Analysis of polyhedral domed sandwich structures

Manos, G. C.

ANALYSIS

OF

POLYHEDRAL DOMED SANDWICH STRUCTURES

by

G.C. Manos

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Department of Engineering Science
University of Durham

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This work is dedicated to my father

"Τά πάντα ρεῖ"

'Ηράκλειτος

ABSTRACT

The finite element method was employed for the analysis of the polyhedral domed sandwich structures.

Two different variational approaches were used for comparison reasons. These are the "displacement formulation" and the "mixed formulation" as they are commonly known.

Initially seven sandwich plate bending models were developed. These models were used to solve a number of problems where a numerical or experimental solution existed and comparisons were made.

The agreement varied from fair to excellent depending on the nature of the model and the type of the solved problem.

As a result of this comparative study four of these models were conseq-uently selected to be extended for the development of the sandwich dome models.

The accuracy of these four sandwich dome models was tested by modelling five polyhedral dome structures. The results derived from each individual model were compared with experimental results obtained by other researchers and by the author himself.

The author's contribution to the experimental work was the design, con-struction and subsequent testing of two full scale prototypes, namely, the 24 faced and the 36 faced domes.

From the whole analysis it was established that the developed numerical models, when selectively applied in the most appropriate way with regard to their special characteristics and the nature of the problem, produce reliable results.

Special problems were investigated arising from the boundary conditions as well as structural details of the joint-lines of the plates forming the polyhedron, and thus a solution was suggested.

Finally, a data generation routine is also described in order to facilitate further application of the various developed models by future users or researchers.

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APPENDICES

1. INTRODUCTION

The present work is the product of the combination of three factors with a contemporary approach in the field of Structural Engineering.

First there is the cost factor which can be countered by mass-production. At the same time we are trying to fulfil two principles of modern Architecture. The first is that the Structure must be functioning in an optimum co-existence with the Environment and the Human (Functionalism). The second is that the Structure must have the flexibility to adapt to new developments and to continuously changing economical and social conditions.(Metabolism). [13,22,50,70,100]

The second factor is the formation and investigation of new structural materials. The main aim in a given Structural application is the optimisation of the use of the material achieved by improving the properties. [5,19,40,46,55,79]

Finally the third and last factor is the mathematical analysis of the problem which involves the modelling of the Structure by using new powerful computerised methods of analysis. [39,59,72,83,84,115]

1.1 Polyhedral domed structures

The polyhedral domed structures approximate to structurally efficient double-curvature surfaces by using flat plates, having at the same time the advantage of easy construction in comparison with the formation of the double-curvature surfaces themselves.

The domed structure is composed from as few types of flat plates as possible, in as far as the dimensions of the plates are concerned, so that the mass-production of the simplest construction-element can be employed.

The solution to a specific problem of space-coverage can be reached by various types of polyhedral domes. This provides considerable flexibility as regards the economic factors as well as aesthetic ones for the final choice.

For the above mentioned reasons it is believed that by the polyhedral domed Structures the Architectural principle of "Functionalism" is well treated.

On the other hand the assemplage of a number of plates (or group of plates) to form the whole structure includes the potentiality of an easy expansion or alteration. This presents an advantageous adaptability to new conditions so that the Architectural principle of "Metabolism" is also well preserved.

More detailed and extensive information about the geometry and construction of the polyedral domed structures is presented in reference [85].

1.2 Sandwich Panels

The present work is exclusively concerned with sandwich panels as construction elements for the polyhedral domed structures.

We define a sandwich panel as one which is a three-layer type of construction. It consists of two thin sheets of high stiffness material which are called the faces of the panel and between them is a thick layer of low average stiffness and density which is called the core of the panel. [3,20,46,85,89]

The most important advantages of sandwich construction are, firstly, that the ratio of high rigidity which can be achieved by the sandwich panels over the total dead weight of the construction is higher in comparison with conventional types of construction; secondly, the panels employed for the structure can easily be made and supplied by the industry in various types and dimensions and thirdly, the structure appears to have good thermal and acoustical insulation.

The materials used for the construction of the experimental prototypes and the work involved is to be presented in chapters 9, 10.

1.3 The Finite element approach

The method which will be used to analyse the behaviour of the polyhedral domed sandwich structures is known as the finite element method. Its basic principle is the idea of piecewise approximating continuous fields.

The method is outlined by reference [84] to Professor Oden's presentation of the differences between the classical and the Finite element approach.

"..... Classically the analysis of continuous systems began with investigations of the properties of small differential elements of the continuum under investigation. Relationships were established among mean values of various quantities associated with the infinitesmal elements and partial differential equations governing the behaviour of the entire domain were obtained by allowing the dimensions of the elements to approach zero as the number of elements become infinitely larger.

In contrast to this classical approach the finite element method begins with investigations of the properties of elements of finite dimensions.

The equations describing the continuum may be employed in order to arrive at the properties of these elements, but the dimensions of the elements remain finite in the analysis, integrations are replaced by finite summations and the partial differential equations of the continuous media are replaced for example by systems of algebraic or ordinary differential equations.

The continuum with infinite degrees of freedom is thus represented by a discrete model which has finite degrees of freedom.

Moreover if certain conditions (to be outlined in Chapter 3) are satisfied, then as the number of elements is increased and their dimensions are decreased the behaviour of the discrete system converges to that of the continuous system.

Many numerical methods were developed before the era of electronic computers and are now adapted for use with these machines.

In contrast, the finite element method is a complete product of the electronic computer age. This is due to the fact that the method possesses certain characteristics that take full advantage of the facilities offered by the high-speed computers so that it can be systematically programmed to accommodate such complex and difficult problems as non-homogeneous materials, non-linear stress-strain behaviour and complicated boundary conditions.

2. CONSTITUTIVE EQUATIONS FOR A SANDWICH PLATE

2.1 Introduction

As was mentioned in Chapter 1 (section 3), first of all the equations describing the continuum must be formed.

We start with the fundamental equations for a plate and include the effects of the sandwich form of the plate taking into account certain assumptions (to be outlined in the next section).

The basic aim is to establish the constitutive equations for a sandwich plate in such a form that together with the variational principles(to be outlined in Chapter 3) we have all that is required for the finite elements analysis

2.2 Mathematical formulation

We consider an infinitesimal element of a sandwich plate and we write the relations between the stress-resultants and the stress tensor (Fig. 2.1) [53,78,102]

$$N_{ij} = \int_{-h/2}^{h/2} t_{ij}\, dz \; , \quad M_{ij} = \int_{-h/2}^{h/2} t_{ij}\, z\, dz \; , \quad Q_i = \int_{-h/2}^{h/2} t_{iz}\, dz \qquad (2.1)$$

$$(i\, ,\, j \;=\; x\, ,\, y)$$

We introduce the following assumptions [4,20,54,71,93,94,101]

1. The displacements u , v , w are constant across the thickness of the plate considering them individually.

2. For $-\frac{c}{2} \leq z \leq \frac{c}{2}$ the stresses $t_{xx}, t_{yy}, t_{xy} = 0$

For $\frac{c}{2} \leq z \leq \frac{c}{2} + f$ the stresses $t_{xz}, t_{yz} = 0$

and $-\frac{c}{2} - f \leq z \leq -\frac{c}{2}$

which means that the contribution of the core to direct stresses is neglected as well as the contribution of the faces to shear stresses.

3. The following equations which relate the strain tensor with the displacements u, v, w (Fig.2.2) are valid (for r, s = x, y). The subscripts after the comma denote derivatives.

$$e_{rs} = -w,_{sr} + \frac{c}{4} ({}_c\gamma_{r,s} + {}_c\gamma_{s,r}) + \varepsilon_{rs} \quad \text{for } \frac{c}{2} \leq z \leq \frac{c}{2} + f$$

$$e_{rs} = -w,_{sr} - \frac{c}{4} ({}_c\gamma_{r,s} + {}_c\gamma_{s,r}) + \varepsilon_{rs} \quad \text{for } -\frac{c}{2} - f \leq z \leq -\frac{c}{2}$$

$$e_{rz} = \frac{1}{2} \; {}_c\gamma_r \qquad \text{for } -\frac{c}{2} \leq z \leq \frac{c}{2} \tag{2.2}$$

where $\varepsilon_{xx} = u,_x + \frac{1}{2} w,_x^2, \quad \varepsilon_{yy} = v,_y + \frac{1}{2} w,_y^2,$

$$\varepsilon_{xy} = u,_y + v,_y + w,_x \, w,_y$$

$$e_{zz} = 0 .$$

4. The thickness of the faces, f, is much smaller than the thickness of the core, c.

5. For both the faces and the core if we consider symmetry with respect to the x-y plane we have the following stress-strain relationship of any point in the plate in a matrix form.

$$\begin{bmatrix} t_{xx} \\ t_{yy} \\ t_{zz} \\ t_{yz} \\ t_{xz} \\ t_{xy} \end{bmatrix} = \begin{bmatrix} C^{xx}_{xx} & C^{xx}_{yy} & C^{xx}_{zz} & \phi. & \phi. & C^{xx}_{xy} \\ & C^{yy}_{yy} & C^{yy}_{zz} & \phi. & \phi. & C^{yy}_{xy} \\ & & C^{zz}_{zz} & \phi. & \phi. & C^{zz}_{xy} \\ & \text{symmetrical} & & C^{yz}_{yz} & C^{yz}_{xz} & \phi. \\ & & & & C^{xz}_{xz} & \phi. \\ & & & & & C^{xy}_{xy} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{yz} \\ e_{xz} \\ e_{xy} \end{bmatrix} \qquad (2.3)$$

Taking into account assumptions 2 and 3 we obtain from equations 2.1 the following

$$N_{ij} = \int_{-c/2-f}^{-c/2} t_{ij}dz + \int_{c/2}^{c/2+f} t_{ij}dz$$

$$M_{ij} = \int_{-c/2-f}^{-c/2} t_{ij}zdz + \int_{c/2}^{c/2+f} t_{ij}zdz \qquad (2.4)$$

$$Q_i = \int_{-c/2}^{c/2} t_{iz}dz$$

Taking into account assumption 3 and equations (2.4) the relationship (2.3) becomes

$$
\begin{bmatrix} t_{xx} \\ t_{yy} \\ t_{xy} \\ t_{xz} \\ t_{yz} \end{bmatrix}
=
\begin{bmatrix}
{}_{f}C^{xx}_{xx} & {}_{f}C^{xx}_{yy} & {}_{f}C^{xx}_{xy} & \phi. & \phi. \\
 & {}_{f}C^{yy}_{yy} & {}_{f}C^{yy}_{xy} & \phi. & \phi. \\
 & & {}_{f}C^{xy}_{xy} & \phi. & \phi. \\
 & \text{symmetrical} & & {}_{c}C^{xz}_{xz} & {}_{c}C^{xz}_{yz} \\
 & & & & {}_{c}C^{yz}_{yz}
\end{bmatrix}
\begin{Bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \\ e_{xz} \\ e_{yz} \end{Bmatrix}
\tag{2.5}
$$

or

$$
\begin{aligned}
t_{ij} &= {}_{f}C^{ij}_{rs} \quad e_{rs} \\
t_{ij} &= {}_{c}C^{iz}_{rz} \quad e_{rz}
\end{aligned}
\tag{2.6}
$$

(for $i, j, r, s = x, y$)

where the prefix of f or c on the elastic constants refers to the faces or the core respectively

Substituting equations (2.6) into equations (2.4) we obtain

$$
\begin{aligned}
N_{ij} &= {}_{f}C^{ij}_{rs} \int_{-c/2-f}^{-c/2} e_{rs}\,dz + {}_{f}C^{ij}_{rs} \int_{c/2}^{c/2+f} e_{rs}\,dz \\
M_{ij} &= {}_{f}C^{ij}_{rs} \int_{-c/2-f}^{-c/2} e_{rs}\,zdz + {}_{f}C^{ij}_{rs} \int_{c/2}^{c/2+f} e_{rs}\,zdz \\
Q_{i} &= {}_{c}C^{iz}_{rz} \int_{-c/2}^{c/2} e_{rz}\,dz
\end{aligned}
\tag{2.7}
$$

Taking into account assumption 3 with regard to equations (2.2) and evaluating of the integrals we obtain the following equations which relate the stress resultants for a sandwich plate to the displacements or the derivative of the displacements of the mid surface of the plate. These will be used in the finite elements analysis.

$$N_{ij} = {}_cC^{ij}_{rs}\, 2f\, \varepsilon_{rs}$$

$$M_{ij} = {}_fC^{ij}_{rs}\left[2w,_{rs}\left(\frac{c^2f}{4} + \frac{cf^2}{2} + \frac{f^3}{3}\right) - (\gamma_{r,s} + \gamma_{s,r})\left(\frac{c^2f}{4} + \frac{cf^2}{2} + \frac{f^3}{4}\right)\right] \qquad (2.8)$$

$$Q_i = {}_cC^{iz}_{rz}\, \frac{c+f}{2}\, \gamma_r$$

where $\gamma_r = \dfrac{c}{c+f}\, {}_c\gamma_r$

At this point, taking the validity of assumption 4 into account, we obtain the following equations in a matrix form

$$\begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \\ Q_x \\ Q_y \\ N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \begin{bmatrix} D^{xx}_{xx} & D^{xx}_{yy} & D^{xx}_{xy}/2 & & & & & \\ D^{yy}_{xx} & D^{yy}_{yy} & D^{yy}_{xy}/2 & & & & & \\ D^{xy}_{xx} & D^{xy}_{yy} & D^{xy}_{xy}/2 & & & & & \\ & & & S^{xz}_{xz} & S^{xz}_{yz} & & & \\ & & & S^{yz}_{xz} & S^{yz}_{yz} & & & \\ & & & & & E^{xx}_{xx} & E^{xx}_{yy} & E^{xx}_{xy} \\ & & & & & E^{yy}_{xx} & E^{yy}_{yy} & E^{yy}_{xy} \\ & & & & & E^{xy}_{xx} & E^{xy}_{yy} & E^{xy}_{xy} \end{bmatrix} \begin{bmatrix} a_{xx} \\ a_{yy} \\ a_{xy} \\ \gamma_x \\ \gamma_y \\ \varepsilon_{xx} \\ \theta_{yy} \\ \varepsilon_{xy} \end{bmatrix} \qquad (2.9)$$

$$a_{xx} = \frac{\partial}{\partial_x} (w,_x - \gamma_x) , \quad a_{yy} = \frac{\partial}{\partial_y} (w,_y - \gamma_y), \quad a_{xy} = \frac{\partial}{\partial_y} (w,_x - \gamma_x) + \frac{\partial}{\partial_x} (w,_y - \gamma_y)$$

(2.10)

$$D^{ij}_{rs} = -{}_fC^{ij}_{rs} \frac{f}{2} (c+f)^2 , \quad S^{iz}_{rz} = {}_cC^{iz}_{rz} \frac{c+f}{2} , \quad E^{ij}_{rs} = {}_fC^{ij}_{rs}\, 2f$$

(2.11)

The constants D^{ij}_{rs} , S^{iz}_{rz} , E^{ij}_{rs} can be evaluated using suitable experimental methods. [20,46]

For orthotropic faces and core the relationship (2.9) becomes

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \\ Q_x \\ Q_y \\ N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} D^{xx}_{xx} & D^{xx}_{yy} & \phi. & & & & & \\ D^{yy}_{xx} & D^{yy}_{yy} & \phi. & & & & & \\ \phi. & \phi. & D^{xy}_{xy}/2 & & & & & \\ & & & S^{xz}_{xz} & & & & \\ & & & & S^{yz}_{yz} & & & \\ & & & & & E^{xx}_{xx} & E^{xx}_{yy} & \phi. \\ & & & & & E^{yy}_{xx} & E^{yy}_{yy} & \phi. \\ & & & & & \phi. & \phi. & E^{xy}_{xy} \end{bmatrix} \begin{Bmatrix} a_{xx} \\ a_{yy} \\ a_{xy} \\ \gamma_x \\ \gamma_y \\ \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} \quad (2.12)$$

So by equations (2.12) we can relate the stress resultants with the strains through an operator which can be written in a shorter matrix form as

$$\{\sigma\} = [D] \quad \{\varepsilon\} \qquad (2.13)$$

$\{\sigma\}$ is the stress resultants vector

$\{\varepsilon\}$ the strain vector which is related through equations (2.2), (2.10) with the displacements of the midsurface u, v, w and their derivatives.

[D] is an operator called the elasticity matrix. The terms of this matrix, as we have already mentioned, can be determined by certain experimental methods

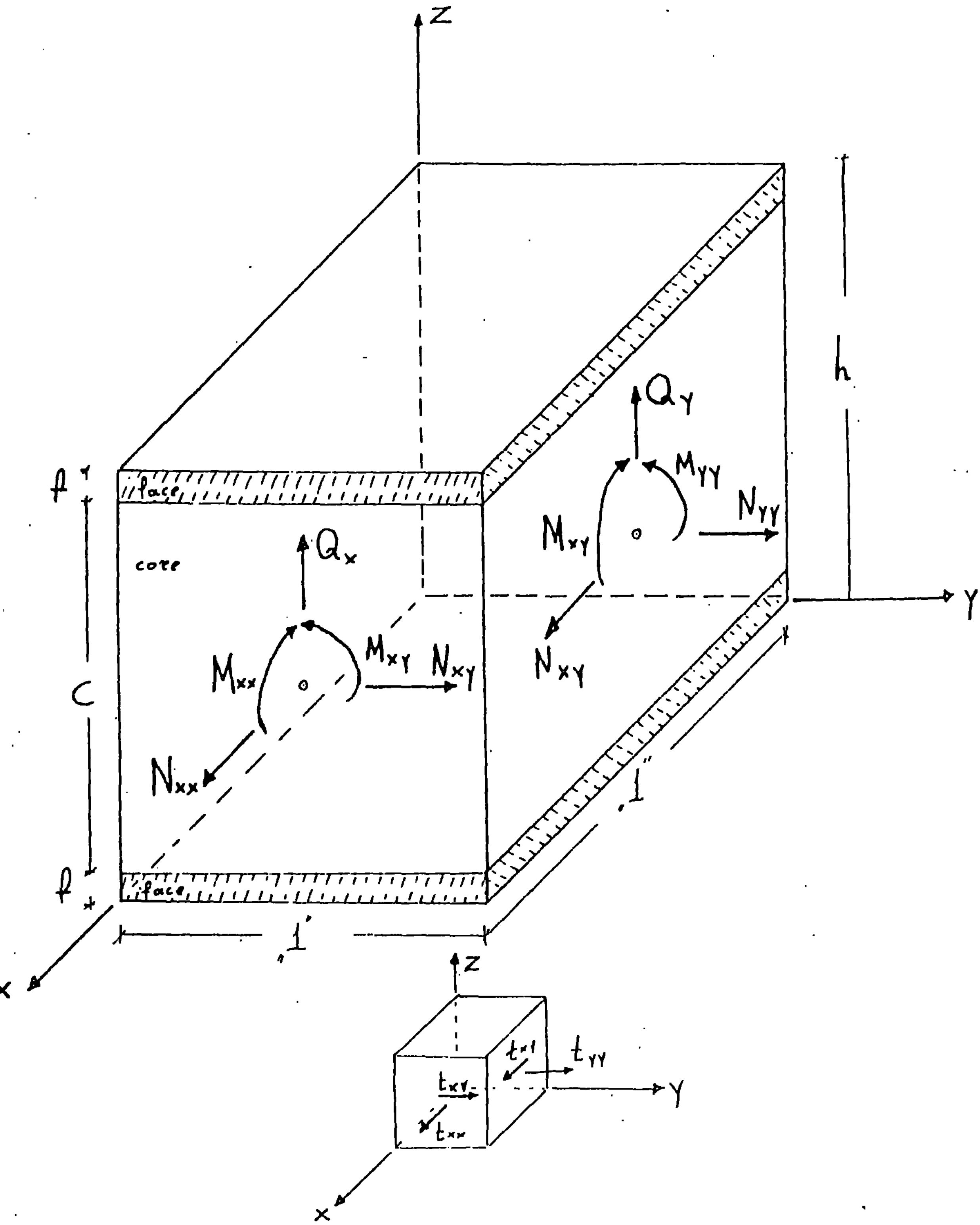


FIG. 2.1.1. SIGN CONVENTION, DISPLACEMENT MODELS

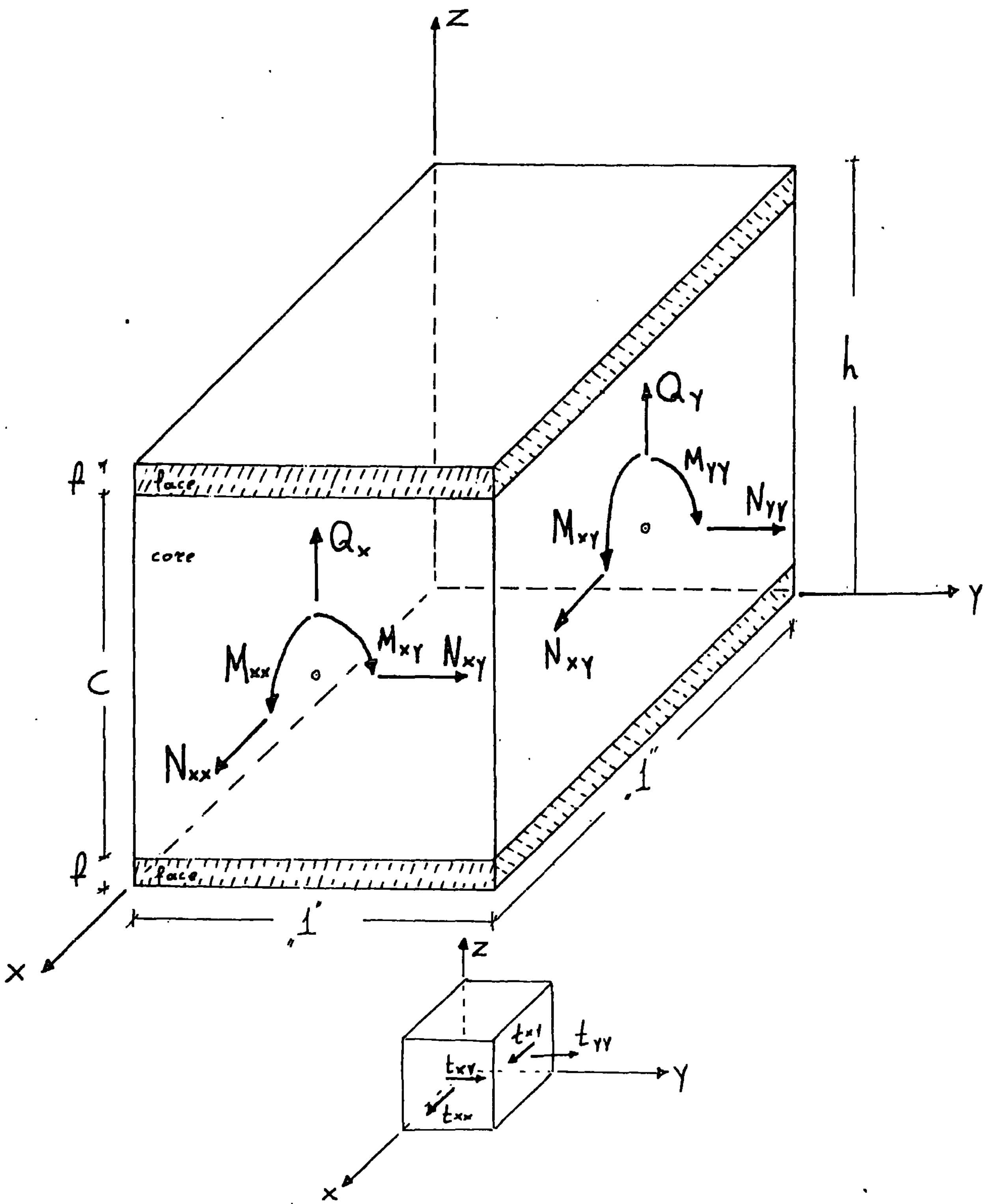


FIG. 2.1.2. SIGN CONVENTION, MIXED MODELS

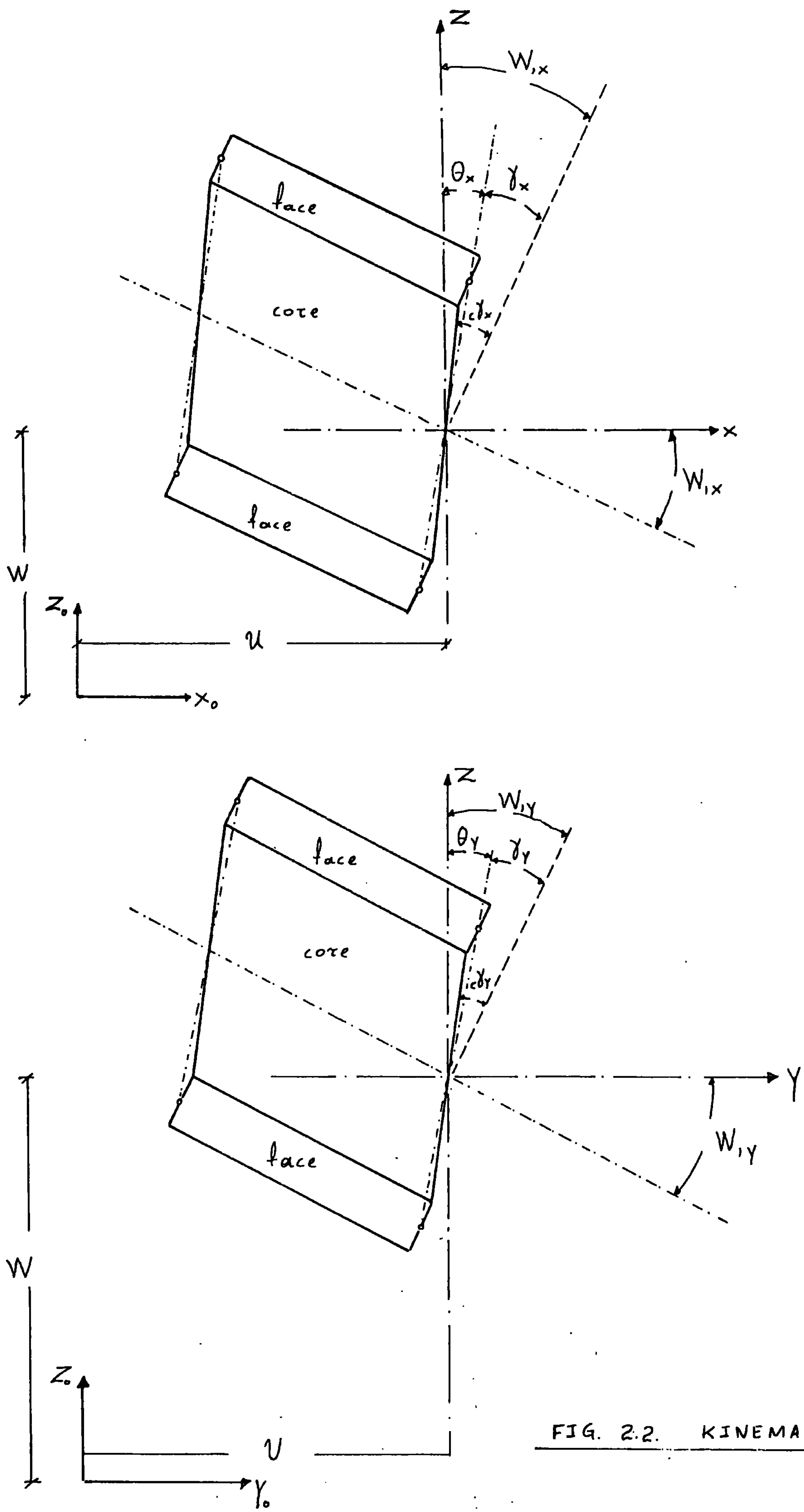


FIG. 2.2. KINEMATICS

3. VARIATIONAL APPROACH

In this chapter some elements of variational calculus are presented. These principles are used in the subsequent analysis.

Consider an expression of the form:-

$$I = \int_{x_1}^{x_2} F(x, w, \frac{dw}{dx}, \frac{d^2w}{dx^2})\, dx \tag{3.1}$$

This expression is generally known as a "functional" and in the analysis of solid continua is an expression with regard to a specific physical state (potential energy, complementary energy etc. to be outlined in Chapter 4). [39,72,80,88,115]

The basic aim is to find a function $w(x)$ satisfying the boundary conditions and being such that the functional is rendered stationary.

This is expressed as follows:

$$\delta I = \phi. \tag{3.2}$$

(where δ is the variation operator)

Following a certain procedure [39,72] we eventually obtain the expressions:

$$\frac{\partial F}{\partial w} - \frac{d}{dx}\left(\frac{\partial F}{\partial (dw/dx)}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial (d^2w/dx^2)}\right) = \phi . \tag{3.3}$$

The above is known as the Euler-Lagrange equation. Also, in addition to the above, the following may be obtained

$$\left[\frac{\partial F}{\partial (dw/dx)} - \frac{d}{dx}\left(\frac{\partial F}{\partial (dw/dx)}\right)\right]_{x_1}^{x_2} = \phi. \quad \text{and} \quad \left[\frac{\partial F}{\partial (d^2w/dx^2)}\right]_{x_1}^{x_2} = \phi. \tag{3.4}$$

This is known as "the natural boundary conditions".

If they are satisfied they are called "free boundary conditions" or else if one of them is not satisfied then a corresponding set of equations must be satisfied instead. The latter are called "geometric boundary conditions" or "forced boundary conditions".

Instead of trying to solve the governing differential equation(3.3) we form a close approximation of the functional which is noted as the $\bar{I}$ (the dash above the symbol indicates the approximate one of the same nature). Hence if we find a solution $\bar{w}(x)$, for the functional $\bar{I}$ it can be assumed that this solution will be close enough to the exact solution w(x) as well. Following the approximate solution approach the analysis proceeds as follows:

Firstly, by assuming a mathematical expression for the unknown function $\bar{w}(x)$, preferably a polynomial of x, so that the functional $\bar{I}$ becomes a function of the unknown coefficients of the polynomial.

Thus $\delta \bar{I} = \phi.$ can be expressed and satisfied by the following set of equations

$$\frac{\partial \bar{I}}{\partial a_i} = \phi. \tag{3.5}$$

(a_i are the coefficients of the polynomial)

The use of a polynomial of x for expressing $\bar{w}$, possesses the advantage of an easy mathematical manipulation.

Secondly, by performing the integration, summing the subintegrals, of the function $\bar{I}$, of a finite number of subdomains which form the whole domain (finite elements).

Thus, combining the finite element method (outlined in Chapter 1 section 3) with the variational approach, the primary functional may be related to the individual element rather than the total domain. Hence the geometry of the overall body and the system of the boundary conditions are not unsolved obstacles, even for highly complex problems, as they were in the classical Rayleigh-Ritz approach, from which the finite element method is derived.

The polynomial for the unknown function mentioned above must be such that certain conditions are satisfied and consequently the convergence towards the exact solution can be achieved.

These conditions vary with the nature of the functional and the variational principle which is to be employed [72,80,81,82,83,84,87, 99,104,115]

These conditions can, however, be described in general as follows:

(a) The number of coefficients (terms) of the polynomal selected to represent the unknown function must be at least equal to the number of the degrees of freedom associated with the element.

(b) The chosen function should provide compatibility of certain quantities across element interfaces.

(c) A rigid body deformation and a constant curvature state should be included in the polynomial.

(d) The assumed function must be continuous and be differentiable to an order consistent with the variational principle expressing the problem.

4. VARIATIONAL PRINCIPLES (SMALL DEFLECTIONS)

The various approaches in the finite element analysis of solid continua are associated with several variational principles of solid mechanics, thus introducing different types of finite element methods.

These types have been classified as follows:

(a) The first derives from the principle of minimum potential
39,41,43,72,74, energy and is based on the assumption of a continuous
80,81,82,87,88, displacement field over the entire solid. The various models
97,104,115] based in this approach are known as "displacement models".

(b) The second derives from the principle of minimum complementary
[5,39,41,43,45, energy and is based on assumed equilibrium stress fields.
72,77,80,81,82, The various models based in this approach are known
86,88,104] as "the equilibrium models".

(c) The third derives from a modified complementary energy
14,34,39,72,80, principle with assumed stress functions within the
81,82,88,104] element and displacement functions at the element interfaces.

The various models based in this approach are known as the "hybrid models".

(d) The fourth derives from Reissners-variational principle
38,39,72,80,81 with assumed continuous displacement field over the
82,88,95,104] entire solid and assumed stress field for individual elements.

The various models based in this approach are known as "the mixed models".

The models used in the present analysis are based either in the first or fourth approach. At the following sections the mathematical formulation of the first and fourth approach are presented in detail

4.1. DEFINITION OF SYMBOLS AND FUNDAMENTAL RELATIONSHIPS

$\{\delta_o^e\}$ overall vector of nodal degrees of freedom for an element

$\{\delta_o^w\}$ vector of nodal degrees of freedom as far as the transverse displacement models are concerned

$\{M_o^e\}$ vector of nodal degrees of freedom as far as the moment-models are concerned

$\{\delta_o^s\}$ vector of nodal degrees of freedom as far as the shear-models are concerned

$\{\delta_o^{uv}\}$ vector of nodal degrees of freedom as far as the in-plane displacement models are concerned

$\{\delta_o^\theta\}$ vector of nodal degrees of freedom as far as the total rotation models are concerned

$\{\delta_o\}$ vector of general displacements within an element corresponding to the nodal degrees of freedom

$[N]$ shape functions matrix relating the general displacement vector $\{\delta_o\}$ with the general vector of nodal degrees of freedom $\{\delta_o^e\}$

$$\{\delta_o\} = [N]\,\{\delta_o^e\} \tag{4.1}$$

$\{\varepsilon\}$ strains vector, as described in Chapter 2, section 2 by the equations (2.9), (2.10), (2.12), (2.13)

[B] Strain-displacement matrix relating the strains vector $\{\varepsilon\}$ with the nodal degreees of freedom vector $\{\delta_o^e\}$

$$\{\varepsilon\} = [B]\ \{\delta_o^e\} \tag{4.2}$$

$\{\sigma\}$ stress-resultants vector as described in Chapter 2, section 2 by the equations (2.1), (2.9), (2.10), (2.12), (2.13), (Fig. 2.1)

[D] elasticity matrix relating the stress-resultants vector $\{\sigma\}$ with the strains vector $\{\varepsilon\}$ as described in Chapter 2, section 2 by the equations (2.9), (2.11), (2.12), (2.13)

$$\{\sigma\} = [D]\ \{\varepsilon\}$$

[C] elasticity matrix relating the strains vector $\{\varepsilon\}$ with the stress-resultants vector $\{\sigma\}$

$$\{\varepsilon\} = [C]\ \{\sigma\} \tag{4.3}$$

$$[C_b] = [D_b]^{-1}\ , \quad D_o = D_{xx}^{xx} \cdot D_{yy}^{yy} - D_{yy}^{xx} \cdot D_{xx}^{yy} \tag{4.4}$$

$[S_n^o]$ stress matrix relating the stress-resultants vector $\{\sigma\}$ with the vector of nodal degrees of freedom $\{\delta_o^e\}$.

$$\{\sigma\} = [S_n^o]\ \{\delta_o^e\} \tag{4.5}$$

w transverse displacement (corresponding to z axis) (Fig.2.2) known otherwise as deflection

u in plane displacement (corresponding to x axis) (Fig. 2.2)

v in plane displacement (corresponding to y axis) (Fig. 2.2)

θ_x , θ_y	total rotation of the cross sections zx, zy respectively (Fig. 2.2)
$w,_x$, $w,_y$ $w,_{xx}$, $w,_{xy}$, $w,_{yy}$	first and second derivatives of the transverse displacement with respect to x or y axis (Physical meaning slopes and curvatures)
ϕ_x, ϕ_y	transverse shear deformation of the zx, zy respectively identical with symbols used before as shear strains γ_x , γ_y for the cross section zx, zy respectively (Fig. 2.2)
$\{M_{ij}\}$ $(i,j = x,y)$	moments vector
$\{Q_i\}$ $(i = x,y)$	Shear forces vector
$\{N_{ij}\}$ $(i,j = x,y)$	in plane forces vector
$\{\bar{R}_o\}$	prescribed nodal force vector (corresponding with the displacement vector $\{\delta_o^e\}$ as far as the work product is concerned)
$\bar{M}$, $\bar{Q}$, $\bar{\theta}$, $\bar{w}$	prescribed quantities of the same nature as the ones noted above
$\bar{P} = \begin{Bmatrix} \bar{P}_x \\ \bar{P}_y \\ \bar{P}_z \end{Bmatrix}$	distributed load vector (corresponding to the axes x, y, z respectively)
A_n	Area of the n^{th} element

$S_{o,n}$	portion of the boundary where (M_{nn}, M_{ns}, Q_n) are prescribed
$S_{n,n}$	portion of the boundary where (w, θ_n, θ_s) are prescribed
$[K_n^o]$	the stiffness matrix of the nth element with respect to the local system
$[R_n^o]$	the load vector of the nth element with respect to the local system
$[K_n]$	the stiffness matrix of the nth element with respect to the global system
$\{R_n\}$	the load vector of the nth element with respect to the global system
$\{\delta^e\}$	the overall nodal degrees of freedom vector
$[K]$	the overall stiffness matrix
$[R]$	the overall load vector

4.2 Displacement-models

For the displacement models the functional which is employed is the potential energy of the continuum. The condition enforced through the variational principle, is such that it minimises the potential energy. The polynomials employed to approximate the unknown functions of the functional, are functions of certain nodal values (degrees of freedom) which from the structural analysis point of view are displacement or derivatives of the displacements.[8,17,21,29, 30,36,44,63,65,74,115] Continuity, compatability and completeness requirements will be discussed for each individual model.

The potential energy for a sandwich plate is

$$I = \sum_{1}^{n} \left\{ \frac{1}{2} \iint_{A_n} \{\varepsilon\}^T \{\sigma\} \, dA - \iint_{A_n} \{\delta_o\}^T \{\bar{P}\} \, dA - \{\delta_o^e\}^T \{\bar{R}_o\} \right\} \quad (4.6)$$

for the finite element approximation all the parameters must be expressed in the functional as functions of the unknown nodal values using the notation of Chapter 4, section 1).
For the diplsacements models the vector $\{\delta_o^e\}$ includes as nodal degrees of freedom displacements and derivatives of the displacements.
After the substitutions the functional has the form:-

$$\bar{I} = \sum_{1}^{n} \left\{ \frac{1}{2} \{\delta_o^e\}^T \iint_{A_n} [B]^T [D] [B] \, dA \, \{\delta_o^e\} - \{\delta_o^e\}^T \iint_{A_n} [N]^T \{\bar{P}\} \, dA - \{\delta_o^e\}^T \{\bar{R}_o\} \right\} \quad (4.7)$$

where Σ is the summation symbol and n the number of the elements).

(Appendices II and III provide more details of the nature of the matrices involved in the above expression in the form they have been developed for the present applications.)

By assuming that the following equations are valid:-

$$[K_n^o] = \iint_{A_n} [B]^T [D][B] \, dA \qquad \{R_n^o\} = \iint_{A_n} [N]^T \{\bar{P}\} \, dA + \{\bar{R}_o\} \qquad (4.8)$$

the expression (4.7) becomes:-

$$\bar{I} = \sum_1^n \left\{ \frac{1}{2} \{\delta_o^e\}^T [K_n^o] \{\delta_o^e\} - \{\delta_o^e\}^T \{R_n^o\} \right\} \qquad (4.9)$$

Applying the variation of the functional $\bar{I}$ in the form

$$\frac{\partial \bar{I}}{\partial \{\delta_o^e\}} = \phi. \qquad (4.10)$$

one obtains:-

$$\sum_1^n \left\{ [K_n^o] \{\delta_o^e\} - \{R_n^o\} \right\} = \phi. \qquad (4.11)$$

or

$$[K] \{\delta^e\} - \{R\} = \phi. \qquad (4.12)$$

4.3 Mixed models

For the mixed models the functional is of a different form than the one used for the displacement models.

The conditions enforced through the variational principle leads to a stationary value of the functional. The polynomials employed to approximate the unknown functions are functions of certain nodal values (degrees of freedom) which are from the structural analysis point of view displacement or derivatives of the displacement as well as stresses. [25,38,56,57,80,81,82,87,90,91,95,105,106,107]

The continuity, compatibility and completeness requirements can vary.

The functional has the form

$$I = \sum_{1}^{n} \left\{ \iint_{A_n} \left(\{\sigma_b\}\{\theta_o\}^T - \frac{1}{2}\{\sigma_b\}^T [C_b] \{\sigma_b\} + \frac{1}{2}\{\varepsilon_{uv}\}^T [D_{uv}]\{\sigma_{uv}\} \right) dA \right.$$

$$- \iint_{A_n} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \{\bar{P}\}^T \, dA \quad - \quad \{\delta_o^e\}^T \quad \{\bar{R}_o\}$$

$$- \int_{S_{o,n}} (\bar{M}_{nn} \, \theta_n + \bar{M}_{ns} \, \theta_s + \bar{Q}_n \, w) \, ds$$

$$\left. - \int_{S_{n,n}} [M_{nn} \, (\theta_n - \bar{\theta}_n) + M_{ns} \, (\theta_s - \bar{\theta}_s) \; [\, Q_n \, (w - \bar{w}) \,] \, ds \right\} \qquad (4.13)$$

(See equations 2.9, 2.10, 2.11, 2.12, 2.13)

where

$$\{\theta_o\}^T = \{-\theta_{x,x}, \; -\theta_{y,y}, -(\theta_{x,y} + \theta_{y,x}), -\theta_x + w_{,x}, -\theta_y + w_{,y}\}$$

$$\{\sigma_b\}^T = \{ M_{xx}, \quad M_{yy}, \quad M_{xy}, \quad Q_x, \quad Q_y \}$$

$$\{\varepsilon_{uv}\}^T = \{ \varepsilon_{xx}, \quad \varepsilon_{yy}, \quad \varepsilon_{xy} \} \qquad (4.14)$$

$$\{\sigma_{uv}\}^T = \{ N_{xx}, \quad N_{yy}, \quad N_{xy} \}$$

$$[C_b] = [D_b]^{-1} = \begin{bmatrix} \frac{D^{yy}_{yy}}{D_o} & -\frac{D^{xx}_{yy}}{D_o} & \phi. & \phi. & \phi. \\ -\frac{D^{yy}_{xx}}{D_o} & \frac{D^{xx}_{xx}}{D_o} & \phi. & \phi. & \phi. \\ \phi. & \phi. & \frac{2}{D^{xy}_{xy}} & \phi & \phi. \\ \phi. & \phi. & \phi. & \frac{1}{S^{xz}_{xz}} & \phi. \\ \phi. & \phi. & \phi. & \phi. & \frac{1}{S^{yz}_{yz}} \end{bmatrix} \quad (4.15)$$

$$[D_{uv}] = \begin{bmatrix} \bar{E}^{xx}_{xx} & \bar{E}^{xx}_{yy} & \phi. \\ E^{yy}_{xx} & E^{yy}_{yy} & \phi. \\ \phi. & \phi. & E^{xy}_{xy} \end{bmatrix} \quad (4.16)$$

The following expression is obtained as the functional using equations (4.13), assuming continuity for M_{nn}, M_{ns}, w, u, v across element interfaces, following the procedure of [25,38,56,57,80,81,82, 87,90,91,95,105,107].

$$I = \sum_{1}^{n} \left\{ \iint_{A_n} \left(\{\sigma_b\}^T\{\theta_1\} - \frac{1}{2}\{\sigma_b\}^T [C_b] \{\sigma_b\} + \frac{1}{2}\{\varepsilon_{uv}\}^T [D_{uv}] \{\sigma_{uv}\} \right) dA \right.$$

$$- \iint_{A_n} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \{\bar{P}\}^T dA - \{\delta_o\}\{\bar{R}_o\}$$

$$\left. - \int_{S_{o,n}} \bar{Q}_n \, w \, ds + \int_{S_{n,n}} (M_{nn}\bar{\theta}_n + M_{ns}\bar{\theta}_s) \, ds \right\} \quad (4.17)$$

where

$$\{\theta_1\}^T = \{ \phi. , \phi. , \phi. , w,_x , w,_y \} \quad (4.18)$$

for the finite elements approximation all the parameters must be expressed in the functional as functions of the unknown nodal values using the notation of Chapter 4, section 1). (For more details see Appendices II and IV).

For the mixed models the nodal degrees of freedom are displacements together with moments.

After the substitutions the functional has the form

$$\bar{I} = \sum_{1}^{n} \left\{ \{M_o^e\}^T [K_n^{mw}] \{\delta_o^w\} + \frac{1}{2} \{M_o^e\}^T [K_\eta^{mq}] \{M_o^e\} + \frac{1}{2} \{\delta_o^{uv}\}^T [K_{uv}] \{\delta_o^{uv}\} \right.$$

$$\left. - \{M_o^e\} \{R_n^m\} - \{\delta_o^w\} \{R_n^w\} - \{\delta_o^{uv}\} \{R_n^{uv}\} \right\} \qquad (4.19)$$

Applying the variation of the functional $\bar{I}$ in the form

$$\left.\begin{aligned} \frac{\partial \bar{I}}{\partial \{\delta_o^{uv}\}} &= \phi. \\ \frac{\partial \bar{I}}{\partial \{\delta_o^{w}\}} &= \phi. \end{aligned}\right\} \text{Force-displacements relationships} \qquad (4.20)$$

$$\frac{\partial \bar{I}}{\partial \{M_o^e\}} = \phi. \quad \text{Equilibrium equations} \qquad (4.21)$$

one obtains

$$\sum_{1}^{n} \left\{ [K_n^o] \{\delta_o^e\} - \{R_n^o\} \right\} = \phi. \qquad (4.22)$$

$$[K_n^o] = \begin{bmatrix} [K_n^{uv}] & [\ \phi.\ ] & [\ \phi.\ ] \\ [\ \phi.\ ] & [\ \phi.\ ] & [K_n^{mw}]^T \\ [\ \phi.\ ] & [K_n^{mw}] & [K_n^{mq}] \end{bmatrix} \qquad (4.23)$$

$$\{\delta_o^e\} = \begin{Bmatrix} \{\delta_o^{uv}\} \\ \{\delta_o^{w}\} \\ \{M_o^e\} \end{Bmatrix} \tag{4.24}$$

$$\{R_n^o\} = \begin{Bmatrix} \{R_n^{uv}\} \\ \{R_n^{w}\} \\ \{R_n^{m}\} \end{Bmatrix} \tag{4.25}$$

or

$$[K] \quad \{\delta_o^e\} \; - \; \{R\} \; = \; \phi. \tag{4.26}$$

Thus a system of linear simultaneous equations is obtained for the mixed models, which are of a similar form to those obtained for the displacement models (4.12).

The stiffness matrix [K] for both the displacements and the mixed models is symmetric and positively definite. When the boundary conditions are introduced the stiffness matrix becomes nonsingular.

Thus a solution can be obtained by using one of the techniques for solving a large system of equations taking advantage of the symmetry and the banded nature of the stiffness matrix [64,75,108]

The various characteristics and advantages of the different techniques are presented in references [39,72,115].

The technique employed in the present analysis is a modification of the frontal solution [20] as it has been developed by BETTESS, compatible with the M.T.S. system (N.U.M.A.C.). There is also a version of the same technique with the same alteration compatible with the OS system in Cambridge.

The frontal solution technique has been proved advantageous due to the nature of the analysed problems involving a very large number of unknowns and complex boundary conditions.

It has been combined with a data generation programme (to be outlined in Chapter 11) which reduces effectively the amount of work required for the solution of a specific problem.

5. MODELS EMPLOYED IN THE FORMATION OF THE SANDWICH PLATE AND SANDWICH DOME MODELS

As it has been presented in Chapter 4 starting from the functionals (4.6), (4.13) and following the variational approach outlined in Chapter 3 a system of linear simultaneous equations can be obtained (4.12), (4.26). According to this approach the different parameters of the functional are approximated with different finite element models which are to be presented in this chapter. The various models for the sandwich plate and sandwich dome problems are composed by suitable combinations of the various basic models which can be classified in the following five groups.

(a) Transverse displacement approximating models (w)

(b) Transverse shear deformation approximating models (ϕ_x, ϕ_y)

(c) In-plane displacements approximating models (u, v)

(d) Moments approximating models (M_{xx}, M_{yy}, M_{xy})

(e) Total rotations approximating models (θ_x, θ_y)

All the above five groups of finite elements in the present analysis are of triangular shape with corner nodes and may also have mid-side or centre nodes (Fig.5.1÷5.15).The triangular shaped models have the potentiality of being applicable to any shape of structure in the interest of the present work.

5.1 Transverse displacement approximating models

Numerous models have been developed for the analysis of classical plate bending (shear deformation neglected). The following have been chosen to be employed in the present analysis as they have been proved successful. [9,10,11,17,18,24,29,30,35, 36,44,48,49,63,92,97,111,112,113,114,115]

5.1.1 Non-conforming triangular finite element in plate bending

This element has been presented first in 1965 [12]

Further investigation of its characeristics has been accomplished by several applications [26,115] Through the conclusions which have been obtained this element has been proved to be simple and successful. The discontinuity of normal slope across the interelement boundaries doesnot prevent the element from yielding accuracy and convergence occurs for regular element sub-divisions.

The transverse displacement w has been given by the relationship

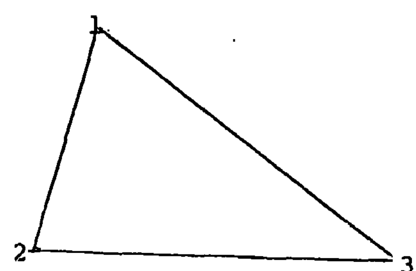


Fig. 5.1

$$w = [N^w] \{\delta_o^w\} \tag{5.1}$$

$$\{\delta_o^w\}^T = [w_1, w_{,x1}, w_{,y1}, w_2, w_{,x2}, w_{,y2}, w_3, w_{,x3}, w_{,y3}] \tag{5.2}$$

subscript x, y indicates derivatives of x, y respectively and the number (1 ÷ 3) indicates the relevant node

$$[N^w] = [N_1^w, N_2^w, N_3^w, N_4^w, N_5^w, N_6^w, N_7^w, N_8^w, N_9^w] \tag{5.3}$$

$$N_1^w = L_1 + L_1^2 L_2 + L_1^2 L_3 - L_1 L_2^2 - L_1 L_3^2 \Longrightarrow N_4^w, N_7^w \text{ from } N_1^w \text{ with}$$

circle-symmetrical substitution of subscripts 1, 2, 3

$$N_2^w = c_3(L_1^2 L_2 + \frac{1}{2} L_1 L_2 L_3) - c_2(L_1^2 L_3 + \frac{1}{2} L_1 L_2 L_3) \Longrightarrow N_5^w, N_6^w \text{ from } N_2^w \text{ as above}$$

$$N_3^w = b_2(L_1^2 L_3 + \frac{1}{2} L_1 L_2 L_3) - b_3(L_1^2 L_2 + \frac{1}{2} L_1 L_2 L_3) \Longrightarrow N_6^w, N_9^w \text{ from } N_3^w \text{ as above} \tag{5.4}$$

(see Appendix I for the geometric symbols in use).

5.1.2 Refined triangular plate bending (eighteen-degrees-of-freedom) finite element

The variation of the transverse displacement for this element is a quintic polynomial of x, y (or L_1, L_2, L_3).
It derives from the full quintic polynomial of 21 terms assuming cubic variation of the normal slope w_{n_i} along the interelement boundaries (see Appendix III). The main advantages of the refined element is that ensuring continuity of the normal slope the convergence is much faster, thus good accuracy can be obtained for coarse mesh idealizations. [18,35,36,37,115]

The discontinuity of the normal curvature and hence bending moment are much smaller than lower order elements. The disadvantage of this element is the difficulty in applying the boundary conditions due to the existence of higher order derivatives of the transverse displacements

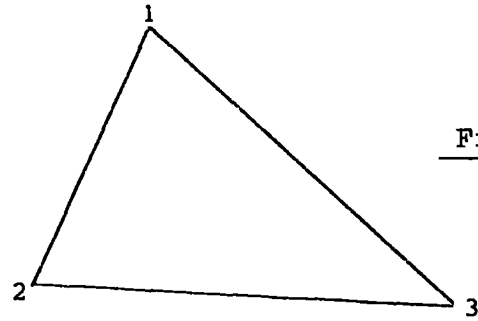


Fig. 5.2

$$w = [N^w] \{\delta_o^w\} \tag{5.5}$$

$$\{\delta_o^w\}^T = [w_i, w_{,xi}, w_{,yi}, w_{,xxi}, w_{,xyi}, w_{,yyi}, \ldots\ldots]_{i = 1 \div 3} \tag{5.6}$$

$$[N^w] = [F] \quad [T]^{-1} \tag{5.7}$$

the formation of the matrices [F] and $[T]^{-1}$ is given in Appendix III.

5.1.3 Linear variation of the transverse displacement

This model is to be used in the mixed formulation [39,59,72,105,115] (See Appendix IV).

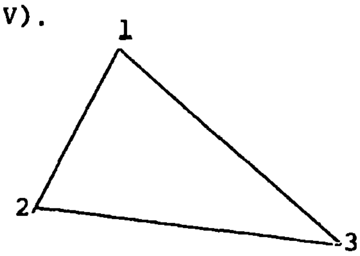


Fig. 5.3

$$w = [N^w] \quad \{\delta_o^w\} \tag{5.8}$$

$$\{\delta_o^w\}^T = [w_1, w_2, w_3] \tag{5.9}$$

$$[N^w] = [L_1, L_2, L_3] \tag{5.10}$$

5.1.4 Quadratic variation of the transverse displacement

This model is also to be used in the mixed formulation [39,59,72,105,115] (See Appendix IV).

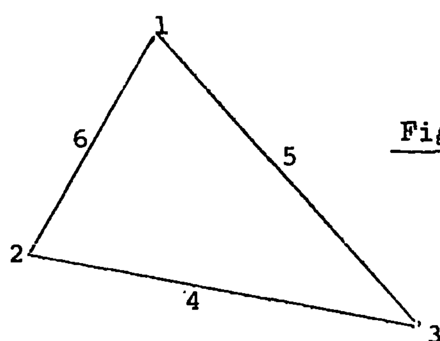


Fig. 5.4

$$w = [N^w] \quad \{\delta_o^w\} \tag{5.11}$$

$$\{\delta_o^w\}^T = [w_1, w_2, w_3, w_4, w_5, w_6] \tag{5.12}$$

$$[N^w] = [(2L_1-1)L_1, \ (2L_2-1)L_2, \ (2L_3-1)L_3, \ 4L_2L_3, \ 4L_1L_3, \ 4L_1L_2] \tag{5.13}$$

5.1.5 Cubic variation of the transverse displacement

This element is to be used in the rotation element (see Chapter 6) [39,59,72,105,115]

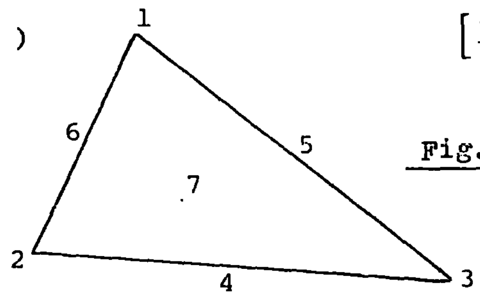


Fig. 5.5

$$w = [N^w] \; \{\delta^w_o\} \tag{5.14}$$

$$\{\delta^w_o\}^T = \{w_i, \; ....\}_{i=1 \div 7} \tag{5.15}$$

$$[N^w] = [N^w_i, \; ....)_{i=1 \div 7} \tag{5.16}$$

$$
\begin{aligned}
N^w_1 &= (2L_1-1)L_1 + 3L_1L_2L_3 \\
N^w_2 &= (2L_2-1)L_2 + 3L_1L_2L_3 \\
N^w_3 &= (2L_3-1)L_3 + 3L_1L_2L_3 \\
N^w_4 &= 4L_2L_3 - 12L_1L_2L_3 \\
N^w_5 &= 4L_1L_3 - 12L_1L_2L_3 \\
N^w_6 &= 4L_1L_2 - 12L_1L_2L_3 \\
N^w_7 &= 27L_1L_2L_3
\end{aligned}
\tag{5.17}
$$

5.2 Transverse shear deformation approximating models. [39,59,72,105,115]

5.2.1 Linear variation of transverse shear deformation.

$$\phi_x = [N^s] \{\delta_1^s\} \tag{5.18}$$

$$\phi_y = [N^s] \{\delta_2^s\} \tag{5.19}$$

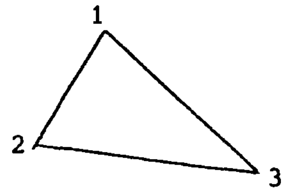


Fig. 5.6

$$\{\delta_1^s\}^T = [\phi_{x1}, \quad \phi_{x2}, \quad \phi_{x3}] \tag{5.20}$$

$$\{\delta_2^s\}^T = [\phi_{y1}, \quad \phi_{y2}, \quad \phi_{y3}] \tag{5.21}$$

$$[N^s] = [L_1, \quad L_2, \quad L_3] \tag{5.22}$$

5.2.2 Quadratic variation of transverse shear deformation

$$\phi_x = [N^s] \{\delta_1^s\} \tag{5.23}$$

$$\phi_y = [N^s] \{\delta_2^s\} \tag{5.24}$$

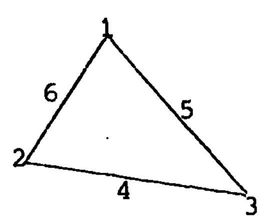


Fig. 5.7

$$\{\delta_1^s\}^T = [\phi_{xi}, \dots\dots] \quad i = 1 \div 6 \tag{5.25}$$

$$\{\delta_2^s\}^T = [\phi_{yi}, \dots\dots] \quad i = 1 \div 6 \tag{5.26}$$

$$[N^s] = [(2L_1-1)L_1, \ (2L_2-1)L_2, \ (2L_3-1)L_3, \ 4L_2L_3, \ 4L_1L_3, \ 4L_1L_2] \tag{5.27}$$

5.2.3 Cubic variation of transverse shear deformation.

$$\phi_x = [N^s] \{\delta_1^s\} \quad (5.28)$$

$$\phi_y = [N^s] \{\delta_2^s\} \quad (5.29)$$

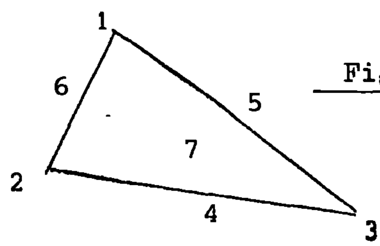


Fig. 5.8

$$\{\delta_1^s\}^T = \{\phi_{xi}, \ldots\ldots\} \quad i = 1 \div 7 \quad (5.30)$$

$$\{\delta_2^s\}^T = \{\phi_{yi}, \ldots\ldots\} \quad i = 1 \div 7 \quad (5.31)$$

$$[N^s] = [N_i^s, \ldots\ldots] \quad i = 1 \div 7 \quad (5.32)$$

$$N_1^s = (2L_1 - 1)L_1 + 3L_1L_2L_3$$

$$N_2^s = (2L_2 - 1)L_2 + 3L_1L_2L_3$$

$$N_3^s = (2L_3 - 1)L_3 + 3L_1L_2L_3$$

$$N_4^s = 4L_2L_3 - 12L_1L_2L_3 \quad (5.33)$$

$$N_5^s = 4L_1L_3 - 12L_1L_2L_3$$

$$N_6^s = 4L_1L_2 - 12L_1L_2L_3$$

$$N_7^s = 27L_1L_2L_3$$

5.3 In plane displacement approximating models

The first successful examples of the application of the finite element method were the two dimensional elastic problems of plane stress. The majority of the various existing finite elements are based on the displacement approach [7,8,28,38,39,42,58,59,72,103,105,111,115] although there are several finite elements based on different approaches

In the present analysis four displacement triangular models have been chosen as the most suitable and simple.

5.3.1 Linear variation of the in plane displacements

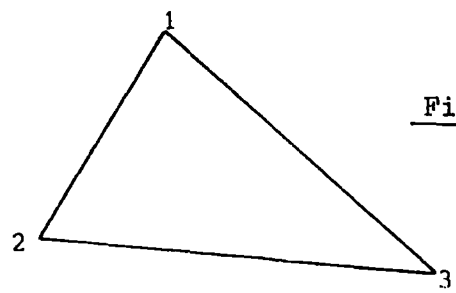


Fig. 5.9

$$u = [N^{uv}] \quad \{\delta_1^{uv}\} \tag{5.34}$$

$$v = [N^{uv}] \quad \{\delta_2^{uv}\} \tag{5.35}$$

$$\{\delta_1^{uv}\}^T = [u_1, u_2, u_3] \tag{5.36}$$

$$\{\delta_2^{uv}\}^T = [v_1, v_2, v_3] \tag{5.37}$$

$$[N^{uv}] = [L_1, L_2, L_3] \tag{5.38}$$

(see Appendix II for the formation of $[B^{uv}]$ matrix.

5.3.2 Quadratic variation of the in plane displacements

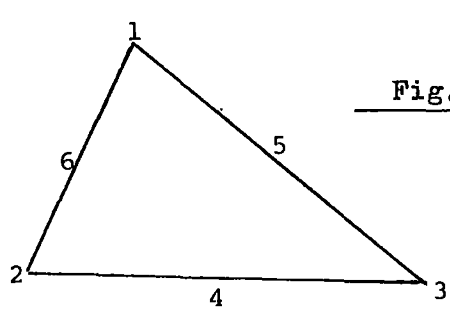


Fig. 5.10

$$u = [N^{uv}] \{\delta_1^{uv}\} \tag{5.39}$$

$$v = [N^{uv}] \{\delta_2^{uv}\} \tag{5.40}$$

$$\{\delta_1^{uv}\}^T = [u_i, \ldots]_{i=1 \div 6} \tag{5.41}$$

$$\{\delta_2^{uv}\}^T = [v_i, \ldots]_{i=1 \div 6} \tag{5.42}$$

$$[N^{uv}] = [(2L_1-1)L_1, (2L_2-1)L_2, (2L_3-1)L_3, 4L_2L_3, 4L_1L_3, 4L_1L_2] \tag{5.43}$$

5.3.3 Cubic variation of the in plane displacements

The variation along the interelement boundaries is quadratic. This element has been developed to cope with transformation difficulties along plate interconnections for the dome structures.

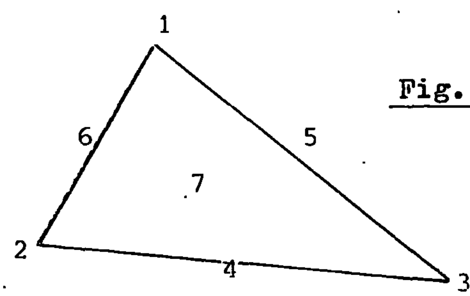


Fig. 5.11

$$u = [N^{uv}] \{\delta_1^{uv}\} \tag{5.44}$$

$$v = [N^{uv}] \{\delta_2^{uv}\} \tag{5.45}$$

$$\{\delta_1^{uv}\}^T = [u_1, u_2, u_3, u_{,SS4}, u_{,SS5}, u_{,SS6}, u_7] \tag{5.46}$$

$$\{\delta_2^{uv}\}^T = [v_1, v_2, v_3, v_{,SS4}, v_{,SS5}, v_{,SS6}, v_7] \tag{5.47}$$

where

$$u_{,SS} = \frac{\partial^2 u}{\partial \bar{s}^2} \quad , \quad v_{,SS} = \frac{\partial^2 v}{\partial \bar{s}^2} \tag{5.48}$$

(where $\bar{s}$ is the vector along the sides $\overrightarrow{23}$, $\overrightarrow{31}$, $\overrightarrow{12}$)

(see also Appendx I (8)).

$$[N^{uv}] = [N_i^{uv}, \ldots]_{i=1 \div 7} \tag{5.49}$$

$$N_1^{uv} = L_1-9L_1L_2L_3 \quad , \quad N_2^{uv} = L_2-9L_1L_2L_3 \quad , \quad N_3^{uv} = L_3-9L_1L_2L_3$$

$$N_4^{uv} = -0.5L_2L_3+1.5L_1L_2L_3 \quad , \quad N_5^{uv} = -0.5L_1L_3+1.5L_1L_2L_3 \quad , \quad N_6^{uv} = 0.5L_1L_2+1.5L_1L_2L_3$$

$$N_7^{uv} = 27L_1L_2L_3 \tag{5.50}$$

5.3.4 Cubic variation of the in plane-displacements.

$$u = [N^{uv}] \{\delta_1^{uv}\} \qquad (5.51)$$

$$v = [N^{uv}] \{\delta_2^{uv}\} \qquad (5.52)$$

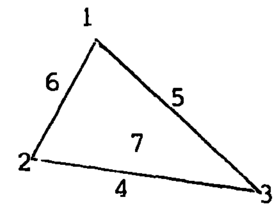


Fig. 5.12

$$\{\delta_1^{uv}\}^T = [u_i , \dots] \quad i = 1 \div 7 \qquad (5.53)$$

$$\{\delta_2^{uv}\}^T = [v_i , \dots] \quad i = 1 \div 7 \qquad (5.54)$$

$$[N^{uv}] = [N_i^{uv}, \dots] \quad i = 1 \div 7 \qquad (5.55)$$

$$N_1^{uv} = (2L_1 - 1)L_1 + 3L_1L_2L_3$$

$$N_2^{uv} = (2L_2 - 1)L_2 + 3L_1L_2L_3$$

$$N_3^{uv} = (2L_3 - 1)L_3 + 3L_1L_2L_3$$

$$N_4^{uv} = 4L_2L_3 - 12L_1L_2L_3 \qquad (5.56)$$

$$N_5^{uv} = 4L_1L_3 - 12L_1L_2L_3$$

$$N_6^{uv} = 4L_1L_2 - 12L_1L_2L_3$$

$$N_7^{uv} = 27L_1L_2L_3$$

5.4 Moments approximating models

For the approximation of moments distribution in the mixed models approach several moments interpolation functions have been used. The nature and the order of these functions is defined by the continuity requirements associated with the functional and the variational principle used for the derivation of the model. [25,56,57,90,91,105,106,107]

The following have been chosen for the present analysis

5.4.1 Linear variation of moments

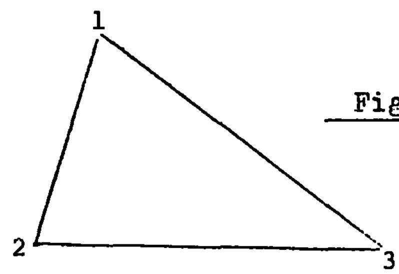


Fig. 5.13

$$M_{xx} = [N^m] \{M_1^e\} \tag{5.57}$$

$$M_{yy} = [N^m] \{M_2^e\} \tag{5.58}$$

$$M_{xy} = [N^m] \{M_3^e\} \tag{5.59}$$

$$\{M_1^e\}^T = [M_{xx1}, M_{xx2}, M_{xx3}] \tag{5.60}$$

$$\{M_2^e\}^T = [M_{yy1}, M_{yy2}, M_{yy3}] \tag{5.61}$$

$$\{M_3^e\}^T = [M_{xy1}, M_{xy2}, M_{xy3}] \tag{5.62}$$

$$[N^m] = [L_1, L_2, L_3] \tag{5.63}$$

(See Appendix IV for more details)

5.4.2 Quadratic variation of moments

$$M_{xx} = [N^m] \quad \{M_1^e\} \tag{5.64}$$

$$M_{yy} = [N^m] \quad \{M_2^e\} \tag{5.65}$$

$$M_{xy} = [N^m] \quad \{M_3^e\} \tag{5.66}$$

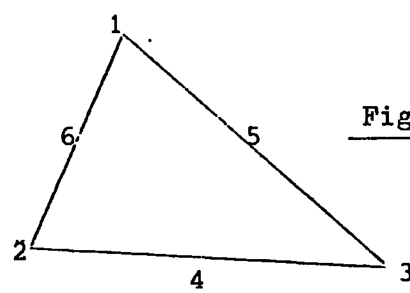


Fig. 5.14

$$\{M_1^e\}^T = [M_{xxi} \, , \, ...] \qquad i = 1 \div 6 \tag{5.67}$$

$$\{M_2^e\}^T = [M_{yyi} \, , \, ...] \qquad i = 1 \div 6 \tag{5.68}$$

$$\{M_3^e\}^T = [M_{xyi} \, , \, ...] \qquad i = 1 \div 6 \tag{5.69}$$

$$[N^m] = [(2L_1 - 1)L_1, \; (2L_2 - 1)L_2, \; (2L_3 - 1)L_3, \; 4L_2L_3, \; 4L_1L_3, \; 4L_1L_2] \tag{5.70}$$

5.5 Total rotation approximating models

For the total rotation finite element formulation (to be outlined in the Chapter 6) the following model has been employed. [39,59,72,105,115]

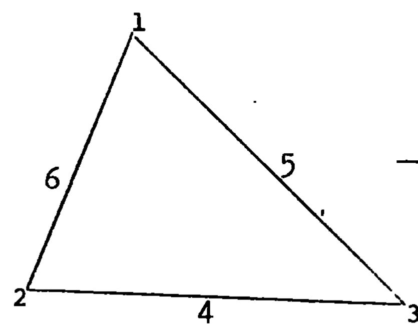


Fig. 5.15

$$\theta_x = [N^{\theta}] \{\delta_1^{\theta}\} \tag{5.71}$$

$$\theta_y = [N^{\theta}] \{\delta_2^{\theta}\} \tag{5.72}$$

$$\{\delta_1^{\theta}\}^T = [\theta_{xi}, \ldots.]_{i=1 \div 6} \tag{5.73}$$

$$\{\delta_2^{\theta}\}^T = [\theta_{yi}, \ldots.]_{i=1 \div 6} \tag{5.74}$$

$$[N^{\theta}] = [(2L_1-1)L_1,\ (2L_2-1)L_2,\ (2L_3-1)L_3,\ 4L_2L_3,\ 4L_1L_3,\ 4L_1L_2] \tag{5.75}$$

6. SANDWICH PLATE BENDING MODELS

A large number of various publications is available as far as the bending of sandwich plate is concerned
Various finite elements have also been developed to solve the problem [1,12,14,15,21,67,68,69,73,85,90,91]

Seven different models have been developed in the present analysis as a first step towards the solution of the polyhedral dome sandwich structures. These models deal with the sandwich plate bending problem and their classification coincides with the classification of the variational principles employed for the development of each individual model respectively, that is displacement models and mixed models (Chapter 4).

6.1 Displacement models

The variational principle outlined in Chapter 4 Section 2 has been employed for the development of the displacement models.

For the finite element approximation the strains and stresses in the functional are expressed as functions of nodal values of displacements (degrees of freedom).

Two different groups of sandwich plate displacement models have been developed depending on the form of strains-nodal displacements relationships.

The models classified in the first group are to be called Deflection-Shear Displacement models.

The strains as formulated in Chapter 2 have the form

$$
\begin{aligned}
a_{xx} &= w,_{xx} - \phi_{x,x} \\
a_{yy} &= w,_{yy} - \phi_{y,y} \\
a_{xy} &= 2w,_{xy} - \phi_{x,y} - \phi_{y,x} \qquad (6.1)\\
\gamma_{x} &= \phi_{x} \\
\gamma_{y} &= \phi_{y}
\end{aligned}
$$

The parameters at the right hand side of equations (6.1) are expressed as functions of nodal transverse displacements and their derivatives and nodal transverse shear deformations independently employing the models outlined in Chapter 5 (section 5.1, 5.2).

The models classified in the second group are to be called Total-Rotation displacement models.

The strains as formulated in Chapter 2 have the form

$$
\begin{aligned}
a_{xx} &= \theta_{x,x} \\
a_{yy} &= \theta_{y,y} \\
a_{xy} &= \theta_{x,y} + \theta_{y,x} \qquad (6.2)\\
\gamma_{x} &= w,_{x} - \theta_{x} \\
\gamma_{y} &= w,_{y} - \theta_{y}
\end{aligned}
$$

The parameters at the right hand side of equations (6.2) are expressed as functions of nodal total rotations and nodal transverse displacements independently, employing the models outlined in Chapter 5 (sections 5.1, 5.5).

The displacement models developed are the following:

6.1.1 Deflection-shear model with 15 degrees of freedom

For reference to this model the symbol PDS 15 has been employed (See Section 6.3 for more details).

This model has been developed employing the non-conforming triangular finite element in plate bending together with the linear variation of shear deformation model (5.1.1., 5.2.1.) for expressing the parameters in equations (6.2), (4.3). More details for the formation of the various matrices are presented in Appendix II

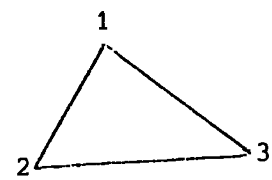


Fig. 6.1

The vector of the nodal degrees of freedom has the form

$$\{\delta_o^e\}^T = \{w_1, w_{,x1}, w_{,y1}, w_2, w_{,x2}, w_{,y2}, w_3, w_{,x3}, w_{,y3}, \phi_{x1}, \phi_{y1}, \phi_{x2}, \phi_{y2}, \phi_{x3}, \phi_{y3}\} \quad (6.3)$$

Employing the transformation relationships (to be outlined in Chapter 8) the transformed stiffness, stress and load matrices are obtained ÷

a) When the vector of the nodal degrees of freedom becomes ÷

$$\{\delta_o^e\}^T = \{w_i, w_{,xi}, w_{,yi}, \phi_{xi}, \phi_{yi}, \ \ldots\ldots\}_{\ i=1 \div 3} \quad (6.4)$$

b) For a node (i) which belongs to a boundary the set of degrees of freedom linked with this node becomes ÷

$$\{w_i,\ \theta_{ni},\ w_{,si},\ \phi_{ni},\ \phi_{si}\} \quad (6.5)$$

(where $\bar{n}$ is the vector normal to the boundary $\bar{s}$)

6.1.2 Deflection-shear model with 21 degrees of freedom

Reference symbol PDS21

This model has been developed employing the non-conforming triangular finite element in plate bending together with the quadratic variation of shear deformation model (5.1.1., 5.2.2) for expressing the parameters in equations (6.1), (4.8). The same procedure for the formation of the various matrices is employed as for the PDS15 model (Appendix II).

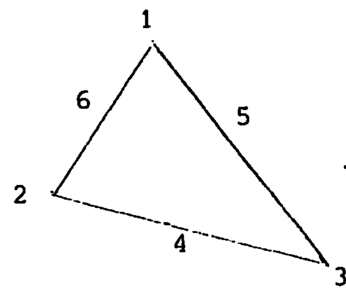


Fig. 6.2

The vector of the nodal degrees of freedom has the form:

$$\{\delta_o^e\}^T = \{w_1, w,_{x1}, w,_{y1}, w_2, w,_{x2}, w,_{y2}, w_3, w,_{x3}, w,_{y3}, \phi_{x1}, \phi_{y1}, \phi_{x2}, \phi_{y2}, \phi_{x3}, \phi_{y3}, \phi_{x4}, \phi_{y4}, \phi_{x5}, \phi_{y5}, \phi_{x6}, \phi_{y6}\} \quad (6.6)$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained.

a) When the vector of the nodal degrees of freedom becomes ÷

$$\{\delta_o^e\}^T = \{w_i, w,_{xi}, w,_{yi}, \phi_{xi}, \phi_{yi}, \ \ldots, \phi_{xj}, \phi_{yj}, \ \ldots\} \quad \begin{matrix} i = 1 \div 3 \\ j = 4 \div 6 \end{matrix} \quad (6.7)$$

b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes ÷

$$\{w_i, \theta_{ni}, w,_{si}, \phi_{ni}, \phi_{si}\} \quad \text{for} \quad i \leq 3 \quad (6.8)$$

$$\{\phi_{ni}, \phi_{si}\} \quad \text{for } i > 3 \quad (6.9)$$

6.1.3 Deflection-shear model with 24 degrees of freedom

Reference symbol PDS24

This model has been developed employing the Refined triangular plate bending finite element (eighteen-degrees-of-freedom) together with the linear variation of shear deformation model (5.1.2., 5.2.1) for expressing the parameters in equations (6.1), (4.8).

More details for the formation of the various matrices are presented in Appendix III.

The vector of the nodal degrees of freedom has the form:

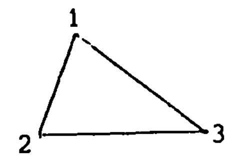


Fig. 6.3

$$\{\delta_o^e\}^T = \{w_i, w,_{xi}, w,_{yi}, w,_{xxi}, w,_{xyi}, w,_{yyi} \ldots\ldots, \phi_{xi}, \phi_{yi}, \ldots\ldots\}_{i=1 \div 3} \quad (6.10)$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained,

a) When the vector of the nodal degrees of freedom becomes

$$\{\delta_o^e\}^T = \{w_i, w,_{xi}, w,_{yi}, w,_{xyi}, w,_{yyi}, \phi_{xi}, \phi_{yi}, \ \ldots\ldots\}_{i=1 \div 3} \quad (6.11)$$

b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes $\div$

$$\{w_i, \theta_{ni}, w,_{si}, w,_{nni}, w,_{sni}, w,_{ssi}, \phi_{ni}, \phi_{si}\} \quad (6.12)$$

6.1.4 Deflection-shear model with 30 degrees of freedom

Reference symbol PDS30

This model has been developed employing the refined triangular plate bending finite element (eighteen-degrees-of-freedom) together with the cubic variation of shear deformation model. (5.1.2, 5.2.3.) for expressing the parameters in equations (6.1), (4.8). The same procedure for the formation of the various matrices is employed as for the PDS24 model (Appendix III).

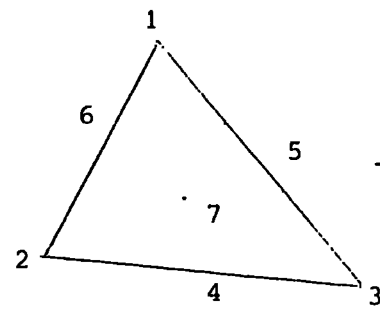


Fig. 6.4

The vector of nodal degrees of freedom after elimination of the degrees of freedom at the centre node (Chapter 8 Section 4) has the form:

$$\{\delta_o^e\}^T = \{w_i, w_{,xi}, w_{,yi}, w_{,xxi}, w_{,xyi}, w_{,yyi}, \ \ldots, \ \phi_{xj}, \phi_{yj}, \ldots\}_{\substack{i=1 \div 3 \\ j=1 \div 7}} \qquad (6.13)$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained,

a) When the vector of the nodal degrees of freedom becomes ÷

$$\{\delta_o^e\}^T = \{w_i, w_{,xi}, w_{,yi}, w_{,xxi}, w_{,xyi}, w_{,yyi}, \phi_{xi}, \phi_{yi}, \ldots \phi_{xj}, \phi_{yj}, \ldots\}_{\substack{i=1 \div 3 \\ j=4 \div 6}} \qquad (6.14)$$

b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes ÷

$$\{w_i, \theta_{ni}, w_{,si}, w_{,nni}, w_{,sni}, w_{,ssi}, \phi_{ni}, \phi_{si}\} \quad \text{for } i \leq 3 \qquad (6.15)$$

$$\{\phi_{ni}, \phi_{si}\} \quad \text{for} \quad i > 3 \qquad (6.16)$$

6.1.5 Total rotation model with 18 degrees of freedom

Reference symbol PRO18

This model has been developed employing the total rotation model together with the cubic variation of transverse displacement model (5.5, 5.1.5) for expressing the parameters in equations (6.2), (4.8)

More details for the formation of the various matrices are presented in appendix V

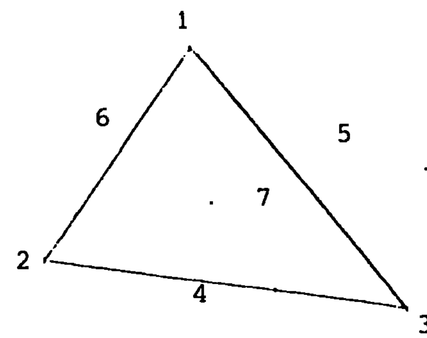


Fig. 6.5

The vector of nodal degrees of freedom after the elimination of the degree of freedom of the centre node (Chapter 8, Section 4) has the form:

$$\{\delta_o^e\}^T = \{\theta_{xi},\theta_{yi},\dots\dots\dots,w_i,\dots\} \quad i = 1 \div 6 \tag{6.17}$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained

a) when the vector of nodal degrees of freedom become ÷

$$\{\delta_o^e\}^T = \{w_i,\theta_{xi},\theta_{yi}, \dots\} \quad i = 1 \div 6 \tag{6.18}$$

(b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes ÷

$$\{w_i,\theta_{ni},\theta_{si}\} \tag{6.19}$$

6.2 Mixed models

The variational principle outlined in Chapter 4, Section 3 has been employed for the development of the mixed models.

For the finite element approximation the various parameters of the functional are expressed as functions of nodal values of moments and displacements.

The mixed models developed are the following:

6.2.1 Mixed model with 12 degrees of freedom

Reference symbol PMX12

This model has been developed employing the linear variation of moments model together with the linear variation of the transverse displacement model (5.4.1 5.1.3) for expressing the various parameters in equations (4.19) (4.22).

The same procedure for the formation of the various matrices is employed as for the PMX24 model which is to be outlined in the next paragraph (see Appendix IV).

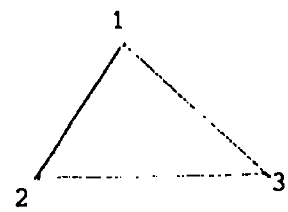


Fig. 6.6

The vector of the nodal degrees of freedom has the form

$$\{\delta_o^e\}^T = \{M_{xxi}, M_{yyi}, M_{xyi}, \;\ldots., w_i, \ldots.\} \quad i = 1 \div 3 \tag{6.20}$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained.

a) when the vector of nodal degrees of freedom becomes

$$\{\delta_o^e\}^T = \{w_i, M_{xxi}, M_{yyi}, M_{xyi}, \ldots. \} \quad i = 1 \div 3 \tag{6.21}$$

b) For a node (i) which belongs to a boundary $(\bar{s})$ the set of degrees of freedom linked with this node becomes $\div$

$$\{ w_i, M_{nni}, M_{ssi}, M_{sni} \} \tag{6.22}$$

6.2.2 Mixed model with 24 degrees of freedom

Reference symbol PMX24

This model has been developed employing the quadratic variation of moments model together with the quadratic variation of the transverse displacement model (5.4.2, 5.1.4) for expressing the various parameters in equations (4.19), (4.22).

More details for the formation of various matrices are presented in Appendix IV

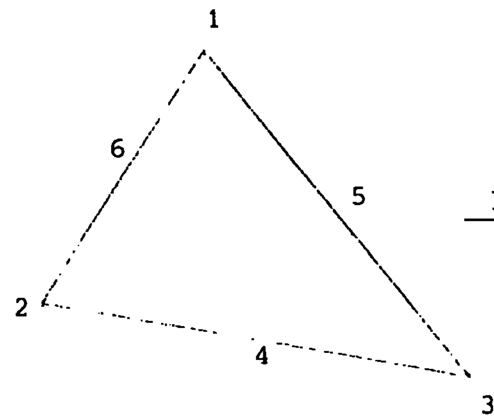


Fig. 6.7

The vector of the nodal degrees of freedom has the form

$$\{\delta_o^e\}^T = \{M_{xxi}, M_{yyi}, M_{xyi}, \ldots\ldots w_i, \ldots\ldots\}_{i = 1 \div 6} \quad (6.23)$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained.

a) when the vector of nodal degrees of freedom becomes:

$$\{\delta_o^e\}^T = \{w_i, M_{xxi}, M_{yyi}, M_{xyi}, \ldots\ldots\} \quad i = 1 \div 6 \quad (6.24)$$

b) For a node which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with the node becomes:

$$\{w_i, M_{nni}, M_{ssi}, M_{sni}\} \quad (6.25)$$

6.3 Symbols for the different elements

The first alphabetic character in the symbol indicates either a Plate element or a Dome element.

The next two alphabetic characters indicate:

a) Deflection-Shear model

b) total ROtation model

c) MiXed model.

The two last characters, the arithmetic ones, indicate the number of degrees of freedom per element.

Thus for the seven sandwich plate bending finite elements presented previously the following symbols have been employed here for each one respectively:

PDS15, PDS21, PDS24, PDS30, PRO18, PMX12, PMX24.

For the dome models which are to be presented in the following chapter (7). The following symbols have been employed in accordance with the above definition, for each one respectively:

DDS21, DDS33, DMX36, DRO30.

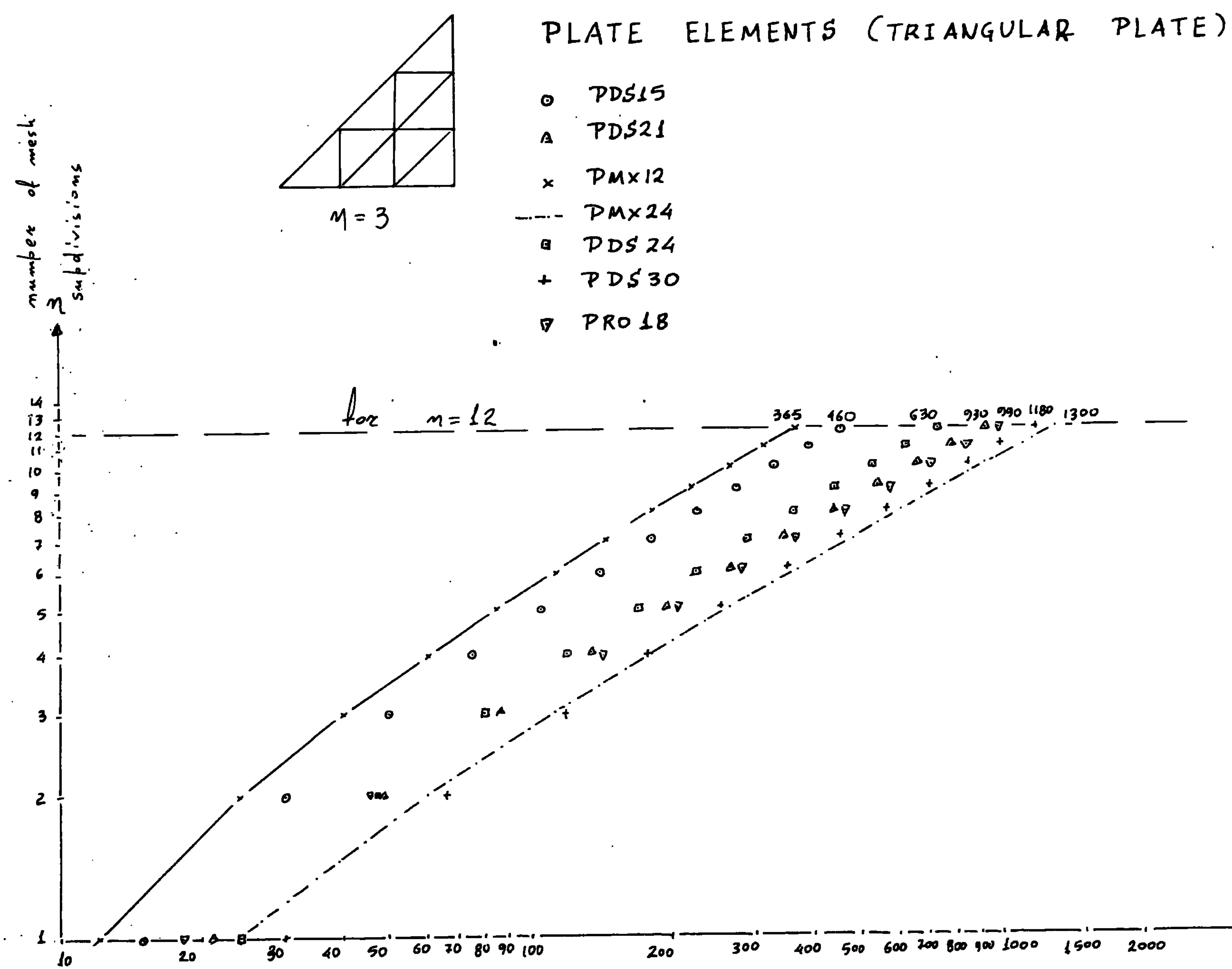
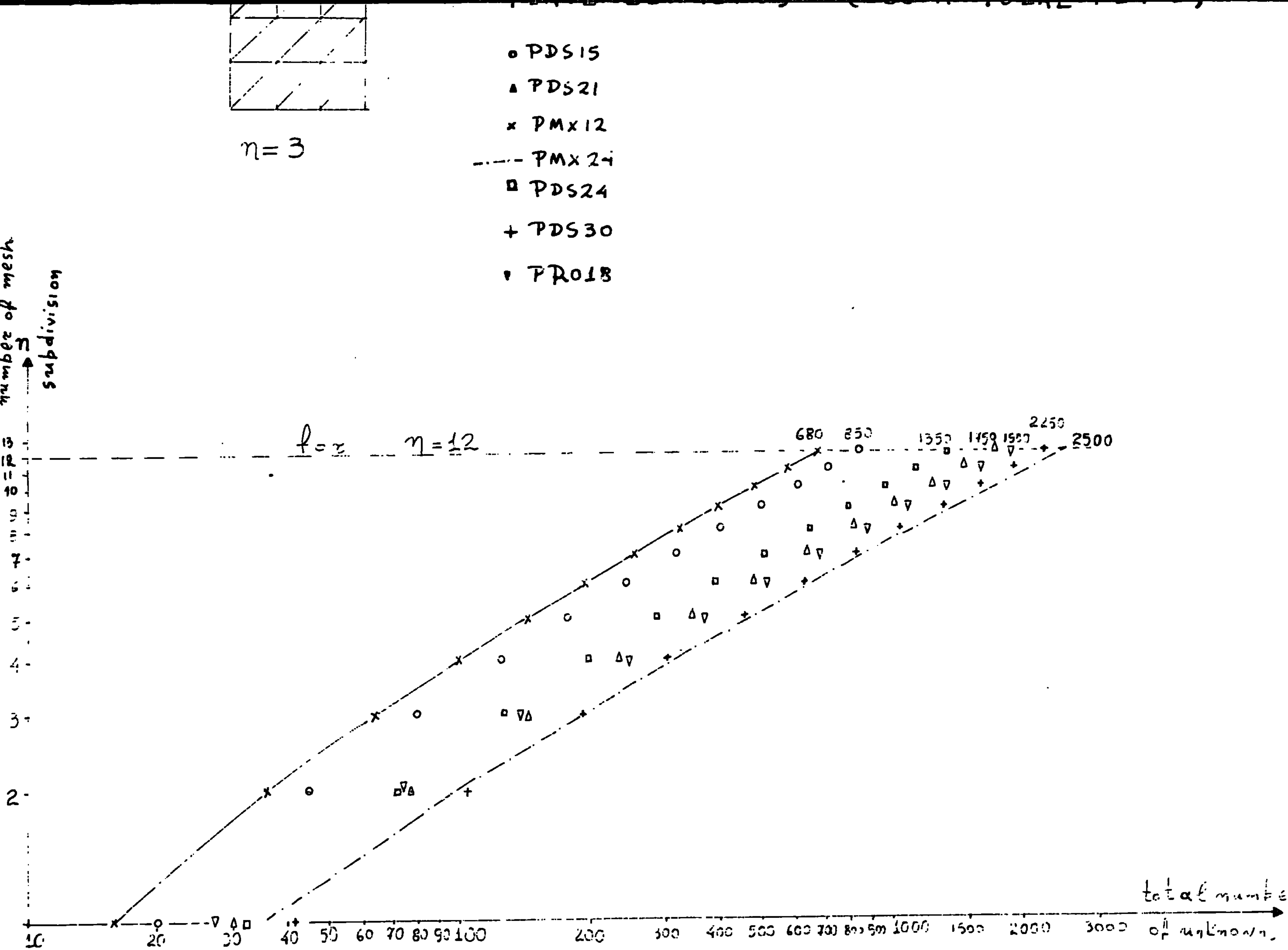


PLATE ELEMENTS (TRIANGULAR PLATE)
PDS15
PDS21
PMx12
PMx24
PDS24
PDS30
PRO18
m = 3
for m = 12
365
460
630
930
990
1180
1300
number of mesh subdivisions m
total number of unknowns
1
2
3
4
5
6
7
8
9
10
11
12
13
14
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
1500
2000

FIG. 6.8.



η = 3
PDS15
PDS21
PMX12
PMX24
PDS24
PDS30
PRO18
number of mesh subdivision
n
η = 12
680
850
1350
1750
1900
2250
2500
total number of unknown
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
1500
2000
3000

FIG. 6.9

7. DOME MODELS

The dome elements to be presented in this chapter are employed for the analysis of the behaviour of the polyhedral dome sandwich structures.

The four models which have been developed, derived from the sandwich plate bending finite elements presented in Chapter 6 in combination with the in-plane displacement models outlined in Chapter 5, section 3.

The choice of four sandwich plate bending finite elements from the total number of seven has been carried out taking into account the following important factors:

a) Accuracy obtained by each individual model for the sandwich plate bending problems in comparison with the number of degrees of freedom involved in it (see Figures 6.8, 6.9, 7.5, 7.6), (Chapter 12)

b) The set of displacements at nodes must be complete so that transformation along the plate interconnections of the polyhedral dome sandwich structure can be carried out (Chapter 8).

c) The ability to apply the necessary boundary condition without too much difficulty.

Further justification for the chosen models with regards to the above factors is to be presented in the conclusions. (Chapter 12)

The variational principles outlined in Chapter 4 are once again employed and consequently three displacement models and one mixed model are derived in a similar way with the one presented for the sandwich plate bending problem (Chapter 6).

The way the reference symbols of the various dome models are generated is presented in Chapter 6, section 3.

For nodes belonging to a boundary, either external or internal (such as plates interconnection) certain transformation and condensation techniques are used (to be described in Chapter 8).

7.1 Displacement models

The two different approaches for expressing the strain-nodal displacement relationship are once again employed (see Chapter 6, section 1). As a result two of the models are referred to as Deflection-shear displacement models and one as a total rotation model.

7.1.1 Deflection shear model with 21 degrees of freedom

Reference symbol DDS21

This model derives from the sandwich plate bending finite element PDS15 (described in 6.1.1) in combination with the linear variation of the in-plane displacements model (described in 5.3.1).

Details relevant to the formation of the various matrices involved are presented in Appendix II.

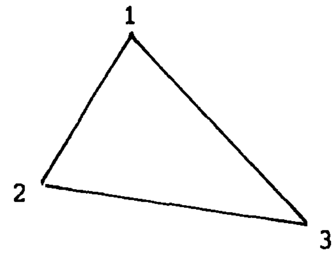


Fig. 7.1

The vector of nodal degrees of freedom through certain transformation formulae becomes:

$$\{\delta_o^e\}^T = \{u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \phi_{xi}, \phi_{yi}, \; \ldots\ldots \} \quad i = 1 \div 3 \qquad (7.1)$$

Consequently the transformed stiffness, stress and load matrices are obtained respectively (Chapter 8).

7.1.2 Deflection-shear model with 33 degrees of freedom

Reference symbol DDS33

This model derives from the sandwich plate bending finite element PDS21 (described in 6.1.2) in combination with the cubic variation of the in-plane displacement model (described in 5.3.3).

The same procedure for the formation of the various matrices is employed as for the DDS21 (Appendix II).

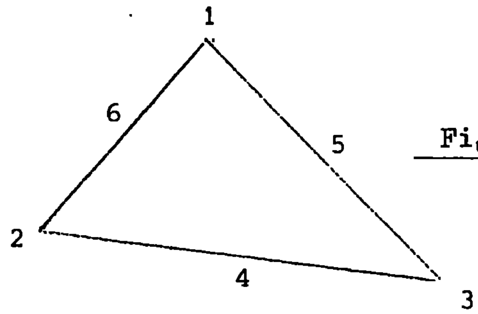


Fig. 7.2

The vector of nodal degrees of freedom through certain transformation formulae as well as elimination of the degrees of freedom at the centre node becomes:

$$\{\delta_o^e\} = \{u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \phi_{xi}, \phi_{yi}, \ \ldots, \phi_{xj}, \phi_{yj}, u_{ssj}, v_{ssj}, \ldots\}_{\substack{i=1\div 3 \\ j=4\div 6}} \qquad (7.2)$$

Consequently the transformed stiffness, stress and load matrices are obtained respectively (Chapter 8).

7.1.3 Total rotation model with 30 degrees of freedom

Reference symbol DRO30

This model derives from the sandwich plate bending finite element PRO18 (described in 6.1.5) in combination with the cubic variation of the in plane displacements model (described in 5.3.4).

Details relevant to the formation of the various matrices involved are presented in Appendix V

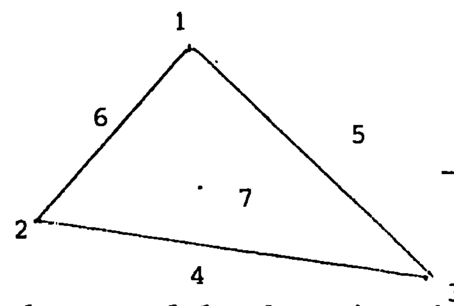


Fig. 7.3

The vector of nodal degrees of freedom, through certain transformation formulae as well as elimination of the degrees of freedom at the centre node, becomes:

$$\{\delta_o^e\} = \{u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \ldots \} \quad i = 1 \div 6 \tag{7.3}$$

Consequently the transformed stiffness, stress and load matrices are obtained respectively (Chapter 8)

7.2 Mixed models

Two models could have been derived, employing the variational principle outlined in Chapter 4, Section 2. as an extension of the sandwich plate bending mixed models, described in Chapter 6, Section 2. That is: First a model which derives from the sandwich plate bending finite element PMX12 (6.2.1) in combination with the linear variation of the in-plane displacements model (5.3.1) DMX18. Secondly, a model which derives from the sandwich plate bending finite element PMX24 (6.2.2) in combination with the quadratic variation of the in plane displacements model (5.3.2) DMX36.

Limitations of the present work with regards to time and space have allowed the derivation of only one.

Accuracy factors have led in the choice of the second model despite the advantage of the low number of degrees of freedom which the first possesses (see Figs. 7.1,7.2). The formation of the first model (DMX18) is similar to the one of the second DMX36 (to be presented next) and the results are believed to be of a similar nature with the results obtained from the application of the DMX36.

It will be seen from the discussion that the former model can easily be formed and applied.

7.2.1 Mixed model with 36 degrees of freedom

Reference symbol DMX36

Details relevant to the formation of the various matrices involved are presented in Appendix IV.

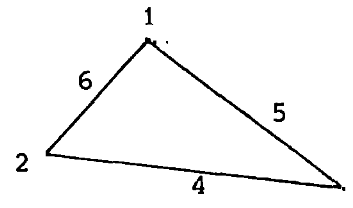
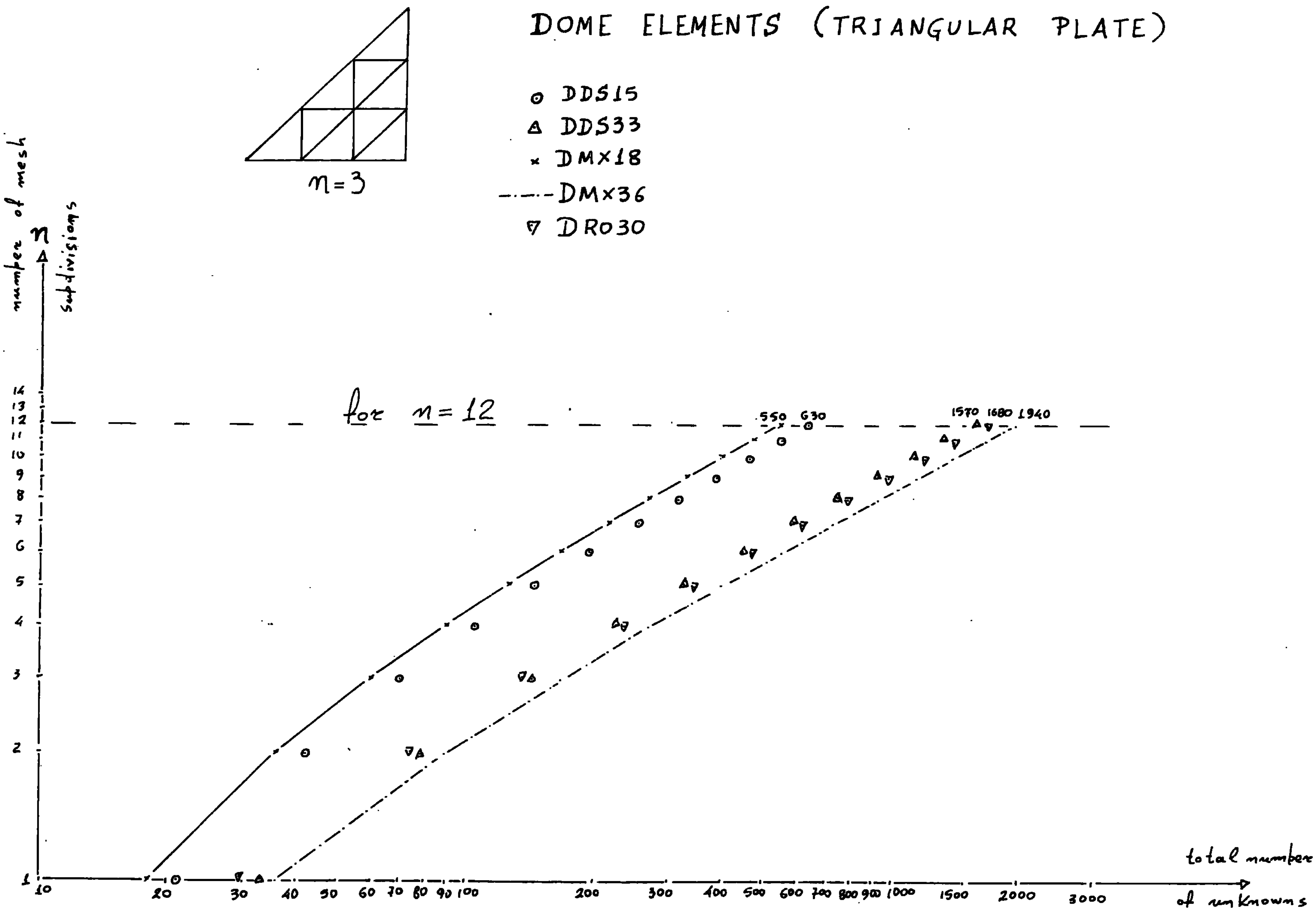


Fig. 7.4

The vector of nodal degrees of freedom through certain transformation formulae becomes:

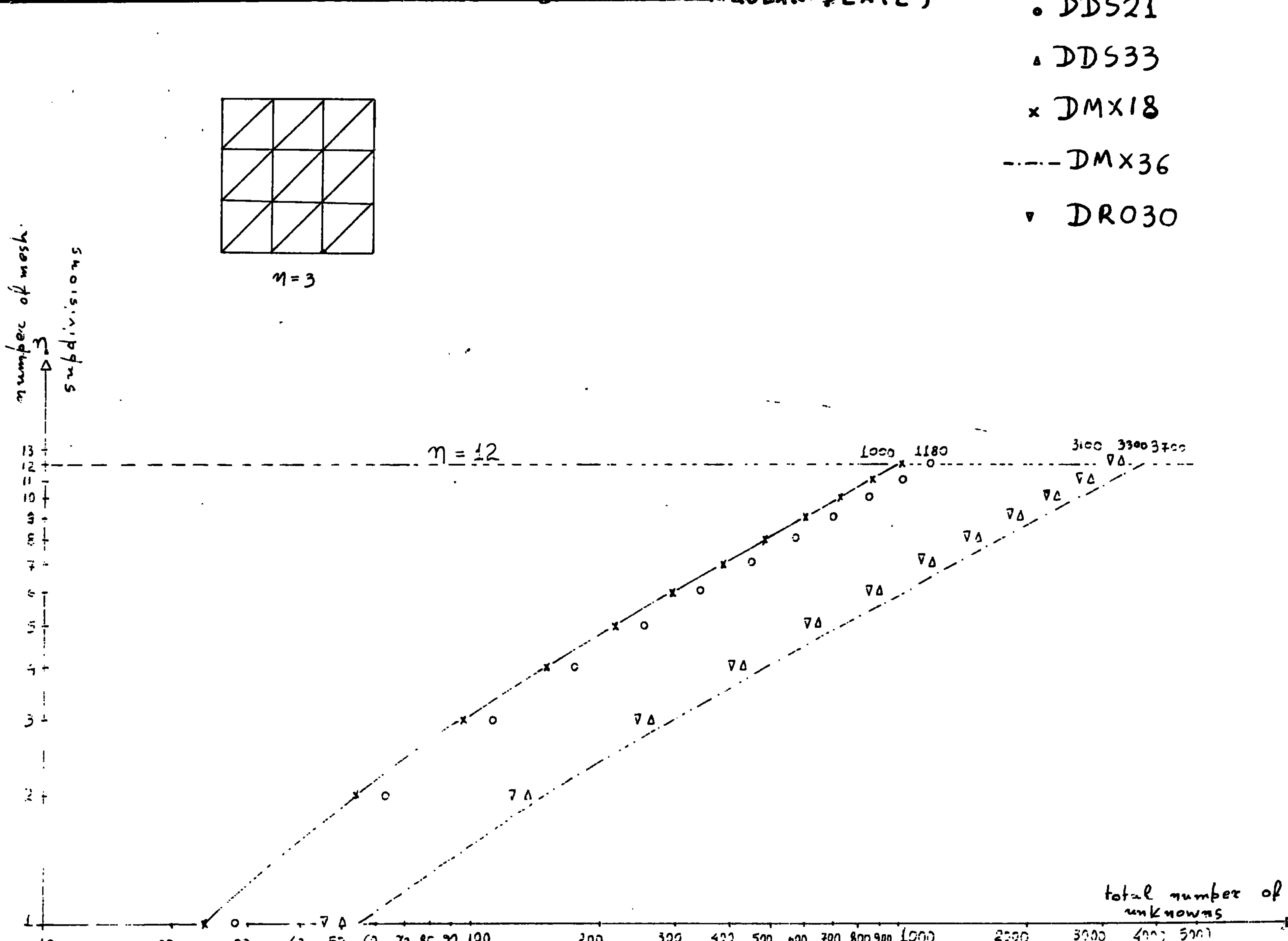
$$\{\delta_o^e\}^T = \{u_i, v_i, w_i, M_{xxi}, M_{yyi}, M_{xyi}, \ \dots \} \quad i = 1 \div 6 \qquad (7.4)$$

Consequently the transformed stiffness, stress and load matrices are obtained respectively (Chapter 8).



DOME ELEMENTS (TRIANGULAR PLATE)
n=3
DDS15
DDS33
DMX18
DMX36
DRO30
for n=12
550
630
1570
1680
1940
number of mesh subdivisions
n
total number of unknowns
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
1500
2000
3000

FIG. 7.5.



• DDS21
▴ DDS33
× DMX18
-·-·- DMX36
▾ DRO30
η = 3
number of mesh subdivisions η
η = 12
1000
1180
3100
3300
3700
total number of unknowns
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
2000
3000
1
2
3
4
5
6
7
8
9
10
11
12
13

FIG. 7.6.

8. TRANSFORMATION - CONDENSATION

To obtain the final set of equations (4.12), (4.26) by the combination of the variational approach and the finite elements approximation (Chapters 3 and 4) it is necessary to assemble the linear equations obtained in a matrix form for each individual element (4.11, 4.12, 4.22, 4.26). These equations are at first evaluated with respect to a local system of cartesian-coordinates (see Appendix I). Thus a common global coordinates system is needed and transformation from the local to the global system must be carried out for all the parameters involved.

For both the plate and the dome finite elements, described in Chapter 6 and 7 respectively, transformation must be introduced in order to assemble the elements and apply the boundary conditions.

8.1 Transformation - formulae

The form of the relationship to be transformed is as follows:

$$\sum_{i=1}^{n} \left\{ [K_n^o] . \{\delta_o^e\} - [R_n^o] \right\} = \phi. \qquad (8.1)$$

$\{\delta_o^e\}$ is the vector of the nodal degrees of freedom in the local system of cartesian coordinates.

The relationship between the nodal degrees of freedom with respect to the local system and the nodal degrees of freedom with respect to the global system has the form

$$\{\delta_o^e\} = [TR] \ \{\delta_g^e\} \qquad (8.2)$$

where $[TR]$ is the transformation matrix

As the corresponding force component must perform the same amount of work in either system the following relationship is obtained

$$\{R_n\}^T \quad \{\delta_g^e\} \quad = \quad \{R_n^o\}^T \quad \{\delta_o^e\} \tag{8.3}$$

where $\{R_n\}$, $\{\delta_g^e\}$ are the load and nodal degrees of freedom vectors with respect to the global system.

Substituting equation (8.2) to equation (8.3) the following equation can be obtained:

$$\{R_n\} \quad = \quad [TR]^T \quad \{R_n^o\} \tag{8.4}$$

Pre-multiplying equation (8.3) by $[TR]^T$ and substituting equation (8.4) the following relationship can be obtained:

$$\sum_1^n \left\{ [TR]^T \; [K_n^o] \; [TR]\{\delta_g^e\} \quad - \quad \{R_n\} \right\} \quad = \quad \phi. \tag{8.5}$$

or

$$\sum_1^n \left\{ [K_n] \; \{\delta_g^e\} \quad - \quad \{R_n\} \right\} \quad = \quad \phi. \tag{8.6}$$

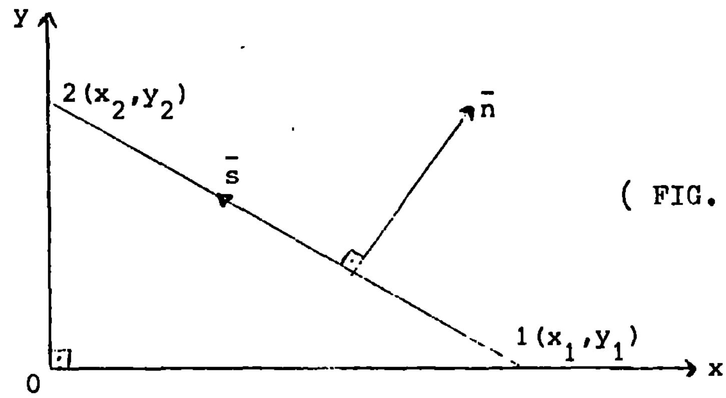
So the stiffness, stress and load matrices $[K_n]$, $[S_n]$, $\{R_n\}$ with respect to a global system can be found from the corresponding matrices with respect to the local system $[K_n^o]$, $[S_n^o]$, $\{R_n^o\}$ through the expressions:

$$\begin{aligned} [K_n] \quad &= \quad [TR]^T \quad [K_n^o] \quad [TR] \\ [S_n] \quad &= \quad [TR] \quad [S_n^o] \\ \{R_n\} \quad &= \quad [TR]^T \quad \{R_n^o\} \end{aligned} \tag{8.7}$$

8.2 Transformation matrices for a plate

The transformation in sandwich plate bending problems is necessary for a node which belongs to boundary ($\bar{s}$) in order to apply the boundary conditions.

The parameters involved have the following form:



(FIG. 8.1)

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} B_2 & G_2 \\ -G_2 & B_2 \end{bmatrix} \begin{Bmatrix} a_n \\ a_s \end{Bmatrix} \qquad (8.8)$$

a_n, a_s are the co-ordinates of the new system $\bar{n}$, $\bar{s}$) .

$$B_2 = \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$G_2 = \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \qquad (8.9)$$

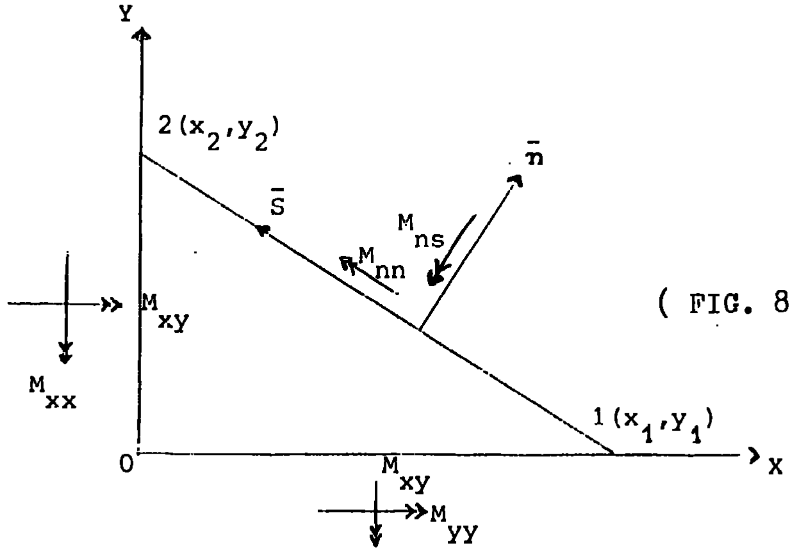
8.2.1 Displacement models

$$\begin{Bmatrix} w,_x \\ w,_y \end{Bmatrix} = \begin{bmatrix} B_2 & G_2 \\ -G_2 & B_2 \end{bmatrix} \begin{Bmatrix} w,_n \\ w,_s \end{Bmatrix}$$

$$\begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} = \begin{bmatrix} B_2 & G_2 \\ -G_2 & B_2 \end{bmatrix} \begin{Bmatrix} \phi_n \\ \phi_s \end{Bmatrix} \qquad \begin{Bmatrix} w,_{xx} \\ w,_{xy} \\ w,_{yy} \end{Bmatrix} = \begin{bmatrix} B_2^2 & 2B_2G_2 & G_2^2 \\ -B_2G_2 & (B_2^2 - G_2^2) & B_2G_2 \\ G_2^2 & -2B_2G_2 & B_2^2 \end{bmatrix} \begin{Bmatrix} w,_{nn} \\ w,_{sn} \\ w,_{ss} \end{Bmatrix} \qquad (8.10)$$

$$\begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} = \begin{bmatrix} B_2 & G_2 \\ -G_2 & B_2 \end{bmatrix} \begin{Bmatrix} \theta_n \\ \theta_s \end{Bmatrix}$$

8.2.2 Mixed models



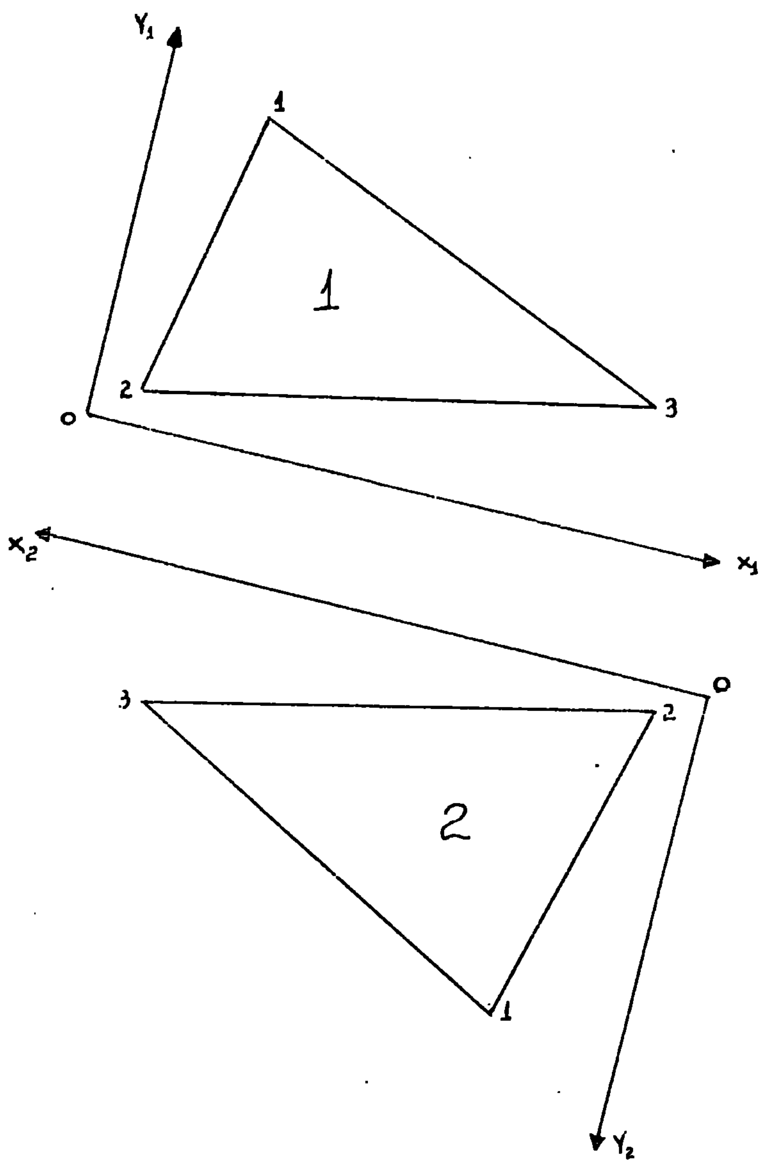
(FIG. 8.2)

$$\begin{Bmatrix} M_{ss} \\ M_{nn} \\ 2M_{sn} \end{Bmatrix} = \begin{bmatrix} G_2^2 & B_2^2 & 2B_2G_2 \\ B_2^2 & G_2^2 & -2B_2G_2 \\ 2B_2G_2 & -2B_2G_2 & 2(B_2^2 - G_2^2) \end{bmatrix} \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} \quad (8.11)$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_2^2 & G_2^2 & 2B_2G_2 \\ G_2^2 & B_2^2 & -2B_2G_2 \\ -B_2G_2 & B_2G_2 & (B_2^2 - G_2^2) \end{bmatrix} \begin{Bmatrix} M_{nn} \\ M_{ss} \\ M_{sn} \end{Bmatrix} \quad (8.12)$$

8.2.3 Rotation of an element

It is very common in many plate and dome cases that an element results from another one of the same dimensions by a single rotation of 180°



(FIG. 8.3)

The various matrices of the second element can be evaluated by a simple transformation of the first element's corresponding matrices.

The following expressions relate the degrees of freedom for the 1st and 2nd element

$$
\begin{aligned}
u_2 &= -u_1 \\
v_2 &= -v_1 \\
w_2 &= w_1 \\
w_{,x2} &= -w_{x1} \\
w_{,y2} &= -w_{,y1} \\
w_{,xx2} &= w_{,xx1} \\
w_{,xy2} &= w_{,xy1} \\
w_{,yy2} &= w_{,yy2} \\
\phi_{,x2} &= -\phi_{,x1} \\
\phi_{,y2} &= -\phi_{,y1} \\
\theta_{,x2} &= -\theta_{,x1} \\
\theta_{,y2} &= -\theta_{,y1} \\
M_{xx2} &= M_{xx1} \\
M_{yy2} &= M_{yy1} \\
M_{xy2} &= M_{xy1}
\end{aligned}
\tag{8.13}
$$

The subscripts 1.2 refer to the coordinate system x_1oy_1 for the first element and the coordinate system x_2oy_2 for the second element with regard to the relevant degree of freedom (Fig. 8.3)

8.3 Transformation matrices for a dome model

As it has been mentioned previously in Chapter 7 a very important factor, which is decisive for the combination of the various bending and membrane models with regard to the derivation of the several dome models, is the completeness of the different sets of nodal degrees of freedom.

The completeness of a set, with respect to the three-dimensions space, is necessary regarding the assemblage of the finite elements with nodes belonging to plates interconnections, and leads to the accomplishment of the transformation.

The above requirement has been fulfilled for all the dome elements presented in Chapter 7 as far as the set of nodal displacements u, v, w is concerned (see Chapter 7).

The set of rotations or moments, however, is incomplete due to the absence of a rotation or a moment, as a degree of freedom, with respect to an axis normal to the plane of the element.

One approach to the problem has been suggested in reference [115] is that additional degrees must be introduced resulting in the completeness of the set.

Some work has been done to determine the form of the part of the stiffness matrix corresponding to these additional degrees of freedom..

A second approach is based on the selection of certain coordinate systems whereby a suitable orientation of the various axes enables the transformation to be performed with a considerable degree of accuracy. [21,31,115]

The second approach has been followed for the present analysis.

First of all the various matrices for a given element are evaluated with respect to a local coordinate system defined as follows:

The x axis of this local system is the intersection of the element's plane with the xoy of the global system of cartesian coordinate Oxyz (Fig.8.4)

The unit vectors of the local system are:

$$\vec{v}_{z'} = \begin{bmatrix} \lambda_{z'x} \\ \lambda_{z'y} \\ \lambda_{z'z} \end{bmatrix} = \frac{1}{DT} \begin{bmatrix} y_{ji}\, z_{mi} - z_{ji}\, y_{mi} \\ z_{ji}\, x_{mi} - x_{ji}\, z_{mi} \\ x_{ji}\, y_{mi} - y_{ji}\, x_{mi} \end{bmatrix} \tag{8.14}$$

$$\vec{v}_{x'} = \begin{bmatrix} \lambda_{x'x} \\ \lambda_{x'y} \\ \phi. \end{bmatrix} = \begin{bmatrix} \lambda_{z'y}/\sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \\ -\lambda_{z'x}/\sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \\ \phi. \end{bmatrix} \tag{8.15}$$

$$\vec{v}_{y'} = \begin{bmatrix} \lambda_{y'x} \\ \lambda_{y'y} \\ \lambda_{y'z} \end{bmatrix} = \begin{bmatrix} \lambda_{z'z}\ \lambda_{z'x}/\sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \\ \lambda_{z'z}\ \lambda_{z'y}/\sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \\ -\sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \end{bmatrix} \tag{8.16}$$

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} \lambda_{x'x} & \lambda_{x'y} & \phi. \\ \lambda_{y'x} & \lambda_{y'y} & \lambda_{y'z} \\ \lambda_{z'x} & \lambda_{z'y} & \lambda_{z'z} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8.17}$$

Thus the transformation matrix from local to global has been obtained.

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} T_{eg} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8.18}$$

Another coordinates system $\bar{x}\ \bar{y}\ \bar{z}$ is defined for an element with a node (m) belonging to a plates-interconnection (1) (2) in the following way. (fig. 8.4)

The $\bar{x}$ axis of the system coincides with the line (1) (2). The $\bar{y}$ axis is normal to the vertical plate through the line (1) (2). And the $\bar{z}$ axis is defined as the cross product of the unit vectors of x and y respectively.

$$\vec{v}_{\bar{x}} = \begin{bmatrix} \mu_{\bar{x}x} \\ \mu_{\bar{x}y} \\ \mu_{\bar{x}z} \end{bmatrix} = \frac{1}{\ell_{12}} \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix} \tag{8.19}$$

$$\ell_{12} = \sqrt{(x_2 - x_1)^2 + (v_2 - v_1)^2 + (z_2 - z_1)^2} \tag{8.20}$$

$$\vec{v}_{\bar{y}} = \begin{bmatrix} \mu_{\bar{y}x} \\ \mu_{\bar{y}y} \\ \mu_{\bar{y}z} \end{bmatrix} = \begin{bmatrix} -\mu_{\bar{x}y} \Big/ \sqrt{\mu^2_{\bar{x}x} + \mu^2_{\bar{x}y}} \\ \mu_{\bar{x}x} \Big/ \sqrt{\mu^2_{\bar{x}x} + \mu^2_{\bar{x}y}} \\ \phi . \end{bmatrix} \tag{8.21}$$

$$\vec{v}_{\bar{z}} = \begin{bmatrix} \mu_{\bar{z}x} \\ \mu_{\bar{z}y} \\ \mu_{\bar{z}z} \end{bmatrix} = \begin{bmatrix} -\mu_{\bar{x}x} \cdot \mu_{\bar{x}z} \Big/ \sqrt{\mu^2_{\bar{x}x} + \mu^2_{\bar{x}y}} \\ -\mu_{\bar{x}y} \cdot \mu_{\bar{x}z} \Big/ \sqrt{\mu^2_{\bar{x}x} + \mu^2_{\bar{x}y}} \\ \sqrt{\mu^2_{\bar{x}x} + \mu^2_{\bar{x}y}} \end{bmatrix} \tag{8.22}$$

$$\begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} = \begin{bmatrix} \mu_{\bar{x}x} & \mu_{\bar{x}y} & \mu_{\bar{x}z} \\ \mu_{\bar{y}x} & \mu_{\bar{y}y} & \mu_{\bar{y}z} \\ \mu_{\bar{z}x} & \mu_{\bar{z}y} & \mu_{\bar{z}z} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8.23}$$

$$\begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} = \begin{bmatrix} T_{\bar{e}g} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8.24}$$

Combining equations (8.18)(8.24) and considering the orthogonal nature of the matrices $[T_{eg}]$, $[T_{\bar{e}g}]$ the following relationships can be obtained

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = [T_{e\bar{e}}] \begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} \tag{8.25}$$

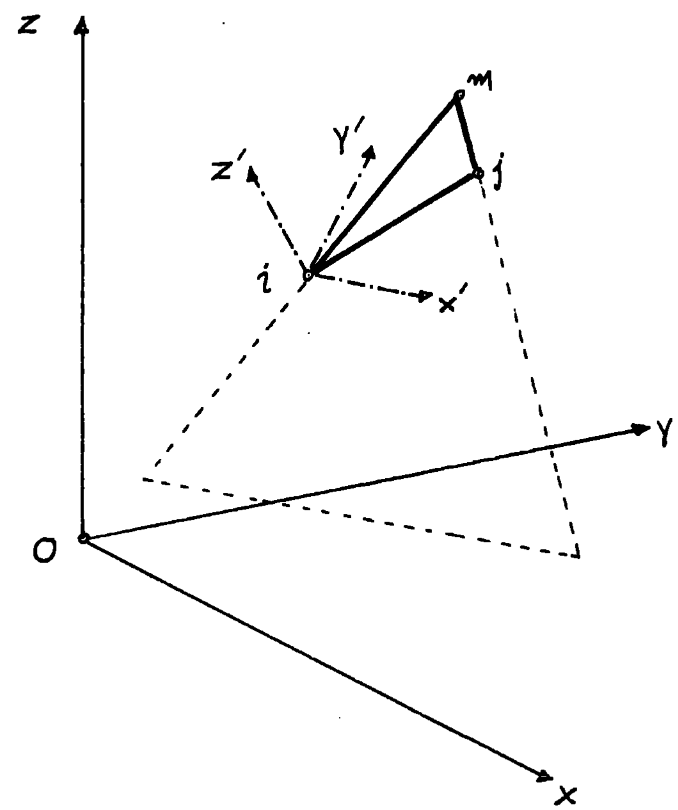
$$[T_{e\bar{e}}] = \begin{bmatrix} v_{x'\bar{x}} & v_{x'\bar{y}} & v_{x'\bar{z}} \\ v_{y'\bar{x}} & v_{y'\bar{y}} & v_{y'\bar{z}} \\ v_{z'\bar{x}} & v_{z'\bar{y}} & v_{z'\bar{z}} \end{bmatrix} \tag{8.26}$$

$$[T_{e\bar{e}}] = [T_{eg}] \quad [T_{\bar{e}g}]^T \tag{8.27}$$

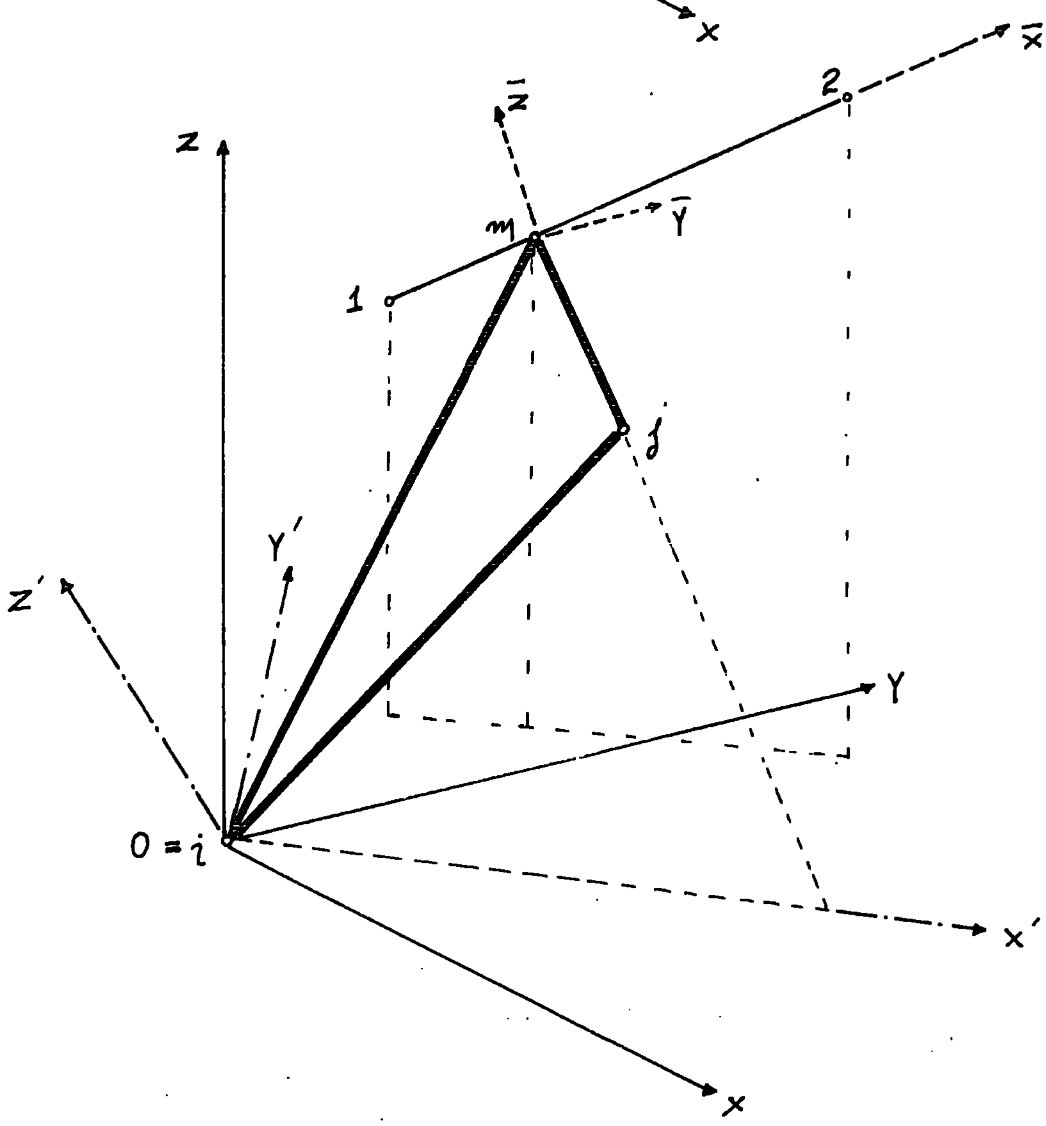
For co-planar nodes the assembling of the equations is peformed in the local co-ordinates system.

For nodes belonging to an interconnection of two plates or to an external boundary the displacements set u', v', w' is transformed employing the matrix $[T_{eg}]$ (8.18) to the global displacements u, v, w together with the relevant transformation procedure for the various matrices involved (8.7). The set of total rotation, transverse shear displacements and moments is transformed employing the matrix $[T_{e\bar{e}}]$ (8.25) to the coordinate system $\bar{x}$ $\bar{y}$ $\bar{z}$ eliminating at the same time the transverse shear displacement relevant to the cross section normal to the $\bar{x}$ axis .

For nodes belonging to an interconnection of more than two plates all the different parameters are transformed to the global system xyz employing the matrix $[T_{eg}]$ eliminating at the same time the transverse shear displacements.



(FIG. 8.4)



8.4 Condensation

When the elimination of a number of nodal degrees of freedom $\{\delta_2^e\}$ is required from the total vector of $\{\delta_1^e\}$ with the remaining vector noted as $\{\delta_1^e\}$ the following relationships can be obtained.

The stiffness matrix can be partitioned with regards to the two sets of nodal degrees of freedom. $\{\delta_1^e\}$, $\{\delta_2^e\}$

$$[K_{11}] \{\delta_1^e\} + [K_{12}] \{\delta_2^e\} = \{R_1\} \tag{8.28}$$

$$[K_{21}] \{\delta_1^e\} + [K_{22}] \{\delta_2^e\} = \{R_2\} \tag{8.29}$$

Solving equation (8.29) with respect to $\{\delta_2^e\}$ and substituting the value for equation (8.28) one obtains:

$$[K_{cd}] \{\delta_1^e\} - \{R_{cd}\} = \phi \tag{8.30}$$

$$[K_{cd}] = [K_{11}] - [K_{12}] [K_{22}]^{-1} [K_{21}] \tag{8.31}$$

$$[R_{cd}] = \{R_1\} - [K_{12}] [K_{22}]^{-1} \{R_2\} \tag{8.32}$$

In the same way the stress matrix after the condensation has the form

$$\{\sigma\} = ([S_1] - [S_2][K_{22}]^{-1}[K_{21}])\{\delta_1^e\} + [S_2][K_{22}]^{-1} \{R_2\} \tag{8.33}$$

9. EXPERIMENTAL WORK

9.1 Introduction

The experimental part of the present work consists of the construction and testing of two polyhedral domed sandwich structures.

(a) The 24 faced dome (see Photogr. a). This dome is formed by six 4 faced pyramidal flat segments (see Chapter 13).

(b) The 36 faced dome (see Photog. b÷e). The second dome is an extension of the first formed by adding six "dormer" sections (see Chapter 13).

The dimensions of both the domes have been obtained through a computer programme (outlined in [85]). They are included in Chap. 13 and are presented in the relevant section. The experimental results obtained from the various loading cases are presented in the same section

9.2 Construction and Materials

The decisive factor for the determination of the dome's dimensions was the size of the basic orthogonal sandwich sheet, readily available from the manufacturers. The panels forming the two domes are all identical as far as their dimensions are concerned and have been formed as part of the basic orthogonal sandwich sheet, as shown in Fig. 9.1 , so that from each sheet two panels can be obtained with the least possible wastage of material.

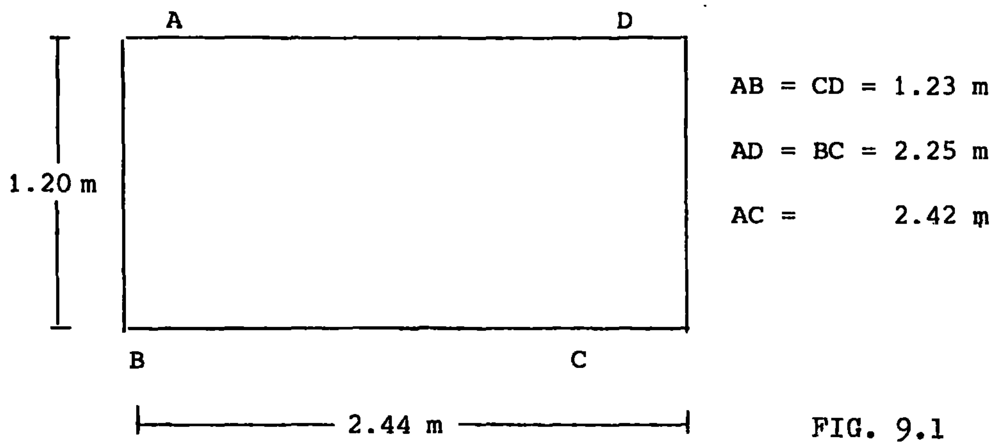


FIG. 9.1

The sandwich panels used were composed from hardboard faces of 4.1 mm thickness and polyurethane core of 50.8mm thickness.

The elasticity moduli for the above mentioned materials, as well as for the composite sandwich structure used for the present analysis, were obtained by other researchers [21,46,85].

The properties of all the various sandwich panels used in the present work are presented in figs. 13.2,13.3.

The construction of the two domes was carried out in the following way:

(a) First the supports and the foundation of the structure were built. Detailed drawings of the foundation are presented in Fig.9.2÷9.7

The foundation must be functioning, at the final stage, in such a way that the displacements, with respect to all three global axes, are zero without constraining the corresponding rotations (pinned-joint function).

For the construction stage a limited amount of displacements with respect to x and y axes have been allowed so that dimensional inaccuracies could be overcome (see detailed drawings). 9.4÷9.6

(b) Second the six 4-faced pyramidal flat segments (Fig. 9.8.1) as well as the six "dormer" sections (Fig. 9.8.2) were built by joining the identical triangular sandwich panels together. The joining technique details are presented in Fig. 9.9

The joining technique has been tested and found satisfactorily strong and economical. Details with regard to the behaviour of the joints are presented in Chapter 10.

(c) At this stage the first dome was formed by erecting the six segments on site and by adjusting the previously mentioned mechanisms at the supports so that all six segments came together. Next, the joints of the adjoining segments, which were built using the same technique mentioned above, were secured by the fitting of the steel cover plates and bolts (See Fig. 9.10)

It is worth mentioning that the construction of the six segments was carried out in 10 days and the erection and formation of the first dome in one day, with the help of three departmental technicians.

(d) After the testing of the first dome was accomplished, the second dome was built by erecting and joining to the existing first dome the six "dormer" sections. The same joining technique was used as previously. (see fig. 9.11)

The second dome was then tested.

9.3 Testing of the Domes

The two domes were tested under concentrated loads acting at the centroids of certain groups of panels.

The first dome was tested under two loading cases:-

1st Concentrated load of 1216 N at all upper panel centroids.

2nd Concentrated load of 1216 N at all bottom panel centroids.

The second dome was tested under three loading cases:-

1st Concentrated load of 1216 Nt at all upper panel centroids.

2nd Concentrated load of 1216 Nt at all bottom panel centroids.

3rd Concentrated load of 1216 Nt at all "dormer" centroids.

The following assumptions made in the analysis of the structure were tested.

(a) The behaviour of the supports was found satisfactory.

The displacements measured at the supports were practically zero. (Approximately two orders of magnitude smaller than the maximum displacement).

(b) The behaviour of the structure under certain concentrated loads was found to be, with regard to the structure as a whole, such that the analytical assumption of an elastic behaviour can be considered as a fair approximation. Visco-elastic indentation of the faces was observed locally for the loaded panels in a limited area surrounding the loading point, which required a little time (1 hour) to recover.

(c) To minimize the effects of temperature and relative humidity changes as well as the wind effects, the testing of the domes under various loading cases was accomplished under as similar weather conditions as possible. In addition, the surface of the panels was covered by two layers of "yacht varnished" for weather protection.

(d) The structure was assumed to be symmetrical. The dimension inaccuracies observed during the construction stage were negligible.

The symmetrical behaviour of the domes was tested next and found to be satisfactory. First, the displacements at certain symmetrical points under a symmetrical loading case were measured and found to be almost symmetrical (with a maximum deviation of 10 per cent). Second, the displacements (of a point on an axis of symmetry), normal to the axis of symmetry, for a symmetrical loading case were measured and found to be negligible.

Due to the above reasons, the assumption of the symmetrical behaviour of the structure under symmetrical load was considered to be valid and consequently only 1/12 of the two domes was numerically modelled for the loading cases (symmetrical) mentioned previously.

(e) Time-dependent behaviour was also observed. To minimize the effects of this behaviour the measurements of the points by the theodolites have been taken in the minimum possible time starting always from the loaded panel where the time-dependent behaviour is expected to occur.

(f) The application of the high density concentrated load of 1216 Nt per loading point, as already mentioned, was dictated by the high rigidity of the structure and the accuracy of the method used for measuring the displacements, to be outlined in the next section. Special steel cylindrical devices were used at the loading points to secure the vertical application and spreading of the load.

9.4 Displacement Measurement

Under a certain loading case the displacements of certain points were measured (see Chapter 13)

For this a combination of two theodolites was used. The theodolites were secured so that they were at the same horizontal level separated by a constant distance.

For each modelling point on the structure eight readings were obtained, four with the structure unloaded and four with the structure loaded. The four readings in each case consist of one horizontal and one vertical angle from both the theodolites.

The global displacements have been obtained for each point from the above measurements by the following mathematical formulation, through a computer program based on it.

The first and second theodolites are considered to be at points 0_1, 0_2 respectively (see Fig. 9.12)

A modelling point, at A_1 before the application of the load, is to be considered.

The coordinates of A_1 with respect to the two cartesian coordinate systems $x_1 0_1 y_1$, $x_2 0_2 y_2$ are (a_1', a_2', a_3'), $(a_1', a_2' + \ell, a_3)$ where ℓ is the constant distance between 0_1 and 0_2 which has already been measured.

The following angles can be measured by the two theodolites:

	For Point A_1		For Point A_1'		
	angle ϕ	+ g ϕ	angle ϕ'	+ g ϕ'	
Horizontal angle	$Y_1O_1B_1$	h_1	$Y_1O_1B_1'$	h_1'	From Point O_1
Vertical angle	$B_1O_1A_1$	v_1	$B_1'O_1A_1'$	v_1'	
Horizontal angle	$Y_2O_2B_1$	h_2	$Y_2O_2B_1'$	h_2'	From Point O_2
Vertical angle	$BB_1O_2A_1$	v_2	$B_1'O_2A_1'$	v_2'	

By the following expressions the coordinates of the points can be related to the measured angles:

$$
\begin{aligned}
a_1 &= a_2 h_1 \\
a_1 &= (a_2 + \ell)\, h_2 \\
a_1' &= a_2'\, h_1' \\
a_1' &= (a_2' + \ell)\, h_2'
\end{aligned}
\qquad (9.1)
$$

From (1) the following can be obtained:

$$
\begin{aligned}
a_1 &- \frac{\ell\, h_1\, h_2}{h_1 - h_2} \\
a_2 &= \frac{\ell\, h_2}{h_1 - h_2} \\
a_1' &= \frac{\ell\, h_1'\, h_2'}{h_1' - h_2'} \\
a_2' &= \frac{\ell\, h_2'}{h_1' - h_2'}
\end{aligned}
\qquad (9.2)
$$

The following expressions related the horizontal coordinates a_1, a_2, a_1', a_2 obtained by equations (2) with the third co-ordinate a_3, a_3' and the vertical angles. (Mid value is being taken)

$$a_3 = \frac{v_1\sqrt{a_1^{\;2} + a_2^{\;2}} + v_2\sqrt{a_1^{\;2} + (a_2 + \ell)^2}}{2}$$

$$a_3' = \frac{v_1'\sqrt{a_1'^{\;2} + a'^{\;2}_2} + v_2'\sqrt{a_1'^{\;2} + (a_2' + \ell)^2}}{2} \qquad (9.3)$$

the displacements for the point A_1 can be obtained with respect to the cartesian system $x_1 0_1 y_1$ (or $x_2 0_2 y_2$)

$$u_{A_1} = a_1' - a_1$$

$$v_{A_1} = a_2' - a_2 \qquad (9.4)$$

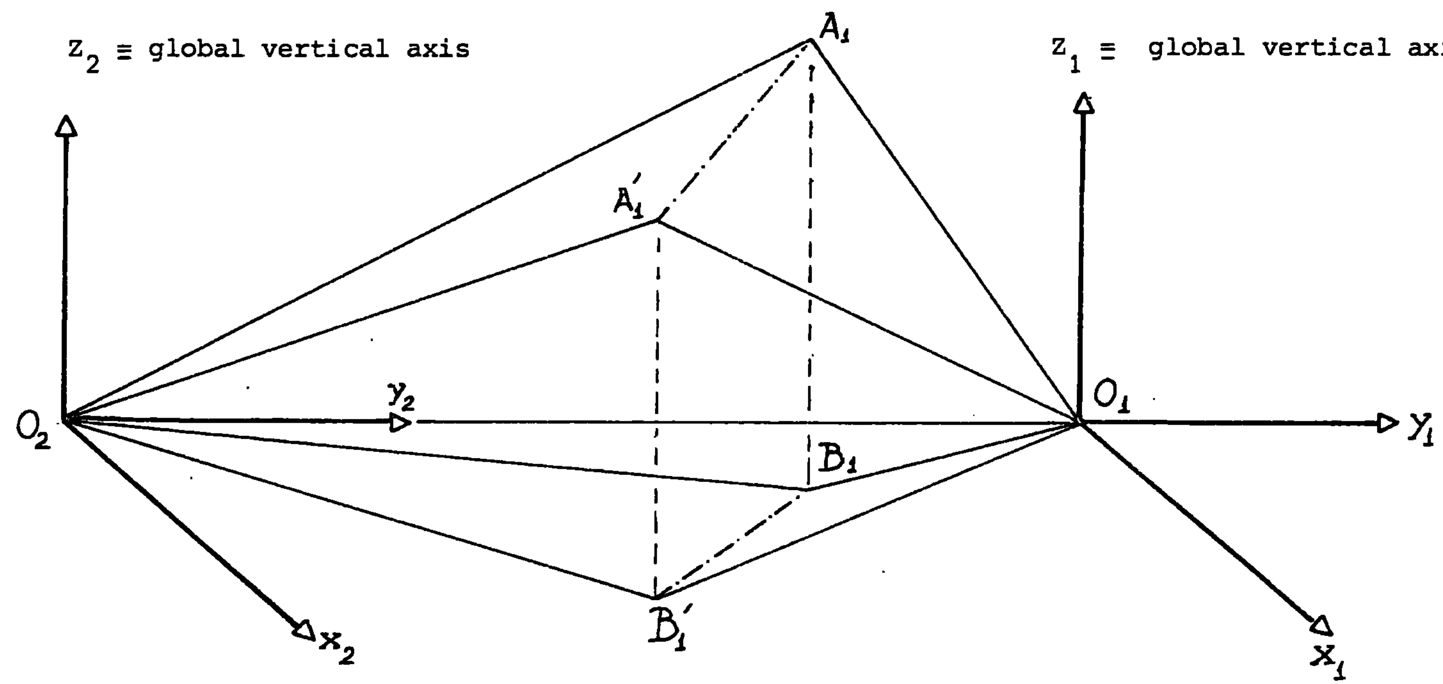
$$w_{A_1} = a_3' - a_3$$

The displacements with respect to any cartesian coordinate system $x_n 0_n y_n$ can be obtained from the u_{A_1}, v_{A_1}, w_{A_1} employing the appropriate transformation expressions.

A_1 a modelling point before loading

$0_1 0_2$ = constant = ℓ

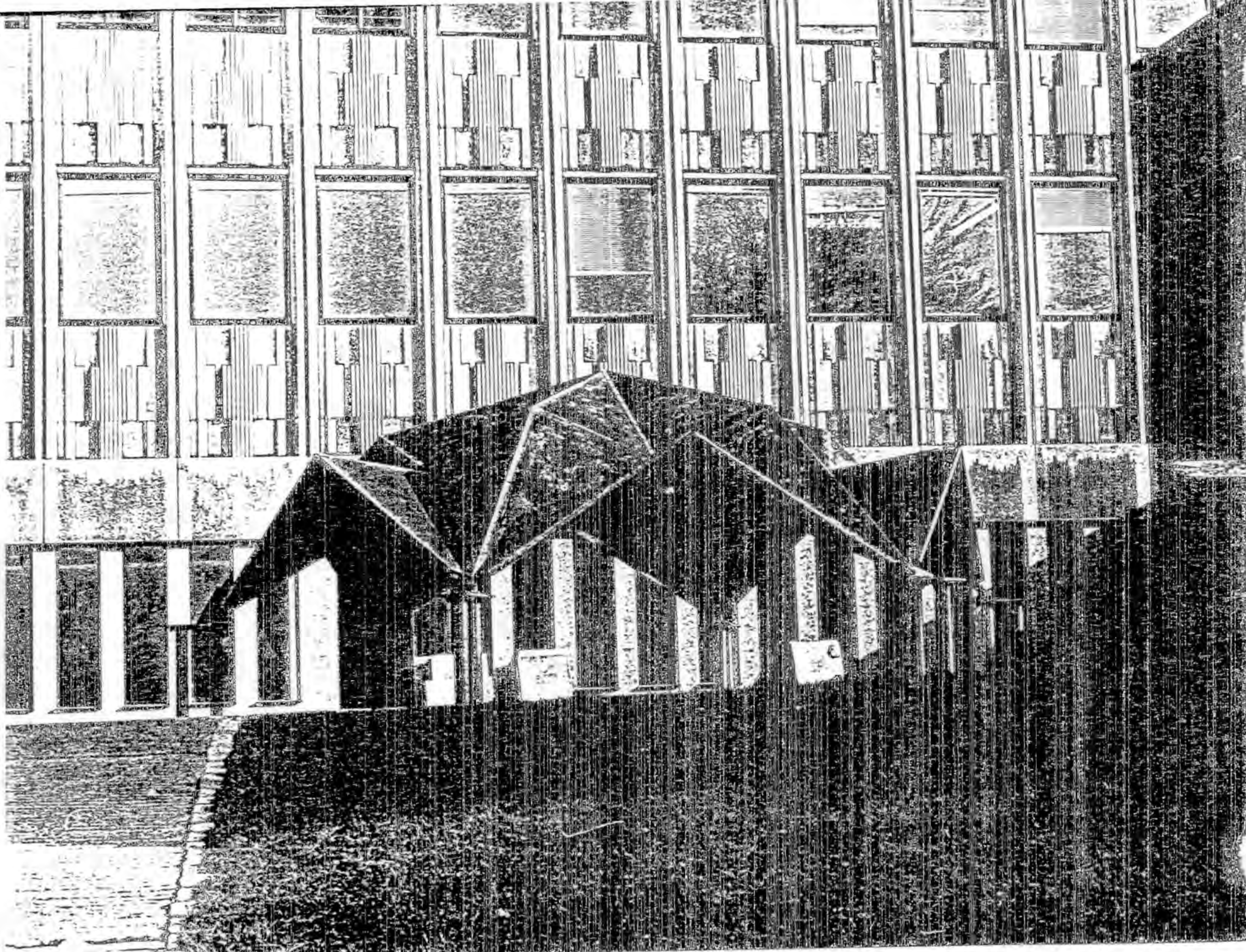
A_1' the same modelling point after loading



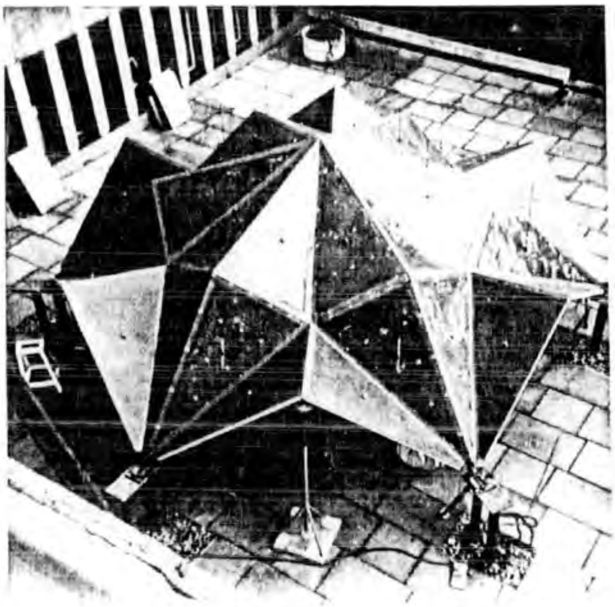
B_1, B_1' the projections of A_1, A_1' respectively on the horizontal plane $x_2 0_2 y_2 \equiv x_1 0_1 y_1$

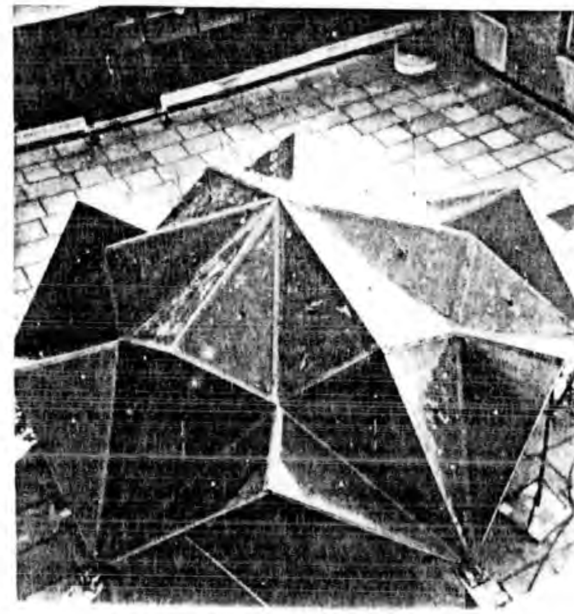
FIG. 9.12

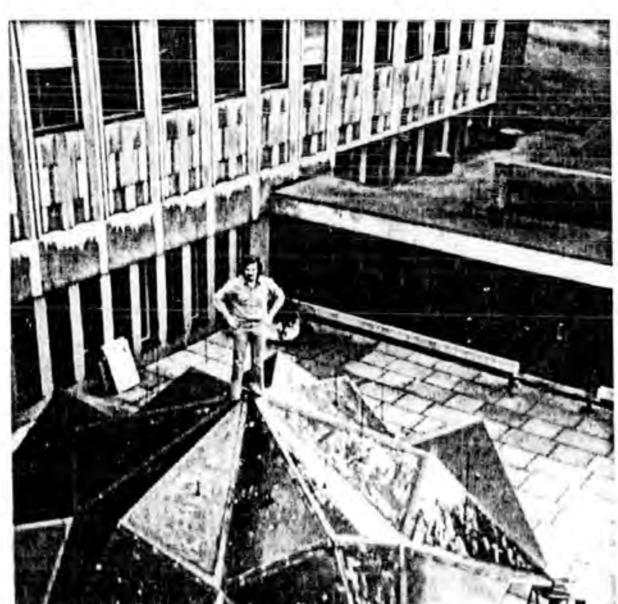
PHOTOGRAPH a.



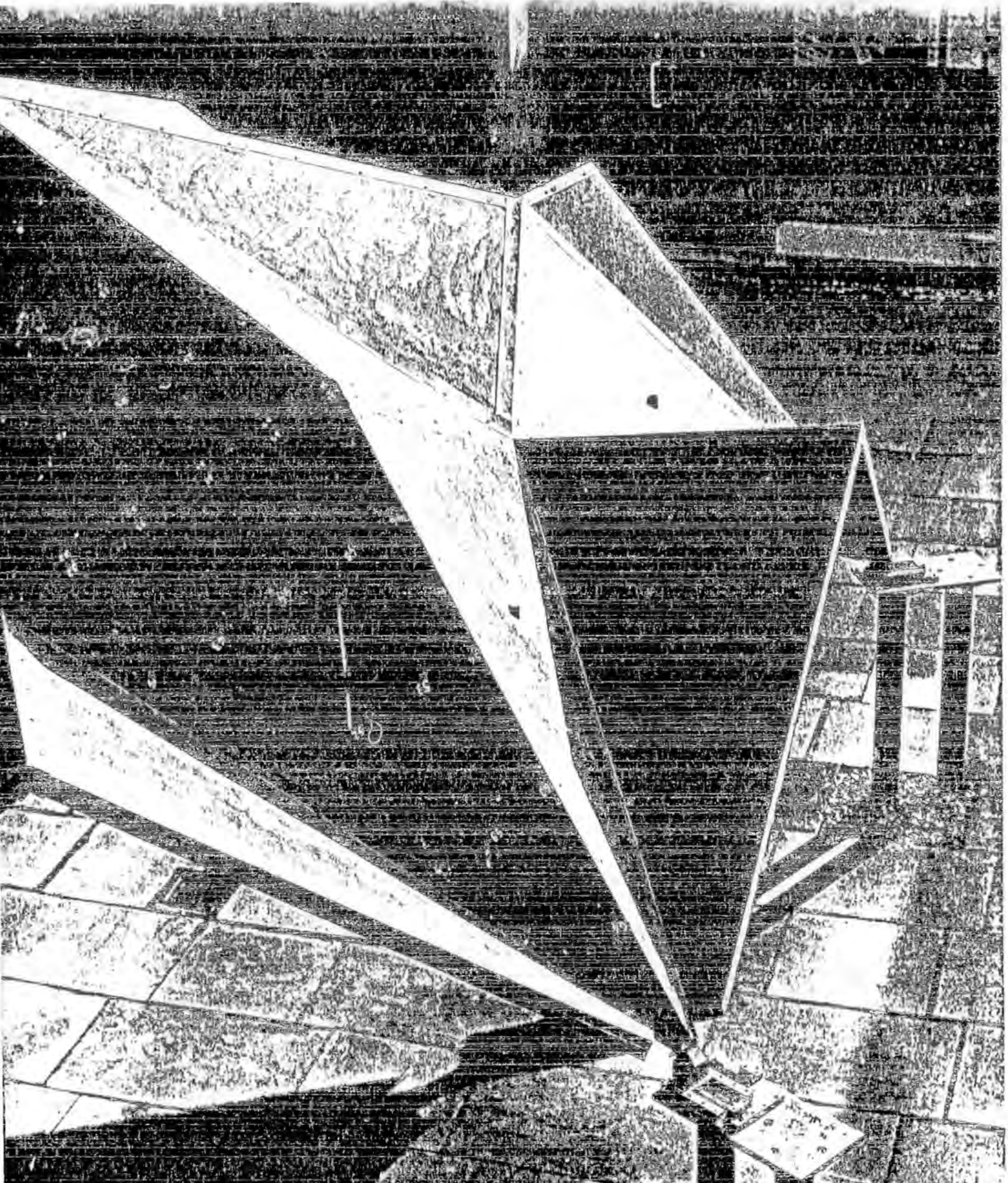
PHOTOGRAPH 6



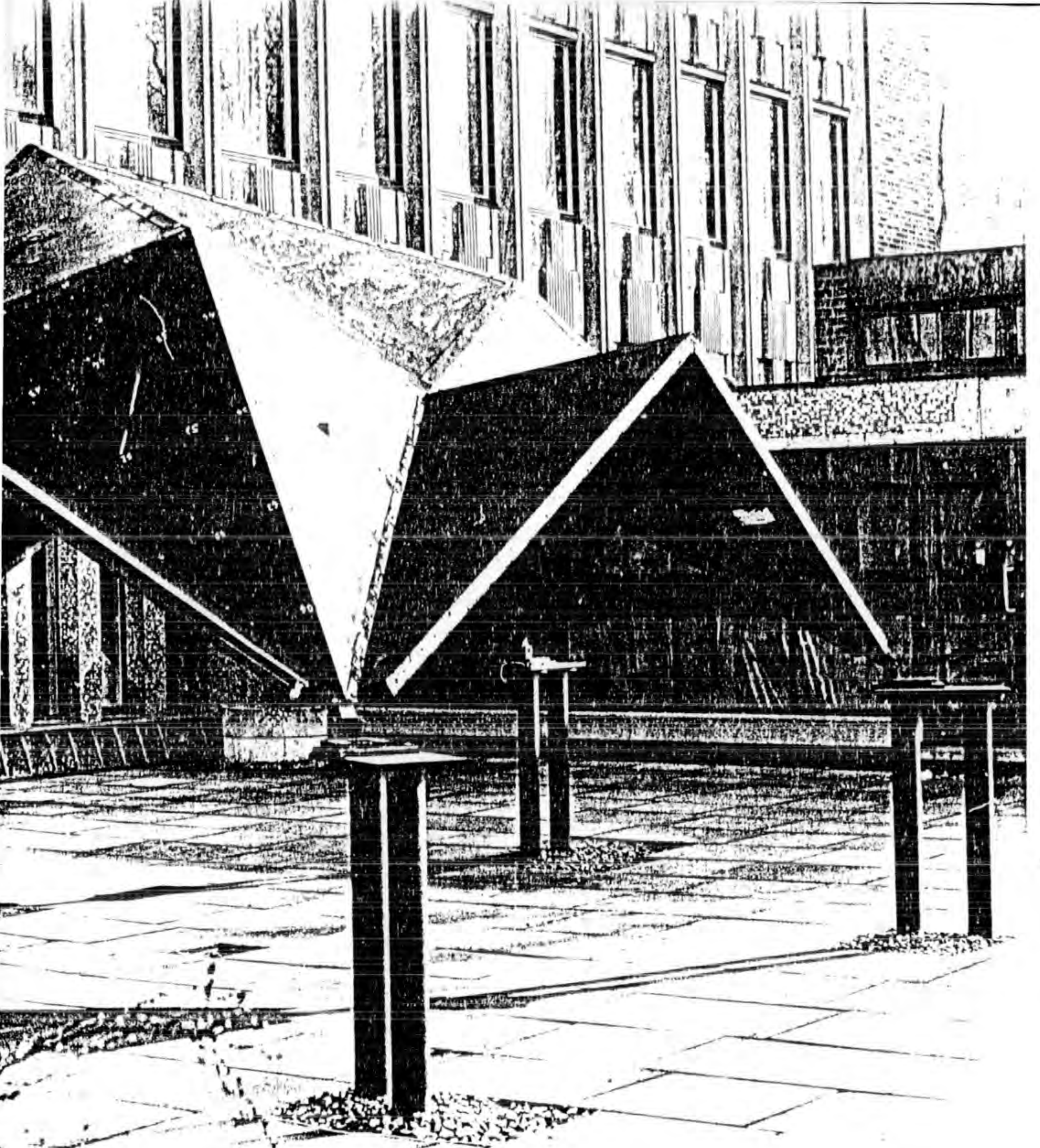




PHOTOGRAPH C

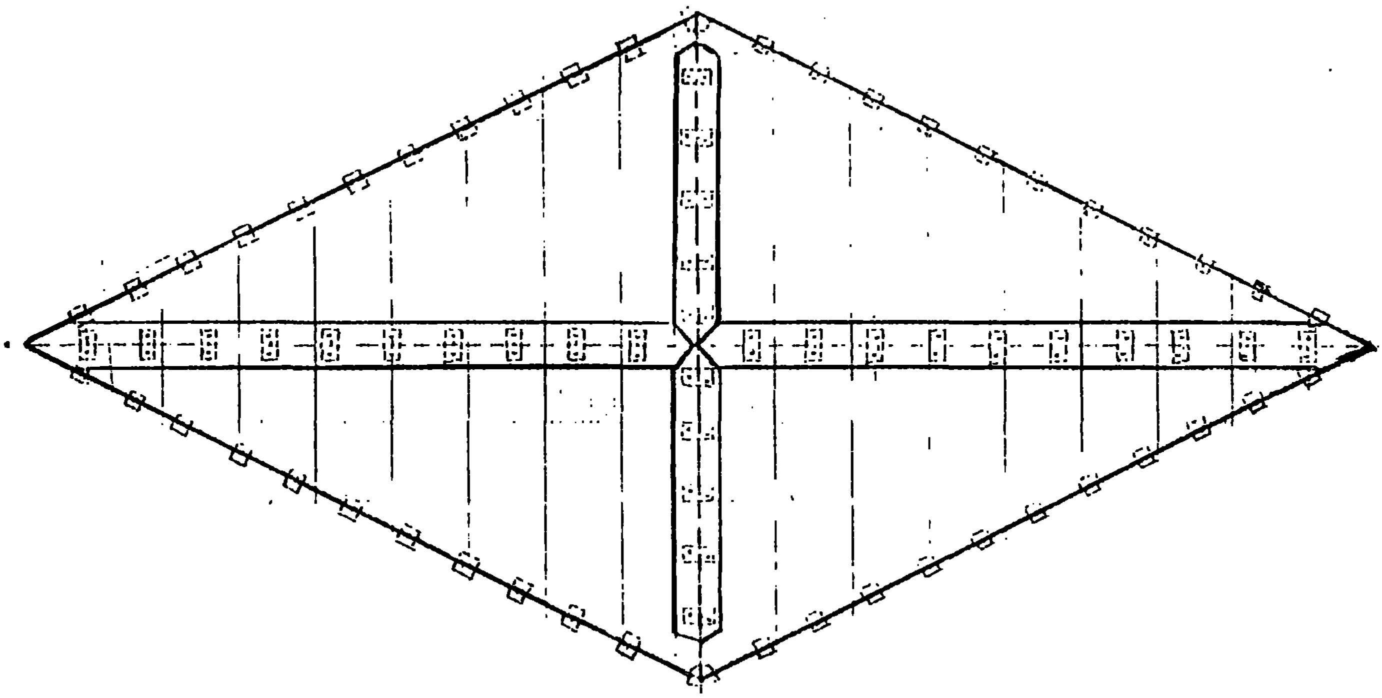


PHOTOGRAPH d.



PHOTOGRAPH e.

9.8.1. 4. FACED PYRAMIDAL SEGMENT.



9.8.2. DORMER

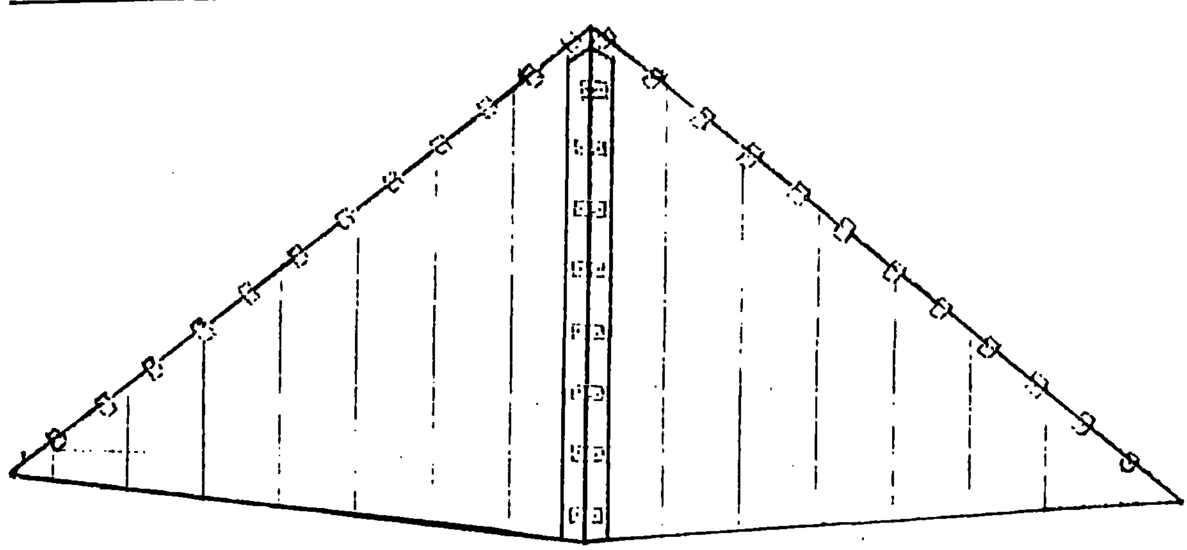


FIG. 9.8.

