Explaining Trends in Household Spending

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Abstract

In this paper we model the changing distribution of household spending in the UK over the period 1978 to 1999 and explore the interpretation of remaining time trends in spending once changes in other observed covariates have been accounted for.

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Summary

- Engel's Law – that the budget share of food falls as living standards increase – is one of the oldest predictions in economics. As a measure of welfare, the budget share of food is independent of any price deflator, unlike, for example, real, per-capita expenditure. Recently, some studies have used discrepancies between the trends in living standards implied by changes in the food share and the trends in real income to make inferences about the price deflators by which income is adjusted, on the basis that such price deflators can be biased measures of the true change in cost-of-living for a number of reasons.
- There have been a variety of trends in the UK other than increasing average incomes which might be thought to have affected average food spending over the time period we examine.
- In this paper we model the changing distribution of household spending in the UK over the period 1978 to 1999 and explore the interpretation of remaining time trends in spending once changes in other observed covariates have been accounted for.
- We find that the coefficients we obtain on time related variables can vary substantially according to the other covariates that we allow for.

1. Introduction

One of the oldest predictions in economics is Engel's Law – that as living standards increase the proportion of the budget spent on food falls. In recent years, the average share of household budgets being spent on food has been steadily falling, whether one talks about spending on food solely for consumption in the home or on all food expenditures more broadly. Documenting and understanding these changes is an interesting exercise in its own right but in addition recent studies have begun to use Engel's Law, in conjunction with documentation of these changes over time, to make statements about changing living standards more generally. Recently, three studies have also used discrepancies between the trends in living standards implied by changes in the food share and the trends in real income to make inferences about the price deflators by which income is adjusted (see Nakamura (1997), Hamilton (2001) and Costa(2001)).

Our analysis considers these issues for the UK and looks at the empirical support for the presence of such price deflator bias once one controls for a number of other factors which may otherwise affect the observed trends in spending patterns. There have been a number of trends in the UK other than increasing average incomes which might be thought to have affected average food spending over the last thirty years, and controlling for these factors will be an important component of our analysis. Amongst these have been a change in household composition, notably a fall in household size; changes in the labour market participation of women and older men; and an increase in the inequality of income and expenditure. Finally, relative prices have not been constant, with recent years being characterised by a constantly falling relative price of food. Only once we can adequately control for these factors (and we have ruled out, for example, changes in the quality of other variables in the data) can we begin to assess the welfare implications of changing food shares and compare these to the trends in real incomes in the population[1].

As some empirical background to the trends occurring in the UK which require explanation, we present some data on the evolution of total spending and food spending using the micro-data that forms the basis of our analysis[2]. Figure 1.1 shows the evolution of per capita expenditure (deflating household spending both by the number of members and the number of adult equivalents) and the share of food in aggregate total expenditure between 1978 and 1999. To check that trends are not due to substitution from food prepared in the home to food prepared outside the home we also present figure 1.2 which includes spending on restaurant and take-away food.

As can be seen, the pattern is similar for food-in and for all food. Both shares have fallen fairly steadily across the period, but per-capita expenditure (the lighter solid line) seems to rise more slowly in the period after 1990 than it does between 1982 and 1990. There also appears to be an anomaly at the start of the period where the share of spending on food falls quite rapidly but per-capita income appears static. As has been found in the US data, then, this UK data

[1] Of course we could always explain any changes we see over time by appealing to the catch-all of changing preferences, which would mean we did not necessarily care about any residual time trend. However, many countries do construct cost-of-living indices and use them to make welfare comparisons over time, all of which does not make much sense unless we assume preferences that are stable over time. So it seems sensible to begin by investigating what may lie behind any changes assuming preferences are stable.

[2] The analysis in this paper uses data from the UK Family Expenditure Survey (FES), which provides detailed information on expenditure, income and family characteristics. Responding households keep a two-weekly expenditure diary which is aggregated into roughly 1,000 categories which can then be assigned to around 80 RPI sub-categories on which we have price indices, and where these groupings are consistent across time. We use data over the period 1978 to 1999. For the purposes of our econometric demand analysis, which we detail in subsequent sections, we undertake some sample selection. Since we use a consistent dataset throughout, we detail this selection now. We select only single adult households or (married or cohabiting) couples, and only those who are not currently self-employed and not resident in Northern Ireland. We drop any household with net income and/or total non-durable expenditure less than zero. In total we have around 67,000 observations distributed across 264 monthly price points.

Figure 1.1: Per-capita expenditure and food-in share over time

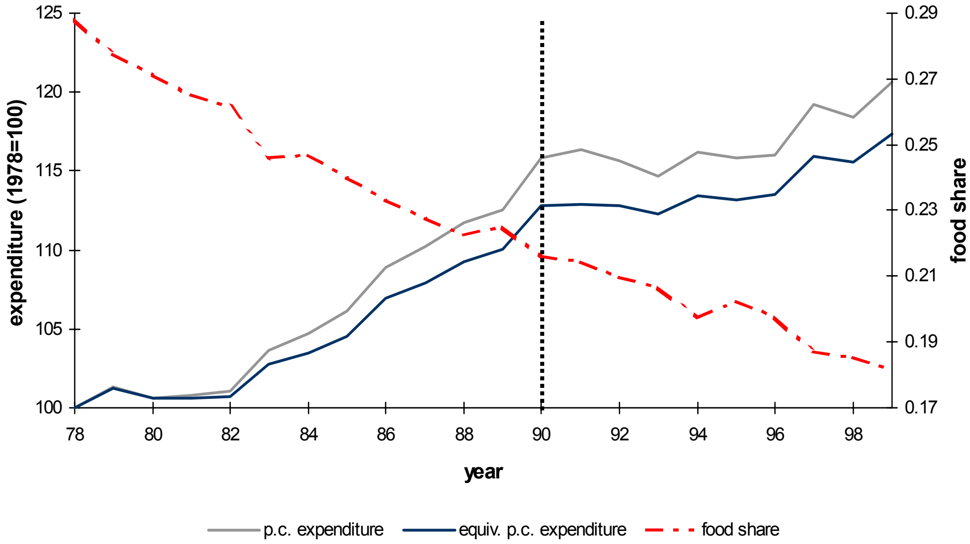
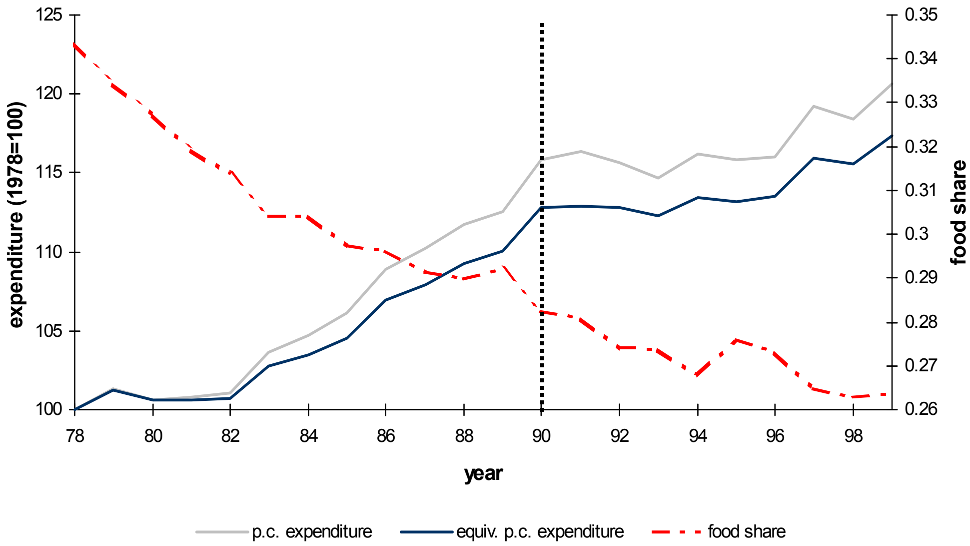


Figure 1.2: Per-capita expenditure and all-food share over time



seems to tell somewhat conflicting stories depending on whether we look at per-capita income or food shares. We know, though, from the discussion above, that there are many observable reasons why this might be so, to do with changing demographics, expenditure distribution and prices. A very crude way to begin to introduce demographic shifts is to look at per-capita equivalised expenditure. We equivalise expenditure using a simple OECD scale[3], and the result is shown by the darker solid line. The basic pattern is the same, although equivalised per-capita expenditure shows less growth than simple per-capita expenditure. The reason for this is that the patterns of compositional change in UK household population over time has resulted in more single households, fewer households (couples) with children, more older households, and fewer extended families and multiple tax-unit households. Equivalising gives each family member more expenditure than simply dividing household expenditure by household size (i.e. looking at per-capita expenditure) but as average household size has fallen over time, the effect of equivalising compared to simple per-capita measures becomes less (if all households were single people then the two would be the same) and hence the growth in equivalised per-capita expenditure is slower than simple per-capita expenditure.

To show the potential importance of composition effects on this trend in average food shares and to motivate the paper more generally, we present a simply counterfactual analysis as a first order approximation to how food shares might have changed had the population change been different. We first estimate the food Engel curve in the first year of our data for different household types, with groups being defined by marital status, pension age and the presence of (and number of) children. Figures 1.3 and 1.4 show the estimated food Engel curves for different household types in 1978, once again for food in only and for all food. The curves were estimated semi-parametrically using cubic B-splines (see appendices A and B for details). The curves are estimated without any restrictions on the shapes except that, for the groups with children, dummies for the number of children were entered as additive terms in the intercept since there are generally not enough observations to sub-divide these groups by the number of children present in the household. Crucially, these figures show that both the slope and the intercept of the Engel curves vary across different household types, and across expenditure levels. This will be one of the central factors motivating our analysis in the sections that follow.

It is clear from the figures that one of main factors that will affect the path of the aggregate food share will be any compositional change that has been occuring in the UK household population across the groups delineated in the figures. In Britain, as with other developed economies, this has been substantial in the last two decades due to factors such as delayed marriage, increased divorce and increased life-expectancies. In our data, for example, the number of single person households rose from 20% to 30% over the period 1978 to 1999, and the number of lone parents rose from only 5% to just under 10%.

Using the Engel curves estimates presented in Figures 1.3 and 1.4 above, we can predict the food shares for FES households in 1979-99 (given observations on family type and total (real) expenditure) as they would have been if households had behaved in accordance with the 1978 Engel curve. We can then use these predicted shares to calculate the share of food in aggregate total expenditure, which will then take account of the demographic change in the population[4]. In this way we begin to control for changes in the expenditure distribution and demographic composition whilst holding preferences (in this case, the shape of the Engel curve) constant. The picture we obtain is what the aggregate food share would have looked like if we allowed the proportions of household types and their total expenditures to change in the way actually observed, but held the Engel curves of each population group at their 1978 shape. This counterfactual can then be compared to the actual aggregate food share over time, and the difference between the two is what

[3]A weight of 1 for the fisrt adult, 0.7 for every subsequent adult, and 0.5 for each child.

