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# Electric field tuning of spin splitting in a quantum dot coupled to a semimagnetic quantum dot

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We develop an approach for tuning the spin splitting and g-factor of a quantum dot by coupling it to semi-magnetic quantum dot and tuning the electric field. We show that spin splittings and g-factors of the states of a non-magnetic quantum dot coupled to semimagnetic quantum dot can be enhanced orders of magnitude. Evaluations are made for coupled CdTe/CdMnTe quantum dots. These effects are caused by electric field control of repulsion of spin sublevels in the non-magnetic dot due to tunnel coupling of quantum dots. Electric field control of spin splittings in quantum dots is of potential interest in connection with spin qubit rotations for quantum computation. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4705287>]

## I. INTRODUCTION

The search for physical implementations for quantum bits (qubits) and quantum gates for quantum information is attracting much research interest. The Zeeman levels of a spin in a magnetic field provide a natural representation for the two quantum states of a qubit. Solid state implementations offer the advantage of scalability to the large number of gates. A spin in a quantum dot is believed to have a relatively long lifetime,<sup>1–10</sup> and quantum dots coherently coupled by exchange have been demonstrated in several systems,<sup>11–14</sup> and electric fields have been used to manipulate single quantum dot spins and spins in coupled quantum dots.<sup>15–20</sup>

Much work is now being directed to developing methods for the spin manipulations needed for single spin gates and two-qubit gates. Fast optical techniques are attractive for these manipulations. In recent work, single spin manipulations,<sup>17,21,22</sup> and also manipulations of two spins in coupled quantum dot systems<sup>23</sup> have been reported.

Tuning spin splitting with magnetic fields is typically relatively slow. It has been suggested that fast spin rotations might be achieved by tuning electron spin splitting and g-factors with electric fields.<sup>17,24–26</sup> This could be carried out rapidly with fields from intense optical pulses.<sup>22</sup> Electric field control of small exciton spin splittings in GaAs/AlGaAs systems has demonstrated for coupled quantum dots.<sup>27,28</sup>

In the present work, we propose an approach for the manipulation of a spin in a quantum dot with an electric field by using an auxiliary semimagnetic quantum dot. It is known that the spin splitting of carrier electron states can be enhanced by orders of magnitude via exchange coupling in Mn doped II-VI materials such as CdMnTe, which are called “semimagnetic” semiconductors,<sup>29,30</sup> or in quantum dots made from them. We propose to use quantum dots made from such semimagnetic materials as auxiliary quantum dots. Then, static magnetic fields are used to bring the spin states of the semimagnetic dot into interaction with those of the non-magnetic (target) dot, and electric fields are used to

modulate their spin splitting by changing the relative energies of the two dots. In this way, we obtain dramatic changes in the spin splitting of the coupled dots.

The mechanism for the increase of spin splitting discussed here in non-magnetic quantum dot is electron tunneling from the non-magnetic onto the semimagnetic quantum dot. Such tunnel coupling is of interest for implementing quantum gates for logic operations. In quantum computing, a challenging task is to implement one qubit operations without affecting other qubits. Auxiliary semimagnetic dots can provide a source of local magnetic fields for individual qubits. Single qubit operations implemented by electron spin resonance pulses could take place in a small external magnetic field, where the electric field shifts the qubit wavefunction onto semimagnetic quantum dot having a large spin splitting. Sizable electron spin splitting also suppresses spin decoherence by suppressing one-phonon processes,<sup>7,8</sup> and large magnetic fields affecting acceptors and nuclei in semimagnetic dots suppress nuclear spin induced decoherence,<sup>31</sup> providing additional advantage for such quantum computing setting.

In earlier work,<sup>32</sup> we showed that the exciton spin states of the two dots can be coupled quantum mechanically and that magnetic fields can tune their couplings in exciton emission experiments on coupled CdTe and CdMnTe quantum dots grown by selective interdiffusion. The present paper is organized as follows: we introduce a model of coupled semimagnetic and nonmagnetic dots in Sec. II. In Sec. III, we discuss electric field spin manipulation, and we summarize the results and discussion in various settings to observe these effects in Sec. IV.

