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Distance in Space Syntax

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Abstract

We explore ways of introducing Euclidean distances associated with street systems represented by axial lines into the two connectivity graphs based on points (or street junctions), and on lines (or streets), the so-called dual and primal representations of the space syntax problem. As the axial line is embedded in the connectivity graph between the points, for the dual problem the specification of Euclidean distance between points is relatively trivial but for the original syntax problem, this is problematic in that it requires us to find a unique point representation for each line. The key is to find the centroids of the lines (of sight or unobstructed movement) between the points on each axial line, and then to use these to form a weighted centroid of centroids. The distances between axial lines which form paths through the connectivity graph between streets, are then computed using these centroids as starting points for each line and routing distance through the street junctions.

There are many issues involving interpretation of these measures. It might be thought that the longer an axial line, the more important it is. But by giving an axial line distance, this suggests that this is a deterrence to interaction, as in spatial interaction theory, with longer axial lines being individually less important, notwithstanding the probability that they are better connected within the overall street system. Clearly in many finer-scale morphologies, this assumption might not be tenable but the measures developed here can be easily adapted to various circumstances. What this focus on distance enables us to do is to treat a 'mixed syntax' problem where we are able to embed truly planar graphs into the axial map. This extends the technique to deal with systems not only comprising streets down which we can see, but also fixed rail lines, subway systems, footpaths and so on which currently are hard to handle in the traditional theory. We illustrate the extended theory for a pure syntax problem, the French village of Gassin, and a mixed syntax problem based on the grid of streets and underground railways in central Melbourne. In conclusion, we introduce the notion that proximity or adjacency at different orders might form more appropriate measures of syntax distance, the proximity of nodes to nodes and lines to lines in the dual and the primal being illustrated for both Gassin and central Melbourne.

1 Accessibility and Distance in Graphs

Any set of relations between a well-defined set of objects or elements can be represented and visualized as graph comprising nodes or vertices – the elements, and arcs or links – the relations between them. In spatial systems, such graphs are often literal representations of elements and their relations embedded in Euclidean space represent crow-fly or shortest route distances between locations in space such as routes along streets or migration paths between regions. If we define the set of locations as $\{j, 1, 2, ... m\}$, then the relations between them are $\{\rho_{jl}\}$ from which we can compute distances $\{d_{jl}\}$ which are Euclidean in some sense. These distances are measures of proximity or nearness called accessibility, and the usual measure is to derive measures of composite accessibility for each location with respect to every other. As distance usually acts a deterrent to movement, these measures incorporate distance or some transformation thereof in inverse fashion, a typical measure of accessibility at location j, V_j, being computed as $V_j \sim \sum_l d_{jl}^{-1}$. There is a long tradition of these kinds of representation in geographical science (see Haggett and Chorley, 1969) and in architectural systems (see March and Steadman, 1971).

In general, the nodes of a graph need not be locations even in spatial problems, and thus they need not be embedded in geographical space. In such cases, the matrix of relations $\{\rho_{jl}\}$ does not generate a distance measure in the Euclidean sense but defines a topological distance. For example, the representation of design problems in architecture by Alexander (1964) deals with topological graphs which relate to spatial problems but whose representation is based on relations between elements of the problem, not the space which it constitutes. In urban morphology however, the simplest graph-theoretic representations are firmly embedded within geographical space and although restrictive, these provide useful and easily visualizable map patterns of relative nearness or accessibility. There are, however, richer variants of urban morphology where relations are measured between elements located in geographical space but whose relationships are topological, not Euclidean measures of association. Space syntax is such as example (Hillier and Hanson, 1984; Hillier, 1996).

Here streets which intersect at junctions or corridors in buildings linking rooms are usually defined in terms of 'how far one can see' or 'how far one can move in an unobstructed manner'. The longer the street or line of sight, the more junctions it is likely to pass, with the number of such junctions being used to count the strength of relationship or accessibility to other streets. In this context, physical distance does not necessarily act as deterrent for it is the relative association of streets through their common junctions or intersections that provides a measure of distance. Streets are usually composed of more than two junctions whereas in the traditional geographical graph problem, a street is always anchored at two junctions and the measure of accessibility then depends on how far it is to other street junctions from a particular junction in question. Space syntax, however, computes accessibility in more abstract terms as a measure of how closely associated any two streets are based on how easily it is to connect them through lines of sight or unobstructed movement.

It is difficult to relate these two types of problem for the traditional geographical problem involves a planar graph where accessibility is measured between nodes whereas in space syntax accessibility is measured between streets or lines. Lines then can be seen as forming the nodes of an association graph. To generalize this conception, in a complementary paper which ideally should be read before this one (Batty, 2004), we introduced a unifying framework where we articulated a generic problem of urban morphology in terms of the relations between any two sets rooted in Euclidean space. In terms of this characterization, these are junctions and their streets – points and lines – which can be represented by an *n x m* matrix $[a_{ij}]$ where

$$a_{ij} = \begin{cases} 1 & if \quad i \Leftrightarrow j \\ 0, & otherwise \end{cases} \quad . \tag{1}$$

$\{i, 1, 2, ..., n\}$ are now streets and $\{j, 1, 2, ..., m\}$ junctions/intersections where the sign $\Leftrightarrow$ means that a street is associated with a junction and vice versa. This is an entirely generic representation which can be extended to any form of urban morphology which specifies relations between two sets. The matrix $[a_{ij}]$ also forms a graph but in this case, it is a bipartite graph; if the two sets of elements can be rooted in Euclidean space, then they can also be represented as a network in such space.

What this representation enables is a basic form which does not privilege any one set over the other. In this sense, we can study how the set $\{i\}$ relates to the set $\{j\}$ or vice versa. If we look at the problem in terms of how streets $\{i\}$ relate to each other which is through $\{j\}$, we have traditional space syntax where streets become the main focus. If we look at the problem in terms of how street junctions $\{j\}$ relate to one another, then this is the traditional geographical graph problem. We call the first problem the *primal problem* and the second the *dual problem.* We introduced this framework in detail in the previous paper but we will briefly restate it below (Batty, 2004). Mathematically neither is more important than the other although in practice, there may be very good reasons for preferring one form over the other. The primal problem thus examines relations between the streets in terms of the junctions, a key measure of which are the out-degrees of the bipartite graph, the number of junctions associated with each street, defined as

$$\ell_i = \sum_j a_{ij} \quad . \tag{2}$$

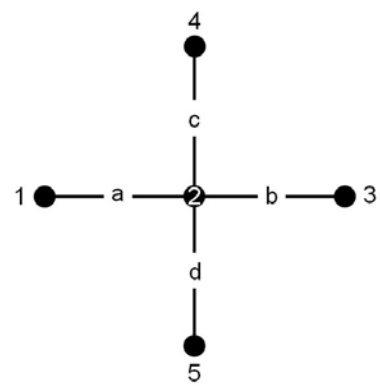
The dual problem examines the relations between the junctions in terms of streets whose key measure is the number of streets associated with each junction (the in-degrees of the bipartite graph) given as

$$\rho_j = \sum_i a_{ij} \quad . \tag{3}$$

However one of the main issues is that the traditional problem is not the dual of the space syntax problem for the matrix $[a_{ij}]$ has a rather different structure for each. The syntax problem is less restrictive in that $\ell_i \geq 2$ and $\rho_j \geq 1$ while the geographical graph problem always constrains the number of junctions associated with a street to 2, that is $\ell_i = 2$ and $\rho_j \geq 1$.

The meaning of these differences is illustrated in Figure 1(a) where for the traditional problem, we show a simple cross-shaped network as a set of five street nodes and four street segments. This is indeed a planar graph which we originally called the geographical graph problem. We also show the basic matrix $[a_{ij}]$ for this graph alongside, from which it is clear that the number of junctions for each street is exactly two, that is $\ell_i = 2$. In Figure 1(b), we have aggregated the two street segments *a* and *b* to form one single line a' and it is now clear that there are three junctions associated with this line. The $[a_{ij}]$ matrix shown alongside now has only 3 lines but still five junctions: in raw physical terms there is no difference to the underlying street network but the space syntax problem produces an abstraction of this which is still coincident with the street map at its basic level. This abstraction is called an 'axial map' and its street components are 'axial lines'.