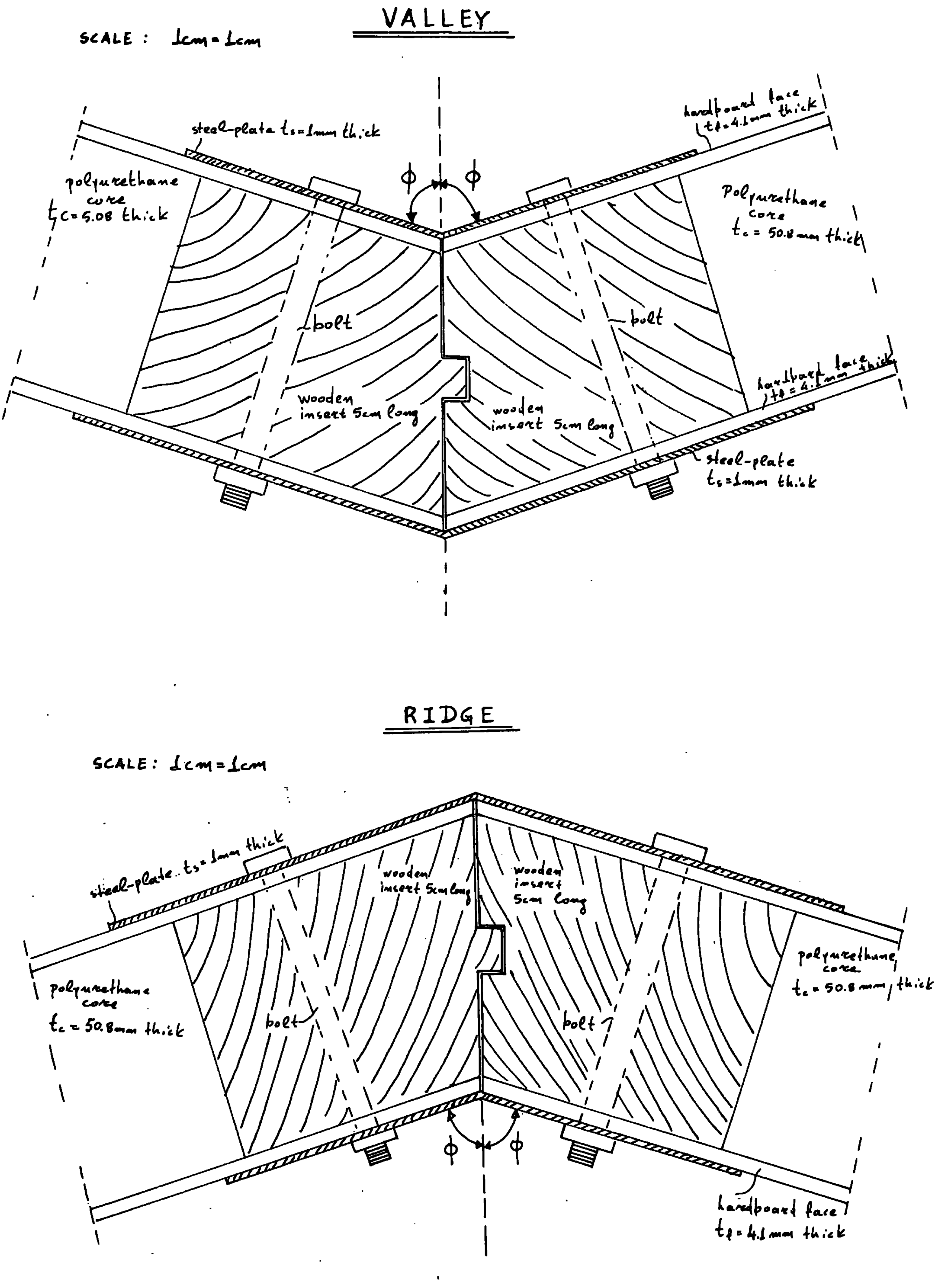
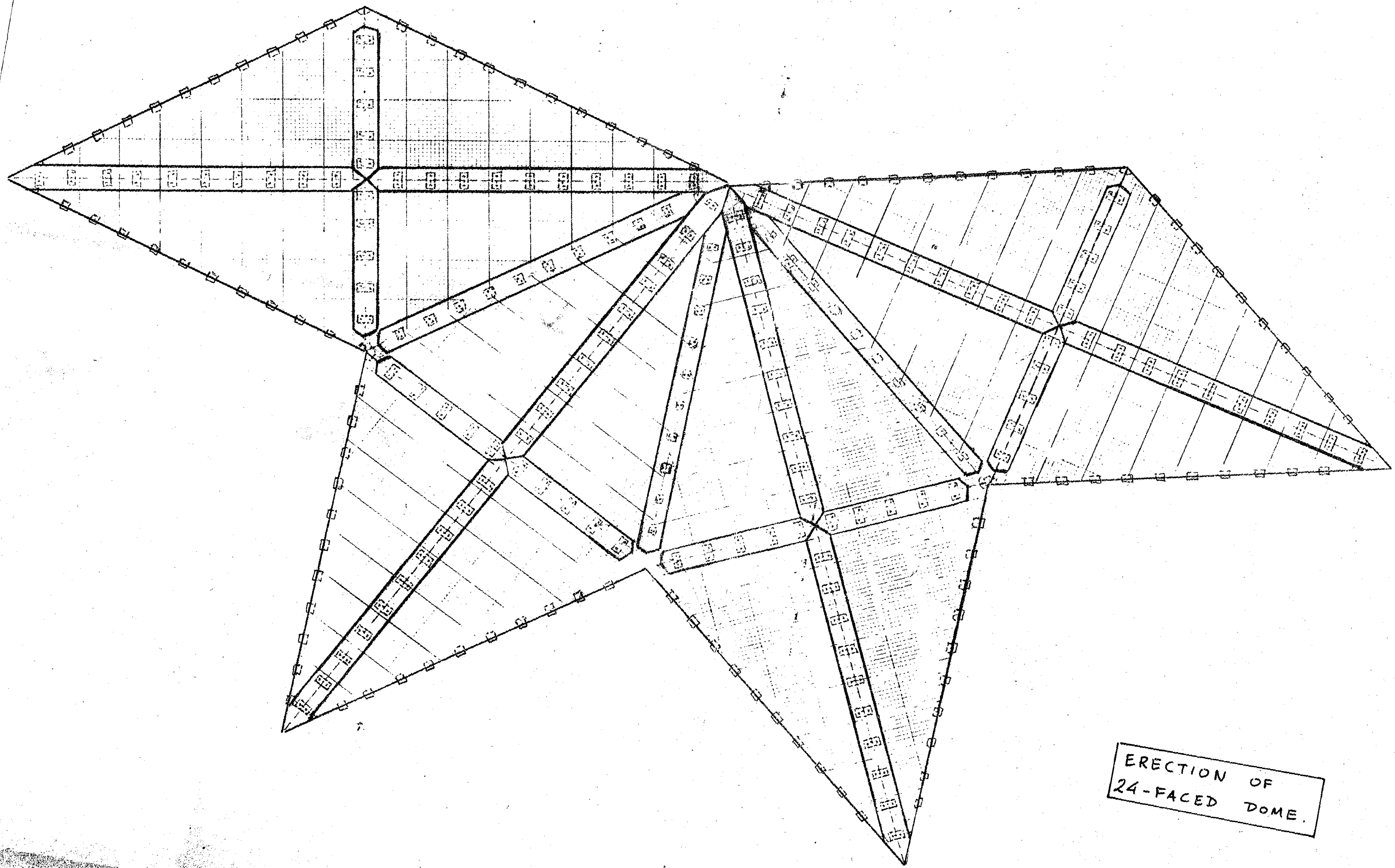

VALLEY
SCALE : 1cm = 1cm
steel-plate ts = 1mm thick
hardboard face tf = 4.1mm thick
polyurethane core tc = 5.08 thick
Polyurethane core tc = 50.8mm thick
ϕ
ϕ
bolt
bolt
wooden insert 5cm long
wooden insert 5cm long
hardboard face tf = 4.1mm thick
steel-plate ts = 1mm thick
RIDGE
SCALE: 1cm = 1cm
steel-plate ts = 1mm thick
wooden insert 5cm long
wooden insert 5cm long
polyurethane core tc = 50.8mm thick
polyurethane core tc = 50.8mm thick
bolt
bolt
ϕ
ϕ
hardboard face tf = 4.1mm thick

FIG. 9.9.



ERECTION OF
24-FACED DOME.

FIG. 9.10.

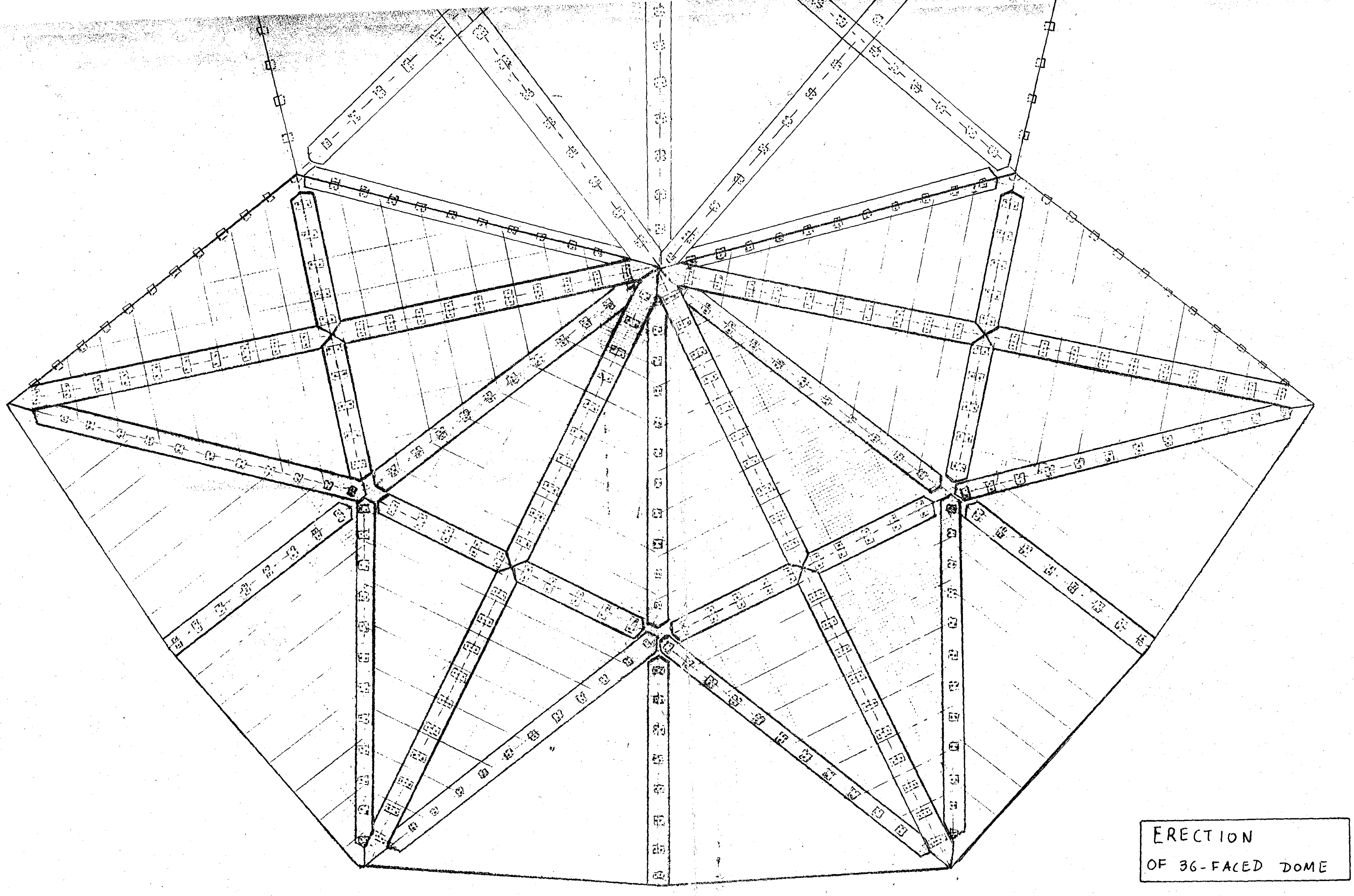


FIG. 9.11.

10. BEHAVIOUR OF THE JOINT

10.1. INTRODUCTION

The behaviour of the joint, as it has been formed for the construction of the two domes, the 24-faced dome and the 36-faced dome, is to be outlined in the present chapter.

The behaviour of the joints belonging to the rest of the domes analysed in the present work is assumed to be as outlined in ref. [85] .

10.2. TESTS

The strength of the joint under axial tension has been evaluated experimentally by a series of tests for various forms of arrangement at the position of the bolts.

It has been established by the results of these tests that the presence of a wooden insert (as shown in figs. 10.1 - 10.4) significantly increases the tensile strength of the joint.

The ultimate axial load of the joint with a wooden cylinder, inserted at the position of the bolts, as shown in fig.10.2 , is 4.33 times greater than the ultimum axial load of the joint without any insert at all.

The ultimum axial load at the joint in the form used in the construction of the two domes, which has a wooden cube as an insert, as shown in fig.10.3 , is 10 times greater than the ultimum axial load at the joint without any insert at all.

There are five wooden inserts per metre of joint length for each of the joints in the two domes. This number has been assumed empirically to be sufficient for the function of the joints with regard to the tests that were to be performed with the two domes.

The function of the joint is to be analysed in the following section.

10.3. ANALYSIS OF THE FUNCTION OF THE JOINT

10.3.1. AXIAL TENSILE STRENGTH

The strength of a unit length sandwich panel in axial tension, assuming constant distribution of the stresses across the thickness at the faces and that the contribution of the core is negligible (see chapter 2). is given by the relationship:

$$P^{s} = 2 \cdot f \cdot \sigma_{f} = 2 \times 4.1\ 10^{-3} \times 2.5\ 10^{7} = 2.05\ 10^{5}\ \text{Nt} \qquad (10.1)$$

(see figs. 10.1 , 10.4)

where P^{s} is the axial tensile load for a sandwich panel of one metre in length, and σ_f the value of the normal stress at the faces.

For a joint line with 5 inserts per metre the axial tensile load is given by the relationship.

$$P^{j} = n_{in} \cdot P^{i} = 5 \times 15\ 10^{3} = 0.75\ 10^{5}\ \text{Nt} \qquad (10.2)$$

(see figs.10.3 , 10.4)

Where n_{in} is the number of inserts per metre and P^{i} the experimental value of the axial load per insert (see fig. 10.4)

10.3.2 STRENGTH UNDER BENDING

The active width $\ell_{ac.}$ of the steel plate at a joint line functioning under bending is assumed to be the one corresponding to the total length of the wooden inserts per metre.

For a bending moment acting at the joint line the stresses which are developed, assuming that the distribution of the stresses across the thickness of the hardboard faces or the steel plate is constant and that the contribution of the core is negligible, are given by the following relationships:

a) For the faces when the sandwich panel only is to be considered

$$M = 1 . f . \sigma_{f} \ (c + f) \qquad (10.3)$$

b) For the steel covering plates only

$$M = \ell_{ac}.\ t_s.\sigma_s\ (c + 2f + t_s) = \ell_{in}\ n_{in}.\ t_s.\sigma_s.\ (c + 2f + t_s) \qquad (10.4)$$

c) At the position of the bolt

$$M = P^c\ n_{in}\ .\ (c + 2f + t_s) \qquad (10.5)$$

$$\text{where } P^c = 0.9P^i \quad \text{(see previous section)} \qquad (10.6)$$

The maximum values of the bending moments which can be applied in each case can be evaluated by substituting at the above relationships the relevant maximum values of stresses or forces as follows

a) For the faces

$$\sigma_f\ max = 2.5\ 10^7\ Nt/m^2 \qquad Mmax = 5.63\ 10^3\ Ntm/m \qquad (10.7)$$

b) For the steel plates

$$\sigma_s\ max = 1.025\ 10^9\ Nt/m^2 \qquad Mmax = 15.38\ 10^3\ Ntm/m \qquad (10.8)$$

c) For the bolt

$$P^i = 15\ 10^3\ Nt \qquad Mmax = 4.12\ 10^3\ Ntm/m \qquad (10.9)$$

10.4. ANGULAR FUNCTION OF THE JOINT UNDER BENDING

For valleys under the action of a bending moment as shown in fig. 10.5.3 the top steel plate can be deformed as shown in fig. 10.5.4.

For a bending moment acting with an opposite sign the presence of the wooden insert restricts such deformation.

The same applies for ridges with a bending moment acting with an opposite sign as shown in figs. 10.5.1 , 10.5.2.

The following analysis has been attempted for evaluating this angular deformation of the joint under bending.

The static system and the applied load are as shown in fig. 10.6.

The resulting stressing condition of the plate is as shown in fig.10.6.

Through this the angular deformation can be evaluated as follows:

$$\omega = 2\Delta\phi = \frac{2\ell^2}{12E_s I_s} \cos\phi \sin\phi \frac{M}{d} \qquad (10.10)$$

$$I_s = \ell ac.t_s^3 /12 \qquad (10.11)$$

$$\omega = \frac{2\ell^2}{E_s \ell ac.t_s^3} \cos\phi \sin\phi \frac{M}{d} \qquad (10.12)$$

$$\frac{\omega}{2\phi} = \varepsilon_{\Delta\phi} = \frac{l^2}{E_s \ell ac.t_s^3 d} \frac{\cos\phi \sin\phi}{\phi} \cdot M \qquad (10.13)$$

$d = 60$ mm $\qquad \ell ac = 0.250$ m $\qquad \ell = 0.025$ m

$$\boxed{\delta_1 = \frac{\cos\phi \sin\phi}{\phi}} \qquad (10.14)$$

For $\varepsilon_{\Delta\phi} = 0.4$ % $\qquad \boxed{M = 20/\delta_1 \text{ Ntm/m}} \qquad (10.15)$

For $\varepsilon_{\Delta\phi} = 0.8$ % $\qquad \boxed{M = 40/\delta_1 \text{ Ntm/m}} \qquad (10.16)$

The values of δ_1 and M are given by the graphs where they are plotted against the value of the corresponding angle

10.5. CONCLUSIONS FROM THE BEHAVIOUR OF THE JOINT

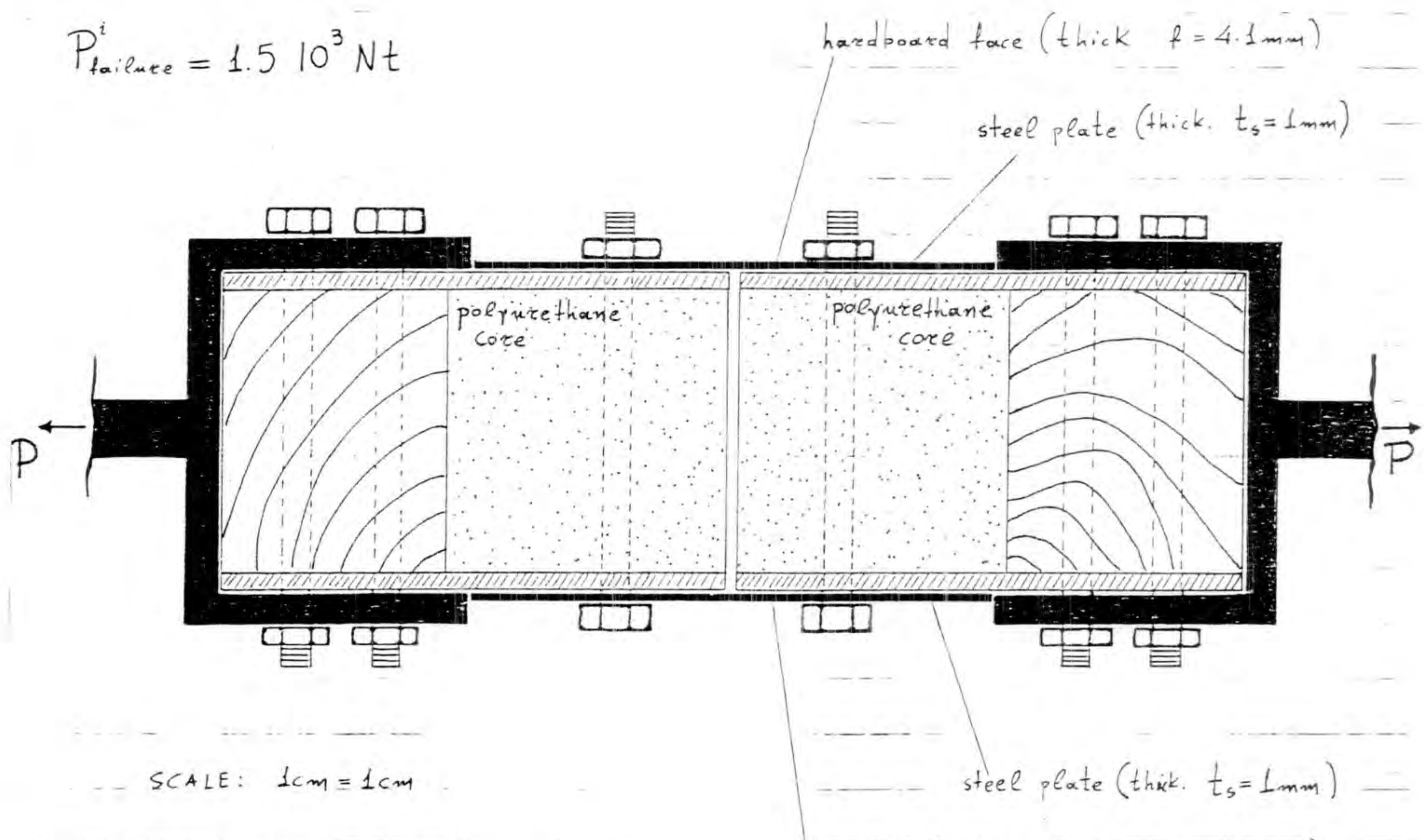
The type of the joint used for the construction of the two domes, the 24-faced dome and the 36-faced dome, is sufficiently strong in comparison with the strength of the sandwich panels joined by it, when the angular function of the joint is negligable due to the sign of the acting bending moment as explained in section 4.

It becomes obvious from the analysis of sections 3. and 4. that when angular deformation of the joint is expected, the joint does not act as an absoloutely fixed joint.

With the joint angle decreasing and/or the distance between the bolt and the joint line increasing the joint tends to act as a hinge.

The values of joint angles which are formed by the various adjacent sandwich panels forming the two domes, the 24-faced dome and the 36-faced dome, are presented in fig. 10.7.

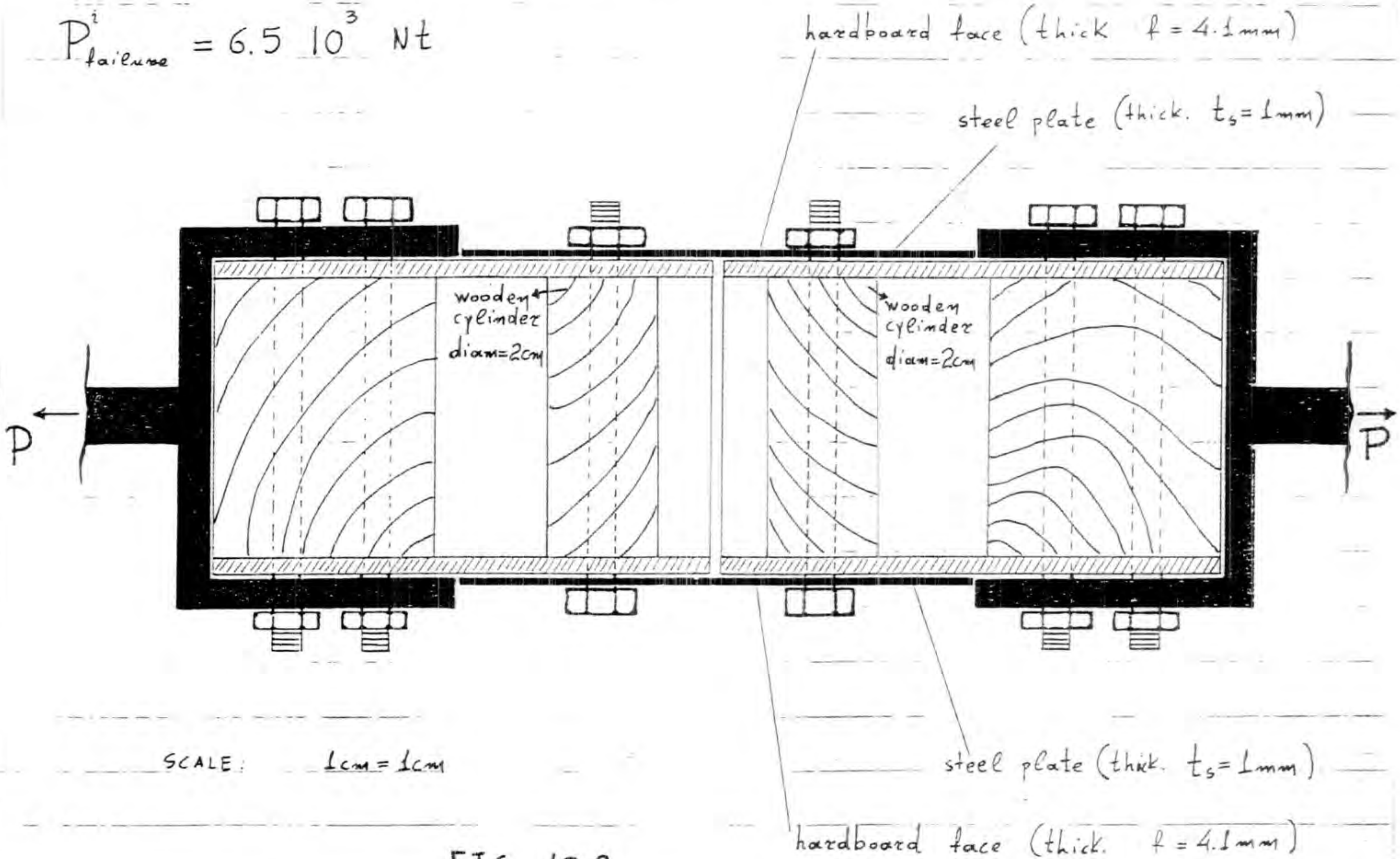
$P^{i}_{failure} = 1.5\ 10^{3}\ Nt$



SCALE: 1cm = 1cm

FIG. 10.1.

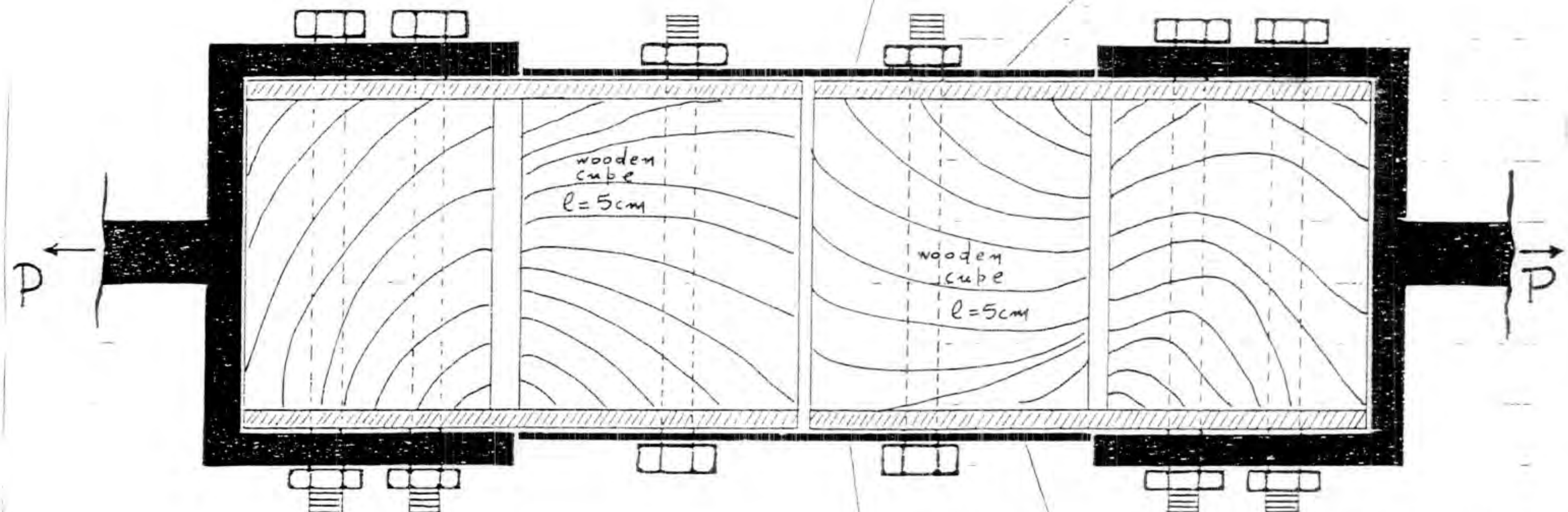
$P^{i}_{failure} = 6.5 \; 10^{3}$ Nt



SCALE: 1cm = 1cm

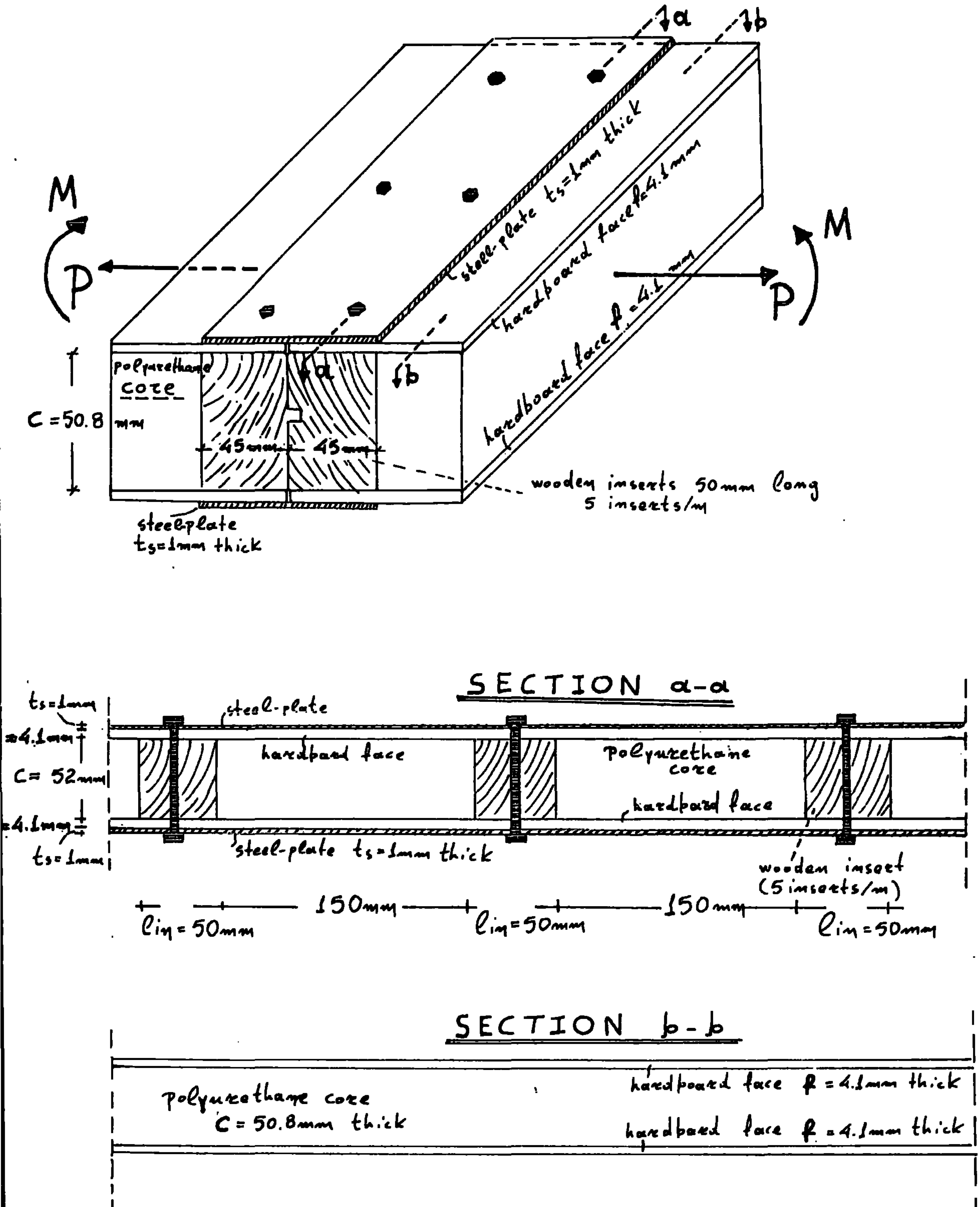
FIG. 10.2.

$P^{i}_{failure} = 15 \cdot 10^{3}$ Nt



SCALE: 1cm = 1cm

FIG. 10.3.



a
b
M
P
steel-plate ts=1mm thick
hardboard face f=4.1mm
hardboard face f=4.1mm
hardboard face f=4.1mm
polyurethane core
C=50.8 mm
45mm
45mm
wooden inserts 50mm long
5 inserts/m
steelplate
ts=1mm thick
SECTION a-a
ts=1mm
f=4.1mm
C= 52mm
f=4.1mm
ts=1mm
steel-plate
hardboard face
polyurethane core
hardboard face
steel-plate ts=1mm thick
wooden insert
(5 inserts/m)
150mm
150mm
lin=50mm
lin=50mm
lin=50mm
SECTION b-b
polyurethane core
C=50.8mm thick
hardboard face f=4.1mm thick
hardboard face f=4.1mm thick

FIG. 10.4.

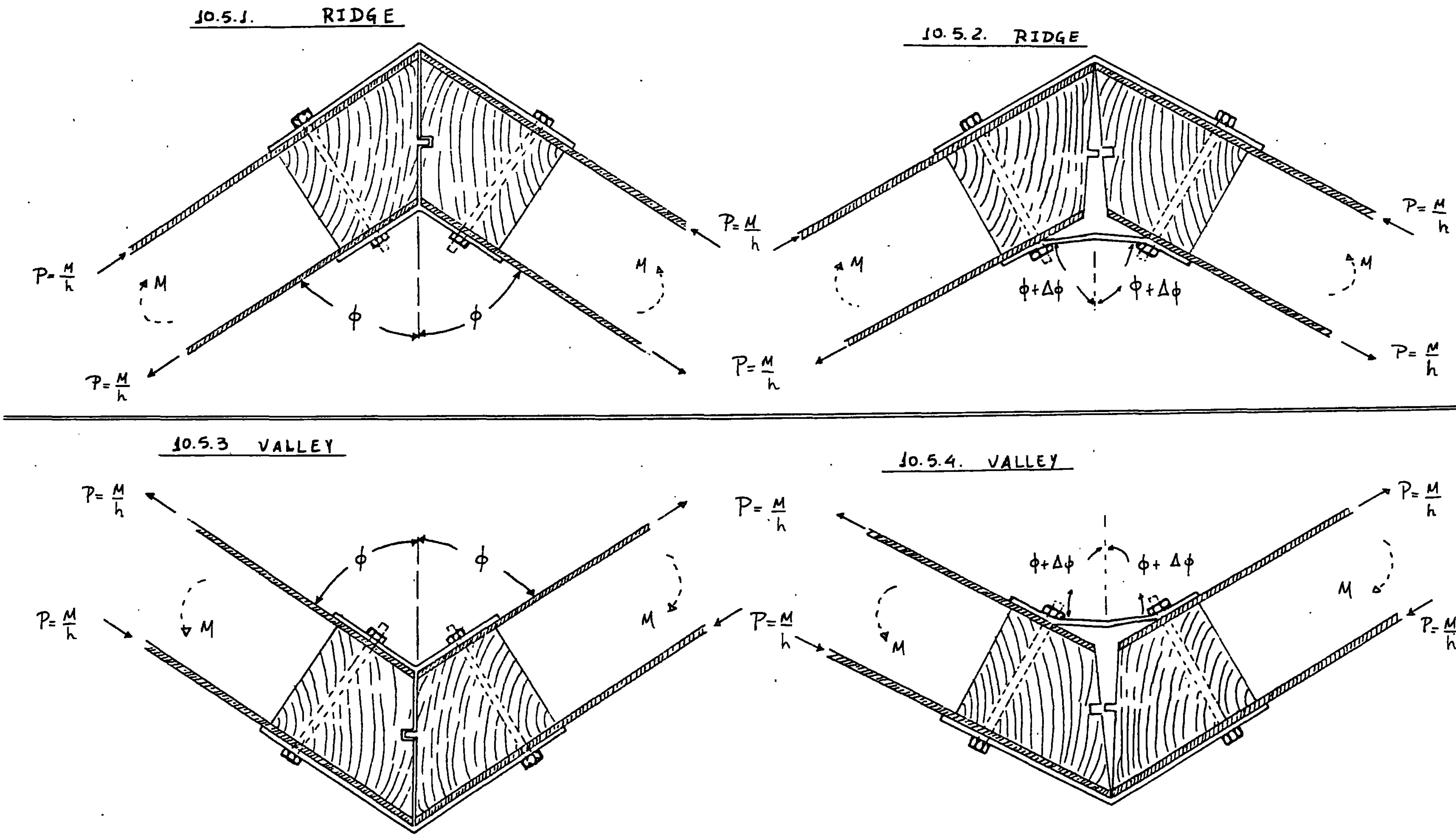
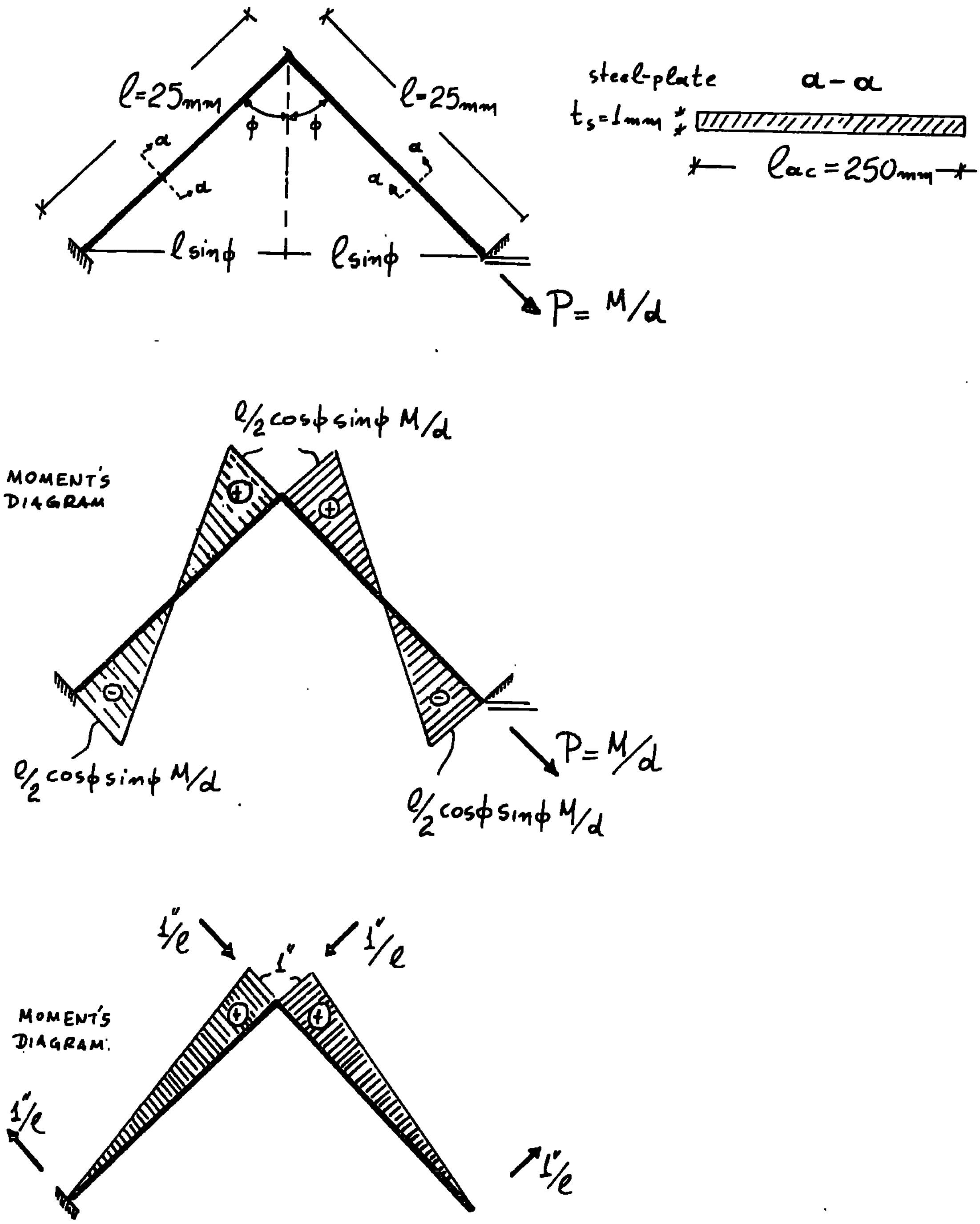


FIG. 10.5.



l=25mm
l=25mm
φ
φ
a
a
l sinφ
l sinφ
P= M/d
steel-plate
a-a
t_s=1mm
l_{ac}=250mm
l/2 cosφ sinφ M/d
MOMENT'S DIAGRAM
l/2 cosφ sinφ M/d
l/2 cosφ sinφ M/d
P= M/d
1"/l
1"/l
1"
MOMENT'S DIAGRAM
1"/l
1"/l

FIG. 10.6.

DESCRIPTION	SYMBOL	degrees
PYRAMID LONG RIDGE	a–a	↑ 61.67
PYRAMID SHORT RIDGE	b–b	↑ 76.41
TOP VALLEY	c–c	↓ 72.93
DORMER RIDGE	d–d	↑ 56.50
DORMER VALLEY	e–e	↓ 56.00

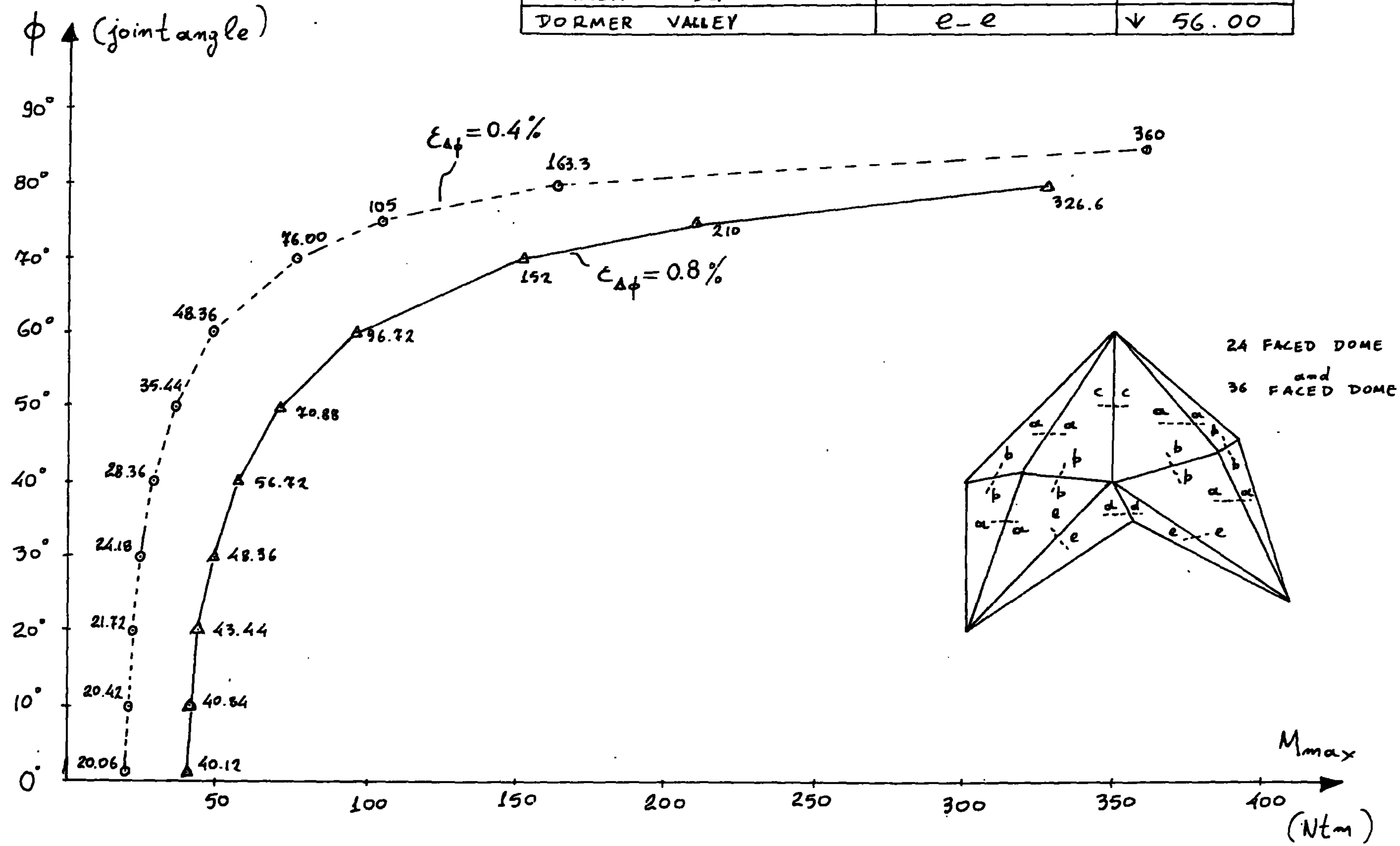


FIG. 10.7.

11. COMPUTER PROGRAMS

11.1 Introduction

For the formation of the stiffness, stress and load matrices (see Chapter 4) eleven separate subroutines have been developed. The first seven deal with the sandwich plate bending models and the remaining four with the dome models each of them corresponding to the relevant model as they were presented in Chapters 6 and 7.

The listing of each individual subroutine is presented in the Appendix.

All eleven subroutines are consistent with the main routine, which is a modified version, by Bettess, of the frontal solution technique for solving a large system of simultaneous equations as it has been developed by Irons.

The main routine assembles the various matrices for the total number of elements (Chapter 4.) and for a given set of boundary conditions proceeds with the solution of the system of simultaneous equations. [20,21,64]

The results for all the various models include for every element: (a) the relevant nodal displacements, (b) the stresses. (See Chapters 6 and 7 and Tables 11.1 ÷ 11.11)

For the displacement and rotation, plate and dome models the stresses are evaluated at the centroid of the element in the following order.

For the sandwich plate bending models:

$$\{M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}\}$$

For the dome models

$$\{M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}, N_{xx_c}, N_{yy_c}, N_{xy_c}\}$$

For the mixed model PMX12 the shear forces are evaluated at the centroid of the element

$$\{ Q_{x_c} , Q_{y_c} \}$$

For the mixed model PMX24 the shear forces are evaluated at the first and second nodes of the element

$$\{ Q_{x1} , Q_{y1} , Q_{x2} , Q_{y2} \}$$

For the mixed model DMX36 the shear forces are evaluated at the first and second nodes and the in-plane forces at the centroid of the element.

$$\{ Q_{x1} , Q_{y1} , Q_{x2} , Q_{y2} , N_{xx_c}, N_{yy_c} , N_{xy_c} \}$$

Due to the considerable amount of work involved for the preparation of the data necessary for the solution of a problem, an additional routine has been developed which can generate the data for a problem suitable for all eleven models.

This routine, for most of the cases, considerably reduces the required amount of work, particularly if a series of solutions with all of the different models involved are to be obtained.

The above mentioned routine has been employed successfully for most of the cases which have been solved as part of the present analysis. It is believed that it could, therefore, be similarly usefully employed by any future users.

The different parameters involved in the data generation routine are outlined in the next section.

11.2 DATA GENERATION ROUTINE INPUT

11.2.1 First the structure has to be divided into triangular elements.

It is very advantageous as far as the computer time and consequently the cost per run are concerned to employ as few types of elements as possible. The term "types of elements" indicates elements of the same nature (one model can be employed for the solution of a problem at one time) but with different dimensions and elasticity moduli.

For an element "similar" to a previous one with regards to the above mentioned parameters (dimensions and elasticity moduli of an element) the various matrices which have already been calculated and stored for the first of the similar elements can be used again for all the remaining similar elements.

For this purpose storage space for two different sets of matrices is allocated.

The polyhedral sandwich dome structures (see Chapter 1) take full advantage of the above mentioned principle.

The numbering of the nodes is then carried out employing single numbers for each node as if the node had only one degree of freedom.

The numbering order is totally insignificant.

If a particular problem is to be solved by more than one model (including mid-side nodes models) it is advisable to number the mid-side nodes together with the corner nodes.

11.2.2 The second step is the numbering of the elements.

It is of great importance that a node which belongs to more than one element appears in such a way that the difference between the smallest number of the element, where the node first appears, and the largest number of the element, where the node last appears, is the minimum possible.

Another important factor which must be taken into account in the numbering of the elements in combination with the above mentioned rule is that one must use the storage facility for similar element matrices in the most efficient way as far as the calculation of the various matrices for an element is concerned (see paragraph 9 INFO(3)).

This can be achieved by numbering the similar elements in consecutive order. Note that a similar element can be obtained by using the option of 180^{o} rotation of an element (see Chapter 8 Section 2 and paragraph 3).

11.2.3 Six numbers are punched via FORMAT (6I5).

The first number (NEIDOS) indicates the code number of the model to be employed (see tables 11.1 ÷ 11.11).

The second number (NELEM) indicates the total number of elements involved in the problem.

The third number (NKIND) indicates the number of different elements as far as the coordinates of the elements are concerned, (maximum 30).

Note that a set of coordinates resulting by pure translation from a previous one is considered identical to the latter.

The fourth number (NSTIF) indicates the number of different stiffness sets as regards the elasticity moduli which are to be considered in the problem, (maximum 10).

The fifth number (NBOUL) indicates the total number of different boundary lines (internal or external) as regards the boundary conditions and the transformation which are to be considered in the problem, (maximum 16).

The sixth number (NBOUP) indicates the number of different individual points with a singularity as far as the boundary conditions and the transformation are concerned so that they have to be considered separately and not as part of one of the previously mentioned boundary lines.

$$(NBOUP + NBOUL < 26)$$

Note: All the above set limits can be increased by changing the size of the relevant arrays in the data generation routine.

For changes which particularly affect the limit of the numbers NSTIF and NBOUL changes in the arrays of the main solving routine have to be introduced.

11.2.4. Nine numbers are to be punched via FORMAT (6D 10.3,/,3D 10.3) for each of the different coordinate sets. (see NKIND paragraph 3).

They represent the coordinates of the three corner nodes of the relevant element with respect to a global cartesian coordinate system in the following order:

$$(x_i , y_i , z_i)_i = 1,3$$

The numerical subscript indicates the 1st, 2nd and 3rd node respectively.

11.2.5. Twelve numbers are to be punched via FORMAT (6D 10.3,/,6D 10.3) for each of the different stiffness sets (see paragraph 3 NSTIF).

They represent the elasticity moduli of a set in the following order:

D^{xx}_{xx} , D^{yy}_{xx} , D^{yy}_{yy} , D^{xx}_{xy} , S^{xz}_{xz} , S^{yz}_{yz} ,

E^{xx}_{xx} , E^{yy}_{xx} , E^{xx}_{yy} , E^{yy}_{yy} , E^{xy}_{xy} , D^{xx}_{yy}

(see Chapter 2, Section 2)

Note:

(a) That D^{xx}_{yy} has to be punched only if it is different from D^{yy}_{xx}

(b) For sandwich plate bending problems the moduli relevant to plane stress (E^{ij}_{rs}) can be substituted by zeros.

11.2.6. A code number (see paragraph 3 NBOUL, NBOUP) must be punched for each individual boundary line and boundary point via FORMAT (26 I 3).

The first one represents the code number for the first boundary line and the (NBOUL)th one represents the code number for the last boundary line.

The (NBOUL+1)th one represents the code number for the first boundary point and the (NBOUL + NBOUD)th one represents the code number for the last boundary point (see tables

1st	2nd	3rd	4th	......	NBOULth	(NBOUL+1)th	......	(NBOUL+NBOUP)th
1st B. Line	2nd B. Line	3rd B. Line	4th B. Line	......	Last B. Line	1st B. Point	......	Last B. Point

NBOUL + NBOUP < 26

11.2.7. A set of six numbers must be punched via FORMAT (6D 10.3) for each of the boundary lines (see paragraph 3 and 6).

They represent the coordinates of two points A_1, A_2 on the relevant boundary line with respect to the global cartesian co-ordinate system in the following order:

$$x_2 , \quad x_1 , \quad y_2 , \quad y_1 , \quad z_2 , \quad z_1$$

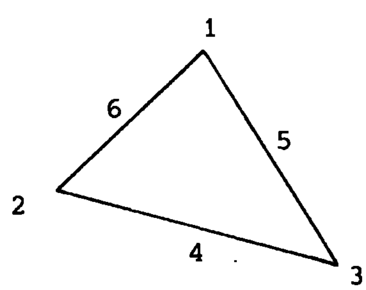
The numerical subscripts 1, 2 indicate point A_1 and A_2 respectively.

11.2 .8. Generally for all eleven models six numbers must be punched next via FORMAT (6IS) for each individual element in the structure from the first to the last element in a consecutive order.

They represent the numbers of the nodes from the 1st to the 6th as they have been set out. (See paragraph 1).

Note: (a) The order of the nodes must be consistent with the setting of their coordinates (see paragraph 4 and Appendix I).

(b) For models with only corner nodes the first three numbers (relevant to the corner nodes) are necessary although all six can be punched as well. (See paragraph 1).



11.2.9. Sixteen numbers must be punched next via FORMAT (16I5) for every element.

<u>The first number</u> (INFO(1)) indicates the relevant stiffness set to be used for the calculation of the various matrices (from the NSTIF sets. See paragraph 3).

Note: Even if the various matrices are to be taken ready from the storage space, in the case of a similar element, the above number must be set greater than zero.

<u>The second number</u> (INFO(2)) indicates the relevant co-ordinate set to be used for the calculations of the various matrices (from the NKIND sets. See paragraph 3).

Note: If set to zero the coordinates of the nodes are to be set as zero. This option can only be used in a case of a similar element.

<u>The third number</u> (INFO(3)) indicates the storage technique to be used and has the following significance with regard to the setting of the value.

If 11: it evaluates the various matrices, stores them in position I of the storage, and uses them.

If 12: it evaluates the various matrices, stores them in position II of the storage and uses them.

If 1: it takes and uses the various matrices stored in position I of the storage.

If 2: it takes and uses the various matrices stored in position II of the storage.

If 0: it evaluates and uses the various matrices without storing them.

If -1: it takes the various matrices from position I of the storage and uses them after transforming them by a rotation of 180° (see Chapter 8, Section 2, paragraph 3).

If -2: it takes the various matrices from position II of the storage and uses them after transforming them by a rotation of 180° (see Chapter 8, Section 2, paragraph 3).

<u>The 4th, 6th, 8th, 10th, 12th, 14th numbers are set as follows</u>

Number	4th	6th	8th	10th	12th	14th	The operations listed below are to be carried out depending on the setting of the corresponding number (as shown)
Refers to	1st node	2nd node	3rd node	4th node	5th node	6th node	
if it is set to	1	2	3	4	5	6	Transformation and boundary conditions
>>	-1	-2	-3	-4	-5	-6	Boundary conditions
>>	0	0	0	0	0	0	Unchanged

The <u>5th, 7th, 9th, 11th, 13th, 15th numbers</u> are set as follows (to be set, if the previously defined six numbers which the present numbers are combined with are different from zero).

Number Refers to	5th 1st node	7th 2nd node	9th 3rd node	11th 4th node	13th 5th node	15th 6th node	Indicates that the relevant node is linked with the Nth. boundary line or point (paragraph 6) as follows
combined with	4th number	6th number	8th number	10th number	12th number	14th number	As previously defined
is set to	N	N	N	N	N	N	WHERE N indicates the nth boundary line or point in the order they have been defined in paragraph 6. The transformation is to be performed with respect to the system defined by the line A_1 A_2
is set to	-N	-N	-N	-N	-N	-N	(only if the corresponding number of the previously defined set is positive) N as above. The transformation is to be performed with respect to the global coordinate system.

See paragraph 7 and Chapter 8.

<u>The 16th number</u> indicates the loading condition for the element and is set in a value as follows

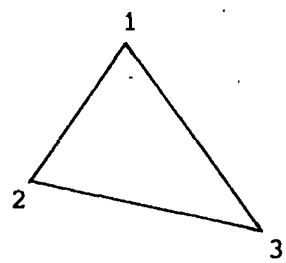
0 If no load is to be applied

n Where n is the number of the non zero terms of the load matrix which is of the following form

1	2	3	4	5	6	7	8	9	10	11	12	
P_x	P_y	P_z	R^1_x	R^1_y	R^1_z	R^2_x	R^2_y	R^2_z	R^3_x	R^3_y	R^3_x	

P_i the component of the uniformly distributed load with respect to i global axis

R^j_i the component of the concentrated load at node j with respect to i global axis



If the 16th number is 0 the next card is the card with the 16 numbers for the next element.

If the 16th number is different from zero the next card (cards) must be inserted in the following way via FORMAT (I5,D 10.3). The first number to be punched indicates the column of the non-zero term in the load matrix and the second number indicates the magnitude of the non-zero term.

PLATE MODEL

REFERENCE SYMBOL
PDS15

CODE NUMBER
1

STRESSES EVALUATED AT CENTROID
$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}$

TOTAL NUMBER OF D.O.F.
15

	NBOUL	NBOUP	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION	DEGREES OF FREEDOM AFTER THE TRANSFORMATION	FREE EDGE	FREE EDGE WITH STIFFENER	SIMPLY SUPPORTED EDGE	SIMPLY SUPPORTED EDGE WITH STIF.	CLAMPED EDGE	CLAMPED EDGE WITH STIF.	AXIS OF SYMMETRY	AXIS OF SYMMETRY WITH STIF.	POINT SUPPORT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT
CORNER NODES	W	W	0	0	1	1	1	1	0	0	1	1	1	0	0	0	1	1
	W_x	Θ_m	0	0	0	0	1	1	1	1	0	1	1	1	1	0	1	0
	W_y	W_s	0	0	1	1	1	1	0	0	0	1	1	1	1	0	1	0
	ϕ_x	ϕ_m	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
	ϕ_y	ϕ_s	0	1	0	1	0	1	0	1	0	0	1	0	1	1	0	1
MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)																		

1 INDICATES CONSTRAINED D.O.F.
0 INDICATES FREE D.O.F.

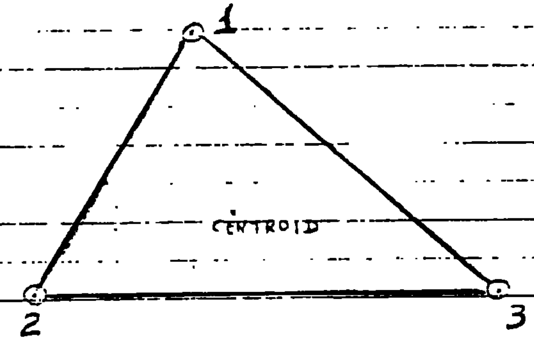
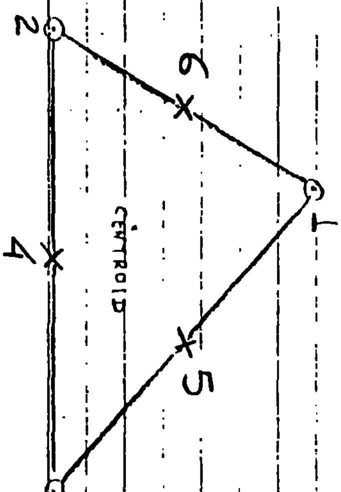


TABLE 11.1.

TABLE 11.2.

DEGREES OF FREEDOM BEFORE THE TRANSFORMATION	Corner nodes: ϕ_y	ϕ_x	W_y	W_x	W	Midside nodes (if different from corner nodes): ϕ_y	ϕ_x
DEGREES OF FREEDOM AFTER THE TRANSFORMATION	ϕ_s	ϕ_n	W_s	Θ_n	W	ϕ_s	ϕ_n
FREE EDGE	0	0	0	0	0	0	0
FREE EDGE WITH STIFFENER	1	0	0	0	0	1	0
SIMPLY SUPPORTED EDGE	0	0	1	0	1	0	0
SIMPLY SUPPORTED EDGE WITH STIF.	1	0	1	0	1	1	0
CLAMPED EDGE	0	1	1	1	1	0	1
CLAMPED EDGE WITH STIF.	1	1	1	1	1	1	1
AXIS OF SYMMETRY	0	1	0	1	0	0	1
AXIS OF SYMMETRY WITH STIF.	1	1	0	1	0	1	1
POINT SUPPORT	0	0	0	0	1		
CORNER POINT	0	0	1	1	1		
CORNER POINT	1	1	1	1	1		
CORNER POINT	0	0	1	1	0		
CORNER POINT	1	1	1	1	0		
CORNER POINT	1	1	0	0	0		
CORNER POINT	0	1	1	1	1		
CORNER POINT	1	1	0	0	1		



1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

REFERENCE SYMBOL PDS21

CODE NUMBER 2

STRESSES EVALUATED AT CENTROID

$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}$

TOTAL NUMBER OF D.O.F. 21

NBOUL, NBOUP		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
DEGREES OF FREEDOM BEFORE THE TRANSFORMATION	DEGREES OF FREEDOM AFTER THE TRANSFORMATION	FREE EDGE	FREE EDGE WITH STIFFENER	SIMPLY SUPPORTED EDGE	SIMPLY SUPPORTED EDGE WITH STIF.	CLAMPED EDGE	CLAMPED EDGE WITH STIF.	AXIS OF SYMMETRY	AXIS OF SYMMETRY WITH STIF.	POINT SUPPORT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT
CORNER NODES: W	W	0	0	1	1	1	1	0	0	1	1	1	0	0	0	1	1
Mxx	Mnn	1	1	1	1	0	0	0	0	1	0	0	0	0	1	0	0
Myy	Mss	0	0	1	1	0	0	0	0	1	0	0	0	0	1	0	0
Mxy	Mns	1	0	1	0	1	0	1	0	0	1	0	1	0	0	1	0
MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)																	

PLATE MODEL

REFERENCE SYMBOL

PMX12

CODE NUMBER

3

STRESSES EVALUATED AT CENTROID

Q_{x_c}, Q_{y_c}

TOTAL NUMBER OF D.O.F.

12

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

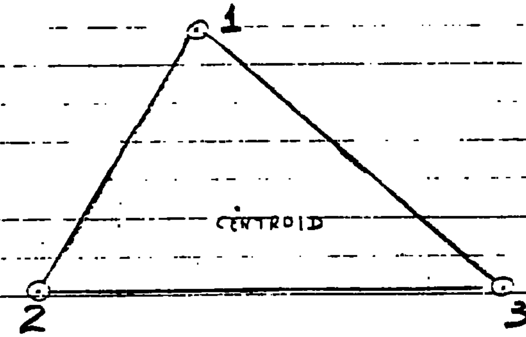


TABLE 11.3.

PLATE MODEL

REFERENCE SYMBOL	CODE NUMBER	STRESSES EVALUATED AT 1st and 2nd NODE.	TOTAL NUMBER OF D.O.F.
PM×24	4	$Q_{x_1}, Q_{y_1}, Q_{x_2}, Q_{y_2}$	24

1 INDICATES CONSTRAINED D.O.F.
0 INDICATES FREE D.O.F.

CORNER NODES

NBOUL, NBOUD	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION / DEGREES OF FREEDOM AFTER THE TRANSFORMATION	W / W	M_{xy} / M_{mm}	M_{yy} / M_{ss}	M_{yy} / M_{ms}
1	FREE EDGE	0	1	0	1
2	FREE EDGE WITH STIFFENER	0	1	0	0
3	SIMPLY SUPPORTED EDGE	1	1	1	1
4	SIMPLY SUPPORTED EDGE WITH STIF.	1	1	1	0
5	CLAMPED EDGE	1	0	0	1
6	CLAMPED EDGE WITH STIF.	1	0	0	0
7	AXIS OF SYMMETRY	0	0	0	1
8	AXIS OF SYMMETRY WITH STIF.	0	0	0	0
9	POINT SUPPORT	1	1	1	0
10	CORNER POINT	1	0	0	1
11	CORNER POINT	1	0	0	0
12	CORNER POINT	0	0	0	1
13	CORNER POINT	0	0	0	0
14	CORNER POINT	0	1	1	0
15	CORNER POINT	1	0	0	1
16	CORNER POINT	1	0	0	0

MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)

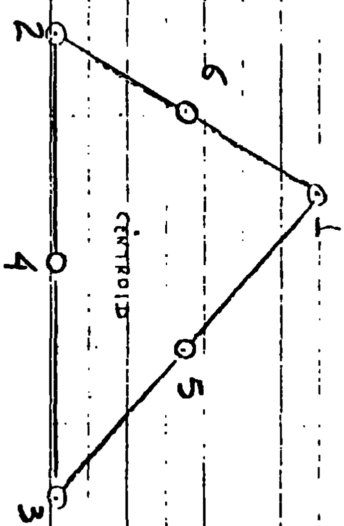


TABLE 11.4.

PLATE MODEL

	NBOUL	NBOUP	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION	DEGREES OF FREEDOM AFTER THE TRANSFORMATION	FREE EDGE	FREE EDGE WITH STIFFENER	SIMPLY SUPPORTED EDGE	SIMPLY SUPPORTED EDGE WITH STIF.	CLAMPED EDGE	CLAMPED EDGE WITH STIF.	AXIS OF SYMMETRY	AXIS OF SYMMETRY WITH STIF.	POINT SUPPORT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT
CORNER NODES	W	W	0	0	1	1	1	1	0	0	1	1	1	0	0	0	1	1
	W_x	θ_M	0	0	0	0	1	1	1	1	0	1	1	1	1	0	1	0
	W_Y	W_S	0	0	1	1	1	1	0	0	0	1	1	1	1	0	1	0
	W_{xx}	W_{MM}	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0
	W_{xY}	W_{SM}	0	0	0	0	0	0	1	1	0	0	0	1	1	0	0	1
	W_{YY}	W_{SS}	0	0	1	1	1	1	0	0	0	1	1	0	0	0	0	0
	ϕ_x	ϕ_M	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
	ϕ_Y	ϕ_S	0	1	0	1	0	1	0	1	0	0	1	0	1	1	0	1
MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)																		

REFERENCE SYMBOL
PDS24

CODE NUMBER
5

STRESSES EVALUATED AT CENTROID

$M_{xxc}, M_{YYc}, M_{xYc}, Q_{xc}, Q_{Yc}$

TOTAL NUMBER OF D.O.F.
24

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

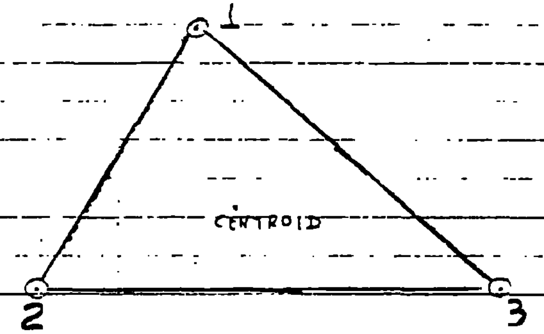


TABLE 11.5.

PLATE MODEL

	NBOUL, NBOUP		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION	DEGREES OF FREEDOM AFTER THE TRANSFORMATION	FREE EDGE	FREE EDGE WITH STIFFENER	SIMPLY SUPPORTED EDGE	SIMPLY SUPPORTED EDGE WITH STIF.	CLAMPED EDGE	CLAMPED EDGE WITH STIF.	AXIS OF SYMMETRY	AXIS OF SYMMETRY WITH STIF.	POINT SUPPORT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT	CORNER POINT
CORNER NODES	W	W	0	0	1	1	1	1	0	0	1	1	1	0	0	0	1	1
	W_x	θ_M	0	0	0	0	1	1	1	1	0	1	1	1	1	0	1	0
	W_Y	W_S	0	0	1	1	1	1	0	0	0	1	1	1	1	0	1	0
	W_{xx}	W_{MM}	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0
	W_{xy}	W_{SM}	0	0	0	0	0	0	1	1	0	0	0	1	1	0	0	1
	W_{YY}	W_{SS}	0	0	1	1	1	1	0	0	0	1	1	0	0	0	0	0
	ϕ_x	ϕ_M	0	0	0	0	1	1	1	1	0	0	1	0	1	1	1	1
	ϕ_Y	ϕ_S	0	1	0	1	0	1	0	1	0	0	1	0	1	1	0	1
MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)	ϕ_x	ϕ_M	0	0	0	0	1	1	1	1								
	ϕ_Y	ϕ_S	0	1	0	1	0	1	0	1								

REFERENCE SYMBOL
PDS30

CODE NUMBER
6

STRESSES EVALUATED AT CENTROID

$M_{xxc}, M_{YYc}, M_{XYc}, Q_{xc}, Q_{Yc}$

TOTAL NUMBER OF D.O.F.
30

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

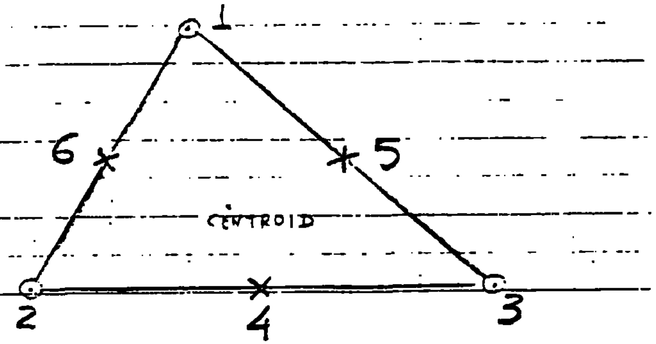


TABLE 11.6.

TABLE 11.7

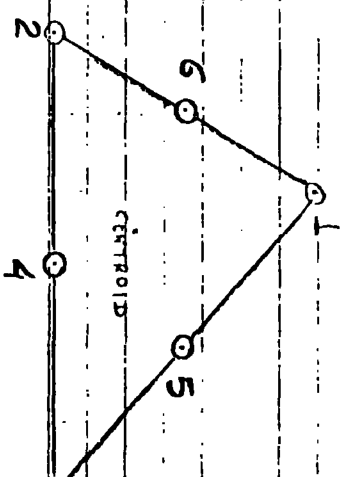
PLATE MODEL

REFERENCE SYMBOL PRO18

CODE NUMBER 7

STRESSES EVALUATED AT CENTROID $M_{xxc}, M_{yyc}, M_{xyc}, Q_{xc}, Q_{yc}$

TOTAL NUMBER OF D.O.F. 18



1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)

CORNER NODES				θ_y / θ_s	θ_x / θ_m	W / W	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION / DEGREES OF FREEDOM AFTER THE TRANSFORMATION	NBOUL, NBOUP
				0	0	0	FREE EDGE	1
				1	0	0	FREE EDGE WITH STIFFENER	2
				0	0	1	SIMPLY SUPPORTED EDGE	3
				1	0	1	SIMPLY SUPPORTED EDGE WITH STIF.	4
				0	1	1	CLAMPED EDGE	5
				1	1	1	CLAMPED EDGE WITH STIF.	6
				0	1	0	AXIS OF SYMMETRY	7
				1	1	0	AXIS OF SYMMETRY WITH STIF.	8
				0	0	1	POINT SUPPORT	9
				1	1	1	CORNER POINT	10
				1	1	1	CORNER POINT	11
				1	1	0	CORNER POINT	12
				1	1	0	CORNER POINT	13
				1	1	0	CORNER POINT	14
				0	1	1	CORNER POINT	15
				1	1	1	CORNER POINT	16

DOME MODEL

REFERENCE SYMBOL	DDS21
CODE NUMBER	8
STRESSES EVALUATED AT CENTROID	$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}, N_{xx_c}, N_{yy_c}, N_{xy_c}$
TOTAL NUMBER OF D.O.F.	21

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

	NBOUL, NBOUP		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION	DEGREES OF FREEDOM AFTER THE TRANSFORMATION	FREE EDGE	FREE EDGE WITH STIF.	AXIS OF SYMMETRY	AXIS OF SYMMETRY WITH ST.	AXIS OF SYMMETRY	AXIS OF SYMMETRY WITH ST.	SIMPLY SUPPORTED EDGE	SIMPLY SUPPORTED WITH STIF.	SIMPLY SUPPORTED EDGE	SIMPLY SUPPORTED WITH STIF.	SIMPLY SUPPORTED EDGE	SIMPLY SUPPORTED WITH STIF.	SIMPLY SUPPORTED EDGE	SIMPLY SUPPORTED WITH STIF.	POINT SUPPORT						PLATE INTERCONNECTION									
CORNER NODES	u^{local}	u^{global}	0	0	1	1	0	0	0	0	1	1	0	0	1	1	1	1	1	0	0	1	0	0	0	0	1	1	1	0	0	0
	v^{ℓ}	v^{g}	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	1	1	0	1	0	0	0	0	0	0	0	1	1	1
	W^{ℓ}	W^{g}	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0
	θ_x^{ℓ}	θ_m	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	θ_y^{ℓ}	θ_s	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	ϕ_x^{ℓ}	ϕ_m	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	ϕ_y^{ℓ}	ϕ_s	0	1	0	1	0	1	0	1	0	1	0	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)																																

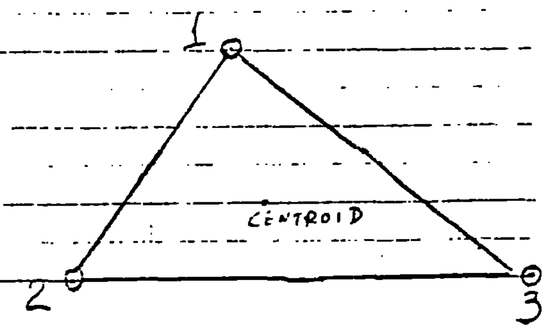


TABLE 11.8.

DOME MODEL

REFERENCE SYMBOL DDS33

CODE NUMBER 9

STRESSES EVALUATED AT CENTROID
$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}$,
$N_{xx_c}, N_{yy_c}, N_{xy_c}$

TOTAL NUMBER OF D.O.F. 33

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)				CORNER NODES								
U_{ss}	u_{ss}	ϕ_y^c	ϕ_x^c	ϕ_y^c	ϕ_x^c	θ_y^e	θ_x^e	W^e	V^e	U^{local}	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION	NBOUL, NBOUP
$\tilde{U}_{ss}$	$\tilde{u}_{ss}$	ϕ_s	ϕ_m	ϕ_s	ϕ_m	Θ_s	Θ_m	W^g	V^g	U^{global}	DEGREES OF FREEDOM AFTER THE TRANSFORMATION	
0	0	0		0	0	0	0	0	0	0	FREE EDGE	1
0	0	1	0	0	1	0	0	0	0	0	FREE EDGE WITH STIF.	2
0	1	0	1	0	1	0	1	0	0	1	AXIS OF SYMMETRY	3
0	1	1	1	1	1	1	1	0	0	1	AXIS OF SYMMETRY WITH ST.	4
1	0	0	1	0	1	0	1	0	1	0	AXIS OF SYMMETRY	5
1	0	1	1	1	1	1	1	0	1	0	AXIS OF SYMMETRY WITH ST.	6
0	0	0	0	0	0	0	0	1	0	0	SIMPLY SUPPORTED EDGE	7
0	0	1	0	1	0	1	0	1	0	0	SIMPLY SUPPORTED WITH STIF.	8
0	1	0	0	0	0	0	0	1	0	1	SIMPLY SUPPORTED EDGE	9
0	1	1	0	1	0	1	0	1	0	1	SIMPLY SUPPORTED WITH STIF.	10
1	0	0	0	0	0	0	0	1	1	0	SIMPLY SUPPORTED EDGE	11
1	0	1	0	1	0	1		1	1	0	SIMPLY SUPPORTED WITH STIF.	12
1	1	0	0	0	0	0	0	1	1	1	SIMPLY SUPPORTED EDGE	13
1	1	1	0	1	0	1	0	1	1	1	SIMPLY SUPPORTED WITH STIF.	14
				1	1	0	0	1	1	1	POINT SUPPORT	15
				1	1	0	0	1	0	1	"	16
				1	1	1	1	0	1	1	"	17
				1	1	0	0	1	1	0	"	18
				1	1	0		1		0	"	19
				1	1	0	0	0	1	1	"	20
				0	0	0		0	0	0	PLATE INTERCONNECTION	21
				0	0	0		0		0	"	22
				0	0	0		0	0	0	"	23
				0	0	0		0	0	0	"	24
				0	0	0		0	0	1	"	25
				0	0	0		0	0	1	"	26
				0	0	0		0		1	"	27
				0	0	0		0	1	0	"	28
				0	0	0		0	1	0	"	29
				0	0	0		0	1	0	"	30

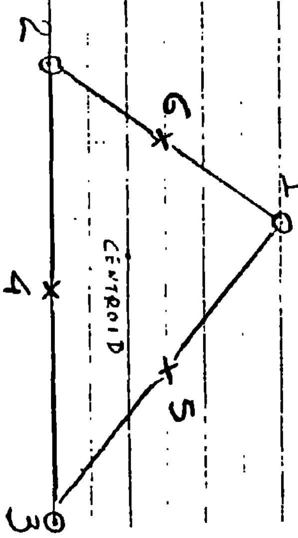
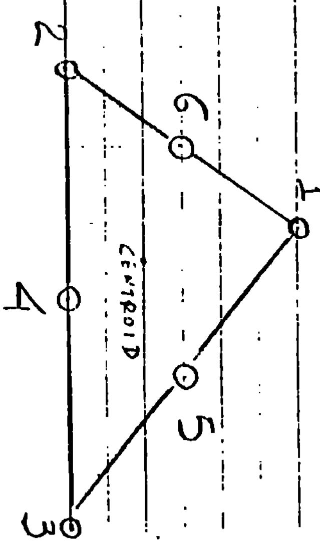


TABLE 11.9.

MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)	CORNER NODES								
		M^e_{xy}	M^e_{yy}	M^e_{xx}	W^e	V^e	u^{local}	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION	NBOUL, NBOUP
		M_{sn}	M_{ss}	M_{nn}	W^g	V^g	u^{global}	DEGREES OF FREEDOM AFTER THE TRANSFORMATION	
		1	0	1	[illegible]	0	0	FREE EDGE	1
		0	0	1	[illegible]	0	0	FREE EDGE WITH STIF.	2
		1	0	0	0	0	1	AXIS OF SYMMETRY	3
		0	0	0	0	0	1	AXIS OF SYMMETRY WITH ST.	4
		1	0	0	0	1	0	AXIS OF SYMMETRY	5
		0	0	0	0	1	0	AXIS OF SYMMETRY WITH ST.	6
		1	1	1	1	0	0	SIMPLY SUPPORTED EDGE	7
		1	1	1	1	0	0	SIMPLY SUPPORTED WITH STIF.	8
		1	1	1	1	0	1	SIMPLY SUPPORTED EDGE	9
		0	1	1	1	0	1	SIMPLY SUPPORTED WITH STIF.	10
		1	1	1	1	1	0	SIMPLY SUPPORTED EDGE	11
		0	1	1	1	1	0	SIMPLY SUPPORTED WITH STIF.	12
		1	1	1	1	1	1	SIMPLY SUPPORTED EDGE	13
		1	1	1	1	1	1	SIMPLY SUPPORTED WITH STIF.	14
		1	1	1	1	1	1	POINT SUPPORT	15
		0	1	1	1	0	1	"	16
		0	0	0	0	1	1	"	17
		0	1	1	1	1	0	"	18
		0	1	1	1		0	"	19
		0	1	1	0	1	1	"	20
		0	0	0	0	0	0	PLATE INTERCONNECTION	21
		0	0	1	0	0	0	"	22
		0	1	1	0	0	0	"	23
		1	0	1	0	0	0	"	24
		0	0	1	0	0	1	"	25
		0	1	1	0	0	1	"	26
		1	0	1	0	0	1	"	27
		0	0	1	0	1	0	"	28
		0	1	1	0	1	0	"	29
		1	0	1	0	1	0	"	30



1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

DOME MODEL

REFERENCE SYMBOL

DMX36

CODE NUMBER

11

STRESSES EVALUATED AT 1st and 2nd NODE and at CENTROID

Q_{x1}, Q_{y1}, Q_{x2}, Q_{y2}

N_{xx}, N_{yy}, N_{xy}

TOTAL NUMBER OF D.O.F.

36

TABLE 11.10.

MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)

NBOUL, NBOUP	DEGREES OF FREEDOM BEFORE THE TRANSFORMATION / AFTER THE TRANSFORMATION	U^e_{local} / U^g_{global}	V^e / V^g	W^e / W^g	Θ_x^e / Θ_m	Θ_y^e / Θ_s
1	FREE EDGE	0	0	0	0	0
2	FREE EDGE WITH STIF.	0	0	0	0	0
3	AXIS OF SYMMETRY	1	0	0	1	0
4	AXIS OF SYMMETRY WITH ST.	1	0	0	1	1
5	AXIS OF SYMMETRY	0	1	0	1	0
6	AXIS OF SYMMETRY WITH ST.	0	1	0	1	1
7	SIMPLY SUPPORTED EDGE	0	0	1	0	0
8	SIMPLY SUPPORTED WITH STIF.	0	0	1	0	1
9	SIMPLY SUPPORTED EDGE	1	0	1	0	0
10	SIMPLY SUPPORTED WITH STIF.	1	0	1	0	1
11	SIMPLY SUPPORTED EDGE	0	1	1	0	0
12	SIMPLY SUPPORTED WITH STIF.	0	1	1	0	1
13	SIMPLY SUPPORTED EDGE	1	1	1	0	0
14	SIMPLY SUPPORTED WITH STIF.	1	1	1	0	1
15	POINT SUPPORT	1	1	1	0	0
16	"	1	0	1	0	0
17	"	1	1	0	1	1
18	"	0	1	1	0	0
19	"	0	0	1	0	0
20	"	1	1	0	0	0
21	PLATE INTERCONNECTION	0	0	0	0	0
22	"	0	0	0	0	0
23	"	0	0	0	0	0
24	"	0	0	0	0	0
25	"	1	0	0	0	0
26	"	1	0	0	0	0
27	"	1	0	0	0	0
28	"	0	1	0	0	0
29	"	0	1	0	0	0
30	"	0	1	0	0	0

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

DOME MODEL

REFERENCE SYMBOL DR030

CODE NUMBER 12

STRESSES EVALUATED AT CENTROID.

$N_{xxc}, M_{xxc}, M_{yyc}, Q_{xc}, Q_{yc}$ — $N_{xyc}, N_{yyc}, N_{xyc}$ —

TOTAL NUMBER OF D.O.F. 30

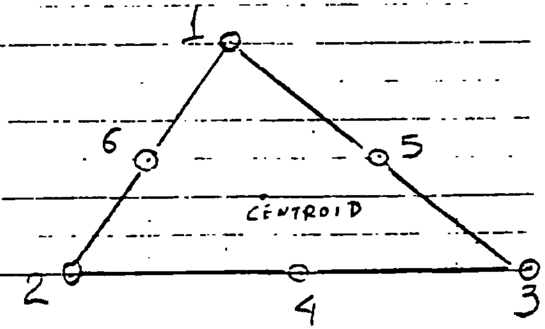


TABLE 11.11.

12. SANDWICH PLATE BENDING RESULTS

The various sandwich plate bending models, (see fig. 12.1), outlined in chapter 6, were tested by solving a series of problems, for which other solutions have already been found by other researchers.

The characteristics of each individual model had to be established by the analysis of the obtained results.

As outlined in chapter 7 the sandwich dome models were formed by extending certain sandwich plate bending models.

The selection of these models was based on the conclusions drawn from the comparative analysis of the results from the plate models presented in this chapter.

The analysed problems are classified as follows.

12.1 Theoretical results.

Most of the existing theoretical results are obtained from numerical solutions of certain sandwich plate bending problems (small deflections).[21]

The following were chosen for the analysis.

12.1.1. Comparison with Dynamic Relaxation Method [16]

Square isotropic sandwich plate under uniformly distributed load.

The plate is simply supported with the twisting moment fixed to zero (Mns = 0) all along the boundaries. (see figs. 12.5, 12.6, CASE 1)

A second solution was obtained with the twisting moment acting (Mns ≠ 0) all along the boundaries. (see figs. 12.5, 12.7, CASE 2)

The deflection curves obtained for the various models compare very well with those of reference [16]. The model PDS21 is more flexible than the rest. The same applies for the models PDS30 to a lesser extent.

For the mixed model PMX24 the agreement is very good; the mixed model PMX12 being less accurate.

The various models are very accurate with regard to the distribution of moment. The agreement with regard to the distribution of shear stresses varies from very good for the models PDS21 and PMX24 to fair for the models PDS15 and PMX12 [16].

12.1.2. Comparison with Finite Difference Method [110]

Square orthotropic sandwich plate under uniformly distributed load.

The plate is simply supported with the twisting moment acting (Mns $\neq$ 0) all along the boundaries.

Six sets of results were obtained by varying the properties of the plate. (see fig. 12.5, CASES 3 ÷ 8)

The results for each case are presented in figs. 12.8 ÷ 12.11

The various models behave in the same way as presented in the previous section 12.1.1.

12.1.3. Comparison with Finite Difference Method [110]

Square orthotropic sandwich plate under uniformly distributed load.

The plate is clamped with the twisting moment acting (Mns $\neq$ 0) all along the boundaries.

Three sets of results were again obtained by varying the properties of the plate. (see fig. 12.5, CASES 9 ÷ 11)

The results obtained for each individual case are presented in figs. 12.12 ÷ 12.20.

The maximum values for the deflection, moment and shear stress obtained through the various models are plotted against the values of shear stiffness for the different cases. (see figs. 12.19, 12.20 CASE 9 ÷ 12)

The deviation of the deflection obtained through the model PDS21 increases for small values of shear stiffness as the deflection due to shear becomes predominant in the increase of the total deflection.

The effect of the shear stiffness on the moments and shear stresses is negligible.

12.1.4. Comparison with Finite Element Models [21, 97]

Skew isotropic sandwich plate under uniformly distributed load.

The plate is simply supported with the twisting moment acting (Mns $\neq$ 0) all along the boundaries. (see figs. 12.21 ÷ 12.24, CASE 1)

A second solution was obtained with the twisting moment fixed to zero (Mns = 0) all along the boundaries. (see figs. 12.21 ÷ 12.27, CASE 2)

The models PMX12, PMX24 appear to be stiff and the models PDS21, PDS30 flexible for CASE 1 with regard to the deflection although the agreement for CASE 2 is very good.

For the distribution of the moment My for CASE 1 the agreement is reasonable for most of the displacement models, although the mixed models differ because the My was fixed to zero at the corner A. For the distribution of the Mx for CASE 1 the mixed models are more accurate.

For CASE 2, all the models are quite accurate with regard to the distribution of the moments Mx, My, the mixed models being superior in this respect.

12.1.5. Comparison with Fourier Series Method [66]

Skew isotropic sandwich plate under uniformly distributed load [66]

The plate is clamped with the twisting moment acting ($Mns \neq 0$) all along the boundaries.

Four sets of results were obtained by varying the skew angle of the plate. (see figs. 12.28 ÷ 12.37, CASES 1 ÷ 4)

For all the cases the mixed models as well as most of the displacement models are seen to be accurate, the mixed models being superior in this respect.

The higher order models PDS24, PDS30 appear to be less accurate.

12.2 Experimental Results

12.2.1. Comparison with Sandwich Plates tested by Bettess

Square sandwich plate under concentrated load at the centre of the plate. [21]

The plate was supported at all four corners and the edges were unstiffened (Mns = 0 all along the boundaries.)

Seven cases were analysed corresponding to seven different sandwich panels tested by Bettess. [21] (see figs. 12.38 ÷ 12.49, CASES 1 ÷ 7)

The above cases proved to be the most difficult modelling tests for the various elements.

The models PDS21, PDS30, PRO18 show an increased flexibility (up to 45%) for most cases excepting CASE 6. (see fig. 12.39)

Due to the above behaviour additional boundary conditions were introduced for the above models, by restraining the rotations at the four supports.

For the quadratic mixed model PMX24 the modelling was performed with the twisting moment acting all along the free edge. Despite this, the model behaves in an increasingly flexible manner for cases 1, 2 and 5, but becomes stiffer for case 6.

The displacement models PDS15 and PDS24 proved more succesful than the rest with the exception of case 5, generally agreeing very well with the experimental results.

12.2.2. Comparison with triangular sandwich plates under concentrated load tested by COLLINS [21, 33, 85]

Six cases were analysed varying the shape, the loading point and the boundary conditions of the plate.

The properties of the plate were the same as Bettes's Plate 3.

For all the models the agreement with the experimental results varies from good to very good. The displacement models PDS15 and PDS24 appear to behave in a less flexible manner in comparison with the experimental results, the latter being closer to the behaviour of the models PDS21 and PDS30 which behave as in the previously analysed cases with an increased flexibility. The behaviour of the mixed models PMX12, PMX24 and the rotation model PRO18 is very satisfactory.

12.3 Conclusions

Seven different models were used in the present analysis to solve sandwich plate bending problems as presented in section 12.1 and 12.2 serving the purpose of comparison between the mixed variational approach and the displacement variational approach. (chapters 4 and 6)

An attempt was made by simulating the theoretical and experimental results obtained by other researchers, to understand the behaviour of the various models and from the comparative study to select the most efficient and appropriate for the sandwich dome problem as explained in chapters 7 and 12.

The following can be concluded from the problems analysed in sections 12.1, 12.2.

From the convergence study presented in figs. 12.2 ÷ 12.5 it is evident that all the models converge, the higher order in a better way as expected.

The improvement in the accuracy obtained from the higher order displacement models PDS24 and PDS30 in comparison with the models PDS15 and PDS21 does not provide enough justification for the considerable increase in computational effort, due to the increase in the total number of unknowns for a problem. (see fig. 6.1, 6.2,12.2 ÷ 12.5)

Another important factor is the boundary conditions for the higher order models PDS24 and PDS30. They include restraintsof second derivations of deflections (W_{nn}, W_{sn}, W_{ss}) which adds one more difficulty to the already delicate problem of simulating the physical boundary restraints.

This proved to have a very great influence in the case of the square plates supported at the four corners under concentrated load, as mentioned in Section 12.2.

The displacement models tend to show better accuracy for problems with few displacement constraints(free edges) than the mixed models. On the other hand for problems with a large number of displacement constraints (clamped edges) but few constraints for the mixed models, the latter are more accurate in their behaviour.

The same is valid from the numerical and computational effort point of view when the boundary constraints are introduced as computer data.

The mixed models proved advantageous with regard to the distribution of the stresses and in particular to the distribution of moments, which is obtained from nodal values, as for the mixed models the moments represent degrees of freedom (see chapters 4, 5 and 6)

As expected the increase in the order of the shear variation for the models PDS21 and PDS30 produce more flexible models because of the increase, as mentioned in section 12.1.1, in the part of the total deflection due to shear.

This was proved to cope in a better way with problems where the ratio shear stiffness over bending stiffness S^{xz}_{xz}/D^{xx}_{xx} (see chapter 2) had small values of the range between 1 ÷ 15, as some of the cases tested experimentally. (12.2)

12.4 The extension from Plate to Dome Models

As already mentioned in chapter 7, four plate models were extended to form the four dome models (figs. 7.1, 7.2 and 13.1). These are PDS15, PDS21, PMX24 and PRO18.

The two higher order elements PDS24 and PDS30 were rejected because, as outlined in section 12.3, the accuracy obtained does not justify the increase in the total number of unknowns. Another decisive factor was the difficulty which the second derivatives present in transformation and boundary conditions.

The mixed models PMX12 was rejected for reasons outlined in chapter 7.

The first models PDS15 (DDS21) has the advantage of possessing the least degrees of freedom as a sandwich dome model, this being of great significance from the computational effort point of view. The behaviour of the model when tested by the different plate bending problems was found, on average to be very satisfactory.

The second model PDS21 (DDS33) although it appeared to behave flexibly in most cases, was included in the set of dome models. The reason for this inclusion was that most of the dome cases to be analysed, presented a ratio of shear stiffness over bending stiffness (S_{xz}/D_{xx}) of a fairly low value. As outlined in the previous section, this particular high shear model proved quite successful in this respect.

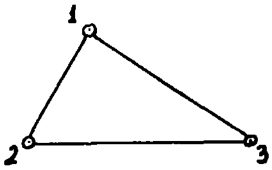
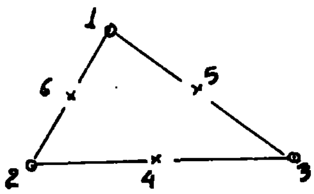
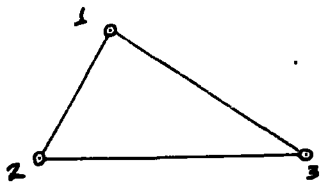
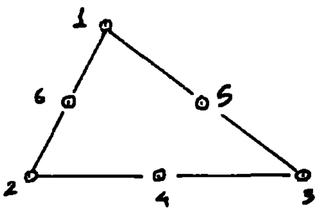
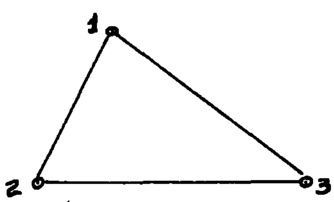
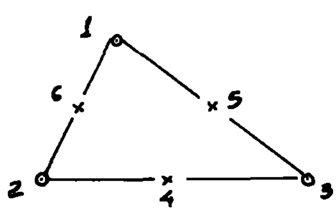
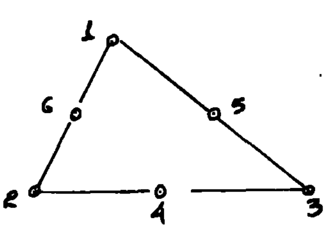
The third model PMX24 (DMX36) was very successful in a large number of cases but not so successful in others, as explained in the previous section. Its application to the dome problem will provide comparative results between the mixed and displacement approach for the dome problem.

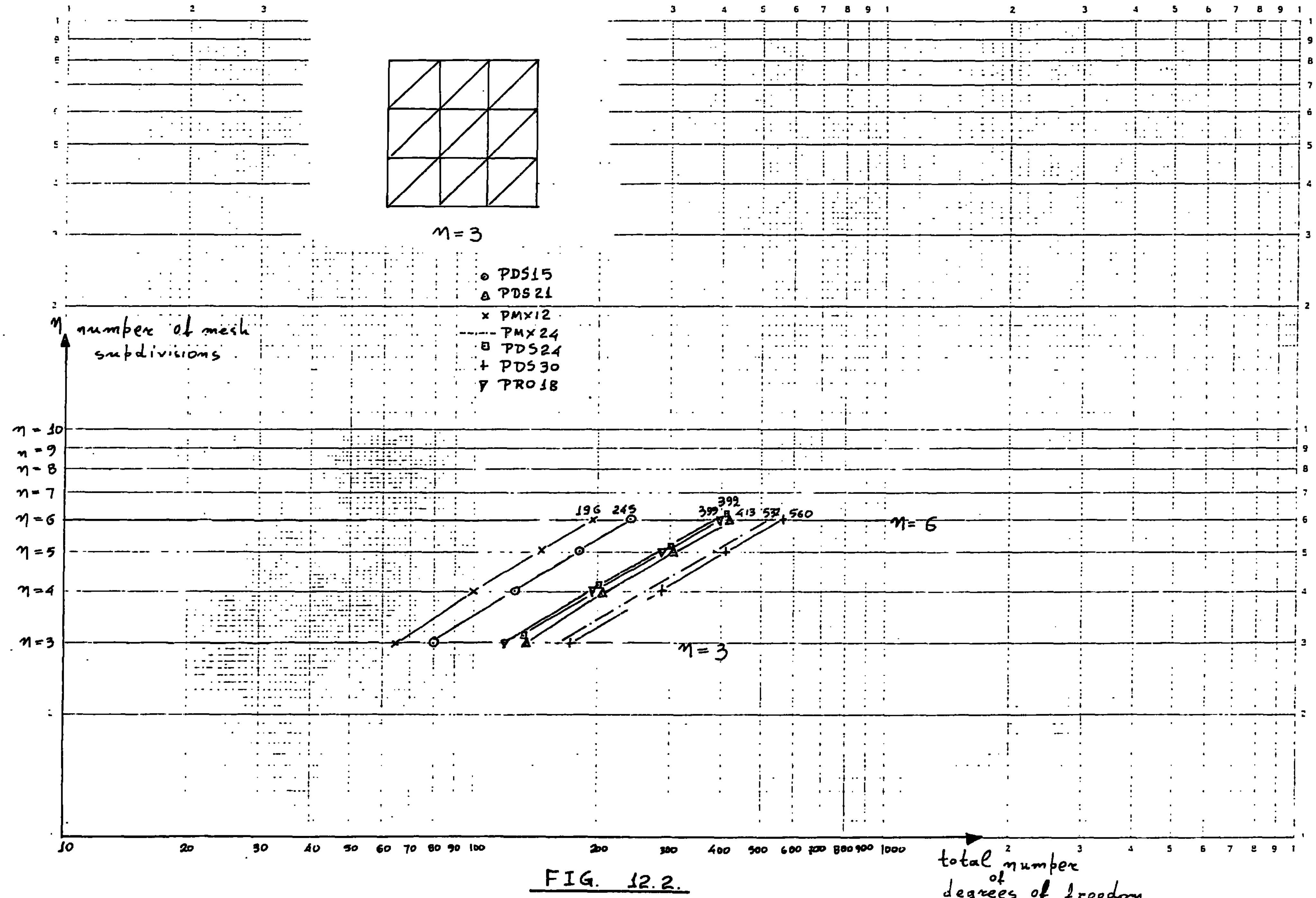
Special advantages of the mixed dome model include the higher variation with regard to plane stress (second order) and the presence of moments as degrees of freedom, the latter being very significant for the analysis of dome structures (see chapters 10 and 13).

The fourth model PRO18 (DRO30) was successful throughout the various sandwich plate bending problems. It was included in order to obtain comparative results from the total rotation model category which it represents.

FIG. 12.1.

SANDWICH PLATE MODELS

REFERENCE SYMBOL / DEGREES OF FREEDOM	SANDWICH – PLATE MODELS	STRESSES
PDS15 $W, W_x, W_y, \phi_x, \phi_y$ ISPLACEMENT MODEL		M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c} at the centroid
PDS21 $w, w_x, w_y, \phi_x, \phi_y$ ϕ_x, ϕ_y ISPLACEMENT MODEL		M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c} at the centroid
PMX12 $w, M_{xx}, M_{yy}, M_{xy}$ IXED MODEL		Q_{x_c}, Q_{y_c} at the centroid
PMX24 $w, M_{xx}, M_{yy}, M_{xy}$ IXED MODEL		Q_{x_1}, Q_{y_1} at 1st node Q_{x_2}, Q_{y_2} at 2nd node
PDS24 $W, W_x, W_y, W_{xx}, W_{xy}, W_{yy}$ ϕ_x, ϕ_y ISPLACEMENT MODEL		M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c} at the centroid
PDS30 $W, W_x, W_y, W_{xx}, W_{xy}, W_{yy}$ ϕ_x, ϕ_y ϕ_x, ϕ_y ISPLACEMENT MODEL		M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c} at the centroid
PRO18 w, θ_x, θ_y DISPLACEMENT MODEL		M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c} at the centroid



n = 3
o PDS15
Δ PDS21
× PMX12
PMX24
□ PDS24
+ PDS30
∇ PRO18
n number of mesh subdivisions
n = 10
n = 9
n = 8
n = 7
n = 6
n = 5
n = 4
n = 3
196
245
399
392
413
537
560
n = 6
n = 3
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
total number of degrees of freedom

FIG. 12.2.

CONVERGENCE TOWARDS SERIES SOLUTION

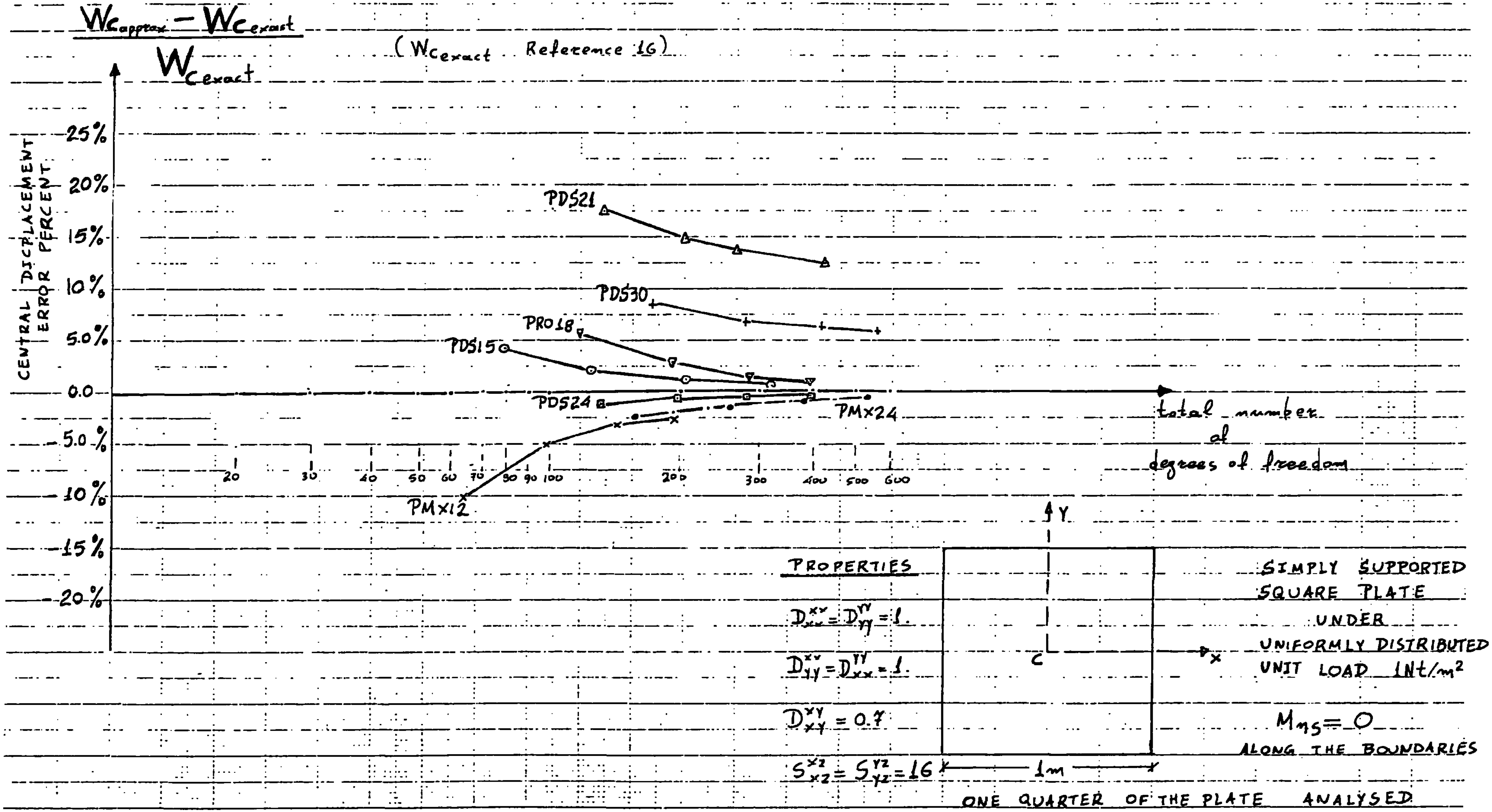
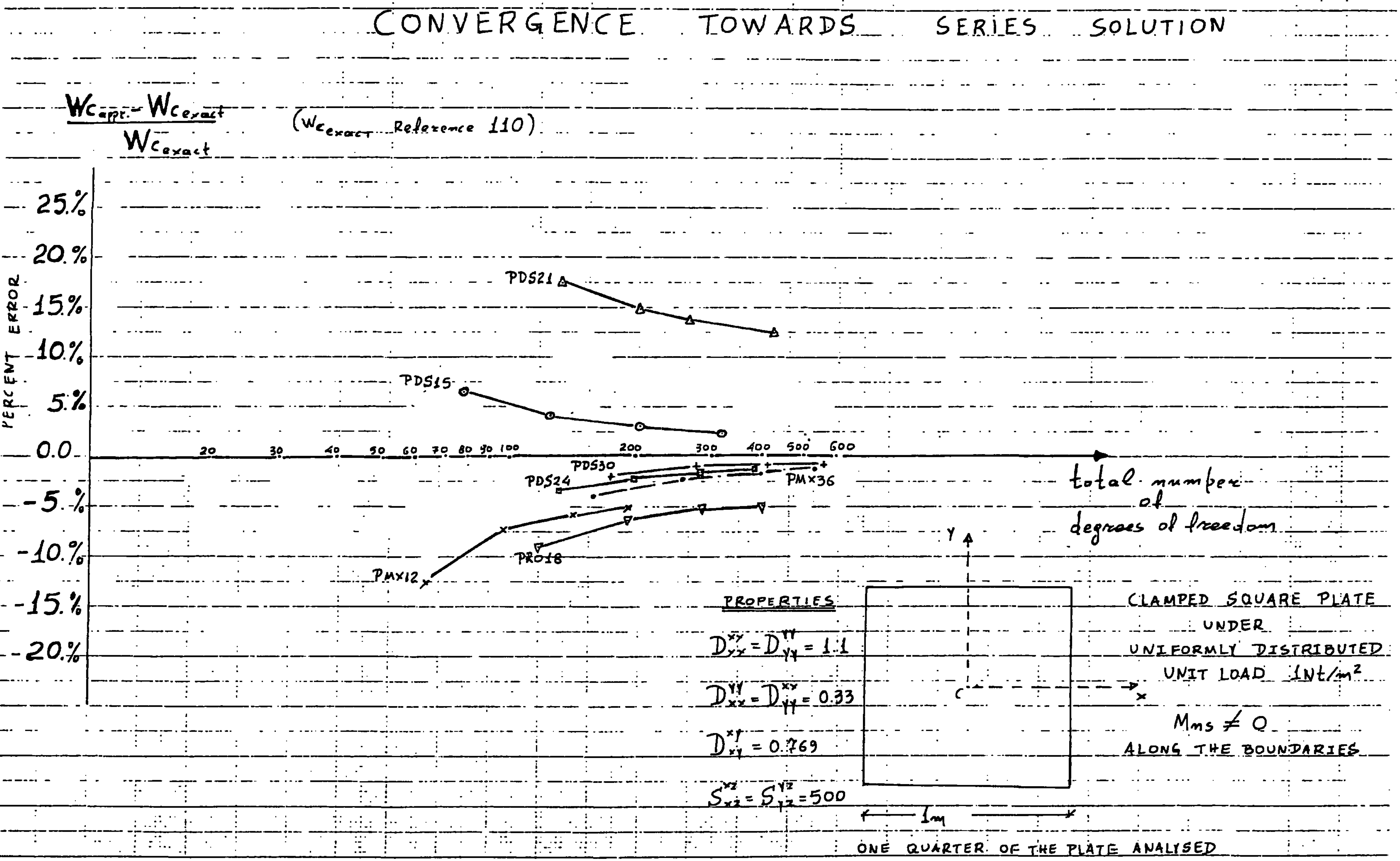
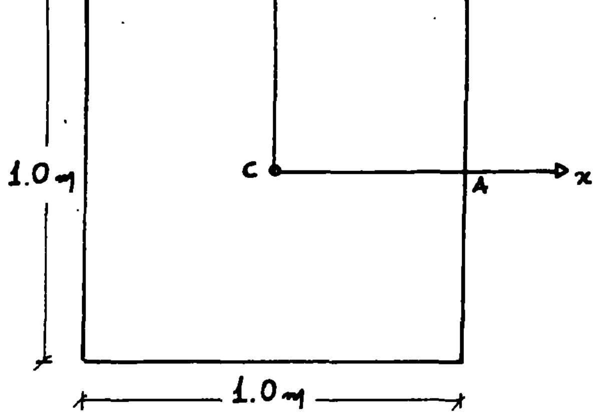


FIG. 12.3.



CONVERGENCE TOWARDS SERIES SOLUTION
Wc appr. - Wc exact / Wc exact
(Wc exact Reference 110)
PERCENT ERROR
25%
20%
15%
10%
5%
0.0
-5%
-10%
-15%
-20%
20
30
40
50
60
70
80
90
100
200
300
400
500
600
PDS21
PDS15
PDS30
PDS24
PMX36
PMX12
PRO18
total number of degrees of freedom
PROPERTIES
$D^{xy}_{yx} = D^{yy}_{yy} = 1.1$
$D^{yy}_{xx} = D^{xy}_{yy} = 0.33$
$D^{xy}_{xy} = 0.769$
$S^{xz}_{xz} = S^{yz}_{yz} = 500$
Y
X
C
1m
ONE QUARTER OF THE PLATE ANALISED
CLAMPED SQUARE PLATE
UNDER
UNIFORMLY DISTRIBUTED
UNIT LOAD 1Nt/m²
$M_{ns} \neq 0$
ALONG THE BOUNDARIES

FIG. 12.4.



SQUARE PLATE

UNDER

UNIFORMLY DISTRIBUTED LOAD

$1\,Nt/m^2$

KEY	
PDS15	∘
PDS21	▵
PMX12	×
PMX24	—·—·—
PDS24	□
PDS30	+
PRO18	▿

CASE	BOUNDARY CONDITIONS	D^{xx}_{xx} (Nt-m)	$D^{yy}_{xx}=D^{xx}_{yy}$ (Nt-m)	D^{yy}_{yy} (Nt-m)	D^{xy}_{xy} (Nt-m)	S^{xz}_{xz} (Nt/m)	S^{yz}_{yz} (Nt/m)	COMPARISON with —+—
1.	SIMPLY SUPPORTED $M_{ns}=0.$ along the supports	1.0	0.3	1.0	0.7	16.	16.	FINITE DIFFERENCES [16] (BASU and DAWSON)
2.	SIMPLY SUPPORTED $M_{ns}\neq 0.$ along the supports	1.0	0.3	1.0	0.7	16.	16.	FINITE DIFFERENCES (BASU and DAWSON) [16]
3.	"	1.1	0.33	1.1	0.769	50.	50.	FINITE DIFFERENCES [110] (CHAPMAN and WILLIAMS)
4.	"	1.08	0.27	0.9	0.631	50.	60.	"
5.	"	1.07	0.229	0.764	0.535	50.	70.	"
6.	"	1.06	0.198	0.663	0.464	50.	80.	"
7.	"	1.055	0.176	0.586	0.409	50.	90.	"
8.	"	1.045	0.157	0.522	0.367	50.	100.	"
9.	CLAMPED $M_{ns}\neq 0.$ along the supports	1.1	0.33	1.1	0.769	50.	50.	"
10.	"	1.1	0.33	1.1	0.769	250.	250.	"
11.	"	1.1	0.33	1.1	0.769	500.	500.	"

FIG. 12.5.

CASE 1.

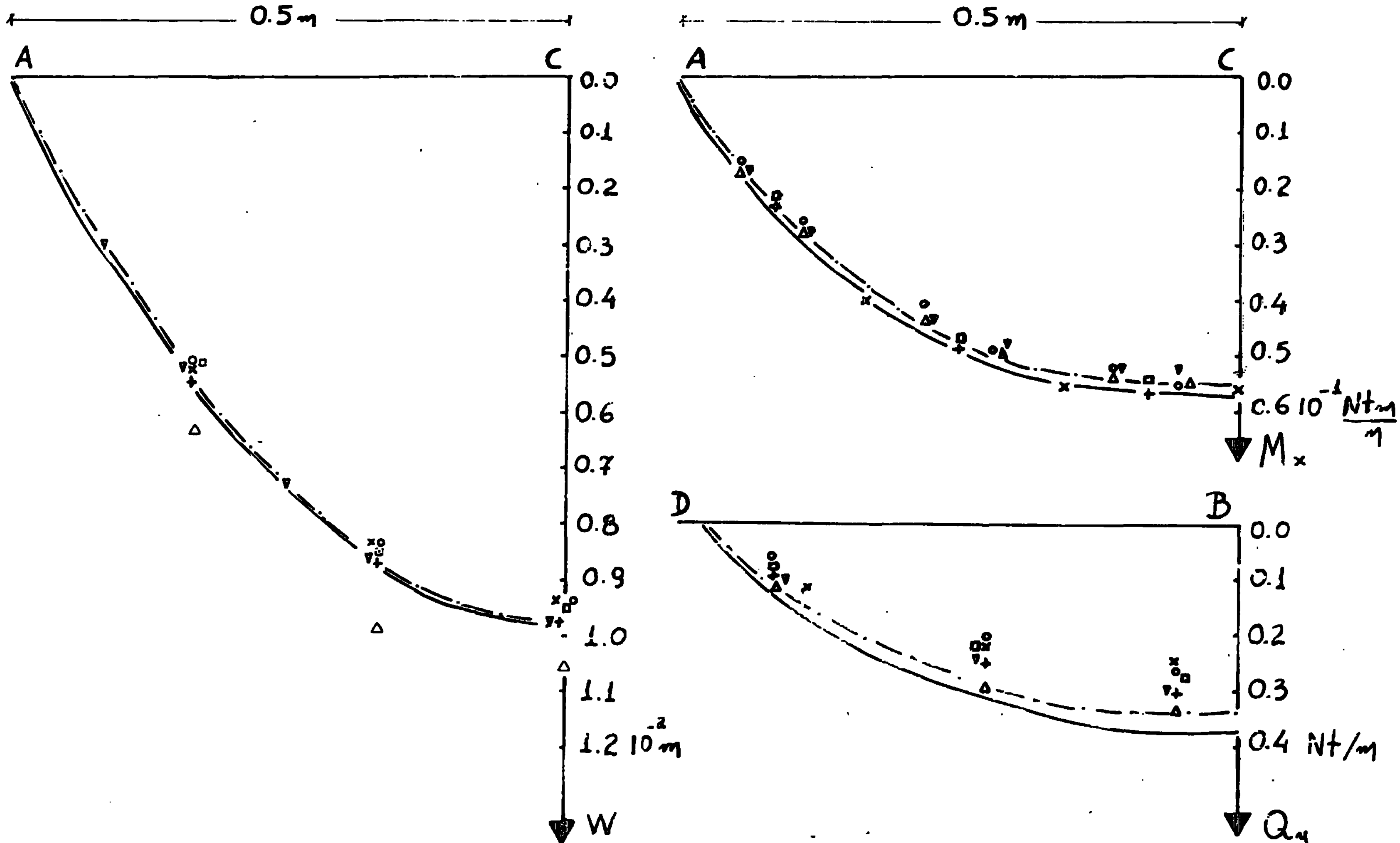


FIG. 12.6.

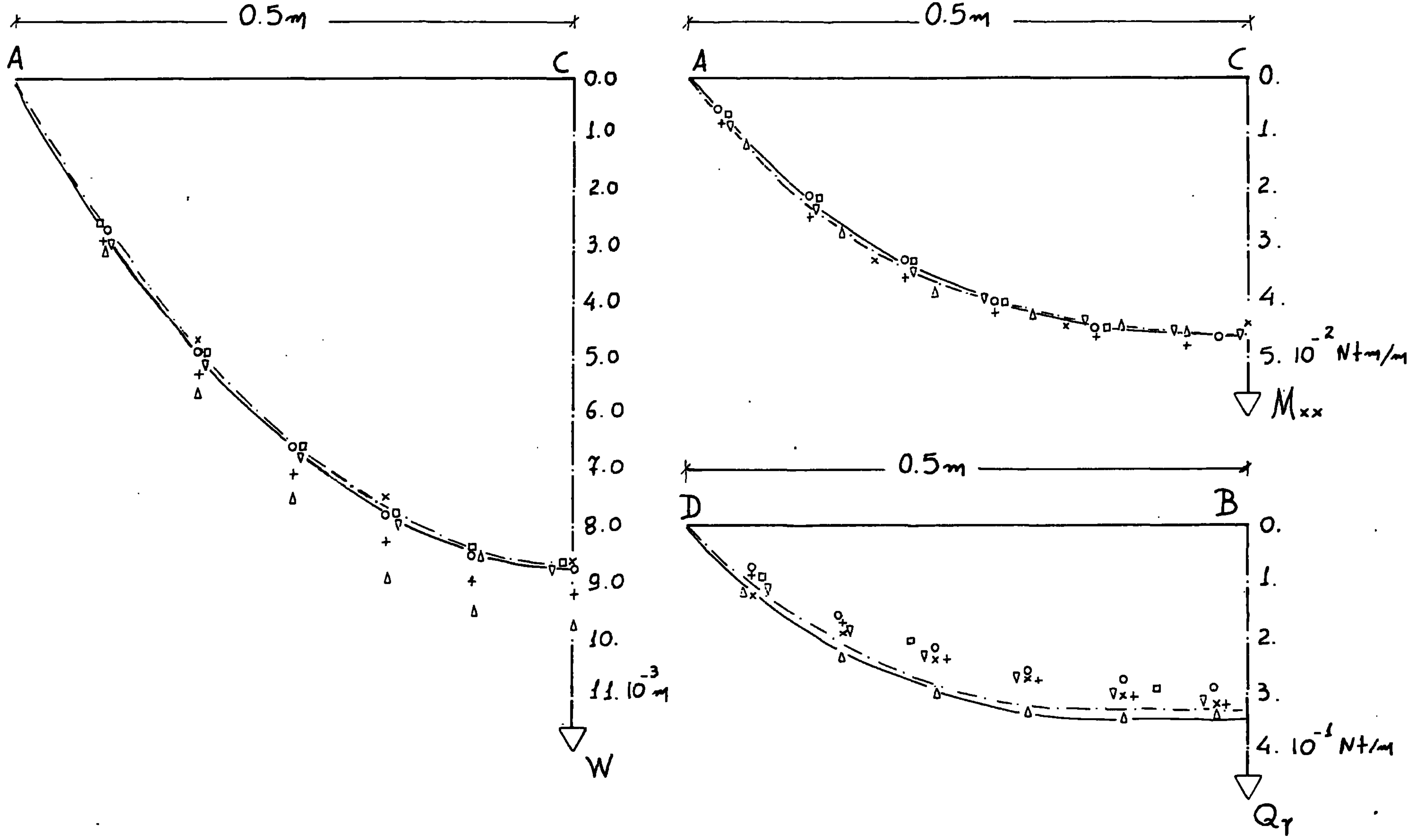

0.5m
A
C
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.
11. 10^-3 m
W
0.5m
A
C
0.
1.
2.
3.
4.
5. 10^-2 Nt-m/m
Mxx
0.5m
D
B
0.
1.
2.
3.
4. 10^-1 Nt/m
Qr

FIG. 12.7.

CASE 3.

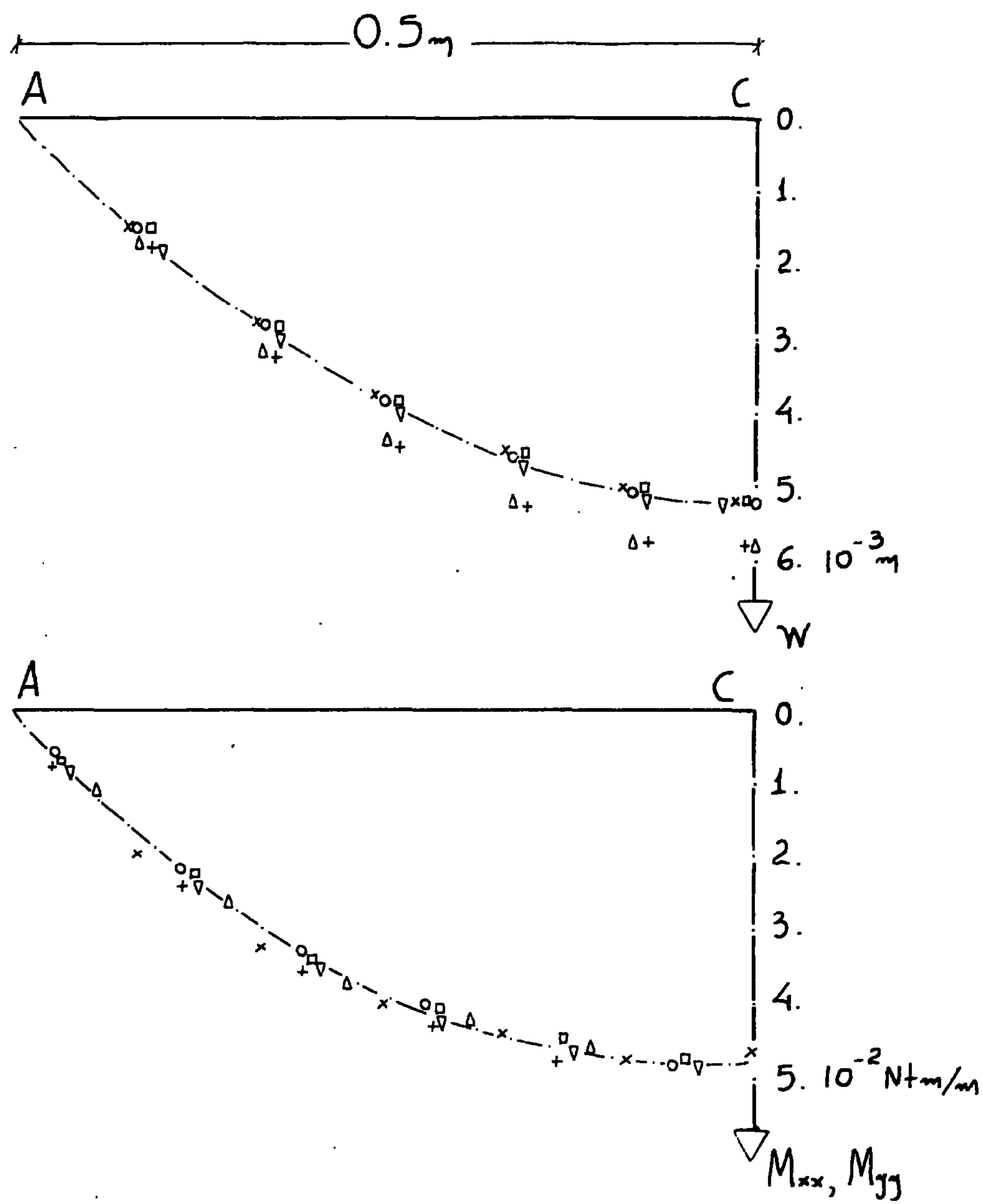


FIG. 12.8.

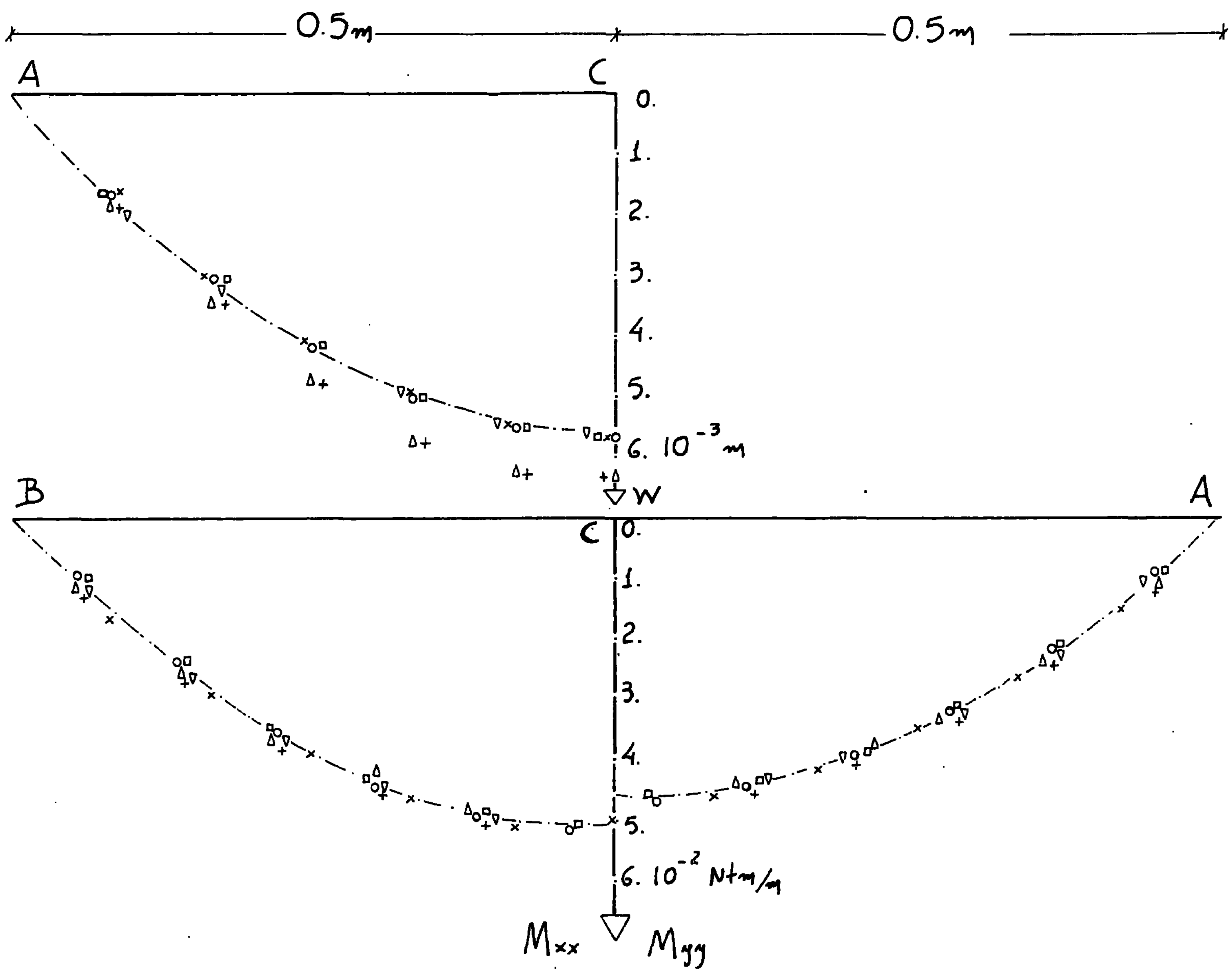
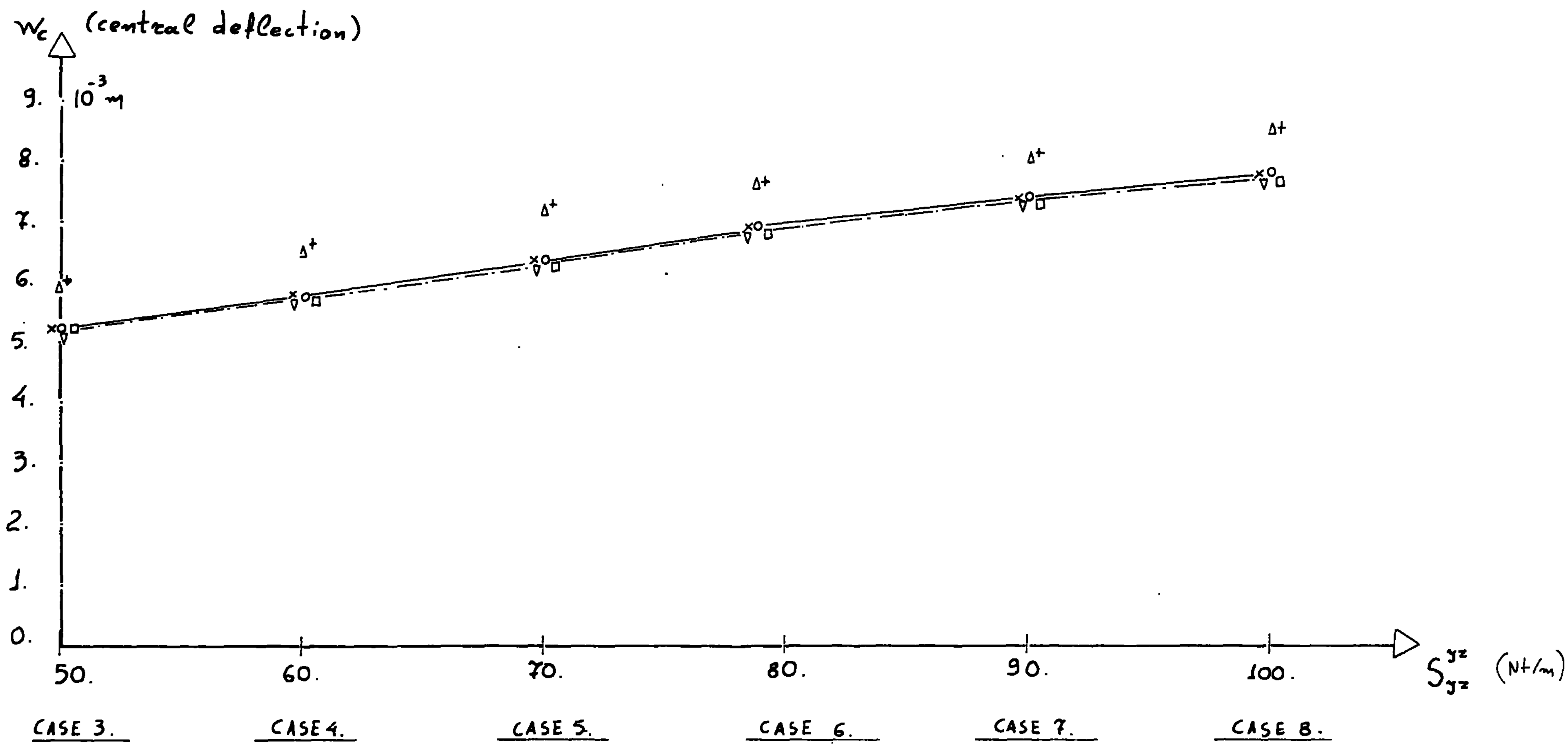
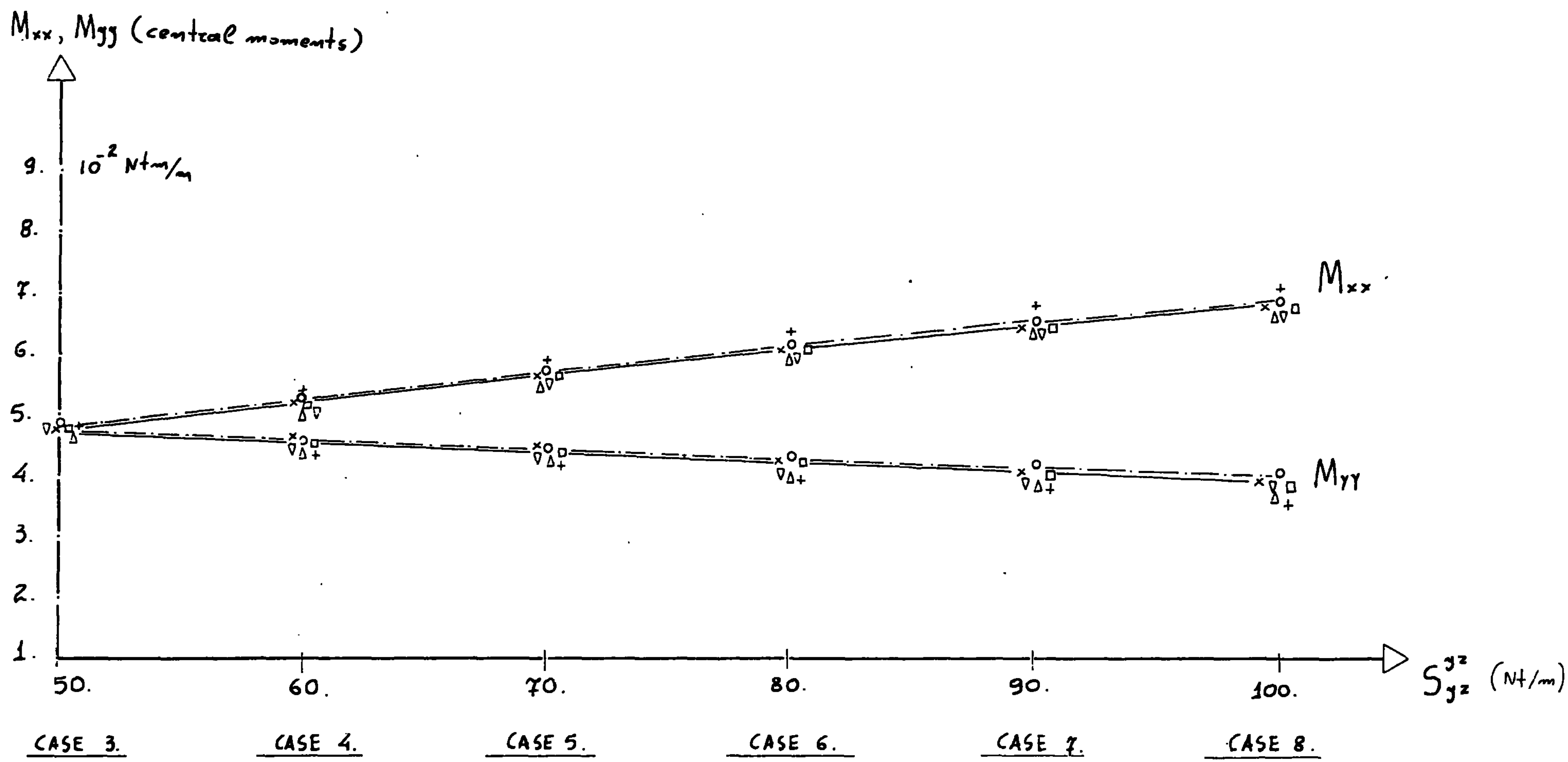


FIG. 12.9.



w_c (central deflection)
10^{-3} m
9.
8.
7.
6.
5.
4.
3.
2.
1.
0.
50.
60.
70.
80.
90.
100.
S_{yz}^{yz} (Nt/m)
CASE 3.
CASE 4.
CASE 5.
CASE 6.
CASE 7.
CASE 8.

FIG. 12.10.



Mxx, Myy (central moments)
10⁻² Ntm/m
9.
8.
7.
6.
5.
4.
3.
2.
1.
50.
60.
70.
80.
90.
100.
Mxx
MYY
Syz (Nt/m)
CASE 3.
CASE 4.
CASE 5.
CASE 6.
CASE 7.
CASE 8.

FIG. 12.11.

CASE 9

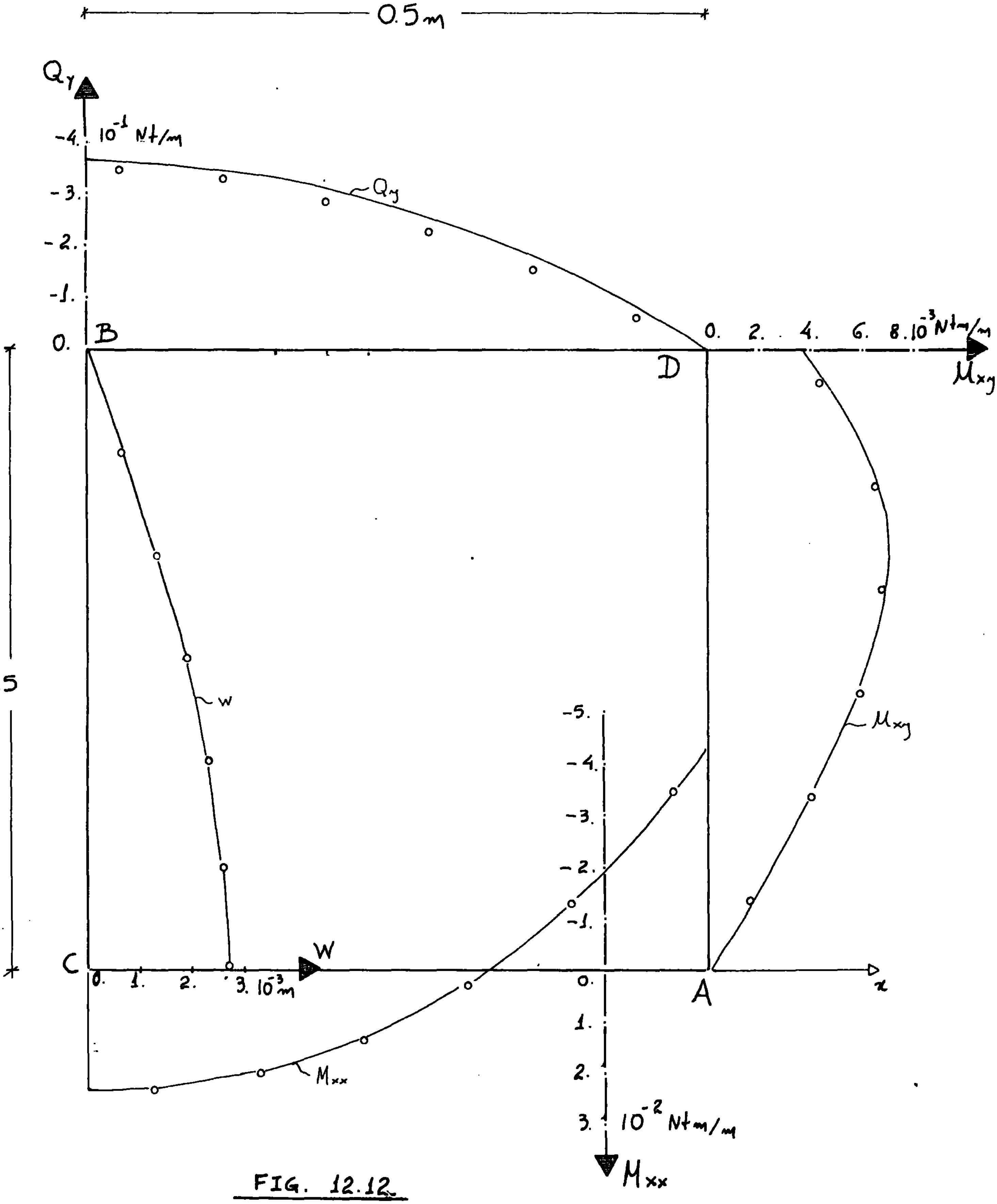


FIG. 12.12

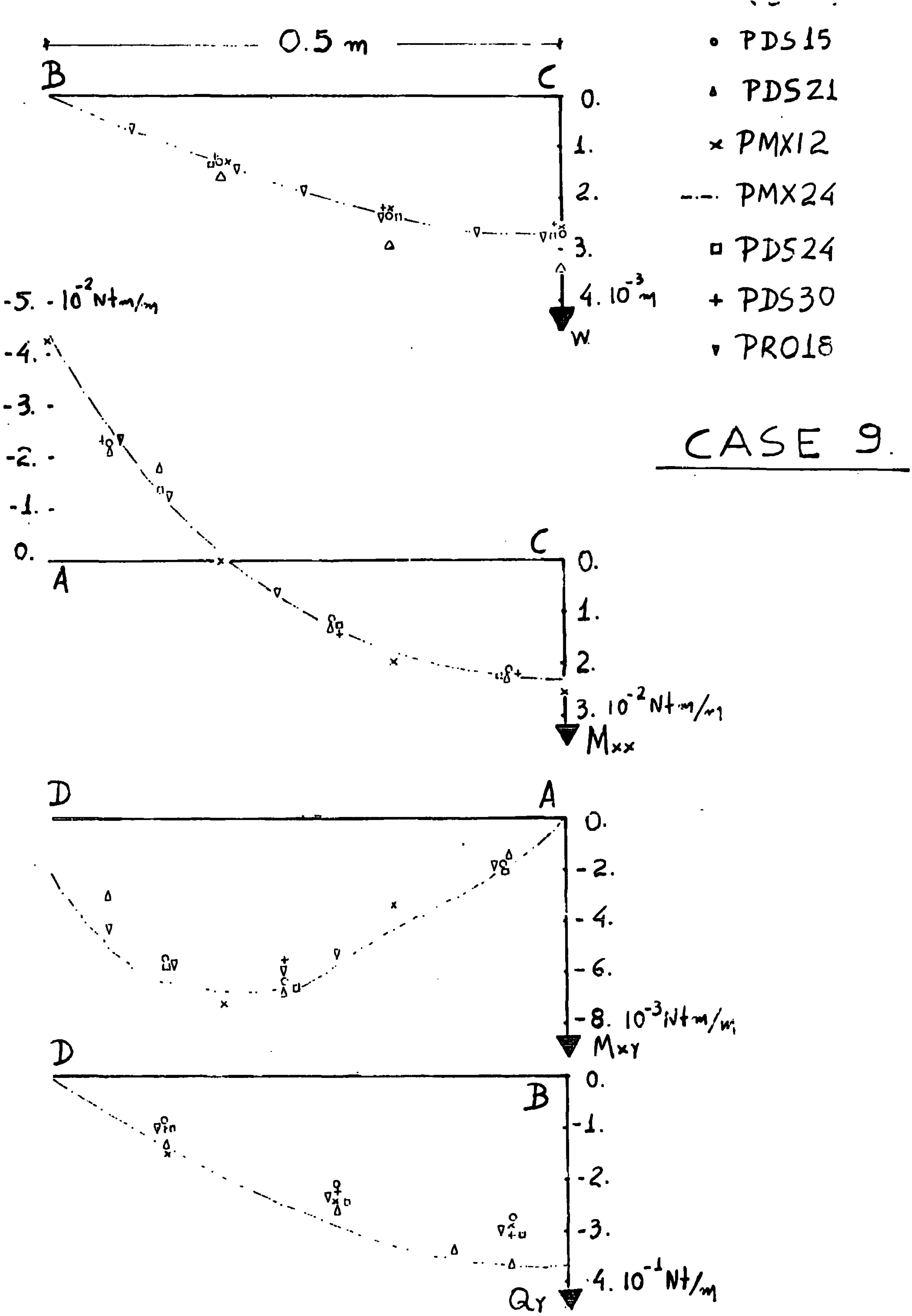

0.5 m
B
C
0.
1.
2.
3.
4. 10⁻³ m
W
PDS15
PDS21
PMX12
PMX24
PDS24
PDS30
PRO18
CASE 9.
-5. 10⁻² Nt·m/m
-4.
-3.
-2.
-1.
0.
A
C
0.
1.
2.
3. 10⁻² Nt·m/m
Mxx
D
A
0.
-2.
-4.
-6.
-8. 10⁻³ Nt·m/m
Mxy
D
B
0.
-1.
-2.
-3.
-4. 10⁻¹ Nt/m
Qy

FIG. 12.13.

CASE 10.

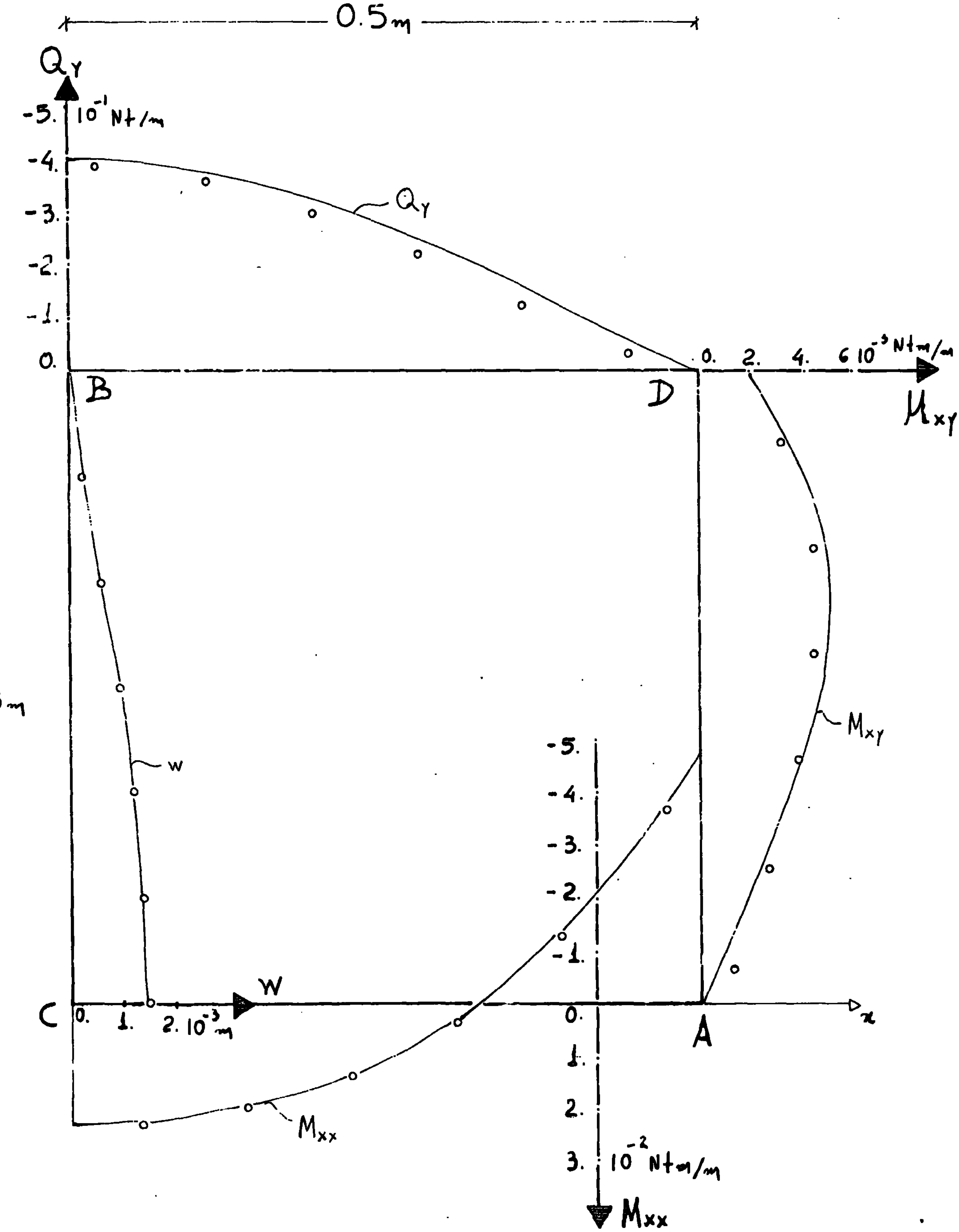


FIG. 12.14.

CASE 10.

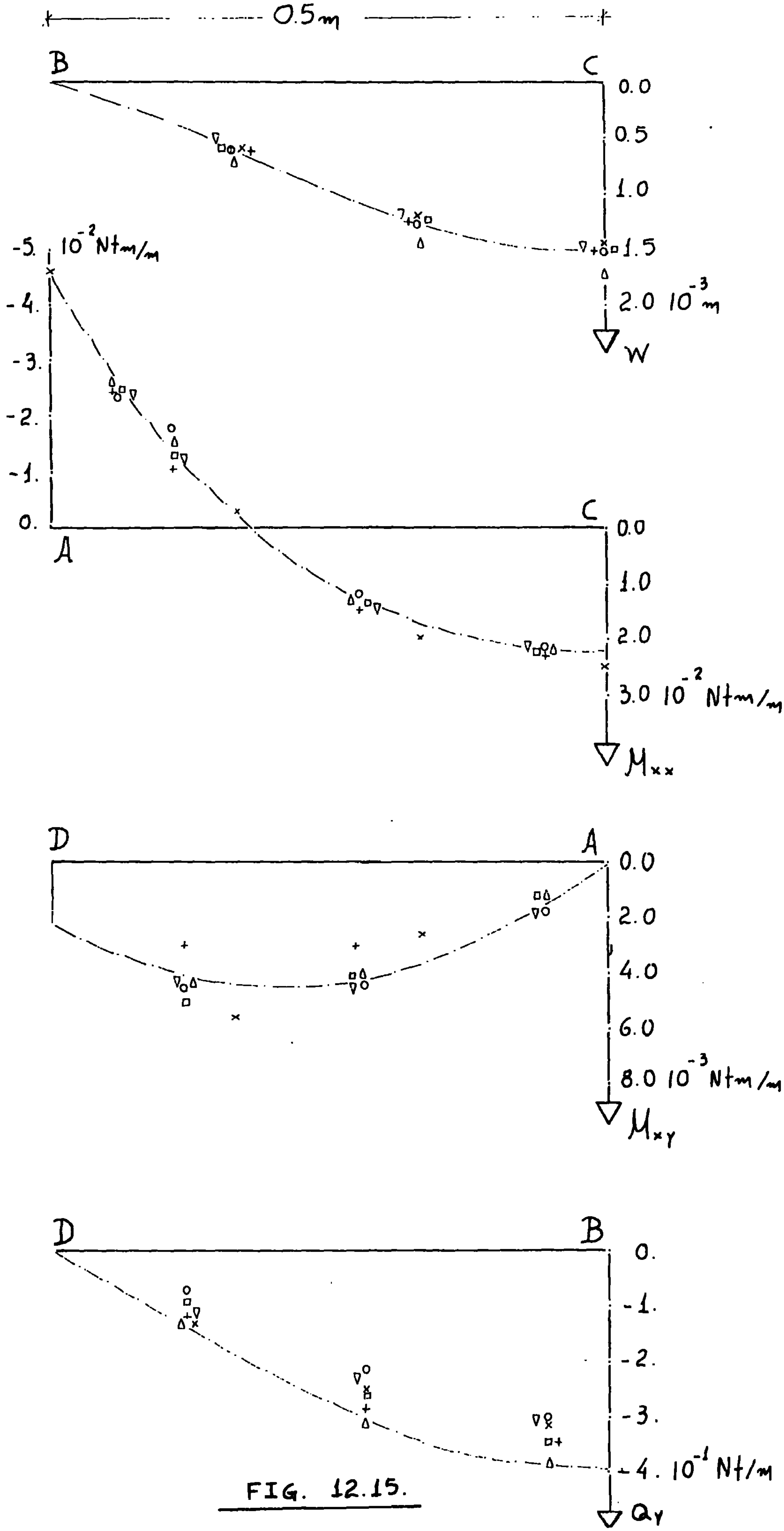


FIG. 12.15.

CASE 11

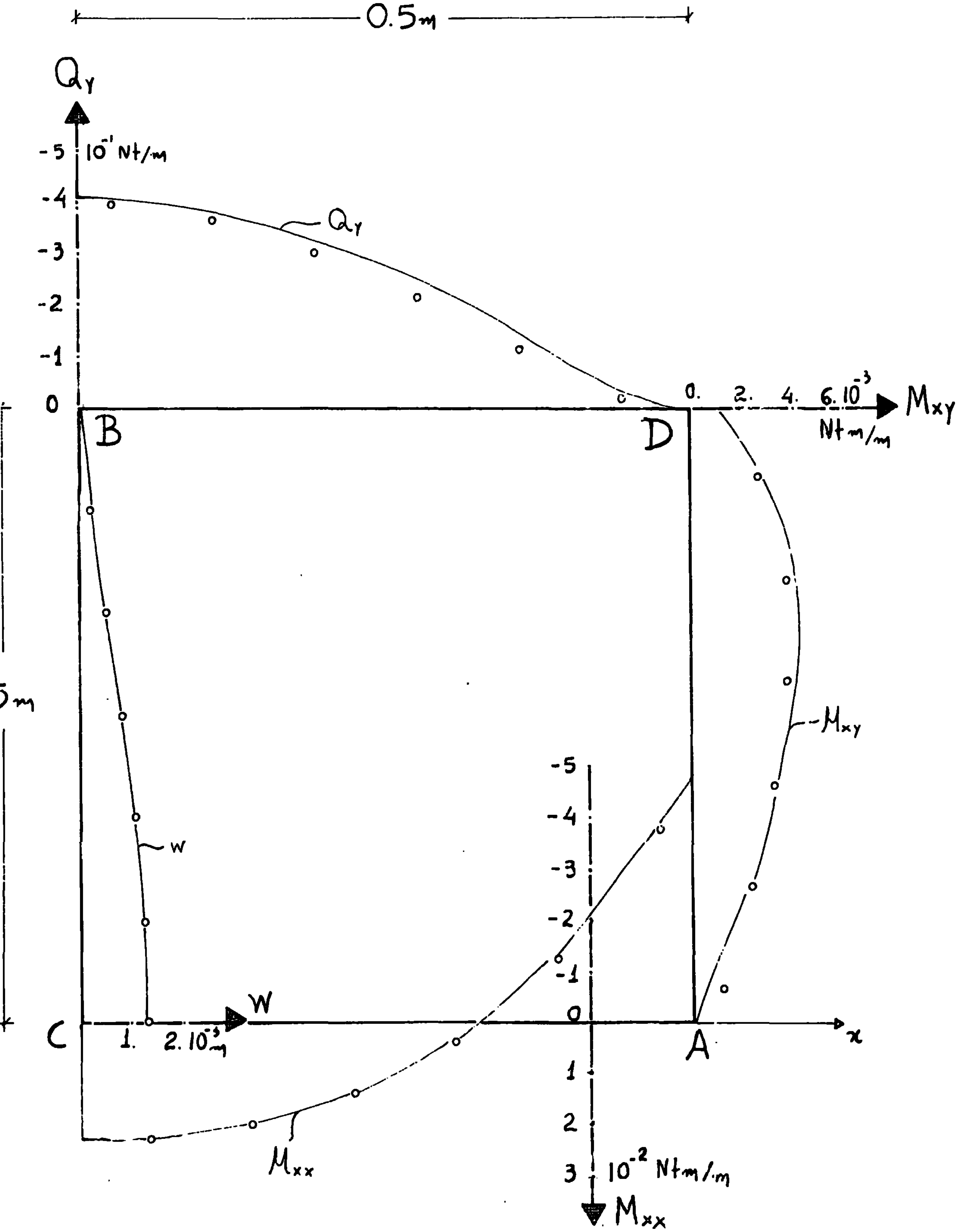


FIG. 12.16.

CASE 11.

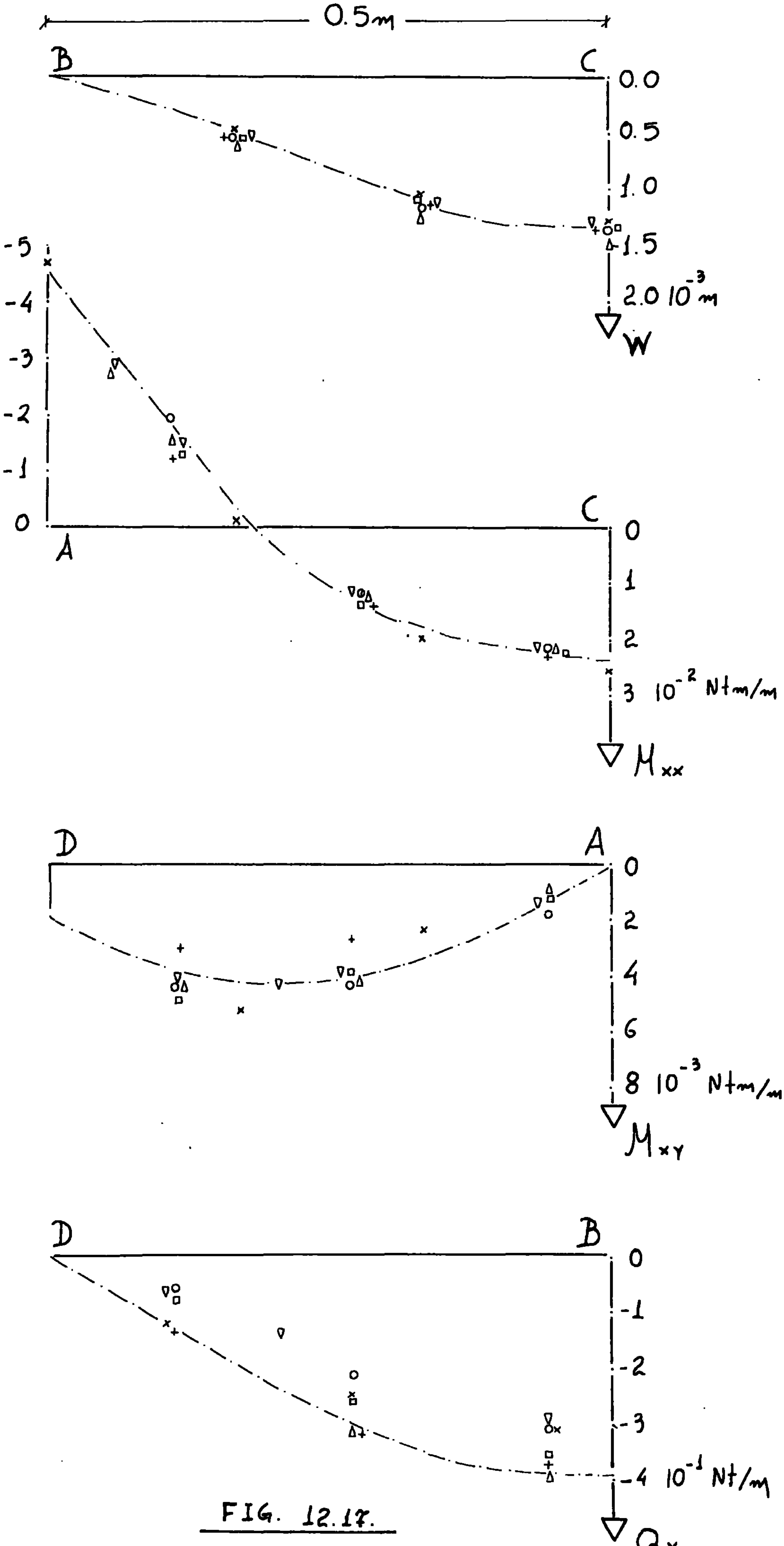


FIG. 12.17.

MESH 3 × 3	CASE 9.					CASE 10.					CASE 11.				
	W_c	$M_{xx(max)}$	$M_{xx(min)}$	$M_{xy(max)}$	$Q_{y(max)}$	W_c	$M_{xx(max)}$	$M_{xx(min)}$	$M_{xy(max)}$	$Q_{y(max)}$	W_c	$M_{xx(max)}$	$M_{xx(min)}$	$M_{xy(max)}$	$Q_{y(max)}$
PDS15	$2.71\ 10^{-3}$	$2.15\ 10^{-2}$	$-2.29\ 10^{-2}$	$6.32\ 10^{-3}$	$-2.74\ 10^{-1}$	$1.53\ 10^{-3}$	$2.17\ 10^{-2}$	$-2.28\ 10^{-2}$	$4.60\ 10^{-3}$	$-3.00\ 10^{-1}$	$1.38\ 10^{-3}$	$2.17\ 10^{-2}$	$-2.28\ 10^{-2}$	$4.28\ 10^{-3}$	$-3.15\ 10^{-1}$
PDS21	$3.41\ 10^{-3}$	$2.23\ 10^{-2}$	$-2.14\ 10^{-2}$	$6.74\ 10^{-3}$	$-3.65\ 10^{-1}$	$1.77\ 10^{-3}$	$2.19\ 10^{-2}$	$-2.56\ 10^{-2}$	$4.42\ 10^{-3}$	$-3.82\ 10^{-1}$	$1.52\ 10^{-3}$	$2.18\ 10^{-2}$	$-2.61\ 10^{-2}$	$4.38\ 10^{-3}$	$-3.93\ 10^{-1}$
PMX12	$2.63\ 10^{-3}$	$2.56\ 10^{-2}$	$-4.24\ 10^{-2}$	$7.24\ 10^{-3}$	$-2.83\ 10^{-1}$	$1.44\ 10^{-3}$	$2.53\ 10^{-2}$	$-4.60\ 10^{-2}$	$5.94\ 10^{-3}$	$-3.01\ 10^{-1}$	$1.28\ 10^{-3}$	$2.53\ 10^{-2}$	$-4.70\ 10^{-2}$	$5.37\ 10^{-3}$	$-3.08\ 10^{-1}$
PMX24	$2.72\ 10^{-3}$	$2.35\ 10^{-2}$	$-4.32\ 10^{-2}$	$6.67\ 10^{-3}$	$-3.68\ 10^{-1}$	$1.49\ 10^{-3}$	$2.26\ 10^{-2}$	$-4.54\ 10^{-2}$	$4.42\ 10^{-3}$	$-3.94\ 10^{-1}$	$1.33\ 10^{-3}$	$2.24\ 10^{-2}$	$-4.56\ 10^{-2}$	$4.13\ 10^{-3}$	$-3.90\ 10^{-1}$
PDS24	$2.74\ 10^{-3}$	$2.25\ 10^{-2}$	$-2.12\ 10^{-2}$	$6.57\ 10^{-3}$	$-3.08\ 10^{-1}$	$1.49\ 10^{-3}$	$2.23\ 10^{-2}$	$-2.50\ 10^{-2}$	$5.15\ 10^{-3}$	$-3.43\ 10^{-1}$	$1.33\ 10^{-3}$	$2.22\ 10^{-2}$	$-2.58\ 10^{-2}$	$4.85\ 10^{-3}$	$-3.53\ 10^{-1}$
PDS30	$2.55\ 10^{-3}$	$2.30\ 10^{-2}$	$-2.34\ 10^{-2}$	$5.49\ 10^{-3}$	$-3.08\ 10^{-1}$	$1.49\ 10^{-3}$	$2.28\ 10^{-2}$	$-2.41\ 10^{-2}$	$2.89\ 10^{-3}$	$-3.43\ 10^{-1}$	$1.34\ 10^{-3}$	$2.27\ 10^{-2}$	$-2.46\ 10^{-2}$	$2.80\ 10^{-3}$	$-3.72\ 10^{-1}$
PRO18	$2.66\ 10^{-3}$	$2.22\ 10^{-2}$	$-2.65\ 10^{-2}$	$5.91\ 10^{-3}$	$-3.00\ 10^{-1}$	$1.46\ 10^{-3}$	$2.16\ 10^{-2}$	$-2.72\ 10^{-2}$	$4.54\ 10^{-3}$	$-2.99\ 10^{-1}$	$1.28\ 10^{-3}$	$2.13\ 10^{-2}$	$-2.71\ 10^{-2}$	$4.14\ 10^{-3}$	$-2.74\ 10^{-1}$
CHAPMAN AND WILLIAMS	$2.70\ 10^{-3}$	$2.30\ 10^{-2}$	$-4.35\ 10^{-2}$	$7.0\ 10^{-3}$	$-3.70\ 10^{-1}$	$1.50\ 10^{-3}$	$2.25\ 10^{-2}$	$-4.70\ 10^{-2}$	$5.00\ 10^{-3}$	$-4.00\ 10^{-1}$	$1.35\ 10^{-3}$	$2.25\ 10^{-2}$	$-5.00\ 10^{-2}$	$4.30\ 10^{-3}$	$-4.10\ 10^{-1}$
(UNITS)	(m)	(Nt-m/m)	(Nt-m/m)	(Nt-m/m)	(Nt/m)	(m)	(Nt-m/m)	(Nt-m/m)	(Nt-m/m)	(Nt/m)	(m)	(Nt-m/m)	(Nt-m/m)	(Nt-m/m)	(Nt/m)

FIG. 12.18.

