

PERFECT SHUFFLE INTERCONNECTIONS USING  
FRESNEL COMPUTER GENERATED  
HOLOGRAMS IN A PLANAR-OPTIC  
CONFIGURATION

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**INTRODUCTION**

Several standard interconnection patterns exist but perhaps the most useful is the perfect shuffle (PS) since multiple identical copies can be cascaded in multistage interconnection networks for use in sorting of optical telecommunication signals, for example. This is not the case for the crossover interconnection network where each stage differs. Perfect shuffles also can be used in algorithms to perform fast Fourier transforms, polynomial evaluation and matrix transposition (Stone(1)). The optical implementation of a perfect shuffle has numerous benefits (Lohmann (2)), not least that data may be arranged in a 2-D fashion and the whole array shuffled in one operation (Lin et al (3)). This operation may be carried out between two arrays of multi quantum well (MQW) modulators (Parry et al (4)), with the modulators and the optical interconnect element all arranged in the same plane (so called planar-optics, Jahns et al (5)). Crossover networks have recently been demonstrated optically in planar-optic configurations (Song et al (6)) The planar-optic configuration has the advantage that the optical paths are folded into a compact and rugged structure and the various components are kept in good alignment. Moreover, the alignment can be performed lithographically so high accuracy and reproducibility can be achieved. This makes the planar optic type of interconnect ideal for interconnecting VLSI or WSI circuits. Interconnects performed electronically are beginning to reach their limits as the chips are progressively increasing in size and speed. One problem is that of distribution of the same clock signal to several points on the wafer surface at the same instant. This has been addressed by the development of novel, self-focusing Fresnel-Dammann gratings, and Fresnel computer generated hologram fanout and focusing elements which have been demonstrated in a slanted axis, multiple reflection, planar optic configuration (Godwin et al (7)). In this paper we concentrate on the task of transferring data from one part of the chip to another in an optical bus i.e. transfer of an image. Moreover, we require the optical interconnection to interchange the pixels between the input and the output in order to perform a perfect shuffle operation. This will

enable multistage routing and sorting networks to be realised in a compact and rugged planar optic form.

**THE 2-D PERFECT SHUFFLE**

The perfect shuffle was originally defined to act upon 1-D data (1), such that the data are divided into two, and the two halves perfectly interleaved, in analogy with shuffling a pack of cards (fig.1). If the data are given addresses, numbered sequentially in binary, it can be seen from fig.1 that the perfect shuffle operation corresponds to a barrel roll of the address bits (ie the data at address abcd goes to address bcda, where a, b, c, d are the address bits, and can be 1 or 0). In order to use the advantages of optical interconnection, however, these 1-D data sets must be mapped into two dimensions (2). (In an electronic circuit this could be a hard-wired mapping.) This is not a unique operation, and there are many possible mapping schemes. The implementation of a perfect shuffle in 2-D may also be defined in many different ways, and the choice of 2-D shuffle must be made compatible with the mapping scheme chosen. In this paper we consider two such mapping schemes, and their corresponding 2-D perfect shuffle operations.

The first mapping scheme we consider was proposed by Stirk et al (8), who termed the corresponding shuffle a folded perfect shuffle. The mapping scheme is a simple one, and allows all of the algorithms designed for 1-D data to be used in 2-D. The 1-D data is written along rows of a 2-D matrix, a new row being started when the previous one is full (see fig.2). This corresponds to the mapping of the 1-D address, [abcd], to the 2-D address, [ab,cd] [row address, column address]. The required 2-D perfect shuffle must therefore move the data at address [ab,cd] to address [bc,da], and this operation may also be seen in fig.2.

The second mapping scheme was described by Taylor (9), and takes [abcd] into [ac,bd] (extending to larger addresses such that [abcdef...] → [ace..., bdf...]). This mapping scheme may be seen in fig.3. The corresponding 2-D perfect shuffle comprises perfect

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therefore refer to this type of 2-D shuffle as a double perfect shuffle):

[abcd]  $\rightarrow$  1-D to 2-D mapping  $\rightarrow$   
 [ac,bd]  $\rightarrow$  double PS  $\rightarrow$   
 [ca,db]  $\rightarrow$  2-D to 1-D mapping  $\rightarrow$  [cdab]

[abcd]  $\rightarrow$  1-D PS  $\rightarrow$   
 [bcda]  $\rightarrow$  1-D PS  $\rightarrow$  [cdab]

This has the advantage that, in routing networks for instance (provided that 4-port switching modules are used), only half the number of interconnection stages are required. The effect of a double perfect shuffle operation could be performed optically by cascading two sets of optical hardware for performing a 1-D perfect shuffle. However, if this can be done using a single stage of special optical hardware it will lead to more compact and more efficient systems. It is worth noting that the reverse is not possible; special double perfect shuffle optical hardware cannot be used to perform a 1-D perfect shuffle. For such a shuffle it is necessary to develop special compact optical hardware to perform a folded perfect shuffle since this has the effect of a single 1-D perfect shuffle. It can be seen, therefore, that there is a requirement for the implementation of different 2-D perfect shuffles in optical hardware.