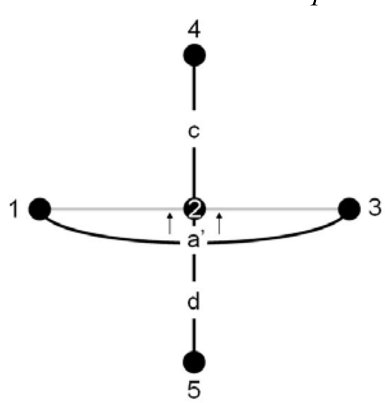
a traditional street network as a planar graph



$[a_{ij}] =$

	1	*2*	*3*	*4*	*5*	ℓ_i
a	1	1	0	0	0	**2**
b	0	1	1	0	0	**2**
c	0	1	0	1	0	**2**
d	0	1	0	0	1	**2**
ρ_j	**1**	**4**	**1**	**1**	**1**	***8***

b the space syntax axial map



$[a_{ij}] =$

	1	*2*	*3*	*4*	*5*	ℓ_i
a'	1	1	1	0	0	**3**
c	0	1	0	1	0	**2**
d	0	1	0	0	1	**2**
ρ_j	**1**	**3**	**1**	**1**	**1**	*7*

Figure 1: A Traditional Planar Graph-Street Network and a Space Syntax Representation

It is easy to guess the relative accessibilities in street maps such as those in Figure 1. For the traditional map and graph in 1(a), it is quite clear that the central junction or node 2 is the most accessible and that as each street line has the same relationship to any other, then the street line accessibility is the same for each. However accessibility is much more difficult to guess in the space syntax problem. As street a' has three junctions and the other two only two each, a' is the most accessible in that it relates directly to both streets. As the junction at the centre of the map is the same, then it seems likely that this node 2 is still the most accessible. We will however compute these relationships exactly, after we have examined the primal and dual problems and stated the various topological and Euclidean distances measures that we will work with here.

In the next section, we briefly introduce the unifying framework (Batty, 2004) and then derive and restate the various topological distance measures associated with the primal and dual problem forms. We will then derive distance measures between the points and between the lines for any form of the generic matrix $[a_{ij}]$ where $\ell_i \geq 2$ and $\rho_j \geq 1$. In fact we can generalize the problem a little further to systems where lines have only one junction associated with them. But at some point in computing distance on a line, we need beginning and end points and although space syntax does deal with lines which have only one junction, other junctions must always be implicit.

We will then examine a pure syntax problem where the streets are lines of sight and where the importance of a place clearly depends on how far one can see. We use the basic example developed by Hillier and Hanson (1984) and reworked by Peponis, et al. (1997), Batty and Rana (2004), Turner (forthcoming), and Carvalho and Batty (2004) for the French village of Gassin where we show that the topological and Euclidean measures of distance and accessibility produce entirely different patterns. We then discuss what we call a 'mixed syntax problem' which involves not only line of sight measures as axial lines but also lines of movement that do not have sight associated with them. This is the case where the technologies involved to move people are usually enclosed: trams, buses, and trains. We illustrate the problem for central Melbourne where the grid of streets lies on top of a heavy rail loop which is underground. This provides us with another perspective on accessibility but it also shows how we can extend space syntax to deal with systems where many kinds of route and mode of transport define the morphology of the city.

2 Primals and Duals: Space Syntax through Lines and Points

The generic representation in the matrix $[a_{ij}]$ allows us to look at the problem in two distinct ways: across each row or line in terms of a count of the points associated with each line, and down each column where each point is associated with a number of lines. These are the primal and dual problems respectively. The number of common points between any two lines forms a network of relations, a weighted graph, whose basic form is computed as

$$\ell_{ik} = \sum_j a_{ij} a_{kj} \qquad , \qquad (4)$$

where $[\ell_{ik}]$ is the number of points in common for any two lines. In space syntax, this matrix is usually sliced to provide a binary form such that

$$Z_{ik} = \begin{cases} 1 \quad if \quad \ell_{ik} > 0, i \neq k \\ 0, \quad otherwise \end{cases} \qquad , \qquad (5)$$

and this means that no weighting is given to the actual number of points that any two lines have in common: association or not thus depends on having at least one point in common. The total number of points in common with respect to all direct associations between one line $\{i\}$ and all other lines is calculated as

$$\tilde{\ell}_i = \sum_k \ell_{ik} \qquad . \qquad (6)$$

This out-degree can be seen as a measure of direct distance, the opposite of nearness, with respect to the line in question. As $[\ell_{ik}]$ is symmetric, then the in-degree $\widetilde{\ell}_k = \widetilde{\ell}_i, k = i$.

This primal form is the classic space syntax problem with the measure of distance $\widetilde{\ell}_i$, the number of streets that lie one depth away from the street in question. The dual problem repeats all this logic on points rather than lines. The relationship matrix $[\rho_{jl}]$ is computed from $[a_{ij}]$ as

$$\rho_{jl} = \sum_i a_{ij} a_{il} \qquad , \qquad (7)$$

where ρ_{jl} is the number of lines that points j and l have in common. The associated measure of direct distance based on the out-degrees is given as

$$\widetilde{\rho}_j = \sum_l \rho_{jl} \qquad , \qquad (8)$$

and the same symmetry conditions on the in-degrees hold. $[\widetilde{\ell}_i]$ and $[\widetilde{\rho}_j]$ are two initial measures of distance just stated although these are really counts of volume, direct nearness, or adjacency, namely the number of points for each line, and the number of lines for each point – the number of points which a line has in common with all other lines, and the number of lines a point has in common with all other points. We now need to develop more refined measures of distance based on any pair of lines and any pair of points computed from the matrices $[\ell_{ik}]$ and $[\rho_{jl}]$ respectively.

The distance measures which take account of all relationships in the graph are computed by deriving all the numbers of lines or points in common for successive path lengths through the two graphs. These graphs are always strongly connected by definition and thus successive path lengths – called step lengths –need only to be computed up to the number of lines or points in the system, no more, and usually the shortest routes will be found well before this size is reached. We will demonstrate this computational mechanism for the primal problem involving the line matrix only, for the dual follows directly. The number of points in common for two steps through the graph from line i to line k is calculated as

$$\ell_{ik}^{2} = \sum_{z} \ell_{iz}^{1} \ell_{zk} \qquad , \tag{9}$$

where $\ell_{ik}^{1} = \ell_{ik}$, and the recursion on equation (9) for any step length $s+1$ thus becomes

$$\ell_{ik}^{s+1} = \sum_{z} \ell_{iz}^{s} \ell_{zk} \qquad . \tag{10}$$

At some point where $s \leq n$, where n is the number of lines in the matrix, this recursion will converge when all paths through the graph become positive, that is when $\ell_{ik}^{s} > 0$.

The first measure of distance $d(\ell)_{ik}$ is based on step length and this is computed at each iteration of equation (10) as

$$d(\ell)_{ik} = s \quad if \quad \ell_{ik}^{s} > 0 \quad and \quad \ell_{ik}^{s-1} = 0 \qquad . \tag{11}$$

This computation eventually converges and two measures of overall accessibility or proximity for each line i can be computed from

$$\ell(d)_{i} = \sum_{k} d(\ell)_{ik}^{-1} \qquad , \qquad and \tag{12}$$

$$\hat{\ell}(d)_{i} = \frac{1}{\sum_{k} d(\ell)_{ik}} \qquad . \tag{13}$$

These measures are likely to produce similar results for the inverse weightings differ only marginally. In the case where inverse distances are computed for each link ik as in equation (12), the inverse power (-1) could be varied and the theory of spatial interaction invoked in terms of the meaning of this scaling.

There are many variants on these measures with different types of normalization often being applied. One which works directly with the number of paths through the graph and the number of points in common for each pair of lines, is based on a linear combination of the different sequential path length matrices $[\ell^s_{ik}]$ and weights these in such a way that successively longer step lengths get successively lesser weighting. This measure is defined as

$$\widetilde{d}(\ell)_{ik} = \sum_s \lambda^s \ell^s_{ik} \qquad , \tag{14}$$

where if we set $0 < \lambda < 1$, each successive term in the sum in equation (14) assumes a lesser importance. In fact, λ must be tuned so that each term in the series reduces in value with a typical value for λ to be in the order of 0.05. The matrix $[\widetilde{d}(\ell)_{ik}]$ is symmetric and thus the in-degrees or out-degrees serve as equivalent measures. As we weight the measures in terms of a decreasing contribution of numbers of points on sequential paths, then the measure is already in accessibility form. An appropriate aggregate is thus

$$\widetilde{d}(\ell)_i = \sum_k \widetilde{d}(\ell)_{ik} \qquad , \tag{15}$$

which we called the weighted accessibility.

We now have five measures of accessibility ℓ_i, $\widetilde{\ell}_i$, $\ell(d)_i$, $\hat{\ell}(d)_i$, and $\widetilde{d}(\ell)_i$ which are repeated for the dual as ρ_j, $\widetilde{\rho}_j$, $\rho(d)_j$, $\hat{\rho}(d)_j$, and $\widetilde{d}(\rho)_j$. These were broadly the measures that we introduced in the previous paper where we built up the theory of the primal and dual space syntax problems (Batty, 2004). What we now intend is to explore how these topological measures can be augmented with measures based on Euclidean distance. The measures which we introduce in the next section are not meant as substitute for the topological measures but as a complement. In fact, Euclidean distance is a somewhat different concept from the topology of relations between lines of sight which is in essence what space syntax is all about. However as the topological measures are close to geographical space for the starting point is the axial map based on its lines and points which are firmly embedded within this space, it makes sense to ask what implications a topological measure of distance between counts of lines and points embedded in geographical networks have for more traditional measures based on Euclidean space. Moreover we also need to explore what the best ways are of visualizing syntax relationships in terms of the primal and the dual are and to this end, an exploration of the properties of axial maps in terms of Euclidean distances is warranted. There are, nevertheless, many other measures of connectivity, distance, and adjacency, and by way of conclusion, we will explore measures of proximity recently introduced by Bera and Claramunt (2002).