[4]The aggregate share may be constructed from household shares by weighting the shares by the household's expenditure share out of aggregate expenditure as discussed below.

Figure 1.3: Engel curves for food in – different household types, 1978

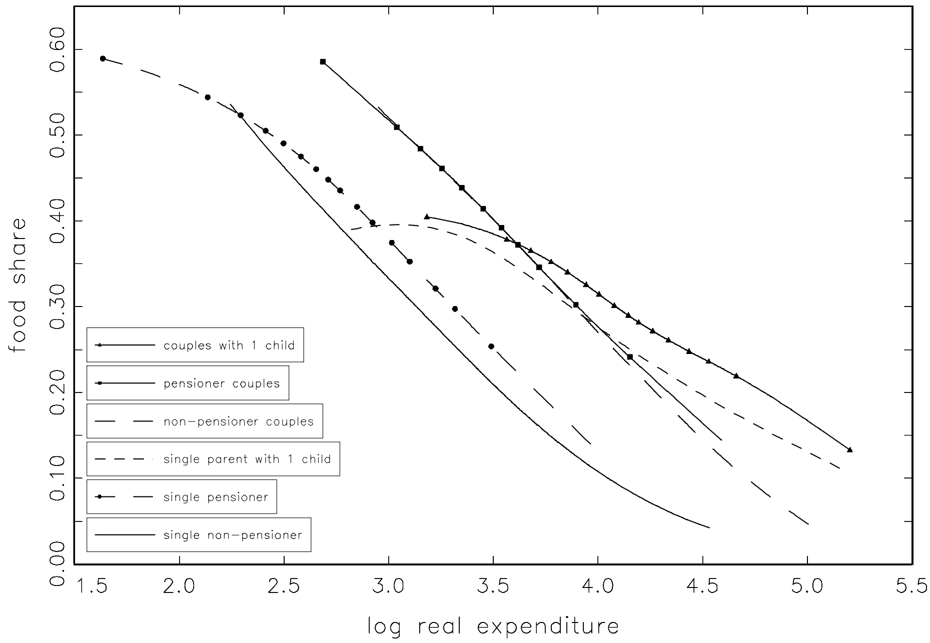


Figure 1.4: Engel curve for all food – different household types, 1978

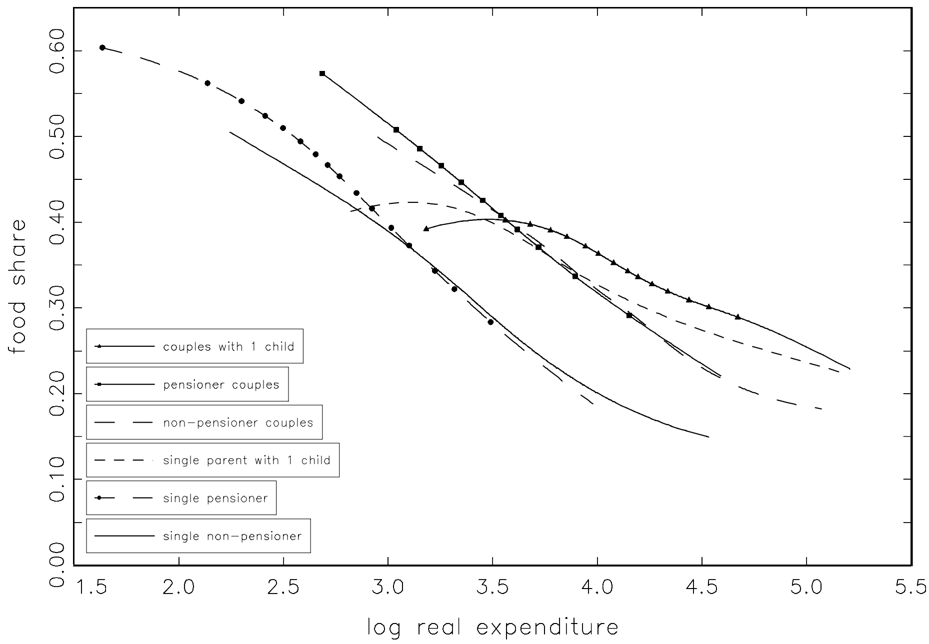
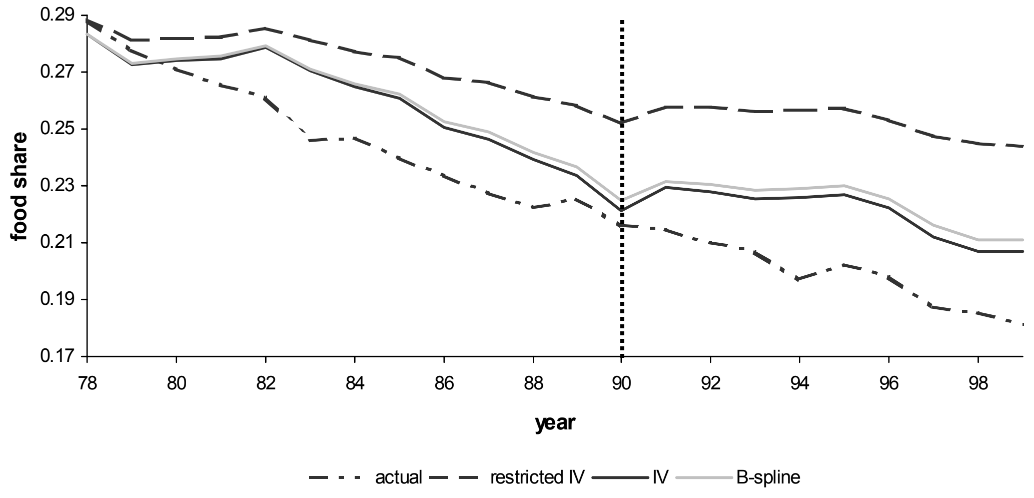


Figure 1.5: Counterfactual for food in – Engel curves held at 1978 estimatess.



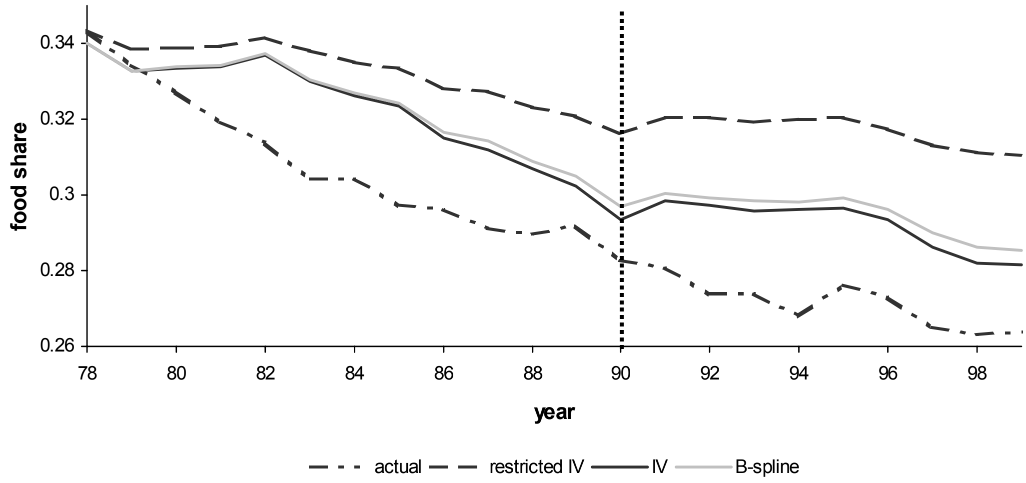
remains to be explained by other factors.

The Engel curves shown in figures 1.3 and 1.4 were used in the way described above to calculate the counterfactuals shown in figures 1.5 and 1.6. For comparative purposes, we also used two other procedures for estimating the household food Engel curves. The curve labelled "IV" uses least squares instrumental variables to estimate Engel curves that are linear in log expenditure, with log expenditure instrumented with household income to control for possible endogeneity due to, for example, purchase infrequency. Although each household type is restricted to have linear (in log expenditure) Engel curves there are no restrictions across household types. The curve labelled "restricted IV" uses Engel curves that are restricted to have the same slope across household types with only the intercept allowed to vary across groups.

There are several points to note from these counterfactual experiments. Firstly, it can be seen that when the Engel curves are allowed to vary freely across groups, the linear in log income assumption does not seem too unreasonable, as indicated by the closeness of the "IV" line and the "B-spline" line, where the shape of the Engel curve is not resticted. Second, the imposition of identical slopes across groups makes a large difference – the "restricted IV" line diverges quite markedly from the results given by the B-spline estimation. Finally, the comparison between the counterfactual path and the actual path bears quite a strong resemblance to the picture given by the path of per-capita expenditures compared to the evolution of the aggregate food share shown in figures 1.1 and 1.2. As noted before, the actual food share falls fairly steadily across the whole period. The counterfactual food share remains pretty much fixed between 1978 and 1982, then falls in line with the actual food share, and then appears to start declining more slowly than the actual path after 1990. This is quite similar to the path of average per-capita expenditures, even after we have accounted for the effects of a changing expenditure distribution and (some) changes in the proportions of household types on the aggregate food share.

It does appear, then, from our counterfactual experiments that, even after controlling for some changes in household characteristics and for the changes in the distribution of household total expenditures over time, some part of the evolution of the aggregate food share remains to be explained. The question is whether the difference can be explained by changes in observable covariates that we have not yet accounted for. Two obvious candidates are more household observable characteristics

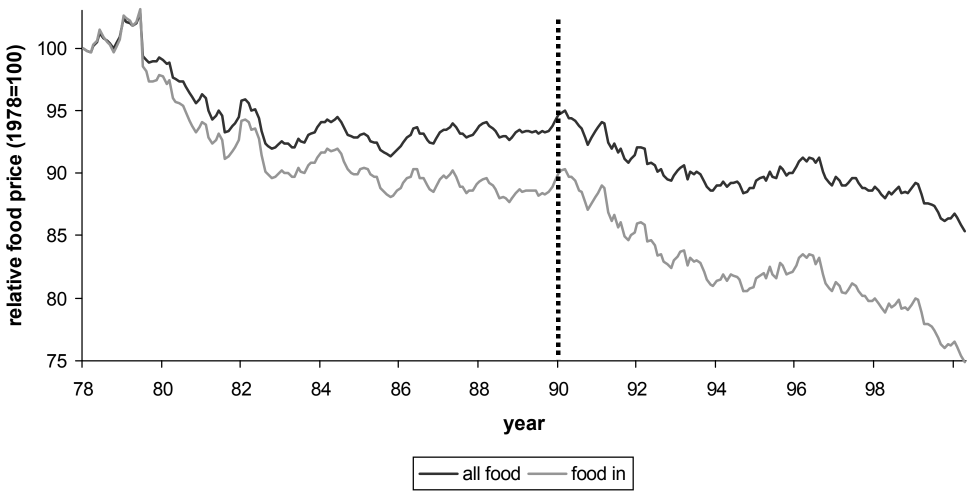
Figure 1.6: Counterfactual for all food – Engel curves held at 1978 estimates.



and to account for the effect that changing relative prices have had on demands. The relative prices of food in and all food are shown in figure 1.7. The recent downwards trend in prices starts in 1990, precisely the same point where real expenditure growth slows down but the food share decline remains similar to previous years.

