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OPTIMAL UNEMPLOYMENT INSURANCE WITH HUMAN CAPITAL DEPRECIATION AND DURATION DEPENDENCE

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Optimal Unemployment Insurance, with Human Capital Depreciation, and Duration Dependence.

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Abstract

This paper studies the effect of human capital depreciation and duration dependence on the design of an optimal unemployment insurance (UI) scheme. Our results partially confirm those obtained in most previous studies: benefits should decrease with unemployment duration. The optimal program also generates two main novel features, which are not present in stationary models. First, if human capital depreciates rapidly enough during unemployment, UI transfers are bounded below by a minimal "assistance" level that arises endogenously in the efficient program. Second, we study the optimality of imposing a history contingent wage tax after reemployment. Our numerical simulations based on the Spanish and US economies show that the wage tax should decrease with the length of worker's previous unemployment spell, and become a *wage subsidy* for long-term unemployed workers. As a by-product of our study, we develop a systematic approach suitable for studying recursively a wide range of dynamic moral-hazard problems, and other models with similar characteristics.

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1 Introduction

Unemployment insurance (UI) programs are an important ingredient of social welfare policies in developed economies. These programs have been widely criticized because of the adverse effects they can have on worker's incentives to search for a new job. This criticism has stimulated extensive research into optimal insurance schemes that take these perverse effects into account.

A series of papers use the dynamic moral hazard model to analyze the trade-off between (unemployment) insurance and (search) incentives. In their seminal work on UI, Shavell and Weiss (1979) establish that, because of moral hazard, benefits must decrease throughout the unemployment spell and approach zero in the limit. Hopenhayn and Nicolini (1997a,b) extend the analysis of Shavell and Weiss by introducing the possibility of contingent wage taxes after reemployment, and they confirm the decreasing benefits result of Shavell and Weiss. In addition, the analysis of Hopenhayn and Nicolini suggests that up to a 30% of overall spending in unemployment compensations could be saved by introducing a tax on the wage the agent receives after reemployment that increases with the length of the previous unemployment spell. In Pavoni (2003), I notice that the optimal schemes based on dynamic moral hazard such as the one proposed by Hopenhayn and Nicolini implicitly assumes that the planner can (and will) inflict infinite punishments on workers.[1] To design an implementable scheme, I consider the possibility that the planner must respect an exogenously given lower bound on the expected discounted utility that the agent can have ex-post, regardless of the previous history. In this case, the optimal contract presents characteristics quite similar to existing unemployment compensation schemes. Among other properties, the optimal wage tax is roughly constant during unemployment.

In all these models, the key features of the environment are that the probability of finding a new job depends *only* on the (unobservable) search effort exerted by the agent, and that the available gross wages distribution is constant throughout unemployment spells.

It is well documented that job opportunities deteriorate during unemployment.[2] In fact,

[1] The key properties of the standard stationary model are reproduced in Proposition 1.

[2] We will extensively revise the literature on wage depreciation, job displacement and duration dependence below in this section. However, some of our calibrations are based on Keane and Wolpin (1997) who use NLSY data and estimated, structurally, an annual human capital depreciation rate for white US males of between 9.6% (for blue collars) and 36.5% (for white collars). In addition, many authors consistently find that displaced US workers face a large and persistent earning loss upon reemployment in the order of $10-25\%$ compared with continuously employed workers (Jacobson el al., 1993; Ruhm, 1987; and Bartel and Borjas, 1981). Finally, Van den Berg and van Ours (1996), after controlling for unobservable heterogeneity, find that the US white male exit probability “genuinely” decreases by 30% after 3 months of unemployment.

many OECD countries propose and apply active labor market policies and wage subsidies for long term unemployed people, mainly because of this adverse change in job opportunities.[3] We thus believe that human capital depreciation and unemployment duration dependence are important elements, that need to be included in the study of an optimal UI designing problem. In this paper, we extend the basic model of unemployment insurance with moral hazard to allow for both the gross wages and the probabilities of reemployment to depend on the length of the worker's unemployment spell. The focus of the present paper is on the design of an optimal unemployment insurance scheme. In particular, we do not study how this designing problem interacts with other welfare and/or labor market policies. However, we do allow the planner to impose history contingent wage taxes or pay wage subsidies upon reemployment.

Our study confirms one important result of most previous studies with stationary models: benefits should decrease with unemployment duration. In fact, this behavior characterizes virtually all existing UI schemes in OECD countries. Moreover, we show that very simple schemes, defined by a low constant benefit payment b and a higher time-invariant wage w, can never be optimal. This result holds for any reasonable range of parameters (no restrictions on the worker's utility function, but concavity, are required), and regardless of the characteristics of the human capital depreciation process. In exchange, we propose a simple necessary characteristic of any optimal unemployment insurance program. A "back of the envelope" check of optimality.

The introduction of human capital depreciation and duration dependence also generates two main novel features in the optimal program. First, provided that human capital depreciates sufficiently rapidly during unemployment, the optimal path for unemployment benefit payments is initially decreasing and then becomes completely flat. The idea is that, for low levels of human capital, the planner does not find worthwhile to induce the agent to supply the high effort level. Unemployment benefits eventually stop decreasing and remain constant forever since the (long-term) unemployed worker is fully insured. This feature of the optimal contract generates an *endogenous lower bound* on worker's expected discounted utility, which provides an alternative way of eliminating the "immiserization" result of Thomas and Worral (1990).[4] This characteristic creates an important link between the characteristics

[3] Although the programmes differ substantially, there are mainly eight OECD countries that have actually introduced major welfare-to-work programmes: the United States, Canada, the United Kingdom, Ireland, Denmark, France, the Nederlands and Sweden. In several others countries, such programmes are under careful consideration.

[4] In the literature, the immiserization result is usually eliminated in stationary models by *exogenously* imposing minimum bounds on expected discounted utility the worker can have ex-post, in the optimal scheme

of the optimal unemployment insurance scheme and the speed of skills depreciation in the economy, which can also be used for positive analysis.

Second, in our model the planner can impose history-contingent wage taxes after reemployment. Although we do not find any main specific qualitative characteristic, the results of our numerical exercises for the Spanish and the US economies show that in an optimal UI scheme the level of wages taxes *decreases* with the length of worker's previous unemployment spell and become a *wage subsidy* for long-term unemployed workers. Recall that the key result of Hopenhayn and Nicolini (1997a) is that because of incentives, in a stationary model, the wage tax should increase during unemployment. In our models this mechanism is at work as well, but there are at least three other reasons that contrast this effect. First, since the planner tends to insure the agent against gross wage depreciation, in absence of incentives the wage tax would decrease during unemployment. Second, a reduction (depreciation) in human capital reduces the effectiveness of the search activity, hence increases incentive costs. This effect tends to widen the difference between unemployment benefits and net wage, decreasing both the UI transfers and the wage tax upon reemployment. These characteristics of an optimal scheme are reinforced by a third effect, which is strictly linked with the exogenous minimum bound analysis of Pavoni (2003). The (now endogenous) presence of a lower bound on worker's expected discounted utility shorthands the effective time-horizon of the problem. This reduces the possibility of giving dynamic incentives, and forces the planner to design a scheme biased toward the "static" component of the incentives. The planner is hence induced to increase the within-period gap between the unemployment insurance benefit and the net wage, further reducing the wage tax level after reemployment.

The optimality of a wage subsidy is a key policy implication of our non stationary search model with moral hazard. At the end of the papers we argue that our results also suggest, implicitly, the adoption of some additional policy measures, especially for long-term unemployed workers. We will discuss how an extended version of this model can be used to study the optimal adoption of more composed labor market programs.

The problem analyzed in this paper required to solve recursively, in a possibly non-stationary framework, the dynamic hidden-action moral hazard problem. This is known to be a non easy task. We find that the value function associated to this recursive problem is, in general, non-concave and non-differentiable. In spite of these non-smoothness problems, we show that the optimal contract can be characterized using the usual first order conditions. To derive the properties of the associated value function we develop a new approach that

(see, for example, Pavoni, 2003; Atkeson and Lucas, 1995; Phelan, 1995). Other authors use overlapping generation models (see, for example Phelan 1994).

uses the generalized envelope theorem of Daskin (1967) and Milgrom and Segal (2002), and which can be extended both to an even more general class of moral-hazard problems and to other models with similar characteristics.

Literature on recursive contracts Our methodology follows the recursive contract literature, and the references most related to our approach are: Abreu, Pearce and Stacchetti (1990), Fudenberg et al. (1990-1994), Spear and Srivastava (1988), Phelan and Townsend (1991) and Atkeson and Lucas (1992). In this paper, we formally study the shape of the value function associated with the dynamic moral hazard model. The approach we propose partially builds on that used by Grossman and Hart (1983) to study the static problem. In their seminal work, Spear and Srivastava (1988) discuss conditions under which the value function of the dynamic model with a continuum of outcome realizations, is concave. Our approach permits to study the case with a finite number of output realizations, which is an inherent characteristic of the unemployment insurance designing problem studied in this paper. Our non-concavity result contrasts with that of Spear and Srivastava. Finally, Phelan and Townsend (1991) allow for lotteries over effort and payments, which imply the (weak) concavity of the value function. As in most applied studies, we focus on the deterministic payments case.

Literature on optimal unemployment insurance The literature on optimal UI is relatively new, yet quite extended. However, most of the papers address questions and/or use approaches that cannot be directly related to our own. The interested reader can refer to the recent summary of Karni (1999). Atkeson and Lucas (1995) use a recursive approach to characterize the optimal contract in a stationary pure adverse-selection setup with temporary (one-period) job offers. They are mainly interested in income distribution, and their approach is closely related to that in Hansen and Imrohoroglu (1992) and Wang and Williamson (2002), where the goal is to quantify the welfare effects of unemployment insurance in general equilibrium. Wang and Williamson (1996) provide calibrated repeated OLG model with moral hazard associated with search effort and job retention. They assume that each new labor-force entrant obtains a prespecified level of ex ante utility and obtain a non monotone unemployment benefits behavior. Hopenhayn and Nicolini (2002) analyze the effects of worker's employment history on the optimal design of unemployment insurance contracts, and Zhao (2001) introduces moral hazard also associated with job retention. Finally, Usami (1983) proposes a finite-horizon model with moral hazard, where the probability of reemployment conditional on search depends on the previous *employment* history. Usami confirms the aforementioned decreasing benefits result, and finds that the worker compensation should be non-decreasing during the employment period. Although our model permits employment history dependence, most of our findings are induced by skill depreciation during *unemployment.* Moreover, Usami studies the problem choosing an "inconvenient" state variable, which prevents a complete analysis. Our recursive formulation results in a manageable value function, which allows us to characterize in detail the optimal scheme, both qualitatively and quantitatively.

Literature on human capital depreciation, job displacement and duration dependence We have already mentioned above that Keane and Wolpin (1997) find important wage depreciation rates during unemployment. In recent years, a great deal of attention has also been paid to the consequences of *worker displacement.* Displacement is usually defined as the involuntary separation of workers from their jobs without cause (i.e. for economic reasons) and without future recall. Research on the effects of worker displacement has grown dramatically in recent years, especially in the in the United States (for surveys, see Hamermesh, 1989; Faber, 1993, 1997; Hall, 1995; Fallick, 1996; and Kletzer, 1998). Using a variety of methods and data sets, the findings are remarkably consistent. Displaced workers face large and persistent earnings losses upon reemployment in the order of $10-25\%$ compared with continuously employed workers (Jacobson el al., 1993; Ruhm, 1987; and Bartel and Borjas, 1981).[5] Evidence for European labor markets is contrasting and not always comparable. For example, recently Burda and Mertens (2001) use self-reported information on job-displacement and estimate an average wage growth reduction of approximately 3.6%, with a peak of 17% (for high pay jobs) for Germany. Lefranc (2002) use micro data from labour force surveys and find wage losses upon displacement in the order of $10-15\%$. In contrast, Bender et al. (1999) use information on plant closing to identify displacement, and estimated near zero losses both in France and Germany.[6]

A number of labor economists interpret wage depreciation as reflecting the destruction of firm-specific or industry-specific human capital associated with tenure. Other authors pointed out that non-observable individual heterogeneity may bias estimated tenures upwards, so that previous tenure might have a positive effect on post-displacement wage rates (see, for example Kletzer, 1989). Another interpretation is simply the destruction of rents associated with good matches, with no return to tenure per se (Mincer and Jovanovic, 1981; Altonji and Shakotko, 1987; Abraham and Faber, 1987; Ruhm, 1990; Altonji and Williams,

[5] In addition to earning losses, displaced workers experience more unemployment than non displaced workers (see - for example - Hall, 1995; Ruhm, 1991; Swain and Podgursky, 1991).

