Homodyne Estimation of Gaussian Quantum Discord

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(Received 9 June 2012; published 2 November 2012)

We address the experimental estimation of Gaussian quantum discord for a two-mode squeezed thermal state, and demonstrate a measurement scheme based on a pair of homodyne detectors assisted by Bayesian analysis, which provides nearly optimal estimation for small value of discord. In addition, though homodyne detection is not optimal for Gaussian discord, the noise ratio to the ultimate quantum limit, as dictated by the quantum Cramer-Rao bound, is limited to about 10 dB.

Quantum correlations are central resources for quantum technology. These tight connections empower the advantages shown by the exploitation of quantum coding in applications to cryptography, computation, and sensing. While at first entanglement was recognized to be the most peculiar form of quantum correlations, novel concepts have been introduced to capture either more specific aspects, such as quantum steering [1,2], or, to the other end of the spectrum, more general occurrences. Quantum discord represents the most successful attempt to observe quantum features within the current picture [3,4]: it is related to the fact that quantum information in a bipartite system cannot be accessed locally without causing an inherent disturbance at a difference with classical probability distributions.

Quantum discord has recently attracted considerable attention, due to its possible, yet controversial, usefulness as a resource in mixed-state quantum computing. There exist in fact architectures for which an exponential improvement over classical resources is obtained [5,6], albeit that the entanglement becomes exponentially small [7,8]. Discord has then been suggested as the empowering resource, while following investigations contested this interpretation [9]. This debate has stimulated an intense effort into looking at protocols where discord acts a resource: it has been demonstrated that discord does play a role in the activation of multipartite entanglement [10], entanglement generation by measurement [11], state merging [12], and for complete positivity of evolutions [13,14].

In the experimental test of such proposed connections, the comparison of discord with relevant figures of merit is clearly connected to the ability of estimating with the best precision allowed by a given amount of resources. A key problem is then to find optimal strategies, and to understand their fundamental limit introducing proper Cramér-Rao bounds (CRB) [15–17]. In fact, experimental

observation of quantum discord has been undertaken either by direct inspection of the density matrix [18–22], or by using a witness [23], however, with no concern about the optimality of the scheme.

Optimal estimation of quantum correlations has been investigated for entanglement [24] and optimal estimators have been experimentally proved to attain the quantum limit for different families of qubit states [25]. For the perspective of quantum metrology, this is highly nontrivial, since there exists no observable directly related to quantum discord. A proper estimator is then needed, which might depend on several characteristic parameters of the quantum state. In such a multiparameter problem, finding an optimized detection scheme might be hard, and could demand complex experimental apparata or heavy postprocessing of the data.

In this Letter we demonstrate homodyne estimation of Gaussian quantum discord in continuous variable systems [26,27], and compare the achieved level of precision with the classical CRB for homodyne detection, and with the quantum CRB, which sets the ultimate precision allowed by quantum mechanics. We found that although homodyne detection is not optimal for Gaussian discord, the noise ratio to the ultimate quantum limit is limited to about 10 dB. Our findings also show how a suitable Bayesian data processing may be employed to improve precision, especially in the estimation of small values of discord.

Quantum discord is defined as the difference between two quantum analogues of classically equivalent expressions of the mutual information in bipartite systems. Its evaluation demands an optimization procedure over the set of all measurements on a given subsystem. For continuous variable system, such minimisation reveals as an extremely complex task; however, in the case of Gaussian states, we can conveniently restrict the search to Gaussian

measurement only [26], obtaining an expression for the Gaussian quantum discord [26,27]. This sets a lower limit to the discord of the state, and also represents an operative figure of merit in those context where, for experimental convenience, only Gaussian measurements are employed.

Our investigation is concerned with an important class of Gaussian states, i.e., the two-mode squeezed thermal states (STS) naturally produced by a non-collinear optical parametric amplifier (OPA). If we introduce the two-mode squeezing operator $S_2(s) = \exp(s(a_0^\dagger a_1^\dagger - a_0 a_1))$, and the thermal state $\nu(N) = \frac{1}{N+1}\sum_n (\frac{N}{N+1})^n |n\rangle\langle n|$, we can write the STS as

$$\varrho(N_s, N_t) = S_2(s)\nu(N_t) \otimes \nu(N_t)S_2(s)^\dagger, \quad (1)$$

and thus can be fully described by the two parameters $N_s = \sinh^2 s$ and N_t, representing, respectively, the effective amount of *squeezing photons* and *thermal photons*. In fact, spurious effects such as unwanted amplification result in a loss of purity of the squeezed state by thermalization, but do not affect the Gaussian character of the emission, so the form of the density matrix (1) provides a fully general description of the output of a realistic OPA [28].