## II. MODEL

We consider two coupled quantum dots,  $\alpha = N, M$ . Dot  $M$  is the semimagnetic (e.g., CdMnTe) and dot  $N$  is the non-magnetic (e.g., CdTe). Dot  $N$  contains a spin due to an excess electron carrier. We consider the two spin states of the lowest confined electron state in each dot, and the Hamiltonian is taken to be

$$H = \begin{pmatrix} E_N + \mu g_z^{(N)} B_z + eFd & 0 & t & 0 \\ 0 & E_N - \mu g_z^{(N)} B_z + eFd & 0 & 0 \\ t & 0 & E_M + \mu g_z^{(M)} B_z - eFd & 0 \\ 0 & 0 & 0 & E_M - \mu g_z^{(M)} B_z - eFd \end{pmatrix}. \quad (1)$$

Here,  $E_\alpha = \varepsilon_\alpha + \delta\varepsilon_\alpha$ , where  $\varepsilon_\alpha$  is the lowest orbital electronic energy when dot  $\alpha$  is isolated, and  $\delta\varepsilon_\alpha$  is the shift of the electron energy of the dot due to the other dot potential,  $\alpha = M, N$ .  $t$  is the amplitude of the quantum tunneling of electrons with the same spin orientation between the dots.  $2d$  is the distance between the two dots, and  $F$  is the electric field along the  $z$  direction separating them.  $g_z^{(M,N)}$  are the longitudinal g-factors along  $z$ , and we include an external field,  $\vec{B}_{ext}$  along the  $z$  direction. The total magnetic field  $\vec{B}$  includes  $\vec{B}_{ext}$  and the internal field from the magnetization of the Mn ions.

Equation (1) includes one orbital level spin-split by the magnetic field for each of the dots. It is assumed that the excited orbital levels are separated by a substantial gap and that the application of electric and magnetic fields is characterized by energy scales (frequencies) smaller than this gap. In the case of 5 nm-wide quantum wells, the characteristic gap associated with  $z$ -direction quantization is at least 0.1 eV, and for in-plane dot sizes  $\sim 10$  nm, the separation between orbital levels from confinement to quantum dots is  $\sim 30$  meV. Therefore, this condition is satisfied for frequencies of external fields of a few GHz, in cases such as the quantum dots used here. We note that for relatively small fields (below  $\sim 0.5$  T), the Hamiltonian may include components of the total magnetic field in the transverse directions due to the fluctuations in orientations of Mn spins.<sup>33</sup> However, these transverse fields have only small effects on the spin splitting, and we do not include those. Furthermore, we also estimate that tunneling accompanied by a spin flips is small in these systems, and do not include spin-flip assisted tunneling here. This tunneling can be due to band structure spin-orbit interactions or to fluctuations in the orientations of Mn spins. We estimate that the characteristic energy scale in the former is  $\sim 0.1$  meV and of the latter is  $\sim 0.5$  meV at a magnetic field of 1 T and a temperature of 4 K. These are considerably smaller than the energy splitting from the alignment of the Mn spins.

The eigenvalues of the Hamiltonian (2) are obtained straightforwardly, and for electric field  $F = 0$  are as follows:

$$\begin{aligned} \tilde{E}_{4,1} &= \frac{E_N + E_M + (\mu g_z^{(N)} + \mu g_z^{(M)}) B_z}{2} \\ &\pm \sqrt{|t|^2 + \left( \frac{E_N - E_M + (\mu g_z^{(N)} - \mu g_z^{(M)}) B_z}{2} \right)^2}, \end{aligned} \quad (2a)$$

$$\begin{aligned} \tilde{E}_{3,2} &= \frac{E_N + E_M - (\mu g_z^{(N)} + \mu g_z^{(M)}) B_z}{2} \\ &\pm \sqrt{|t|^2 + \left( \frac{E_N - E_M - (\mu g_z^{(N)} - \mu g_z^{(M)}) B_z}{2} \right)^2}. \end{aligned} \quad (2b)$$