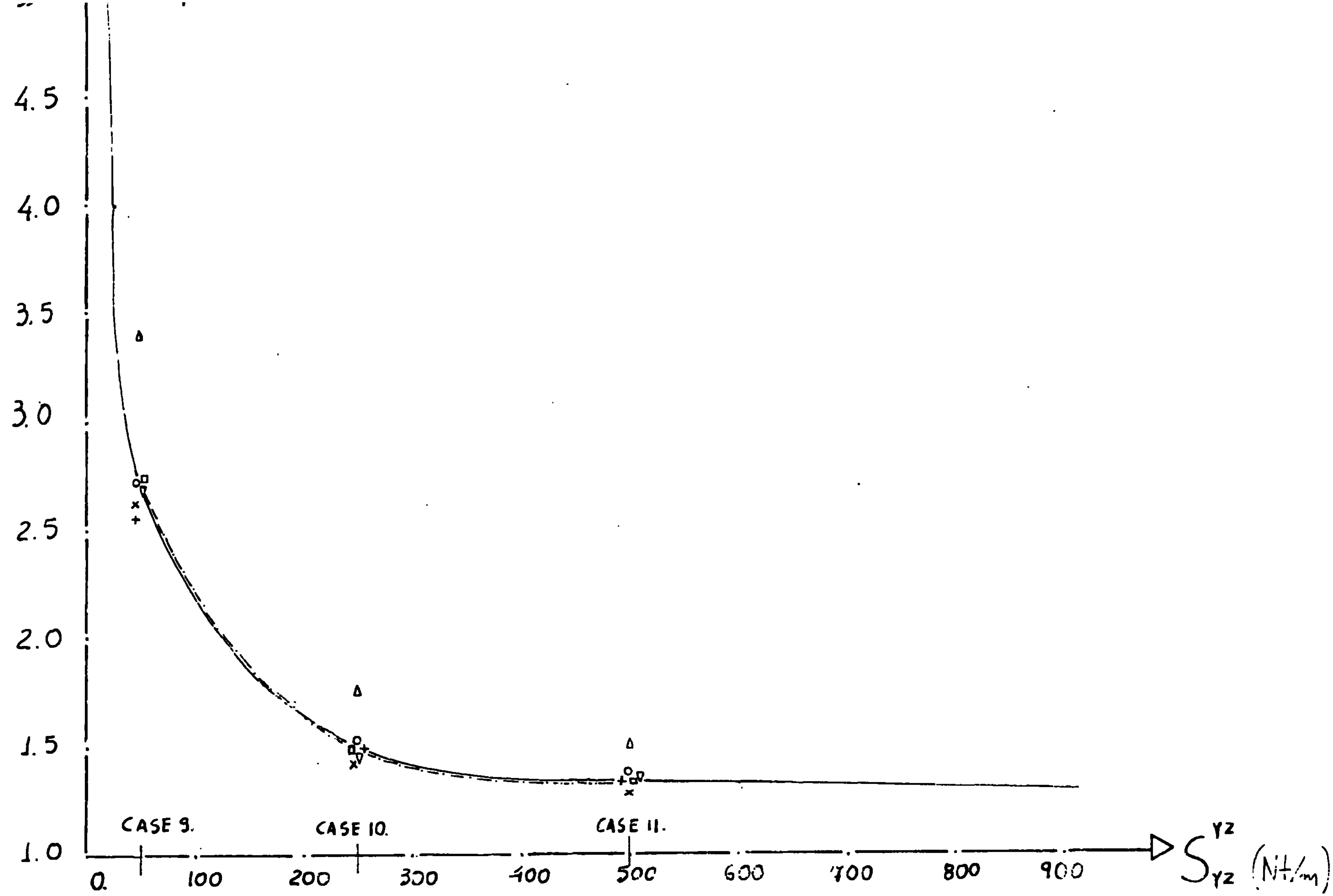
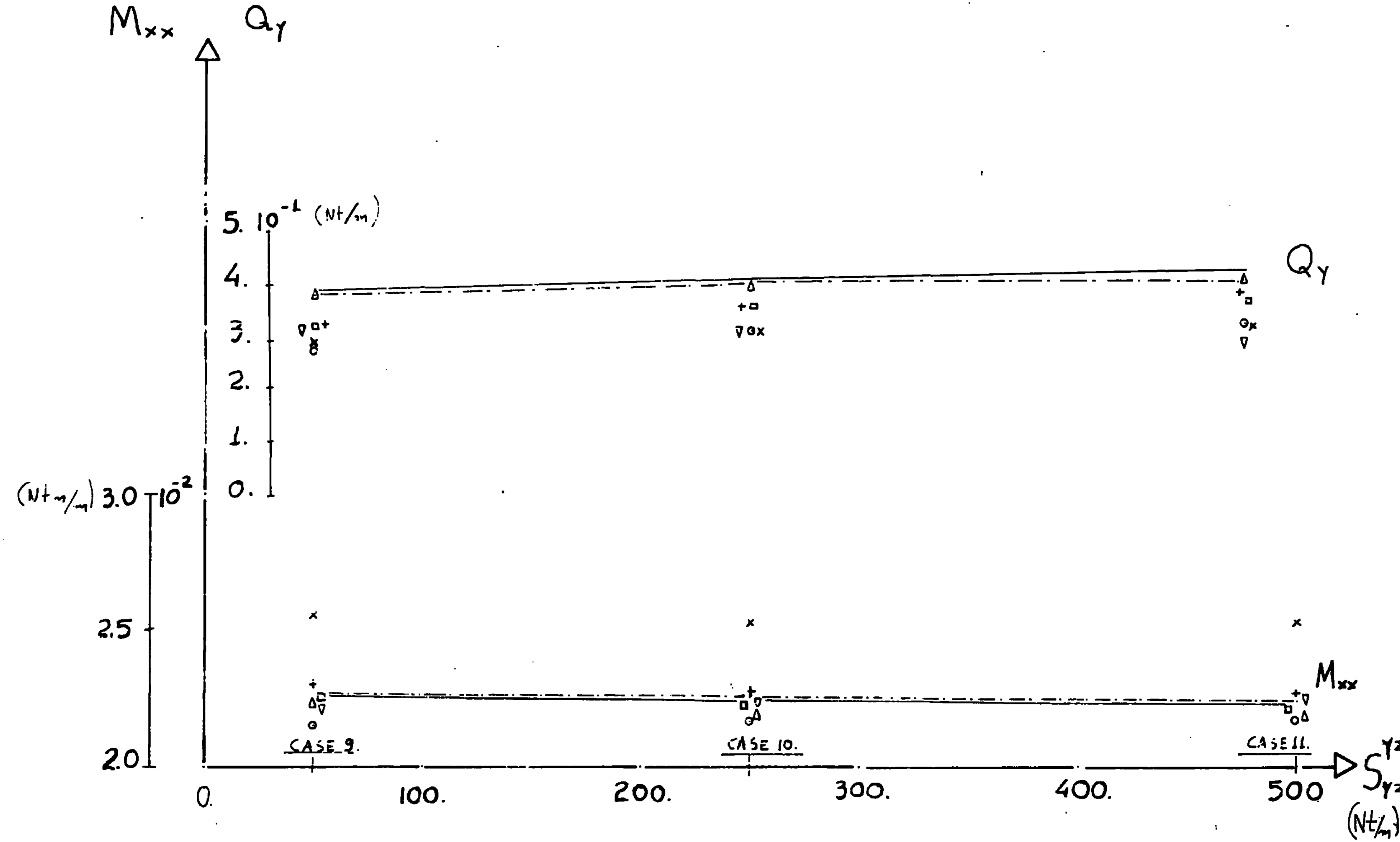

4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
CASE 9.
CASE 10.
CASE 11.
0.
100
200
300
400
500
600
700
800
900
S_{YZ}^{YZ} (Nt/m)

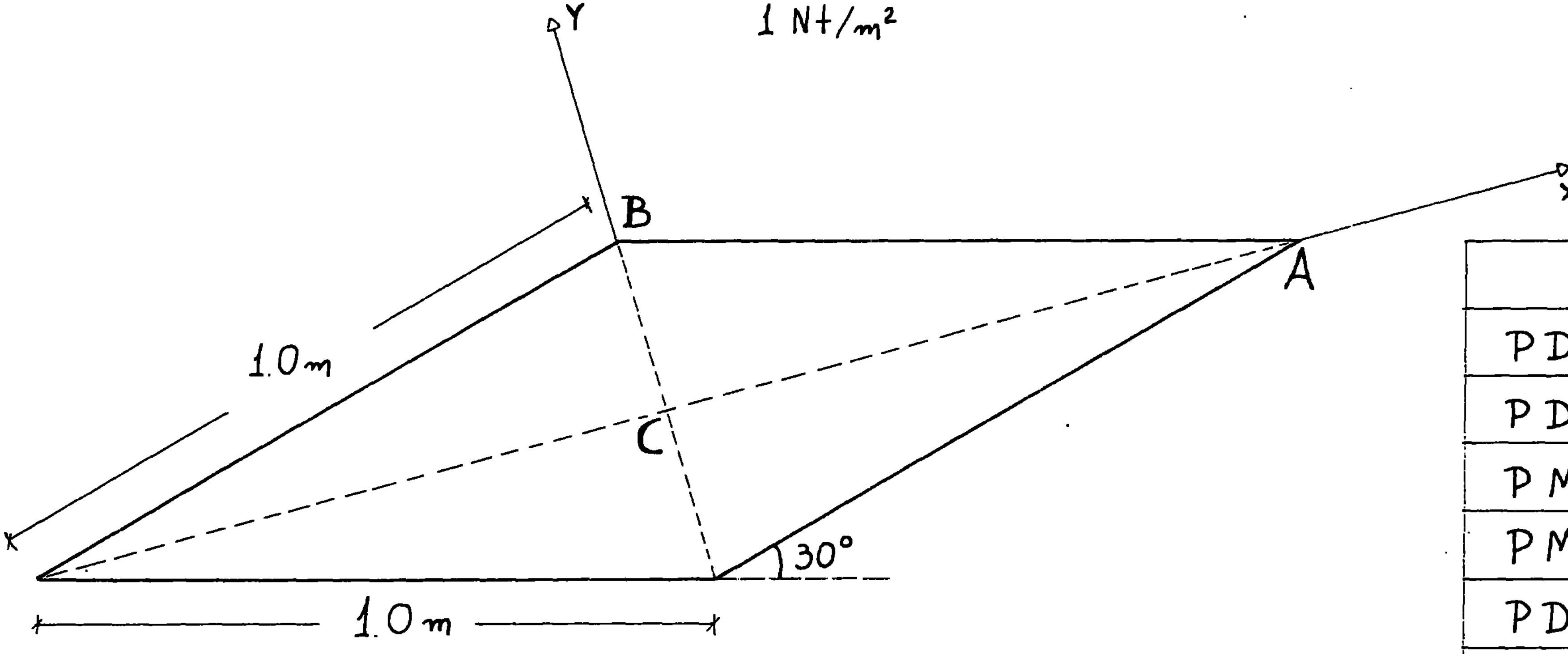
FIG. 12.19.



Mxx
Qy
5. 10^-1 (Nt/m)
4.
3.
2.
1.
0.
(Nt-m/m) 3.0 10^-2
2.5
2.0
Qy
Mxx
CASE 9.
CASE 10.
CASE 11.
0.
100.
200.
300.
400.
500
(Nt/m)

FIG. 12.20.

SKEW SIMPLY SUPPORTED PLATE
UNDER
UNIFORMLY DISTRIBUTED LOAD
1 Nt/m^2



KEY	
PDS15	○
PDS21	△
PMX12	×
PMX24	—·—·—
PDS24	□
PDS30	+
PRO18	▽

CASE	BOUNDARY CONDITIONS	D^{xx}_{xx} (Nt-m)	$D^{YY}_{xx}=D^{xx}_{YY}$ (Nt-m)	D^{YY}_{YY} (Nt-m)	D^{xY}_{xY} (Nt-m)	S^{xz}_{xz} (Nt/m)	S^{Yz}_{Yz} (Nt/m)	COMPARISON with	
1.	SIMPLY SUPPORTED $M_{ns}\neq 0.$ along the boundaries	1.0	0.3	1.0	0.7	218.4	218.4	BETTESS —+— [21]	SANDERS ——— [97]
2.	SIMPLY SUPPORTED $M_{ns}=0.$ along the boundaries	1.0	0.3	1.0	0.7	218.4	218.4	BETTESS —+— [21]	SANDERS ——— [97]

FIG. 12.21.

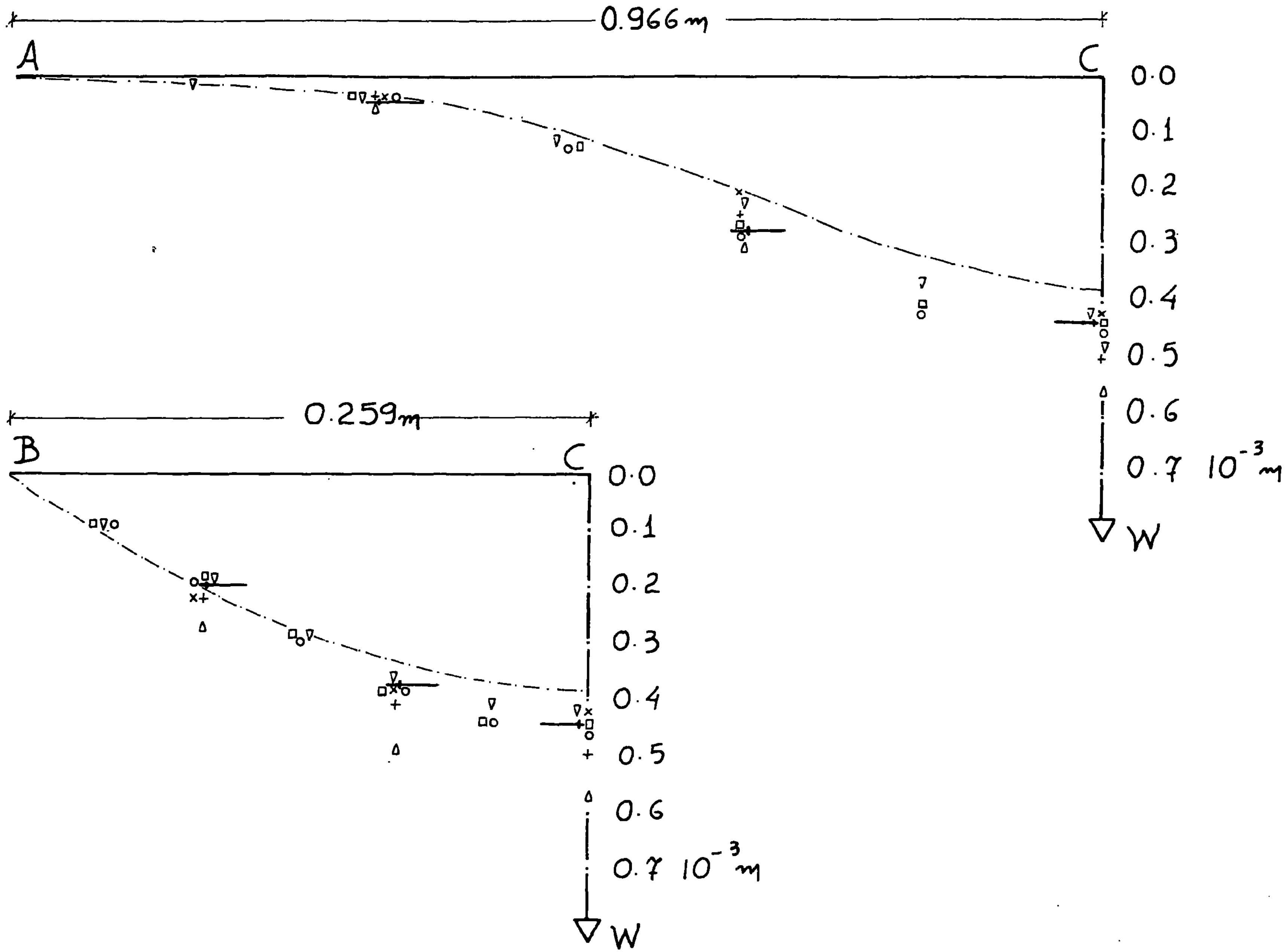

0.966m
A
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7 10^-3 m
W
0.259m
B
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7 10^-3 m
W

FIG. 12.22.

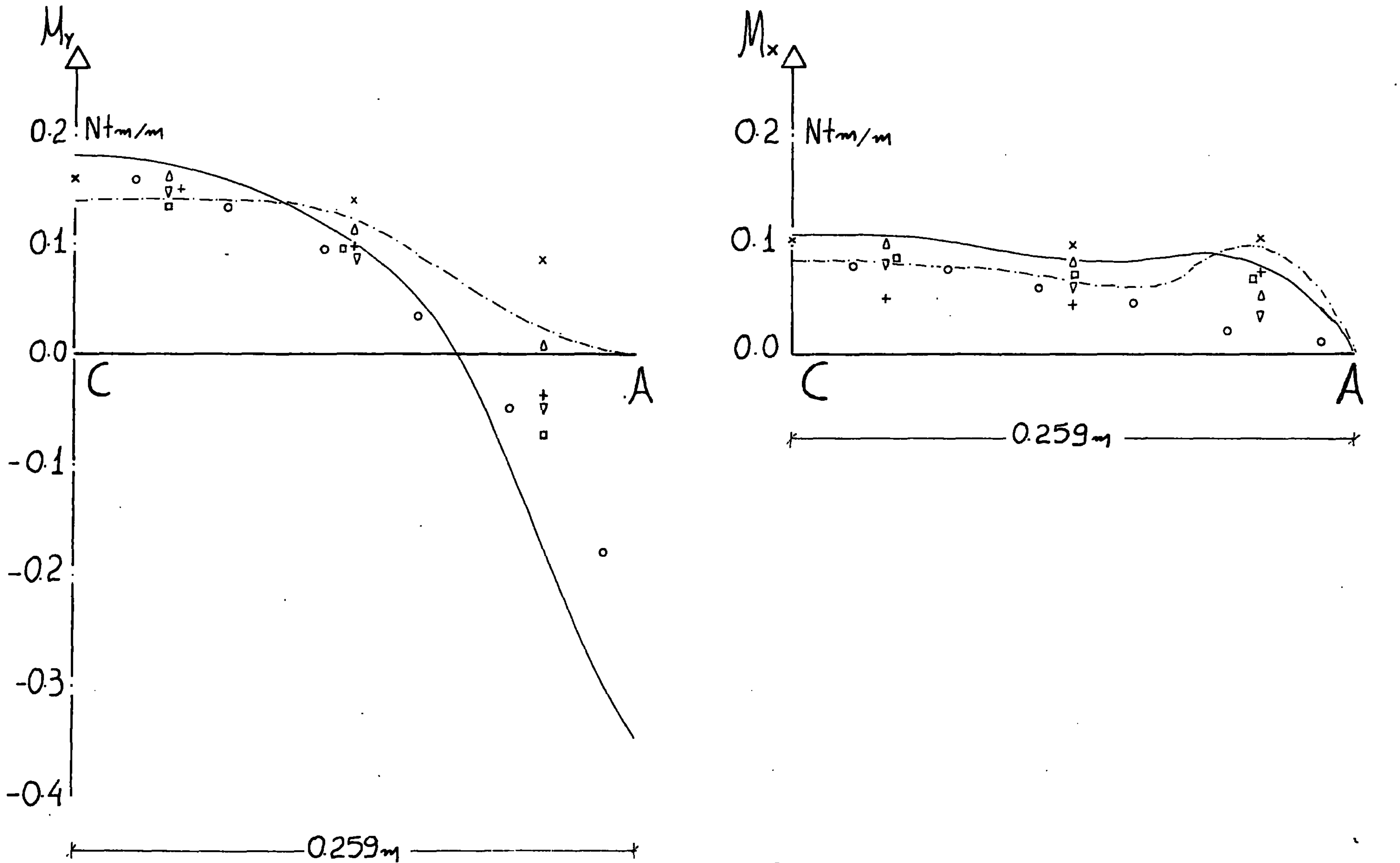


FIG. 12.23.

CASE I.

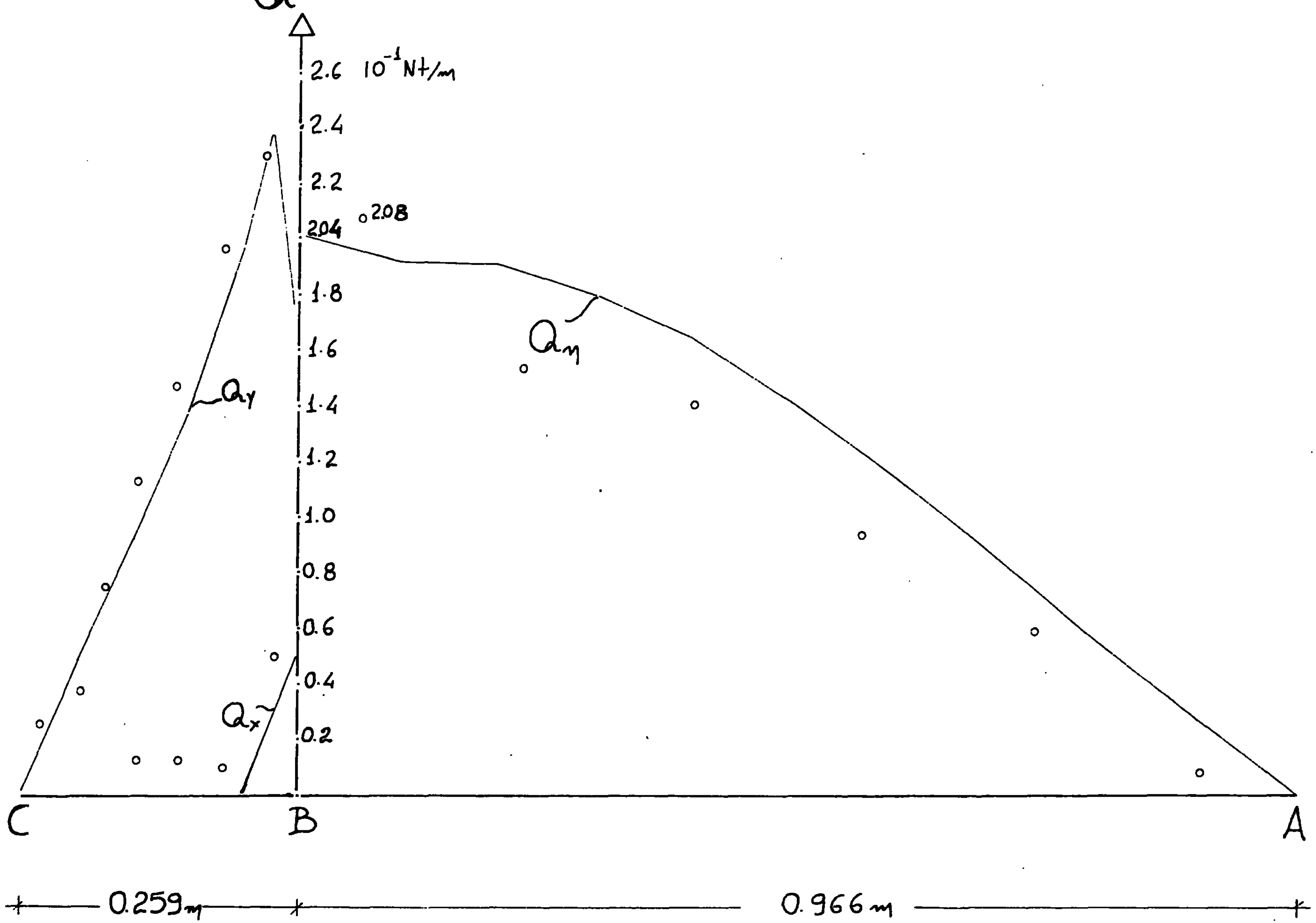


FIG. 12.24.

CASE 2.

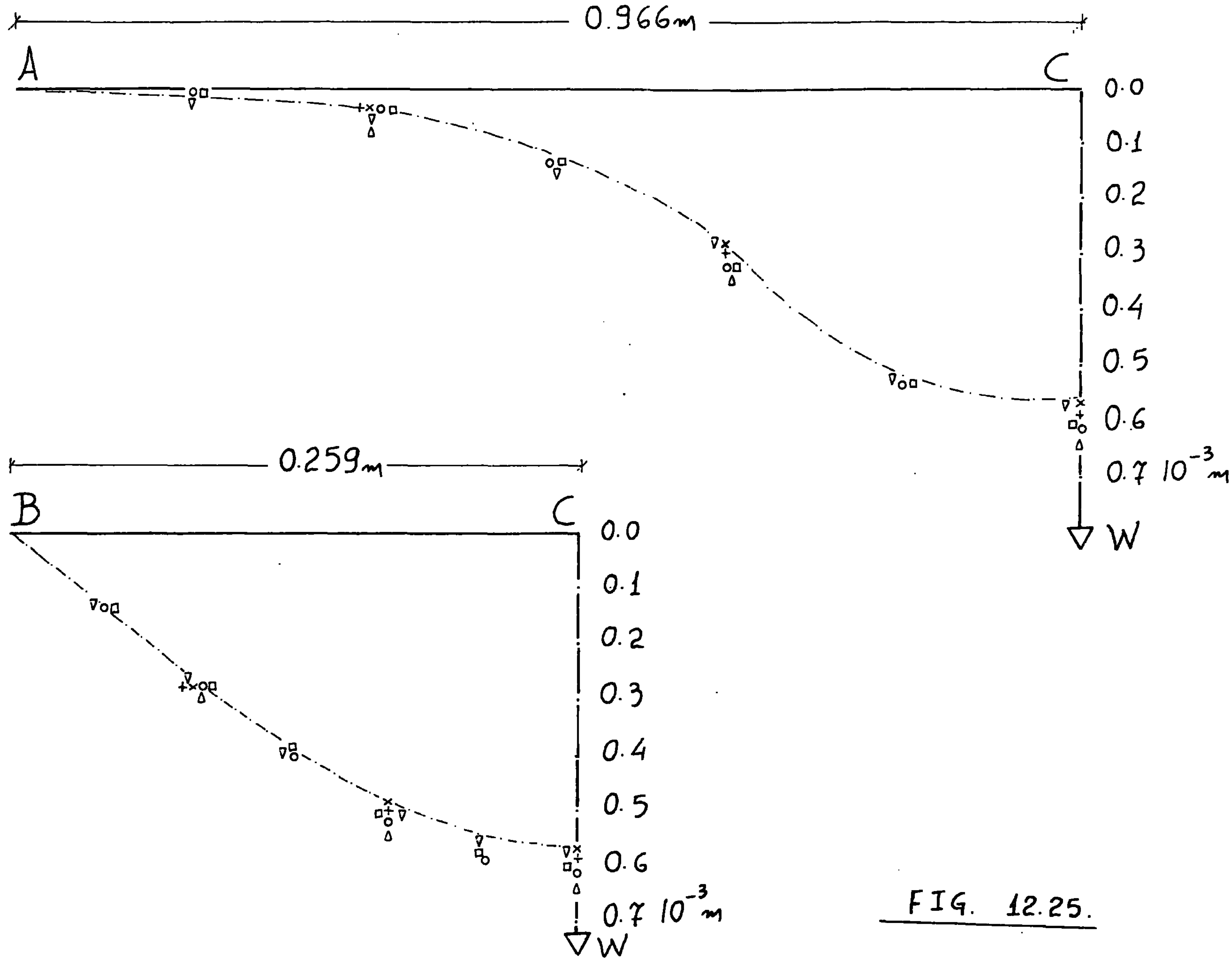


FIG. 12.25.

CASE 2.

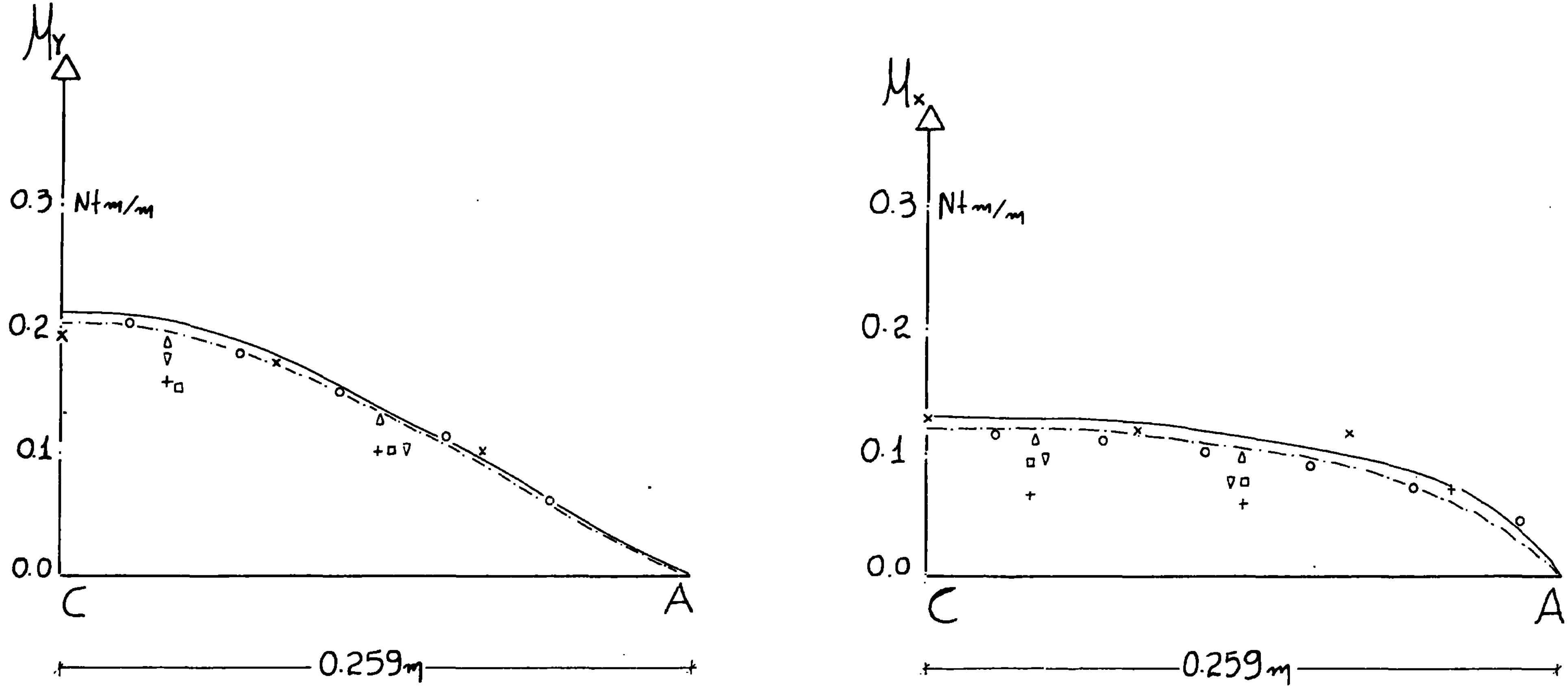
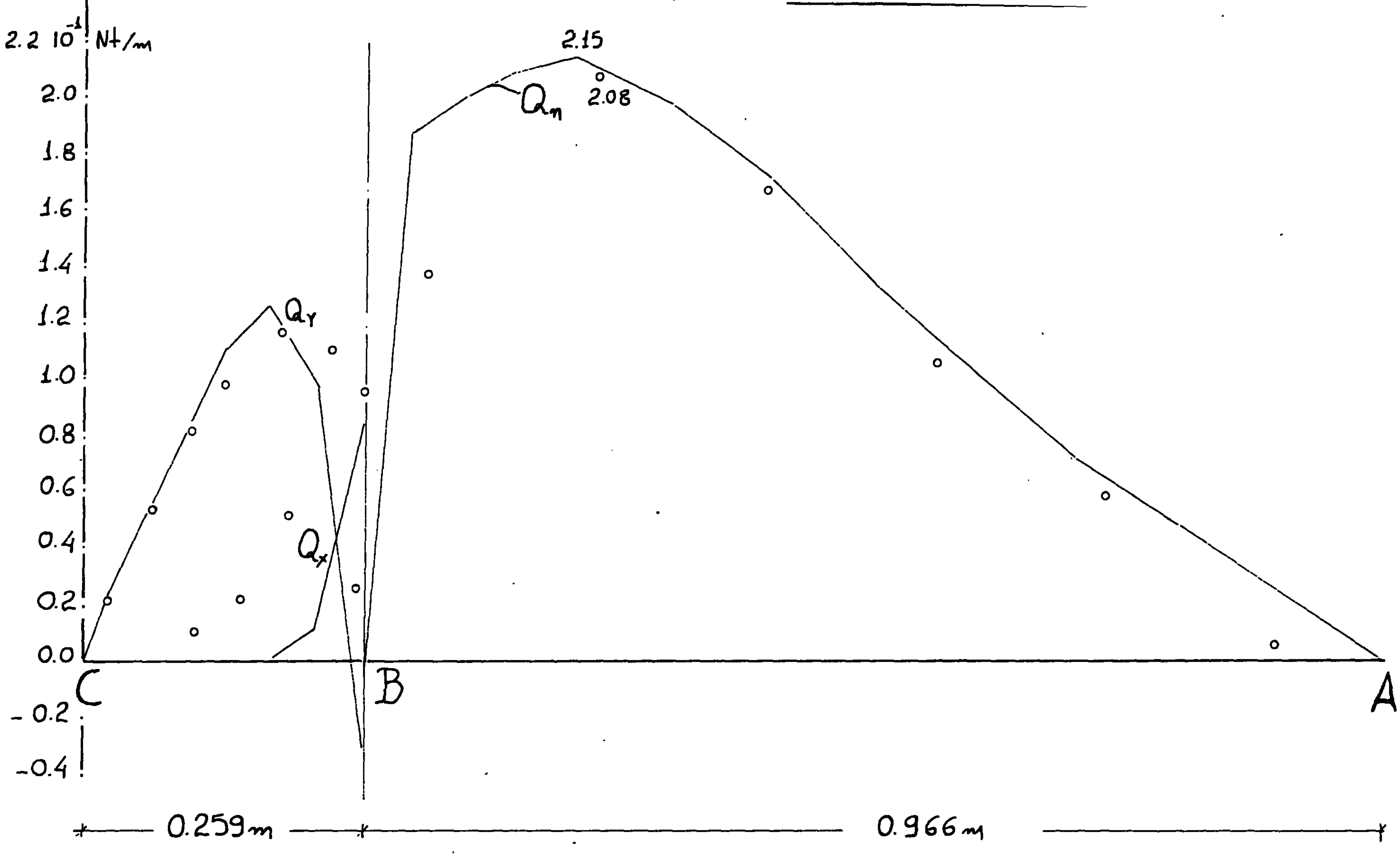


FIG. 12.26.

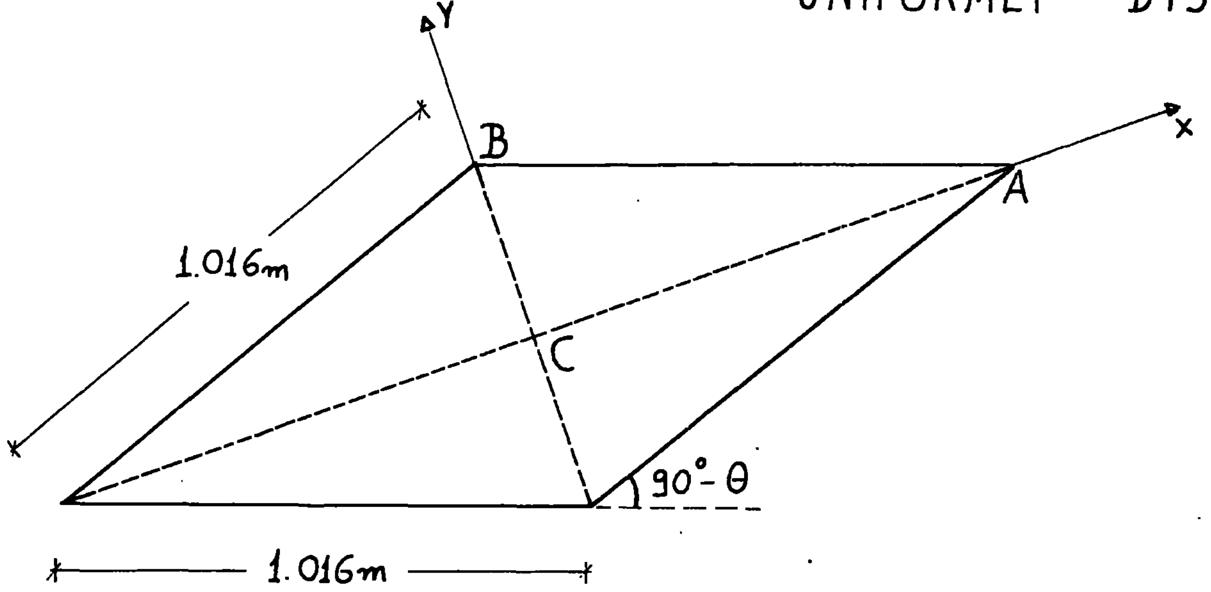


2.2 10⁻¹ Nt/m
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
2.15
2.08
Qn
Qy
Qx
C
B
A
0.259m
0.966m

FIG. 12.27.

UNDER

UNIFORMLY DISTRIBUTED LOAD 1Nt/m²



KEY	
PDS15	∘
PDS21	▵
PMX12	×
PMX24	—·—
PDS24	□
PDS30	------ or +
PRO18	▿

CASE	BOUNDARY CONDITIONS	Θ (dg.)	D^{xx}_{xx} (Ntm/m)	$D^{xx}_{YY}=D^{YY}_{xx}$ (Ntm/m)	D^{YY}_{YY} (Ntm/m)	D^{xY}_{xY} (Ntm/m)	S^{xz}_{xz} (Nt/m)	S^{Yz}_{Yz} (Nt/m)	COMPARISON with —+—
1.	CLAMPED $M_{ns}=0$ along the boundaries	15°	1.6527 10^4	0.5289 10^4	1.6527 10^4	1.1238 10^4	0.8979 10^5	0.8979 10^5	KENNEDY [66]
2.	»	30°	»	»	»	»	»	»	»
3.	»	45°	»	»	»	»	»	»	»
4.	»	60°	»	»	»	»	»	»	»

FIG. 12.28.

CASE 1.

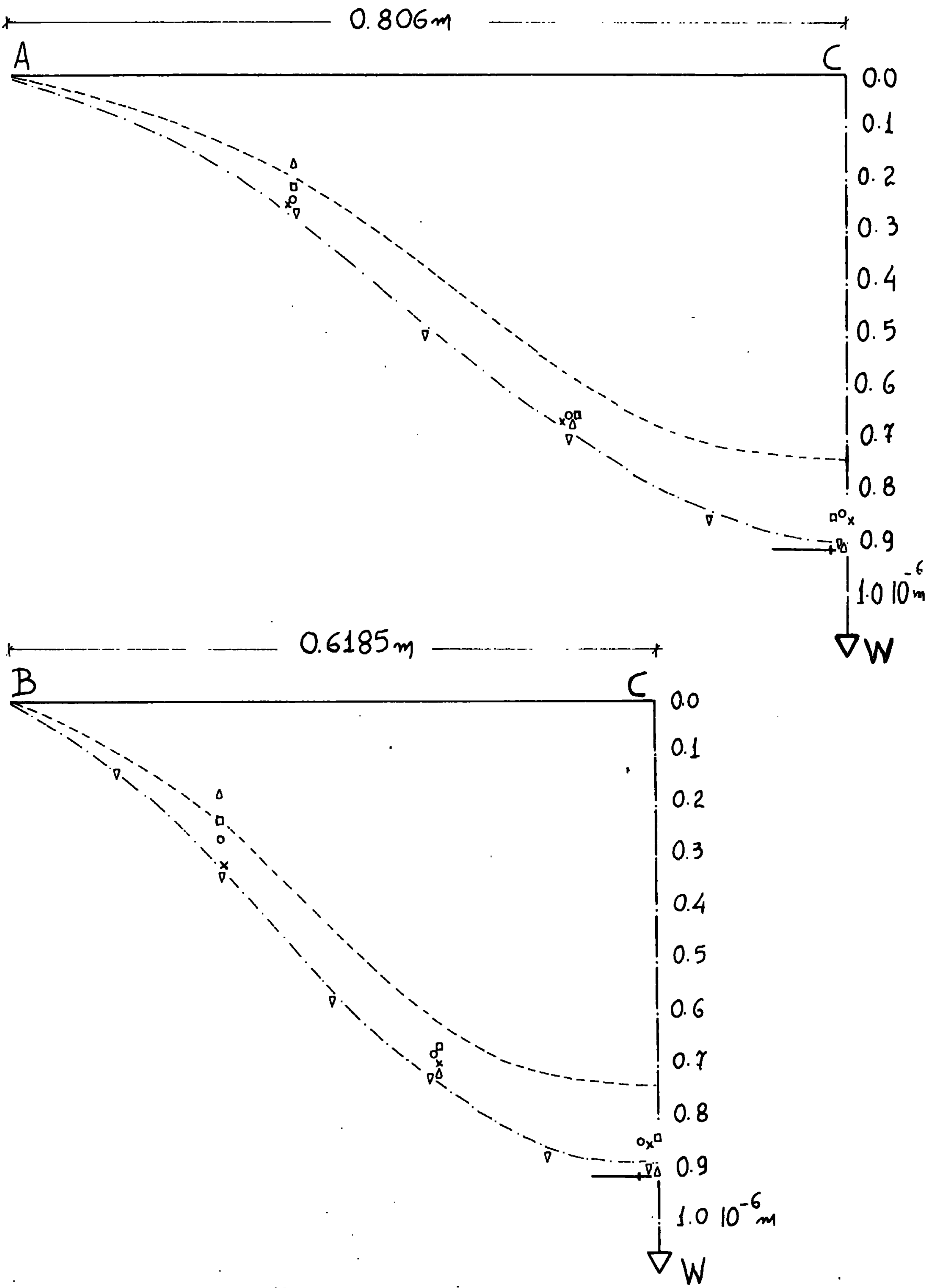


FIG. 12.29.

CASE 1.

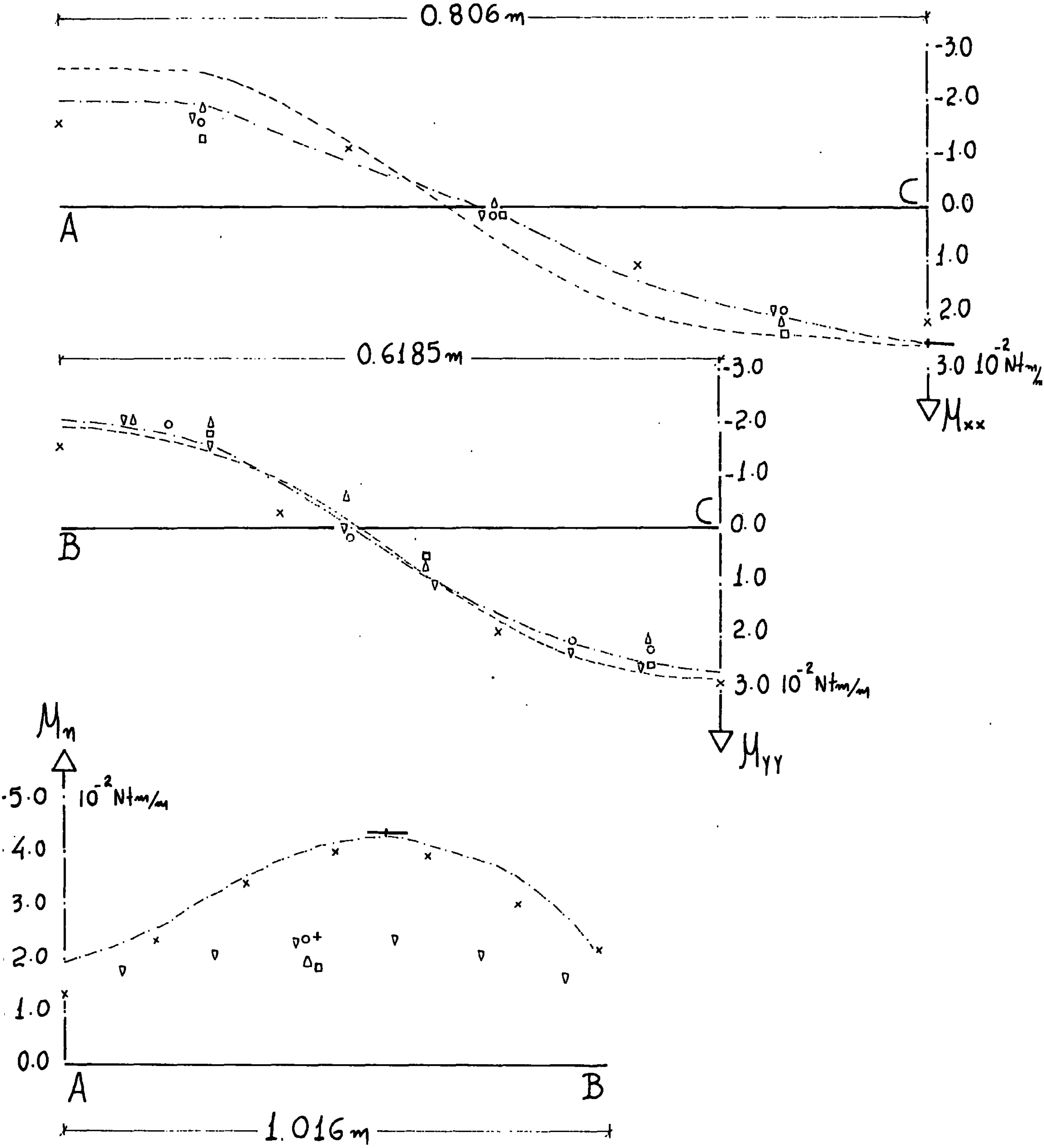


FIG. 12.30.

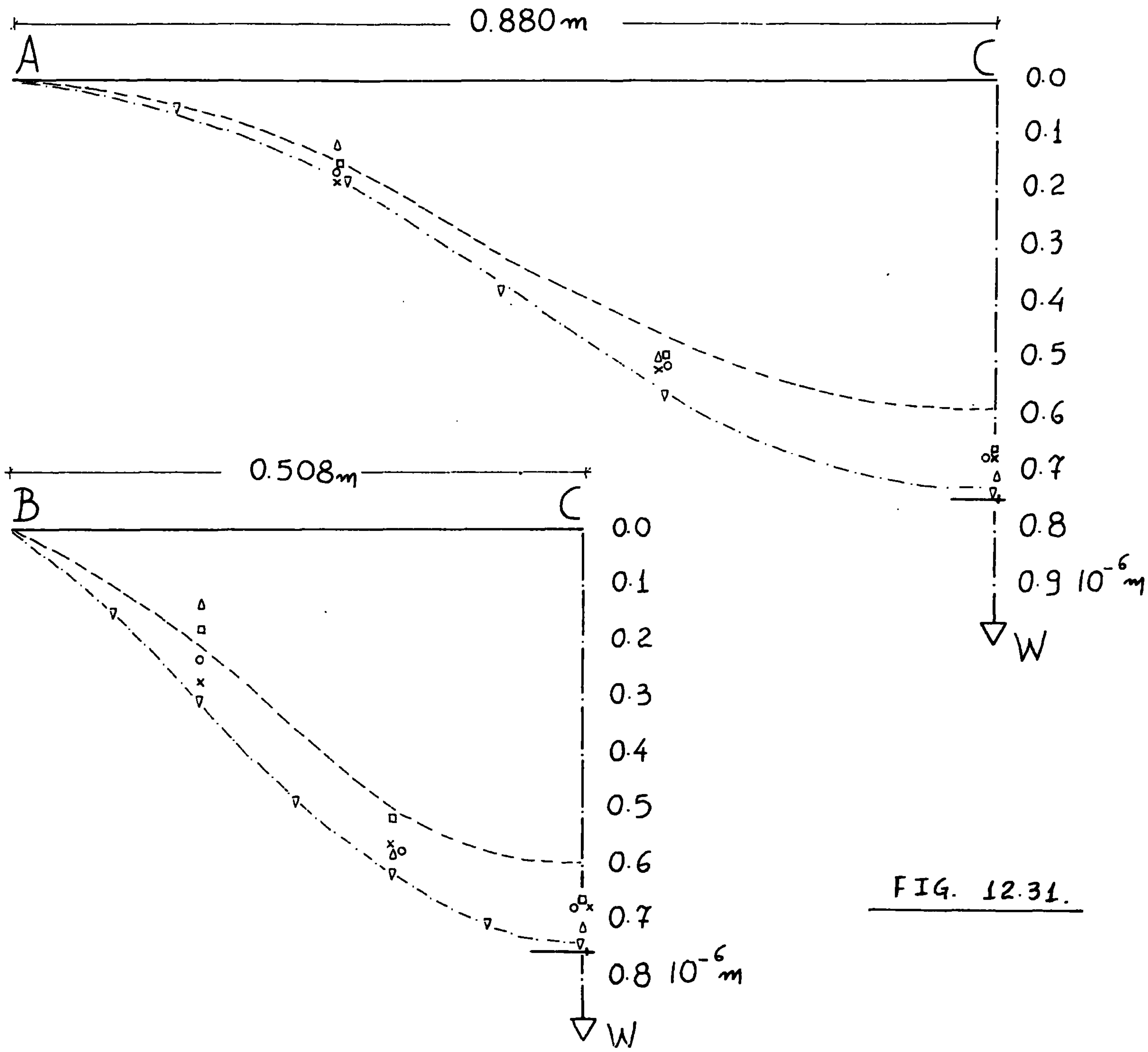

0.880m
A
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9 10^-6 m
W
0.508m
B
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8 10^-6 m
W

FIG. 12.31.

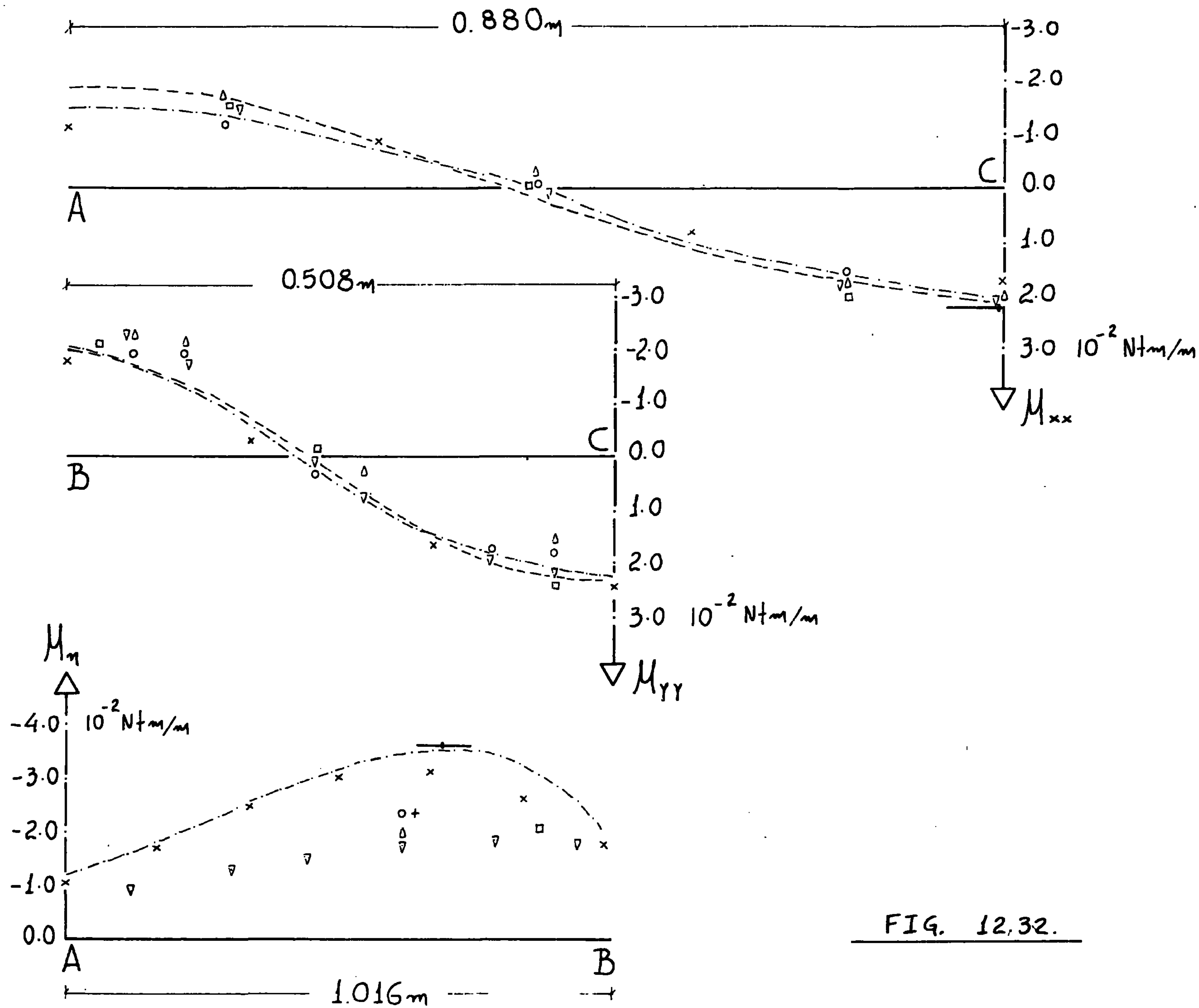

0.880m
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
10^{-2} Ntm/m
M_{xx}
A
C
0.508m
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
10^{-2} Ntm/m
M_{YY}
B
C
M_n
10^{-2} Ntm/m
-4.0
-3.0
-2.0
-1.0
0.0
A
B
1.016m

FIG. 12.32.

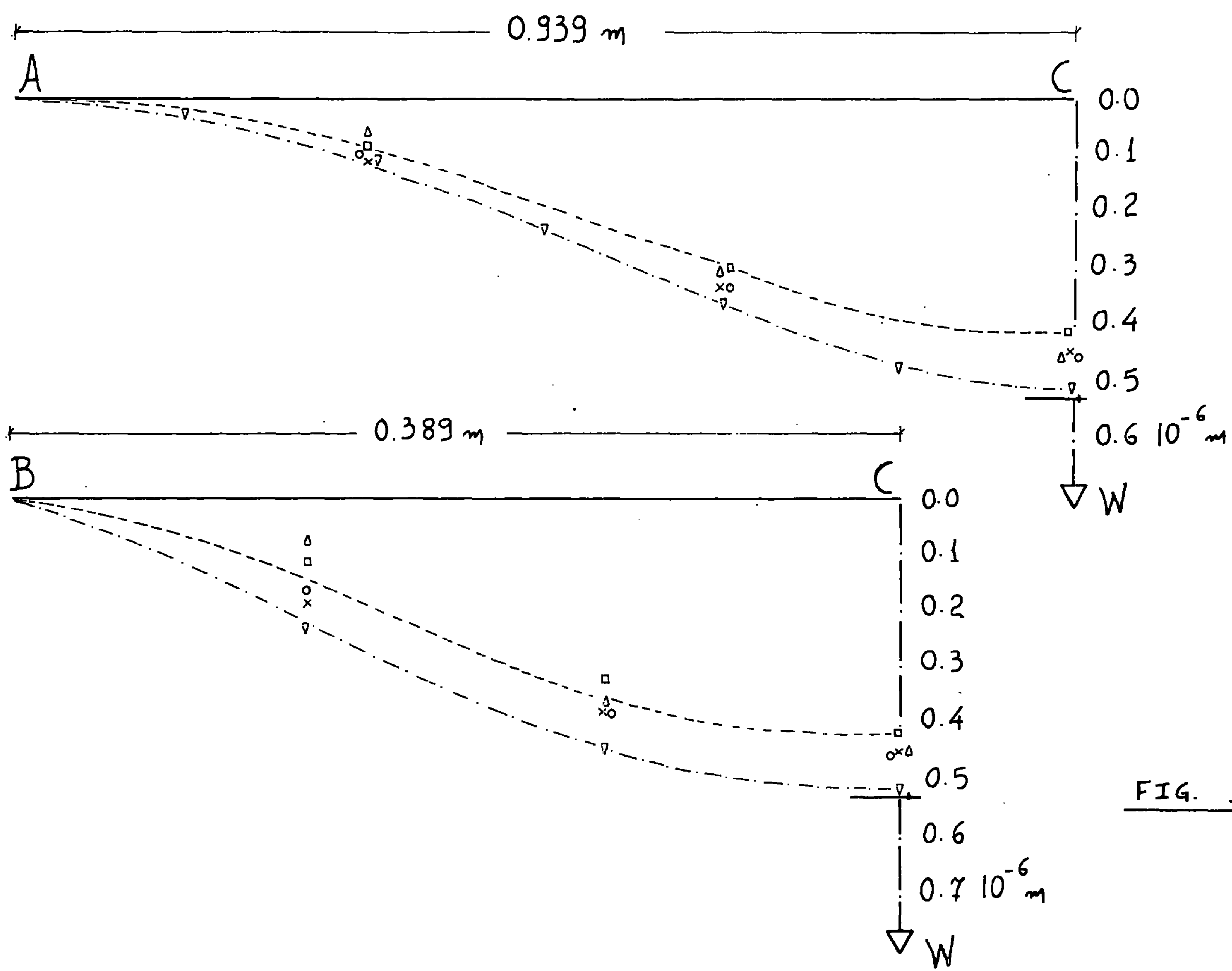

0.939 m
A
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6 10^{-6} m
W
0.389 m
B
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7 10^{-6} m
W

FIG. 12.33.

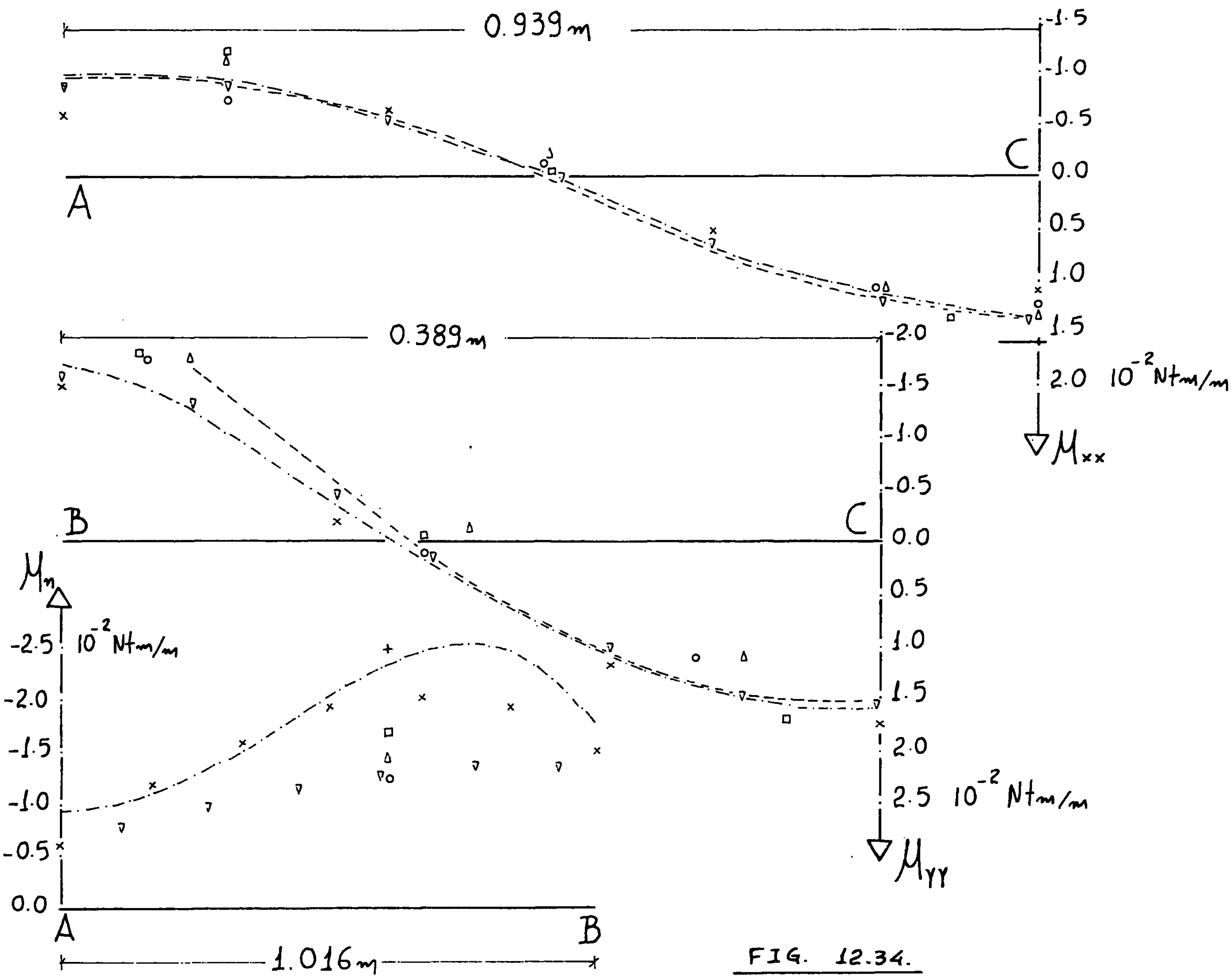

0.939 m
A
C
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
10^{-2} Ntm/m
M_{xx}
0.389 m
B
C
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
10^{-2} Ntm/m
M_{YY}
M_n
10^{-2} Ntm/m
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
A
B
1.016 m

FIG. 12.34.

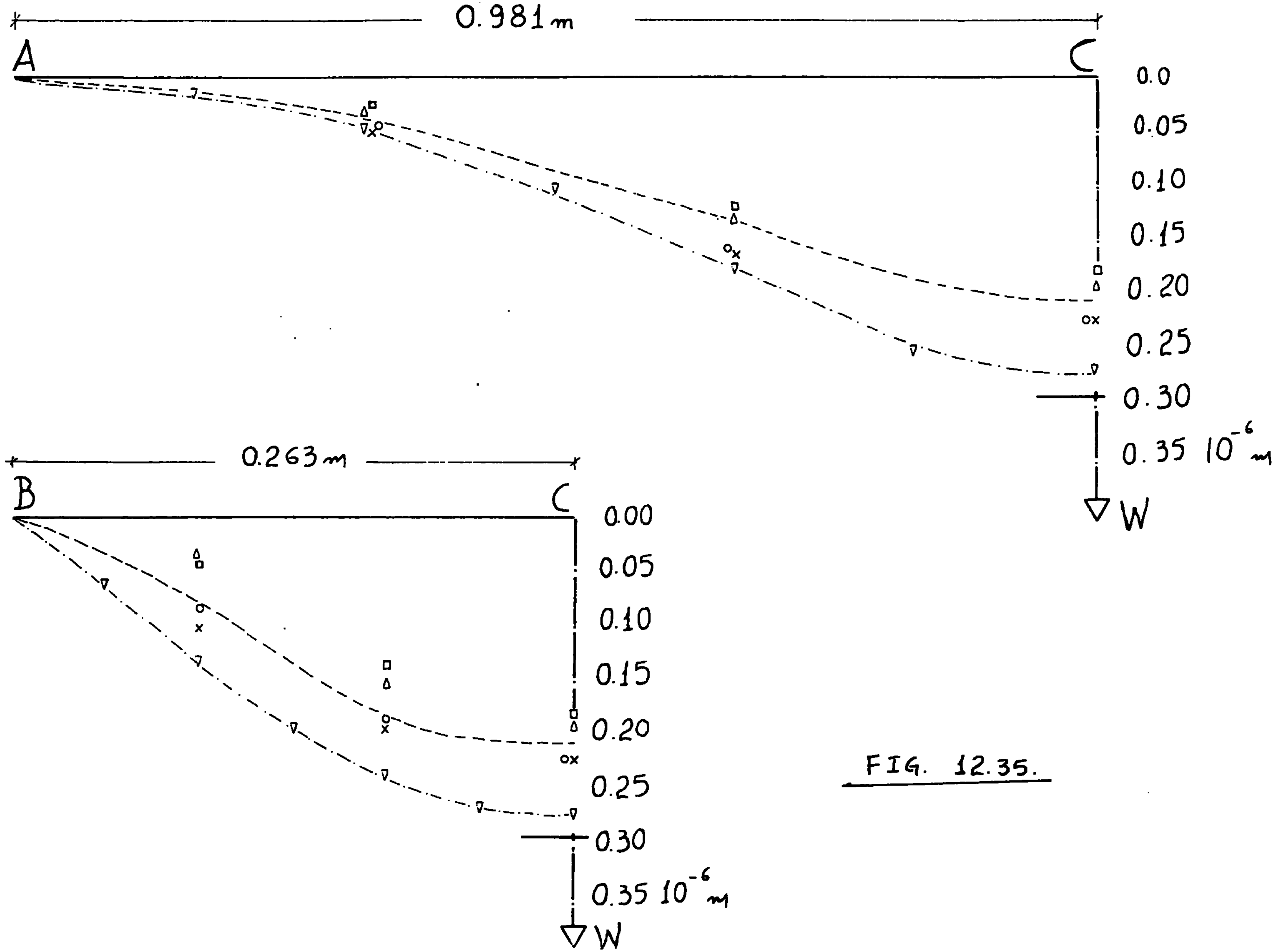

0.981 m
A
C
0.0
0.05
0.10
0.15
0.20
0.25
0.30
0.35 10^{-6} m
W
0.263 m
B
C
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35 10^{-6} m
W

FIG. 12.35.

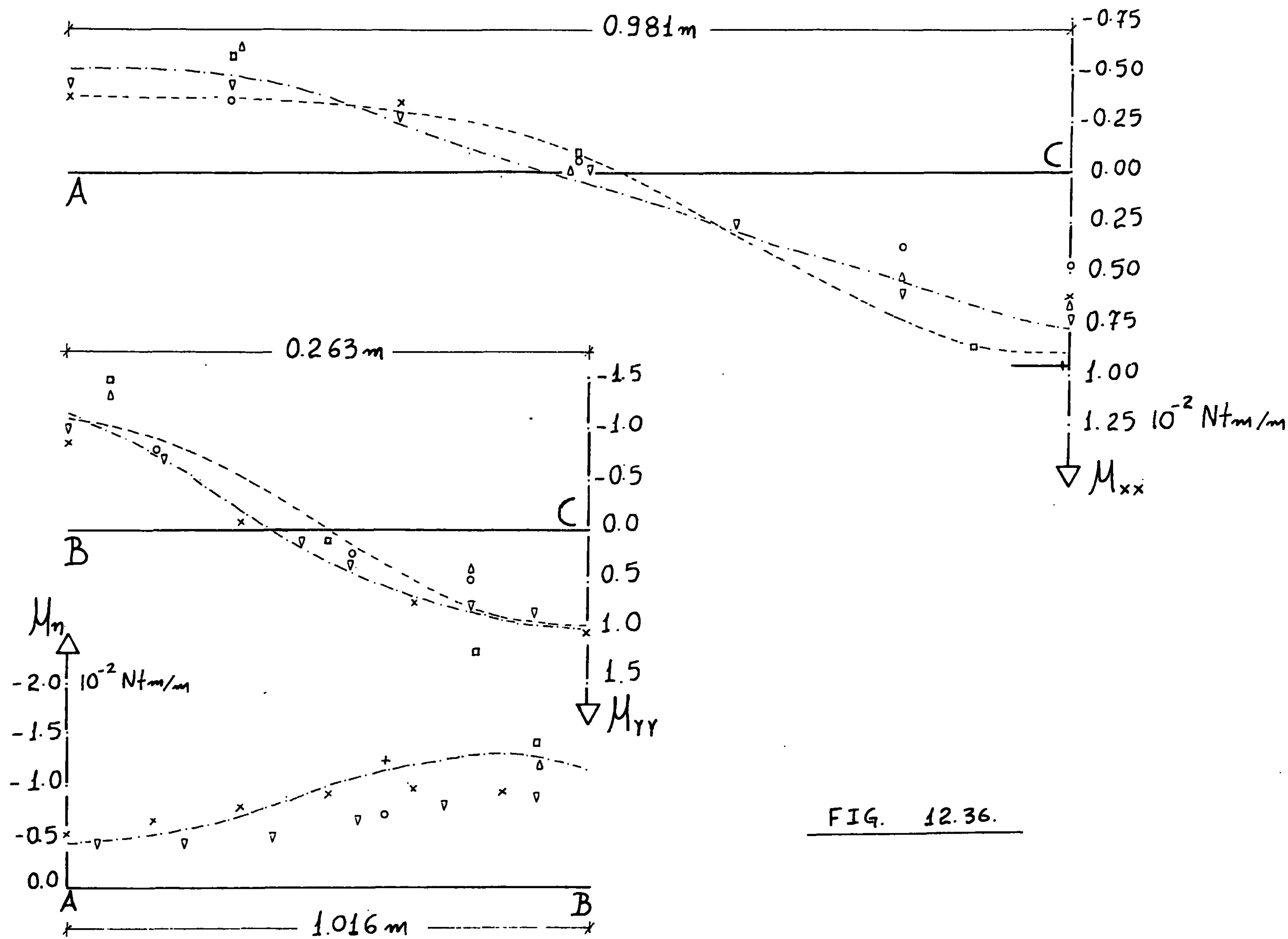
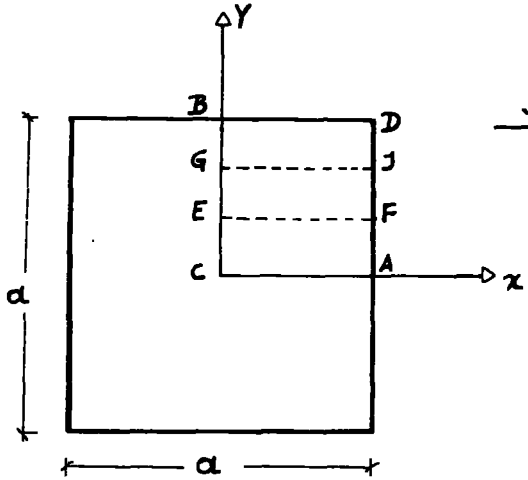

0.981 m
-0.75
-0.50
-0.25
0.00
0.25
0.50
0.75
1.00
1.25 10^-2 Ntm/m
Mxx
A
C
0.263 m
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
MYY
B
C
Mn
10^-2 Ntm/m
-2.0
-1.5
-1.0
-0.5
0.0
A
B
1.016 m

FIG. 12.36.

	CASE 1. (θ=15°)			CASE 2. (θ=30°)			CASE 3. (θ=45°)			CASE 4. (θ=60°)		
	W_c	M_{max}	M_{edge}	W_c	M_{max}	M_{edge}	W_c	M_{max}	M_{edge}	W_c	M_{max}	M_{edge}
PDS15	0.841	2.22	-2.35	0.674	1.74	-2.30	0.448	1.25	-1.21	0.218	0.703	-0.721
PDS21	0.899	2.33	-1.85	0.707	2.06	-1.90	0.442	1.37	-1.41	0.188	0.643	-1.25
PMX12	0.851	2.86	-3.99	0.673	2.43	-3.10	0.445	1.76	-1.99	0.221	0.964	-0.940
PMX24	0.888	2.65	-4.30	0.732	2.24	-3.54	0.509	1.64	-2.53	0.270	0.934	-1.35
PDS24	0.848	2.56	-1.78	0.657	2.44	-1.91	0.411	2.15	-1.71	0.173	1.120	-1.66
PDS30	0.729	2.66	-2.40	0.588	2.22	-2.37	0.398	1.52	-2.35	0.204	0.881	-1.95
PRO18	0.885	2.65	-2.35	0.729	2.18	-1.90	0.507	1.54	-1.47	0.269	0.828	-0.950
KENNEDY	0.912	2.53	-4.36	0.748	2.19	-3.66	0.524	1.63	-3.59	0.282	0.956	-2.15
	10^{-6} m	10^{-2} Ntm/m	10^{-2} Ntm/m	10^{-6} m	10^{-2} Ntm/m	10^{-2} Ntm/m	10^{-6} m	10^{-2} Ntm/m	10^{-2} Ntm/m	10^{-6} m	10^{-2} Ntm/m	10^{-2} Ntm/m

FIG. 12.37.



SQUARE PLATE WITH FOUR CORNER SUPPORTS

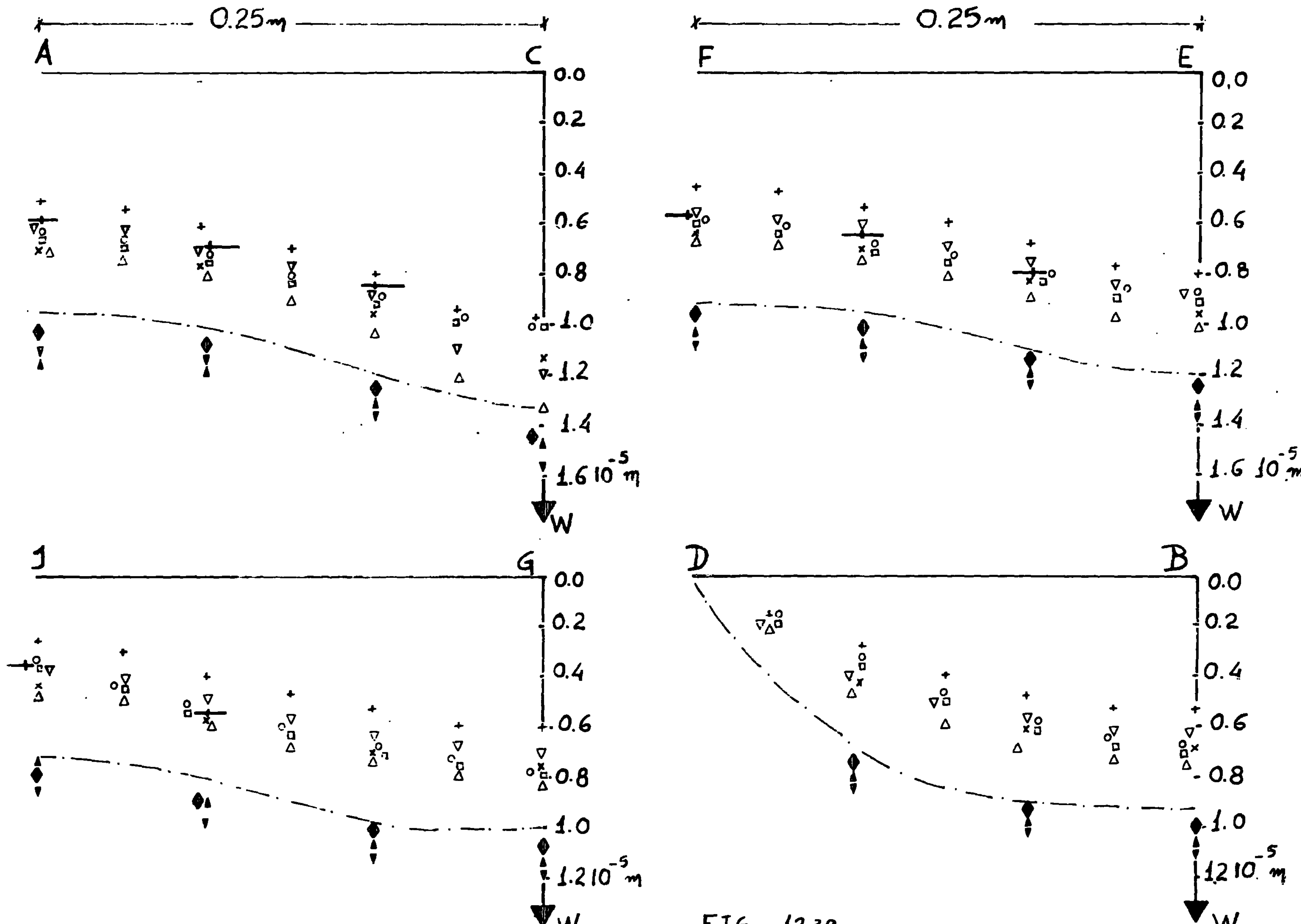
UNDER

1 Nt. CENTRAL LOAD

KEY	
PDS15	∘
PDS21 with stif-corners	△
PMX12	×
PMX24	------
PDS24	□
PDS30 with stif-corners	+
PRO18 with stif-corners	▽
PDS21	▲
PDS30	◆
PRO18	▼

CASE	DESCRIPTION OF THE PLATE	a (m)	D^{xx}_{xx} (Nt-m)	$D^{xx}_{yy} = D^{yy}_{xx}$ (Nt-m)	D^{yy}_{yy} (Nt-m)	D^{xy}_{xy} (Nt-m)	S^{xz}_{xz} (Nt/m)	S^{yz}_{yz} (Nt/m)	COMPARISON WITH —+—
1.	PLYWOOD PLATE (BETTES PLATE 1.) $\frac{c}{f} = 15.7$	0.5	9700	2600	6800	7100	59000	59000	BETTESS EXPERIMENT [21]
2.	PLYWOOD PLATE (BETTES PLATE 3.) $\frac{c}{f} = 7.5$	0.5	2540	690	1800	1900	32000	32000	"
3.	FIBREGLASS PLATE (BETTES PLATE 4.)	0.5	$0.29\ 10^4$	$0.86\ 10^3$	$0.29\ 10^4$	$0.20\ 10^4$	$2.5\ 10^5$	$2.5\ 10^5$	"
4.	FIBREGLASS PLITE (BETTES PLATE 5.)	0.5	$0.15\ 10^4$	$0.46\ 10^3$	$0.15\ 10^4$	$0.11\ 10^4$	$0.18\ 10^6$	$0.18\ 10^6$	"
5.	FIBREGLASS PLATE (BETTES PLATE 6.)	0.5	$0.59\ 10^3$	$0.18\ 10^3$	$0.59\ 10^3$	$0.41\ 10^3$	$1.0\ 10^5$	$1.0\ 10^5$	"
6.	ALUMINIUM PLATE (BETTES PLATE 7.) $\frac{c}{f} = 22.6$	1.0	$0.29\ 10^4$	$0.72\ 10^3$	$0.29\ 10^4$	$0.17\ 10^4$	$0.37\ 10^6$	$0.37\ 10^6$	"
7.	HARDBOARD PLATE (BETTES PLATE 8.) $\frac{c}{f} = 15.3$	1.0	$0.26\ 10^5$	$0.79\ 10^4$	$0.26\ 10^5$	$0.92\ 10^4$	$1.4\ 10^5$	$1.4\ 10^5$	"

FIG. 12.38.



0.25m
A
C
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6 10^{-5} m
W
0.25m
F
E
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6 10^{-5} m
W
J
G
0.0
0.2
0.4
0.6
0.8
1.0
1.2 10^{-5} m
W
D
B
0.0
0.2
0.4
0.6
0.8
1.0
1.2 10^{-5} m
W

FIG. 12.39.

CASE 2.

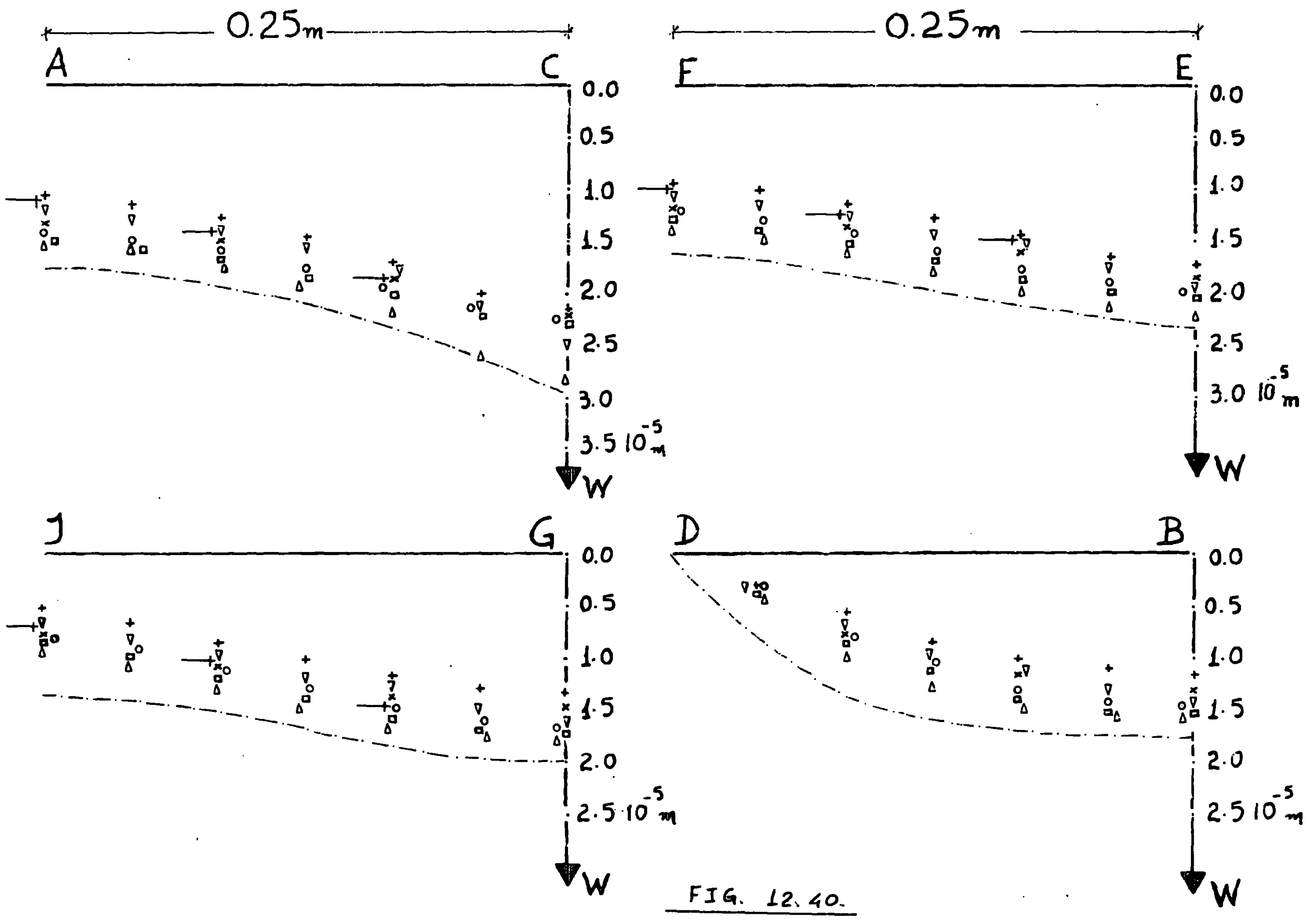


FIG. 12.40.

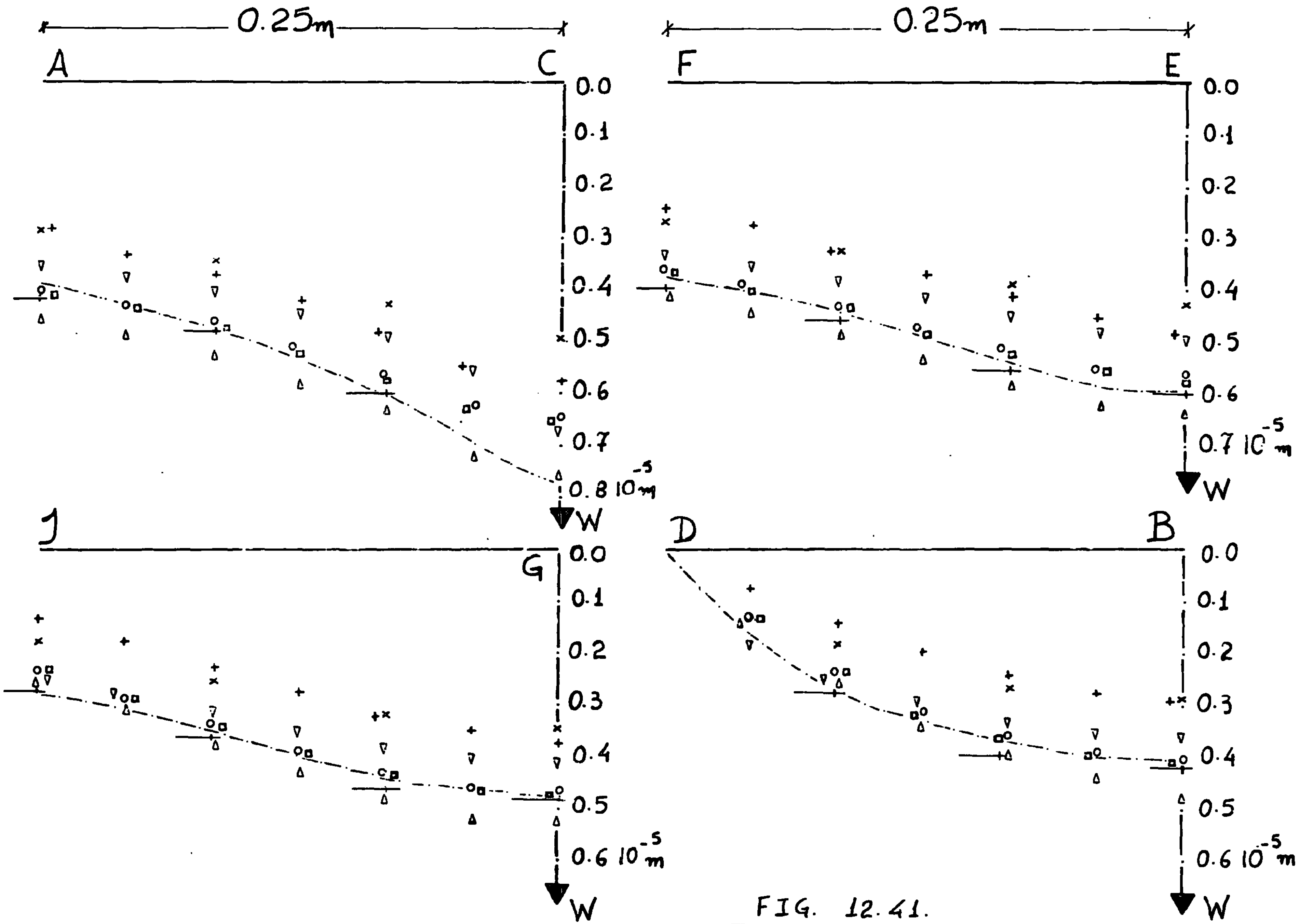

0.25m
0.25m
A
C
F
E
J
G
D
B
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8 10^-5 m
0.7 10^-5 m
0.6 10^-5 m
0.6 10^-5 m
W
W
W
W

FIG. 12.41.

CASE 4.

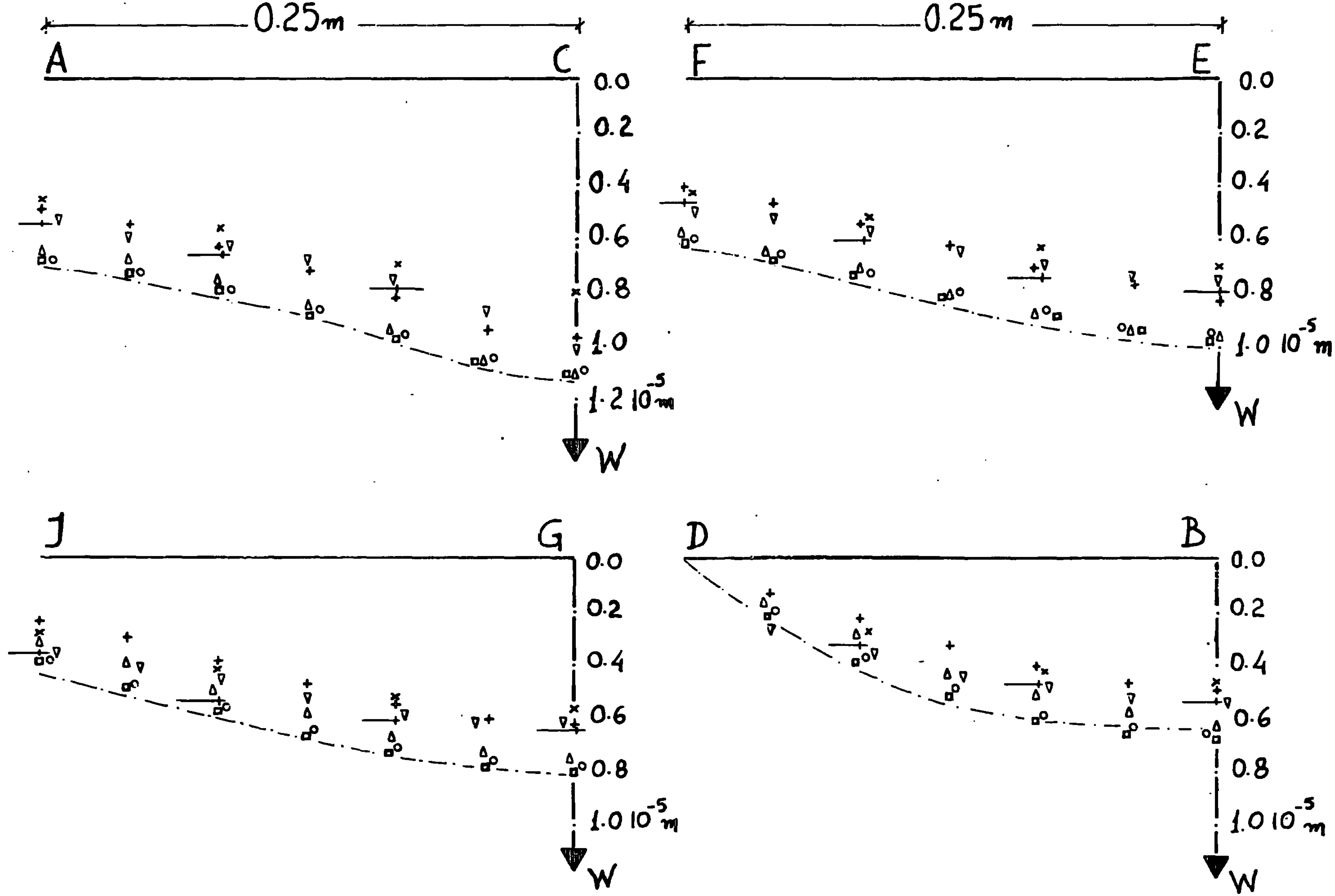


FIG. 12.42.

CASE 5.

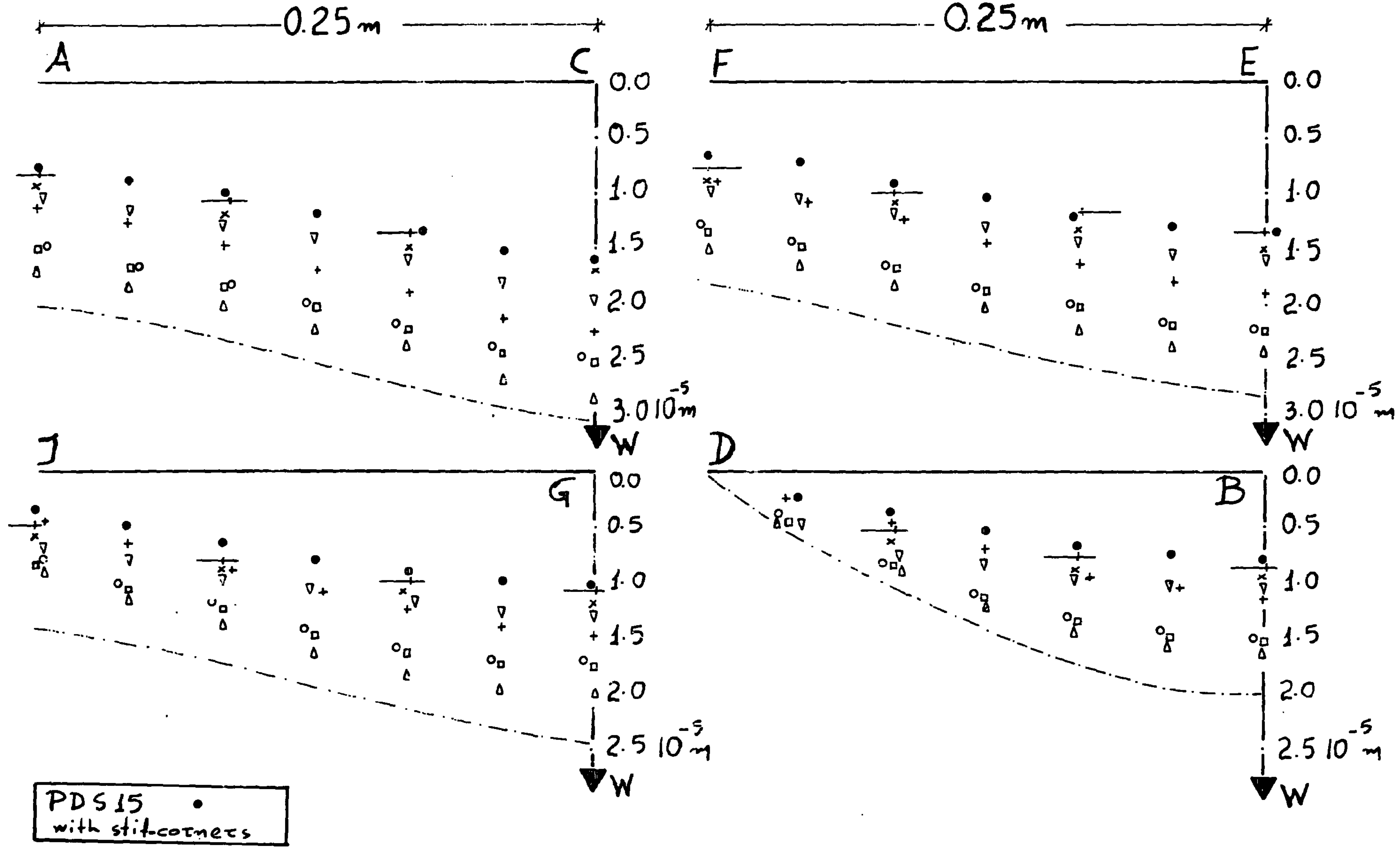


FIG. 12.43.

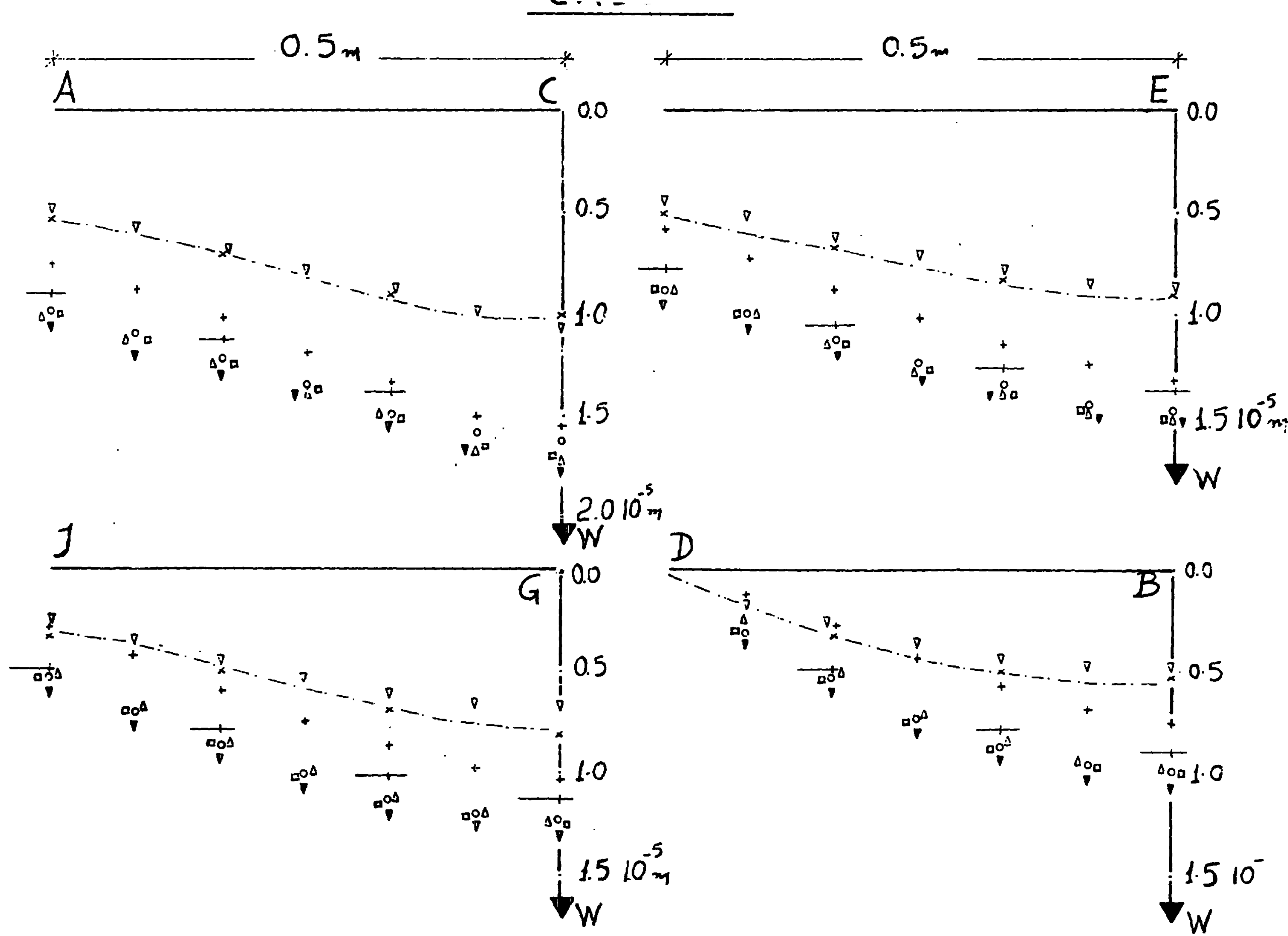


FIG. 12.44.

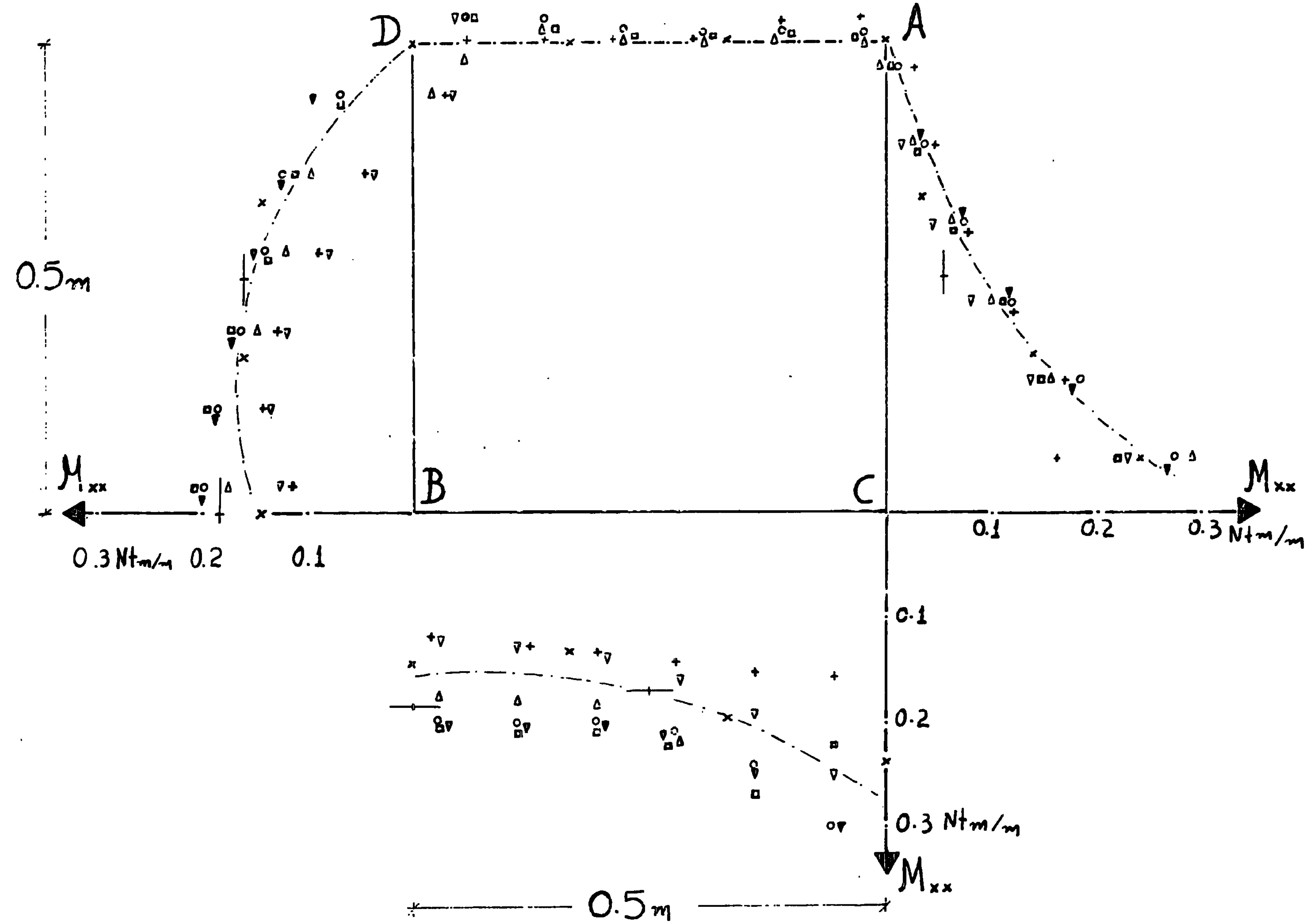


FIG. 12.45.

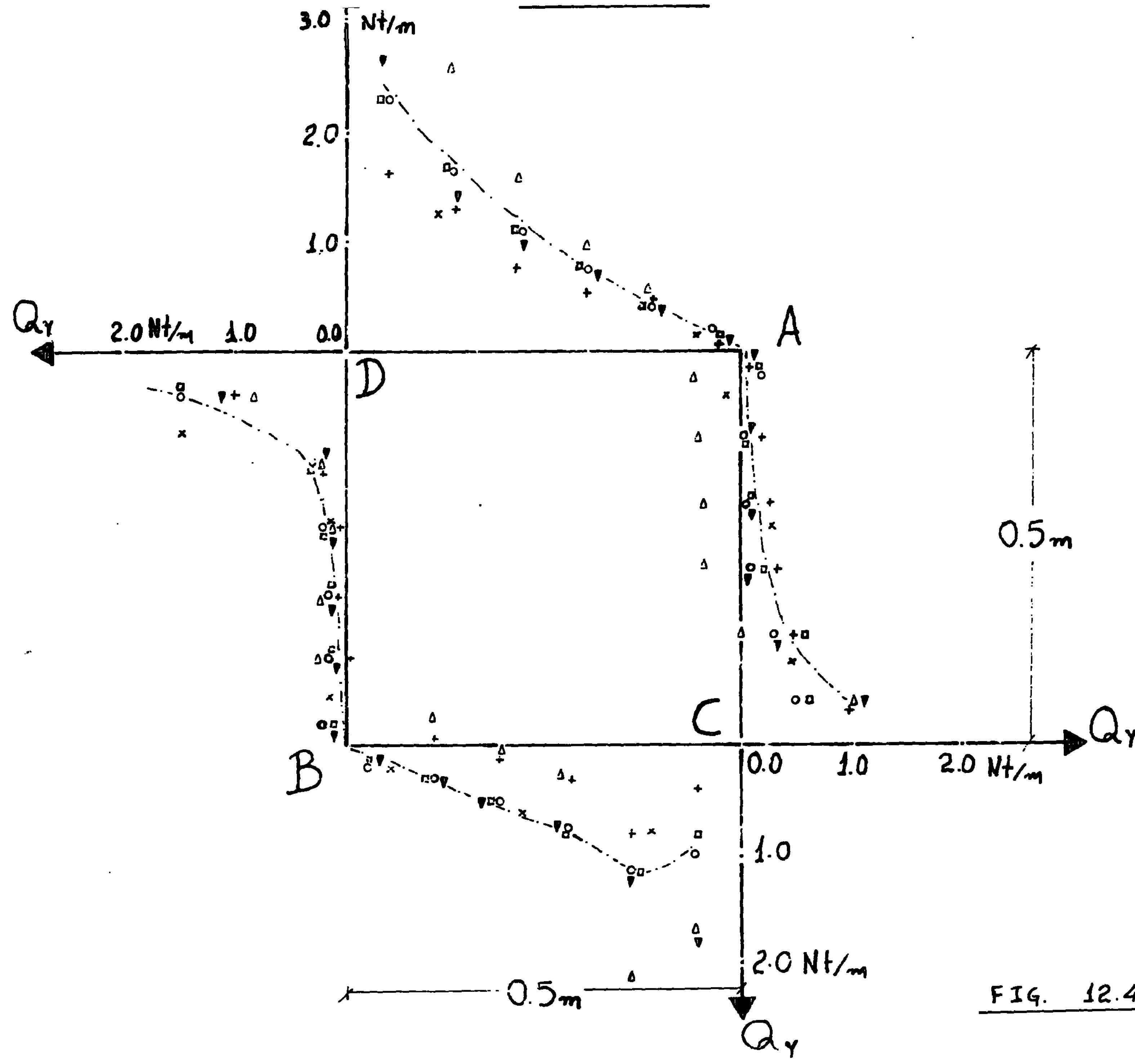

3.0
Nt/m
2.0
1.0
0.0
Qy
2.0 Nt/m
1.0
0.0
A
D
B
C
0.5m
Qy
0.0
1.0
2.0 Nt/m
1.0
2.0 Nt/m
0.5m
Qy

FIG. 12.46.

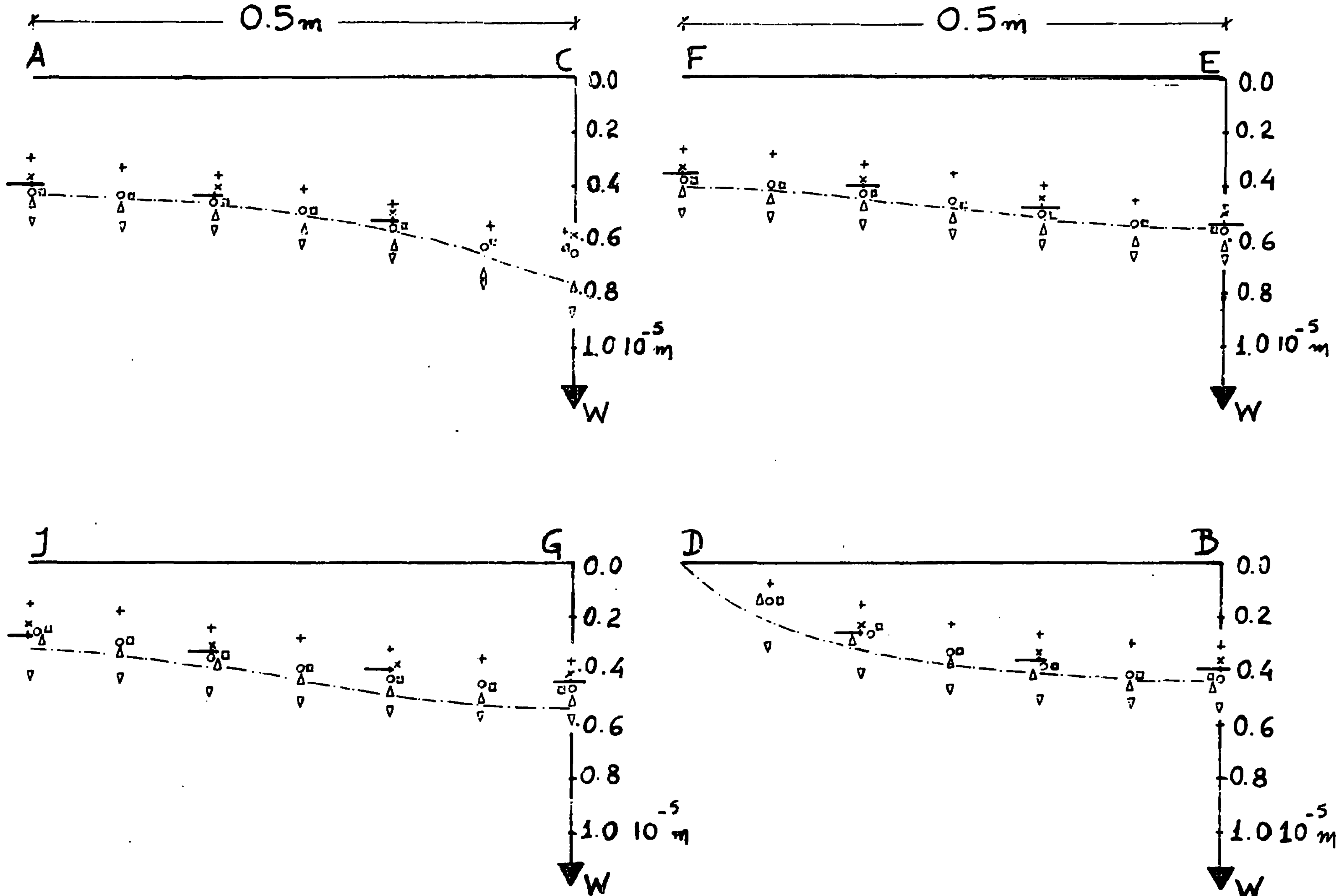
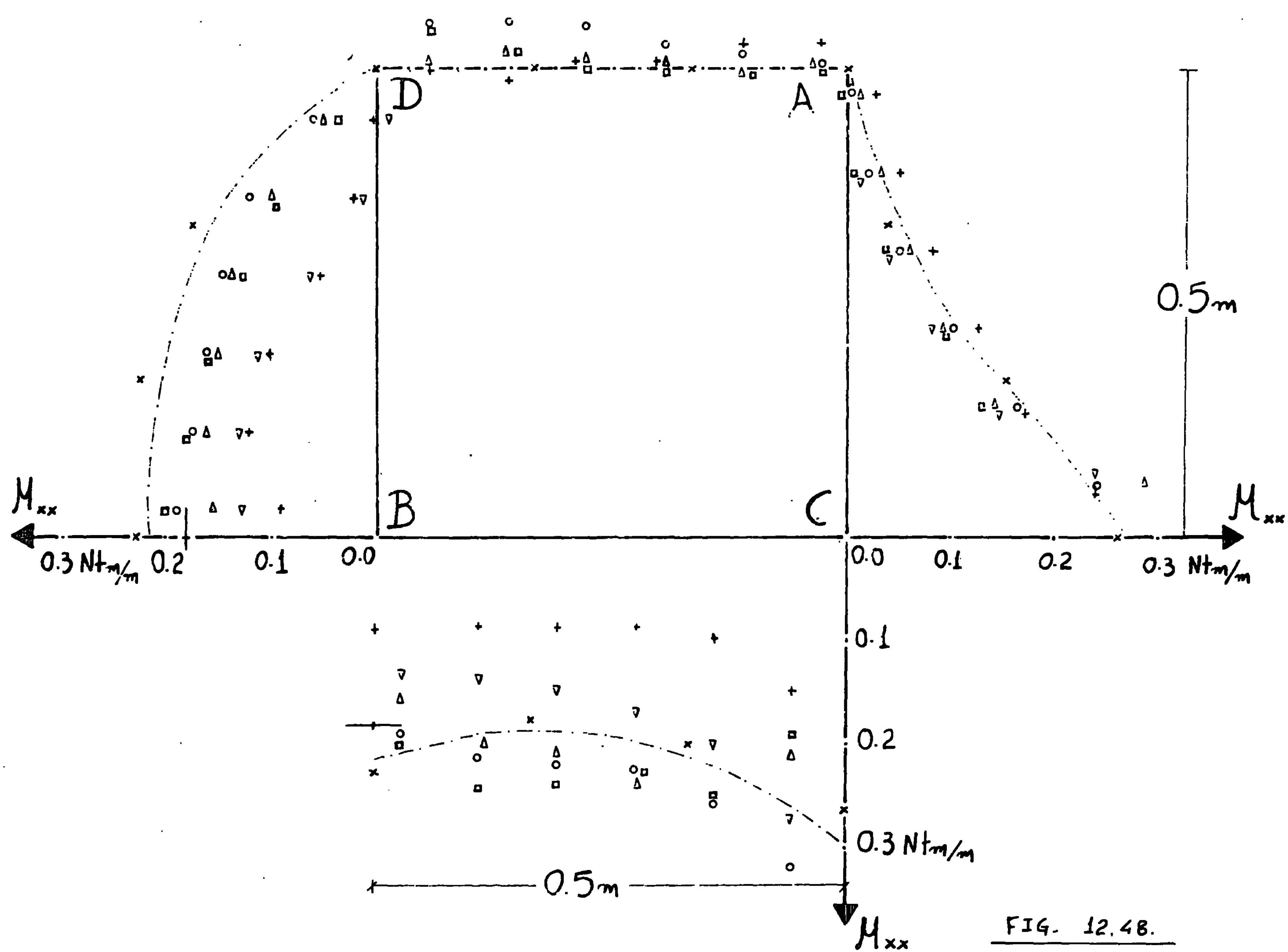


FIG. 12.47.



D
A
B
C
0.5m
M_{xx}
0.3 Ntm/m
0.2
0.1
0.0
0.1
0.2
0.3 Ntm/m
0.0
0.1
0.2
0.3 Ntm/m
0.5m
M_{xx}
M_{xx}

FIG. 12.48.

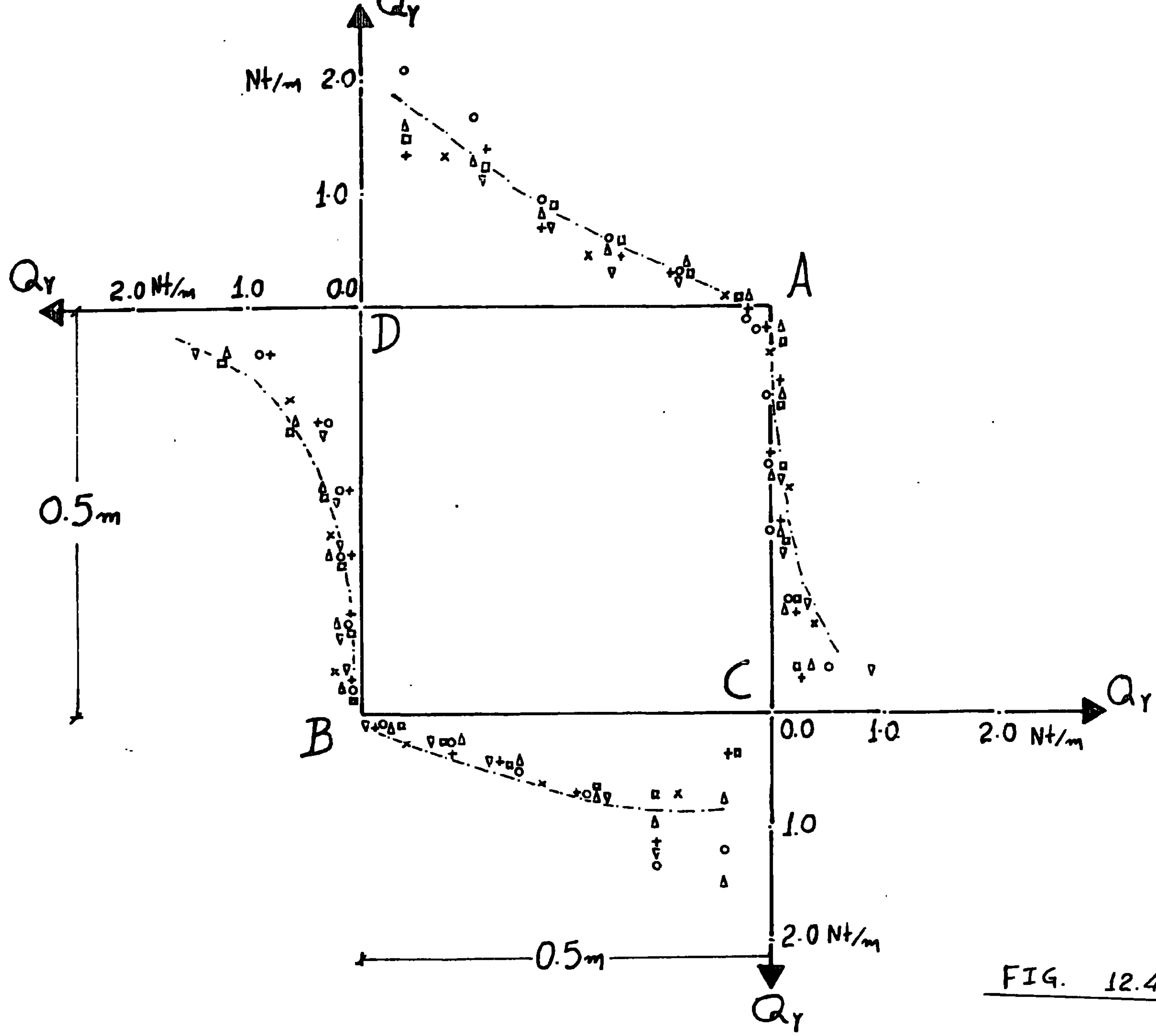
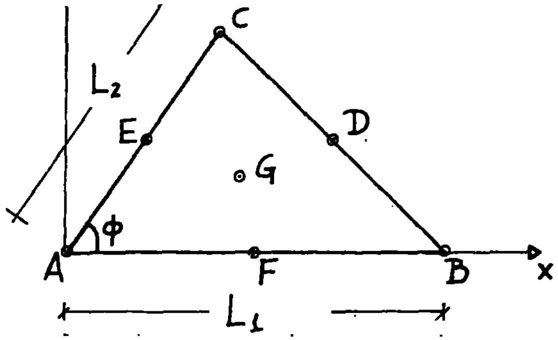

Qy
Nt/m 2.0
1.0
0.0
Qy
2.0 Nt/m
1.0
A
D
0.5m
C
Qy
B
0.0
1.0
2.0 Nt/m
1.0
2.0 Nt/m
0.5m
Qy

FIG. 12.49.



$D_{xx}^{xx} = 0.254\ 10^4\ Nt\,m$, $D_{xx}^{YY} = D_{YY}^{xx} = 0.690\ 10^3\ Nt\,m$

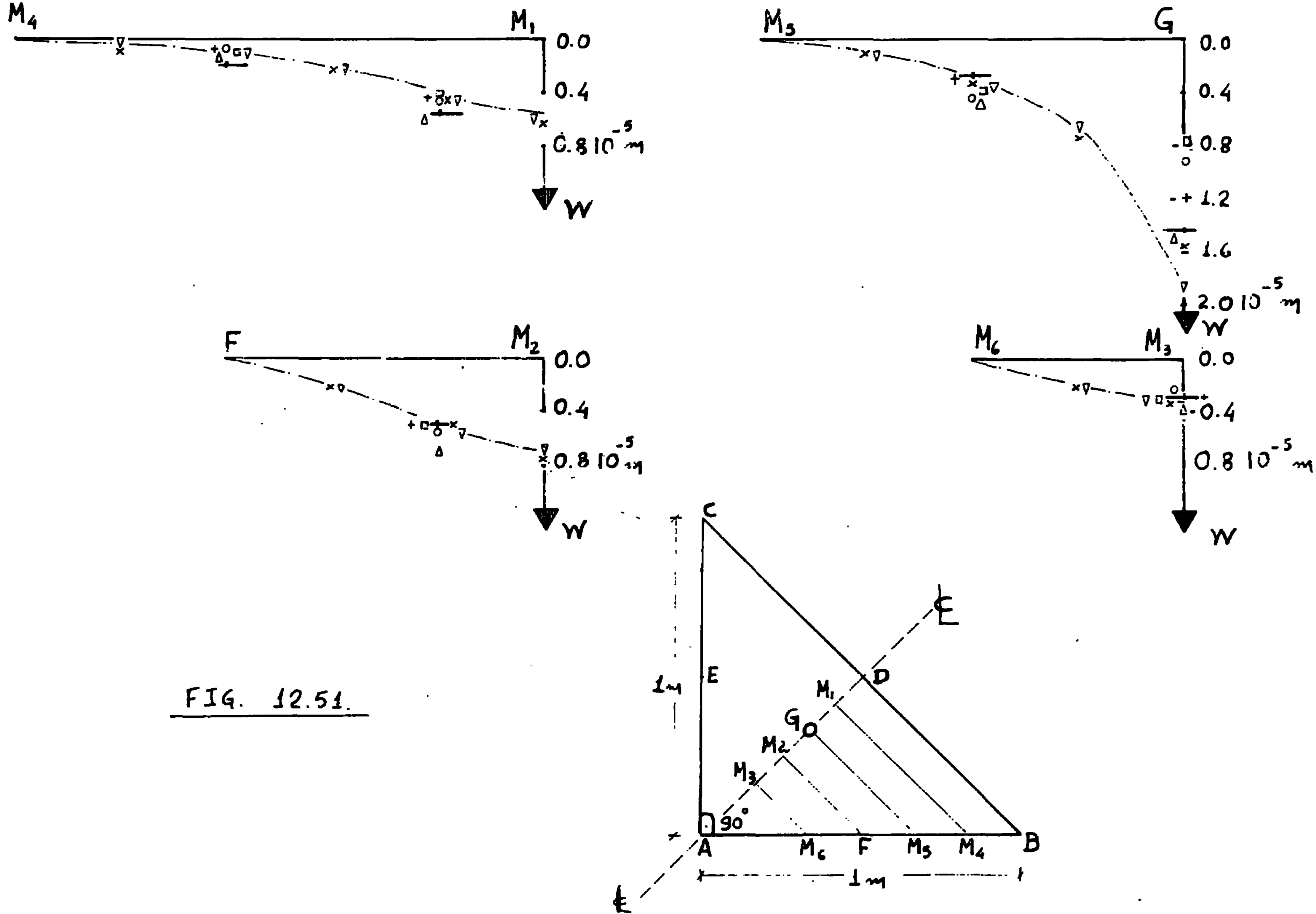
$D_{YY}^{YY} = 0.180\ 10^4\ Nt\,m$, $D_{xY}^{xY} = 0.190\ 10^4\ Nt\,m$

$D_{xz}^{xz} = D_{Yz}^{Yz} = 0.320\ 10^5\ Nt/m$

KEY	
PDS15	∘
PDS21	•
PMX12	×
PMX24	-------
PDS24	□
PDS30	+
PR018	▾

CASE	L_1 (m)	L_2 (m)	φ (dg.)	BOUNDARY CONDITION OF THE SIDE AB	BOUNDARY CONDITION OF THE SIDE BC	BOUNDARY CONDITION OF THE SIDE CA	1. Nt. CON. LOAT AT THE POINT	COMPARISON WITH —+—
1.	1.	1.	90°	simply supported	simply supported	simply supported	G	COLLINS [33] EXPERIMENT
2.	1.	1.	90°	>>	simply supported	free	E	>>
3.	1.	1.	90°	>>	free	simply supported	G	>>
4.	0.75	0.75	60°	>>	simply supported	simply supported	G	>>
5.	0.75	0.75	60°	>>	free	simply supported	D	>>
6.	0.75	0.75	60°	>>	free	simply supported	G	>>

FIG. 12.50.



M4
M1
0.0
0.4
0.8 10⁻⁵ m
W
M5
G
0.0
0.4
0.8
1.2
1.6
2.0 10⁻⁵ m
W
F
M2
0.0
0.4
0.8 10⁻⁵ m
W
M6
M3
0.0
0.4
0.8 10⁻⁵ m
W
C
E
D
M1
G
M2
M3
90°
A
M6
F
M5
M4
B
1 m
1 m

FIG. 12.51.

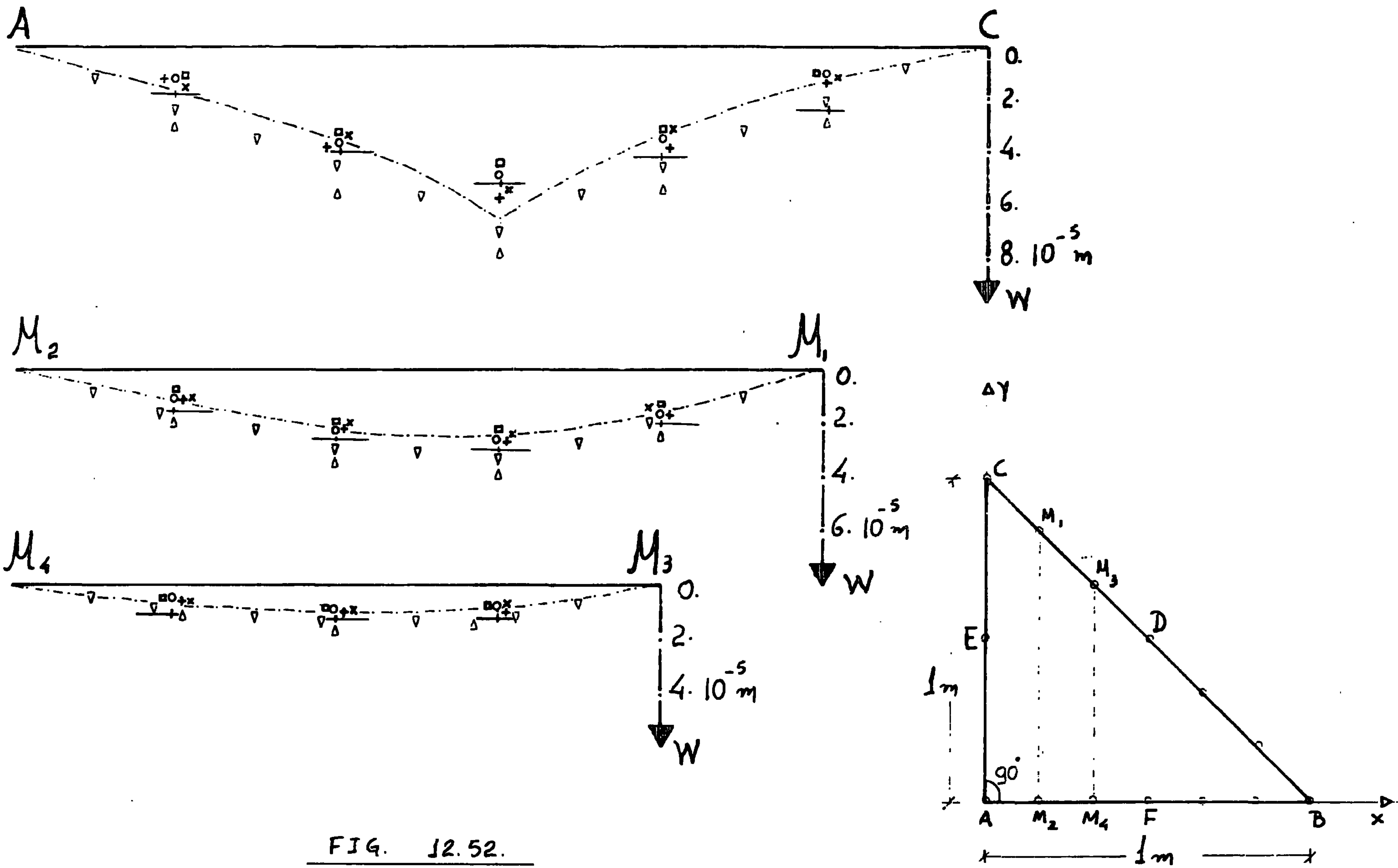

A
C
0.
2.
4.
6.
8. 10⁻⁵ m
W
M2
M1
0.
2.
4.
6. 10⁻⁵ m
W
M4
M3
0.
2.
4. 10⁻⁵ m
W
ΔY
C
M1
M3
D
E
1m
90°
A
M2
M4
F
B
x
1m

FIG. 12.52.

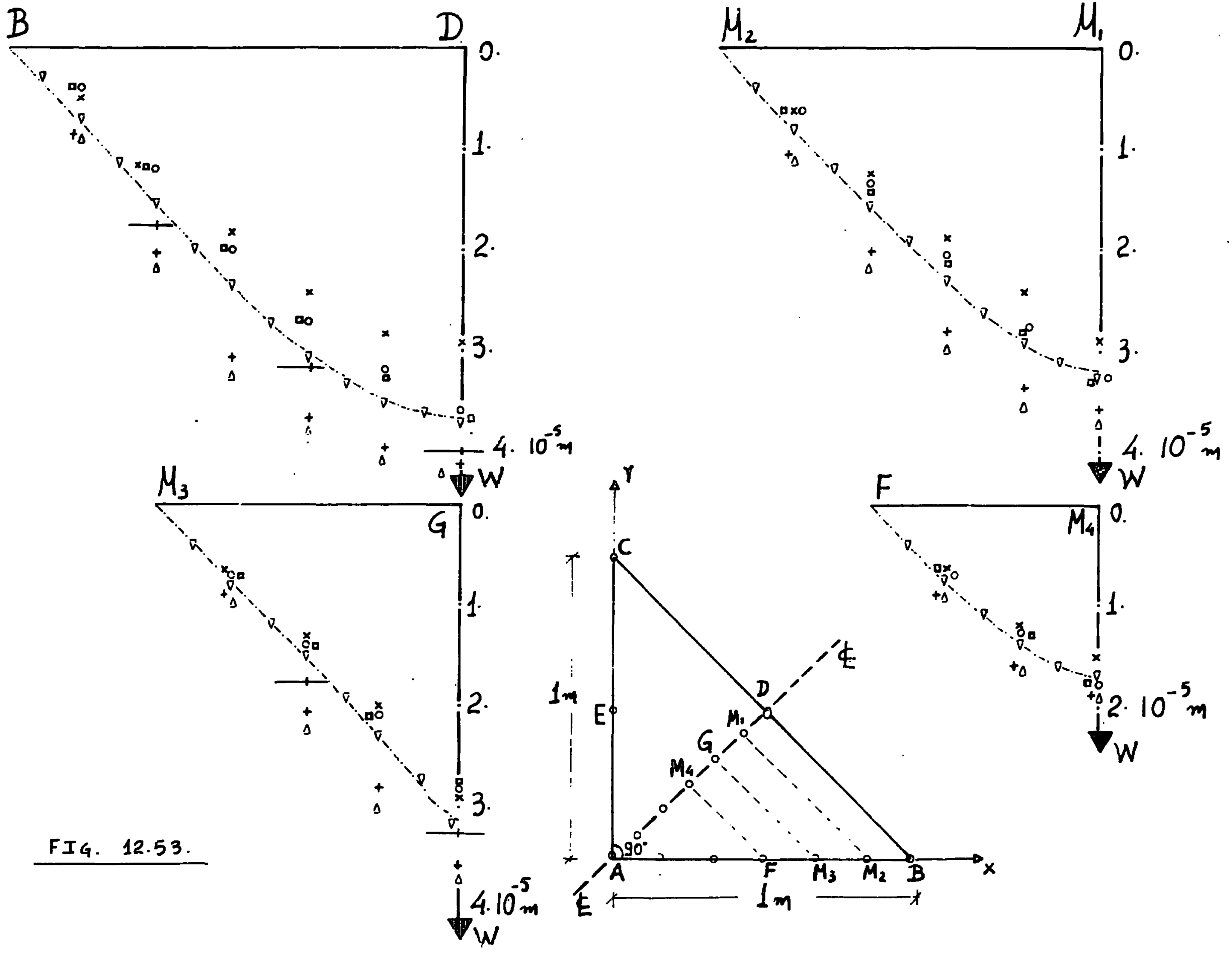


FIG. 12.53.

CASE 4.

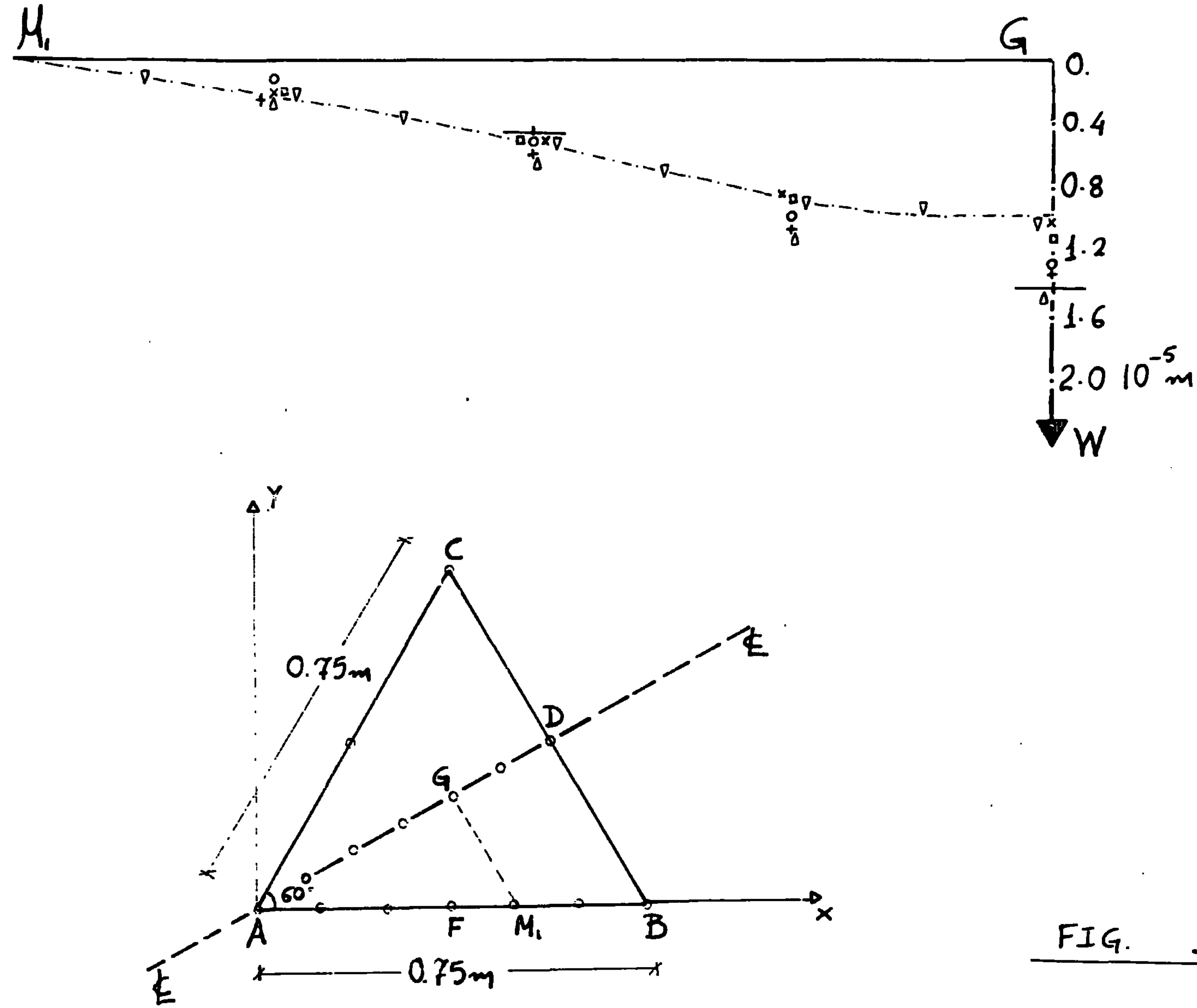


FIG. 12.54.

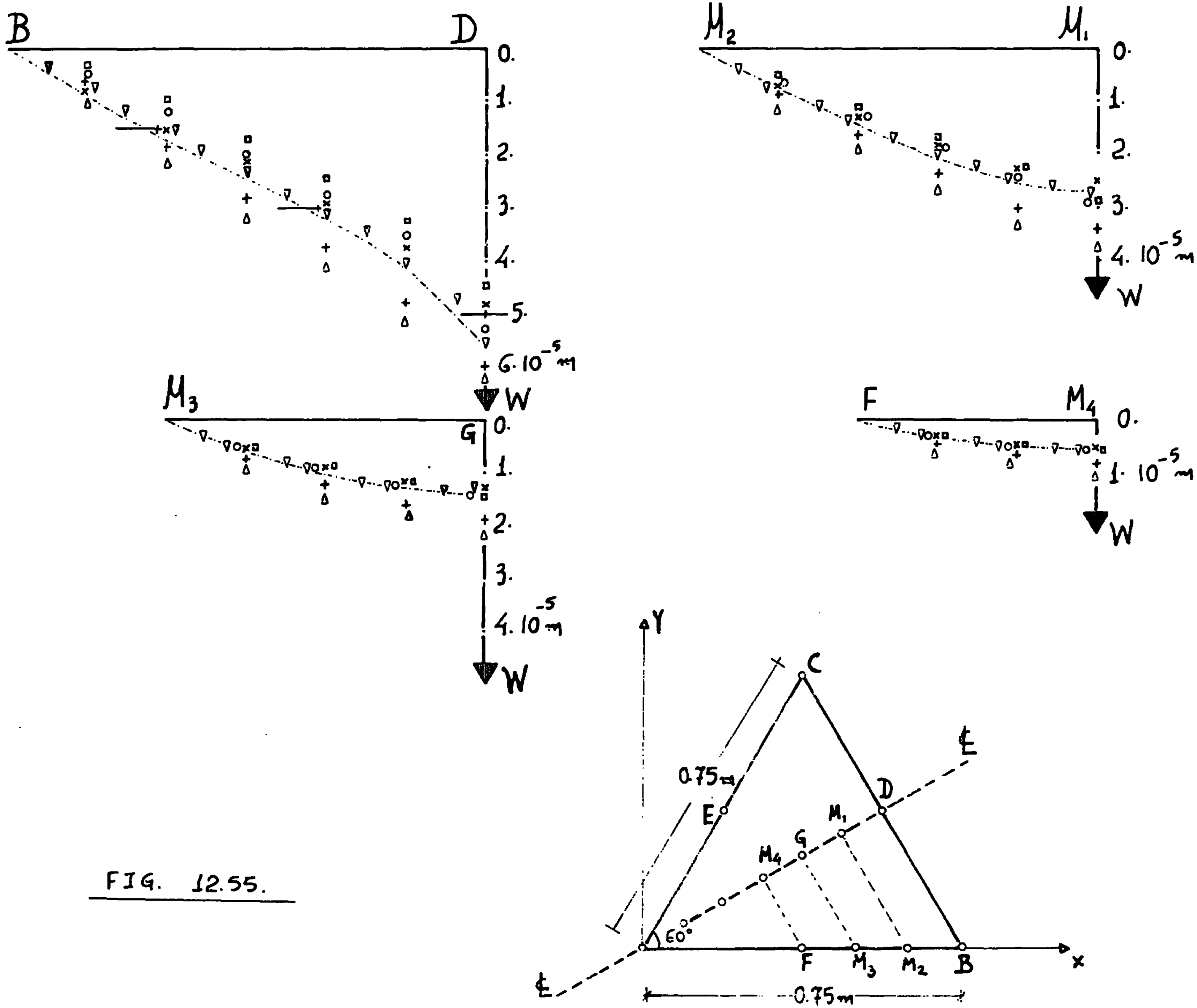

B
D
M2
M1
0.
1.
2.
3.
4.
5.
6. 10^-5 m
4. 10^-5 m
W
M3
G
F
M4
1. 10^-5 m
Y
x
C
E
0.75m
M1
M4
60°
F
M3
M2
0.75m

FIG. 12.55.

CASE [illegible]

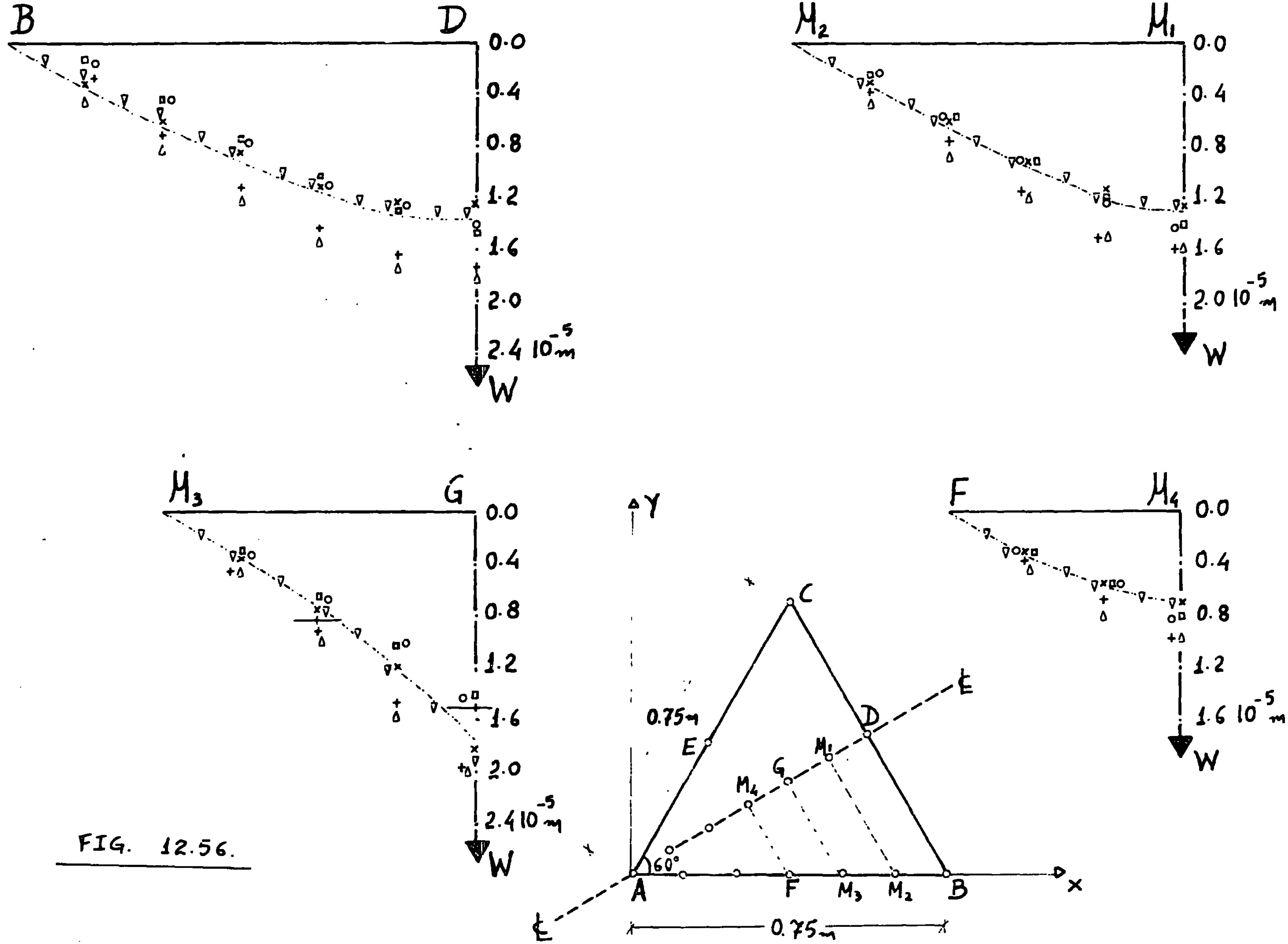


FIG. 12.56.

13. RESULTS FROM THE SANDWICH DOME MODELS

As presented in the previous chapter (see section 12.4), four sandwich dome models were developed.

They were tested with a series of sandwich dome problems and the obtained results were compared with the experimental results obtained for the same problems by previous researchers as well as by the author himself.

These analysed problems are as follows:

(a)	Tetrahedral dome	(fig. 13.6 ÷ 13.17)	[85]
(b)	Square Pyramid	(fig. 13.18 ÷ 13.32)	[21]
(c)	16-faced dome	(fig. 13.33 ÷ 13.33)	[85]
(d)	24-faced dome	(fig. 13.59 ÷ 13.87)	(see chapter 9)
(e)	36-faced dome	(fig. 13.88 ÷ 13.132)	(see chapter 9)

The elastic properties of the sandwich panels from which the above structures were constructed are listed in fig. 13.3.

The elastic properties of the materials, from which the sandwich panels themselves are composed, are outlined in fig. 13.2 (see also references [21, 85]).

13.1 Tetrahedral Dome (figs. 13.6 ÷ 13.17) [85]

The shape, support conditions and dimensions of the structure are shown in figures 13.6 and 13.7. (For a more detailed presentation see references [21, 85]).

This dome was analysed for one loading case. The structure was loaded with 1Nt vertical concentrated load applied at the centroid of face ABC as shown in figures 13.6 and 13.7. Due to symmetry one half of the structure was modelled.

The displacements normal to the face are plotted against experimental results [85], for the loaded panel. (figs. 13.13, 13.13).

It is evident that the presence of a free edge at the loaded panel increases the deformations due to shear, especially as the sandwich panels forming the tetrahedral dome have low shear rigidity. (see fig. 13.3)

It is for this reason that the results obtained from the higher shear variation model, DDS33, appear more accurate when compared with the experimental results.

The horizontal displacements obtained from the various models are plotted in figures 13.8 ÷ 13.11, for half of the whole structure.

The moments and shear forces distribution of the loaded panel ABC, are shown in figures 13.14 ÷ 13.17. The results obtained from the various models when compared with each other appear to agree fairly well in their respective distribution form.

For the mixed model DMX36, the moment M_{nn}, where n is the vector normal to the relevant joint line (see fig. 8.2), was fixed to zero all along the various joint lines of the structure, with the result that the joints acted like hinges.

This constraint of the moment M_{nn} was justified by the way these joints were constructed. (see chapter 10 and reference [85]).

13.2 Square Pyramid (figs. 13.18 ÷ 13.32) [21]

The shape, support conditions and dimensions of the structure are shown in figures 13.18 and 13.19. (For a more detailed presentation see reference [21]).

This structure was analysed when loaded with 1 Nt vertical concentrated load at all four face centroids, as shown in figures 13.18 and 13.19. Due to symmetry only one half of the structure was modelled.

The horizontal displacements for half of the whole structure and the in-plane stresses at specific points of the loaded face EBC are shown in figures 13.20 ÷ 13.23.

The agreement between the results obtained from the various models and the experiment with regard to the in-plane stresses is very good. The models with higher order variation of the in-plane displacements (DDS33, DMX36, DRO30) show increased values of horizontal displacements, as expected, when compared with the linear variation model with regard to the in-plane displacements. (DDS21).

Displacements normal to the face are plotted against experimental results [21], for the loaded panel EBC. (see figs. 13.24, 13.27). The agreement between the experimental results and the result obtained from the various models is good, especially for the models DDS21 and DMX36.

The free edge does not influence the behaviour of the loaded panel in the same way as outlined in the previous section.

This is due to the high shear rigidity of the aluminium sandwich panels from which the square pyramid is composed. (see fig. 13.3).

The distribution of moments for the loaded panel EBC is shown in figures 13.28 ÷ 13.32.

The results obtained from the various models, when compared with each other, appear to agree very well in their respective distribution form. Moreover, the agreement with the experimental results is also very good.

For the mixed model DMX36, the moment M_{nn}, where n is the vector normal to the relevant joint line (see fig. 8.2), was fixed to zero all along the various joint lines so that the joints acted like hinges. This constraint of the moment M_{nn} was justified by the way these joints were constructed. (see chapter 10 and reference [85]).

13.3 16-faced domes (figs. 13.33 ÷ 13.58) [85]

The shape, support conditions and dimensions of the structure are shown in figures 13.33, 13.34. (For a more detailed presentation see reference [85]).

This structure was analysed for two loading cases. Due to symmetry only $^1/_8$ of the structure was modelled.

The first loading case consisted of 1Nt vertical concentrated load acting at all eight upper panel centroids. (see figs. 13.35 ÷ 13.46).

For the second case the load was 1 Nt vertical concentrated load applied at all eight bottom panel centroids. (see figs. 13.47 ÷ 13.58).

The experimental results are presented in reference [85]. For each loading case, the horizontal and vertical displacements for $^1/_8$ of the whole structure as well as the normal displacements of the loaded face, obtained by employing the four individual dome models, are plotted against the experimental results. [85]. (see figs. 13.39 ÷ 13.42, 13.51 ÷ 13.54)

The agreement between the results obtained from the various models and the experimental results varies from quite good for the first model (DDS21) to very good for the remaining models (DDS33, DMX36, DRO30). (see figs. 13.39 ÷ 13.42, 13.51 ÷ 13.54)

The reason for this is the linear variation of the in-plane displacements for the first model (DDS21) in comparison with the higher order variation for the same parameters for the other models:

For the second loading case the presence of the free edge and the low shear rigidity of the sandwich panels forming the structure, resulted in the same behaviour, with regard to the higher order shear model DDS33, as the one observed in the tetrahedral dome (see section 13.1), in that the latter model showed increased flexibility.

For the mixed model, DMX36, the moment M_{nn}, where n is the vector normal to the relevant joint line (see fig. 8.2), was fixed to zero all along the various joint lines of the structure, with the result that the joints acted like hinges.

This constraint of the moment M_{nn} was justified by the way these joints were constucted. (see chapter 10 and reference [85]).

13.4 <u>24-faced dome</u> (figs. 13.59 ÷ 13.87)

The shape, support conditions and dimensions of the structure are shown in figs. 13.59, 13.60.

Details relevant to the construction and testing of this dome are presented in chapter 9.

This structure was analysed for two loading cases. Due to symmetry, only $^{1}/_{12}$ of the structure was modelled.

The first loading case consisted of 1 Nt vertical concentrated load acting at all twelve upper panel centroids. (see figs. 13.62 ÷ 13.74)

In the second loading case the vertical concentrated load of 1 Nt was applied at all twelve bottom panel centroids. (see figs. 13.75 ÷ 13.87).

The experimental results are presented in figures 13.62 and 13.75 for each loading case respectively.

The horizontal and vertical displacements for $^{1}/_{12}$ of the whole structure as well as the normal displacements of the loaded face were obtained for both loading cases by employing all four individual dome models.

These results are plotted against experimental results obtained by testing the structure as presented in chapter 9.

For both loading cases, the higher order variation of the in plane displacement models DDS33, DMX36 and DRO30, yield better accuracy than the linear variation of the in plane displacement model DDS21.

The accuracy obtained is satisfactory. All the models agree in their deformation pattern with the experimental results. The higher order shear variation model DDS33 agrees well with the experimental results especially for the second loading case, where the presence of the free edge at the loaded panel influences the results in the same way as explained previously. (see sections 13.1, 13.3).

For the mixed model DMX36, the moment M_{nn}, where n is the vector normal to the joint-line AB (see fig. 8.2) was constrained to zero all along the line for both loading cases. This was enforced in order to simulate the joint action as outlined in chapter 10.

Some further conclusions drawn from the analysis of the 24-faced dome are as follows:

First, with regard to the joint-lines, and, in particular those unaffected by the joint action described in chapter 10, the covering steel plates (see chapters 9. 10) have a stiffening effect on the structure in the longitudinal direction of the joint. As a result, a more detailed analysis must be carried out by modelling the area adjacent to the joint lines with elements of a higher stiffness.

The second conclusion refers to the support conditions of the overall structure. In the analysis it was assumed that the structure is supported at points with all three displacements u, v, w as well as the moments M_z, and M_x (where x, and z the gloval axes, see figs. 13.59, 13.60) fixed to zero.

The supports at the actual structure were constructed in such a way that the above assumption is fulfilled (see chapter 9).

However, due to the considerable stiffness of those parts of the supports which are joined to the sandwich panels the assumption of points supports is not valid.

Thus for a more correct representation of the supports' behaviour a line support instead of a point support is recommended.

In addition, the elements adjacent to this line support must be considered with an increased stiffness.

13.5 <u>36-faced dome</u> (see figs. 13.88 ÷ 13.132)

The shape, support conditions and dimensions of the structure are shown in figures 13.88 ÷ 13.90

Details relevant to the construction and testing of this dome are presented in chapter 9. This structure was analysed for three loading cases. Due to symmetry only $^{1}/_{12}$ of the whole structure was modelled.

The first loading case consisted of 1 Nt vertical concentrated load acting at all twelve upper panel centroids. (see figs. 13.91 ÷ 13.104)

In the second loading case a vertical concentrated load of 1 Nt was applied at all twelve bottom panel centroids. (see figs. 13.105 ÷ 13.118)

For the third loading case the same load was applied at all twelve centroids of the panels forming the dormer sections. (see figs. 13.119 ÷ 13.132)

The horizontal and vertical displacements for $^1/_{12}$ of the whole structure as well as the normal displacements of the loaded face, were obtained for each individual loading case, employing the four dome models.

These results are plotted against the experimental results obtained by testing the structure as presented in chapter 9. The same conclusions as stated in the previous section are again valid for this dome.

The models DDS33, DMX36 and DRO30 are more accurate than the model DDS21. All the models agree in their deformation pattern with the experimental results.

The higher shear order element, DDS33, agrees well with the experimental results especially for the third loading case where the presence of the free edges on the dormer section panels influences the results in the same way as discussed in sections 13.1 and 13.3.

For the mixed model, DMX36, the moment M_{nn}, where n is a vector normal to a line (see fig. 8.2), was constrained to zero all along the joint-lines, AB and BD, for every loading case.

This was enforced in order to simulate the joint action as explained in chapter 10.

An investigation of the distribution of the normal moment, M_{nn}, all along the joint lines for the different models, was carried out from the obtained results.

It is evident from the cases, that when the load is applied on a panel with a joint in the form of a valley adjacent to it then this joint will experience normal moments M_{nn}, of a considerable magnitude exceeding the limits found in the analysis of chapter 10 with regard to the joint action.

Further investigation and adaptation of the various computer programs, developed in the present work, which will incorporate the function of the joints, is therefore recommended. The mixed model, DMX36, has the most potential in that respect due to the presence of moments as nodal degrees of freedom.

A solution to the problem would be to introduce elastic constraints so that the normal moments could be transferred at a joint line between adjacent panels only when its magnitude does not exceed a certain limit. This limit can be decided on by a semi-analytical method, (chapter 10) or by purely experimental investigations.

14. CONCLUSIONS

Four finite element models were developed for the analysis of the polyhedral dome sandwich structures.

After a comparative study between the results obtained from these four dome models and the experimental results of the five specific analysed domes (see previous sections) it is evident that the accuracy obtained from each model varies from fair to very good depending on the nature of the problem.

A convergency study carried out for the square pyramid (fig. 13.5) indicates that the various models converge in a monotonic manner.

All the examined structures were analysed, using a subdivision of 100 triangular elements for each panel of the polyhedron.

Each individual model carries particular features which makes it more accurate for certain cases. These are as follows:

The first model,DDS21, has the advantage of possessing the lowest number of degrees of freedom.

For problems where the bending action is predominant, when compared with the membrane action, this model yields accurate results.

For structures where the membrane action becomes predominant, as in the case of the last three polyhedra, the finite element models, with a variation of the in plane displacements of an order higher than linear, yield better results. This is due to the slow convergency of the linear variation of the in-plane displacement finite element which is part of the first dome model DDS21. As a result the model becomes relatively stiffer and produces less accurate results.

As stated for the examined sandwich plate bending, as well as the sandwich dome problems, structures with free edges experience high deformations of the free edge when loaded at the relevant panel.

This is particularly valid for structures composed of sandwich panels with core of low shear rigidity. In this case a representation of the shear deformation with a variation higher than linear yields better results.

As discussed previously, the joint action could be of considerable significance for the dome structures. Further research must be carried out either analytically or experimentally in this direction as suggested.

This can be achieved numerically by modifying the models developed in the present work so that they can accommodate the desired joint action. The mixed formulation is more succesful in this respect.

The boundary conditions is another important factor and has to be treated with great care as it has a very decisive effect on the results.

Point supports, as mentioned for the sandwich plate bending and sandwich dome problems, present difficulties which can lead to incorrect physical representation of the structure. They can also lead to numerical inadequacies.

Depending on the degree of accuracy desired and the funds available a more detailed study can be performed including all these factors. The developed programs are able to accommodate such options.

For a complete understanding of the behaviour of the structures analysed in the present work the effects of geometric nonlinearities (large deflections) and the time dependent phenomena (creep) must be considered.

These structures when loaded for a long period become subject to considerable deformation which is largely due to the creep effect. This is evident from the experiments performed by Parton as well as from those carried out by the author. [85]

The first model DDS21 has the lowest number of degrees of freedom and consequently is the most efficient for use in any future analysis which includes the creep effects.

This is due to the iteration procedure which must be employed in order to solve the time dependent equations.

The author investigated these aspects but more research in this direction is necessary.

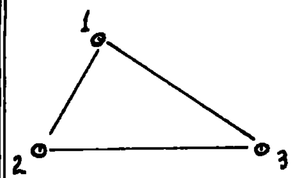
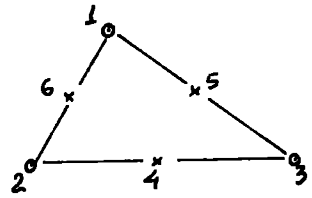
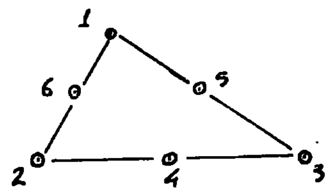
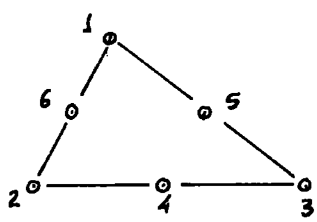
REFERENCE SYMBOL DEGREES OF FREEDOM	SANDWICH – DOME MODELS	STRESSES
DDS21 ⊙ $u, v, w, w_x, w_y, \phi_x, \phi_y$ DISPLACEMENT MODEL		$M_{xx_c}, M_{yy_c}, M_{xy_c}$ Q_{x_c}, Q_{y_c} $N_{xx_c}, N_{yy_c}, N_{xy_c}$ } at the centroid
DDS33 ⊙ $u, v, w, w_x, w_y, \phi_x, \phi_y$ × $\phi_x, \phi_y, u_{sy}, v_{BS}$ DISPLACEMENT MODEL		$M_{xx_c}, M_{yy_c}, M_{xy_c}$ Q_{x_c}, Q_{y_c} $N_{xx_c}, N_{yy_c}, N_{xy_c}$ } at the centroid
DMX36 ⊙ $u, v, w, M_{xx}, M_{yy}, M_{xy}$ MIXED MODEL		Q_{x_1}, Q_{y_1} at 1st node Q_{x_2}, Q_{y_2} at 2nd node $N_{xx_c}, N_{yy_c}, N_{xy_c}$ at the centroid
DRO30 ⊙ $u, v, w, \theta_x, \theta_y$ DISPLACEMENT MODEL		$M_{xx_c}, M_{yy_c}, M_{xy_c}$ Q_{x_c}, Q_{y_c} $N_{xx_c}, N_{yy_c}, N_{xy_c}$ } at the centroid

FIG. 13.1. SANDWICH DOME MODELS

	MATERIALS (REFERENCE [21])			thickness measured (m)	ELASTIC PROPERTIES (Nt/m²)					
					$\frac{C^{xx}_{xx}}{10^{10}}$	$\frac{C^{yy}_{yy}}{10^{10}}$	$\frac{C^{xx}_{yy}}{10^{10}}$	$\frac{C^{xy}_{xy}}{10^{10}}$	$\frac{C^{yz}_{yz}}{10^{7}}$	$\frac{C^{xz}_{xz}}{10^{7}}$
1	PLYWOOD	FACE	ORTHOTROPIC	0.00166	1.2÷1.4	0.78÷0.91	0.32÷0.38	0.55		
2	ALUMINIUM	FACE	ISOTROPIC	0.000486	6.8	6.8	2.0	4.8		
3	FIBREGLASS	FACE	>>	VARIED						
4	HARDBOARD (ICI)	FACE	>>	0.00408	0.56	0.56	0.168	0.392		
5	HARDBOARD	FACE	>>	0.00340	0.42	0.42	0.126	0.294		
6	EXPANDED POLYURETHANE	CORE	>>	0.0260					0.2	0.2
7	EXPANDED POLYURETHANE	CORE	>>	0.0190					0.2	0.2
8	EXPANDED POLYURETHANE	CORE	>>	0.0125					0.2	0.2
9	EXP. POLYURETHANE BONDED TO THE HARDBOARD	CORE	>>	0.0260					0.2	0.2
10	EXPANDED POLYVINYL CHLORIDE	CORE	>>	0.0110					1.0	1.0
11	EXPANDED POLYURETHANE	CORE	>>	0.0190					0.14	0.14

FIG. 13.2. ELASTIC PROPERTIES

	Polyhedral dome constructed by sandwich panels	Composed from materials	Elastic propepties of sandwich panels (Reference [21])										
			D^{xx}_{xx}	$D^{xy}_{yy}=D^{yy}_{xx}$	D^{yy}_{yy}	D^{xy}_{xy}	S^{xz}_{xz}	S^{yz}_{yz}	E^{xx}_{xx}	E^{xx}_{yy}	E^{yy}_{xx}	E^{yy}_{yy}	E^{xy}_{xy}
			10^4 Nt m	10^3 Nt m	10^4 Nt m	10^4 Nt m	10^5 Nt/m	10^5 Nt/m	10^8 Nt/m	10^7 Nt/m	10^7 Nt/m	10^8 Nt/m	10^8 Nt/m
1	Tetrahedral dome plates (1) $h=0.031$ m $c/\ell=15.66$	1 and 6	0.97	2.6	0.68	0.71	0.59	0.59	0.46	1.3	1.3	0.30	0.37
2	Tetrahedral dome plates (2) $h=0.023$ m $c/\ell=11.44$	1 and 7	0.54	1.5	0.38	0.39	0.45	0.45	0.46	1.3	1.3	0.30	0.37
3	Tetrahedral dome plates (3) $h=0.016$ m $c/\ell=7.5$	1 and 8	0.254	0.69	0.18	0.19	0.32	0.32	0.46	1.3	1.3	0.30	0.37
4	Square pyramid $h=0.013$ m $c/\ell=22.63$	2 and 10	0.29	0.72	0.29	0.17	3.7	3.7	0.66	1.9	1.9	0.66	0.47
5	16 faced dome $h=0.016$ m $c/\ell=7.5$	1 and 8	0.19	0.52	0.135	0.14	0.32	0.32	0.35	0.98	0.98	0.225	0.27
6	24 faced dome $h=0.058$ m $c/\ell=12.75$	4 and 9	2.6	7.9	2.6	0.92	1.4	1.4	0.41	1.2	1.2	0.41	0.29
7	36 faced dome $h=0.058$ $c/\ell=12.75$	4 and 9	2.6	7.9	2.6	0.92	1.4	1.4	0.41	1.2	1.2	0.41	0.29
8	Hexagoual dome $h=0.032$ $c/\ell=7.03$	4 and 9	0.57	1.7	0.57	0.4	0.99	0.99	0.41	1.2	1.2	0.41	0.29

FIG. 13.3. ELASTIC PROPEPTIES OF SANDWICH PANELS

DOME ELEMENTS (POLYHEDRAL STUCTURE OF TWO PLATES

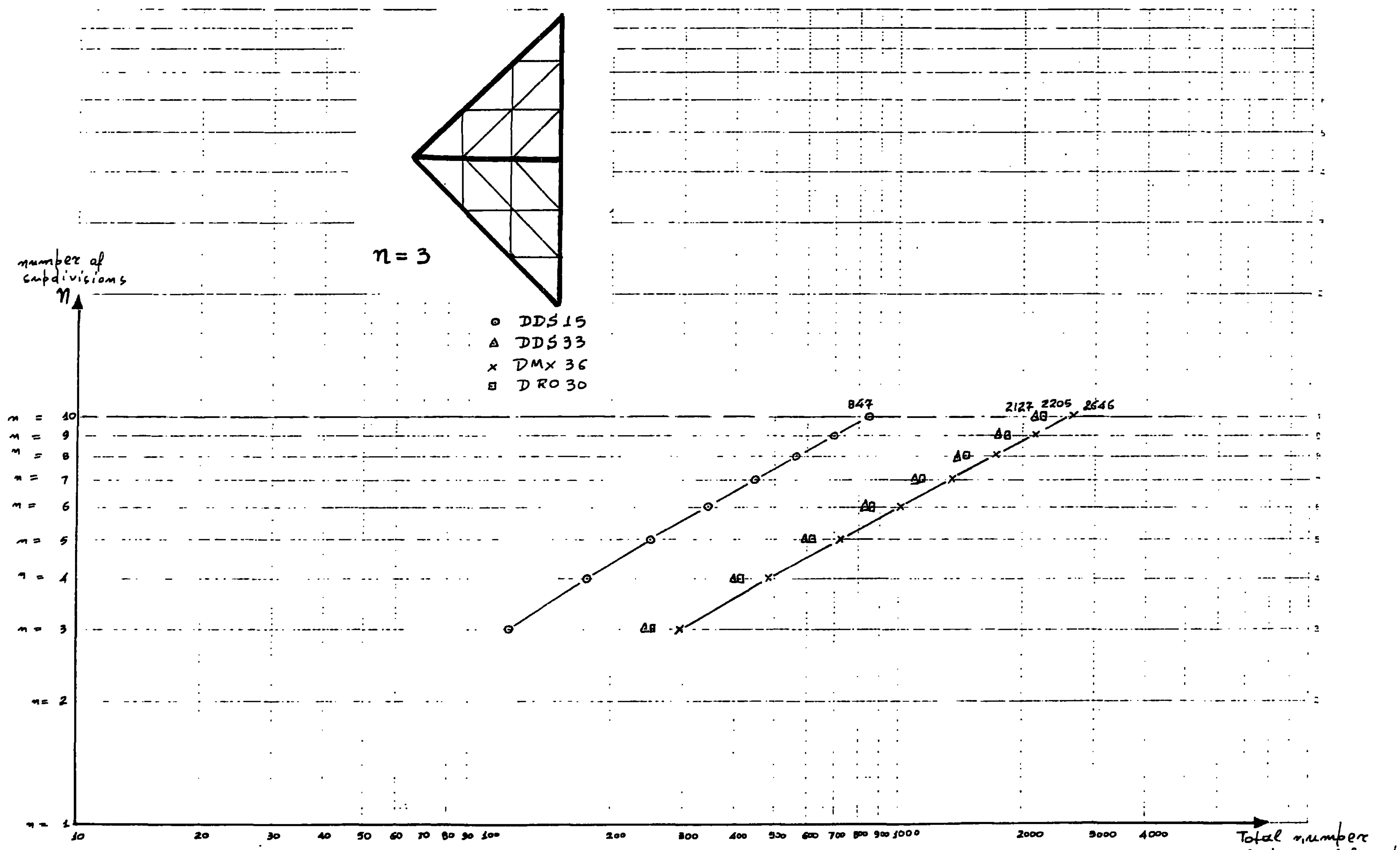
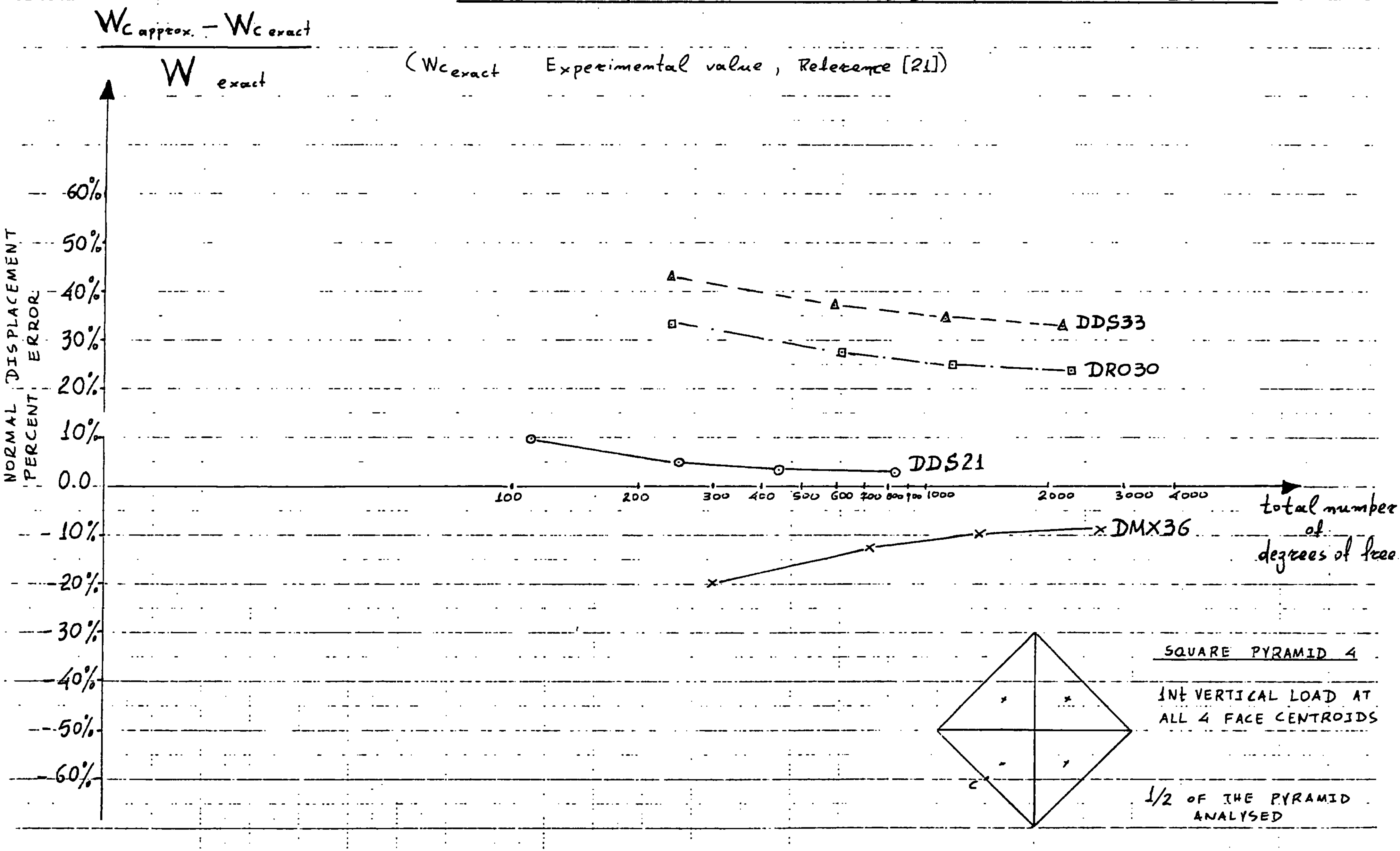


FIG. 13.4.