3 Euclidean Distance in Space Syntax Graphs

We will assume as in space syntax that we are dealing with a geographical system which is composed of straight lines – axial lines or straight line segments between nodes for which have coordinates from which we can computer straight line distances. In fact our treatment easily extends to 'curved lines' which can be approximated at some level of resolution by finer straight line segments but we will not invoke such generalizations here. We will begin with the dual problem which is straightforward and for which we have coordinate pairs $\{x_j, y_j\}$ and $\{x_l, y_l\}$ for the points *j* and *l* defining a relevant line segment. Noting that the direct distance elements $[\rho_{jl}]$ which define the primal graph, can only be equal to 1 or 0 for any line *i* between *j* and *l*, $j \neq l$ due to the fact that there can be no more than 1 line between any two points, then the direct Euclidean distance d_{jl} is

$$d_{jl} = \rho_{jl}\left[(x_j - x_l)^2 + (y_j - y_l)^2\right]^{\frac{1}{2}} \quad , \qquad (16)$$

where the self-distances d_{jl} are clearly zero. Shortest routes between any points *j* and *l* can now be defined using the standard Dijkstra algorithm which in the form used here based on step lengths is formulated as follows:

$$if \quad \rho_{jl}^{s} > 0 \quad and \quad \rho_{jl}^{s-1} = 0 \quad then \quad d_{jl} = \min_{z}\left\{ d_{jz} + d_{zl} \right\} \ . \qquad (17)$$

Recursion on equation (17) occurs until all step lengths in the graph become positive or until the number of iterations s approaches the number of points *m* in the graph.

In general, the matrix $[\rho_{jl}]$ is different from that for a planar graph which we can write as $[p_{jl}]$. As a line can be associated with more than 2 nodes which is not the case in a planar graph, in the general case some elements of $[\rho_{jl}]$ are positive and equal to 1 in contrast to $[p_{jl}]$ which is a more parsimonious structure as is clear from Figures 1(a) and (b). We defined a measure of the deviation from planarity in terms of the number of points associated with each line in the previous paper (Batty, 2004) as $\Psi(\ell) = \sum_i \ell_i \big/ 2n$ but other measures based on the graph distances might be

$$\Psi(\rho) = \frac{\sum_{jl} \left| \rho_{jl} - p_{jl} \right|}{\sum_{jl} p_{jl}} \qquad \text{and} \qquad \Psi(d) = \frac{\sum_{jl} \left| d_{jl} - d(p)_{jl} \right|}{\sum_{jl} d(p)_{jl}} \ , \qquad (18)$$

where $d(p)_{jl}$ is the measure of distance computed for the planar graph which is associated with the dual syntax graph (which can be easily pruned from $[\rho_{jl}]$). It is also possible to generate trip lengths as in spatial interaction theory from these distances. If we have a loading of trips or movement volumes $\{T_{jl}\}$ on each link, then the standard mean trip length for the system can be computed as

$$T(\rho) = \frac{\sum_{jl} T_{jl}\, d_{jl}}{\sum_{jl} T_{jl}} \qquad \text{or} \qquad \overline{T}(\rho) = \frac{\sum_{jl} d_{jl}}{m^2} \ , \qquad (19)$$

where the second equation in (19) represents the case where the movements on each link are absent, hence set to unity.

Trip lengths can of course be computed for each point or node simply by summing equations (19) over l not j or vice versa but the more appropriate measures are the inverse forms for the out-degrees of the distance matrix: the sum of the inverse distances, and the inverse of the sum of the distances with respect to each point. These are true Euclidean distance potentials defined as

$$e(\rho)_j = \sum_l d_{jl}^{-1} \qquad , \qquad and \tag{20}$$

$$\hat{e}(\rho)_j = \frac{1}{\sum_l d_{jl}} \qquad . \tag{21}$$

These are measures of locational access. They could be weighted by the mass of the points as in traditional social physics and spatial interaction theory (Wilson, 1970) but in this context we will not confuse the problem. Note that the inverse sum in equation (21) is proportional to the inverse unweighted mean trip length associated with the location j.

Euclidean distance measures for the primal problem are trickier in that we need to compute centroids associated with each axial line. In essence, an axial line can relate to more than 2 points and thus there is a centroid for every such line of sight associated with the line. For example in Figure 1(b), the axial line a' is associated with three lines of sight from 1 to 2, from 2 to 3 and from 1 to 3. Thus it is logical to compute a centroid from these centroids using a simple averaging although again variable weighting might be considered. We first compute a centroid for each line of sight associated with the axial line i as

$$\left.\begin{aligned} \bar{x}_{ijl} &= a_{ij} a_{il} (x_j + x_l)/2 \\ \bar{y}_{ijl} &= a_{ij} a_{il} (y_j + y_l)/2 \end{aligned}\right\} \qquad . \tag{22}$$

These centroids need to be averaged and this is accomplished by

$$
\left.\begin{aligned}
\bar{x}_i &= \sum_{\substack{j \neq l \\ l > j}} \frac{x_{ijl}}{m(m-1)/2} \\
\bar{y}_i &= \sum_{\substack{j \neq l \\ l > j}} \frac{y_{ijl}}{m(m-1)/2}
\end{aligned}\right\} \qquad , \tag{23}
$$

where the summations are over all pairs of points associated with the line in question which is equivalent to averaging the coordinates of all relevant points.

This simple averaging could be augmented with differential weights if views along an axial line to different points reflected differing degrees of importance but here we will stick with the non-weighted form. We can now compute distance between any two axial lines as i and k by taking the distance from the centroid of line *i*, say, to the point which is common to the line *k* to which it is being linked. This distance is

$$
d_{ik} = \ell_{ik} \left\{ \left[(\bar{x}_i - x_j)^2 + (\bar{y}_i - y_k)^2 \right]^{\frac{1}{2}} + \left[(\bar{x}_i - x_j)^2 + (\bar{y}_i - y_k)^2 \right]^{\frac{1}{2}} \right\} , \tag{24}
$$

due the fact that axial lines are straight. This operation could easily be generalized to non-straight lines by replacing them with finer scale straight lines and operating recursively on equation (24). As each line has a mass – that is, it has finite length – then it is possible to compute an intra-line distance, a self-distance which must be specified as

$$
d_{ii} = \sum_{\substack{j \neq l \\ l > j}} a_{ij} a_{il} \frac{\left[(x_j - x_l)^2 + (x_j - x_l)^2 \right]^{\frac{1}{2}}}{\left[m(m-1)/2 \right]} \qquad . \tag{25}
$$

In this paper, we will set $d_{ii} = 0$ as we follow the tradition of space syntax but other arguments can clearly be made for keeping this self-distance as a positive deterrent to mobility.

We are now in a position to compute the shortest routes between lines and we do this in exactly the same manner we did for distances between points which we illustrated in equation (17), that is

$$if \quad \ell_{ik}^{s} > 0 \quad and \quad \ell_{ik}^{s-1} = 0 \quad then \quad d_{ik} = \min_{z}\left\{ d_{iz} + d_{zk} \right\} \quad , \tag{26}$$

where convergence is guaranteed by the time $s = n$. We can now compute the line accessibilities from $[d_{ik}]$ in the form of the sum of the inverse or the inverse of the sum. Then in analogy to equations (20) and (21)

$$e(\ell)_i = \sum_k d_{ik}^{-1} \; , \qquad and \tag{27}$$

$$\hat{e}(\ell)_i = \frac{1}{\sum_k d_{ik}} \; . \tag{28}$$

All the other measures involving trip lengths which we noted in equations (19) apply and the lines can be weighted with trip volumes if required. However as in space syntax, we assume that this is not necessary for problems involving lines of sight, and all we require is an overall measure of line distance which we define as

$$\overline{T}(\ell) = \frac{\sum_{ik} d_{ik}}{n^2} \qquad . \tag{29}$$

We now have two more measures of distance to add to our arsenal. To illustrate the subtle differences in meaning of these measures, we have computed all of them for the dual and primal problems which emanate from the planar street network graph in Figure 1(a) and the axial street map in Figure 1(b).

For the street network in Figure 1(a) which is a planar graph in terms of the dual problem formulation, where each line or street has exactly two points or junctions associated with it, the accessibilities of any point to any other is obvious by inspection. Point or node 2 is clearly the most central and in a commonsense way has the highest, while the four others have lower but equal accessibility. What we have done is interpolated between these point accessibilities producing a surface in the time-honored way and this is shown in Figure 2(a). There are clear edge effects in such surfaces which are hard to control for but in general, the pattern of point accessibility varies inversely with distance from the central point for all measures, step-length measures, as well as those based on Euclidean distances. For the primal problem involving accessibility from any line to all others, each line would appear to have the same accessibility due to the nature of the symmetry. This indeed is the case as we show in Figure 2(b) where the pattern of accessibility is uniform over the entire space. Once again, this applies to all measures. In the planar street network case, it would appear from this simple example and from our intuition that all the measures co-vary with one another, notwithstanding differences in distribution.