[6] Leonard and van Audenrode (1995) find results similar to the ones of Bender et al. for Belgium, and so does Ackum (1991) for Sweden.

1992).

One of the distinctive features of many current European labor markets is the high proportion of workers that remain unemployed for a long period of time. This feature of the European labor market is widely regarded as a serious problem and has attracted a lot of attention both for efficiency and equity reasons. The *negative duration dependence* in the exit rate from unemployment is felt as one of the major causes for long-term unemployment.

In the literature, labor economists suggest different justifications for the observed negative duration dependence in the exit probability from unemployment. A first stream of research supports a purely statistical explanation for this phenomenon. It is argued that the hazard rate estimations are affected by an unobservable heterogeneity effect, which induces a fictitious negative duration dependence in the estimated hazard rates. The idea is that aggregate hazard rates decrease since the sample characteristics are changing through unemployment. In particular, the pool of long-term unemployed, is dominated by the share of people whose individual hazard rates are lower than the average.

However, a number of studies document that, even after controlling for unobservable heterogeneity, a non negligible negative duration dependence still remains. For the US, van den Berg and van Ours (1996) use CPS data and find that white male exit probability "genuinely" decreases by 30% after 3 months of unemployment. In Europe, van den Berg and van Ours (1999) find that for French young workers the exit probability decreases by $30 - 35\%$ after 2.5 years of unemployment. For the UK, Nickel (1979) find a 50% decrease after 60 weeks of unemployment. More recently, van den Berg and van Ours (1994) find that the British male exit rate of unemployment decreases by 20% after one quarter and by more than 30% after 6 months of unemployment (for the UK see also Lynch, 1985; for Spain see Bover et al., 1997).[7]

A second stream of research attempts to give a theoretical explanation for the mentioned "true" negative duration of the hazard rate. It is indeed empirically documented that during unemployment there is a negative time dependence in the arrival rate of job-opportunities. Heckman and Borjas (1980) call this phenomenon "occurrence dependence". The analysis of the genuine duration dependence can be divided in two categories.

On one hand, we have what we might call *supply side* explanations. A long-term unemployed worker finds it more difficult to know the existence of vacancies, either because the worker loses valuable social contacts,[8] or because the long-term unemployed worker suffers some sort of stigmatization by the other workers in the supply side of the market (Gregg and

[7]In other European countries the result are again somehow contrasting. For a summary on a number of OECD countries, see Machin and Manning (1999).

[8]See, for example, Calvó-Armengol (2003) for a theoretical analysis of this aspects of the labor market.

Wadsworth, 1996). The stock-flow approach to search of Coles and Smith (1994) and Gregg and Petrongolo (1997) gives another supply side explanation for the negative duration dependence phenomenon. The idea is that the observed occurrence dependence is in fact only apparent. What really happens is that, at the beginning of unemployment the worker faces and evaluates a stock of vacancies while, in latter periods of unemployment, only the newly open vacancies are processed. Finally, a third motivation is that the hazard rate decreases during unemployment simply because the worker reduces his search effort level. This may be due to a sort of discouraged worker effect that may induce long term unemployed to remain in the labor market, but actually looking almost passively for new jobs,[9] or because skills and work habits atrophy during unemployment (Sinfield, 1981).

We then have a few *demand side* - or firm-hiring behavior - explanations for the negative duration dependence. Recently, a lot of attention has been devoted to study the so called "stigma effect". The idea behind this approach is the following. It is assumed that firms imperfectly test workers prior to hiring them. If some firms hire only workers who pass the test, then there is an informational externality; unemployment duration is a signal of workers' productivity and firms tend to avoid to even test for hiring long-term unemployed workers (Vishwanath, 1989; Lockwood, 1991; Belzil, 1995; and Yoshiaki, 1997). Finally, Blanchard and Diamond (1994) propose a "ranking" model, where it is simply assumed that, independently from any payoff-relevant motivation, firms hire the workers with the lowest unemployment duration levels.

Outline of the paper In Section 2, we present the dynamic moral-hazard model with human capital depreciation. In Section 3, we describe our approach to characterize the optimal contract. In Section 4, we calibrate the model with the US and Spanish economies, and simulate the optimal scheme. Section 5 concludes and consider future extensions.

2 Model

The model is the natural extension of the framework proposed by Hopenhayn and Nicolini (1997a,b) to study the design of and optimal UI program. Consider a risk-neutral planner who must design an optimal unemployment compensation scheme for a risk-averse worker. In any given period, the worker has time invariant preferences of the following separable

[9] Think for example of a worker who is learning about his ability to find a new job. A long period of unemployment will obviously imply a downward biased perception of his hazard rate.

form

$$u(c) - v(a)$$

where c is consumption and a is search effort. We assume $u(\cdot)$ to be strictly increasing, strictly concave and continuously differentiable, with inverse u^{-1} bounded.[10]

In any period, the worker can be either employed (e) or unemployed (u). If the worker is employed, he earns a gross wage $S(h)$ which is assumed to be an increasing and bounded function of the worker's human capital endowment h. Moreover, we assume that h follows the following stochastic law of motion:

$$h' = m_z(h),\ z = u, e; \quad \text{with } m_u(h) \leq h \leq m_e(h); \quad \text{and } m_z(\cdot) \text{ continuous,} \tag{1}$$

where h' is the next period human capital level and z is the worker's employment state. The idea is that during unemployment $(z = u)$ h depreciates, whereas during employment there can be human capital accumulation due, for example, to on-the-job training.[11]

The timing of the model in the unemployment state is reported in Figure 1. While unemployed, the worker can either search ($a = 1$) or non-search ($a = 0$) for a new job, i.e. $a \in A = \{0, 1\}$. The search activity is costly, hence $v(1) = v > v(0) = 0$. The search effort a affects the transition probability between employment states, according to a hazard rate function $\pi(a, h)$, with $\pi(1, h) \equiv \pi(h) \geq 0$ increasing with h. When the worker does not search, the job finding probability is zero, i.e. $\pi(0, h) = 0$ for any h. The situation where the planner requires the agent to stop searching for a job can be interpreted as the one of "social assistance" or "early retirement." To simplify the analysis we assume that this state is an absorbing one.[12]

The crucial assumption of the model that we keep throughout the paper is that the planner cannot observe the worker's search effort a. Thus, during unemployment there is a moral-hazard problem. This means that unemployment benefits are not paid only as insurance, but must also play the role of giving incentives for search. We assume there are no informational problems related with h.[13]

[10] This latter assumption is merely a technical one. It allow us to simplify considerably the proof of Proposition 3. In Pavoni and Violante (2003) we show that it can easily be relaxed.

[11] In order to have a simple tractable recursive formulation, we call "human capital" the aggregate variable h, which affects both available gross wages and reemployment probabilities. However, the (state) variable h can well be multidimensional, capturing - for example - different levels of specificity on human capital. In fact, the nature of h should be considered more broadly than mere skill or ability.

[12] In the numerical exercise, we allow for a non absorbing social assistance state with $\pi(0, h) > 0$.

[13] Note that, since the laws m_z are known, and the realized employment states z are perfectly observable, it suffices to assume that the planner knows the initial endowment h_0.

Following the recursive contracts literature, we characterize the contract using the following recursive formulation. We consider first the unemployment state case. Let U and h be the discounted utility promised to the agent in period t, and its human capital endowment respectively. Given (U, h), the planner's value function in the unemployment state V is defined as follows:[14]

$$V(U,h) = \max_{a\in\{0,1\}} \{V_a(U,h)\}. \tag{2}$$

The function V_1 describes the planner's value in the case where the agent is required to actively search for a job, and solves

$$\begin{aligned} V_1(U,h) &= \sup_{b,U^u,U^e} -b+\beta\left[\pi(h)W(U^e,h')+(1-\pi(h))V(U^u,h')\right] && (3)\\ \text{s.t.} &: \\ U &= u(b)-v+\beta\left[(1-\pi(h))U^u+\pi(h)U^e\right], && (4)\\ U &\geq u(b)+\beta U^u, \text{ and} && (5)\\ h' &= m_u(h). \end{aligned}$$

Equation (4) requires the contract to deliver the promised level of discounted utility to the worker, and is called the promise keeping constraint and plays the role of law of motion for the state variable U. Constraint (5) is the incentive compatibility constraint ensuring the agent is willing to deliver the amount of effort called for in the contract. When the worker is required to not search ($a = 0$), the planner's value is

$$\begin{aligned} V_0(U,h) &= V_0(U) = \max_{b,U^u} -b+\beta V_0(U^u) && (6)\\ s.t. &: \\ U &= u(b)+\beta U^u. \end{aligned}$$

In the "assistance" state the planner pays the worker a constant benefit transfer equal to $b = u^{-1}\left((1-\beta)U\right)$ forever. Hence $V_0(U) = -\frac{u^{-1}((1-\beta)U)}{1-\beta}$.

The function $W(U, h)$ denotes the planner's net return in the employment state when the worker is required to receive a lifetime utility level of U, and is endowed with human capital stock h. During this state, the planner satisfies the promise keeping restriction

$$U = u(w) - l + \beta U^e \tag{7}$$

by transferring a net wage w, after imposing the tax (or paying the subsidy) $\tau \equiv S(h) - w$ on the gross wage. We denote $l \geq 0$ the effort cost of working. In the model jobs are permanent,

[14] The starting value U_0 will be given by the time-zero participation constraint, and may depend on the initial level of human capital endowment h_0.

and we assume that there are no incentive problems in the employment state. It is hence easy to see that while employed the worker is fully insured, and that - by using (7) - the planner's value is

$$W(U,h) = \frac{S_e(h) - w(U)}{1-\beta} = \frac{S_e(h) - u^{-1}((1-\beta)U + l)}{1-\beta}, \tag{8}$$

where $S_e(h)$ is the average discounted gross wage, and it is given by the gross wage sequence induced by the wage function $S(\cdot)$ and the accumulation law m_e.[15] The properties of u and S, guarantee that W is bounded, strictly decreasing, strictly concave and continuously differentiable in U.

2.1 The Stationary Benchmark

In a stationary model $\pi(h) \equiv \pi > 0$ and $S(h) \equiv S > 0$ do not depend on the level of human capital h. Similarly to the general case, the value of unemployment V is defined as

$$V(U) = \max_{a \in \{0,1\}} \left\{ -\frac{u^{-1}((1-\beta)U)}{1-\beta},\ V_1(U) \right\} \tag{9}$$

where V_1 is the stationary analogous of (3)-(5), and $W(U) = \frac{S - u^{-1}((1-\beta)U)}{1-\beta}$.

Proposition 1 *Assume that in period $t = 0$ the planner decides to implement $a_0 = 1$. Then* (i) *the agent will never be required to stop searching;* (ii) *both U_t, b_t and w_t are strictly decreasing during unemployment; and* (iii) *if u is unbounded below then for any initial U_0 and arbitrarily low level of utility $\underline{U}$, there exists a finite unemployment spell duration T, such that $U_T < \underline{U}$.*

All proofs are reported in Appendix A.

When U_0 is very large, even compensating the agent for the search effort cost becomes too costly, hence the planner suggests $a_0 = 0$ (social assistance). For more moderate levels of initial lifetime utility, in a stationary model there is no role for social assistance: the worker is always required to search for a job, and the level and UI benefit transfers never stop decreasing. In addition, the reemployment wage tax $\tau_t = S - w_t$ increases with unemployment duration. This is the key results of Hopenhayn and Nicolini (1997a).

[15] S_e can be computed recursively as follows

$$S_e(h) = (1-\beta)S(h) + \beta S_e(m_e(h)).$$

Proposition 1 *(ii)* also emphasizes a typical feature of dynamic models with information asymmetries. In order to spread out incentive costs, the planner reduces the agent's expected discounted utility U_t through time. This property, always true in stationary models, have sometimes unpleasant consequences. Point *(iii)* shows that the optimal contract implied by repeated moral hazard models creates a weaker form of the "immiserization result": efficiency requires that worker's expected discounted utility falls, with positive probability, below any arbitrary negative level. The infinite punishments result is questionable in some circumstances. For example, it may be impossible for the planner to enforce, ex-post, such punitive plans because these would imply excessive social conflict costs. Similarly, excessive punishments may induce the worker to opt out of the insurance scheme. To design an implementable UI scheme, one may consider the possibility that the planner must respect a lower bound on the expected discounted utility that the agent can have ex-post. In Pavoni (2003) I find that in the stationary case with exogenous utility bounds, UI benefits should stop decreasing after finitely many periods, and that reemployment wage taxes are typically constant. Below we show that when human capital depreciates rapidly enough the optimal scheme generates an endogenous lower bound on payments, hence on lifetime utilities.