FIG. 13.5. CONVERGENCE TOWARDS SERIES SOLUTION



TETRAHEDRAL DOME (PLATES 3)

SCALE : $1 cm = 10^{-1} m$

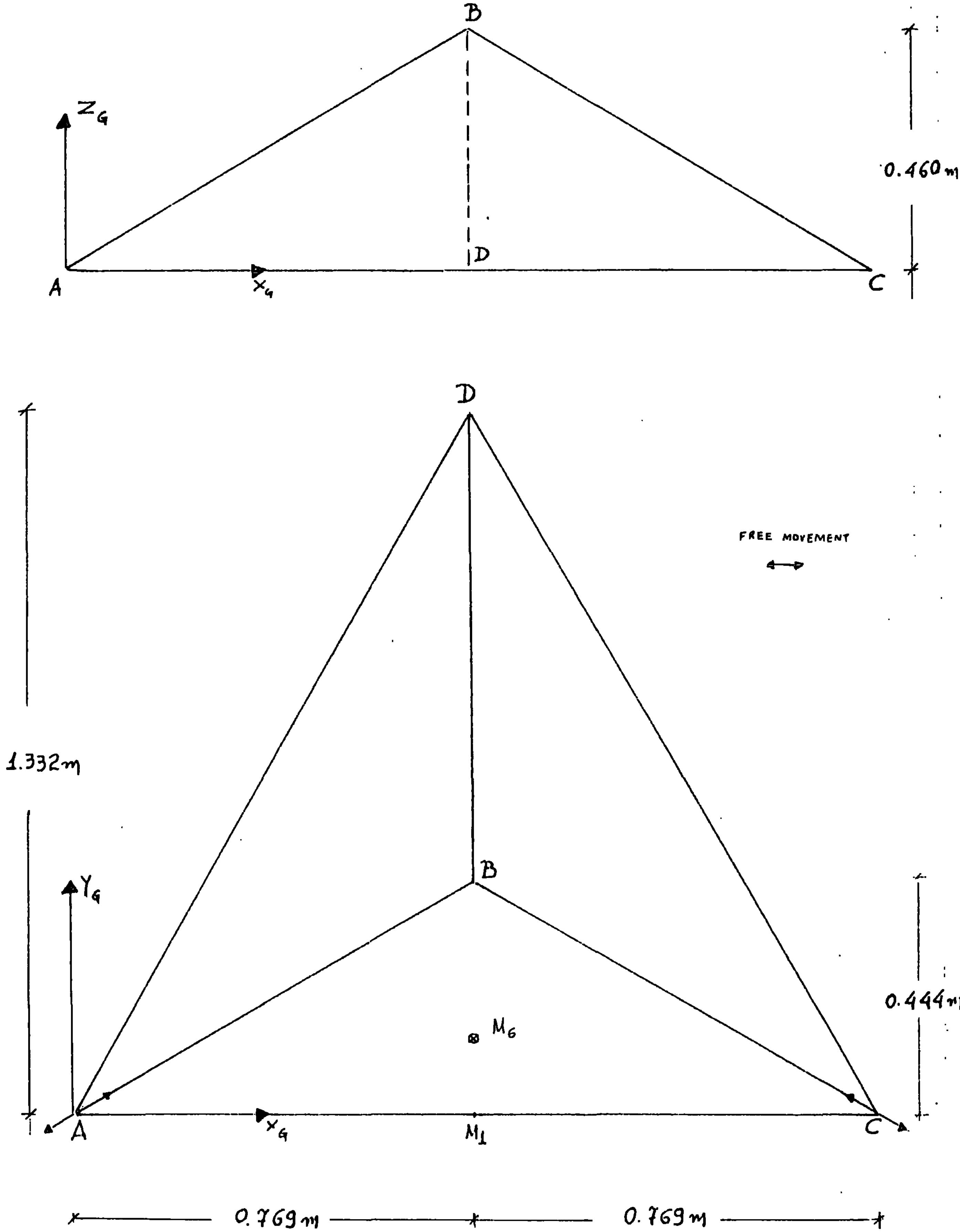


FIG. 13.6. GENERAL ARRANGEMENT

TETRAHEDRAL DOME (PLATES 3)

1Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

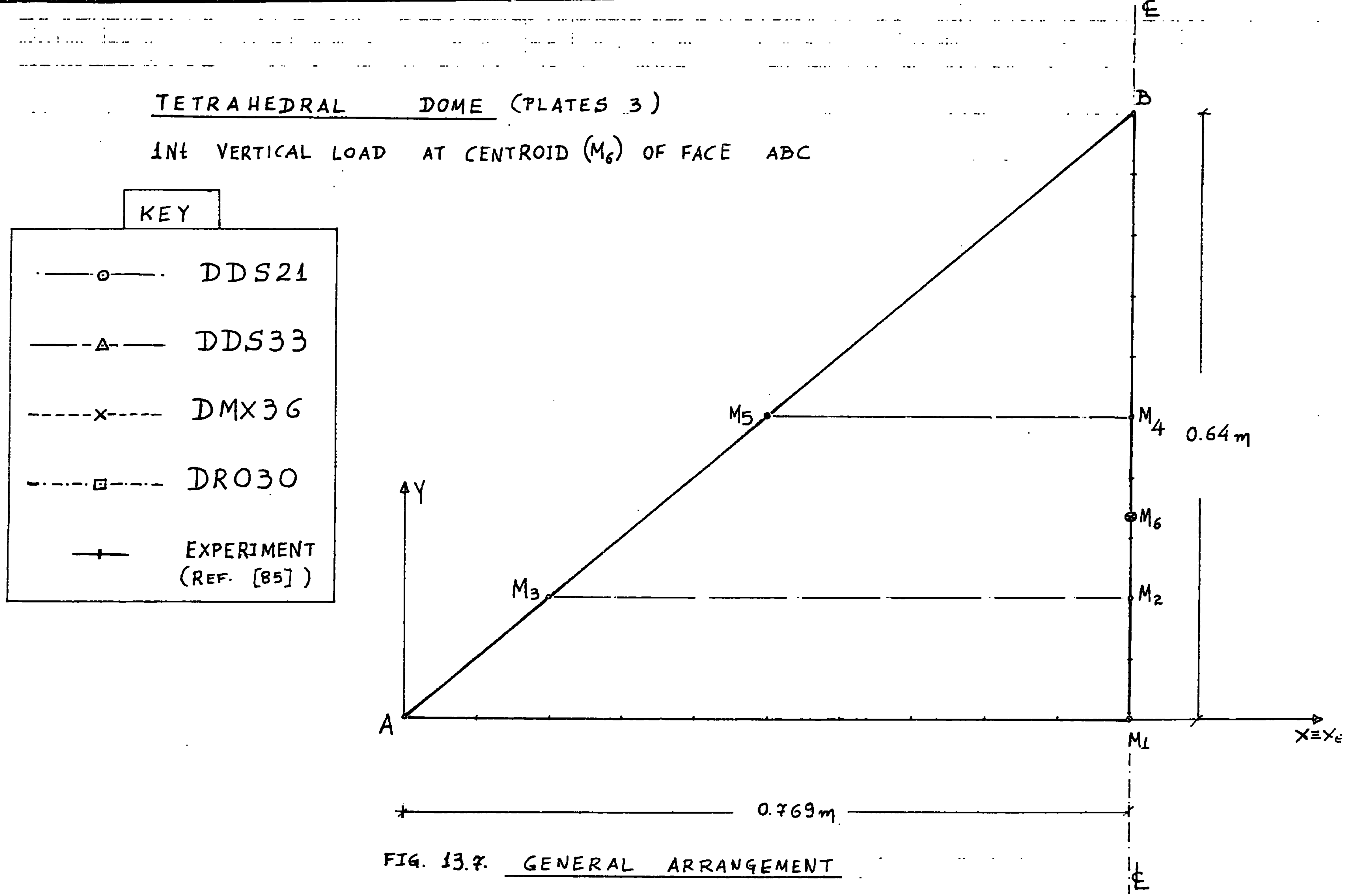


FIG. 13.7. GENERAL ARRANGEMENT

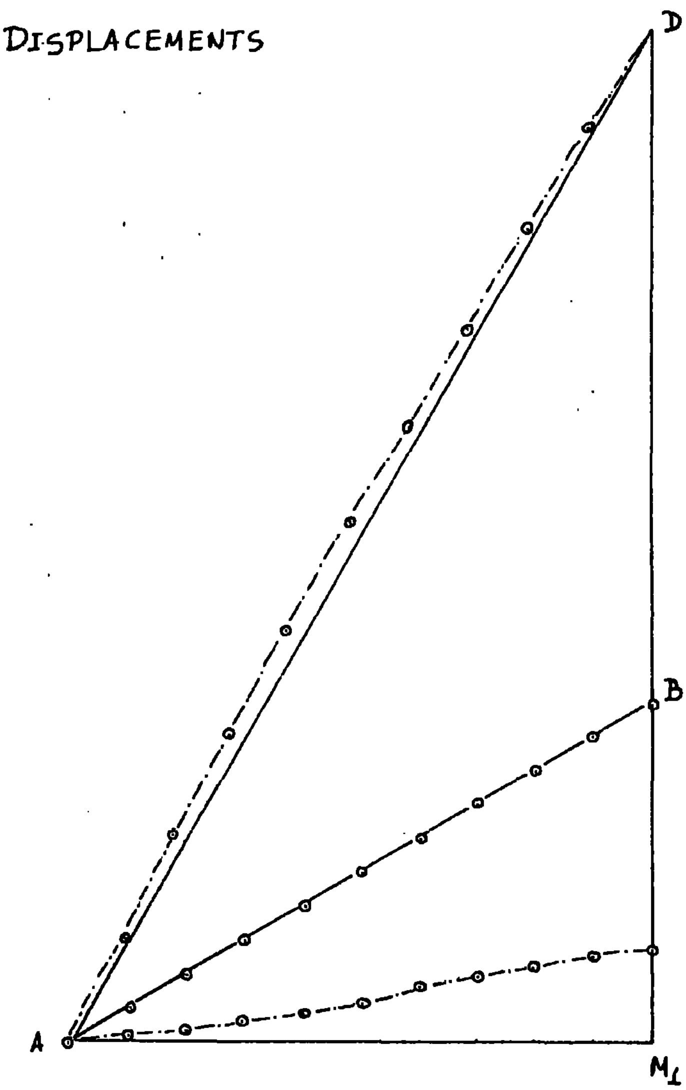
1Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

HORIZONTAL DISPLACEMENTS

SCALES:

$1 cm = 10^{-1} m$ LENGTH

$1 cm = 10^{-5} m$ DISPLACEMENTS



TETRAHEDRAL DOME (PLATES 3)

FIG. 13.8.

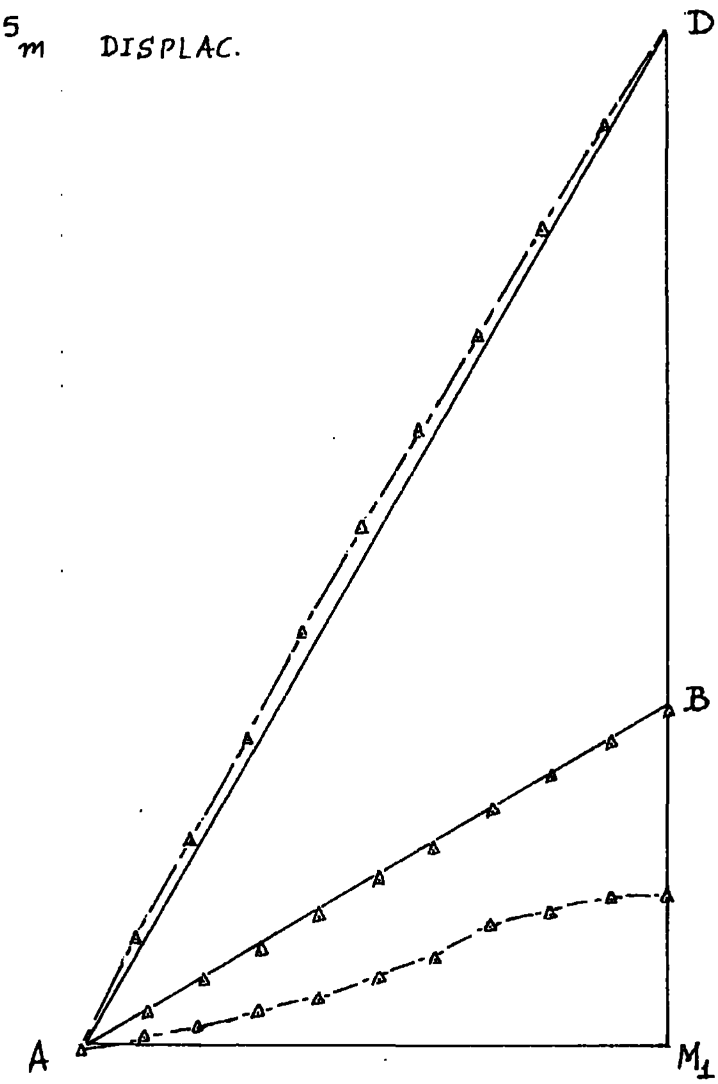
1Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

HORIZONTAL DISPLACEMENTS

SCALES :

$1cm = 10^{-1}m$ LENGTH

$1cm = 10^{-5}m$ DISPLAC.



TETRAHEDRAL DOME (PLATES 3)

FIG. 13.9.

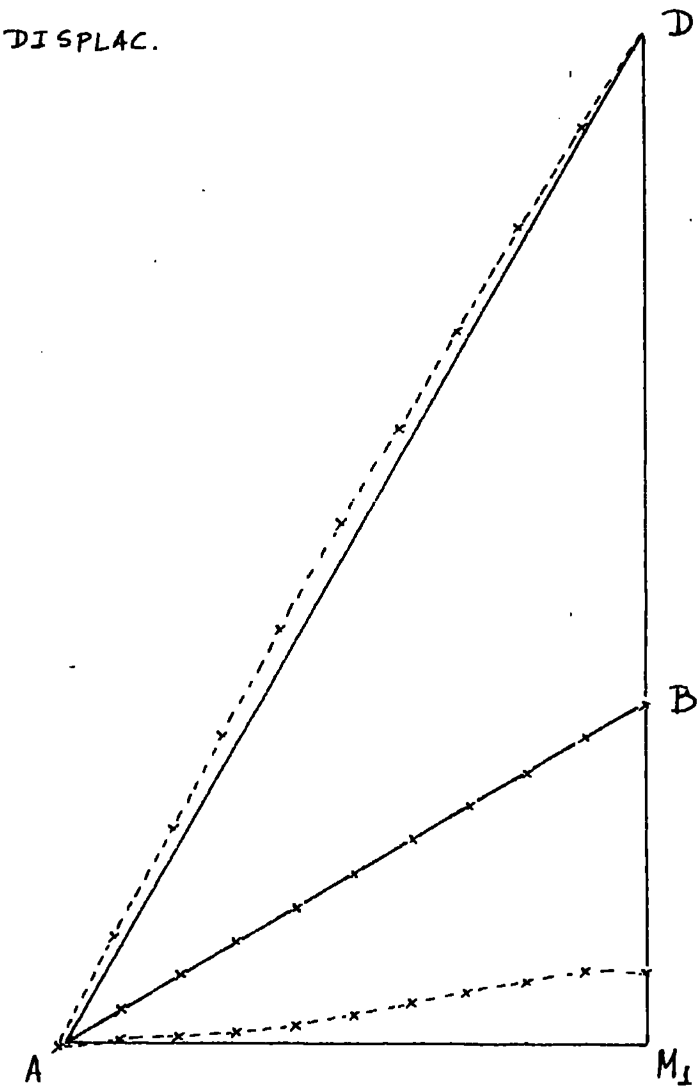
1 Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

HORIZONTAL DISPLACEMENTS

SCALES:

1cm = 10^{-1}m LENGTH

1cm = 10^{-5}m DISPLAC.



TETRAHEDRAL DOME (PLATES 3)

FIG. 13.10.

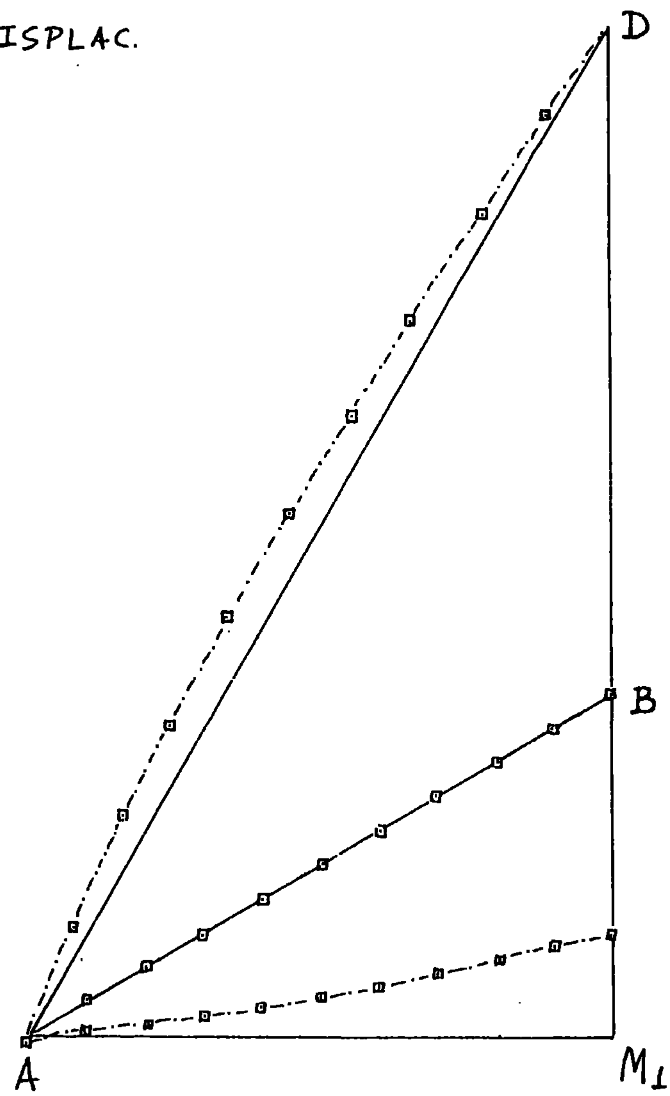
1Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

HORIZONTAL DISPLACEMENTS

SCALES:

1cm = 10^{-1} m LENGTH

1cm = 10^{-5} m DISPLAC.



TETRAHEDRAL DOME (PLATES 3)

FIG. 13.11.

1Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

NORMAL DISPLACEMENTS OF LOADED FACE

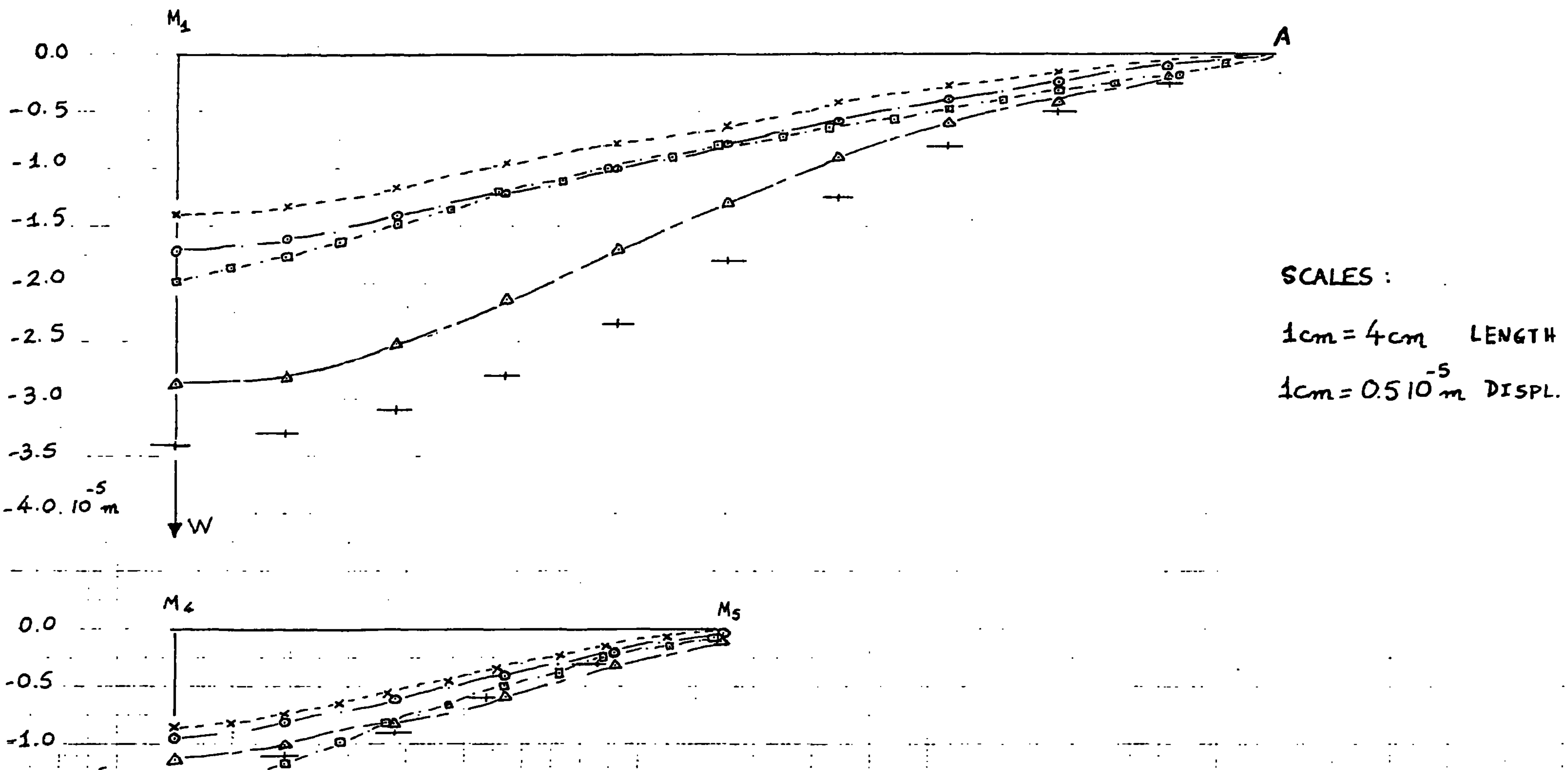
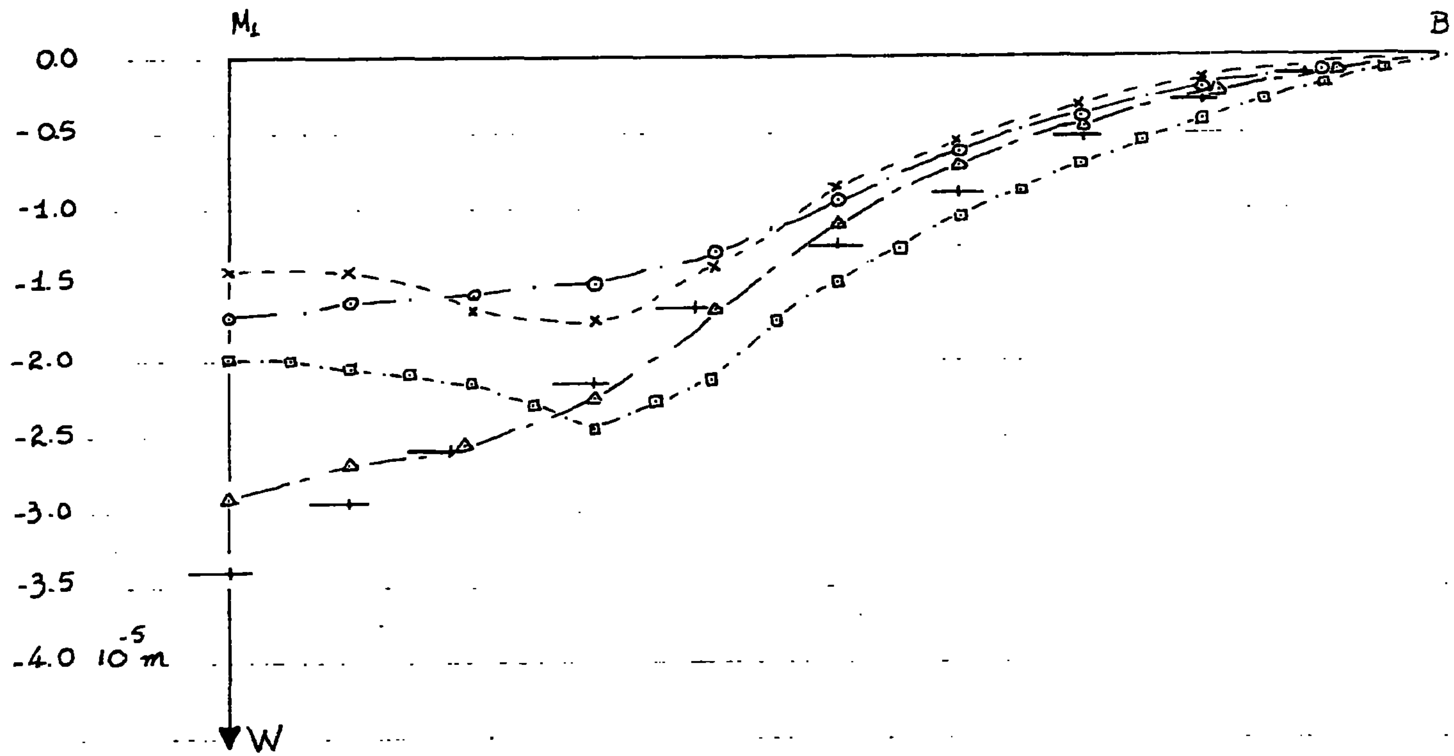


FIG. 13.12. TETRAHEDRAL DOME (PLATES 3)

1Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

NORMAL DISPLACEMENTS OF LOADED FACE



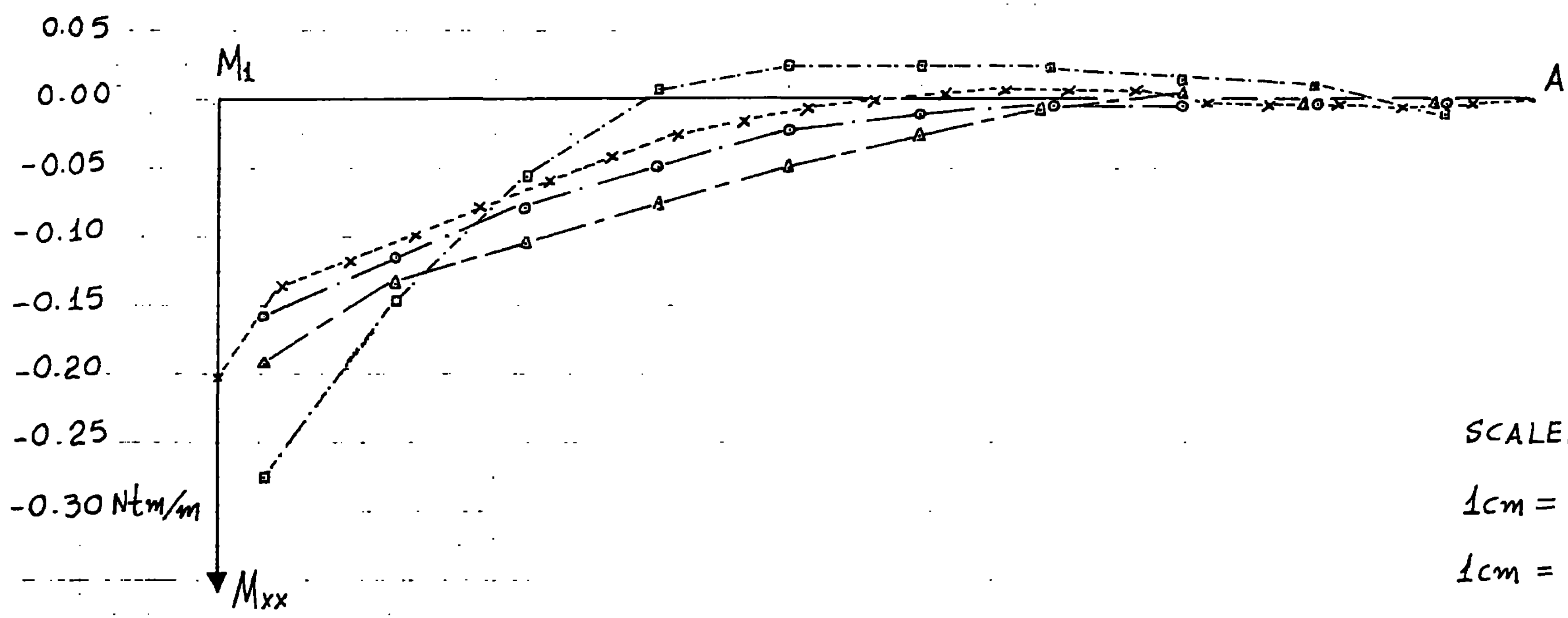
SCALES:

1cm = 4cm LENGTH

1cm = 0.5 10^{-5}m DISPLAC.

FIG. 13.13. TETRAHEDRAL DOME (PLATES 3)

1Nt VERTICAL LOAD AT CENTROID (M_c) OF FACE ABC



SCALES

1cm = 4cm LENGTH

1cm = $5 \cdot 10^{-2}$ Ntm/m MOMENTS

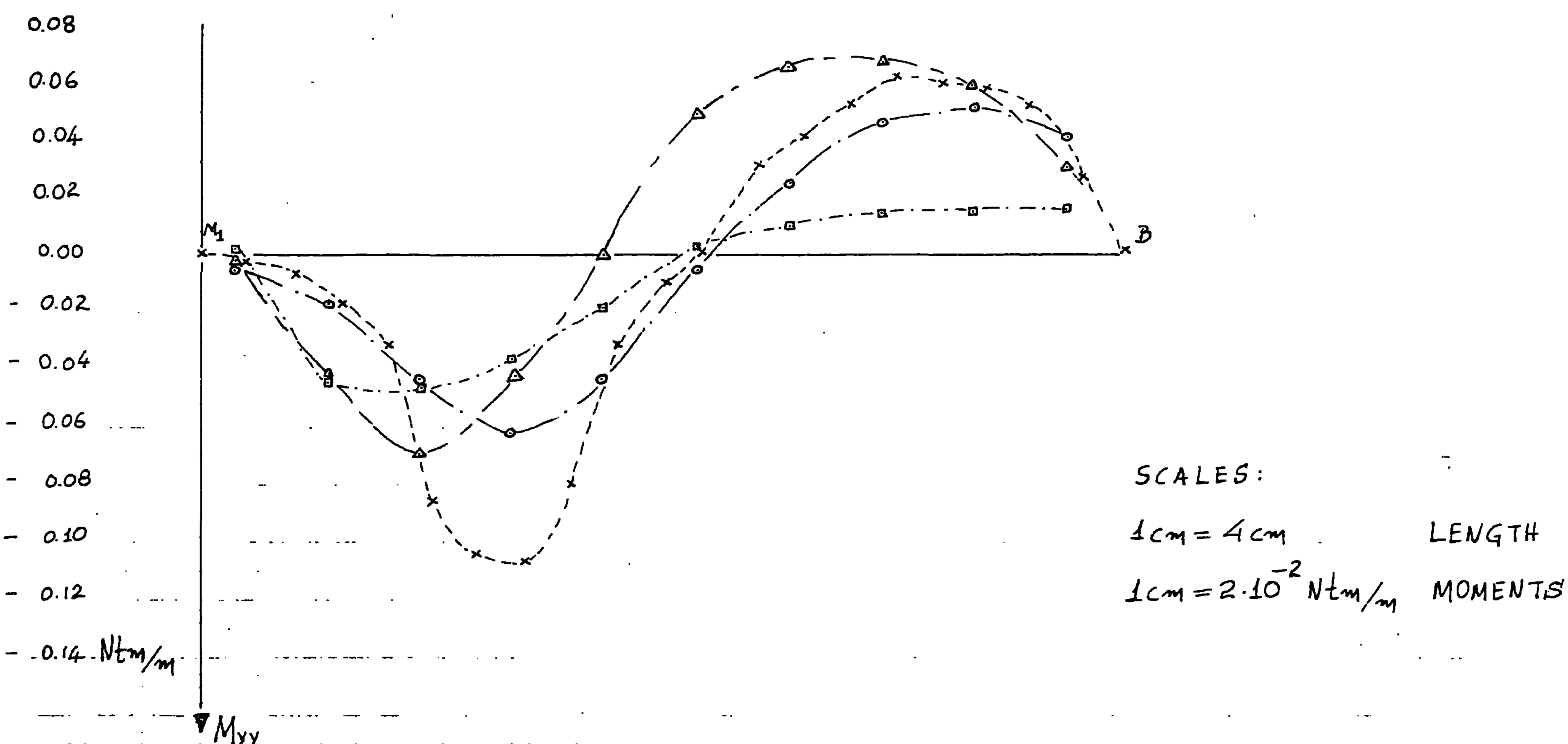
MOMENTS M_{xx} OF LOADED FACE

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.14.

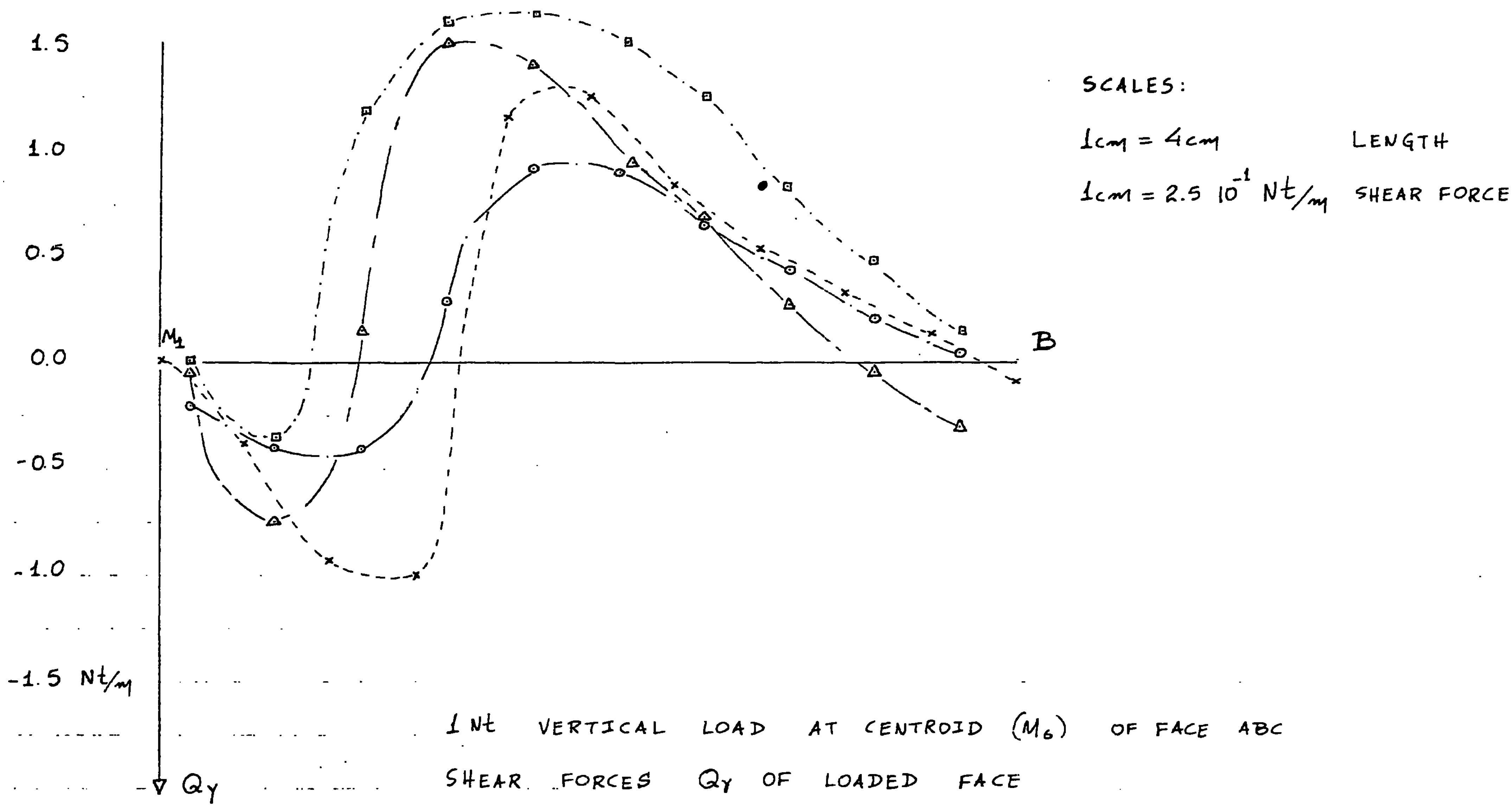
1Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

MOMENTS M_{yy} OF LOADED FACE



TETRAHEDRAL DOME (PLATES 3)

FIG. 13.15.



1 Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

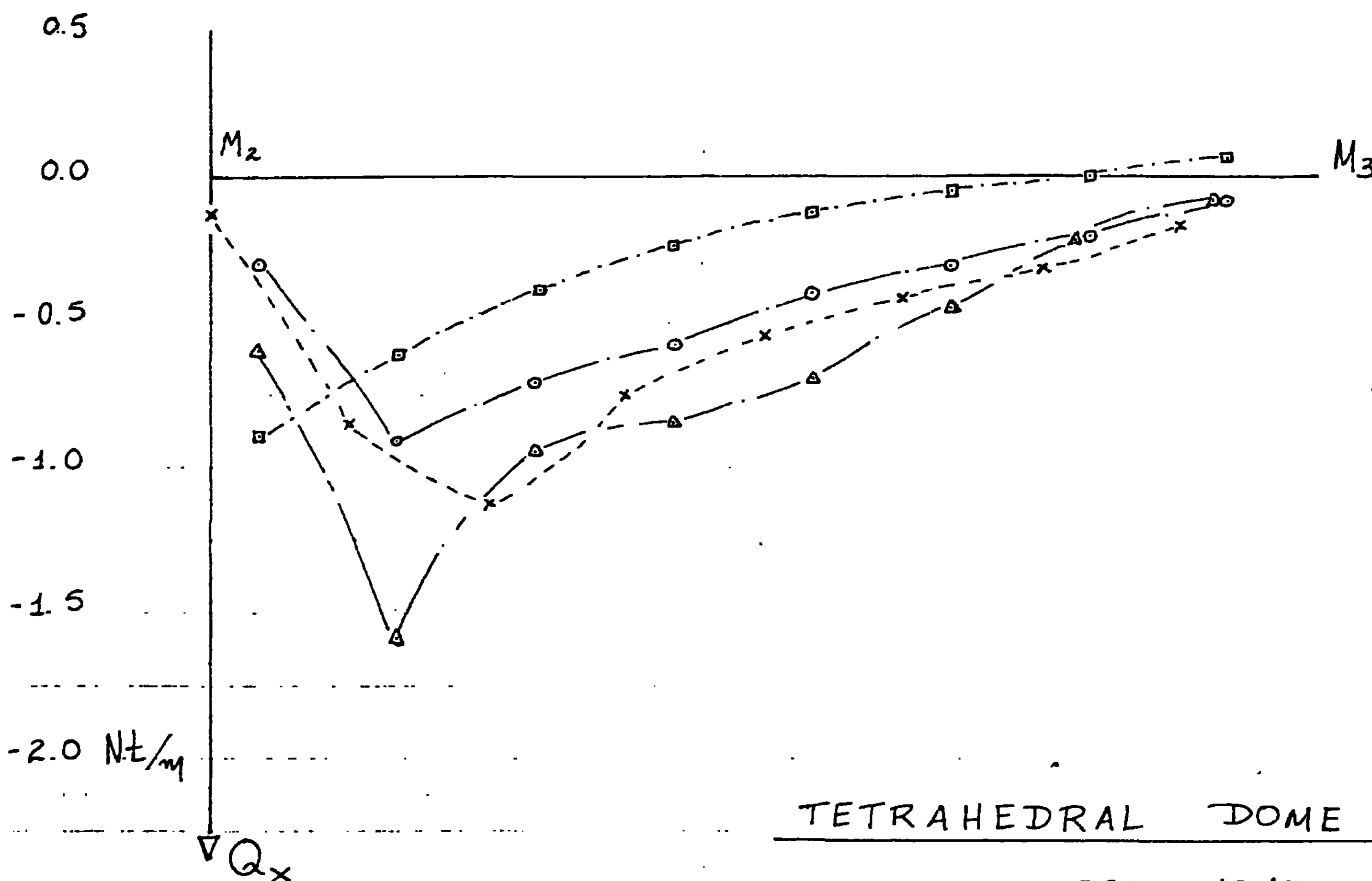
SHEAR FORCES Q_Y OF LOADED FACE

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.16.

1 Nt VERTICAL LOAD AT CENTROID (M_6) OF FACE ABC

SHEAR FORCES Q_x OF LOADED FACE



SCALES :

1cm = 4cm LENGTH

1cm = 2.5 10^{-1} Nt/m SHEAR FORCE

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.17

SQUARE PYRAMID

GENERAL ARRANGEMENT

SCALE: 1cm = 10⁻¹m

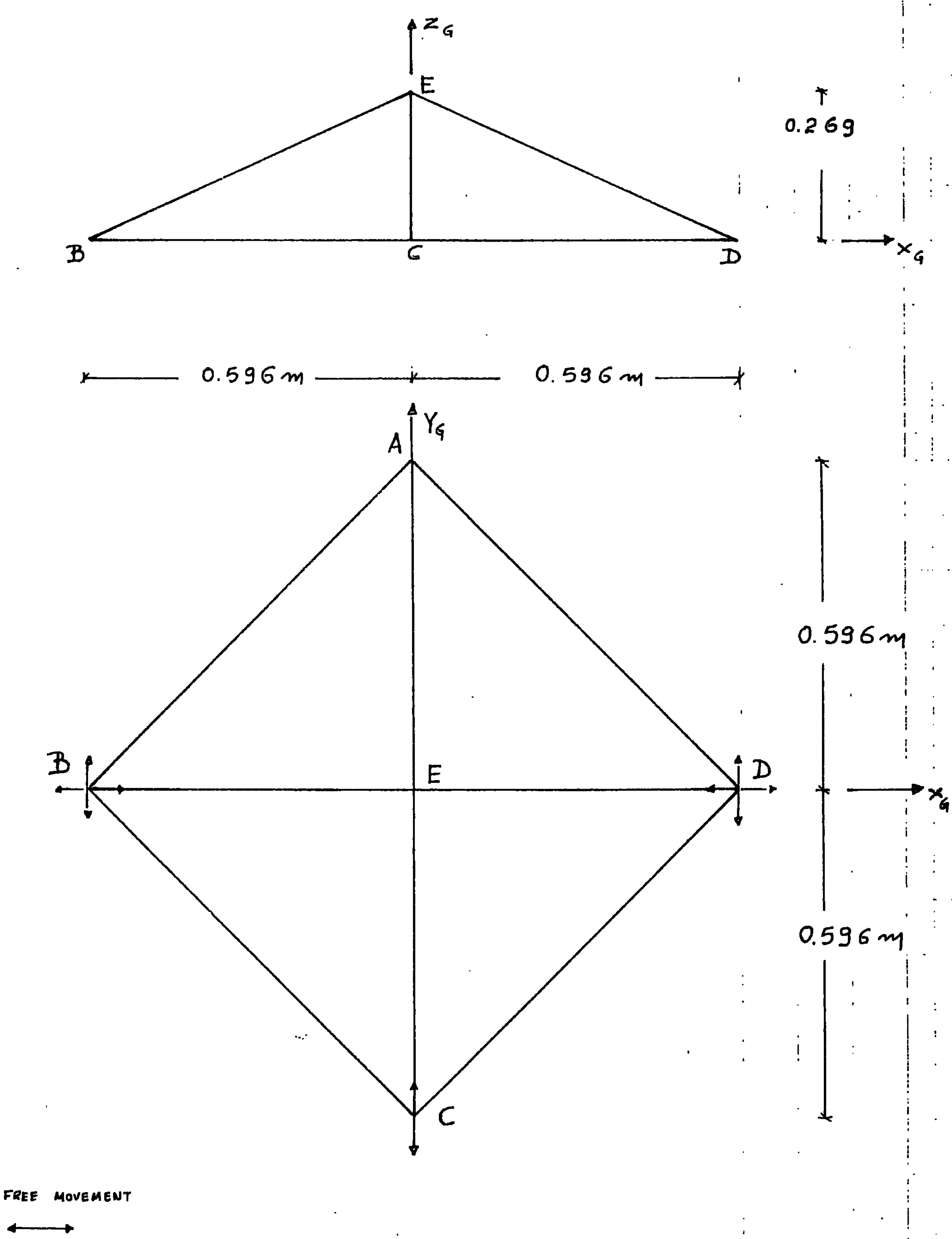


FIG. 13.18.

SQUARE PYRAMID

GENERAL ARRANGEMENT

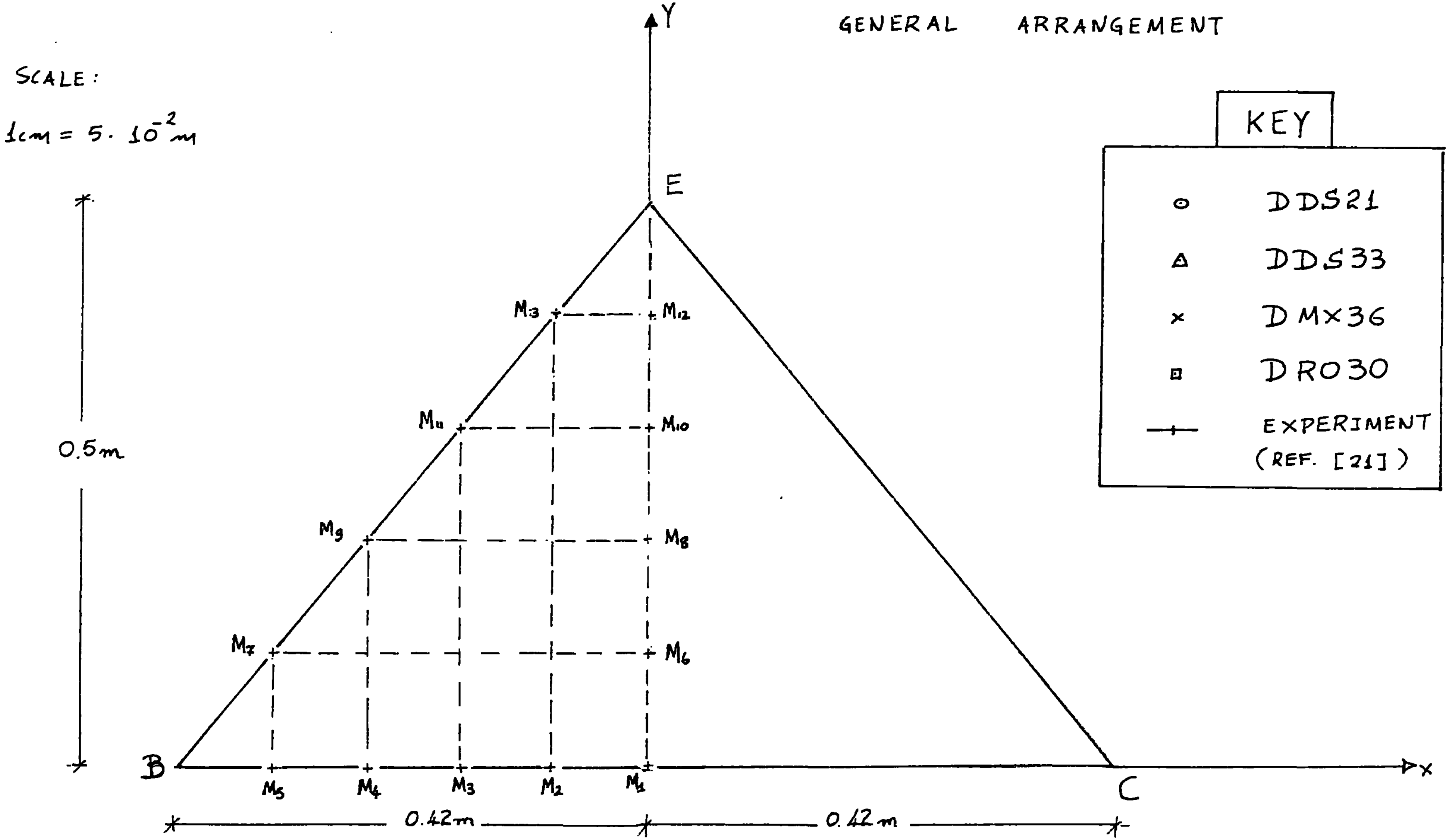
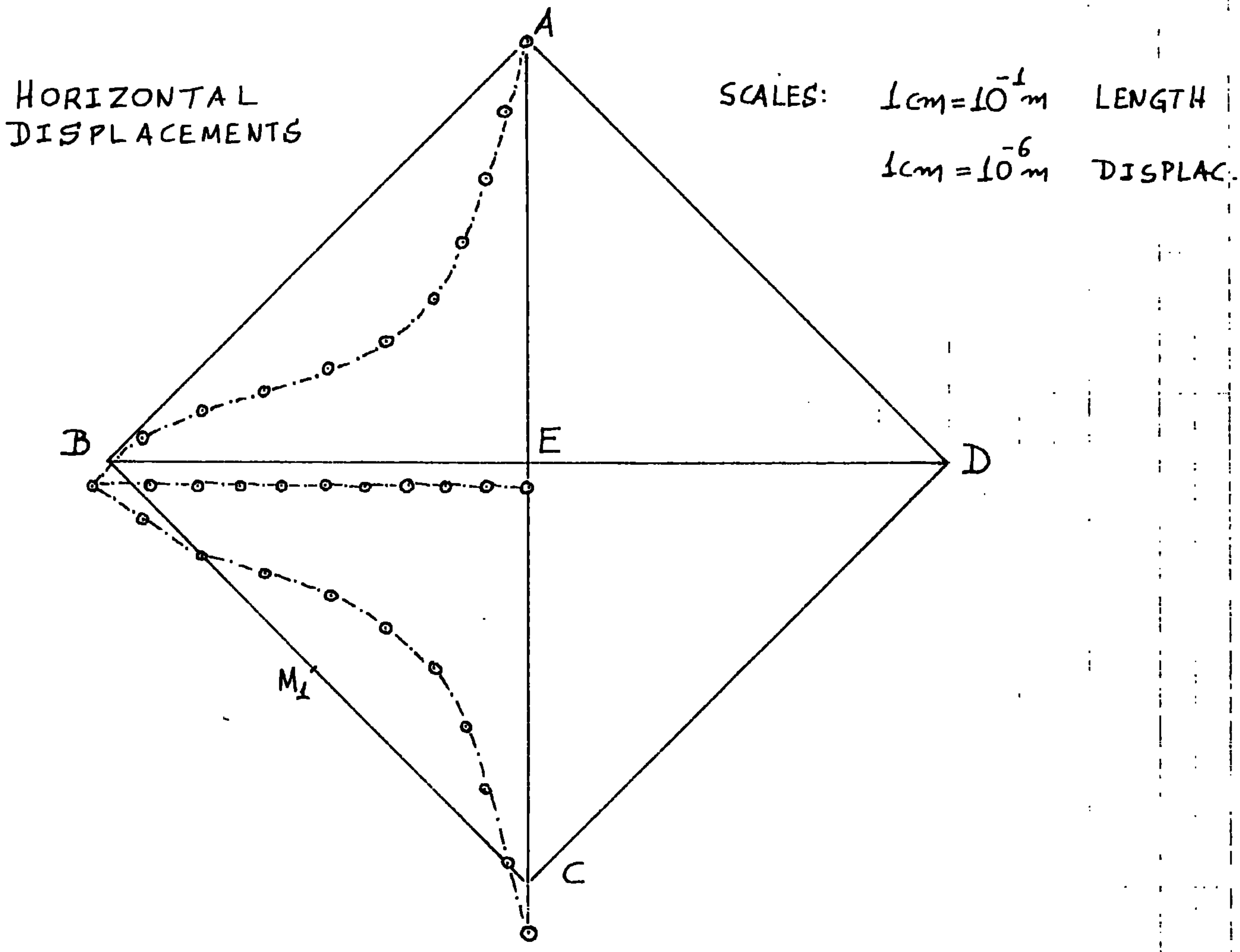


FIG. 13.19.

SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS



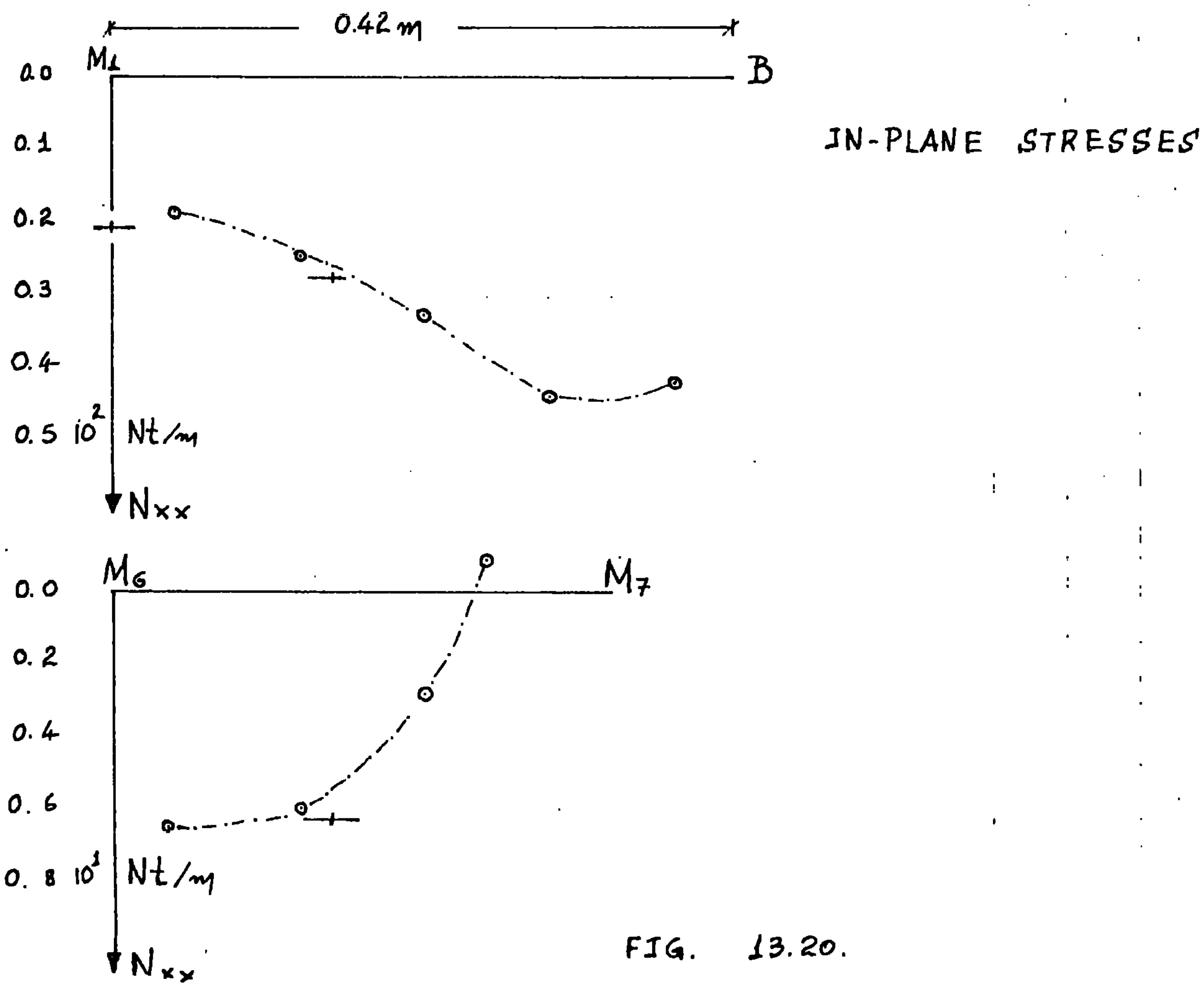
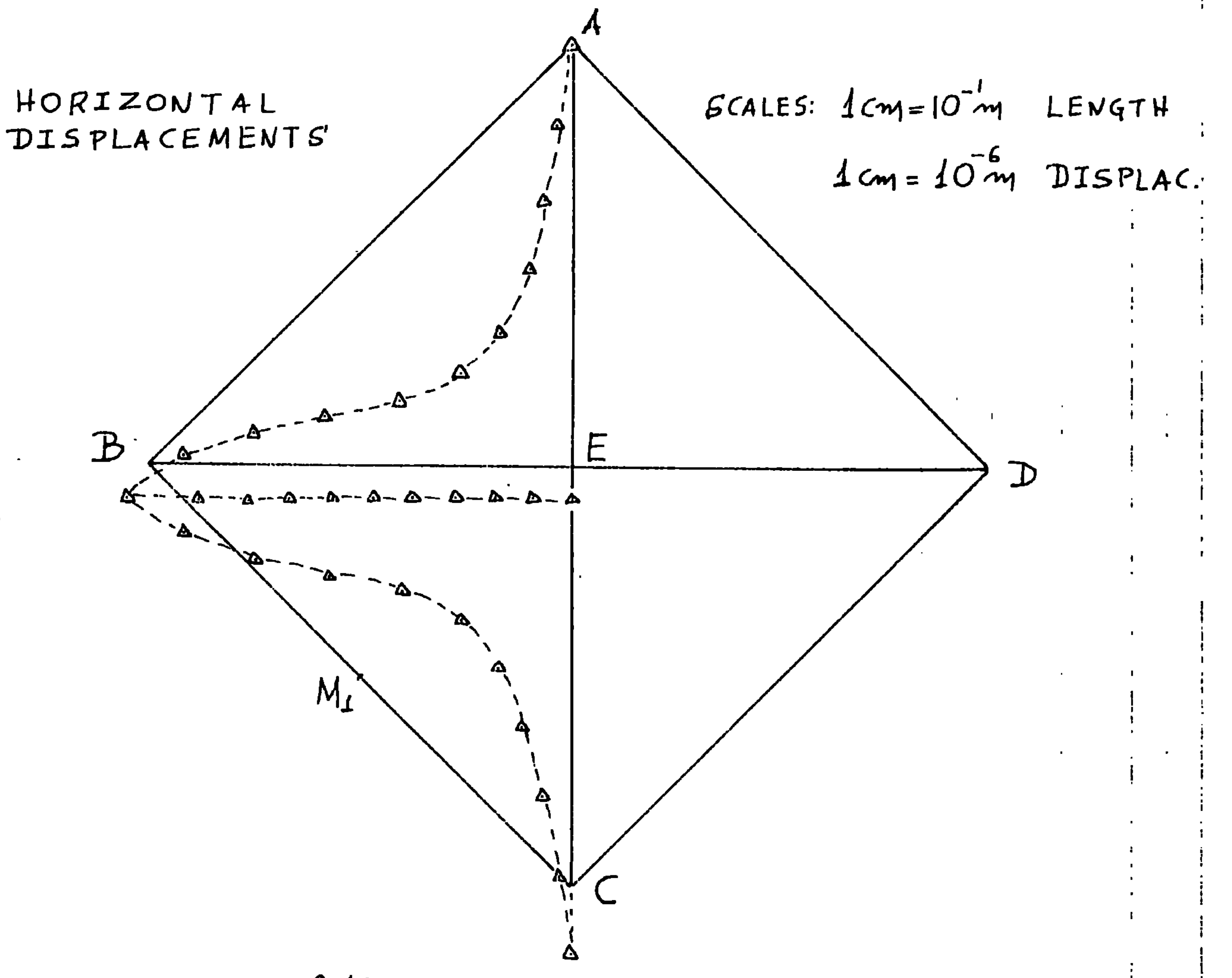
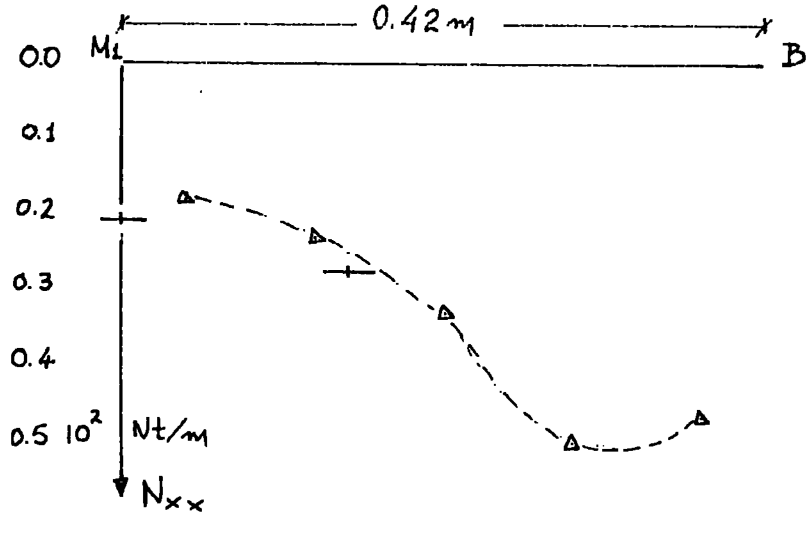


FIG. 13.20.

SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS





IN-PLANE STRESSES

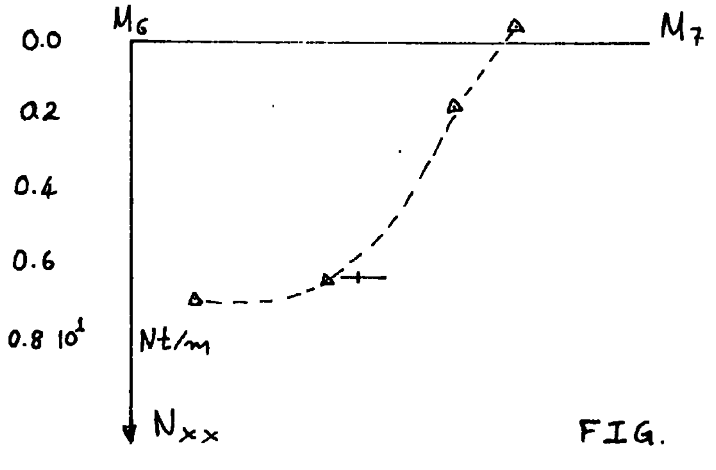
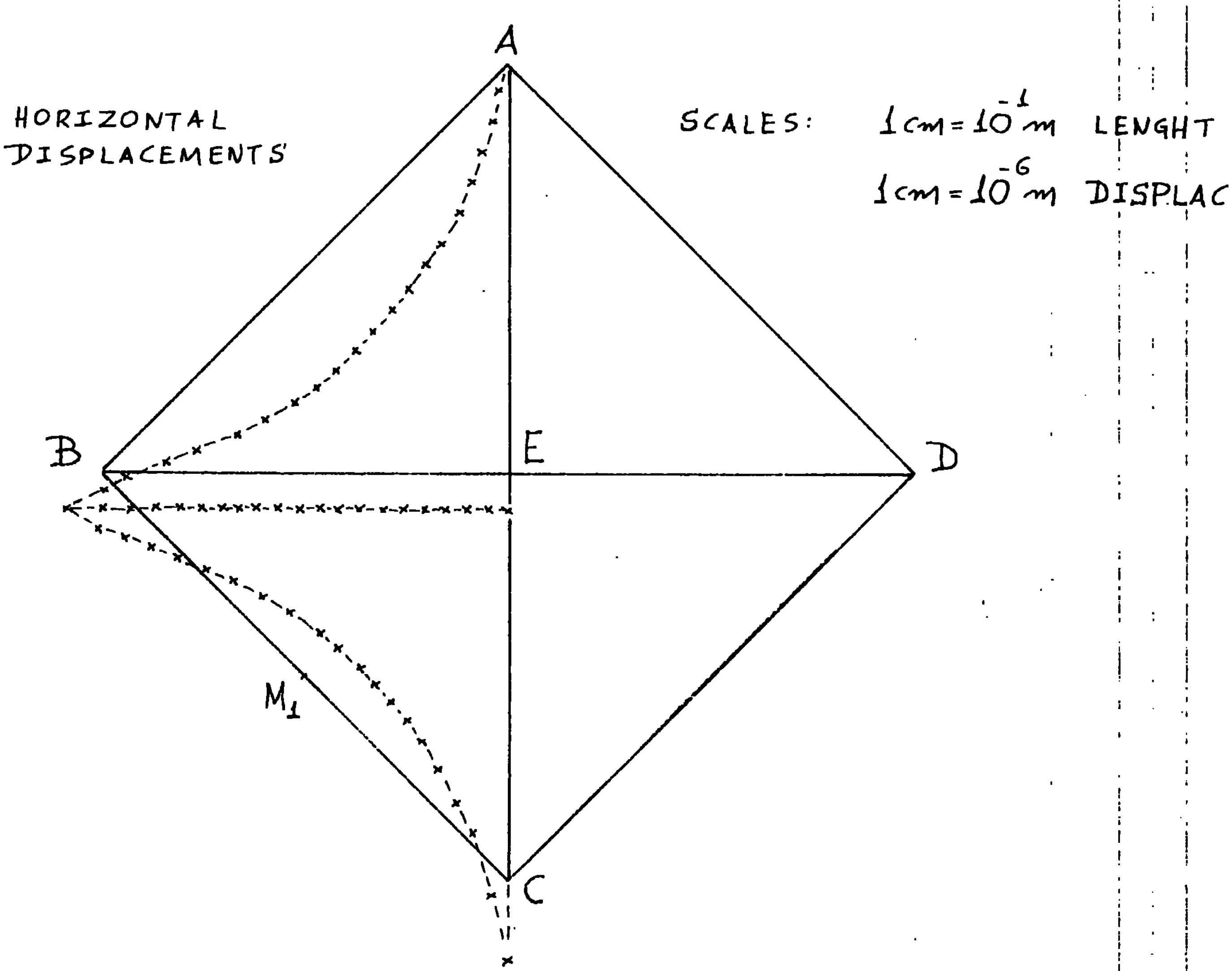


FIG. 13.21.

SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS



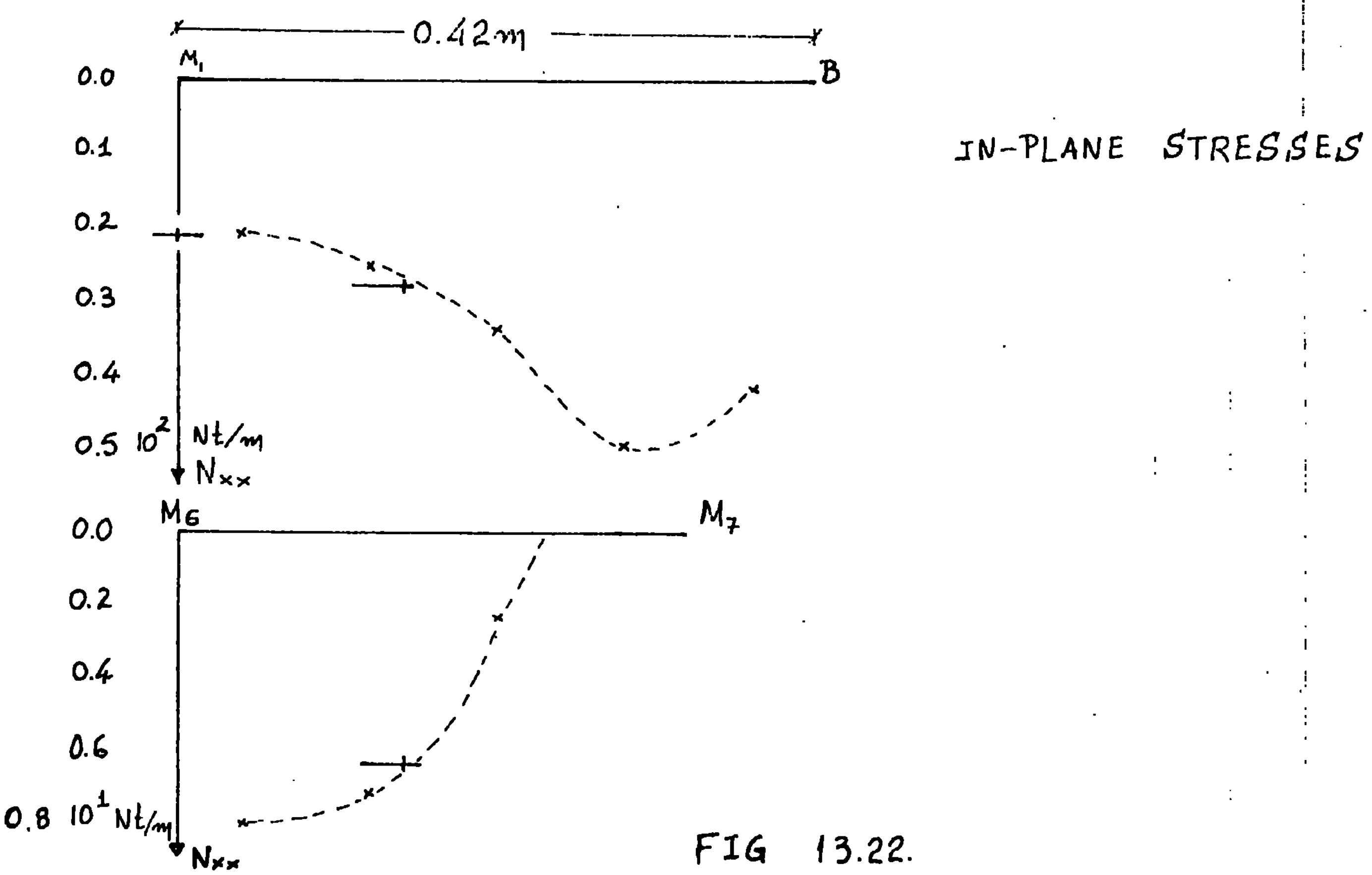


FIG 13.22.

SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

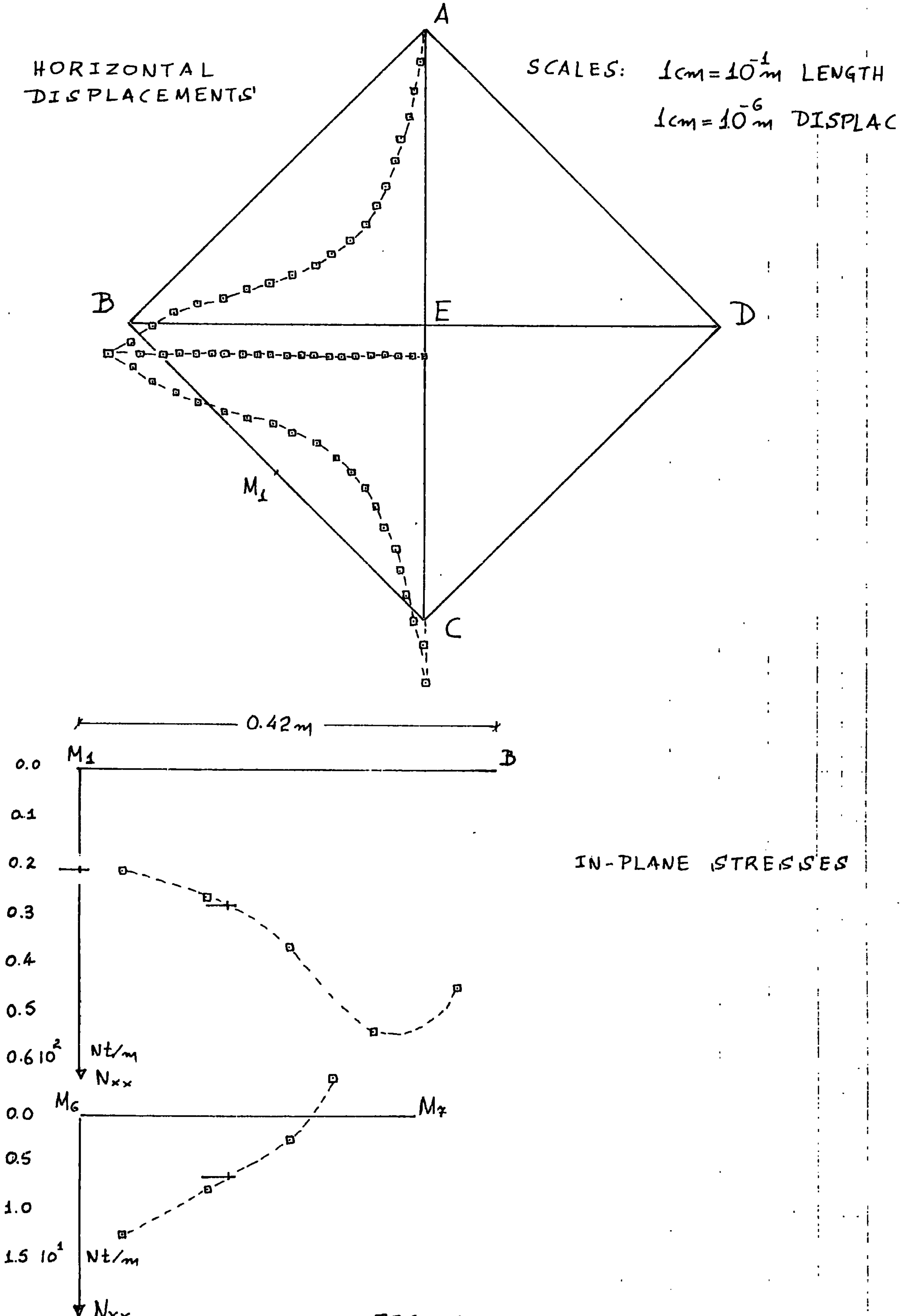


FIG. 13.23.

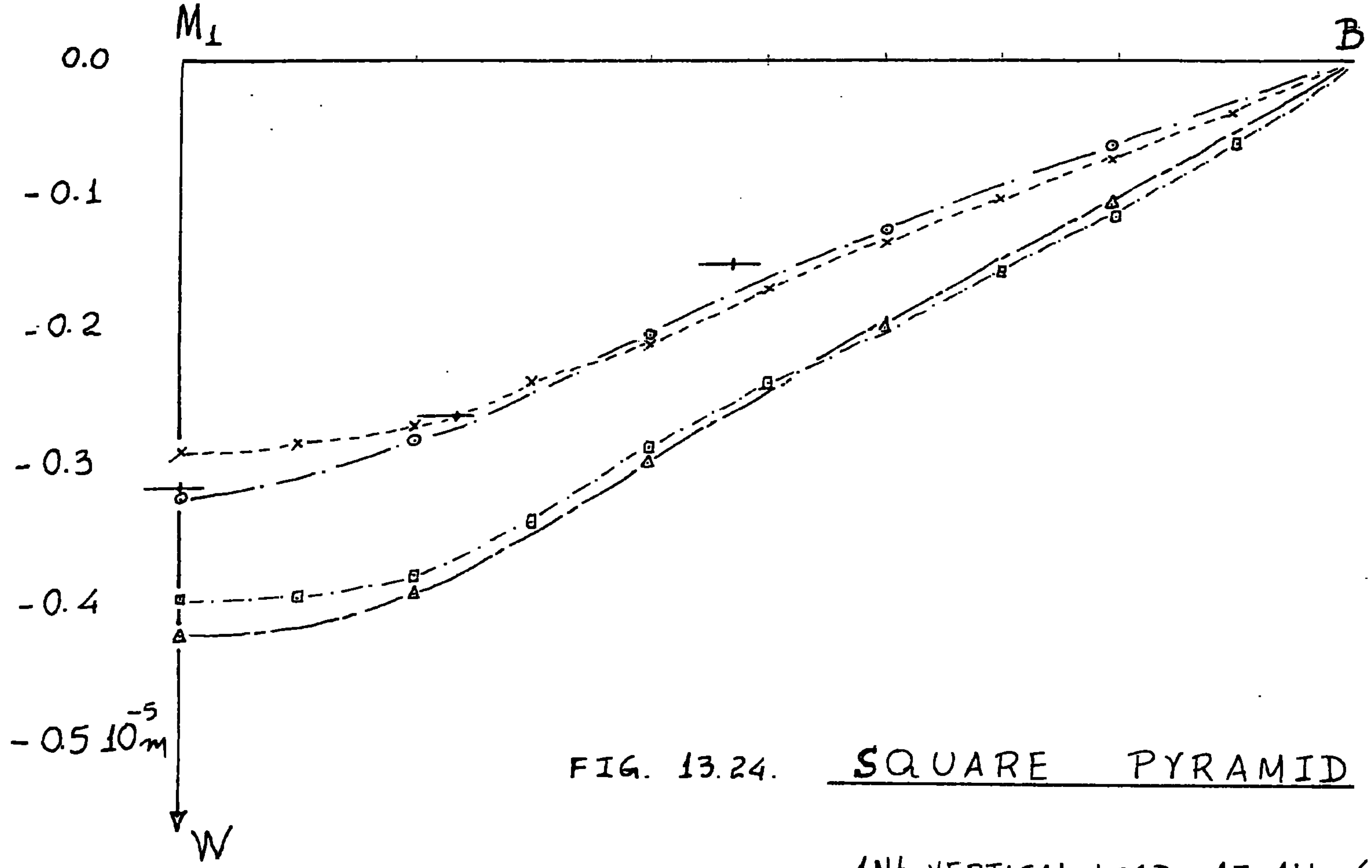


FIG. 13.24. **SQUARE PYRAMID**

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES: $1cm = 2.5\ 10^{-2} m$ LENGTH

$1cm = 0.5\ 10^{-6} m$ DISPLAC.

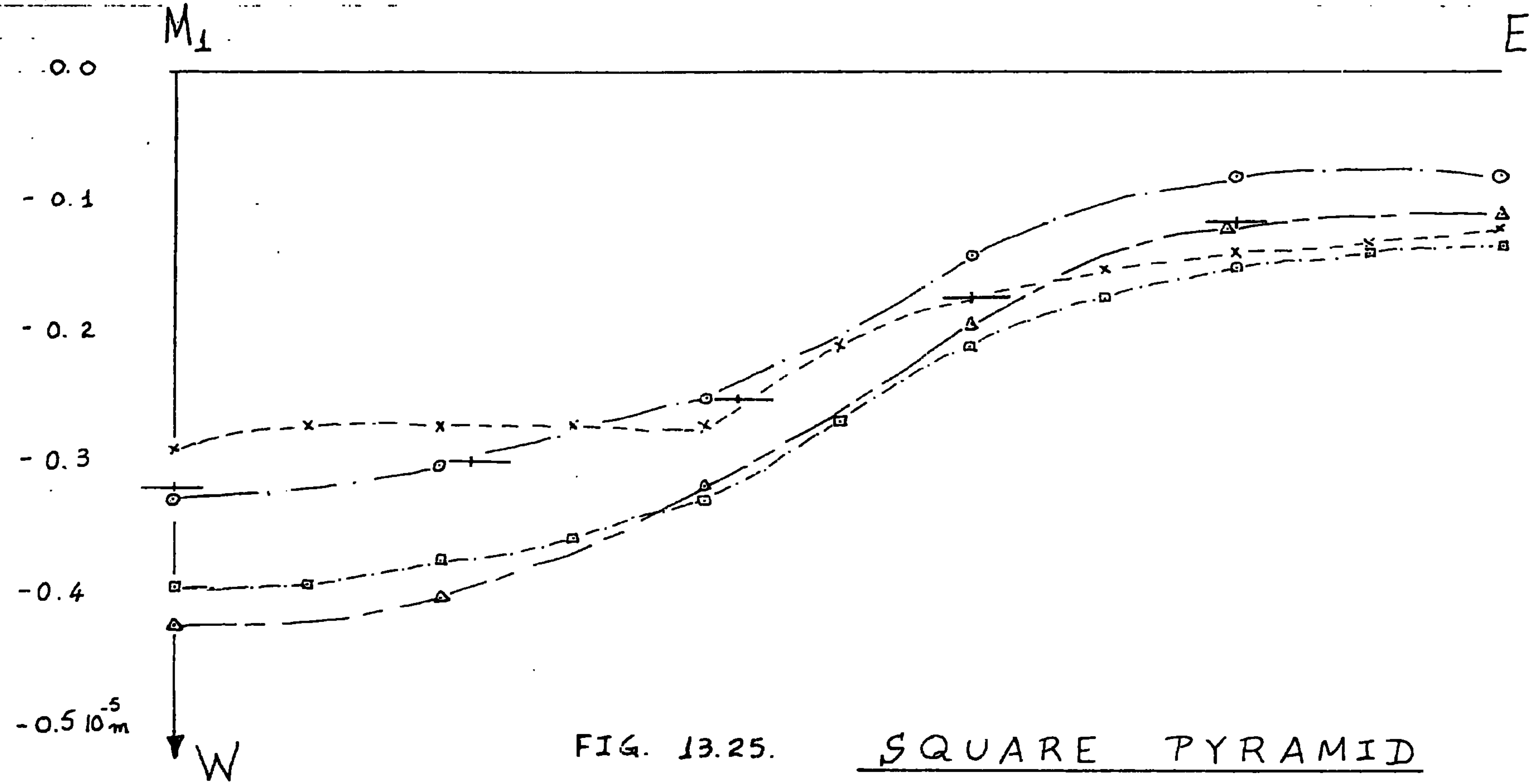


FIG. 13.25. SQUARE PYRAMID

1 Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES : 1cm = 2.5 10^{-2} m LENGTH

1cm = 0.5 10^{-6} m DISPLAC.

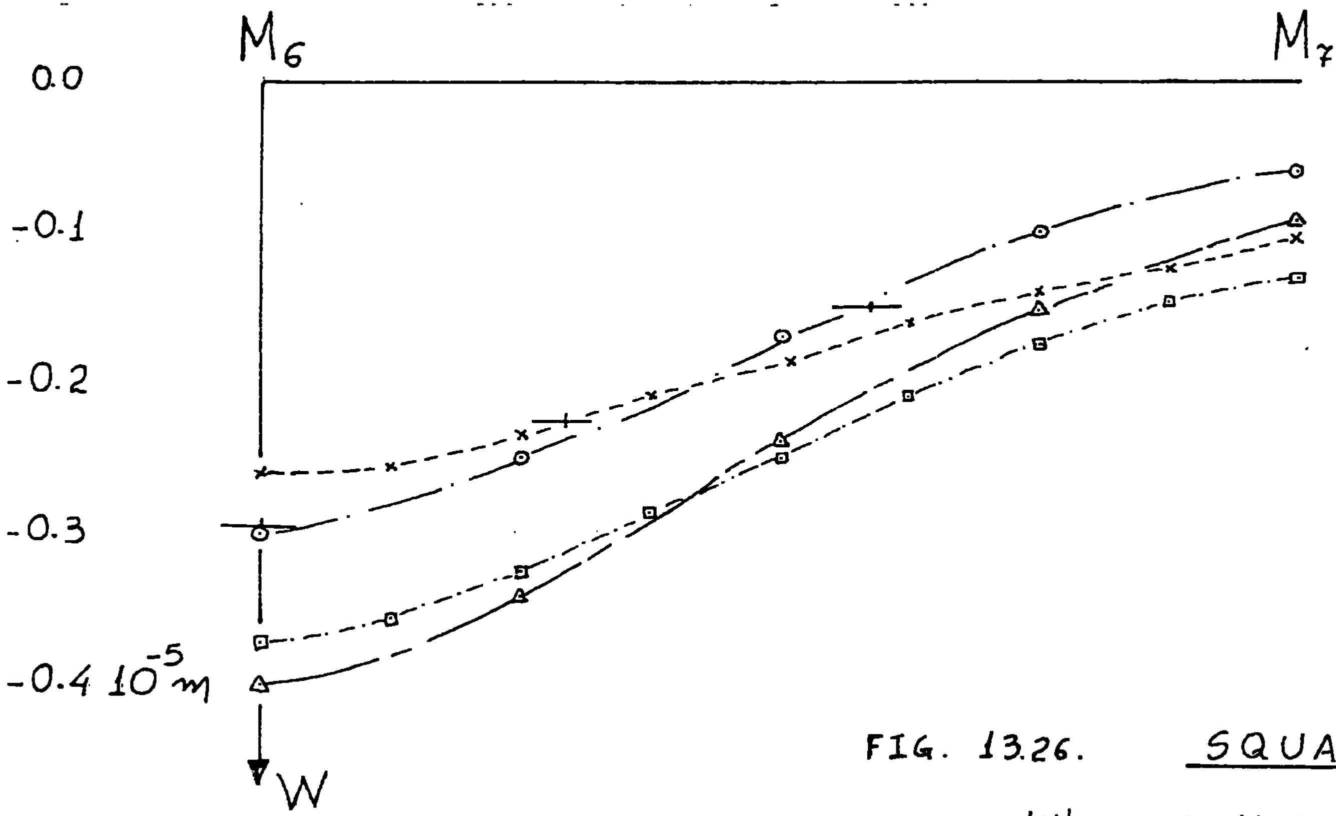


FIG. 13.26. SQUARE PYRAMID

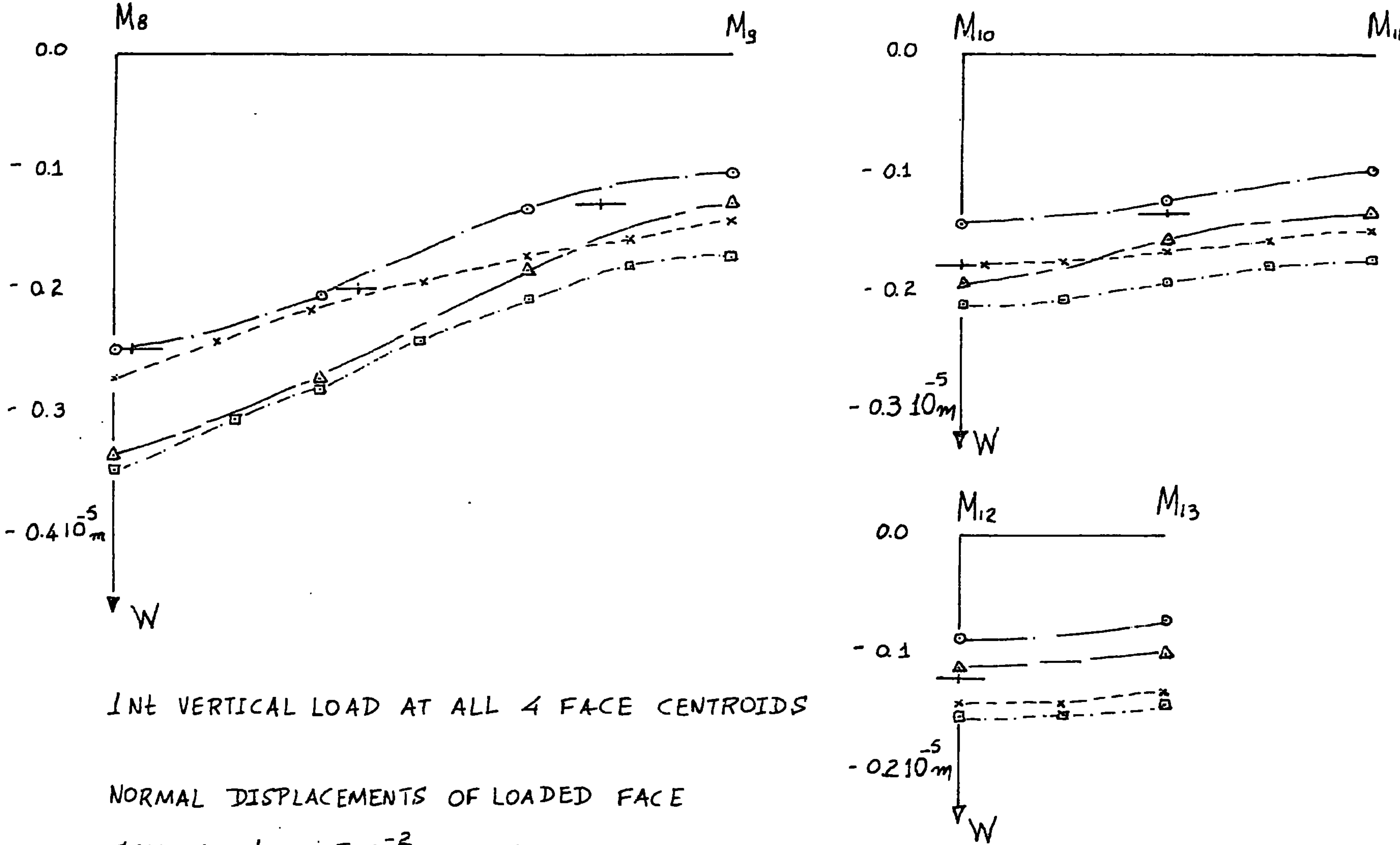
1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES: $1 cm = 5 \cdot 10^{-2} m$ LENGTH

$1 cm = 0.5\ 10^{-6} m$ DISPLAC.

FIG. 13.27. SQUARE PYRAMID



1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES: $1 cm = 5 \cdot 10^{-2} m$ LENGTH

$1 cm = 0.5\ 10^{-6} m$ DISPLAC.

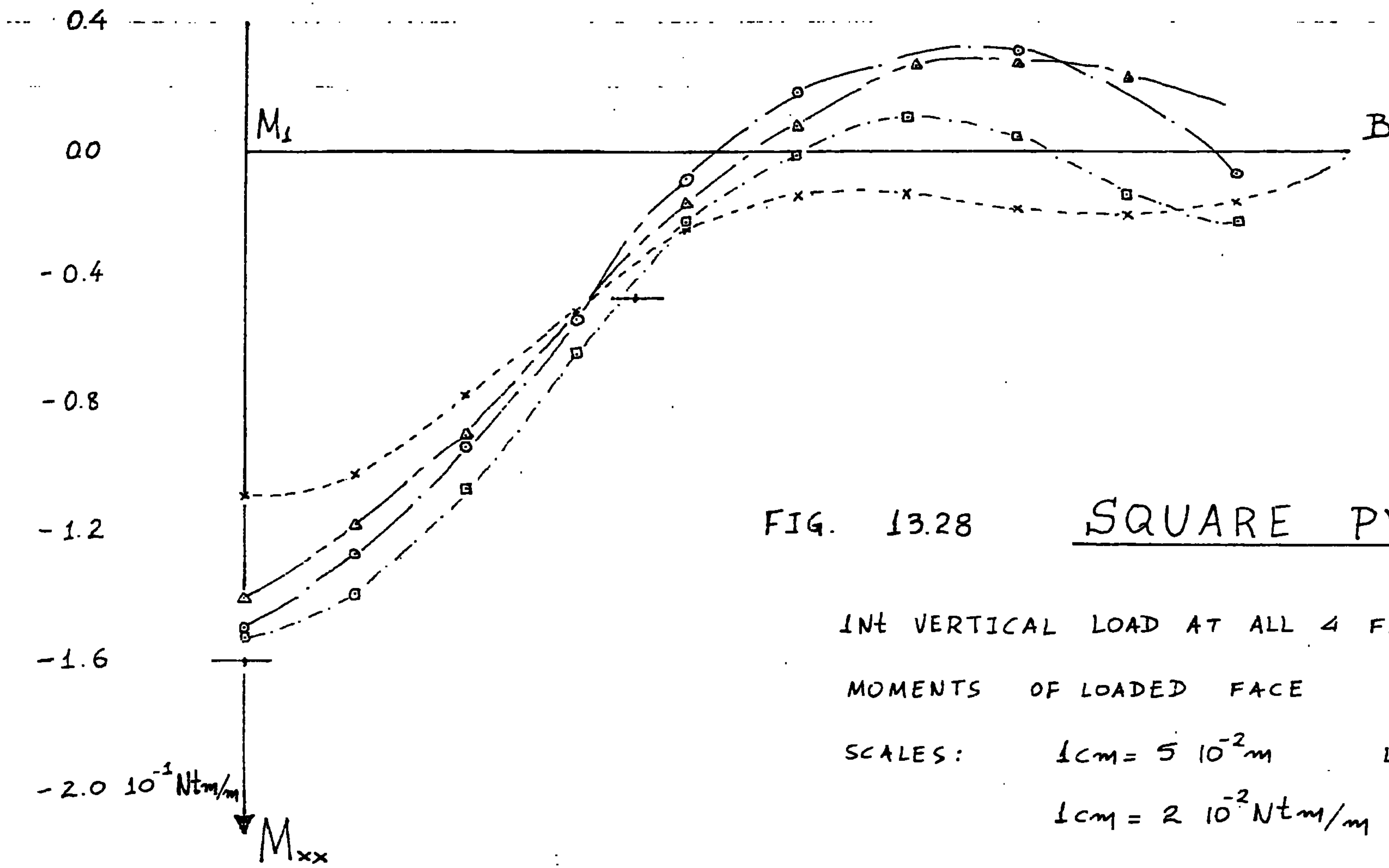


FIG. 13.28 SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES: 1cm = 5 10^{-2}m LENGTH

1cm = 2 10^{-2} Ntm/m MOMENTS

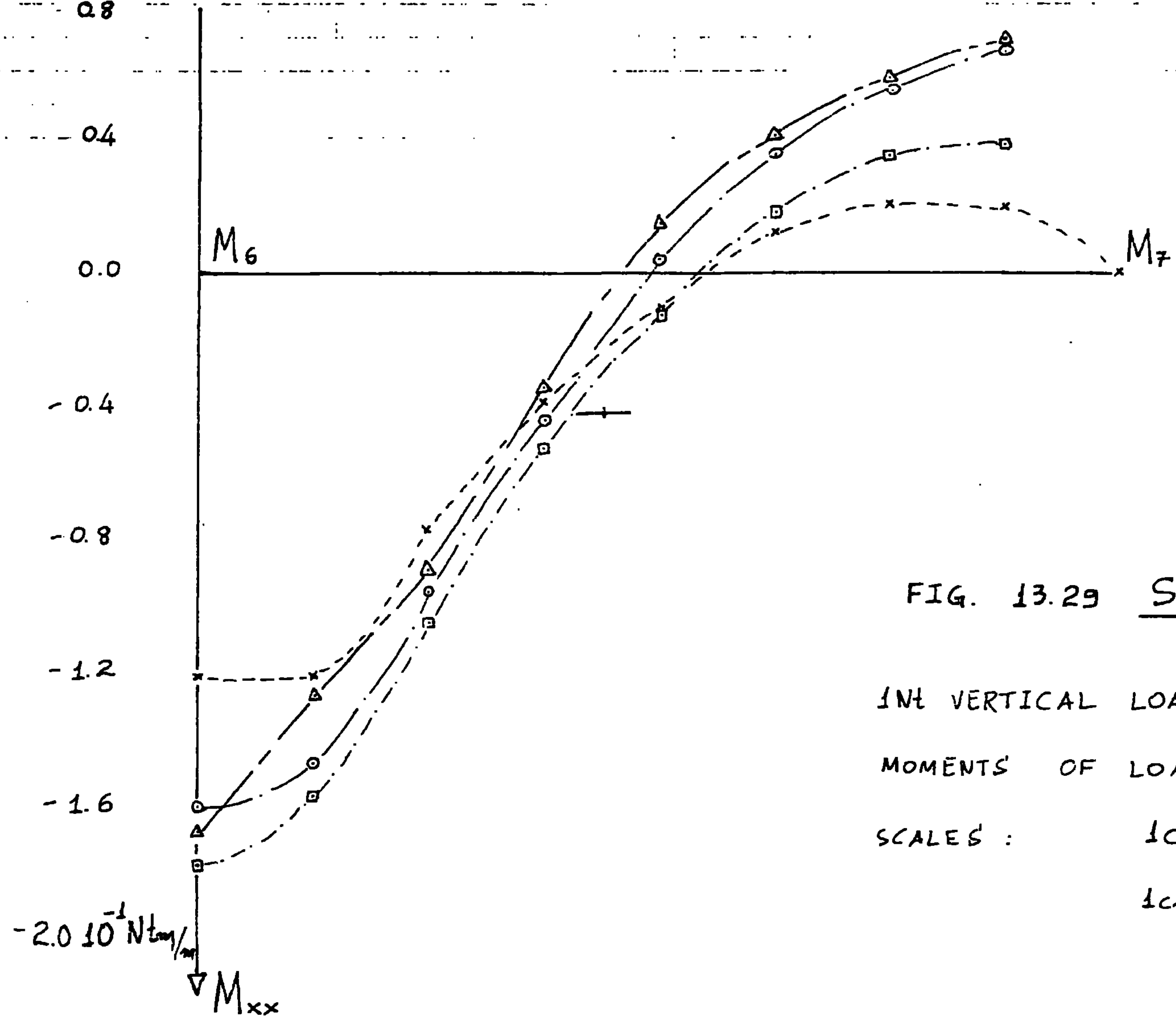


FIG. 13.29 SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES : 1cm = 5 10^{-2} m LENGTH

1cm = 2 10^{-2} Ntm/m MOMENTS

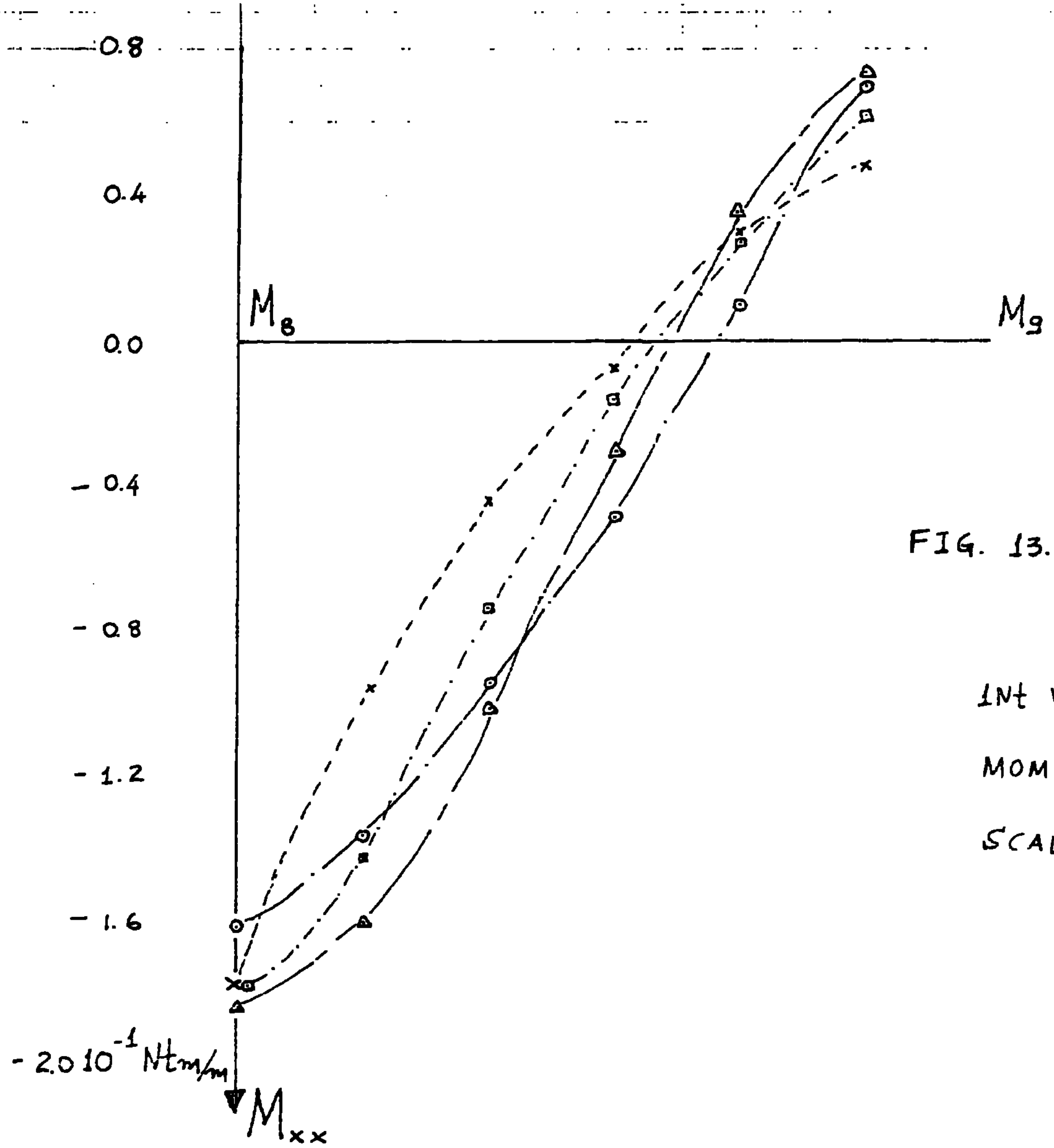


FIG. 13.30. SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES 1cm = $5 \cdot 10^{-2}$ m LENGTH

1cm = $2 \cdot 10^{-2}$ Ntm/m MOMENTS

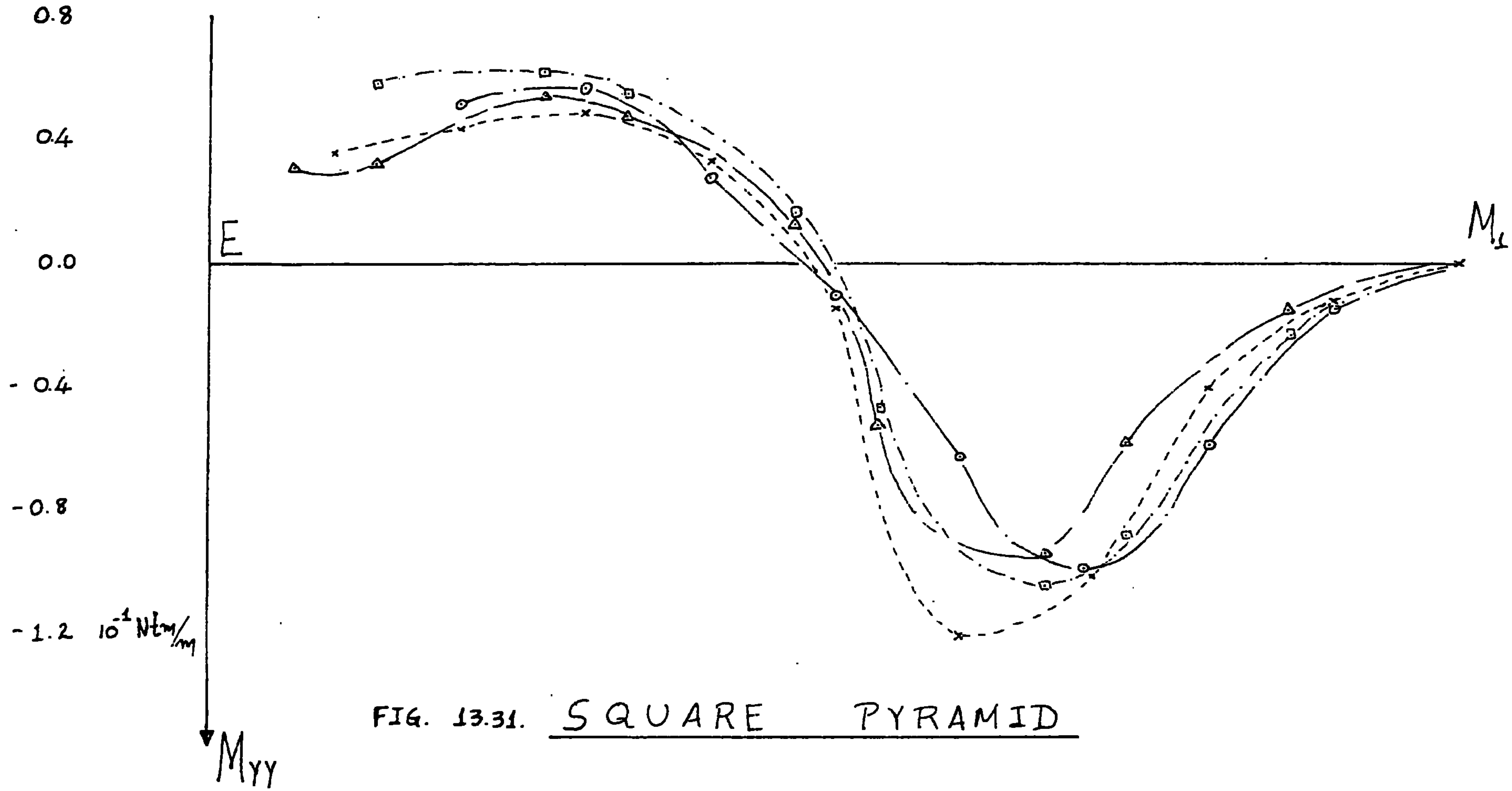


FIG. 13.31. SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES : $1\,cm = 5 \cdot 10^{-2}\,m$ LENGTH

$1\,cm = 2 \cdot 10^{-2}\,Ntm/m$ MOMENTS

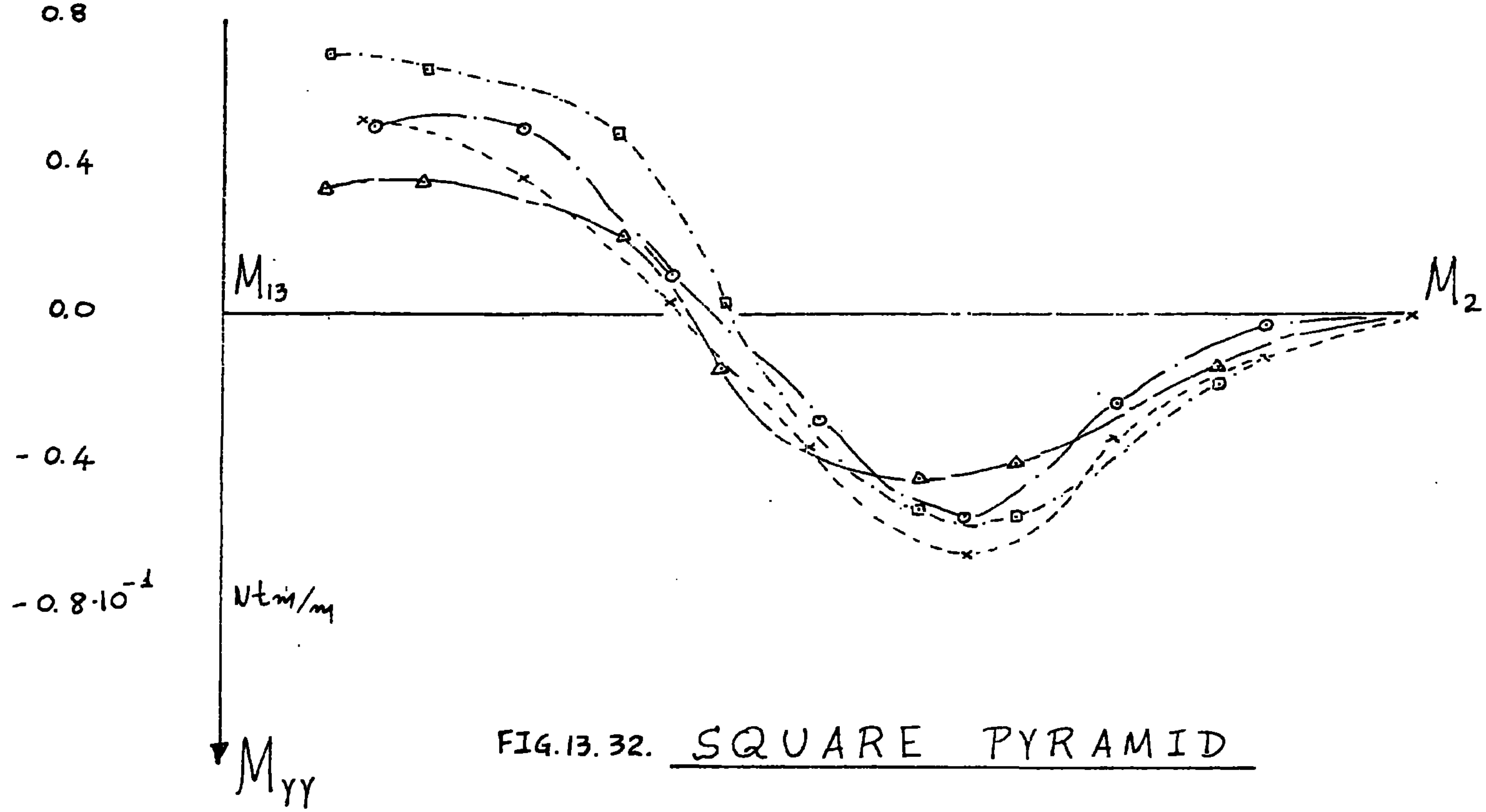


FIG. 13. 32. SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES: $1cm = 5 \cdot 10^{-2} m$ LENGTH

$1cm = 2 \; 10^{-2}$ Ntm/m MOMENTS

16 FACED DOME

GENERAL ARRANGEMENT

SCALE: 1m = 25m

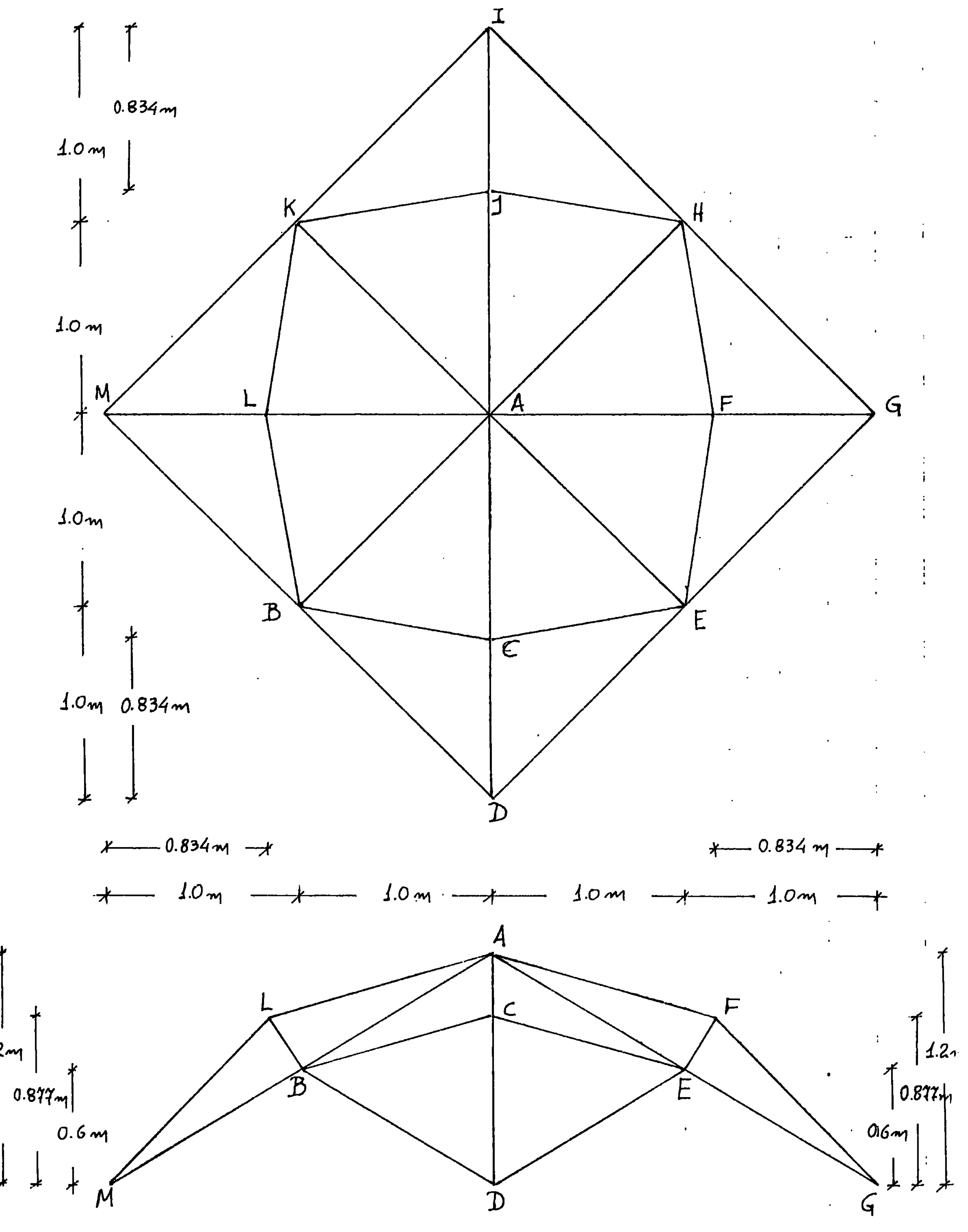


FIG 13.33.

16 FACED DOME

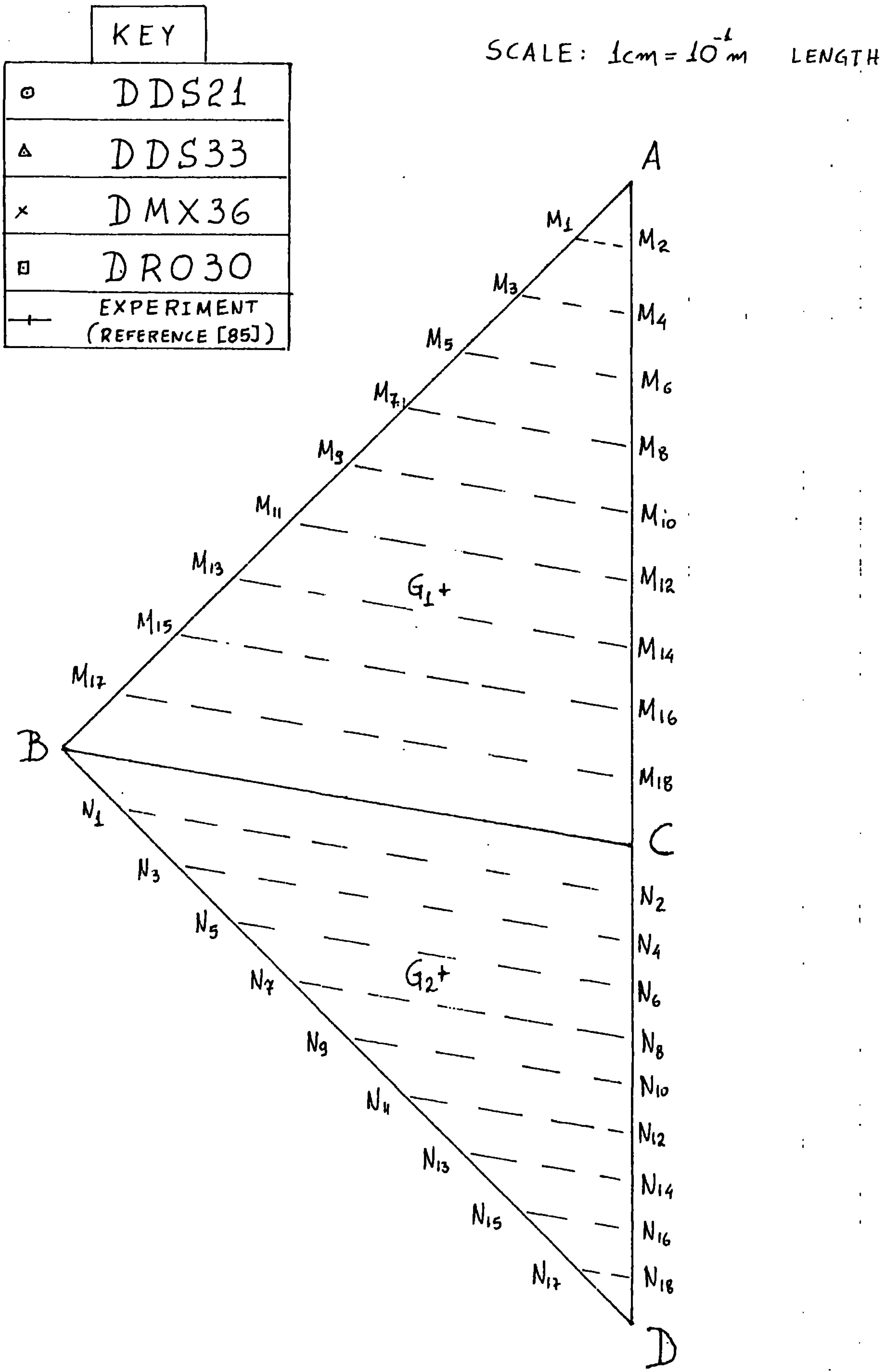
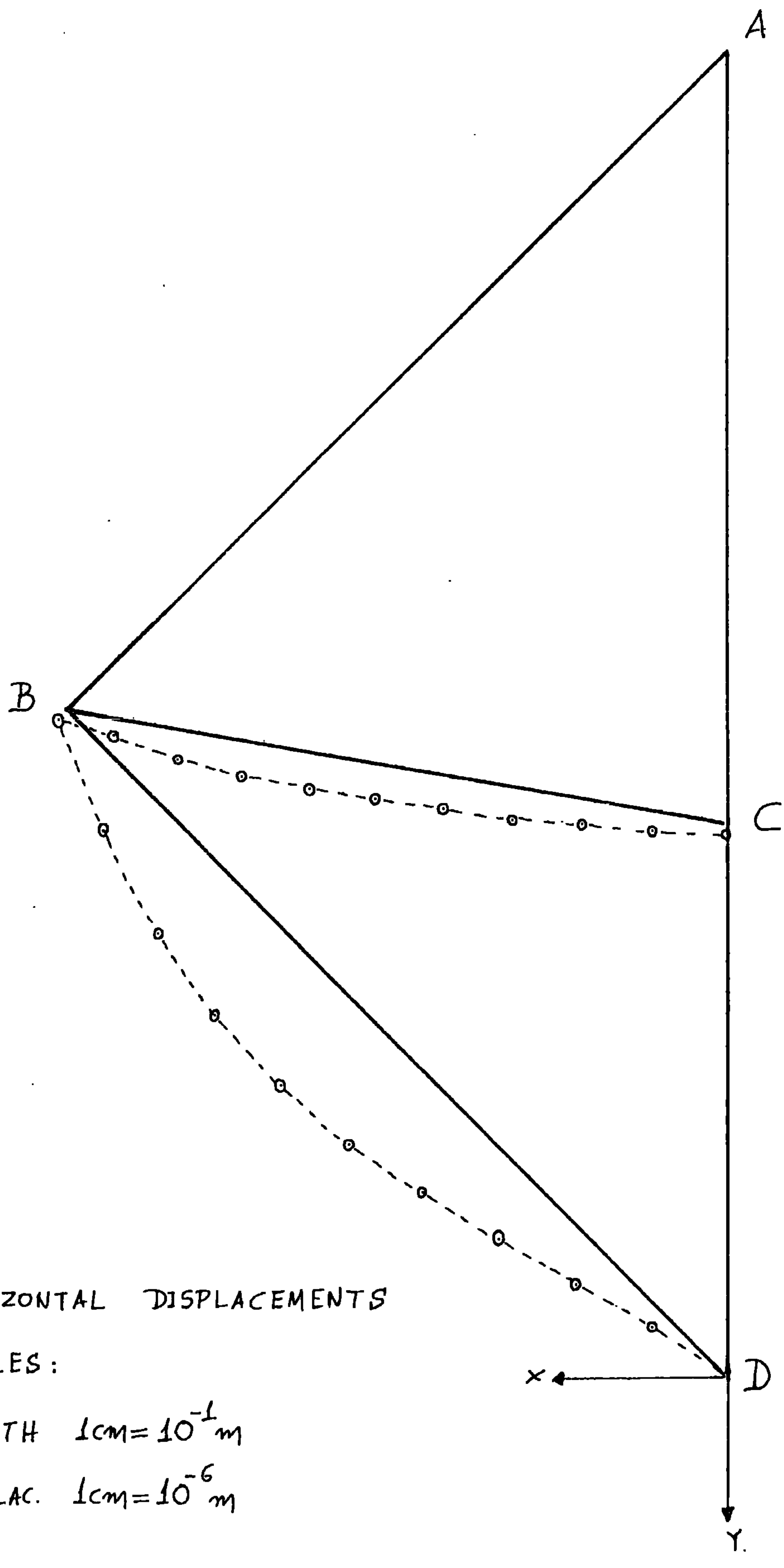


FIG. 13.34. GENERAL ARRANGEMENT

FIG. 13.35. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

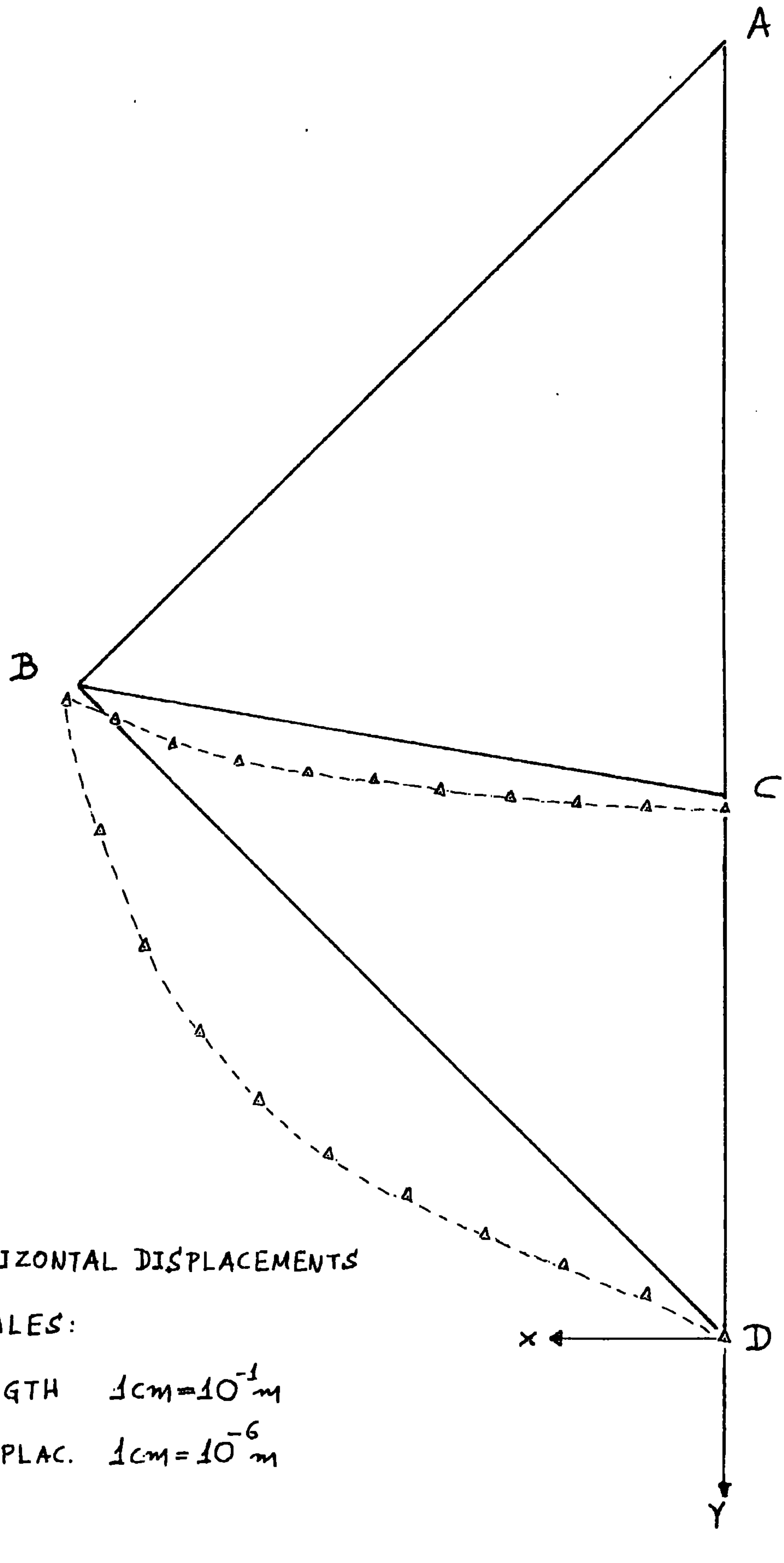
SCALES:

LENGTH 1cm = 10^{-1} m

DISPLAC. 1cm = 10^{-6} m

FIG. 13.36. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH 1cm = 10^{-1} m

DISPLAC. 1cm = 10^{-6} m

FIG. 13.37. 16 FACED DOME

1 Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

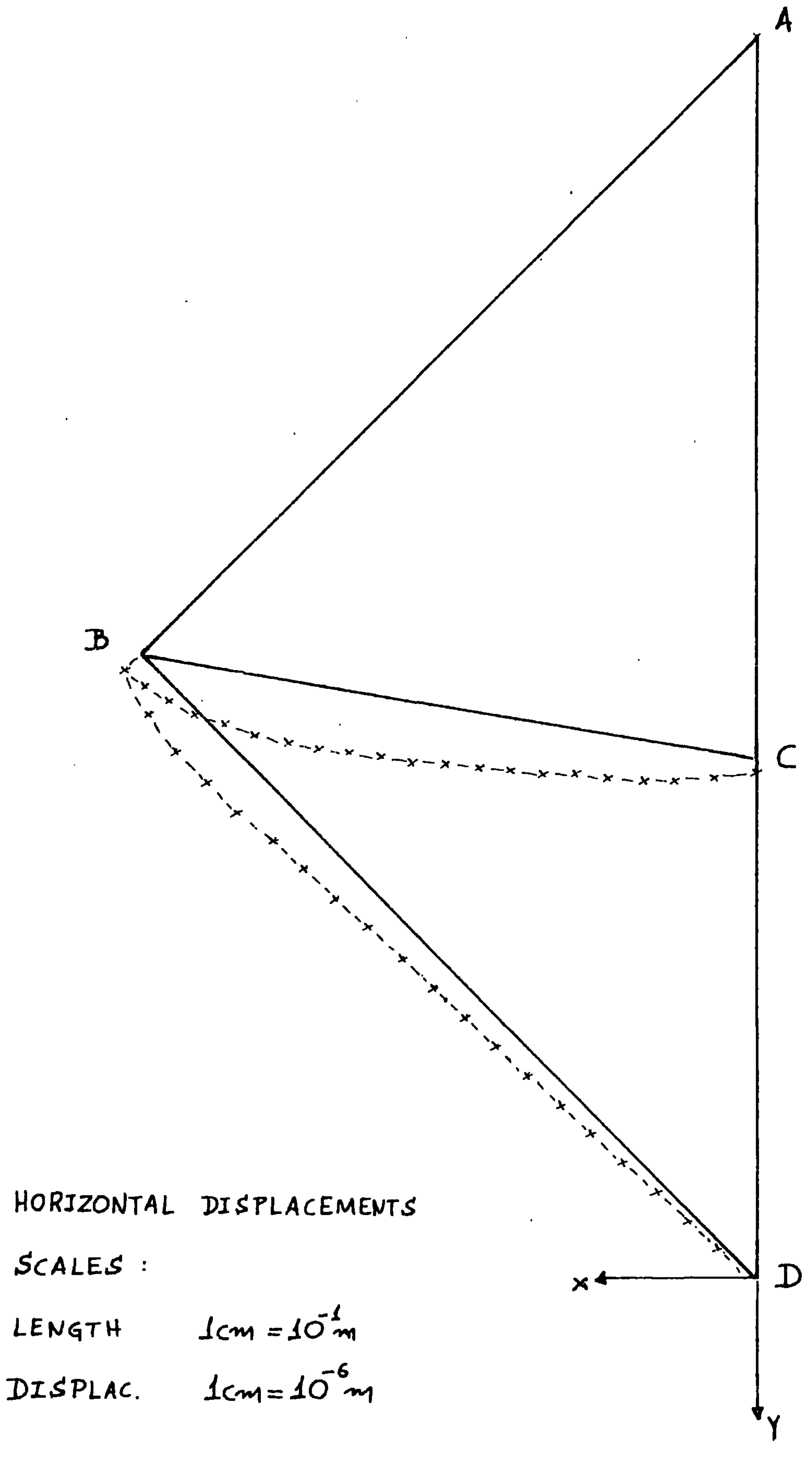
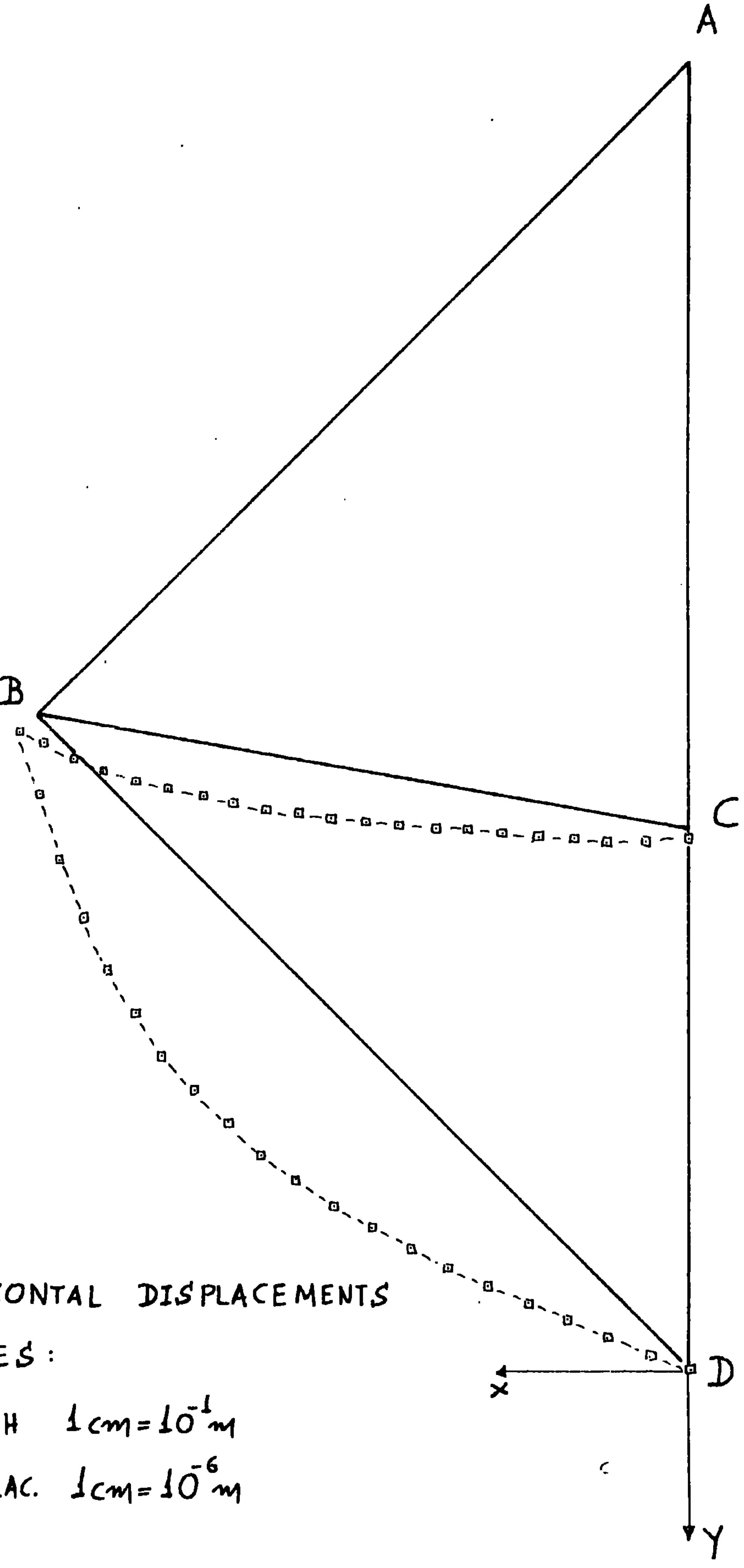


FIG. 13.38. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH $1 cm = 10^{-1} m$

DISPLAC. $1 cm = 10^{-6} m$

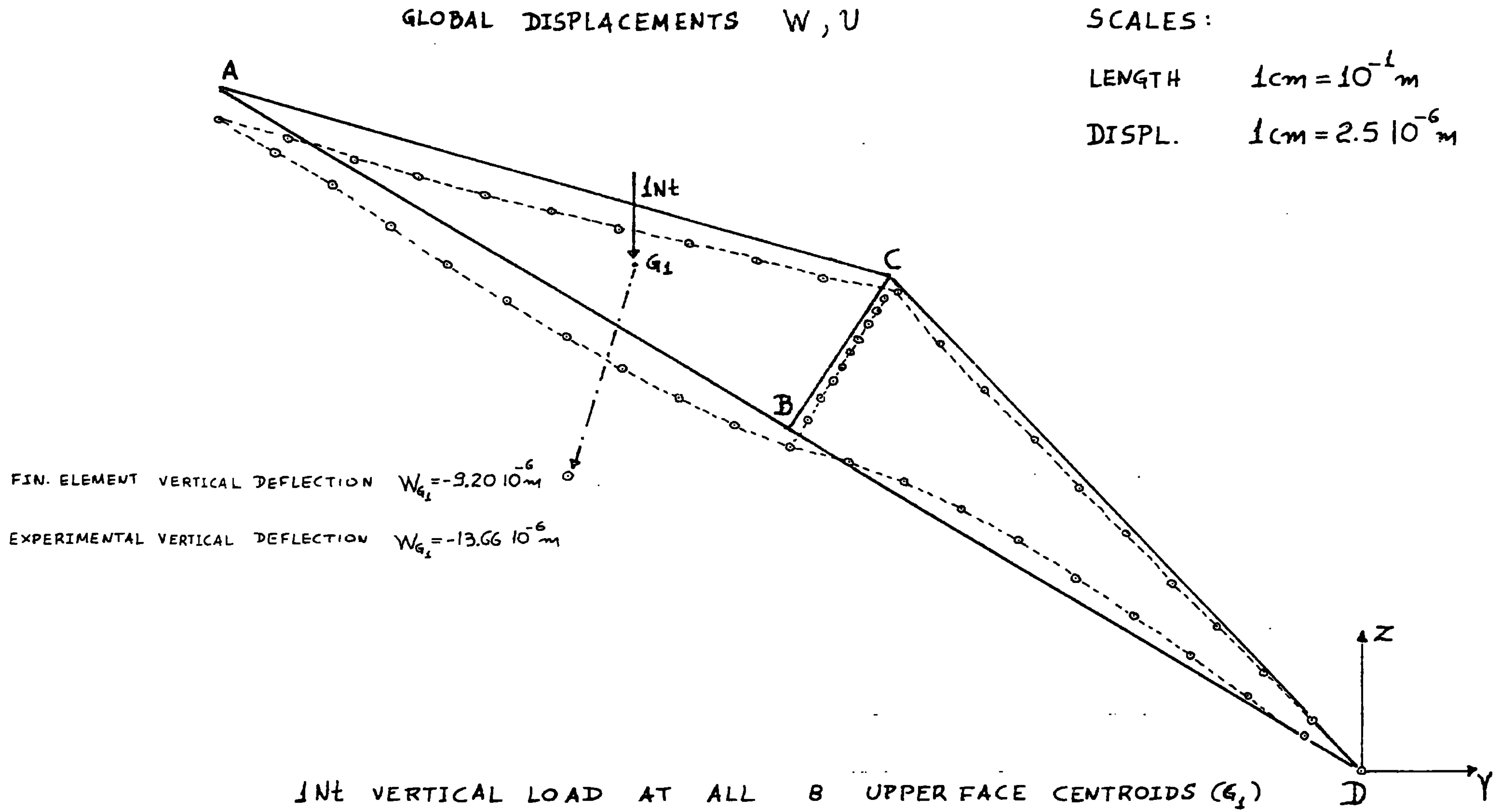
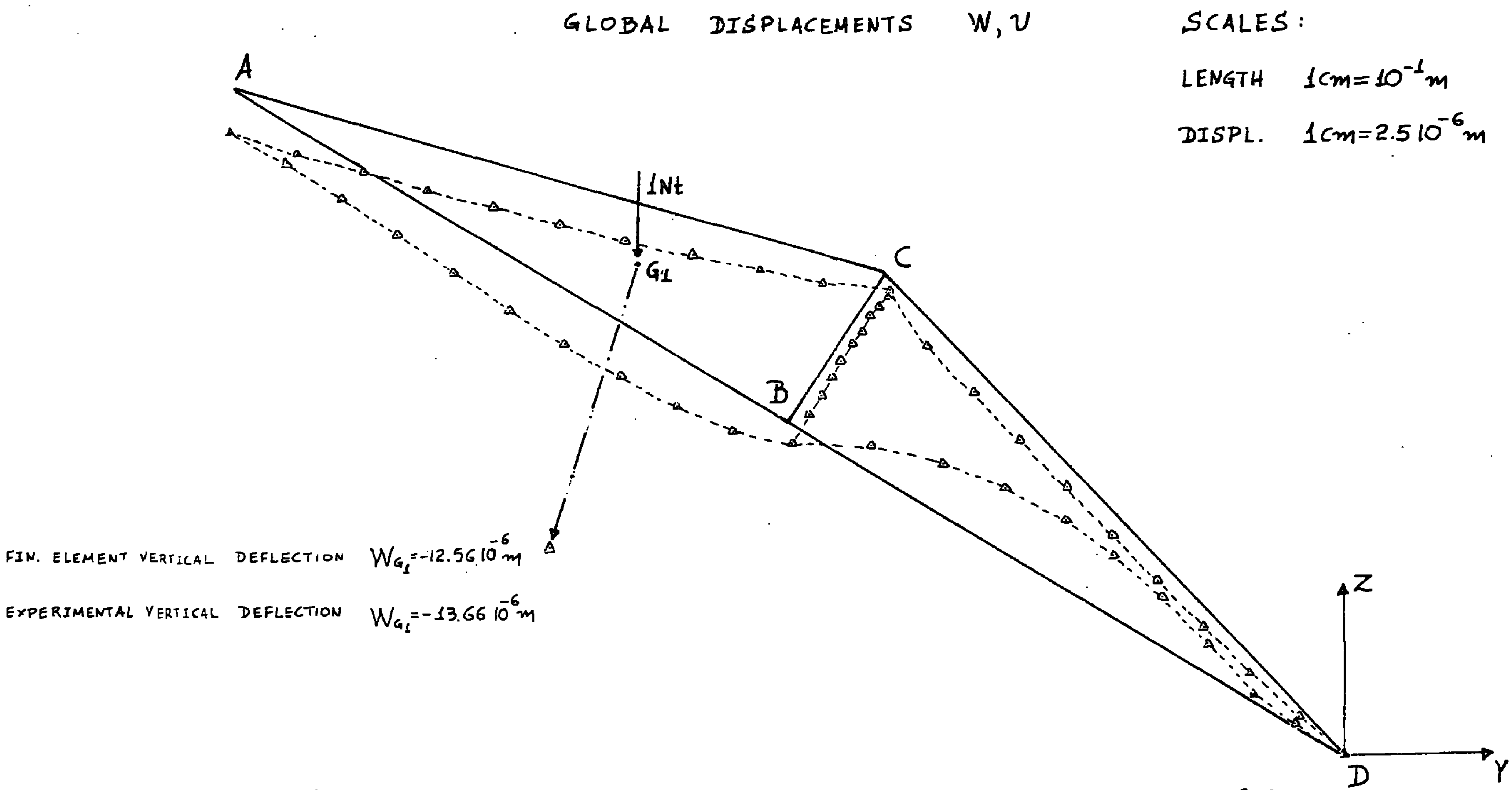
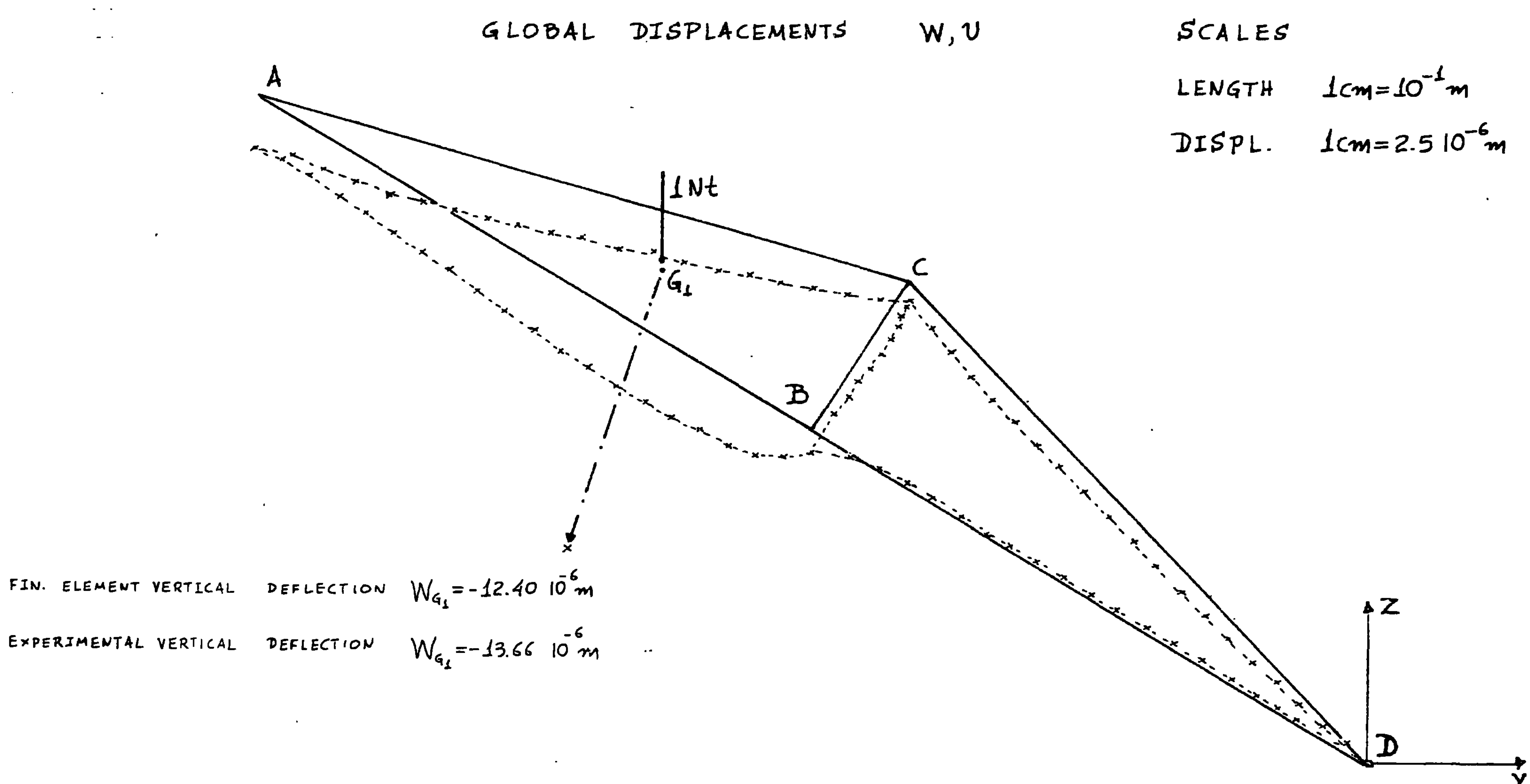


FIG. 13.39. 16 FACED DOME



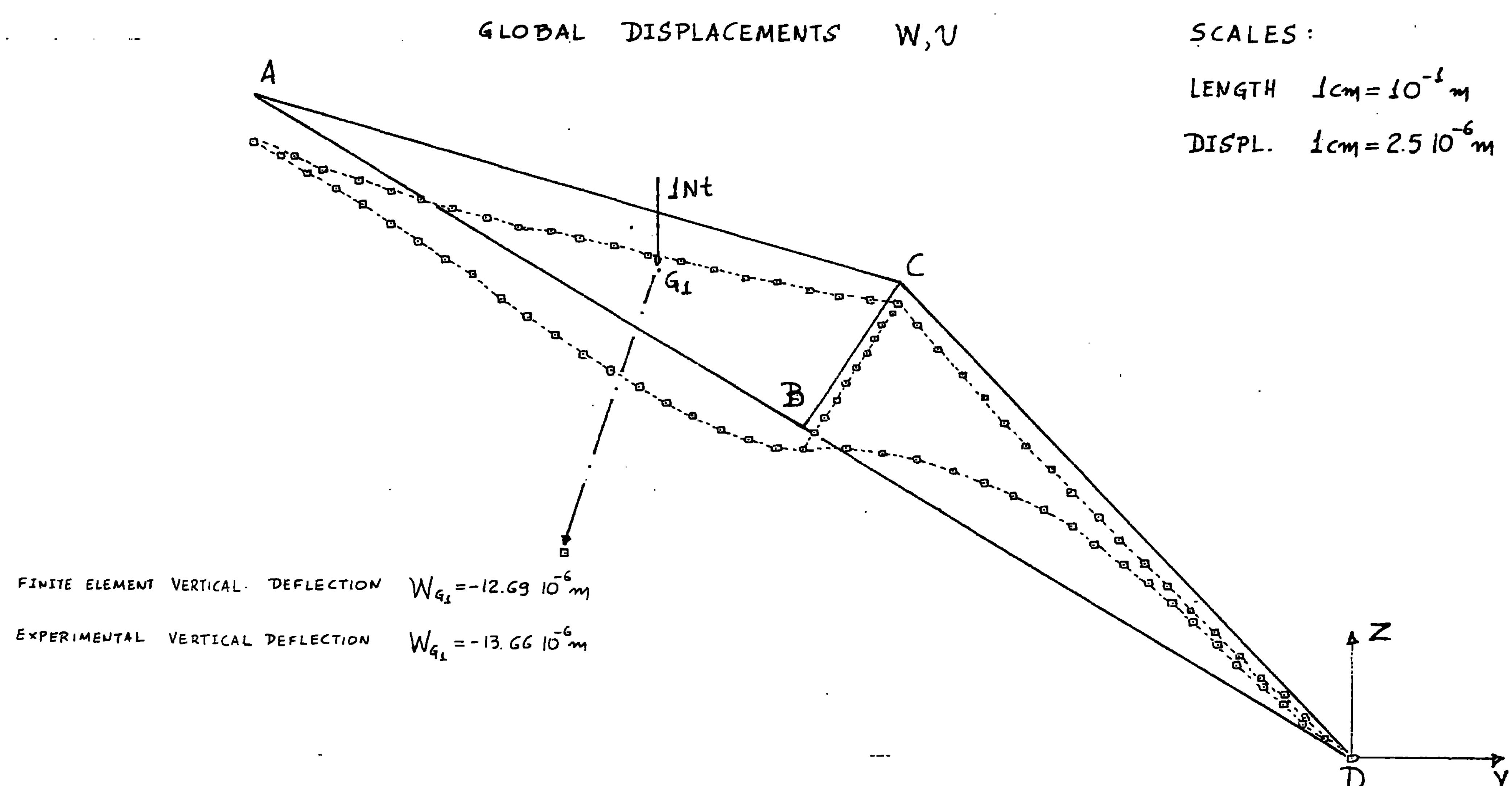
1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS (G_1)

FIG. 13.40. 16 FACED DOME



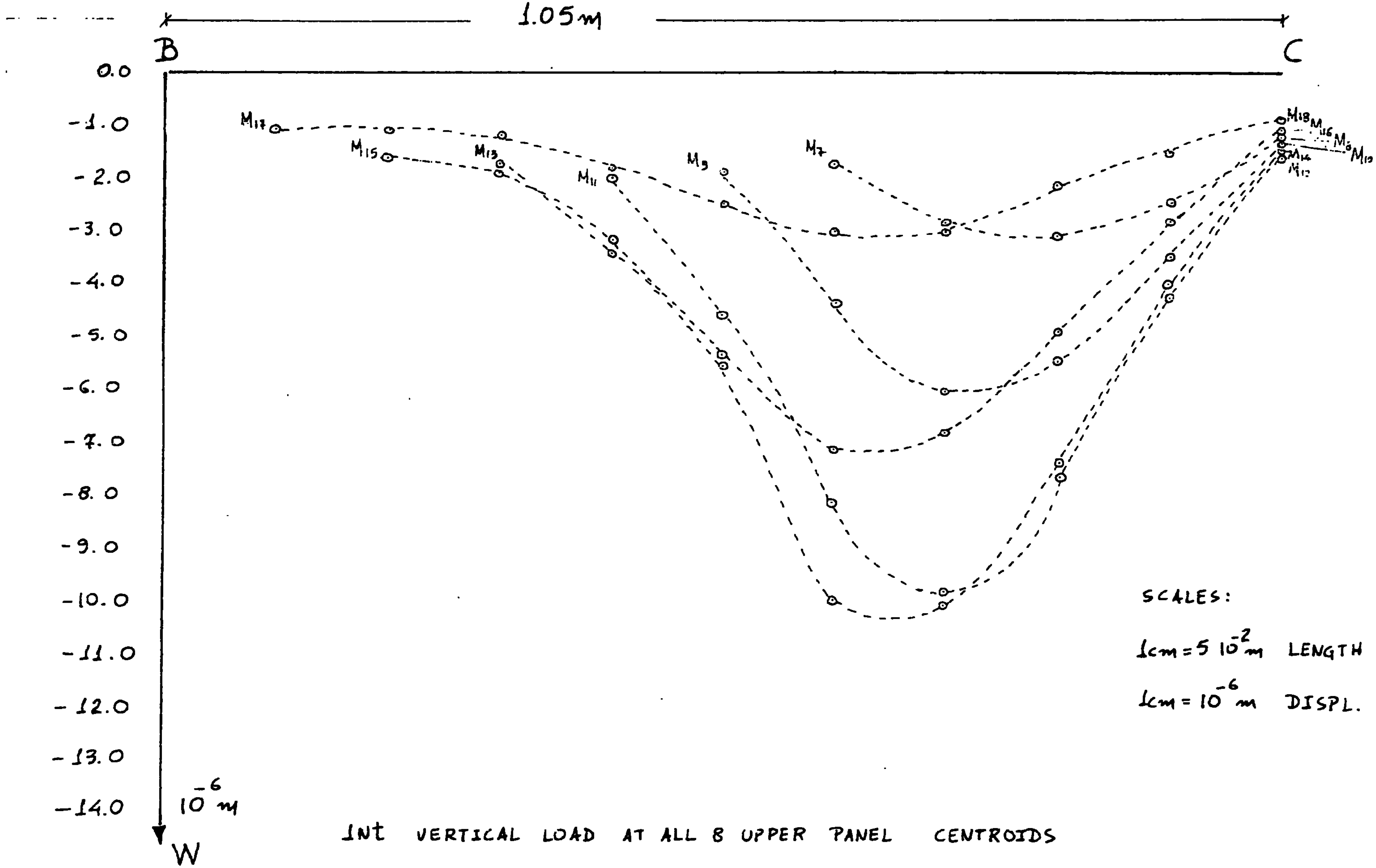
1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS (G_1)

FIG. 13.41. 16 FACED DOME



1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS (G_1)

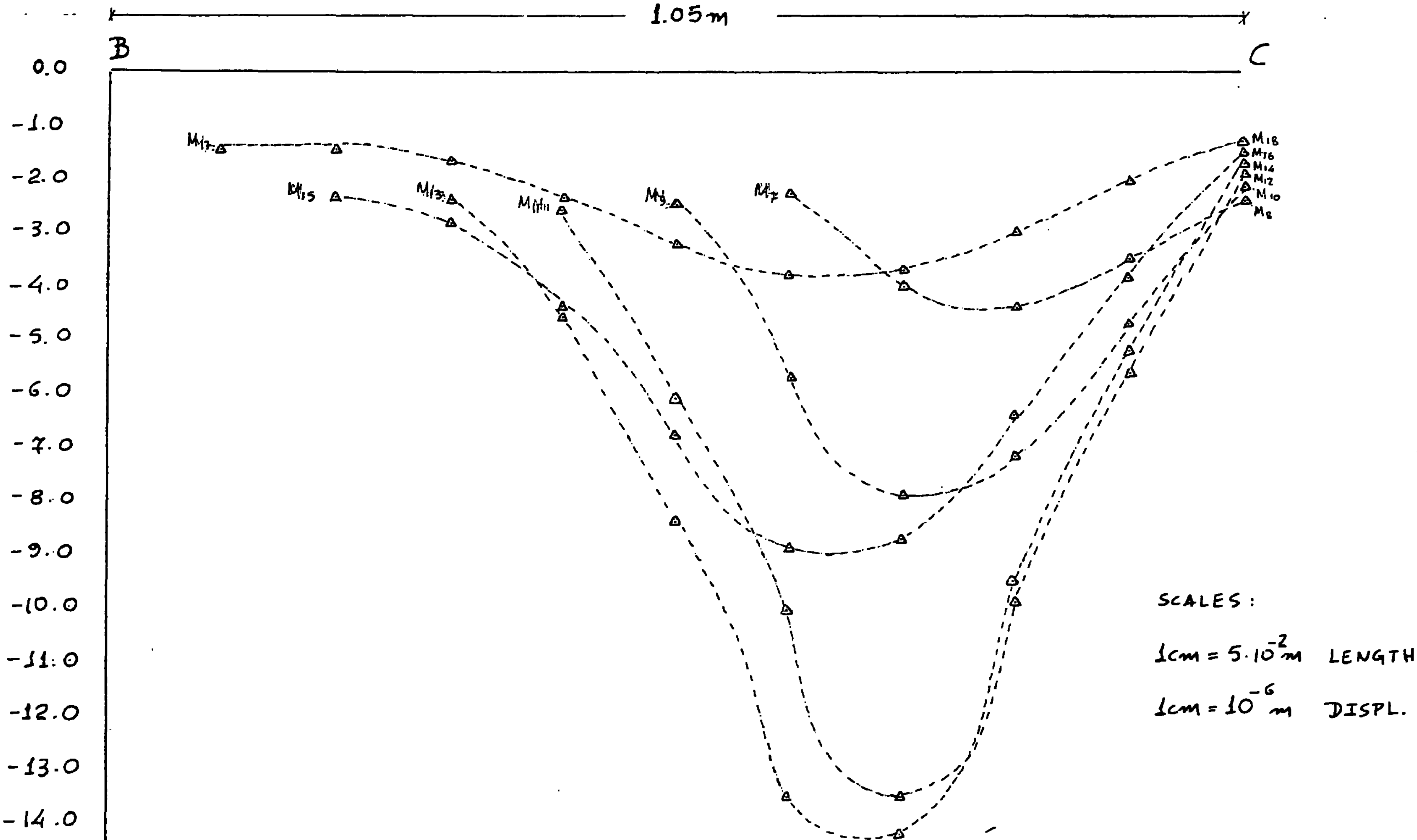
FIG. 13.42. 16 FACED DOME



1nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

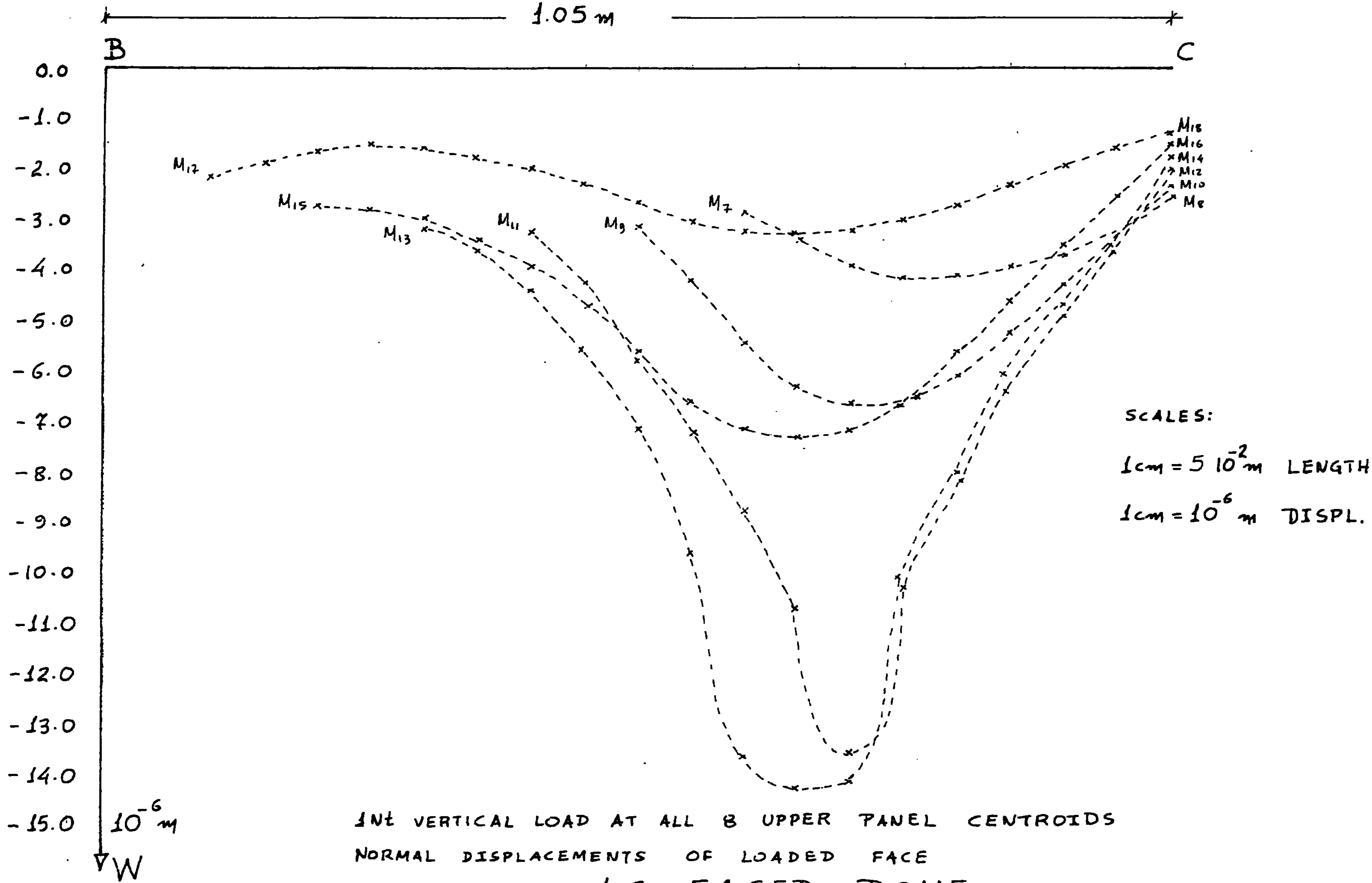
FIG. 13.43. 16 FACED DOME



1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

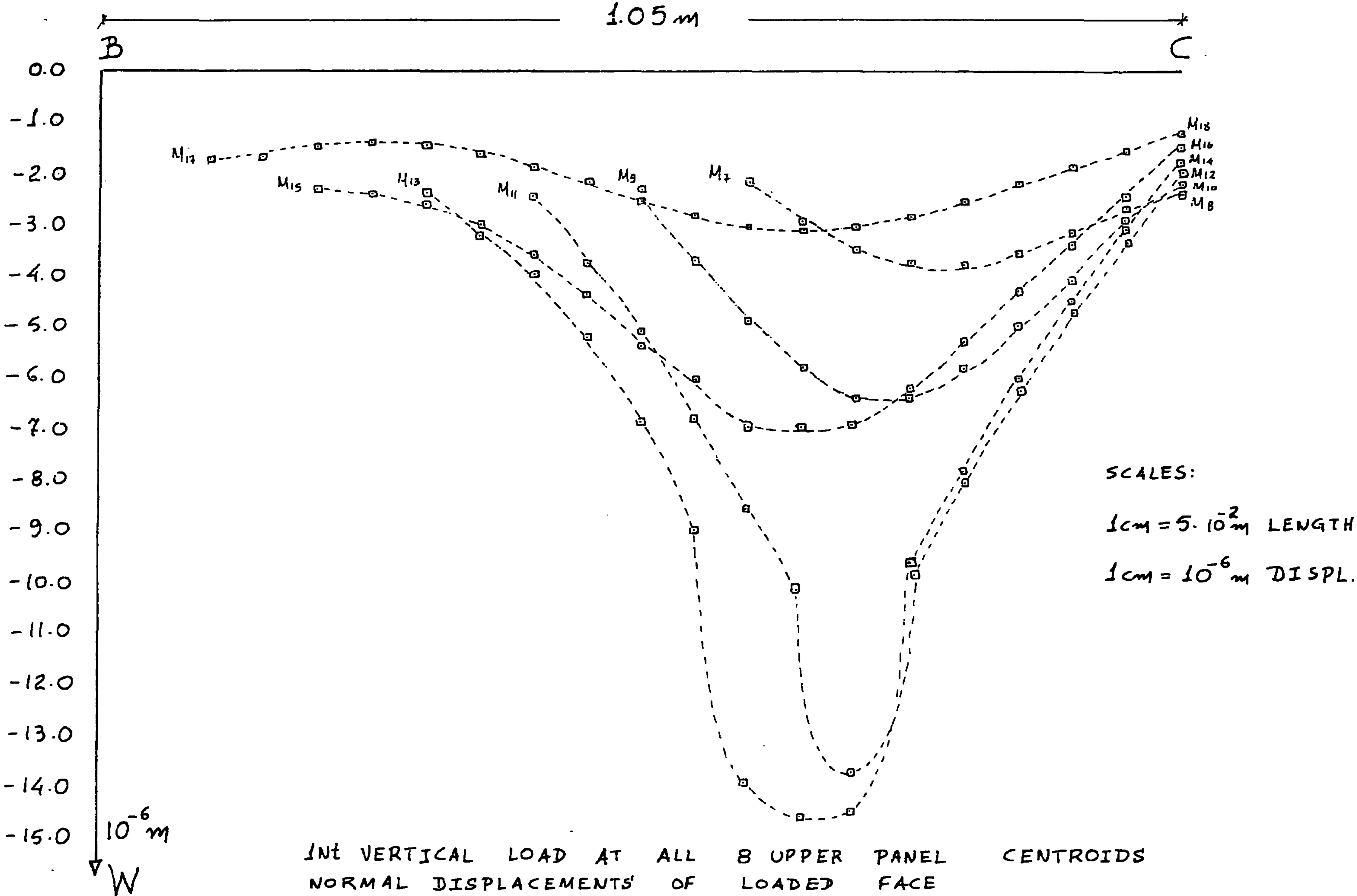
FIG. 13.44. <u>16 FACED DOME</u>



1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.45. 16 FACED DOME

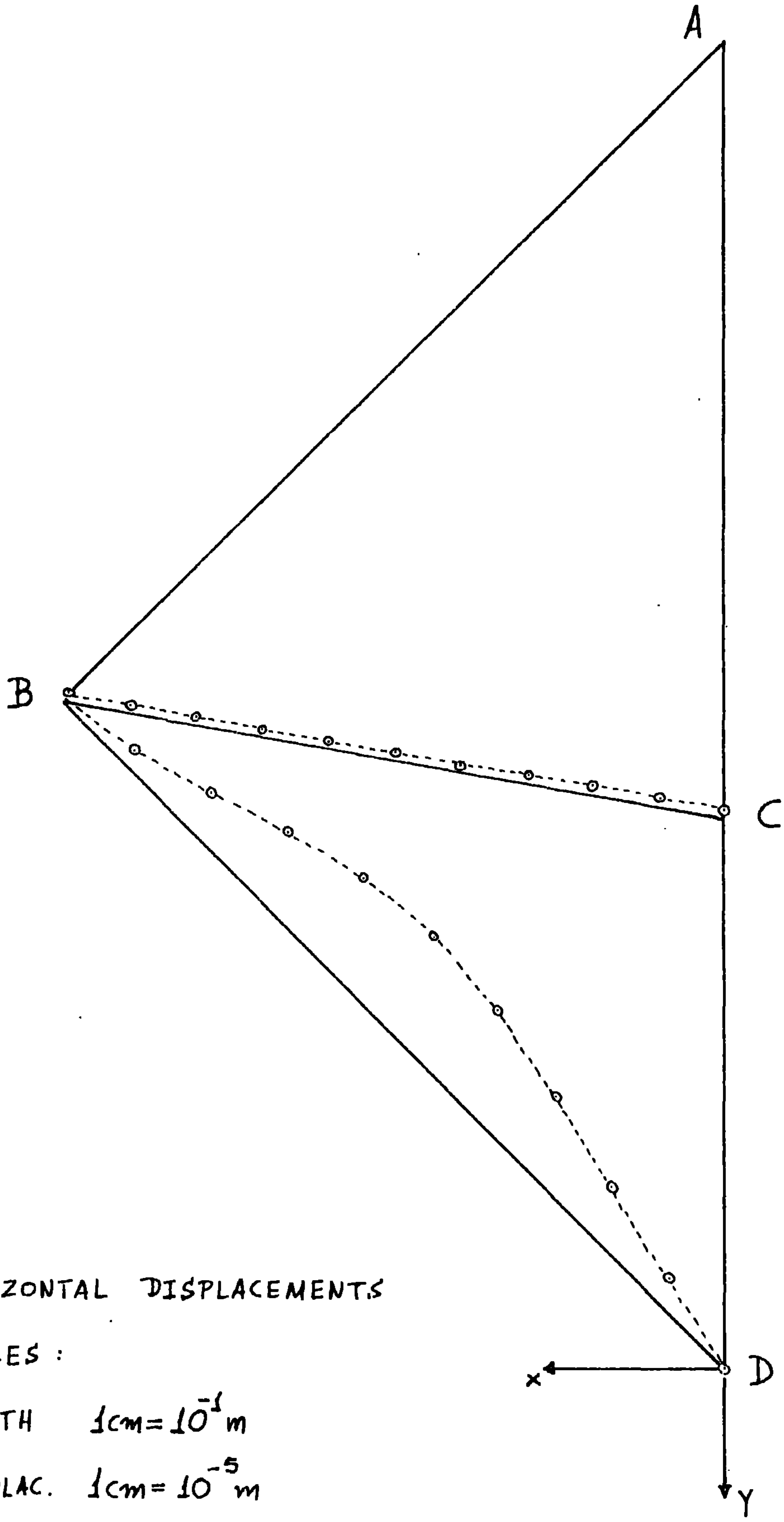


1nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS
NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.46. 16 FACED DOME

FIG. 13.47. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

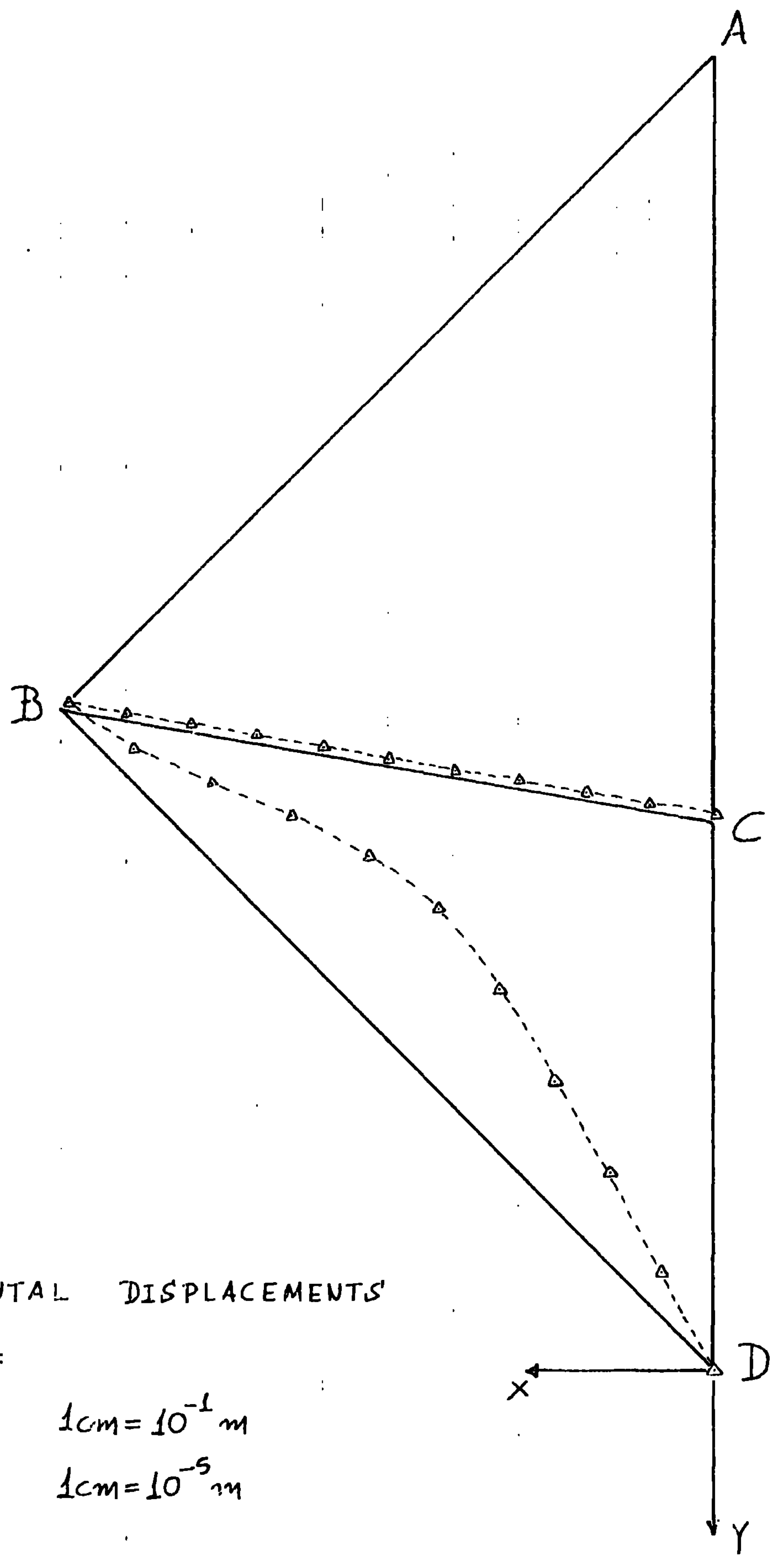
SCALES :