A full non-parametric estimation of the effect of many more household characteristics would entail dividing our sample into groups that are too small for any sensible analysis, and this, plus the fact that modelling price effects non-parametrically is problematic means that we will employ

Figure 1.7: Relative prices of food in and all food



a flexible parametric model of demand in what follows. We begin by revisiting the theory of household spending patterns in the context of welfare measurement and go on to discuss the theory behind using trends in spending patterns to identify changes in welfare and hence bias in price indices. We then present a full empirical specification which allows us to control in a flexible manner for other changes in the household population and discuss the estimation of this model. As suggested by the cross-sectional Engel curves presented above, we show the importance of controlling for demographic variables in the slopes as well as the intercepts of this model. Finally, we use our preferred specification to estimate bias in price deflators, finding that estimated bias is much reduced when one controls adequately for other factors that have been changing in the population.

2. Trends in food spending, bias in price indices and welfare

Some recent papers have begun to use the evolution over time in the share of spending on food to make statements about changing living standards more generally, in the context of Engel's law. For example, in an article in the Business Review of the Federal Reserve Bank of Philadelphia in 1997, Leonard Nakamura noted the paradox that in an era of rapid technological progress, official statistics for the US show that real GDP growth has been rather low post-1973 when compared to historical trends – the average per capita growth between 1959 and 1974 was 2.5% whereas it was 1.7% between 1974 and 1994. Nakamura suggested looking at the share of certain types of expenditure out of total expenditure to obtain an alternative indicator of economic growth.

"A systematic way of testing for the presence of economic growth is to examine the rate at which basic economic necessities, such as food and clothing and household operations, are shrinking as a proportion of total expenditures"

This idea is based on Engel's law, an empirical regularity first noted by Engel as far back as 1857 that the share of spending on certain goods such as food and fuel (i.e. necessities) falls as household total expenditure increases, whereas the budget share of other types of goods, such as recreational goods and services (known as luxury goods), increases. Thus we would expect a predictable relationship between total expenditure and spending allocations across different goods, and hence Nakamura's suggestion of looking at spending patterns as an alternative indicator of consumption growth. Nakamura finds that the change in spending patterns over time appears to tell a different story to the consumption growth figures.

"From 1959 to 1974, according to the official statistics, real income per person grew 45 percent. In the longer period from 1974 to 1994, real income per person grew 39 percent. If these numbers are accurate, one would expect that the share of necessities in total expenditures should have shrunk by about the same amount in the two periods (or perhaps a bit less in the second period). In fact, the proportion of the average budget spent on food fell from 27.3 percent in 1959 to 23.1 percent in 1974, or 4.2 percentage points, but fell substantially more – 7.1 percentage points – from 1974 to 1994."

An essential ingredient for calculating the real rate of growth of consumer income or spending is the availability of a reliable measure of the change in cost-of-living. Such a price deflator, however, is not required when looking at shares of different goods in total expenditure. Nakamura suggested that, if official price deflators are upwards biased, and the bias is worse in the second period that in the first, then this could explain the anomally between growth measures that rely on a cost-of-living index (per capita income, expenditure, GDP) and those that do not (the share of food expenditure, for example). There has recently been much interest in (and concern over) the possibility that offical price indices such as the US Consumer Price index (CPI) and the UK Retail Prices Index

(RPI) are biased measures of the true change in the cost-of-living, and, indeed, this is primarily what motivates Nakamura's analysis. Bias in offical price indices can arise for a variety of reasons; for example *substitution bias* arises because indices such as the RPI and CPI measure the changing cost of buying a fixed basket of goods and thus fail to account for the fact that consumers may respond to relative price changes by altering their spending patterns (substituting away from goods whose relative prices have increased). *Quality bias* and *new goods bias* arise when the price index fails to take account of the changing quality and range of goods available. In a recent report of the results of a study of bias in the US CPI (the Boskin Report) it was suggested that bias is probably higher now than historically because a large part of the problem is to do with quality improvements and innovations, which are more important now than in previous years. This adds some support to Nakamura's findings.

This approach draws conclusions from data on average expenditure shares and average expenditure per person, and this may be problematic. In the language of standard consumer demand analysis, Engel's law simply tells us that items such as food have an expenditure elasticity of less than one, so that, among households with the same demographic composition facing given prices, the expenditure share for food decreases as total expenditure increases. Thus the share of expenditure on necessities can be thought of as some kind of (inverse) indicator of total expenditure for a given household type facing given prices. Looking at a single hypothetical household over time, we might be surprised if their food share dropped dramatically but their real expenditure did not rise by much, but we really need to know the precise relationship between food share, total expenditure and (unless relative prices stayed constant over time) prices to be able to assess whether the food share has declined 'too much' compared to the increase in real expenditure. Engel's law really only applies within a given price regime – it would be perfectly possible for a household to increase its budget share of food between two periods and still be better off if relative prices had changed sufficiently.

To be able to apply the same analysis to the relationship between population average food share and per-capita expenditure as we would to a single household requires some very stringent restrictions to be met. Households differ both in their total budget and in their demographic composition, and we would expect both factors to affect how they allocate spending across different goods. There are very limited circumstances under which we can expect a society made up of different households with different budgets to behave as a single or representative consumer with average budget, and, as a result, the conditions under which we can interpret changes in average shares over time as being representative of the population at large are limited. If the budget share of food for household type h, w_f^h, is some function $f^h\left(m^h\right)$ of total household expenditure, m^h, then the share of aggregate spending on food in aggregate total expenditure, M, will be $\sum_h \mu^h f^h\left(m^h\right)$ (where μ^h is household h's share in aggregate expenditure, m^h/M). To be able to write this as a function of average household expenditure, M/H, (or average per capita expenditure M/N where N is the number of people rather than households), would require that $f^h\left(m^h\right) = k$ for all households – i.e. that all households have an identical food budget share regardless of budget or demographic composition, and hence that the budget share of food tells us nothing about welfare. We can ask, instead, when we can write the average share as some function of a representative level of expenditure, $\tilde{m}$, which depends on the distribution of household expenditures, and where we can expect this relationship to remain constant over time so we can look at aggregate time series data. Muellbauer (1975, 1976) shows that if the relationship between budget shares, prices and expenditure at time t is

$$w_{ft}^h = a\left(p_t\right) + b\left(p_t\right)\left(\frac{m_t^h}{s_t^h}\right)^{-\alpha} \tag{2.1}$$

which he calls price-independent-generalised-linearity (PIGL), then we can write

$$\bar{w}_{ft} = a(p_t) + b(p_t)\,\tilde{m}_t^{-\alpha}$$

where

$$\tilde{m}_t = \left[\sum_h \mu_t^h \left(\frac{m_t^h}{s_t^h}\right)^{-\alpha}\right]^{-1/\alpha}$$

which is linearly homogenous in m^h and so can be written as $\theta_t \overline{m}_t$, where $\overline{m}_t = M_t/H_t$ (or it could just as well be M_t/N_t), and

$$\theta_t = \left[\sum_h \mu_t^h \left(\frac{m_t^h/\bar{m}_t}{s_t^h}\right)^{-\alpha}\right]^{-1/\alpha}$$

which summarises the effects of nonlinear Engel curves, unequal distribution of expenditures and the effect of demographic composition on Engel curves. Thus, only if the distribution of expenditures and of demographics remain constant over time (and, in addition, s^h, and therefore θ, is independent of prices) so that θ remains constant, can the average share be calculated as a function of average expenditure without there being aggregation bias (but note that the parameter recovered on expenditure from estimating this relationship will be $b\theta^{-\alpha}$ and not b). Differences across households are reflected by s^h which can be thought of as an equivalence scale – demographics enter as a deflator on expenditure, and a household of type h has a reference household equivalent expenditure of m^h/s^h, that is, each member of household h with expenditure m^h is as well off as the a reference household with expenditure m^h/s^h[5].

We might also want to be careful in applying a welfare intepretation to the aggregate share of food expenditure[6]. For the food share to identify equal welfare levels across different household types requires the following restriction on the household expenditure function $c(u, \mathbf{p}, \mathbf{z})$ (which gives the minimum cost of achieving welfare level u for a household with demographic characterictics $\mathbf{z}$)

$$c(u, \mathbf{p}, \mathbf{z}) = \bar{c}(u, \mathbf{p})\, s\left(\mathbf{p}^{-}, \mathbf{z}\right)$$

where $\bar{c}(u, \mathbf{p})$ is the expenditure function for the reference household and $\mathbf{p}^{-}$ denotes the vector of prices of all goods except food. In general, the equivalence scale is $s(u, \mathbf{p}, \mathbf{z})$, and so the restriction here is that it is independent of the utility level and of the price of food. We can then write

$$\begin{aligned} w_f(u, \mathbf{p}, \mathbf{z}) &\equiv \frac{\partial \ln c(u, \mathbf{p}, \mathbf{z})}{\partial \ln p_f} = \frac{\partial \ln \bar{c}(u, \mathbf{p})}{\partial \ln p_f} + \frac{\partial \ln s(\mathbf{p}^{-}, \mathbf{z})}{\partial \ln p_f} \\ &= \bar{w}_f(u, \mathbf{p}) \end{aligned}$$

(since $s(\mathbf{p}^{-}, z)$ is independent of p_f). To get shares in terms of (log) expenditure rather than utility, we substitute back the level of expenditure which achieves utility level u. So if we choose $\ln m^h =$

[5]The identification of equivalence scales has been the subject of much discussion – briefly, since an expenditure function of the form $c(\varphi(u, \mathbf{z}), \mathbf{p}, \mathbf{z})$ gives rise to the same observable demands as $c(u, \mathbf{p}, \mathbf{z})$, then the equivalence scale cannot be identified from demand data alone. However, this is not really less objectionable than the idea of making straight interpersonal comparisons – for any utility function, observed demands are invariant to a monotonic transformation of the function (i.e. a relabelling of the indifference curves) and so comparing money welfare even across observably identical individuals ignores the fact that one may simply be a better "utility producer" than the other.

[6]Hamilton (2001), for example, (see below for further discussion of this paper) states that, in estimating CPI bias, his "fundamental approach is to infer well-being by observing food's share".