In terms of the syntax problem in Figure 1(b), here two of the lines in planar network case are collapsed into one, a' being formed from a and b. In this case intuition suggests that the point accessibility involving the nodes is much the same as before but as the two north-south lines c and d involve more step lengths to reach line a', then the two north-south nodes 4 and 5 are slightly less accessible in term of step lengths $\rho(d)_j, \rho(d)_j$ as Figure 2(c) shows. For the direct step lengths, weighted distance and Euclidean distance accessibilities $\rho_j, \widetilde{\rho}_j, \widetilde{d}(\rho)_j, e(\rho)_j, \hat{e}(\rho)_j$, then the nodes have the same pattern of accessibility as the planar street network as Figure 2(e) shows. In terms of lines, the binary step length and weighted distance measures $\widetilde{\ell}_i, \ell(d)_i, \hat{\ell}(d)_i, \widetilde{d}(\ell)_i$ show each line with equal accessibility [Figure 2(d)] in contrast to the other measures $\ell_i, e(\ell)_i, \hat{e}(\ell)_i$ where the merged line a' is more accessible than the other lines c and d [Figure 2(f)]. This indicates that the number of lines of sight associated with a line like a' does reinforce the importance of the line especially as this line is central to the morphology. However as we will see, this is not a straightforward issue as in systems where there are many short lines with some long ones dominating, then the pattern of Euclidean accessibility can be quite different from the step-length accessibilities.

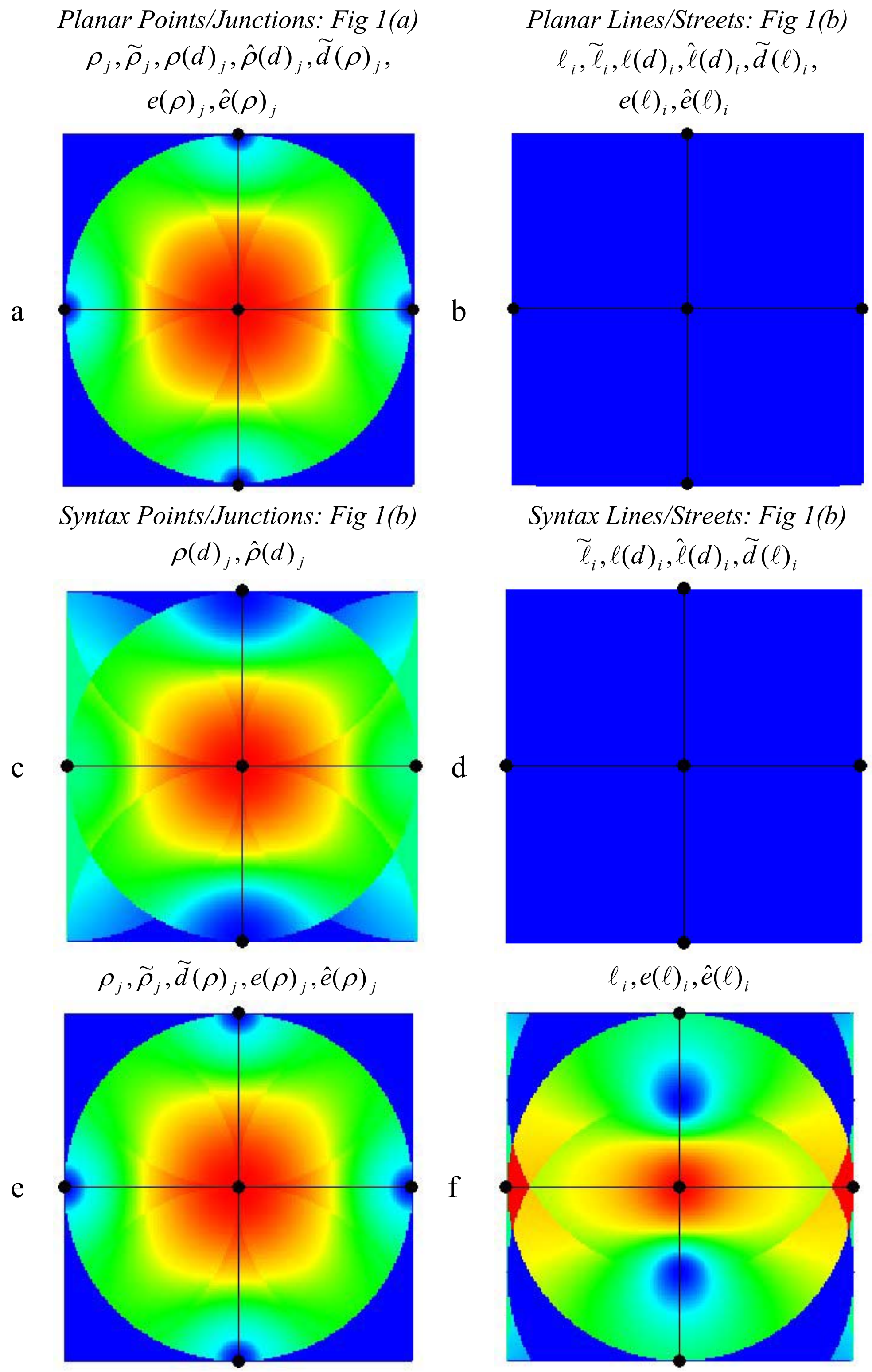


Figure 2: Accessibility Surfaces for the Primal and Dual Problems from the Simple Planar and Axial Maps shown in Figures 1(a) and 1(b)

Before we move to a realistic problem, it is worth noting the way in which surface distributions are produced from the point and line estimates. A technique of spatial averaging using a kernel which is centered over each cell in question, diffuses the value of that cell over a given search radius and does this until all the cells are filled in one pass. For a system of five nodes say, each value at each node is generalized this way with the diffusion being based on an inverse distance allocation from the points in question. The technique is stopped at the boundary of the system in this case where there is an abrupt cut-off, hence the skew introduced into the diffusion at the edge. Basically this could be controlled by putting the boundary further out and this diffusion would become smooth but we prefer to visualize this elementary problem in its most basic form.

4 The Pure Syntax Problem: Applications Once Again to Gassin

We are not going to repeat the data for Gassin because this has been reproduced in a number of papers as a benchmark example of space syntax. It was originally presented by Hillier and Hanson (1984), reworked by Peponis et al. (1997), Batty and Rana (2004), Turner (forthcoming), and Carvalho and Batty (2004). Nor will we generate all forms of syntax map for the primal and dual problems avoiding representing the weighted lines across the standard color range for the line accessibilities or pie charts for the measures of accessibility at each point. We will stick to the surface representations as these provide good impressionistic pictures of the variations in accessibility over the map of streets. For the primal maps, we will overlay the street line, and for the dual, the street junctions/points.

We refer to Gassin as a 'pure space syntax' problem in that there was no intention in the original application of measuring accessibility in terms of Euclidean distance. Axial lines are lines of sight and the longer the line of sight, the more likely the line to intersect with other lines of sight at junctions. But the longer the line of sight, the longer the distance associated with that line. Although we will not set a measure of Euclidean distance for the relation of each line to itself [as identified above in equation (25)], a long axial line is given a centroid which reflects its length. All other things being equal, it would be more distant, hence less accessible to other lines in the system. In contrast in terms of measuring nearness to other lines without taking distance into account, its length would not be a factor *per se*. However it would intersect with more lines, all other things being equal, and hence its accessibility to all lines would be greater. In Figure 1(b), this was not the case because the symmetry of the system and the equal distance of the elemental line segments ensured that the axial line was best connected in both step length and Euclidean distance terms.

In Gassin shown below in terms of its axial lines in Figure 3, we have several long lines of sight but in general these are not in the area of the village where there is the greatest concentration of streets. Thus in terms of distances, the cluster of lines which mark the village core are very close to one another in distance terms and in general we might expect that accessibility computed between these lines would be much higher than that between the longer lines. As we will show, this indeed is the case and in Gassin, the space syntax interpretation is qualitatively different from that based on Euclidean distance. The same difference is reflected in the clustering of the street intersections which again reflects the clustering of the lines and as we have noted before (Batty, 2004), Gassin is close to planarity with $\Psi = \sum_i \ell_i \big/ 2n = 1.065$, hence the close association of street line accessibility with street junction accessibility.