LENGTH 1cm = 10^{-1} m

DISPLAC. 1cm = 10^{-5} m

FIG. 13.48. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

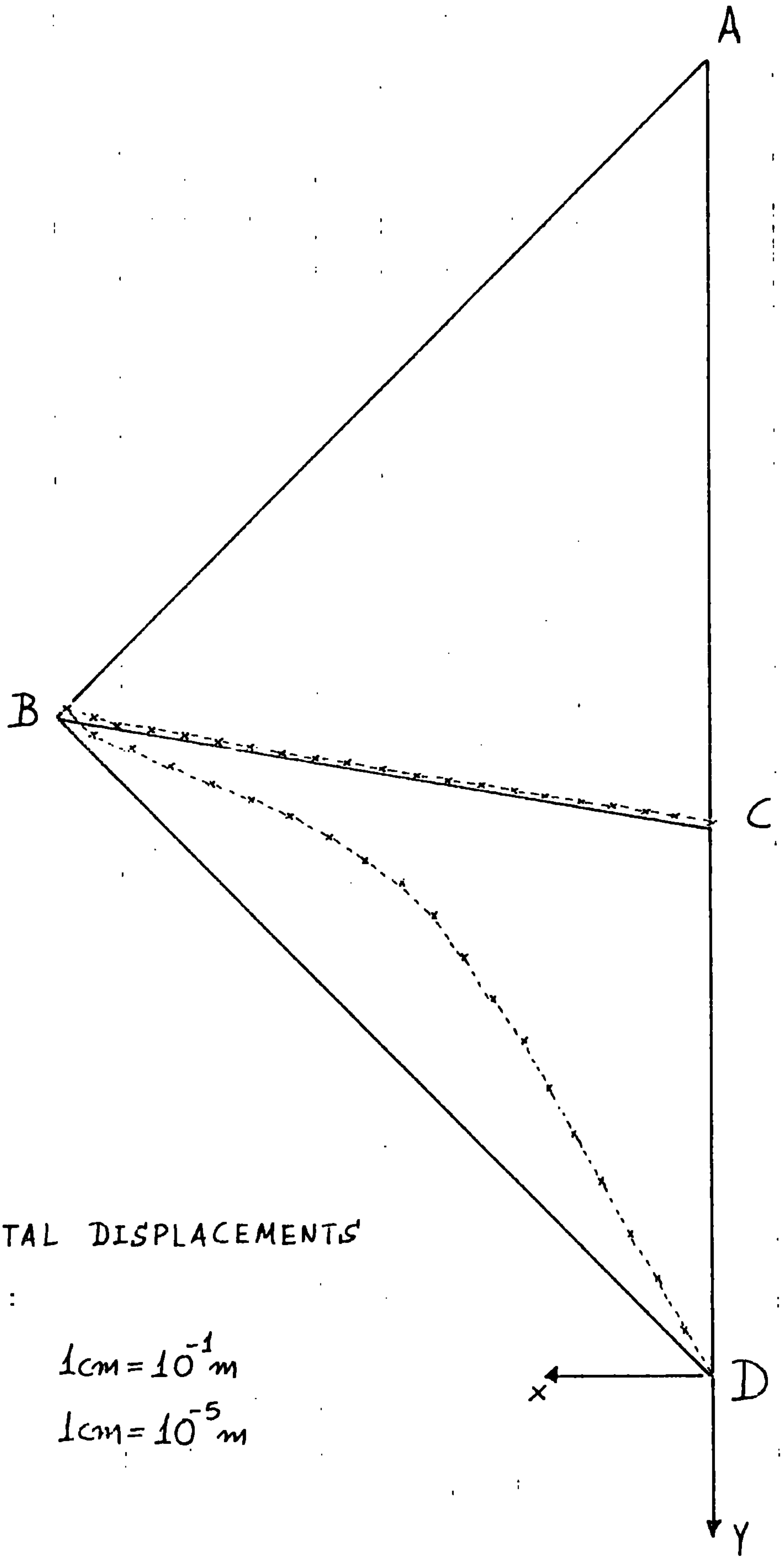
SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$

FIG. 13.49. <u>16 FACED DOME</u>

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

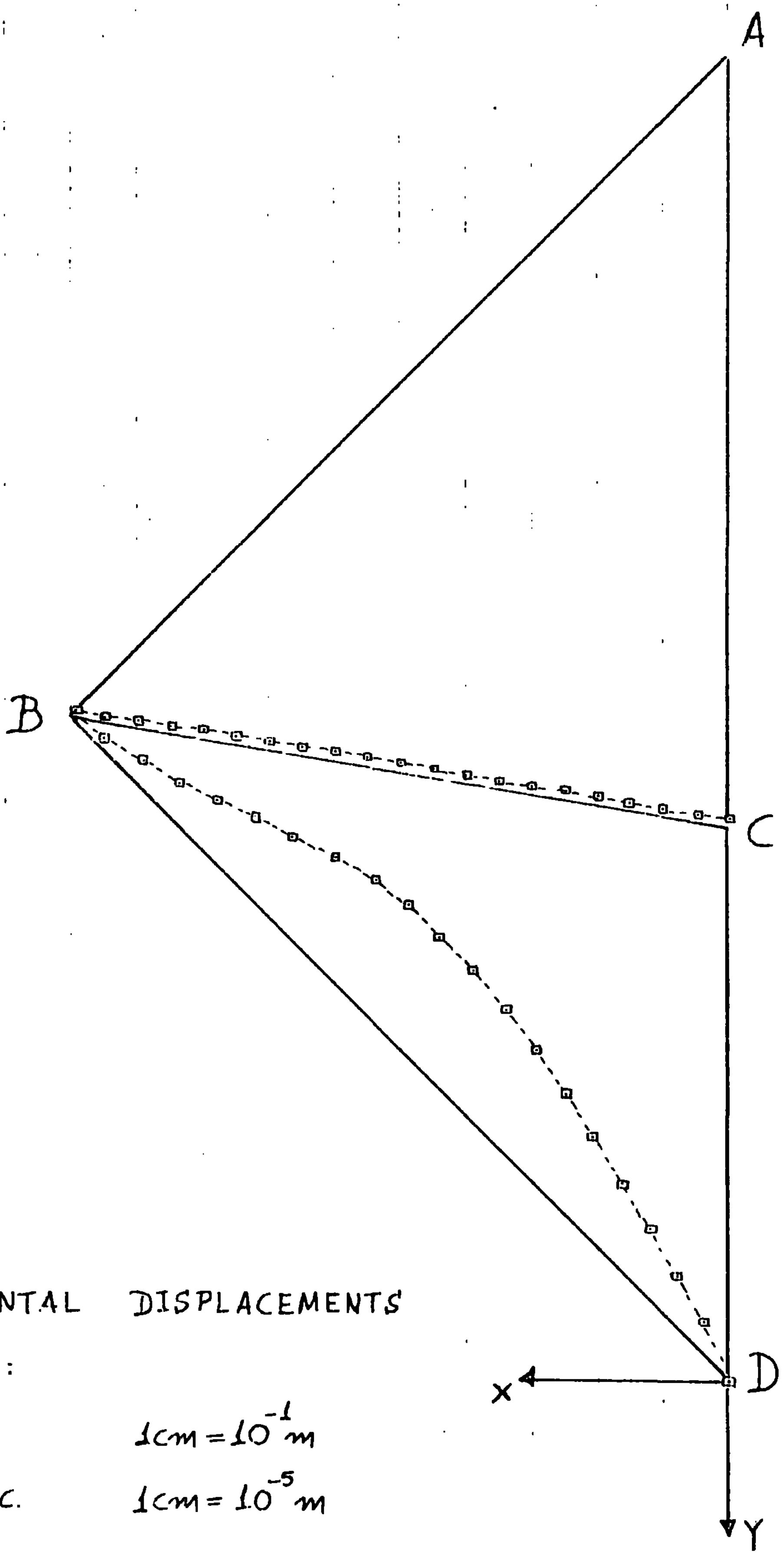
SCALES:

LENGTH 1cm = 10^{-1} m

DISPLAC. 1cm = 10^{-5} m

FIG. 13.50. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

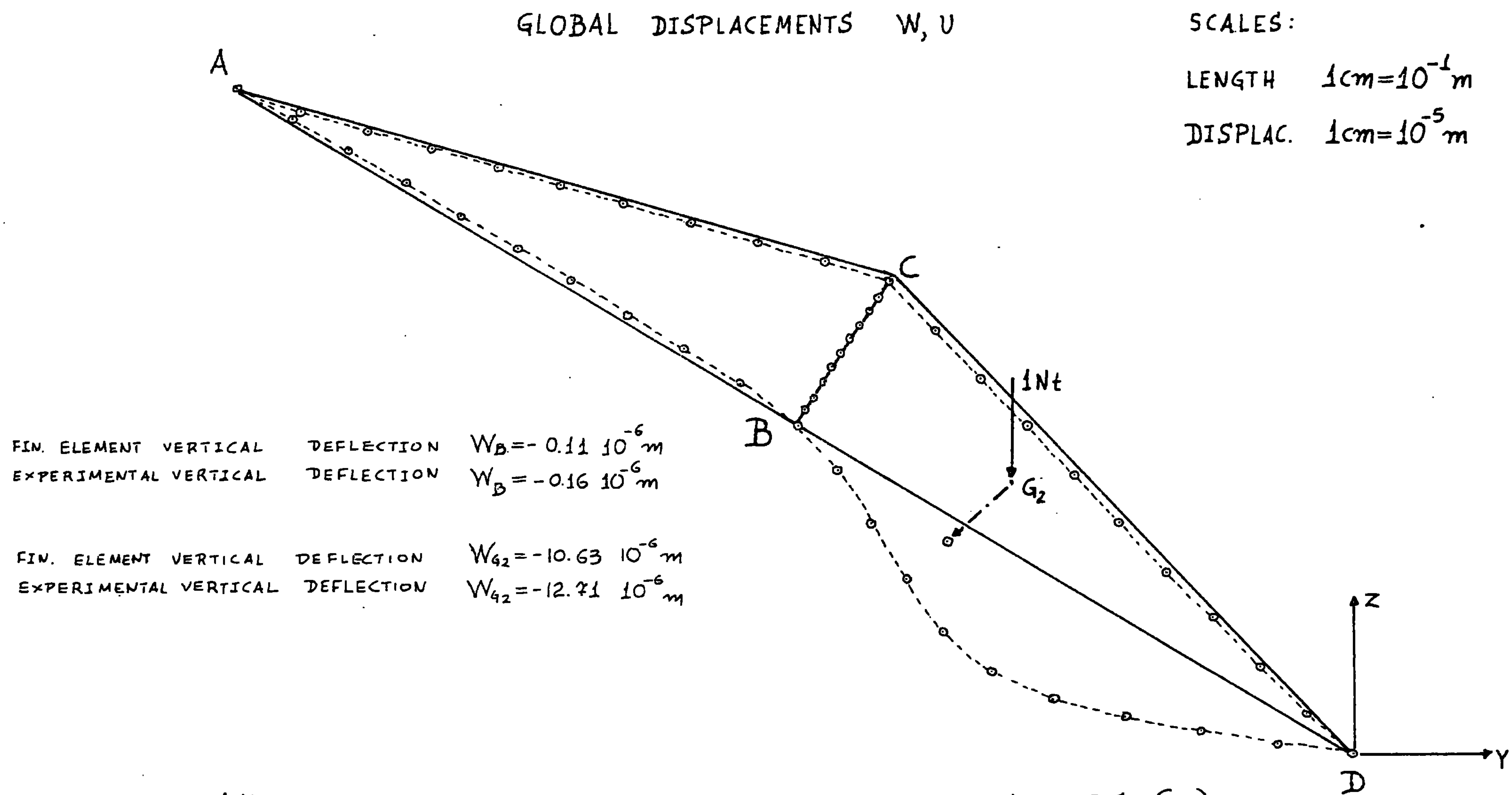


HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$



1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS (G_2)

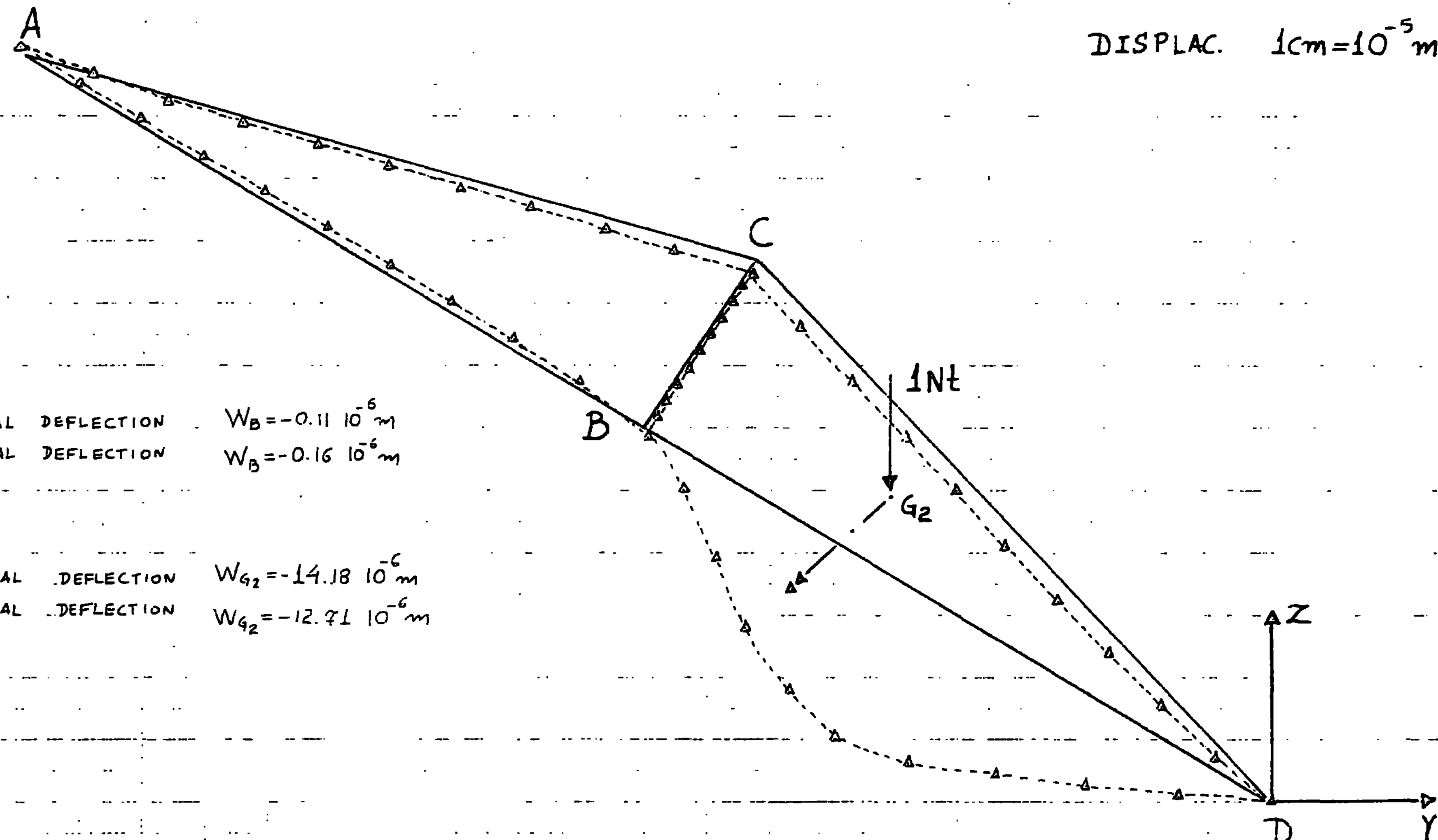
FIG. 13.51. 16 FACED DOME

GLOBAL DISPLACEMENTS W, v

SCALES:

LENGTH 1cm = 10^{-1} m

DISPLAC. 1cm = 10^{-5} m



FIN. ELEMENT VERTICAL DEFLECTION $W_B = -0.11 \; 10^{-6}$ m

EXPERIMENTAL VERTICAL DEFLECTION $W_B = -0.16 \; 10^{-6}$ m

FIN. ELEMENT VERTICAL DEFLECTION $W_{G_2} = -14.18 \; 10^{-6}$ m

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_2} = -12.71 \; 10^{-6}$ m

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS (G_2)

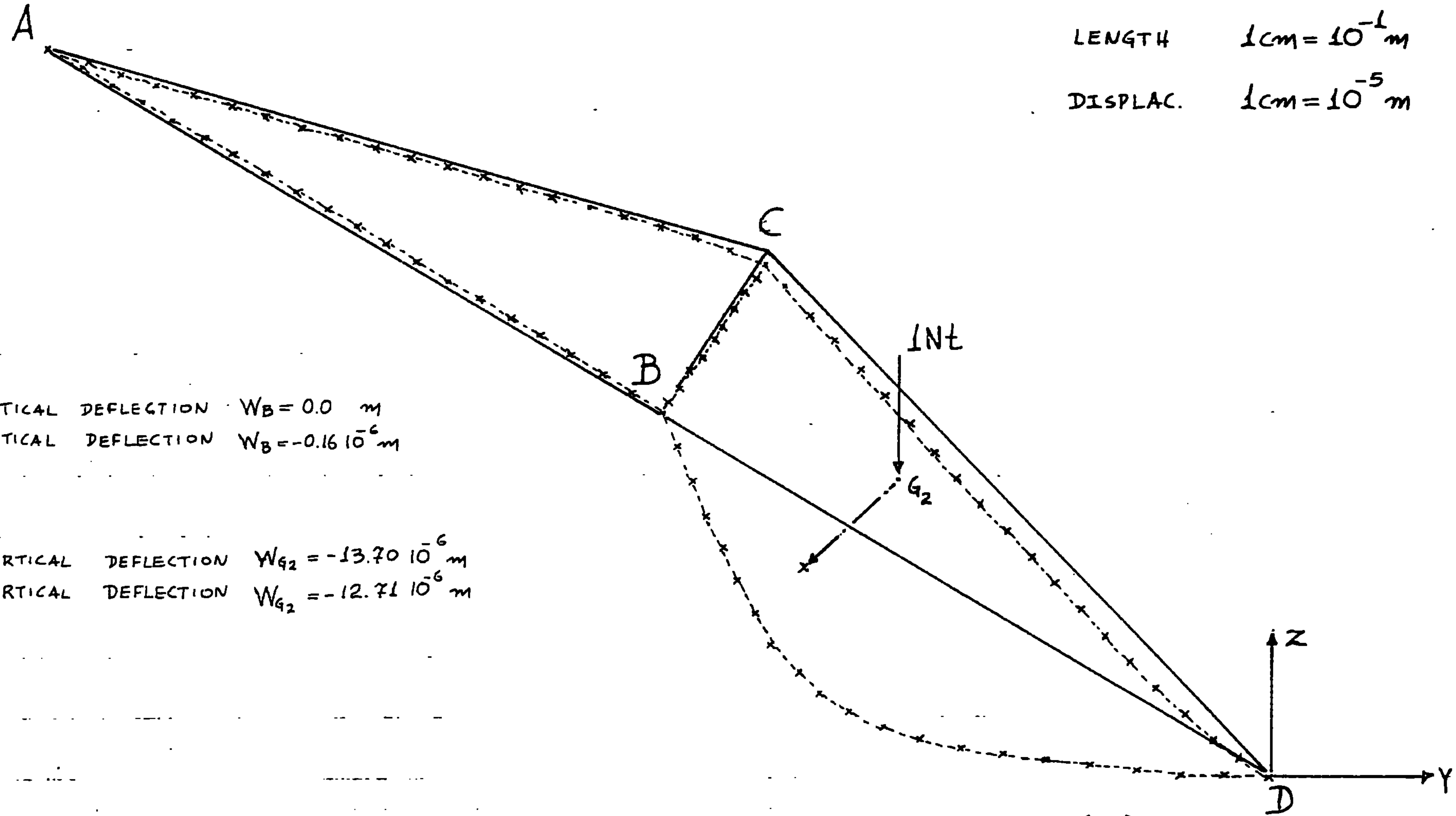
FIG. 13.52. 16 FACED DOME

GLOBAL DISPLACEMENTS W, v

SCALES:

LENGTH $1 cm = 10^{-1} m$

DISPLAC. $1 cm = 10^{-5} m$



FIN. ELEMENT VERTICAL DEFLECTION $W_B = 0.0$ m

EXPERIMENTAL VERTICAL DEFLECTION $W_B = -0.16\ 10^{-6}$ m

FIN. ELEMENT VERTICAL DEFLECTION $W_{G_2} = -13.70\ 10^{-6}$ m

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_2} = -12.71\ 10^{-6}$ m

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS (G_2)

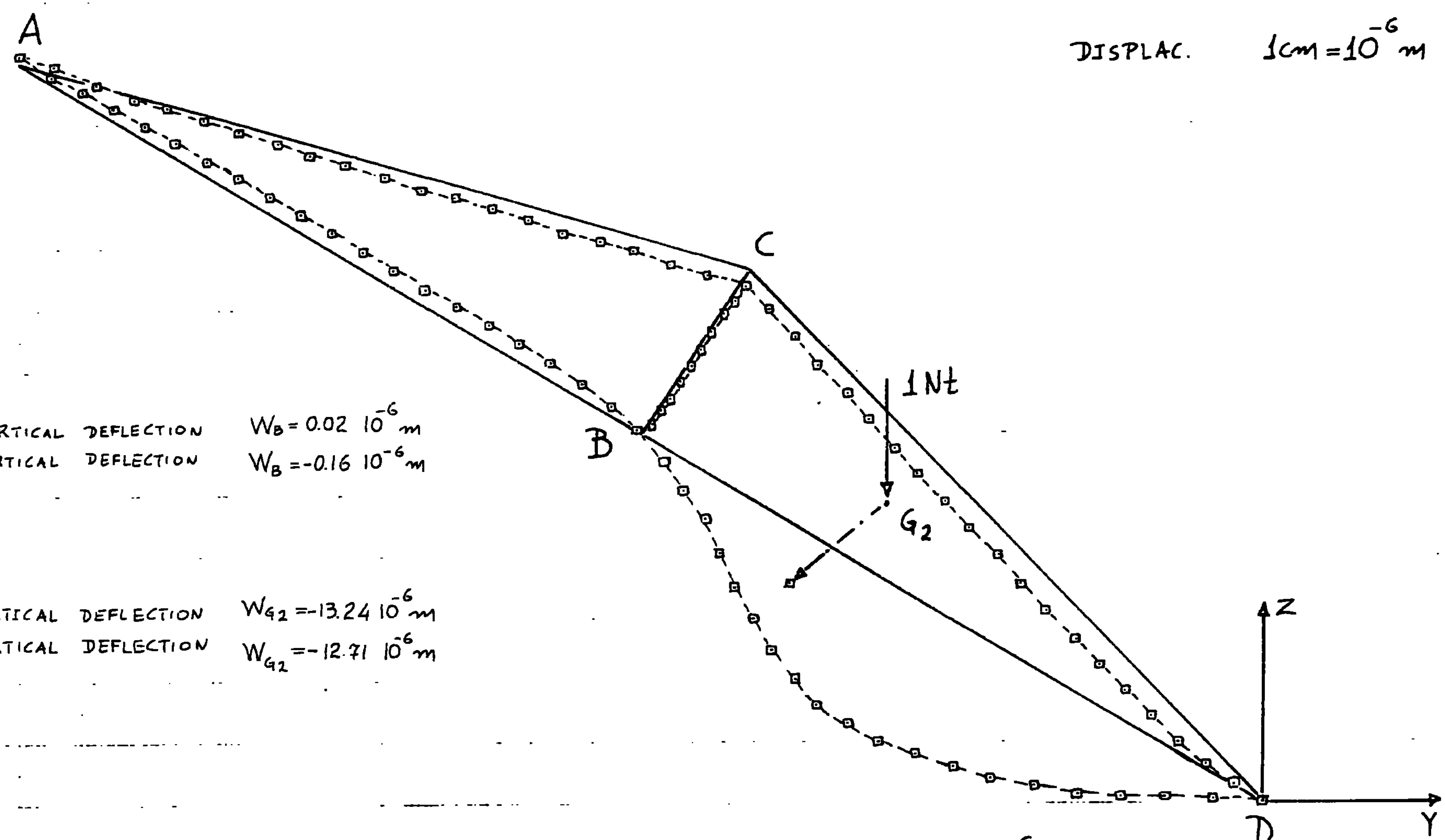
FIG. 13.53. 16 FACED DOME

GLOBAL DISPLACEMENTS W, v

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-6} m$



FIN. ELEMENT VERTICAL DEFLECTION $W_B = 0.02\ 10^{-6} m$

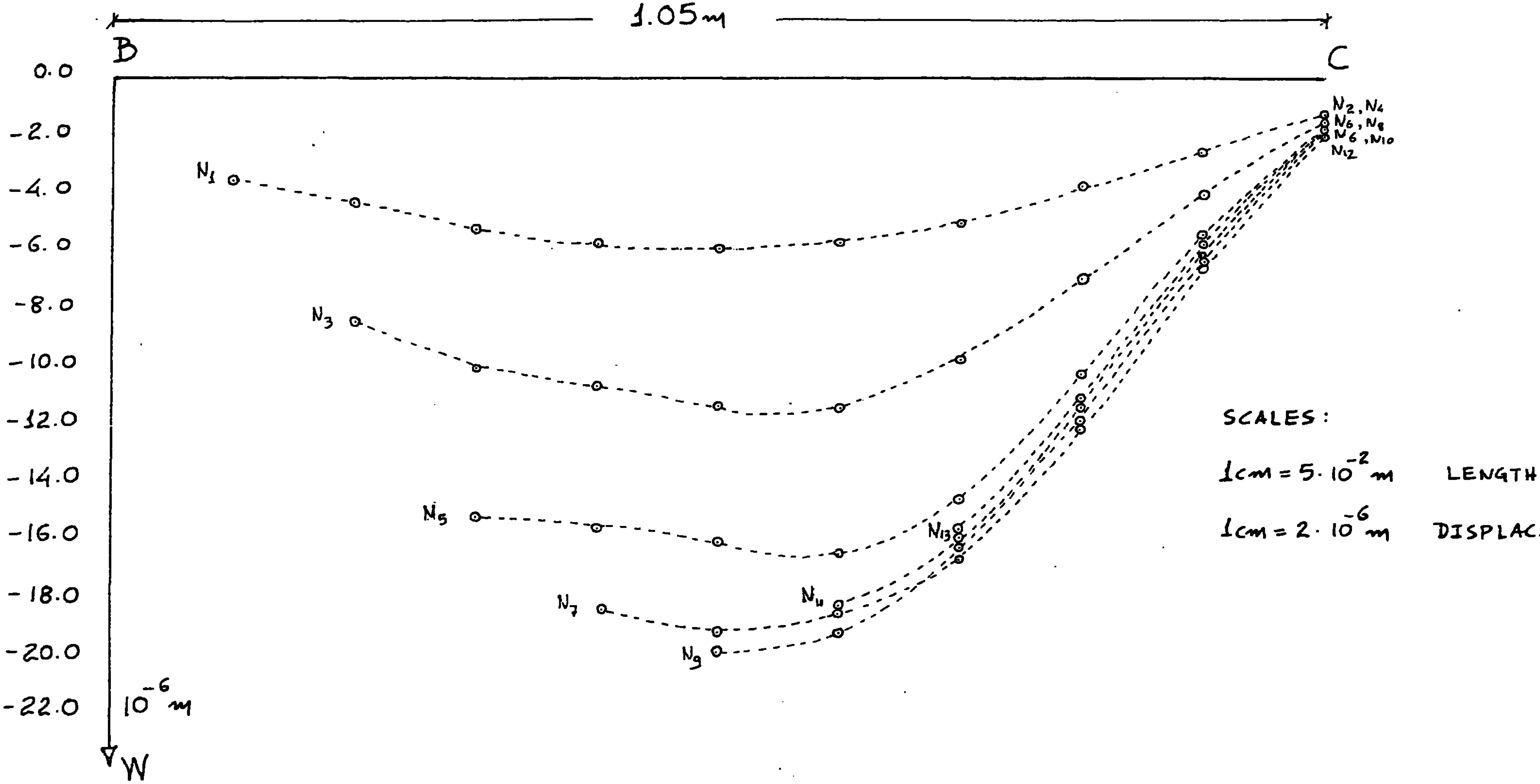
EXPERIMENTAL VERTICAL DEFLECTION $W_B = -0.16\ 10^{-6} m$

FIN. ELEMEN VERTICAL DEFLECTION $W_{G2} = -13.24\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_{G2} = -12.71\ 10^{-6} m$

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS (G_2)

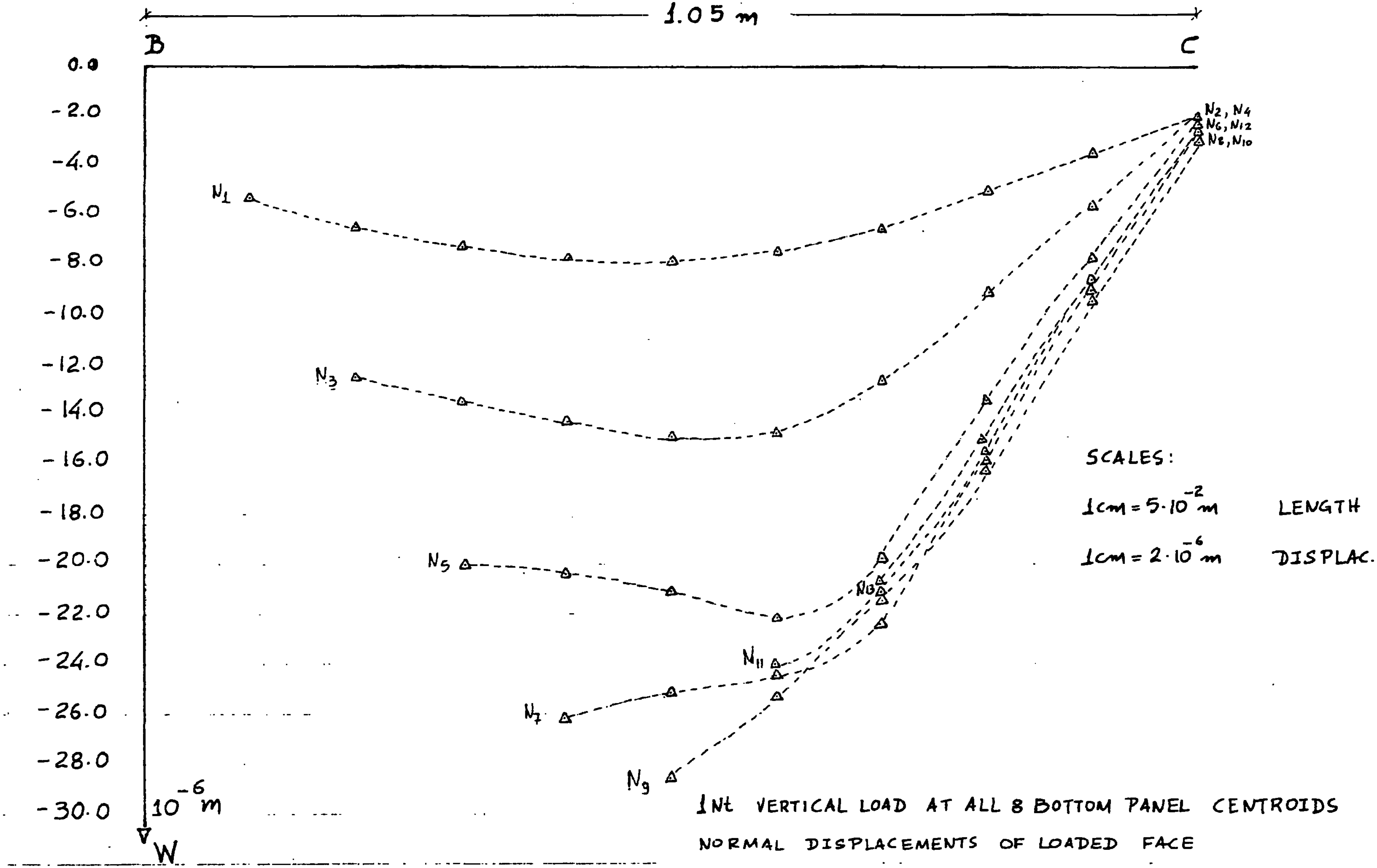
FIG. 13.54. 16 FACED DOME



1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROID

NORMAL DISPLACEMENTS OF LOADED FACE

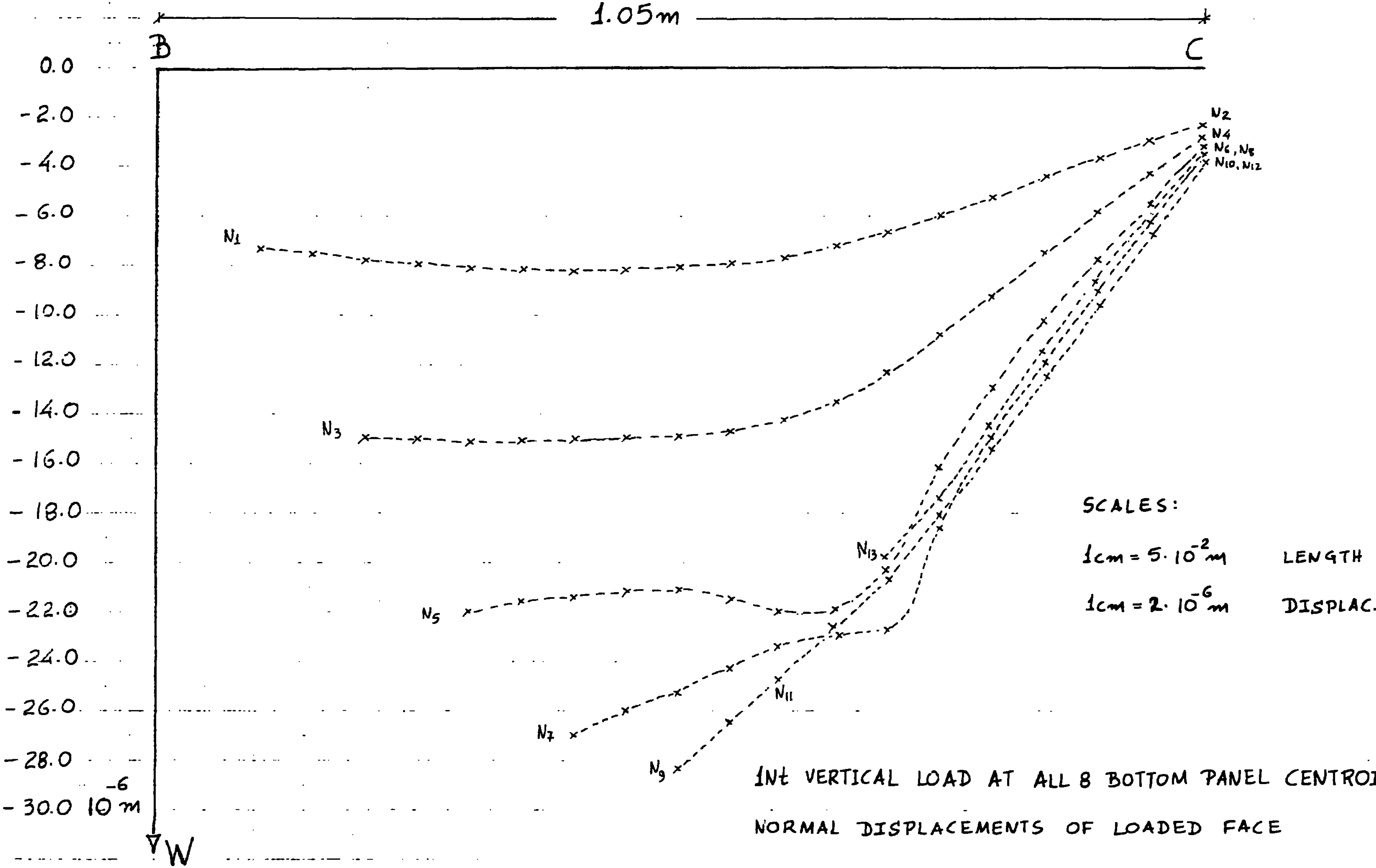
FIG. 13.55. 16 FACED DOME



1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

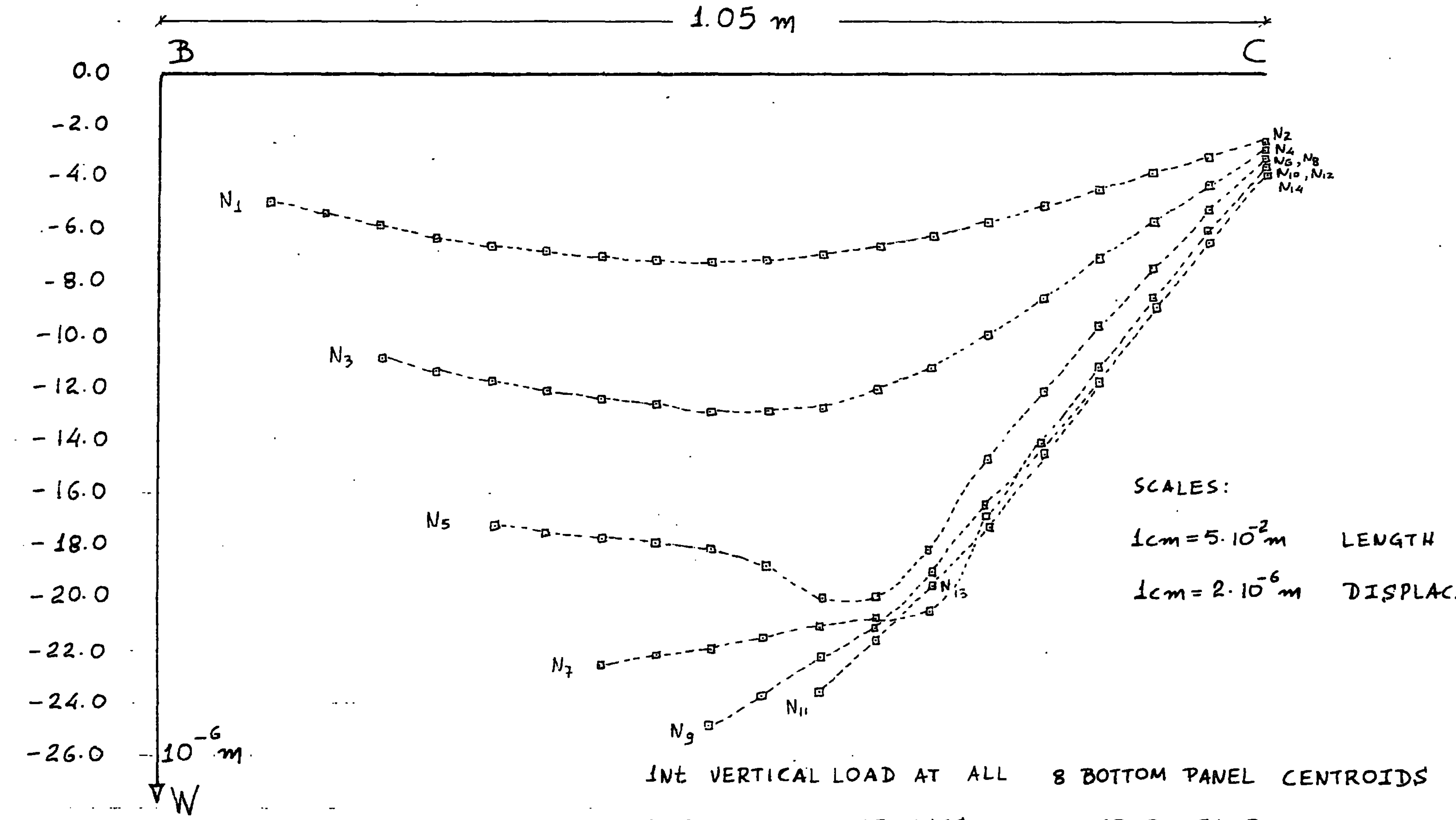
FIG. 13.56. 16 FACED DOME



1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.57. 16 FACED DOME



1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

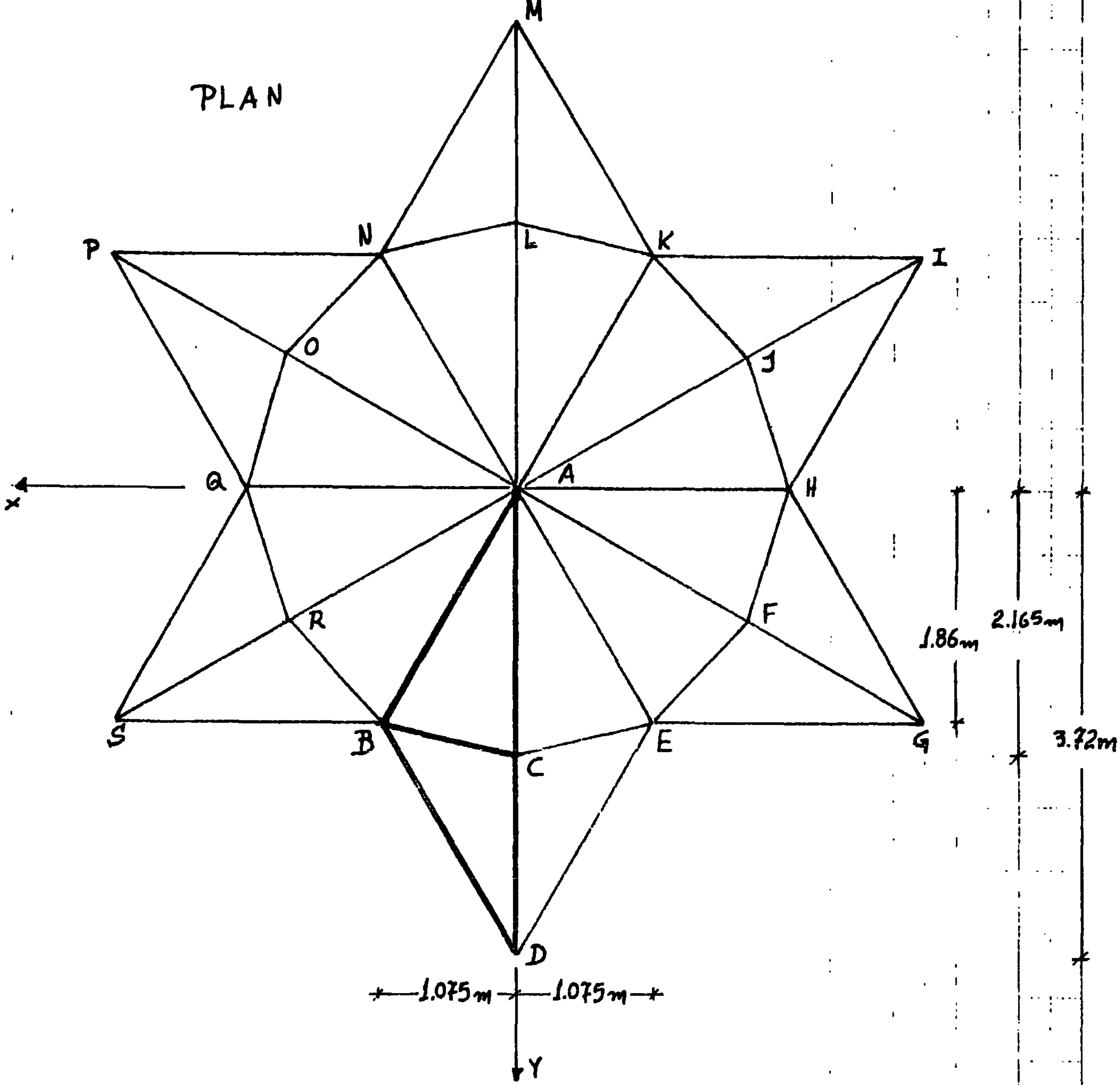
FIG. 13.58. 16 FACED DOME

FIG. 13.59. 24. FACED DOME

GENERAL ARRANGEMENT

SCALE:
1cm = 0.5m LENGTH

PLAN



ELEVATION

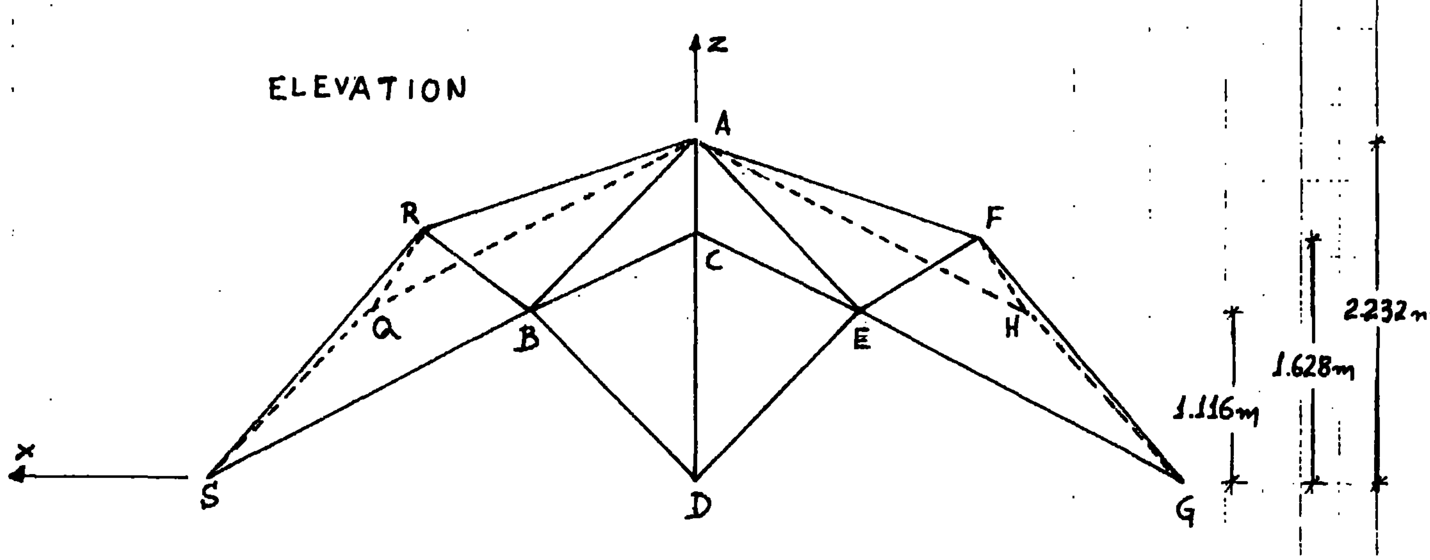
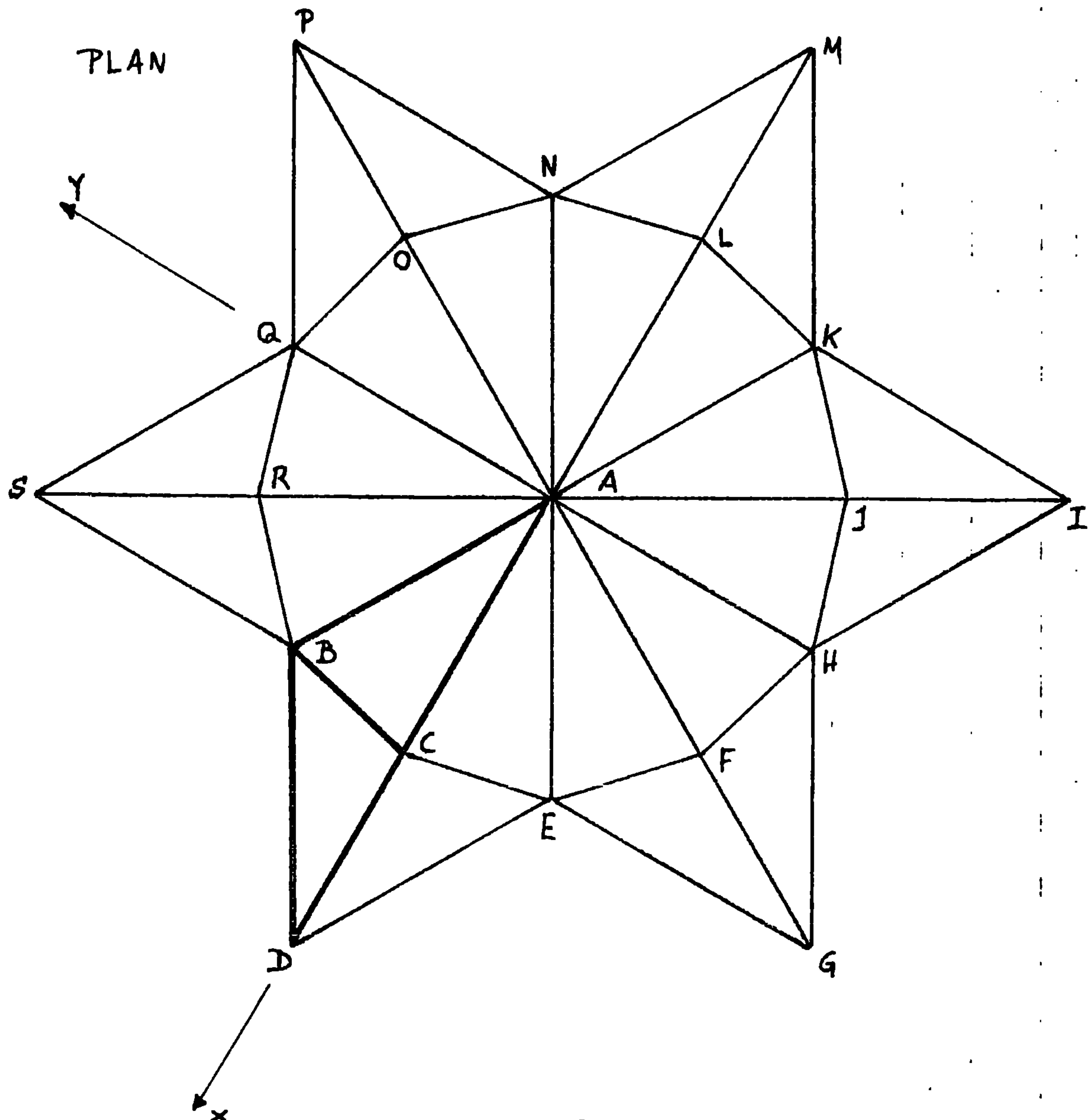


FIG. 13.60. 24. FACED DOME

GENERAL ARRANGEMENT

SCALE
1cm = 0.5 m LENGTH

PLAN



ELEVATION

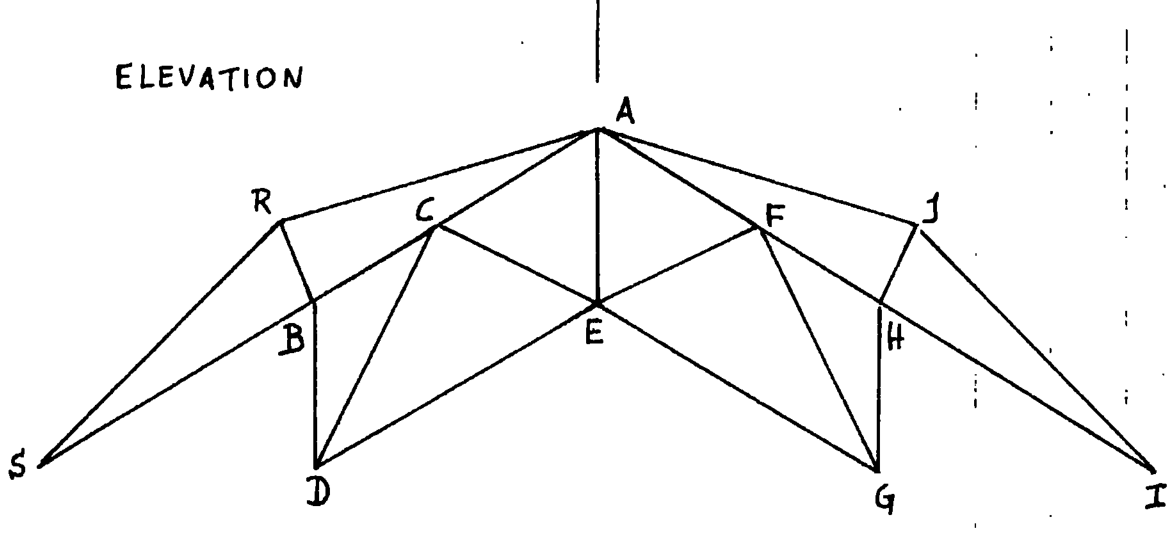
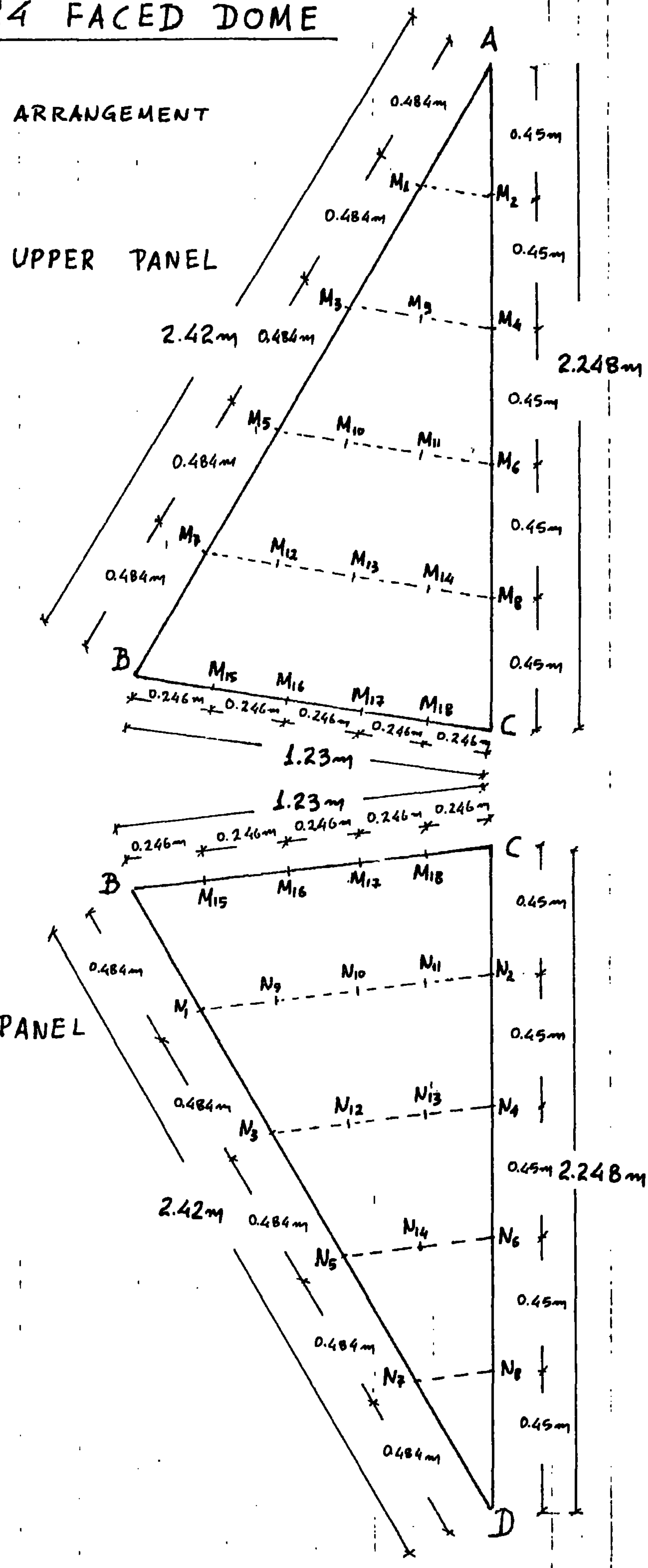


FIG. 13.61 24 FACED DOME

GENERAL ARRANGEMENT

SCALE :

$1cm = 2 \cdot 10^{-1} m$ LENGTH



KEY	
DDS21	⊙
DDS33	△
DMX36	×
DRO30	⊡
EXPERIMENT	—+—

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

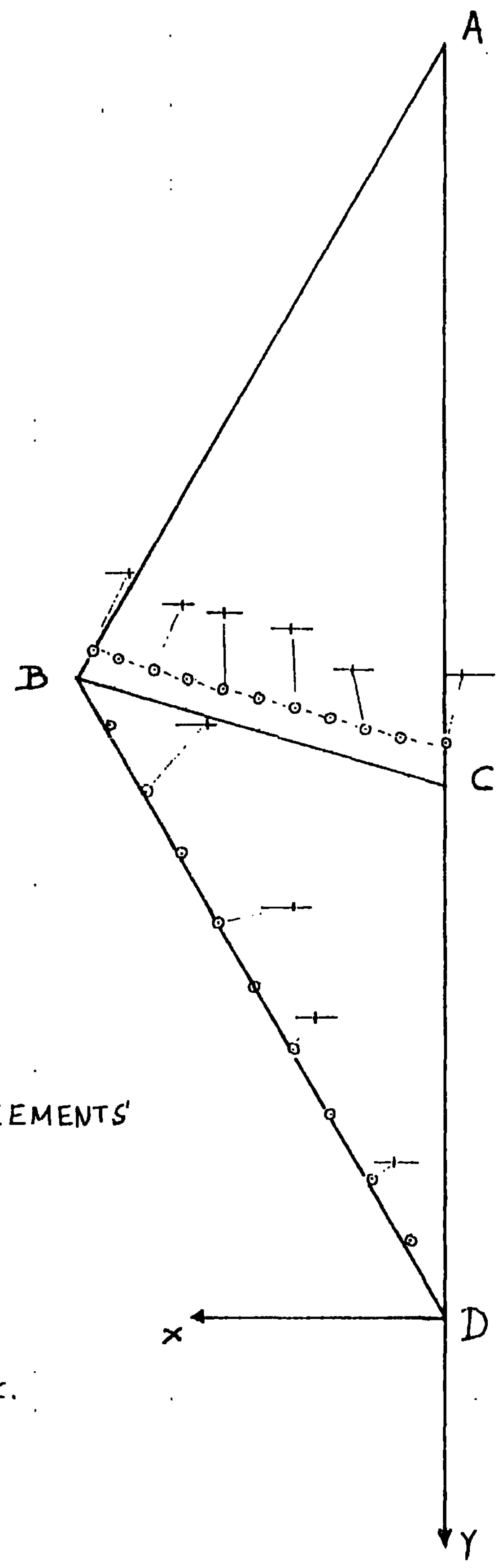
EXPERIMENTAL VALUES OF GLOBAL DISPL. u, v, w

POINT	u 10^{-6}m	v 10^{-6}m	w 10^{-6}m	POINT	u 10^{-6}m	v 10^{-6}m	w 10^{-6}m	POINT	u 10^{-6}m	v 10^{-6}m	w 10^{-6}m
A	-0.05	-0.07	-0.66	M_{13}	-1.15	-1.60	-3.50	N_{11}	0.00	-1.32	-2.13
M_1	-1.30	-1.89	-2.14	M_{14}	0.25	-0.66	-3.19	N_2	0.05	-1.06	-3.28
M_2	-0.32	-1.07	-1.89	M_8	-0.34	-2.30	-2.46	N_3	-0.98	-0.41	-1.31
M_3	-0.74	-1.31	-3.45	B	-0.82	-1.56	-3.28	N_{12}	-1.15	-0.82	-2.21
M_9	0.00	0.00	-3.03	M_{15}	-0.47	-1.39	-2.87	N_{13}	-0.51	-0.65	-1.56
M_4	-0.25	-1.64	-2.29	M_{16}	0.00	-1.56	-2.63	N_4	-0.08	-0.82	-1.39
M_5	-0.71	-1.43	-4.26	M_{17}	0.08	-1.64	-1.97	N_5	-0.33	-0.57	-1.06
M_{10}	-1.97	-1.39	-4.01	M_{18}	0.30	-1.31	-1.80	N_{14}	-0.65	-0.41	-1.23
M_{11}	-0.98	-2.71	-3.93	C	-0.25	-1.64	-2.38	N_6	-0.08	-1.01	-0.98
M_6	0.08	-1.31	-3.12	N_1	-0.82	-1.48	-2.54	N_7	-0.33	-0.41	-0.33
M_7	-0.62	-1.31	-3.61	N_9	-1.48	-1.48	-2.13	N_8	0.00	-0.51	0.00
M_{12}	-1.15	-1.47	-3.93	N_{10}	-1.23	-1.72	-1.88				

FIG. 13.62. 24 FACED DOME

FIG. 13.63. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

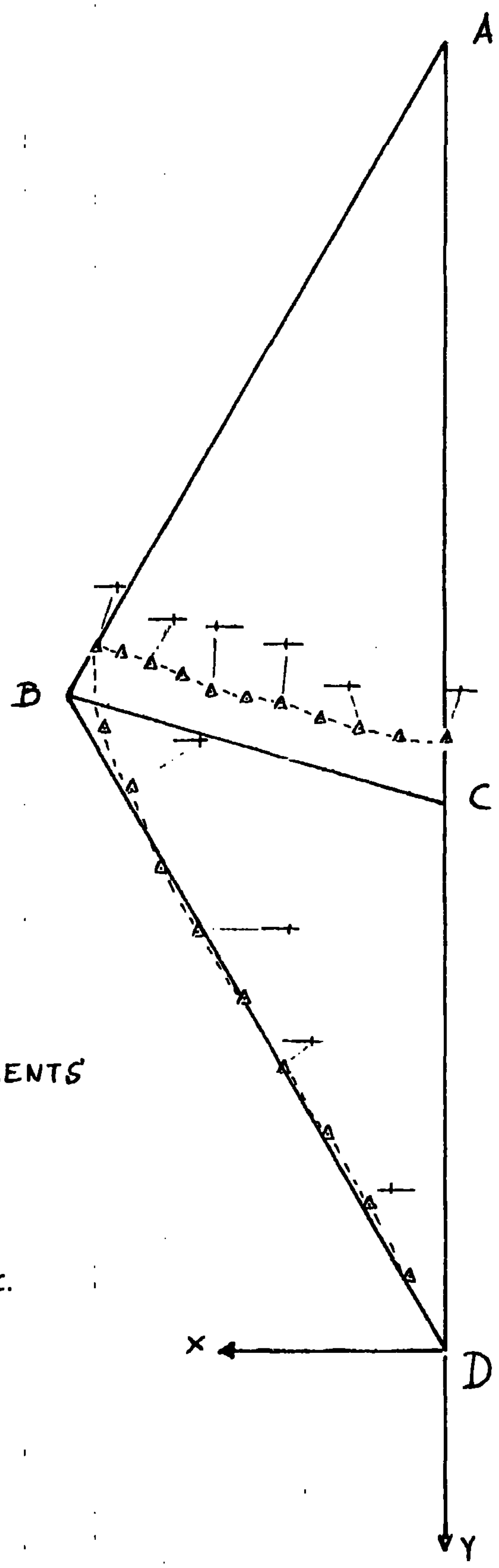
SCALES :

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

FIG. 13.64. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

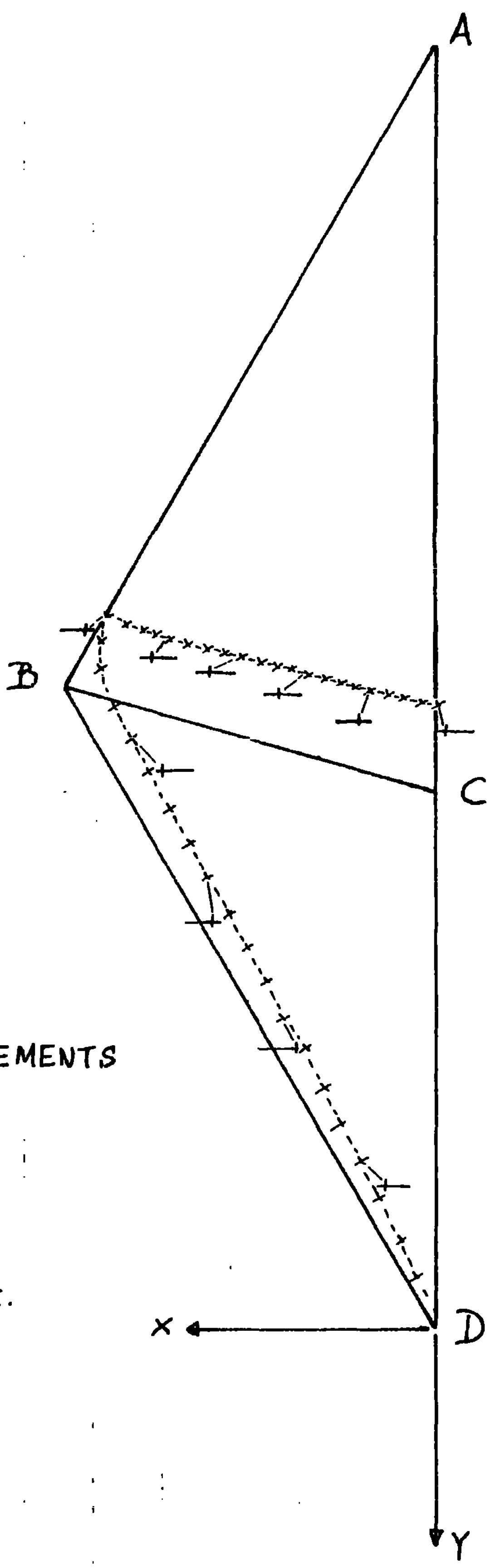
SCALES :

$1 cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 10^{-6} m$ DISPLAC.

FIG. 13.65. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

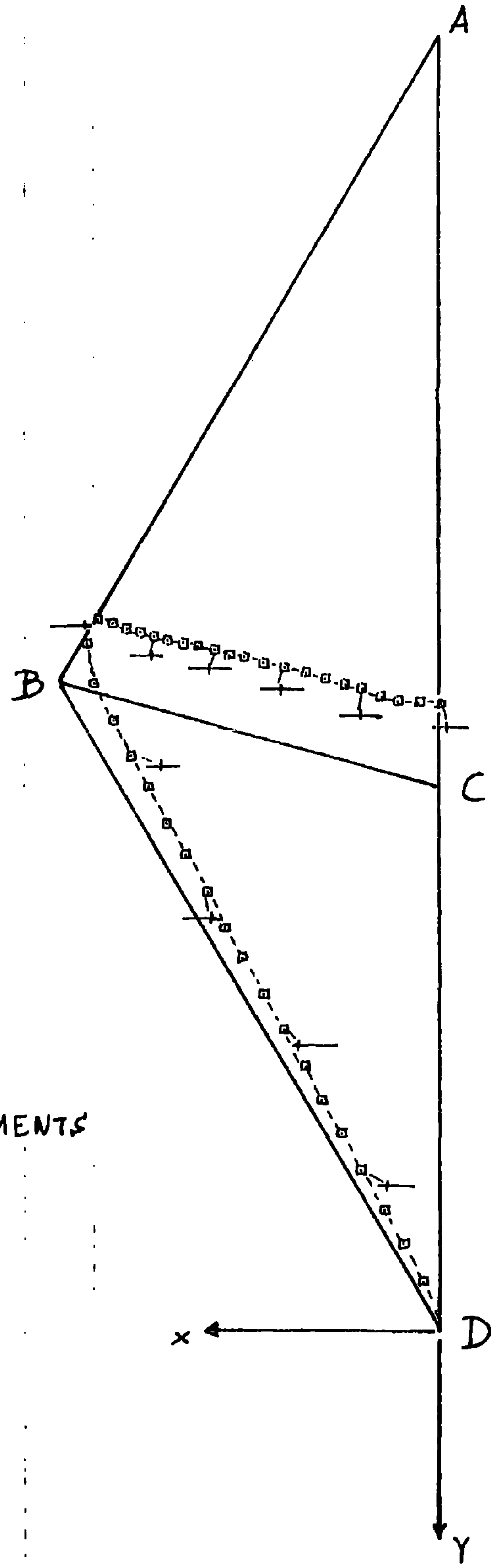
SCALES:

$1cm = 2\cdot10^{-1}m$ LENGTH

$1cm = 2\cdot10^{-6}m$ DISPLAC.

FIG. 13.66. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

SCALES:

$1 cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 2 \cdot 10^{-6} m$ DISPLAC.

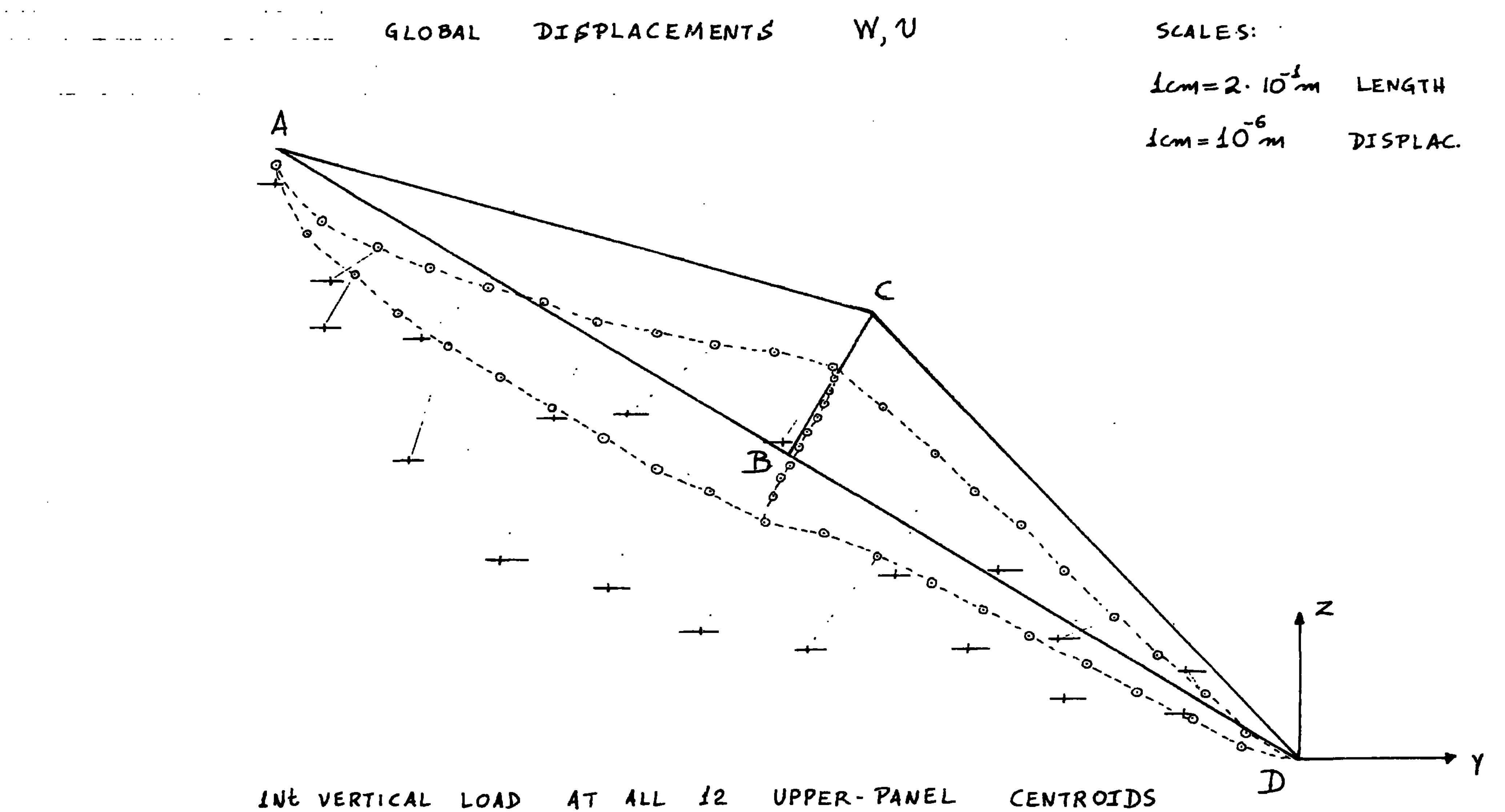
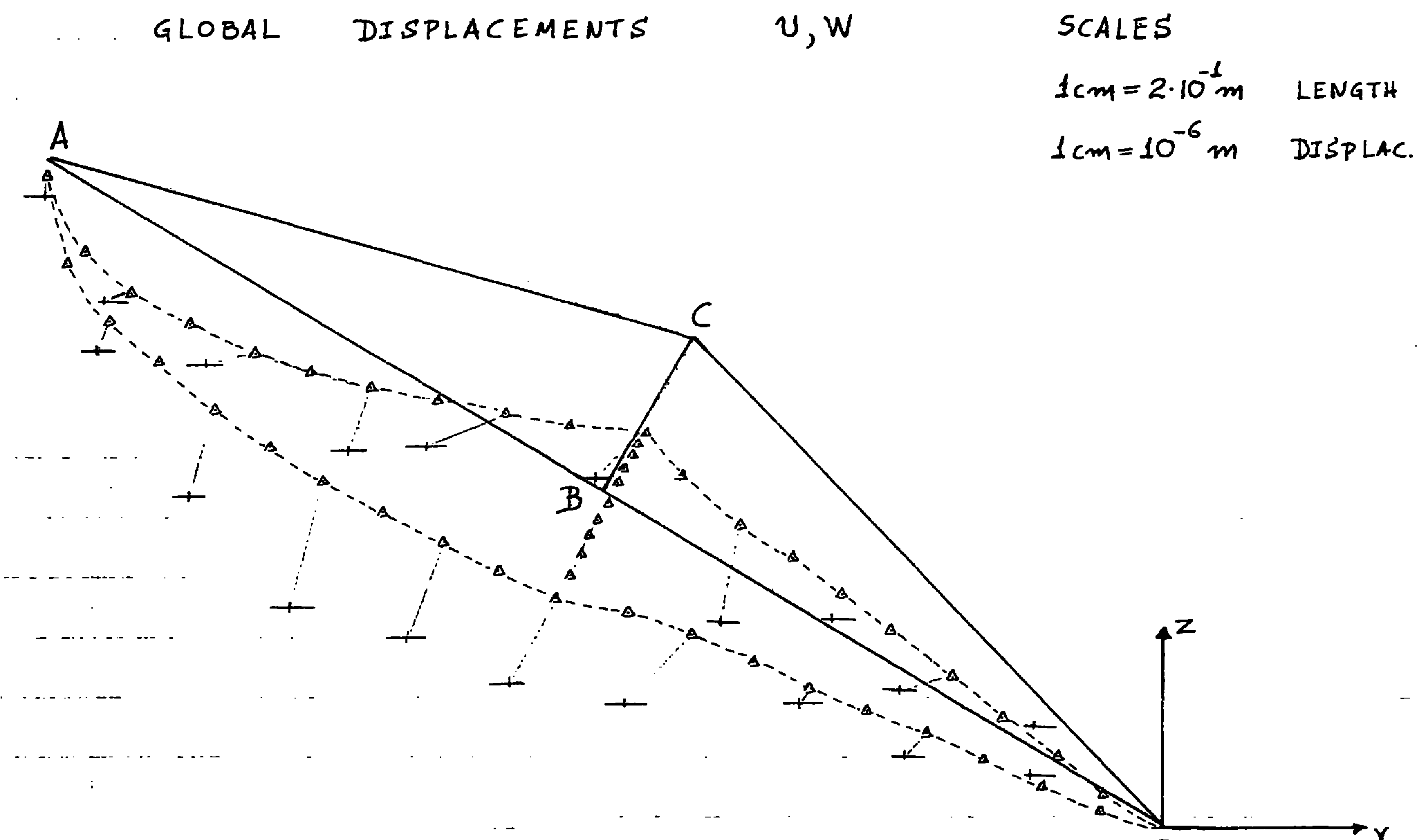


FIG. 13.67. 24 FACED DOME



1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

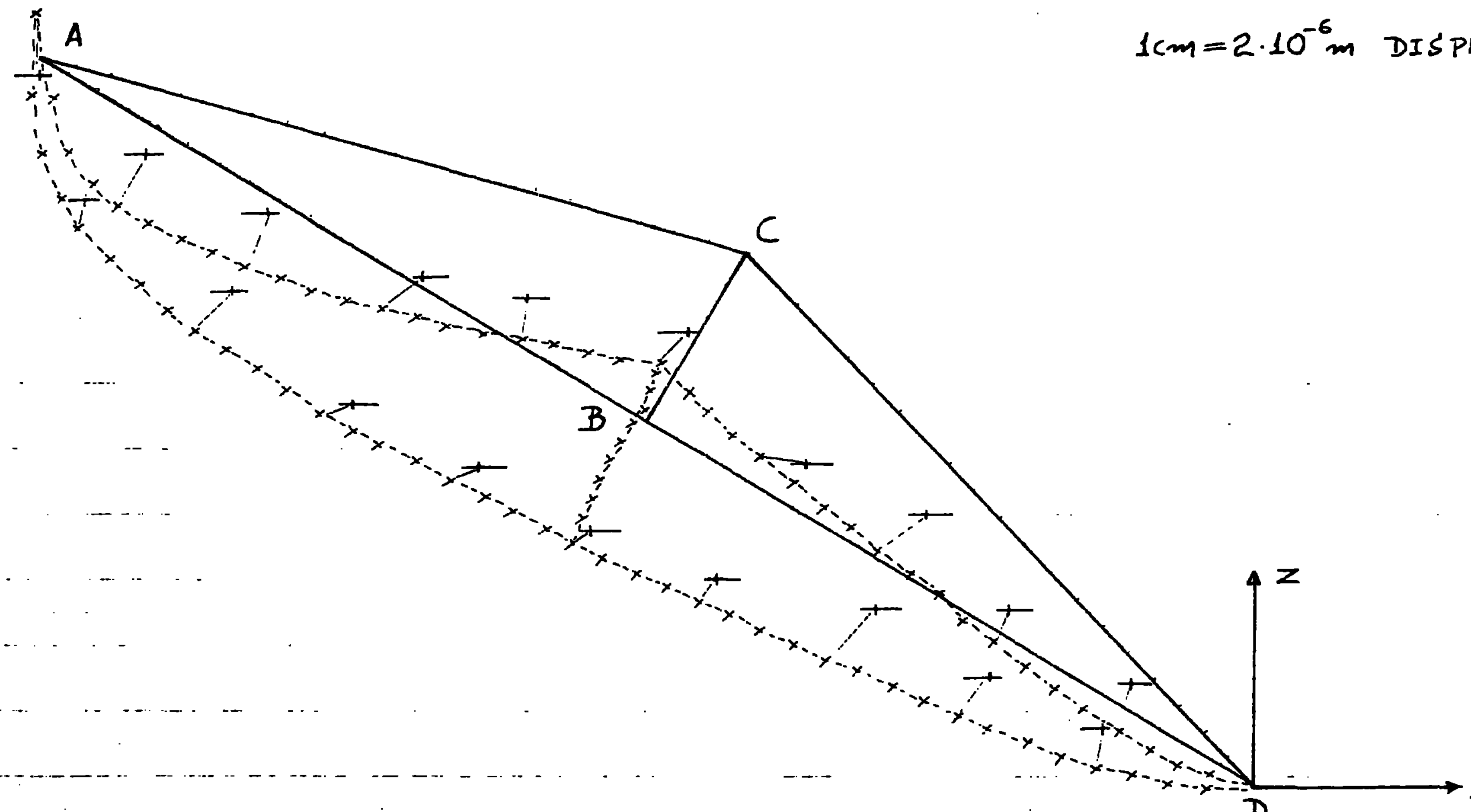
1Nt FIG. 13.68. 24 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

1cm = 2·10⁻¹ m LENGTH

1cm = 2·10⁻⁶ m DISPLAC.



1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

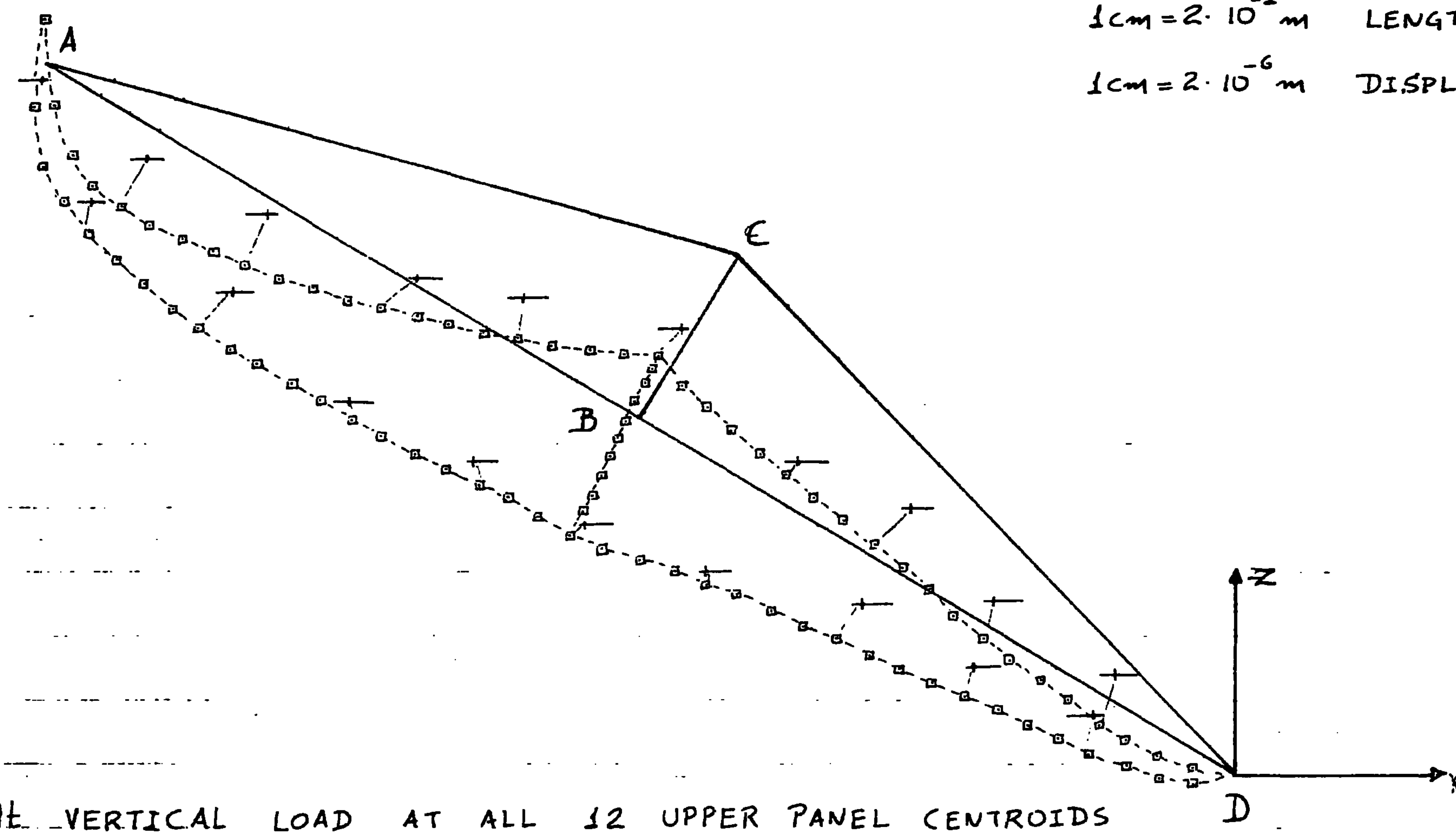
FIG. 13.69. 24 FACED DOME

GLOBAL DISPLACEMENTS W, V

SCALES:

1cm = 2· 10^{-1} m LENGTH

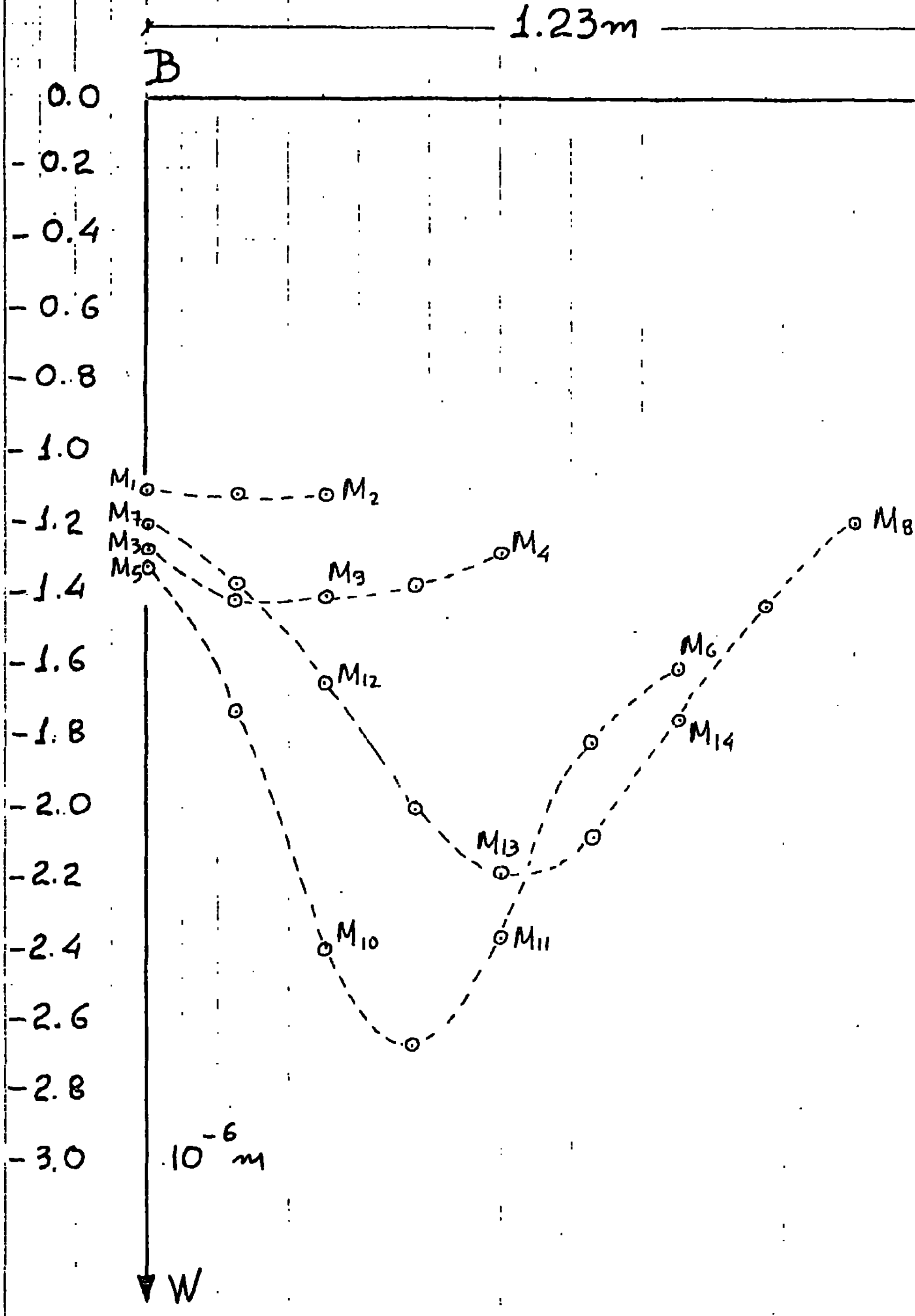
1cm = 2· 10^{-6} m DISPLAC.



1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

FIG. 13.70. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



NORMAL DISPLACEMENTS OF LOADED FACE

SCALES:

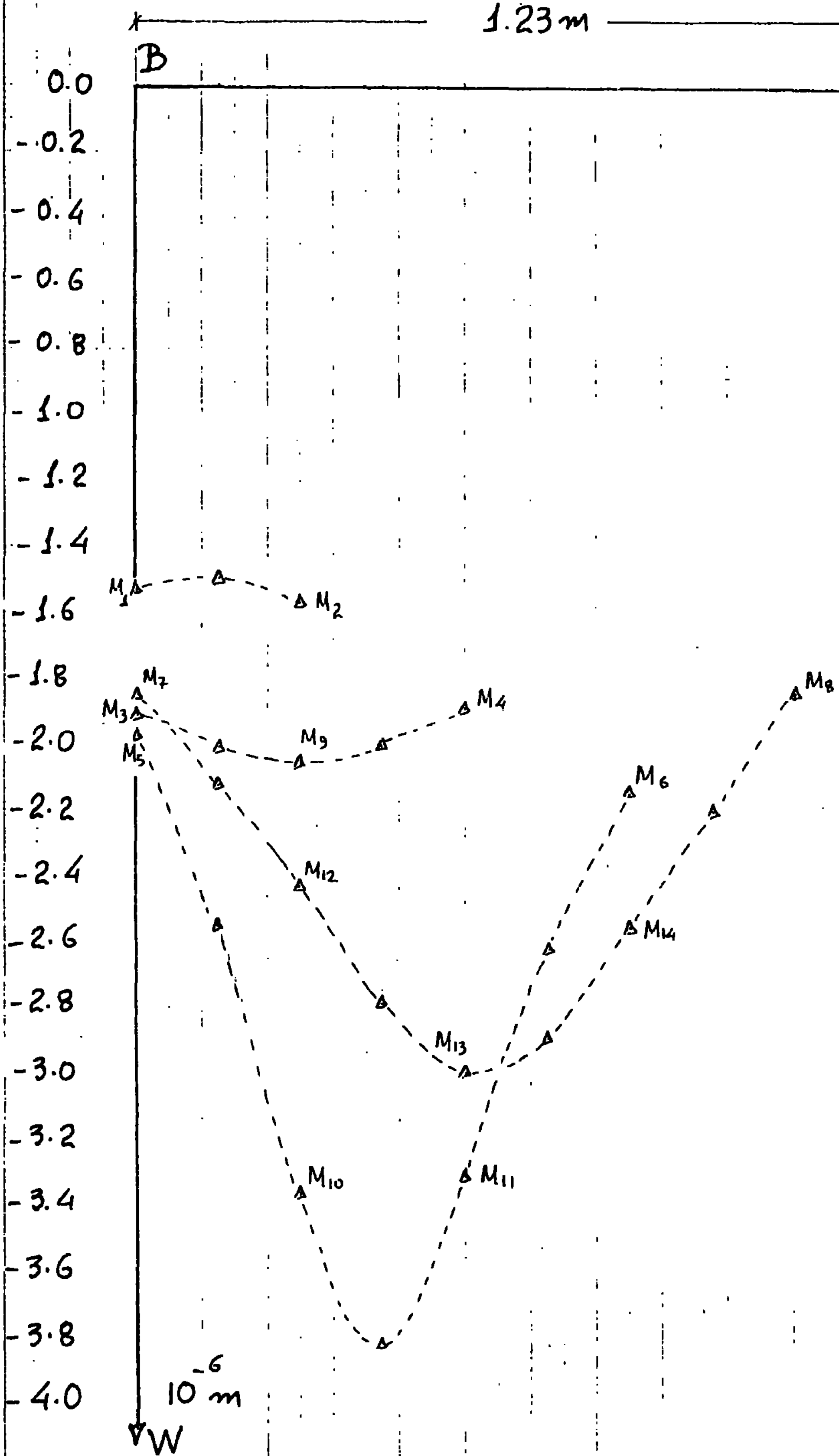
1cm = 10^{-1} m LENGTH

1cm = $2 \cdot 10^{-7}$ m DISPLAC.

	FINITE ELEMENT (DDS21) VALUE OF NORMAL DISPLAC. W (10^{-6}m)	EXPERIMENTAL VALUE OF NORMAL DISPLAC. W (10^{-6}m)
M_1	-1.10	-2.26
M_2	-1.12	-1.85
M_3	-1.28	-3.23
M_9	-1.40	-2.57
M_4	-1.28	-2.23
M_5	-1.32	-3.95
M_{10}	-2.39	-3.73
M_{11}	-2.36	-3.97
M_6	-1.60	-2.95
M_7	-1.20	-3.37
M_{12}	-1.65	-3.68
M_{13}	-2.18	-3.35
M_{14}	-1.65	-2.86
M_8	-1.19	-2.63

FIG. 13.71. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



NORMAL DISPLACEMENTS OF LOADED FACE

	FINITE ELEMENT (DDS33) VALUE OF NORMAL DISPLAC W (10^{-6} m)	EXPERIMENTAL VALUE OF NORMAL DISPLAC. W (10^{-6} m)
M_1	-1.53	-2.26
M_2	-1.58	-1.85
M_3	-1.91	-3.23
M_9	-2.06	-2.57
M_4	-1.89	-2.23
M_5	-1.97	-3.95
M_{10}	-3.36	-3.73
M_{11}	-3.31	-3.97
M_6	-2.15	-2.95
M_7	-1.85	-3.37
M_{12}	-2.43	-3.68
M_{13}	-3.00	-3.35
M_{14}	-2.56	-2.86
M_8	-1.84	-2.63

SCALES:

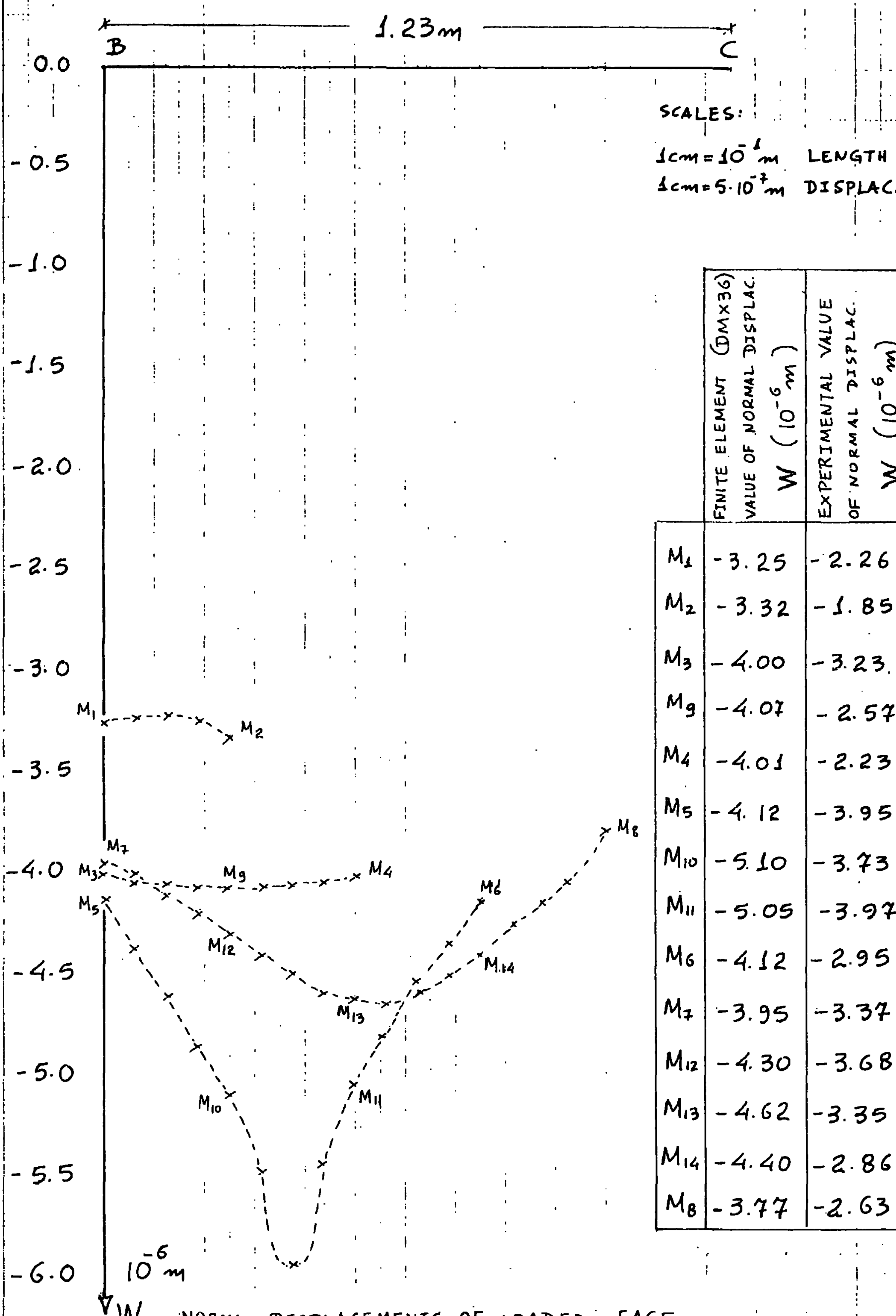
1cm = 10^{-1} m LENGTH

1cm = $2 \cdot 10^{-7}$ m DISPLAC.

FIG. 13.72. 24 FACED DOME

FIG. 13.73. 24 FACED DOME

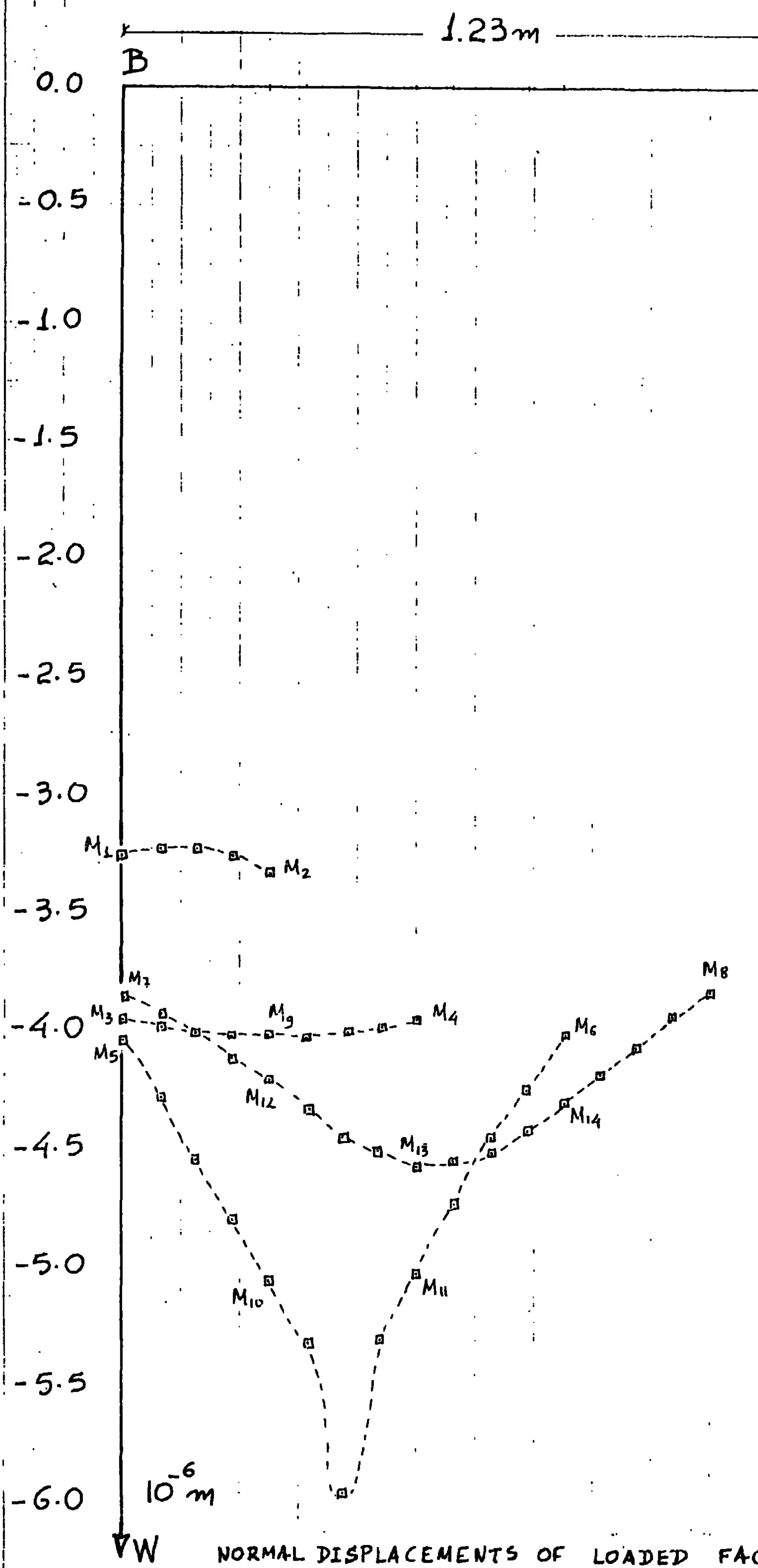
1 Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



	FINITE ELEMENT (DMx36) VALUE OF NORMAL DISPLAC. W (10^{-6}m)	EXPERIMENTAL VALUE OF NORMAL DISPLAC. W (10^{-6}m)
M_1	-3.25	-2.26
M_2	-3.32	-1.85
M_3	-4.00	-3.23
M_9	-4.07	-2.57
M_4	-4.01	-2.23
M_5	-4.12	-3.95
M_{10}	-5.10	-3.73
M_{11}	-5.05	-3.97
M_6	-4.12	-2.95
M_7	-3.95	-3.37
M_{12}	-4.30	-3.68
M_{13}	-4.62	-3.35
M_{14}	-4.40	-2.86
M_8	-3.77	-2.63

NORMAL DISPLACEMENTS OF LOADED FACE

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



	FINITE ELEMENT (DRO30) VALUE OF NORMAL DISPLAC. W $(10^{-6} m)$	EXPERIMENTAL VALUE OF NORMAL DISPLAC. W $(10^{-6} m)$
M_1	-3.25	-2.26
M_2	-3.32	-1.85
M_3	-3.95	-3.23
M_9	-4.01	-2.57
M_4	-3.95	-2.23
M_5	-4.04	-3.95
M_{10}	-5.06	-3.73
M_{11}	-5.03	-3.97
M_6	-4.01	-2.95
M_7	-3.85	-3.37
M_{12}	-4.21	-3.68
M_{13}	-4.57	-3.35
M_{14}	-4.30	-2.86
M_8	-3.82	-2.63

FIG. 13.74. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

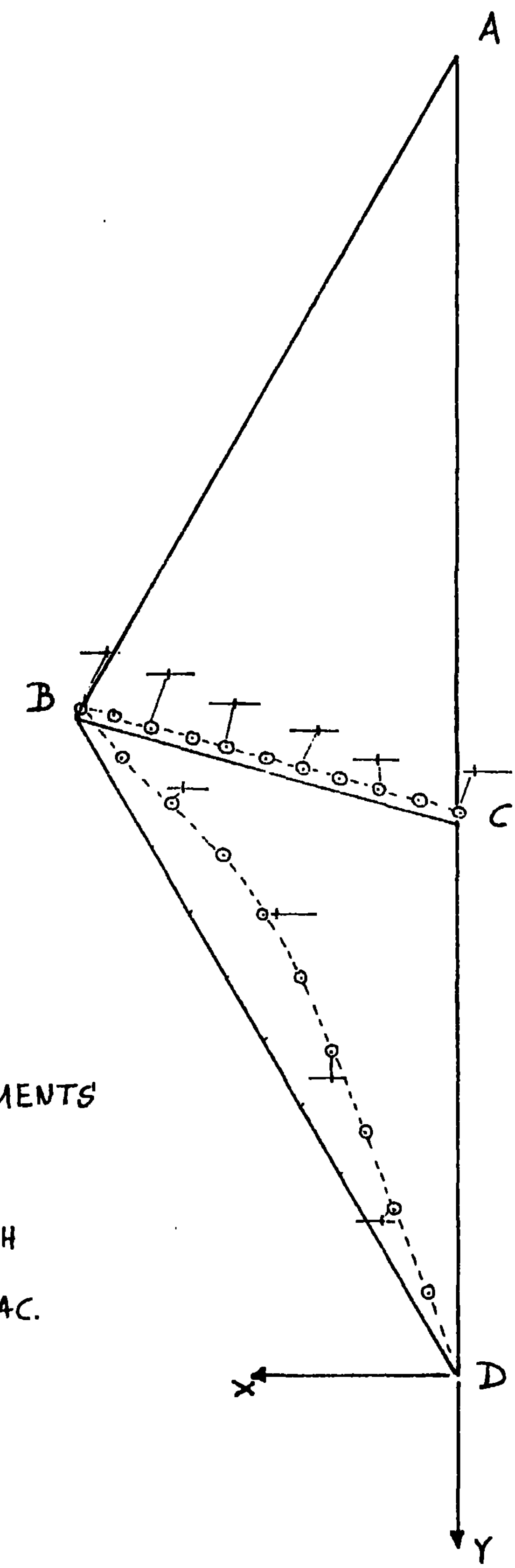
EXPERIMENTAL VALUES OF GLOBAL DISPL. u, v, w

POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m	POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m	POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m
A	-0.08	-0.21	3.45	M_{13}	-0.74	-1.89	-1.15	N_{11}	-0.33	-2.13	-2.62
M_1	-0.33	-1.39	2.13	M_{14}	-0.33	-1.15	-1.56	N_2	0.09	-0.82	-3.11
M_2	-0.35	-1.07	2.54	M_8	-0.16	-1.80	-0.98	N_3	-2.75	-3.44	-5.14
M_3	-0.57	-0.90	0.33	B	-1.89	-3.75	-4.02	N_{12}	-1.80	-3.85	-4.47
M_9	-0.16	-1.31	1.23	M_{15}	-1.23	-3.52	-3.85	N_{13}	-0. 5	-2. 5	-
M_4	-0.33	-1.64	1.07	M_{16}	-0.25	-3.03	-3.11	N_4	0.08	-1.00	-1.80
M_5	-0.74	-3.11	-1.39	M_{17}	-1.05	-2.98	-2.62	N_5	-0.65	-2.66	-3.66
M_{10}	-0.36	-1.15	-0.98	M_{18}	-0.18	-2.38	-2.54	N_{14}	-0.16	-1.89	-2.29
M_{11}	0.00	-1.89	-0.41	C	-0.75	-3.03	-2.29	N_6	0.16	-0.74	-1.25
M_6	-0.33	-2.87	0.25	N_1	-1.72	-3.46	-4.65	N_7	0.00	-0.75	-1.90
M_7	-1.80	-3.52	-2.62	N_9	-1.39	-3.85	-3.85	N_8	0.21	0.00	-0.45
M_{12}	-1.07	-3.11	-2.46	N_{10}	-0.66	-2.38	-3.52				

FIG. 13.75 24 FACED DOME

FIG. 13.76. 24 FACED DOME

1nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

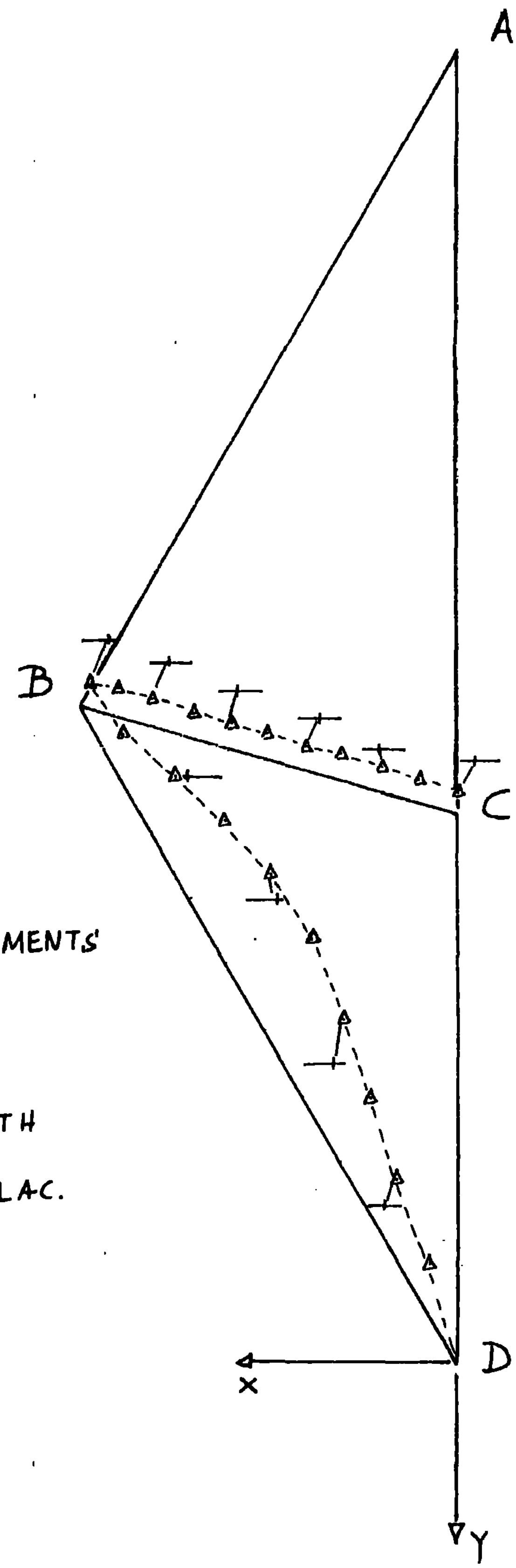
SCALES :

1cm = $2 \cdot 10^{-1}$ m LENGTH

1cm = $4 \cdot 10^{-6}$ m DISPLAC.

FIG. 13.77. <u>24 FACED DOME</u>

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

SCALES:

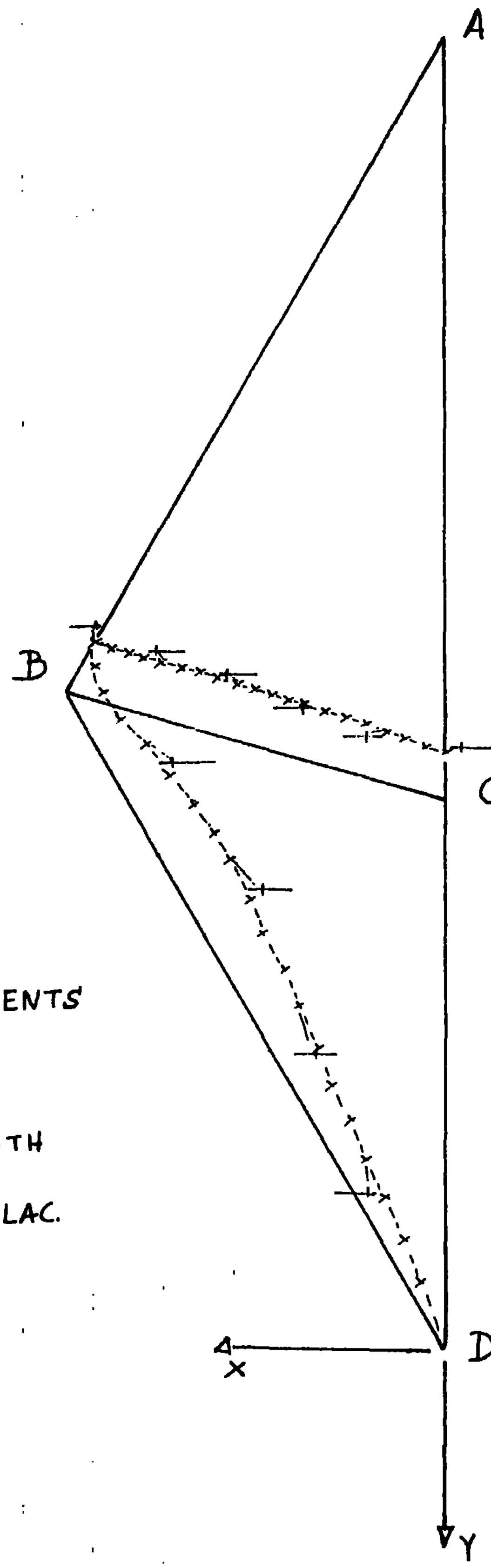
$1 cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 4 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.78.

24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

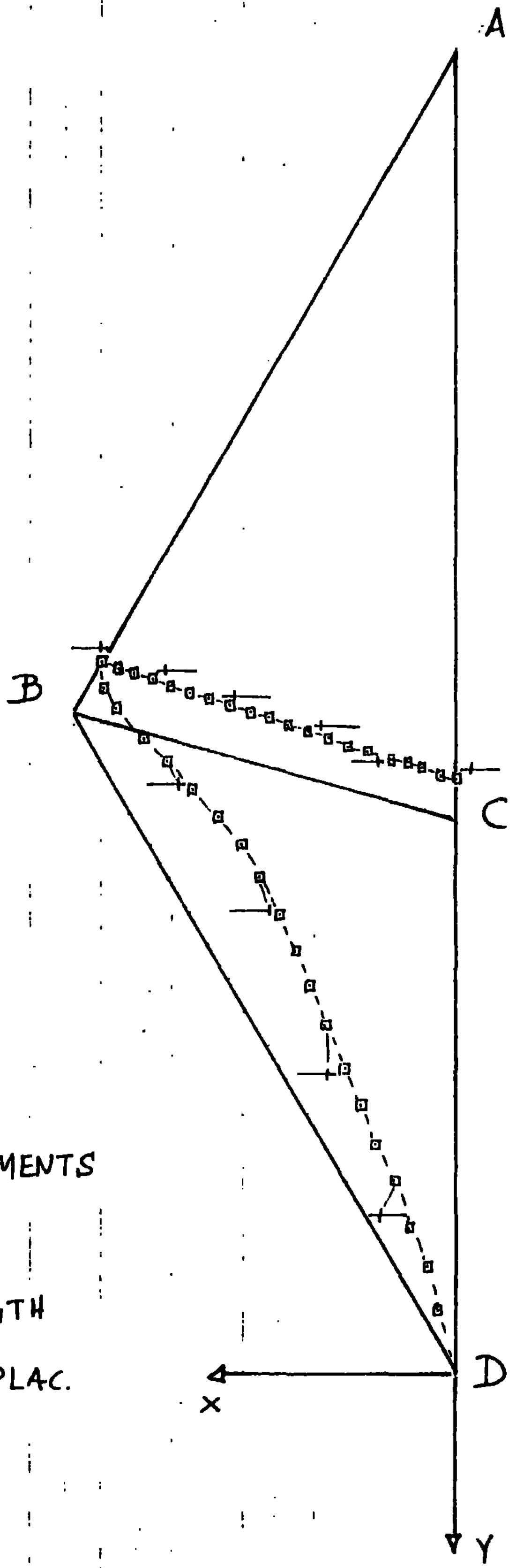
SCALES :

$1\,cm = 2 \cdot 10^{-1}\,m$ LENGTH

$1\,cm = 4 \cdot 10^{-6}\,m$ DISPLAC.

FIG. 13.79. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

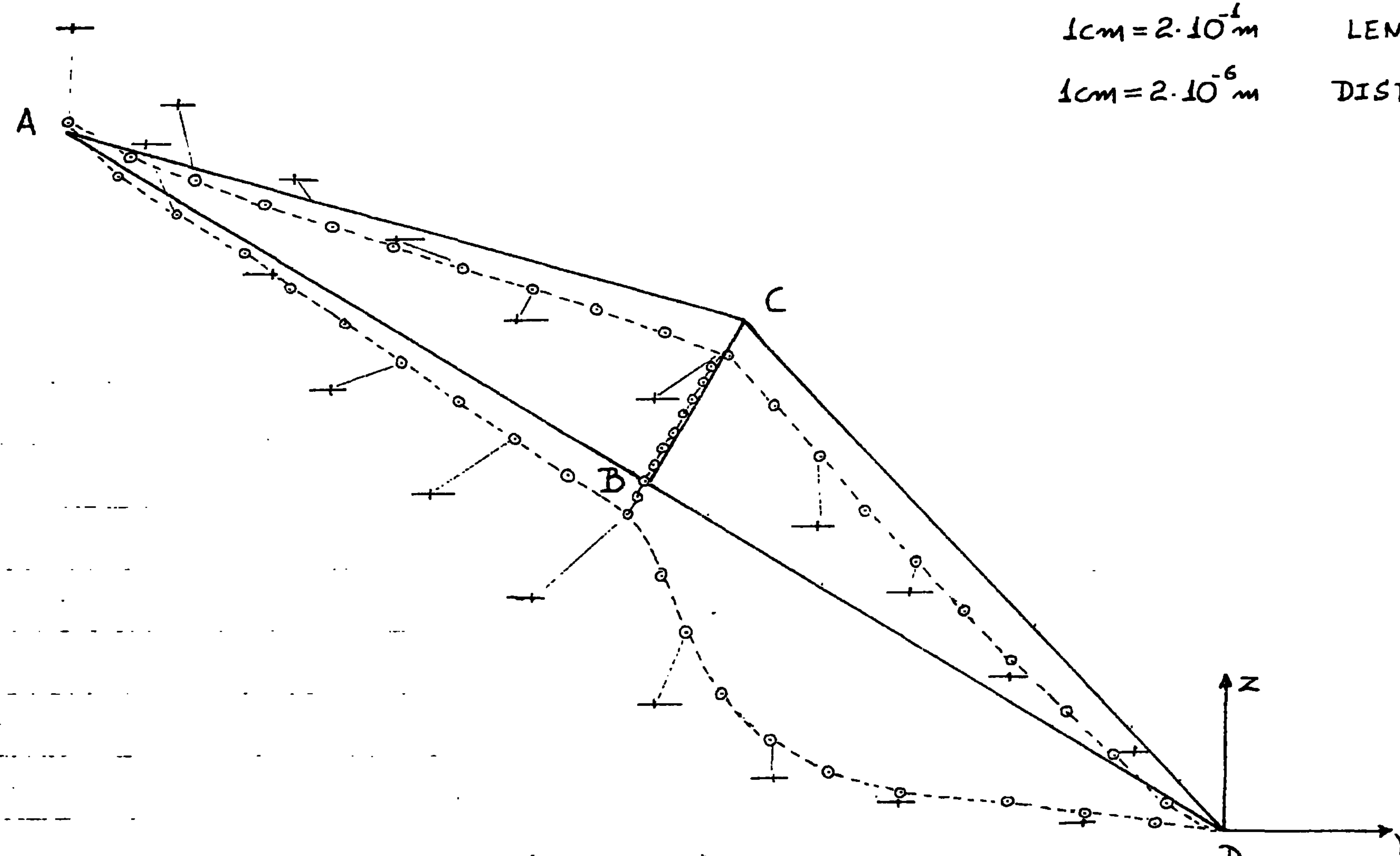
$1cm = 4 \cdot 10^{-6} m$ DISPLAC.

GLOBAL DISPLACEMENTS W, U

SCALES :

1cm = $2 \cdot 10^{-1}$ m LENGTH

1cm = $2 \cdot 10^{-6}$ m DISPLAC.



1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

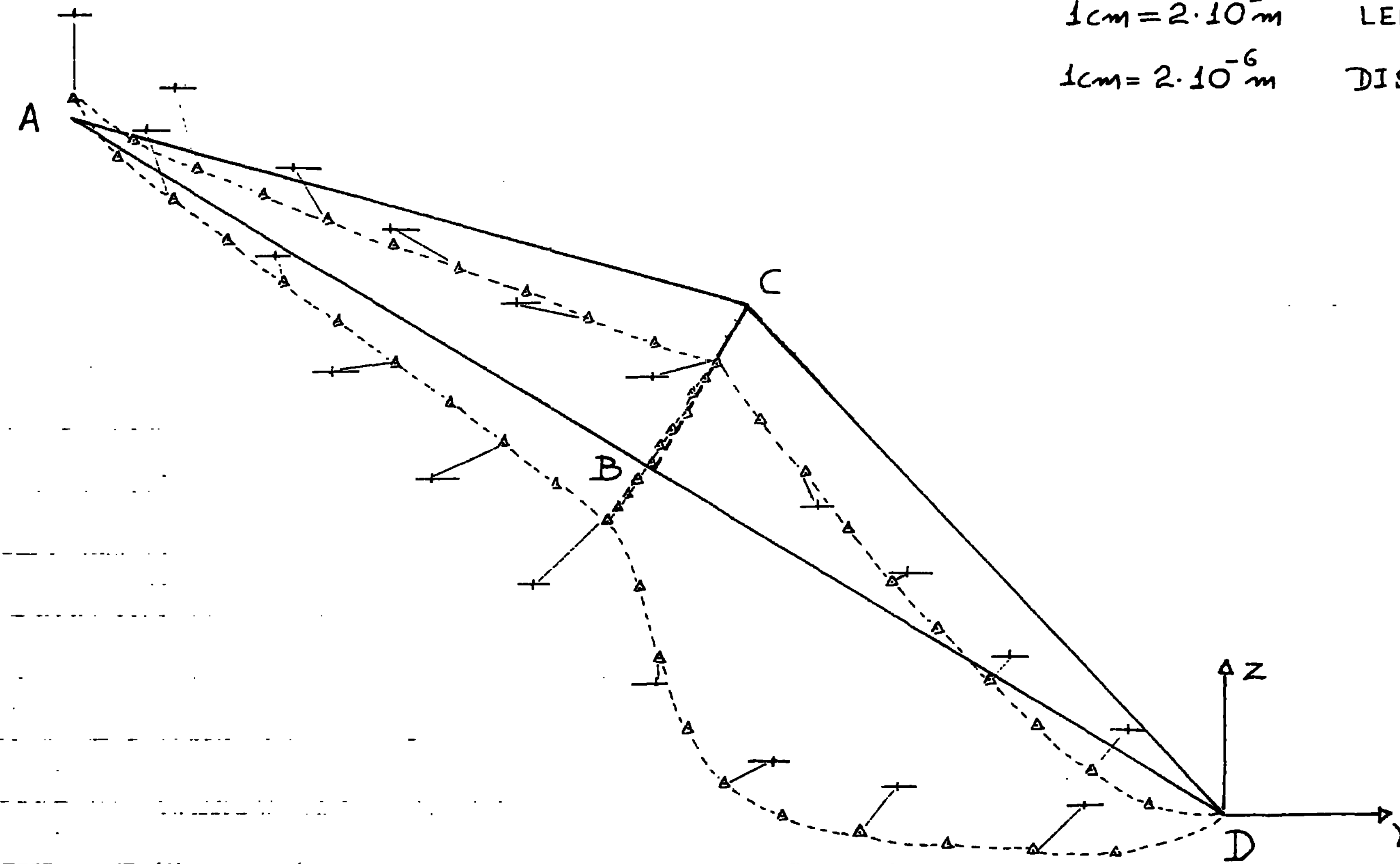
FIG. 13.80. 24 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES

1cm = $2 \cdot 10^{-1}$ m LENGTH

1cm = $2 \cdot 10^{-6}$ m DISPLAC



1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.81. 24 FACED DOME

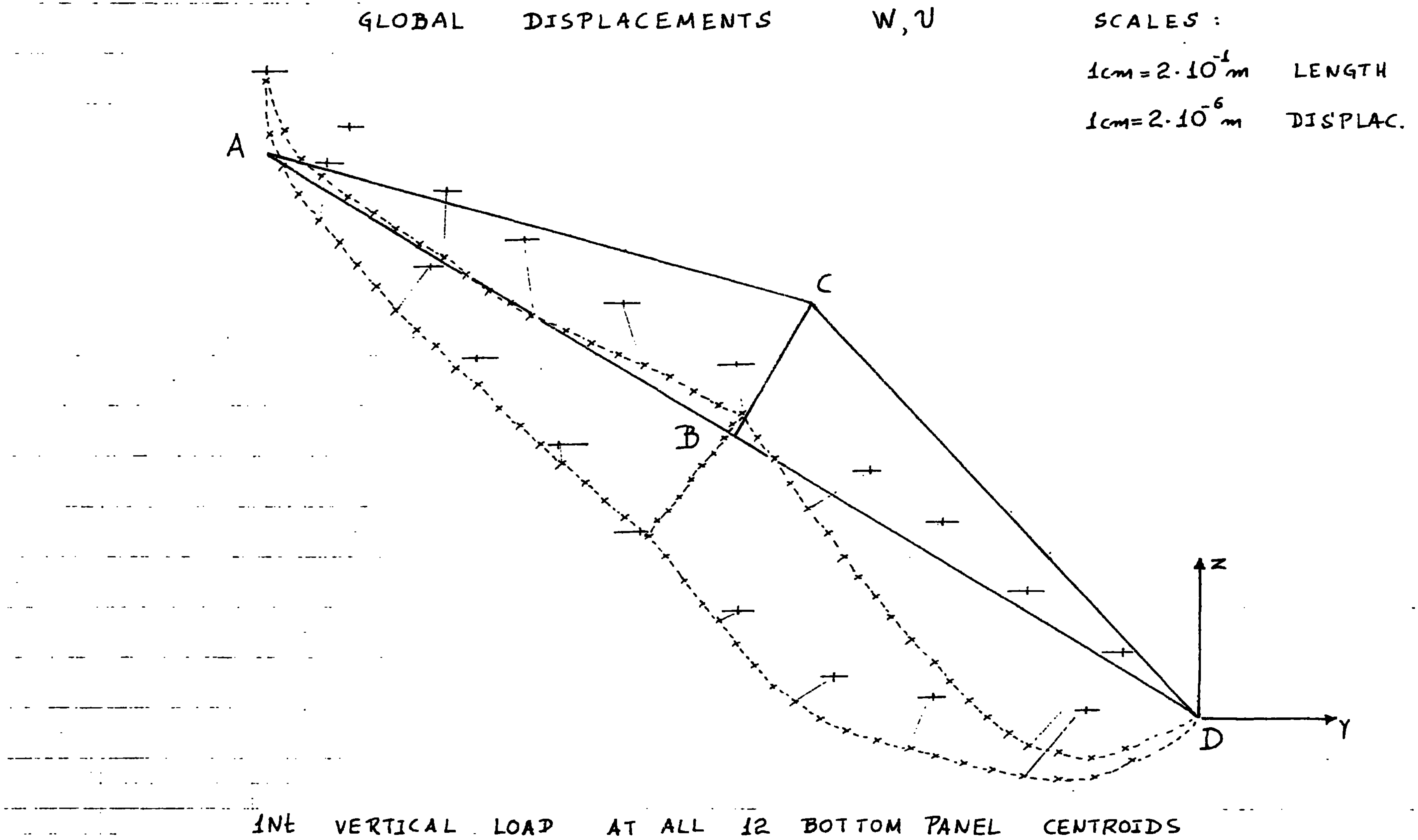


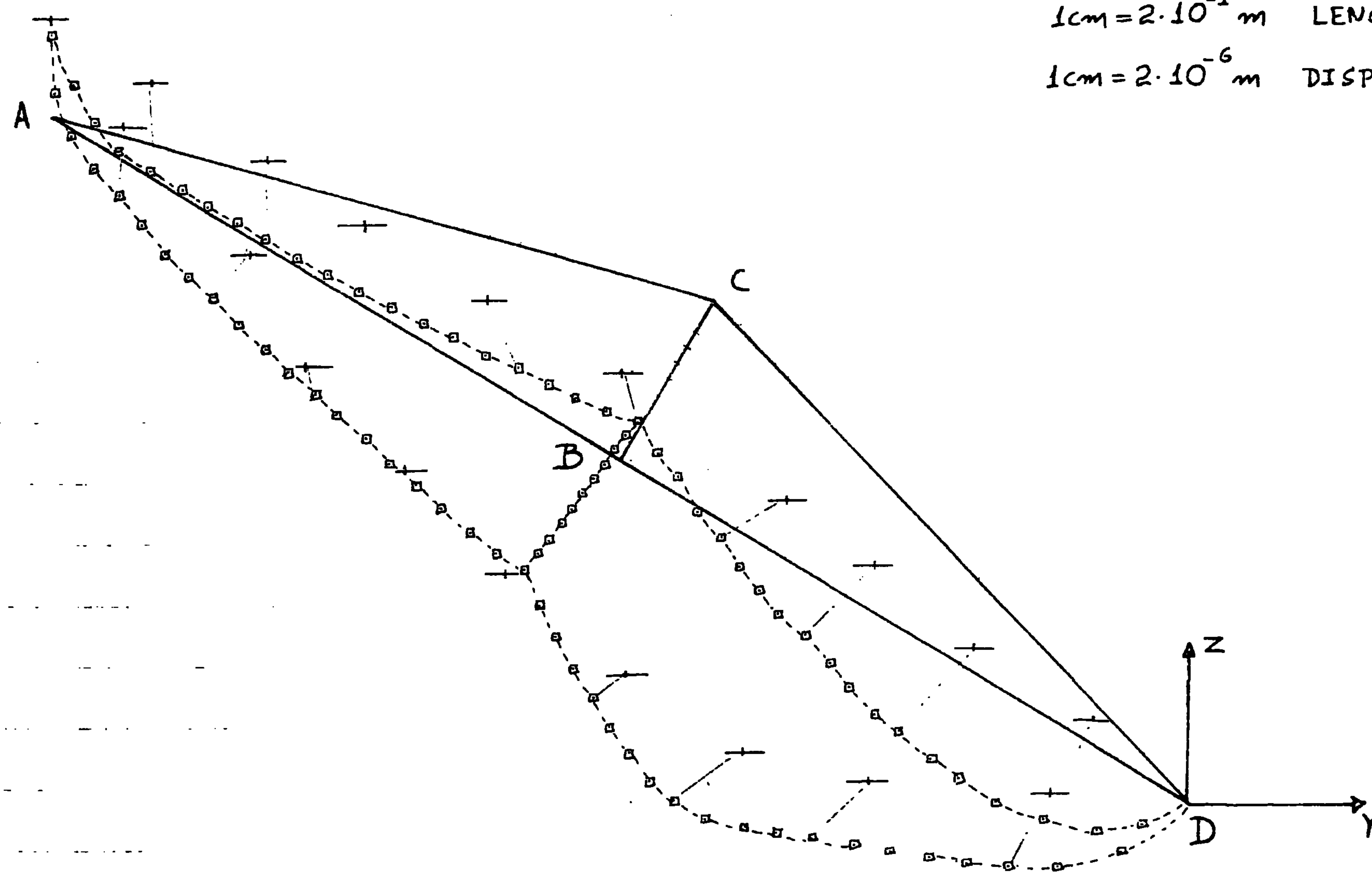
FIG. 13.82. 24 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

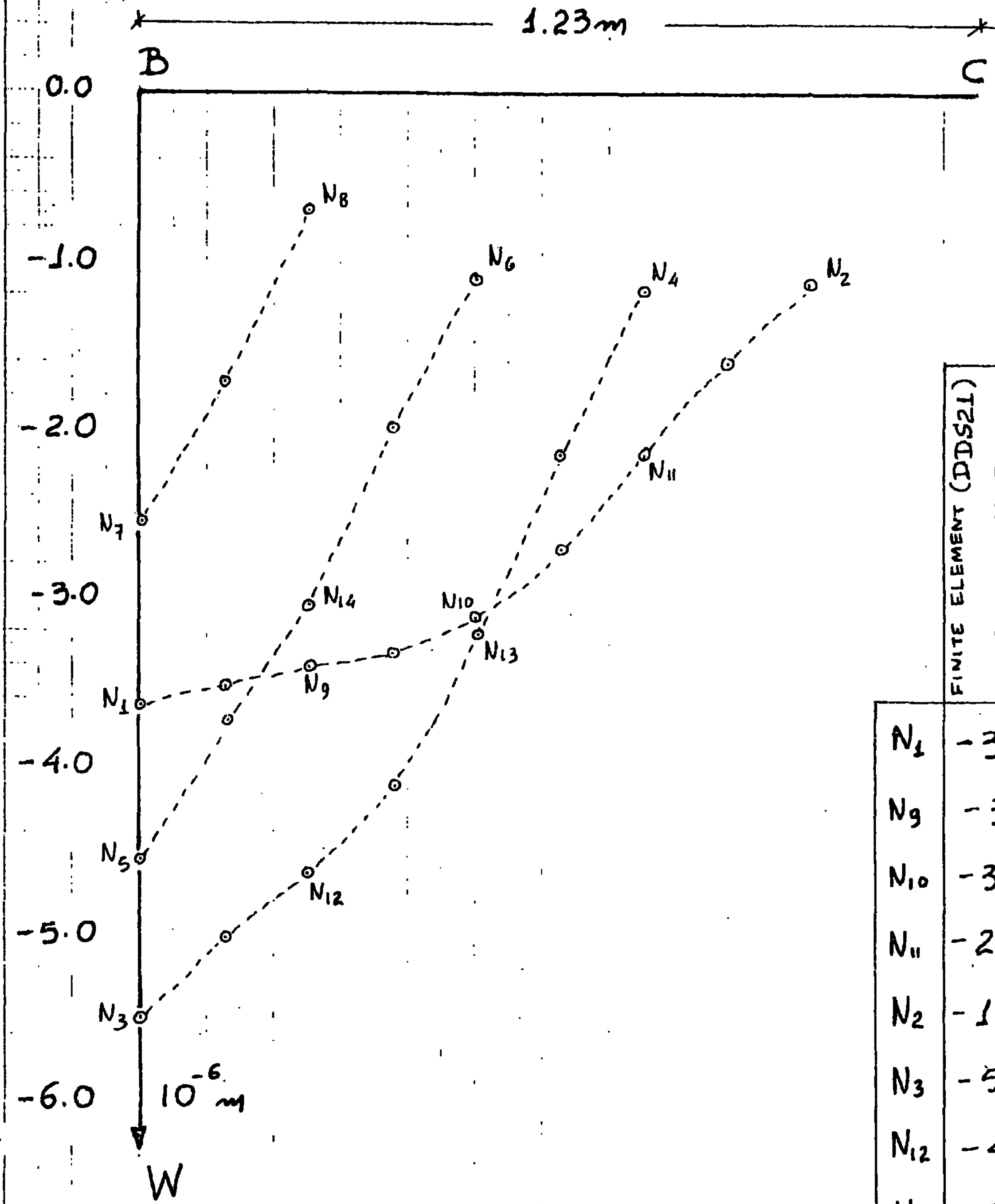
$1cm = 2 \cdot 10^{-6} m$ DISPLAC.



1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.83. 24 FACED DOME

FIG. 13.84. 24 FACED DOME



NORMAL DISPLACEMENTS OF LOADED FACE

SCALES:

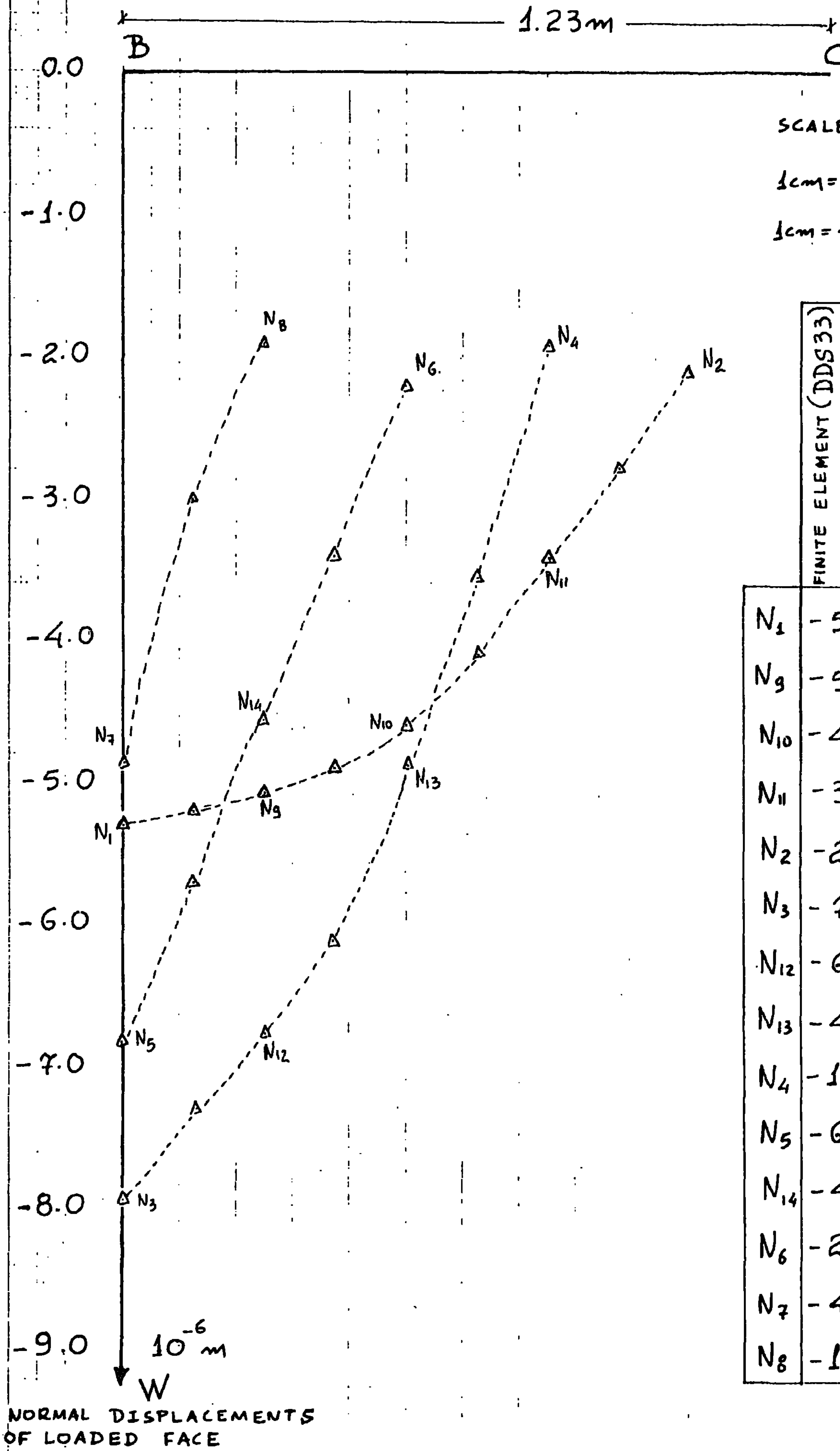
1cm = 10^{-1} m LENGTH

1cm = $4 \cdot 10^{-7}$ m DISPLAC.

	FINITE ELEMENT (DDS21) VALUE OF NORMAL DISPLAC. W (10^{-6} m)	EXPERIMENTAL VALUE OF NORMAD DISPLAC. W (10^{-6} m)
N_1	-3.64	-5.84
N_9	-3.43	-5.45
N_{10}	-3.13	-3.97
N_{11}	-2.14	-3.10
N_2	-1.12	-2.37
N_3	-5.51	-6.62
N_{12}	-4.64	-6.03
N_{13}	-3.20	-4.23
N_4	-1.18	-1.70
N_5	-4.56	-423
N_{14}	-2.85	-2.67
N_6	-1.11	-1.16
N_7	-2.56	-1.64
N_8	-0.70	-0.18

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.85. 24 FACED DOME



SCALES:

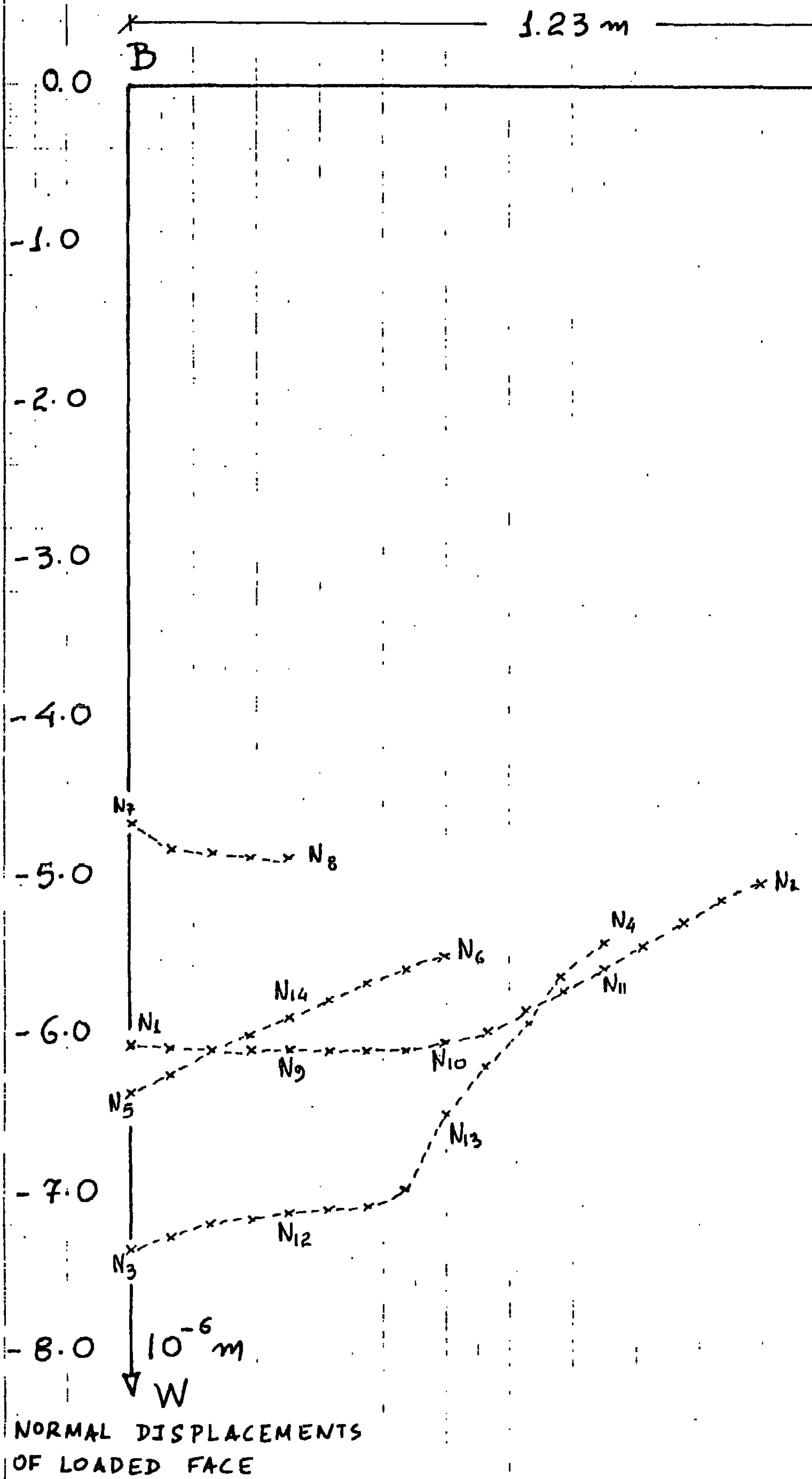
1cm = 10^{-1} m LENGTH

1cm = $4 \cdot 10^{-7}$ m DISPLAC.

	FINITE ELEMENT (DDS33) VALUE OF NORMAL DISPLAC. W (10^{-6} m)	EXPERIMENTAL VALUE OF NORMAL DISPLAC. W (10^{-6} m)
N_1	-5.30	-5.84
N_9	-5.07	-5.45
N_{10}	-4.60	-3.97
N_{11}	-3.41	-3.10
N_2	-2.09	-2.37
N_3	-7.94	-6.62
N_{12}	-6.76	-6.03
N_{13}	-4.87	-4.23
N_4	-1.91	-1.70
N_5	-6.83	-4.23
N_{14}	-4.56	-2.67
N_6	-2.20	-1.16
N_7	-4.86	-1.64
N_8	-1.90	-0.18

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.86. <u>24 FACED DOME</u>

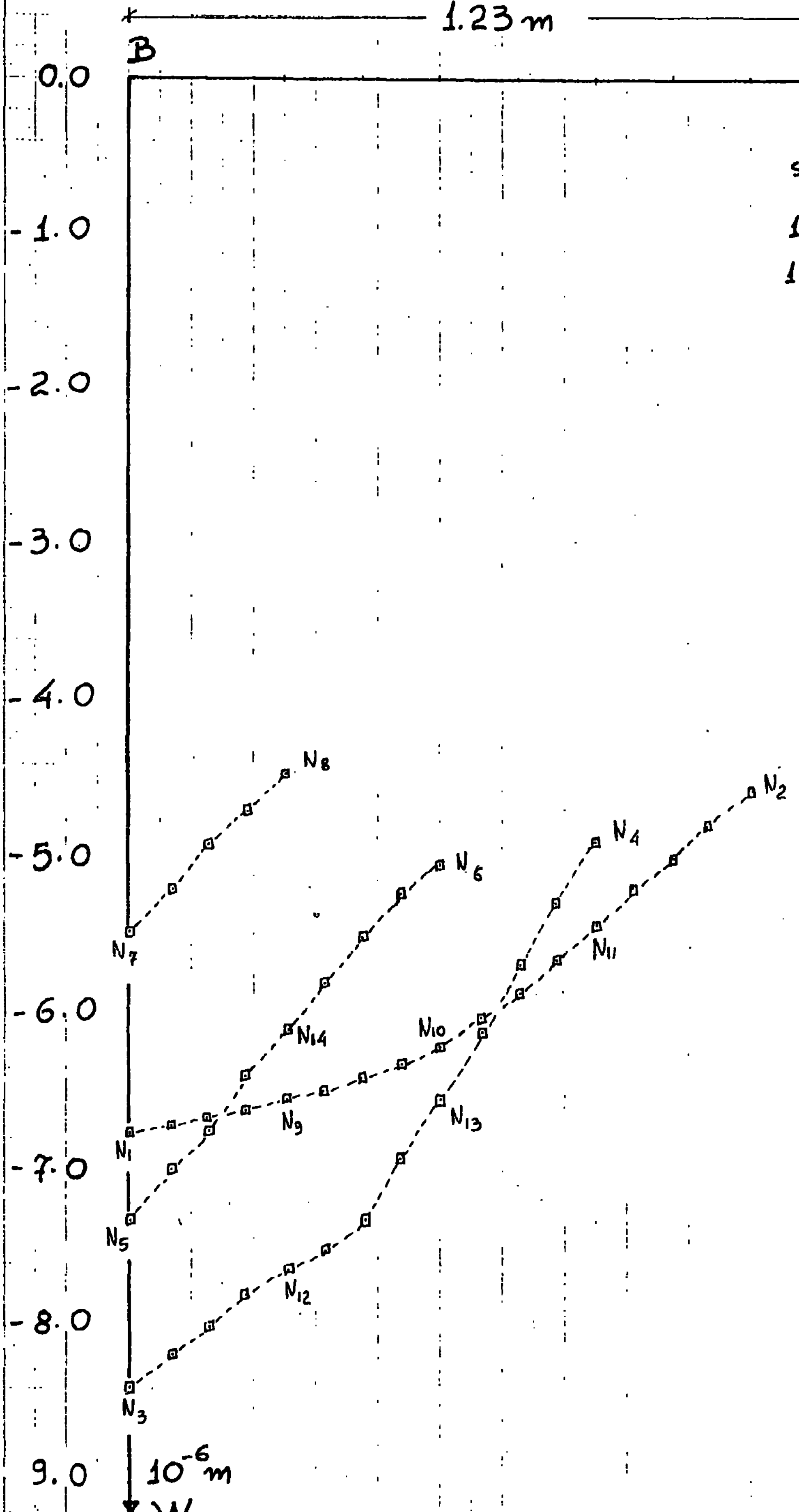


SCALES:
1cm = 10^{-1} m LENGTH
1cm = $4 \cdot 10^{-7}$ m DISPLAC.

	FINITE ELEMENT (DMX36) VALUE OF NORMAL DISPLAC. W (10^{-6} m)	EXPERIMENTAL VALUE OF NORMAL DISPLAC. W (10^{-6} m)
N_1	-6.07	-5.84
N_9	-6.13	-5.45
N_{10}	-6.05	-3.97
N_{11}	-5.58	-3.10
N_2	-5.04	-2.37
N_3	-7.36	-6.62
N_{12}	-7.16	-6.03
N_{13}	-6.45	-4.23
N_4	-5.42	-1.70
N_5	-6.36	4.23
N_{14}	-5.89	-2.67
N_6	-5.49	-1.16
N_7	-4.66	-1.64
N_8	-4.89	-0.18

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.87 24 FACED DOME



SCALES:

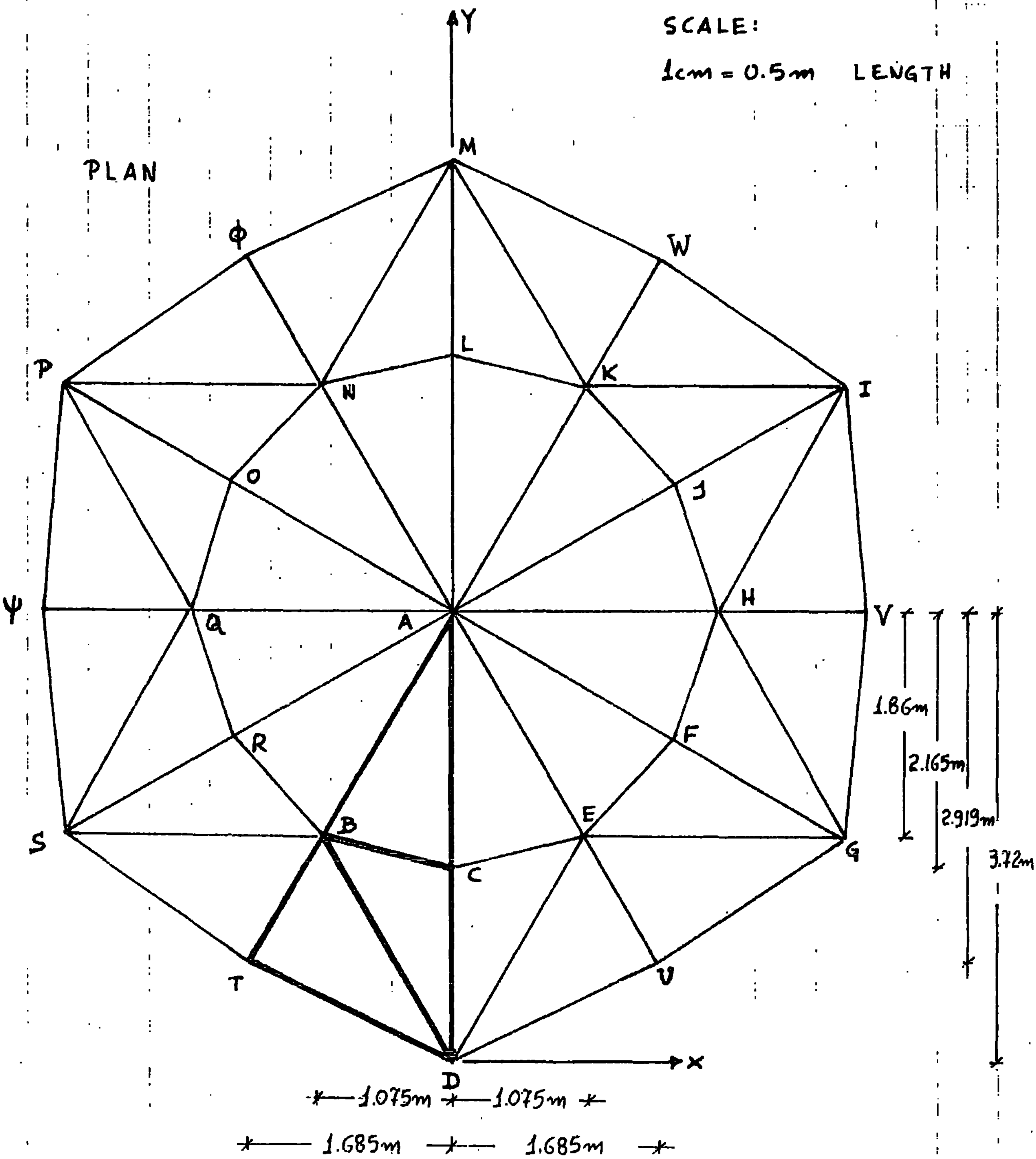
1cm = 10^-1 m LENGTH

1cm = 4·10^-7 m DISPLAC.

	FINITE ELEMENT (DRO30) VALUE OF NORMAL DISPL. W (10^{-6} m)	EXPERIMENTAL VALUE OF NORMAL DISPLAC. W (10^{-6} m)
N_1	-6.76	-5.84
N_9	-6.55	-5.45
N_{10}	-6.21	-3.97
N_{11}	-5.42	-3.10
N_2	-4.56	-2.37
N_3	-8.41	-6.62
N_{12}	-7.63	-6.03
N_{13}	-6.56	-4.23
N_4	-4.88	-1.70
N_5	-7.32	4.23
N_{14}	-6.11	-2.67
N_6	-5.06	-1.16
N_7	-5.48	-1.64
N_8	-4.46	-0.18

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.88. **36 FACED DOME**



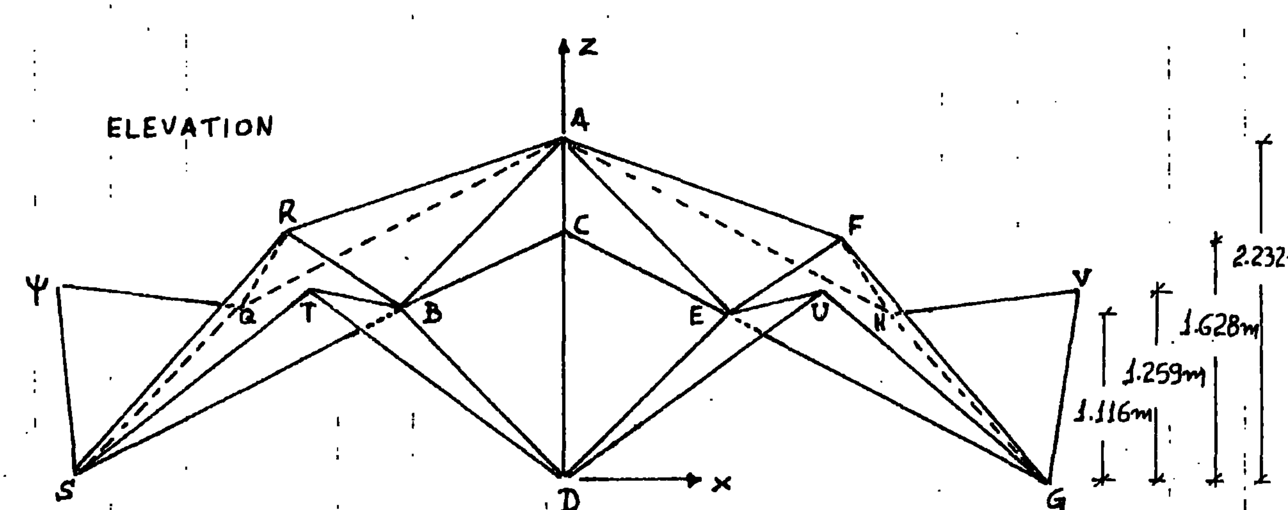
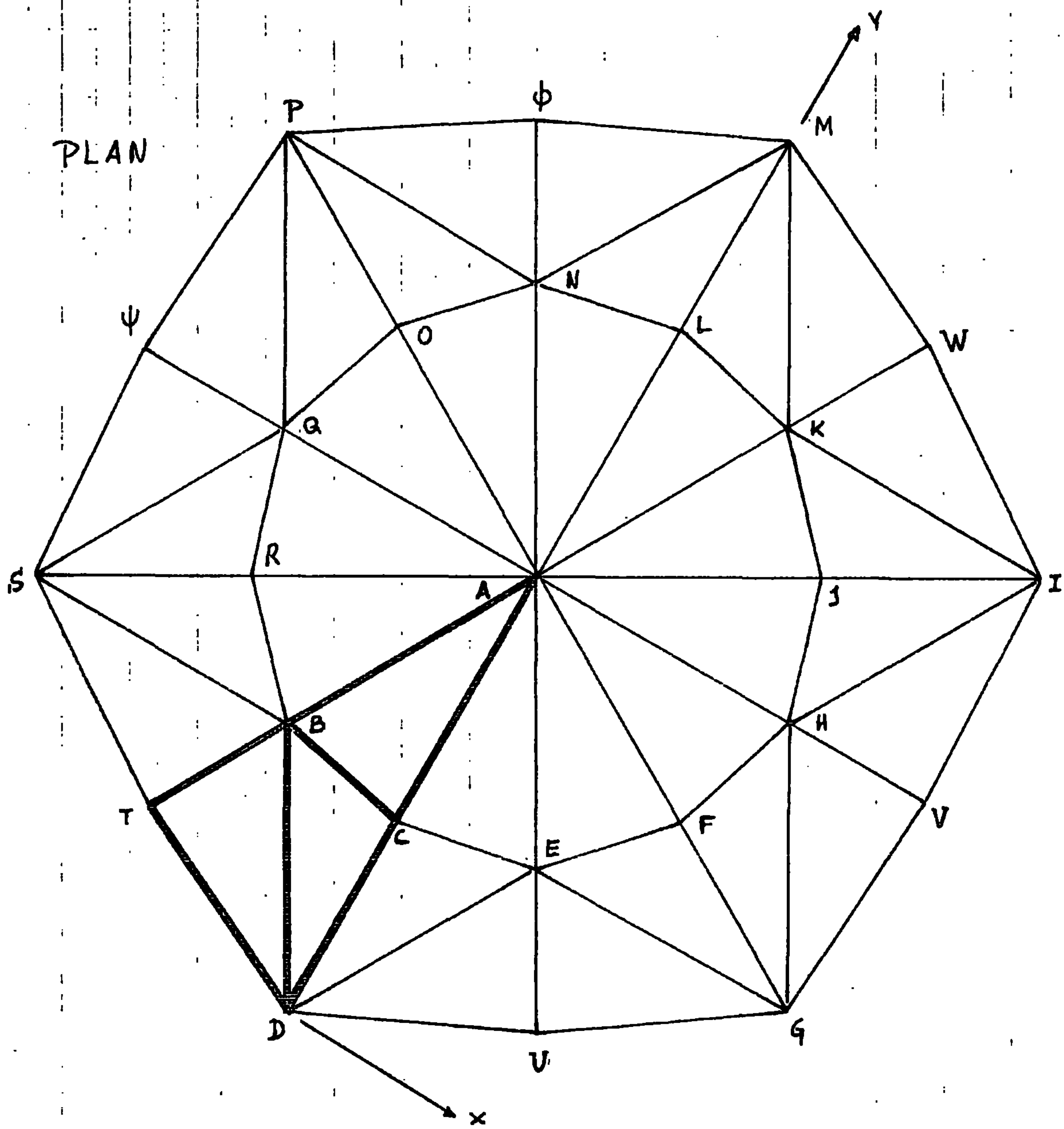


FIG. 13.89. 36 FACED DOME

SCALE:

1cm = 0.5m LENGTH



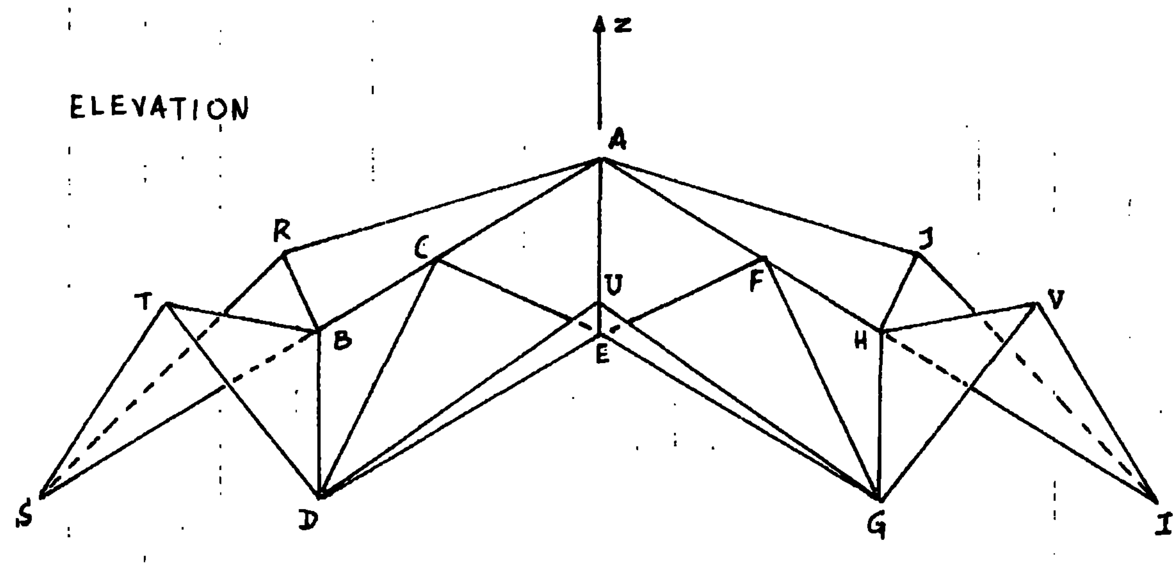


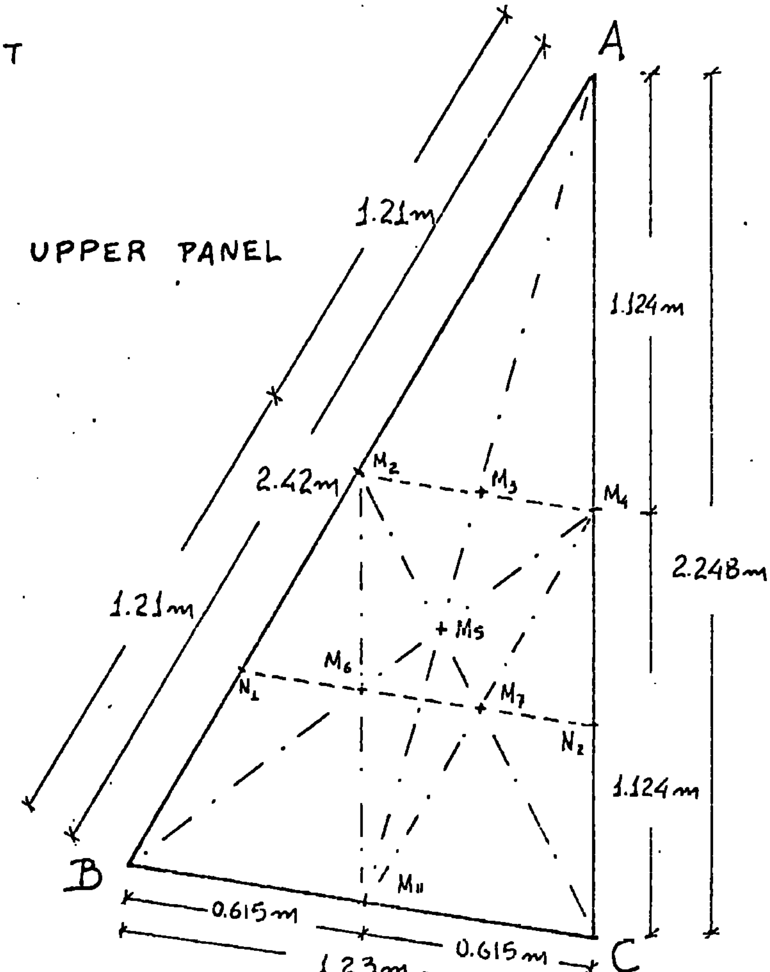
FIG. 13.90. 36 FACED DOME

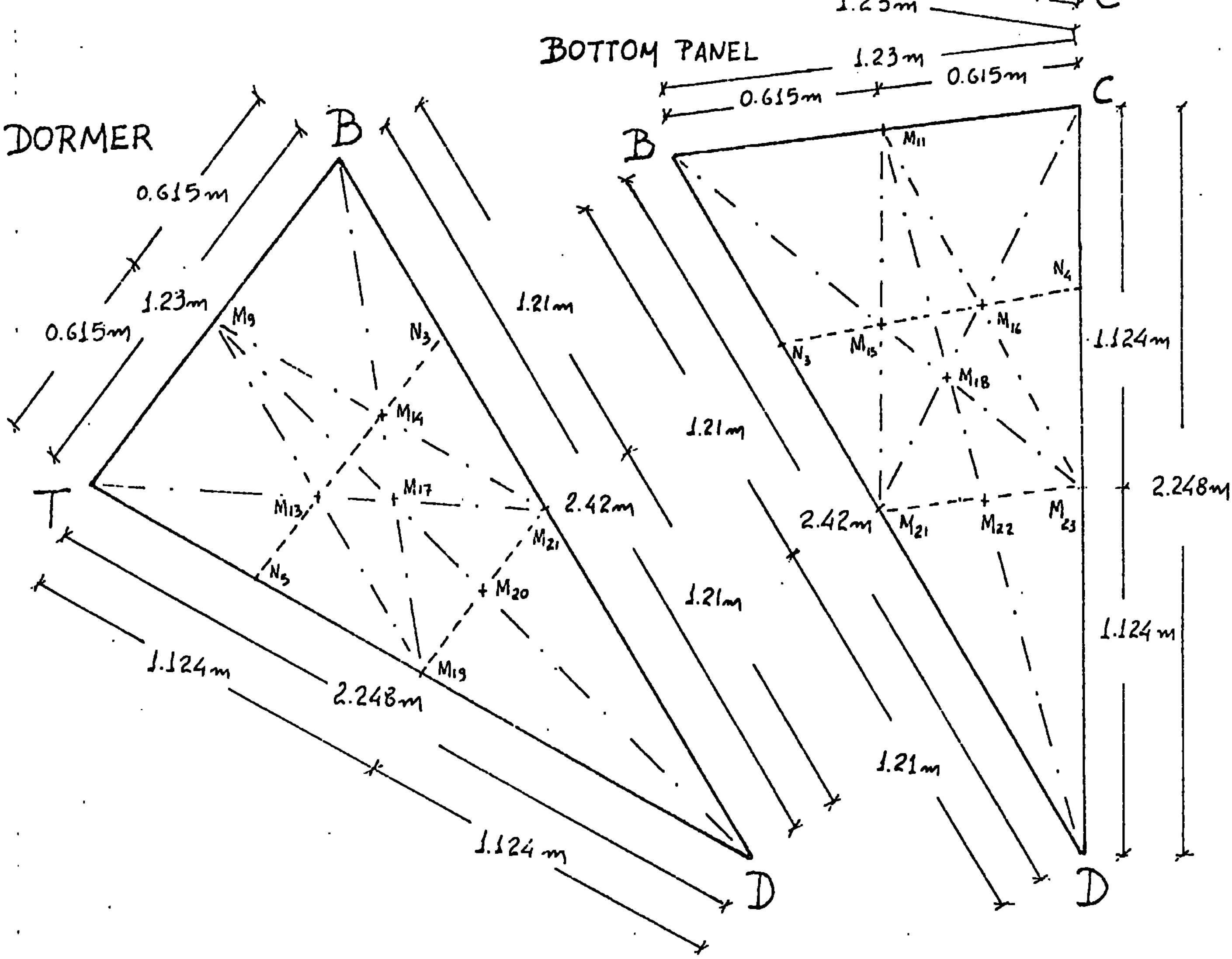
GENERAL ARRANGEMENT

SCALE :

1cm = 2·10^{-1}m LENGTH

KEY	
DDS21	⊙
DDS33	△
DMX36	×
DR030	⊡
EXPERIMENT	—+—





1 Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

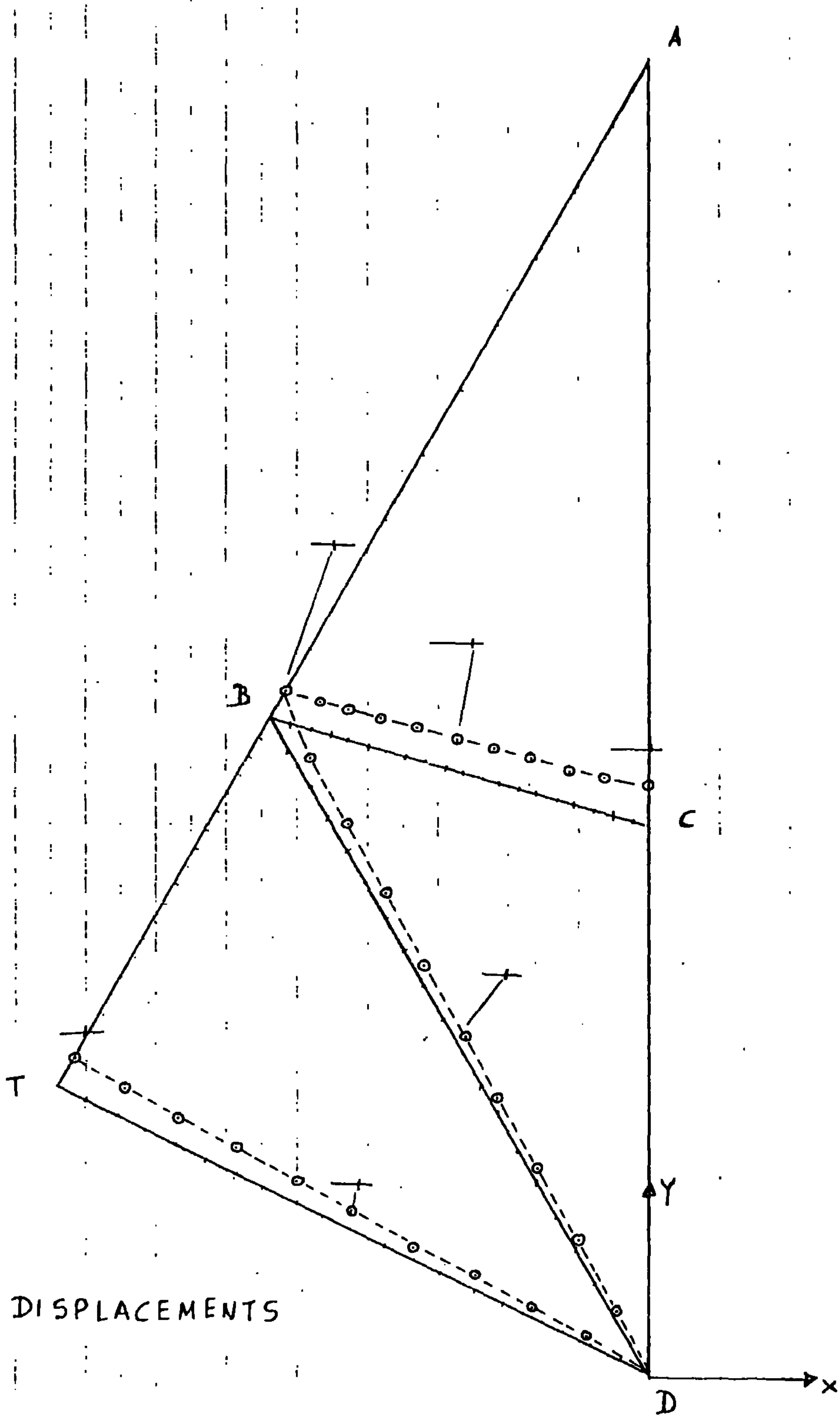
EXPERIMENTAL VALUES OF GLOBAL DISPLAC. u, v, w

POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m	POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m
A	-0.04	-0.06	-0.53	M_{13}	0.16	0.57	-0.94
M_2	-0.08	0.52	-1.64	M_{14}	0.04	1.15	-1.43
M_3	+0.28	0.72	-2.28	M_{15}	-0.08	1.05	-1.07
M_4	-0.13	0.48	-1.73	M_{16}	0.04	1.15	-1.07
M_5	+0.78	0.98	-4.34	M_{17}	0.45	0.41	-0.78
M_6	0.49	1.80	-2.42	M_{18}	0.04	0.16	-0.78
M_7	0.61	1.48	-2.05	M_{19}	0.16	0.57	-0.82
T	0.33	0.86	-0.98	M_{20}	0.29	0.90	-0.91
M_9	0.37	1.15	-1.54	M_{21}	0.25	0.80	-1.25
B	0.66	2.21	-2.25	M_{22}	0.45	0.16	-0.66
M_{11}	0.21	1.97	-1.64	M_{23}	0.15	0.51	-0.66
C	-0.08	1.11	-1.19				

FIG. 13.91. 36 FACED DOME

FIG. 13.92 36 FACED DOME

1N VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

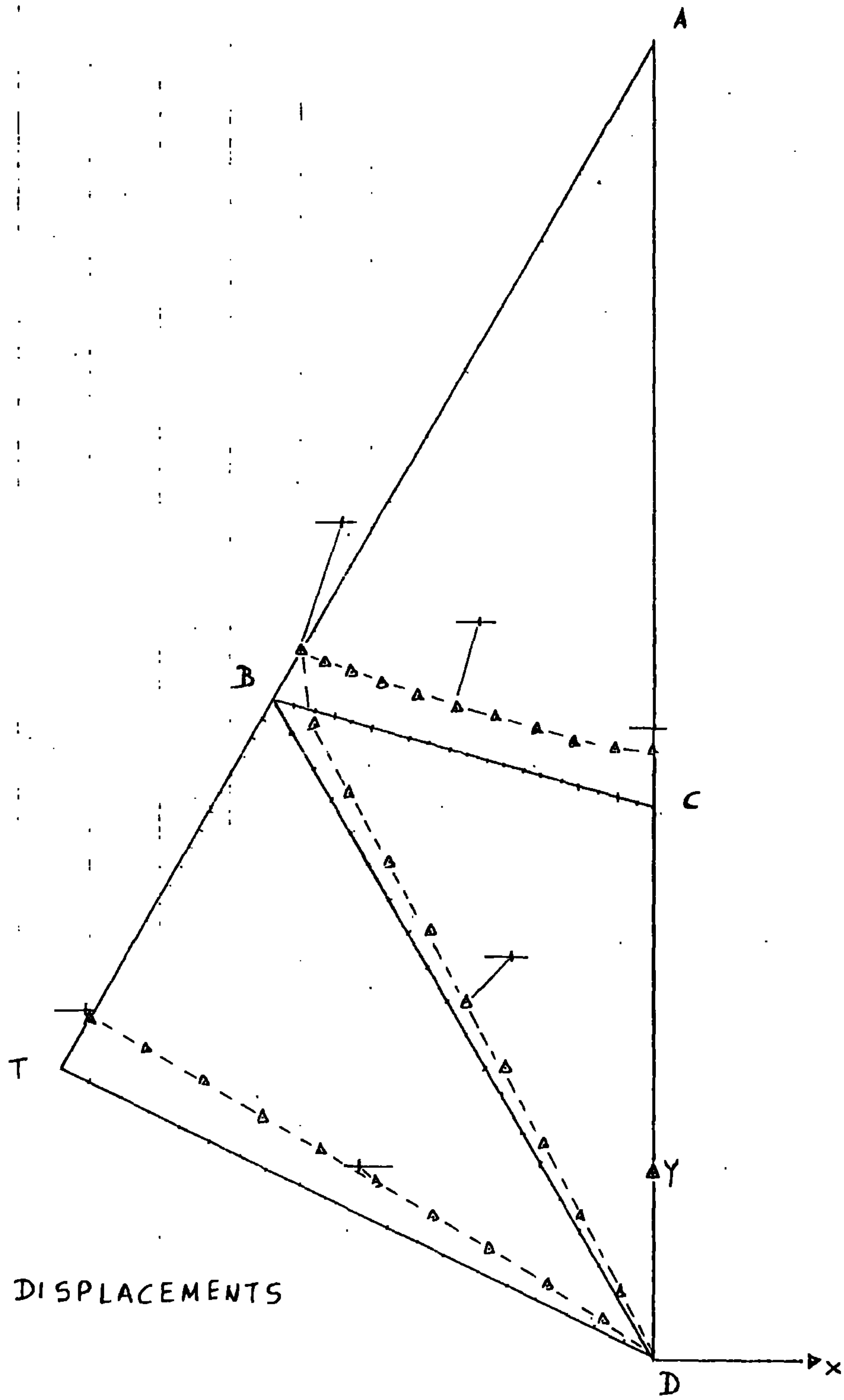
SCALES:

$1cm = 2.10^{-1}m$ LENGTH

$1cm = 1.10^{-6}m$ DISPLAC.