The qualitative features of the eigenstates  $\tilde{E}_j$ ,  $j=1\dots 4$ , from Eq. (2) are illustrated in the sketch in Fig. 1. Energy levels 3 and 4 originate from non-magnetic dot, and levels 1 and 2 originate from magnetic dot. We will be interested in systems for which the energies  $E_M$  and  $E_N$  differ by several meVs, as indicated in Fig. 1, where the semimagnetic dot  $M$  is taken to have the higher energy at  $B_{ext} = 0$ . Such energy differences are common for quantum dots and can originate from different sizes, shapes, or compositions. At non-zero  $B_{ext}$ , the spin splitting of the semimagnetic dot is greater than that of the non-magnetic dot. In such systems, the g-factors of the non-magnetic and semimagnetic quantum dots often have opposite signs, leading to an opposite ordering of their spin states in small magnetic fields as illustrated in Fig. 1. There we use a positive g-factor for magnetic quantum dot  $M$  and a negative g-factor for non-magnetic quantum dot  $N$ . For high enough  $B_{ext}$ , the spin-down state of dot  $M$  ( $\tilde{E}_2$ ) approaches the spin-down state of dot  $N$  ( $\tilde{E}_3$ ), and they interact by the tunneling  $t$ . The effective interaction of the two spin-down states ( $\tilde{E}_2, \tilde{E}_3$ ) is greater than that of the two spin-up states ( $\tilde{E}_1, \tilde{E}_4$ ), because  $\tilde{E}_2$  and  $\tilde{E}_3$  are closer in energy, and this leads to a greater repulsion between the spin down states. As we shall see, this repulsion can result in crossing of levels  $\tilde{E}_3$  and  $\tilde{E}_4$ , so that the spin down level  $\tilde{E}_3$  becomes the lowest state as magnetic field increases.

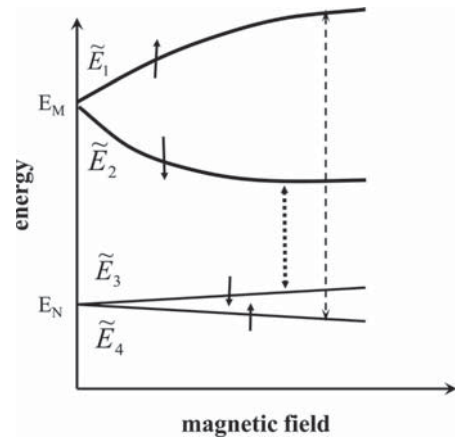


FIG. 1. Sketch of energy levels of coupled quantum dots.  $E_M$  and  $E_N$  are energies of isolated semimagnetic and non-magnetic dots, respectively.  $\tilde{E}_j$ ,  $j = 1, 2, 3, 4$  are the energies of the four states of coupled dots deriving from the spin doublets of lowest orbital states of each quantum dot in the absence of tunneling. Vertical dashed lines indicate dominant couplings of states, which are due to tunneling between levels with like spin states. Arrows indicate spin orientations of the four states. The two spin down levels repel more strongly, which leads to state 3 becoming the ground state at moderate magnetic fields. As discussed in the text, an electric field can be used to change the splitting  $E_M - E_N$ , modifying the coupling of the two quantum dots.

### III. ELECTRIC FIELD CONTROL

An electric field can change the relative energies of the two isolated dots,  $E_M - E_N$ , and this controls the coupling of the spin states and the mixing of states originating from the semimagnetic and non-magnetic dots. In this way, the electric field can modulate the spin splitting and g-factors.