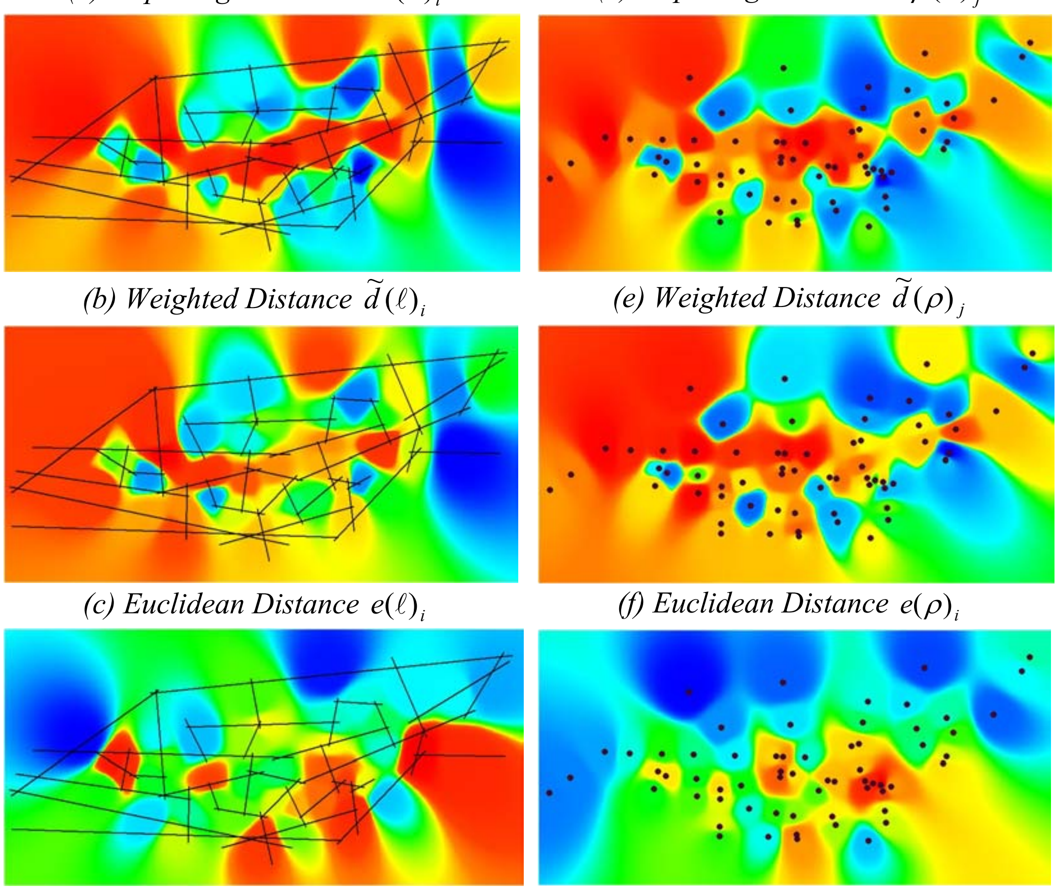
We have computed the seven distance measures for both the primal and dual problems and we show the correlations between them in Table 1. Those for the primal problem in Table 1(a) which deals with accessibility between the lines partitions very cleanly into two sets (which we show in **bold** and *italic* type). The measures based on step length which do not have any implication for Euclidean distances are highly correlated. The two Euclidean measures are highly correlated but the correlations between these two sets are low and negative. This effects the qualitative difference in what is being measured; lines of sight in Gassin do not correlate very well with the clusters of junctions that provide the most accessible central areas in terms of Euclidean distance. In terms of the dual problem, the points are more highly correlated but the same distinction exists between non-Euclidean and Euclidean measures. In fact for the raw out-degrees data, the correlations with all other measures are very low and this simply implies that the distribution of the out-degrees for the points – that is the number of lines associated with each point is either 2 or 3 implying a step-like function.

(a) Line Access	ℓ_i	$\tilde{\ell}_i$	$\ell(d)_i$	$\hat{\ell}(d)_i$	$\tilde{d}(\ell)_i$	$e(\ell)_i$	$\hat{e}(\ell)_i$
ℓ_i	•	**0.971**	**0.770**	**0.839**	**0.967**	-0.088	-0.099
$\tilde{\ell}_i$		•	**0.807**	**0.897**	**0.984**	-0.094	-0.098
$\ell(d)_i$			•	**0.973**	**0.810**	-0.275	-0.272
$\hat{\ell}(d)_i$				•	**0.889**	-0.223	-0.222
$\tilde{d}(\ell)_i$					•	-0.122	-0.112
$e(\ell)_i$						•	*0.970*
$\hat{e}(\ell)_i$							•

(b) Point Access	ρ_j	$\tilde{\rho}_j$	$\rho(d)_j$	$\hat{\rho}(d)_j$	$\tilde{d}(\rho)_j$	$e(\rho)_j$	$\hat{e}(\rho)_j$
ρ_j	•	0.393	0.156	0.215	0.297	-0.092	-0.087
$\tilde{\rho}_j$		•	**0.792**	**0.900**	**0.984**	-0.006	-0.085
$\rho(d)_j$			•	**0.973**	**0.806**	-0.122	-0.169
$\hat{\rho}(d)_j$				•	**0.912**	-0.093	-0.151
$\tilde{d}(\rho)_j$					•	-0.060	-0.141
$e(\rho)_j$						•	*0.949*
$\hat{e}(\rho)_j$							•

Table 1: Correlations between the Seven Distance Measures for Gassin

We would expect all these differences to be reflected in the surfaces associated with the spatial distribution of these measures. In Figure 3, we show these for three of the accessibility measures which we consider are the best reflectors of the difference between the distributions, namely the step lengths $\ell(d)_i$ and $\rho(d)_j$ which are the basic space syntax measures, the weighted distance measures $\tilde{d}(\ell)_i$ and $\tilde{d}(\rho)_j$ which are alternative measures of the syntax, and the Euclidean measures $e(\ell)_i$ and $e(\rho)_j$ which measure accessibility over the physical network. The patterns shown in Figure 3 bear out the differences that we suggested at the beginning of this section. The step length and weighted measures generate the same surfaces as those we illustrated in the previous but complementary paper (Batty, 2004) with any slight differences due to the fact that we use a narrower range of colors and a smaller exponent of spatial averaging here. For both the primal and dual problems, we derive surfaces where the central axis of the village is the area where streets and their junctions are most accessible to each other with the west of the village more accessible than the north, south, or east [Figures 3(a) and (d), and (b) and (e)]. The Euclidean distance measures in Figures 3(c) and (f) produce a quite different picture. The most accessible street and their junctions are in the areas where the streets are shortest and the lines densest. These do not correlate with the longest lines of sight and this the picture is one where the clusters of high accessibility are broken up along the central axis but with a tendency towards highest accessibility in the south east of the village. This is about all we can say for Gassin: that space syntax is very different from the street distance accessibility and that this in itself is the basis of informed speculation as to how the visual quality of the town and the location of its key land uses, the movement patterns therein, all relate to these different measures of accessibility.



The Primal Lines/Streets Problem — *The Dual Points/Junctions Problem*

(a) Step-Length Distance $\ell(d)_i$ — *(d) Step-Length Distance* $\rho(d)_j$

(b) Weighted Distance $\tilde{d}(\ell)_i$ — *(e) Weighted Distance* $\tilde{d}(\rho)_j$

(c) Euclidean Distance $e(\ell)_i$ — *(f) Euclidean Distance* $e(\rho)_i$

Figure 3: Key Accessibility Measures for the Primal and Dual Pure Syntax Analysis of Gassin

5 The 'Mixed Syntax' Problem: Systems with Overground and Underground Routes

What we have in these two types of distance measure is a mix of accessibility indices based on lines of sight and physical travel distance. There are however many systems where travel is based not only on streets down which one can see but also on routes where one cannot see, as in underground railways, or even on routes where sight is of much lesser importance such as on buses or trams. To conclude this paper, we will apply these ideas to systems where we can easily define such mode differences which in turn reflect a mixture of axial lines based on lines of sight or unobstructed movement, and route segments which reflect planarity. Our application is to central Melbourne which is laid out on a grid but around which there is a heavy rail loop, buried underground. We show the axial map/planar route network in Figure 4 where we distinguish between the two types of route. There is a much denser morphology of routes in the CBD than we shown in Figure 4 and the central area is criss-crossed by surface level trams. But the really distinctive structure is the underground railway which connects to the street level at some 5 key stations. If one wishes to loop around the CBD, then the fastest way to do this and the shortest is using this railway, so it is certain that this will make an impact on accessibility if Euclidean distance is taken account of. If you want to see places within the CBD, then the long straight streets provide perfect axiality and thus the contrast between getting to a place fast and seeing the same place, immediately, could not be greater.

