HELMHOLTZ GEMEINSCHAFT

AWI

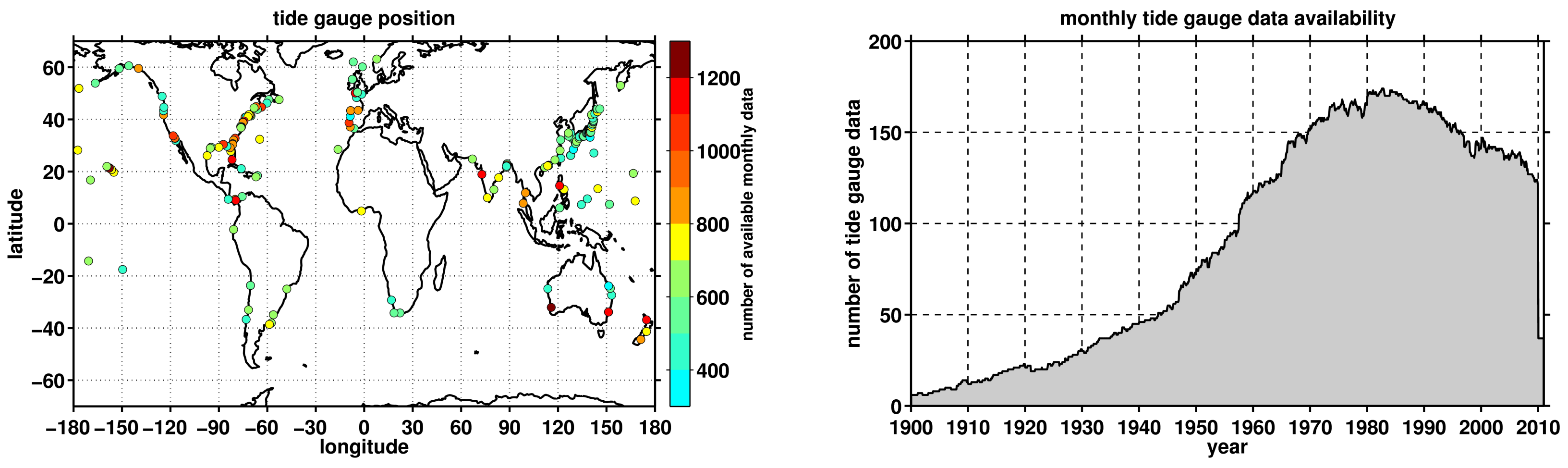
Reconstruction of Global Sea Level Variations from Tide Gauges and Altimetry

Manfred Wenzel and Jens Schröter

Introduction

Global sea level anomaly fields are reconstructed from tide gauges for the period 1900-2009 in a two step procedure. First we present an improved way to train a neural network to fill data gaps in time-series, e.g. from tide gauges. In the paper of Wenzel and Schröter (2010) the network used for this purpose was trained using only time steps that have complete data. Here we describe a method that can deal with arbitrarily distributed missing values even during the training phase.
Sea level anomaly are then calculated from these completed tide gauge records. This is done by estimating their projection onto the principal components from the EOF decomposition of the altimetry data.

Selected Tide Gauges

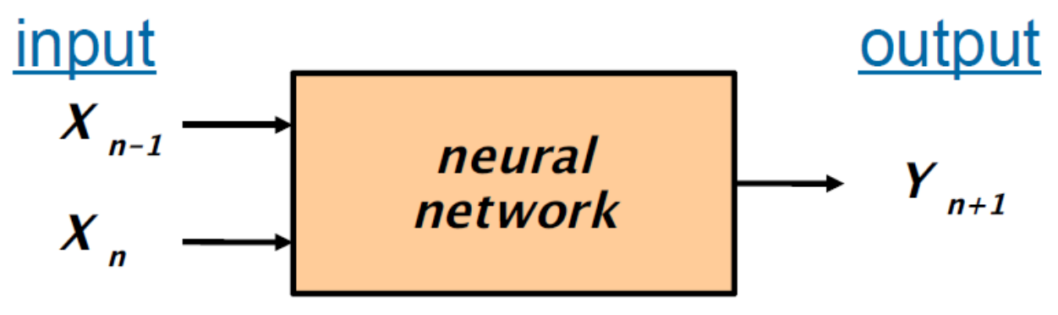


The left graph shows the position of the selected tide gauges.The color coding gives the amount of available monthly data at the corresponding tide gauges, while the right graph shows the monthly availability of tide gauge data.

For the purpose of this work 178 tide gauges are selected from the PSMSL database (RLR, monthly) in the latitudinal band 65°S-65°N that have at least 30 annual mean values given for the years after 1950. It is obvious that many data are missing especially prior to 1950 and that there is no month that has complete data.