We now give specific calculations to illustrate the electric field tuning of spin splitting. The parameters are taken from experiment on bulk systems. We take dot  $M$  to be composed of  $\text{Cd}_{0.98}\text{Mn}_{0.02}\text{Te}$  with  $g_z^{(M)} = 40$  and dot  $N$  to be composed of  $\text{CdTe}$  with  $g_z^{(N)} = -2$ . Each dot is taken to be 5 nm high with a radius of 8 nm, and the width of the barrier between two dots is 5 nm. The dots are taken to be surrounded by  $\text{Cd}_{0.8}\text{Mg}_{0.2}\text{Te}$  barriers in the vertical direction. The energy separation is taken to be  $E_M - E_N = 20$  meV, and resulting tunneling amplitude is  $t = 3$  meV, which describe well the data on photoluminescence experiments in magnetic field in a system of coupled magnetic and non-magnetic dots.<sup>32</sup>

Figure 2(a) gives the spin splitting of the upper spin doublet at  $F = 0$ . At non-zero  $B_{ext}$ , the spin splitting becomes large, but there is no noticeable modification of the dispersion with  $B_{ext}$  from the coupling between the two dots in the range shown. There are dramatic effects in the spin splitting of the lower doublet shown in Fig. 2(b), however.  $\tilde{E}_3$  is driven down by the coupling to  $\tilde{E}_2$ , and it passes through  $\tilde{E}_4$  at  $B_{ext} \approx 3.3$  T. As a result, the spin splitting and g-factor of the lower doublet go to zero and then reverse its sign. The large effect of the coupling between the dots on the lower doublet is due to the strong dispersion (large g-factor) of the states of the upper doublet. The splitting of the lower state, on the other hand, is small and has a negligible effect on the upper doublet. The changes in the spin splitting of the lower doublet arise from the mixing with the semimagnetic dot.

A small anticrossing could occur in place of the crossing of energy levels  $\tilde{E}_3$  and  $\tilde{E}_4$  at  $B_{ext} \approx 3.3$  T in Fig. 2(b) as a result of spin-orbit interaction and transverse spin fluctuations. We estimate that this anticrossing is small, and we do not include it here. Transverse fluctuations can lead to observable effects at small magnetic fields.<sup>33</sup> However, for fields on the order of 1 T or higher, the Zeeman splitting due to the alignment of the Mn spins in the external field is dominant.<sup>32,33</sup>

An electric field along the growth direction  $z$  can change the splitting  $E_M - E_N$ , which in turn modifies the effects of

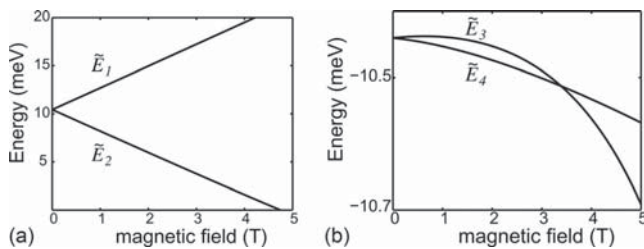


FIG. 2. (a) Energies of the upper doublet of spin states vs. external magnetic field. (b) Energies of the lower doublet of spin states vs. external magnetic field. Energies are labeled as in Fig. 1. At magnetic field  $\sim 3.3$  T, state 3 becomes the ground state because of the strong repulsion of levels 2 and 3, reversing the order of states in lower doublet compared to Fig. 1.

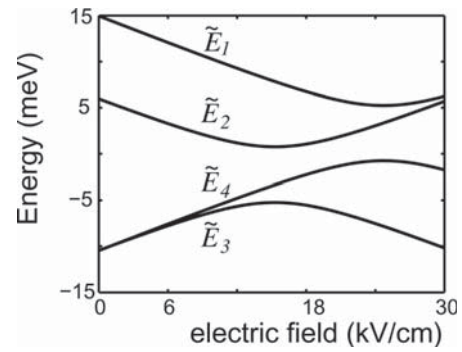


FIG. 3. Energies of four states of coupled quantum dots vs. electric field at external magnetic field slightly above magnetic field 3.3 T corresponding to crossing of Fig. 2(b).