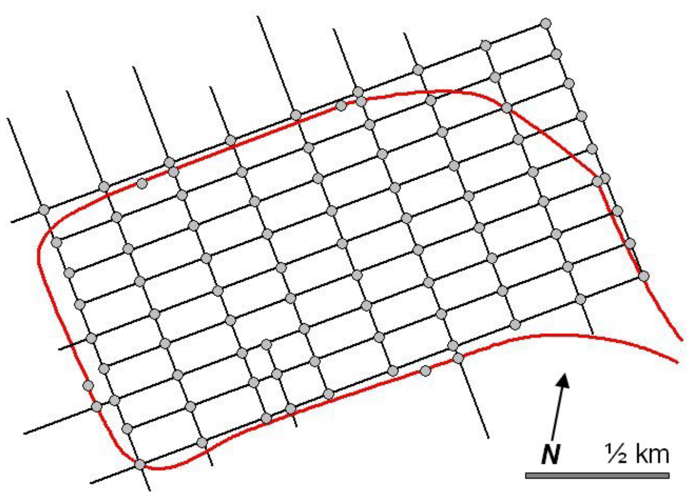


Figure 4: The Street Grid for Central Melbourne with the Underground Rail Loop

(a) Line Access	ℓ_i	$\tilde{\ell}_i$	$\ell(d)_i$	$\hat{\ell}(d)_i$	$\tilde{d}(\ell)_i$	$e(\ell)_i$	$\hat{e}(\ell)_i$
ℓ_i	•	**0.993**	**0.899**	**0.938**	**0.994**	-0.055	0.124
$\tilde{\ell}_i$		•	**0.934**	**0.967**	**0.999**	-0.098	0.100
$\ell(d)_i$			•	**0.991**	**0.928**	-0.170	0.039
$\hat{\ell}(d)_i$				•	**0.962**	-0.143	0.070
$\tilde{d}(\ell)_i$					•	-0.089	0.103
$e(\ell)_i$						•	*0.849*
$\hat{e}(\ell)_i$							•

(b) Point Access	ρ_j	$\tilde{\rho}_j$	$\rho(d)_j$	$\hat{\rho}(d)_j$	$\tilde{d}(\rho)_j$	$e(\rho)_j$	$\hat{e}(\rho)_j$
ρ_j	•	0.169	0.075	0.079	0.145	-0.012	-0.207
$\tilde{\rho}_j$		•	**0.916**	**0.944**	**0.999**	-0.062	-0.193
$\rho(d)_j$			•	**0.996**	**0.922**	0.057	-0.110
$\hat{\rho}(d)_j$				•	**0.949**	0.028	-0.129
$\tilde{d}(\rho)_j$					•	-0.059	-0.191
$e(\rho)_i$						•	*0.823*
$\hat{e}(\rho)_i$							•

Table 2: Correlations between the Seven Distance Measures for Central Melbourne

In Table 2, we measure the correlations between the seven accessibility measures which we have computed for both the primal and dual problems. The structure of these bears a remarkable similarity to those in Table 1 for Gassin with the out-degree, step-length and weighted measures being highly correlated with one another in contrast to the Euclidean measures which in turn are highly correlated but not with the first set of measures. As the distribution of points in each line has greater variability than the distribution of lines over each point (the planarity measure is $\Psi = 1.033$ which shows that the map is very nearly planar in these terms), these raw out-degree measures form the first set of dense correlations. In the dual problem however which involves the points, correlations between these out-degrees and the other two sets of measures are low. In short what we have here even before we begin to explore the spatial distribution of these measures, is consistency between the primal and dual in terms of a major difference between the step-length type measures and the Euclidean. Step-length measures which pick up syntax as nearness in terms of the way lines of sight are close or far from one another, is dramatically different from the way lines and points are near to each other in terms of their physical distance. This is also clear from Figure 4 for the railway has few points of contact with the street system but its relative accessibility to the streets where it touches, is much higher than the more even distribution of street junctions and the lines that link these.

We will examine the surfaces for our three key measures $\ell(d)_i$, $\tilde{d}(\ell)_i$ and $e(\ell)_i$ in Figure 5 where we also show the basic out-degrees for the lines ℓ_i which are illustrated in terms of line thickness. Note how the railway has very few points for each of its lines/track. The patterns for step-length and weighted distance accessibility in Figures 5(b) and 5(c) generate the highest accessibility in terms of the nearness to different lines of sight, broadly in the centre of the CBD. The area to the south west of the physical center of the CBD map reveals a pocket of low accessibility – lines of sight with few common points, where the major grid is permeated by a couple of local narrower streets. However when we look at the Euclidean distance in Figure 5(d), the stations along the rail routes are pockets of lower accessibility because again at those points, there are much lesser number of lines of sight that you can reach.

The dual problem which involves accessibility between points or street junctions is even clearer in its distinctions between the step-length distance and Euclidean measures. In Figure 6(a), we show the distribution of lines for each point as simple pie charts and this reveals that out of 93 junctions, there are only 5 which have more than 2 lines associated with them and in those 5 cases there are only 3 such lines. This indicates how close the network is in terms of planarity. The step-length measures in Figure 6(b) and the weighted measure in Figure 6(c) are highly correlated and both show that it is the station areas that have the lowest accessibility. This is because there are less points from which to see long vistas. In contrast, Figure 6(d) shows exactly the opposite: the stations are the high points of accessibility and form the heartland of the CBD where many roads intersect near to stations. In all cases however the areas on the very edge of the map have lowest accessibility as one might expect from the imposition of arbitrary boundaries on the problem.

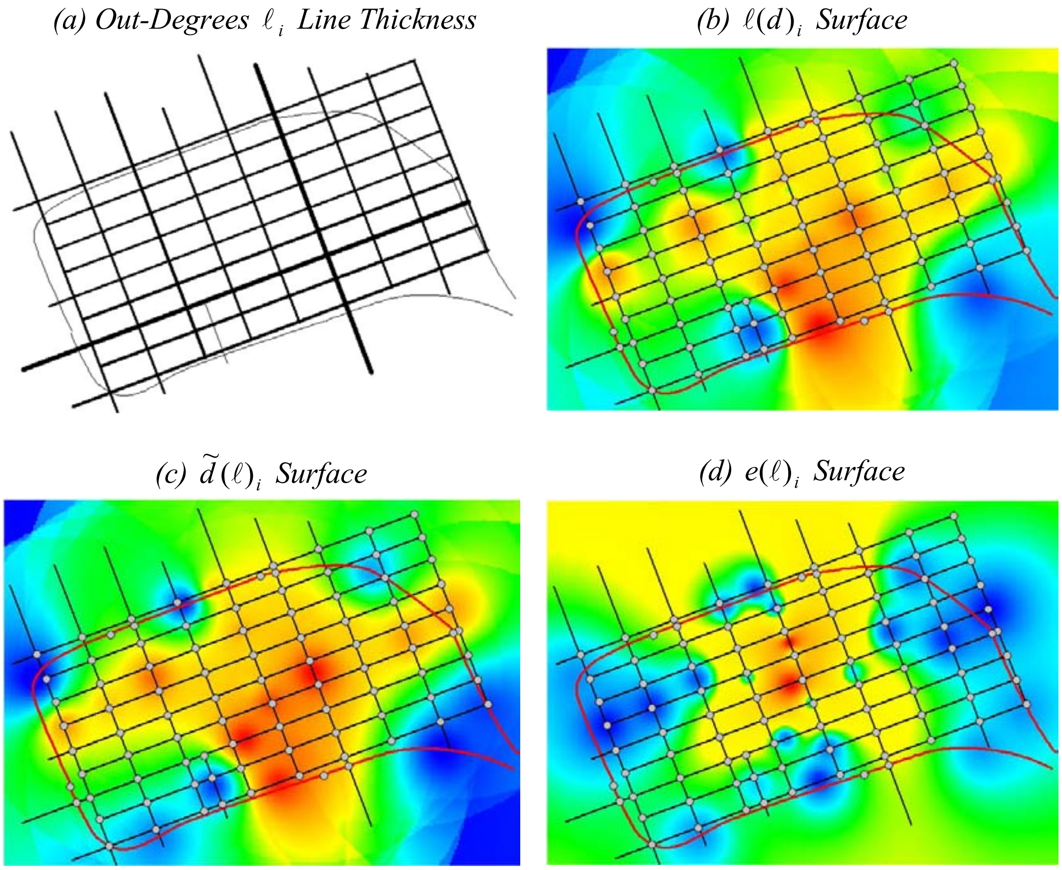


Figure 5: Line Accessibility Surfaces Based on the Out-Degrees (a), Step-Distances (b), Weighted Distances (c), and Euclidean Distances (d)

A comparison of Figures 3 and 6 for Gassin and Melbourne is instructive for there are many interesting comparisons to explore further. The wealth of interpretations which come from these two types of distance measure suggest that the way forward involves many syntaxes, rather than one, with the consequent challenge that the diversity of indices and surfaces associated with such multiple syntaxes needs to be integrated.

6 Proximity: Extending the Measures of Step-Length Distance

The critical difference between space syntax and geographical graph representations involves the nature and meaning of distance in the two types of problem. In syntax, the starting point is a topological representation of relationships between streets as lines while in geographical problems the relationships constitute physical measures of distance between nodes. The fact that the two types of problem are rooted in the same underlying geometry of the street system generates a confusion of purpose that has plagued all critical debate abut the meaning of space syntax since its inception. Although this and related papers are an attempt to clarify this difference, the fact remains that space syntax does not concern itself with geometric distance but simply with whether or not there is a relationship between two points or lines in space, not the physical distance between these points or lines. Thus proximity as it is measured in terms of adjacency in the bipartite graph, in the dual and primal topological graphs that are generated from the bipartite representation, or in the step-length distances that are generated from these graphs form the core analytical tools for dissecting the syntactical structure of urban space.