#### HOLOGRAPHIC IMPLEMENTATION

If we have a regular,  $n \times n$ , square array of small pixels (fig.4), on a pitch of  $d$ , we can get a perfect shuffled arrangement of these signals by copying the array four times and superimposing the copies. The first step is to copy and superimpose the array such that the four quadrants of the array overlap perfectly (fig.5). This happens when the centres of the copies are placed at the corners of a square, having the same orientation as the array, and of side  $nd/2$ . The second step is to shift the centres of each of the copies by  $d/4$  in both the  $x$ - and  $y$ -directions (ie, diagonally by  $d/2\sqrt{2}$ ). The sense of these final shifts, as applied to each of the copies, determines the type of perfect shuffle. A maximum of 24 different shuffles may be formed by shifting the centres in different ways (there are four centres, each of which may be shifted in one of four directions, but a given shift direction may only be used once - this gives a total of  $4 \times 3 \times 2 \times 1 = 24$  possible combinations). For instance, the folded perfect shuffle (fig.6a) is achieved by moving the centres of the top right and bottom left arrays apart along the diagonal joining them, and the centres of the top left and bottom right together. The double perfect shuffle (fig.6b) is formed by pushing all of the centres together along the diagonals (note that the centres move by  $d/4$  parallel to the square edge, and so the centre spacing decreases by  $d/2$ ).

From the above considerations it is possible to see that a perfect shuffle may be formed by using a 1:4 fanout

hologram (Carey et al (10)). This is a lenslike element, but with four foci instead of one. If it is used to image an array of spots, four copy arrays will appear, the centres of which will coincide with the hologram's four focal spots. If the distance between these spots is chosen according to steps one and two above, a perfect shuffle results. Two holograms were chosen for this work. One produced a rhombic array of four spots, and can be used to perform a folded perfect shuffle, whilst the other gave a square array of four spots and could be used to perform a double perfect shuffle.

#### GENERATION OF THE HOLOGRAMS

The holograms were designed to operate at  $30^\circ$ , in the planar optic configurations shown in fig.7. They were generated by calculating the Fresnel interference pattern, at the hologram plane, between waves diverging from a point source as a reference, and waves converging to an array of four spots (with the spacing calculated to produce the desired perfect shuffle). The interference pattern can be represented by the following equation (Kawai and Kohga (11)):

$$\begin{aligned} \delta\phi(x,y) &= \phi_r(x,y) - \phi_o(x,y) \\ &= \phi_r(x,y) - \tan^{-1} \left\{ \frac{\sum_i \sin \phi_i(x,y)}{\sum_i \cos \phi_i(x,y)} \right\} \quad (1) \end{aligned}$$

where  $\delta\phi$  is the phase difference between reference and image waves,  $x$  and  $y$  are coordinates in the hologram plane, and all  $\phi$ s represent phases in the hologram plane;  $\phi_r$  is the phase of the reference beam,  $\phi_o$  is the phase of the image array taken as a whole, and the  $\phi_i$  are the phases of the individual light spots,  $i$ , in the image ( $i$  runs from 1-4 in this case). The hologram is stored as a binary pattern such that, when  $\cos \delta\phi \geq 0$ , 1 is stored, and when  $\cos \delta\phi < 0$ , 0 is stored. The hologram can be manufactured as a phase grating such that, on replay, a 1 represents a  $\pi$  phase delay, and 0 a 0 phase delay.

Part of the computer generated hologram pattern that creates a rhombic array can be seen in fig.8a, and that for a square array can be seen in fig.8b. The arrays generated by these holograms when illuminated by a white light source (in the configuration of fig.7b) can be seen in fig.9. A perfect shuffle that has been generated holographically can be seen in fig.10.

#### CONCLUSIONS

We have described the design and experimental performance of two binary Fresnel holograms which, when illuminated by a point source, generate a square and a rhombic array of light spots respectively. The

square array hologram can be used to double perfect shuffle an input array, and the rhombic hologram performs a folded perfect shuffle. Depending upon the relative spacings of the holographic array and the array to be shuffled, 24 different shuffles can be performed. Their utility would depend upon the mapping of data from 1-D into 2-D, and the function that the particular perfect shuffle operation is designed to perform. The holograms were demonstrated operating at 30 degree oblique incidence in a planar optic configuration suitable for integration with VLSI or WSI circuits.