$\ln c(u, \mathbf{p}, \mathbf{z})$ for a household with characteristics $\mathbf{z}$, then we get $\ln \bar{c}(u, \mathbf{p}) \equiv \ln m^h - \ln s(\mathbf{p}^-, \mathbf{z})$ for the reference household, therefore

$$w_f\left(\ln m^h, \mathbf{p}, \mathbf{z}\right) = \bar{w}_f\left(\ln m^h - \ln s(\mathbf{p}^-, \mathbf{z}), \mathbf{p}\right)$$

which, if $\bar{w}_f$ takes the form $a(p) + b(p)(m)^{-\alpha}$, is precisely the condition required for non-linear aggregation. Taking the PIGLOG form of equation (2.1) the share of aggregate food spending in aggregate total expenditure at time t, $\overline{w}_{ft}$, is given by

$$\overline{w}_{ft} = a(p_t) + b(p_t)\sum_{h=1}^{H_t}\frac{m_t^h}{M_t}\ln m_t^{eh}$$

This can be written as

$$\overline{w}_{ft} = a(p_t) + b(p_t)\ln\left(\kappa_t \overline{m_t^e}\right)$$

where

$$\ln \kappa_t = \frac{1}{H_t}\sum_{h=1}^{H_t}\frac{m_t^h}{\overline{m}_t}\ln\left(\frac{m_t^{eh}}{\overline{m_t^e}}\right)$$

which is similar to the Theil entropy measure of inequality[7]. The value of $\ln \kappa_t$ increases as inequality increases, being equal to zero under total equality and $\ln\left(N/n^h\right)$ under maximum inequality (i.e. where one household has all expenditure, there are n^h members of this household and N members of the population in total). Thus, higher inequality would tend to reduce the share of food in aggregate expenditure (to take an extreme example, if one household had all national expenditure, their share of food spending, which also equals the aggregate share, would be very low) whereas a more equal distribution (for the same aggregate expenditure) would increase the food share. The average food share as a welfare indicator would actually be giving the wrong message (assuming the social welfare function is inequality averse). If the expenditure distribution, $a(p_t)$ and $b(p_t)$ remained constant, then there would be a linear (negative) relationship between food share and average *equivalised* expenditure, but this is unlikely to be the case. The widening in the expenditure distribution in the UK over time has been documented in, for example, Goodman, Johnson and Webb (1997). and the changing relative price of food, for example, was shown in figure 1.7.

In summary

1. The restrictions required on household behaviour to enable us to treat aggregate (or average) shares as if they were generated by a single, welfare maximising consumer with aggregate (or average) expenditure/income are very demanding. For most plausible specifications of consumer preferences, average shares will (at the very least) depend on the distribution of expenditure across the population as well as on average expenditure, and so a changing expenditure distribution over time could affect the observed average shares.

[7] The actual Theil entropy measure of dispersion for household equivalised expenditures would be

$$\frac{1}{H_t}\sum_{h=1}^{H_t}\frac{m_t^{eh}}{\overline{m_t^e}}\ln\left(\frac{m_t^{eh}}{\overline{m_t^e}}\right)$$

2. Depending on the shape of the Engel curve for a good, there is no reason to expect the relationship between percentage changes in total expenditure and percentage changes in budget shares to be constant even for an individual household. Only if shares are linear in log expenditure would this be the case at the household level, and this would extend to the aggregate only if a given percentage increase in aggregate expenditure stemmed from an identical percentage increase in each household's expenditure, so that their share in aggregate expenditure was unchanged.

3. The relationship between total household budget and budget shares for each good varies across different types of household – for example it may depend on family size, and within that the number of adults versus children, and their age and sex, and on other factors such as female labour force participation. Therefore, the relationship between aggregate expenditure and expenditure shares on different goods will depend on the proportions of different family types which make up the population, and we have already dicussed how demographics have been changing over time.

4. Finally, we would expect a particular relationship between total expenditure and the budget share of food, say, to hold only within a particular price regime. Relative prices have been changing over time, though, which may affect the relationshipship we see *over time* between total expenditure and expenditure shares.

Thus, any observed anomaly in the relationship between aggregate expenditure growth over time and the expenditure shares on different types of good could simply be to do with changes in the distribution of expenditures, changes in demographics or changes in relative prices.

3. Estimation

The importance of changes in demographics, prices and other relevant covariates in explaining observed changes in expenditure patterns can be explored by analysing data at the household level. In this approach, the relationship between household spending decisions and demographics, prices and total household budget is modelled explicitly, which in turn allows estimation of the extent to which changing expenditure patterns over time can be explained by changes in observable covariates. This is the route followed in two recent papers, Hamilton (2001) and Costa (2001), where US household level data is analysed to allow for the effects that changes in the distribution of total expenditure, changes in demographics, and so on may have on consumption patterns. After having controlled for a (fairly limited) number of covariates, they attribute any remaining unexplained time effects to bias in the CPI.

3.1. Hamilton's methodology

As described above, a full analysis of whether the different pictures given by the evolution of per-capita expenditure and aggregate food share can be explained by the evolution of demographics, prices and other relevant covariates involves modelling demand at the household level.

Hamilton's analysis (which Costa subsequently applied to her data) of household expenditure data is based on a Working-Leser type demand structure for estimating the budget share of food eaten at home:

$$w_{frt}^h = \alpha' Z_{rt}^h + \gamma \ln\left(\frac{p_{frt}}{p_{nrt}}\right) + \beta \ln\left(\frac{y_{rt}^h}{p_{rt}}\right) \tag{3.1}$$

where w^h_{frt} denotes the share of food in household *income* (he does not use total expenditure) for household h in year t and in region r, y^h_{rt} is household (nominal) income, Z^h_{rt} is a vector of household/regional covariates, p_{frt} is the price of food in year t and region r, p_{nrt} is the (similarly indexed) price of non-food, p_{rt} is the general price level (a weighted average of p_{frt} and p_{nrt}). Now suppose that true prices, p_{irt}, $i = f, nf$, are not observed, and that instead biased prices $q_{irt} = p_{irt}e_{it}$ are observed, where e_{it} represent the bias in terms of how much the true price must be multiplied to get biased prices. Thus $(e_{it} - 1) \times 100$ would give the percentage bias. Note that bias is being assumed to be the same across regions. Also denote the overall level of price bias by e_t (a weighted average of e_{frt} and e_{nrt}), so that $q_{rt} = p_{rt}e_t$. Since $\ln p_{irt} = (\ln q_{irt} - \ln e_{it})$ equation 3.1 can be written

$$w^h_{frt} = \alpha' Z^h_{rt} + \gamma \ln\left(\frac{q_{frt}}{q_{nrt}}\right) - \gamma \ln\left(\frac{e_{ft}}{e_{nt}}\right) + \beta \ln\left(\frac{y^h_{rt}}{q_{rt}}\right) - \beta \ln e_t \tag{3.2}$$

Hamilton uses data from the American Panel Study of Income Dynamics (PSID) from 1974–1991 and, for a small subsample of the data, the CPI has information on geographical price variation as well as temporal variation. With price variation across both region and time, the empircal version of equation 3.2 for estimation becomes

$$w^h_{ft} = \alpha' Z^h_{rt} + \gamma \ln\left(\frac{q_{frt}}{q_{nrt}}\right) + \beta \ln\left(\frac{y^h_{rt}}{q_{rt}}\right) + \sum \delta_t D_t + u^h_{rt} \tag{3.3}$$

where u^h_{rt} is a classical error term, D_t are time dummies and Z^h_{rt} includes region dummies. Comparing equations 3.2 and 3.3, and assuming that all unexplained time effects are due to CPI bias, the interpretation of the coefficients on the time dummies becomes clear, and is

$$\delta_t = -\gamma \ln\left(\frac{e_{ft}}{e_{nt}}\right) - \beta \ln e_t$$

Further assuming that food and non-food price measurements are equally biased ($e_{ft} = e_{nt} = e_t$, so that $\ln(e_{ft}/e_{nt}) = 0$), Hamilton obtains

$$\ln e_t = -\frac{\delta_t}{\beta}$$

To use the full sample, Hamilton must lose the geographical price variation, and so the estimating equation is

$$w^h_{ft} = \alpha' Z^h_{rt} + \beta \ln\left(\frac{y^h_{rt}}{q_t}\right) + \sum \delta_t D_t + u^h_{rt}$$

where now (still assuming that food and non-food are equally biased)

$$\delta_t = \gamma \ln\left(\frac{q_{ft}}{q_{nt}}\right) - \beta \ln e_t$$

Finally, to calculate the bias, Hamilton borrows the estimate of γ from the sample with geographical price variation.

Costa follows Hamilton's methodology using consumer expenditure data collected by the US Department of Labour over various periods of time between 1888 and 1994. Her selection criteria are similar to Hamilton's, however Costa follows the more usual share estimation approach of looking at expenditure shares out of total expenditure and not out of total income. Hamilton and

Costa experiment to a limited degree with functional form, mainly by including an expenditure or income squared term, but, interestingly, appear not to fully exploit the interactions that arise with an expenditure squared term which might help identification[8]. In addition, we have seen from the preliminary data analysis above and also know from previous research (see for example Blundell, Pashardes and Weber (1993)) that the slopes of Engel curves can vary a great deal with household composition. Therefore, it may be of crucial importance in this analysis to investigate the effects of different demand specifications, otherwise a large part of what the time dummies pick up may simply be aggregation biases with respect to compositional changes in the household population.

3.2. Full modelling

In the introduction to this paper we illustrated, using semiparametric techniques on one year of data, how both the slope and the intercept of the Engel curves vary across different household types. To fully investigate the importance of functional form over a longer period of time we switch to a flexible quadratic model, since controlling for prices in a theory consistent way totally nonparametrically would be extremely complex. In this paper we estimate a quadratic Almost Ideal (QUAIDS) demand system. The QUAIDS model derives from the following Quadratic Logarithmic specification of the consumer cost function, $\ln C^h$

$$\ln C^h = \ln a^h(p) + \frac{ub^h(p)}{1 - ug^h(p)}$$

For the price indices $\ln a^h(p)$ and $b^h(p)$ we use the forms typically employed in the AI demand system – a translog form for $\ln a^h(p)$ and a Cobb-Douglas form for $b^h(p)$. For $g^h(p)$ we follow the specification used in Banks, Blundell and Lewbel (1997) which completes the QUAIDS specification. Thus:

$$\begin{aligned}
\ln a^h(p_t) &= \alpha_0 + \sum_i \alpha_i^h \ln p_{it} + \frac{1}{2}\sum_i\sum_j \gamma_{ij} \ln p_{it} \ln p_{jt} \\
b^h(p_t) &= \prod_i p_{it}^{\beta_i^h} \\
g^h(p_t) &= \sum_i \lambda_i^h \ln p_{it}
\end{aligned}$$

where

$$\begin{aligned}
\alpha_i^h &= \alpha_{i0} + \sum_k \alpha_{ik} z_k^{\alpha} \\
\beta_i^h &= \beta_{i0} + \sum_k \beta_{ik} z_k^{\beta} \\
\lambda_i^h &= \lambda_{i0} + \sum_k \lambda_{ik} z_k^{\lambda}
\end{aligned}$$

in which the z's denote demographic characteristics such as age of head, number of children, and so on.