For the class of states in Eq. (1) the Gaussian quantum discord is given by

$$D(N_s, N_t) = h(\kappa_1) - 2h(\kappa_2) + h(\kappa_3),$$

where $h(x) = (x + 1/2)\log(x + 1/2) - (x - 1/2) \times \log(x - 1/2)$ is the binary entropy and $\kappa_1 = (1 + 2N_s) \times (1 + 2N_t)$, $\kappa_2 = (N_t + 1/2)$, $\kappa_3 = (1 + N_s + N_t)(N_t + 1/2)/(1 + N_s + N_t + 2N_sN_t)$. We can estimate the discord from N_s and N_t as we varied the pump power of our OPA [29]. For each power setting, these two parameters are extracted by the outcome of two homodyne detectors, one on each mode, which measure pairs of quadratures $\{X_0, X_1\}$ and $\{P_0, P_1\}$ (Fig. 1). From these, we can evaluate the four linear combinations

$$Q^{(1/2)} = \frac{X_0 \pm X_1}{\sqrt{2}}, \qquad Q^{(3/4)} = \frac{P_0 \pm P_1}{\sqrt{2}}, \quad (2)$$

where $Q^{(1)}$ and $Q^{(4)}$ are squeezed quadratures, while $Q^{(2)}$ and $Q^{(3)}$ are antisqueezed; in particular, M_q measurement outcomes are recorded for each one of the four quadratures. The corresponding variances, $\sigma^2(Q_{sq})$ and $\sigma^2(Q_{asq})$, that can be obtained from the experimental data, can be rewritten as a function of N_s and N_t as follows

$$\sigma^2(Q_{sq/asq}) = [1 + 2N_s \mp 2\sqrt{N_s(1 + N_s)}](1 + 2N_t). \quad (3)$$

The expressions obtained can be then inverted to obtain the experimental estimate N_s^{inv} and N_t^{inv}, along with the relative uncertainties $\sigma^2(N_s^{inv})$ and $\sigma^2(N_t^{inv})$. These values can be used in the expression for discord [29] to calculate its value D^{inv}, and the uncertainty $\sigma^2(D^{inv})$. The uncertainties on these quantities are then obtained by a Monte Carlo procedure [29]. One can use the same data and refine the
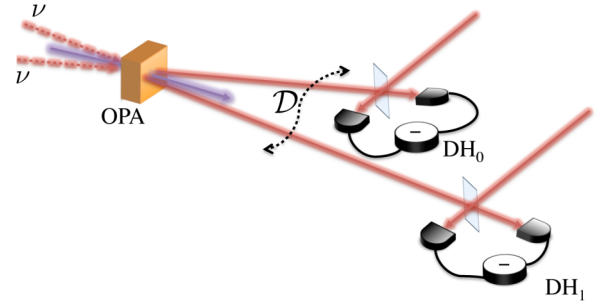

FIG. 1 (color online). Conceptual layout of the discord estimation. Our non-collinear OPA is based on a nonlinear KNbO₃ crystal, pumped by a frequency-doubled Ti:sapphire laser (repetition rate 800 kHz, wavelength $\lambda_p = 425$ nm, pulse duration 120 fs). This produces a STS with discord D depending on the pump power, i.e., on their average photon number. The two modes are measured by two homodyne detectors DH₀ and DH₁. The relative phase between the local oscillators is locked by mapping them to different polarizations on the same spatial mode. In this way, we can record blocks of 20 000 values of pairs of quadratures for $\{X_0, X_1\}$ and the same for $\{P_0, P_1\}$.

estimation by using a Bayesian analysis. As described above, each data sample corresponds to $M_q = 2 \times 10^4$ measurement of each of the four quadratures. The total sample, thus corresponds to $M_T = 4M_q$ homodyne outcomes

$$\mathcal{X} = \{q_1^{(1)}, ..., q_{M_q}^{(1)}, q_1^{(2)}, ..., q_{M_q}^{(2)}, q_1^{(3)}, ..., q_{M_q}^{(3)}, q_1^{(4)}, ..., q_{M_q}^{(4)}\}.$$