In Figure 3, we report a parametrized example of the closed form of $V(U)$ derived in Pavoni (2002) for the case where the worker has logarithmic utility. The value function $V(U)$ is represented by the (solid line) upper envelope of the two functions. The flatter dotted line represents $V_0(U)$: the planner's return in case he decides to fully insures the worker. The steeper dotted line corresponds to $V_1(U)$, the case where the planner decides to ask the worker to always search for a job. The crossing point between the two curves is where the planner "switches" between the two regimes. From the picture it is clear that at the switching point the value function $V(U)$ is neither concave nor differentiable. We will see that this is a typical feature of the problem.

2.2 The Emergence of Endogenous Lower Bounds: A Closed Form Example

We now take advantage of the closed form derived in Pavoni (2002) and show sufficient conditions on the speed of human capital depreciation that guarantee the emergence of a lower bound on worker's lifetime utility. When the utility of the agent takes the logarithmic form, i.e. $u(c) = \ln(c)$, it can be shown that the following solution to the functional equation (9) is the "true" value function:

$$V_1(U;S) = \frac{SK}{1-\beta} - \frac{B\exp\{(1-\beta)U\}}{1-\beta} \text{ if } U \leq M, \tag{10}$$

$$\begin{aligned} V_0(U) &= -\frac{\exp\{(1-\beta)U\}}{1-\beta} \qquad \text{if } U \geq M; \text{ and} &(11)\\ W(U;S) &= \frac{S}{1-\beta} - \frac{\exp\{l\}\exp\{(1-\beta)U\}}{1-\beta} \text{ for all } U; &(12)\end{aligned}$$

where $M = \frac{\ln\left(\frac{KS}{B-1}\right)}{1-\beta}$, $K = \frac{\beta\pi}{1-(1-\pi)\beta}$ and B solves[16]

$$\ln\left[B^{\frac{1}{\beta}} - (1-\pi)B\right] = \ln\pi + \frac{1-\beta}{\beta}\frac{v}{\pi} + l. \tag{13}$$

During unemployment, lifetime utility decreases according to:[17]

$$U^u(U) = U - \frac{\ln B}{\beta}$$

Since $\beta < 1$ (13) implies that $\frac{1-\beta}{\beta}\frac{v}{\pi} + l$ constitutes an upper bound for $U - U^u(U)$.

Now assume that during unemployment S in fact depreciates at a rate δ, i.e. $S_{t+1} = (1-\delta)\,S_t$. For simplicity, assume that during employment S_t remains constant. It is easy to see that if $a_t = 1$ for any t as it would be the case for the stationary model, the planner would insure the agent against gross wage depreciation and the ratio between $V_1(U_t;\ S_t)$ and $V_0(U_t)$ would satisfy

$$\frac{V_1(U_t;\ S_t)}{V_0(U_t)} \leq \frac{S_0K\left(\frac{1-\delta}{1-\gamma}\right)^t - B\exp\{(1-\beta)U_0\}}{-\exp\{(1-\beta)U_0\}} \tag{14}$$

where $1-\gamma = \exp\left\{-(1-\beta)\left[\frac{1-\beta}{\beta}\frac{v}{\pi} + l\right]\right\}$. Notice that when $(1-\gamma) \geq (1-\delta)$, as $t \to \infty$ the right hand side of (14) tends to $-B < -1$. As a consequence, the conjecture that $a_t = 1$ for any t is false. The program must have an endogenous lower bound on U, since by continuity there must exist a $T < \infty$ such that $V_0(U_T) \geq V_1(U_T;\ S_T)$. Hence, we have the following:

Proposition 2 *Let δ the depreciation rate of S. If $(1-\delta) \leq \exp\left\{-(1-\beta)\left[\frac{1-\beta}{\beta}\frac{v}{\pi} + l\right]\right\}$, the scheme generates an endogenous lower bound on lifetime utilities, hence on payments.*

[16] It can be shown that $B > 1$.

[17] We also have

$$\begin{aligned} U^e(U) &= U + \frac{\frac{v}{\pi} - \ln B}{\beta}. \\ u\left(b(U)\right) &= (1-\beta)U + \ln B \end{aligned}$$

Moreover, the employment state policy is simply a constant utility for each period, guaranteed by a within-period net wage equal to

$$u\left(w(U)\right) = (1-\beta)U + l.$$

If we normalize $S_0 = 100$, a reasonable value for the effort and work costs v and l would be for example $v = l = 1$.[18] Moreover, the value of 0.08 constitutes a lower bound for the US weakly hazard rate π.[19] Easy numerical computations, after setting $\beta = 0.999$, suggest that a weakly human capital depreciation rate of 0.1% would guarantee the emergence of lower bounds. And according to Keane and Wolpin (1997), the US weakly value for δ varies between 0.2 and 0.8%.

3 Qualitative Analysis

It is well known that hidden-action moral-hazard models do not typically describe concave problems (Grossman and Hart, 1983, and Phelan and Townsend, 1991). There are four main reasons why this may prove to be problematic, especially in a dynamic environment. First of all, non concavity might also lead to the non differentiability of the problem. Second, even assuming differentiability, first-order conditions may not longer be sufficient for local maxima. Third, the analysis might become more complicate since we are actually looking for a global maximum. Finally, and more importantly, the usual envelope theorems[20] cannot be applied, and this may reduce considerably the usefulness of our recursive formulation. In this section we develop a systematically recursive approach that allows for these complications.

First, we somehow confirm the above mentioned difficulties. We find that "in most cases" the associated value function is neither concave nor differentiable. We however find also a positive result: the optimal contract can still be characterized to a great extent by using the familiar first order conditions.

The idea of our approach is as follows. The complication involved by the recursive study of the dynamic moral-hazard problem comes from the incentive constraint. This prevents a direct approach to the study of the concavity and differentiability of the value function V. We thus first reformulate the problem to make it suitable for such analysis. We define a collection of concave and continuously differentiable functions (*the conditional functions*), of which the value function V is the upper envelope. We then apply the extended envelope theorem of Daskin (1967) to this problem to show that V is almost everywhere differentiable.[21]

[18] Since $\ln S_0 = \ln 100 = 4.6$, an effort cost of 1 is between one fourth and one fifth of this value.

[19] According to Meyer (1990) for example, $\pi = 0.1$.

[20] For envelope theorems we refer to theorems that describe conditions under which the value function of a parametrized optimization problem is a differentiable function of parameter.

[21] In the economic literature, this result was rediscovered by Kim (1993), Sah and Zhao (1998), Milgrom (1999), and recently extended by Milgrom and Segal (2002). However, to our best knowledge, none of these papers mention that the Daskin's result can be applied to solve the dynamic moral hazard problem. We

Our successive step is to study the "switching points," that is, the utility levels at which the upper envelope function V switches between two different conditional functions of the above mentioned class. Those points are indeed the only problematic ones. However, given the characteristics of our class of functions, each switching point possesses a very nice characteristic: either the V function is in fact differentiable at this point, or the point is never reached in equilibrium. The fact that the points of non-differentiability cannot be reached in equilibrium allows us to disregard them while characterizing the optimal contract.

3.1 The sequence of efforts formulation and the existence result

Consider the space $\mathcal{A}$ of all the sequences of efforts $\mathbf{a} = \{a(n)\}_{n=0}^{\infty}$ $a(n) \in A$, implementable during unemployment. For any human capital endowment $h \in \mathcal{H}$, effort sequence $\mathbf{a} \in \mathcal{A}$ and utility level $U \in \mathcal{U} \subset \mathrm{I\!R}$ we can define

$$\begin{aligned} V(\mathbf{a}, U, h) &= \sup_{b, U^e, U^u} -b + \beta\left[\pi(a,h)W(U^e, h') + (1 - \pi(a,h))V({}_1\mathbf{a}, U^u, h')\right] \\ & \quad \text{s.t. (1), and} \\ \text{if } a &= 1, \text{ (4) and (5);} \quad \text{if } a = 0,\ U = u(b) + \beta U^u. \end{aligned} \tag{15}$$

The function(al) $V(\mathbf{a}, U, h)$ represents the planner's optimal payoffs *conditional on* a given sequence of efforts, when the worker is unemployed. The symbol ${}_1\mathbf{a} = \{a(n)\}_{n=1}^{\infty}$ stands for the one step ahead continuation of $\mathbf{a}$.

It would be easy to show that the value function of the sequential problem satisfies (15).[22] In the next Proposition we show that also the converse is true.

Proposition 3 *The Bellman operator implied by (15) defines a contraction in the space of continuous and bounded functions with the sup norm. Thus V exists and is unique, and*

$$\|V\|_{\infty} = \sup_{y \in Y = \mathcal{A} \times \mathcal{U} \times \mathcal{H}} |V(y)| < \infty.$$

In the next Proposition, we show that each conditional function $V(\mathbf{a}, U, h)$ has nice properties, very similar to the ones of the employment state value function W.

Proposition 4 *Consider a sequence of efforts $\mathbf{a} \in \mathcal{A}$ and an endowment level h, together with a law m. Let $V(\mathbf{a}, U_0, h) = -b_0 + \beta\left[\pi(a_0,h)W(U_0^e, h') + (1 - \pi(a_0,h))V({}_1\mathbf{a}, U_0^u, h')\right]$ with*

also show that in dynamic models the characteristics of the value function permits the use of the first order conditions.

[22] The technical reader should be reassured by the fact that we do not have problems of measurability, since we have only two outcomes.

U_0 in the interior of the effective domain of $V(\mathbf{a},\cdot,h)$, and with b_0 belongs to the interior of the domain of the agent's utility function u. The conditional function $V(\mathbf{a},\cdot,h)$ is concave and continuously differentiable in U at any such U_0, and

$$V'(\mathbf{a},U_0,h) \equiv \frac{\partial V(\mathbf{a},U_0,h)}{\partial U} = -\frac{1}{u'(b_0)} < 0 \tag{16}$$

The problem (15) of implementing optimally (minimizing costs) a *given* sequence of efforts $\mathbf{a}$ is concave, with linear constraints. The associate value function $V(\mathbf{a},U,h)$ is thus concave as well. Given concavity, differentiability can be shown by applying the Benveniste and Scheinkman (1979) Lemma in the standard way.

We finish this subsection with an important result. The maximization with respect to $\mathbf{a}$ is always well defined. Hence, the value function $V(U,h)$ defined can be written as the upper envelope of the collections of conditional functions $V(\mathbf{a},U,h)$.

Proposition 5 *The set $\mathcal{A}$ of sequences of efforts is compact and $V(\mathbf{a},U,h)$ is continuous in $\mathcal{A}$ for any (U,h). Thus, a maximum exists for any (U,h), and we can define*

$$V(U,h) = \max_{\mathbf{a}\in\mathcal{A}} V(\mathbf{a},U,h). \tag{17}$$

3.2 The shape of the value function

Proposition 5 defines $V(U,h)$ as the upper envelope of the collection of the conditional functions $V(\mathbf{a},U,h)$. The approach we propose exploits this interpretation for $V(U,h)$. However, we must first eliminate the possibility of weird behaviors at the infinite, due to the non-stationarity of the problem. We decided to make the following regularity assumption.

> **Assumption A1** For any endowment h, there is a time horizon $T(h) < \infty$ such that $\forall\, t \geq T(h)$ both $S(m^t(0,h)) = \underline{S}$, and $\pi(m^t(0,h)) = \underline{\pi}$.

Assumption **A1** can be seen as a restriction on the law m, or on the functions $S(\cdot)$ and $\pi(\cdot)$, or on both. For example, the condition on S can be interpreted as a minimum wage condition. It should be noted that, the above conditions allow $T(h)$ to be arbitrarily large (provided that it remains finite).[23]

[23] Proposition 6 is based on the Daskin's envelope theorem. In order to apply this result to our problem we need to guarantee that the derivative $V'(\mathbf{a},U,h) \equiv \frac{\partial V(\mathbf{a},U,h)}{\partial U}$ is jointly continuous in $(\mathbf{a},U)$. Hence, the main task is to show that this assumption is true for our model. Our line of proof is quite simple. Notice that any finite periods version of our model satisfies the joint continuity requirement of the partial derivative $V'(\mathbf{a},U,h)$. The reason is simple. If the time horizon is finite, then the set of all possible paths of efforts $\mathcal{A}$ is

Proposition 6 *Consider a pair U, h, with U as in Proposition 4 and assume that* ***A1*** *is satisfied. Then $V(U,h)$ possesses both right and left derivative in the first argument, with $V_+(U,h) \geq V_-(U,h)$. Moreover, $V(\cdot,h)$ is almost everywhere differentiable for any h, and where it is differentiable we have*

$$V'(U,h) = V'(\mathbf{a},U,h) \text{ for any } \mathbf{a} \in \mathcal{A}^*(U,h), \tag{18}$$

where $\mathcal{A}^*(U,h)$ is the (non-empty) set of efforts solving the maximization (17) defined in Proposition 5.