Tide Gauge Reconstruction

The first task will be to fill these data gaps in an appropriate way. For this task a neural network is used as an time stepping operator as outlined below:



$X_n = \{x_{n,k}\}$: input vector $\qquad Y_n = \{y_{n,k}\}$: output vector

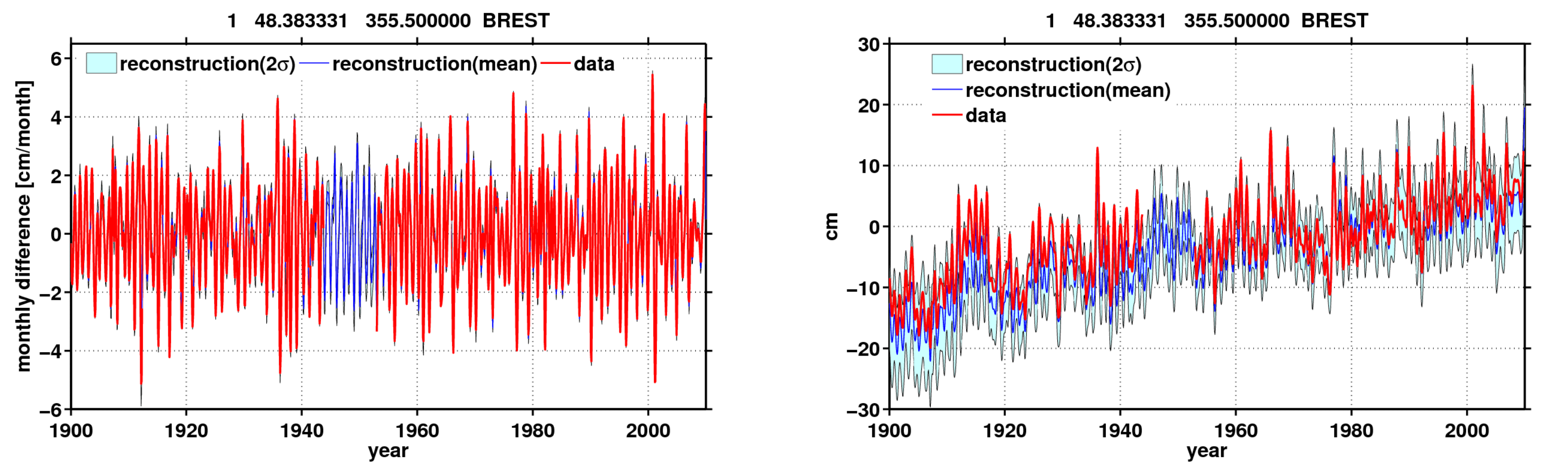
$$x_{n,k} = \begin{cases} y_{n,k} & \text{if } d_{n,k} = \text{undef.} \\ d_{n,k} & \text{otherwise} \end{cases} \qquad D_n = \{d_{n,k}\} : \text{data}$$

at which the initial conditions, Y_0 and Y_1 , are unknown

The unknown matrices ***H, O*** and the bias terms $\boldsymbol{b_H}$ and $\boldsymbol{b_O}$ of the neural network as well as missing values in the initial conditions are estimated by minimizing a weighted least square cost function:

$$K = \sum\sum w_{n,k} (y_{n,k} - d_{n,k})^2 + C_r \cdot n_{dat} \cdot [1/n_O \sum\sum (o_{ij})^2 + 1/n_H \sum\sum (h_{ij})^2]$$