The overall sample probability can be evaluated as

$$p(\mathcal{X}|N_s, N_t) = \prod_{k=1}^{4} \prod_{j=1}^{M_q} p_k(q_j^{(k)}|N_s, N_t), \quad (4)$$

where the probability of obtaining the outcome $q_j^{(k)}$ by measuring the quadrature $Q^{(k)}$ is a Gaussian distribution

$$p_k(q_j^{(k)}|N_s, N_t) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{(q_j^{(k)})^2}{2\sigma_k^2}\right). \quad (5)$$

For squeezed quadratures ($k = \{1, 4\}$) we substitute $\sigma_k^2 = \sigma^2(Q_{sq})$, while for an antisqueezed quadrature ($k = \{2, 3\}$) $\sigma_k^2 = \sigma^2(Q_{asq})$. By means of the Bayes theorem, we obtain the *a posteriori* probability

$$p(N_s, N_t|\mathcal{X}) = \frac{1}{\mathcal{N}} p(\mathcal{X}|N_s, N_t)p_0(N_s)p_0(N_t), \quad (6)$$

$$\mathcal{N} = \int dN_s dN_t p(\mathcal{X}|N_s, N_t)p_0(N_s)p_0(N_t), \quad (7)$$

where the $p_0(N_s)$ and $p_0(N_t)$ are the so-called *a priori* probability distributions for the two parameters. In our procedure, we use the results of the inversion estimation to construct these *a priori* distributions. That is, we consider $p_0(N_s)$ and $p_0(N_t)$ as Gaussian functions with,

respectively, mean values equal to N_s^{inv} and N_t^{inv}, and variances equal to $\sigma^2(N_s^{\text{inv}})$ and $\sigma^2(N_t^{\text{inv}})$. Then, we can use the *a posteriori* probability distribution evaluated as in Eq. (6) to obtain an estimate of the two parameters and of their variances. In formula, (for $j = s, t$)

$$N_j^{\text{bay}} = \int dN_s dN_t N_j p(N_s, N_t | \mathcal{X}), \quad (8)$$

$$\sigma^2(N_j^{\text{bay}}) = \int dN_s dN_t (N_j - N_j^{\text{bay}})^2 p(N_s, N_t | \mathcal{X}). \quad (9)$$

By using the formula of the discord for two-mode STS in Ref. [29] and by propagating the errors, we then obtain an estimate D^{bay} for the discord, along with its variance $\sigma^2(D^{\text{bay}})$.

The value of discord depends on both the squeezing and thermal photons. Consequently, its estimation is inherently a multiparameter problem, and we have to identify the relevant physical parameters to evaluate the correct CRB. In the multiparameter scenario, the quantum Fisher information (QFI) associated to a vector of parameters $\bar{\lambda} = \{\lambda_i\}_{0 \leq i \leq n}$ is in the form of a matrix $\boldsymbol{H}$. This sets a lower bound on the covariance $\sigma_{ij}^2 = \langle \lambda_i \lambda_j \rangle - \langle \lambda_i \rangle \langle \lambda_j \rangle$ after M repetitions on the experiment:

$$\sigma_{ij}^2 \geq \frac{1}{M} (\boldsymbol{H}^{-1})_{ij}. \quad (10)$$

In the specific case of our experiment, we can bound the uncertainty on the discord D of the states we prepare as $\sigma^2(D) \geq \frac{1}{M}(\boldsymbol{H}^{-1})_{DD}$. While our measurement strategy has the advantage of being simple, it is not expected to be optimal, i.e., to saturate the quantum CRB. In order to assess the estimator, i.e., the data processing, we also need to compare it to the classical CRB associated to our specific measurement, which is analogously described by a classical Fisher information (FI) matrix $\boldsymbol{F}$.

In the evaluation of the correct bound, we need a suitable parametrization of the state, so that in the expression (10) one parameter only actually varies, while the others are kept fixed: this cannot be the case for the number of thermal and squeezing photons, as both of them change with the pump power. Therefore, we need to reshape the QFI matrices for different couples of parameters, so to consider those which are more directly connected to the experimental conditions. We start by considering the first couple $\boldsymbol{\lambda}_1 = \{N_s, N_t\}$; by using the formulas described in Ref. [29] we obtain

$$\boldsymbol{H}^{(1)} = \text{diag}\left(\frac{(1 + 2N_t)^2}{N_s(1 + N_s)(1 + 2N_t + 2N_t^2)}, \frac{1}{N_t(1 + N_t)} \right). \quad (11)$$