Notice that from (16) and (18), when V is differentiable, we can recover the usual envelope theorem. This property will be used below.

3.3 Characterization of the optimal contract

Notice that the property $V'_+(U,h) \geq V'_-(U,h)$ is not a characteristic of concave functions. In fact, when the directional derivatives differ, $V(U,h)$ cannot be concave in any interval containing U. This confirms the analysis of the stationary model (see Figure 3 and the closed form (10)-(11)). However, notice that the "kink" at the switching point in Figure 3 has a particular nature: it is an "inward" one. This is a good news since a simple graphical check should convince the reader that the optimal choice of the continuation utility U^u will never be at the switching point. It turns out that this is true in general for our problem, which implies that problem (3) is differentiable at all "relevant" points. But then the usual first-order conditions, although obviously non sufficient, become necessary for optimality. This is indeed our main finding in Proposition 8.

Let f be any continuous functions having both left and right hand derivatives at a point U_0. A necessary condition for U_0 to be a maximum is $f'_-(U_0) \geq 0 \geq f'_+(U_0)$ so we have the following.[24]

Lemma 7 *Assume that f is a continuous function that admits both right and left derivatives in an interior point U_0. If we have $f'_-(U_0) < f'_+(U_0)$ then U_0 cannot be a maximum.*

finite. From Proposition 4 we know that $V'(\mathbf{a},U,h)$ is continuous in U alone, but finiteness of $\mathcal{A}$ implies joint continuity in $(\mathbf{a},U)$. We show that under assumption **A1** the infinite version of our model can be reduced to the case with a finite set of effort sequences.

Hence, a direct implication of our line of proof, guarantees that for any finite periods model, the result holds in great generality, for a much more general class of dynamic moral hazard models.

[24] To see in more detail why this is the case, write for example, the incremental ratio for the left derivative $f'_-(U_0)$. If $\frac{f(U)-f(U_0)}{U-U_0} < 0$ for $U - U_0 < 0$, with U sufficiently closed to U_0, we must have $f(U) > f(U_0)$, which is a contradiction to U_0 being a maximum. A similar argument can, of course, be used for the right derivative.

Proposition 8 *Assume **A1** and interiority. The optimal contract necessarily satisfies*

$$V'(U,h) = -\frac{1}{u'(b^*)} \tag{19}$$

$$W'(U^{e*},h') = -\frac{1}{u'(b^*)} - \mu \qquad \mu \geq 0 \tag{20}$$

$$V'(U^{u*},h') = -\frac{1}{u'(b^*)} + \frac{\pi(a^*,h)}{1-\pi(a^*,h)}\mu, \tag{21}$$

with $\mu = 0$ *if either* $a^* = 0$ *or (5), and is satisfied with strict inequality. Moreover, (19) can possibly fail only in the first period. In addition, we have*

$$V'(U,h) = \left[\pi(a^*,h)W'(U^{e*},h) + (1-\pi(a^*,h))V'(U^{u*},h)\right]. \tag{22}$$

The implications for the optimal scheme are not yet transparent. By rearranging the above derived first-order conditions and using envelope theorem we get the following:[25]

Corollary 9 *Under the conditions of the previous proposition we have the following*

$$\frac{1}{u'(b_t^*)} = \pi(a_t^*,h_t)\frac{1}{u'(w_{t+1}^*)} + (1-\pi(a_t^*,h_t))\frac{1}{u'(b_{t+1}^*)}. \tag{23}$$

Moreover, (i) $w_{t+1}^* \geq b_t^* \geq b_{t+1}^*$; *and* (ii) *either is true that* $w_{t+1}^* > b_t^* > b_{t+1}^*$, *or* $w_{t+1}^* = b_t^* = b_{t+1}^*$.

Result *(i)* above confirms, for a general, possibly non stationary framework, one key finding of Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). The optimal unemployment insurance scheme requires UI benefits to decrease with the duration of unemployment. On the other hand, notice that the wage tax behavior remains indeterminate. In Section 4, we take advantage of the recursive formulation to perform computer simulations of the optimal contract for the US and the Spanish economy. We anticipate that in our numerical exercises we find that an optimal scheme typically generates a plan of wage taxes $\tau_t^* = S_t - w_t^*$ which decreases with the length of previous unemployment spell. This is of course in contrast with the result of the stationary model of Hopenhayn and Nicolini (1997a), where the optimal reemployment wage tax τ_t^* increases during unemployment.

The second part of Corollary 9 has another important implication. In many studies, unemployment insurance programs are modelled in a very simple way. Only two parameters are used to define the scheme. It is assumed to be a time invariant unemployment benefit

[25] Condition (23) in Corollary 9 recalls that derived by Rogerson (1985) using a variational approach. Notice however that condition (23) alone would not typically allow us to say much about the monotonicity of the payments for example. This would be especially true in a model with more than two outcomes.

payment b, which - usually because of job-search incentives - is strictly lower than a time invariant wage payment w. We can then ask the following question. Could this simple scheme be optimal, for some combination of wage depreciation and duration dependence? Part *(ii)* gives a clear negative answer to this question. Then we could ask whether there exists a simple way of describing an optimal scheme, or at least some of its characteristics. We believe that we can answer positively to this question.

To make our results ready to use for policy purposes, we rearrange (23) and obtain

$$\frac{1}{u'(b_t^*)} - \frac{1}{u'(b_{t+1}^*)} = \pi(a_t^*, h_t)\left[\frac{1}{u'(w_{t+1}^*)} - \frac{1}{u'(b_{t+1}^*)}\right],$$

which, for the case with logarithmic utility, reduces to

$$b_t^* - b_{t+1}^* = \pi(a_t^*, h_t)\left[w_{t+1}^* - b_{t+1}^*\right]. \tag{24}$$

Because of its graphical representation, shown in Figure 2, we may name the above equation as *the triangle rule*. According to (24), for small $\pi(a_t^*, h_t)$ optimality suggests almost flat UI schemes, with a relatively large difference between net wage and unemployment insurance benefits. And vice versa for high hazard rates. This gives to (24) a quite appealing economic interpretation. According to this condition, workers facing relatively low hazard rates should be motivated to search for new jobs mainly through rewards: in case they find a new job, they should receive a high net wage w_{t+1}^*. In contrast, to those workers facing high probabilities of reemployment, search incentives are mainly given by the use of punishments: in case of failure in the job-search process, on those workers the planner imposes a considerable drop in the unemployment benefit payment b_{t+1}^*.

The triangle rule can be used as a very simple, “back of the envelope”, test for optimality of any unemployment insurance scheme. For that consider an existing scheme. For each period t, using today and tomorrow’s benefit payment levels and tomorrow’s net wage, it is always possible to draw a triangle as the one in Figure 2. The test also needs a reliable point estimate of the hazard rate $\pi(a_t^*, h_t)$ associated with unemployment duration of length t. Then using $\pi(a_t^*, h_t)$ one can easily check whether the two parts that form the segment $w_{t+1}^* - b_{t+1}^*$ have the proportions required by (24). When this is not the case, the trade-off between insurance and incentives is not exploited optimally, and there is room for a budget saving reform.

Alternatively, we could consider condition (24) as a valuable tool for the optimal unemployment insurance scheme designing problem. A hypothetical planner could indeed compute the unemployment scheme restricting himself to satisfy “at least” the triangle rule derived above.[26]

[26] Of course, this exercises requires a full knowledge of the hazard rate functions $\pi(\cdot, \cdot)$.

Finally, note that the triangle rule is not satisfied if we introduce other constraints into the designing problem, such as the exogenous lower bound on worker's utility I imposed in Pavoni (2003). Thus - assuming a maximizing government behavior - the triangle rule could also be used to test for the existence of exogenous minimum bounds; which is a way to discriminate between the unrestricted model of Hopenhayn and Nicolini (1997a,b) and the restricted one with utility bounds of Pavoni (2003).

A further quantitative assessment would regard the degree of approximation - toward the fully optimal scheme - that the use of the triangle rule would involve for utility functions different from the logarithmic one. This analysis is left for future research.[27]

4 Quantitative Analysis

In this sections, we aim at determining some additional characteristics of the optimal UI scheme. For that we specifically examine the effects of imposing strict monotonicity on reemployment wages and hazard rates respectively. Human capital depreciation and duration dependence clearly have the effects of increasing the planner's incentive costs for any *given* level of worker's lifetime utility. However, the analysis of the stationary model shows that there is an additional force that contrasts the one induced by the deterioration of worker's job opportunities. Because of dynamic incentive provision, the worker's expected discounted utility is decreasing during unemployment, and low lifetime utilities imply lower incentive and effort compensation costs. As a consequence, in a stationary environment an optimal UI program requires worker to always search actively for new jobs. Obviously, which one of these two forces dominates - human capital depreciation or decrease in lifetime utility - in a non stationary model is a quantitative issue.

More than giving a detailed policy advise, the aim of this section is to illustrate the effect of both wage depreciation and hazard rate duration dependence on incentives, and - as a consequence - on the shape of the optimal UI scheme. To better understand the forces in act, we disentangle the effects on the optimal UI program of each one of these two consequences of human capital depreciation. We pursue this task using a couple of calibration exercises. The first example refers to Spain. Spanish data on wages are often

[27]The logarithmic case seems to be a good approximation for an average level of relative risk aversion. Attanasio and Weber (1993) use UK cohort data to estimate the intertemporal elasticity of substitution. Assuming CRRA preferences, the results of Attanasio and Weber imply a constant risk aversion parameter between 1.3 and 1.5 (where 1 corresponds to the log-case). This is consistent with many other previous studies. For example, Mehra and Prescott (1985) cite various empirical studies that provide support for a constant relative risk aversion parameter between 1 and 2.

poor or difficult to interpret. In contrast, most empirical studies document a clear negative duration dependence in the reemployment probability. We thus use the Spanish economy to study how the optimal unemployment insurance scheme is affected by duration dependence in the probability finding a new job *alone*. In our second example, we use the US economy to study the implications of wage depreciation.

Our numerical methodology is based on value function iteration. We approximate the value function with Chebyshev polynomials.[28] Value function iteration involves a global maximization step. Although first-order conditions were successfully used to characterize qualitatively the contract, given the non-concavity of the problem, we are forced to use a numerical maximization procedure to determine the optimal choices at each value function iteration.

In both exercises we assume CRRA worker's preferences

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

with $\sigma = 1$, i.e. log-utility. Moreover, in both cases we set the search cost equal to the cost of labor, i.e. $v = l > 0$.

4.1 The Spanish Example: the effects of the hazard rate duration dependence.

To calibrate the model with the Spanish economy we assumed a *constant* gross wage S. Hence the calibration exercise consists of choosing 3 parameters: $[S, \beta, l]$, together with the hazard rate paths $\{\pi(a, h_t)\}_{t=0}^{T}$ for $a \in \{0, 1\}$ and the initial utility level U_0. We normalize the wage S to 100, and interpret each period as a month by setting $\beta = 0.996$. Finally, we set $l = 1$, which is between one fourth and one fifth of the utility the agent would receive from consuming the gross wage $u(S) = \ln(100) = 4.6$. In summary

$$[S,\ \ \beta,\ \ l] = [100,\ \ 0.996,\ \ 1].$$

The initial level of worker's utility U_0 is computed backward in accordance with the existing scheme. Since $S = 100$, the current insurance system can be represented by a contract that has not taxes or transfers when unemployed ($w = S$), and pays a first benefit level b^1 of 70 for the first six months of unemployment, from the 7th to the 24th month the benefit level b^2 is set equal to 60, and from the 25th onward we assume the worker receives

[28] Chebyshev polynomials have several mathematical and practical advantages as shown by Judd (1998).

an assistance level of benefits $b^3 = 20$.[29] The corresponding expected discounted utility value U_0 for an unemployed worker, can be calculated backward as follows. When a worker finds a job, his lifetime utility is

$$U_{work} = \frac{u(S) - l}{(1-\beta)} = \frac{4.6 - 1}{0.004} = 900,$$

which represents the utility of working forever and receiving the gross wage $S = 100$. Moreover, note that from period T onward the worker's problem is stationary. Both the unemployment benefits and the probability of finding a job are at their minimum level. Thus, under our parametrization, the worker will be searching for a job ($a = 1$), which will be found with probability $\pi(1, h_T) = \underline{\pi}$. And jobs are permanent. The value of his expected discounted utility U_T at period T can be computed as follows

$$U_T = \frac{u(b^3) - 1 + \beta\underline{\pi}U_{work}}{1 - \beta(1 - \underline{\pi})},$$

where $b^3 = 20$ is the non-contributive assistance level of unemployment benefits. For any $0 \leq t \leq T$ we can now define the value U_{T-t} recursively by

$$U_{T-t} = u(b_t) - v(a_t^*) + \beta\left[\pi(a_t^*, h_t)U_{work} + (1 - \pi(a_t^*, h_t))\, U_{T-(t-1)}\right],$$

where the period t benefit level $b_t \in \{b^1, b^2, b^3\}$ is computed according to the three steps scheme described above, and a_t^* denotes the effort level chosen optimally by the worker in period t.[30]

Our calibration of hazard rates is based on Bover, Arellano and Bentolila (1997). According to our interpretation of the data, an increase in unemployment duration seems to

[29] In Spain, the replacement ratio is equal to 70% during the first six months of unemployment and 60% thereafter, subject of a floor of 75% of the minimum wage. Benefit duration is one-third of the last job's tenure, with a maximum of two years. The assistance system pays, for up to two years, 75% of the minimum wage to (unemployed) workers, with dependant, whose average family income is precisely below that amount. In 1998, the minimum wage was around 70,000 pesetas ($280) (Guia Laboral 1998 y de Asuntos Sociales (1998)). The amount of the non-contributive assistance level of transfers varies across different Autonomous Communities between 30,000 to 45,000 pesetas ($150/180), is means-tested, and there is no fixed duration (See Lopez (1996)). The Bulletin of Labor Statistics (1999) reports as 300,000 pesetas ($1,200) per month the 1998 average wage in non-agricultural activities. Following the common assumption that workers subject to severe unemployment risk face a wage that is two thirds of the average national wage, we consider the assistance level of benefits as $\frac{1}{5} = 20\%$ of the gross wage S.