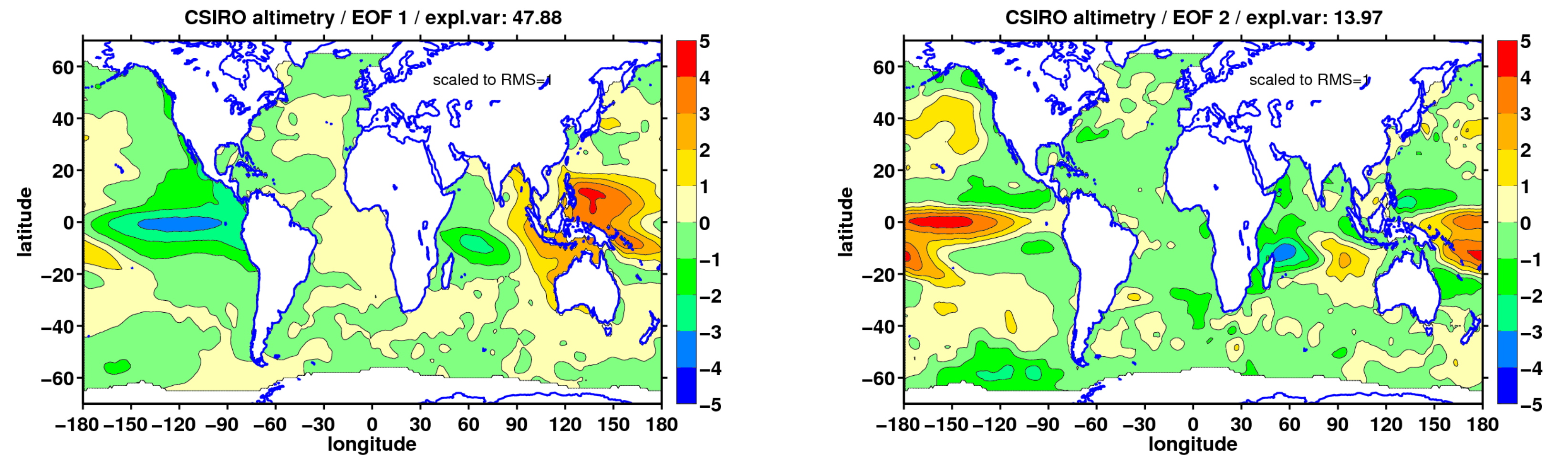
that includes a *ridge regression constraint* $\mathbf{c}_r$ to minimize/suppress less importent entrees in the matrices. Eight realizations of the network are trained using different prior estimated/optimized weights $\mathbf{c}_r$.



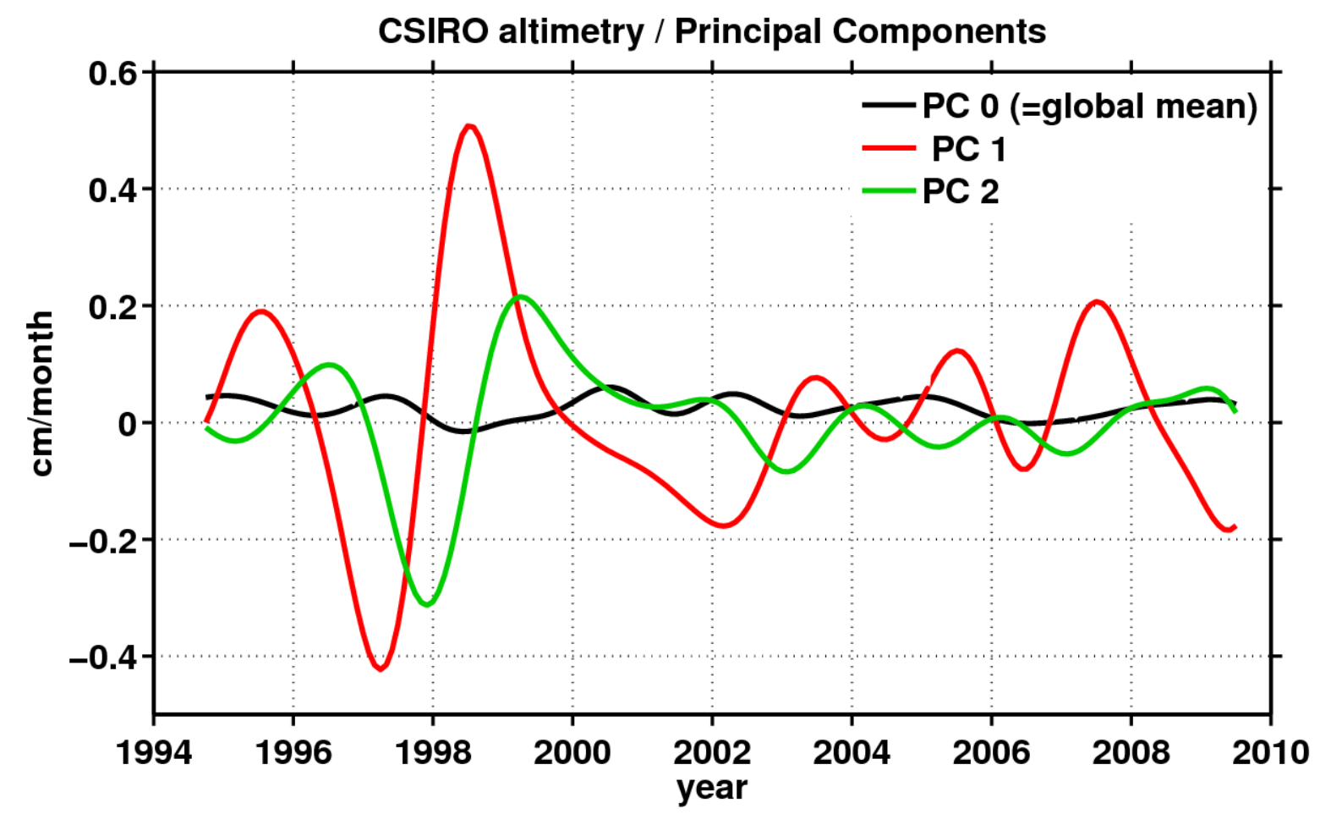
Retrieved monthly sea level differences (left, scaled) and the resulting sea level variations (right) at tide gauge **BREST**. *For the monthly differences the ensemble mean and standard deviation from eight differently trained networks are shown. For the sea level variations the mean and the error are estimated from an ensemble (25 members) created by adding Gaussian noise to the monthly differences.*