tunnel coupling on the spin states, because quantum states of magnetic and non-magnetic dots move closer to resonance. In Eqs. (2a) and (2b), the energy detuning of semimagnetic and non-magnetic dot  $E_M - E_N$  at zero electric field becomes  $E_M - E_N - 2eFd$  in non-zero electric field. In Fig. 3, the energies of the four spin states are shown as functions of electric field  $F$  at  $H_{ext} \approx 3.3$  T, which is near (slightly above) the magnetic field giving the crossing of the lower doublet in Fig. 2(b). By applying the magnetic field that brings system close to crossing, we obtain an especially pronounced effect. At  $F = 0$ , the lower doublet ( $\tilde{E}_3, \tilde{E}_4$ ) has nearly zero splitting, and the upper doublet ( $\tilde{E}_1, \tilde{E}_2$ ) has a large splitting.  $\tilde{E}_2$  and  $\tilde{E}_3$  have the same spin direction, and with increasing  $F$  they repel one another and anticross at  $F \approx 14$  kV/cm. Similarly,  $\tilde{E}_1$  and  $\tilde{E}_4$  have the same spin direction and anticross at  $F \approx 26$  kV/cm. As a result, the spin splitting of the lower doublet increases from zero to a large value, and the splitting of the upper doublet decreases to nearly zero. In effect, the ground state doublet of the coupled dots ( $\tilde{E}_3, \tilde{E}_4$ ), which originated from the non-magnetic dot at zero magnetic and electric fields, acquires the character of the magnetic dot with increasing  $F$  due to hybridization between the two doublets. This corresponds to shifting the weight of the wavefunction onto the semimagnetic quantum dot. Thus, we find that an electric field can control dramatically the spin splitting and g-factors of the ground state of these coupled dots.

The weights of the two lowest lying states (3 and 4) on the magnetic and non-magnetic dots (M, N) are shown in Fig. 4. At  $F = 0$ , both functions are centered primarily on the

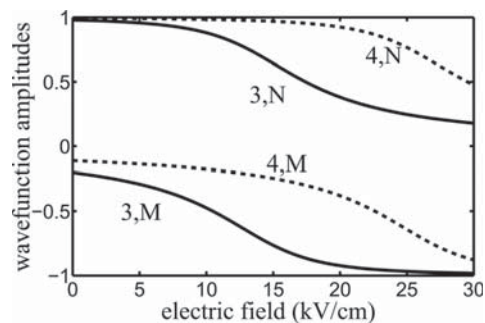


FIG. 4. The weights of the wavefunctions of the two lowest energy levels (3 and 4) on the magnetic (M) and non-magnetic quantum dots as functions of electric field.  $B = 3.3$  T.

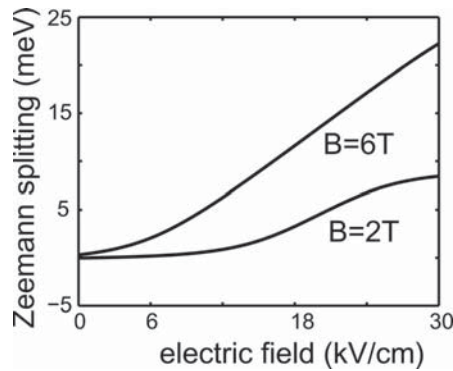


FIG. 5. Dependence of spin splitting of lowest lying spin doublet vs. electric field for external magnetic fields of 2 T and 6 T.

non-magnetic dot. We observe that the amplitudes of being on magnetic and non-magnetic dot are equal in magnitude at  $F \sim 14$  kV/cm, when state 3 with spin down anticrosses with the higher lying state 2, and for  $F \sim 26$  kV/cm, when the state 4 with spin up anticrosses with the higher lying state 1 with the same spin. For large fields, the weights are primarily on the magnetic M dot. Thus, we see that for increasing  $F$ , the N and M character of the wavefunctions become mixed and change from mainly N at low fields to mainly M at high fields.