#### ACKNOWLEDGEMENTS

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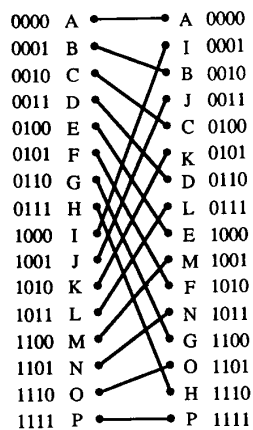


Figure 1: 1-D perfect shuffle

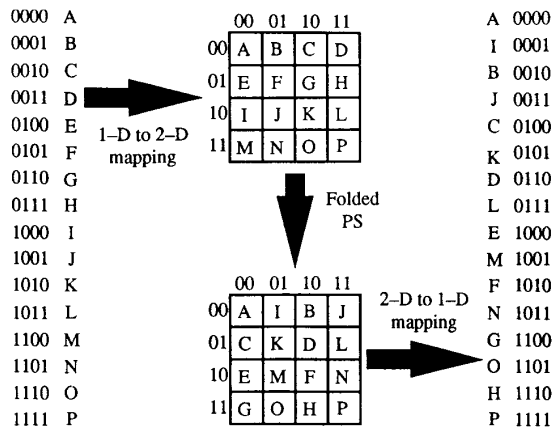


Figure 2: 1-D to 2-D mapping and shuffle for folded perfect shuffle

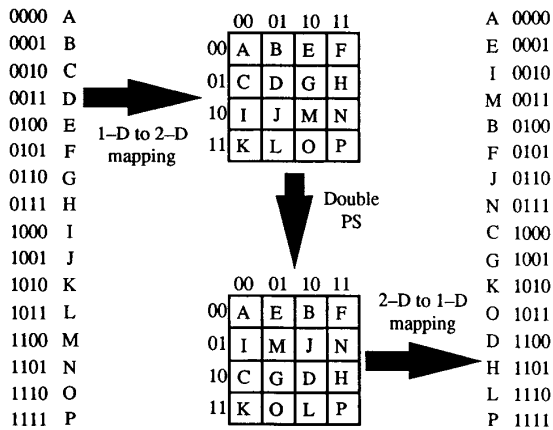


Figure 3: 1-D to 2-D mapping and shuffle for double perfect shuffle

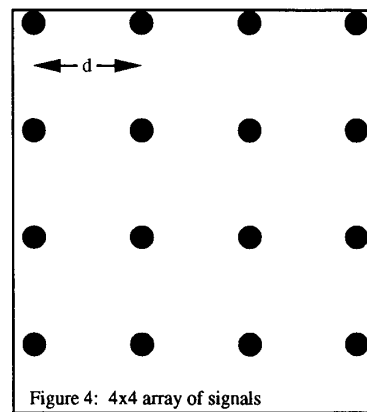


Figure 4: 4x4 array of signals

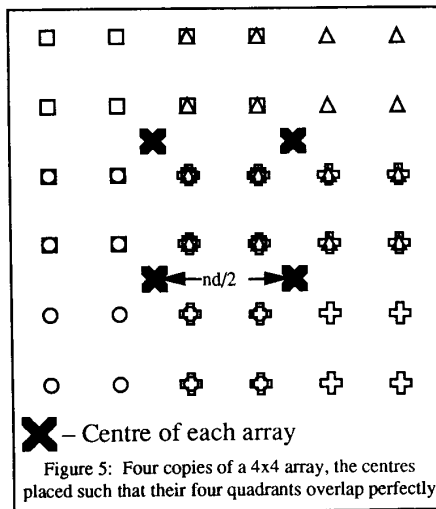
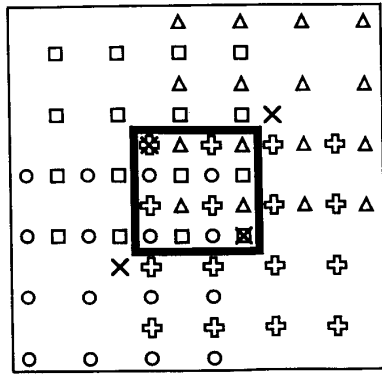
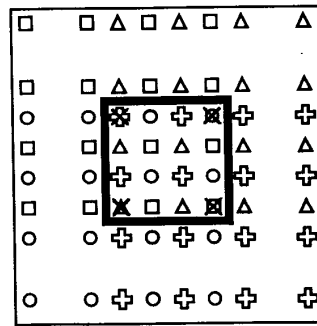


Figure 5: Four copies of a 4x4 array, the centres placed such that their four quadrants overlap perfectly