There are however new measures of proximity being devised which appear to have important advantages over the traditional step-length distances in graphs. In spatial systems, a proposal by Bera and Claramunt (2002) depends on a subtle manipulation of the concept of adjacency which is based on weighted sum of a direct measure – whether or not a line (or point) is linked to another, and the commonality between the set from which the link originates and the set associated with the adjacent destination. We can use several measures to express direct adjacency such as that used earlier in equation (5) as $Z_{ik} = 1 \;\; if \;\; \ell_{ik} > 0$, otherwise $Z_{ik} = 0$. We now need to extend this definition, notating it with respect to lines and points, and this equation (5) then becomes

$$Z(\ell)_{ik} = \begin{cases} 1 & if \quad \ell_{ik} > 0, i \neq k \\ 0, & otherwise \end{cases}, \quad Z(\rho)_{jl} = \begin{cases} 1 & if \quad \rho_{jl} > 0, j \neq l \\ 0, & otherwise \end{cases}. \tag{30}$$

We need to define the out-degrees (and in-degrees) of these measures for these define the size of the set associated with lines and points. From equation (30), then

$$Z(\ell)_i = \sum_k Z(\ell)_{ik} \,, \quad \text{and} \quad Z(\rho)_j = \sum_l Z(\rho)_{ijl} \qquad , \tag{31}$$

where it is clear from our previous definitions that the in-degrees and out-degrees are symmetric due to the fact that $Z(\ell)_{ik} = Z(\ell)_{ki}, i = k$ and $Z(\rho)_{jl} = Z(\rho)_{jl}, j = l$.

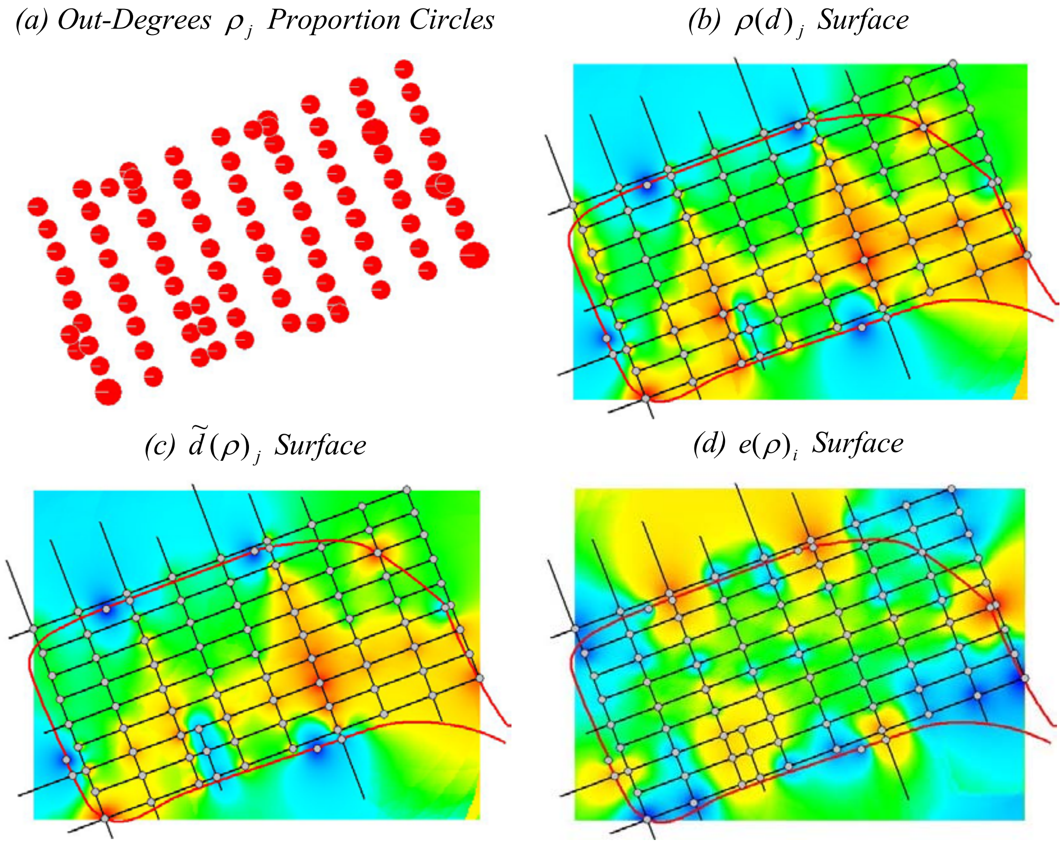


Figure 6: Points Accessibility Surfaces Based on the Out-Degrees (a), Step-Distances (b), Weighted Distances (c), and Euclidean Distances (d)

The new measure can be defined for lines (points follow by analogy) as

$$R(\ell)_{ik} = \alpha \; Z(\ell)_{ik} + (1-\alpha) \sum_{z \in \Omega_i} \frac{R(\ell)_{zk}}{Z(\ell)_i} \qquad , \qquad (32)$$

where the two components on the RHS of equation (32) are weighted by the parameter $0 \le \alpha \le 1$. The first component is simply the adjacency index as defined in equation (30) which gives a unit link from i to k if a link from one element to an adjacent one exists. The second is a relative measure which compares the number of elements adjacent to the origin set Ω_i to those which are linked to the destination set associated with k. This is a little like a first-order clustering coefficient similar to that used to define clustering in small world graphs by Watts (1999). The weighted sum essentially compares the direct link between i and k to the number of common intermediate links between i and k through elements z which are common to i. If there are few in common between *i* and *k*, then the measure will reduce in its impact where, for example, we have equal weighting. The weights themselves of course control the strength of the direct and indirect adjacencies.

Bera and Claramunt (2002) show that this system of linear equations (32) has a unique solution for certain minimal constraints on the adjacency matrix (such as strong connectivity, for example). We have solved this system using iteration and for the size of problems involved here – Gassin with 41 lines and 63 points, and central Melbourne with 25 and 119 – the procedure is fast, taking no more than 20 iterations for the dual or the primal in each application. A key feature of the solution is that the resultant matrix of relative adjacencies $[R(\ell)_{ik}]$ and $[R(\rho)_{jl}]$ is not symmetric with the measure picking up the fact that the set of streets or junctions which are accessible from *i* to *k* is not the same as that from *k* to *i*. This however does not involve any uni-directional links for essentially the basic adjacency graphs are not directed and this implies that we need to examine both the out-degrees and in-degrees of the relevant matrices. For lines (and points follow directly), these are defined as

$$R(\ell)_i = \sum_k R(\ell)_{ik} \;, \quad R(\ell)_k = \sum_i R(\ell)_{ki} \;, \text{ and } R(\ell)_i \neq R(\ell)_k \,, \; i = k \,. \quad (33)$$

In fact, we might expect these measures to be quite close to one another because the adjacent sets considered in the formula are only one step removed. This suggests that other measures incorporating higher-order adjacencies at larger and larger step lengths might be constructed. Although Bera and Claramunt (2002) do not extend their measures in this way, they do show how the measure can be weighted by variables that reflect geometric properties such as distance, perimeter, and area, thus suggesting as we do here, how Euclidean distance information might be handled.

We have reworked the Gassin and central Melbourne examples with these proximity measures and we show the correlations between these and the seven measures used in Tables 1 and 2 in Tables 3(a) and 3(b) respectively. The same structure as we displayed in Tables 1 and 2 is revealed by this comparison in that the proximity measures have high correlations with all the traditional measures and low correlations with the Euclidean measures for the two examples, with respect to both the dual and primal problems. As in the previous tests, correlations with the point distributions for the relative adjacency out-degree measures in both examples have low correlations because the out-degrees of the points have hardly any structure, reflecting the near planarity of each example. Correlations between the relative adjacency in-degrees and out-degrees are quite high and a more graphic demonstration of this is illustrated in the surface representations of these two measures shown in Figures 7(a) and (b) for lines and points in Gassin and in Figures 8(a) and (b) in central Melbourne.

Table 3(a) Gassin

Line Access	$R(\ell)_i$	$R(\ell)_k$	**Point Access**	$R(\rho)_j$	$R(\rho)_l$
ℓ_i	**0.868**	**0.887**	ρ_j	0.165	0.222
$\tilde{\ell}_i$	**0.945**	**0.966**	$\tilde{\rho}_j$	**0.953**	**0.982**
$\ell(d)_i$	**0.862**	**0.852**	$\rho(d)_j$	**0.831**	**0.818**
$\hat{\ell}(d)_i$	**0.949**	**0.941**	$\hat{\rho}(d)_j$	**0.928**	**0.918**
$\tilde{d}(\ell)_i$	**0.938**	**0.952**	$\tilde{d}(\rho)_j$	**0.985**	**0.989**
$e(\ell)_i$	-0.175	-0.135	$e(\rho)_i$	-0.040	-0.005
$\hat{e}(\ell)_i$	-0.165	-0.123	$\hat{e}(\rho)_i$	-0.120	-0.091
$R(\ell)_i$	•	*0.983*	$R(\rho)_j$	•	*0.982*
$R(\ell)_k$	*0.983*	•	$R(\rho)_l$	*0.982*	•