FIG. 13.93 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER - PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

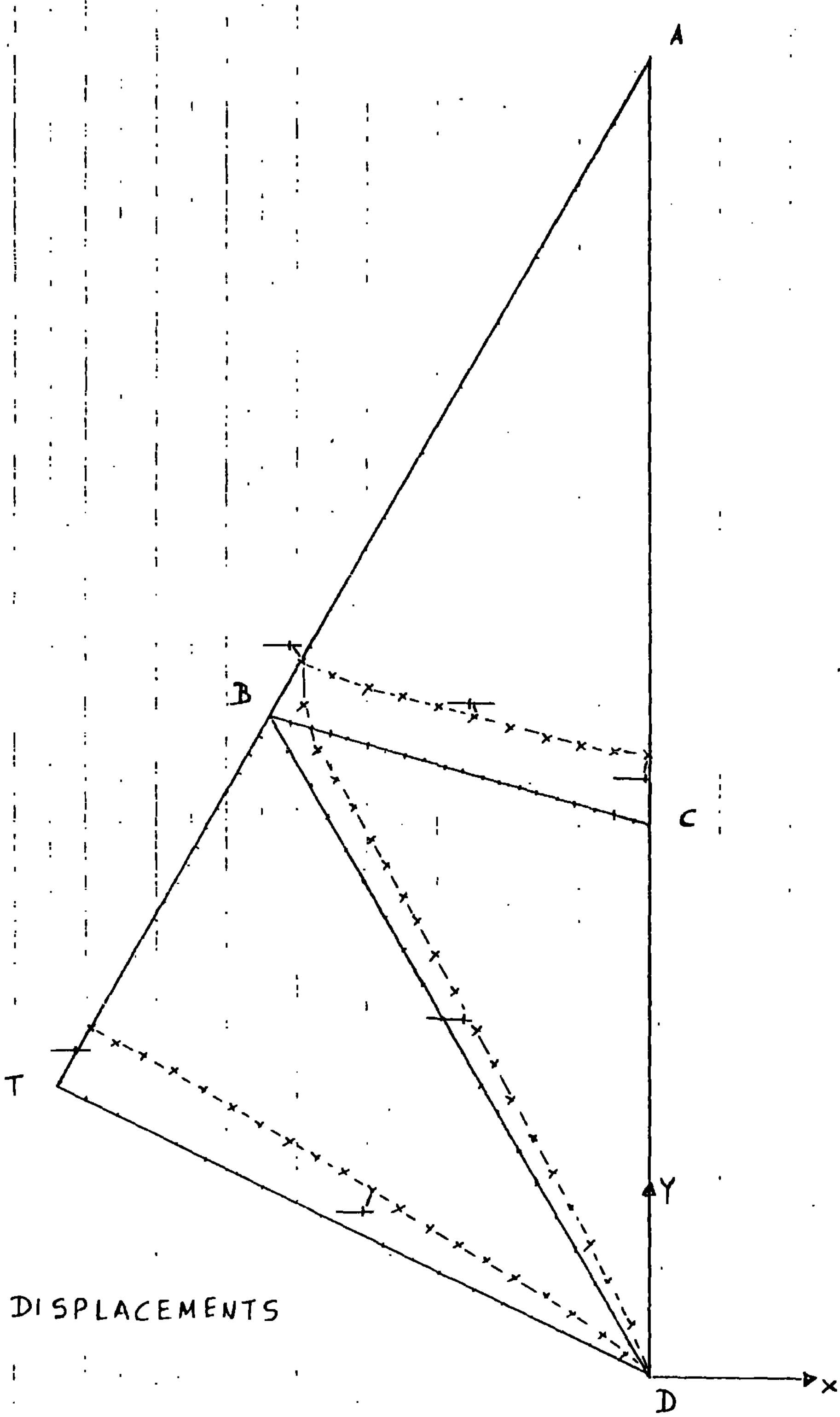
SCALES:

1cm = 2.10^{-1} m LENGTH

1cm = 1. 10^{-6} m DISPLAC.

FIG. 13.94 36 FACED DOME

1N VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

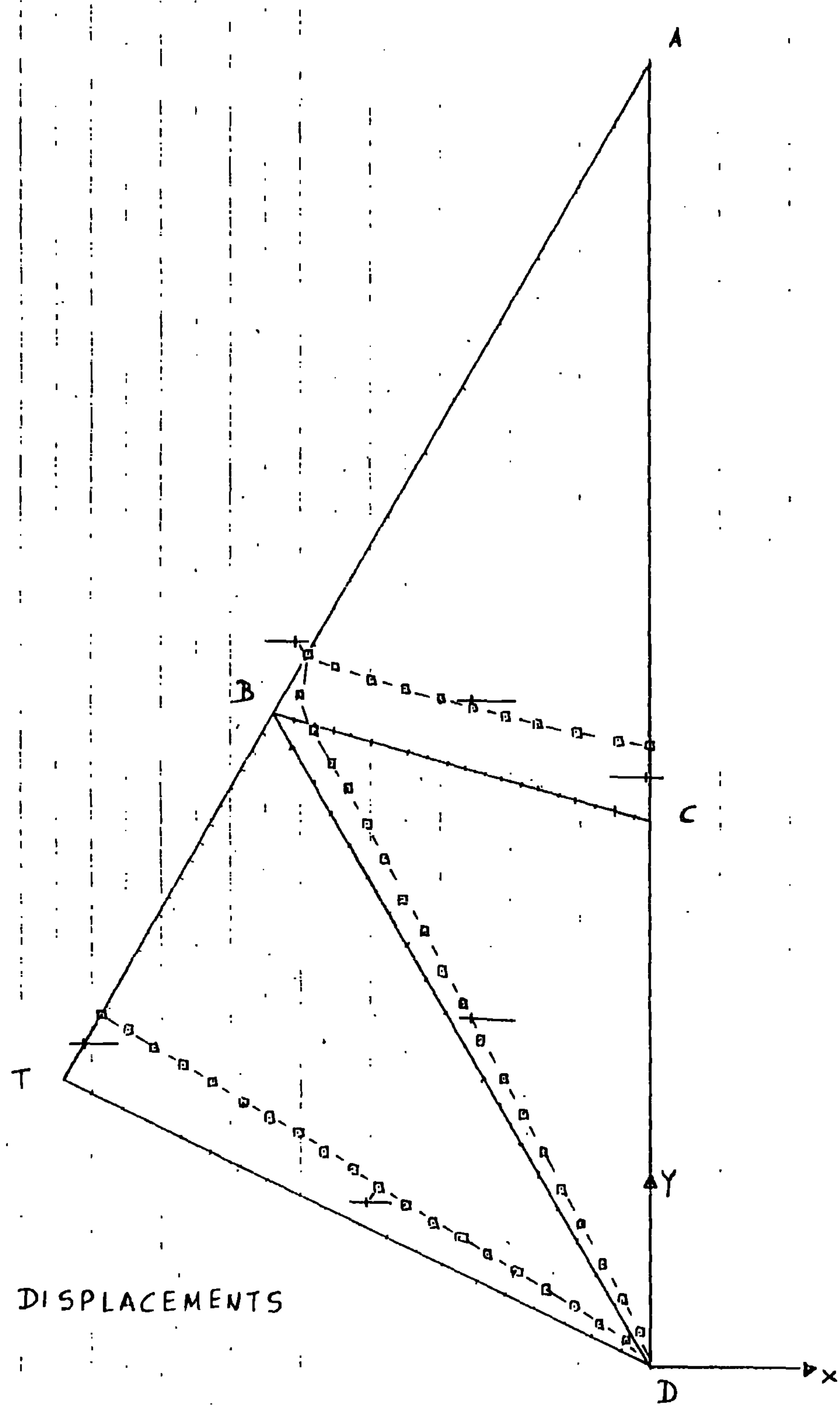
SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 2.10^{-6} m$ DISPLAC.

FIG. 13.95 36 FACED DOME

1N VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

SCALES:

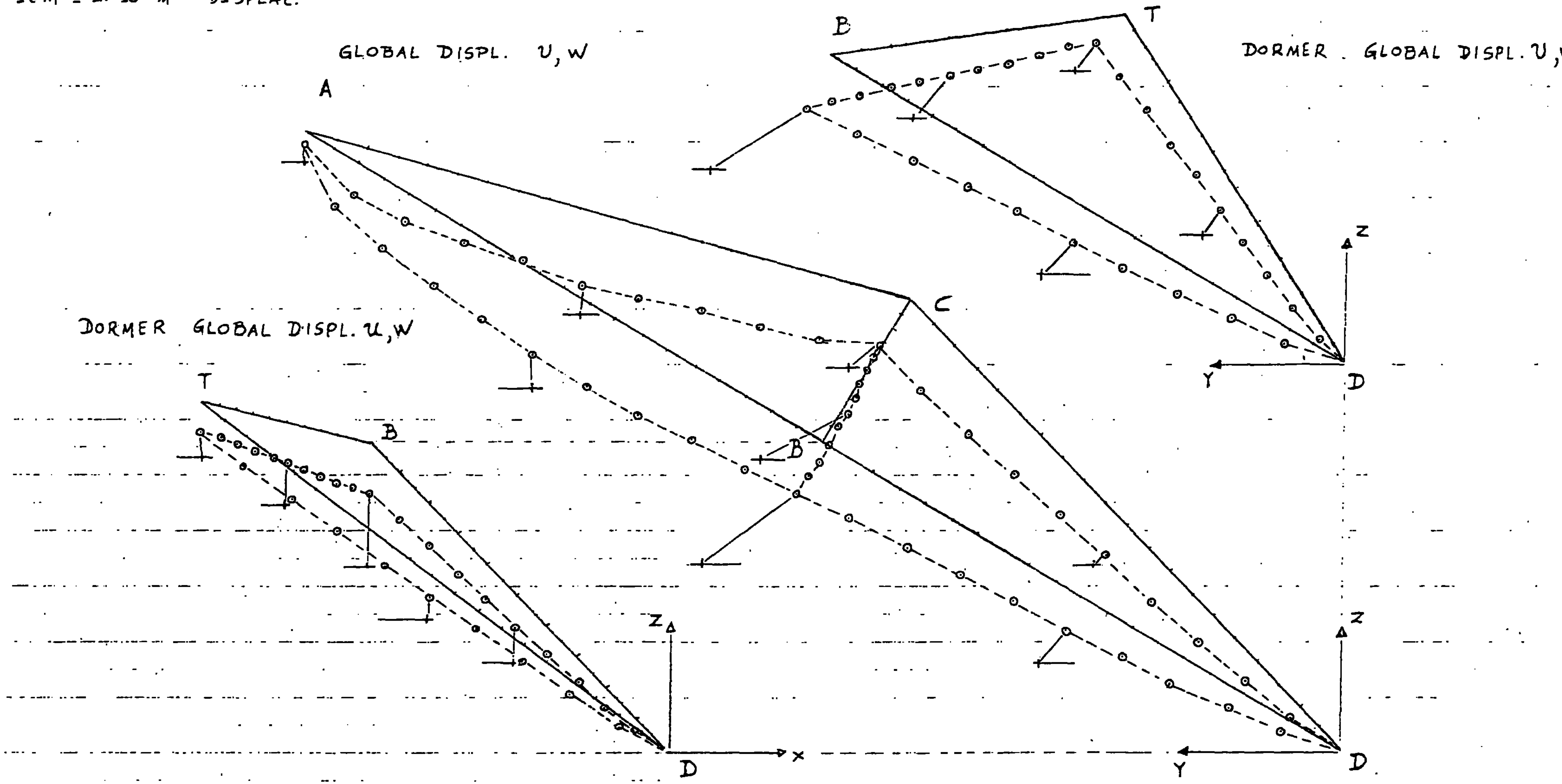
$1cm = 2.10^{-1}m$ LENGTH

$1cm = 2.10^{-6}m$ DISPLAC.

SCALES:

$1 cm = 2.10^{-1} m$ LENGTH

$1 cm = 1.10^{-6} m$ DISPLAC.



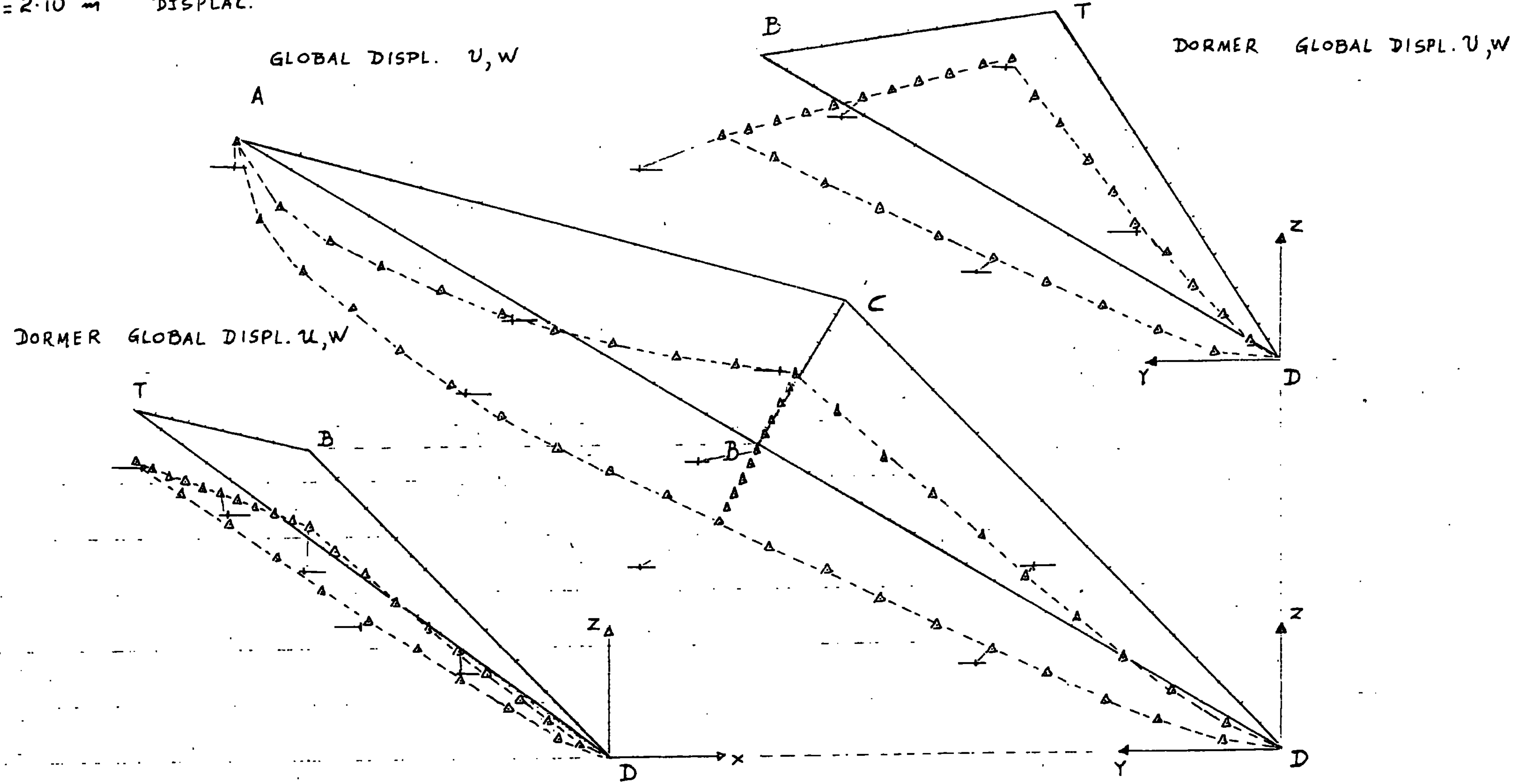
1Nt VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS

FIG. 13.96 36 FACED DOME

SCALES:

1cm = 2.10^{-1} m LENGTH

1cm = 2.10^{-6} m DISPLAC.



1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

FIG. 13.97 <u>36 FACED DOME</u>

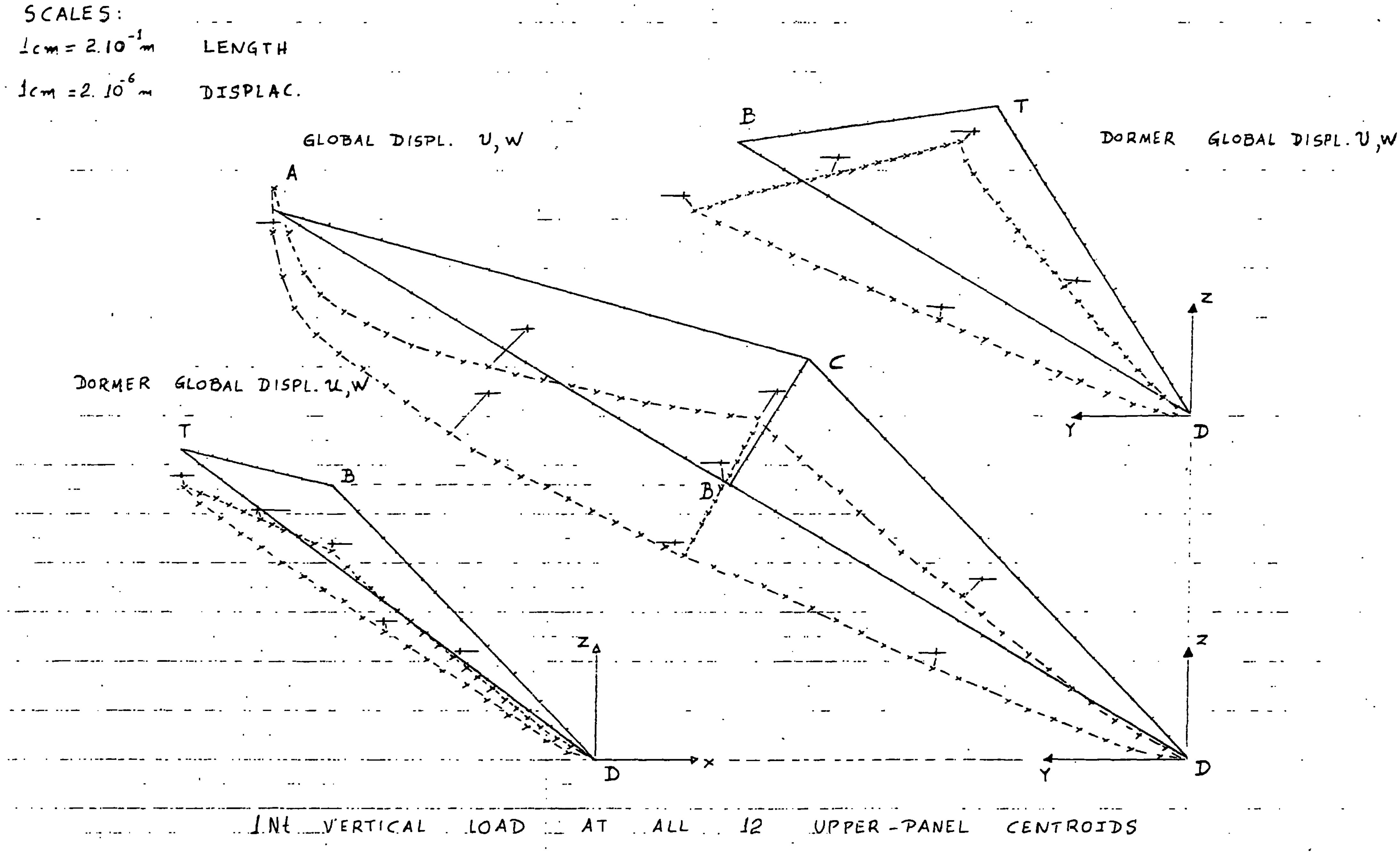


FIG. 13.98 36 FACED DOME

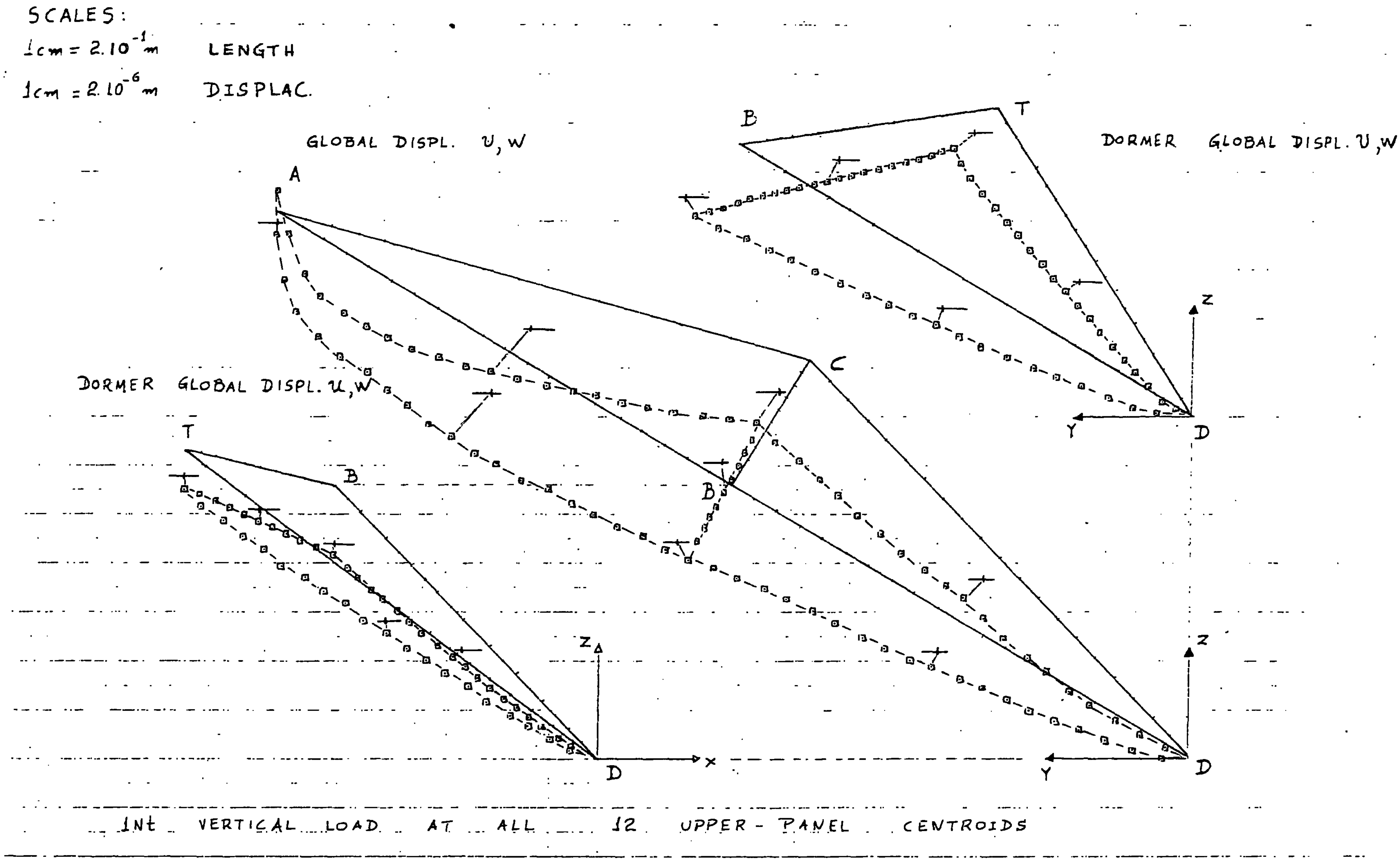
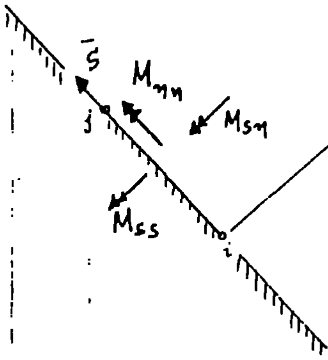


FIG. 13.99 36 FACED DOME

FIG. 13.100 56 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

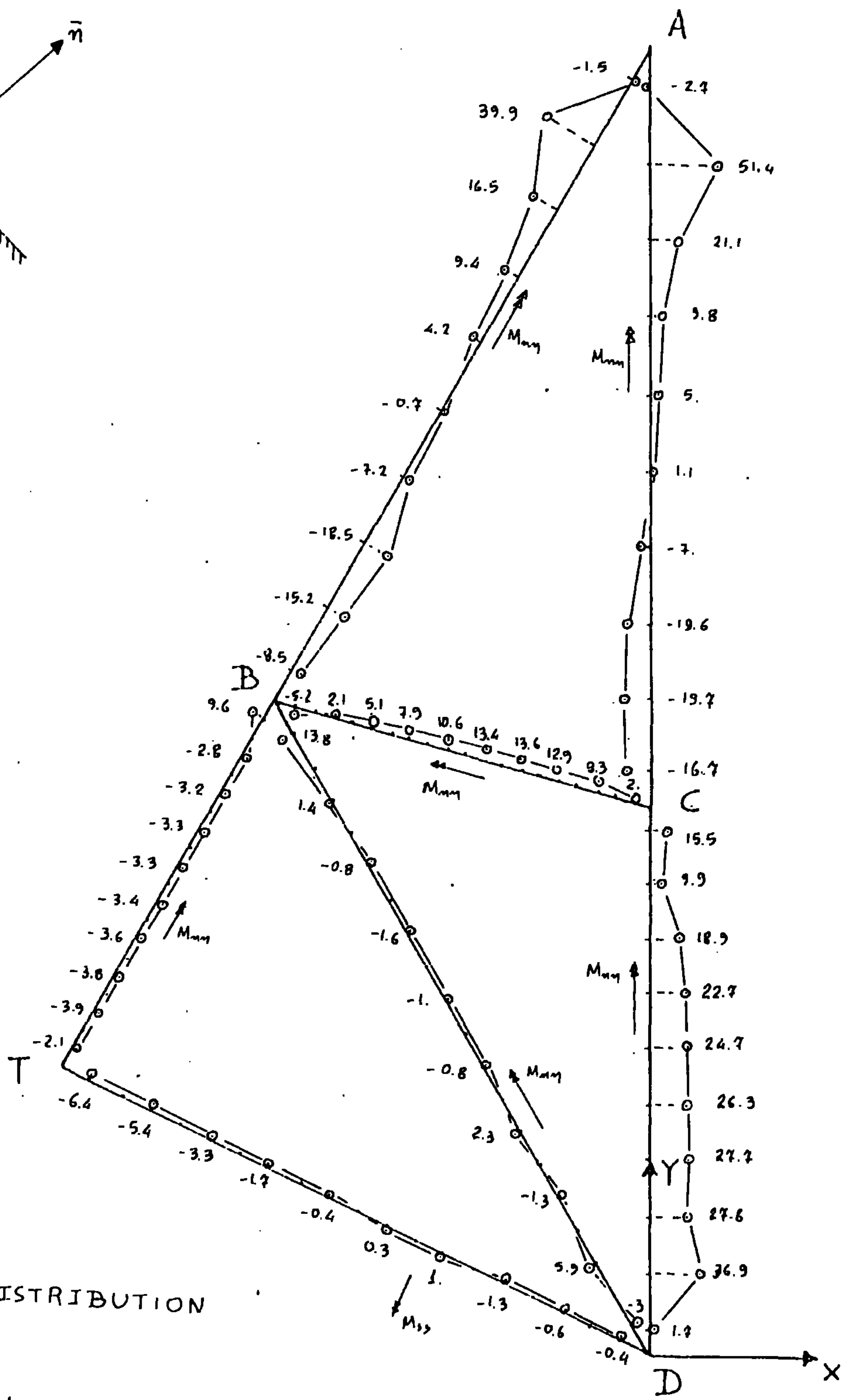


SIGN CONVENTION

LINE VECTORS:

$\vec{CA}$, $\vec{BA}$, $\vec{CB}$

$\vec{DC}$, $\vec{DB}$, $\vec{DT}$, $\vec{TB}$



MOMENTS DISTRIBUTION

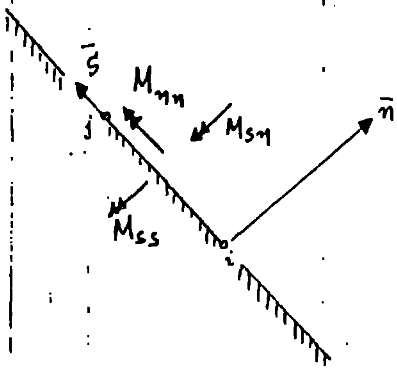
SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 50 Ntm/m$ MOMENTS

FIG. 13. 101 **36 FACED DOME**

1 KNt VERTICAL LOAD AT ALL UPPER PANEL CENTROIDS

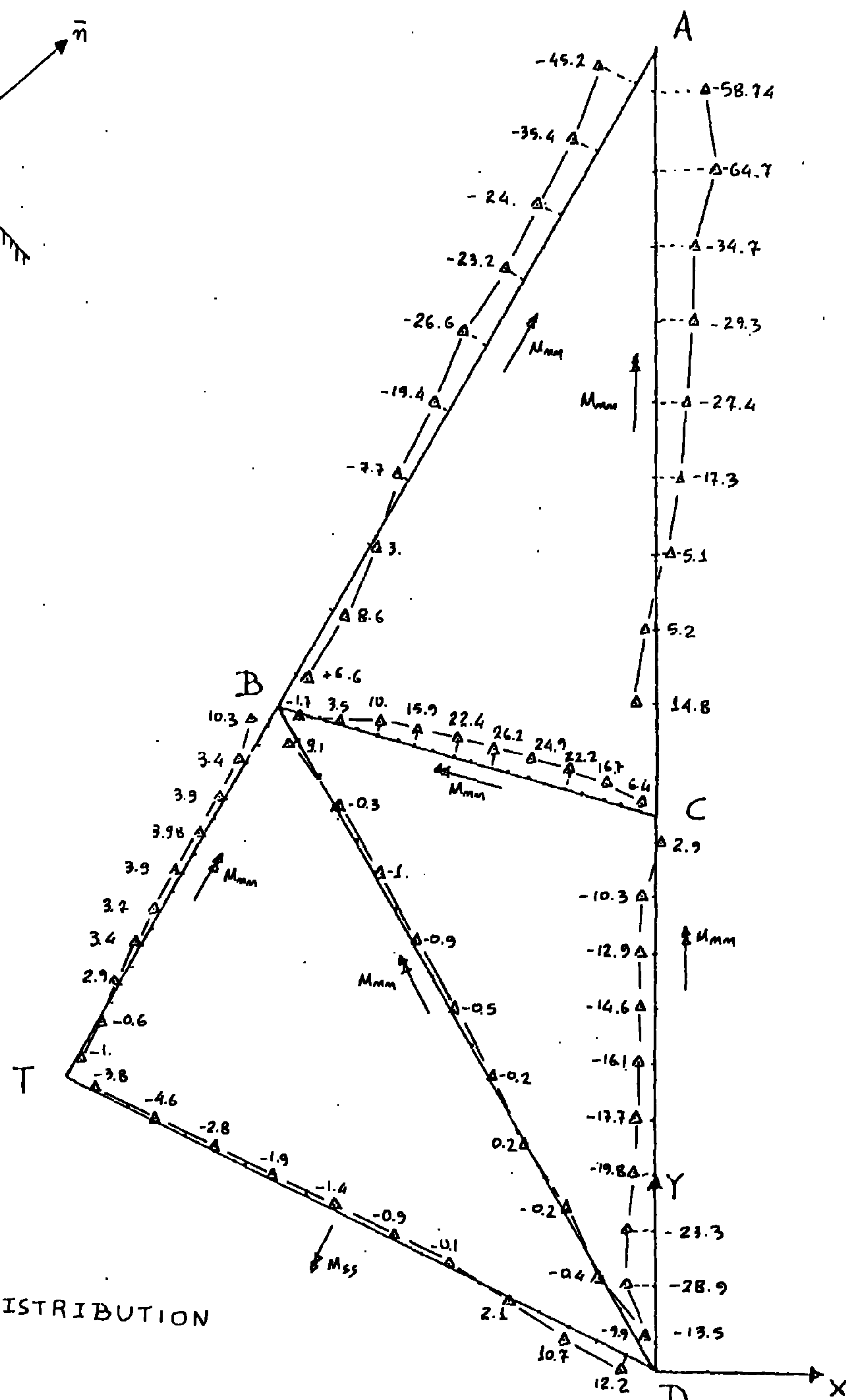


SIGN CONVENTION

LINE VECTORS:

$\vec{CA}$, $\vec{BA}$, $\vec{CB}$

$\vec{DC}$, $\vec{DB}$, $\vec{DT}$, $\vec{TB}$



MOMENTS DISTRIBUTION

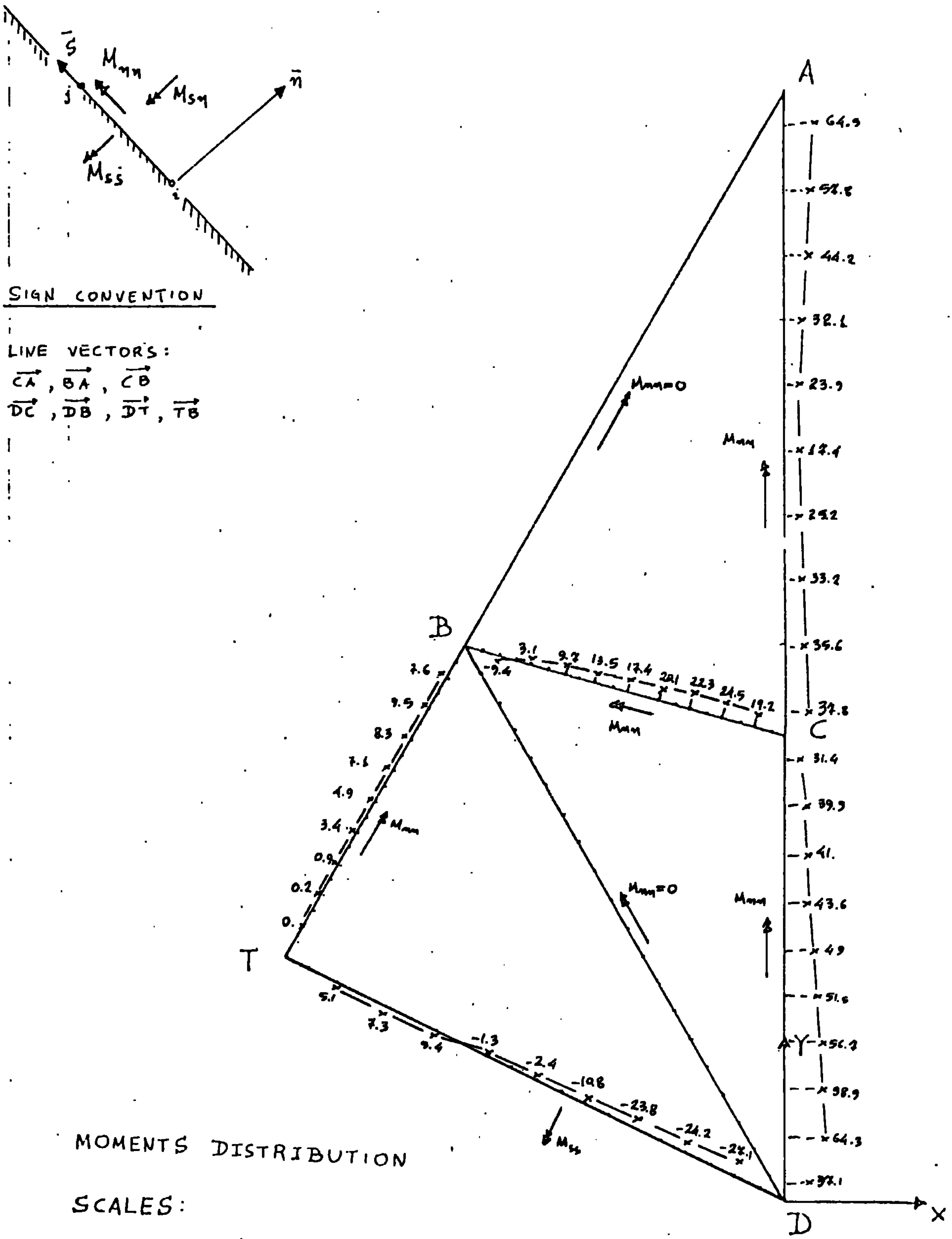
SCALES:

$1cm = 2\cdot 10^{-1} m$ LENGTH

1cm = 50 Ntm/m MOMENTS

FIG. 13.102 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS



MOMENTS DISTRIBUTION

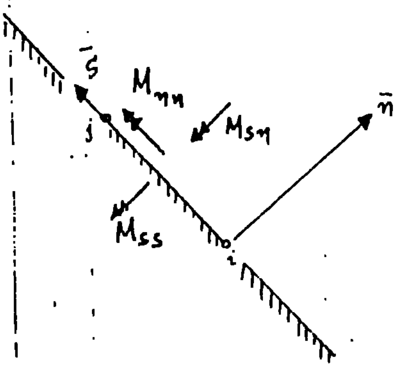
SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 50$ Ntm/m MOMENTS

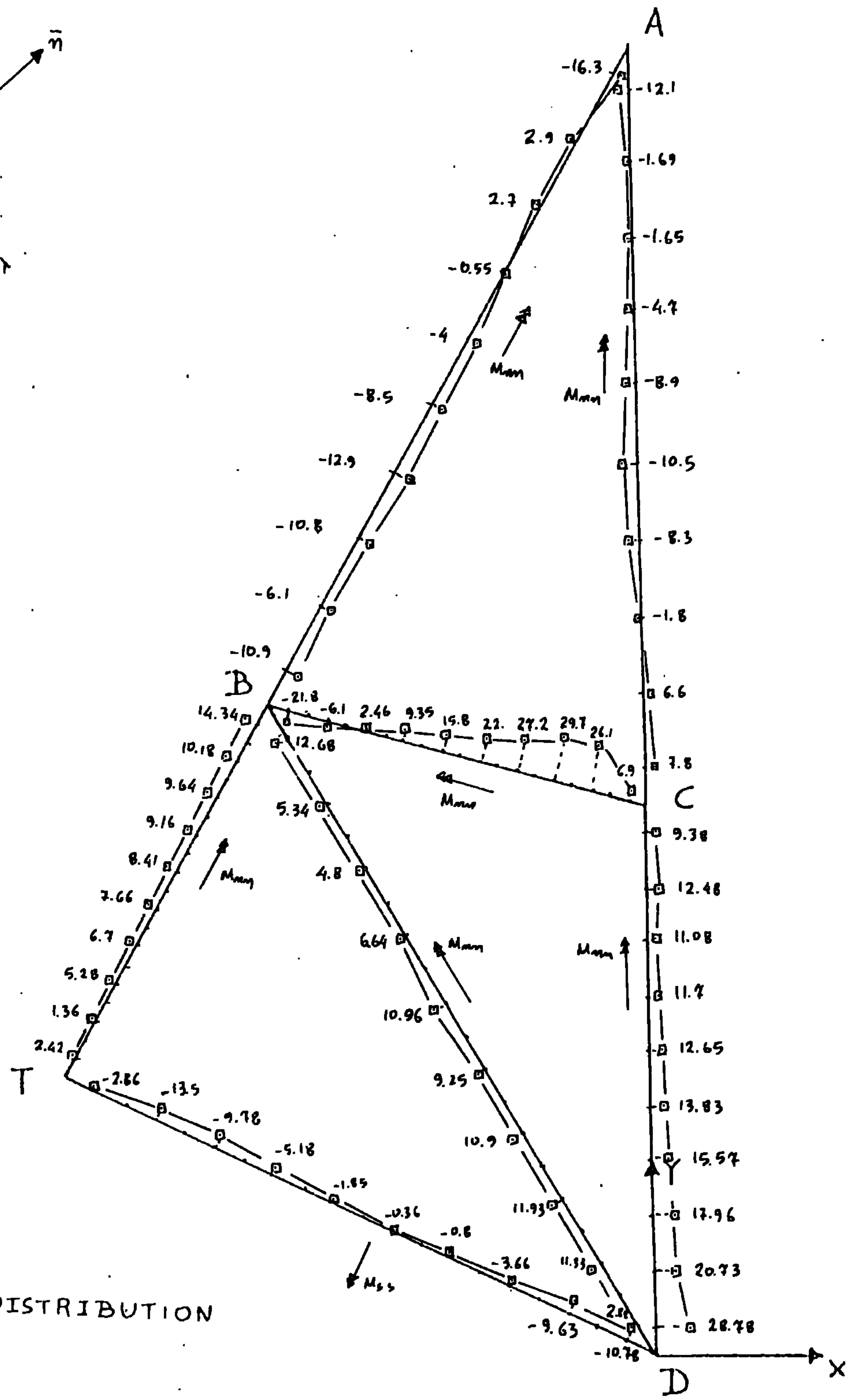
FIG. 13.103 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL UPPER PANEL CENTROIDS



SIGN CONVENTION

LINE VECTORS:
$\vec{CA}$, $\vec{BA}$, $\vec{CB}$
$\vec{DC}$, $\vec{DB}$, $\vec{DT}$, $\vec{TB}$



MOMENTS DISTRIBUTION

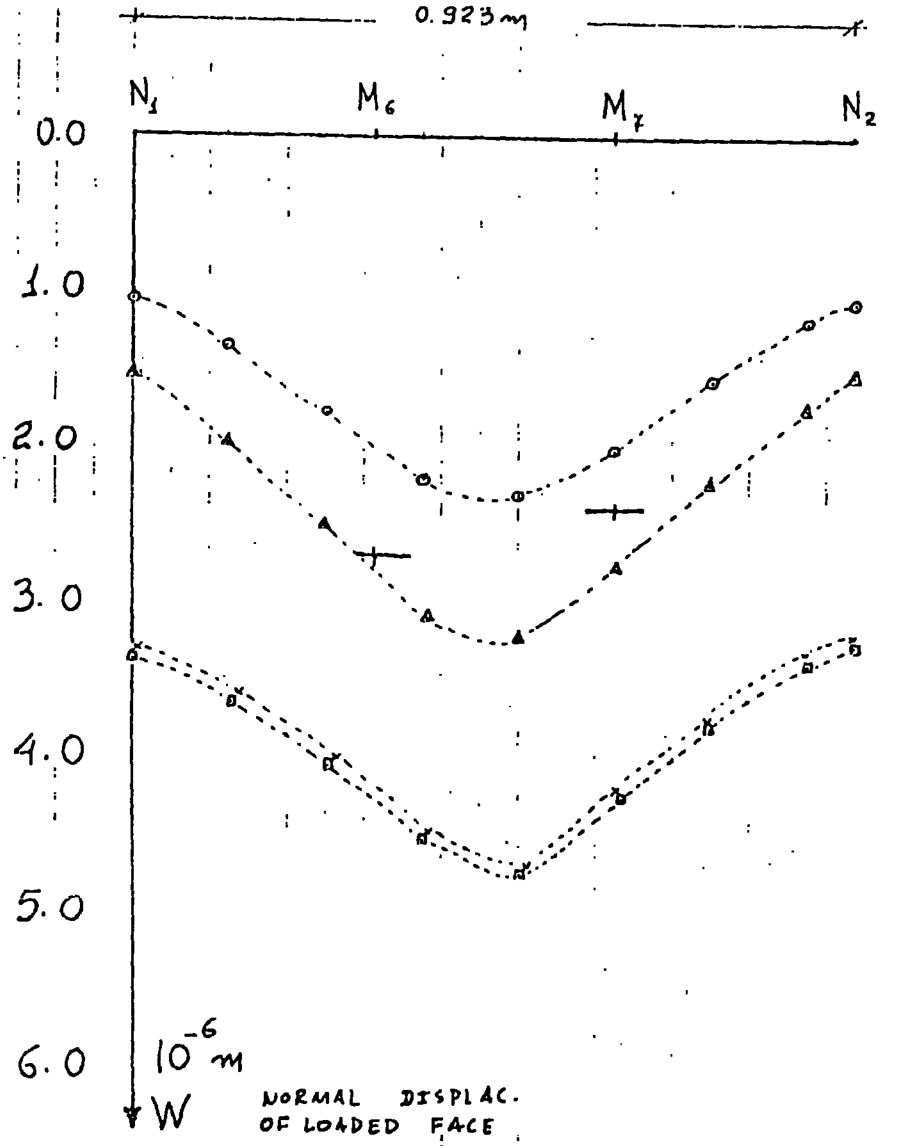
SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

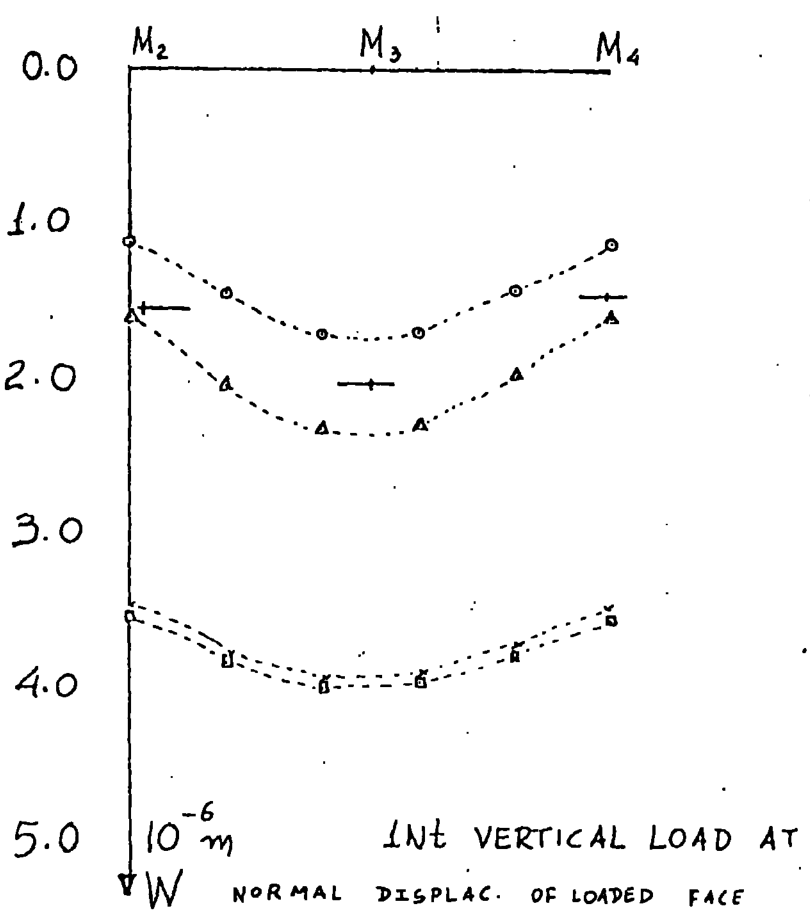
$1cm = 50 Ntm/m$ MOMENTS

Fig. 13.104

36 FACED DOME



SCALES:
1cm = 10^{-1} m LENGTH
1cm = $5\cdot10^{-7}$ m DISPLAC.



	+ EXPERIMENTAL VALUE OF NORMAL DISPLACEMENT W	⊙ FINITE ELEMENT VALUE (DDS21) OF NORMAL DISPLACEMENT W	Δ FINITE ELEMENT VALUE (DDS33) OF NORMAL DISPLACEMENT W	× FINITE ELEMENT VALUE (DMX36) OF NORMAL DISPLACEMENT $M_{nn}=0$ along AB, BD W	□ FINITE ELEMENT VALUE (DR030) OF NORMAL DISPLACEMENT W
M_2	-1.55	-1.11	-1.62	-3.51	-3.55
M_3	-2.05	-1.70	-2.30	-3.92	-3.97
M_4	-1.44	-1.11	-1.61	-3.50	-3.54
M_6	-2.71	-1.99	-2.80	-4.24	-4.27
M_7	-2.38	-2.02	-2.78	-4.20	-4.21
	10^{-6} m	10^{-6} m	10^{-6} m	10^{-6} m	10^{-6} m

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

1 Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

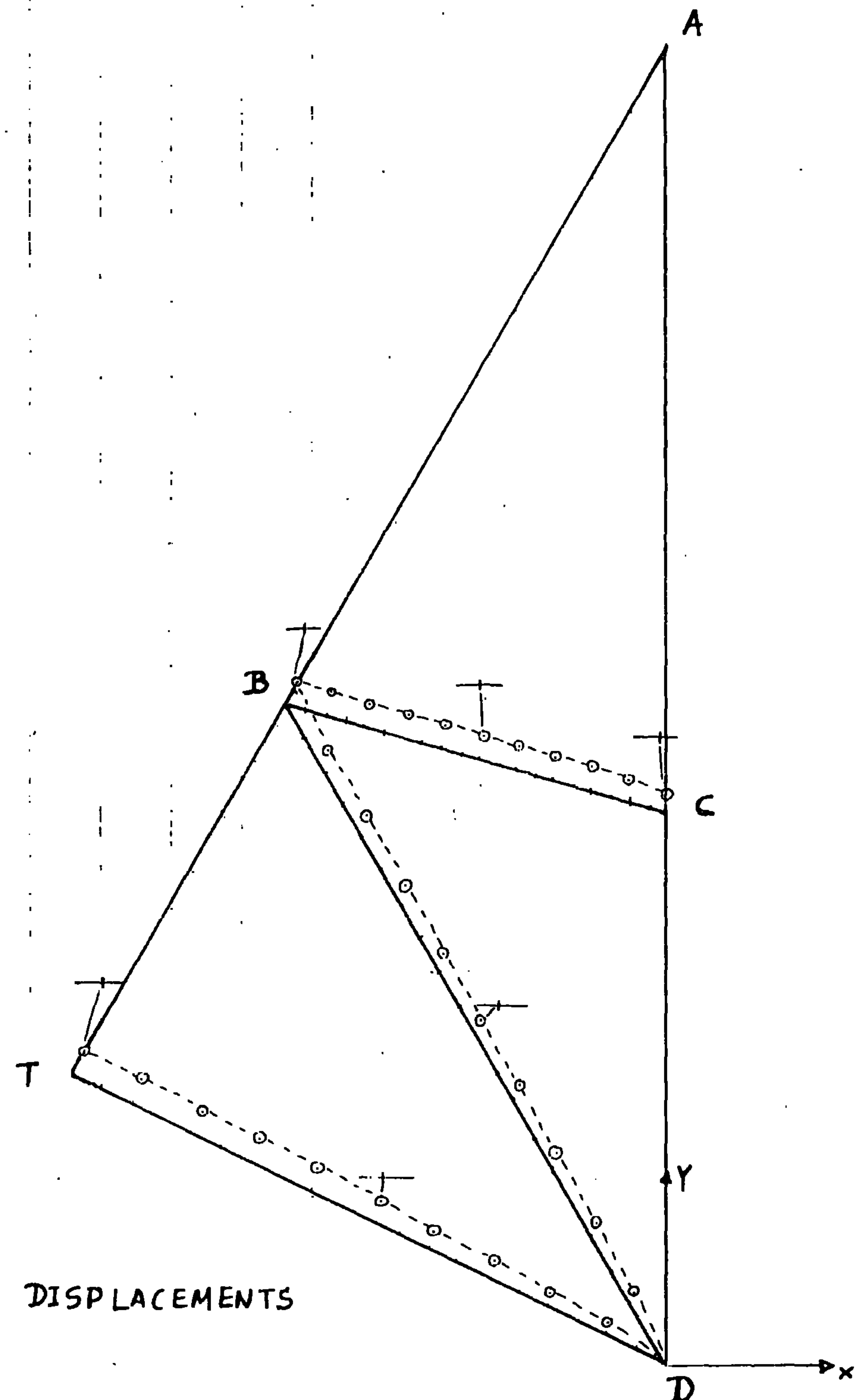
EXPERIMENTAL VALUES OF GLOBAL DISPLAC. u, v, w

POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m	POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m
A	0.00	0.02	0.37	M_{13}	0.62	2.38	-1.48
M_2	0.61	1.15	-0.12	M_{14}	0.62	2.13	-1.72
M_3	0.33	1.15	-0.53	M_{15}	0.86	2.54	-2.62
M_4	0.21	1.39	-0.53	M_{16}	0.45	1.89	-1.84
M_5	0.25	1.31	-0.98	M_{17}	0.74	2.13	-1.35
M_6	0.37	1.80	-1.52	M_{18}	1.56	3.07	-2.79
M_7	-0.04	0.74	-0.70	M_{19}	0.29	1.19	-0.70
T	0.90	2.38	-0.62	M_{20}	0.49	1.80	-1.56
M_9	0.61	2.66	-1.35	M_{21}	0.66	0.86	-1.27
B	0.49	2.21	-2.54	M_{22}	0.49	1.48	-1.56
M_{11}	0.03	2.21	-1.89	M_{23}	0.04	1.15	-0.94
C	-0.12	2.13	-1.56				

FIG. 13.105 <u>36 FACED DOME</u>

FIG. 13.106 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM-PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

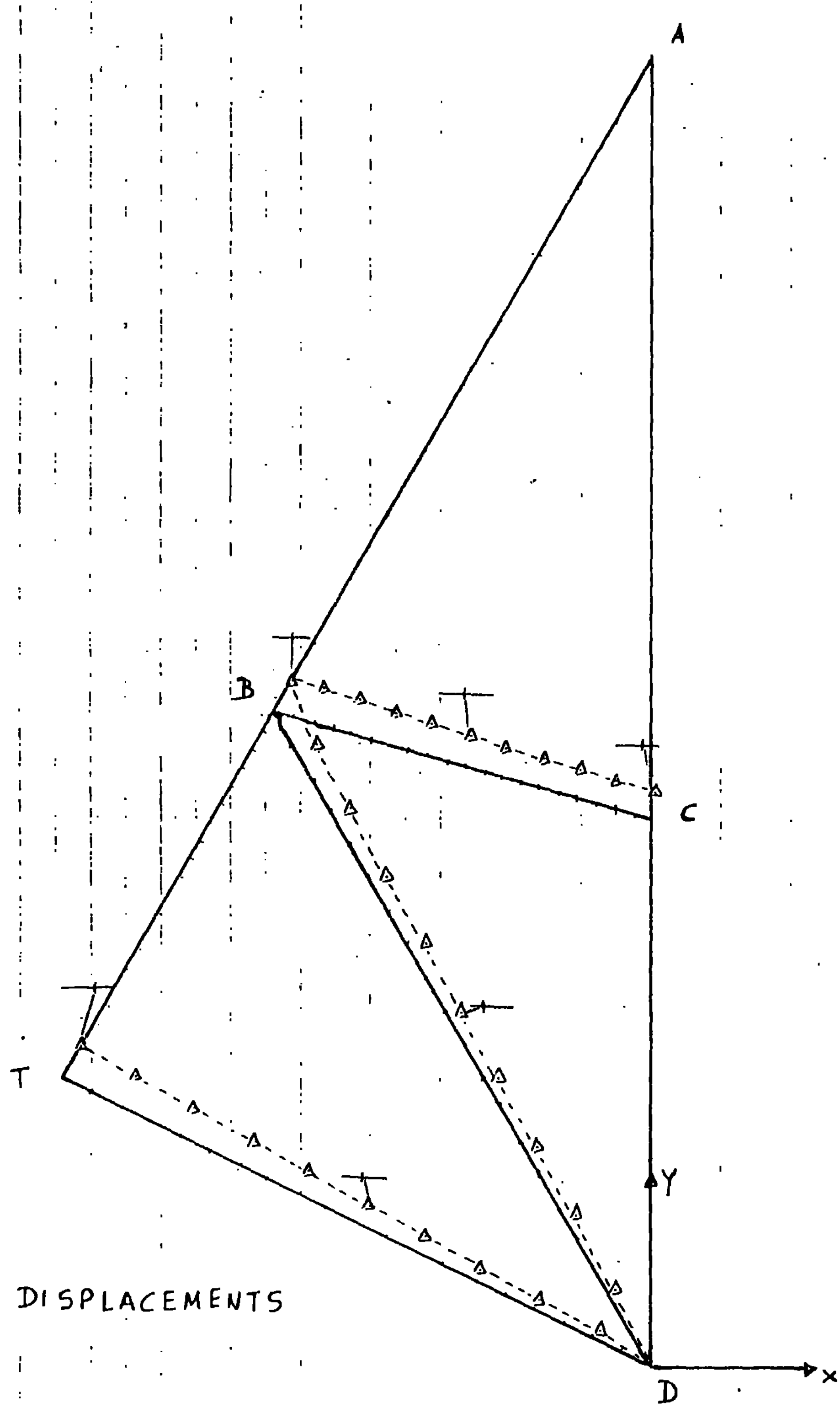
SCALES:

$1cm = 2\cdot10^{-1} m$ LENGTH

$1cm = 2\cdot10^{-6} m$ DISPLAC.

FIG. 13.107 <u>36 FACED DOME</u>

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

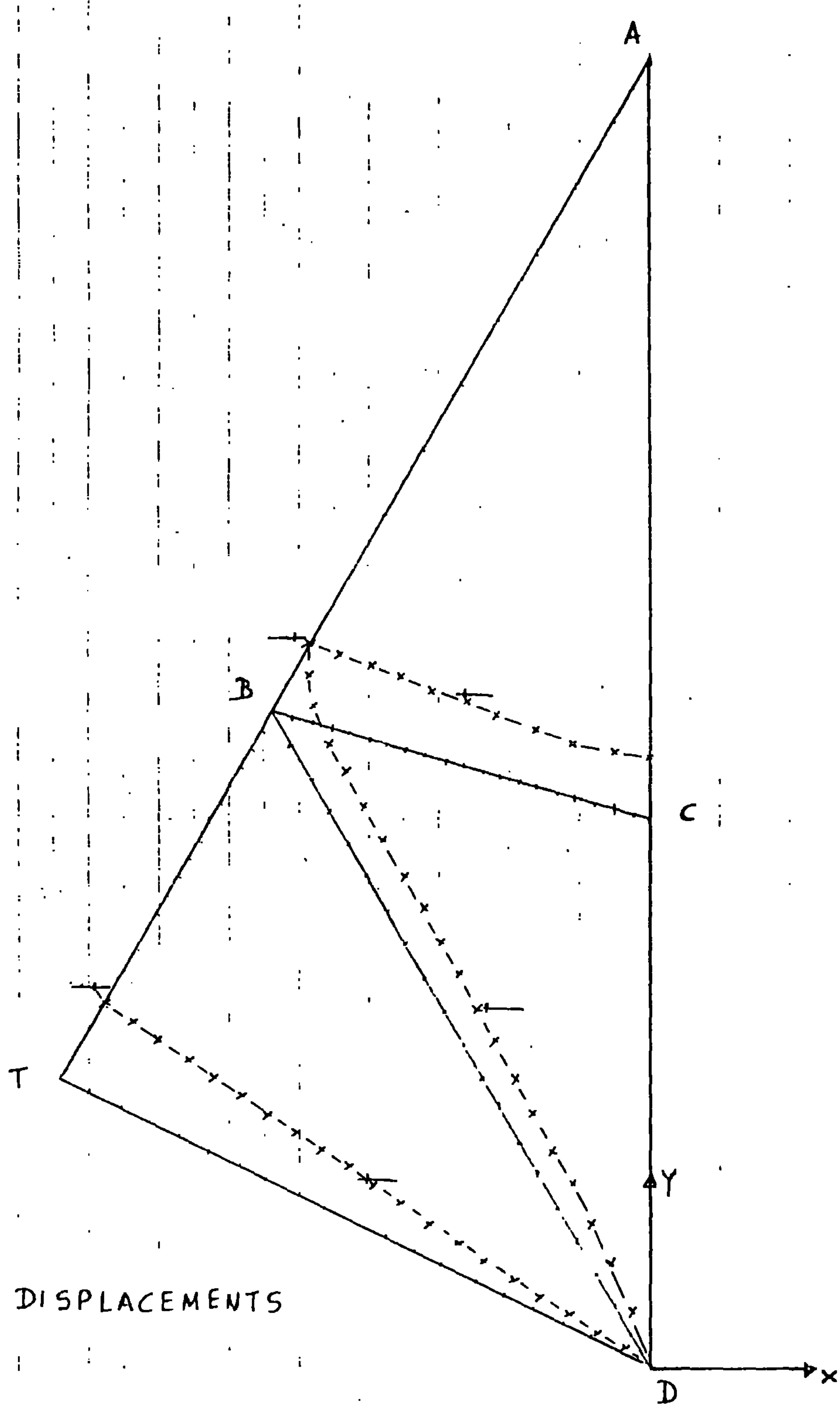
SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.108   36 FACED DOME

1N/ VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

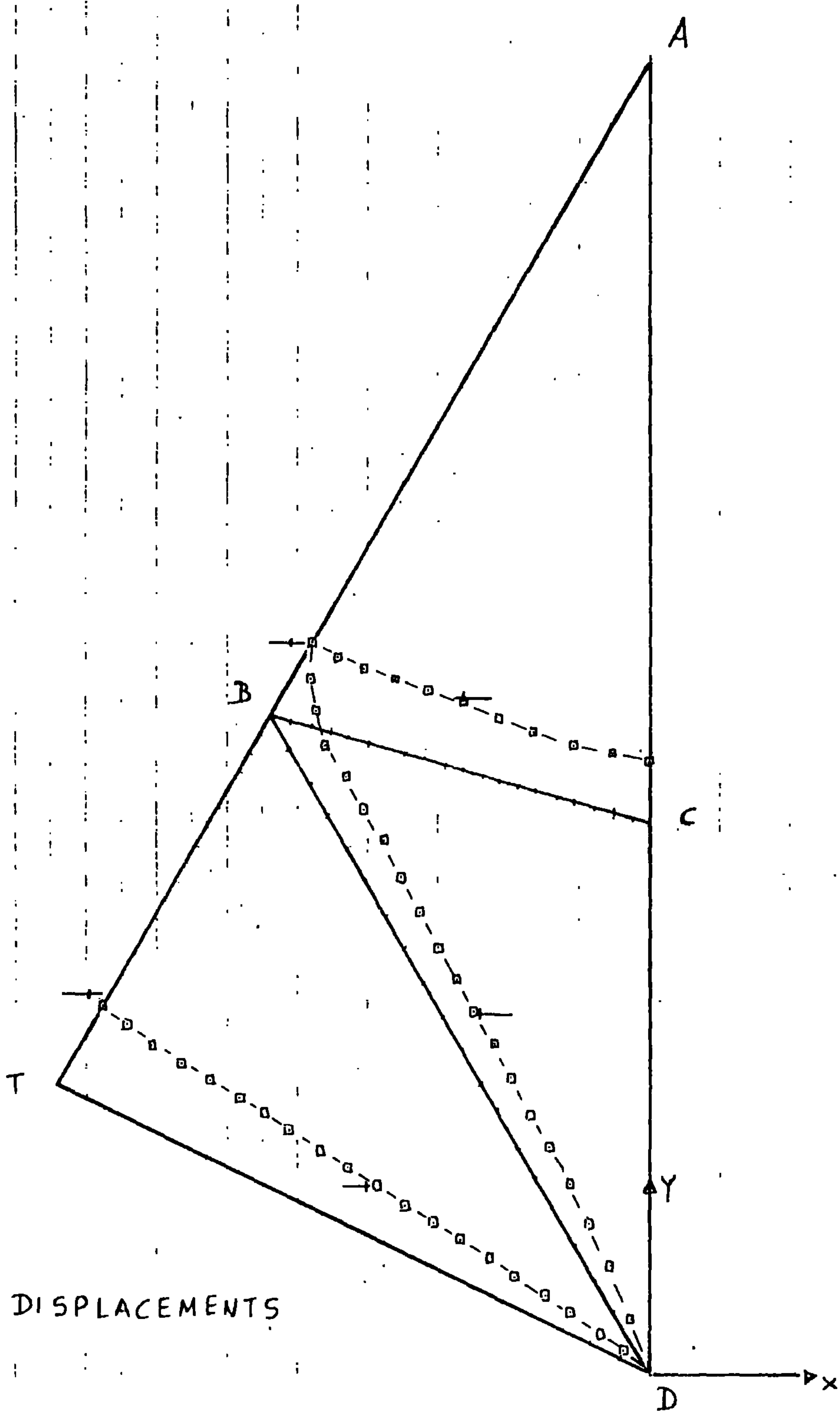
SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.109 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



HORIZONTAL DISPLACEMENTS

SCALES:

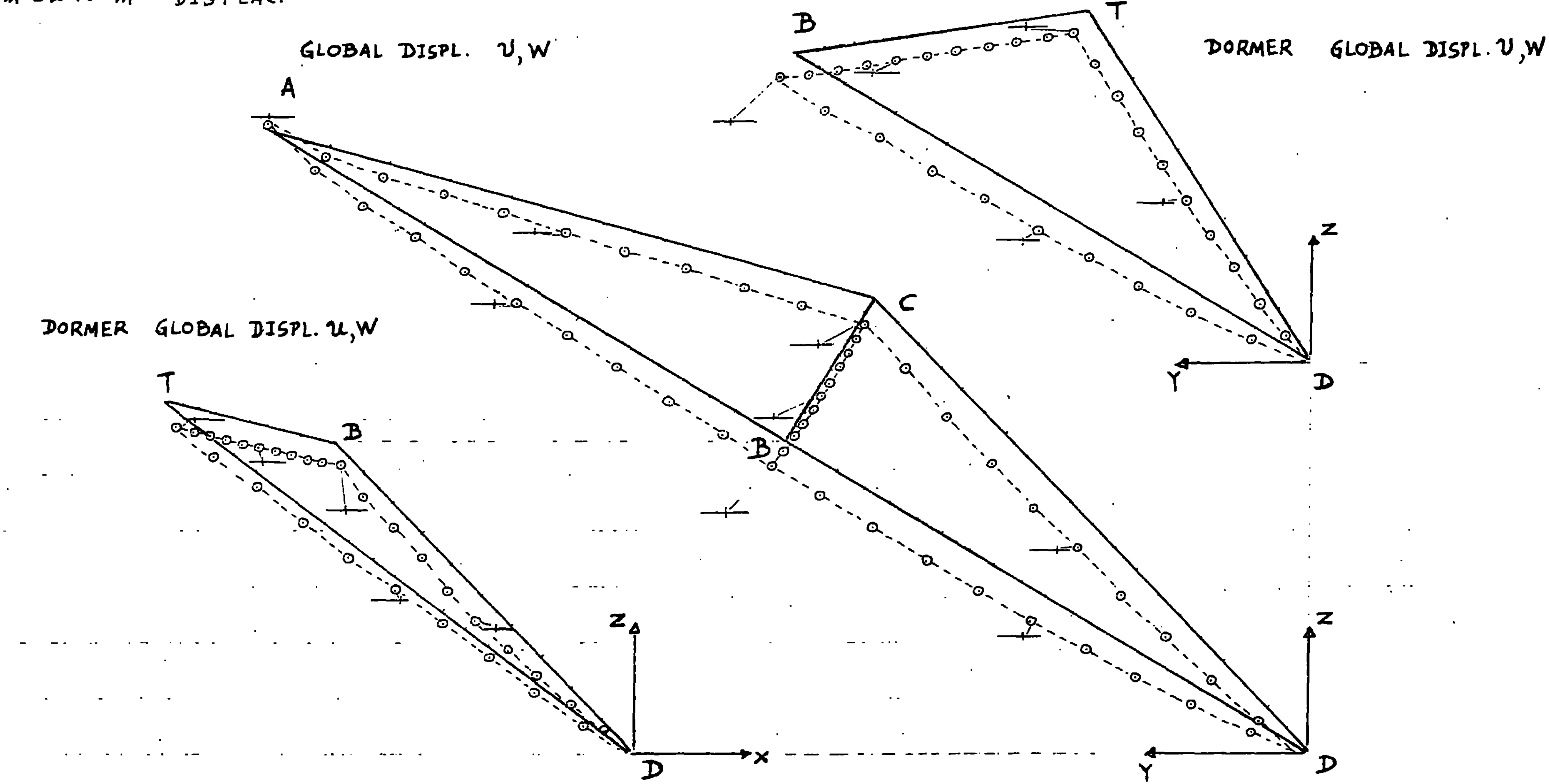
$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

SCALES:

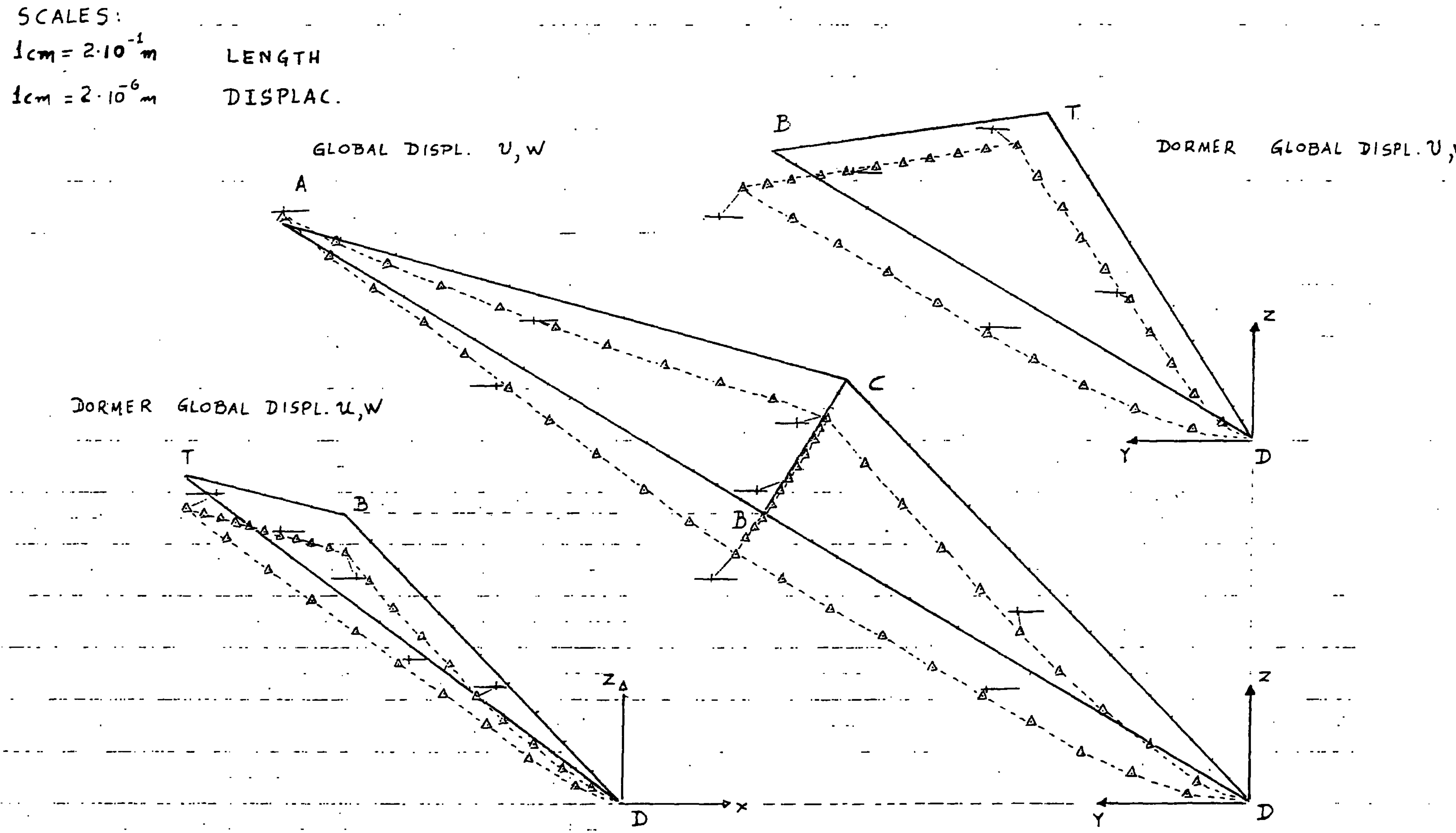
1cm = 2·10⁻¹ m LENGTH

1cm = 2·10⁻⁶ m DISPLAC.



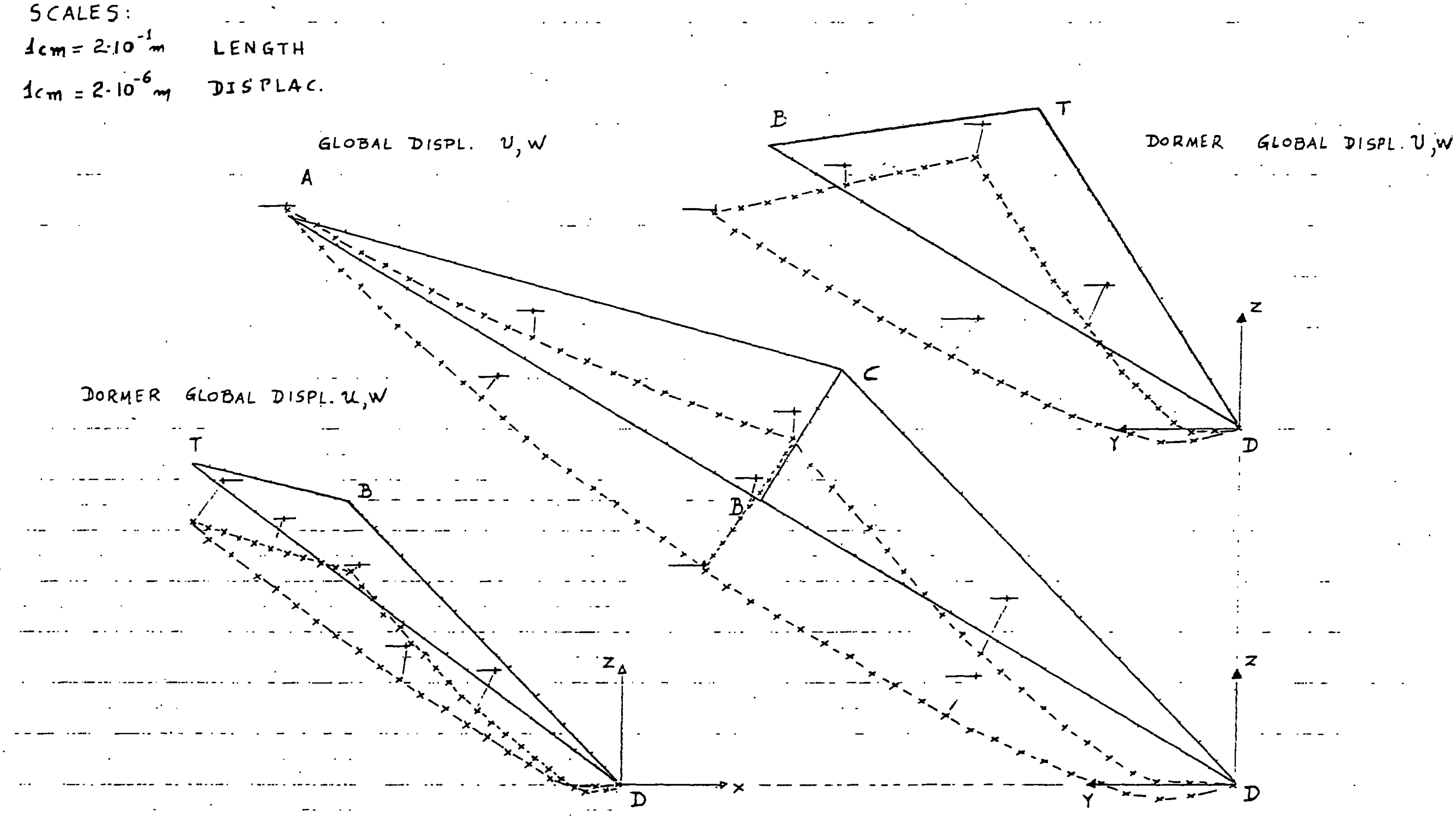
1Nt VERTICAL LOAD AT ALL 12 BOTTOM-PANEL CENTROIDS

FIG. 13.110 36 FACED DOME



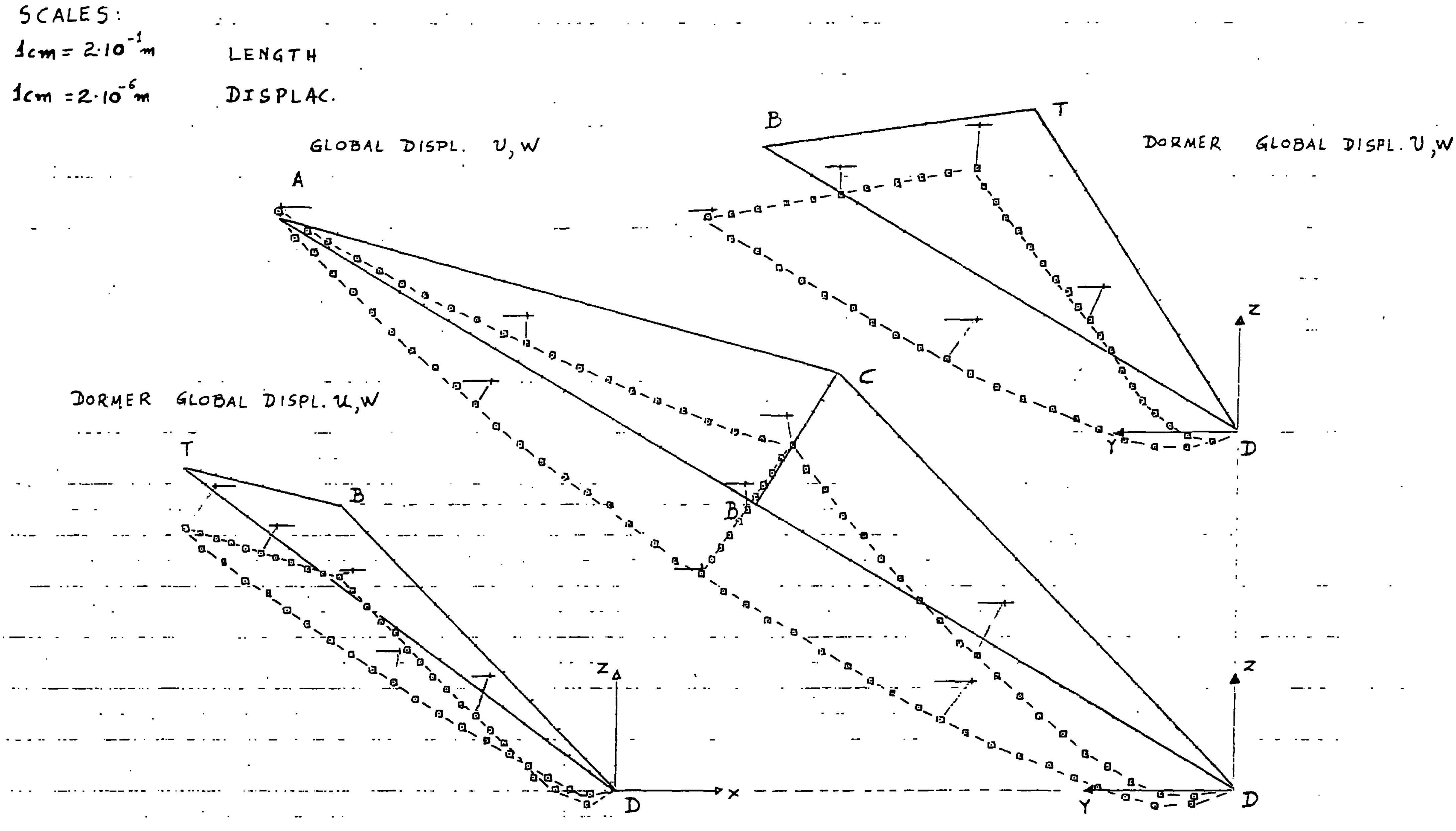
1Nt VERTICAL LOAD AT ALL 12 BOTTOM-PANEL CENTROIDS

FIG. 13.111 36 FACED DOME



1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.112 <u>36 FACED DOME</u>

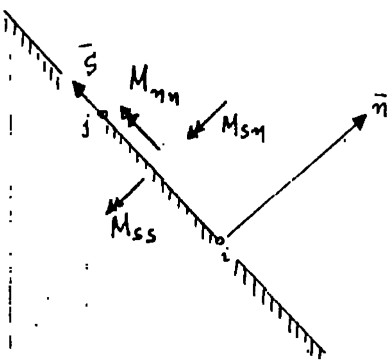


1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.113 36 FACED DOME

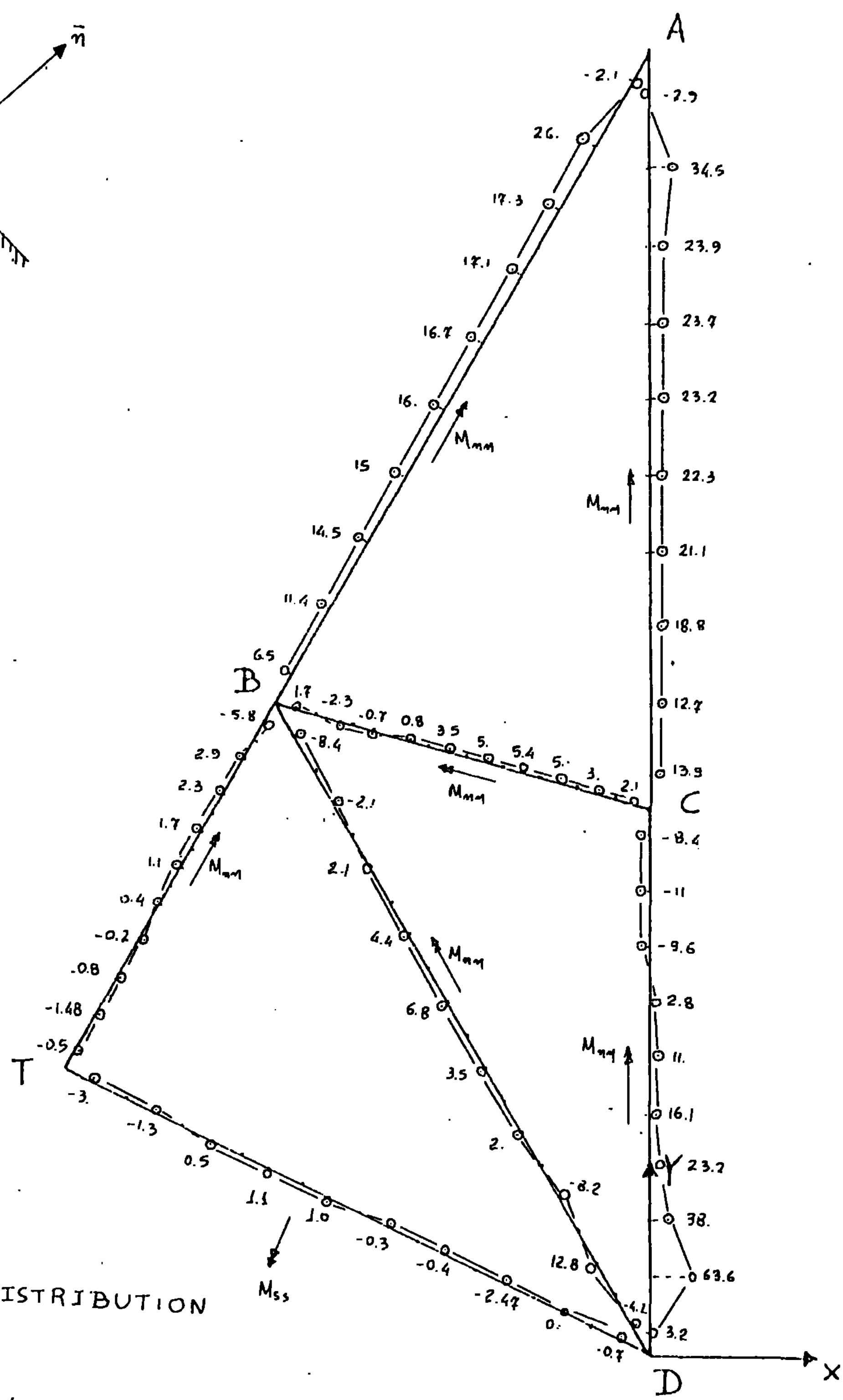
FIG. 13.114 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 BOTTOM-PANEL CENTROIDS



SIGN CONVENTION

LINE VECTORS:
$\vec{CA}$, $\vec{BA}$, $\vec{CB}$
$\vec{DC}$, $\vec{DB}$, $\vec{DT}$, $\vec{TB}$



MOMENTS DISTRIBUTION

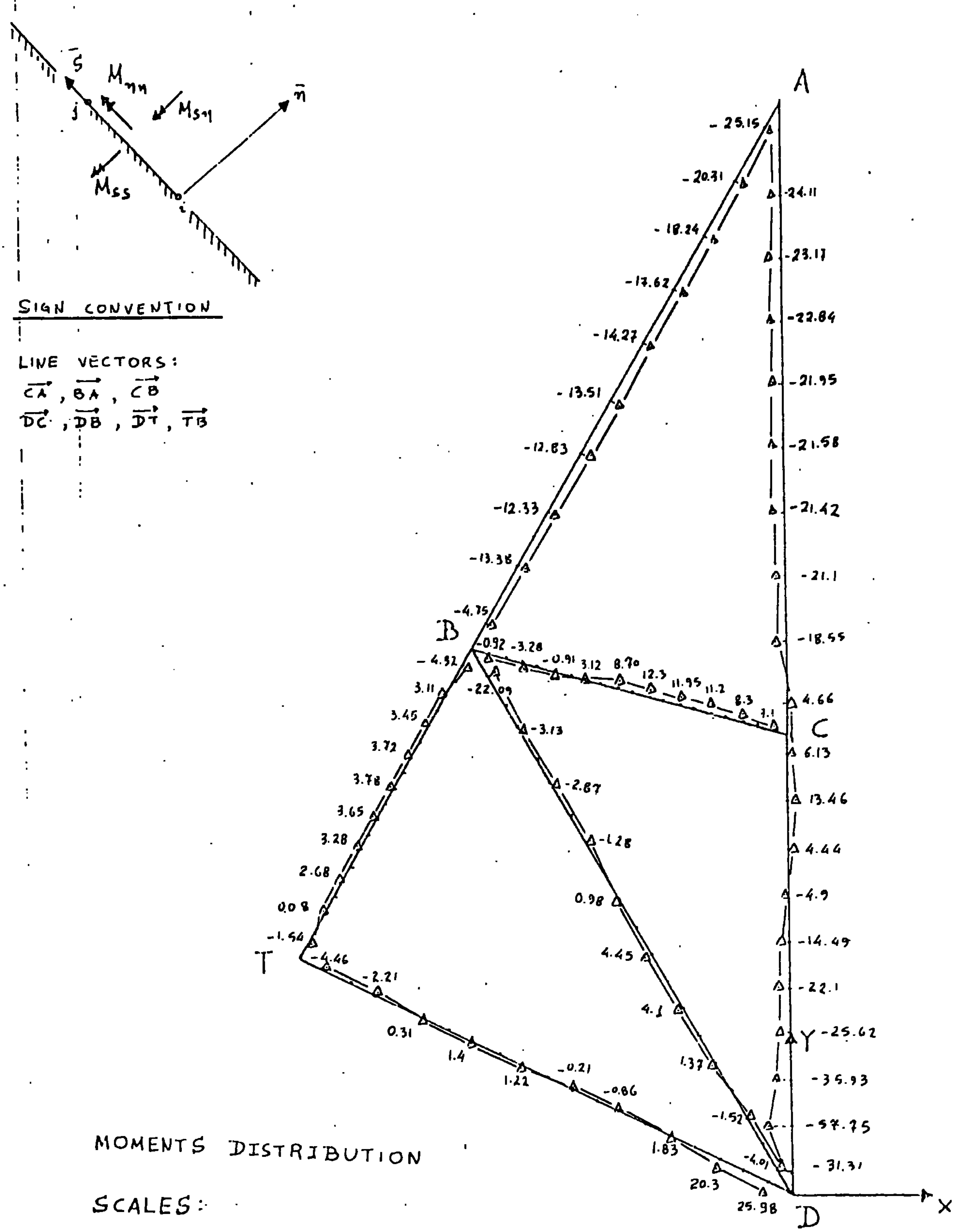
SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 100 Ntm/m$ MOMENTS

FIG. 13.115 **36 FACED DOME**

1 KNt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



MOMENTS DISTRIBUTION

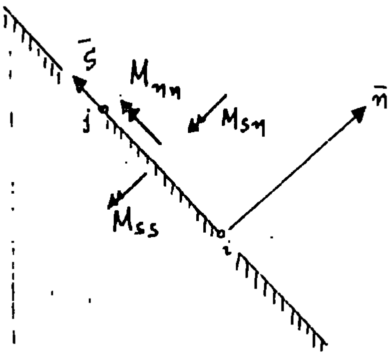
SCALES:

$1cm = 2 \cdot 10^{-1}$ m LENGTH

1 cm = 100 Nt m/m MOMENTS

FIG 13.116 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 BOTTOM - PANEL CENTROIDS

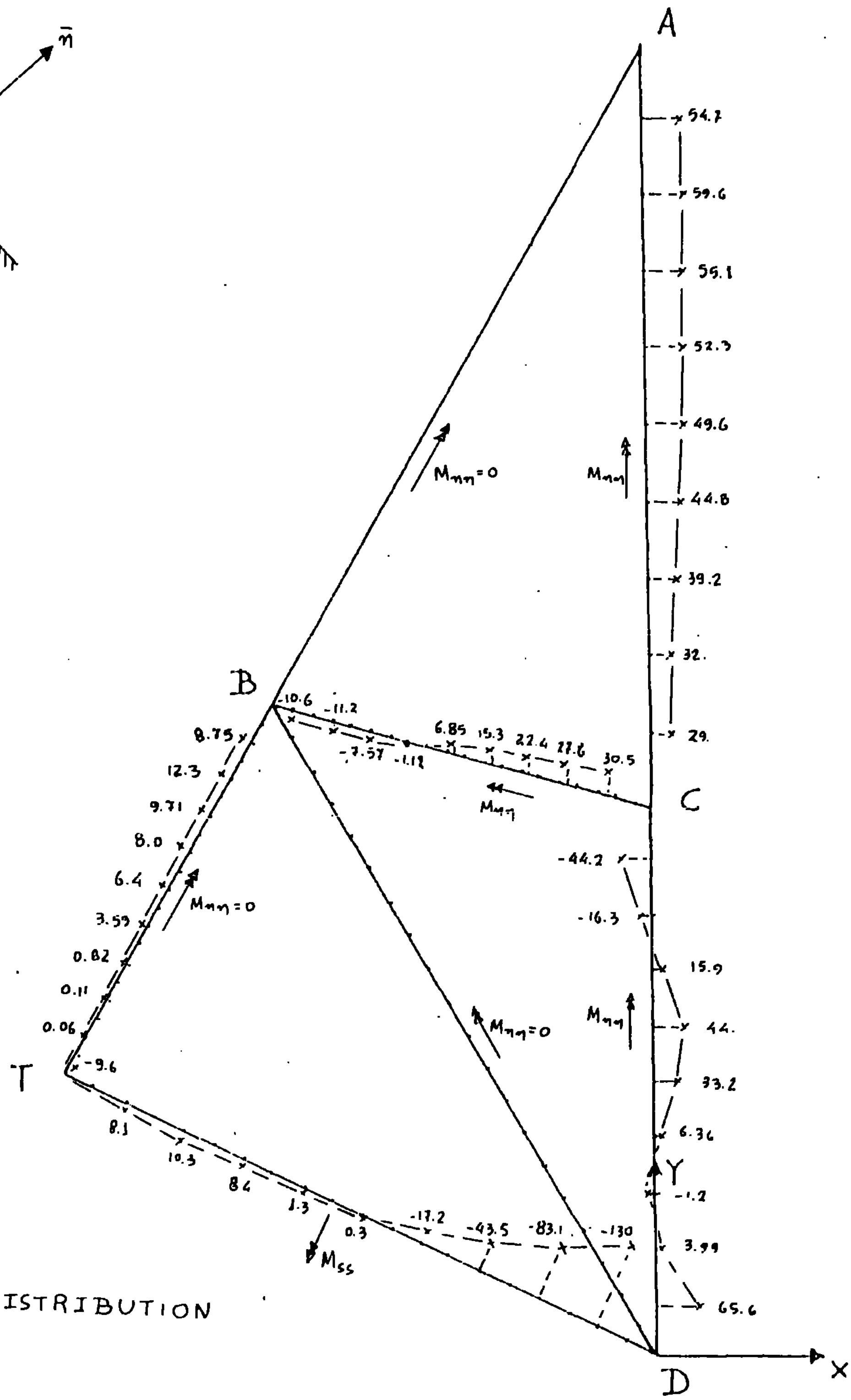


SIGN CONVENTION

LINE VECTORS:

$\vec{CA}$, $\vec{BA}$, $\vec{CB}$

$\vec{DC}$, $\vec{DB}$, $\vec{DT}$, $\vec{TB}$



MOMENTS DISTRIBUTION

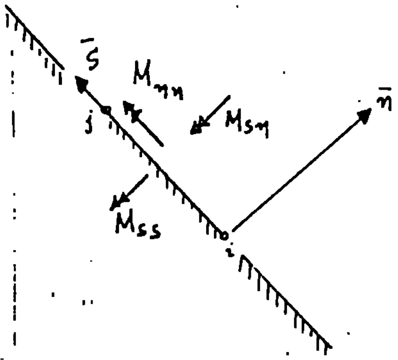
SCALES:

$1 cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 100$ Ntm/m MOMENTS

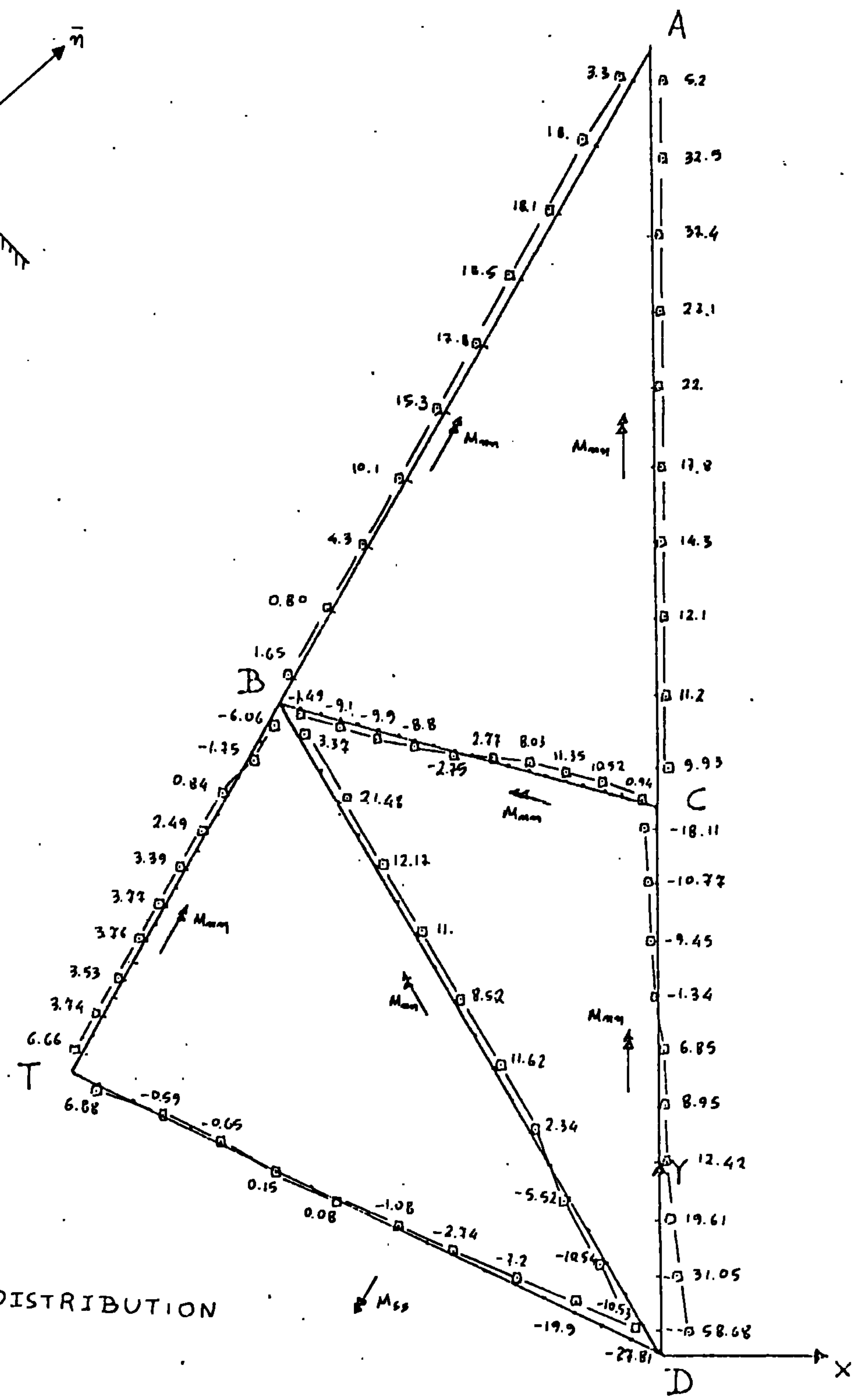
FIG. 13.117 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS



SIGN CONVENTION

LINE VECTORS:
$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$
$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$



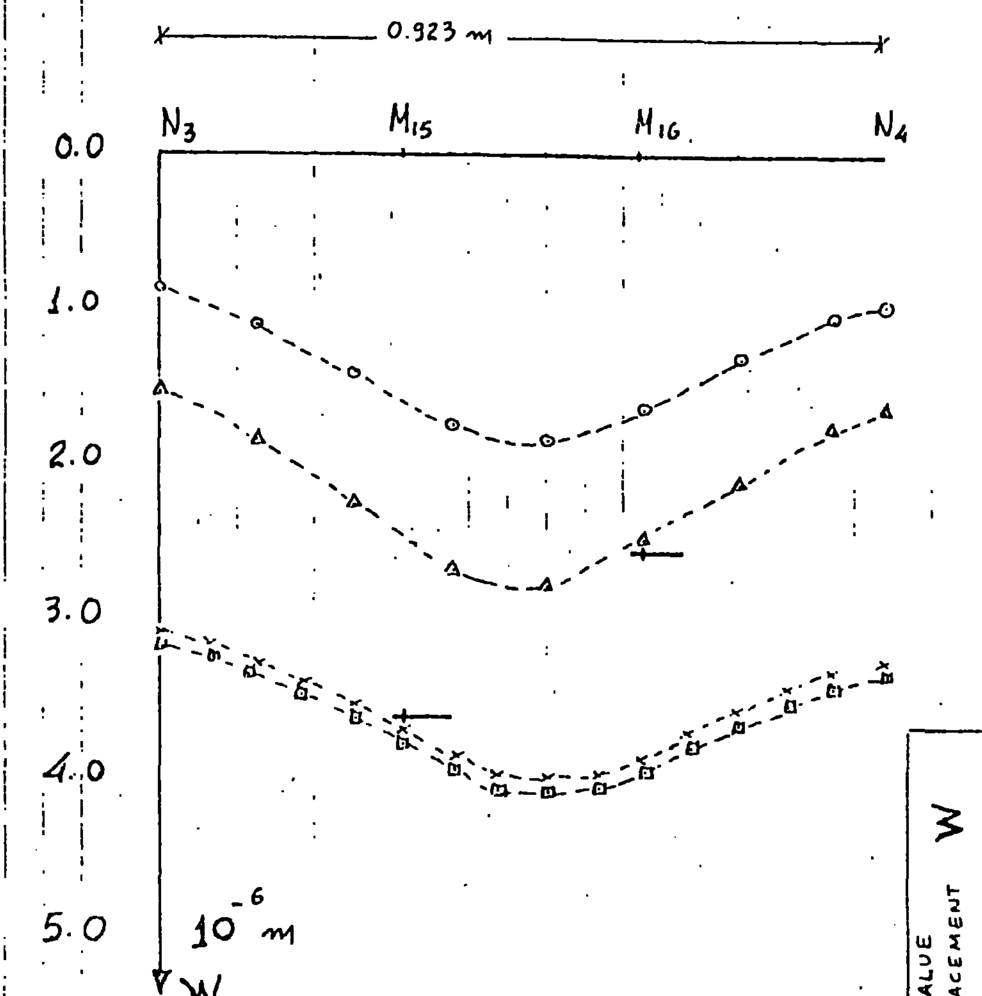
MOMENTS DISTRIBUTION

SCALES:

$1 cm = 2\cdot 10^{-1} m$ LENGTH

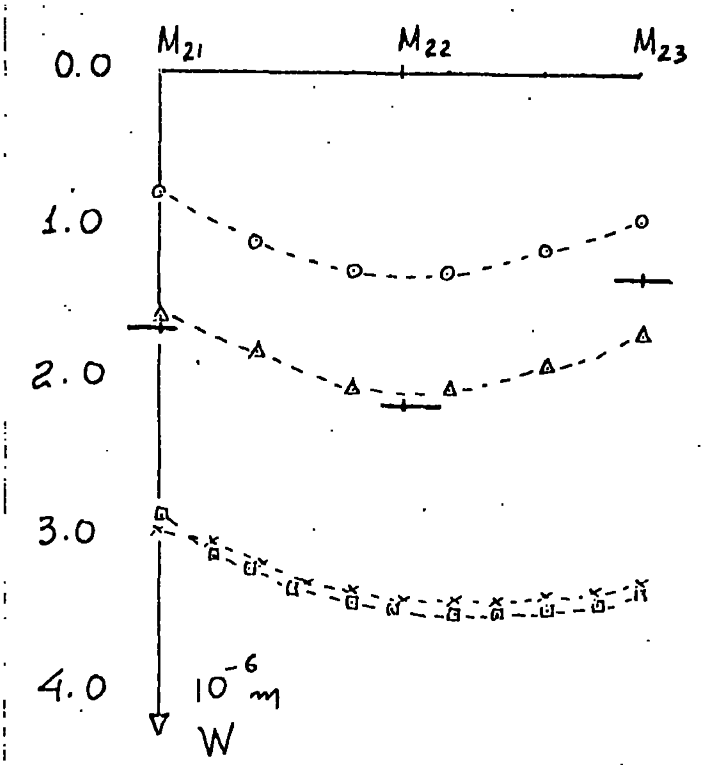
$1 cm = 100 Ntm/m$ MOMENTS

FIG. 13.118 36 FACED DOME



NORMAL DISPLACEMENTS OF LOADED FACE

SCALES:
1cm = 10^{-1} m LENGTH
1cm = $5 \cdot 10^{-7}$ m DISPLAC.



NORMAL DISPLACEMENTS OF LOADED FACE

	+ EXPERIMENTAL VALUE OF NORMAL DISPLACEMENT W	⊙ FINITE ELEMENT VALUE (DDS21) OF NORMAL DISPLACEMENT W	△ FINITE ELEMENT VALUE (DDS33) OF NORMAL DISPLACEMENT W	× FINITE ELEMENT VALUE (DMx36) OF NORMAL DISPLACEMENT $M_{nn}=0$ α_{0-1} AB, BD W	⊡ FINITE ELEMENT VALUE (DRO30) OF NORMAL DISPLACEMENT W
M_{15}	-3.62	-1.58	-2.46	-3.72	-3.75
M_{16}	-2.54	-1.65	-2.49	-3.88	-3.92
M_{21}	-1.63	-0.80	-1.57	-2.93	-2.88
M_{22}	-2.13	-1.29	-2.05	-3.42	-3.44
M_{23}	-1.33	-0.96	-1.69	-3.30	-3.34
	10^{-6} m	10^{-6} m	10^{-6} m	10^{-6} m	10^{-6} m

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

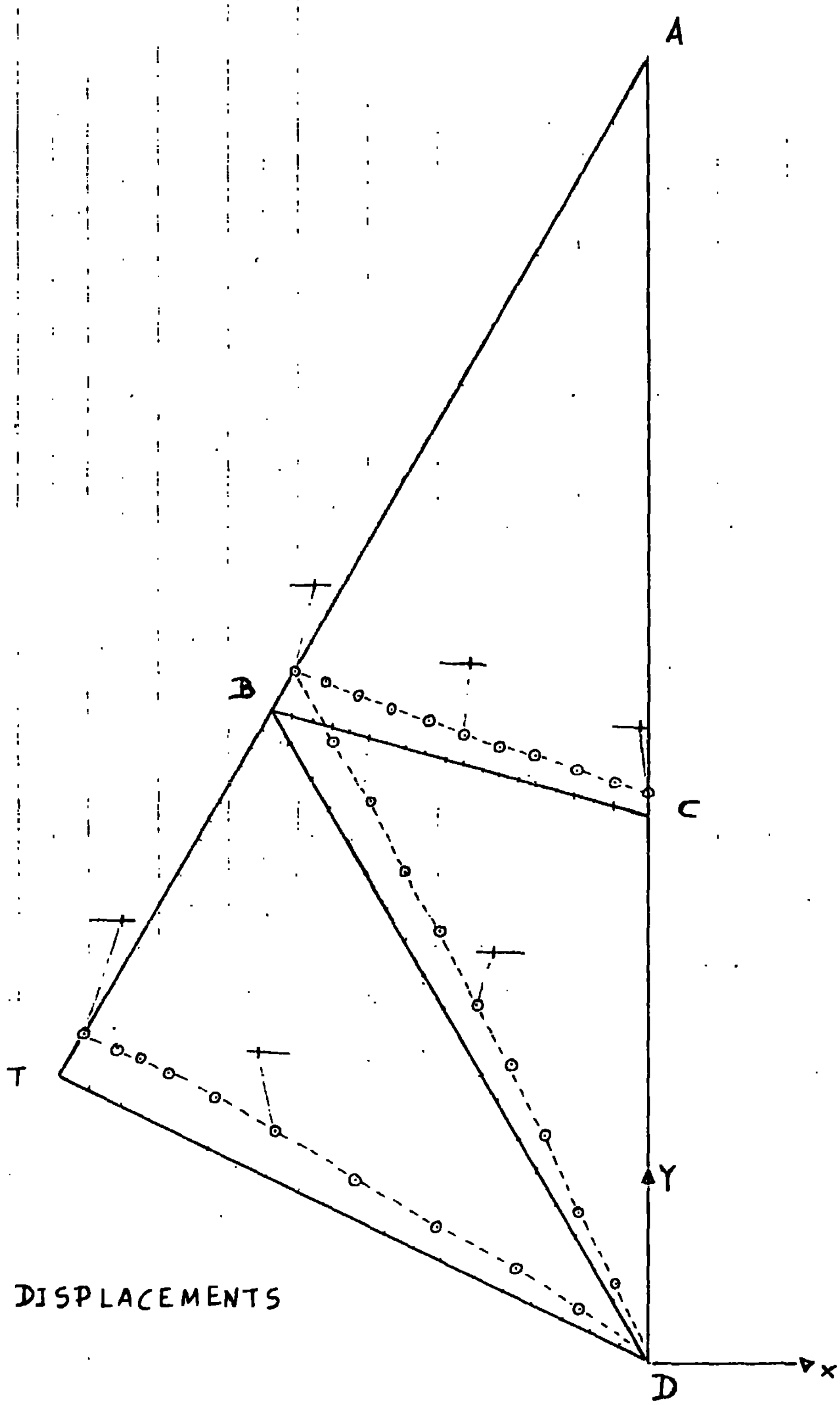
EXPERIMENTAL VALUES OF GLOBAL DISPL. u, v, w

POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m	POINT	u 10^{-6} m	v 10^{-6} m	w 10^{-6} m
A	-0.12	-0.10	0.16	M_{13}	-1.37	1.73	-3.79
M_2	-0.33	1.23	-1.39	M_{14}	-0.49	2.54	-3.21
M_3	0.08	0.95	-0.41	M_{15}	0.25	1.64	-2.71
M_4	0.00	1.15	-0.61	M_{16}	-0.08	1.60	-2.62
M_5	0.08	1.37	-1.85	M_{17}	-1.89	3.28	-5.90
M_6	0.10	1.52	-1.95	M_{18}	0.00	1.47	-1.45
M_7	0.00	1.47	-1.52	M_{19}	-1.56	2.30	-3.85
T	0.90	2.35	-1.45	M_{20}	-0.57	2.87	-3.28
M_9	0.82	2.04	-1.85	M_{21}	0.49	1.31	-1.64
B	0.57	1.96	-1.95	M_{22}	0.41	0.33	-1.47
M_{11}	0.00	1.50	-1.46	M_{23}	0.14	0.64	-0.96
C	-0.16	0.76	-1.31				

FIG. 13.119 36 FACED DOME

FIG. 13.120 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS



HORIZONTAL DISPLACEMENTS

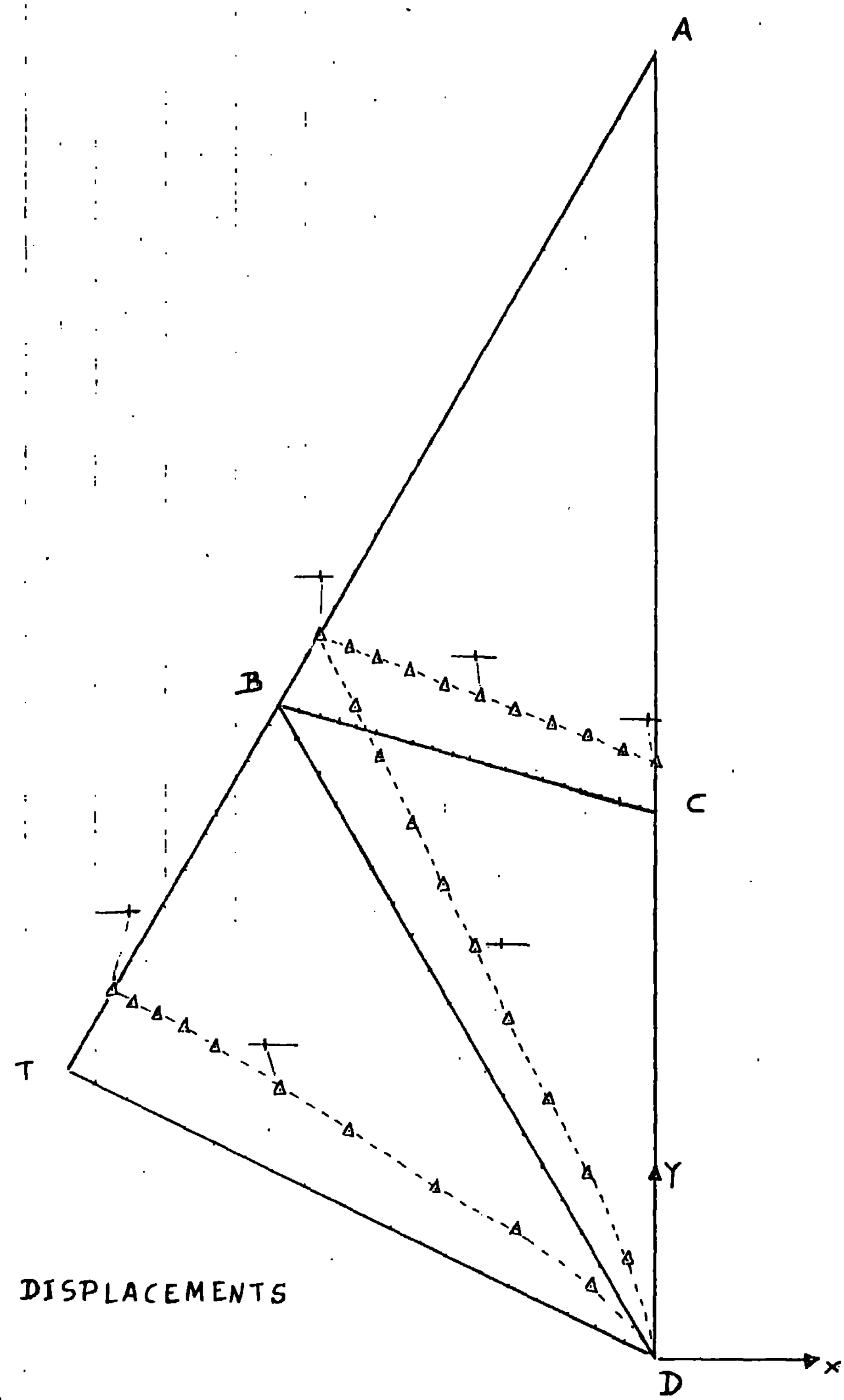
SCALES:

$1cm = 2\cdot10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

FIG. 13.121 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS



HORIZONTAL DISPLACEMENTS

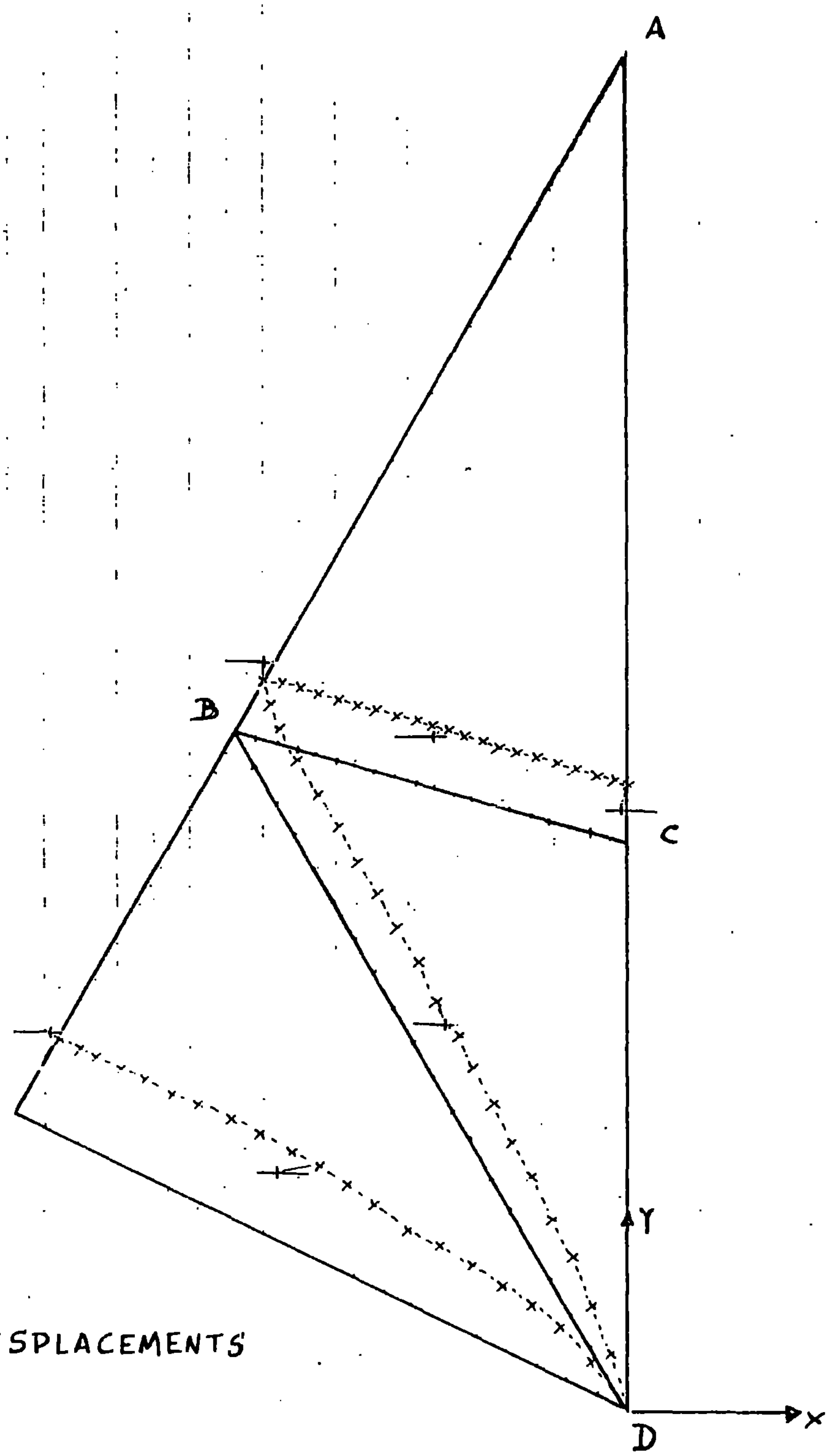
SCALES:

$1cm = 2\cdot10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

FIG. 13.122 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS



HORIZONTAL DISPLACEMENTS

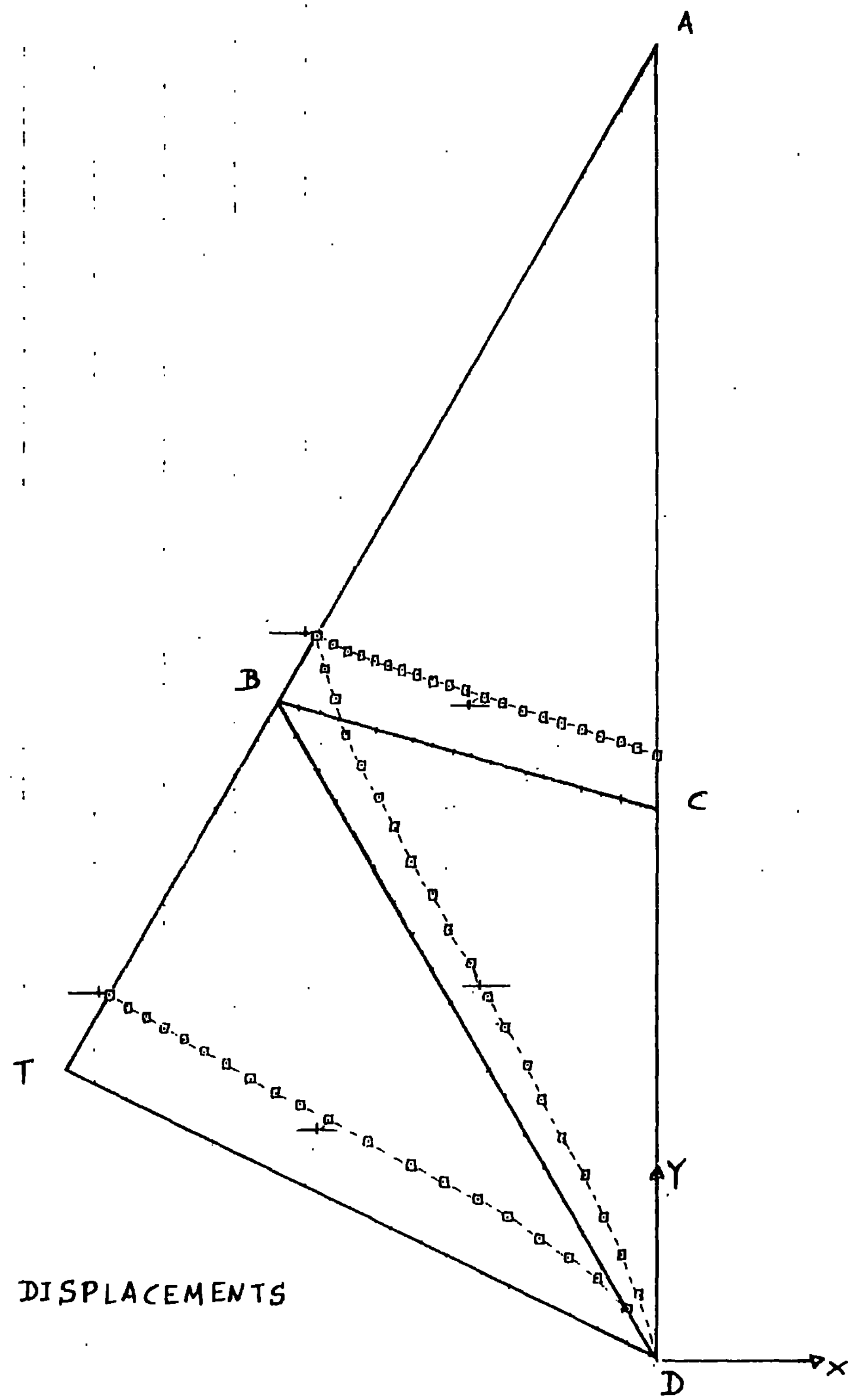
SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.123 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

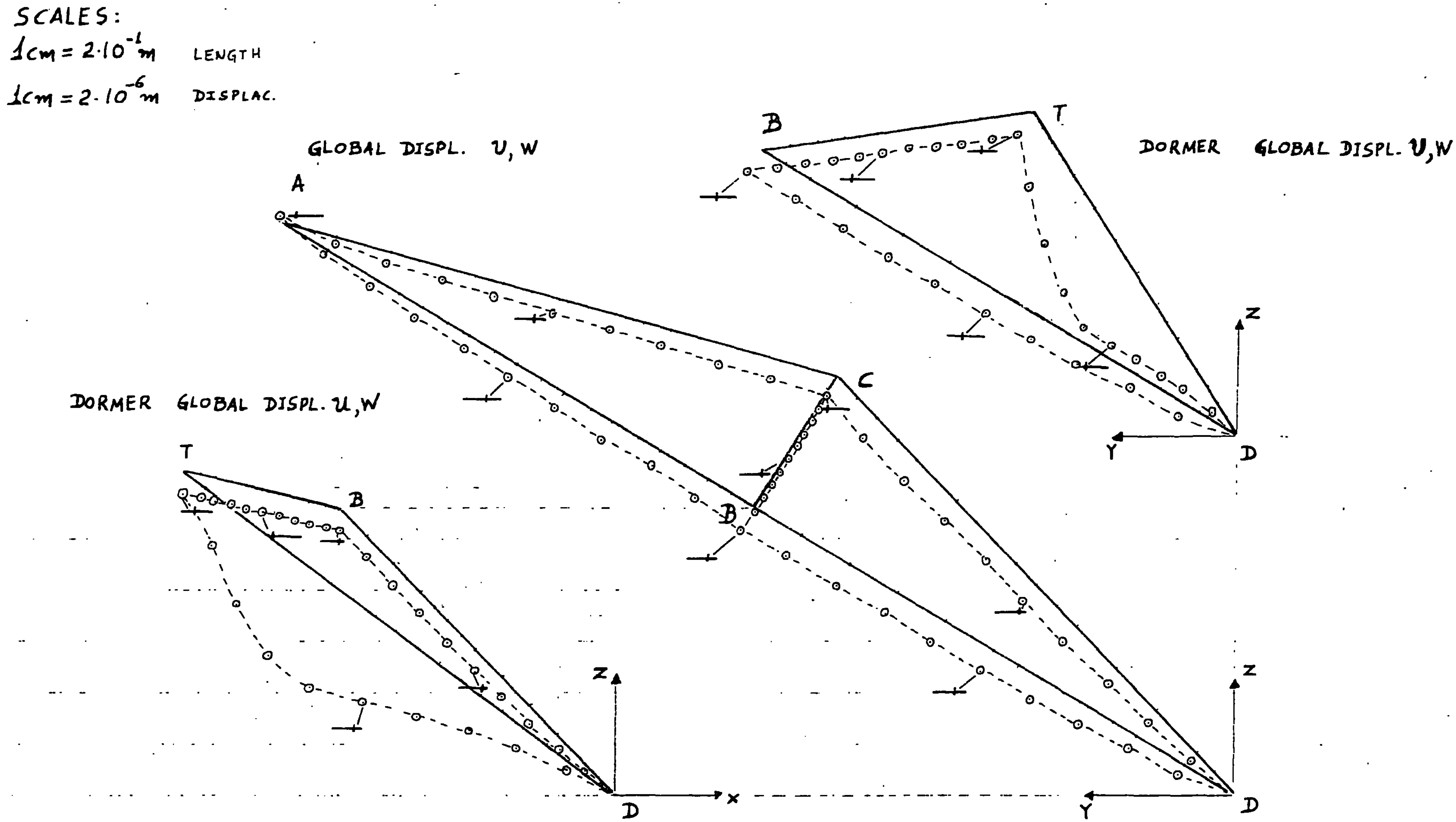


HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2\cdot10^{-1} m$ LENGTH

$1cm = 2\cdot10^{-6} m$ DISPLAC.



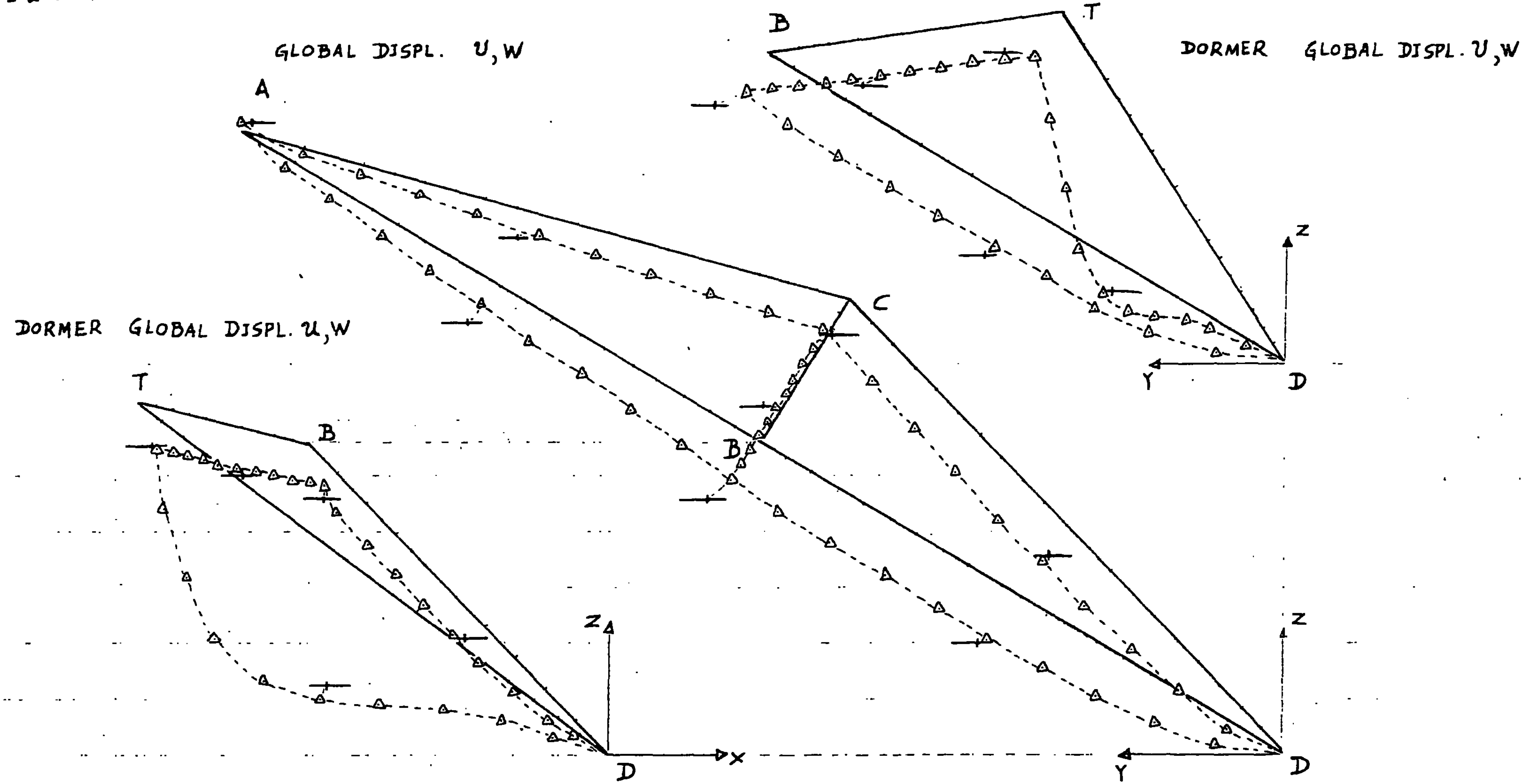
1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

FIG. 13.124 36 FACED DOME

SCALES:

$1 cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 2 \cdot 10^{-6} m$ DISPLAC.



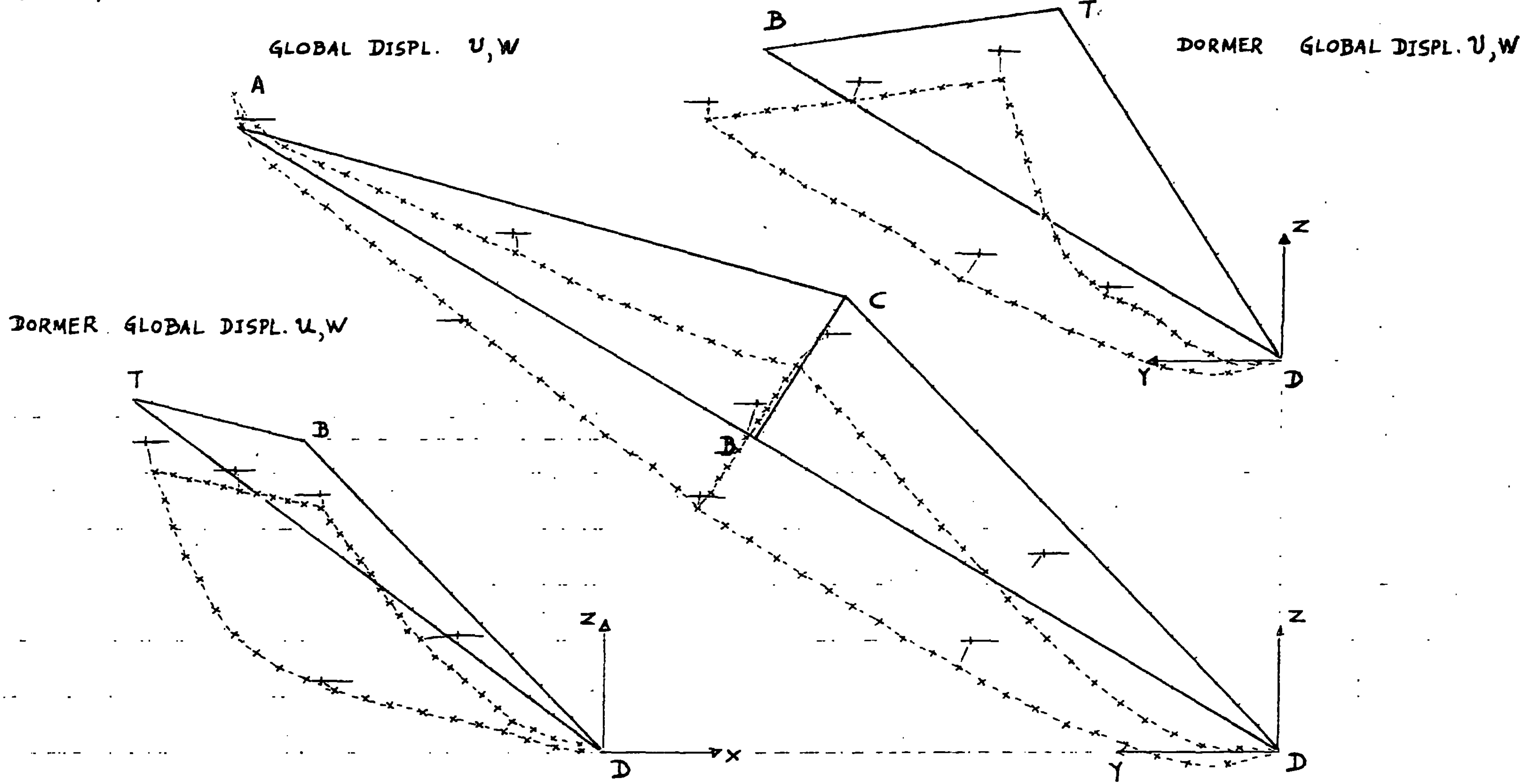
1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

FIG. 13.125 36 FACED DOME

SCALES:

1cm = 2·10^-1 m LENGTH

1cm = 2·10^-6 m DISPLAC.



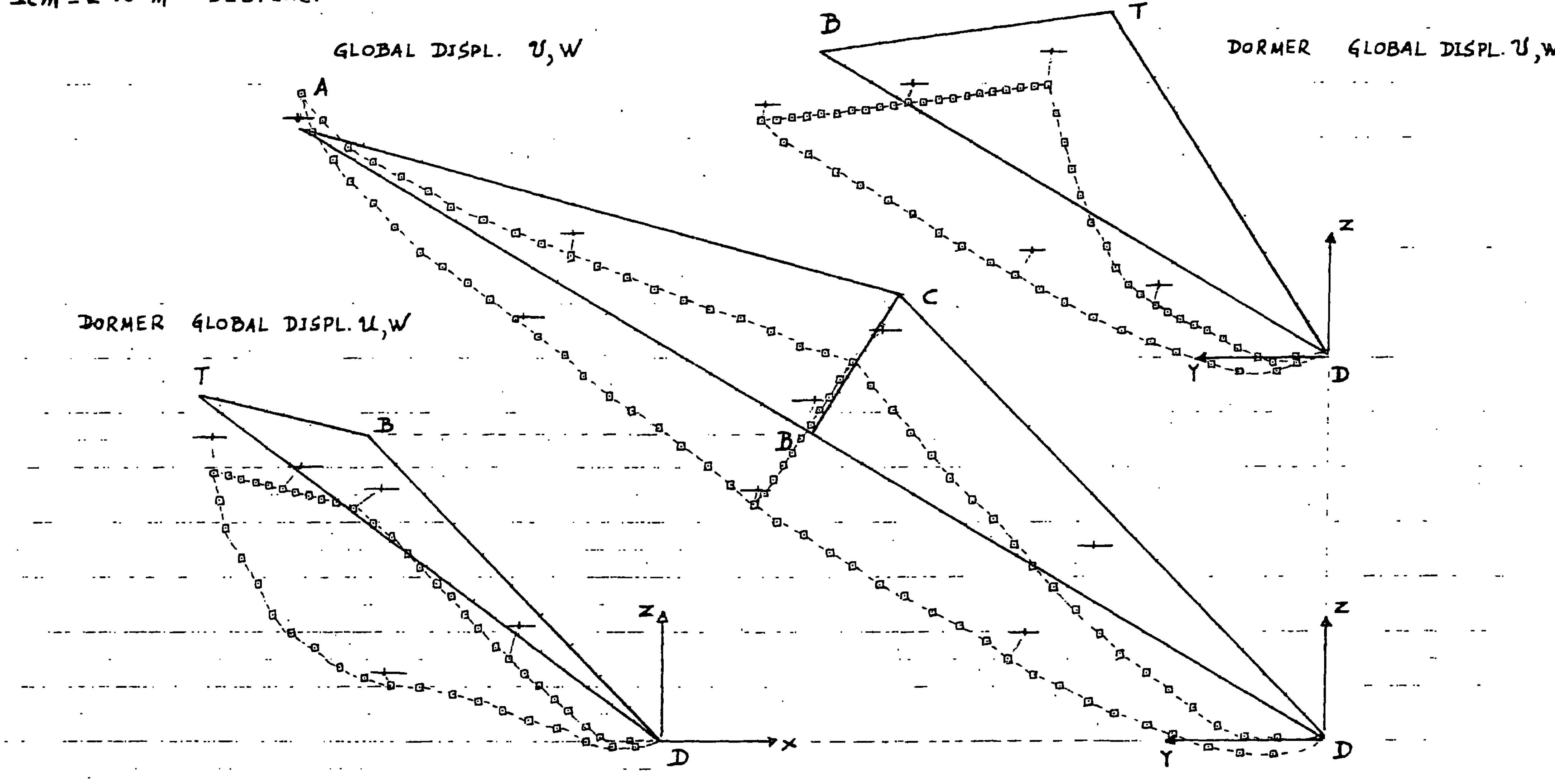
1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

FIG. 13.126 36 FACED DOME

SCALES:

$1 cm = 2\cdot10^{-1} m$ LENGTH

$1 cm = 2\cdot10^{-6} m$ DISPLAC.



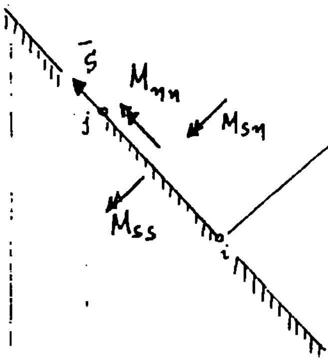
1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

FIG. 13.127 36 FACED DOME

FIG. 13.128

36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

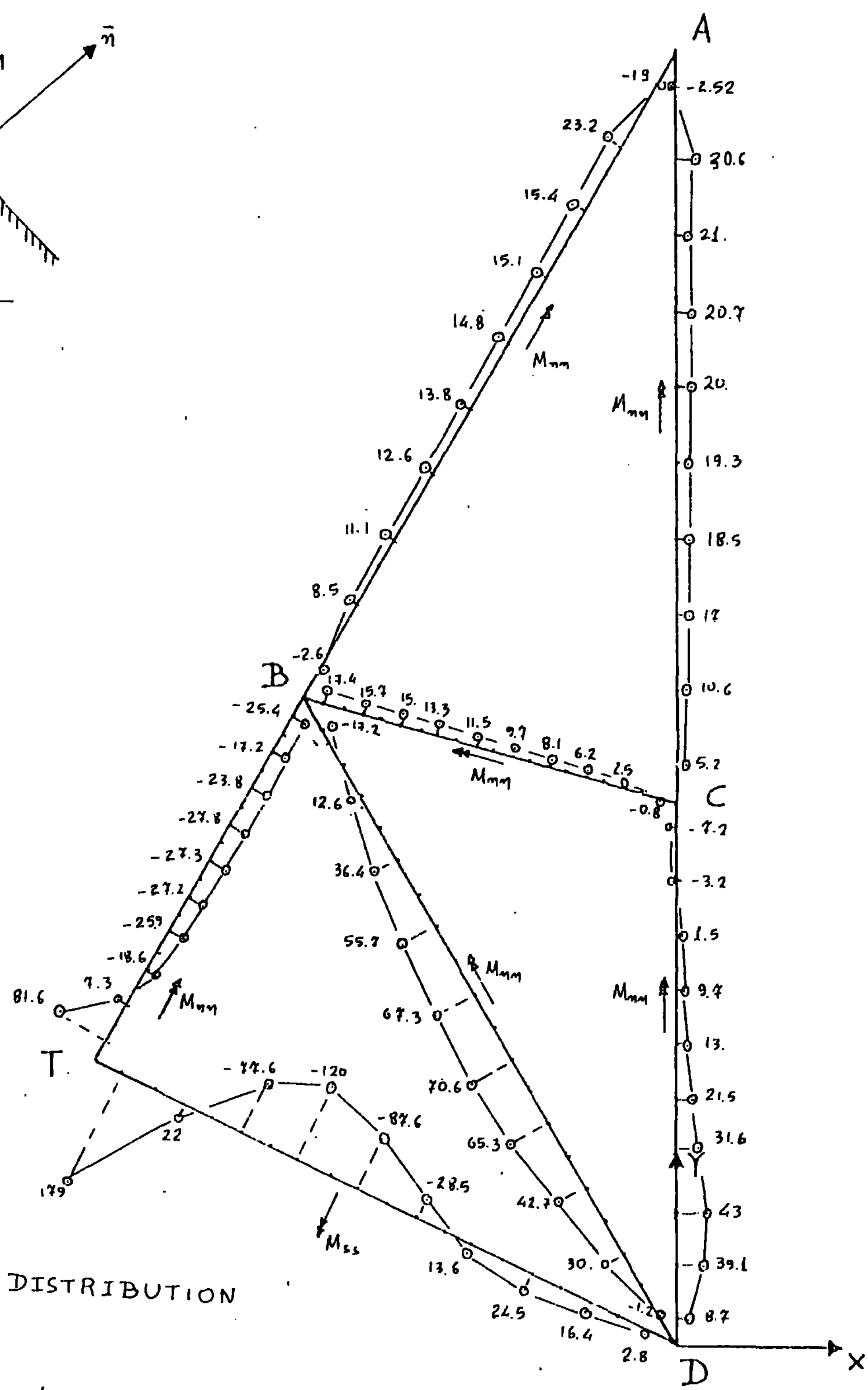


SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$



MOMENTS DISTRIBUTION

SCALES:

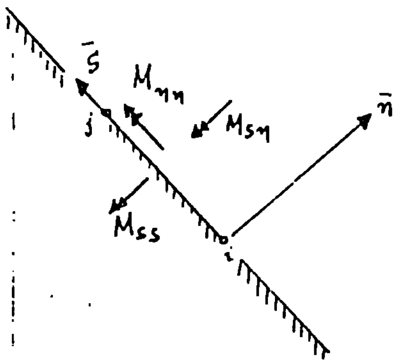
$1cm = 2 \cdot 10^{-1}\,m$ LENGTH

$1cm = 100\,Ntm/m$ MOMENTS

FIG. 13.129

36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

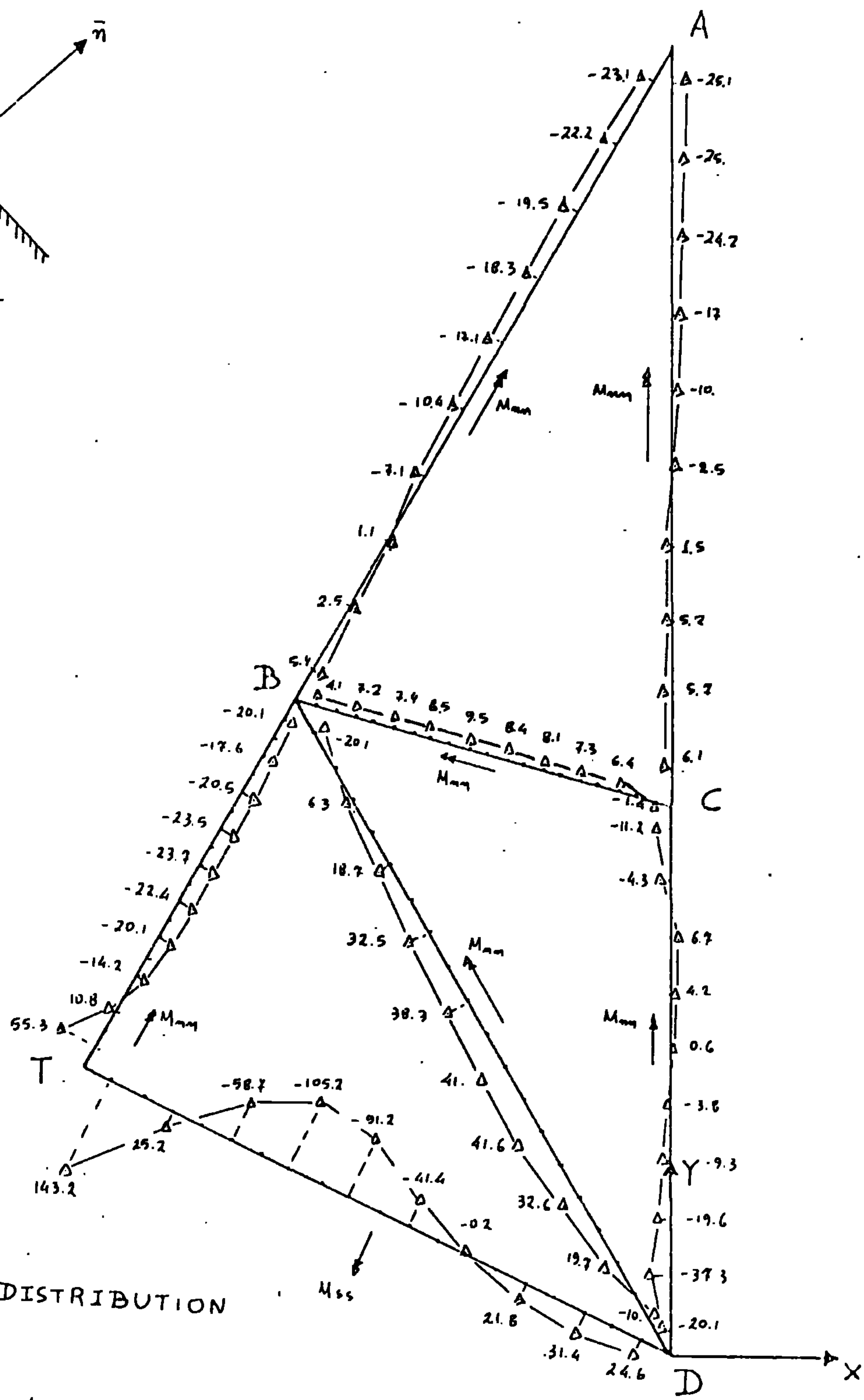


SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$



MOMENTS DISTRIBUTION

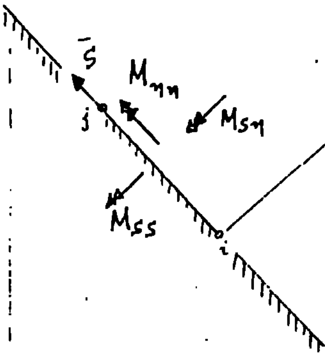
SCALES:

1cm = $2 \cdot 10^{-1}$ m LENGTH

1cm = 100 Ntm/m MOMENTS

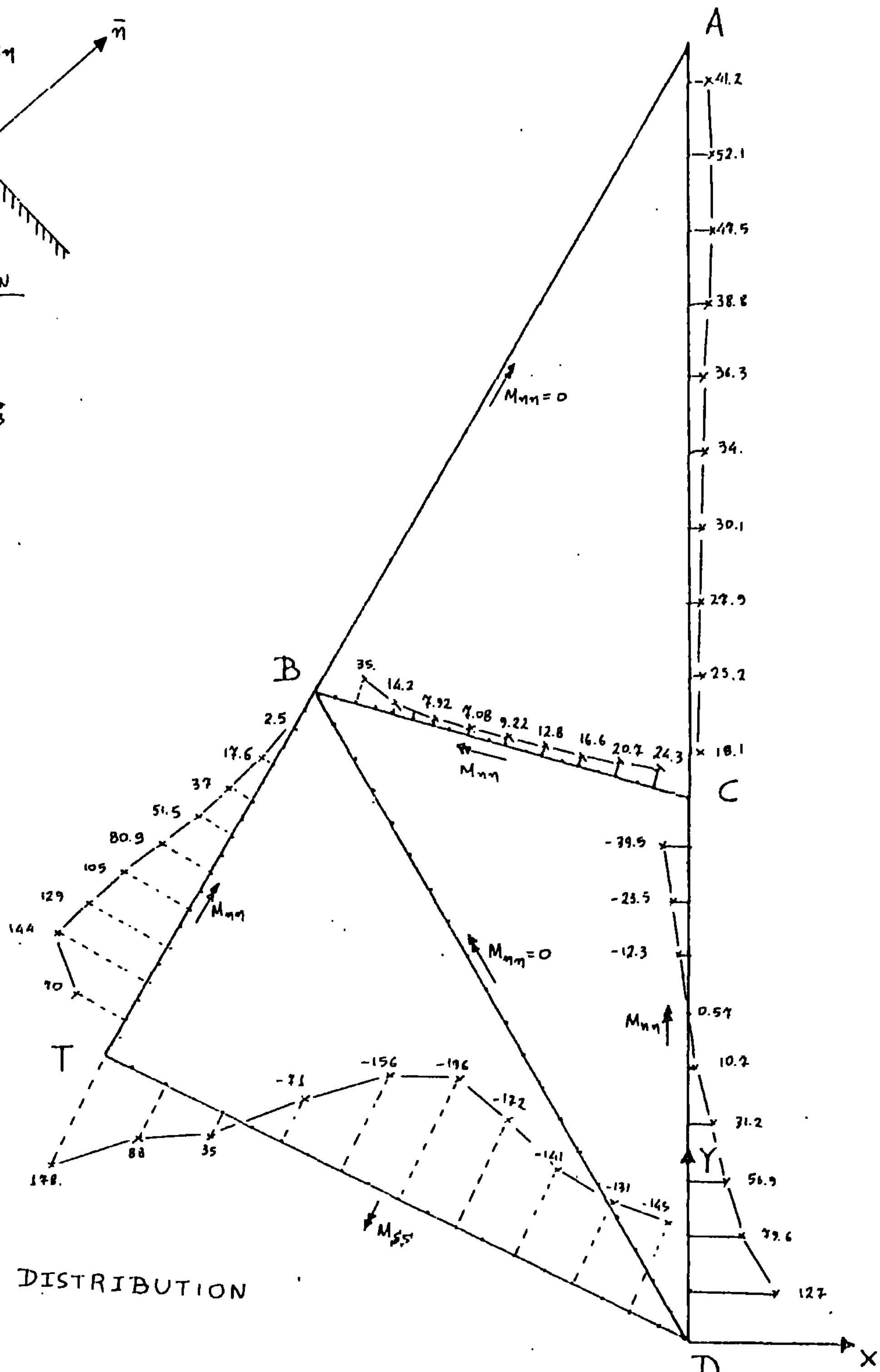
FIG. 13.130 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS



SIGN CONVENTION

LINE VECTORS:
$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$
$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$



MOMENTS DISTRIBUTION

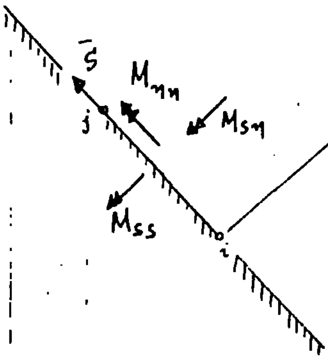
SCALES:

$1cm = 2\cdot 10^{-1} m$ LENGTH

$1cm = 100\ Ntm/m$ MOMENTS

FIG. 13.131 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

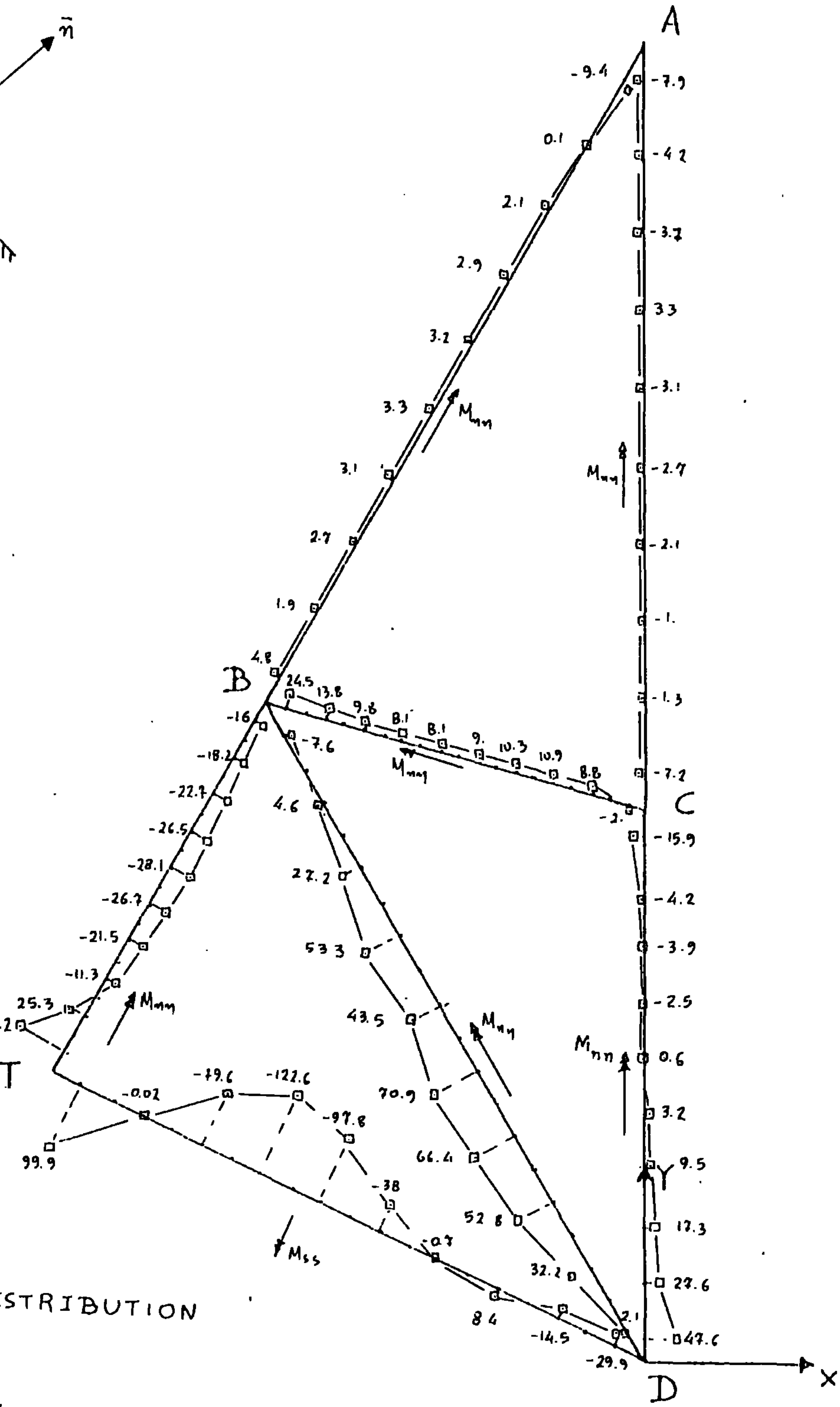


SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$



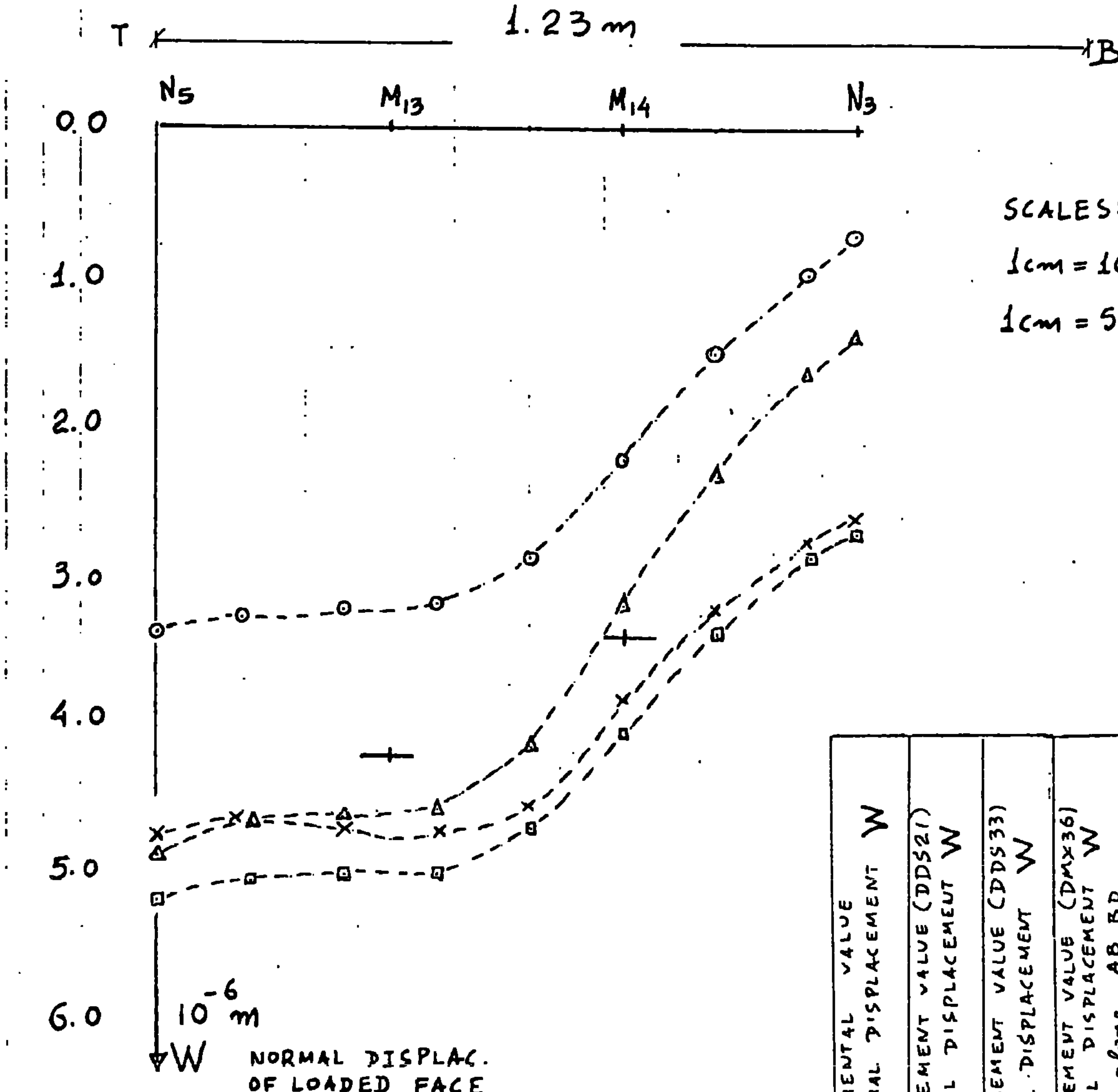
MOMENTS DISTRIBUTION

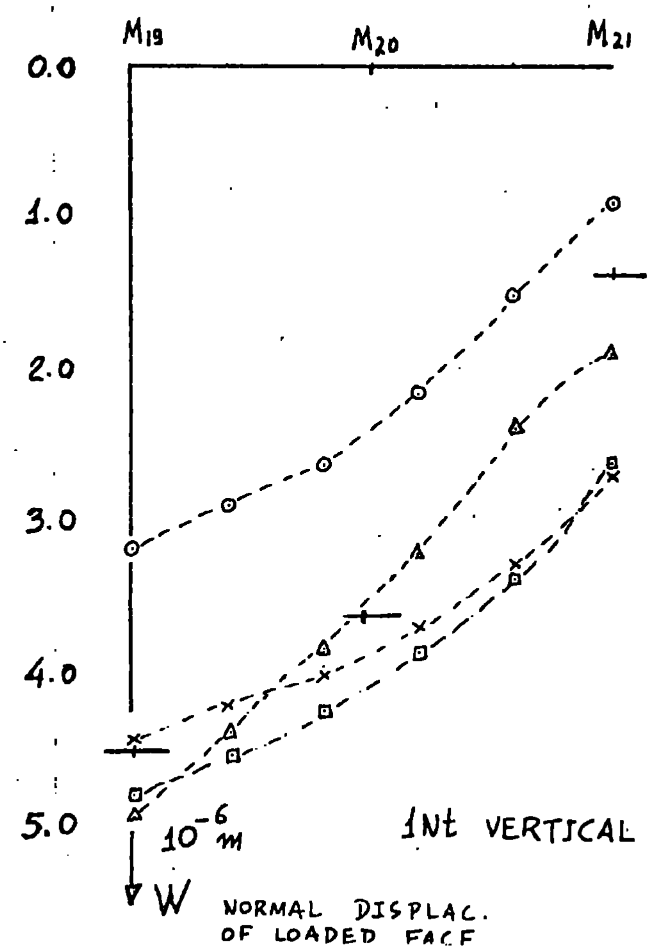
SCALES:

$1 cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 100 \; Ntm/m$ MOMENTS

FIG. 13.132 36 FACED DOME





	+ EXPERIMENTAL VALUE OF NORMAL DISPLACEMENT W	⊙ FINITE ELEMENT VALUE (DDS21) OF NORMAL DISPLACEMENT W	△ FINITE ELEMENT VALUE (DDS33) OF NORMAL DISPLACEMENT W	× FINITE ELEMENT VALUE (DMX36) OF NORMAL DISPLACEMENT W $M_{nn}=0$ along AB, BD	⊡ FINITE ELEMENT VALUE (DRO30) OF NORMAL DISPLACEMENT W
M_{13}	-4.19	-3.23	-4.57	-4.67	-4.97
M_{14}	-3.40	-2.20	-3.20	-3.82	-4.02
M_{19}	-4.45	-3.15	-4.90	-4.38	-4.76
M_{20}	-3.57	-2.35	-3.45	-3.82	-4.02
M_{21}	-1.35	-0.85	-1.85	-2.65	-2.60
	10^{-6} m	10^{-6} m	10^{-6} m	10^{-6} m	10^{-6} m

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

NOTATION

a_i, b_i, c_i ; a'_i, b'_i, c'_i $(i = 1,3)$	Parameters for natural coordinates system.	APPENDIX I-V
a_i $(i = 1 \div n)$	Polynomial coefficients	CHAPTER 3 APPENDIX III
a_n, a_s	Skew coordinates with respect to system $\bar{n}, \bar{s}$.	CHAPTER 8
A_n	Area of the n^{th} element	
a_{ij} $(i,j = x,y)$	Bending strains	
a_i, a'_i $(i = 1 \div 3)$	Parameters related to the angles measured by the theodolite	CHAPTER 9
$\{B\}$	Vector of polynomial coefficients	APPENDIX III
B_i $(i = 1 \div 18)$	Polynomial coefficients	APPENDIX III
$[B]$	Strains-displacements matrix	
B_{ij}	Elements of the [B] matrix	APPENDICES II-V
$[B^w]$	Bending strains-transverse displacements matrix	APPENDICES II,III
$[B^{s1}]$	Bending strains - shear displacements matrix	APPENDICES II,III
$[B^{s2}]$	Shear strains - shear displacements matrix	APPENDICES II,III

$[B_1^\theta]$	Bending strains - total rotations matrix	APPENDIX V
$[B_2^\theta]$	Shear strains - total rotations matrix	APPENDIX V
$[B^w]$	Shear strains - transverse displacements matrix	APPENDIX V
$[B^{uv}]$	In-plane strains - in-plane displacements matrix	APPENDICES II - V
B_2, G_2	Parameters for the transformation with respect to the skew coordinates system $\bar{n}$, $\bar{s}$.	CHAPTER 8
C	Core thickness	
C_{rs}^{ij} $(i,j,r,s = x,y)$	Cartesian elasticity tensor. When with a prefix f or c refers to the faces or the core respectively.	
[C]	Elasticity matrix relating the strains vector to the stress resultants vector	CHAPTER 4
$[C_b]$	Elasticity matrix relating the bending and shear stress resultants vector	CHAPTER 4 and APPENDIX 4
d_i $(i = 1 \div 3)$	Perpendiculars of a triangle	APPENDIX I
DT	Twice the area of a triangle	
[D]	Elasticity matrix relating the stress resultants vector to the strains vector	
D_{rs}^{ij} $(i,k,r,s = x,y)$	Elements of the elasticity matrix [D] with respect to the bending strains.	