[8] With time dummies for the error terms an interaction between expenditure and these dummies arises, and with the time trend specification that Hamilton moves to (to reduce the number of variables) one should obtain a trend squared term and an interaction between the trend and expenditure.

This gives the following form for the share equations[9]

$$w_{it}^h = \alpha_i^h + \sum_j \gamma_{ij} \ln p_{jt} + \beta_i^h \ln \left[\frac{m_t^h}{a^h(p_t)}\right] + \frac{\lambda_i^h}{b^h(p_t)} \ln \left[\frac{m_t^h}{a^h(p_t)}\right]^2 \tag{3.4}$$

which is a complicated non-linear function of prices. The estimation procedure in papers such as Banks, Blundell and Lewbel (1997) exploits the linearity of the share equation *given* $a^h(p_t)$ and $b^h(p_t)$. The procedure is iterative – first, the price and expenditure parameters are estimated for given values of $a^h(p_t)$ and $b^h(p_t)$, then $a^h(p_t)$ and $b^h(p_t)$ are updated using the estimated values of α_i^h, γ_{ij} and β_i^h, and then the procedure is repeated using the updated price indices, continuing until the difference between the current and previous estimates is negligible[10].

Using the full sample and estimating the QUAIDS model without trying to understand possible bias in the price indices given in equation 3.4 we show that it is important to control for demographic variation in the slopes as well as the intercepts of the Engel curves, consistent with the analysis in section 1.

We classify expenditures into six groups, and get RPI indices for each. The groups are food (we use both food in and all food), fuel, clothing, alcohol, travel and other non-durables. In addition to our original classification into household types, we control for age, education, number and age of children, home ownership status, durable ownership, smoking behaviour, work status of head and spouse, and, of course, relative prices. We impose homogeneity by expressing all prices relative to 'other non-durables'.

Having established the importance of including demographics, we can move on to look at what the implications of such a model would be for the bias calculations that are in Costa (2001) and Hamilton (2001). An economic true cost-of-living index is generally defined as $c(\overline{u}, \mathbf{p}_{t+1}, \mathbf{z}) / c(\overline{u}, \mathbf{p}_t, \mathbf{z})$, the minimum cost of achieving welfare level $\overline{u}$ compared across price regimes $\mathbf{p}_t$ and $\mathbf{p}_{t+1}$. Except under special circumstances we can expect this index to be potentially different for every household in our sample depending on their demographics and reference utility. We discussed in section 2 how the RPI could suffer from, for example, substitution bias compared to a true cost-of-living index, but, in addition, one RPI figure is constructed for everyone based on average spending patterns, which may be far removed from the actual spending patterns of any one individual (although with full QUAIDS estimation, price indices $a^h(\mathbf{p}_t)$ and $b^h(\mathbf{p}_t)$ are constructed for each household based on the RPI price sub-indices within each group of goods, the sub-indices themselves are constructed from average spending data, and so suffer from the same problem). Thus we can expect the total deviation of the RPI from actual cost-of-living to differ across different households. Obviously we cannot identify a separate measure for each household – we could use the entire sample and calculate some "average" bias or conduct the analysis separately for smaller, more homogenous groups (which automatically allows for some variation in covariate parameters according to household type). In this analysis we restrict our sample to households where the head and spouse (where present) are of working age, and the head is in full-time employment (a group similar to that of Costa and Hamilton) to illustrate the variation of the bias estimate with different specifications.

Recall that we assume that prices are measured with error, e_{it}, such that $q_{it} = p_{it}e_{it}$ where q_{it} is the measured price and p_{it} is the true price. Supposing that all prices are equally biased, $e_{it} = e_t$ $\forall\, i$, so that $q_{it}/q_{jt} = p_{it}/p_{jt}$, then, in the two good food/non-food example we obtain the following equation for the food share in terms of measured prices

$$w_{ft}^h = \alpha' Z_t^h + \gamma \ln\left(\frac{q_{ft}}{q_{nt}}\right) + \beta_i^h \left[\ln\left(\frac{m_t^h}{a^h(\mathbf{q}_t)}\right) + \ln e_t\right] + \frac{\lambda_i^h}{b^h(\mathbf{q}_t)} \left[\ln\left(\frac{m_t^h}{a^h(\mathbf{q}_t)}\right) + \ln e_t\right]^2 + u_t^h \tag{3.5}$$

[9] The share, w_{it}^h, of the *ith* good for household h in time period t is given by $\partial \ln C^h / \partial \ln p_{it}$.

[10] The consistency and asymptotic efficiency of these estimators is described in Blundell and Robin (1999).

since, from homogeneity, $\ln a^h(\mathbf{p}_t) = \ln a^h(\mathbf{q}_t) - \ln e_t$ and $b^h(\mathbf{p}_t) = b^h(\mathbf{q}_t)$. We can estimate $\ln e_t$ by estimating

$$w_{ft}^h = \alpha' Z_t^h + \gamma \ln\left(\frac{q_{ft}}{q_{nt}}\right) + \beta_i^h \ln\left[\frac{m_t^h}{a^h(\mathbf{q}_t)} + \sum \delta_t D_t\right] + \frac{\lambda_i^h}{b^h(\mathbf{q}_t)} \ln\left[\frac{m_t^h}{a^h(\mathbf{q}_t)} + \sum \delta_t D_t\right]^2 + u_t^h$$

where $\{D_t\}$ are year dummies, so $\widehat{\delta}_t = \widehat{\ln e_t}$ assuming that all unaccounted for year effects of this form are interpretable as RPI bias[11]. Here we are assuming that the price error is constant across a year and, since we have monthly price variation, we can identify all the $\{\delta_t\}$ even with a linear specification and no demographic interection in slopes. However, moving to a fuller specification results in interaction of the time dummies with demographics and expenditures, and corresponding restrictions on the coefficients, and so we are not simply relying on monthly price variation for identification.

4. Results

All models have a full set of intercept terms, including: age of head, age of head squared, age of spouse, numbers of children aged 0–2, 3–10 and 11–18, number of non-dependent children, a single adult household dummy, a dummy for head remaining in education beyond compulsary schooling age, regional and seasonal dummies, car ownership dummy, spouse's labour force participation, a dummy for the presence of a smoker in the household, as well, of course, as prices. We estimate three models which differ in the way that log expenditure is included, as detailed below.

- Model 1 – log expenditure and log expenditure squared. Estimation by instrumental variables to account for possible endogeneity of expenditure, car ownership and smoking[12].
- Model 2 – as model 1 plus interactions between log expenditure terms and seasonal variables.
- Model 3 – as model 3 plus interactions between log expenditure terms and demographic and seasonal variables.

After experimenting with slope and intercept terms, we arrived at our preferred full specification in model 3. The results illustrate the importance of allowing for Engel curves that are quadratic in log expenditure, and also allowing the slopes with respect to log expenditure to vary with demographic and seasonal variables.

We estimated two versions of the demand system, one in which the food expenditure includes both food eaten inside and outside the home, and the second where food expenditure is just food eaten inside the home, with food eaten out being included in the omitted 'other nondurable expenditure' category.

Table 4.1 explains the notation used in the two tables (and those in appendix C) which indicate the sign and significance of the estimates of the slope coefficients for the two versions of the estimated

[11] As mentioned in the introduction, even if we were totally confident that we had included all relevant covariates, any 'drift' we find in the ability of our model to explain spending behaviour could simply be rationalised under the umbrella of 'changing preferences' rather than attributed to mismeasurement in prices. However, the whole concept of a cost-of-living index rests on consumers having stable preferences over time – if they did not, it would not make sense to ask how much money we would need to give someone to make them as well off today as they were yesterday. This is how official price indices are used all the time (uprating of benefits and so on), and so operating within the paradigm of a cost-of-living index, it makes little sense to attribute unexplained variation to random preference changes.

[12] Instruments include log income and log income squared and the relevent interactions of these with other covariates, regional unemployment rates, lagged car and tobacco prices, population density and years of formal education of the head and spouse.

demand system. Results are shown in tables 4.2 and 4.3. For simplicity, the values of the coefficients are shown for the simplest model when they are significant at a 95% level of confidence or above. For the rest of the table, a single (double) plus sign indicates a positive coefficient with significance at a 95% (99%) level of confidence, and similarly for a single or double minus sign.

Table 4.1: Description of variable names.

$\ln m, (\ln m)^2$	log expenditure and log expenditure squared
q_1, q_2, q_3	year-quarter dummies (reference is last quarter, i.e. October–December)
$k02$	number of children aged 0–2
$k310$	number of children aged 3–10
$k1118$	number of children aged 11–18
knd	number of non-dependent children
ed_h	dummy for head remaining in education beyond compulsary schooling age
age_h	age of head (divided by 10)
$single$	dummy for single adult (reference is couple)

Next we show the results from using a food and 'other expenditure' demand system to estimate the error in the RPI using several different specifications. To calculate the standard error on $\widehat{e}_t$ which we need to calculate confidence intervals around our estimates we use the delta method, which gives us $var\left(\widehat{e}_t\right) = \exp\left(2\widehat{\ln e_t}\right) var\left(\widehat{\ln e_t}\right)$. Again, we estimate several different model specifications to investigate the effect this has on our estimates of $\widehat{e}_t$, which are listed as *error*_1979, *error*_1980 and so on in the full results which we give in Appendix C – we only present results for food eaten inside and outside the home as the results for food inside the home only were very similar. Here we simply give a graphical representation of our results. The different specifications are as follows (again, all specifications have a full set of intercept terms):

- Model 1 – log expenditure, expenditure deflated by RPI
- Model 2 – as model 1, but using instrumental variables to account for possible endogeneity of expenditure, car ownership and smoking.
- Model 3 – as model 2 plus log expenditure squared.
- Model 4 – as model 3 plus interactions between log expenditure and log expenditure squared and demographic and seasonal variables.
- Model 5 – as model 4 but with expenditure deflated by Stone price index.
- Model 6 – full QUAIDS estimation.

Figure 4.1 shows the percentage price errors estimated from some of the different models and figure 4.2 shows how these translate into a revised (non-durable good) RPI series. On all figures, the lighter lines depict 95% confidence intervals for the estimates.

Obviously the main conclusion to be drawn from this exercise is how much the estimated "price error" can change according to the specification used. It is interesting to see that in the food in and out version the error actually increases between models 1 and 3 (not surprisingly, the increase

Table 4.2: Estimates of slope coefficients for demand system (food in and out).