[30] That is, a_t^* solves

$$\max_{a_t \in \{0,1\}} u(b_t) - v(a_t) + \beta\left[\pi(a_t, h_t)U_{work} + (1 - \pi(a_t, h_t))\, U_{T-(t-1)}\right].$$

reduce the effectiveness of the job-search activity. However, since the search effort level a is obviously not reported by the study we had to perform an "identification" exercise. Here is our strategy. Bover et al. (1997) use data from the Spanish Labour Force Survey (EPA) to estimate the Spanish hazard rates both for workers receiving unemployment insurance benefits and for those who do not receive any UI benefit transfer.[31] This for 30 consecutive months of unemployment duration. As expected, for any duration level of unemployment the former hazard rates (the one related to workers receiving UI benefits) are always lower then the latter ones. Moreover, the difference between the two functions decreases considerably with unemployment duration, and approaches zero after 2 years. We believe it is reasonable to assume that workers not receiving benefits supply a higher effort level than workers receiving benefits. We need to assume that this is the case at any level of unemployment duration. These considerations induce us to interpret in our two effort framework the decrease in the difference between the hazard rates of the two groups of workers as a decrease in the effectiveness of the search activity. As a consequence, we calibrate the hazard rate paths $\{\pi(a, h_t)\}_{t=0}^{T}$ by linearly interpolating the estimations of Bover et al. (1997) as follows. We set $T = 30$, $\pi(1, h_0) = 0.21$ and $\pi(1, h_T) = 0.03$ with a time constant decrease in the hazard rate of 0.06 each period. Furthermore, consistently with the approximate stationarity of the estimated lower hazard rate function, we set the "passive" hazard rate $\pi(0, h_t) = \hat{\pi} = 0.01$ at a constant level for any unemployment duration t.

The results of our simulations of the efficient scheme are reported in Figure 4. First, the interested reader can verify that the scheme always obeys to the triangle rule. Second, we observe that the optimal path for unemployment benefit payments (the dotted lower-level line) is initially decreasing an then becomes completely flat. The reason is that, when human capital depreciates rapidly enough during unemployment, the planner looses the incentive to induce the agent to supply the high effort level. Thus, benefits eventually stop decreasing because the agent is fully insured. The second part of the benefit path (the flat one) is particularly important since induces an endogenous lower bound on workers' expected discounted utilities. Finally, note that the net wage w_t - represented by the upper-level solid line - is initially roughly constant and then increases with unemployment duration. Moreover, it presents an important downward jump for long term unemployed, when the worker is asked to stop searching actively, and he is fully insured by the planner.

The wage tax behavior is one key policy implication of the our non stationary model. In fact, we already mentioned that this prediction contrasts with the one of stationary models such as Hopenhayn and Nicolini. Finally, notice that after 24th months the simulated optimal

[31] Workers non receiving benefits include those who received benefits at some previous period.

scheme presents a sharp downward jump of the net wage, that is, a sharp increase in the wage tax. However, it seems reasonable to assume that while working agents have additional costs with respect to the situation where they are unemployed and do not search for new jobs: consider for example transportation costs, or the cost of having lunch in a restaurant during the working days, an so on. Thus, in order to compensate the worker for these additional costs, full-insurance actually implies a net wage level well above the unemployment insurance benefits.

4.2 The US Example: the consequences of wage depreciation.

Our second numerical example is a calibration with the US economy, and focuses on the effect of wage depreciation. We thus assume that the hazard rates do not vary with h, that is, $\pi(1,h) = \pi$ and $\pi(0,h) = \hat{\pi}$ for any h. To allow for wage depreciation, we consider a very simple relationship between human capital endowment and gross wage[32]

$$S(h_t) = \omega h_t.$$

Moreover, we assume a geometric depreciation rule m_u for human capital:

$$h_{t+1} = (1-\delta)h_t.$$

This formulation implies that during unemployment the gross wage $S(h_t)$ decreases at the exogenous and constant rate δ, equal to that of human capital. For simplicity, we assume that when the worker is employed his human capital endowment remains constant, i.e. $m_e(h) = h$ for any h. To complete our calibration exercise we have six parameters to choose: $[S_0,\ l,\ \beta,\ \pi,\ \hat{\pi},\ \delta]$, together with the initial utility level U_0. We set $S(h_0) = S_0 = 100$, and $l = 1$. To be consistent with the US payment system, we choose one week as reference period, and accordingly calibrate $\beta = 0.999$. Using the results in Meyer (1990) and partially following Hopenhayn and Nicolini (1997a), we set the weakly US hazard rate $\pi = 0.1$, the "passive" hazard rate $\hat{\pi} = 0.01$, and we computed the initial utility level U_0 in a way similar to the previous example.[33] Finally, the depreciation rate parameter δ is calibrated following

[32] This formulation can be motivated by an aggregate technology in which skill units h are perfect substitute. In this case, ω is equal to skill's marginal product.

[33] According to Meyer (1990), the average level of UI benefits received in the sample is 66% of the average value of the pre-unemployment wage $S(h_0) = 100$, and lasted - again in average - 34 weeks. To have a finite value for U_0 with log utility, we assumed that after 34 weeks of UI benefit payments the worker continues to receive 0.1% of his gross wage. The latter can be justified - for example - by the existence of assistance programs such as the Temporary Assistance for Needy Families (TANF) program; or by the existence of charity institutions which should provide minimal levels of cash financial support to individuals.

the results of Keane and Wolpin (1997). Using NLSY data, they provide (structural) econometric estimates for rates of skill depreciation during periods of unemployment. Keane and Wolpin estimate an annual human capital depreciation rate for white USA males of between 9.6% (for blue collars) and 36.5% (for white collars). We set our weekly level of $\delta = 0.005$, which corresponds to an intermediate annual depreciation rate of 23%. In summary, our choice are as follows

$$\begin{bmatrix} S_0 & l = v & \beta & \pi & \hat{\pi} & \delta \\ 100 & 1 & 0.999 & 0.1 & 0.01 & 0.005 \end{bmatrix}.$$

The results of our computer simulations are reported in Figure 5. Three lines are displayed in this figure as a function of unemployment duration t: the UI benefit payments b_t (represented by the thick dotted lower-level line), the gross wage $S_t = S(h_t)$ (represented by the homogeneously decreasing solid line), and the net wage $w_t = S_t - \tau_t$ (the thin dotted upper-level line). The optimal path for unemployment benefit payments b_t presents qualitatively the same characteristics as the Spanish case: it is initially decreasing an then - approximately after 60 weeks - becomes completely flat. Notice that in this example both π and $\hat{\pi}$ are constant, hence the incentive costs of implementing the high effort level are constant. The planner's expected returns are however decreasing, since the gross wage $S(h)$ decreases with unemployment duration. The resulting effect is similar to the one of previous example: at some point the planner releases the agent from the search duty, and simply transfers him some income. The agent is hence fully insured and the unemployment insurance benefits stop decreasing. The net wage schedule w_t is surprisingly very flat at a level of 91/93. As it is clear from the figure, the flatness of the w_t schedule implies that the reemployment wage tax $\tau_t = S(h_t) - w_t$ is strictly decreasing during unemployment, and for unemployed durations between 12 and 60 weeks is optimal to pay a wage subsidy after reemployment.[34]

The net wage w_t presents again an important downward jump after a sufficiently long period of unemployment. This occurs when the worker is asked to stop searching actively and is fully insured by the planner. However, with decreasing gross wage $S(h_t)$ - and recalling the discussion about the existence of some additional costs in the working state, we made at the end of the previous example for Spain - what seems mainly to happen in this US example is that the government simply stops paying the wage subsidy.

[34] For example, recall that the parameter δ is calibrated such that after $t = 54$ weeks, the gross wage is $S(h_{54}) = 76$. This implies that, after approximately one year of unemployment, the worker should receive a $\left(\frac{92}{76} - 1\right) 100 = 21\%$ wage subsidy after unemployment.

5 Conclusions

In the present paper we extend previous studies on optimal unemployment insurance to incorporate the effects of human capital depreciation and duration dependence in this mechanism-design problem.

Our results partially confirm those obtained with stationary models, namely that benefits should decrease with unemployment duration. However, the introduction of human capital depreciation and duration dependence generates some novel features on the optimal program. First, if human capital depreciates sufficiently rapidly during unemployment, the planner loses the incentives to induce the agent to supply the high search effort level. Consequentially unemployment benefit payments are initially decreasing and then eventually become completely flat, since the long-term unemployed worker is fully insured by the planner. This creates an endogenous lower bound on worker's expected discounted utility, providing an alternative way of eliminating the immiserization result of Thomas and Worral (1990). Second, we find that the increasing wage tax result of Hopenhayn and Nicolini (1997a) is not robust to this extension. Our simulation results both for the US and Spanish economies show that although it is optimal to impose a wage tax after reemployment on short-term unemployed workers, the optimal level of wage tax should decrease with the length of worker's previous unemployment spell, eventually becoming a wage subsidy for the long-term unemployed. The optimality of a *wage subsidy* is in fact a key policy implication of our non stationary search model with moral hazard.

Our analysis also has an independent theoretical interest, in that we develop a new approach that allows us to study recursively the properties of the dynamic moral hazard model in a systematic way. We find that the associated value function is in general non-concave and non-differentiable. In spite of these non-smoothness problems, we show that the optimal contract can be characterized by using the usual first order conditions. The technique we developed in this paper uses the Daskin's envelope theorem, and can be easily extended both to a more general class of moral-hazard problems and to other problems with similar characteristics.

One aspect of our characterization of the contract is a simple necessary characteristic of any optimal unemployment insurance scheme, which we call it the triangle rule. We believe that the simplicity of the triangle rule deserves an accurate analysis. We plan to study the degree of optimality of an unemployment scheme computed using the triangle rule, when the worker's utility function is different from the logarithmic case, within a range of parameters consistent with microeconomic empirical estimations.

In addition to a wage subsidy, the results in this paper suggest that, by extending the policy instruments available to the planner, some other policy implications could be derived, especially for long-term unemployed workers. In both examples of Section 4, a worker who stays unemployed for a sufficiently long period is required to stop actively searching for a job, and is fully insured. The long-term unemployed worker is a net cost for the government, especially when he has no hope of finding a new job in the future. In this case, an alternative active labor market policy may become the only valuable alternative to rescue the worker from his state. The fact that the planner's value function V represents the monetary cost of the unemployed worker suggests that our recursive formulation might be a natural way to analyze the choice of alternative active labour market policies. In Pavoni and Violante (2003) we build on the model developed here, we extend the instruments available to the planned to include retraining programs and/or job-search activities, and study the optimal sequence of programs and payments.