Fig. 5 gives the absolute value of the spin splitting of the lower doublet ( $\tilde{E}_3, \tilde{E}_4$ ) with increasing electric field for  $B_{ext} = 2T$  and for  $B_{ext} = 6T$ . At  $F = 0$ , the spin splitting of non-magnetic ground state doublet of the dots is small. However, we see that the spin splitting can be strongly tuned with an electric field by using the field to change the dot relative energies. The electric field tuning of spin splitting obtained here is much larger than those obtained by moving the electron function between materials as in Refs. 26–28. This is because the electric field here not merely shifts the weights of the wavefunction into material with larger g-factor as in Refs. 26–28 but does this near resonance between states of the two quantum dots.

Fluctuations in the orientation of Mn spins in semimagnetic semiconductors can give rise to variations in Zeeman splittings and also to electron spin dephasing. However, in magnetic fields  $\sim 1$  T or higher the fluctuations are suppressed. There also can be deviations from the linear dependence of spin splittings on field in bulk semimagnetic materials at sufficiently high fields ( $\sim 7$  T) due to the saturation of the polarization of the Mn spins.<sup>33</sup> However, for the fields studied here these effects are small, and they are not included here.

#### IV. CONCLUSION

Here, we have developed an approach for the tuning of spin splitting in single nonmagnetic quantum dots. A semimagnetic quantum dot gives large spin splitting in the non-magnetic quantum dot, and an electric field can tune the splitting. This approach may provide opportunities for designing spin rotation operations in spin qubit gates. For example, we could envision a chain of non-magnetic quantum dots each coupled to an auxiliary semimagnetic dot with

varying Mn concentrations. In this way, qubit operations might be carried out on different pairs of quantum dots with a single electric field.

Electric field induced tunneling onto semimagnetic dots could provide opportunities for relatively fast qubit operations. One can envision an array of semimagnetic quantum dots each vertically coupled to a dot from an array of non-magnetic dots. Pairs of coupled dots can vary in their parameters making their switching characteristics individual. Both arrays are placed in a Schottky diode, which can provide fast switching electric field. The Schottky diode scheme is one of the configurations already used for studies of coupled quantum dots in electric field.<sup>34,35</sup> The switching frequency for the electric field in this case can reach from 2 MHz to several GHz, as in conventional Schottky diode.<sup>36</sup> This gives nanosecond to ten picoseconds time scale for qubit and gate operations. This is even faster than in recent experiments on lateral electrostatic quantum dots.<sup>10</sup> Furthermore, using resonant tunneling diodes, one potentially can reach switching frequency 700 GHz (Ref. 37) (corresponding to picosecond time scale) for electric fields with amplitude as high as 40 kV/cm. In optical settings, the fast electric fields can come from the AC stark effect in optical manipulations. However, speed of manipulations by electric field is limited by the need to operate within the spin-split ground state, at characteristic frequencies smaller than the gaps, and GHz scale appears to be the limit for quantum dots systems such as described here.

Systems of the kind discussed in this work could be fabricated in several materials. Coupled CdMnTe and CdTe dots have been made by selective interdiffusion.<sup>32</sup> Mn can be doped into GaAs,<sup>38</sup> and GaMnAs dots could be used to control GaAs dots in vertical configuration AlGaAs/GaMnAs/GaAs/AlGaAs. Also, the effect we discuss here can be used to control spin splitting in two coupled non-magnetic dots, one of which has a g-factor bigger than the other. For example, the g-factor of bulk InAs is approximately  $-15$ , whereas narrow quantum wells of InAs have g-factors approximately  $-1$ . Thus, quantum dots from wide InAs quantum wells could be used to control dots confined to narrower quantum wells.

#### ACKNOWLEDGMENTS

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