D_o	Elastic constant	CHAPTER 4
$[D_b]$	Part of the elasticity matrix [D] with regard to the bending strains and shear strains	
$[D_{uv}]$	Part of the elasticity matrix [D] with regard to the in-plane strains	
E^{ij}_{rs} $(i,j,r,s=x,y)$	Elements of the elasticity matrix [D] with respect to the in plane strains	
e_{rs} $(r,s=x,y,z)$	Cartesian strain tensor	
f	Face thickness	
F( )	Function	
[F], $[F_{gen}]$, [GC]	Matrices for the formation of the shape functions matrix	CHAPTER 5 and APPENDIX III
h	Plate thickness	
h_1, v_1, h_1', v_1'	Parameters related to the angles measured by the theodolite	CHAPTER 9
I	Functional	
$\bar{I}$	Approximate expression of the functional	CHAPTER 3
$[K_n^o]$	The stiffness matrix of the n^{th} element with repect to a local coordinates system	

$[K_n]$	The stiffness matrix of the n^{th} element with respect to a global coordinate system	
$[K]$	The overall stiffness matrix	
$[K_n^{uv}]$	Part of the stiffness matrix referring to the in-plane displacements of the n^{th} element	CHAPTER 4 and APPENDIX IV
$[K_n^{mw}]$	Part of the stiffness matrix (for the mixed formulation) referring to the transverse displacement of the n^{th} element	CHAPTER 4 and APPENDIX IV
$[K_n^{mq}]$	Part of the stiffness matrix (for the mixed formulation) referring to the moments as generalised displacements of the n^{th} element	CHAPTER 4 and APPENDIX IV
K_i $(i = 1 \div 3)$	Parameters for the formation of various matrices	APPENDIX III
L_i $(i = 1 \div 3)$	Natural coordinates for a triangle	
ℓ_{in}	Length of the side in	CHAPTER 8
ℓ_{in}	Length of an insert at a joint	CHAPTER 10
ℓ_{ac}	Active length of a joint	CHAPTER 10
m_i $(i = 1 \div 5)$	Parameters for the formation of the matrix $[F_{gen}]^{-1}$	APPENDIX III
M_{ij} $(i,j = x,y$ or $n,s)$	Bending srress resultants (Moments)	

$\{M_o^e\}$	Vector of nodal degrees of freedom with regard to the moments models
$\{M_i^e\}$ $(i = 1 \div 3)$	Vector of nodal degrees of freedom with regard to the moments M_{xx}, M_{yy}, M_{xy} respectively
$\bar{M}_{ij}$ $(i,j = x,y$ or $n,s)$	Moments as prescribed quantities
N_{ij} $(i,j = x,y)$	In-plane stress resultants
$[N]$	Shape functions matrix relating the general displacements vector to the general vector of degrees of freedom for an element
N_i	Element of the shape functions matrix referring to the i^{th} node
$[N^w]$	Shape functions matrix with respect to the transverse displacements models
$[N^s]$	Shape functions matrix with respect to the shear models
$[N^\theta]$	Shape functions matrix with respect to the total rotations models
$[N^m]$	Shape functions matrix with respect to the moments models
$[N_m^o]$	Shape functions matrix (mixed model) APPENDIX IV
$[N_m^1]$	Shape functions matrix (mixed model) APPENDIX IV
$[N^{uv}]$	Shape functions matrix with respect to the in-plane models

$\bar{n},\bar{s}$	Coordinates of skewed coordinates system	
n_{in}	Number of inserts per metre of joint-line	CHAPTER 10
P^i	Load carried through an insert at a joint	CHAPTER 10
P^j	Load carried by the inserts per metre of joint-line	CHAPTER 10
P^s	Load carried by the hardboard faces per metre of joint-line	CHAPTER 10
P^c	Reduced by 10% value of load carried by an insert	CHAPTER 10
$\bar{P}$	Distributed load vector	
Q_i (i,j = x,y or n,s)	Shear stress resultants (Shear Forces)	
$\bar{Q}_i$ (i,j = x,y or n,s)	Shear forces prescribed quantities	
$\{\bar{R}_o\}$	Prescribed load vector, corresponding to the general displacements vector $\{\delta_o^e\}$	
$\{R_n^o\}$	Generalised load vector for the n^{th} element with respect to a local coordinates system corresponding to the vector of nodal degrees of freedom $\{\delta_o^e\}$	

$\{R_n\}$	Generalised load vector for the n^{th} element with respect to a global coordinates system corresponding to the vector of nodal d.o.f. $\{\delta_g^e\}$	
$\{R\}$	Overall load vector	
$\{R_n^w\}$	Load vector corresponding to the nodal values of transverse displacements as d.o.f. for the n^{th} element	CHAPTER 4 and APPENDIX IV
$\{R_n^m\}$	Load vector corresponding to the nodal values of moments as d.o.f. for the n^{th} element	CHAPTER 4 and APPENDIX IV
$\{R_n^{uv}\}$	Load vector corresponding to the nodal values of in-plane displacements as d.o.f. for the n^{th} element	CHAPTER 4 and APPENDIX IV
S_{jz}^{iz} $(i,j=x,y)$	Elements of the elasticity matrix [D] with respect to the shear strains	
$[S_n^o]$	Stress matrix with respect to a local coordinates system for the n^{th} element	
$[S_n]$	Stress matrix with respect to a global coordinates system for the n^{th} element	
$S_{o,n}$	Part of the boundary where M_{nn}, M_{ns}, Q_n are prescribed	

$S_{n,n}$	Part of the boundary where w, θ_n, θ_s are prescribed	
t_s	Steel plate thickness	CHAPTER 10
t_{ij} $(i,j = x,y,z)$	Cartesian stress tensor	
$[T]^{-1}$	Matrix for the formation of the shape functions matrix	CHAPTER 5 and APPENDIX III
[TR]	Transformation matrix from a local to a global coordinates system	CHAPTER 8
$[T_{\bar{e}g}]$	Transofmration matrix relating the x', y', z' and x,y,z sets of coordinates	CHAPTER 8
$[T_{\bar{e}e}]$	Transformation matrix relating the x',y',z' and $\bar{x},\bar{y},\bar{z}$ sets of coordinates	CHAPTER 8
u	In-plane displacement corresponding to x axis	
u_i	Nodal value of the in-plane displacement u at the i^{th} node	
$u_{,i}$ $(i = x,y$ or $n,s)$	First derivatives of the in-plane displacement u with respect to the i axis	
$u_{,ij}$ $(i,j = x,y$ or $n,s)$	Second derivatives of the in-plane displacement u with respect to the i,j axes	

v	In-plane displacement corresponding to the y axis	
v_i	Nodal value of the in-plane displacement v at the i^{th} node	
$v,_i$ ($i = x,y$ or n,s)	First derivatives of the in-plane displacement v with respect to the i^{th} axis	
$v,_{ij}$ ($i,j = x,y$ or n,s)	Second derivatives of the in-plane displacement with respect to the i,j axes	
$v_{i,\bar{j}}$ ($i,j = x,y,z$)	Elements of the transformation matrix $[T_{e\bar{e}}]$	
$w_{(x)}$	Function of x	
$\bar{w}_{(x)}$	Approximate expression of the above function	CHAPTER 3
w	Transverse displacement corresponding to the z axis (deflection)	
$\bar{w}$	Prescribed quantity of the transverse displacement	CHAPTER 4
w_i	Nodal value of the transverse displacement at the i^{th} node	
$w,_i$ ($i = x,y$ or n,s)	First derivatives of the transverse displacement with respect to the i^{th} axis	
$w,_{ir}$ ($i = x,y$ $r = 1 \div 3$)	The above quantities at the r^{th} node	

$w,_{ij}$ $(i,j = x,y \text{ or } n,s)$	Second derivatives of the transverse displacement with respect to i,j axes	
$w,_{ijr}$ $(i,j = x,y \text{ or } n,s,$ $r = 1 \div 3)$	The above quantities at the r^{th} node	
x,y,z	Cartesian coordinates	
x,y,z	Global coordinates system	CHAPTER 8
x',y',z'	Local coordinates system	CHAPTER 8
$\bar{x},\bar{y},\bar{z}$	Plates-interconnection coordinates system	CHAPTER 8
γ_i $(i = x,y)$	Shear strains. With a prefix c refers to the core	
δ	Variational operator	CHAPTER 3
δ_1	Trigonometric function	CHAPTER 10
$\{\delta o\}$	General displacements vector	
$\{\delta^e\}$	Overall nodal generalised displacements (degrees of freedom) vector	
$\{\delta_o^e\}$	Nodal generalised displacements (degrees of freedom) vector for an element with respect to a local coordinates system	
$\{\delta_g^e\}$	Nodal generalised displacements (d.o.f.) vector for an element with respect to a global system	CHAPTER 8

$\{\delta^{w}\}$	Vector of nodal degrees of freedom with regard to the transverse displacement models
$\{\delta^{s}\}$	Vector of nodal d.o.f. with regard to the shear models
$\{\delta_i^{s}\}$ $(i=1,2)$	The above quantities referring to the shear angles ϕ_x, ϕ_y respectively
$\{\delta^{\theta}\}$	Vector of nodal d.o.f. with regard to the total rotation models
$\{\delta_i^{\theta}\}$ $(i=1,2)$	The above quantities referring to the total rotations θ_x, θ_y respectively
$\{\delta^{uv}\}$	Vector of nodal d.o.f. with regard to the in-plane displacements models
$\{\delta_i^{uv}\}$ $(i=1,2)$	The above quantities referring to the in-plane displacements u,v, respectively
$\varepsilon_{\Delta\phi}$	Strain referring to the change of angle for a joint — CHAPTER 10
$\{\varepsilon\}$	Strains vector
$\{\varepsilon_{uv}\}$	In-plane strains vector
$\{\varepsilon_{b}\}$	Bending and shear strains vector
θ_i $(i=x,y \text{ or } n,s)$	Total rotations
$\theta_{i,r}$ $(i=x,y \text{ or } r=1\div 6)$	Total rotations at the r^{th} node
$\bar{\theta}_i$ $(i=x,y \text{ or } n,s)$	Prescribed quantities of the total rotations

$\lambda_{i,j}$ $(i,j = x,y,z)$	Elements of the transformation matric $[T_{eg}]$	CHAPTER 8
$\mu_{\bar{i},j}$ $(i,j = x,y,z)$	Elements of the transformation matrix $[T_{\bar{e}g}]$	CHAPTER 8
$\{\sigma\}$	Stress resultants vector	
$\{\sigma_b\}$	Vector of bending and shear stress resultants	
$\{\sigma_{uv}\}$	Vector of in-plane stress resultants	
σ_s	Stress of the steel plate	CHAPTER 10
σ_f	Stress of the hardboard face	CHAPTER 10
τ_i $(i = 1 \div 3)$	Distance of α point P within a triangle from the sides of the triangle	APPENDIX I
$0, \phi$.	Zero	
Φ	Function	
ϕ	Angle	
ϕ_i $(i = x,y$ or $n,s)$	Shear angle	
$\phi_{i,r}$ $(i = x,y$ or n,s $r = 1 \div 6)$	Shear angle at the r^{th} node	
ω	Change of angle at a joint	

Σ	Indicates summation
∇	Hamilton's operator
{ }	Vector
[ ]	Matrix of two dimensions
$[\ ]^{-1}$	Inverse of a matrix
$[\ ]^T$, $\{\ \}^T$	Transpose of a matrix or a vector
$[n_1, n_2, n_3 \ldots]$	Where n_i are numbers indicating the references
Nt	Newton weight units

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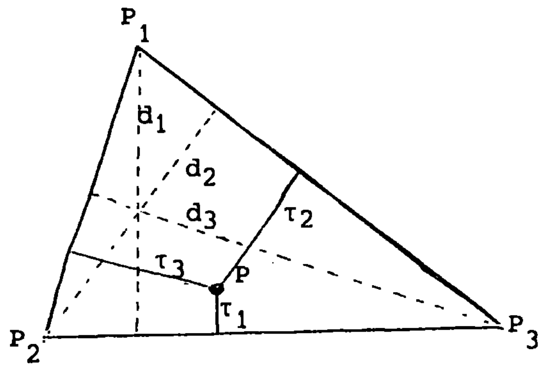
APPENDIX I

GEOMETRICAL RELATIONSHIPS FOR A TRIANGLE

The area coordinates L_1, L_2, L_3 can be defined as follows:- [39,52,59,63 105,115]

$$L_1 = \frac{\tau_1}{d_1}, \qquad L_2 = \frac{\tau_2}{d_2}, \qquad L_3 = \frac{\tau_3}{d_3} \tag{AI.1}$$



If (x_1,y_1), (x_2,y_2), (x_3,y_3) are the cartesian sets of coordinates for the vertices P_1, P_2, P_3 respectively, and (x,y), (L_1, L_2, L_3) the cartesian and area coordinates respectively for the point P the following relationships are valid:

$$\begin{aligned} x_1L_1 + x_2L_2 + x_3L_3 &= x \\ y_1L_1 + y_2L_2 + y_3L_3 &= y \\ L_1 + L_2 + L_3 &= 1 \end{aligned} \tag{AI.2}$$

The above give the cartesian set (x,y) for any set of area coordinates L_1, L_2, L_3.

In reverse we can evaluate the area coordinates L_1, L_2, L_3 for any set of cartesian coordinates (x,y) through the relationships

$$L_i = \frac{1}{DT} (a_i + b_i x + c_i y_i) \quad i = 1 \div 3 \tag{AI.3}$$

$$\left.\begin{aligned} a_1 &= x_2y_3 - x_3y_2 \\ b_1 &= y_2 - y_3 \\ c_1 &= x_3 - x_2 \end{aligned}\right\} \tag{AI.4}$$

a_2, b_2, c_2

a_3, b_3, c_3

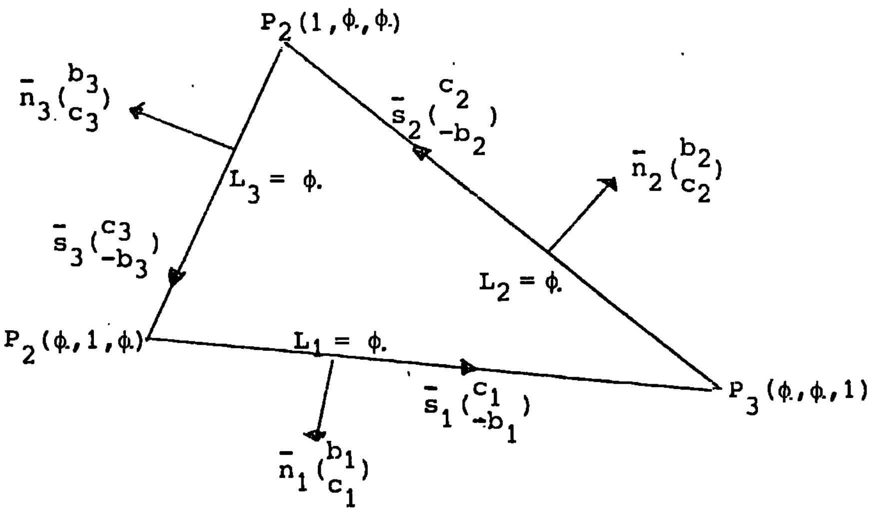
from a_1, b_1, c_1 with circle symmetrical substitution of the subscripts 1, 2, 3

DT = twice the area of the triangle $= b_1c_2 - b_2c_1 = b_2c_3 - b_3c_2 = b_3c_1 - b_1c_3$

(AI.5)

From (AI.3) $\Longrightarrow$

$$\frac{\partial L_i}{\partial_x} = \frac{b_i}{DT}, \qquad \frac{\partial L_i}{\partial_y} = \frac{c_i}{DT} \qquad i = 1 \div 3 \tag{AI.6}$$



We have a function of the form $\Phi(L_1, L_2, L_3)$ and we want to express the derivatives of Φ at the direction $\bar{n}_1, \bar{n}_2, \bar{n}_3$ which are vectors normal to the sides $\overline{23}$ $(L_1 = \phi)$, $\overline{31}$ $(L_2 = \phi)$, $\overline{12}$ $(L_3 = \phi)$ respectively.

For easier formulation we have

$$b'_i = \frac{b_i}{DT}, \qquad c'_i = \frac{c_i}{DT} \qquad i = 1 \div 3 \tag{AI.7}$$

$$\frac{\partial \Phi}{\partial_{n_i}} = \bar{n}_i \nabla\Phi = \bar{n}_i \frac{\partial \Phi}{\partial L_1} \nabla L_1 + \frac{\partial \Phi}{\partial L_2} \nabla L_2 + \frac{\partial \Phi}{\partial L_3} \nabla L_3) \tag{AI.8}$$

where ∇ is Hamilton's operator. For two dimensions has the form

$$\nabla F = \frac{\partial F}{\partial x}\bar{i} + \frac{\partial F}{\partial y}\bar{j}$$

(i,j the unit vectors at x,y)

so

$$\nabla L_1 = \begin{bmatrix} b'_1 \\ c'_1 \end{bmatrix} , \quad \nabla L_2 = \begin{bmatrix} b'_2 \\ c'_2 \end{bmatrix} , \quad \nabla L_3 = \begin{bmatrix} b'_3 \\ c'_3 \end{bmatrix} \qquad \text{(AI.9)}$$

Substituting the (A.9) to (A.8) and naming as

$$\phi, i = \frac{\partial \phi}{\partial L_i} \qquad \text{(AI.10)}$$

we obtain -

$$\frac{\partial \phi}{\partial n_i} = \bar{n}_i \left\{ \phi,_1 \begin{bmatrix} b'_1 \\ c'_1 \end{bmatrix} + \phi,_2 \begin{bmatrix} b'_2 \\ c'_2 \end{bmatrix} + \phi,_3 \begin{bmatrix} b'_3 \\ c'_3 \end{bmatrix} \right\} \qquad \text{(AI.11)}$$

so taking the right values for $\bar{n}_1$, $\bar{n}_2$, $\bar{n}_3$ we can write

$$\frac{\partial \phi}{\partial \bar{n}_1} = \left[\phi,_1 (b_1^2 + c_1^2) + \phi,_2 (b_1 b_2 + c_1 c_2) + \phi,_3 (b_1 b_3 + c_1 c_3) \right] / DT \times (P_2 P_3)$$

$$\frac{\partial \phi}{\partial \bar{n}_2} = \left[\phi,_1 (b_1 b_2 + c_1 c_2) + \phi,_2 (b_2^2 + c_2^2) + \phi,_3 (b_2 b_3 + c_2 c_3) \right] / DT \times (P_1 P_3) \qquad \text{(AI.12)}$$

$$\frac{\partial \phi}{\partial \bar{n}_3} = \left[\phi,_1 (b_1 b_3 + c_1 c_3) + \phi,_2 (b_2 b_3 + c_2 c_3) + \phi,_3 (b_3^2 + c_3^2) \right] / DT \times (P_1 P_2)$$

APPENDIX II

FORMATION OF MATRICES (Reference to DDS15, DDS21)

Formation of [B]

$$\{\varepsilon\} = \begin{Bmatrix} a_{xx} \\ a_{yy} \\ a_{xy} \\ \gamma_x \\ \gamma_y \\ \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = [B] \quad \{\delta_o^e\} \qquad \text{(AII.1)}$$

$$\left.\begin{array}{lllll} a_{xx} = w,_{xx} - \phi_{x,x} & & & & \\ a_{yy} = w,_{yy} - \phi_{y,y} & & & & \\ a_{xy} = 2W,_{xy} - \phi_{x,y} - \phi_{y,x} & & & & \\ \gamma_x = \phi_x, & \gamma_y = \phi_y & & & \\ \varepsilon_{xx} = u,_x & \varepsilon_{yy} = v,_y & & & \\ \varepsilon_{xy} = u,_y + u,_x & & & & \end{array}\right\} \qquad \text{(AII.2)}$$

$$\{\delta_o^e\} = \begin{Bmatrix} \delta_o^w \\ \delta_o^s \\ \delta_o^{uv} \end{Bmatrix} \qquad \text{(AII.3)}$$

$\{\delta_o^w\}$ deflections and derivatives of the deflections as degrees of freedom

$\{\delta_o^s\}$ shear angles as degrees of freedom

$\{\delta_o^{uv}\}$ membrane moves as degrees of freedom.

$$\{\delta_o^w\}^T = \{w_1, w_{,x1}, w_{,y1}, w_2, w_{,x2}, w_{,y2}, w_3, w_{,x3}, w_{,y3}\} \quad (AII.4)$$

$$\{\delta_o^s\}^T = \{\phi_{x1}, \phi_{y1}, \phi_{x2}, \phi_{y2}, \phi_{x3}, \phi_{y3}\} \quad (AII.5)$$

$$\{\delta_o^{uv}\}^T = \{u_1, v_1, u_2, v_2, u_3, v_3\} \quad (AII.6)$$

the subscript $,x, y$ indicates derivatives of x, y respectively and the number (1 ÷ 6) indicates the relevant node.

$$\{\varepsilon\} = \begin{bmatrix} [B^w] & [B^{s1}] & [\phi] \\ [\phi] & [B^{s2}] & [\phi] \\ [\phi] & [\phi] & [B^{uv}] \end{bmatrix} \begin{Bmatrix} \{\delta_o^w\} \\ \{\delta_o^s\} \\ \{\delta_o^{uv}\} \end{Bmatrix} \quad (AII.7)$$

<u>Formation of $[B^w]$ matrix</u>

$[B^w]$ is a matrix of 3 rows and 9 columns. We will indicate as B^w_{ij} on of its elements at the i row and j column.

$$B^w_{1,1} = \frac{1}{(DT)^2}\left[-2b_1(L_1b_1+L_2b_2+L_3b_3)+2(b_1-b_2)(L_2b_1+L_1b_2)+2(b_1-b_3)(L_1b_3+L_3b_1)\right]$$

$B^w_{1,4}$, $B^w_{1,7}$ from the above will circle symmetrical substitution of the subscript 1,2,3.

$$B^w_{1,2} = \frac{1}{(DT)^2}\left[2b_1^2(c_3L_2-c_2L_3)+4L_1b_1(b_2c_3-b_3c_2)+(c_3-c_2)(b_1b_2L_3+p_1p_3L_2+b_2b_3L_1)\right]$$

$B^w_{1,5}$, $B^w_{1,8}$ from the above with circle-symmetrical substitution of the subscripts 1,2,3.

$$B^{w}_{1,3}=\frac{1}{(DT)^2}\left[2b_1^2(b_2L_3-b_3L_2)+(b_2-b_3)(b_1b_2L_3+b_1b_3L_2+b_2b_3L_1)\right]$$

$B^{w}_{1,6}$, $B^{w}_{1,9}$ from the above will circle symmetrical substitution of the subscripts 1,2,3.

$$B^{w}_{2,1}=\frac{1}{(DT)^2}\left[-2c_1(L_1c_1+L_2c_2+L_3c_3)+2(c_1-c_2)(L_2c_1+L_1c_2)+2(c_1-c_3)(L_1c_3+L_3c_1)\right]$$

$B^{w}_{2,4}$, $B^{w}_{2,7}$ from the above as previously.

$$B^{w}_{2,2}=\frac{1}{(DT)^2}\left[2c_1^2(c_3L_2-c_2L_3)+(c_3-c_2)(c_1c_2L_3+c_1c_3L_2+c_2c_3L_1)\right]$$

$B^{w}_{2,5}$, $B^{w}_{2,8}$ from the above as previously.

$$B^{w}_{2,3}=\frac{1}{(DT)^2}\left[2c_1^2(b_2L_3-b_3L_2)+4L_1c_1(c_3b_2-c_2b_3)+(b_2-b_3)(c_1c_2L_3+c_1c_3L_2+c_2c_3L_1)\right]$$

$B^{w}_{2,6}$, $B^{w}_{2,9}$ from the above as previously.

$$B^{w}_{3,1}=\frac{1}{(DT)^2}\left[-2b_1(c_1L_1+c_2L_2+c_3L_3)+2(b_1-b_2)(L_2c_1+L_1c_2)+2(b_1-b_3)(L_1c_3+L_3c_1)\right]$$

$B^{w}_{3,4}$, $B^{w}_{3,7}$ from the above as previously.

$$B^{w}_{3,2}=\frac{1}{(DT)^2}\left\{2b_1c_1(c_3L_2-c_2L_3)+2c_1L_1(b_2c_3-b_3c_2)+\frac{c_3-c_2}{2}\left[L_1(c_3b_2+c_2b_3)+L_2(c_1b_3+c_3b_1)+L_3(c_1b_2+c_2b_1)\right]\right\}$$

$B^{w}_{3,5}$, $B^{w}_{3,8}$ from the above as previously.

$$B^{w}_{3,3}=\frac{1}{(DT)^2}\left\{2b_1c_1(b_2L_3-b_3L_2)+2b_1L_1(c_3b_2-b_3c_2)+\frac{b_2-b_3}{2}\left[L_1(c_3b_2+c_2b_3)+L_2(c_1b_3+c_3b_1)+L_3(c_1b_2+c_2b_1)\right]\right\}$$

$B^{w}_{3,6}$, $B^{w}_{3,9}$ from the above as previously.

Formation of $[B^{s1}]$ Matrix

$[B^{s1}]$ is a matrix of 3 rows and 6 columns.

$[B^{s1}] = \frac{1}{DT}$

$-b_1$	$\phi.$	$-b_2$	$\phi.$	$-b_3$	$\phi.$
$\phi.$	$-c_1$	$\phi.$	$-c_2$	$\phi.$	$-c_3$
$-c_1$	$-b_1$	$-c_2$	$-b_2$	$-c_3$	$-b_3$

Formation of $[B^{s2}]$ Matrix

$[B^{s2}]$ is a matrix of 2 rows and 6 columns

$[B^{s2}] =$

L_1	$\phi.$	L_2	$\phi.$	L_3	$\phi.$
$\phi.$	L_1	$\phi.$	L_2	$\phi.$	L_3

Formation of $[B^{uv}]$ Matrix

$[B^{uv}]$ is a matrix of 3 rows and 6 columns

$[B^{uv}] = \frac{1}{DT}$

b_1	$\phi.$	b_2	$\phi.$	b_3	$\phi.$
$\phi.$	c_1	$\phi.$	c_2	$\phi.$	c_3
c_1	b_1	c_2	b_2	c_3	b_3

The stiffness, stress and load matrices $[K_n]$, $[S]$, $\{R_n\}$ are obtained through equations (4.8), (4.5) employing the numerical integration formulae of Appendix VI.

APPENDIX III

FORMATION OF MATRICES (Reference to PDS24)

The deflection is a complete 5th order polynomial with 21 terms

$$w \equiv \phi(L_1,L_2) = a_1 + a_2L_1 + a_3L_2 + a_4L_1^2 + a_6L_2^2 + a_7L_1^3 + a_8L_1^2L_2 + a_9L_1L_2^2 +$$

$$+ a_{10}L_2^3 + a_{11}L_1^4 + a_{12}L_1^3L_2 + a_{13}L_1^2L_2^2 + a_{14}L_1L_2^3 + a_{15}L_2^4 + a_{16}L_1^3 + a_{17}L_1^4L_2 + a_{18}L_1^3L_2^2 \pm$$

$$+ a_{19}L_1^2L_2^3 + a_{20}L_1L_2^4 + a_{21}L_2^5 ,$$

with $\phi,_3 = \phi.$ the relationships (AI.1) Appendix I .

$$\frac{\partial\phi}{\partial\bar{n}_1} = \left[(b_1^2+c_1^2)\phi,_1 + (b_1b_2+c_1c_2)\phi,_2\right]/(P_2P_3)DT$$

$$\frac{\partial\phi}{\partial\bar{n}_2} = \left[(b_1b_2+c_1c_2)\phi,_1 + (b_2^2+c_2^2)\phi,_2\right]/(P_1P_3)DT$$

$$\frac{\partial\phi}{\partial\bar{n}_3} = \left[(b_1b_3+c_1c_3)\phi,_1 + (b_2b_3+c_2c_3)\phi,_2\right]/(P_1P_2)DT$$

$$k_1 = b_1b_2+c_1c_2$$
$$k_2 = b_2b_3+c_2c_3$$
$$k_3k_3 = b_1b_3+c_1c_3$$
$$k_1+k_2 = -(b_2^2+c_2^2) \qquad \text{(AIII.1)}$$
$$k_2+k_3 = -(b_3^2+c_3^2)$$
$$k_1+k_3 = -(b_1^2+c_1^2)$$

4th Order

$$\phi,_1 = 5a_{16}L_1^4 + 4a_{17}L_1^3L_2 + 3a_{18}L_1^2L_2^2 + 2a_{19}L_1L_2^3 + a_{20}L_2^4$$

4th Order

$$\phi,_2 = a_{17}L_1^4 + 2a_{18}L_1^3L_2 + 3a_{19}L_1^2L_2^2 + 4a_{20}L_1L_2^3 + a_{21}L_2^4$$

side $\overline{23}$ $L_1 = \phi$. $\frac{\partial w}{\partial \bar{n}_1}$ cubic $\Longrightarrow$ $5k_1a_{21}-(k_1+k_3)a_{20} = \phi$.

side $\overline{31}$ $L_2 = \phi$. $\frac{\partial w}{\partial \bar{n}_2}$ cubic $\Longrightarrow$ $5k_1a_{16}-(k_1+k_2)a_{17} = \phi$.

(AIII.2)

side $\overline{12}$ $L_1+L_2 = 1$. $\frac{\partial w}{\partial \bar{n}_3}$ cubic $\Longrightarrow$

$$5k_3a_{16}+(k_2-4k_3)a_{17}+(3k_3-2k_2)a_{18}+(3k_2-2k_3)a_{19}+(k_3-4k_2)a_{20}+5k_2a_{21} = \phi.$$

from the above three equations we can express the coefficients a_{20}, a_{17}, a_{19} as functions of the rest a_{16}, a_{18}, a_{21} if we substitute these expressions in the initial polynomial form we will have a new polynomial with 18 terms which will fulfil the conditions $\frac{\partial w}{\partial n_i} = \phi$. $i = 1 \div 3$ so the normal slope will be continuous across interelement boundaries. if $\{w_i, w_{xi}, w_{yi}, w_{xxi}, w_{xyi}, w_{yyi}\}$ $i = 1,3$ at three corner modes are chosen as degrees of freedom the polynomial is of the form

$$w = F(L_1,L_2) = B_1+B_2L_1+B_3L_2+B_4L_1^2+B_5L_1L_2+B_6L_2^2+B_7L_1^3+B_8L_1^2L_2+B_9L_1L_2^2+$$
$$+B_{10}L_2^3+B_{11}L_1^4+B_{12}L_1^3L_2+B_{13}L_1^2L_2^2+B_{14}L_1L_2^3+B_{15}L_2^4+B_{16}[L_1^5+m_1L_1^4L_2+m_2L_1^2L_2^3]+ \quad \text{(AIII.3)}$$
$$+B_{17}[L_1^3L_2^2+m_3L_1^2L_2^3]+B_{18}[L_2^5+m_4L_1L_2^4+m_5L_1^2L_2^3]$$

$$m_1 = \frac{5k_1}{k_1+k_2} \qquad m_2 = \frac{5(k_3k_2+k_1k_2-3k_1k_3)}{(k_1+k_2)(2k_3-3k_2)} \qquad m_3 = \frac{3k_3-2k_3}{2k_3-3k_2}$$

(AIII.4)

$$m_4 = \frac{5k_1}{k_1+k_3} \qquad m_5 = \frac{5(k_3k_2+k_1k_3-3k_1k_2)}{(k_1+k_3)\ (2k_3-3k_2)}$$

are in a matrix form $w = [F]\{B\}$, $[F] = [F_i]$ $i = 1 \div 18$ a row matrix with 18 elements and $\{B\}$ a vector with 18 elements.

The elements of matrix [F] are as follows

$$F_1 = 1,\ F_2 = L_1,\ F_3 = L_2,\ F_4 = L_1^2,\ F_5 = L_1 L_2,\ F_6 = L_2^2,\ F_7 = L_1^3,\ F_8 = L_1^2 L_2$$

$$F_9 = L_1 L_2^2,\ F_{10} = L_2^3,\ F_{11} = L_1^4,\ F_{12} = L_1^3 L_2,\ F_{13} = L_1^2 L_2^2,\ F_{14} = L_1 L_2^3,\ F_{15} = L_2^4 \qquad \text{(AIII.5)}$$

$$F_{16} = L_1^5 + m_1 L_1^4 L_2 + m_2 L_1^2 L_3^2,\quad F_{17} = L_1^3 L_2^2 + m_3 L_1^2 L_3^2,\quad F_{18} = L_2^5 + m_4 L_1 L_2^4 + m_s L_1^2 L_2^3$$

The displacement within the element can be expressed in the form

$$\begin{Bmatrix} w \\ w_{,x} \\ w_{,y} \\ w_{,xx} \\ w_{,xy} \\ w_{,yy} \end{Bmatrix} = \begin{bmatrix} 1 & \phi. & \phi. & \phi. & \phi. & \phi. \\ \phi. & b_1/DT & b_2/DT & \phi. & \phi. & \phi. \\ \phi & c_1/DT & c_2/DT & \phi. & \phi. & \phi. \\ \phi. & \phi. & \phi. & b_1^2/(DT)^2 & 2b_1 b_2/(DT)^2 & b_2^2/(DT)^2 \\ \phi. & \phi. & \phi. & b_1 c_1/(DT)^2 & (b_1 c_2 + b_2 c_1)/(DT)^2 & b_2 c_2/(DT)^2 \\ \phi. & \phi. & \phi. & c_1^2/(DT)^2 & 2c_1 c_2/(DT)^2 & c_2^2/(DT)^2 \end{bmatrix} \begin{bmatrix} [F] \\ [F]_{,1} \\ [F]_{,2} \\ [F]_{,11} \\ [F]_{,21} \\ [F]_{,22} \end{bmatrix} \{B\}$$

(AIII.6)

the subscripts at [F] indicates derivatives of L_1, L_2 respectively so

$[F]_{,1} = \partial [F]/\partial L_1$ $\qquad$ $[F]_{,12} = \partial^2 [F]/\partial L_1 \partial L_2$ etc.

thus

$$\begin{Bmatrix} w \\ w_{,x} \\ w_{,y} \\ w_{,xx} \\ w_{,xy} \\ w_{,yy} \end{Bmatrix} = [GC]\ [Fgen]\ \{B\} \qquad \text{(AIII.7)}$$

$$\{\delta_o^W\} = \begin{bmatrix} [GC] & [\phi.] & [\phi.] \\ [\phi.] & [GC] & [\phi.] \\ [\phi.] & [\phi.] & [GC] \end{bmatrix} \begin{bmatrix} [Fgen]^{L_1=1}_{L_2=0} \\ [Fgen]^{L_1=0}_{L_2=1} \\ [Fgen]^{L_1=0}_{L_2=0} \end{bmatrix} \{B\} \qquad \text{(AIII.8)}$$

$$\{\delta_o^W\}^T = \left\{ W_i,\ W_{,xi},\ W_{,yi},\ W_{,xxi},\ W_{,xyi},\ W_{,yyi},\ \ldots\ldots \right\}_{i=1 \div 3} \qquad \text{(AIII.9)}$$

$$\{\delta_o^W\}^T = [GCdiag]\ [Fgen]_{all}\ \{B\} \qquad \text{(AIII.10)}$$

$$\{B\} = [Fgen]^{-1}_{all}\ [GCdiag]^{-1}\ \{\delta_o^W\} = [T]^{-1}\ \{\delta_o^W\} \qquad \text{(AIII.11)}$$

$$[GCdiag]^{-1} = \begin{bmatrix} [GC]^{-1} & [\phi.] & [\phi.] \\ [\phi.] & [GC]^{-1} & [\phi.] \\ [\phi.] & [\phi.] & [GC]^{-1} \end{bmatrix} \qquad \text{(AIII.12)}$$

$$[GC]^{-1} = \begin{bmatrix} 1 & \phi. & \phi. & \phi. & \phi. & \phi. \\ \phi. & c_2 & -b_2 & \phi. & \phi. & \phi. \\ \phi. & -c_1 & b_1 & \phi. & \phi. & \phi. \\ \phi. & \phi. & \phi. & c_2^2 & -2c_2b_2 & b_2^2 \\ \phi. & \phi. & \phi. & -c_1c_2 & c_1b_2 + c_2b_1 & -b_1b_2 \\ \phi. & \phi. & \phi. & c_1^2 & -2c_1b_1 & b_1^2 \end{bmatrix} \qquad \text{(AIII.13)}$$

$[Fgen]_{all}^{-1}$ is a matrix of 18 x 18 size. We will indicate as (i,j) the element of the ith row and jth column. The non-mentioned elements are equal zero.

$(1,13)=1$

$(2,14)=1$

$(3,15)=1$

$(4.16)=0.5$

$(5,17)=1$

$(6,18)=0.5$

$(7,1)=10$, $(7,2)=-4$, $(7,4)=0.5$, $(7,13)=-10$, $(7,14)=-6$, $(7,16)=-1.5$

$(8,1)=6m_1$, $(8,2)=-3m_1$, $(8,3)=3$, $(8,4)=0.5m_1$, $(8,5)=-1$, $(8,13)=-6m_1$

$(8,14)=-3m_1$, $(8,15)=-3$, $(8,16)=0.5m_1$, $(8.17)=-2$

$(9,7)=6m_4$, $(9,8)=3$, $(9,9)=-3m_4$, $(9,11)=-1$, $(9,12)=0.5m_4$, $(9,13)=-6m_4$

$(9,14)=-3$, $(9,15)=-3m_4$, $(9,17)=-2$, $(9,18)=-0.5m_4$

$(10,7)=10$, $(10,9)=-4$, $(10,12)=0.5$, $(10,13)=-10$, $(10,15)=-6$, $(10,18)=-1.5$

$(11,1)=-15$, $(11,2)=7$, $(11,4)=-1$, $(11,13)=15$, $(11,14)=8$, $(11,16)=1.5$

$(12,1)=-12m_1$, $(12,2)=6m_1$, $(12,3)=-2$, $(12,4)=-m_1$, $(12,5)=1$, $(12,13)=12m_1$

$(12,14)=6m_1$, $(12,15)=2$, $(12,16)=m_1$, $(12,17)=1$

$(13,1)=-6(m_1+m_2)/(1-m_3)$, $(13,2)=3(m_1+m_2)/(1-m_3)$, $(13,3)=-3/(1-m_3)$

$(13,4)=-(m_1+m_2)/2(1-m_3)$, $(13,5)=1/(1-m_3)$, $(13,6)=-m_3/2(1-m_3)$, $(13,7)=6(m_3m_4-m_5)/(1-m_3)$

$(13,8)=3m_3/(1-m_3)$, $(13,9)=3(m_3-m_3m_4)/(1-m_3)$, $(13,10)=1/2(1-m_3)$, $(13,11)=-m_3/(1-m_3)$

$(13,12)=(m_3m_4-m_5)/2(1-m_3)$, $(13,13)=6(m_1+m_2+m_3m_4)/(1-m_3)$

$(13,14)=3(m_1+m_2-m_3)/(1-m_3)$, $(13,15)=3(1+m_5-m_3m_4)/(1-m_3)$

$(13,16)=(m_1+m_2-1)/2(1-m_3)$, $(13,17)=2$, $(13,18)=(m_3+m_3-m_3m_4)/2(1-m_3)$

$(14,7)=-12m_4$, $(14,8)=-2$, $(14,9)=6m_4$, $(14,11)=1$, $(14,12)=-m_4$

$(14,13)=12m_4$, $(14,14)=2$, $(14,15)=6m_4$, $(14,17)=1$, $(14,18)=m_4$

$(15,7)=-15$, $(15,9)=7$, $(15,12)=-1$, $(15,13)=15$, $(15,15)=8$, $(15,18)=1.5$

$(16,1)=6$, $(16,2)=-3$, $(16,4)=0.5$, $(16,13)=-6$, $(16,14)=-3$, $(16,16)=-0.5$

$(17,1)=6(m_1+m_2)/(1-m_3)$, $(17,2)=-3(m_1+m_2)/(1-m_3)$, $(17,3)=3/(1-m_3)$

$(17,4)=(m_1+m_2)/2(1-m_3)$, $(17,5)=-1/(1-m_3)$, $(17,6)=1/2(1-m_3)$

$(17,7)=6(m_3-m_4)/(1-m_3)$, $(17,8)=-3/(1-m_3)$, $(17,9)=3(m_4-m_5)/(1-m_3)$

$(17,10)=-1/2(1-m_3)$, $(17,11)=1/(1-m_3)$, $(17,12)=(m_3-m_4)/2(3-m_3)$

$(17,13)=6(m_4-m_5-m_1-m_2)/(1-m_3)$, $(17,14)=3(1-m_1-m_2)/(1-m_3)$

$(17,15)=3(m_4-m_5-1)/(1-m_3)$, $(17,16)=(1-m_1-m_2)/2(1+m_3)$

$(17,18)=(m_4-m_5-1)/1(1-m_3)$

$(18,7)=6$, $(18,9)=-3$, $(18,12)=0.5$, $(18,13)=-6$, $(18,15)=-3$, $(18,18)=-0.5$

$$1 - m_3 = \frac{k_2 + K_3}{3k_2 - 2k_3} = - \frac{b_3^2 + c_3^2}{b_3(3b_2 - 2b_1) + c_3(3c_2 - 2c_1)}$$

has always a real value different from zero for a triangle.

So the inverse matrix $[Fgen]^{-1}_{all}$ can be always evaluated

$$\{\varepsilon\} = \begin{bmatrix} [B^w] & [B^{s1}] \\ \phi. & [B^{s2}] \end{bmatrix} \begin{bmatrix} \{\delta^w_o\} \\ \{\delta^s_o\} \end{bmatrix} \qquad \text{(AIII.14)}$$

$[B^{s1}]$, $[B^{s2}]$ identical with the ones defined for PDS15 - DDS21.

$[B^w]$ is a matrix of 18 x 18 size defined by the following relationships.

As it has been written at (AIII.6) the second derivatives (part of the strains) are given by the form

$$\begin{Bmatrix} w,_{xx} \\ w,_{yy} \\ 2w,_{xy} \end{Bmatrix} = \begin{bmatrix} b_1^2/(DT)^2 & 2b_1b_2/(DT)^2 & b_2^2/(DT)^2 \\ c_1^2/(DT)^2 & 2c_1c_2/(DT)^2 & c_2^2/(DT)^2 \\ 2b_1c_1/(DT)^2 & 2(b_1c_2 + b_2c_1)/(DT)^2 & 2b_2c_2/(DT)^2 \end{bmatrix} \begin{bmatrix} [F],_{11} \\ [F],_{21} \\ [F],_{22} \end{bmatrix} \{B\}$$

(AIII.15)

$[F],_{11} = \partial^2[F]/\partial L_1^2$ etc. as defined previously.

Thus:

$$\begin{Bmatrix} w,_{xx} \\ w,_{yy} \\ 2w,_{xy} \end{Bmatrix} = [B^w_1] \quad \{B\} = [B^w_1] \quad [T]^{-1} \quad \{\delta^w_o\} \qquad \text{(AIII.16)}$$

$$\text{with } [B^w] = [B^w_1] \; [T]^{-1} \qquad \text{(AIII.17)}$$

The stiffness, stress and load matrices $[k_n]$, $[S]$, $\{R_n\}$ are obtained through equations (4.8), (4.5) employing the numerical integration formulae of Appendix VI.

APPENDIX IV

Formation of Stiffness Matrix for the model PMX24

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = [N_m^o]\ \{M_o^e\}, \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{Bmatrix} M_{xx,x} + M_{xy,y} \\ M_{yy,y} + M_{xy,z} \end{Bmatrix} = [N_m^1]\{M_o^e\}, \begin{Bmatrix} W_{,x} \\ W_{,y} \end{Bmatrix} = [N_w^2]\ \{\delta_o^w\}$$

$$\{M_o^e\}^T = \{M_{xxi},\ M_{yyi},\ M_{xyi}\}_{i=1\div 6} \qquad \{\delta_o^w\}^T = \{w_i\}_{i=1\div 6}$$

$[N_m^o]$ =

N_1	φ.	φ.	N_2	φ.	φ.	N_3	φ.	φ.	N_4	φ.	φ.	N_5	φ.	φ.	N_6	φ.	φ.
φ.	N_1	φ.	φ.	N_2	φ.	φ.	N_3	φ.	φ.	N_4	φ.	φ.	N_5	φ.	φ.	N_6	φ.
φ.	φ.	N_1	φ.	φ.	N_2	φ.	φ.	N_3	φ.	φ	N_4	φ.	φ.	N_5	φ.	φ.	N_6

$$N_1 = (2L_1 - 1)L_1, \quad N_2 = (2L_2 - 1)L_2, \quad N_3 = (2L_3 - 1)L_3$$

$$N_4 = 4L_2L_3, \quad N_5 = 4L_1L_3, \quad N_6 = 4L_1L_2.$$

$[N_m^1]$ =

$N_{,x1}$	φ.	$N_{,y1}$	$N_{,x2}$	φ.	$N_{,y2}$	$N_{,x3}$	φ.	$N_{,y3}$	$N_{,x4}$	φ.	$N_{,y4}$	$N_{,x5}$	φ.	$N_{,y5}$	$N_{,x6}$	φ.	$N_{,y6}$
φ.	$N_{,y1}$	$N_{,xi}$	φ.	$N_{,y2}$	$N_{,x2}$	φ.	$N_{,y3}$	$N_{,x3}$	φ.	$N_{,y4}$	$N_{,x4}$	φ.	$N_{,y5}$	$N_{,x5}$	φ.	$N_{,x6}$	$N_{,x6}$

$[N_w^2]$ =

$N_{,x1}$	$N_{,x2}$	$N_{,x3}$	$N_{,x4}$	$N_{,x5}$	$N_{,x6}$
$N_{,y1}$	$N_{,y2}$	$N_{,y3}$	$N_{,y4}$	$N_{,y5}$	$N_{,y6}$

$$\{\sigma b\} = \begin{bmatrix} [N_m^o] \\ [N_m^1] \end{bmatrix} \{M_o^e\} = [N_{mq}]\ \{M_o^e\}$$

$N_{,x1} = b_1(4L_1 - 1)/DT$ $\qquad N_{,y1} = c_1(4L_1 - 1)/DT$

$N_{,x2} = b_2(4L_2 - 1)/DT$ $\qquad N_{,y2} = c_2(4L_2 - 1)/DT$

$N_{,x3} = b_3(4L_3 - 1)/DT$ $\qquad N_{,y3} = c_3(4L_3 - 1)/DT$

$N_{,x4} = 4(b_2L_3 + b_3L_2)/DT$ $\qquad N_{,y4} = 4(c_2L_3 + c_3L_2)/DT$

$N_{,x5} = 4(b_1L_3 + b_3L_1)/DT$ $\qquad N_{,y5} = 4(c_1L_3 + c_3L_1)/DT$

$N_{,x6} = 4(b_2L_1 + b_1L_2)/DT$ $\qquad N_{,y6} = 4(c_2L_1 + c_2L_2)/DT$

$$[K_n^{mw}] = \iint_{A_n} [N_m^1]^T \, [N_w^2] \, dA$$

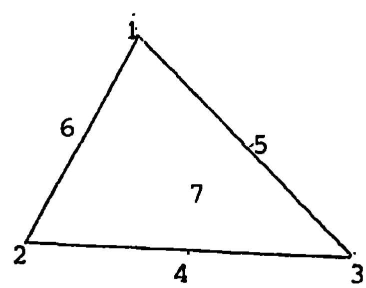
$$[K_n^{mq}] = -\iint_{A_n} [N_{mq}]^T \, [C_b] \, [N_{mq}] \, dA$$

The integrals are evaluated employing the numerical integration formulae of Appendix VI.

<u>APPENDIX V</u>

FORMATION OF MATRICES (Reference to PRO18, DRO30)

Formation of [B]



$$\{\varepsilon\} = \begin{Bmatrix} a_{xx} \\ a_{yy} \\ a_{xy} \\ \gamma_x \\ \gamma_y \\ \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = [B] \quad \{\delta_o^e\} \qquad \text{(AV.1)}$$

$$\left.\begin{aligned} a_{xx} &= \theta_{x,x} \\ a_{yy} &= \theta_{y,y} \\ a_{xy} &= \theta_{x,y} + \theta_{y,x} \\ \gamma_x &= w_{,x} - \theta_x \\ \gamma_y &= w_{,y} - \theta_y \\ \varepsilon_{xx} &= u_{,x} \\ \varepsilon_{yy} &= v_{,y} \\ \varepsilon_{xy} &= u_{,y} + V_{,x} \end{aligned}\right\} \qquad \text{(AV.2)}$$

$$\{\delta_o^e\} = \begin{Bmatrix} \delta_o^\theta \\ \delta_o^w \\ \delta_o^{uv} \end{Bmatrix} \qquad \text{(AV.3)}$$

$\{\delta_o^\theta\}$ vector of nodal degrees of freedom with nodal total rotation s.

$\{\delta_o^w\}$ vector of nodal degrees of freedom with nodal transverse displacements.

$\{\delta_o^{uv}\}$ vector of nodal degrees of freedom with nodal in plane displacements.

$$\{\delta_o^\theta\}^T = [\theta_{xi}, \theta_{yi}, \ldots\ldots] \quad i = 1 \div 6 \qquad \text{(AV.4)}$$

$$\{\delta_o^w\}^T = [w_i, \ldots\ldots\ldots\ldots] \quad i = 1 \div 7 \qquad \text{(AV.5)}$$

$$\{\delta_o^{uv}\}^T = [u_i, v_i, \ldots\ldots] \quad i = 1 \div 7 \qquad \text{(AV.6)}$$

$$\{\varepsilon\} = \begin{bmatrix} [B_1^\theta] & [\phi.] & [\phi.] \\ [B_2^\theta] & [B^w] & [\phi.] \\ [\phi.] & [\phi.] & [B^{uv}] \end{bmatrix} \begin{bmatrix} \{\delta_o^\theta\} \\ \{\delta_o^w\} \\ \{\delta_o^{uv}\} \end{bmatrix} \qquad \text{(AV.7)}$$

Formation of $[B_1^\theta]$ matrix

$[B_1^\theta]$ is a matrix of 3 rows and]2 columns.

$B^\theta_{1,1} = b_1(4L_1-1)/DT$ — $B^\theta_{1,3}$, $B^\theta_{1,5}$ form $B^\theta_{1,1}$ with circle symmetrical substitution of the subscripts 1, 2, 3.

$B^\theta_{3,2} = B^\theta_{1,1}$ — $B^\theta_{3,4}$, $B^\theta_{3,6}$ from $B^\theta_{3,2}$ as previously

$B^\theta_{1,7} = (4b_2L_3 + 4b_3L_2)/DT$ — $B^\theta_{1,9}$, $B^\theta_{1,11}$ from $B^\theta_{1,7}$ as previously

$B^\theta_{3,8} = B^\theta_{17}$ — $B^\theta_{3,10}$, $B^\theta_{3,12}$ from $B^\theta_{3,8}$ as previously

$B^\theta_{2,2} = c_i(4L_i - 1)/DT$ — $B^\theta_{2,4}$, $B^\theta_{2,6}$ from $B^\theta_{2,2}$ as previously

$B^\theta_{3,1} = B^\theta_{2,2}$ — $B^\theta_{3,3}$, $B^\theta_{3,5}$ from $B^\theta_{3,1}$ as previously

$B^\theta_{2,8} = (4C_2L_3 + 4C_3L_2)/DT$ — $B^\theta_{2\ 10}$, $B^\theta_{2,12}$ from $B^\theta_{2,8}$ as previously

$B^\theta_{3,7} = B^\theta_{2,8}$ — $B^\theta_{3,9}$, $B^\theta_{3,11}$ from $B^\theta_{3,7}$ as previously.

The rest of the elements in the matrix are zéro.

$[B_2^{\theta}] = -$

$(2L_1-1)L_1$	$\phi.$	$(2L_2-1)L_2$	$\phi.$	$(2L_3-1)L_3$	$\phi.$	$4L_2L_3$	$\phi.$	$4L_1L_3$	$\phi.$	$4L_1L_2$	$\phi.$
$\phi.$	$(2L_1-1)L_1$	$\phi.$	$(2L_2-1)L_2$	$\phi.$	$(2L_3-1)L_3$	$\phi.$	$4L_2L_3$	$\phi.$	$4L_1L_3$	$\phi.$	$4L_1L_2$

$DIM2 = (b_1L_2L_3 + b_2L_1L_3 + b_3L_1L_2)/DT \qquad DIM3 = (c_1L_2L_3 + c_2L_1L_3 + c_3L_1L_2)/DT$

$N_{x1} = b_1(4L_1 - L)/DT + 3DIM2$, $N_{x2} = b_2(4L_2-1)/DT + 3DIM2$, $N_{x3} = b_3(4L_3 - 1)/DT + 3DIM2$

$N_{x4} = 4(b_2L_3 + b_3L_2)/DT - 12DIM2$, $N_{x5} = 4(b_1L_3 + b_3L_1)/DT - 12DIM2$, $N_{x6} = 4(b_1L_2 + b_2L_1)/DT - 12DIM2$

$N_{x7} = 27DIM2$

$N_{y1} = c_1(4L_1 - 1)/DT + 3DIM3$, $N_{y2} = c_2(4L_2 - 1)/DT + 3DIM3$, $N_{y3} = c_3(4L_3 - 1)/DT + 3DIM3$

$N_{y4} = 4(c_2L_3 + c_3L_2)/DT - 12DIM3$, $N_{y5} = 4(c_1L_3 + c_3L_1)/DT - 12DIM3$, $N_{y6} = 4(c_1L_2 + c_2L_1)/DT - 12DIM3$

$N_{y7} = 27DIM3$

$[B^w] =$

N_{x1}	N_{x2}	N_{x3}	N_{x4}	N_{x5}	N_{x6}	N_{x7}
N_{y1}	N_{y2}	N_{y3}	N_{y4}	N_{y5}	N_{y6}	N_{y7}

$[B^{uv}] =$

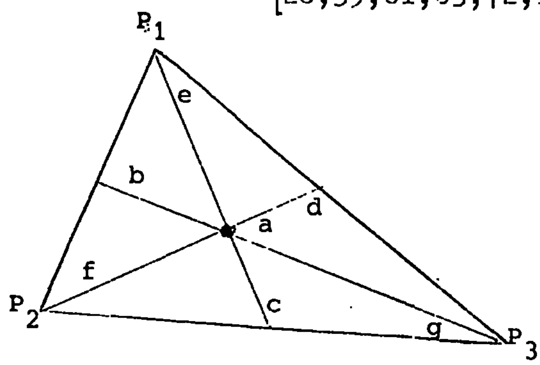
N_{x1}	$\phi.$	N_{x2}	$\phi.$	N_{x3}	$\phi.$	N_{x4}	$\phi.$	N_{x5}	$\phi.$	N_{x6}	$\phi.$	N_{x7}	$\phi.$
$\phi.$	N_{y1}	$\phi.$	N_{y2}	$\phi.$	N_{y3}	$\phi.$	N_{y4}	$\phi.$	N_{y5}	$\phi.$	N_{y6}	$\phi.$	N_{y7}
N_{y1}	N_{x1}	N_{y2}	N_{x2}	N_{y3}	N_{x3}	N_{y4}	N_{x4}	N_{y5}	N_{x5}	N_{y6}	N_{x6}	N_{y7}	N_{y7}

The stiffness, stress and load matrices $[K_n]$, $[S]$, $\{R_n\}$ are obtained through equations (4.8), (4,5) employing the numberical integration formulae of Appendix VI.

APPENDIX VI

NUMERICAL INTEGRATION

The quintic numerical integration formulae have been employed as follows [28,39,61,63,72,115]



	L_1	L_2	L_3
a	1/3	1/3	1/3
b	α_1	β_1	β_1
c	α_1	α_1	β_1
d	β_1	β_1	α_1
e	α_2	β_2	β_2
f	β_2	α_2	β_2
g	β_2	β_2	α_2

$\alpha_1 = 0.05971587$

$\beta_1 = 0.47014206$

$\alpha_2 = 0.79742699$

$\beta_2 = 0.10128651$

where it was possible the integration has been applied straightforward by the use of the formulae

$$\iint_{P_1P_2P_3} L_1^a \; L_2^b \; L_3^c \; dx\,dy = \frac{a!\,b!\,c!}{(a+b+c+2)!} \, DT$$

DT = twice the area of the triangle $P_1P_2P_3$

SANDWICH PLATE BENDING MODELS

COMPUTER LISTING

	Reference symbol
1.	PDS15
2.	PDS21
3.	PMX12
4.	PMX24
5.	PDS24
6.	PDS30
7.	PRO18

1. <u>Reference symbol PDS15</u>

```
 P=1 . PRCLTF=CURH CCPIES=4
= UNIVERSITY, BATCH
 WAS: 10:26:39
SIGNED ON AT  11:36:1  ON MON SEP 22/75
PRINT:
  *** PLATE DISPLAC.T  MODEL  15 DEGR. OF FREEDOM W, MX, MY, FX, FY  3 NODES**
***** STRESSES  AT THE CENTROID  MX, MY, MXY, QX, QY   ****************
*****    TRANSFORMATION  TO  W, MN, WS, FN, FS  ****************************
****  CUBIC  VARIATION  OF W   LINEAR OF FX,FY  *****************************
      SUBROUTINE STIFF
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,2 ),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(20)
      COMMON/MAN/BOL(2,36,36),COL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
      COMMON/PAT/CO2(3,3),XO(3),YO(3)
      DIMENSION TRAN(15,15),SMK(15,15),SMK1(15,15),STR(5,15),TIK(5,15),
     1DANG(15),XM(16,2),YM(16,2),CO3(4,4)
      DO 100 I=1,36
      DO 100 J=1,36
 100  STUCK(J,I)= .
      DO 101 I=1,36
      DO 101 J=1,8
 101  STICK(J,I)= .
      DO 102 I=1,36
 102  FORCE(I)=0.
      DO 103 I=1,3
      XO(I)=X(1,I)
 103  YO(I)=X(2,I)
      IF(NGRAM.EQ.0) GO TO 517
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
 516  YM(M,I)=GRAM(M,I+2)
 517  CONTINUE
      NLAK=INFO(1)
      IF(NLAK.EQ.1.OR.NLAK.EQ.2) GO TO 1 4
      CALL SUBTI
      IF(NLAK.EQ.0) GO TO 105
      NLAK=NLAK-1
      DO 106 I=1,15
      DO 106 J=1,15
 106  BOL(NLAK,J,I)=STUCK(J,I)
      DO 107 I=1,15
      DO 107 J=1,5
 107  COL(NLAK,J,I)=STICK(J,I)
      DO 108 I=1,15
 108  DOL(NLAK,I)=FORCE(I)
      GO TO 105
 104  DO 109 I=1,15
      DO 1 9 J=1,15
 109  STUCK(J,I)=BOL(NLAK,J,I)
      DO 110 I=1,15
      DO 11  J=1,5
 110  STICK(J,I)=COL(NLAK,J,I)
      DO 111 I=1,15
 111  FORCE(I)=DOL(NLAK,I)
 105  CONTINUE
      NLIK=INFO(2)
      IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
      DO 113 I=1,15
```

```
      DO 113 J=1,15
  113 SMK(J,I)=STUCK(J,I)
      DO 114 I=1,15
      DO 114 J=1,5
  114 STR(J,I)=STICK(J,I)
      DO 115 I=1,NIK
      K=(I-1)*2+3
      L=INEC(K)
      K1=INEC(K+1)
      DO 583 II=1,15
      DO 584 IJ=1,15
  584 TRAN(IJ,II)= .
  583 TRAN(II,II)=1.
      CALL TRANL(XN,YN,K1,CC3)
      KK=(L-1)*5
      DO 587 IN=1,4
      DO 587 IL=1,4
  587 TRAN(KK+IN+1,KK+IL+1)=CC3(IN,IL)
      CALL TIMES(SMK,TRAN,SMK1,15,15,15,1)
      CALL TIMES(TRAN,SMK1,SMK,15,15,15,2)
      CALL TIMES(STR,TRAN,TIK,5,15,15,1)
      DO 116 NI=1,15
      DO 116 NJ=1,5
  116 STR(NJ,NI)=TIK(NJ,NI)
  115 CONTINUE
      DO 117 I=1,15
      DO 117 J=1,15
  117 STUCK(J,I)=SMK(J,I)
      DO 118 I=1,15
      DO 118 J=1,5
  118 STICK(J,I)=STR(J,I)
  112 CONTINUE
      BRRO=0.
      DO 604 J=6,8
      DO 604 I=1,3
  604 IF(X(I,J).NE.0)BREC=1.
      IF(BRRO.NE.1.)GO TO 560
      DO 273 I=1,15
  273 DADG(I)=0.
      DADG(1)=X(3,6)
      DADG(6)=X(3,7)
      DADG(11)=X(3,8)
      DO 274 I=1,15
  274 FORCE(I)=FORCE(I)+DADG(I)
  560 CONTINUE
      RETURN
      END
      SUBROUTINE TRANL(X,Y,K,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(16,2),Y(16,2),TRL(4,4),X2(2),Y2(2)
      DO 515 J=1,2
      X2(J)=X(K,J)
  515 Y2(J)=Y(K,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      GLEN=DSQRT(G2**2+B2**2)
      DO 100 I=1,4
      DO 1    J=1,4
  100 TRL(I,J)=0.
      IF(GLEN.EQ.0) WRITE(6,700)
```

```
      TRL(1,1)=B2/GLEN
      TRL(1,2)=G2/GLEN
      TRL(1,3)=B2/GLEN
      TRL(2,1)=-G2/GLEN
      TRL(2,2)=B2/GLEN
      TRL(2,3)=-G2/GLEN
      TRL(3,3)=B2/GLEN
      TRL(3,4)=G2/GLEN
      TRL(4,3)=-G2/GLEN
      TRL(4,4)=B2/GLEN
 50   FORMAT(/,'      ****ERROR****  SUBROUTINE  TRAN1    GLEN----',/)
      RETURN
      END
      *********         ME      THN     VOHTHIA      TOY      THEORY   ********
      *********         ME      THN     VOHTHIA      TOY      THEORY   ********
         SUBPROGRAMMS        **********************
         SUBPROGRAMMS        **********************
      THE   STIFFNESS    MATRIX    ST
      THE   STRESS       MATRIX    S
      THE   LOAD         MATRIX    END
      INPUT   FOR   EACH POINT A THE AXIA COORDINATES,P THE VALUES OF LOAD
      INPUT   FOR   EACH ELEMENT THE COEFFICIENTS  B(3) ,G(3)
      INPUT   FOR   EACH ELEMENT THE ELASTICITY  MATRIX  E(9)
      SUBROUTINE  SHAPEF(A,B,C,DT,NEK)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/JON/ST(15,15),S(5,15),D(5,5)
      COMMON/JAC/ENP(15,1),PIN(15),P(5),PI(15)
      DIMENSION A(3),B(3),G(3),BE(5,15),EB(5,15)
      DIMENSION SW(3),SX(3),SY(3),SWX(3),SXX(3),SYX(3),SWY(3),SXY(3),SYY
     1(3),SWXX(3),SXXX(3),SYXX(3),SWYY(3),SXYY(3),SYYY(3),SWXY(3),SXXY(3
     2),SYXY(3),SU(3),SV(3),SFX(3),SFY(3),SUX(3),SUY(3),SVX(3),SVY(3)
      DO 2 1 I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      SW(I)=A(I)+A(I)**2*A(K)+A(I)**2*A(J)-A(I)*A(K)**2-A(I)*A(J)**2
      SY(I)=B(K)*(A(I)**2*A(J)+0.5*A(I)*A(K)*A(J))-B(J)*(A(K)*A(I)**2+
     10.5*A(I)*A(K)*A(J))
      SX(I)=G(J)*(A(I)**2*A(K)+ .5*A(I)*A(K)*A(J))-G(K)*(A(J)*A(I)**2+
     10.5*A(I)*A(K)*A(J))
      SWX(I)=(1./DT)*(2.*B(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
     1A(K)*(B(I)-B(K))+2.*A(I)*A(J)*(B(I)-B(J)))
      SYX(I)=(1./DT)*(2.*B(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+0.5*(B(J)-B(K))
     1*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)*A(I)*A(K)))*(-1. )
      SXX(I)=(1./DT)*(2.*B(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+A(I)**2*(B(K)*
     1G(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+
     2B(J)*A(I)*A(K)))
      SWY(I)=(+1./DT)*(2.*G(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
     1A(K)*(G(I)-G(K))+2.*A(I)*A(J)*(G(I)-G(J)))
      SYY(I)=(-1./DT)*(2.*G(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+A(I)**2*(G(K)*
     1B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+
     2G(J)*A(I)*A(K)))
      SXY(I)=(+1./DT)*(2.*G(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+0.5*(G(J)-G(K)
     1)*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+G(J)*A(I)*A(K)))
      SU(I)=A(I)
      SV(I)=A(I)
      SFY(I)=A(I)
      SFX(I)=A(I)
      SWXX(I)=(+1./DT**2)*(-2.*B(I)*(A(I)*B(I)+A(K)*B(K)+A(J)*B(J))+
```

```
12.*(B(I)-B(K))*(A(K)*B(I)+A(I)*B(K))+2.*(B(I)-B(J))*(A(I)*B(J)+
2A(J)*B(I)))
 SYXX(I)=(-1./DT**2)*(2.*B(I)**2*(B(J)*A(K)-B(K)*A(J))+(B(J)-B(K)
1)*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B(K)*B(J)*A(I)))
 SXXX(I)=(+1./DT**2)*(2.*B(I)**2*(G(J)*A(K)-G(K)*A(J))+4.*A(I)*B(I)
1*(B(K)*G(J)-B(J)*G(K))+(G(J)-G(K))*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+
2B(K)*B(J)*A(I)))
 SWYY(I)=(+1./ DT**2)*(-2.*G(I)*(A(I)*G(I)+A(K)*G(K)+A(J)*G(J))+2.*
1(G(I)-G(K))*(A(K)*G(I)+A(I)*G(K))+2.*(G(I)-G(J))*(A(I)*G(J)+A(J)*
2G(I)))
 SYYY(I)=(-1./DT**2)*(2.*G(I)**2*(B(J)*A(K)-B(K)*A(J))+4.*A(I)*G(I)
1*(G(K)*B(J)-G(J)*B(K))+(B(J)-B(K))*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+
2G(K)*G(J)*A(I)))
 SXYY(I)=(+1./DT**2)*(2.*G(I)**2*(G(J)*A(K)-G(K)*A(J))+(G(J)-G(K)
1)*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+G(K)*G(J)*A(I)))
 SWXY(I)=(2./DT**2)*(-2.*B(I)*(G(I)*A(I)+G(K)*A(K)+G(J)*A(J))+2.*
1(B(I)-B(K))*(A(K)*G(I)+A(I)*G(K))+2.*(B(I)-B(J))*(A(I)*G(J)+A(J)*
2G(I)))
 SYXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(B(J)*A(K)-B(K)*A(J))+2.*B(I)*
1A(I)*(G(K)*B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(A(I)*(G(J)*B(K)+G(K)*
2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
3*(-1.0)
 SXXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(G(J)*A(K)-G(K)*A(J))+2.*G(I)*
1A(I)*(B(K)*G(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(A(I)*(G(J)*B(K)+G(K)*
2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
 SUX(I)=(1./DT)*B(I)
 SVX(I)=(1./DT)*B(I)
 SUY(I)=(1./DT)*G(I)
 SVY(I)=(1./DT)*G(I)
201 CONTINUE
    DO 401 I=1,15
    DO 401 J=1,5
401 BE(J,I)=0.
    DO 202  J=1,7,3
    I=J
    IF(I.LE.3)K=I
    IF(I.GT.3.AND.I.LE.6)K=I-2
    IF(I.GT.6)K=I-4
    BE(1,J)=SWXX(K)
    BE(2,J)=SWYY(K)
    BE(3,J)=SWXY(K)
    BE(1,J+1)=SYXX(K)
    BE(2,J+1)=SYYY(K)
    BE(3,J+1)=SYXY(K)
    BE(1,J+2)=SXXX(K)
    BE(2,J+2)=SXYY(K)
    BE(3,J+2)=SXXY(K)
202 CONTINUE
    DO 203 J=10,14,2
    I=J-9
    IF(I.LE.2)K=I
    IF(I.GT.2.AND.I.LE.4)K=I-1
    IF(I.GT.4)K=I-2
    BE(1,J)=-SUX(K)
    BE(3,J)=-SUY(K)
    BE(4,J)=SFX(K)
    BE(2,J+1)=-SVY(K)
    BE(3,J+1)=-SVX(K)
    BE(5,J+1)=SFY(K)
203 CONTINUE
```

```
     STIFNESS MATRIX         BY MULTIPLICATION  B *D*B
     STIFNESS MATRIX    SM
     STRESS   MATRIX    S  BY  MULTIPLICATION    D*B
    CALL TIMES(D,BE,S,5,5,15,1)
    CALL TIMES(BE,S,SM,15,5,15,2)
    DO 402 I=1,15
    DO 4 2 J=1,5
02  EN(J,I)=0.
    PROGRAMM    FOR  THE  SHAPE  FUNCTION  MATRIX   N
    DO  213  J=1,7,3
    I=J
    IF(I.LE.3)K=I
    IF(I.GT.3.AND.I.LE.6)K=I-2
    IF(I.GT.6)K=I-4
    EN(1,J)=SW(K)
    EN(2,J)=SWY(K)
    EN(3,J)=SWX(K)
    EN(1,J+1)=SY(K)
    EN(2,J+1)=SYY(K)
    EN(3,J+1)=SYX(K)
    EN(1,J+2)=SX(K)
    EN(2,J+2)=SXY(K)
    EN(3,J+2)=SXX(K)
13  CONTINUE
    DO 214 J=10,14,2
    I=J-9
    IF(I.LE.2)K=I
    IF(I.GT.2.AND.I.LE.4)K=I-1
    IF(I.GT.4)K=I-2
    EN(4,J)=SFX(K)
    EN(5,J+1)=SFY(K)
14  CONTINUE
    CALL TIMES(EN,P,ENP,15,5,1,2)
    RETURN
    END
    SUBROUTINE  SUBTI
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),ELMOD(12),STUCK(36,36),STICK(8,36),FORCE(36)
   1,INFO(20)
    COMMON/PAT/CO2(3,3),XI(3),Y(3)
    COMMON/JON/SM(15,15),S(5,15),D(5,5)
    COMMON/JAC/ENP(15,1),PIN(15),P(5),PI(15)
    DO 100 I=1,5
    DO 100 J=1,5
    D(J,I)=0.
    D(1,1)=ELMOD(1)
    D(1,2)=ELMOD(2)
    D(2,2)=ELMOD(3)
    D(3,3)=ELMOD(4)/2.
    D(4,4)=ELMOD(5)
    D(5,5)=ELMOD(6)
    FORM   THE  WHOLE  SYMMETRIC   MATRIX  D
    DO 21   J=1,5
    DO 210 I=1,5
    D(I,J)=D(J,I)
1   CONTINUE
    DODO=ELMOD(12)
    IF(DODO.NE.0.) D(2,1)=DODO
    DO 224 I=1,5
24  P(I)=0.
```

```
   P(1)=X(3,5)
   DIMENSION A(3),B(3),G(3),LENG(3),W(7),A1(7),A2(7),A3(7),ENK(15),
  1SMK(15,15),TIK(5,15),M1(15),SMK1(15,15)
   DO 200  I=1,3
   K=I+1
   IF(K.GT.3)K=K-3
   J=I+2
   IF(J.GT.3)J=J-3
   B(I)=Y(K)-Y(J)
   G(I)=XI(J)-XI(K)
   LENG(I)=DSQRT(B(I)**2+G(I)**2)
00 CONTINUE
   DT=B(1)*G(2)-B(2)*G(1)
QUINTIC  INTEGRATION  FOR STIFFNESS MATRIX  STUCK
   AREA   COORDINATES   L1  L2  L3    AREA         A(I)
 WEIGHTS  OF  INTEGRATION           W(I)
   DATA A1/ .33333333D  ,  . 59715870 00,2*0.47014206D 00,0.79742699D
  1 00,2*0.10128651D 00/,A3/0.33333333D 00,2*0.47014206D 00,0.0597158
  27D 00,2*0.10128651D 00,0.79742699D  /,W/ .225      D    ,3* .132394
  315D 00,3*0.12593918D 00/
   DO 211  I=1,15
   DO 211  J=1,15
11  SMK(J,I)=0.
   DO 222  I=1,15
22  ENK(I)= .
   DO 212 K=1,7
   A(1)=A1(K)
   A(3)=A3(K)
   A(2)=1.-A(1)-A(3)
   CALL SHAPEF(A,B,G,DT, )
   IF(K.NE.1) GO TO 801
   DO 802 I=1,15
   DO 8 2 J=1,5
02  TIK(J,I)=S(J,I)
01  CONTINUE
   DO 221 I=1,15
21  ENK(I)=ENK(I)+DT/2.*W(K)*ENP(I,1)
   DO 215 I=1,15
   DO 215 J=1,15
15  SMK(J,I)=SMK(J,I)+DT/2.*W(K)*SM(J,I)
12  CONTINUE
   DATA M1/1,3,2,10,11,4,6,5,12,13,7,9,8,14,15/
   DIMENSION TOT(15,15)
   DO 247 I=1,15
   DO 247 J=1,15
47  TOT(J,I)=0.
   DO 248 I=1,15
48  TOT(M1(I),I)=1.
   CALL TIMES(SMK,TOT,SMK1,15,15,15,1)
   CALL TIMES(TOT,SMK1,SMK,15,15,15,2)
   CALL TIMES(TIK,TOT,S,5,15,15,1)
   CALL TIMES(TOT,ENK,ENP,15,15,1,2)
   DO 216 I=1,15
   DO 216 J=1,15
16  STUCK(J,I)=SMK(J,I)
   DO 217 I=1,15
   DO 217 J=1,5
17  STICK(J,I)=S(J,I)
   DO 218 I=1,15
18  FORCE(I)=ENP(I,1)
```

```
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),R(1)
      KOK=1   A(N,M) , B(M,L) , R(N,L)     REGULAR     A*B
      KOK=2   A(M,N) , B(M,L) , R(N,L)     TRANSPOSE A'*B
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
  101 CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
  103 R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
  102 CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
  104 R(IR)=R(IR)+A(IA)*B(IB)
  100 IR=IR+1
      RETURN
      END
```

2. <u>Reference symbol PDS21</u>

```
) P=100 RECUTE=PUPE COPIES=4
= UNIVERSITY, BATCH
WAS: 11:36:10
SIGNED ON AT 11:36:26 ON MON SEP 22/75
PRINT*
  *** PLATE DISPLAC.MODEL 21 DEG. OF FREED.  W,WX,WY,FX,FY  3 CORNER NODES ***
  ****   +   FX,FY   3 MIDSIDE NODES                              *********************
  ****CUBIC VARIATION FOR W ,  QUADRATIC FOR FX,FY           *********************
  *** STRESSES  AT CENTROID  MXX,MYY,MXY,QX,QY    *************************
  *** TRANSFORMATION  W, PN, WS, FN, FS            *********************
     SUBROUTINE STIFF
     IMPLICIT REAL*8 (A-H,O-Z)
     COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
    1INFO(20)
     COMMON/MAN/BOL(2,36,36),COL(2,9,36),DOL(2,36),GRAM(16,16),NGRAM
     COMMON/PAT/CO2(3,3),XO(3),YO(3)
     DIMENSION TRAN(21,21),SMK(21,21),SMK1(21,21),STR(5,21),TIK(5,21),
    1DAUG(21),XM(16,2),YM(16,2),CO3(4,4)
     DO 100 I=1,36
     DO 100 J=1,36
100  STUCK(J,I)=0.
     DO 101 I=1,36
     DO 101 J=1,8
101  STICK(J,I)=0.
     DO 102 I=1,36
102  FORCE(I)= .
     DO 103 I=1,3
     XO(I)=X(1,I)
103  YO(I)=X(2,I)
     IF(NGRAM.EQ.0) GO TO 517
     DO 516 M=1,NGRAM
     DO 516 I=1,2
     XM(M,I)=GRAM(M,I)
516  YM(M,I)=GRAM(M,I+2)
517  CONTINUE
     NLAK=INFO(1)
     IF(NLAK.EQ.1.OR.NLAK.EQ.2) GO TO 104
     CALL SUBTI
     IF(NLAK.EQ.0) GO TO 105
     NLAK=NLAK-10
     DO 106 I=1,21
     DO 106 J=1,21
106  BOL(NLAK,J,I)=STUCK(J,I)
     DO 107 I=1,21
     DO 107 J=1,5
107  COL(NLAK,J,I)=STICK(J,I)
     DO 108 I=1,21
108  DOL(NLAK,I)=FORCE(I)
     GO TO 105
104  DO 109 I=1,21
     DO 109 J=1,21
109  STUCK(J,I)=BOL(NLAK,J,I)
     DO 110 I=1,21
     DO 110 J=1,5
110  STICK(J,I)=COL(NLAK,J,I)
     DO 111 I=1,21
111  FORCE(I)=DOL(NLAK,I)
105  CONTINUE
     NLIK=INFO(2)
     IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
```

```
      DO 113 I=1,21
      DO 113 J=1,21
113   SMK(J,I)=STUCK(J,I)
      DO 114 I=1,21
      DO 114 J=1,5
114   STR(J,I)=STICK(J,I)
      DO 115 I=1,NLINK
      K=(I-1)*2+2
      L=INFO(K)
      K1=INFO(K+1)
      DO 583 II=1,21
      DO 584 IJ=1,21
584   TRAN(IJ,II)=0.
583   TRAN(II,II)=1.
      CALL TRANL(XM,YM,K1,COB)
      IF(L.GT.3) GO TO 119
      KK=(L-1)*5
      DO 587 IN=1,4
      DO 587 IL=1,4
587   TRAN(KK+IN+1,KK+IL+1)=COB(IN,IL)
      GO TO 120
119   KK=2*L+7
      DO 121 IN=1,2
      DO 121 IL=1,2
121   TRAN(KK+IN,KK+IL)=COB(IN,IL)
120   CONTINUE
      CALL TIMES(SMK,TRAN,SMK1,21,21,21,1)
      CALL TIMES(TRAN,SMK1,SMK,21,21,21,2)
      CALL TIMES(STR,TRAN,TIK,5,21,21,1)
      DO 116 NI=1,21
      DO 116 NJ=1,5
116   STR(NJ,NI)=TIK(NJ,NI)
115   CONTINUE
      DO 117 I=1,21
      DO 117 J=1,21
117   STUCK(J,I)=SMK(J,I)
      DO 118 I=1,21
      DO 118 J=1,5
118   STICK(J,I)=STR(J,I)
112   CONTINUE
      BRPO=0.
      DO 604 J=6,8
      DO 604 I=1,3
604   IF(X(I,J).NE.0)BRPO=1.
      IF(BRPO.NE.1.)GO TO 56
      DO 273 I=1,21
273   DACG(I)=0.
      DACG(1)=X(3,6)
      DACG(6)=X(3,7)
      DACG(11)=X(3,8)
      DO 274 I=1,21
274   FORCE(I)=FORCE(I)+DACG(I)
56    CONTINUE
      RETURN
      END
      SUBROUTINE TRANL(X,Y,K,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(16,2),Y(16,2),TRL(4,4),X2(2),Y2(2)
      DO 515 J=1,2
      X2(J)=X(K,J)
```

```
15 Y2(J)=Y(K,J)
   G2=X2(2)-X2(1)
   B2=Y2(2)-Y2(1)
   GLEN=DSQRT(G2**2+B2**2)
   DO 100 I=1,4
   DO 1    J=1,4
00 TRL(I,J)=0.
   IF(GLEN.EQ.0) WRITE(6,700)
   TRL(1,1)=B2/GLEN
   TRL(1,2)=G2/GLEN
   TRL(1,3)=B2/GLEN
   TRL(2,1)=-G2/GLEN
   TRL(2,2)=B2/GLEN
   TRL(2,3)=-G2/GLEN
   TRL(3,3)=B2/GLEN
   TRL(3,4)=G2/GLEN
   TRL(4,3)=-G2/GLEN
   TRL(4,4)=B2/GLEN
00 FORMAT(/,'        ****ERROR****  SUBROUTINE   TRANL    GLEN----',/)
   RETURN
   END
    *** ****       ME      THN      VOHTHIA        TOY        THEOY    ********
    *** ****       ME      THN      VOHTHIA        TOY        THEOY    ********
      SUBPOGRAMMS          ********************
      SUBPOGRAMMS          ********************
   THE   STIFFNESS   MATRIX    SM
   THE   STRESS      MATRIX    S
   THE   LOAD        MATRIX    FNP
   INPUT   FOR   EACH POINT A THE AREA COORDINATES,P THE VALUES OF LOAD
   INPUT   FOR   EACH ELEMENT THE COEFFICIENTS  B(3) ,G(3)
   INPUT   FOR   EACH ELEMENT THE ELASTICITY  MATRIX  D(8)
   SUBROUTINE  SHAPEF(A,B,G,DT,NOK)
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/JON/SM(21,21),S(5,21),D(5,5)
   COMMON/JAC/FNP(21,1),PIN(15),P(5),PI(15)
   DIMENSION A(3),B(3),G(3),BF(5,21),EN(5,21)
   DIMENSION SW(3),SX(3),SY(3),SWX(3),SXX(3),SYX(3),SWY(3),SXY(3),SYY
  1(3),SWXX(3),SXYX(3),SYXX(3),SWYY(3),SXYY(3),SYYY(3),SWXY(3),SXXY(3
  2),SYXY(3),SF(6),SFX(6),SFY(6)
   DO 201 I=1,3
   K=I+1
   IF(K.GT.3)K=K-3
   J=I+2
   IF(J.GT.3)J=J-3
   SW(I)=A(I)+A(I)**2*A(K)+A(I)**2*A(J)-A(I)*A(K)**2-A(I)*A(J)**2
   SY(I)=B(K)*(A(I)**2*A(J)+0.5*A(I)*A(K)*A(J))-B(J)*(A(K)*A(I)**2+
  10.5*A(I)*A(K)*A(J))
   SX(I)=G(J)*(A(I)**2*A(K)+0.5*A(I)*A(K)*A(J))-G(K)*(A(J)*A(I)**2+
  1 .5*A(I)*A(K)*A(J))
   SWX(I)=(1./DT)*(2.*B(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
  1A(K)*(B(I)-B(K))+2.*A(I)*A(J)*(B(I)-B(J)))
   SYX(I)=(1./DT)*(2.*B(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+0.5*(B(J)-B(K))
  1*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)*A(I)*A(K)))*(-1.0)
   SXX(I)=(1./DT)*(2.*B(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+A(I)**2*(B(K)*
  1G(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+
  2B(J)*A(I)*A(K)))
   SWY(I)=(+1./DT)*(2.*G(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
  1A(K)*(G(I)-G(K))+2.*A(I)*A(J)*(G(I)-G(J)))
   SYY(I)=(-1./DT)*(2.*G(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+A(I)**2*(G(K)*
  1B(J)-G(J)*B(K))+ .5*(B(J)-B(K))*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+
```

```
  2G(J)*A(I)*A(K)))
   SXY(I)=(+1./DT)*(2.*G(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+ .5*(G(J)-G(K)
  1)*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+G(J)*A(I)*A(K)))
   SWXX(I)=(+1./DT**2)*(-2.*B(I)*(A(I)*B(I)+A(K)*B(K)+A(J)*B(J))+
  12.*(B(I)-B(K))*(A(K)*B(I)+A(I)*B(K))+2.*(B(I)-B(J))*(A(I)*B(J)+
  2A(J)*B(I)))
   SYXX(I)=(-1./DT**2)*(2.*B(I)**2*(B(J)*A(K)-B(K)*A(J))+(B(J)-B(K)
  1)*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B(K)*B(J)*A(I)))
   SXXX(I)=(+1./DT**2)*(2.*B(I)**2*(G(J)*A(K)-G(K)*A(J))+4.*A(I)*B(I)
  1*(B(K)*G(J)-B(J)*G(K))+(G(J)-G(K))*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+
  2B(K)*B(J)*A(I)))
   SWYY(I)=(+1./ DT**2)*(-2.*G(I)*(A(I)*G(I)+A(K)*G(K)+A(J)*G(J))+2.*
  1(G(I)-G(K))*(A(K)*G(I)+A(I)*G(K))+2.*(G(I)-G(J))*(A(I)*G(J)+A(J)*
  2G(I)))
   SYYY(I)=(-1./DT**2)*(2.*G(I)**2*(B(J)*A(K)-B(K)*A(J))+4.*A(I)*G(I)
  1*(G(K)*B(J)-G(J)*B(K))+(B(J)-B(K))*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+
  2G(K)*G(J)*A(I)))
   SXYY(I)=(+1./DT**2)*(2.*G(I)**2*(G(J)*A(K)-G(K)*A(J))+(G(J)-G(K)
  1)*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+G(K)*G(J)*A(I)))
   SWXY(I)=(2./DT**2)*(-2.*B(I)*(G(I)*A(I)+G(K)*A(K)+G(J)*A(J))+2.*
  1(B(I)-B(K))*(A(K)*G(I)+A(I)*G(K))+2.*(B(I)-B(J))*(A(I)*G(J)+A(J)*
  2G(I)))
   SYXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(B(J)*A(K)-B(K)*A(J))+2.*B(I)*
  1A(I)*(G(K)*B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(A(I)*(G(J)*B(K)+G(K)*
  2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
  3*(-1.0)
   SXXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(G(J)*A(K)-G(K)*A(J))+2.*G(I)*
  1A(I)*(B(K)*G(J)-B(J)*G(K))+ .5*(G(J)-G(K))*(A(I)*(G(J)*B(K)+G(K)*
  2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
   SF(I)=(2.*A(I)-1.)*A(I)
   SF(I+3)=4.*A(K)*A(J)
   SFX(I)=B(I)*(4.*A(I)-1.)/DT
   SFX(I+3)=(4.*B(K)*A(J)+4.*B(J)*A(K))/DT
   SFY(I)=G(I)*(4.*A(I)-1.)/DT
201 SFY(I+3)=(4.*G(K)*A(J)+4.*G(J)*A(K))/DT
   DO 401 I=1,21
   DO 401 J=1,5
  1 BE(J,I)=0.
   DO 202  J=1,7,3
   I=J
   IF(I.LE.3)K=1
   IF(I.GT.3.AND.I.LE.6)K=I-2
   IF(I.GT.6)K=I-4
   BE(1,J)=SWXX(K)
   BE(2,J)=SWYY(K)
   BE(3,J)=SWXY(K)
   BE(1,J+1)=SYXX(K)
   BE(2,J+1)=SYYY(K)
   BE(3,J+1)=SYXY(K)
   BE(1,J+2)=SXXX(K)
   BE(2,J+2)=SXYY(K)
   BE(3,J+2)=SXXY(K)
02 CONTINUE
   DO 105 I=1,6
   J=8+2*I
   BE(1,J)=-SFX(I)
   BE(3,J)=-SFY(I)
   BE(4,J)=SF(I)
   BE(2,J+1)=-SFY(I)
   BE(3,J+1)=-SFX(I)
```

```
 5 BF(5,J+1)=SF(I)
    STIFNESS MATRIX         BY MULTIPLICATION   B *D*B
    STIFNESS MATRIX    SM
    STRESS    MATRIX    S   BY   MULTIPLICATION      D*B
   CALL TIMES(D,BF,S,5,5,21,1)
   CALL TIMES(BF,S,SM,21,5,21,2)
   DO 402 I=1,21
   DO 402 J=1,5
 2 FN(J,I)= .
   PROGRAMM      FOR   THE   SHAPE   FUNCTION  MATRIX    N
   DO  213   J=1,7,3
   I=J
   IF(I.LE.3)K=I
   IF(I.GT.3.AND.I.LE.6)K=I-2
   IF(I.GT.6)K=I-4
   FN(1,J)=SM(K)
   FN(2,J)=SMY(K)
   FN(3,J)=SMX(K)
   FN(1,J+1)=SY(K)
   FN(2,J+1)=SYY(K)
   FN(3,J+1)=SYX(K)
   FN(1,J+2)=SX(K)
   FN(2,J+2)=SXY(K)
   FN(3,J+2)=SXX(K)
13 CONTINUE
   DO 110 I=1,6
   J=8+2*I
   FN(4,J)=SF(I)
10 FN(5,J+1)=SF(I)
   CALL TIMES(FN,P,FNP,21,5,1,2)
   RETURN
   END
   SUBROUTINE   SUBTI
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/STI/X(3,20),ELMOD(12),STUCK(36,36),STICK(8,36),FORCE(36)
  1,INFO(20)
   COMMON/PAT/CO2(3,3),XI(3),Y(3)
   COMMON/JCN/SM(21,21),S(5,21),D(5,5)
   COMMON/JAC/FNP(21,1),PIN(15),P(5),PI(15)
   DO 100 I=1,5
   DO 100 J=1,5
   D(J,I)=0.
   D(1,1)=ELMOD(1)
   D(1,2)=ELMOD(2)
   D(2,2)=ELMOD(3)
   D(3,3)=ELMOD(4)/2.
   D(4,4)=ELMOD(5)
   D(5,5)=ELMOD(6)
   FORM    THE   WHOLE   SYMMETRIC    MATRIX  D
   DO 210 J=1,5
   DO 210 I=1,5
   D(I,J)=D(J,I)
1  CONTINUE
   IF(ELMOD(12).NE.0.) D(2,1)=ELMOD(12)
   DO 224 I=1,5
24 P(I)=0.
   P(1)=X(3,5)
   DIMENSION A(3),B(3),C(3),LENG(3),H(7),A1(7),A2(7),A3(7),FNK(21),
  1SMK(21,21),TIK(5,21),M1(21),SMK1(21,21)
   DO 200  I=1,3
```

```
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      B(I)=Y(K)-Y(J)
      C(I)=XI(J)-XI(K)
      LENG(I)=DSQRT(B(I)**2+G(I)**2)
00    CONTINUE
      DT=B(1)*G(2)-B(2)*G(1)
C     QUINTIC   INTEGRATION   FOR STIFFNESS MATRIX   STUCK
C     AREA    COORDINATES    L1   L2   L3     ARCC          A(I)
C     WEIGHTS   OF   INTEGRATION              W(I)
      DATA A1/0.33333333D 00,0.05971587D 00,2*0.47014206D 00,2*0.79742699D
     1   ,2*0.10128651D   /,A3/ .33333333D 00,2*0.47014206D 00,0.0597158
     27D 00,2*0.10128651D 00,0.79742699D 00/,W/0.22500000D 00,3*0.132394
     315D 00,3*0.12593918D 00/
      DO 211 I=1,21
      DO 211 J=1,21
11    SMK(J,I)=0.
      DO 222 I=1,21
22    FNK(I)=0.
      DO 212 K=1,7
      A(1)=A1(K)
      A(3)=A3(K)
      A(2)=1.-A(1)-A(3)
      CALL SHAPEF(A,B,G,DT,0)
      IF(K.NE.1) GO TO 801
      DO 802 I=1,21
      DO 802 J=1,5
802   TIK(J,I)=S(J,I)
801   CONTINUE
      DO 221 I=1,21
221   FNK(I)=FNK(I)+DT/2.*W(K)*FNP(I,1)
      DO 215 I=1,21
      DO 215 J=1,21
15    SMK(J,I)=SMK(J,I)+DT/2.*W(K)*SM(J,I)
12    CONTINUE
      DATA M1/1,3,2,1 ,11,4,6,5,12,13,7,9,8,14,15,16,17,18,19,20,21/
      DIMENSION TOT(21,21)
      DO 247 I=1,21
      DO 247 J=1,21
47    TOT(J,I)=0.
      DO 248 I=1,21
48    TOT(M1(I),I)=1.
      CALL TIMES(SMK,TOT,SMK1,21,21,21,1)
      CALL TIMES(TOT,SMK1,SMK,21,21,21,2)
      CALL TIMES(TIK,TOT,S,5,21,21,1)
      CALL TIMES(TOT,FNK,FNP,21,21,1,2)
      DO 216 I=1,21
      DO 216 J=1,21
16    STUCK(J,I)=SMK(J,I)
      DO 217 I=1,21
      DO 217 J=1,5
17    STICK(J,I)=S(J,I)
      DO 218 I=1,21
18    FORCE(I)=FNP(I,1)
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KCK)
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
      DIMENSION A(1),B(1),R(1)
      KOK=1   A(N,M) , B(M,L) , R(N,L)    REGULAR    A*B
      KOK=2   A(M,N) , B(M,L) , R(N,L)    TRANSPOSE A*B
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
01    CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
03    R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
2     CONTINUE
      DO 104 I=1,M
      IA=N*(J-1)+I
      IB=M*(K-1)+I
04    R(IR)=R(IR)+A(IA)*B(IB)
      IR=IR+1
      RETURN
      END
```

3. Reference symbol PMX12

```
 P=100 PRCUTE=DURH COPIES=4
= UNIVERSITY, BATCH
WAS: 11:36:26
IGNED ON AT  11:37:14 ON MON SEP 22/75
RINT
 ***** MIXED  MODEL  12 DEGREES OF FREEDOM W, MXX, MYY, MXY  3 NODES *****
     ***** STRESSES   CONSTANT   WITHIN  AN ELEMENT  QX ,QY **********
    ****** TRANSFORMATION   W , MNN , MSS , MNS      ****************
    ****** LINEAR   VARIATION    OF THE  FUNCTIONS    ****************
    SUBROUTINE  STIFF
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(20)
    COMMON/MAN/BOL(2,36,36),COL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
    COMMON/COOR/X1(3),Y1(3)
    DIMENSION TRAN(12,12),SMK(12,12),SMK1(12,12),STR(2,12),
   1TIK(2,12),DADG(12),XM(16,2),YM(16,2),CC3(3,3)
    DO 100 I=1,3
    XL(I)=X(1,I)
    Y1(I)=X(2,I)
    DO 101 I=1,36
    DO 101 J=1,36
 1  STUCK(J,I)=0.
    DO 102 I=1,36
    DO 102 J=1,8
102 STICK(J,I)=0.
    DO 103 I=1,36
 3  FORCE(I)= .
    IF(NGRAM.EQ.0) GO TO 513
    DO 516 M=1,NGRAM
    DO 516 I=1,2
    XM(M,I)=GRAM(M,I)
16  YM(M,I)=GRAM(M,I+2)
13  CONTINUE
    NLAK=INFO(1)
    IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 1 4
    CALL SUBTI
    IF(NLAK.EQ.0)GO TO 105
    NLAK=NLAK-1
    DO 106 I=1,12
    DO 106 J=1,12
 6  BOL(NLAK,J,I)=STUCK(J,I)
    DO 113 I=1,12
    DO 113 J=1,2
13  COL(NLAK,J,I)=STICK(J,I)
    DO 108 I=1,12
08  DOL(NLAK,I)=FORCE(I)
    GO TO 105
04  DO 109 I=1,12
    DO 109 J=1,12
09  STUCK(J,I)=BOL(NLAK,J,I)
    DO 114 I=1,12
    DO 114 J=1,2
14  STICK(J,I)=COL(NLAK,J,I)
    DO 111 I=1,12
11  FORCE(I)=DOL(NLAK,I)
05  CONTINUE
    NLIK=INFO(2)
    IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
    DO 213 I=1,12
```

```
      DO 213 J=1,12
213   SMK(J,I)=STUCK(J,I)
      DO 214 I=1,12
      DO 214 J=1,2
214   STR(J,I)=STICK(J,I)
      DO 115 I=1,NLIK
      K=(I-1)*2+3
      L=INFO(K)
      KI=INFO(K+1)
      DO 583 II=1,12
      DO 584 IJ=1,12
584   TRAN(IJ,II)= .
583   TRAN(II,II)=1.
      CALL TRANL(XM,YM,KI,CC3)
      KK=(L-1)*4
      DO 587 IN=1,3
      DO 587 IL=1,3
587   TRAN(KK+IN+1,KK+IL+1)=CC3(IN,IL)
      CALL TIMES(SMK,TRAN,SMK1,12,12,12,1)
      CALL TIMES(TRAN,SMK1,SMK,12,12,12,2)
      CALL TIMES(STR,TRAN,TIK,2,12,12,1)
      DO 116 NI=1,12
      DO 116 NJ=1,2
116   STR(NJ,NI)=TIK(NJ,NI)
115   CONTINUE
      DO 117 I=1,12
      DO 117 J=1,12
117   STUCK(J,I)=SMK(J,I)
      DO 118 I=1,12
      DO 118 J=1,2
118   STICK(J,I)=STR(J,I)
112   CONTINUE
      BRPO=0.
      DO 604 J=6,8
      DO 6 4 I=1,3
604   IF(X(I,J).NE.0.)BRPO=1.
      IF(BRPO.NE.1.) GO TO 560
      DO 273 I=1,12
273   DAOG(I)=0.
      DAOG(1)=X(3,6)
      DAOG(5)=X(3,7)
      DAOG(9)=X(3,8)
      DO 274 I=1,12
274   FORCE(I)=FORCE(I)+DAOG(I)
560   CONTINUE
      RETURN
      END
      SUBROUTINE SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(2 )
      COMMON/COOR/XL(3),YL(3)
      COMMON/TREA/B(3),G(3),EL(3),DT
      DIMENSION FNM1(2,9),FNW2(2,3),FFB(3,3),FFS(2,2),STNM(9,9),FNM(2,9
     1),STMO(9,9),STN(9,9),STMM(9,3),TT(12,12),SIM(12,12),M1(12),STR(2,
     212),STR1(2,12),GE(12,12)
      DO 100 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
```

```
      IF(J.GT.3)J=J-3
      B(I)=YL(K)-YL(J)
      C(I)=XL(J)-XL(K)
 00   EL(I)=B(I)**2+C(I)**2
      DT=B(1)*C(2)-B(2)*C(1)
      D11=YOUNG(1)
      D12=YOUNG(2)
      D22=YOUNG(3)
      D33=YOUNG(4)/2.
      D44=YOUNG(5)
      D55=YOUNG(6)
      D21=D12
      DDDD=YOUNG(12)
      IF(DDDD.NE.0.) D21=DDDD
      DO 101 I=1,3
      DO 1 1 J=1,3
 01   EEB(J,I)=0.
      DO 102 I=1,2
      DO 102 J=1,2
 02   EES(J,I)=0.
      DDT=D11*D22-D12*D21
      IF(DDT.EQ.0.)WRITE(6,700)
 00   FORMAT(/,'        *****  ERROR    ****  ELASTICITY  MATRIX ',/)
      EEB(1,1)=D22/DDT
      EEB(1,2)=-D12/DDT
      EEB(2,1)=-D21/DDT
      EEB(2,2)=D11/DDT
      EEB(3,3)=1./D33
      EES(1,1)=1./D44
      EES(2,2)=1./D55
      DO 103 I=1,9
      DO 1 3 J=1,2
 03   ENM1(J,I)=0.
      DO 104 I=1,3
      J=3*I-2
      ENM1(1,J)=B(I)/DT
      ENM1(2,J+1)=C(I)/DT
      ENM1(1,J+2)=G(I)/DT
 04   ENM1(2,J+2)=B(I)/DT
      DO 1 5 I=1,3
      ENW2(1,I)=B(I)/DT
 05   ENW2(2,I)=G(I)/DT
      DO 106 I=1,9,3
      DO 106 J=1,9,3
      AS=DT/24.
      K=I-J
      IF(K.LT.3.AND.K.GT.-3) AS=DT/12.
      STMM(I,J)=EEB(1,1)*AS
      STMM(I+1,J)=EEB(2,1)*AS
      STMM(I+2,J)=EEB(3,1)*AS
      STMM(I,J+1)=EEB(1,2)*AS
      STMM(I+1,J+1)=EEB(2,2)*AS
      STMM(I+2,J+1)=EEB(3,2)*AS
      STMM(I,J+2)=EEB(1,3)*AS
      STMM(I+1,J+2)=EEB(2,3)*AS
    6 STMM(I+2,J+2)=EEB(3,3)*AS
      CALL TIMES(EES,ENM1,ENM,2,2,9,1)
      CALL TIMES(ENM1,ENM,STMG,9,2,9,2)
      CALL TIMES(ENM1,ENW2,STMH,9,2,3,2)
      DO 107 I=1,9
```

```
      DO 1 7 J=1,9
107  STM(J,I)=STMM(J,I)+DT/2.*STMO(J,I)
      DO 110 I=1,12
      DO 11  J=1,12
      TT(J,I)=0.
110  SIM(J,I)=0.
      DO 111 I=1,3
      DO 111  J=1,9
111  SIM(J+3,I)=STMW(J,I)*DT/2.
      DO 112 I=1,9
      DO 112 J=1,3
112  SIM(J,I+3)=STMW(I,J)*DT/2.
      DO 113 I=1,9
      DO 113 J=1,9
113  SIM(J+3,I+3)=-STM(J,I)
      DO 114 I=1,12
      DO 114 J=1,2
114  STR(J,I)=0.
      DO 115 I=1,9
      DO 115 J=1,2
115  STR(J,I+3)=FNM1(J,I)
      DATA M1/1,4,5,6,2,7,3,9,8,10,11,12/
      DO 116 I=1,12
116  TT(M1(I),I)=1.
      CALL TIMES(SIM,TT,GE,12,12,12,1)
      CALL TIMES(TT,GE,SIM,12,12,12,2)
      CALL TIMES(STR,TT,STR1,2,12,12,1)
      DO 117 I=1,12
      DO 117 J=1,12
117  STUCK(J,I)=SIM(J,I)
      DO 118 I=1,12
      DO 118 J=1,2
118  STICK(J,I)=STR1(J,I)
      PG=X(3,5)
      FORCE(1)=DT/6.*PG
      FORCE(5)=DT/6.*PG
      FORCE(9)=DT/6.*PG
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),R(1)
      KOK=1  A(N,M) , B(M,L) , R(N,L)    REGULAR    A*B
      KOK=2  A(M,N) , B(M,L) , R(N,L)    TRANSPOSE A'*B
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
101  CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
103  R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
102  CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
104  R(IR)=R(IR)+A(IA)*B(IB)
```

```
CO  IR=IR+1
    RETURN
    END
    SUBROUTINE TRANL(X,Y,K1,TRL)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION X(16,2),Y(16,2),X2(2),Y2(2),TRL(3,3)
    DO 515 J=1,2
    X2(J)=X(K1,J)
15  Y2(J)=Y(K1,J)
    G2=X2(2)-X2(1)
    B2=Y2(2)-Y2(1)
    GLEN=DSQRT(B2**2+G2**2)
    IF(GLEN.EQ.0.) WRITE(6,700)
    COB=B2/GLEN
    SIB=-G2/GLEN
    TRL(1,1)=COB**2
    TRL(1,3)=-2.*COB*SIB
    TRL(1,2)=SIB**2
    TRL(2,1)=SIB**2
    TRL(2,3)=2.*COB*SIB
    TRL(2,2)=COB**2
    TRL(3,1)=COB*SIB
    TRL(3,3)=COB**2-SIB**2
    TRL(3,2)=-COB*SIB
7   FORMAT(/,'        ***** ERROR  ******  GLEN   SUB. TRANL',/)
    RETURN
    END
```

4. <u>Reference symbol PMX24</u>

```
C P=100 PRCUTF=DURH CCPIES=4
= UNIVERSITY, BATCH
 WAS: 11:37:14
SIGNED ON AT  11:37:19 CN MON SEP 22/75
PRINT*
   **********    MIXED  MODEL  24   DEGREES OF  FREEDOM  W, MX, MY, MXY,  6 NO
   **********   STRESSES  QX, QY AT FIRST AND SECOND NODES  *****************
   **********     TRANSFORMATION   TO U,  MNN,  MSS,  MSN   *********************
   **********      QUADRATIC  VARIATION   OF THE FUNCTIONS        ************
    SUBROUTINE  STIFF
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(20)
    COMMON/MAN/BCL(2,36,36),CCL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
    COMMON/COOR/X1(3),Y1(3)
    DIMENSION TRAN(24,24),SMK(24,24),SMK1(24,24),STP(4,24),TIK(4,24),
   1DANG(24),XM(16,2),YM(16,2),CO3(3,3)
    DO 100 I=1,3
    X1(I)=X(1,I)
100 Y1(I)=X(2,I)
    DO 101 I=1,36
    DO 101 J=1,36
101 STUCK(J,I)=0.
    DO 102 I=1,36
    DO 102 J=1,8
102 STICK(J,I)=0.
    DO 103 I=1,36
103 FORCE(I)=0.
    IF(NGRAM.EQ.0) GO TO 517
    DO 516 M=1,NGRAM
    DO 516 I=1,2
    XM(M,I)=GRAM(M,I)
516 YM(M,I)=GRAM(M,I+2)
517 CONTINUE
    NLAK=INFO(1)
    IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
    CALL SUBTI
    IF(NLAK.EQ.0)GO TO 105
    NLAK=NLAK-10
    DO 106 I=1,24
    DO 106 J=1,24
106 BCL(NLAK,J,I)=STUCK(J,I)
    DO 113 I=1,24
    DO 113 J=1,4
113 CCL(NLAK,J,I)=STICK(J,I)
    DO 108 I=1,24
108 DCL(NLAK,I)=FORCE(I)
    GO TO 105
104 DO 109 I=1,24
    DO 109 J=1,24
109 STUCK(J,I)=BCL(NLAK,J,I)
    DO 214 I=1,24
    DO 214 J=1,4
214 STICK(J,I)=CCL(NLAK,J,I)
    DO 111 I=1,24
111 FORCE(I)=DCL(NLAK,I)
105 CONTINUE
    NLIK=INFO(2)
    IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
    DO 213 I=1,24
```

```
   CO 213 J=1,24
13 SMK(J,I)=STUCK(J,I)
   CO 114 I=1,24
   CO 114 J=1,4
14 STR(J,I)=STICK(J,I)
   CO 115 I=1,NLIK
   K=(I-1)*2+3
   L=INFO(K)
   K1=INFO(K+1)
   DO 583 II=1,24
   DO 584 IJ=1,24
84 TRAN(IJ,II)=0.
83 TRAN(II,II)=1.
   CALL TRANL(XM,YM,K1,CO3)
   KK=(L-1)*4
   DO 587 IN=1,3
   CO 587 IL=1,3
87 TRAN(KK+IN+1,KK+IL+1)=CO3(IN,IL)
   CALL TIMES(SMK,TRAN,SMK1,24,24,24,1)
   CALL TIMES(TRAN,SMK1,SMK,24,24,24,2)
   CALL TIMES(STR,TRAN,TIK,4,24,24,1)
   CO 116 NI=1,24
   CO 116 NJ=1,4
16 STR(NJ,NI)=TIK(NJ,NI)
15 CONTINUE
   CO 117 I=1,24
   CO 117 J=1,24
17 STUCK(J,I)=SMK(J,I)
   DO 118 I=1,24
   DO 118 J=1,4
18 STICK(J,I)=STR(J,I)
12 CONTINUE
   BRPO=0.
   CO 604 J=6,11
   DO 604 I=1,3
04 IF(X(I,J).NE.0.)BRPO=1.
   IF(BRPO.NE.1.) GO TO 560
   CO 273 I=1,24
73 DAOG(I)=0.
   DAOG(1)=X(3,6)
   DAOG(5)=X(3,7)
   DAOG(9)=X(3,8)
   DAOG(13)=X(3,9)
   DAOG(17)=X(3,10)
   DAOG(21)=X(3,11)
   DO 274 I=1,24
74 FORCE(I)=FORCE(I)+DAOG(I)
60 CONTINUE
   RETURN
   END
   SUBROUTINE SUBTI
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/STI/X(3,20),YO(12),STUCK(36,36),STICK(8,36),FORCE(36),
  1INFO(20)
   COMMON/TRDA/B(3),G(3),EL(3),DT
   COMMON/OLCEP/EMM(18,18),EMO(18,18),STMW(18,6),STR(4,24)
   COMMON/COOR/XL(3),YL(3)
   DIMENSION STM(18,18),TT(24,24),M1(24),SIM(24,24),CE(24,24)
  1,STR1(4,24)
   DO 1  I=1,3
```

```
    K=I+1
    J=I+2
    IF(K.GT.3)K=K-3
    IF(J.GT.3)J=J-3
    B(I)=YL(K)-YL(J)
    G(I)=XL(J)-XL(K)
00  FL(I)=B(I)**2+G(I)**2
    DT=B(1)*G(2)-B(2)*G(1)
    DO 111 I=1,24
    DO 111 J=1,4
11  STR(J,I)=0.
    CALL INTSU
    DO 101 I=1,18
    DO 101 J=1,18
01  STM(J,I)=EMM(J,I)+EMG(J,I)
    DO 102 I=1,24
    DO 102 J=1,24
    TT(J,I)=0.
02  SIM(J,I)=0.
    DO 103 I=1,6
    DO 103 J=1,18
03  SIM(J+6,I)=STMW(J,I)
    DO 104 I=1,18
    DO 104 J=1,6
04  SIM(J,I+6)=STMW(I,J)
    DO 105 I=1,18
    DO 105 J=1,18
05  SIM(J+6,I+6)=-STM(J,I)
    DATA M1/1,7,8,9,2,10,11,12,3,13,14,15,4,16,17,18,5,19,20,21,
   16,22,23,24/
    DO 107 I=1,24
07  TT(M1(I),I)=1.
    CALL TIMES(SIM,TT,GE,24,24,24,1)
    CALL TIMES(TT,GE,SIM,24,24,24,2)
    CALL TIMES(STR,TT,STR1,4,24,24,1)
    DO 106 I=1,24
    DO 106 J=1,24
06  STUCK(J,I)=SIM(J,I)
    DO 112 I=1,24
    DO 112 J=1,4
12  STICK(J,I)=STR1(J,I)
    PG=X(3,5)
    FORCE(13)=DT/6.*PG
    FORCE(17)=DT/6.*PG
    FORCE(21)=DT/6.*PG
    RETURN
    END
    SUBROUTINE TRANL(X,Y,K1,TRL)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION X(16,2),Y(16,2),X2(2),Y2(2),TRL(3,3)
    DO 515 J=1,2
    X2(J)=X(K1,J)
15  Y2(J)=Y(K1,J)
    G2=X2(2)-X2(1)
    B2=Y2(2)-Y2(1)
    GLEN=DSQRT(B2**2+G2**2)
    IF(GLEN.EQ.0.) WRITE(6,700)
    COB=B2/GLEN
    SIB=-G2/GLEN
    TRL(1,1)=COB**2
```

```
      TRL(1,3)=-2.*COB*SIB
      TRL(1,2)=SIB**2
      TRL(2,1)=SIB**2
      TRL(2,3)=2.*COB*SIB
      TRL(2,2)=COB**2
      TRL(3,1)=COB*SIB
      TRL(3,3)=COB**2-SIB**2
      TRL(3,2)=-COB*SIB
      FORMAT(/,'        ***** ERROR  ******  GLEN   SUB. TRANL',/)
      RETURN
      END
      SUBROUTINE INTSU
      IMPLICIT REAL*8 (A-H,O-Z)
      D21=YOUNG(12)
      COMMON/TREA/B(3),C(3),EL(3),DT
      COMMON/OLDEP/EMM(18,18),EMQ(18,18),EMW(18,6),STR(4,24)
      COMMON/STI/X(3,2 ),YOUNG(12),ST(36,36),SF(8,36),FCR(36),INFO(20)
      DIMENSION A1(7),A2(7),A3(7),W(7),EFB(3,3),EFS(2,2),A(3),
     1FN(6),EE(6),FF(6),ENMO(3,18),ENW2(2,6),ENM1(2,18)
     2,EMW1(18,6),EMM1(3,18),EMM2(18,18),EMQ1(2,18),EMQ2(18,18)
      DATA A1/0.33333333D 00,0.05971587D 00,2*0.47014206D 00,0.79742699
     1   ,2* .1 128651D   /,A3/ .33333333D   ,2* .47 142 6D   , . 597158
     27D 00,2*0.10128651D 00,0.79742699D 00/,W/0.22500000D 00,3*0.132394
     315D 00,3*0.12593918D 00/
      D11=YOUNG(1)
      D12=YOUNG(2)
      D22=YOUNG(3)
      D33=YOUNG(4)/2.
      D44=YOUNG(5)
      D55=YOUNG(6)
      D21=D12
      DODO=YOUNG(12)
      IF(DODO.NE. .)D21=DODO
      DO 100 I=1,3
      DO 100 J=1,3
      EFB(J,I)=0.
      DO 101 I=1,2
      DO 1 1 J=1,2
01    EFS(J,I)=0.
      DOT=D11*D22-D12*D21
      IF(DOT.EQ. .)WRITE(6,7  )
00    FORMAT(/,'        *****  ERROR   ****  ELASTICITY  MATRIX ',/)
      EFB(1,1)=D22/DOT
      EFB(1,2)=-D12/DOT
      EFB(2,1)=-D21/DOT
      EFB(2,2)=D11/DOT
      EFB(3,3)=1./D33
      EFS(1,1)=1./D44
      EFS(2,2)=1./D55
      DO 107 I=1,18
      DO 107 J=1,18
      EMM(J,I)=0.
07    EMQ(J,I)=0.
      DO 1 8 I=1,6
      DO 108 J=1,18
08    EMW(J,I)=0.
           INTEGRATION
      DO 102 K=1,9
      IF(K.EQ.8) A(1)=1.
      IF(K.EQ.8) A(2)=0.
```

```
      IF(K.EQ.8) A(3)=0.
      IF(K.EQ.9) A(1)=0.
      IF(K.EQ.9) A(2)=1.
      IF(K.EQ.9) A(3)=0.
      IF(K.GT.7)GO TO 112
      A2(K)=1.-A1(K)-A3(K)
      A(1)=A1(K)
      A(2)=A2(K)
      A(3)=A3(K)
 12   CONTINUE
      EN(1)=(2.*A(1)-1.)*A(1)
      EN(2)=(2.*A(2)-1.)*A(2)
      EN(3)=(2.*A(3)-1.)*A(3)
      EN(4)=4.*A(2)*A(3)
      EN(5)=4.*A(1)*A(3)
      EN(6)=4.*A(1)*A(2)
      EE(1)=B(1)*(4.*A(1)-1.)/DT
      EE(2)=B(2)*(4.*A(2)-1.)/DT
      EE(3)=B(3)*(4.*A(3)-1.)/DT
      EE(4)=4.*(B(2)*A(3)+B(3)*A(2))/DT
      EE(5)=4.*(B(1)*A(3)+B(3)*A(1))/DT
      EE(6)=4.*(B(1)*A(2)+B(2)*A(1))/DT
      FF(1)=G(1)*(4.*A(1)-1.)/DT
      FF(2)=G(2)*(4.*A(2)-1.)/DT
      FF(3)=G(3)*(4.*A(3)-1.)/DT
      FF(4)=4.*(G(2)*A(3)+G(3)*A(2))/DT
      FF(5)=4.*(G(1)*A(3)+G(3)*A(1))/DT
      FF(6)=4.*(G(1)*A(2)+G(2)*A(1))/DT
      DO 103 I=1,18
      DO 103 J=1,3
 03   ENM (J,I)=0.
      DO 203 I=1,18
      DO 203 J=1,2
  3   ENM1(J,I)=0.
      DO 104 J=1,6
      DO 104 I=1,3
      M=3*J+I-3
 04   ENM0(I,M)=EN(J)
      DO 105 J=1,18,3
      M=(J+2)/3
      ENM1(1,J)=EE(M)
      ENM1(1,J+2)=FF(M)
      ENM1(2,J+1)=FF(M)
 05   ENM1(2,J+2)=EE(M)
      IF(K.GT.7)GO TO 113
      DO 106 J=1,6
      ENW2(1,J)=EE(J)
 06   ENW2(2,J)=FF(J)
      CALL TIMES(ENM1,ENW2,EMW1,18,2,6,2)
      CALL TIMES(EEB,ENM ,ENM1,3,3,18,1)
      CALL TIMES(ENM0,ENM1,ENM2,18,3,18,2)
      CALL TIMES(EES,ENM1,EMO1,2,2,18,1)
      CALL TIMES(ENM1,EMO1,EMO2,18,2,18,2)
      DO 109 I=1,18
      DO 109 J=1,18
      EMM(J,I)=EMM(J,I)+W(K)*EMM2(J,I)*DT/2.
 09   EMO(J,I)=EMO(J,I)+W(K)*EMO2(J,I)*DT/2.
      DO 11  I=1,6
      DO 110 J=1,18
 10   EMW(J,I)=EMW(J,I)+W(K)*EMW1(J,I)*DT/2.
```

```
13  CONTINUE
    IF(K.LT.8) GO TO 102
    L=0
    IF(K.EQ.9) L=2
    DO 114 I=1,18
    DO 114 J=1,2
14  STR(L+J,I+6)=ENM1(J,I)
02  CONTINUE
    RETURN
    END
    SUBROUTINE TIMES(A,B,R,N,M,L,KCK)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION A(1),B(1),R(1)
    KCK=1   A(N,M)  ,  B(M,L)  ,  R(N,L)     REGULAR     A*B
    KCK=2   A(M,N)  ,  B(M,L)  ,  R(N,L)     TRANSPOSE  A'*B
    IR=1
    DO 100 K=1,L
    DO 100 J=1,N
    R(IR)=0.
    GO TO(101,102),KCK
01  CONTINUE
    DO 103 I=1,M
    IA=N*(I-1)+J
    IB=M*(K-1)+I
03  R(IR)=R(IR)+A(IA)*B(IB)
    GO TO 100
02  CONTINUE
    DO 104 I=1,M
    IA=M*(J-1)+I
    IB=M*(K-1)+I
04  R(IR)=R(IR)+A(IA)*B(IB)
00  IR=IR+1
    RETURN
    END
```

5. <u>Reference symbol PDS24</u>

```
 P=100 "PROUTE=PURE COPIES=4
= UNIVERSITY, BATCH
WAS: 11:37:15
IGNED CN AT  11:48:40 CN MON SEP 22/75
RINTS
   **** PLATE  DISPL. ELEMENT 24 DC. CF FREEDOM AT 3 NODES *****
   ****  QUINTIC  VARIATION OF W ( CUBIC VAR. OF WN ALONG THE SIDES )****
   ****  LINEAR   VARIATION CF SHEAR ANGLE  FX,FY          *****
   ****  STRESSES MXX,MYY,MXY,QX,QY   AT CENTROID          *******
    **** MODULI OF ELASTISITY THROUGH  YOUNG(12)     ********
   ****TRANSFORMATION TO  W, WN, WS, WNN, WSN, WSS, FN, FS  **************
    SUBROUTINE TRANL(X,Y,Z,K,TRL)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(7,7),X2(2),Y2(2)
    DO 515 J=1,2
    X2(J)=X(K,J)
15  Y2(J)=Y(K,J)
    C2=X2(2)-X2(1)
    B2=Y2(2)-Y2(1)
    GLEN2=DSQRT(C2**2+B2**2)
    IF(GLEN2.LT. .1D-12.AND.GLEN2.GT.- .1D-12) WRITE(6,  )
    B2=B2/GLEN2
    G2=C2/GLEN2
    DO 516 I=1,7
    DO 516 J=1,7
16  TRL(J,I)=0.
    TRL(1,1)=B2
    TRL(1,2)=G2
    TRL(1,6)=B2
    TRL(2,1)=-G2
    TRL(2,2)=B2
    TRL(2,6)=-G2
    TRL(3,3)=B2**2
    TRL(3,4)=2.*B2*G2
    TRL(3,5)=G2**2
    TRL(4,3)=-B2*G2
    TRL(4,4)=B2**2-G2**2
    TRL(4,5)=B2*G2
    TRL(5,3)=G2**2
    TRL(5,4)=-2.*B2*G2
    TRL(5,5)=B2**2
    TRL(6,6)=B2
    TRL(6,7)=G2
    TRL(7,6)=-G2
    TRL(7,7)=B2
00  FORMAT('     *****     ERROR     TRANL     *****')
    RETURN
    END
    SUBROUTINE  STIFF
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,2 ),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(20)
    COMMON/MAN/BCL(2,36,36),COL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
    COMMON/CLOR/CO2(3,3),X0(3),Y0(3),CGM(3,3)
    DIMENSION TRAN(24,24),SMK(24,24),STR(5,24),DANG(24),
   1XM(16,2),YM(16,2),ZM(16,2),X1(3),Y1(3),Z1(3),CC3(7,7)
   2,C1(3),B1(3),C1(3),STU(24,24),TIK(5,24),FOR(24)
    DO 100 I=1,3
    X0(I)=X(1,I)
    DO 101 I=1,36
```

```
    DO 101 J=1,36
101 STUCK(J,I)=0.
    DO 102 I=1,36
    DO 102 J=1,3
102 STICK(J,I)=0.
    DO 103 I=1,36
103 FORCE(I)=0.
    DO 607 I=1,24
607 DAOG(I)=0.
    IF(NGRAM.EQ.0) GO TO 517
    DO 516 M=1,NGRAM
    DO 516 I=1,2
    XM(M,I)=GRAM(M,I)
    ZM(M,I)=GRAM(M,I+4)
516 YM(M,I)=GRAM(M,I+2)
517 CONTINUE
    CNLD=0.
    DO 432 I=1,3
    DO 432 J=6,8
432 IF(X(I,J).NE. .) CNLD=1.
    IF(CNLD.NE.1.)GO TO 437
    DO 433 I=6,8
    J=8*(I-6)+1
433 DAOG(J)=X(3,I)
437 CONTINUE
    NLAK=INFO(1)
    IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
    CALL SUBTI
    IF(NLAK.EQ.0)GO TO 105
    NLAK=NLAK-10
    DO 106 I=1,24
    DO 106 J=1,24
106 BCL(NLAK,J,I)=STUCK(J,I)
    DO 113 I=1,24
    DO 113 J=1,5
113 CCL(NLAK,J,I)=STICK(J,I)
    DO 108 I=1,24
108 DCL(NLAK,I)=FORCE(I)
    GO TO 105
104 DO 109 I=1,24
    DO 109 J=1,24
109 STUCK(J,I)=BCL(NLAK,J,I)
    DO 214 I=1,24
    DO 214 J=1,5
214 STICK(J,I)=CCL(NLAK,J,I)
    DO 111 I=1,24
111 FORCE(I)=DCL(NLAK,I)
105 CONTINUE
    DO 438 I=1,24
438 FORCE(I)=FORCE(I)+DAOG(I)
    NLIK=INFO(2)
    IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 115
    DO 601 I=1,24
    DO 601 J=1,24
601 STU(J,I)=STUCK(J,I)
    DO 602 I=1,24
    DO 602 J=1,5
602 TIK(J,I)=STICK(J,I)
    DO 603 I=1,24
603 FOR(I)=FORCE(I)
```

```
    DO 116 I=1,NLIK
    K=(I-1)*2+3
    L=INEC(K)
    K1=INEC(K+1)
    DO 583 II=1,24
    DO 584 IJ=1,24
584 TRAN(IJ,II)=0.
583 TRAN(II,II)=1.
    KK=(L-1)*8
    CALL TRANI(XM,YM,ZM,K1,CO3)
    DO 222 IL=1,7
    DO 222 JL=1,7
222 TRAN(KK+IL+1,KK+JL+1)=CO3(IL,JL)
    CALL TIMES(STU,TRAN,SMK,24,24,24,1)
    CALL TIMES(TRAN,SMK,STU,24,24,24,2)
    CALL TIMES(TIK,TRAN,STR,5,24,24,1)
    CALL TIMES(TRAN,FOR,DACC,24,24,1,2)
    DO 232 IL=1,24
    DO 232 JL=1,5
232 TIK(JL,IL)=STR(JL,IL)
    DO 233 IL=1,24
233 FOR(IL)=DACC(IL)
116 CONTINUE
    DO 604 I=1,24
    DO 604 J=1,24
604 STUCK(J,I)=STU(J,I)
    DO 605 I=1,24
    DO 605 J=1,5
605 STICK(J,I)=TIK(J,I)
    DO 606 I=1,24
606 FORCE(I)=FOR(I)
615 CONTINUE
    RETURN
    END
    SUBROUTINE  SHAPEF(A,B,G,DT)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/JCN/SM(24,24),S(5,24),D(5,5),P(8),END(24,1)
    DIMENSION  BE(5,24),BET(3,18),CET(3,3),A(3),B(3),G(3)
   1,BED(3,18),CBI(6,6),TEM(6,18),CUN(6,18),EN(8,24)
    COMMON/DORA/CM(5),OK(3)
    CET(1,1)=(B(1)/DT)**2
    CET(1,2)=2.*B(1)*B(2)/DT**2
    CET(1,3)=(B(2)/DT)**2
    CET(2,1)=(G(1)/DT)**2
    CET(2,2)=2.*G(1)*G(2)/DT**2
    CET(2,3)=(G(2)/DT)**2
    CET(3,1)=2.*G(1)*B(1)/DT**2
    CET(3,2)=2.*(G(1)*B(2)+G(2)*B(1))/DT**2
    CET(3,3)=2.*G(2)*B(2)/DT**2
    DO 100 I=1,3
    DO 100 J=1,18
    BET(I,J)=0.
100 CONTINUE
    DO 101 I=1,5
    DO 101 J=1,24
    BE(I,J)=0.
101 CONTINUE
    BET(1,4)=2.
    BET(1,7)=6.*A(1)
    BET(1,8)=2.*A(2)
```

```
      BET(1,11)=12.*A(1)**2
      BET(1,12)=6.*A(1)*A(2)
      BET(1,13)=2.*A(2)**2
      BET(1,16)=20.*A(1)**3+12.*OM(1)*A(1)**2*A(2)+2.*OM(2)*A(2)**3
      BET(1,17)=6.*A(1)*A(2)**2+2.*OM(3)*A(2)**3
      BET(1,18)=2.*A(2)**3*OM(5)
      BET(2,5)=1.
      BET(2,8)=2.*A(1)
      BET(2,9)=2.*A(2)
      BET(2,12)=3.*A(1)**2
      BET(2,13)=4.*A(1)*A(2)
      BET(2,14)=3.*A(2)**2
      BET(2,16)=4.*OM(1)*A(1)**3+6.*OM(2)*A(1)*A(2)**2
      BET(2,17)=6.*A(1)**2*A(2)+6.*OM(3)*A(1)*A(2)**2
      BET(2,18)=6.*A(1)*A(2)**2*OM(5)+4.*OM(4)*A(2)**3
      BET(3,6)=2.
      BET(3,9)=2.*A(1)
      BET(3,1 )=6.*A(2)
      BET(3,13)=2.*A(1)**2
      BET(3,14)=6.*A(1)*A(2)
      BET(3,15)=12.*A(2)**2
      BET(3,16)=6.*OM(2)*A(1)**2*A(2)
      BET(3,17)=2.*A(1)**3+6.*OM(3)*A(1)**2*A(2)
      BET(3,18)=20.*A(2)**3+12.*OM(4)*A(1)*A(2)**2+6.*OM(5)*A(1)**2*A(2)
      CALL TIMES(CET,BET,BED,3,3,18,1)
      DO 1 2 I=1,3
      DO 102 J=1,18
      BE(I,J)=-BED(I,J)
 .2   CONTINUE
      DO 103 J=1,3
      K=(J-1)*2+1
      BE(1,18+K)=B(J)/DT
      BE(2,19+K)=C(J)/DT
      BE(3,18+K)=C(J)/DT
      BE(3,19+K)=B(J)/DT
  3   CONTINUE
      BE(4,19)=A(1)
      BE(4,21)=A(2)
      BE(4,23)=A(3)
      BE(5,20)=A(1)
      BE(5,22)=A(2)
      BE(5,24)=A(3)
      CALL TIMES(D,BE,S,5,5,24,1)
      CALL TIMES(BE,S,SM,24,5,24,2)
      DO  104 I=1,6
      DO  104 J=1,6
      CBI(I,J)=0.
 04   CONTINUE
      DO  1 5 I=1,6
      DO  105 J=1,18
      TEN(I,J)=0.
 .5   CONTINUE
      DO  106 I=1,8
      DO  106 J=1,24
      EN(I,J)=0.
 06   CONTINUE
      CBI(1,1)=1.
      CBI(2,2)=B(1)/DT
      CBI(2,3)=B(2)/DT
      CBI(3,2)=C(1)/DT
```

```
      CBI(3,3)=G(2)/DT
      CBI(4,4)=(B(1)/DT)**2
      CBI(4,5)=2.*B(1)*B(2)/DT**2
      CBI(4,6)=(B(2)/DT)**2
      CBI(5,4)=G(1)*B(1)/DT**2
      CBI(5,5)=(G(2)*B(1)+G(1)*B(2))/DT**2
      CBI(5,6)=G(2)*B(2)/DT**2
      CBI(6,4)=(G(1)/DT)**2
      CBI(6,5)=2.*G(1)*G(2)/DT**2
      CBI(6,6)=(G(2)/DT)**2
      TEM(1,1)=1.
      TEM(1,2)=A(1)
      TEM(1,3)=A(2)
      TEM(1,4)=A(1)**2
      TEM(1,5)=A(1)*A(2)
      TEM(1,6)=A(2)**2
      TEM(1,7)=A(1)**3
      TEM(1,8)=A(1)**2*A(2)
      TEM(1,9)=A(1)*A(2)**2
      TEM(1,10)=A(2)**3
      TEM(1,11)=A(1)**4
      TEM(1,12)=A(1)**3*A(2)
      TEM(1,13)=A(1)**2*A(2)**2
      TEM(1,14)=A(1)*A(2)**3
      TEM(1,15)=A(2)**4
      TEM(1,16)=A(1)**5+CM(1)*A(1)**4*A(2)+CM(2)*A(1)**2*A(2)**3
      TEM(1,17)=A(1)**3*A(2)**2+CM(3)*A(1)**2*A(2)**3
      TEM(1,18)=A(2)**5+CM(4)*A(1)*A(2)**4+CM(5)*A(1)**2*A(2)**3
      TEM(2,2)=1.
      TEM(2,4)=2.*A(1)
      TEM(2,5)=A(2)
      TEM(2,7)=3.*A(1)**2
      TEM(2,8)=2.*A(1)*A(2)
      TEM(2,9)=A(2)**2
      TEM(2,11)=4.*A(1)**3
      TEM(2,12)=3.*A(1)**2*A(2)
      TEM(2,13)=2.*A(1)*A(2)**2
      TEM(2,14)=A(2)**3
      TEM(2,16)=5.*A(1)**4+4.*CM(1)*A(1)**3*A(2)+2.*CM(2)*A(1)*A(2)**3
      TEM(2,17)=3.*A(1)**2*A(2)**2+2.*CM(3)*A(1)*A(2)**3
      TEM(2,18)=CM(4)*A(2)**4+2.*CM(5)*A(1)*A(2)**3
      TEM(3,3)=1.
      TEM(3,5)=A(1)
      TEM(3,6)=2.*A(2)
      TEM(3,8)=A(1)**2
      TEM(3,9)=2.*A(1)*A(2)
      TEM(3,10)=3.*A(2)**2
      TEM(3,12)=A(1)**3
      TEM(3,13)=2.*A(1)**2*A(2)
      TEM(3,14)=3.*A(1)*A(2)**2
      TEM(3,15)=4.*A(2)**3
      TEM(3,16)=CM(1)*A(1)**4+3.*CM(2)*A(1)**2*A(2)**2
      TEM(3,17)=2.*A(1)**3*A(2)+3.*CM(3)*A(1)**2*A(2)**2
      TEM(3,18)=5.*A(2)**4+4.*CM(4)*A(1)*A(2)**3+3.*CM(5)*A(1)**2*A(2)**
     12
      TEM(4,4)=2.
      TEM(4,7)=6.*A(1)
      TEM(4,8)=2.*A(2)
      TEM(4,11)=12.*A(1)**2
      TEM(4,12)=6.*A(1)*A(2)
```

```
      TEN(4,13)=2.*A(2)**2
      TEN(4,16)=20.*A(1)**3+12.*OM(1)*A(1)**2*A(2)+2.*OM(2)*A(2)**3
      TEN(4,17)=6.*A(1)*A(2)**2+2.*OM(3)*A(2)**3
      TEN(4,18)=2.*OM(5)*A(2)**3
      TEN(5,5)=1.
      TEN(5,8)=2.*A(1)
      TEN(5,9)=2.*A(2)
      TEN(5,12)=3.*A(1)**2
      TEN(5,13)=4.*A(1)*A(2)
      TEN(5,14)=3.*A(2)**2
      TEN(5,16)=4.*OM(1)*A(1)**3+6.*OM(2)*A(1)*A(2)**2
      TEN(5,17)=6.*A(1)**2*A(2)+6.*OM(3)*A(1)*A(2)**2
      TEN(5,18)=4.*OM(4)*A(2)**3+6.*OM(5)*A(1)*A(2)**2
      TEN(6,6)=2.
      TEN(6,9)=2.*A(1)
      TEN(6,10)=6.*A(2)
      TEN(6,13)=2.*A(1)**2
      TEN(6,14)=6.*A(1)*A(2)
      TEN(6,15)=12.*A(2)**2
      TEN(6,16)=6.*OM(2)*A(1)**2*A(2)
      TEN(6,17)=2.*A(1)**3+6.*OM(3)*A(1)**2*A(2)
      TEN(6,18)=20.*A(2)**3+12.*OM(4)*A(1)*A(2)**2+6.*OM(5)*A(1)**2*A(2)
      CALL TIMES(CBT,TEN,DEN,6,6,18,1)
      DO 107  I=1,6
      DO 107  J=1,18
      EN(I,J)=DEN(I,J)
107   CONTINUE
      EN(7,19)=A(1)
      EN(7,21)=A(2)
      EN(7,23)=A(3)
      EN(8,20)=A(1)
      EN(8,22)=A(2)
      EN(8,24)=A(3)
      CALL TIMES(EN,P,ENP,24,8,1,2)
      RETURN
      END
      SUBROUTINE  SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,2 ),ELMOD(12),STUCK(36,36),STICK(8,36),FORCE(36)
     1,INFO(20)
      COMMON/JCN/SM(24,24),S(5,24),D(5,5),P(8),ENP(24,1)
      COMMON/DOPA/OM(5),CK(3)
      COMMON/COCP/CO2(3,3),XO(3),YO(3),COM(3,3)
      DO 208 I=1,5
      DO 208 J=1,5
208   D(J,I)=0.
      D(1,1)=ELMOD(1)
      D(1,2)=ELMOD(2)
      D(2,2)=ELMOD(3)
      D(3,3)=ELMOD(4)/2.
      D(4,4)=ELMOD(5)
      D(5,5)=ELMOD(6)
      D(2,1)=D(1,2)
      IF(ELMOD(12).NE.0.)D(2,1)=ELMOD(12)
      DATA ALF/0.33333333D00/,ALF1/0.05971587D00/,BET1/0.47014206D00/
      DATA ALF2/0.79742699D00/,BET2/0.10128651D00/
      DATA WE1/0.225D000/,WE2/0.13239415D00/,WE3/0.12593918D00/
      DIMENSION  B(3),G(3),ARCO(2,7),W(7),A(3),DAOI(24,1),LENG(3),
     1T(24,24),SOM(24,24),FOF(24),TIK(5,24),V1(24)
      ARCO(1,1)=ALF
```

```
      ARCO(2,1)=ALF
      ARCO(1,2)=BET1
      ARCO(2,2)=BET1
      ARCO(1,3)=ALF1
      ARCO(2,3)=BET1
      ARCO(1,4)=BET1
      ARCO(2,4)=ALF1
      ARCO(1,5)=BET2
      ARCO(2,5)=BET2
      ARCO(1,6)=ALF2
      ARCO(2,6)=BET2
      ARCO(1,7)=BET2
      ARCO(2,7)=ALF2
      W(1)=WE1
      W(2)=WE2
      W(3)=WE2
      W(4)=WE2
      W(5)=WE3
      W(6)=WE3
      W(7)=WE3
      DO 223 I=1,8
 23   P(I)=0.
      P(1)=X(3,5)
      DO 200 I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      B(I)=YO(K)-YO(J)
      G(I)=XO(J)-XO(K)
      LENG(I)=DSQRT(B(I)**2+G(I)**2)
 00   CONTINUE
      OK(1)=B(1)*B(2)+G(1)*G(2)
      OK(2)=B(2)*B(3)+G(2)*G(3)
      OK(3)=B(3)*B(1)+G(3)*G(1)
      DT=B(1)*G(2)-B(2)*G(1)
      OM(1)=5.*OK(1)/(OK(1)+OK(2))
      OM(2)=5.*(OK(3)*OK(2)+OK(1)*OK(2)-3.*OK(1)*OK(3))/
     1((OK(1)+OK(2))*(2.*OK(3)-3.*OK(2)))
      OM(3)=(3.*OK(3)-2.*OK(2))/(2.*OK(3)-3.*OK(2))
      OM(4)=5.*OK(1)/(OK(1)+OK(3))
      OM(5)=5.*(OK(3)*OK(2)+OK(1)*OK(3)-3.*OK(1)*OK(2))/
     1((OK(1)+OK(3))*(2.*OK(3)-3.*OK(2)))
      CALL ANTI(B,G,DT,T)
      DO 214 I=1,24
      DO 214 J=1,24
      SOM(I,J)=0.
 14   CONTINUE
      DO 314 I=1,24
      PACL(I,1)=0.
 14   CONTINUE
      DO 213  K=1,7
      A(1)=ARCO(1,K)
      A(2)=ARCO(2,K)
      A(3)=1.-A(1)-A(2)
      CALL   SHAPEF(A,B,G,DT)
      IF(K.NE.1)GO TO 510
      DO 511 I=1,24
      DO 511 J=1,5
 11   TIK(J,I)=S(J,I)
```

```
1  CONTINUE
   DO 313 I=1,24
   DACL(I,1)=DACL(I,1)+DT/2.*W(K)*ENP(I,1)
13 CONTINUE
   DO 213 J=1,24
   DO 213 I=1,24
   SOM(I,J)=SOM(I,J)+DT/2.*W(K)*SM(I,J)
13 CONTINUE
   CALL  TIMES(SOM,T,SM,24,24,24,1)
   CALL  TIMES(T,SM,SOM,24,24,24,2)
   CALL  TIMES(TIK,T,S,5,24,24,1)
   CALL  TIMES(T,DACL,FRE,24,24,1,2)
   DATA M1/1,2,3,4,5,6,19,20,7,8,9,10,11,12,21,22,
  113,14,15,16,17,18,23,24/
   DO 224 I=1,24
   DO 224 J=1,24
24 T(J,I)= .
   DO 222 I=1,24
22 T(M1(I),I)=1.
   CALL  TIMES(SOM,T,SM,24,24,24,1)
   CALL  TIMES(T,SM,SOM,24,24,24,2)
   CALL  TIMES(S,T,TIK,5,24,24,1)
   CALL  TIMES(T,FRE,DACL,24,24,1,2)
   DO 550 I=1,24
   DO 55  J=1,24
50 STUCK(J,I)=SOM(J,I)
   DO 551 I=1,24
   DO 551 J=1,5
51 STICK(J,I)=TIK(J,I)
   DO 552 I=1,24
52 FORCE(I)=DACL(I,1)
   RETURN
   END
   SUBROUTINE  ANTI(B,G,DT,T)
   IMPLICIT REAL*8 (A-H,O-Z)
   DIMENSION TI(18,18),BCI(18,18),TO(18,18),T(24,24),B(3),C(3)
   COMMON/DORA/OM(5),OK(3)
   DO 100 I=1,18
   DO 1    J=1,18
   TI(I,J)=0.
00 CONTINUE
   DO 11  I=1,24
   DO 110 J=1,24
   IF(I.EQ.J.AND.I.GT.18) GO TO 111
   T(I,J)=0.
   GO TO 110
11 T(I,J)=1.
10 CONTINUE
   OMI=1.-OM(3)
   IF(OMI.EQ. )GO TO 1 7
   TI(1,13)=1.
   TI(2,14)=1.
   TI(3,15)=1.
   TI(4,16)=0.5
   TI(5,17)=1.
   TI(6,18)=0.5
   TI(7,1)=10.
   TI(7,2)=-4.
   TI(7,4)=0.5
   TI(7,13)=-10.
```

```
      TI(7,14)=-6.
      TI(7,16)=-1.5
      TI(8,1)=6.*OM(1)
      TI(8,2)=-3.*OM(1)
      TI(8,3)=3.
      TI(8,4)=0.5*OM(1)
      TI(8,5)=-1.
      TI(8,13)=-6.*OM(1)
      TI(8,14)=-3.*OM(1)
      TI(8,15)=-3.
      TI(8,16)=-.5*OM(1)
      TI(8,17)=-2.
      TI(9,7)=6.*OM(4)
      TI(9,8)=3.
      TI(9,9)=-3.*OM(4)
      TI(9,11)=-1.
      TI(9,12)=0.5*OM(4)
      TI(9,13)=-6.*OM(4)
      TI(9,14)=-3.
      TI(9,15)=-3.*OM(4)
      TI(9,17)=-2.
      TI(9,18)=-0.5*OM(4)
      TI(10,7)=10.
      TI(10,9)=-4.
      TI(10,12)=0.5
      TI(10,13)=-10.
      TI(10,15)=-4.
      TI(10,18)=-1.5
      TI(11,1)=-15.
      TI(11,2)=7.
      TI(11,4)=-1.
      TI(11,13)=15.
      TI(11,14)=8.
      TI(11,16)=1.5
      TI(12,1)=-12.*OM(1)
      TI(12,2)=6.*OM(1)
      TI(12,3)=-2.
      TI(12,4)=-OM(1)
      TI(12,5)=1.
      TI(12,13)=12.*OM(1)
      TI(12,14)=6.*OM(1)
      TI(12,15)=2.
      TI(12,16)=OM(1)
      TI(12,17)=1.
      TI(13,1)=-6.*(OM(1)+OM(2))/OMI
      TI(13,2)=3.*(OM(1)+OM(2))/OMI
      TI(13,3)=-3./OMI
      TI(13,4)=-(OM(1)+OM(2))/(2.*OMI)
      TI(13,5)=1./OMI
      TI(13,6)=-OM(3)/(2.*OMI)
      TI(13,7)=6.*(OM(3)*OM(4)-OM(5))/OMI
      TI(13,8)=3.*OM(3)/OMI
      TI(13,9)=3.*(OM(5)-OM(3)*OM(4))/OMI
      TI(13,10)=1./(2.*OMI)
      TI(13,11)=-OM(3)/OMI
      TI(13,12)=(OM(3)*OM(4)-OM(5))/(2.*OMI)
      TI(13,13)=6.*(OM(1)+OM(2)+OM(5)-OM(3)*OM(4))/OMI
      TI(13,14)=3.*(OM(1)+OM(2)-OM(3))/OMI
      TI(13,15)=3.*(1.+OM(5)-OM(3)*OM(4))/OMI
      TI(13,16)=(OM(1)+OM(2)-1.)/(2.*OMI)
```

```
      TI(13,17)=2.
      TI(13,18)=(CM(5)+CM(3)-CM(3)*CM(4))/(2.*CMI)
      TI(14,7)=-12.*CM(4)
      TI(14,8)=-2.
      TI(14,9)=6.*CM(4)
      TI(14,11)=1.
      TI(14,12)=-CM(4)
      TI(14,13)=12.*CM(4)
      TI(14,14)=2.
      TI(14,15)=6.*CM(4)
      TI(14,17)=1.
      TI(14,18)=CM(4)
      TI(15,7)=-15.
      TI(15,9)=7.
      TI(15,12)=-1.
      TI(15,13)=15.
      TI(15,15)=8.
      TI(15,18)=1.5
      TI(16,1)=6.
      TI(16,2)=-3.
      TI(16,4)=0.5
      TI(16,13)=-6.
      TI(16,14)=-3.
      TI(16,16)=-0.5
      TI(17,1)=6.*(CM(1)+CM(2))/CMI
      TI(17,2)=-3.*(CM(1)+CM(2))/CMI
      TI(17,3)=3./CMI
      TI(17,4)=(CM(1)+CM(2))/(2.*CMI)
      TI(17,5)=-1./CMI
      TI(17,6)=1./(2.*CMI)
      TI(17,7)=6.*(CM(5)-CM(4))/CMI
      TI(17,8)=-3./CMI
      TI(17,9)=3.*(CM(4)-CM(5))/CMI
      TI(17,10)=-1./(2.*CMI)
      TI(17,11)-1./CMI
      TI(17,12)=(CM(5)-CM(4))/(2.*CMI)
      TI(17,13)=6.*(CM(4)-CM(5)-CM(1)-CM(2))/CMI
      TI(17,14)=3.*(1.-CM(1)-CM(2))/CMI
      TI(17,15)=3.*(CM(4)-CM(5)-1.)/CMI
      TI(17,16)=(1.-CM(1)-CM(2))/(2.*CMI)
      TI(17,18)=(CM(4)-CM(5)-1.)/(2.*CMI)
      TI(18,7)=6.
      TI(18,9)=-3.
      TI(18,12)= .5
      TI(18,13)=-6.
      TI(18,15)=-3.
      TI(18,18)=-0.5
      DO 105 J=1,18
      DO 1 5 I=1,18
      BCI(I,J)=0.
05    CONTINUE
      BCI(1,1)=1.
      BCI(2,2)=G(2)
      BCI(2,3)=-B(2)
      BCI(3,2)=-G(1)
      BCI(3,3)=B(1)
      BCI(4,4)=G(2)**2
      BCI(4,5)=-2.*G(2)*B(2)
      BCI(4,6)=B(2)**2
      BCI(5,4)=-G(1)*G(2)
```

```
   BCI(5,5)=G(2)*B(1)+G(1)*B(2)
   BCI(5,6)=-B(1)*B(2)
   BCI(6,4)=G(1)**2
   BCI(6,5)=-2.*G(1)*B(1)
   BCI(6,6)=B(1)**2
   DO 106 K=6,12,6
   DO 106 J=1,6
   DO 106 I=1,6
   BCI(K+J,K+I)=BCI(J,I)
06 CONTINUE
   CALL TIMES(TI,BCI,TC,18,18,18,1)
   DO  109 I=1,18
   DO  109 J=1,18
   T(I,J)=TC(I,J)
09 CONTINUE
   GO TO 108
07 WRITE(6,600)
08 CONTINUE
   FORMAT('*******      ERROR     ANTISTRCFIS     *******')
   RETURN
   END
   SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
   IMPLICIT REAL*8 (A-H,O-Z)
   DIMENSION A(1),B(1),R(1)
   KOK=1  A(N,M) , B(M,L) , R(N,L)    REGULAR    A*B
   KOK=2  A(M,N) , B(M,L) , R(N,L)    TRANSPOSE A'*B
   IR=1
   DO 100 K=1,L
   DO 100 J=1,N
   R(IR)=0.
   GO TO(101,102),KOK
 1 CONTINUE
   DO 103 I=1,M
   IA=N*(I-1)+J
   IB=M*(K-1)+I
03 R(IR)=R(IR)+A(IA)*B(IB)
   GO TO 100
02 CONTINUE
   DO 104 I=1,M
   IA=M*(J-1)+I
   IB=M*(K-1)+I
04 R(IR)=R(IR)+A(IA)*B(IB)
   IR=IR+1
   RETURN
   END
```

6. Reference symbol PDS30

```
 P=100 PRCUTE=PUPH CCPILS=4
= UNIVERSITY, PATCH
WAS: 11:48:4
IGNED ON AT  11:48:46 ON MON SEP 22/75
RINT*
  **** PLATE  DISPL. ELEMENT 3  DC. OF FREEDOM AT 6 NODES *****
  ****  QUINTIC  VARIATION OF W ( CUBIC VAR. OF WN ALONG THE SIDES )****
  ****  CUBIC    VARIATION OF SHEAR ANGLE  FX,FY              *****
  ****  STRESSES MXX,MYY,MXY,QX,QY    AT CENTROID             ********
    **** MODULI OF ELASTISITY THROUGH  YOUNG(12)      ********
  ****TRANSFORMATION TO  W, WN, WS, WNN, WSN, WSS, FN, FS  **************
    SUBROUTINE TRANL(X,Y,Z,K,TRL)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(7,7),X2(2),Y2(2)
    DO 515 J=1,2
    X2(J)=X(K,J)
15  Y2(J)=Y(K,J)
    G2=X2(2)-X2(1)
    B2=Y2(2)-Y2(1)
    GLEN2=DSQRT(G2**2+B2**2)
    IF(GLEN2.LT.0.1D-12.AND.GLEN2.GT.-0.1D-12) WRITE(6,700)
    B2=B2/GLEN2
    G2=G2/GLEN2
    DO 516 I=1,7
    DO 516 J=1,7
16  TRL(J,I)=0.
    TRL(1,1)=B2
    TRL(1,2)=G2
    TRL(1,6)=B2
    TRL(2,1)=-G2
    TRL(2,2)=B2
    TRL(2,6)=-G2
    TRL(3,3)=B2**2
    TRL(3,4)=2.*B2*G2
    TRL(3,5)=G2**2
    TRL(4,3)=-B2*G2
    TRL(4,4)=B2**2-G2**2
    TRL(4,5)=B2*G2
    TRL(5,3)=G2**2
    TRL(5,4)=-2.*B2*G2
    TRL(5,5)=B2**2
    TRL(6,6)=B2
    TRL(6,7)=G2
    TRL(7,6)=-G2
    TRL(7,7)=B2
00  FORMAT('      *****      ER302     TRANL     *****')
    RETURN
    END
    SUBROUTINE CNDNS(K)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/CON/ST(32,32),S(5,32),FC2(32)
    DIMENSION B(32)
    A=1./ST(K,K)
    DO 40 I=1,32
40  B(I)=ST(K,I)
    DO 41 J=1,32
    DO 41 I=1,32
41  ST(J,I)=ST(J,I)-B(I)*B(J)*A
    DO 42 I=1,5
    D=S(I,K)
```

```
   DO 42 J=1,32
42 S(I,J)=S(I,J)-P(J)*A*D
   D=FOP(K)
   DO 43 I=1,32
43 FOP(I)=FOP(I)-P(I)*D*A
   RETURN
   END
   SUBROUTINE  STIFF
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/STI/X(3,2 ),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
  1INFO(20)
   COMMON/MAN/BOL(2,36,36),COL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
   COMMON/COOR/CO2(3,3),XO(3),YO(3),CCM(3,3)
   DIMENSION TRAN(30,30),SMK(30,30),STR(5,30),DAOG(30),
  1XM(16,2),YM(16,2),ZM(16,2),X1(3),Y1(3),Z1(3),CO3(7,7)
  2,A1(3),B1(3),C1(3),STU(30,30),TIK(5,30),FOR(30)
   DO 100 I=1,3
   XO(I)=X(1,I)
00 YO(I)=X(2,I)
   DO 101 I=1,36
   DO 101 J=1,36
01 STUCK(J,I)=0.
   DO 102 I=1,36
   DO 102 J=1,8
02 STICK(J,I)=0.
   DO 103 I=1,36
 3 FORCE(I)= .
   DO 607 I=1,30
07 DAOG(I)=0.
   IF(NGRAM.EQ.0) GO TO 517
   DO 516 M=1,NGRAM
   DO 516 I=1,2
   XM(M,I)=GRAM(M,I)
   ZM(M,I)=GRAM(M,I+4)
16 YM(M,I)=GRAM(M,I+2)
17 CONTINUE
   CNLD=0.
   DO 432 I=1,3
   DO 432 J=6,8
32 IF(X(I,J).NE.0.) CNLD=1.
   IF(CNLD.NE.1.)GO TO 437
   DO 433 I=6,8
   J=8*(I-6)+1
33 DAOG(J)=X(3,I)
37 CONTINUE
   NLAK=INFO(1)
   IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
   CALL SUBTI
   IF(NLAK.EQ. )GO TO 105
   NLAK=NLAK-10
   DO 106 I=1,30
   DO 106 J=1,30
06 BOL(NLAK,J,I)=STUCK(J,I)
   DO 113 I=1,3
   DO 113 J=1,5
13 COL(NLAK,J,I)=STICK(J,I)
   DO 108 I=1,30
08 DOL(NLAK,I)=FORCE(I)
   GO TO 105
04 DO 109 I=1,30
```

```
   DO 109 J=1,30
.9 STUCK(J,I)=BOL(NLAK,J,I)
   DO 214 I=1,30
   DO 214 J=1,5
14 STICK(J,I)=COL(NLAK,J,I)
   DO 111 I=1,30
11 FORCE(I)=DOL(NLAK,I)
05 CONTINUE
   DO 438 I=1,30
38 FORCE(I)=FORCE(I)+DAGG(I)
   NLIK=INFO(2)
   IF(NLIK.EQ. .OR.NLIK.EQ.00) GO TO 115
   DO 601 I=1,3
   DO 601 J=1,30
01 STU(J,I)=STUCK(J,I)
   DO 602 I=1,30
   DO 6 2 J=1,5
02 TIK(J,I)=STICK(J,I)
   DO 603 I=1,30
.3 FOR(I)=FORCE(I)
   DO 116 I=1,NLIK
   K=(I-1)*2+3
   L=INFO(K)
   K1=INFO(K+1)
   DO 583 II=1,3
   DO 584 IJ=1,30
84 TRAN(IJ,II)=0.
83 TRAN(II,II)=1.
   CALL TRANL(XM,YM,ZM,K1,CO3)
   IF(L.GT.3) GO TO 608
   KK=(L-1)*8
   DO 222 IL=1,7
   DO 222 JL=1,7
22 TRAN(KK+IL+1,KK+JL+1)=CO3(IL,JL)
   GO TO 609
.8 KK=24+(L-4)*2
   DO 610 IL=1,2
   DO 610 JL=1,2
10 TRAN(KK+IL,KK+JL)=CO3(IL,JL)
.9 CONTINUE
   CALL TIMES(STU,TRAN,SMK,30,30,30,1)
   CALL TIMES(TRAN,SMK,STU,30,30,30,2)
   CALL TIMES(TIK,TRAN,STR,5,30,30,1)
   CALL TIMES(TRAN,FOR,DAGG,30,30,1,2)
   DO 232 IL=1,3
   DO 232 JL=1,5
32 TIK(JL,IL)=STR(JL,IL)
   DO 233 IL=1,3
33 FOR(IL)=DAGG(IL)
16 CONTINUE
   DO 6 4 I=1,3
   DO 604 J=1,30
04 STUCK(J,I)=STU(J,I)
   DO 605 I=1,30
   DO 605 J=1,5
05 STICK(J,I)=TIK(J,I)
   DO 606 I=1,30
06 FORCE(I)=FOR(I)
15 CONTINUE
   RETURN
```

```
      END
      SUBROUTINE  SHAPEF(A,B,G,DT)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/JON/SM(32,32),S(5,32),D(5,5),P(3),ENP(32,1)
      DIMENSION  BE(5,32),BET(3,18),CET(3,3),A(3),B(3),G(3)
     1,BED(3,18),CBI(6,6),TEN(6,18),DEN(6,18),EN(8,32)
     2,SF(7),SFX(7),SFY(7)
      COMMON/SO3A/OM(5),OK(3)
      CET(1,1)=(B(1)/DT)**2
      CET(1,2)=2.*B(1)*B(2)/DT**2
      CET(1,3)=(B(2)/DT)**2
      CET(2,1)=(G(1)/DT)**2
      CET(2,2)=2.*G(1)*G(2)/DT**2
      CET(2,3)=(G(2)/DT)**2
      CET(3,1)=2.*G(1)*B(1)/DT**2
      CET(3,2)=2.*(G(1)*B(2)+G(2)*B(1))/DT**2
      CET(3,3)=2.*G(2)*B(2)/DT**2
      DO 100 I=1,3
      DO 100 J=1,18
      BET(I,J)= .
00    CONTINUE
      DO 101 I=1,5
      DO 101 J=1,32
      BE(I,J)=0.
01    CONTINUE
      BET(1,4)=2.
      BET(1,7)=6.*A(1)
      BET(1,8)=2.*A(2)
      BET(1,11)=12.*A(1)**2
      BET(1,12)=6.*A(1)*A(2)
      BET(1,13)=2.*A(2)**2
      BET(1,16)=20.*A(1)**3+12.*OM(1)*A(1)**2*A(2)+2.*OM(2)*A(2)**3
      BET(1,17)=6.*A(1)*A(2)**2+2.*OM(3)*A(2)**3
      BET(1,18)=2.*A(2)**3*OM(5)
      BET(2,5)=1.
      BET(2,8)=2.*A(1)
      BET(2,9)=2.*A(2)
      BET(2,12)=3.*A(1)**2
      BET(2,13)=4.*A(1)*A(2)
      BET(2,14)=3.*A(2)**2
      BET(2,16)=4.*OM(1)*A(1)**3+6.*OM(2)*A(1)*A(2)**2
      BET(2,17)=6.*A(1)**2*A(2)+6.*OM(3)*A(1)*A(2)**2
      BET(2,18)=6.*A(1)*A(2)**2*OM(5)+4.*OM(4)*A(2)**3
      BET(3,6)=2.
      BET(3,9)=2.*A(1)
      BET(3,10)=6.*A(2)
      BET(3,13)=2.*A(1)**2
      BET(3,14)=6.*A(1)*A(2)
      BET(3,15)=12.*A(2)**2
      BET(3,16)=6.*OM(2)*A(1)**2*A(2)
      BET(3,17)=2.*A(1)**3+6.*OM(3)*A(1)**2*A(2)
      BET(3,18)=20.*A(2)**3+12.*OM(4)*A(1)*A(2)**2+6.*OM(5)*A(1)**2*A(2)
      CALL TIMES(CET,BET,BED,3,3,18,1)
      DO 102 I=1,3
      DO 102 J=1,18
      BE(I,J)=-BED(I,J)
02    CONTINUE
      DIM1=A(1)*A(2)*A(3)
      DIM2=(B(1)*A(2)*A(3)+B(2)*A(1)*A(3)+B(3)*A(1)*A(2))/DT
      DIM3=(G(1)*A(2)*A(3)+G(2)*A(1)*A(3)+G(3)*A(1)*A(2))/DT
```

```
      DO 108 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      SF(I)=(2.*A(I)-1.)*A(I)+3.*DIM1
      SF(I+3)=4.*A(K)*A(J)-12.*DIM1
      SFX(I)=B(I)*(4.*A(I)-1.)/DT+3.*DIM2
      SFX(I+3)=(B(K)*4.*A(J)+B(J)*4.*A(K))/DT-12.*DIM2
      SFY(I)=G(I)*(4.*A(I)-1.)/DT+3.*DIM3
 8    SFY(I+3)=(G(K)*4.*A(J)+G(J)*4.*A(K))/DT-12.*DIM3
      SF(7)=27.*DIM1
      SFX(7)=27.*DIM2
      SFY(7)=27.*DIM3
      DO 103 J=19,31,2
      K=(J-19)/2+1
      BE(1,J)=SFX(K)
      BE(2,J+1)=SFY(K)
      BE(3,J)=SFY(K)
      BE(3,J+1)=SFX(K)
      BE(4,J)=SF(K)
03    BE(5,J+1)=SF(K)
      CALL TIMES(C,BE,S,5,5,32,1)
      CALL TIMES(BE,S,SM,32,5,32,2)
      DO 104 I=1,6
      DO 104 J=1,6
      CBI(I,J)=0.
04    CONTINUE
      DO 105 I=1,6
      DO 105 J=1,18
      TEN(I,J)=0.
05    CONTINUE
      DO 106 I=1,8
      DO 106 J=1,32
      SM(I,J)=0.
06    CONTINUE
      CBI(1,1)=1.
      CBI(2,2)=B(1)/DT
      CBI(2,3)=B(2)/DT
      CBI(3,2)=G(1)/DT
      CBI(3,3)=G(2)/DT
      CBI(4,4)=(B(1)/DT)**2
      CBI(4,5)=2.*B(1)*B(2)/DT**2
      CBI(4,6)=(B(2)/DT)**2
      CBI(5,4)=G(1)*B(1)/DT**2
      CBI(5,5)=(G(2)*B(1)+G(1)*B(2))/DT**2
      CBI(5,6)=G(2)*B(2)/DT**2
      CBI(6,4)=(G(1)/DT)**2
      CBI(6,5)=2.*G(1)*G(2)/DT**2
      CBI(6,6)=(G(2)/DT)**2
      TEN(1,1)=1.
      TEN(1,2)=A(1)
      TEN(1,3)=A(2)
      TEN(1,4)=A(1)**2
      TEN(1,5)=A(1)*A(2)
      TEN(1,6)=A(2)**2
      TEN(1,7)=A(1)**3
      TEN(1,8)=A(1)**2*A(2)
      TEN(1,9)=A(1)*A(2)**2
      TEN(1,10)=A(2)**3
```

```
      TEM(1,11)=A(1)**4
      TEM(1,12)=A(1)**3*A(2)
      TEM(1,13)=A(1)**2*A(2)**2
      TEM(1,14)=A(1)*A(2)**3
      TEM(1,15)=A(2)**4
      TEM(1,16)=A(1)**5+CM(1)*A(1)**4*A(2)+CM(2)*A(1)**2*A(2)**3
      TEM(1,17)=A(1)**3*A(2)**2+CM(3)*A(1)**2*A(2)**3
      TEM(1,18)=A(2)**5+CM(4)*A(1)*A(2)**4+CM(5)*A(1)**2*A(2)**3
      TEM(2,2)=1.
      TEM(2,4)=2.*A(1)
      TEM(2,5)=A(2)
      TEM(2,7)=3.*A(1)**2
      TEM(2,8)=2.*A(1)*A(2)
      TEM(2,9)=A(2)**2
      TEM(2,11)=4.*A(1)**3
      TEM(2,12)=3.*A(1)**2*A(2)
      TEM(2,13)=2.*A(1)*A(2)**2
      TEM(2,14)=A(2)**3
      TEM(2,16)=5.*A(1)**4+4.*CM(1)*A(1)**3*A(2)+2.*CM(2)*A(1)*A(2)**3
      TEM(2,17)=3.*A(1)**2*A(2)**2+2.*CM(3)*A(1)*A(2)**3
      TEM(2,18)=CM(4)*A(2)**4+2.*CM(5)*A(1)*A(2)**3
      TEM(3,3)=1.
      TEM(3,5)=A(1)
      TEM(3,6)=2.*A(2)
      TEM(3,8)=A(1)**2
      TEM(3,9)=2.*A(1)*A(2)
      TEM(3,10)=3.*A(2)**2
      TEM(3,12)=A(1)**3
      TEM(3,13)=2.*A(1)**2*A(2)
      TEM(3,14)=3.*A(1)*A(2)**2
      TEM(3,15)=4.*A(2)**3
      TEM(3,16)=CM(1)*A(1)**4+3.*CM(2)*A(1)**2*A(2)**2
      TEM(3,17)=2.*A(1)**3*A(2)+3.*CM(3)*A(1)**2*A(2)**2
      TEM(3,18)=5.*A(2)**4+4.*CM(4)*A(1)*A(2)**3+3.*CM(5)*A(1)**2*A(2)**
     12
      TEM(4,4)=2.
      TEM(4,7)=6.*A(1)
      TEM(4,8)=2.*A(2)
      TEM(4,11)=12.*A(1)**2
      TEM(4,12)=6.*A(1)*A(2)
      TEM(4,13)=2.*A(2)**2
      TEM(4,16)=2 .*A(1)**3+12.*CM(1)*A(1)**2*A(2)+2.*CM(2)*A(2)**3
      TEM(4,17)=6.*A(1)*A(2)**2+2.*CM(3)*A(2)**3
      TEM(4,18)=2.*CM(5)*A(2)**3
      TEM(5,5)=1.
      TEM(5,8)=2.*A(1)
      TEM(5,9)=2.*A(2)
      TEM(5,12)=3.*A(1)**2
      TEM(5,13)=4.*A(1)*A(2)
      TEM(5,14)=3.*A(2)**2
      TEM(5,16)=4.*CM(1)*A(1)**3+6.*CM(2)*A(1)*A(2)**2
      TEM(5,17)=6.*A(1)**2*A(2)+6.*CM(3)*A(1)*A(2)**2
      TEM(5,18)=4.*CM(4)*A(2)**3+6.*CM(5)*A(1)*A(2)**2
      TEM(6,6)=2.
      TEM(6,9)=2.*A(1)
      TEM(6,10)=6.*A(2)
      TEM(6,13)=2.*A(1)**2
      TEM(6,14)=6.*A(1)*A(2)
      TEM(6,15)=12.*A(2)**2
      TEM(6,16)=6.*CM(2)*A(1)**2*A(2)
```

```
      TEN(6,17)=2.*A(1)**3+6.*OM(3)*A(1)**2*A(2)
      TEN(6,18)=2.*A(2)**3+12.*OM(4)*A(1)*A(2)**2+6.*OM(5)*A(1)**2*A(2)
      CALL TIMES(CBT,TEN,DEN,6,6,18,1)
      DO 107  I=1,6
      DO 107  J=1,18
      FM(I,J)=DEN(I,J)
107   CONTINUE
      DO 109  J=19,31,2
      K=(J-19)/2+1
      FM(7,J)=SF(K)
  109 FM(8,J+1)=SF(K)
      CALL TIMES(FM,P,FNP,32,8,1,2)
      RETURN
      END
      SUBROUTINE   SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),ELMOD(12),STUCK(36,36),STICK(8,36),FORCE(36)
     1,INFO(20)
      COMMON/JCN/SM(32,32),S(5,32),D(5,5),P(3),FNP(32,1)
      COMMON/CCRA/CM(5),OK(3)
      COMMON/COCP/CO2(3,3),XC(3),YC(3),CCM(3,3)
      COMMON/CON/STT(32,32),TIK(5,32),DACL(32)
      DO 208  I=1,5
      DO 208  J=1,5
208   D(J,I)=0.
      D(1,1)=ELMOD(1)
      D(1,2)=ELMOD(2)
      D(2,2)=ELMOD(3)
      D(3,3)=ELMOD(4)/2.
      D(4,4)=ELMOD(5)
      D(5,5)=ELMOD(6)
      D(2,1)=D(1,2)
      IF(ELMOD(12).NE.0.)D(2,1)=ELMOD(12)
      DATA ALF/0.33333333D00/,ALF1/0.05971587D00/,BET1/0.47014206D00/
      DATA ALF2/ .79742699D  /,BET2/ .1 1286510  /
      DATA WE1/0.225D00/,WE2/0.13239415D00/,WE3/0.12593918D00/
      DIMENSION   B(3),G(3),ARCO(2,7),W(7),A(3),LENG(3),
     1T(32,32),FSF(32),S1(32)
      ARCO(1,1)=ALF
      ARCO(2,1)=ALF
      ARCO(1,2)=BET1
      ARCO(2,2)=BET1
      ARCO(1,3)=ALF1
      ARCO(2,3)=BET1
      ARCO(1,4)=BET1
      ARCO(2,4)=ALF1
      ARCO(1,5)=BET2
      ARCO(2,5)=BET2
      ARCO(1,6)=ALF2
      ARCO(2,6)=BET2
      ARCO(1,7)=BET2
      ARCO(2,7)=ALF2
      W(1)=WE1
      W(2)=WE2
      W(3)=WE2
      W(4)=WE2
      W(5)=WE3
      W(6)=WE3
      W(7)=WE3
      DO 223 I=1,8
```

```
23  P(I)=0.
    P(1)=X(3,F)
    DO 200 I=1,3
    K=I+1
    IF(K.GT.3)K=K-3
    J=I+2
    IF(J.GT.3)J=J-3
    B(I)=YO(K)-YO(J)
    G(I)=XO(J)-XO(K)
    LENG(I)=DSQRT(B(I)**2+G(I)**2)
00  CONTINUE
    CK(1)=B(1)*B(2)+G(1)*G(2)
    CK(2)=B(2)*B(3)+G(2)*G(3)
    CK(3)=B(3)*B(1)+G(3)*G(1)
    DT=B(1)*G(2)-B(2)*G(1)
    CM(1)=5.*CK(1)/(CK(1)+CK(2))
    CM(2)=5.*(CK(3)*CK(2)+CK(1)*CK(2)-3.*CK(1)*CK(3))/
   1((CK(1)+CK(2))*(2.*CK(3)-3.*CK(2)))
    CM(3)=(3.*CK(3)-2.*CK(2))/(2.*CK(3)-3.*CK(2))
    CM(4)=5.*CK(1)/(CK(1)+CK(3))
    CM(5)=5.*(CK(3)*CK(2)+CK(1)*CK(3)-3.*CK(1)*CK(2))/
   1((CK(1)+CK(3))*(2.*CK(3)-3.*CK(2)))
    CALL ANTI(B,G,DT,T)
    DO 214 I=1,32
    DO 214 J=1,32
    SCM(I,J)=0.
14  CONTINUE
    DO 314 I=1,32
    DACL(I)= .
14  CONTINUE
    DO 213  K=1,7
    A(1)=ARCO(1,K)
    A(2)=ARCO(2,K)
    A(3)=1.-A(1)-A(2)
    CALL   SHAPEF(A,B,G,DT)
    IF(K.NE.1)GO TO 510
    DO 511 I=1,32
    DO 511 J=1,5
11  TIK(J,I)=S(J,I)
1   CONTINUE
    DO 313 I=1,32
    DACL(I)=DACL(I)+DT/2.*W(K)*FNP(I,1)
13  CONTINUE
    DO 213 J=1,32
    DO 213 I=1,32
    SCM(I,J)=SCM(I,J)+DT/2.*W(K)*SM(I,J)
13  CONTINUE
    CALL TIMES(SCM,T,SM,32,32,32,1)
    CALL TIMES(T,SM,SCM,32,32,32,2)
    CALL TIMES(TIK,T,S,5,32,32,1)
    CALL TIMES(T,DACL,FRF,32,32,1,2)
    DATA M1/1,2,3,4,5,6,19,2 ,7,8,9,10,11,12,21,22,
   113,14,15,16,17,18,23,24,25.26,27,28,29,30,31,32/
    DO 224 I=1,32
    DO 224 J=1,32
24  T(J,I)=0.
    DO 222 I=1,32
22  T(M1(I).I)=1.
    CALL TIMES(SCM,T,SM,32,32,32,1)
    CALL TIMES(T,SM,SCM,32,32,32,2)
```

```
      CALL TIMES(S,T,TIK,5,32,32,1)
      CALL TIMES(T,FRS,FACL,32,32,1,2)
      CALL CADNS(32)
      CALL CADNS(31)
      DO 550 I=1,3
      DO 550 J=1,30
550   STUCK(J,I)=SCM(J,I)
      DO 551 I=1,3
      DO 551 J=1,5
551   STICK(J,I)=TIK(J,I)
      DO 552 I=1,3
552   FORCE(I)=FACL(I)
      RETURN
      END
      SUBROUTINE  ANTI(B,G,DT,T)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION TI(18,18),BCI(18,18),TO(18,13),T(32,32),B(3),G(3)
      COMMON/DOSA/OM(5),OK(3)
      DO 100  I=1,18
      DO 100 J=1,18
      TI(I,J)=0.
      CONTINUE
      DO 110 I=1,32
      DO 110 J=1,32
      IF(I.EQ.J.AND.I.GT.18) GO TO 111
      T(I,J)=0.
      GO TO 110
111   T(I,J)=1.
110   CONTINUE
      CMI=1.-OM(3)
      IF(OMI.EQ.0)GO TO 107
      TI(1,13)=1.
      TI(2,14)=1.
      TI(3,15)=1.
      TI(4,16)=0.5
      TI(5,17)=1.
      TI(6,18)=0.5
      TI(7,1)=10.
      TI(7,2)=-4.
      TI(7,4)=0.5
      TI(7,13)=-1 .
      TI(7,14)=-6.
      TI(7,16)=-1.5
      TI(8,1)=6.*OM(1)
      TI(8,2)=-3.*OM(1)
      TI(8,3)=3.
      TI(8,4)=0.5*OM(1)
      TI(8,5)=-1.
      TI(8,13)=-6. *OM(1)
      TI(8,14)=-3.*OM(1)
      TI(8,15)=-3.
      TI(8,16)=- .5*OM(1)
      TI(8,17)=-2.
      TI(9,7)=6.*OM(4)
      TI(9,8)=3.
      TI(9,9)=-3.*OM(4)
      TI(9,11)=-1.
      TI(9,12)=0.5*OM(4)
      TI(9,13)=-6.*OM(4)
      TI(9,14)=-3.
```

```
TI(9,15)=-3.*OM(4)
TI(9,17)=-2.
TI(9,18)=-0.5*OM(4)
TI(10,7)=10.
TI(10,9)=-4.
TI(10,12)=0.5
TI(10,13)=-10.
TI(10,15)=-6.
TI(10,18)=-1.5
TI(11,1)=-15.
TI(11,2)=7.
TI(11,4)=-1.
TI(11,13)=15.
TI(11,14)=8.
TI(11,16)=1.5
TI(12,1)=-12.*OM(1)
TI(12,2)=6.*OM(1)
TI(12,3)=-2.
TI(12,4)=-OM(1)
TI(12,5)=1.
TI(12,13)=12.*OM(1)
TI(12,14)=6.*OM(1)
TI(12,15)=2.
TI(12,16)=OM(1)
TI(12,17)=1.
TI(13,1)=-6.*(OM(1)+OM(2))/OMI
TI(13,2)=3.*(OM(1)+OM(2))/OMI
TI(13,3)=-3./OMI
TI(13,4)=-(OM(1)+OM(2))/(2.*OMI)
TI(13,5)=1./OMI
TI(13,6)=-OM(3)/(2.*OMI)
TI(13,7)=6.*(OM(3)*OM(4)-OM(5))/OMI
TI(13,8)=3.*OM(3)/OMI
TI(13,9)=3.*(OM(5)-OM(3)*OM(4))/OMI
TI(13,10)=1./(2.*OMI)
TI(13,11)=-OM(3)/OMI
TI(13,12)=(OM(3)*OM(4)-OM(5))/(2.*OMI)
TI(13,13)=6.*(OM(1)+OM(2)+OM(5)-OM(3)*OM(4))/OMI
TI(13,14)=3.*(OM(1)+OM(2)-OM(3))/OMI
TI(13,15)=3.*(1.+OM(5)-OM(3)*OM(4))/OMI
TI(13,16)=(OM(1)+OM(2)-1.)/(2.*OMI)
TI(13,17)=2.
TI(13,18)=(OM(5)+OM(3)-OM(3)*OM(4))/(2.*OMI)
TI(14,7)=-12.*OM(4)
TI(14,8)=-2.
TI(14,9)=6.*OM(4)
TI(14,11)=1.
TI(14,12)=-OM(4)
TI(14,13)=12.*OM(4)
TI(14,14)=2.
TI(14,15)=6.*OM(4)
TI(14,17)=1.
TI(14,18)=OM(4)
TI(15,7)=-15.
TI(15,9)=7.
TI(15,12)=-1.
TI(15,13)=15.
TI(15,15)=8.
TI(15,18)=1.5
TI(16,1)=6.
```

```
      TI(16,2)=-3.
      TI(16,7)=0.5
      TI(16,13)=-6.
      TI(16,14)=-3.
      TI(16,16)=-0.5
      TI(17,1)=6.*(CM(1)+CM(2))/CMI
      TI(17,2)=-3.*(CM(1)+CM(2))/CMI
      TI(17,3)=3./CMI
      TI(17,4)=(CM(1)+CM(2))/(2.*CMI)
      TI(17,5)=-1./CMI
      TI(17,6)=1./(2.*CMI)
      TI(17,7)=6.*(CM(5)-CM(4))/CMI
      TI(17,8)=-3./CMI
      TI(17,9)=3.*(CM(4)-CM(5))/CMI
      TI(17,10)=-1./(2.*CMI)
      TI(17,11)=1./CMI
      TI(17.12)=(CM(5)-CM(4))/(2.*CMI)
      TI(17,13)=6.*(CM(4)-CM(5)-CM(1)-CM(2))/CMI
      TI(17,14)=3.*(1.-CM(1)-CM(2))/CMI
      TI(17,15)=3.*(CM(4)-CM(5)-1.)/CMI
      TI(17,16)=(1.-CM(1)-CM(2))/(2.*CMI)
      TI(17,18)=(CM(4)-CM(5)-1.)/(2.*CMI)
      TI(18,7)=6.
      TI(18,9)=-3.
      TI(18,12)=0.5
      TI(18,13)=-6.
      TI(18,15)=-3.
      TI(18,18)=-0.5
      DO 105 J=1,18
      DO 105 I=1,18
      BCI(I,J)=0.
    5 CONTINUE
      BCI(1,1)=1.
      BCI(2,2)=G(2)
      BCI(2,3)=-B(2)
      BCI(3,2)=-G(1)
      BCI(3,3)=B(1)
      BCI(4,4)=G(2)**2
      BCI(4,5)=-2.*G(2)*B(2)
      BCI(4,6)=B(2)**2
      BCI(5,4)=-G(1)*G(2)
      BCI(5,5)=G(2)*B(1)+G(1)*B(2)
      BCI(5,6)=-B(1)*B(2)
      BCI(6,4)=G(1)**2
      BCI(6,5)=-2.*G(1)*B(1)
      BCI(6,6)=B(1)**2
      DO 106 K=6,12,6
      DO 106 J=1,6
      DO 106 I=1,6
      BCI(K+J,K+I)=BCI(J,I)
   06 CONTINUE
      CALL TIMES(TI,BCI,TO,18,18,18,1)
      DO 109 I=1,18
      DO 1 9 J=1,18
      T(I,J)=TO(I,J)
    9 CONTINUE
      GO TO 108
   07 WRITE(6,600)
   08 CONTINUE
   00 FORMAT('*******      ERROR      ANTISTROFIS       ******')
```

```
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KCK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),R(1)
      KCK=1   A(N,M)  ,  B(M,L)  ,  R(N,L)      REGULAR     A*B
      KCK=2   A(M,N)  ,  B(M,L)  ,  R(N,L)      TRANSPOSE  A'*B
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KCK
  01  CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
  03  R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
  02  CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
  4   R(IR)=R(IR)+A(IA)*B(IB)
  00  IR=IR+1
      RETURN
      END
```