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6 Appendix A: Proofs

Proof of Proposition 1 *(i)* If at $t = 0$ $a_0 = 1$ then we have that $V_1(U_0) \geq V_0(U_0)$. For any $n \geq 0$ let us now define the set of conditional functions as follows

$$\begin{aligned} V(U, n+1) &= \max_{b,U^e,U^u} -b + \beta\left[\pi W(U^e) + (1-\pi)V(U^u, n)\right] \\ U &= u(b) - v + \beta\left[\pi U^e + (1-\pi)U^u\right] \\ U &\geq u(b) + \beta U^u, \end{aligned}$$

with $V(U, 0) \equiv V_0(U)$. Notice they are all strictly concave and continuously differentiable. By the contraction mapping, also $V(U, \infty)$ is concave. It is not difficult to see that $V(U, \infty)$ is continuously differentiable.

Lemma 10 *$V'(U, n+1) \leq V'(U, n)$ for any n with strict inequality at least for some n.*

Proof. The proof for $n = 0$ is as follows and then we can follow an inductive argument. So let consider the problem

$$\begin{aligned} V(U, 1) &= \max_{b,U^e,U^u} -g(z) + \beta\left[\pi W(U^e) + (1-\pi)V_0(U^u)\right] \\ U &= z - v + \beta\left[\pi U^e + (1-\pi)U^u\right] \\ U &\geq z + \beta U^u. \end{aligned}$$

It is easy to see that incentive compatibility is binding in this case (since $W'(U) = V_0'(U)$ if the incentive compatibility is slack we have $U^u = U^e$). Hence from the first order conditions we have

$$-g'\left(U - \beta U^u\right) = V'(U, 1) > V_0'(U^u) = -g'\left((1-\beta)U^u\right)$$

which implies $U > U^u$. But then $U - \beta U^u > (1-\beta)U$. From the convexity of g we have that $V_0'(U) = -g'\left((1-\beta)U\right) > V'(U, 1)$. Now assume that $V'(U, n) \leq V'(U, n-1)$ and solve the problems in the two cases. From first order conditions we get

$$\begin{aligned} V'(U_n^u, n-1) &= -g'\left(U - \beta U_n^u\right) + \mu_n \frac{\pi}{1-\pi} \\ W'(U_n^e) &= -g'\left(U - \beta U_n^u\right) - \mu_n \\ V'(U_{n+1}^u, n) &= -g'\left(U - \beta U_{n+1}^u\right) + \mu_{n+1} \frac{\pi}{1-\pi} \\ W'(U_{n+1}^e) &= -g'\left(U - \beta U_{n+1}^u\right) - \mu_{n+1} \end{aligned}$$

Now assume $\mu_{n+1} > \mu_n \geq 0$. Then by the induction argument we must have $U_n^u \geq U_{n+1}^u$ (just assume that $U_{n+1}^u > U_n^u$ and you get a contradiction), and by the incentive compatibility constraint we have $U_n^e \geq U_{n+1}^e$. But then $U - \beta U_n^u \leq U - \beta U_{n+1}^u$and that $\mu_{n+1} > \mu_n$ implies that $-g'\left(U - \beta U_{n+1}^u\right) - \mu_{n+1} < -g'\left(U - \beta U_n^u\right) - \mu_n$ which is in contradiction with the fact that by concavity $W'(U_{n+1}^e) \geq W'(U_n^e)$. Hence it must be that $\mu_n \geq \mu_{n+1} \geq 0$. Similarly as before, the induction argument and incentive compatibility imply $U_n^u \leq U_{n+1}^u$ and by envelope $V'(U, n+1) \leq V'(U, n)$. **Q.E.D.**

Of course, the limit function $V(U, \infty)$ is such that $V'(U, \infty) \leq V'(U, n)$. Now notice that we have also shown that if $a_0 = 1$ then $U^u \leq U$. As a consequence Lemma 10 implies that if $a_0 = 1$ will never choose $a_t = 0$ in the future and $V_1(U) = V(U, \infty)$.

(ii) Now since V_1 is concave, at any t, from the first order conditions we have $U_t > U_t^u = U_{t+1}$, (Pavoni, 2003, Lemma 1 shows that the incentive constraint must be binding) hence U_t is decreasing. From the envelope condition b_t decreases as well, and from the incentive compatibility constraint we have U_t^e decreasing and from envelope $u(w_t) = (1-\beta)U_t^e$ so w_t decreases as well. *(iii)* If we let $\delta \equiv \min_{U \leq U_0} \{U - U^u(U)\}$ then the minimization is well defined by the theorem of the maximum $\delta > 0$, and T is such that $\delta^T \geq U_0 - \underline{U}$. **Q.E.D.**

Proof of Proposition 3 First of all we should define a topology on $\mathcal{A}$. In fact, we do much more than this. We define the $\delta-$metric on $\mathcal{A}$ as follows

$$d_\delta(\mathbf{a}, \mathbf{a}') = \sum_{n=0}^{\infty} \delta^n \left|a(n) - a'(n)\right|, \ \delta \in (0,1).$$

Second, let us simplify the notation by eliminating the h indexation. It will become clear below that the continuity of $m_u(\cdot)$, together with Lemma 9.5 of Stokey and Lucas (1989), allows us to make this simplification, at this stage.

Now consider a generic $s = (\mathbf{a}, U)$ with $\mathbf{a} = \{a(0), a(1), a(2).....\} = \{a(0), {}_1\mathbf{a}\} \in \mathcal{A}$. Recall that the distance between two such points can be derived as follows $\|s - s'\| = \|\mathbf{a} - \mathbf{a}'\|_\delta + |U - U'|$. Using the promise keeping constraint we rewrite the Bellman operator T as follows

$$(TV)(s) = \sup_{U^u, U^e;\ sub(5)} -u^{-1}\left(U - v(a(0)) - \beta\left[\pi(a(0))U^e + (1 - \pi(a(0)))U^u\right]\right) +$$
$$+\beta\left[\pi(a(0))W(U^e) + (1 - \pi(a(0)))\, V({}_1\mathbf{a}, U^u)\right]$$

Now we show that the operator T maps bounded and continuous functions into itself. From the definition of continuity, we must verify that for each given point s and for each $\varepsilon > 0$, there exists a $\gamma > 0$ such that

$$\text{if } \|s - s'\| < \gamma \quad \text{then} \quad |(TV)(s) - (TV)(s')| < \varepsilon.$$

To this extent, we rewrite the previous condition using the definition of the Bellman operator T

$$|\sup_{U^u, U^e;\ sub(5)} g(U, U^u, U^e, a(0)) + \beta\left(1 - \pi(a(0))\right) V({}_1\mathbf{a}, U^u) -$$

$$- \sup_{U^u, U^e;\, sub(5)} g(U', U^u, U^e, a'(0)) + \beta\, (1 - \pi(a'(0)))\, V({}_1\mathbf{a}', U^u)\ | < \varepsilon \tag{25}$$

where

$$g(U, U^u, U^e, a(0)) = -u^{-1}\left(U - v(a(0)) - \beta\left[\pi(a(0))U^e + (1 - \pi(a(0)))U^u\right]\right) + \beta\pi(a(0))W(U^e)$$

and

$$W(U^e) = \frac{S - u^{-1}((1-\beta)U^e + l)}{1 - \beta}.$$

Now consider the two cases.

Case 1: Suppose that $a(0) = a'(0)$. In this case, we can assume $a(0)$ as a parameter of the problem, and apply the Maximum Theorem to the problem

$$\begin{aligned} G(U,{}_1\mathbf{a}) &= \sup_{U^u, U^e} g(U, U^u, U^e, a(0)) + \beta\, (1 - \pi(a_0))\, V({}_1\mathbf{a}, U^u) \\ sub &: (5),\ U^u, U^e \in \Gamma(U) \end{aligned} \tag{26}$$

to show continuity of G in $(U,{}_1\mathbf{a})$. The auxiliary constraint $\Gamma(U)$ is imposed in order to guarantee the constraint correspondence to be compact valued. A possibility is the following. The incentive compatibility constraint (5) requires $U^e \geq U^u + f(a(0))$, so we can always choose appropriately two constants $k_1, k_2 > 0$, and add to (5) the constraints $U^u \geq U - k_1$ and $U^e \leq U + k_2$. The continuity of G implies that we can always find a γ such that (25) is verified.

Case 2: Now suppose $a(0) \neq a'(0)$. The idea here is that we do not check for continuity in this case, that is, we set γ such that, whenever $a(0) \neq a'(0)$, then $\|s - s'\| > \gamma$. This can always be done since in this case $|a(0) - a'(0)| = 1$. In summary, the choice of γ is done according to the continuity properties of G, with the restriction $\gamma \leq 1$.

Since u^{-1} is bounded, if we start from a bounded V then TV will remain bounded.

Finally, one can checked directly that the operator satisfies the Blackwell's sufficient conditions, thus T defines a contraction in the complete metric space of the bounded and continuous functions with the sup norm, in the "reduced" space $S = \mathcal{A} \times \mathcal{U}$. The continuity of the low $m_u(\cdot)$, allows us to complete the proof by applying Lemma 9.5 and Theorem 9.6 of Stokey and Lucas (1989) which guarantee that the contraction mapping result is still true in the original space $\mathcal{A} \times \mathcal{U} \times \mathcal{H}$, with h as exogenous sate variable. ■

Proof of Proposition 4 The presence of the index h creates only notational complications, so we fix h and eliminate the h index in what follows. Following Grossman and Hart (1983) and changing the variable by defining $z \equiv u(b)$, the problem becomes

$$V(\mathbf{a}, U) = \sup_{z, U^u, U^e} -u^{-1}(z) + \beta\left[\pi(a)W(U^e) + (1 - \pi(a))V({}_1\mathbf{a}, U^u)\right] \tag{27}$$

$$\begin{aligned} s.t. \quad &: \quad z - v(a) + \beta\left[(1-\pi(a))U^u + \pi(a)U^e\right] \geq z - v(\hat{a}) + \beta\left[(1-\pi(\hat{a}))U^u + \pi(\hat{a})U^e\right] \\ U \quad &= \quad z - v(a) + \beta\left[(1-\pi(a))U^u + \pi(a)U^e\right] \end{aligned}$$

where a is the first element in the sequence $\mathbf{a} = \{a(n)\}$. Notice that the problem satisfies all the conditions required to apply Theorems 4.7 and 4.8 of Stokey, Lucas and Prescott (1989). To see why the problem is monotone, use the promise keeping constraint and notice that since u is increasing, the planner's objective function $-u^{-1}\left(U - v(a) + \beta\left[(1-\pi(a))U^u + \pi(a)U^e\right]\right)$ is strictly decreasing in U. In particular, notice that interiority is important here, it guarantees that $z = U - v(a) + \beta\left[(1-\pi(a))U^u + \pi(a)U^e\right]$ can indeed be modified to satisfy promise keeping without affecting incentive compatibility. Finally, note that if V is concave the planner objective function is concave (since u^{-1} is convex), and the constraints set is convex (linear), as a consequence (27) is a concave problem. This proves concavity.

Differentiability can be shown as follows. Given the value function is concave we can use Lemma 2 in Benveniste and Scheinkman (1979). For a fixed level of promised utility U_0 we are looking for a differentiable concave function $G(\mathbf{a}, U)$ such that it is well defined in an interval I around U_0 and such that for any $U \in I$ we have $G(\mathbf{a}, U) \leq V(\mathbf{a}, U)$ and $G(\mathbf{a}, U_0) = V(\mathbf{a}, U_0)$. We claim that

$$G(\mathbf{a}, U) = -u^{-1}\left(U + v(a) - \beta\left[(1-\pi(a))U_0^u + \pi(a)U_0^e\right]\right) + \beta\left[pW\left(U_0^e\right) + (1-p)V\left({}_1\mathbf{a}, U_0^u\right)\right]$$

is the function we are looking for. Indeed, the optimal values U_0^e and U_0^u satisfy the incentive compatibility and (by interiority) the promise keeping constraint can always be satisfied by varying the benefit transfer b, so the function G is well defined and we have $G(\mathbf{a}, U) \leq V(\mathbf{a}, U)\ \forall\ U \in I$ as required. The properties of u imply the concavity and differentiability of $-u^{-1}$. So G is concave and differentiable, and this implies that V is differentiable in U_0 and $V'(\mathbf{a}, U_0) = -\frac{1}{u'(b_0)}$. Since V is concave, it is continuously differentiable. ■

Proof of Proposition 5 Given the continuity result we obtained in Proposition 3, to show Proposition 5 it suffices to show the compactness of $\mathcal{A}$. That is

Lemma 11 *$\mathcal{A}$ is compact in topology induced by the metric $d_\delta(x, y)$ for any $\delta \in (0, 1)$.*

Proof Noting that the set of all infinite sequences of zeros and ones corresponds to the Cantor set $\Delta \equiv \{0, 1\}^{\mathbb{N}}$, which is known to be compact in the topology induced by the metric $d_\delta(x, y)$ for $\delta = \frac{1}{3}$.[35] From this we can easily show that the Cantor set is topologically

[35] If we start by endowing the set $\{0, 1\}$ with the discrete topology (which is both compact and Hausdorff), then it is well known that the metric $d_\delta(x, y) = \sum_{n=0}^{\infty} \delta^n |x(n) - y(n)|$ with $\delta = \frac{1}{3}$ induces the product topology on Δ (see, for example, Aliprantis and Border 1994, page 93) so from the properties of the Hausdorff spaces and using the Tychonoff Product Theorem we have that Δ is both Hausdorff and Compact.

equivalent to the same set endowed with the topology induced by a generic $\delta \in (0,1)$. Finally notice that the set $\mathcal{A}$ is a closed set of such set of sequences, hence it is compact. **Q.E.D.**

Given that existence is proved, it is then easy to combine the continuity and boundedness results of Proposition 5 to show the equivalence between the sequential and the recursive choice of efforts. ■

Proof of Proposition 6 Our line of proof if based on the Daskin's envelope theorem. To state the result in terms of our model we fist need some definitions.