Altimetry Data - EOF Decomposition

For the reconstruction of the global and regional sea level we use altimetry data provided on the CSIRO sea level web site. From the available versions the one with no IB correction applied has been chosen. The altimetry data are processed further as follows: ***i)*** take the monthly differences, ***ii)*** filter the local time series to exclude the annual cycle and ***iii)*** subtract the global mean value. The latter will be treated as the given zero'th principal component (PC) of the following empirical orthogonal function (EOF) decomposition, that results in 27 EOF's, whereof 16 are needed to explain 98% of the variance.



*The figures above show the two leading EOF's from the filtered CSIRO altimetry data (monthly differences). The corresponding PC's are given in the graph on the right (red and green line) together with the global mean (=***PC 0***; black line) that was subtracted prior to the decomposition.*



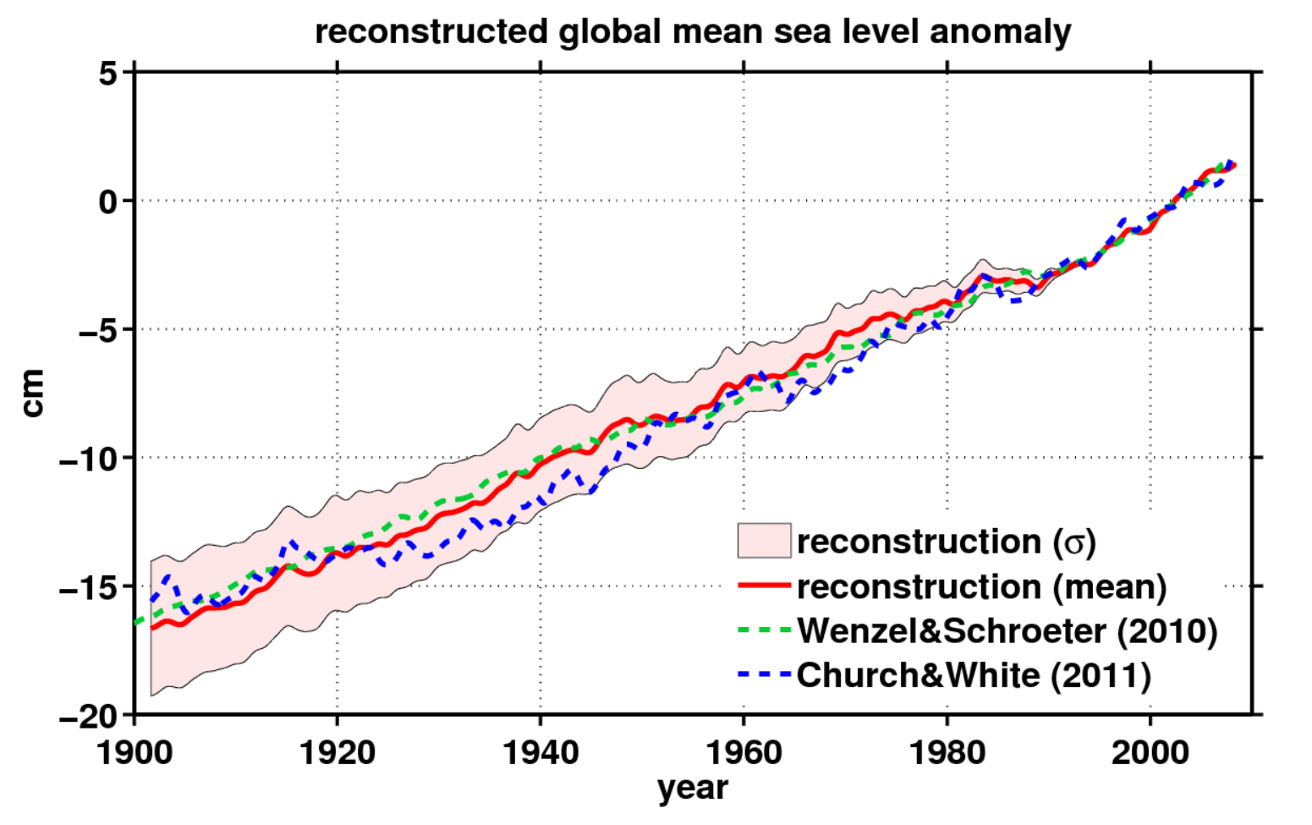
Results

The principal components **PC 0** (=global mean) to **PC16** are reconstructed from the accordingly filtered TG data to give global sea level anomaly fields from 1900 onwards. Each of these principal components, $\mathbf{PC_k(t)}$, is reconstructed from the TG data **TG(t)** by estimating a transfer vector $\mathbf{M_k}$ that provides