Table 3(b) Central Melbourne

Line Access	$R(\ell)_i$	$R(\ell)_k$	**Point Access**	$R(\rho)_j$	$R(\rho)_l$
ℓ_i	**0.957**	**0.975**	ρ_j	0.067	0.089
$\tilde{\ell}_i$	**0.975**	**0.993**	$\tilde{\rho}_j$	**0.985**	**0.996**
$\ell(d)_i$	**0.946**	**0.953**	$\rho(d)_j$	**0.960**	**0.921**
$\hat{\ell}(d)_i$	**0.977**	**0.982**	$\hat{\rho}(d)_j$	**0.980**	**0.948**
$\tilde{d}(\ell)_i$	**0.976**	**0.992**	$\tilde{d}(\rho)_j$	**0.989**	**0.998**
$e(\ell)_i$	-0.087	-0.127	$e(\rho)_i$	-0.034	-0.050
$\hat{e}(\ell)_i$	0.112	0.085	$\hat{e}(\rho)_i$	-0.167	-0.173
$R(\ell)_i$	•	*0.987*	$R(\rho)_j$	•	*0.989*
$R(\ell)_k$	*0.987*	•	$R(\rho)_l$	*0.989*	•

Table 3: Correlations between the Nine Measures for Gassin and Central Melbourne

Figures 7 and 8 reveal that the relative adjacency measures in these two examples are near symmetric as might be expected in systems where surfaces are completely covered by streets. In the applications discussed by Bera and Claramunt (2002) where there is real structure in terms of what is adjacent to what, between countries, for example, the lack of symmetry is much more significant but in these examples, this is not the case. In fact, what these proximity indices show is that the proximities of lines in the primal and nodes in the dual problem in Gassin are quite close, apart from the importance of the northern axis which is stronger in the primal than the dual. In Melbourne there is a little less correspondence between the primal and dual; the line-based primal problem reveals proximity measures which are much more spread over the central area than in the case of the nodes in the dual where the north-south axes produce striations in the accessibility surface distorting the spread. Unlike the Euclidean distance measures, the position of the rail stations and their relative inaccessibility to the street system in terms of their remoteness to lines of sight explains the relative lack of proximity of these points within the central area.

7 Conclusions: Next Steps

What this paper has shown in quite graphic terms is that the accessibility measures associated with axial lines and their intersections are quite different from that which result from measuring physical distance between such points and lines. The two types of measure imply two different problems but the problems are only separate in conceptual terms because they are defined with respect to the same network of physical relations: the street network which contains the axial map. In the case where we have a pure syntax problem – where axial lines are defined solely with respect to lines of sight and where they are then used to interpret physical distance – we have two largely separable problems but with an ability to compare line of sight accessibility with accessibility based on physical distance, How these accessibilities interact with each other to produce the kind of morphologies that emerge in cities is part of the problematic as the theory of space syntax suggests that such accessibilities are instrumental in generating the forms that we see around us (Hillier, 1996). The question thus posed is: which accessibilities should we define to show that this is actually the way cities develop, is there combination of them that will achieve this, and what are these multiple accessibilities?

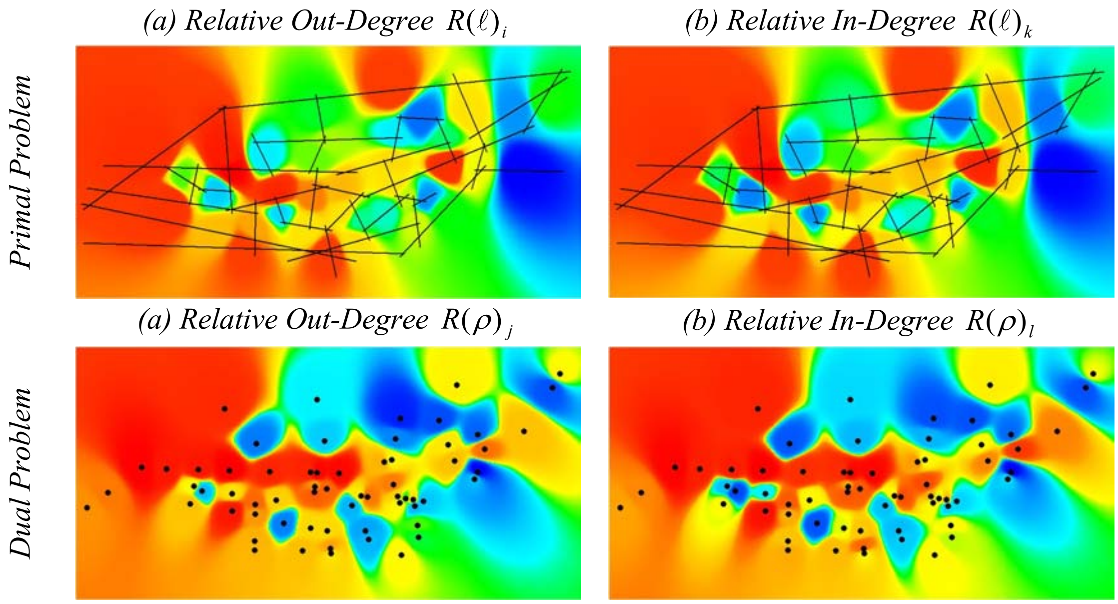


Figure 7: Relative Proximities for the Primal and Dual Pure Syntax Analysis of Gassin

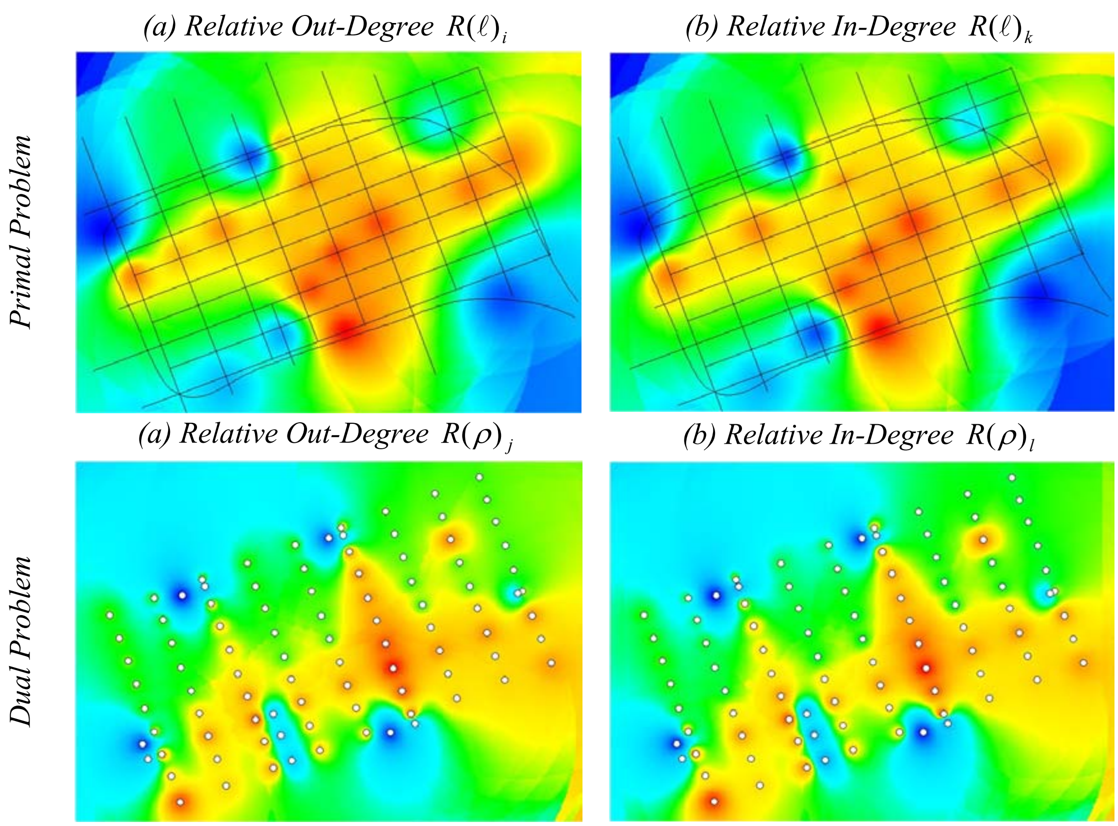


Figure 8: Relative Proximities for the Primal and Dual Analysis of Central Melbourne

The mixed syntax problem complicates the picture even further. In this case it is not possible to see physical and topological accessibilities as being separable. The street network can be based on lines of sight, but there is always a part of the network which does not have lines of sight and where physical accessibility is paramount. How can one compare different step-length accessibilities with physical distance accessibilities when the actual routes defining such accessibilities are themselves mixed? Probably the answer lies in identifying different types of street network for different purposes and reorienting the analysis this way. The task remains however of integrating the measures that are derived for each problem in some way once they have been generated. This then constitutes the next challenge – to actually work out whether accessibility measures for the primal and dual problems can somehow be partitioned to be associated with Euclidean or with line of sight step-length distances. The challenge remains of how to generate different measures and use them where the lines and points involved are associated with one or the other or a mixture of these.

Our digression into proximity measures which clearly correlate highly with traditional space syntax distances based on step length, reinforces the need to look at adjacency rather than geometric measures, and this suggests that the material of this paper far from providing the last word on distance in space syntax is just the beginning. So what began with some coherence about primals and duals, lines and points, has emerged into a debate about topological, Euclidean and proximity distances and a mix of these in systems where topology and physical distance are clearly all appropriate, but probably reflecting different purposes. It is these distinctions of purpose in characterizing urban morphology that future comparisons of distance and accessibility must address.

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