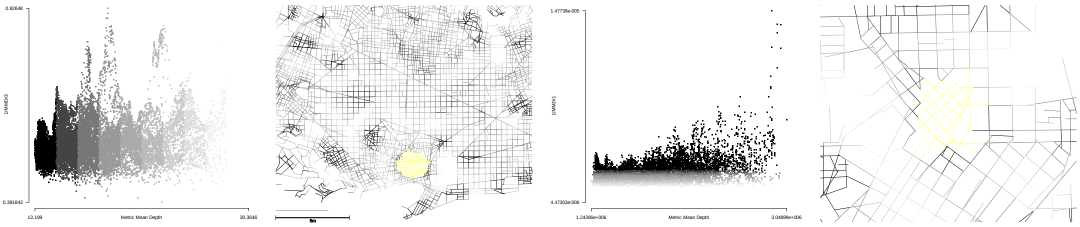


Figure 17 The centres of Barcelona (left) and Atlanta (right) are the left most peak in both cases.

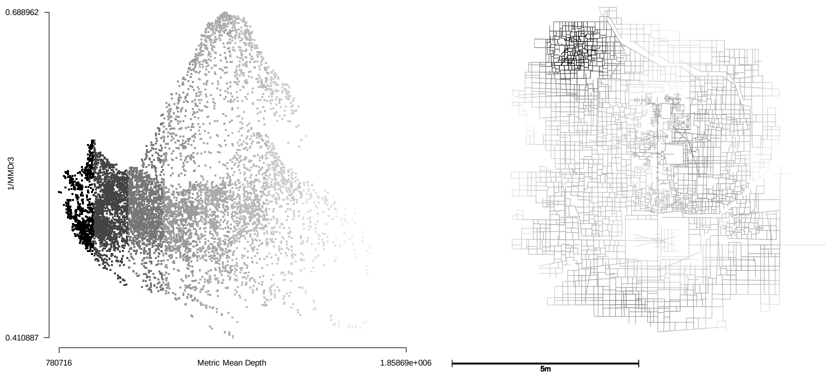


Figure 18 The central peak is the original settlement at Teotihuacan