X - Centre of each array

Figure 6a: Shifts required for a folded perfect shuffle



X - Centre of each array

Figure 6b: Shifts required for a double perfect shuffle

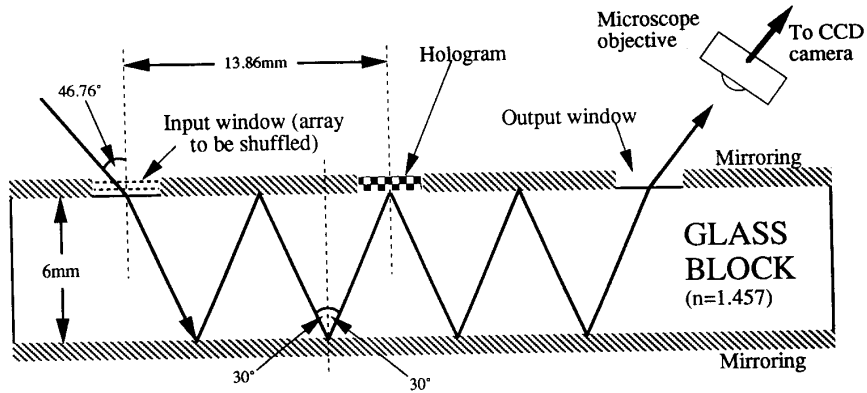


Figure 7a: Planar-optic configuration for performing a perfect shuffle on an input array at 30°

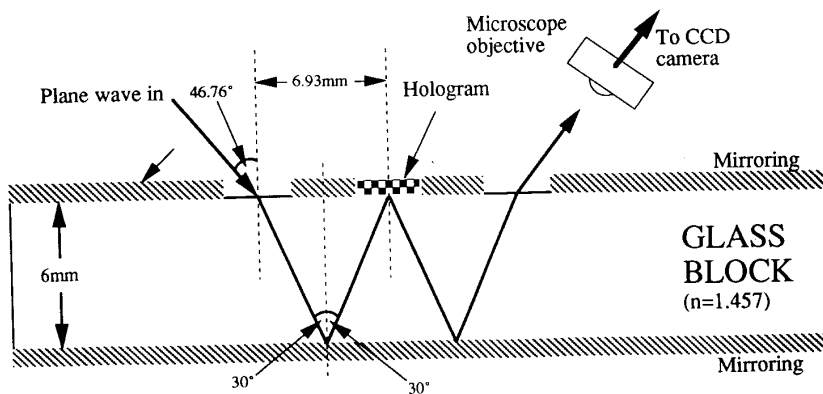


Figure 7b: Planar-optic configuration for testing a holographic fanout element at 30°

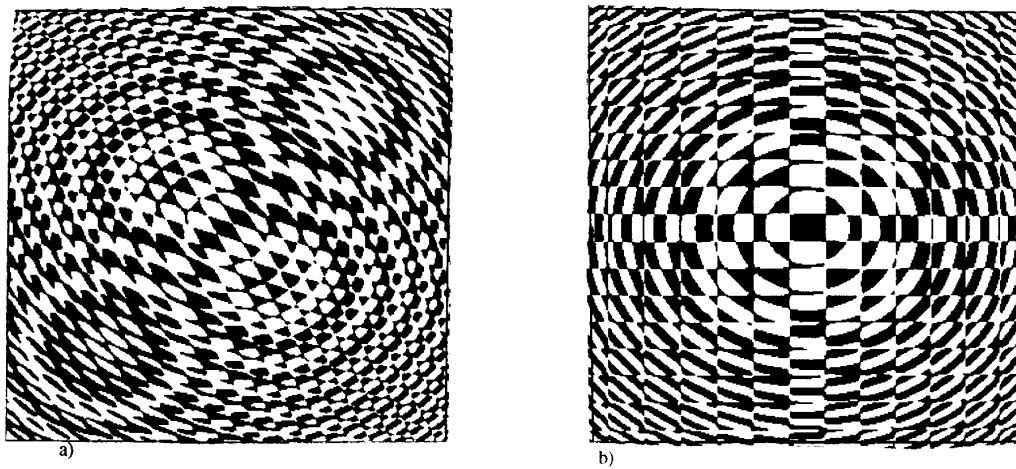


Figure 8: Part of the hologram patterns for the 2x2 array generators  
a) rhombic  
b) square

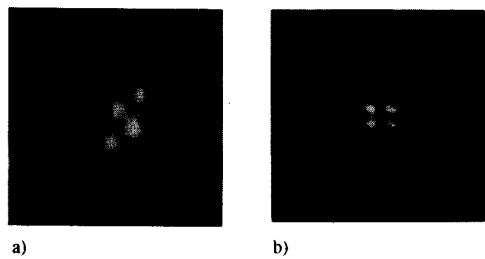


Figure 9: Output of 2x2 array generators  
a) rhombic  
b) square



Figure 10: Holographically generated perfect shuffle