Definition 1 *For each U and h, define the non-empty set $\mathcal{A}^*(U,h) = \arg\max_{\mathbf{a}\in\mathcal{A}} V(\mathbf{a},U,h)$, moreover we call $\mathcal{A}^*(h) = \bigcup_U \mathcal{A}^*(U,h)$ the set of all possible maximizers.*

Here below we state the Daskin's envelope theorem and give a version of the proof adapted from Milgrom (1999).

Lemma 12 *Assume that (i) $\mathcal{A}^*(h)$ is non-empty and compact and that for any U, h, the function $V(\cdot,U,h) : \mathcal{A}^*(h) \to I\!R$ is continuous. Moreover assume that (ii) $V'(\mathbf{a},U,h) \equiv \frac{\partial V(\mathbf{a},U,h)}{\partial U}$ exists and is continuous in ($\mathbf{a}$,U). Then, for any h, the value function $V(U,h)$ has always both right and left derivative in U, and these are given by the formulas*

$$
\begin{aligned}
V'_+(U,h) &= \max_{\mathbf{a}\in\mathcal{A}^*(U,h)} V'(\mathbf{a},U,h) \\
V'_-(U,h) &= \min_{\mathbf{a}\in\mathcal{A}^*(U,h)} V'(\mathbf{a},U,h),
\end{aligned}
$$

moreover $V(U,h)$ is almost everywhere differentiable in U, and whenever the derivative exists then

$$V'(U,h) = V'(\mathbf{a}^*,U,h) \quad \textit{for any} \quad \mathbf{a}^* \in \mathcal{A}^*(U,h).$$

Proof of the lemma. To simplify the notation we neglect the dependence on h. Hence $\mathcal{A}^*(U,h)$ becomes $\mathcal{A}^*(U)$, and $\mathcal{A}^*(h)$ becomes $\mathcal{A}^*$.

Let us show first the right hand derivative. From our assumptions $\mathcal{A}^*(U)$ is non-empty and u.h.c., so for each U we can take $\mathbf{a}(U) \in \arg\max_{\mathbf{a}\in\mathcal{A}^*(U)} V'(\mathbf{a},U)$. This is indeed a well defined procedure since $V'(\mathbf{a},U)$ is continuous. Now consider $U' > U$ and write the incremental ratio

$$\frac{V(U') - V(U)}{U' - U} = \frac{V(\mathbf{a}(U'),U') - V(\mathbf{a}(U),U)}{U' - U} \geq \frac{V(\mathbf{a}(U),U') - V(\mathbf{a}(U),U)}{U' - U}$$

the last inequality comes from the fact that $\mathbf{a}(U') \in \mathcal{A}^*(U')$ so any other choice will reduce the value of $V(\mathbf{a}(U'),U') = V(U')$. Now from the properties of the *conditional* functions as

$U' \to U$ with $U' > U$ the right hand side converges to the derivative. So we have that

$$\liminf_{\substack{U' \to U \\ U' > U}} \frac{V(U') - V(U)}{U' - U} \geq V'(\mathbf{a}(U), U).$$

And we are done for the first part. We now want to show that

$$\limsup_{\substack{U' \to U \\ U' > U}} \frac{V(U') - V(U)}{U' - U} \leq V'(\mathbf{a}(U), U)$$

where I recall that $\mathbf{a}(U)$ is the maximizer of the partial derivative $V'(\mathbf{a}, U)$ over the no-empty and compact set $\mathcal{A}^*(U)$. To this extent, notice that we have the following

$$\frac{V(U') - V(U)}{U' - U} \leq \frac{V(\mathbf{a}(U'), U') - V(\mathbf{a}(U'), U)}{U' - U} = V'(\mathbf{a}(U'), \xi). \tag{28}$$

The first inequality in (28) is guaranteed by a similar reasoning as the one we made before. Namely, because $\mathbf{a}(U')$ does not in general maximizes V when the utility is at U instead of at U'. The last equality, i.e. the existence of a point $\xi \in (U', U)$ such that it is true is guaranteed by the by the Mean Value Theorem. Now, since ξ and the value of the derivative is calculated for a given $\mathbf{a}(U')$, which in turn depends on U', we write this number $\xi(U')$. From the continuity of $V'(\mathbf{a}, U)$ we know that for any U', the partial derivative in the right hand side $V'(\mathbf{a}(U'), \xi(U'))$ is a real number, hence the number

$$R = \limsup_{\substack{U' \to U \\ U' > U}} V'(\mathbf{a}(U'), \xi(U'))$$

is well defined. Since R is an accumulation point, there exists a converging subsequence $R_n \to R$. Moreover, we take a sequence of U_n such that this sequence is reproduced by points of the type $V'(\mathbf{a}(U_n), \xi(U_n))$. The reason is that $V'(\mathbf{a}, U)$ is continuous jointly in $(\mathbf{a}, U)$. Since R_n converges, there must exist a converging sequence $(\mathbf{a}(U_n), \xi(U_n)) \to (\overline{\mathbf{a}}, U)$. The sequence of $\xi(U_n)$ must converge to U since for any n $\xi(U_n) \in (U_n, U)$ and $U_n \to U$. Moreover, by construction, the sequence of $\mathbf{a}(U_n)$ is such that for any n, $\mathbf{a}(U_n) \in \mathcal{A}^*(U_n)$. If the correspondence $\mathcal{A}^*(U)$ were upper-hemicontinuous, then we could be sure that $\overline{\mathbf{a}} \in \mathcal{A}^*(U)$. This is nothing more then the definition of upper-hemicontinuity: an upper-hemicontinuous correspondences is defined by the fact that $\mathbf{a}(U_n) \in \mathcal{A}^*(U_n)$ for any n implies that $\overline{\mathbf{a}} = \lim_{n\to\infty} \mathbf{a}(U_n)$ is such that $\overline{\mathbf{a}} \in \mathcal{A}^*(\lim_{n\to\infty} U_n) = \mathcal{A}^*(U)$. But notice that since the partial derivative $V'(\mathbf{a}, U)$ is continuous in U, $V(\mathbf{a}, U)$ is continuous also in U (not necessary jointly in $(\mathbf{a}, U)$). So the Theorem of the Maximum (or Berge's Theorem) guarantees that $\mathcal{A}^*(U) = \arg\max_{\mathbf{a} \in K} V(\mathbf{a}, U)$ is upper-hemicontinuous correspondence. We then have

$$\limsup_{\substack{U' \to U \\ U' > U}} \frac{V(U') - V(U)}{U' - U} \leq \limsup_{\substack{U' \to U \\ U' > U}} V'(\mathbf{a}(U'), \xi(U')) = V'(\overline{\mathbf{a}}, U) \leq V'(\mathbf{a}(U), U)$$

where the first inequality comes from the first part of equation (28) by taking the sup on both sides. The second equality comes from our assumptions and the last inequality holds because $\overline{\mathbf{a}} \in \mathcal{A}^*(U)$ and $\mathbf{a}(U)$ is the maximizer of $V'(\mathbf{a}, U)$ in this set. ■

Proof of Proposition 6 Notice that after period $T(h)$ the problem becomes stationary. Hence from Proposition 1 we know that the optimal path of actions $\mathbf{a}^*$ is necessarily such that either *(I)* for all $t \geq T(h)$ we have $a(t) = 0$ or *(II)* $a(t) = 1$ for all $t \geq T(h)$. In both cases the set of optimal efforts $\mathcal{A}^*(h)$ for the period-zero problem, is a subset of the *finite* set of all the sequences of efforts which end either with the sequence $\mathbf{0} = \{0, 0, 0, 0.....\}$ or with the sequence $\mathbf{a} = \mathbf{1}$ after $T(h) < \infty$.

From Proposition 5 the set $\mathcal{A}^*(h)$ is non-empty. Moreover, it is also trivially compact, since it is finite. This guarantees that assumption *(i)* of Lemma 12 is satisfied. Finally, notice that from Proposition 4 $V'(\mathbf{a}, U, h) \equiv \frac{\partial V(\mathbf{a},U,h)}{\partial U}$ exists and it is continuous in U. Since $\mathcal{A}^*(U, h)$ is finite (and non-empty), $V'(\mathbf{a}, U, h)$ is also jointly continuous in $(\mathbf{a}, U)$. Hence also assumption *(ii)* is satisfied, and we can apply Lemma 12 to our problem. ■

Proof of Proposition 8 It is immediate to see that (19), (20) and (21) are the first order conditions for the proposed problem. Moreover, notice that the existence of $V'(U, h)$ is justified by Proposition 4. However, we must show: *(i)* first, that the differentiability conditions for taking the fist order conditions are indeed satisfied, *(ii)* second, that $\mu \geq 0$ as claimed.

(i) Since the case with $a^* = 0$ is obvious, we will consider only $a^* = 1$. When $a^* = 1$, the incentive constraint (5) can be rewritten as follows

$$U^e - U^u \geq \frac{v}{\beta\pi(1, h)}. \tag{29}$$

We can have two cases. *Case 1:* At the optimum the incentive constraint (29) is satisfied with equality. If we rewrite the objective function using (29) with equality and use (4), we can rewrite the problem as a function of U^u alone:

$$\begin{aligned}&\sup_{U^u} -u^{-1}\left(U + v - \beta\left[U^u + \frac{\pi(1,h)v}{\beta\pi(1,h)}\right]\right) + \\ &+\beta\left[\pi(1,h)W\left(U^u + \frac{v}{\beta\pi(1,h)}\right) + (1-\pi(1,h))\,V(U^u, h')\right].\end{aligned}$$

The problem is now a free maximization whose objective function is a weighted sum between the differentiable functions u^{-1} and W, and the function $V(U^u, h')$. We can directly apply Lemma 7 to this problem and obtain the desired result. *Case 2:* The optimum is such that the incentive constraint (29) is slack. In this case, we can use (4) and rewrite the problem as a function of both U^u and U^e as follows

$$\sup_{U^u,U^e} -u^{-1}\left(U+v(a^*)-\beta\left[U^u+\pi(a^*,h)\left(U^e-U^u\right)\right]\right)+ \\ +\beta\left[\pi(a^*,h)W(U^e,h)+\left(1-\pi(a^*,h)\right)V(U^u,h')\right].$$

Notice that in the objective function the two choice variables U^u and U^e interact in a very peculiar way. Either they are part of a linear mapping into a differentiable function (this is the case if the first term of the objective function, inside u^{-1}) or they enter into two different function which are related linearly among them. This feature guarantees that when taking the directional derivative for optimality we can separate the two variables. The choice of U^e is clearly well defined since both u^{-1} and W are differentiable everywhere. Moreover, for any given choice of U^e, the optimal level U^u is now computed by solving again a *free* maximization over a weighted sum between the differentiable function u^{-1} and the function $V(U^u,h')$, thus Lemma 7 also applies to this case.