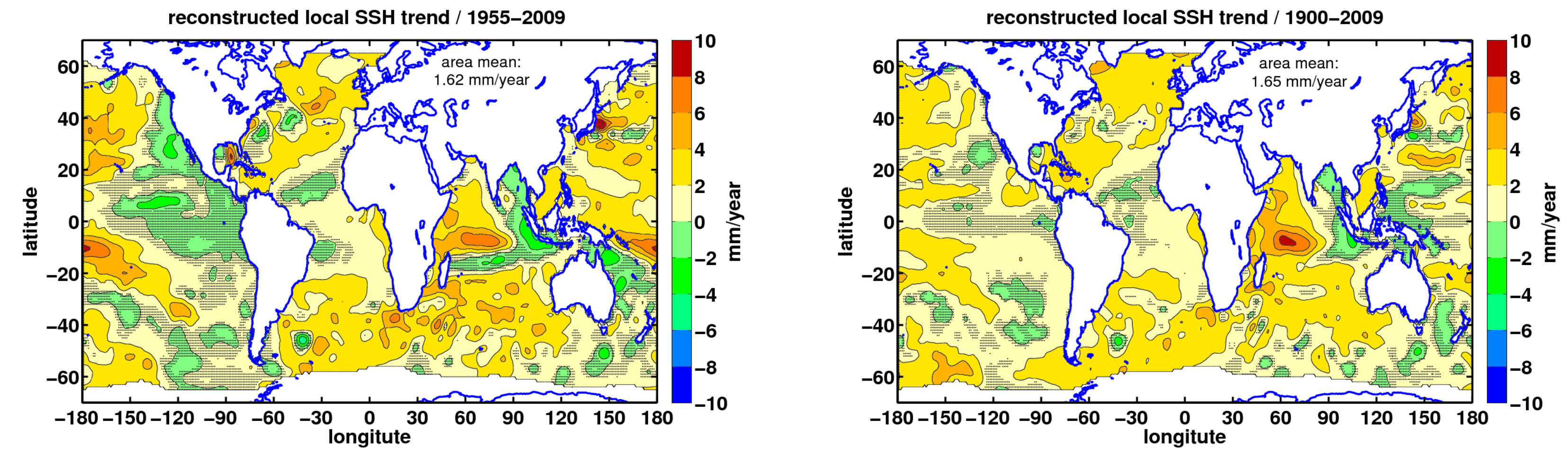
$$\mathbf{PC_k(t) = < M_k , TG(t) >}$$

i.e. the PC values are the weighted sum of the TG values. The vector $\mathbf{M_k}$ is estimated from the period where PC data exist via a least square fit and assumed to be valid for the whole period starting from 1900.
Eight estimates are performed for each PC that differ in whether or not: ***i)*** errors in the tide gauge data are accounted for; ***ii)*** a correction of the TG trend is applied to compensate the effect of vertical land movement that is not inherent in altimetry; ***iii)*** a ridge regression constraint is applied to the transfer vector $\mathbf{M_k}$, that reduces the influence of TG's with low absolute correlation between tide gauge data and PC. Finally the global sea level anomaly fields are reconstructed by combining the estimated PC's with the altimetry EOF's.

Reconstructed global mean sea level anomaly (=cumulative sum of **PC 0***). Shown are the mean and standard deviation σ from the ensemble of estimates. The centennial trend is estimated to*
1.65 ± 0.25 mm/year right ⟶



below ↓
Reconstructed local sea level trend for the periods **[1955-2009]** *(left) and* **[1900-2009]** *(right).*



Climate Dynamics Section, Alfred-Wegener-Institute for Polar & Marine Research
Manfred.Wenzel@awi.de