	Food	**Fuel**	**Clothing**	**Alcohol**	**Travel**
Model 1					
$\ln m$	-0.0839	-0.0502	0.0264	0.0258	-0.0345
$(\ln m)^2$	-0.0178	0.0165		-0.0115	-0.0136
Model 2					
$\ln m$	$--$	$--$	$++$	$++$	$--$
$\ln m \times q_1$		$-$	$--$		
$\ln m \times q_2$			$-$		
$\ln m \times q_3$		$+$		$-$	$-$
$(\ln m)^2$	$--$	$++$		$--$	$-$
$(\ln m)^2 \times q_1$					
$(\ln m)^2 \times q_2$					
$(\ln m)^2 \times q_3$				$+$	
Model 3					
$\ln m$	$--$	$--$			$--$
$\ln m \times q_1$			$--$		
$\ln m \times q_2$			$-$		
$\ln m \times q_3$		$++$		$-$	
$\ln m \times k02$		$--$		$++$	$++$
$\ln m \times k310$	$-$	$--$		$++$	$+$
$\ln m \times k1118$	$--$	$--$		$++$	$++$
$\ln m \times knd$	$-$	$--$	$-$	$++$	$+$
$\ln m \times age\_h$	$--$			$++$	$+$
$\ln m \times ed\_h$		$++$			
$\ln m \times single$	$++$	$+$		$+$	
$(\ln m)^2$	$++$	$+$			
$(\ln m)^2 \times q_1$					
$(\ln m)^2 \times q_2$					
$(\ln m)^2 \times q_3$				$+$	
$(\ln m)^2 \times k02$	$--$			$--$	$-$
$(\ln m)^2 \times k310$		$++$		$--$	$-$
$(\ln m)^2 \times k1118$		$++$		$--$	
$(\ln m)^2 \times knd$		$++$		$--$	$-$
$(\ln m)^2 \times age\_h$	$-$	$+$			
$(\ln m)^2 \times ed\_h$	$-$	$--$	$-$		
$(\ln m)^2 \times single$				$--$	$--$

Table 4.3: Estimates of slope coefficients for demand system (food in).

	Food	**Fuel**	**Clothing**	**Alcohol**	**Travel**
Model 1					
$\ln m$	-0.0551	-0.0502	0.0269	0.0257	-0.0350
$(\ln m)^2$	-0.0274	0.0163		-0.0120	-0.0134
Model 2					
$\ln m$	$--$	$--$	$++$	$++$	$--$
$\ln m \times q_1$		$-$	$--$		
$\ln m \times q_2$	$-$		$-$		
$\ln m \times q_3$		$+$		$-$	$-$
$(\ln m)^2$	$--$	$++$		$--$	$-$
$(\ln m)^2 \times q_1$					
$(\ln m)^2 \times q_2$					
$(\ln m)^2 \times q_3$				$+$	
Model 3					
$\ln m$	$--$	$--$			$--$
$\ln m \times q_1$			$--$		
$\ln m \times q_2$	$-$		$-$		
$\ln m \times q_3$		$++$		$-$	
$\ln m \times k02$		$--$		$++$	$++$
$\ln m \times k310$	$--$	$--$		$++$	$+$
$\ln m \times k1118$	$--$	$--$		$++$	$++$
$\ln m \times knd$	$-$	$--$	$-$	$++$	$+$
$\ln m \times age\_h$	$--$			$++$	$+$
$\ln m \times ed\_h$	$+$	$++$			
$\ln m \times single$	$++$	$+$		$+$	
$(\ln m)^2$	$+$	$+$			
$(\ln m)^2 \times q_1$					
$(\ln m)^2 \times q_2$	$+$				
$(\ln m)^2 \times q_3$				$+$	
$(\ln m)^2 \times k02$				$--$	$-$
$(\ln m)^2 \times k310$	$++$	$++$		$--$	$-$
$(\ln m)^2 \times k1118$	$++$	$++$		$--$	$-$
$(\ln m)^2 \times knd$				$--$	$-$
$(\ln m)^2 \times age\_h$	$-$	$+$			
$(\ln m)^2 \times ed\_h$	$--$	$--$	$--$		
$(\ln m)^2 \times single$				$--$	$--$

Figure 4.1: Price error estimates from selected models – food in and out version

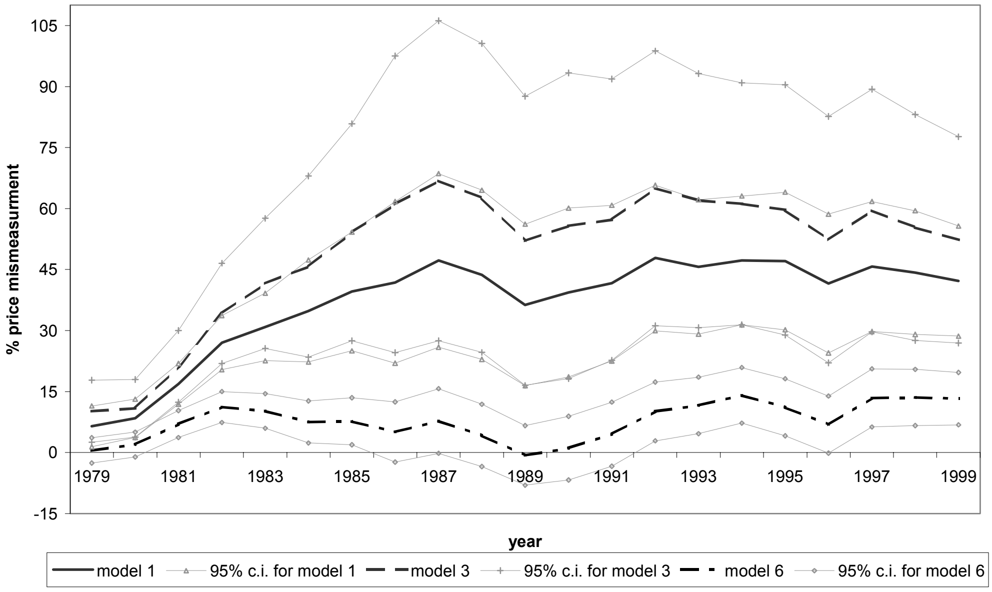
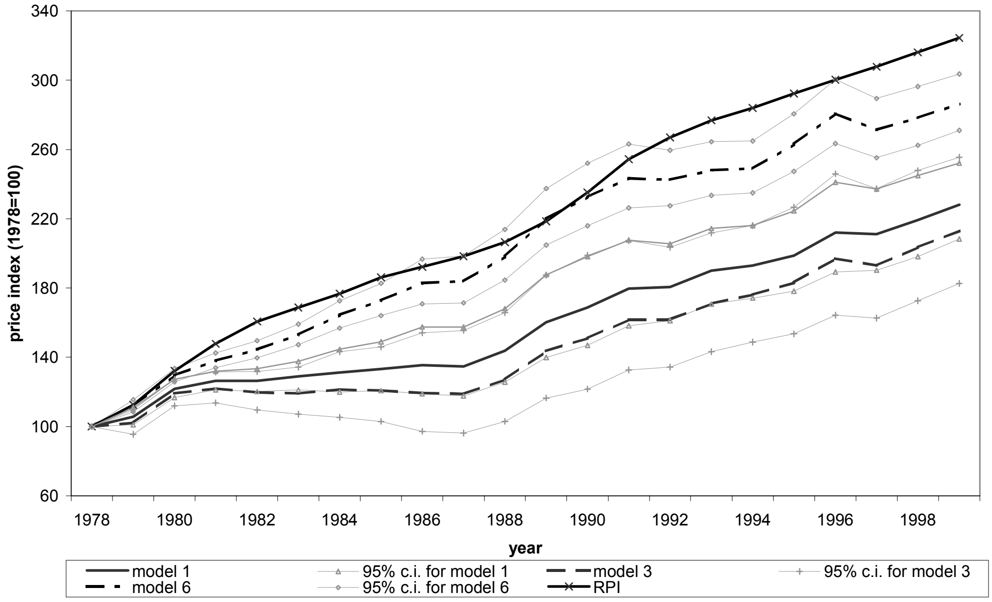


Figure 4.2: RPI and 'corrected' price indices from selected models – food in and out version



occurs in the move to instrumental variables estimation between models 1 and 2). Using the simplest model could lead us to the conclusion that, between 1978 and 1999, although measured prices increased by about 225% (or an average annual inflation rate of around 5.8%), the "true" price increase was only around 110% (albeit with a fairly large standard error). Moving to the fuller set of demographic interactions, which we have seen to be an important determinant of household demands, reduces this figure a good deal to a "true" increase of around 185% (an average annual inflation rate of around 5.1%).

5. Extending the model

We may be concerned that the assumption of equal price bias for food and "all other goods" is not a very good one, and be interested in how this assumption affects the average estimated time effects. Suppose we wish to relax the assumption that the price errors are uniform across goods and we want to estimate the errors acoss a set of goods. *Conditional on* $a^h(p_t)$ and $b^h(p_t)$ *being correct* we have

$$w_{it}^h = \alpha_i^h + \sum_j \gamma_{ij} \ln q_{jt} - \sum_j \gamma_{ij} \ln e_{jt} + \beta_i^h \ln \left[\frac{m_t^h}{a^h(p_t)}\right] + \frac{\lambda_i^h}{b^h(p_t)} \ln \left[\frac{m_t^h}{a^h(p_t)}\right]^2 + u_t^h \tag{5.1}$$

which we could estimate using, for example

$$w_{it}^h = \alpha_i^h + \sum_j \gamma_{ij} \ln q_{jt} + \beta_i^h \ln \left[\frac{m_t^h}{a^h(p_t)}\right] + \frac{\lambda_i^h}{b^h(p_t)} \ln \left[\frac{m_t^h}{a^h(p_t)}\right]^2 + \sum \delta_{it} D_t$$

where $\delta_{it} = -\sum_j \gamma_{ij} \ln e_{jt}$, and recover the error at time t with the following relationship

$$E_t = \Gamma^{-1} \Delta_t \tag{5.2}$$

where E_t is the vector $(e_{1t}, ..., e_{nt})'$, Γ is the matrix with typical element γ_{ij} and Δ_t is the vector $(\delta_{1t}, ..., \delta_{nt})'$. Theoretically, by homogeneity and adding up, Γ is a singular matrix, and therefore cannot be inverted. Separate identification of all the price errors using some method which gave us a non-singular, and therefore invertible, Γ matrix would be relying on theoretically incorrect results, and we do not wish to follow this approach.