(ii) Once we have shown that the problem must be differentiable at the optimum, we can use the (local) Kuhn-Tucker theorem. For this, notice that the incentive constraint (29) is linear, hence satisfies the constraint qualification requirement needed to apply the Kuhn-Tucker theorem. Hence, if μ is the multiplier associated to the incentive constraint, μ is non-negative as claimed. ■

Proof of Corollary 9 The first part of the corollary is easily derived from the last result of Proposition 8. It suffice to use Proposition (4) and rewrite $V'(U,h)=-\frac{1}{u'(b_t^*)}$, $W'\left(U^{e*},h'\right)=-\frac{1}{u'(w_{t+1}^*)}$, and $V'\left(U^{u*},h'\right)=-\frac{1}{u'(b_{t+1}^*)}$. To show the second part, notice that since $\pi(1,h)>0$, both (i) and (ii) results can be easily derived from (19), (20), (21) $\mu\geq 0$ and the strict concavity of u. Obviously, if $a_t^*=0$ then $w_{t+n}^*=b_t^*=b_{t+n}^*$ for $n\geq 1$. ■

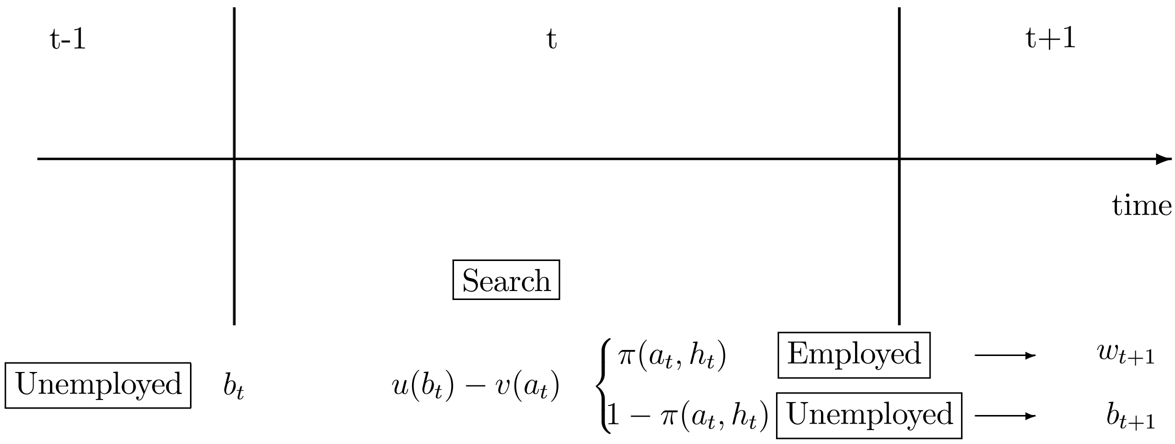

Figure 1: **The timing of the model in the unemployment state.** At the beginning of period t the unemployed worker receives the unemployment benefit b_t and is required to supply the costly job-search effort a_t. This effort affects the probability of being employed at the beginning of the next period $\pi(a_t, h_t)$. If at the beginning of next period the worker is employed he receives a net wage $w_{t+1} = S(h_{t+1}) - \tau_{t+1}$ otherwise, if he still unemployed, the worker receives the unemployment benefit b_{t+1}, and so on.

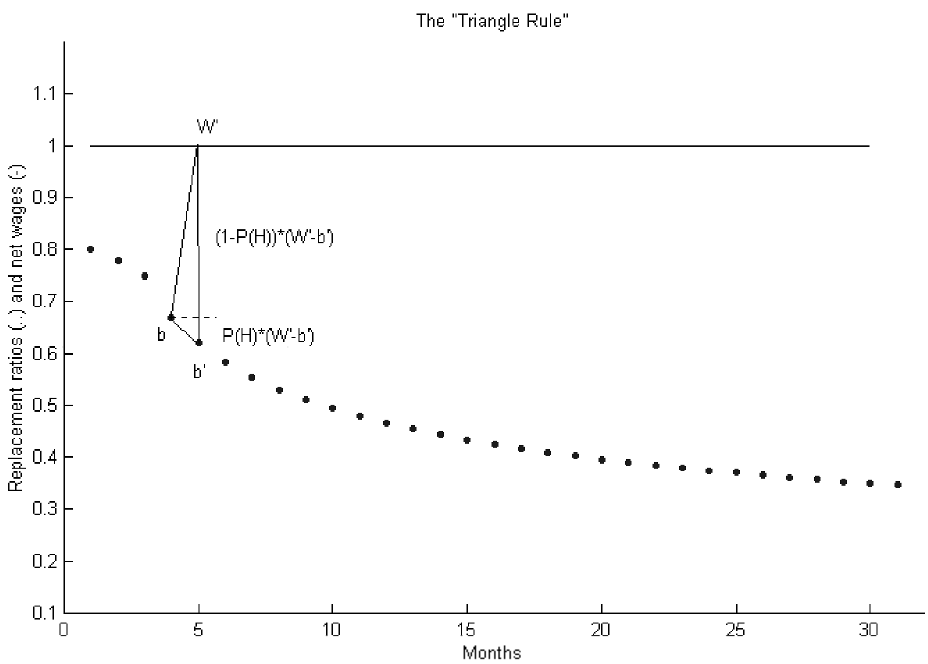

Figure 2: An example of the triangle rule.

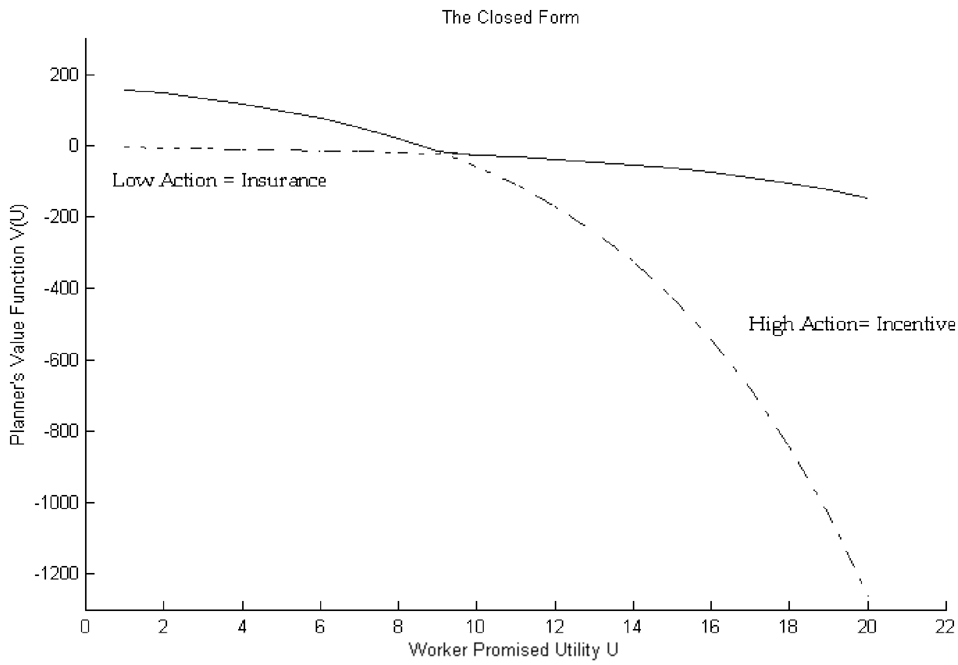

Figure 3: A parametrized example of the Closed form

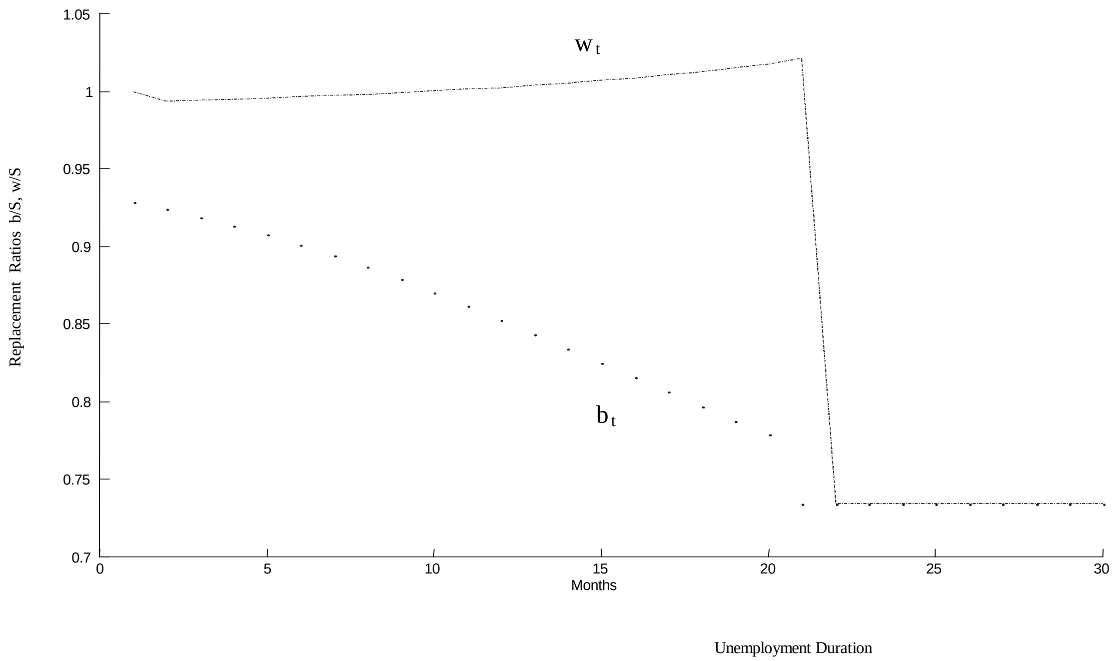

Figure 4: **The Spanish Example: the effects of the hazard rate duration dependence.** In this figure, the dotted lower-level line represents UI benefit payments b_t, and the upper-level line represents net wage $w_t = S - \tau_t$, as a function of unemployment duration. From the figure b_t is clearly decreasing in t and given S is constant in our example, τ_t is initially decreasing in t, and them presents a last downward jump.

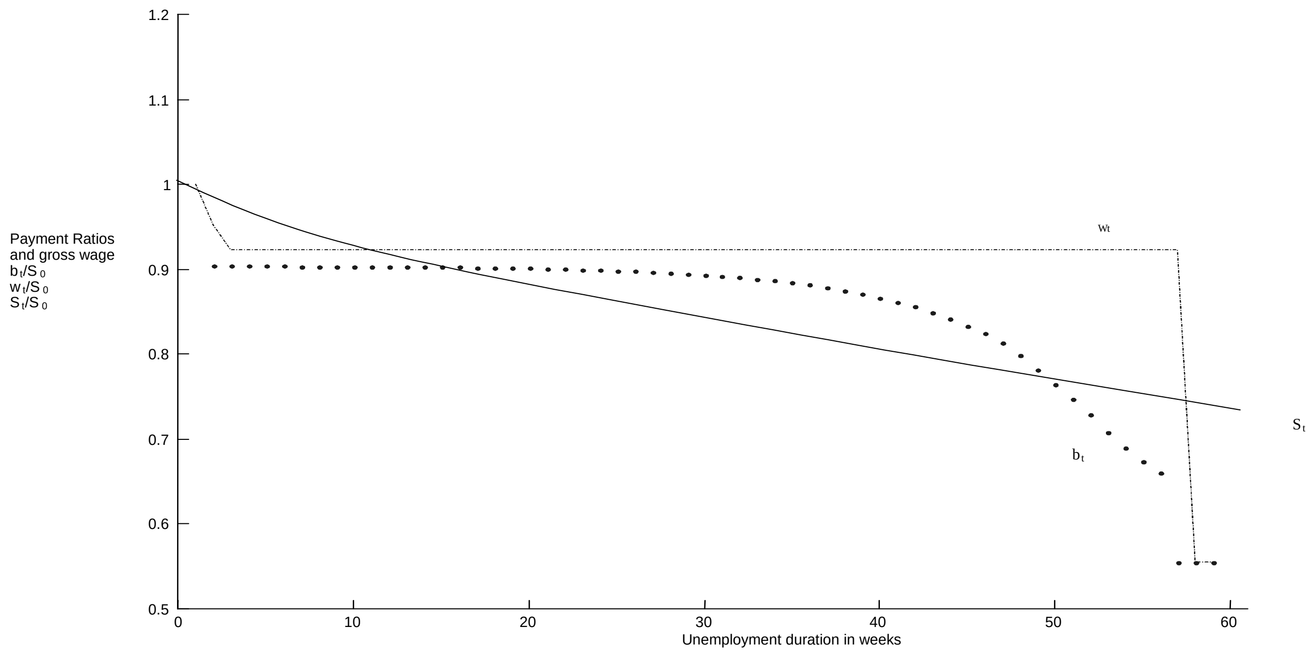

Figure 5: **The US Example: the consequences of wage depreciation**. In this figure, the thick dotted lower-level line represents UI benefit payments b_t, the homogeneously decreasing solid line represents the gross wage $S_t = S(h_t)$, and the thin dotted upper-level line represents net wage $w_t = S_t - \tau_t$, as a function of unemployment duration. From the figure, b_t is decreasing in t and the reemployment wage tax τ_t steadily decreases until the $58th$ week, where it is negative, i.e. it is actually a substantial subsidy. Then the subsidy jumps down.

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