As explained above, thermal photons appear because of imperfections in the operation of the OPA and because of loss. When the squeezing is not too low, we can reparametrize our state by taking in to consideration the effective

squeezing strength, r, and a parasite amplification with strength γr [30,31]. The overall homodyne detection can be separately calibrated, obtaining $\eta = 0.62$. Thus, we can rewrite the matrix (11) in terms of the two unknown physical parameters $\boldsymbol{\lambda}_2 = \{r, \gamma\}$ via the expression $\boldsymbol{H}^{(2)} = B_{12}\boldsymbol{H}^{(1)}B_{12}^T$, where B_{12} is the transfer matrix for this change of variables [29]. Next, since the physical parameter that changes during our experiment, resulting in the variation of the amount of discord, is the squeezing parameter r (while γ and η can be considered to remain constant), we perform the last change of variable, by considering $\boldsymbol{\lambda}_3 = \{D, \gamma\}$. Again the QFI matrix can be obtained as $\boldsymbol{H}^{(3)} = B_{23}\boldsymbol{H}^{(2)}B_{23}^T$, and the bound on the variance for the quantum discord can be easily evaluated as described in Eq. (10).

We also want to derive the classical CRB for quantum discord, that we obtain if we consider as measurement homodyne detection of squeezed and antisqueezed quadratures of a two-mode squeezed thermal state. Let us start by considering the Fisher information matrix we obtain if we want to estimate the two parameters $\boldsymbol{\lambda}_1 = \{N_s, N_t\}$ by means of homodyne detection on a certain quadrature Q_ϕ. Since the state is a Gaussian state, the conditional probability distribution of measuring a value x is a Gaussian function, with zero mean, and variance $\boldsymbol{\sigma}^2(Q_\phi)$. By using the formulas in Ref. [29] and evaluating some Gaussian integrals, one easily obtains the following formula for the Fisher matrix elements:

$$\boldsymbol{F}_{\mu\nu} = \frac{1}{2\sigma^2(Q_\phi)} \frac{\partial \sigma^2(Q_\phi)}{\partial \lambda_\mu} \frac{\partial \sigma^2(Q_\phi)}{\partial \lambda_\nu}, \quad (12)$$

where $\lambda_\mu = \{N_s, N_t\}$. If one considers measuring the squeezed or the antisqueezed quadratures one obtains the following FI matrices:

$$\boldsymbol{F}^{\text{sq/asq}} = \begin{pmatrix} \frac{1}{2N_s + 2N_s^2} & \mp \frac{1}{\sqrt{N_s(1 + N_s)(1 + 2N_t)}} \\ \mp \frac{1}{\sqrt{N_s(1 + N_s)(1 + 2N_t)}} & \frac{2}{(1 + 2N_t)^2} \end{pmatrix}.$$

If we perform a fixed number of measurements, where half of them are done on the squeezed quadratures, and the remaining ones on the antisqueezed quadratures, the overall FI matrix which will give the CRB for the two parameters $\boldsymbol{\lambda}_1 = \{N_s, N_t\}$ is obtained as

$$\boldsymbol{F}^{(1)} = \frac{1}{2}(\boldsymbol{F}^{\text{sq}} + \boldsymbol{F}^{\text{asq}}) = \text{diag}\left(\frac{1}{2N_s + 2N_s^2}, \frac{2}{(1 + 2N_t)^2} \right). \quad (13)$$

To obtain the CRB for homodyne detection of Gaussian discord, we can proceed as we showed for the quantum CRB, simply replacing the QFI matrices, with the FI ones. The values of the discord obtained using our Bayesian estimation are shown in Fig. 2: the points indicate the experimental data, while the solid line describes the model (1), where the homodyne efficiency η and the relative
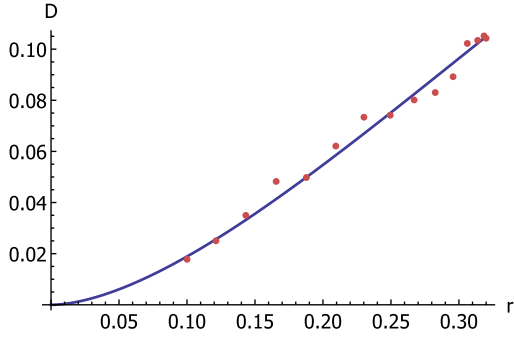
FIG. 2 (color online). Experimental values of Gaussian quantum discord from homodyne data and Bayesian estimation. The points correspond to the estimated experimental values, while the solid line is the theoretical prediction for $\eta = 0.62$ and $\gamma = 0.73$ (the value of γ has been extracted from a best-fit of the points). Uncertainties are within the point size.