We can impose homogeneity in our estimation process by choosing a good, good 1, say, and imposing the restriction that $\gamma_{i1} = -\sum_{j=2}^n \gamma_{ij}\ \forall i$. This restriction is easy to impose in the estimation process by simply using prices expressed relative to the price of good 1 (just as we did in section 3) since the homogeneity restriction means that the share equations can be expressed as (where the summations now run from goods 2 to n)

$$w_{it} = \alpha_i^h + \sum_j \gamma_{ij} \ln \left(\frac{q_{jt}}{q_{1t}}\right) + \beta_i^h \ln \left(\frac{m_t}{a(p_t)}\right) + \frac{\lambda_i^h}{b(p_t)} \left[\ln \left(\frac{m_t}{a(p_t)}\right)\right]^2 + \sum \delta_{it} D_t \tag{5.3}$$

and the price indices can also be expressed in the following manner

$$\begin{aligned} \ln a(p) &= \alpha_0 + \sum_i \alpha_i \ln \left(\frac{p_i}{p_1}\right) + \ln p_1 + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln \left(\frac{p_i}{p_1}\right) \ln \left(\frac{p_j}{p_1}\right) \\ b(p) &= \prod_i \left(\frac{p_i}{p_1}\right)^{\beta_i} \end{aligned}$$

since theoretical restrictions also imply that $\alpha_{10} = 1 - \sum_{i=2}^{n} \alpha_{i0}$, $\alpha_{1k} = -\sum_{i=2}^{n} \alpha_{ik}\ \forall k \neq 0$ and $\beta_{1k} = -\sum_{i=2}^{n} \beta_{ik}\ \forall k$. This means that $\delta_{it} = -\sum_{j=2}^{n} \gamma_{ij} \ln(e_{jt}/e_{1t})$ and can be used to recover $(e_{2t}/e_{1t}, ..., e_{nt}/e_{1t})$.

Now, of course, we do not know the correct $a^h(p_t)$ and $b^h(p_t)$, but rewriting the price indices in terms of q and e gives

$$\begin{aligned} \ln a^h(p_t) &= \alpha_0 + \sum_i \alpha_i^h \ln\left(\frac{q_{it}}{q_{1t}}\frac{e_{1t}}{e_{it}}\right) + (\ln q_{1t} - \ln e_{1t}) + \frac{1}{2}\sum_i\sum_j \gamma_{ij} \ln\left(\frac{q_{it}}{q_{1t}}\frac{e_{1t}}{e_{it}}\right)\ln\left(\frac{q_{jt}}{q_{1t}}\frac{e_{1t}}{e_{jt}}\right) \\ b^h(p_t) &= \prod_i \left(\frac{q_{it}}{q_{1t}}\frac{e_{1t}}{e_{it}}\right)^{\beta_i^h} \end{aligned}$$

which means that, with each iteration's estimates of $\ln(e_{it}/e_{1t})$, $b^h(p_t)$ can be calculated using the current estimates of E_t, but we can only calculate $\ln a^h(p_t) + \ln e_{1t}$ since the time dummies only give us estimates of the relative errors $\ln(e_{it}/e_{1t})$ and not the absolute error $\ln e_{1t}$. Denoting $\ln a^h(p_t) + \ln e_{1t}$ as $\ln \widetilde{a}^h(p_t)$, equation (5.1) can be written as

$$\begin{aligned} w_{it}^h &= \alpha_i^h + \sum_j \gamma_{ij} \ln\left(\frac{q_{jt}}{q_{1t}}\right) - \sum_j \gamma_{ij} \ln\left(\frac{e_{jt}}{e_{1t}}\right) + \beta_i^h \ln\left(\frac{m_t^h}{\tilde{a}^h(p_t)}\right) + \frac{\lambda_i^h}{b^h(p_t)}\left[\ln\left(\frac{m_t^h}{\tilde{a}^h(p_t)}\right)\right]^2 \\ &\quad + \beta_i^h \ln e_{1t} + 2\frac{\lambda_i^h}{b^h(p_t)} \ln\left(\frac{m_t^h}{\tilde{a}^h(p_t)}\right) \ln e_{1t} + \frac{\lambda_i^h}{b^h(p_t)}\left[\ln e_{1t}\right]^2 \qquad (5.4) \end{aligned}$$

With the quadratic term in log expenditure, $\ln e_{1t}$ becomes interacted with $\ln m_t^h$ and, with more than one equation a cross-equation restiction arises since $\ln e_1$ is the same in all share equations. Hence $\ln e_{1t}$ can be estimated as well as $\ln(e_{it}/e_{1t})$. Again, conditional on $b^h(p_t)$ and $\widetilde{a}^h(p_t)$ this is a linear estimation problem, and so the Blundell and Robin (1999) results on consistency should apply. Naturally, if all prices are equally biased, $\ln(e_{it}/e_{1t}) = 0\ \forall\ i$ and equation (5.4) reduces to a multiple good version of equation (3.5).

The time dummy specification is a very general way of including the price error term and proves rather difficult to estimate because of the large number of dummy variables and interactions that arise. To implement the estimation we move to a specification where there is a constant amount of price mismeasurement every year, which we denote by ε_i. This obviously gets compounded over time, so that after t years, the total bias is $e_{it} = (\varepsilon_i)^t$. So if ε_i is 1.03, for example, the price of good i is measured with 3% error each year, and after ten years e_{it} is around 1.34, or a 34% cumulative bias. Hence the log error follows a trend over time, $\ln e_{it} = t \ln \varepsilon_i$. For comparative purposes, we re-ran the most complex version (model 6) of the food/all other goods, equal price bias model with a time trend instead of time dummies. Recall that the time dummy version reduced average inflation over the period from around 5.8% to 5.1%. With a time trend, we obtain a value for ε of about 1.007 – which gives a very similar averaged result (since $1.058/1.007 \simeq 1.051$).

We chose a fairly disaggregated grouping of goods, namely food in and out, fuel, clothing, alcohol, travel, and leisure goods and services (the omitted group is other household and personal services). The number of interactions and restrictions in the model that occur if we have many demographics covariates means the regression becomes very time consuming to run and so we carry out the estimation on a relatively homogenous subsection of the population. We chose employed couples with no children. As discussed previously, there is no reason to expect that any bias in the RPI should be the same across different household groups, since the RPI is one average measure for everyone, but different households may buy different mixtures of goods. If we run the food/all other goods equal price bias model of section 4 for this narrower group of households we obtain a higher coefficient on the time trend than for the wider group, of the order of 1.014. If we interpret this as

RPI bias then this result seems plausible – childless, working couples are better off on average than the wider group, and we may expect them to be buying more of the types of goods and services that have been subject to quality improvement, for example.

Some results are given in table 5.1. It may seem surprising that the coefficient for leisure goods does not indicate a positive price bias, but, as the estimation is done over nondurables, leisure goods excludes large items such as audio-visual goods that, *a priori,* we might expect to have had the greatest increase in quality in this group. In addition, of course, it may simply be wrong to interpret this trend solely as a measure of price bias. Based on the RPI weights for the goods, the individual errors would give a typical combined average error in a year of 1.012, or a reduction in average inflation over the period from about 5.8% to 4.6%. If we then run the same regressions but assuming an equal bias in all the prices we obtain a coefficient of about 1.018 – higher than when we use just the food/all other goods specification.

Table 5.1: Estimates of good-varying price errors.

Good	**Error estimate**
food in	0.997
food out	1.026
fuel	0.995
clothing	0.985
alcohol	1.030
travel	1.024
leisure goods	0.992
leisure services	1.029
h'hold and personal services	1.030

6. Conclusions

Engel's Law – that the budget share of food falls as living standards increase – is one of the oldest predictions in economics. As a measure of welfare, the budget share of food is independent of any price deflator, unlike, for example, real, per-capita expenditure. Recently, some studies in the US have used discrepancies between the trends in living standards implied by changes in the food share and the trends in real income to make inferences about the price deflators by which income is adjusted, on the basis that such price deflators can be biased measures of the true change in cost-of-living for a number of reasons.

Engel's law applies to a given household facing given prices. To analyse the changing share of food spending on aggregate over time we ideally need to control for all the other factors which may otherwise affect the observed trends in spending patterns, two of the most obvious being changes in household composition and changes in relative prices. We have shown that when we include time related variables into the estimation of budget shares on micro-data, then the coefficients we obtain can vary substantially according to the other covariates that we allow for. We have also shown how interactions with the quadratic expenditure term in the QUAIDS model and a multiple good specification can, in theory, allow the identification of separate "price error" terms across different goods. In our example, this affected the etimate of price bias compared to the assumption of equal bias and the estimation of only food shares, but not to a great degree. However, given the degree to which the time related coefficients can vary depending on the specification, we would have to be fairly confident that we had controlled for all factors that could affect spending patterns and that changes in the quality of our data could not lead to a time trend before we could confidently interpret these coefficients as a measure of price bias.

Appendices

Appendix A

Estimating the partially linear model

We want to estimate the following partially linear Engel curve

$$w_i^h = f_i\left(\ln x^h\right) + Z^h\beta + \epsilon_i^h$$

where $E\left(\epsilon_i^h | \ln x^h, Z^h\right) = 0$ so

$$E\left(w_i^h | \ln x^h, Z^h\right) = f_i\left(\ln x^h\right) + Z^h\beta \tag{6.1}$$

Noting that

$$E\left(w_i^h | \ln x^h\right) = f_i\left(\ln x^h\right) + E\left(Z^h | \ln x^h\right)\beta \tag{6.2}$$

we can subtract 6.2 from 6.1 and write

$$\xi_w^h = \xi_Z^h\beta + \epsilon_i^h \tag{6.3}$$

where $\xi_w^h = w_i^h - E\left(w_i^h | \ln x^h\right)$ and $\xi_Z^h = Z^h - E\left(Z^h | \ln x^h\right)$.

If we knew ξ_w^h and ξ_Z^h then we could estimate β by least squares, however $E\left(w_i^h | \ln x^h\right)$ and $E\left(Z^h | \ln x^h\right)$ are not known, and so to implement the estimation of β in equation 6.3 we need to replace $E\left(w_i^h | \ln x^h\right)$ in ξ_w^h and $E\left(Z^h | \ln x^h\right)$ in ξ_Z^h by their estimated values. Robinson (1988) shows that this will give an asymptotically efficient estimate of β (assuming that $E\left(\left[\epsilon_i^h\right]^2 | \ln x^h, Z^h\right) = E\left(\epsilon^2\right) = \sigma^2 < \infty$). Denote our nonparametric estimators of $E\left(w_i^h | \ln x^h\right)$ and $E\left(Z^h | \ln x^h\right)$ by $\hat{g}_w\left(\ln x^h\right)$ and $\hat{g}_Z\left(\ln x^h\right)$, respectively, then $\hat{\xi}_w^h = w_i^h - \hat{g}_w\left(\ln x^h\right)$ and $\hat{\xi}_Z^h = Z^h - \hat{g}_Z\left(\ln x^h\right)$ and our estimate of β is

$$\hat{\beta} = \left(\hat{\xi}_Z'\hat{\xi}_Z\right)^{-1}\hat{\xi}_Z'\hat{\xi}_w$$

where $\hat{\xi}_Z$ and $\hat{\xi}_w$ are the vectors of $\hat{\xi}_Z^h$ and $\hat{\xi}_w^h$.

Since $f\left(\ln x^h\right) = E\left(w_i^h - Z^h\beta | \ln x^h\right)$ we can estimate $f_i\left(\ln x^h\right)$ as the nonparametric regression of $w_i^h - Z^h\widehat{\beta}$ on $\ln x^h$, or get the identical estimate from the earlier regressions as

$$\hat{f}\left(\ln x^h\right) = \hat{g}_w\left(\ln x^h\right) - \hat{g}_Z\left(\ln x^h\right)\widehat{\beta}$$

Dealing with endogeneity of total expenditure

To adjust for endogeneity we adapt the control function or augmented regression technique (see Holly and Sargan (1982), for example) to the semiparametric Engel curve framework. To avoid cluttered notation we drop the demographic variables, Z, in the following discussion. Suppose $\ln x^h$ is endogenous in that $E(\epsilon_i^h | \ln x^h) \neq 0$. In this case the nonparametric estimator will not be consistent for the function of interest. Following Blundell, Duncan, Pendakur (1998) and Blundell, Browning and Crawford (2000) we take the log of disposable income as the excluded instrumental variable for log total expenditure, and assume that this instrumental variable, $\ln y^h$, is such that

$$\ln x^h = \eta \ln y^h + v^h \text{ with } E(v^h | \ln y^h) = 0 \tag{6.4}$$

and that

$$E(w_i^h|\ln x^h, \ln y^h) = E(w_i^h|\ln x^h, v^h) \tag{6.5}$$
$$= f_i(\ln x^h) + v^h\rho_i \tag{6.6}$$

This implies the augmented regression model

$$w_i^h = f_i(\ln x^h) + v^h\rho_i + \epsilon_i^h \tag{6.7}$$

with

$$E(\epsilon_i^h|\ln x^h) = 0 \tag{6.8}$$

so $v^h\rho_i$ can just be added into the partially linear framework.

Note that the unobservable error component v in (6.7) is unknown. In estimation v is replaced with the first stage reduced form residuals

$$\widehat{v}^h = \ln x^h - \widehat{\eta}' \ln y^h \tag{6.9}$$

where $\widehat{\eta}$ is the least squares estimator of η. This is a semi-parametric version of the idea proposed in Newey, Powell and Vella (1999).

Appendix B

Cubic B-splines

For $w_i = f(x_i) + \epsilon_i$ we estimate $f(x)$ using a cubic b-spline basis in the following way.

1. Choose K knots, then $K+7$ points such that: $t_1 = t_2 = t_3 = t_4 = a < t_5 < .... < t_{K+3} < b = t_{K+4} = t_{K+5} = t_{K+6} = t_{K+7}$, where $[a, b]$ is the range of the x data.
2. Construct the zero-degree b-spline $B_k^0(x) = I(t_k \leq x < t_{k+1})$ for $k = 1, ..., K+6$
3. For $n = 1, 2, 3$, construct

$$B_k^n(x) = \left(\frac{x - t_k}{t_{k+n} - t_k}\right) B_k^{n-1}(x) + \left(\frac{t_{k+n+1} - x}{t_{k+n+1} - t_{k+1}}\right) B_{k+1}^{n-1}(x)$$

so the final iteration gives us the required cubic splines. Denote the matrix of cubic splines by $\mathbf{B}^3$.

4. We want $\hat{f}(x) = \mathbf{B}^3\hat{\gamma}$ so we choose γ to minimise

$$\left(w - \hat{f}(x)\right)' \left(w - \hat{f}(x)\right) + \lambda \int_a^b \left(\hat{f}''(x)\right)^2 dx$$

where λ is the trade-off between closeness and smoothness

5. Hence $\hat{\gamma} = \left(\mathbf{B}^{3\prime}\mathbf{B}^3 + \lambda\mathbf{S}\right)^{-1} \mathbf{B}^{3\prime} w$, where $S_{ij} = \int_a^b \left(B_i^{3\prime\prime}(x) B_j^{3\prime\prime}(x)\right) dx$

Appendix C

Full results

Table 6.1: Food in and out – price and slope coefficient estimates.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
$\ln pfood$	0.1500	0.0668	0.1393	0.0401	0.0360	0.0341
	(0.0330)	(0.0394)	(0.0360)	(0.0207)	(0.0210)	(0.0193)
$\ln m$	−0.1099	−0.0509	−0.1388	−0.6225	−0.4976	−0.1140
	(0.0009)	(0.0057)	(0.0301)	(0.1440)	(0.1409)	(0.0149)
$\ln m \times q_1$				0.0077	−0.0056	0.0065
				(0.0553)	(0.0535)	(0.0055)
$\ln m \times q_2$				−0.1215	−0.1247	−0.0166
				(0.0656)	(0.0643)	(0.0064)
$\ln m \times q_3$				−0.0639	−0.0687	−0.0111
				(0.0599)	(0.0586)	(0.0063)
$\ln m \times k02$				0.1162	0.0865	−0.0055
				(0.0639)	(0.0626)	(0.0081)
$\ln m \times k310$				0.0081	−0.0042	−0.0169
				(0.0305)	(0.0295)	(0.0041)
$\ln m \times k1118$				−0.0308	−0.0369	−0.0304
				(0.0302)	(0.0281)	(0.0048)
$\ln m \times knd$				−0.1239	−0.1116	−0.0392
				(0.0425)	(0.0405)	(0.0074)
$\ln m \times ed\_h$				0.0455	0.0366	−0.0045
				(0.0245)	(0.0240)	(0.0022)
$\ln m \times age\_h$				0.0848	0.0492	0.0124
				(0.0516)	(0.0507)	(0.0047)
$\ln m \times single$				0.1145	0.1086	0.0600
				(0.0748)	(0.0746)	(0.0077)
$(\ln m)^2$			0.0065	0.0634	0.0482	0.0528
			(0.0028)	(0.0161)	(0.0158)	(0.0140)
$(\ln m)^2 \times q_1$				−0.0008	0.0005	−0.0071
				(0.0062)	(0.0060)	(0.0052)
$(\ln m)^2 \times q_2$				0.0131	0.0134	0.0117
				(0.0073)	(0.0071)	(0.0058)
$(\ln m)^2 \times q_3$				0.0069	0.0073	0.0082
				(0.0066)	(0.0065)	(0.0058)
$(\ln m)^2 \times k02$				−0.0150	−0.0115	−0.0145
				(0.0070)	(0.0069)	(0.0065)
$(\ln m)^2 \times k310$				−0.0027	−0.0013	−0.0002
				(0.0033)	(0.0032)	(0.0031)
$(\ln m)^2 \times k1118$				0.0008	0.0016	0.0040
				(0.0032)	(0.0030	(0.0030)
$(\ln m)^2 \times knd$				0.0106	0.0096	0.0096
				(0.0043)	(0.0042)	(0.0039)
$(\ln m)^2 \times ed\_h$				−0.0062	−0.0050	−0.0042
				(0.0028)	(0.0027)	(0.0025)
$(\ln m)^2 \times age\_h$				−0.0097	−0.0054	−0.0113
				(0.0058)	(0.0057)	(0.0050)
$(\ln m)^2 \times single$				−0.0071	−0.0073	−0.0071
				(0.0086)	(0.0085)	(0.0076)

Table 6.2: Food in and out – price error estimates.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
*error*_1979	1.0649	1.1370	1.1018	1.0069	1.0018	1.0055
	(0.0255)	(0.0706)	(0.0389)	(0.0160)	(0.0161)	(0.0158)
*error*_1980	1.0847	1.1473	1.1090	1.0219	1.0163	1.0202
	(0.0238)	(0.0654)	(0.0360)	(0.0159)	(0.0157)	(0.0155)
*error*_1981	1.1691	1.3277	1.2118	1.0738	1.0696	1.0703
	(0.0253)	(0.0843)	(0.0447)	(0.0177)	(0.0174)	(0.0169)
*error*_1982	1.2704	1.4847	1.3424	1.1175	1.1108	1.1122
	(0.0338)	(0.1165)	(0.0626)	(0.0207)	(0.0204)	(0.0194)
*error*_1983	1.3089	1.6010	1.4161	1.1081	1.0992	1.1024
	(0.0421)	(0.1536)	(0.0811)	(0.0235)	(0.0229)	(0.0216)
*error*_1984	1.3483	1.5553	1.4573	1.0774	1.0648	1.0755
	(0.0638)	(0.1981)	(0.1131)	(0.0279)	(0.0278)	(0.0263)
*error*_1985	1.3963	1.6652	1.5417	1.0809	1.0646	1.0771
	(0.0740)	(0.2398)	(0.1356)	(0.0314)	(0.0313)	(0.0294)
*error*_1986	1.4183	1.6626	1.6105	1.0502	1.0263	1.0507
	(0.1008)	(0.3101)	(0.1850)	(0.0392)	(0.0392)	(0.0375)
*error*_1987	1.4724	1.7239	1.6683	1.0766	1.0481	1.0778
	(0.1082)	(0.3327)	(0.1996)	(0.0423)	(0.0417)	(0.0403)
*error*_1988	1.4371	1.6433	1.6260	1.0413	1.0109	1.0426
	(0.1055)	(0.3156)	(0.1927)	(0.0408)	(0.0401)	(0.0388)
*error*_1989	1.3629	1.5143	1.5206	0.9914	0.9639	0.9934
	(0.1008)	(0.2906)	(0.1804)	(0.0386)	(0.0384)	(0.0372)
*error*_1990	1.3936	1.5617	1.5576	1.0064	0.9744	1.0112
	(0.1053)	(0.3066)	(0.1906)	(0.0410)	(0.0404)	(0.0397)
*error*_1991	1.4166	1.6048	1.5727	1.0419	1.0086	1.0455
	(0.0973)	(0.2886)	(0.1755)	(0.0389)	(0.0401)	(0.0401)
*error*_1992	1.4785	1.7696	1.6498	1.1020	1.0656	1.1014
	(0.0908)	(0.2947)	(0.1713)	(0.0387)	(0.0375)	(0.0367)
*error*_1993	1.4565	1.7455	1.6198	1.1177	1.0759	1.1161
	(0.0839)	(0.2747)	(0.1585)	(0.0376)	(0.0357)	(0.0353)
*error*_1994	1.4725	1.7350	1.6118	1.1405	1.0989	1.1408
	(0.0804)	(0.2590)	(0.1509)	(0.0370)	(0.0351)	(0.0347)
*error*_1995	1.4711	1.6600	1.5966	1.1102	1.0716	1.1115
	(0.0859)	(0.2590)	(0.1562)	(0.0369)	(0.0358)	(0.0356)
*error*_1996	1.4156	1.5385	1.5234	1.0647	1.0272	1.0691
	(0.0866)	(0.2474)	(0.1538)	(0.0366)	(0.0358)	(0.0357)
*error*_1997	1.4576	1.6956	1.5949	1.1337	1.0848	1.1346
	(0.0812)	(0.2566)	(0.1514)	(0.0371)	(0.0357)	(0.0362)
*error*_1998	1.4427	1.6352	1.5534	1.1344	1.0889	1.1357
	(0.0771)	(0.2371)	(0.1411)	(0.0356)	(0.0345)	(0.0351)
*error*_1999	1.4221	1.6332	1.5231	1.1362	1.0876	1.1329
	(0.0687)	(0.2181)	(0.1287)	(0.0336)	(0.0320)	(0.0326)

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