FIG. 3 (color online). The noise ratio K_M as a function of discord. Circles and triangles correspond, respectively, to the quantum and the classical CRB. Solid points refer to Bayesian estimation, while empty ones correspond to estimation by inversion. The uncertainties for the inversion method are estimated using a Monte Carlo procedure on 10^6 points. Notice that the number of resources for Bayesian estimation is $M = N_b M_T$, while for the inversion method $M = M_T$.

parasite gain γ are kept to a constant value. Our model is in satisfactory agreement with the data, so we can be confident of that the CRB calculated after the matrix (11) reliably describes the ultimate limit for precision. In the formulae of the Bayes rule (6), we need to multiply several probabilities (5), which rapidly give a number hardly manageable by reasonable computing power: this sets a limit to the number of quadrature values one can effectively use in about 800 points. In order to use larger samples, we have divided our data in $N_b = 10^2$ blocks of 200 points for each of the quadratures (2), calculated the Bayesian estimation of the discord for each block, then considered the average weighted on the associated uncertainties. We notice that the *a priori* probabilities (5) are calculated from the whole set of data containing M_T values: as they intervene in the evaluation for each block, the overall number of resources to be considered is $M = N_b M_T$.

The comparison between our experimental uncertainties and both the Cramér-Rao limit for our detection (12) and the quantum Cramér-Rao limit is shown in Fig. 3, where we report the quantity $K_M = M\sigma^2(D)/(F^{-1})_{DD}$ (or the analogue quantity involving the QFI) expressed in dB. K_M is the variance of the discord estimator from homodyne data multiplied by the number of resources and divided by the relevant elements of the (quantum) inverse Fisher matrix. For K_M equal to unity we have optimal estimation. Solid points refer to Bayesian estimation while empty ones correspond to estimation by inversion. We notice that for low values of discord, the Bayesian technique provides a nearly optimal estimator for the chosen measurement strategy, whereas estimation by inversion is noisier. We also notice that the point corresponding to the lowest value of the discord is slightly below the quantum CRB: this confirms that for low values of the squeezing of the pump, the model we use is not as accurate as in other regimes. For increasing values, the observed variances depart from the optimum by less than an order of magnitude: as Bayesian estimation rapidly converges to optimal, we can attribute
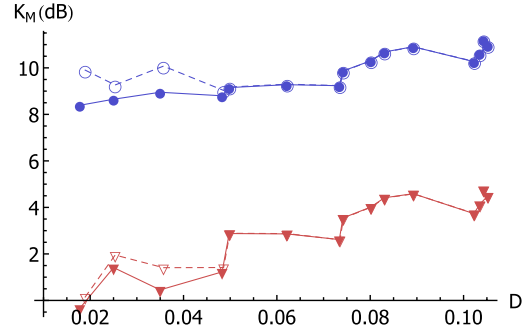
this trend to actual variations of the value of the discord in the experiment, becoming more important than statistical fluctuations when the discord increases.

The measurement we have adopted has the considerable advantage of being the simplest experimental option; however, simplicity always comes at a price, and we do not expect it to deliver the best estimator for discord as established by the quantum CRB. In the limit of low discord, we measure a ratio of about 10 dB, which tells us that the price we have to pay is quite reasonable. The departure from the quantum CRB then slightly increases with discord.

In conclusion, we have presented the experimental estimation of Gaussian quantum discord for a two-mode squeezed state. Our scheme is based on homodyne detection assisted by Bayesian analysis. Our results are in good agreement with the theoretical model, and this allows us to perform a reliable precision analysis. We found that homodyne estimation shows about 10 dB of added noise compared to the ultimate bound imposed by the quantum Fisher information, with Bayesian analysis that slighlty improves performances for small values of discord. We have also compared our results with the CRB for homodyne detection and found that the estimation is nearly optimal for small values of discord. The usefulness of quantum discord as a resource for quantum technology is a heavily debated topic, and a definitive answer may only come from experiments involving carefully prepared quantum states. Our results contribute to the precise characterization of Gaussian discord and illustrate how a suitable data processing may decrease the uncertainty when optimal detection schemes are not available.

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