7. Reference symbol PRO18

```
 P=100 PRCUTF=DURF CCPIES=4
= UNIVERSITY, FATCH
WAS: 11:48:46
IGNED CN AT  11:48:53 CN MON SEP 22/75
RINT*
  *** PLATE ROTATION ELEMENT WITH 13 DEGREES OF FR.  AT 6 NODES W,RX,RY*****
  **** CUBIC VARIATICN FOR THE DIS. W QUADRATIC FOR THE ROTATIONS       ******
  **** 5  STRESSES AT THE CENTROID    MXX,MYY,MXY,OX,CY                  ******
  ****  TRANSFORMATICN  TO  RN  RS                                        ******
  ****  ELASTICITY  MODULI  VIA  YOUNG(12)                                ******
    SUBROUTINE STIFF
    IMPLICIT REAL*8 (A-H,O-Z)
    CCMMCN/STI/X(3,2 ),YCUNG(12),STUCK(36,36),STICK(8,36),FORCE(36)
   1,INFO(20)
    CCMMCN/MAN/BCL(2,36,36),CCL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
    CCMMCN/COOR/X1(3),Y1(3)
    DIMENSION TRAM(18,18),SMK(18,18),SMK1(18,18),STR(5,18),TIK(5,18),
   1LEAG(18),XM(16,2),YM(16,2),C03(2,2)
    DO 100 I=1,3
    X1(I)=X(1,I)
00  Y1(I)=X(2,I)
    DO 101 I=1,36
    DO 101 J=1,36
 1  STUCK(J,I)= .
    DO 102 I=1,36
    DO 102 J=1,8
 2  STICK(J,I)=0.
    DO 103 I=1,36
03  FORCE(I)=0.
    IF(NGRAM.EQ.0) GO TO 517
    DO 516 M=1,NGRAM
    DO 516 I-1,2
    XM(M,I)=GRAM(M,I)
16  YM(M,I)=GRAM(M,I+2)
17  CONTINUE
    NLAK=INFO(1)
    IF(NLAK.LO.1.OR.NLAK.EQ.2) GO TO 104
    CALL SUBTI
    IF(NLAK.EQ. ) GO TO 1 5
    NLAK=NLAK-10
    DO 106 I=1,18
    DO 106 J=1,18
06  BCL(NLAK,J,I)=STUCK(J,I)
    DO 113 I=1,18
    DO 113 J=1,5
13  CCL(NLAK,J,I)=STICK(J,I)
    DO 1 8 I=1,18
08  DCL(NLAK,I)=FORCE(I)
    GO TO 105
04  DO 109 I=1,18
    DO 109 J=1,18
.9  STUCK(J,I)=BCL(NLAK,J,I)
    DO 214 I=1,18
    DO 214 J=1,5
14  STICK(J,I)=CCL(NLAK,J,I)
    DO 111 I=1,18
11  FORCE(I)=DCL(NLAK,I)
05  CONTINUE
    NLIK=INFO(2)
    IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
```

```
      DO 213 I=1,18
      DO 213 J=1,18
 13   SMK(J,I)=STUCK(J,I)
      DO 114 I=1,18
      DO 114 J=1,5
 14   STR(J,I)=STICK(J,I)
      DO 115 I=1,NLIK
      K=(I-1)*2+3
      L=INEC(K)
      K1=INEC(K+1)
      DO 583 II=1,18
      DO 584 IJ=1,18
 84   TRAN(IJ,II)=0.
 83   TRAN(II,II)=1.
      CALL TRANL(XM,YM,K1,CO3)
      KK=(L-1)*3
      DO 587 IN=1,2
      DO 587 IL=1,2
 87   TRAN(KK+IL+1,KK+IN+1)=CO3(IL,IN)
      CALL TIMES(SMK,TRAN,SMK1,18,18,18,1)
      CALL TIMES(TRAN,SMK1,SMK,18,18,18,2)
      CALL TIMES(STR,TRAN,TIK,5,18,18,1)
      DO 116 NI=1,18
      DO 116 NJ=1,5
 16   STR(NJ,NI)=TIK(NJ,NI)
 15   CONTINUE
      DO 117 I=1,18
      DO 117 J=1,18
 17   STUCK(J,I)=SMK(J,I)
      DO 118 I=1,18
      DO 118 J=1,5
 18   STICK(J,I)=STR(J,I)
 12   CONTINUE
      BRPO=0.
      DO 604 J=6,11
      DO 604 I=1,3
 04   IF(X(I,J).NE. .)BRPO=1.
      IF(BRPO.NE.1) GO TO 560
      DO 273 I=1,18
 73   DAOG(I)= .
      DAOG(1)=X(3,6)
      DAOG(4)=X(3,7)
      DAOG(7)=X(3,8)
      DAOG(10)=X(3,9)
      DAOG(13)=X(3,1 )
      DAOG(16)=X(3,11)
      DO 274 I=1,18
 74   FORCE(I)=FORCE(I)+DAOG(I)
 60   CONTINUE
      RETURN
      END
      SUBROUTINE TRANL(X,Y,K1,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(16,2),Y(16,2),X2(2),Y2(2),TRL(2,2)
      DO 515 J=1,2
      X2(J)=X(K1,J)
 15   Y2(J)=Y(K1,J)
      C2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      GLEN2=DSQRT(C2**2+B2**2)
```

```
      IF(GLEN2.LT. .1D-12.AND.GLEN2.GT.- .1D-12) WRITE(6,700)
      B2=B2/GLEN2
      G2=G2/GLEN2
      TRL(1,1)=B2
      TRL(1,2)=G2
      TRL(2,1)=-G2
      TRL(2,2)=B2
700   FORMAT('        ******    ERROR   TRANL        *******')
      RETURN
      END
      SUBROUTINE CONDS(K)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/CDN/ST(19,19),T(5,19)
      DIMENSION B(19)
      A=1./ST(K,K)
      DO 40 I=1,19
40    B(I)=ST(K,I)
      DO 41 J=1,19
      DO 41 I=1,19
41    ST(J,I)=ST(J,I)-B(I)*B(J)*A
      DO 42 I=1,5
      D=T(I,K)
      DO 42 J=1,19
42    T(I,J)=T(I,J)-B(J)*A*D
      RETURN
      END
      SUBROUTINE SHAP(A,B,G,DT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3),B(3),G(3),BE(5,19),SR(6),SRX(6),SRY(6),
     1SWX(7),SWY(7)
      COMMON/JCN/SM(19,19),S(5,19),D(5,5)
      DIM1=A(1)*A(2)*A(3)
      DIM2=(B(1)*A(2)*A(3)+B(2)*A(1)*A(3)+B(3)*A(1)*A(2))/DT
      DIM3=(G(1)*A(2)*A(3)+G(2)*A(1)*A(3)+G(3)*A(1)*A(2))/DT
      DO 100 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      SR(I)=(2.*A(I)-1.)*A(I)
      SR(I+3)=4.*A(K)*A(J)
      SRX(I)=B(I)*(4.*A(I)-1.)/DT
      SRX(I+3)=(B(K)*4.*A(J)+B(J)*4.*A(K))/DT
      SRY(I)=G(I)*(4.*A(I)-1.)/DT
      SRY(I+3)=(G(K)*4.*A(J)+G(J)*4.*A(K))/DT
      SWX(I)=SRX(I)+3.*DIM2
      SWX(I+3)=SRX(I+3)-12.*DIM2
      SWY(I)=SRY(I)+3.*DIM3
      SWY(I+3)=SRY(I+3)-12.*DIM3
100   CONTINUE
      SWX(7)=27.*DIM2
      SWY(7)=27.*DIM3
      DO 101 I=1,19
      DO 101 J=1,5
101   BE(J,I)=0.
      DO 102 I=1,11,2
      K=(I+1)/2
      BE(1,I)=SRX(K)
      BE(2,I+1)=SRY(K)
      BE(3,I)=SRY(K)
```

```
      BE(3,I+1)=SRX(K)
      BE(4,I)=-SR(K)
      BE(5,I+1)=-SR(K)
 02   CONTINUE
      DO 1 3 I=13,19
      K=I-12
      BE(4,I)=SWX(K)
 03   BE(5,I)=SWY(K)
      CALL TIMES(D,BE,S,5,5,19,1)
      CALL TIMES(BE,S,SM,19,5,19,2)
      RETURN
      END
      SUBROUTINE SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36)
     1,INFO(20)
      COMMON/JON/SM(19,19),S(5,19),D(5,5)
      COMMON/COOR/XO(3),YO(3)
      COMMON/COM/ST(19,19),TIK(5,19)
      DIMENSION A1(7),A2(7),A3(7),W(7),B(3),G(3),A(3),M1(19),T(19,19)
      DATA A1/ .33333333D    , . 5971587D    ,2* .47 142 6D   , .79742699D
     1 00,2*0.10128651D 00/,A3/0.33333333D 00,2*0.47014206D 00,0.05971587
     2 7D 00,2*0.10128651D 00,0.79742699D 00/,W/0.22500000D 00,3*0.132394
     3 15D    ,3* .12593918D    /
      DO 200 I=1,5
      DO 200 J=1,5
      D(J,I)= .
      D(1,1)=YOUNG(1)
      D(1,2)=YOUNG(2)
      D(2,1)=D(1,2)
      D(2,2)=YOUNG(3)
      D(3,3)=YOUNG(4)/2.
      D(4,4)=YOUNG(5)
      D(5,5)=YOUNG(6)
      IF(YOUNG(12).NE. .) D(2,1)=YOUNG(12)
      DO 210 I=1,19
      DO 21  J=1,19
 1    ST(J,I)= .
      DO 211 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      B(I)=YO(K)-YO(J)
 11   G(I)=XO(J)-XO(K)
      DT=B(1)*G(2)-B(2)*G(1)
      DO 212 K=1,7
      A2(K)=1.-A1(K)-A3(K)
      A(1)=A1(K)
      A(2)=A2(K)
      A(3)=A3(K)
      CALL SHAP(A,B,G,DT)
      IF(K.NE.1) GO TO 213
      DO 214 I=1,19
      DO 214 J=1,5
 14   TIK(J,I)=S(J,I)
 13   CONTINUE
      DO 215 I=1,19
      DO 215 J=1,19
 15   ST(J,I)=ST(J,I)+W(K)*SM(J,I)*DT/2.
```

```
12 CONTINUE
   DO 217 I=1,19
   DO 217 J=1,19
17 T(J,I)=0.
   DATA M1/13,1,2,14,3,4,15,5,6,16,7,8,17,9,10,18,11,12,19/
   DO 218 I=1,19
18 T(M1(I),I)=1.
   CALL TIMES(ST,T,SM,19,19,19,1)
   CALL TIMES(T,SM,ST,19,19,19,2)
   CALL TIMES(TIK,T,S,5,19,19,1)
   DO 219 I=1,19
   DO 219 J=1,5
19 TIK(J,I)=S(J,I)
   CALL CNDNS(19)
   DO 220 I=1,18
   DO 220 J=1,18
20 STUCK(J,I)=ST(J,I)
   DO 221 I=1,18
   DO 221 J=1,5
21 STICK(J,I)=TIK(J,I)
   PG=X(3,5)
   FORCE(10)=PG*DT/6.
   FORCE(13)=PG*DT/6.
   FORCE(16)=PG*DT/6.
   RETURN
   END
   SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
   IMPLICIT REAL*8 (A-H,O-Z)
   DIMENSION A(1),B(1),R(1)
   KOK=1 A(N,M) , B(M,L) , R(N,L)  REGULAR A*B=R
   KOK=2 A(M,N) , B(M,L) , R(N,L) TRANSPOSE  A'*B=R
   IR=1
   DO 100 K=1,L
   DO 100 J=1,N
   R(IR)=0.
   GO TO(101,102),KOK
01 CONTINUE
   DO 103 I=1,M
   IA=N*(I-1)+J
   IB=M*(K-1)+I
3  R(IR)=R(IR)+A(IA)*B(IB)
   GO TO 100
02 CONTINUE
   DO 104 I=1,M
   IA=M*(J-1)+I
   IB=M*(K-1)+I
04 R(IR)=R(IR)+A(IA)*B(IB)
00 IR=IR+1
   RETURN
   END
```

SANDWICH DOME MODELS

COMPUTER LISTING

	Reference symbol
1.	DDS21
2.	DDS33
3.	DMX36
4.	DRO30

1. Reference symbol DDS21

```
C P=100 PRCLTE=PUPH COPIES=4
 = UNIVERSITY, BATCH
 WAS: 15:52:34
SIGNED ON AT  15:52:40 ON MON SEP 22/75
PRINT*
   **** DOME ELEMENT  21 DG. OF FREEDOM AT 3 CORNER NODES ********
      ****   CUBIC   VARIATION OF W (W,WX,WY)   AND LINEAR OF U,V,FX,FY ******
   *****  TRANSFORMATION AT GLOBAL SET OF U,V,W AND TOTAL ROTATIONS  *******
   **** STRESSES AT CENTROID  MXX,MYY,MXY QX,QY,NXX,NYY,NXY   *********
      SUBROUTINE  STIFF
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(2 )
      COMMON/MAR/BOL(2,36,36),COL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
      COMMON/PAT/CO2(3,3),XO(3),YO(3)
      DIMENSION X1(3),Y1(3),Z1(3),G1(3),B1(3),C1(3),CO1(3,3),CO3(3,3)
      DIMENSION XM(16,2),YM(16,2),ZM(16,2),TRAN(21,21),MCL(3)
      DIMENSION CO4(3,3),SOK(21,21),SIK(8,21),DAOL(21),FAOG(21)
      COMMON/COND/SOK1(21,21),SIK1(8,21),SEK1(21)
      DATA MCL/3,10,17/
      DO 599 I=1,21
599   FORCE(I)=0.
      X1 , Y1 , Z1   GLOBAL  COORDINATES  OF  THE  VERTICES  (1,2,3)
      DO 415 I=1,3
      X1(I)=X(1,I)
      Y1(I)=X(2,I)
415   Z1(I)=X(3,I)
  XM , YM , ZM (M,2)  CLOBAL  COORDIN.  OF THE JOINT LINE (DIRECTION 1,2)
  M= THE CODE NUMBER OF THE JOINT LINE (MAX=16)
      IF(NGRAM.EQ.0)GO TO 517
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
      YM(M,I)-GRAM(M,I+2)
516   ZM(M,I)=GRAM(M,I+4)
517   CONTINUE
      DO 400 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      G1(I)=X1(J)-X1(K)
      B1(I)=Y1(J)-Y1(K)
400   C1(I)=Z1(J)-Z1(K)
  CO2(3,3)   TRANSF.   MATRIX FROM GLOBAL TO LOCAL  D' = < CO2 > D
  D'  =  LOCAL    D  =  GLOBAL
      CALL  TRANS(G1,B1,C1)
  XO(3) , YO(3) THE LOCAL COORDIN.  OF THE VERTICES (1,2,3)
      DO 592 I=1,3
      DO 592 J=1,3
592   CO4(I,J)=X(I,J)
      CALL TIMES(CO2,CO4,CO1,3,3,3,1)
      DO 414 J=1,3
      XO(J)=CO1(1,J)
414   YO(J)=CO1(2,J)
      NLAK=INFO(1)
      METRO=0
      IF(NLAK.EQ.-1.OR.NLAK.EQ.-2) GO TO 840
      IF(NLAK.EQ.1.OR.NLAK.EQ.2) GO TO 575
      CALL SUBTI
```

```
      IF(NLAK.EQ.0) GO TO 58
      NLAK=NLAK-10
      DO 900 I=1,21
9     DOL(NLAK,I)=FORCE(I)
      DO 576 J=1,21
      DO 576 I=1,21
576   BOL(NLAK,I,J)=STUCK(I,J)
      DO 577 J=1,21
      DO 577 I=1,8
577   COL(NLAK,I,J)=STICK(I,J)
      GO TO 580
840   NLAK=-NLAK
      METRO=-1
575   DO 578 J=1,21
      DO 578 I=1,21
578   STUCK(I,J)=BOL(NLAK,I,J)
      DO 579 J=1,21
      DO 579 I=1,8
579   STICK(I,J)=COL(NLAK,I,J)
      DO 901 I=1,21
901   FORCE(I)=DOL(NLAK,I)
      IF(METRO.NE.-1) GO TO 580
      DO 861 I=1,21
      DO 861 J=1,21
      E1=0.
      IF(I.NE.J) GO TO 862
      E1=-1.
      DO 863 II=1,3
      IF(I.EQ.NOL(II)) E1=1.
863   CONTINUE
862   TRAN(J,I)=E1
861   CONTINUE
      DO 1 I=1,21
      DO 1 J=1,21
  1   SOK1(J,I)=STUCK(J,I)
      DO 2 I=1,21
      DO 2 J=1,8
  2   SIK(J,I)=STICK(J,I)
      DO 3 I=1,21
  3   DADL(I)=FORCE(I)
      CALL TIMES(SOK1,TRAN,SOK,21,21,21,1)
      CALL TIMES(TRAN,SOK,SOK1,21,21,21,2)
      CALL TIMES(SIK,TRAN,SIK1,8,21,21,1)
      CALL TIMES(TRAN,DADL,SEK1,21,21,1,2)
      DO 4 I=1,21
      DO 4 J=1,21
  4   STUCK(J,I)=SOK1(J,I)
      DO 5 I=1,21
      DO 5 J=1,8
  5   STICK(J,I)=SIK1(J,I)
      DO 6 I=1,21
  6   FORCE(I)=SEK1(I)
580   CONTINUE
   INFO(1)=0   FINDS LOCAL STIFF. AND STRESS MATR. WITOUT STORING
   INFO(1)=1   TAKES LOCAL STIFF. AND STRESS MATR. OF THE 1ST. STORAGE
   INFO(1)=2   TAKES LOCAL STIFF AND STRESS MATR.  OF THE 2ND. STORAGE
   INFO(1)=11  FINDS LOCAL STIFF. AND STRESS MATR. AND STORES IN THE 1ST.
   INFO(1)=12  FINDS LOCAL STIFF. AND STRESS MATR. AND STORES IN THE 2ND.
   INFO(2)=0   KEEPS LOCAL WITHOUT  TRANSFORMATION
   INFO(2)=X   X  NO NODES IN THE ELEMENT NEED TRANSFORMATION (MAX=3)
```

```
      IF(INFC(2).EQ. .OR.INFC(2).EQ.99) GO TO 581
      DO 801 I=1,21
      DO 801 J=1,21
801   SOK1(J,I)=STUCK(J,I)
      DO 802 I=1,21
      DO 8 2 J=1,8
802   SIK1(J,I)=STICK(J,I)
      DO 8 6 I=1,21
8 6   SFK1(I)=FORCE(I)
      NLIK=INFO(2)
      DO 582 I=1,NLIK
C   INFO(3) AYXCN ARITHMES TIS NODE
C   INFC(4) CODE NUMBER OF NODE--JOINT LINE(MAX=16) (IF.EQ.0 ALL GLOBAL,
C   IF.EQ.-1 ONLY U,V,W TRANSF)
C   INFO(5),INFO(7) THE SAME AS INFO(3)
C   INFC(6),INFC(8) THE SAME AS INFC(4)
      K=(I-1)*2+3
      L=INFC(K)
      K1=INFC(K+1)
      DO 583 II=1,21
      DO 584 IS=1,21
584   TRAN(IS,II)=0.
583   TRAN(II,II)=1.
      KK=(L-1)*7
      DO 585 IN=1,3
      DO 585 IL=1,3
585   TRAN(KK+IN,KK+IL)=CO2(IN,IL)
      IF(K1.EQ.-1) GO TO 582
      IF(K1.EQ.0) GO TO 586
C   INTERCONECTION OF TWO PLATES
      CALL TRANL(XM,YM,ZM,K1,CO3)
C   X AXE OF THE NEW SYSTEM THE JOINT LINE
C   FY  SHEAR ANGLE CONDENSED OUT
      DO 587 IN=1,2
      DO 587 IL=1,2
      TRAN(KK+IN+3,KK+IL+3)=CO3(IN,IL)
587   TRAN(KK+IN+5,KK+IL+5)=CO3(IN,IL)
      CALL TIMES(SOK1,TRAN,SOK,21,21,21,1)
      CALL TIMES(TRAN,SOK,SOK1,21,21,21,2)
      DO 902 IM=1,21
902   DACL(IM)=SFK1(IM)
      DO 593 JM=1,21
      DO 593 IM=1,8
593   SIK(IM,JM)=SIK1(IM,JM)
      CALL TIMES(SIK,TRAN,SIK1,8,21,21,1)
      CALL TIMES(TRAN,DACL,SFK1,21,21,1,2)
      KUK=KK+7
      CALL CNDNS(KUK)
      SOK1(KUK,KUK)=1.
      GO TO 582
C   INTERCONECTION OF MORE THAN TWO PLATES
586   CALL CNDNS(KK+6)
      CALL CNDNS(KK+7)
      SOK1(KK+7,KK+7)=1.
      DO 588 IN=1,3
      DO 588 IL=1,3
588   TRAN(KK+IN+3,KK+IL+3)=CO2(IN,IL)
      CALL TIMES(SOK1,TRAN,SOK,21,21,21,1)
      CALL TIMES(TRAN,SOK,SOK1,21,21,21,2)
      DO 903 IM=1,21
```

```
903 DAOL(IM)=SEK1(IM)
    DO 594 JM=1,21
    DO 594 IM=1,8
594 SIK(IM,JM)=SIK1(IM,JM)
    CALL TIMES(SIK,TRAN,SIK1,8,21,21,1)
    CALL TIMES(TRAN,DAOL,SEK1,21,21,1,2)
582 CONTINUE
    DO 803 I=1,21
    DO 803 J=1,21
803 STUCK(J,I)=SOK1(J,I)
    DO 804 I=1,21
    DO 804 J=1,8
804 STICK(J,I)=SIK1(J,I)
    DO 805 I=1,21
805 FORCE(I)=SEK1(I)
581 CONTINUE
    DO 600 I=1,21
600 DAOL(I)=0.
    BRPO=0.
    DO 604 J=6,8
    DO 604 I=1,3
604 IF(X(I,J).NE.0.)BRPO=1.
    IF(BRPO.NE.1.) GO TO 56
    DO 273 I=1,21
273 DAOG(I)=0.
    KN=0
    DO 274 I=1,15,7
    KN=KN+1
    DAOG(I)=X(1,5+KN)
    DAOG(I+1)=X(2,5+KN)
274 DAOG(I+2)=X(3,5+KN)
    DO 595 I=1,21
    DO 596 J=1,21
596 TRAN(J,I)=0.
595 TRAN(I,I)=1.
    DO 812 KM=1,15,7
    L=KM-1
    KN=L/7+1
    GL=0.
    IF(NLIK.EQ.0)GO TO 810
    DO 811 IN=1,NLIK
    K=(IN-1)*2+3
811 IF(KN.EQ.INFO(K))GL=1.
    IF(GL.EQ.1.)GO TO 812
810 CONTINUE
    DO 597 I=1,3
    DO 597 J=1,3
597 TRAN(I+L,J+L)=CO2(I,J)
812 CONTINUE
    CALL TIMES(TRAN,DAOG,DAOL,21,21,1,1)
560 CONTINUE
    DO 602 I=1,21
602 FORCE(I)=FORCE(I)+DAOL(I)
    RETURN
    END
    SUBROUTINE TRANL(X,Y,Z,K,TRL)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/PAT/TRG(3,3),XG(3),YG(3)
    DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(3,3),TR(3,3)
    DIMENSION X2(3),Y2(3),Z2(3)
```

```
      DO 515 J=1,2
      X2(J)=X(K,J)
      Y2(J)=Y(K,J)
515   Z2(J)=Z(K,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      C2=Z2(2)-Z2(1)
      GLEN2=DSQRT(G2**2+B2**2+C2**2)
      IF(GLEN2.EQ.0.)WRITE(6,700)
      TR(1,1)=G2/GLEN2
      TR(2,1)=B2/GLEN2
      TR(3,1)=C2/GLEN2
      TR(1,2)=TRG(3,2)*TR(3,1)-TRG(3,3)*TR(2,1)
      TR(2,2)=TRG(3,3)*TR(1,1)-TRG(3,1)*TR(3,1)
      TR(3,2)=TRG(3,1)*TR(2,1)-TRG(3,2)*TR(1,1)
      TR(1,3)=TRG(3,1)
      TR(2,3)=TRG(3,2)
      TR(3,3)=TRG(3,3)
      CALL TIMES(TRG,TR,TRI,3,3,3,1)
700   FORMAT(10X,'      **** ERROR ****  GLEN2',/)
      RETURN
      END
      SUBROUTINE CNDNS(K)
      IMPLICIT REAL*8 (A-H,O-Z)
C     CONDENSES OUT THE  K TH.  DEGREE OF FREEDOM
      COMMON/COND/SIM(21,21),TIM(8,21),VIM(21)
      DIMENSION B(21)
      A=1./SIM(K,K)
      DO 40 I=1,21
 4    B(I)=SIM(K,I)
      DO 41 J=1,21
      DO 41 I=1,21
 41   SIM(I,J)=SIM(I,J)-B(I)*B(J)*A
      DO 42 I=1,8
      D=TIM(I,K)
      DO 42 J=1,21
 42   TIM(I,J)=TIM(I,J)-B(J)*A*D
      D=VIM(K)
      DO 43 I=1,21
 43   VIM(I)=VIM(I)-B(I)*D*A
      RETURN
      END
C      *********         ME    THN    VCHTHIA      TCY       THEOY    ********
C     THE  STIFFNESS  MATRIX   SM
C     THE  STRESS     MATRIX   S
C     THE  LOAD       MATRIX   ENP
C     INPUT  FOR  EACH POINT A THE AREA COORDINATES,P THE VALUES OF LOAD
C     INPUT  FOR  EACH ELEMENT THE COEFFICIENTS  B(3) ,G(3)
C     INPUT  FOR  EACH ELEMENT THE ELASTICITY  MATRIX  D(8)
      SUBROUTINE  SHAPEF(A,B,G,DT,NOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/JON/SM(21,21),S(8,21),D(8,8),ENP(21,1),P(7)
      DIMENSION A(3),B(3),G(3),BE(8,21),FN(7,21)
      DIMENSION SW(3),SX(3),SY(3),SWX(3),SXX(3),SYX(3),SWY(3),SXY(3),SYY
     1(3),SWXX(3),SXXX(3),SYXX(3),SWYY(3),SXYY(3),SYYY(3),SWXY(3),SXXY(3
     2),SYXY(3),SU(3),SV(3),SFX(3),SFY(3),SUX(3),SUY(3),SVX(3),SVY(3)
      DO 201 I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
```

```
      IF(J.GT.3)J=J-3
      SW(I)=A(I)+A(I)**2*A(K)+A(I)**2*A(J)-A(I)*A(K)**2-A(I)*A(J)**2
      SY(I)=B(K)*(A(I)**2*A(J)+0.5*A(I)*A(K)*A(J))-B(J)*(A(K)*A(I)**2+
     1 .5*A(I)*A(K)*A(J))
      SX(I)=G(J)*(A(I)**2*A(K)+0.5*A(I)*A(K)*A(J))-G(K)*(A(J)*A(I)**2+
     10.5*A(I)*A(K)*A(J))
      SWX(I)=(1./DT)*(2.*B(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
     1A(K)*(B(I)-B(K))+2.*A(I)*A(J)*(B(I)-B(J)))
      SYX(I)=(1./DT)*(2.*B(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+ .5*(B(J)-B(K))
     1*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)*A(I)*A(K)))*(-1.0)
      SXX(I)=(1./DT)*(2.*B(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+A(I)**2*(B(K)*
     1G(J)-B(J)*G(K))+ .5*(G(J)-G(K))*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+
     2B(J)*A(I)*A(K)))
      SWY(I)=(+1./DT)*(2.*G(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
     1A(K)*(G(I)-G(K))+2.*A(I)*A(J)*(G(I)-G(J)))
      SYY(I)=(-1./DT)*(2.*G(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+A(I)**2*(G(K)*
     1B(J)-G(J)*B(K))+ .5*(B(J)-B(K))*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+
     2G(J)*A(I)*A(K)))
      SXY(I)=(+1./DT)*(2.*G(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+0.5*(G(J)-G(K)
     1)*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+G(J)*A(I)*A(K)))
      SU(I)=A(I)
      SV(I)=A(I)
      SFY(I)=A(I)
      SFX(I)=A(I)
      SWXX(I)=(+1./DT**2)*(-2.*B(I)*(A(I)*B(I)+A(K)*B(K)+A(J)*B(J))+
     12.*(B(I)-B(K))*(A(K)*B(I)+A(I)*B(K))+2.*(B(I)-B(J))*(A(I)*B(J)+
     2A(J)*B(I)))
      SYXX(I)=(-1./DT**2)*(2.*B(I)**2*(B(J)*A(K)-B(K)*A(J))+(B(J)-B(K)
     1)*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B(K)*B(J)*A(I)))
      SXXX(I)=(+1./DT**2)*(2.*B(I)**2*(G(J)*A(K)-G(K)*A(J))+4.*A(I)*B(I)
     1*(B(K)*G(J)-B(J)*G(K))+(G(J)-G(K))*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+
     2B(K)*B(J)*A(I)))
      SWYY(I)=(+1./ DT**2)*(-2.*G(I)*(A(I)*G(I)+A(K)*G(K)+A(J)*G(J))+2.*
     1(G(I)-G(K))*(A(K)*G(I)+A(I)*G(K))+2.*(G(I)-G(J))*(A(I)*G(J)+A(J)*
     2G(I)))
      SYYY(I)=(-1./DT**2)*(2.*G(I)**2*(B(J)*A(K)-B(K)*A(J))+4.*A(I)*G(I)
     1*(G(K)*B(J)-G(J)*B(K))+(B(J)-B(K))*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+
     2G(K)*G(J)*A(I)))
      SXYY(I)=(+1./DT**2)*(2.*G(I)**2*(G(J)*A(K)-G(K)*A(J))+(G(J)-G(K)
     1)*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+G(K)*G(J)*A(I)))
      SWXY(I)=(2./DT**2)*(-2.*B(I)*(G(I)*A(I)+G(K)*A(K)+G(J)*A(J))+2.*
     1(B(I)-B(K))*(A(K)*G(I)+A(I)*G(K))+2.*(B(I)-B(J))*(A(I)*G(J)+A(J)*
     2G(I)))
      SYXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(B(J)*A(K)-B(K)*A(J))+2.*B(I)*
     1A(I)*(G(K)*B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(A(I)*(G(J)*B(K)+G(K)*
     2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
     3*(-1.0)
      SXXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(G(J)*A(K)-G(K)*A(J))+2.*G(I)*
     1A(I)*(B(K)*G(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(A(I)*(G(J)*B(K)+G(K)*
     2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
      SUX(I)=(1./DT)*B(I)
      SVX(I)=(1./DT)*B(I)
      SUY(I)=(1./DT)*G(I)
      SVY(I)=(1./DT)*G(I)
  201 CONTINUE
C     FORM    MATRIX    B
      DO 301 J=1,21
      DO 301 I=1,8
      BE(I,J)=0.
  301 CONTINUE
```

```
      DO 2 2 K=1,3
      J=3*K-2
      BE(1,J)=SWXX(K)
      BE(2,J)=SWYY(K)
      BE(3,J)=SWXY(K)
      BE(1,J+1)=SYXX(K)
      BE(2,J+1)=SYYY(K)
      BE(3,J+1)=SYXY(K)
      BE(1,J+2)=SXXX(K)
      BE(2,J+2)=SXYY(K)
      BE(3,J+2)=SXXY(K)
2 2   CONTINUE
      DO 203 K=1,3
      J=2*K+8
      BE(1,J)=-SUX(K)
      BE(3,J)=-SUY(K)
      BE(4,J)=SFX(K)
      BE(2,J+1)=-SVY(K)
      BE(3,J+1)=-SVX(K)
      BE(5,J+1)=SFY(K)
203   CONTINUE
      DO 204 K=1,3
      J=2*K+14
      BE(6,J)=SUX(K)
      BE(8,J)=SUY(K)
      BE(7,J+1)=SVY(K)
      BE(8,J+1)=SVX(K)
204   CONTINUE
       STIFNESS    MATRIX          BY MULTIPLICATION  B *D*B
       STIFNESS    MATRIX    SM
       STRESS      MATRIX    S     BY   MULTIPLICATION      D*B
      CALL TIMES(D,BE,S,8,8,21,1)
      CALL TIMES(BE,S,SM,21,3,21,2)
      PROGRAMA     FOR  THE  SHAPE  FUNCTION  MATRIX    N
      DO 300 J=1,21
      DO 300 I=1,7
      EN(I,J)=0.
300   CONTINUE
      DO 213 K-1,3
      J=3*K-2
      EN(1,J)=SW(K)
      EN(2,J)=SWY(K)
      EN(3,J)=SWX(K)
      EN(1,J+1)=SY(K)
      EN(2,J+1)=SYY(K)
      EN(3,J+1)=SYX(K)
      EN(1,J+2)=SX(K)
      EN(2,J+2)=SXY(K)
      EN(3,J+2)=SXX(K)
213   CONTINUE
      DO 214 K=1,3
      J=2*K+8
      EN(4,J)=SFX(K)
      EN(5,J+1)=SFY(K)
214   CONTINUE
      DO 215 K=1,3
      J=2*K+14
      EN(6,J)=SU(K)
      EN(7,J+1)=SV(K)
215   CONTINUE
```

```
    FORM THE  MATRIX  FNP   BY MULTIPLICATION    FN*P
    FN    SHAPE  FUNCTION  MATRIX
    P    LOAD   MATRIX  WITH SEVEN COMPONENTS  AT EACH POINT
    CALL TIMES(FN,P,FNP,21,7,1,2)
    RETURN
    END
    FORM  MATRIX  D     PROPERTIES  OF MATERIAL
       SANDWICH          PLATES
    ISOTROPIC   MATERIAL      FACES
    ORTHOTROPIK     MATERIAL   CORE
    EF  ELASTICITY   MODULUS  OF THE FACES
    ENF   POISSON   RATIO    OF   THE FACES
   GF     MODULUS   OF   RIGITY  OF  THE   FACES
   GCY   MODULUS  OF  RIGITY  OF  THE  CORE   IN  THE    YZ    LEVEL
   GCX   MODULUS  OF  RIGITY  OF   CORE    IN  THE     XZ   LEVEL
    THF    THICKNESS     OF  THE  FACE
    THC    THICKNESS     OF  THE  CORE
    A2M    MOMENT   OF AREA   OF  THE SANDWICH   SECTION  OF   THE   FACES
    SUBROUTINE  SUBTT
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),ELMOD(12),SMK(36,36),TIK(8,36),PILK(36)
   1,INFO(2 )
    COMMON/JON/SM(21,21),S(8,21),D(8,8),FNP(21,1),P(7)
    COMMON/PAT/VAV(3,3),XI(3),Y(3)
    DIMENSION A(3),B(3),G(3),GLE(3),W(7),W1(4),W2(4)
    DIMENSION PG(7),PL(7),VAV1(7,7),TOT(21,21),SMK1(21,21)
    DO 8   I=1,8
    DO 800 J=1,8
800 D(J,I)=0.
    D(1,1)=ELMOD(1)
    D(1,2)=ELMOD(2)
    D(2,2)=ELMOD(3)
    D(3,3)=ELMOD(4)/2.
    D(4,4)=ELMOD(5)
    D(5,5)=ELMOD(6)
    D(6,6)=ELMOD(7)
    D(6,7)=ELMOD(8)
    D(7,7)=ELMOD(1 )
    D(8,8)=ELMOD(11)
    FORM   THE  WHOLE  SYMMETRIC   MATRIX  D
    DO 210 J=1,8
    DO 210 I=1,8
    D(I,J)=D(J,I)
210 CONTINUE
    D(7,6)=ELMOD(9)
    IF(ELMOD(12).NE. .) D(2,1)=ELMOD(12)
       PG(7)     THE     GLOBAL   DISTRIBUTED   LOAD
    DO 224 I=1,7
224 PG(I)= .
    PG(1)=X(1,5)
    PG(2)=X(2,5)
    PG(3)=X(3,5)
    DO 430 I=1,7
    DO 43  J=1,7
430 VAV1(I,J)=0.
    DO 431 I=1,3
    DO 431 J=1,3
431 VAV1(I,J)=VAV(I,J)
    DO 432 K=3,5,2
    DO 432 I=1,2
```

```
      DO 432 J=1,2
  432 VAV1(I+K,J+K)=VAV(I,J)
      CALL TIMES(VAV1,PG,PL,7,7,1,1)
      DO 435 I=1,5
  435 P(I)=PL(I+2)
      P(6)=PL(1)
      P(7)=PL(2)
C   COORDINATES OF VERTICES OF TRIANGLE   X(I,7)   I=1,6
      DIMENSION ENK(21),ARCO(2,7),ARCO2(4),S1(8,21)
      DO 200  I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      B(I)=Y(K)-Y(J)
      G(I)=XI(J)-XI(K)
      CLE(I)=DSQRT(B(I)**2+G(I)**2)
  200 CONTINUE
      CT=B(1)*G(2)-B(2)*G(1)
C   QUINTIC  INTEGRATION  FOR STIFFNESS MATRIX  STUCK
C     AREA   COORDINATES   L1  L2  L3    ARCO        A(I)
C    WEIGHTS  OF  INTEGRATION          W(I)
      DATA ALF/0.33333333D  /,ALF1/0.05971587D00/,BET1/0.47014206D00/
      DATA ALF2/0.79742699D00/,BET2/0.10128651D00/
      DATA WF1/0.22500D00/,WF2/0.13239415D  /,WF3/ .12593918D  /
      DATA ALFA1/ .50  /,ALFA2/ . D  /,WF/0.33333333D00/
      ARCO(1,1)=ALF
      ARCO(2,1)=ALF
      ARCO(1,2)=ALF1
      ARCO(2,2)=BET1
      ARCO(1,3)=BET1
      ARCO(2,3)=ALF1
      ARCO(1,4)=BET1
      ARCO(2,4)=BET1
      ARCO(1,5)=ALF2
      ARCO(2,5)=BET2
      ARCO(1,6)=BET2
      ARCO(2,6)=ALF2
      ARCO(1,7)=BET2
      ARCO(2,7)=BET2
      W(1)=WF1
      W(2)=WF2
      W(3)=WF2
      W(4)=WF2
      W(5)=WF3
      W(6)=WF3
      W(7)=WF3
      DO 222 I=1,21
  222 ENK(I)=0.
      DO 211 J=1,21
      DO 211 I=1,21
      SMK1(I,J)=0.
  211 CONTINUE
      DO 212  K=1,7
      A(1)=ARCO(1,K)
      A(2)=ARCO(2,K)
      A(3)=1.-A(1)-A(2)
      CALL SHAPEF(A,B,G,CT, )
      IF(K.NE.1) GO TO 801
      DO 802 I=1,21
```

```
    DO 8 2 J=1,8
802 S1(J.I)=S(J,I)
801 CONTINUE
    DO 221 I=1,21
221 ENK(I)=ENK(I)+DT/2.*W(K)*ENP(I,1)
    DO 212 J=1,21
    DO 212 I=1,21
    SMK1(I,J)=SMK1(I,J)+DT/2.*W(K)*SM(I,J)
212 CONTINUE
 AREA  COORDINATES   L1  L2  L3   ARCC1        A(I)
 WEIGHTS   OF  INTEGRATION         W1(I)
      P(1,7)   REPRESENTS  DISTRIBUTED  LOADS  CONSTAND IN  ELEM  AREA DT/M2
    ENK(21)  REPRESENTS THE SAME LOADS IN  THE THREE NODES  1,2,3  AS  STUCK
     RECLASSIFICATION    OF   THE  DEGREES  OF  FREEDOM
    DO 425 J=1,21
    DO 425 I=1,21
425 TOT(I,J)=0.
    DO 45  K1=1,7,3
    K2=7*(K1+2)/3-4
    TOT(K1,K2)=1.
    DO 450 K3=1,3,2
    TOT(K1+1,K2+K3+1)=1.
450 TOT(K1+2,K2+K3)=1.
    DO 451 K1=6,20,7
    K2=2*(K1+1)/7+8
    DO 451 K3=1,6,5
    K4=6*(K3-1)/5
    TOT(K2+K4,K1-K3+1)=1.
451 TOT(K2+K4+1,K1-K3+2)=1.
    CALL TIMES(SMK1,TOT,SM,21,21,21,1)
    CALL TIMES(TOT,SM,SMK1,21,21,21,2)
    CALL TIMES(S1,TOT,S,8,21,21,1)
    CALL TIMES(TOT,ENK,ENP,21,21,1,2)
    DO 5 1 I=1,21
    DO 501 J=1,21
501 SMK(J,I)=SMK1(J,I)
    DO 5 2 I=1,21
    DO 502 J=1,8
502 TIK(J,I)=S(J,I)
    DO 803 I=1,21
803 PINK(I)=ENP(I,1)
    RETURN
    END
    SUBROUTINE TRANS(G1,B1,C1)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/PAT/TRMA(3,3),XO(3),YO(3)
    DIMENSION B1(3),G1(3),C1(3)
   FORM  THE TRANSFORMATION MATRIX   LOCAL=(TRMA)*GLOBAL
    DO 10 I=1,3
    DO 1  J=1,3
 10 TRMA(I,J)=0.
    AA=B1(2)*C1(3)-B1(3)*C1(2)
    AB=C1(3)*C1(2)-G1(2)*C1(3)
    AC=B1(3)*G1(2)-G1(3)*B1(2)
    IF((AA.GT.-.1D-12.AND.AA.LT.0.1D-12).AND.(AB.GT.-.1D-12.AND.AB.
   1  LT. .1D-12)) GO TO 20
    AD=DSQRT(AA**2+AB**2+AC**2)
    AE=DSQRT(AA**2+AB**2)
    TRMA(3,1)=AA/AD
    TRMA(3,2)=AB/AD
```

```
      TRMA(3,3)=AC/AD
      TRMA(1,1)=-AB/AE
      TRMA(1,2)=AA/AE
      TRMA(2,1)=-AA*AC/(AE*AD)
      TRMA(2,2)=-AB*AC/(AE*AD)
      TRMA(2,3)=(AA**2+AB**2)/(AE*AD)
      RETURN
   20 DO 30 I=1,3
   30 TRMA(I,I)=1.
      RETURN
      END
      SUBROUTINE    TIMES(A,B,R,N,M,L,KCK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION   A(1),B(1),R(1)
C     KCK=1   A(N,M),B(M,L),R(N,L)     REGULAR     A*B
C     KCK=2   A(M,N),B(M,L),R(N,L)     TRANSPOSE   A*B
      IR=1
      DO   100    K=1,L
      DO   100    J=1,N
      R(IR)=0.
      GO TO(101,102),KCK
  101 CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
  103 R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
  102 CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
  104 R(IR)=R(IR)+A(IA)*B(IB)
  100 IR=IR+1
      RETURN
      END
```

2. Reference symbol DDS33

```
0 P=100 PRCUTE=DUPH CCPIFS=4
 = UNIVERSITY, BATCH
 WAS: 15:52:4
SIGNED ON AT  15:52:46 ON MON SEP 22/75
PRINT
   ***** DCMF ELEMENT  23 DG. OF FREEDOM  AT 6 NODES  *****
   **** CUBIC VARIATION OF W(U,WX,WY) AND QUADRATIC OF U,V,FX,FY *********
   *****  TRANSFORMATION AT GLOBAL SET OF U,V,W AND TOTAL ROTATIONS  *******
   **** STRESSES AT CENTROID  MXX,MYY,MXY QX,QY,NXX,NYY,NXY    **********
    SUBROUTINE STIFF
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(20)
    COMMON/MAN/BOL(2,36,36),CCL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
    COMMON/PAT/CO2(3,3),XO(3),YO(3)
    DIMENSION X1(3),Y1(3),Z1(3),G1(3),B1(3),C1(3),CO1(3,3),CO3(3,3),
   1XM(16,2),YM(16,2),ZM(16,2),TRAN(36,36),CO4(3,3),SCK(36,36),STK(8,3
   26),STI(36),DAOL(36),MOL(3)
    DATA MOL/3,1 ,17/
    X1,Y1,Z1  GLOBAL COORDINATES OF THE VERTICES  (1,2,3)
    DO 100 I=1,3
    X1(I)=X(1,I)
    Y1(I)=X(2,I)
100 Z1(I)=X(3,I)
    XM,YM,ZM (M,2) GLOBAL COORD  OF THE JOINT LINE  (DIRECTION  1,2)
    M=  THE  CORR  NUMBER OF  THE   JOINT   (MAX=16)
    IF(NGRAM.EQ. )GO TO 1 1
    DO 102 I=1,2
    DO 102 M=1,NGRAM
    XM(M,I)=GRAM(M,I)
    YM(M,I)=GRAM(M,I+2)
102 ZM(M,I)=GRAM(M,I+4)
101 CONTINUE
    DO 103 I=1,3
    K=I+1
    J=I+2
    IF(K.GT.3)K=K-3
    IF(J.GT.3)J=J-3
    C1(I)=X1(J)-X1(K)
    B1(I)=Y1(J)-Y1(K)
1 3 C1(I)=Z1(J)-Z1(K)
    CO2(3,3) TRANSF. MATRIX FROM GLOBAL TO LOCAL  D' = < CO2 > D
    D' = LOCAL    D  = GLOBAL
    XO(3),YO(3)  THE  LOCAL  COORDINATES OF THE VERTICES (1,2,3)
    CALL TRANS(G1,B1,C1)
    DO 1 4 J=1,3
    DO 104 I=1,3
104 CO4(I,J)=X(I,J)
    CALL  TIMES(CO2,CO4,CO1,3,3,3,1)
    DO 105 J=1,3
    XO(J)=CO1(1,J)
1 5 YO(J)=CO1(2,J)
    DO 106  I=1,36
106 FORCE(I)=0.
    INFO(1)=-1 TAKES LOCAL STIF. STR, FROM 1ST STORE AND REVERSES
    THE APPROPRIATE D.O.F
    INFO(1)=-2 THE SAME AS BEFORE FROM THE 2ND STORA
    INFO(1)=0  FINDS LOCAL STIFF. STR. AND D.LOAD  WITHOUT STORING
    INFO(1)=1  TAKES LOCAL STIFF. STR. AND D.LOAD  FROM THE  1ST  STORAGE
    INFO(1)=2  TAKES LOCAL STIFF. STR. AND D.LOAD  FROM THE  2ND  STORAGE
```

```
      INFO(1)=11 FINDS LOCAL STIFF. STR. AND D.LOAD  AND  STORES AT 1ST STORAGE
      INFO(1)=12 FINDS LOCAL STIFF. STR. AND D.LOAD  AND  STORES AT 2ND STORAGE
      NLAK=INFO(1)
      METRC=0
      IF(NLAK.EQ.-1.OR.NLAK.EQ.-2) GO TO 340
      IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 107
      CALL SUBTI
      IF(NLAK.EQ.0)GO TO 108
      NLAK=NLAK-10
      DO 109 I=1,33
  109 DOL(NLAK,I)=FORCE(I)
      DO 140 I=1,33
      DO 140 J=1,8
  140 COL(NLAK,J,I)=STICK(J,I)
      DO 141 I=1,33
      DO 141 J=1,33
  141 BOL(NLAK,J,I)=STUCK(J,I)
      GO TO 108
  340 NLAK=-NLAK
      METRC=-1
  107 DO 111 I=1,33
  111 FORCE(I)=DOL(NLAK,I)
      DO 142 I=1,33
      DO 142 J=1,8
  142 STICK(J,I)=COL(NLAK,J,I)
      DO 143 I=1,33
      DO 143 J=1,33
  143 STUCK(J,I)=BOL(NLAK,J,I)
      IF(METRC.NE.-1) GO TO 108
      DO 861 I=1,36
      DO 861 J=1,36
      E1=0.
      IF(I.NE.J) GO TO 862
      E1=-1.
      DO 863 II=1,3
      IF(I.EQ.MOL(II)) E1=1.
  863 CONTINUE
  862 TRAN(J,I)=E1
  861 CONTINUE
      CALL TIMES(STUCK,TRAN,SOK,36,36,36,1)
      CALL TIMES(TRAN,SOK,STUCK,36,36,36,2)
      CALL TIMES(STICK,TRAN,SIK,3,36,36,1)
      CALL TIMES(TRAN,FORCE,DACL,36,36,1,2)
      DO 5 I=1,33
      DO 5 J=1,8
    5 STICK(J,I)=SIK(J,I)
      DO 6 I=1,33
    6 FORCE(I)=DACL(I)
  108 CONTINUE
      INFO(2)=0 KEEPS LOCAL  WITHOUT  TRANSFORMATION
      INFO(2)=X X NO  NODES  OF THE   ELEMENT  NEED   TRANSFORMATION  (MAX=5)
      NLIK=INFO(2)
      IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 120
      DO 120 I=1,NLIK
      INFO(3)    NUMBER  OF THE NODE  (1 6)
      INFO(4)    CODE NO OF NODE--JOINT LINE (MAX=16) (IF.EQ.0 ALL GLOBAL,
      IF.EQ.-1  ONLY   U , V , W   TRANSF)
      INFO(5), INFO(7), INFO(9), INFO(11)  THE SAME AS INFO(3)
      INFO(6), INFO(8), INFO(10),INFO(12)  THE SAME AS INFO(4)
      K=(I-1)*2+3
```

```
      L=INDEX(K)
      K1=INEE(K+1)
      DO 117 IS=1,36
      DO 116 IT=1,36
116   TRAN(IT,IS)=0.
117   TRAN(IS,IS)=1.
      IF(L.GT.3)GO TO 118
      KK=(L-1)*7
      DO 119 IL=1,3
      DO 119 IN=1,3
119   TRAN(KK+IN,KK+IL)=CO2(IN,IL)
      IF(K1.EQ.-1)GO TO 12
      IF(K1.EQ.0) GO TO 121
      CALL TRANL(XM,YM,ZM,K1,CO3)
C     X  AXE OF THE NEW  SYSTEM  THE  JOINT LINE
C     FX   SHEAR   ANGLE  IS  CONDENSED  OUT INTERCONECTION OF TWO PLATES
      DO 122 IL=1,2
      DO 122 IN=1,2
      TRAN(KK+IN+3,KK+IL+3)=CO3(IN,IL)
122   TRAN(KK+IN+5,KK+IL+5)=CO3(IN,IL)
      CALL TIMES(STUCK,TRAN,SOK,36,36,36,1)
      CALL TIMES(TRAN,SOK,STUCK,36,36,36,2)
      DO 123 JM=1,36
123   SIL(JM)=FORCE(JM)
      DO 144 JM=1,36
      DO 144 IM=1,8
144   SIK(IM,JM)=STICK(IM,JM)
      CALL TIMES(SIK,TRAN,STICK,8,36,36,1)
      CALL TIMES(TRAN,SIL,FORCE,36,36,1,2)
      KUK=KK+7
      CALL CNDNS(KUK)
      STUCK(KUK,KUK)=1.
      GO TO 120
C     INTERCONECTION  OF  MORE  THAN  TWO PLATES
121   CALL CNDNS(KK+6)
      CALL CNDNS(KK+7)
      STUCK(KK+7,KK+7)=1.
      DO 124 IL=1,3
      DO 124 IN=1,3
124   TRAN(KK+IN+3,KK+IL+3)=CO2(IN,IL)
      CALL TIMES(STUCK,TRAN,SOK,36,36,36,1)
      CALL TIMES(TRAN,SOK,STUCK,36,36,36,2)
      DO 125 JM=1,36
125   SIL(JM)=FORCE(JM)
      DO 145 JM=1,36
      DO 145 IM=1,8
145   SIK(IM,JM)=STICK(IM,JM)
      CALL TIMES(SIK,TRAN,STICK,8,36,36,1)
      CALL TIMES(TRAN,SIL,FORCE,36,36,1,2)
      GO TO 12
C     MIDSIDE  NODES  TRANSFORMATION  AT INTRCONECTION  OF TWO PLATES
C     X AXE OF THE NEW SYSTEM  THE JOINT-LINE  SHEAR ANGE FY  IS CONDENSED OUT
118   KK=(L-4)*4+21
      CALL TRANL(XM,YM,ZM,K1,CO3)
      DO 126 IL=1,2
      DO 126 IN=1,2
126   TRAN(KK+IN,KK+IL)=CO3(IN,IL)
      CALL TIMES(STUCK,TRAN,SOK,36,36,36,1)
      CALL TIMES(TRAN,SOK,STUCK,36,36,36,2)
      DO 127 JM=1,36
```

```
127 SIL(JM)=FORCE(JM)
    DO 146 JM=1,36
    DO 146 IM=1,8
146 SIK(IM,JM)=STICK(IM,JM)
    CALL TIMES(SIK,TRAN,STICK,8,36,36,1)
    CALL TIMES(TRAN,SIL,FORCE,36,36,1,2)
    KUK=KK+2
    CALL CNDNS(KUK)
    STUCK(KUK,KUK)=1.
120 CONTINUE
    CNSL= .
    DO 131 I=1,36
131 CACL(I)=0.
    DO 129 J=6,8
    DO 129 I=1,3
129 IF(X(I,J).NE.0.)CNSL=1.
    IF(CNSL.NE.1.)GO TO 128
    DO 130 I=1,36
130 SIL(I)=0.
    KO=0
    DO 132 I=1,15,7
    KO=KO+1
    SIL(I)=X(1,5+KO)
    SIL(I+1)=X(2,5+KO)
132 SIL(I+2)=X(3,5+KO)
    DO 134 I=1,36
    DO 133 J=1,36
133 TRAN(J,I)=0.
134 TRAN(I,I)=1.
    DO 135 KM=1,15,7
    L=KM-1
    KN=L/7+1
    GL=0.
    IF(NLIK.EQ.0)GO TO 137
    DO 136 IN=1,NLIK
    K=(IN-1)*2+3
136 IF(KN.EQ.INFO(K))GL=1.
    IF(GL.EQ.1.)GO TO 135
137 CONTINUE
    DO 138 J=1,3
    DO 138 I=1,3
138 TRAN(I+L,J+L)=CC2(I,J)
135 CONTINUE
    CALL TIMES(TRAN,SIL,CACL,36,36,1,1)
128 DO 139 I=1,36
139 FORCE(I)=FORCE(I)+CAOL(I)
    RETURN
    END
    SUBROUTINE  SHAP(A,B,G,DT,M)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION  SW(3),SWX(3),SWY(3),SWXX(3),SWYY(3),SWXY(3),SX(3),SXX(3
   1),SXY(3),SXXX(3),SXYY(3),SXXY(3),SY(3),SYX(3),SYY(3),SYXX(3),SYYY(
   23),SYXY(3),SU(7),SUX(7),SUY(7),SF(6),SFX(6),SFY(6),A(3),B(3),G(3),
   3BE(8,35),EN(7,35)
    COMMON/JON/SM(35,35),S(8,35),D(8,8),P(7),ENP(35,1)
    D1=A(1)*A(2)*A(3)
    D2=B(1)*A(2)*A(3)+B(2)*A(1)*A(3)+B(3)*A(1)*A(2)
    D3=G(1)*A(2)*A(3)+G(2)*A(1)*A(3)+G(3)*A(1)*A(3)
    DO  100 I=1,3
    K=I+1
```

```
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      SW(I)=A(I)+A(I)**2*A(K)+A(I)**2*A(J)-A(I)*A(K)**2-A(I)*A(J)**2
      SY(I)=B(K)*(A(I)**2*A(J)+.5*A(I)*A(K)*A(J))-B(J)*(A(K)*A(I)**2+ .
     15*A(I)*A(K)*A(J))
      SX(I)=G(J)*(A(I)**2*A(K)+0.5*A(I)*A(K)*A(J))-G(K)*(A(J)*A(I)**2+0.
     15*A(I)*A(K)*A(J))
      SWX(I)=(1./DT)*(2.*B(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*A(
     1K)*(B(I)-B(K))+2.*A(I)*A(J)*(B(I)-B(J)))
      SYX(I)=(1./DT)*(2.*B(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+0.5*(B(J)-B(K))
     1*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)*A(I)*A(K)))*(-1. )
      SXX(I)=(1./DT)*(2.*B(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+A(I)**2*(B(K)*G
     1(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)
     2*A(I)*A(K)))
      SWY(I)=(1./DT)*(2.*G(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*A(
     1K)*(G(I)-G(K))+2.*A(I)*A(J)*(G(I)-G(J)))
      SYY(I)=(-1./DT)*(2.*G(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+A(I)**2*(G(K)*
     1B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+G(J
     2)*A(I)*A(K)))
      SXY(I)=(1./DT)*(2.*G(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+0.5*(G(J)-G(K))
     1*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+G(J)*A(I)*A(K)))
      SWXX(I)=(1./DT**2)*(-2.*B(I)*(A(I)*B(I)+A(K)*B(K)+A(J)*B(J))+2.*(B
     1(I)-B(K))*(A(K)*B(I)+A(I)*B(K))+2.*(B(I)-B(J))*(A(I)*B(J)+A(J)*B(I
     2)))
      SYXX(I)=(-1./DT**2)*(2.*B(I)**2*(B(J)*A(K)-B(K)*A(J))+(B(J)-B(K))*
     1(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B(K)*B(J)*A(I)))
      SXXX(I)=(1./DT**2)*(2.*B(I)**2*(G(J)*A(K)-G(K)*A(J))+4.*A(I)*B(I)*
     1(B(K)*G(J)-B(J)*G(K))+(G(J)-G(K))*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B
     2(K)*B(J)*A(I)))
      SWYY(I)=(1./DT**2)*(-2.*G(I)*(A(I)*G(I)+A(K)*G(K)+A(J)*G(J))+2.*(G
     1(I)-G(K))*(A(K)*G(I)+A(I)*G(K))+2.*(G(I)-G(J))*(A(I)*G(J)+A(J)*G(I
     2)))
      SYYY(I)=(-1./DT**2)*(2.*G(I)**2*(B(J)*A(K)-B(K)*A(J))+4.*A(I)*G(I)
     1*(G(K)*B(J)-G(J)*B(K))+(B(J)-B(K))*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+
     2G(K)*G(J)*A(I)))
      SXYY(I)=(1./DT**2)*(2.*G(I)**2*(G(J)*A(K)-G(K)*A(J))+(G(J)-G(K))*(
     1G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+G(K)*G(J)*A(I)))
      SWXY(I)=(2./DT**2)*(-2.*B(I)*(G(I)*A(I)+G(K)*A(K)+G(J)*A(J))+2.*(B
     1(I)-B(K))*(A(K)*G(I)+A(I)*G(K))+2.*(B(I)-B(J))*(A(I)*G(J)+A(J)*G(I
     2)))
      SYXY(I)=(-2./DT**2)*(2.*B(I)*G(I)*(B(J)*A(K)-B(K)*A(J))+2.*B(I)*A(
     1I)*(G(K)*B(J)-G(J)*B(K))+ .5*(B(J)-B(K))*(A(I)*(G(J)*B(K)+G(K)*B(J
     2))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
      SXXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(G(J)*A(K)-G(K)*A(J))+2.*G(I)*A(I
     1)*(B(K)*G(J)-B(J)*G(K))+ .5*(G(J)-G(K))*(A(I)*(G(J)*B(K)+G(K)*B(J)
     2)+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
      SU(I)=A(I)-9.*D1
      SU(I+3)=-0.5*A(K)*A(J)+1.5*D1
      SUX(I)=(1./DT)*(B(I)-9.*D2)
      SUX(I+3)=(1./DT)*(- .5*(B(K)*A(J)+B(J)*A(K))+1.5*D2)
      SUY(I)=(1./DT)*(G(I)-9.*D3)
      SUY(I+3)=(1./DT)*(-0.5*(G(K)*A(J)+G(J)*A(K))+1.5*D3)
      SF(I)=(2.*A(I)-1.)*A(I)
      SF(I+3)=4.*A(K)*A(J)
      SFX(I)=(1./DT)*(B(I)*(4.*A(I)-1.))
      SFX(I+3)=(1./DT)*(4.*B(K)*A(J)+4.*B(J)*A(K))
      SFY(I)=(1./DT)*(G(I)*(4.*A(I)-1.))
      SFY(I+3)=(1./DT)*(4.*G(K)*A(J)+4.*G(J)*A(K))
      SU(7)=27.*D1
```

```
      SUX(7)=27.*D2
      SUY(7)=27.*D3
      FORM   MATRIX   BE(8,35)
      DO 103 I=1,35
      DO 103 J=1,8
  103 BE(J,I)=0.
      DO 104  I=1,3
      J=3*I-2
      BE(1,J)=SWXX(I)
      BE(2,J)=SWYY(I)
      BE(3,J)=SWXY(I)
      BE(1,J+1)=SXXX(I)
      BE(2,J+1)=SXYY(I)
      BE(3,J+1)=SXXY(I)
      BE(1,J+2)=SYXX(I)
      BE(2,J+2)=SYYY(I)
  104 BE(3,J+2)=SYXY(I)
      DO 105 I=1,6
      K1=8
      IF(I.GT.3)K1=14
      J=K1+2*I
      BE(1,J)=-SFX(I)
      BE(3,J)=-SFY(I)
      BE(4,J)=SF(I)
      BE(2,J+1)=-SFY(I)
      BE(3,J+1)=-SFX(I)
  105 BE(5,J+1)=SF(I)
      DO 106 I=1,7
      K1=14
      IF(I.GT.3)K1=20
      J=K1+2*I
      BE(6,J)=SUX(I)
      BE(8,J)=SUY(I)
      BE(7,J+1)=SUY(I)
  106 BE(8,J+1)=SUX(I)
      CALL TIMES(D,BE,S,8,8,35,1)
      CALL TIMES(BE,S,SM,35,8,35,2)
      FORM  MATRIX   EN    SHAPE  FUNCTIONS
      DO 108 J=1,35
      DO 108 I=1,7
  108 EN(I,J)=0.
      DO 109 I=1,3
      J=3*I-2
      EN(1,J)=SW(I)
      EN(2,J)=SWX(I)
      EN(3,J)=SWY(I)
      EN(1,J+1)=SX(I)
      EN(2,J+1)=SXX(I)
      EN(3,J+1)=SXY(I)
      EN(1,J+2)=SY(I)
      EN(2,J+2)=SYX(I)
  109 EN(3,J+2)=SYY(I)
      DO 110 I=1,6
      K1=8
      IF(I.GT.3)K1=14
      J=K1+2*I
      EN(4,J)=SF(I)
  110 EN(5,J+1)=SF(I)
      DO 111 I=1,7
      K1=14
```

```
      IF(I.GT.3)K1=20
      J=K1+2*I
      EN(6,J)=SU(I)
  111 EN(7,J+1)=SU(I)
      CALL TIMES(EN,P,ENP,35,7,1,2)
      RETURN
      END
      SUBROUTINE SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),ELMOD(12),SMK(36,36),TIK(3,36),PINK(36),
     1INFO(20)
      COMMON/JCN/SM(35,35),S(8,35),D(8,8),P(7),ENP(35,1)
      COMMON/PAT/VAV(3,3),XL(3),YL(3)
      DIMENSION A(3),B(3),C(3),W(7),ARCO(2,7),SMK1(35,35),FNK(35)
      DO 100 J=1,8
      DO 100 I=1,8
  100 D(I,J)=0.
      D(1,1)=ELMOD(1)
      D(1,2)=ELMOD(2)
      D(2,2)=ELMOD(3)
      D(3,3)=ELMOD(4)/2.
      D(4,4)=ELMOD(5)
      D(5,5)=ELMOD(6)
      D(6,6)=ELMOD(7)
      D(6,7)=ELMOD(8)
      D(7,7)=ELMOD(10)
      D(8,8)=ELMOD(11)
      DO 101 J=1,8
      DO 101 I=1,8
  101 D(I,J)=D(J,I)
      D(7,6)=ELMOD(9)
      IF(ELMOD(12).NE.0.) D(2,1)=ELMOD(12)
      DIMENSION PG(7),PL(7),VAV1(7,7),TOT(35,35),S1(8,35)
      PG(7)    THE  GLOBAL DISTRIBUTED  LOAD
      DO 111 I=1,7
  111 PG(I)=0.
      PG(1)=X(1,5)
      PG(2)=X(2,5)
      PG(3)=X(3,5)
      DO 112 J=1,7
      DO 112 I=1,7
  112 VAV1(I,J)=0.
      DO 113 J=1,3
      DO 113 I=1,3
  113 VAV1(I,J)=VAV(I,J)
      DO 114  K=3,5,2
      DO 114  I=1,2
      DO 114  J=1,2
  114 VAV1(I+K,J+K)=VAV(I,J)
      CALL TIMES(VAV1,PG,PL,7,7,1,1)
      DO 115  I=1,5
  115 P(I)=PL(I+2)
      P(6)=PL(1)
      P(7)=PL(2)
      DATA ALF/0.33333333D00/,ALF1/0.05971587D00/,BET1/0.47014206D00/,
     1ALF2/ .79742699D  /,BET2/ .1 12865LD  /,WE1/ .225 D  /,WE2/ .13239
     2415DCO/,WE3/0.12593918D00/,ALFA1/0.5D00/
      ARCO(1,1)=ALF
      ARCO(2,1)=ALF
      ARCO(1,2)=ALF1
```

```
      ARCO(2,2)=BET1
      ARCO(1,3)=BET1
      ARCO(2,3)=ALF1
      ARCO(1,4)=BET1
      ARCO(2,4)=BET1
      ARCO(1,5)=ALF2
      ARCO(2,5)=BET2
      ARCO(1,6)=BET2
      ARCO(2,6)=ALF2
      ARCO(1,7)=BET2
      ARCO(2,7)=BET2
      W(1)=WF1
      W(2)=WF2
      W(3)=WF2
      W(4)=WF2
      W(5)=WF3
      W(6)=WF3
      W(7)=WF3
      DO 102 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      B(I)=YL(K)-YL(J)
102   C(I)=XL(J)-XL(K)
      DT=B(1)*C(2)-B(2)*C(1)
      DO 103 I=1,35
103   ENK(I)=0.
      DO 121 I=1,35
      DO 121 J=1,35
121   SMK1(J,I)=0.
      DO 122 K=1,7
      A(1)=ARCO(1,K)
      A(2)=ARCO(2,K)
      A(3)=1.-A(1)-A(2)
      CALL SHAP(A,B,C,DT,2)
      IF(K.NE.1) GO TO 502
      DO 501 I=1,35
      DO 501 J=1,8
501   S1(J,I)=S(J,I)
502   CONTINUE
      DO 104  I=1,35
104   ENK(I)=ENK(I)+DT/2.*W(K)*ENP(I,1)
      DO 122 I=1,35
      DO 122 J=1,35
122   SMK1(J,I)=SMK1(J,I)+DT/2.*W(K)*SM(J,I)
      LOCAL  RECLASSIFICATION  OF THE  DEGREES OF FREEDOM
      DO 105 J=1,35
      DO 105 I=1,35
105   TOT(I,J)=0.
      DO 106 K1=1,7,3
      K2=7*(K1+2)/3-4
      TOT(K1,K2)=1.
      DO 106 K3=1,3,2
      TOT(K1+1,K2+K3)=1.
106   TOT(K1+2,K2+K3+1)=1.
      DO 107 K1=6,20,7
      K2=2*(K1+1)/7+8
      DO 107 K3=1,6,5
      K4=6*(K3-1)/5
```

```
      TOT(K2+K4,K1-K3+1)=1.
  107 TOT(K2+K4+1,K1-K3+2)=1.
      DO 108 K1=22,30,4
      K2=K1/2+11
      DO 108 K3=1,3,2
      K4=6*(K3-1)/2
      TOT(K2+K4,K1+K3-1)=1.
  108 TOT(K2+K4+1,K1+K3)=1.
      TOT(34,34)=1.
      TOT(35,35)=1.
      CALL TIMES(SMK1,TOT,SM,35,35,35,1)
      CALL TIMES(TOT,SM,SMK1,35,35,35,2)
      CALL TIMES(SL,TOT,S,9,35,35,1)
      CALL TIMES(TOT,ENK,ENP,35,35,1,2)
      DO 109 I=1,35
  109 PINK(I)=ENP(I,1)
      DO 110 I=1,35
      DO 110 J=1,9
  110 TIK(J,I)=S(J,I)
      DO 124 I=1,35
      DO 124 J=1,35
  124 SMK(J,I)=SMK1(J,I)
      CALL CNDNS(35)
      CALL CNDNS(34)
      RETURN
      END
      SUBROUTINE TRANS(G1,B1,C1)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PAT/TRMA(3,3),XO(3),YO(3)
      DIMENSION B1(3),G1(3),C1(3)
      FORM  THE TRANSFORMATION MATRIX LOCAL=(TRMA)*GLOBAL
      DO 10 J=1,3
      DO 10 I=1,3
   10 TRMA(I,J)= .
      AA=B1(2)*C1(3)-B1(3)*C1(2)
      AB=G1(3)*C1(2)-G1(2)*C1(3)
      AC=B1(3)*G1(2)-G1(3)*B1(2)
      IF((AA.GT.-.1D-12.AND.AA.LT.0.1D-12).AND.(AB.GT.-.1D-12.AND.AB.LT.
     1 .1D-12))GO TO 2
      AD=DSQRT(AA**2+AB**2+AC**2)
      AE=DSQRT(AA**2+AB**2)
      TRMA(3,1)=AA/AD
      TRMA(3,2)=AB/AD
      TRMA(3,3)=AC/AD
      TRMA(1,1)=-AB/AE
      TRMA(1,2)=AA/AE
      TRMA(2,1)=-AA*AC/(AE*AD)
      TRMA(2,2)=-AB*AC/(AE*AD)
      TRMA(2,3)=(AA**2+AB**2)/(AE*AD)
      RETURN
   20 DO 30 I=1,3
   30 TRMA(I,I)=1.
      RETURN
      END
      SUBROUTINE  TRANL(X,Y,Z,K,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PAT/TRG(3,3),XG(3),YG(3)
      DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(3,3),TR(3,3),X2(3),Y2(3),Z2(
     13)
      DO 100 J=1,2
```

```
      X2(J)=X(K,J)
      Y2(J)=Y(K,J)
      Z2(J)=Z(K,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      C2=Z2(2)-Z2(1)
      GLEN2=DSQRT(G2**2+B2**2+C2**2)
      IF(GLEN2.LT. .1D-12)WRITE(6,7  )
      TR(1,1)=G2/GLEN2
      TR(2,1)=B2/GLEN2
      TR(3,1)=C2/GLEN2
      TR(1,2)=TRG(3,2)*TR(3,1)-TRG(3,3)*TR(2,1)
      TR(2,2)=TRG(3,3)*TR(1,1)-TRG(3,1)*TR(3,1)
      TR(3,2)=TRG(3,1)*TR(2,1)-TRG(3,2)*TR(1,1)
      TR(1,3)=TRG(3,1)
      TR(2,3)=TRG(3,2)
      TR(3,3)=TRG(3,3)
      CALL TIMES(TRG,TR,TRL,3,3,3,1)
    7 FORMAT(1 X,'      **** ERROR   GLEN2 ****',/)
      RETURN
      END
      SUBROUTINE CONDS(K)
      IMPLICIT REAL*8 (A-H,O-Z)
      CONDENSES OUT  THE  K TH. DEGREE OF FREEDOM
      COMMON/STI/X(3,2 ),YOUNG(12),SIM(36,36),TIM(8,36),VIM(36),INFO(20)
      DIMENSION B(36)
      A=1./SIM(K,K)
      DO 40  I=1,35
   40 B(I)=SIM(K,I)
      DO 41  J=1,35
      DO 41  I=1,35
   41 SIM(I,J)=SIM(I,J)-B(I)*B(J)*A
      DO 42  I=1,8
      D=TIM(I,K)
      DO 42  J=1,35
   42 TIM(I,J)=TIM(I,J)-B(J)*A*D
      D=VIM(K)
      DO 43 I=1,35
   43 VIM(I)=VIM(I)-B(I)*D*A
      RETURN
      END
      SUBROUTINE   TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  A(1),B(1),R(1)
      KOK=1  A(N,M),B(M,L),R(N,L)    REGULAR     A*B
      KOK=2  A(M,N),B(M,L),R(N,L)    TRANSPOSE   A'*B
      IR=1
      DO  100   K=1,L
      DO  100   J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
  101 CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
  103 R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
  102 CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
```

```
      IB=M*(K-1)+I
104   B(IB)=B(IB)+A(IA)*B(IB)
1     IB=IB+1
      RETURN
      END
```

3. Reference symbol DMX36

```
=10 P=100 PRCUTE=?URF COPIES=4
E = UNIVERSITY, FATCH
N WAS: 13:45:12
SIGNED ON AT 15:51:57 ON MON SEP 22/75
*PRINT*
      ****** DONE MIXED ELEMENT 36 DG. OF FREEDOM AT 6 NODES ******
      *****  QUADRATIC VARIATION OF U,V,W,MXX,MYY,MXY         *********
      ***** TRANSFORMATION  TO GLOBAL SET OF  U,V,W,    AND MNN,MSS,MNS**
      *****  STRESSES  AT 1ST VERTIX QX1,QY1, 2ND VERTIX QX2,QY2 ******
      ******  STRESSES AT CENTROID    MXX,NYY,NXY              *************
      ******   MODULI OF ELASTISITY    THROUGH  YOUNG(12)      ********
       SUBROUTINE TRANS(G1,B1,C1)
       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON/COOR/TRDF(3,3),XG(3),YG(3),TRM(3,3)
       DIMENSION B1(3),G1(3),C1(3),CN(3,3)
C   TRDF   TRANSFORMATION MATRIX FOR THE DEFLACTION   'LOCAL'=TRDF*'GLOBAL'
C   TRM    TRANSFORMATION MATRIX FOR THE MOMENTS      'LOCAL'=TRM *'GLOBAL'
       DO 10 I=1,3
       DO 10 J=1,3
       TRM(J,I)=0.
   10  TRDF(J,I)=0.
       AA=B1(2)*C1(3)-B1(3)*C1(2)
       AB=C1(3)*G1(2)-G1(2)*C1(3)
       AC=B1(3)*G1(2)-G1(3)*B1(2)
       IF((AA.GT.-.1D-12.AND.AA.LT.0.1D-12).AND.(AB.GT.-.1D-12.AND.AB.LT.
      10.1D-12)) GO TO 20
       AD=DSQRT(AA**2+AB**2+AC**2)
       AE=DSQRT(AA**2+AB**2)
       TRDF(3,1)=AA/AD
       TRDF(3,2)=AB/AD
       TRDF(3,3)=AC/AD
       TRDF(1,1)=-AB/AE
       TRDF(1,2)=AA/AE
       TRDF(2,1)=-AA*AC/(AE*AD)
       TRDF(2,2)=-AB*AC/(AE*AD)
       TRDF(2,3)=(AA**2+AB**2)/(AE*AD)
       GO TO 4
   20  DO 30 I=1,3
   30  TRDF(I,I)=1.
   4   A2=TRDF(1,1)
       A1=DSIN(DARCOS(A2))
       A3=TRDF(3,3)
       TRM(1,1)=A1**2
       TRM(1,2)=A2**2*A3
       TRM(1,3)=2.*A1*A2*A3
       TRM(2,1)=A2**2
       TRM(2,2)=A1**2*A3
       TRM(2,3)=-2.*A1*A2*A3
       TRM(3,1)=-A1*A2
       TRM(3,2)=A1*A2*A3
       TRM(3,3)=(A1**2-A2**2)*A3
       RETURN
       END
       SUBROUTINE TRANL(X,Y,Z,K,TRM)
       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON/COOR/TRG(3,3),XG(3),YG(3),CCM(3,3)
       DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(3,3),TR(3,3),TRM(3,3)
      1,X2(3),Y2(3),Z2(3)
       DO 100 J=1,2
       X2(J)=X(K,J)
```

```
      Y2(J)=Y(K,J)
  100 Z2(J)=Z(K,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      C2=Z2(2)-Z2(1)
      GLEN2=DSQRT(G2**2+B2**2+C2**2)
      IF(GLEN2.LT.0.1D-12) WRITE(6,700)
      TR(1,1)=G2/GLEN2
      TR(2,1)=B2/GLEN2
      TR(3,1)=C2/GLEN2
      TR(1,2)=TRG(3,2)*TR(3,1)-TRG(3,3)*TR(2,1)
      TR(2,2)=TRG(3,3)*TR(1,1)-TRG(3,1)*TR(3,1)
      TR(3,2)=TRG(3,1)*TR(2,1)-TRG(3,2)*TR(1,1)
      TR(1,3)=TRG(3,1)
      TR(2,3)=TRG(3,2)
      TR(3,3)=TRG(3,3)
      CALL TIMES(TRG,TR,TRL,3,3,3,1)
  700 FORMAT(10X,'     ****  ERROR   TRANL  ****      ',/)
      A2=TRL(1,1)
      A1=DSIN(DARCOS(A2))
      A3=TRL(3,3)
      TRM(1,1)=A1**2
      TRM(1,2)=A2**2*A3
      TRM(1,3)=2.*A1*A2*A3
      TRM(2,1)=A2**2
      TRM(2,2)=A1**2*A3
      TRM(2,3)=-2.*A1*A2*A3
      TRM(3,1)=-A1*A2
      TRM(3,2)=A1*A2*A3
      TRM(3,3)=(A1**2-A2**2)*A3
      RETURN
      END
     ****    MIXED  MODEL  24   DEGREES OF  FREEDOM  W, MX, MY, MXY,  6 NODES**
     **********   STRESSES  QX, QY AT FIRST AND SECOND NODES*******************
     **********        TRANSFORMATION   TO W,   MNN,   MSS,   MSN*****************
     *** QUADRATIC  VARIATION  OF ALL  THE FUNCTIONS  W, MXX, MYY, MXY *****
      SUBROUTINE  STIFF
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(20)
      COMMON/MAN/BOL(2,36,36),COL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
      COMMON/COOR/CO2(3,3),XO(3),YO(3),COM(3,3)
      DIMENSION  TRAN(36,36),SMK(36,36),STR(8,36),DAOL(36),MOL(12),
     1DAOC(36),XM(16,2),YM(16,2),ZM(16,2),X1(3),Y1(3),Z1(3),CO3(3,3)
     2,G1(3),B1(3),C1(3),CO1(3,3),CO4(3,3)
      DATA MOL/1,2,7,8,13,14,19,20,25,26,31,32/
      DO 100 I=1,3
      X1(I)=X(1,I)
      Z1(I)=X(3,I)
  100 Y1(I)=X(2,I)
      DO 101 I=1,36
      DO 101 J=1,36
  101 STUCK(J,I)=0.
      DO 102 I=1,36
      DO 102 J=1,8
  102 STICK(J,I)=0.
      DO 103 I=1,36
      DAOC(I)=0.
  103 FORCE(I)=0.
      IF(NGRAM.EQ.0) GO TO 517
```

```
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
      ZM(M,I)=GRAM(M,I+4)
  516 YM(M,I)=GRAM(M,I+2)
  517 CONTINUE
      DO 203 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      G1(I)=X1(J)-X1(K)
      B1(I)=Y1(J)-Y1(K)
  203 C1(I)=Z1(J)-Z1(K)
      CALL TRANS(G1,B1,C1)
      DO 204 I=1,3
      DO 204 J=1,3
  204 CO4(J,I)=X(J,I)
      CALL TIMES(CO2,CO4,CO1,3,3,3,1)
      DO 205 J=1,3
      XO(J)=CO1(1,J)
  205 YO(J)=CO1(2,J)
      CNLD=0.
      DO 432 I=1,3
      DO 432 J=6,11
  432 IF(X(I,J).NE.0.) CNLD=1.
      IF(CNLD.NE.1.)GO TO 437
      DO 433 I=6,11
      J=6*(I-6)+1
      DACG(J)=X(1,I)
      DACG(J+1)=X(2,I)
  433 DACG(J+2)=X(3,I)
      DO 434 I=1,36
      DO 435 J=1,36
  435 TRAN(J,I)=0.
  434 TRAN(I,I)=1.
      DO 436 K=1,36,6
      DO 436 I=1,3
      DO 436 J=1,3
  436 TRAN(K+J-1,K+I-1)=CO2(J,I)
      CALL TIMES(TRAN,DACG,DACL,36,36,1,1)
  437 CONTINUE
      NLAK=INFO(1)
      METRO=
      IF(NLAK.EQ.-1.OR.NLAK.EQ.-2) GO TO 840
      IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
      CALL SUBTI
      IF(NLAK.EQ.0)GO TO 105
      NLAK=NLAK-1
      DO 106 I=1,36
      DO 106 J=1,36
  106 BOL(NLAK,J,I)=STUCK(J,I)
      DO 113 I=1,36
      DO 113 J=1,8
  113 COL(NLAK,J,I)=STICK(J,I)
      DO 108 I=1,36
  108 DOL(NLAK,I)=FORCE(I)
      GO TO 105
  840 NLAK=-NLAK
      METRO=-1
```

```
104 DO 109 I=1,36
    DO 109 J=1,36
109 STUCK(J,I)=BOL(NLAK,J,I)
    DO 214 I=1,36
    DO 214 J=1,8
214 STICK(J,I)=COL(NLAK,J,I)
    DO 111 I=1,36
111 FORCE(I)=DOL(NLAK,I)
    IF(METEO.NE.-1) GO TO 105
    DO 841 I=1,36
    DO 841 J=1,8
    E1=1.
    DO 842 II=1,12
842 IF(I.EQ.MOL(II)) E1=-1.
841 STICK(J,I)=STICK(J,I)*E1
105 CONTINUE
    DO 438 I=1,36
438 FORCE(I)=FORCE(I)+DAOL(I)
    NLIK=INFO(2)
    IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 115
    DO 115 I=1,NLIK
    K=(I-1)*2+3
    L=INFO(K)
    K1=INFO(K+1)
    DO 583 II=1,36
    DO 584 IJ=1,36
584 TRAN(IJ,II)=0.
583 TRAN(II,II)=1.
    KK=(L-1)*6
    DO 219 IL=1,3
    DO 219 JL=1,3
219 TRAN(KK+IL,KK+JL)=CO2(IL,JL)
    IF(K1.EQ.-1) GO TO 212
    IF(K1.EQ.0) GO TO 215
    CALL TRANL(XM,YM,ZM,K1,CO3)
    DO 222 IL=1,3
    DO 222 JL=1,3
222 TRAN(KK+IL+3,KK+JL+3)=CO3(IL,JL)
    GO TO 212
215 DO 216 IL=1,3
    DO 216 JL=1,3
216 TRAN(KK+IL+3,KK+JL+3)=COM(IL,JL)
212 CONTINUE
    CALL TIMES(STUCK,TRAN,SMK,36,36,36,1)
    CALL TIMES(TRAN,SMK,STUCK,36,36,36,2)
    CALL TIMES(STICK,TRAN,STR,8,36,36,1)
    CALL TIMES(TRAN,FORCE,DAOG,36,36,1,2)
    DO 232 IL=1,36
    DO 232 JL=1,8
232 STICK(JL,IL)=STR(JL,IL)
    DO 233 IL=1,36
233 FORCE(IL)=DAOG(IL)
115 CONTINUE
    RETURN
    END
    SUBROUTINE SUBTI
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),YO(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(2 )
    COMMON/TRDA/B(3),G(3),EL(3),DT
```

```
      COMMON/CLOLP/FMM(18,18),FMO(18,18),STMN(18,6),STR(8,36)
     1,STUV(12,12)
      COMMON/COSP/CO2(3,3),XL(3),YL(3),COM(3,3)
      DIMENSION STM(18,18),TT(36,36),M1(36),SIM(36,36),CF(36,36),PG(3),
     1PL(3)
      DO 100 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      F(I)=YL(K)-YL(J)
      G(I)=XL(J)-XL(K)
100   EL(I)=F(I)**2+G(I)**2
      DT=F(1)*G(2)-F(2)*G(1)
      DO 111 I=1,36
      DO 111 J=1,8
111   STR(J,I)= .
      CALL INTSU
      DO 101 I=1,18
      DO 1 1 J=1,18
101   STM(J,I)=FMM(J,I)+FMO(J,I)
      DO 102 I=1,36
      DO 102 J=1,36
      TT(J,I)=0.
1 2   SIM(J,I)= .
      DO 103 I=1,6
      DO 103 J=1,18
1 3   SIM(J+6,I)=STMN(J,I)
      DO 104 I=1,18
      DO 104 J=1,6
104   SIM(J,I+6)=STMN(I,J)
      DO 105 I=1,18
      DO 1 5 J=1,18
105   SIM(J+6,I+6)=-STM(J,I)
      DO 200 I=1,12
      DO 2   J=1,12
200   SIM(J+24,I+24)=STUV(J,I)
      DATA M1/25,26,1,7,8,9,27,28,2,10,11,12,29,30,3,13,14,15,31,32,4
     1,16,17,18,33,34,5,19,20,21,35,36,6,22,23,24/
      DO 107 I=1,36
1 7   TT(M1(I),I)=1.
      CALL TIMES(SIM,TT,CF,36,36,36,1)
      CALL TIMES(TT,CF,STUCK,36,36,36,2)
      CALL TIMES(STR,TT,STICK,8,36,36,1)
      PG(1)=X(1,5)
      PG(2)=X(2,5)
      PG(3)=X(3,5)
      CALL TIMES(CO2,PG,PL,3,3,1,1)
      FORCE(19)=PL(1)*DT/6.
      FORCE(20)=PL(2)*DT/6.
      FORCE(21)=PL(3)*DT/6.
      FORCE(25)=PL(1)*DT/6.
      FORCE(26)=PL(2)*DT/6.
      FORCE(27)=PL(3)*DT/6.
      FORCE(31)=PL(1)*DT/6.
      FORCE(32)=PL(2)*DT/6.
      FORCE(33)=PL(3)*DT/6.
      RETURN
      END
      SUBROUTINE INTSU
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
      D21=YOUNG(12)
      COMMON/TRPA/B(3),G(3),EL(3),DT
      COMMON/OLCEP/EMM(18,18),EMO(18,18),EMW(18,6),STR(8,36),STUV(12,12)
      COMMON/STI/X(3,2 ),YOUNG(12),ST(36,36),SF(8,36),FCP(36),INFO(20)
      DIMENSION A1(7),A2(7),A3(7),W(7),EEB(3,3),EES(2,2),A(3),
     1EN(6),FE(6),FF(6),ENMO(2,18),ENM2(2,6),ENM1(2,18)
     2,EMW1(18,6),EMM1(3,18),EMM2(18,18),EMO1(2,18),EMO2(18,18)
     3,SUV(3,12),BUV(3,12),D(3,3),EUV(12,12)
      DATA A1/0.33333333D 00,0.05971587D 00,2*0.47014206D 00,0.79742699D
     1    ,2* .1 128651D    /,A3/ .33333333D    ,2* .47 142 6D    , . 597158
     27D 00,2*0.10128651D 00,0.79742699D 00/,W/0.22500000D 00,3*0.132394
     315D 00,3*0.12593918D 00/
      D11=YOUNG(1)
      D12=YOUNG(2)
      D22=YOUNG(3)
      D33=YOUNG(4)/2.
      D44=YOUNG(5)
      D55=YOUNG(6)
      D21=D12
      DODO=YOUNG(12)
      IF(DODO.NE. .)D21=DODO
      D66=YOUNG(7)
      D67=YOUNG(8)
      D76=YOUNG(9)
      D77=YOUNG(10)
      D88=YOUNG(11)
      DO 100 I=1,3
      DO 100 J=1,3
1.    EEB(J,I)=0.
      DO 101 I=1,2
      DO 101 J=1,2
1 1   EES(J,I)=0.
      DO 200 I=1,3
      DO 2    J=1,3
200   D(J,I)=0.
      DET=D11*D22-D12*D21
      IF(DET.EQ. .)WRITE(6,7  )
700   FORMAT(/,'        ***** ERROR    **** ELASTICITY  MATRIX ',/)
      EEB(1,1)=D22/DET
      EEB(1,2)=-D12/DET
      EEB(2,1)=-D21/DET
      EEB(2,2)=D11/DET
      EEB(3,3)=1./D33
      EES(1,1)=1./D44
      EES(2,2)=1./D55
      D(1,1)=D66
      D(1,2)=D67
      D(2,1)=D76
      D(2,2)=D77
      D(3,3)=D88
      DO 107 I=1,18
      DO 107 J=1,18
      EMM(J,I)=0.
107   EMO(J,I)=0.
      DO 108 I=1,6
      DO 108 J=1,18
108   EMW(J,I)=0.
      DO 208 I=1,12
      DO 208 J=1,12
```

```
 208 STUV(J,I)=0.
C         INTEGRATION
      DO 102 K=1,9
      IF(K.EQ.8) A(1)=1.
      IF(K.EQ.8) A(2)=0.
      IF(K.EQ.8) A(3)=0.
      IF(K.EQ.9) A(1)= .
      IF(K.EQ.9) A(2)=1.
      IF(K.EQ.9) A(3)=0.
      IF(K.GT.7)GO TO 112
      A2(K)=1.-A1(K)-A3(K)
      A(1)=A1(K)
      A(2)=A2(K)
      A(3)=A3(K)
 112  CONTINUE
      EN(1)=(2.*A(1)-1.)*A(1)
      EN(2)=(2.*A(2)-1.)*A(2)
      EN(3)=(2.*A(3)-1.)*A(3)
      EN(4)=4.*A(2)*A(3)
      EN(5)=4.*A(1)*A(3)
      EN(6)=4.*A(1)*A(2)
      EE(1)=B(1)*(4.*A(1)-1.)/DT
      EE(2)=B(2)*(4.*A(2)-1.)/DT
      EE(3)=B(3)*(4.*A(3)-1.)/DT
      EE(4)=4.*(B(2)*A(3)+B(3)*A(2))/DT
      EE(5)=4.*(B(1)*A(3)+B(3)*A(1))/DT
      EE(6)=4.*(B(1)*A(2)+B(2)*A(1))/DT
      FF(1)=G(1)*(4.*A(1)-1.)/DT
      FF(2)=G(2)*(4.*A(2)-1.)/DT
      FF(3)=G(3)*(4.*A(3)-1.)/DT
      FF(4)=4.*(G(2)*A(3)+G(3)*A(2))/DT
      FF(5)=4.*(G(1)*A(3)+G(3)*A(1))/DT
      FF(6)=4.*(G(1)*A(2)+G(2)*A(1))/DT
      DO 103 I=1,18
      DO 103 J=1,3
 103  ENMO(J,I)=0.
      DO 303 I=1,18
      DO 303 J=1,2
 303  ENM1(J,I)=0.
      DO 203 I=1,12
      DO 203 J=1,3
 203  BUV(J,I)=0.
      DO 205 J=1,12,2
      M=(J+1)/2
      BUV(1,J)=EE(M)
      BUV(2,J+1)=FF(M)
      BUV(3,J)=FF(M)
 205  BUV(3,J+1)=EE(M)
      CALL TIMES(D,BUV,SUV,3,3,12,1)
      CALL TIMES(BUV,SUV,EUV,12,3,12,2)
      DO 104 J=1,6
      DO 104 I=1,3
      M=3*J+I-3
 104  ENMO(I,M)=EN(J)
      DO 105 J=1,13,3
      M=(J+2)/3
      ENM1(1,J)=EE(M)
      ENM1(1,J+2)=FF(M)
      ENM1(2,J+1)=FF(M)
 105  ENM1(2,J+2)=EE(M)
```

```
      IF(K.GT.7)GO TO 113
      DO 106 J=1,6
      ENW2(1,J)=EE(J)
  106 ENW2(2,J)=EE(J)
      CALL TIMES(ENM1,ENW2,EMW1,18,2,6,2)
      CALL TIMES(AE6,ENM0,ENM1,3,3,18,1)
      CALL TIMES(ENM ,ENM1,ENM2,18,3,18,2)
      CALL TIMES(EES,ENM1,EMO1,2,2,18,1)
      CALL TIMES(ENM1,EMO1,EMO2,18,2,18,2)
      DO 109 I=1,18
      DO 109 J=1,18
      ENM(J,I)=ENM(J,I)+W(K)*ENM2(J,I)*DT/2.
  109 EMO(J,I)=EMO(J,I)+W(K)*EMO2(J,I)*DT/2.
      DO 110 I=1,6
      DO 110 J=1,18
  110 EMW(J,I)=EMW(J,I)+W(K)*EMW1(J,I)*DT/2.
      DO 210 I=1,12
      DO 210 J=1,12
  210 STUV(J,I)=STUV(J,I)+W(K)*EUV(J,I)*DT/2.
  113 CONTINUE
      IF(K.NE.1)GO TO 224
      DO 225 I=1,12
      DO 225 J=1,3
  225 STR(J+4,I+24)=SUV(J,I)
  224 CONTINUE
      IF(K.LT.8) GO TO 102
      L=0
      IF(K.EQ.9) L=2
      DO 114 I=1,18
      DO 114 J=1,2
  114 STR(L+J,I+6)=ENM1(J,I)
  102 CONTINUE
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),R(1)
C     KOK=1   A(N,M) , B(M,L) , R(N,L)     REGULAR     A*B
C     KOK=2   A(M,N) , B(M,L) , R(N,L)     TRANSPOSE  A'*B
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
  101 CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
  103 R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
  102 CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
  104 R(IR)=R(IR)+A(IA)*B(IB)
  100 IR=IR+1
      RETURN
      END
```

4. Reference symbol DR030

```
10 P=100 PROUTE=NUPF COPIES=4
E = UNIVERSITY, BATCH
N WAS: 15:51:57
 SIGNED ON AT  15:52:26 ON MON SEP 22/75
*PRINT*
     *** DOME  ROTATION ELEMENT WITH 30 DEG. OF FR. AT 6 NODES U,V,W,RX,RY****
     **** CUBIC VARIATION FOR THE DIS. W QUADRATIC FOR THE ROTATIONS     ******
     **** 5  STRESSES AT THE CENTROID    MXX,MYY,MXY,QX,QY               ******
     ****  TRANSFORMATION  TO  RN  RS                                    ******
     ****  ELASTICITY  MODULI  VIA  YOUNG(12)                            ******
      SUBROUTINE STIFF
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36)
     1,INEF(20)
      COMMON/MAN/BCL(2,36,36),CCL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
      COMMON/COOR/CO2(3,3),XO(3),YO(3)
      DIMENSION TRAN(30,30),SMK(30,30),SMK1(30,30),STR(8,30),TIK(8,30),
     1DAOG(30),XM(16,2),YM(16,2),CO3(2,2),DAOL(30),MOL(6),FOR(30)
     2,ZM(16,2),A1(3),B1(3),C1(3),X1(3),Y1(3),Z1(3),CO4(3,3),CO1(3,3)
      DATA MOL/3,8,13,18,23,28/
      DO 100 I=1,3
      X1(I)=X(1,I)
      Z1(I)=X(3,I)
 100  Y1(I)=X(2,I)
      DO 101 I=1,36
      DO 1 1 J=1,36
 101  STUCK(J,I)=0.
      DO 102 I=1,36
      DO 102 J=1,8
 102  STICK(J,I)=0.
      DO 1 3 I=1,36
 103  FORCE(I)=0.
      DO 460 I=1,30
      DAOL(I)=0.
 460  DAOG(I)=0.
      IF(NGRAM.EQ.0) GO TO 517
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
      ZM(M,I)=GRAM(M,I+4)
 516  YM(M,I)=GRAM(M,I+2)
 517  CONTINUE
      DO 400 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      A1(I)=X1(J)-X1(K)
      B1(I)=Y1(J)-Y1(K)
 4    C1(I)=Z1(J)-Z1(K)
      CALL  TRANS(A1,B1,C1)
      DO 401 I=1,3
      DO 4 1 J=1,3
 401  CO4(I,J)=X(I,J)
      CALL  TIMES(CO2,CO4,CO1,3,3,3,1)
      DO 4 2 J=1,3
      XO(J)=CO1(1,J)
 402  YO(J)=CO1(2,J)
      CMLD=0.
      DO 432 I=1,3
```

```
      DO 432 J=6,8
432   IF(X(I,J).NE.0.)CNLD=1.
      IF(CNLD.NE.1.) GO TO 437
      DO 683 II=1,30
      DO 684 IJ=1,30
684   TRAN(IJ,II)=0.
683   TRAN(II,II)=1.
      DO 433 I=6,8
      J=5*(I-6)+1
      DAOG(J)=X(1,I)
      DAOG(J+1)=X(2,I)
      DAOG(J+2)=X(3,I)
433   CONTINUE
      DO 436 K=1,30,5
      DO 436 I=1,3
      DO 436 J=1,3
436   TRAN(K+J-1,K+I-1)=CC2(J,I)
      CALL TIMES(TRAN,DAOG,DAOL,30,30,1,1)
437   CONTINUE
      NLAK=INFO(1)
      METRO=0
      IF(NLAK.EQ.-1.OR.NLAK.EQ.-2) GO TO 840
      IF(NLAK.EQ.1.OR.NLAK.EQ.2) GO TO 104
      CALL SUBTI
      IF(NLAK.EQ.0) GO TO 105
      NLAK=NLAK-10
      DO 106 I=1,30
      DO 106 J=1,30
106   BOL(NLAK,J,I)=STUCK(J,I)
      DO 113 I=1,30
      DO 113 J=1,8
113   COL(NLAK,J,I)=STICK(J,I)
      DO 108 I=1,30
108   DOL(NLAK,I)=FORCE(I)
      GO TO 105
840   NLAK=-NLAK
      METRO=-1
104   DO 109 I=1,30
      DO 109 J=1,30
109   STUCK(J,I)=BOL(NLAK,J,I)
      DO 214 I=1,30
      DO 214 J=1,8
214   STICK(J,I)=COL(NLAK,J,I)
      DO 111 I=1,30
111   FORCE(I)=DOL(NLAK,I)
      IF(METRO.NE.-1) GO TO 105
      DO 861 I=1,30
      DO 861 J=1,30
      E1=0.
      IF(I.NE.J) GO TO 862
      E1=-1.
      DO 863 II=1,6
      IF(I.EQ.MOL(II)) E1=1.
863   CONTINUE
862   TRAN(J,I)=E1
861   CONTINUE
      DO 1 I=1,30
      DO 1 J=1,30
    1 SMK1(J,I)=STUCK(J,I)
      DO 2 I=1,30
```

```
      DO 2 J=1,8
    2 TIK(J,I)=STICK(J,I)
      DO 3 I=1,30
    3 DACG(I)=FORCE(I)
      CALL TIMES(SMK1,TRAN,SMK,30,30,30,1)
      CALL TIMES(TRAN,SMK,SMK1,3 ,3 ,3 ,2)
      CALL TIMES(TIK,TRAN,STR,8,30,30,1)
      CALL TIMES(TRAN,DACG,FOR,30,30,1,2)
      DO 4 I=1,3
      DO 4 J=1,30
    4 STUCK(J,I)=SMK1(J,I)
      DO 5 I=1,3
      DO 5 J=1,8
    5 STICK(J,I)=STR(J,I)
      DO 6 I=1,3
    6 FORCE(I)=FOR(I)
  105 CONTINUE
      DO 438 I=1,30
  438 FORCE(I)=FORCE(I)+DACL(I)
      NLIK=INFO(2)
      IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
      DO 213 I=1,30
      DO 213 J=1,3
  213 SMK(J,I)=STUCK(J,I)
      DO 114 I=1,30
      DO 114 J=1,8
  114 STR(J,I)=STICK(J,I)
      DO 439 I=1,3
  439 DACL(I)=FORCE(I)
      DO 115 I=1,NLIK
      K=(I-1)*2+3
      L=INFO(K)
      K1=INFO(K+1)
      DO 583 II=1,30
      DO 584 IJ=1,30
  584 TRAN(IJ,II)= .
  583 TRAN(II,II)=1.
      KK=(L-1)*5
      DO 588 IN=1,3
      DO 588 JN=1,3
  588 TRAN(KK+IN,KK+JN)=CO2(IN,JN)
      IF(K1.LT.0) GO TO 589
      IF(K1.EQ. ) GO TO 59
      CALL TRANI(XM,YM,K1,CO3)
      DO 587 IN=1,2
      DO 587 IL=1,2
  587 TRAN(KK+IL+3,KK+IN+3)=CO3(IL,IN)
      GO TO 589
  590 DO 591 IN=1,2
      DO 591 JN=1,2
  591 TRAN(KK+JN+3,KK+IN+3)=CO2(JN,IN)
  589 CONTINUE
      CALL TIMES(SMK,TRAN,SMK1,30,30,30,1)
      CALL TIMES(TRAN,SMK1,SMK,3 ,3 ,3 ,2)
      CALL TIMES(STR,TRAN,TIK,8,30,30,1)
      CALL TIMES(TRAN,DACL,DACG,30,30,1,2)
      DO 116 NI=1,3
      DO 116 NJ=1,8
  116 STR(NJ,NI)=TIK(NJ,NI)
      DO 440 II=1,30
```

```
440 CACL(II)=CACG(II)
115 CONTINUE
    DO 117 I=1,30
    DO 117 J=1,3)
117 STUCK(J,I)=SMK(J,I)
    DO 118 I=1,30
    DO 118 J=1,8
118 STICK(J,I)=STP(J,I)
    DO 441 I=1,30
441 FORCE(I)=CACL(I)
112 CONTINUE
    RETURN
    END
    SUBROUTINE TRANL(X,Y,K1,TPL)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION X(16,2),Y(16,2),Z(16,2),Z2(2),X2(2),Y2(2),TPL(2,2)
   1,TR(3,3),TR1(3,3)
    COMMON/COOP/TRG(3,3),X0(3),Y0(3)
    DO 515 J=1,2
    X2(J)=X(K1,J)
    Z2(J)=Z(K1,J)
515 Y2(J)=Y(K1,J)
    G2=X2(2)-X2(1)
    B2=Y2(2)-Y2(1)
    C2=Z2(2)-Z2(1)
    CLEN2=DSQRT(G2**2+B2**2+C2**2)
    IF(CLEN2.LT. .1D-12.AND.CLEN2.GT.- .1D-12) WRITE(6,700)
    B2=B2/CLEN2
    G2=G2/CLEN2
    C2=C2/CLEN2
    TR(1,1)=G2
    TR(2,1)=B2
    TR(3,1)=C2
    TR(1,2)=TRG(3,2)*TR(3,1)-TRG(3,3)*TR(2,1)
    TR(2,2)=TRG(3,3)*TR(1,1)-TRG(3,1)*TR(3,1)
    TR(3,2)=TRG(3,1)*TR(2,1)-TRG(3,2)*TR(1,1)
    TR(1,3)=TRG(3,1)
    TR(2,3)=TRG(3,2)
    TR(3,3)=TRG(3,3)
    CALL TIMES(TRG,TR,TR1,3,3,3,1)
    DO 10 I=1,2
    DO 10 J=1,2
10  TPL(J,I)=TR1(J,I)
700 FORMAT('      ******    ERROR   TRANL      *******')
    RETURN
    END
    SUBROUTINE CNDNS(K)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/CON/ST(33,33),T(8,33)
    DIMENSION B(33)
    A=1./ST(K,K)
    DO 40 I=1,33
40  B(I)=ST(K,I)
    DO 41 J=1,33
    DO 41 I=1,33
41  ST(J,I)=ST(J,I)-B(I)*B(J)*A
    DO 42 I=1,8
    D=T(I,K)
    DO 42 J=1,33
42  T(I,J)=T(I,J)-B(J)*A*D
```

```
      RETURN
      END
      SUBROUTINE SHAP(A,B,G,DT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3),B(3),G(3),BE(8,33),SR(6),SRX(6),SRY(6),
     1SWX(7),SWY(7)
      COMMON/JCN/SM(33,33),S(8,33),D(8,8)
      DIM1=A(1)*A(2)*A(3)
      DIM2=(B(1)*A(2)*A(3)+B(2)*A(1)*A(3)+B(3)*A(1)*A(2))/DT
      DIM3=(G(1)*A(2)*A(3)+G(2)*A(1)*A(3)+G(3)*A(1)*A(2))/DT
      DO 100 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      SR(I)=(2.*A(I)-1.)*A(I)
      SR(I+3)=4.*A(K)*A(J)
      SRX(I)=B(I)*(4.*A(I)-1.)/DT
      SRX(I+3)=(B(K)*4.*A(J)+B(J)*4.*A(K))/DT
      SRY(I)=G(I)*(4.*A(I)-1.)/DT
      SRY(I+3)=(G(K)*4.*A(J)+G(J)*4.*A(K))/DT
      SWX(I)=SRX(I)+3.*DIM2
      SWX(I+3)=SRX(I+3)-12.*DIM2
      SWY(I)=SRY(I)+3.*DIM3
      SWY(I+3)=SRY(I+3)-12.*DIM3
  100 CONTINUE
      SWX(7)=27.*DIM2
      SWY(7)=27.*DIM3
      DO 101 I=1,33
      DO 101 J=1,8
  101 BE(J,I)=0.
      DO 102 I=1,11,2
      K=(I+1)/2
      BE(1,I)=SRX(K)
      BE(2,I+1)=SRY(K)
      BE(3,I)=SRY(K)
      BE(3,I+1)=SRX(K)
      BE(4,I)=-SR(K)
      BE(5,I+1)=-SR(K)
  102 CONTINUE
      DO 103 I=13,19
      K=I-12
      BE(4,I)=SWX(K)
  103 BE(5,I)=SWY(K)
      DO 104 I=20,33,2
      K=(I-18)/2
      BE(6,I)=SWX(K)
      BE(7,I+1)=SWY(K)
      BE(8,I)=SWY(K)
  104 BE(8,I+1)=SWX(K)
      CALL TIMES(D,BE,S,8,8,33,1)
      CALL TIMES(BE,S,SM,33,8,33,2)
      RETURN
      END
      SUBROUTINE SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,2 ),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36)
     1,INFO(20)
      COMMON/JCN/SM(33,33),S(8,33),D(8,8)
      COMMON/COOR/CO2(3,3),XO(3),YO(3)
```

```
      COMMON/CON/ST(33,33),TIK(8,33)
      DIMENSION A1(7),A2(7),A3(7),W(7),B(3),G(3),A(3),M1(33),T(33,33)
     1,PG(3),PL(3)
      DATA A1/0.33333333D 00,0.05971587D 00,2*0.47014206D 00,0.79742699D
     1 00,2*0.10128651D 00/,A3/0.33333333D 00,2*0.47014206D 00,0.0597158
     27D 00,2*0.10128651D 00,0.79742699D 00/,W/0.22500000D 00,3*0.132394
     315D 00,3*0.12593918D 00/
      DO 200 I=1,8
      DO 200 J=1,8
  200 D(J,I)=0.
      D(1,1)=YOUNG(1)
      D(1,2)=YOUNG(2)
      D(2,1)=D(1,2)
      D(2,2)=YOUNG(3)
      D(3,3)=YOUNG(4)/2.
      D(4,4)=YOUNG(5)
      D(5,5)=YOUNG(6)
      D(6,6)=YOUNG(7)
      D(6,7)=YOUNG(8)
      D(7,6)=YOUNG(9)
      D(7,7)=YOUNG(10)
      D(8,8)=YOUNG(11)
      IF(YOUNG(12).NE.0.) D(2,1)=YOUNG(12)
      DO 210 I=1,33
      DO 210 J=1,33
  210 ST(J,I)=0.
      DO 211 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      B(I)=YO(K)-YO(J)
  211 G(I)=XO(J)-XO(K)
      DT=B(1)*G(2)-B(2)*G(1)
      DO 212 K=1,7
      A2(K)=1.-A1(K)-A3(K)
      A(1)=A1(K)
      A(2)=A2(K)
      A(3)=A3(K)
      CALL SHAP(A,B,G,DT)
      IF(K.NE.1) GO TO 213
      DO 214 I=1,33
      DO 214 J=1,8
  214 TIK(J,I)=S(J,I)
  213 CONTINUE
      DO 215 I=1,33
      DO 215 J=1,33
  215 ST(J,I)=ST(J,I)+W(K)*SM(J,I)*DT/2.
  212 CONTINUE
      DO 217 I=1,33
      DO 217 J=1,33
  217 T(J,I)=0.
      DATA M1/20,21,13,1,2,22,23,14,3,4,24,25,15,5,6,26,27,16,7,8,
     128,29,17,9,10,30,31,18,11,12,32,33,19/
      DO 218 I=1,33
  218 T(M1(I),I)=1.
      CALL TIMES(ST,T,SM,33,33,33,1)
      CALL TIMES(T,SM,ST,33,33,33,2)
      CALL TIMES(TIK,T,S,8,33,33,1)
      DO 219 I=1,33
```

```
      DO 219 J=1,8
219   TIK(J,I)=S(J,I)
      CALL CNDNS(33)
      CALL CNDNS(32)
      CALL CNDNS(31)
      DO 220 I=1,30
      DO 220 J=1,30
220   STUCK(J,I)=ST(J,I)
      DO 221 I=1,30
      DO 221 J=1,8
221   STICK(J,I)=TIK(J,I)
      PG(1)=X(1,5)
      PG(2)=X(2,5)
      PG(3)=X(3,5)
      CALL TIMES(CC2,PG,PL,3,3,1,1)
      FORCE(16)=PL(1)*DT/6.
      FORCE(17)=PL(2)*DT/6.
      FORCE(18)=PL(3)*DT/6.
      FORCE(21)=PL(1)*DT/6.
      FORCE(22)=PL(2)*DT/6.
      FORCE(23)=PL(3)*DT/6.
      FORCE(26)=PL(1)*DT/6.
      FORCE(27)=PL(2)*DT/6.
      FORCE(28)=PL(3)*DT/6.
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),R(1)
      KOK=1 A(N,M) , B(M,L) , R(N,L)   REGULAR A*B=R
      KOK=2 A(M,N) , B(M,L) , R(N,L) TRANSPOSE  A'*B=R
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
101   CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
103   R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
102   CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
104   R(IR)=R(IR)+A(IA)*B(IB)
100   IR=IR+1
      RETURN
      END
      SUBROUTINE TRANS(G1,B1,C1)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/COOR/TR(3,3),XO(3),YO(3)
      DIMENSION B1(3),G1(3),C1(3)
      DO 10 I=1,3
      DO 10 J=1,3
10    TR(J,I)=0.
      AA=B1(2)*C1(3)-B1(3)*C1(2)
      AB=G1(3)*C1(2)-G1(2)*C1(3)
      AC=B1(3)*G1(2)-G1(3)*B1(2)
```

```
      IF((AA.GT.-.1D-12.AND.AA.LT.0.1D-12).AND.(AB.GT.-.1D-12.AND.AB.
     1LT.0.1D-12)) GO TO 20
      AD=DSQRT(AA**2+AB**2+AC**2)
      AE=DSQRT(AA**2+AB**2)
      TR(3,1)=AA/AD
      TR(3,2)=AB/AD
      TR(3,3)=AC/AD
      TR(1,1)=-AB/AE
      TR(1,2)=AA/AE
      TR(2,1)=-AA*AC/(AE*AD)
      TR(2,2)=-AB*AC/(AE*AD)
      TR(2,3)=(AA**2+AB**2)/(AE*AD)
      RETURN
   20 DO 30 I=1,3
   30 TR(I,I)=1.
      RETURN
      END
```

DATA GENERATION ROUTINE

```
$SIG ESN6 T=10 P=100 PROUTE=CURF CCPIES=4
CHARGING RATE = UNIVERSITY, BATCH
**LAST SIGNCN WAS: 10:24:0C
 USER "ESN6" SIGNEC CN AT  11:17:C8 CN TUE CCT 14/75
$C GENE(3C01) TO *PRINT*
C        ******    CATA GENERATICN RLTINE  **********
C       ******  FOR 7 PLATE- ELEMENTS  ANC  5 DCME-ELEMENTS  ****
C       ******     NEIDOS  FRCM  1-12  RESPECTIVELY   ***********
      IMPLICIT REAL*8 (A-H,O-Z)
      CCMMCN/CNE/NEIDCS,NSTIF,YCUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(3C,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NCF2,
     2M1(8,30),M2(4,30),NF(26),KK
      READ(5,601) NEIDCS,NELEM,NKINC,NSTIF,NBCUL,NBCUP
C      ALLCWED 30  CIFFERENT  COORDINATE  SETS
      DC 10C  I=1,NKINC
      READ(5,6C7)((X(L,M),L=1,3),M=1,3)
      DO 100 J=1,3
      DO 10C K=1,3
 1C0  COOR(I,J,K)=X(J,K)
C      ALLCWED 10  CIFFERENT  STIFFNESS  SETS
      READ(5,606)((YCUNG(J,I),I=1,12),J=1,NSTIF)
C      ALLOWED 26  DIFFERENT  BOUNCARY   SETS  IN  A PRCBLEM
      NBCU=NBCUL+NBCUP
      READ(5,609)(KBCU(I),I=1,NBCU)
      REAC(5,606)((XB(I,J),J=1,6),I=1,NBCUL)
      CALL SFCRM
      DO 1C1 KK=1,NELEM
      REAC(5,602)(N(I),I=1,6)
      DO 511 MCM=1,5
      MOL=MCM+1
      NANA=N(MCM)
      DO 51C MCN=MCL,6
      IF(NANA.EC.N(MCN)) GO TO 512
 510  CCNTINUE
 511  CONTINUE
      CO 102 J=1,6
 102  N(J)=N(J)*10+1
      WRITE(6,602)(N(J),J=1,6)
      WRITE(6,608)
 1C1  CCNTINUE
      WRITE(6,608)
      WRITE(6,6C8)
      DO 103 KK=1,NELEM
      REAC(5,601)(INFO(I),I=1,16)
      IF(KK.GT.1) GC TC 209
      CALL BCON
 209  CCNTINUE
      DO 1C9 I=1,26
 1C9  INF(I)=0
      L=3
      DO 108 I=4,NIN,2
 1C8  IF(INFO(I).GT.0)L=L+2
      WRITE(6,603) INFC(1),L
      KIND=INFO(2)
      IF(KIND.EC.0)GO TO 205
      WRITE(6,606)((CCOR(KIND,J,M3),J=1,3),M3=1,3)
      GO TO 2C6
 205  WRITE(6,608)
      WRITE(6,6C8)
 2C6  CONTINUE
```

```
      IF(INFO(16).EQ.0)GO TO 105
      CALL LFORM
      GO TO 106
  105 WRITE(6,608)
      WRITE(6,608)
  106 CONTINUE
      INF(1)=INFO(3)
      INF(2)=L/2-1
      L1=1
      DO 110 I=4,NIN,2
      IF(INFO(I).LT.0.AND.INFO(I+1).LT.0) GO TO 517
      IF(INFO(I).LE.0)GO TO 110
      L1=L1+2
      INF(L1)= INFO(I)
      IF(INFO(I+1).LT.0)GO TO 210
      INF(L1+1)=INFO(I+1)
      GO TO 110
  210 INF(L1+1)=0
      INFO(I+1)=-INFO(I+1)
  110 CONTINUE
      L2=L1+2
      DO 112 I=1,26
  112 INF1(I)=0
      L1=0
      DO 114 I=4,NIN,2
      IF(INFO(I).EQ.0)GO TO 114
      IF(INFO(I).LT.0)INFO(I)=-INFO(I)
      L=INFO(I)
      K1=INFO(I+1)
      IF(L.NE.0.AND.K1.EQ.0) GO TO 515
      K=KBOU(K1)
      IF(L.GT.3.AND.NDF2.NE.0) GO TO 116
      DO 115  J=1,NDF1
      NF(J)=M1(J,K)*(J+(L-1)*NDF1)
      IF(NF(J).EQ.0) GO TO 115
      L1=L1+1
      INF1(L1)=NF(J)
  115 CONTINUE
      GO TO 114
  116 DO 117 J=1,NDF2
      NF(J)=M2(J,K)*(3*NDF1+J+(L-4)*NDF2)
      IF(NF(J).EQ.0) GO TO 117
      L1=L1+1
      INF1(L1)=NF(J)
  117 CONTINUE
  114 CONTINUE
      IF(L1.EQ.0) INF(L2)=0
      IF(L1.NE.0) INF(L2)=99
      WRITE(6,609)(INF(I),I=1,L2)
      IF(L1.EQ.0) GO TO 103
      WRITE(6,611) L1,(INF1(I),I=1,L1)
  103 CONTINUE
      WRITE(6,608)
      GO TO 520
  512 WRITE(6,612) KK
      GO TO 521
  515 WRITE(6,614) KK
      GO TO 521
  517 WRITE(6,618) KK
  521 A=1.
```

```
      B=0.
      C=A/B
      GO TO 520
  601 FORMAT(16I5)
  602 FORMAT(6I5)
  603 FORMAT('  999',I5,'     ',I5)
  606 FORMAT(6D10.3)
  607 FORMAT(6D10.3,/,3D10.3)
  608 FORMAT('     ')
  609 FORMAT(26I3)
  611 FORMAT(I3,/,26I3)
  612 FORMAT(10X,' ****  ERROR  1 ****  NICKNAMES OF ELEMENT ',I10,//)
  614 FORMAT(10X,' ****  ERROR  2 **** INFORMATION OF ELEMENT ',I10,/,
     1'    INFO(3) NOT 0  INFO(4) = 0 FOR  NODE 1  ',/,
     2'     (5)-(6) FOR NODE 2  (7)-(8) FOR NODE 3 ETC.    ')
  618 FORMAT(10X,' ****  ERROR  3 **** INFORMATION OF ELEMENT ',I10,/,
     1'            INFO(3) INFO(4) BOTH NEGATIVE   FOR NODE 1',/,
     2'     (5)-(6) FOR NODE 2  (7)-(8) FOR NODE 3 ETC.    ')
  520 STOP
      END
      SUBROUTINE BCON
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      IF(KK.GT.1) RETURN
      DO 16 I=1,8
      DO 16 J=1,30
   16 M1(I,J)=0
      DO 17 I=1,4
      DO 17 J=1,30
   17 M2(I,J)=0
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12),NEIDOS
    1 NIN=8
      NDF1=5
      NDF2=0
      CALL CA1
      GO TO 15
    2 NIN=14
      NDF1=5
      NDF2=2
      CALL CA2
      GO TO 15
    3 NIN=8
      NDF1=4
      NDF2=0
      CALL CA34
      GO TO 15
    4 NIN=14
      NDF1=4
      NDF2=0
      CALL CA34
      GO TO 15
    5 NIN=8
      NDF1=8
      NDF2=0
      CALL CA5
      GO TO 15
    6 NIN=14
      NDF1=8
```

```
      NDF2=2
      CALL CA6
      GO TO 15
    7 NIN=14
      NDF1=3
      NDF2=0
      CALL CA7
      GO TO 15
    8 NIN=8
      NDF1=7
      NDF2=0
      CALL CA8
      GO TO 15
    9 NIN=14
      NDF1=7
      NDF2=4
      CALL CA8
      GO TO 15
   10 NIN=8
      NDF1=6
      NDF2=0
      CALL CA10
      GO TO 15
   11 NIN=14
      NDF1=6
      NDF2=0
      CALL CA10
      GO TO 15
   12 NIN=14
      NDF1=5
      NDF2=0
      CALL CA8
   15 CONTINUE
      RETURN
      END
      SUBROUTINE SFORM
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDCS,NSTIF,YOUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      WRITE(6,701)
      WRITE(6,702)
      WRITE(6,703)
      GO TO(101,102,103,104,105,106,107,108,109,110,111,112),NEIDCS
  101 WRITE(6,704)
      WRITE(6,706)
      WRITE(6,707)
      WRITE(6,708)
      WRITE(6,709)
      WRITE(6,710)
      WRITE(6,711) NSTIF
      GO TO 114
  102 WRITE(6,705)
      WRITE(6,706)
      WRITE(6,707)
      WRITE(6,708)
      WRITE(6,712)
      WRITE(6,710)
      WRITE(6,713) NSTIF
      GO TO 114
```

```
103 WRITE(6,704)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,717)
    WRITE(6,715)
    WRITE(6,718) NSTIF
    GO TO 114
104 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,714)
    WRITE(6,715)
    WRITE(6,716) NSTIF
    GO TO 114
108 WRITE(6,704)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,723)
    WRITE(6,724)
    WRITE(6,725) NSTIF
    GO TO 114
109 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,726)
    WRITE(6,724)
    WRITE(6,727) NSTIF
    GO TO 114
111 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,728)
    WRITE(6,724)
    WRITE(6,729) NSTIF
    GO TO 114
105 WRITE(6,704)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,731)
    WRITE(6,710)
    WRITE(6,730) NSTIF
    GO TO 114
106 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,733)
    WRITE(6,710)
    WRITE(6,732) NSTIF
    GO TO 114
107 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
```

```
      WRITE(6,735)
      WRITE(6,710)
      WRITE(6,734) NSTIF
      GO TO 114
  110 WRITE(6,704)
      WRITE(6,706)
      WRITE(6,707)
      WRITE(6,708)
      WRITE(6,737)
      WRITE(6,724)
      WRITE(6,736) NSTIF
      GO TO 114
  112 WRITE(6,705)
      WRITE(6,706)
      WRITE(6,707)
      WRITE(6,708)
      WRITE(6,739)
      WRITE(6,724)
      WRITE(6,738) NSTIF
  114 WRITE(6,720)((YOUNG(J,I),I=1,12),J=1,NSTIF)
      WRITF(6,721) NBCUL
      WRITE(6,720)((XB(I,J),J=1,6),I=1,NBCUL)
      WRITE(6,722)
  701 FORMAT(1X,'(13I3)')
  702 FORMAT(1X,'(6D10.3,/,6D10.3)')
  703 FORMAT(1X,'(6I5)')
  704 FORMAT(1X,'(3I5,/,12I5)')
  705 FORMAT(1X,'(6I5,/,12I5)')
  706 FORMAT(1X,'(I5,5(I5,D10.3))')
  707 FORMAT(1X,'(4I5,4(/,6D10.3))')
  708 FORMAT(1X,'(25I3)')
  709 FORMAT(1X,'(3(1X,5D10.3,/))')
  710 FORMAT(1X,'(/,1X,5D10.3)')
  711 FORMAT('  1  3  8  3',I3,' 12  5  5  5  5')
  712 FORMAT(1X,'(3(1X,5D10.3,/),1X,6D10.3,/)')
  713 FORMAT('  2  6  8  3',I3,' 12  5  5  5  5  2  2  2')
  714 FORMAT(1X,'(6(1X,4D10.3,/))')
  715 FORMAT(1X,'(/,1X,2D10.3)')
  716 FORMAT('  4  6  8  3',I3,' 12  4  4  4  4  4  4  4')
  717 FORMAT(1X,'(3(1X,4D10.3,/))')
  718 FORMAT('  3  3  8  3',I3,' 12  2  4  4  4')
  719 FORMAT('        ')
  720 FORMAT(6D10.3)
  721 FORMAT(25I3)
  722 FORMAT('    1    1    0    1 1000 1000')
  723 FORMAT(1X,'(3(1X,7D10.3,/))')
  724 FORMAT(1X,'(/,1X,8D10.3)')
  725 FORMAT('  8  3  8  3',I3,' 12  8  7  7  7')
  726 FORMAT(1X,'(3(1X,7D10.3,/),3(1X,4D10.3,/))')
  727 FORMAT('  9  6  8  3',I3,' 12  8  7  7  7  4  4  4')
  728 FORMAT(1X,'(6(1X,6D10.3,/))')
  729 FORMAT(' 11  6  8  3',I3,' 12  8  6  6  6  6  6  6')
  730 FORMAT('  5  3  8  3',I3,' 12  5  8  8  8')
  731 FORMAT(1X,'(3(1X,8D10.3,/))')
  732 FORMAT('  6  6  8  3',I3,' 12  5  8  8  8  2  2  2')
  733 FORMAT(1X,'(3(1X,8D10.3,/),1X,6D10.3,/)')
  734 FORMAT('  7  6  8  3',I3,' 12  5  3  3  3  3  3  3')
  735 FORMAT(1X,'(6(1X,3D10.3,/))')
  736 FORMAT(' 10  3  8  3',I3,' 12  8  6  6  6')
  737 FORMAT(1X,'(3(1X,6D10.3,/))')
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738 FORMAT(' 12  6  8  3',I3,' 12  8  5  5  5  5  5  5')
739 FORMAT(1X,'(6(1X,5D10.3,/))')
    RETURN
    END
    SUBROUTINE LFORM
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFC(16),
   1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
   2M1(8,30),M2(4,30),NF(26),KK
    DIMENSION P(12)
    K=INFC(16)
    DO 100 I=1,12
100 P(I)=0.
    DO 101 J=1,K
    READ(5,780) I,A
101 P(I)=A
    WRITE(6,781)(P(I),I=1,12)
780 FORMAT(I5,1D10.3)
781 FORMAT(6D10.3)
    RETURN
    END
    SUBROUTINE DA1
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFC(16),
   1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
   2M1(8,30),M2(4,30),NF(26),KK
    DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
   1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
   2,MODE(12)
    DATA MODE/9,9,9,5,8,0,0,0,0,0,0,0/
    DATA N1/3,4,5,6,9,10,11,15,16,21*0/
    DATA N2/5,6,7,8,10,11,12,13,15,21*0/
    DATA N3/3,4,5,6,10,11,12,13,15,21*0/
    DATA N4/11,13,14,15,16,25*0/
    DATA N5/2,4,6,8,11,13,14,16,22*0/
    DATA N6/30*0/
    DATA N7/30*0/
    DATA N8/30*0/
    DATA N9/30*0/
    DATA N10/30*0/
    DATA N11/30*0/
    DATA N12/30*0/
    DO 210 LL=1,12
    KOK=MODE(LL)
    IF(KOK.LT.1) GO TO 211
    GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
301 DO 401 I=1,KOK
401 M1(1,N1(I))=1
    GO TO 211
302 DO 402 I=1,KOK
402 M1(2,N2(I))=1
    GO TO 211
303 DO 403 I=1,KOK
403 M1(3,N3(I))=1
    GO TO 211
304 DO 404 I=1,KOK
404 M1(4,N4(I))=1
    GO TO 211
305 DO 405 I=1,KOK
405 M1(5,N5(I))=1
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      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
      SUBROUTINE DA2
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBCU,NBOUL,XB(16,6),INFC(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MODE(12)
      DATA MODE/9,9,9,9,8,0,0,0,4,4,0,0/
      DATA N1/3,4,5,6,9,10,11,15,16,21*0/
      DATA N2/5,6,7,8,10,11,12,13,15,21*0/
      DATA N3/3,4,5,6,10,11,12,13,15,21*0/
      DATA N4/5,6,7,8,11,13,14,15,16,21*0/
      DATA N5/2,4,6,8,11,13,14,16,22*0/
      DATA N6/30*0/
      DATA N7/30*0/
      DATA N8/30*0/
      DATA N9/5,6,7,8,26*0/
      DATA N10/2,4,6,8,26*0/
      DATA N11/30*0/
      DATA N12/30*0/
      DO 210 LL=1,12
      KCK=MODE(LL)
      IF(KOK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
  301 DO 401 I=1,KCK
  401 M1(1,N1(I))=1
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
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404 M1(4,N4(I))=1
    GO TO 211
305 DO 405 I=1,KCK
405 M1(5,N5(I))=1
    GO TO 211
306 DO 406 I=1,KCK
406 M1(6,N6(I))=1
    GO TO 211
307 DO 407 I=1,KCK
407 M1(7,N7(I))=1
    GO TO 211
308 DO 408 I=1,KCK
408 M1(8,N8(I))=1
    GO TO 211
309 DO 409 I=1,KCK
409 M2(1,N9(I))=1
    GO TO 211
310 DO 410 I=1,KCK
410 M2(2,N10(I))=1
    GO TO 211
311 DO 411 I=1,KCK
411 M2(3,N11(I))=1
    GO TO 211
312 DO 412 I=1,KCK
412 M2(4,N12(I))=1
211 CONTINUE
210 CONTINUE
    RETURN
    END
    SUBROUTINE DA34
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/ONE/NEIDCS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFO(16),
   1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
   2M1(8,30),M2(4,30),NF(26),KK
    DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
   1N7(30),N8(30 ),N9(30),N10(30),N11(30),N12(30)
   2,MODE(12)
    DATA MODE/9,6,4,7,0,0,0,0,0,0,0,0/
    DATA N1/3,4,5,6,9,10,11,15,16,21*0/
    DATA N2/1,2,3,4,9,14,24*0/
    DATA N3/3,4,9,14,26*0/
    DATA N4/1,3,5,7,10,12,15,23*0/
    DATA N5/30*0/
    DATA N6/30*0/
    DATA N7/30*0/
    DATA N8/30*0/
    DATA N9/30*0/
    DATA N10/30*0/
    DATA N11/30*0/
    DATA N12/30*0/
    DO 210 LL=1,12
    KCK=MODE(LL)
    IF(KCK.LT.1) GO TO 211
    GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
301 DO 401 I=1,KCK
401 M1(1,N1(I))=1
    GO TO 211
302 DO 402 I=1,KCK
402 M1(2,N2(I))=1
    GO TO 211
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303 DO 403 I=1,KOK
403 M1(3,N3(I))=1
    GO TO 211
304 DO 404 I=1,KOK
404 M1(4,N4(I))=1
    GO TO 211
305 DO 405 I=1,KOK
405 M1(5,N5(I))=1
    GO TO 211
306 DO 406 I=1,KOK
406 M1(6,N6(I))=1
    GO TO 211
307 DO 407 I=1,KOK
407 M1(7,N7(I))=1
    GO TO 211
308 DO 408 I=1,KOK
408 M1(8,N8(I))=1
    GO TO 211
309 DO 409 I=1,KOK
409 M2(1,N9(I))=1
    GO TO 211
310 DO 410 I=1,KOK
410 M2(2,N10(I))=1
    GO TO 211
311 DO 411 I=1,KOK
411 M2(3,N11(I))=1
    GO TO 211
312 DO 412 I=1,KOK
412 M2(4,N12(I))=1
211 CONTINUE
210 CONTINUE
    RETURN
    END
    SUBROUTINE DA5
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
   1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
   2M1(8,30),M2(4,30),NF(26),KK
    DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
   1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
   2,MODE(12)
    DATA MODE/9,9,9,3,5,6,5,8,0,0,0,0/
    DATA N1/3,4,5,6,9,10,11,15,16,21*0/
    DATA N2/5,6,7,8,10,11,12,13,15,21*0/
    DATA N3/3,4,5,6,10,11,12,13,15,21*0/
    DATA N4/10,11,15,27*0/
    DATA N5/7,8,12,13,16,25*0/
    DATA N6/3,4,5,6,10,11,24*0/
    DATA N7/11,13,14,15,16,25*0/
    DATA N8/2,4,6,8,11,13,14,16,22*0/
    DATA N9/30*0/
    DATA N10/30*0/
    DATA N11/30*0/
    DATA N12/30*0/
    DO 210 LL=1,12
    KOK=MODE(LL)
    IF(KOK.LT.1) GO TO 211
    GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
301 DO 401 I=1,KOK
401 M1(1,N1(I))=1
```

```
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
      SUBROUTINE DA6
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFC(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MODE(12)
      DATA MODE/9,9,9,3,5,6,9,8,4,4,0,0/
      DATA N1/3,4,5,6,9,10,11,15,16,21*0/
      DATA N2/5,6,7,8,10,11,12,13,15,21*0/
      DATA N3/3,4,5,6,10,11,12,13,15,21*0/
      DATA N4/10,11,15,27*0/
      DATA N5/7,8,12,13,16,25*0/
      DATA N6/3,4,5,6,10,11,24*0/
      DATA N7/5,6,7,8,11,13,14,15,16,21*0/
      DATA N8/2,4,6,8,11,13,14,16,22*0/
      DATA N9/5,6,7,8,26*0/
      DATA N10/2,4,6,8,26*0/
      DATA N11/30*0/
      DATA N12/30*0/
      DO 210 LL=1,12
      KCK=MODE(LL)
```

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      IF(KOK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
  301 DO 401 I=1,KCK
  401 M1(1,N1(I))=1
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
      SUBROUTINE DA7
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDCS,NSTIF,YOUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NCF1,NCF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MCDE(12)
      DATA MCDE/9,11,10,0,0,0,0,0,0,0,0,0/
      DATA N1/3,4,5,6,9,10,11,15,16,21*0/
      DATA N2/5,6,7,8,10,11,12,13,14,15,16,19*0/
      DATA N3/2,4,6,8,10,11,12,13,14,16,20*0/
      DATA N4/30*0/
      DATA N5/30*0/
      DATA N6/30*0/
      DATA N7/30*0/
      DATA N8/30*0/
      DATA N9/30*0/
      DATA N10/30*0/
```

```
      CATA N11/30*C/
      DATA N12/30*0/
      DC 210 LL=1,12
      KCK=MCDE(LL)
      IF(KOK.LT.1) GC TC 211
      GO TO (301,3C2,303,304,3C5,3C6,3C7,3C8,309,310,311,312),LL
  3C1 DC 401 I=1,KCK
  401 M1(1,N1(I))=1
      GC TC 211
  3C2 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GC TC 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GC TC 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GC TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GC TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  4C7 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  4C8 M1(8,N8(I))=1
      GO TO 211
  309 DC 409 I=1,KCK
  409 M2(1,N9(I))=1
      GC TO 211
  310 DC 410 I=1,KCK
  410 M2(2,N10(I))=1
      GC TC 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GC TC 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CCNTINUE
  210 CCNTINUE
      RETURN
      END
      SUBROUTINE DA8
      IMPLICIT REAL*8 (A-H,O-Z)
      CCMMCN/CNE/NEICCS,NSTIF,YCUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),CCOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NCF1,NCF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(3C),N1C(3C),N11(3C),N12(3C)
     2,MCDE(12)
      DATA MCDE/13,13,12,5,7,10,13,0,4,7,6,6/
      CATA N1/3,4,9,10,13,14,15,16,17,20,25,26,27,17*C/
      CATA N2/5,6,11,12,13,14,15,17,18,20,28,29,30,17*0/
      DATA N3/7,8,9,10,11,12,13,14,15,16,18,19,18*0/
      CATA N4/3,4,5,6,17,25*0/
      DATA N5/4,6,8,10,12,14,17,23*0/
      CATA N6/3,4,5,6,15,16,17,18,19,2C,2C*C/
```

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      DATA N7/2,4,6,8,10,12,14,15,16,17,18,19,20,17*0/
      DATA N8/30*0/
      DATA N9/3,4,5,6,26*0/
      DATA N10/2,4,6,8,10,12,14,23*0/
      DATA N11/3,4,9,10,13,14,24*0/
      DATA N12/5,6,11,12,13,14,24*0/
      DO 210 LL=1,12
      KCK=MODE(LL)
      IF(KCK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
  301 DO 401 I=1,KCK
  401 M1(1,N1(I))=1
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
      SUBROUTINE DA10
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NELCOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NOF1,NOF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MODE(12)
      DATA MODE/13,13,12,24,16,10,0,0,0,0,0,0/
      DATA N1/3,4,9,10,13,14,15,16,17,20,25,26,27,17*0/
      DATA N2/5,6,11,12,13,14,15,17,18,20,28,29,30,17*0/
```
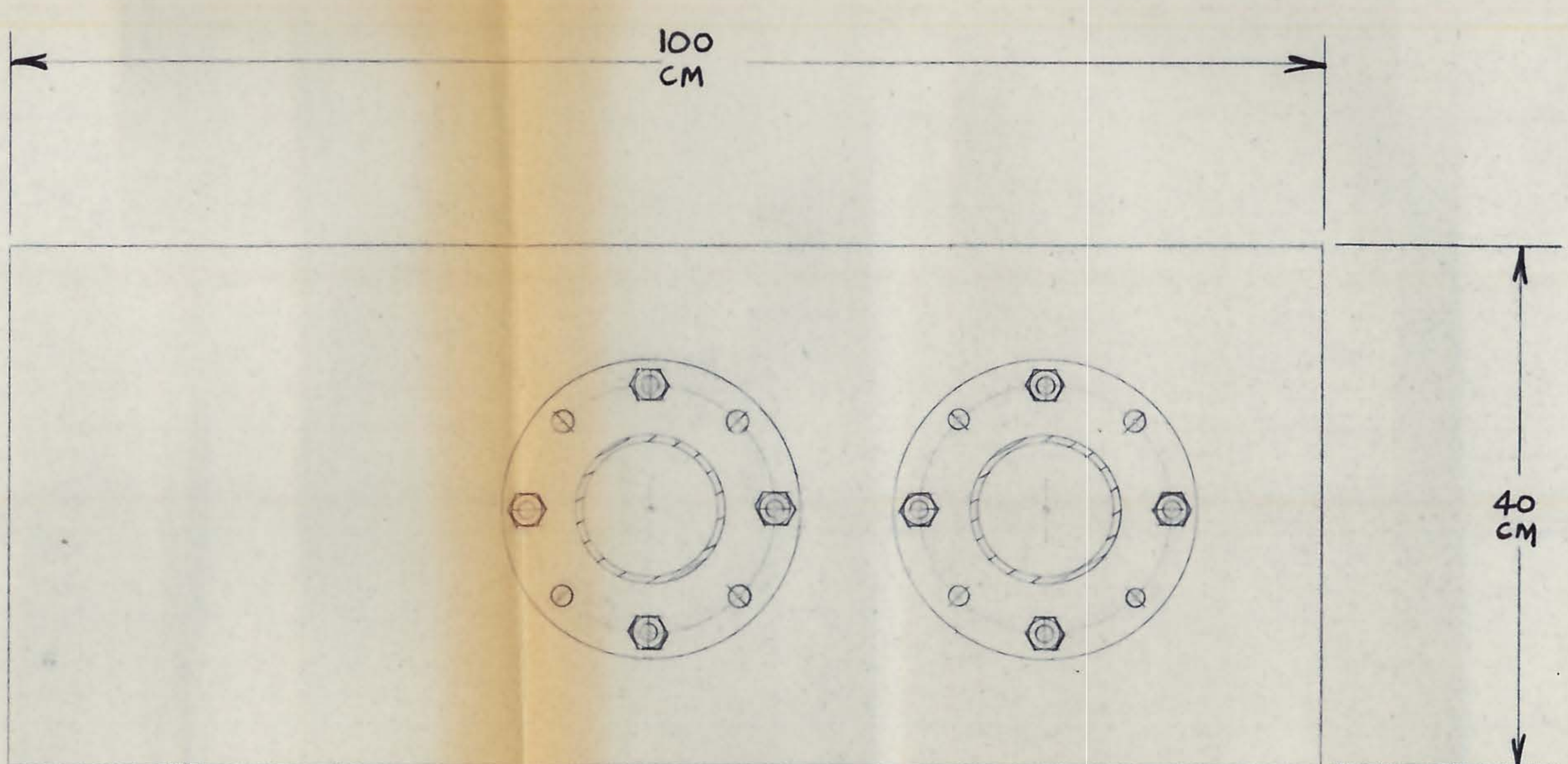

```
      DATA N3/7,8,9,10,11,12,13,14,15,16,18,19,18*0/
      DATA N4/1,2,7,8,9,10,11,12,13,14,15,16,18,19,20,22,23,24,25,26,27,
     128,29,30,6*0/
      DATA N5/7,8,9,10,11,12,13,14,15,16,18,19,20,23,26,29,14*0/
      DATA N6/1,3,5,7,9,11,13,24,27,30,20*0/
      DATA N7/30*0/
      DATA N8/30*0/
      DATA N9/30*0/
      DATA N10/30*0/
      DATA N11/30*0/
      DATA N12/30*0/
      DO 210 LL=1,12
      KCK=MODE(LL)
      IF(KCK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
  301 DO 401 I=1,KCK
  401 M1(1,N1(I))=1
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
$SIG OFF
```

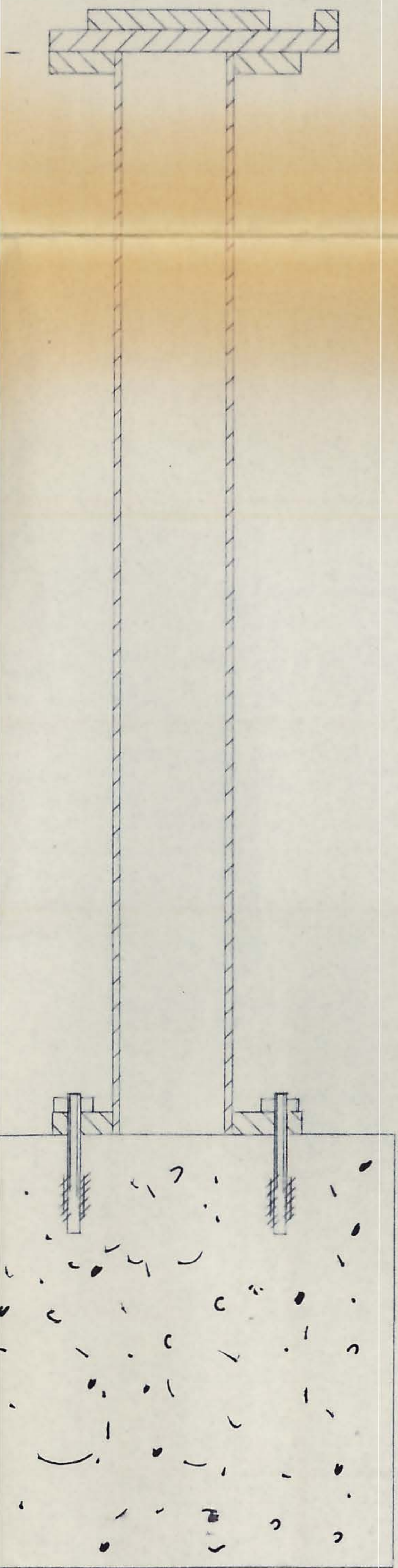
DETAILS OF SUPPORTS
FOR
24 and 36 FACED DOMES

SCALE : 1cm = 5cm

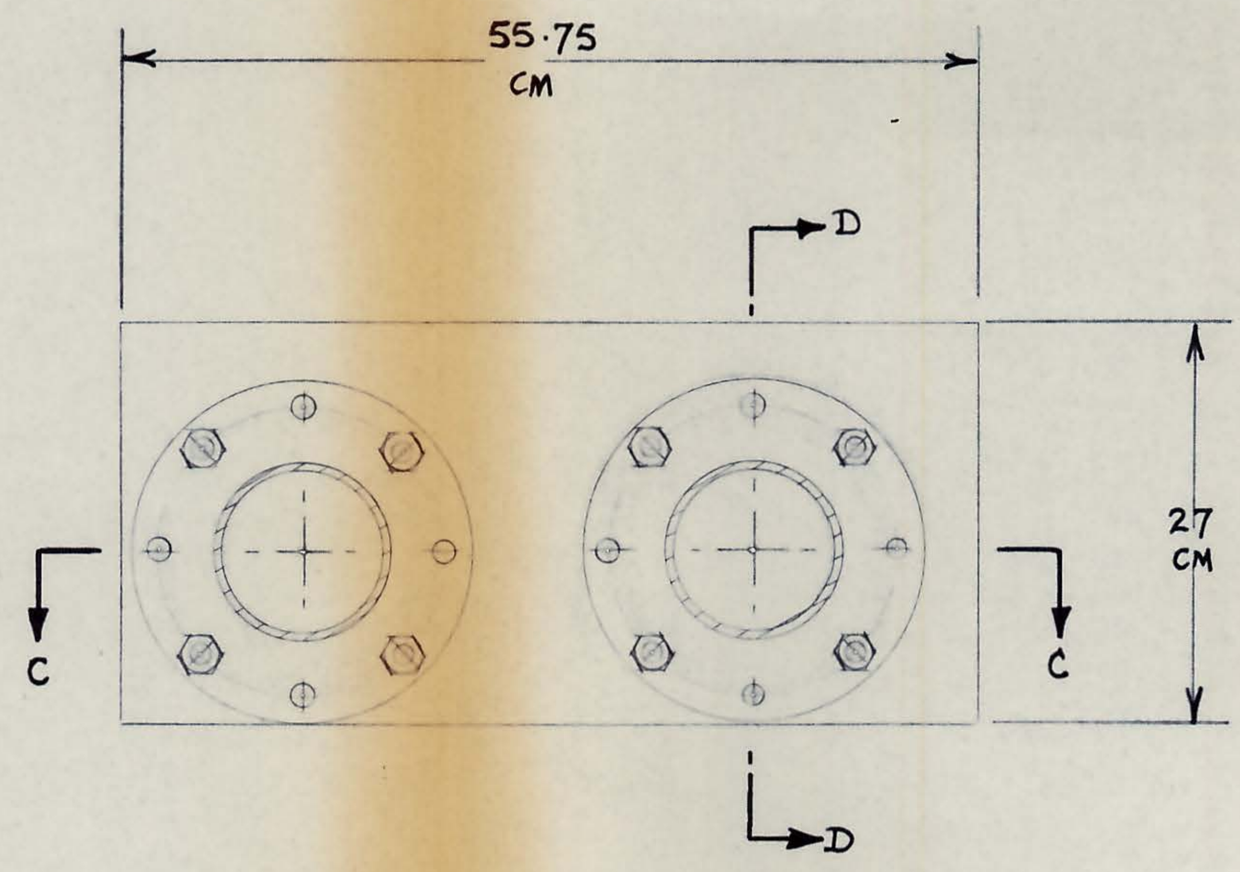
FIG. 9.3.



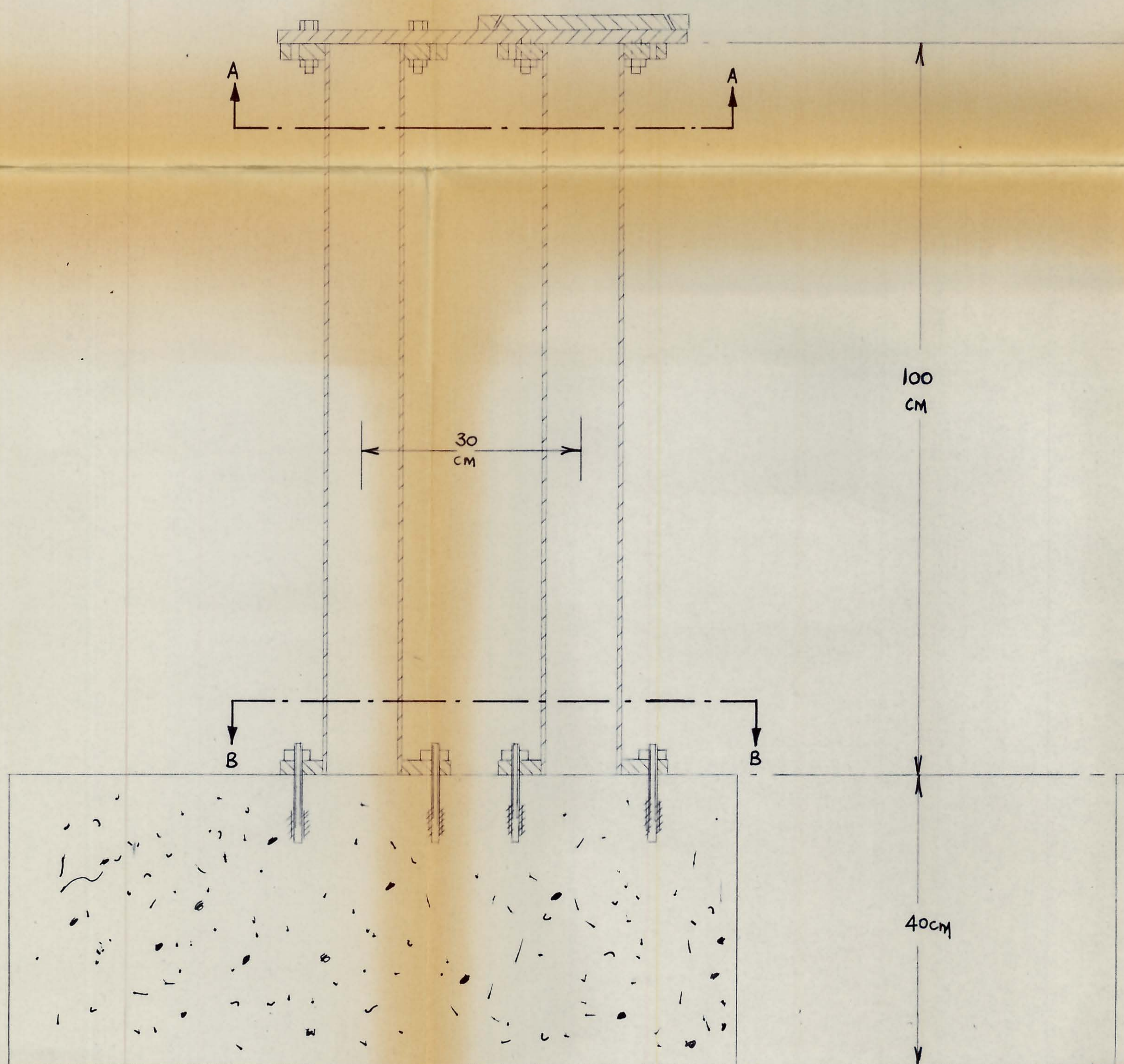
SECTION B-B



SECTION D-D.



SECTION A - A



SECTION C - C

PLAN SCALE: 1cm = 5mm

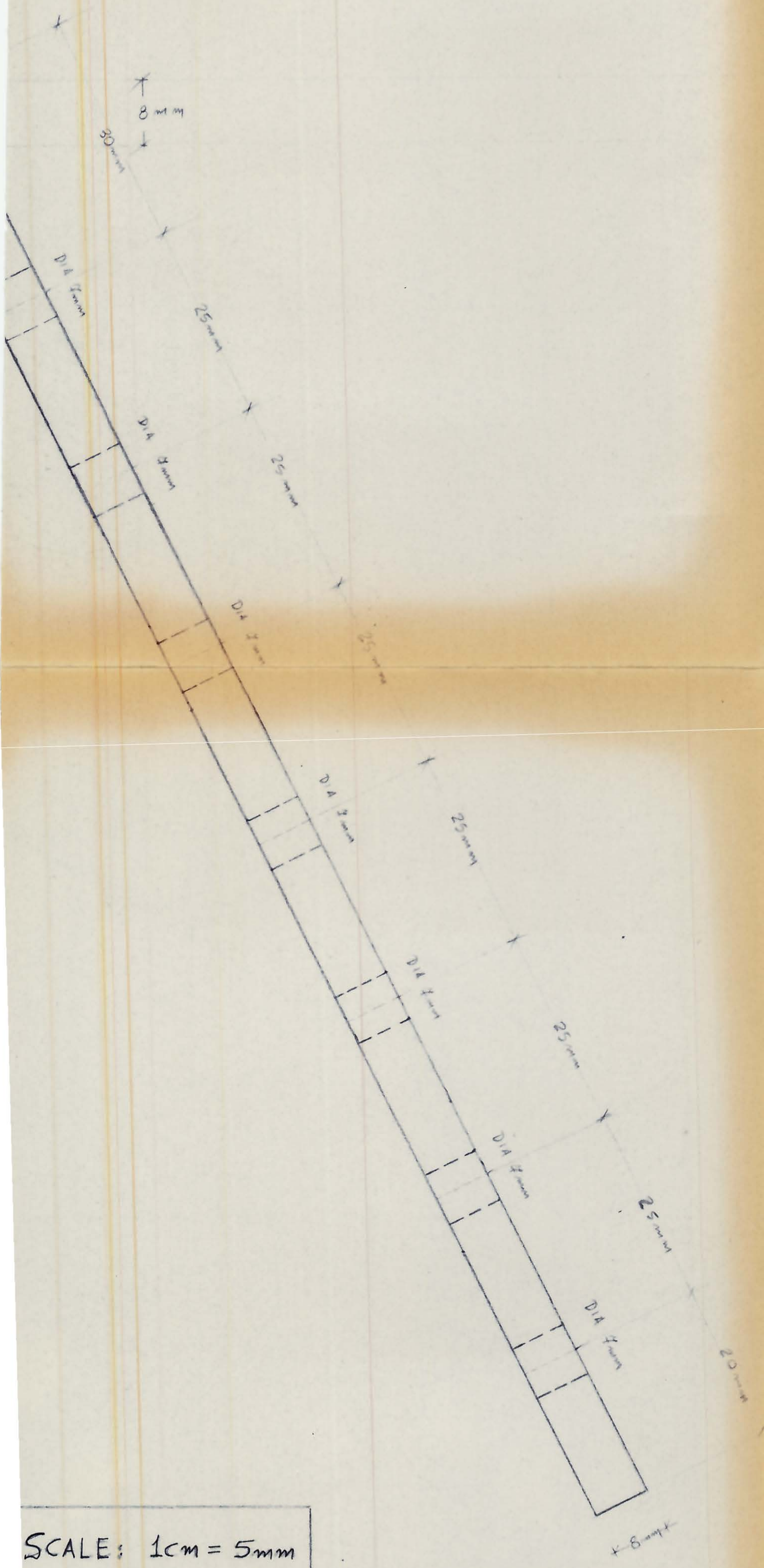
DETAILS OF SUPPORTS FOR 24 and 36 FACED DOMES | FIG. 9.4. | G. MANOS

SECTION A-A SCALE: 1cm = 5mm

DETAILS OF SUPPORTS FOR 24 and 36 FACED DOMES

G. MANOS

FIG. 9.5.



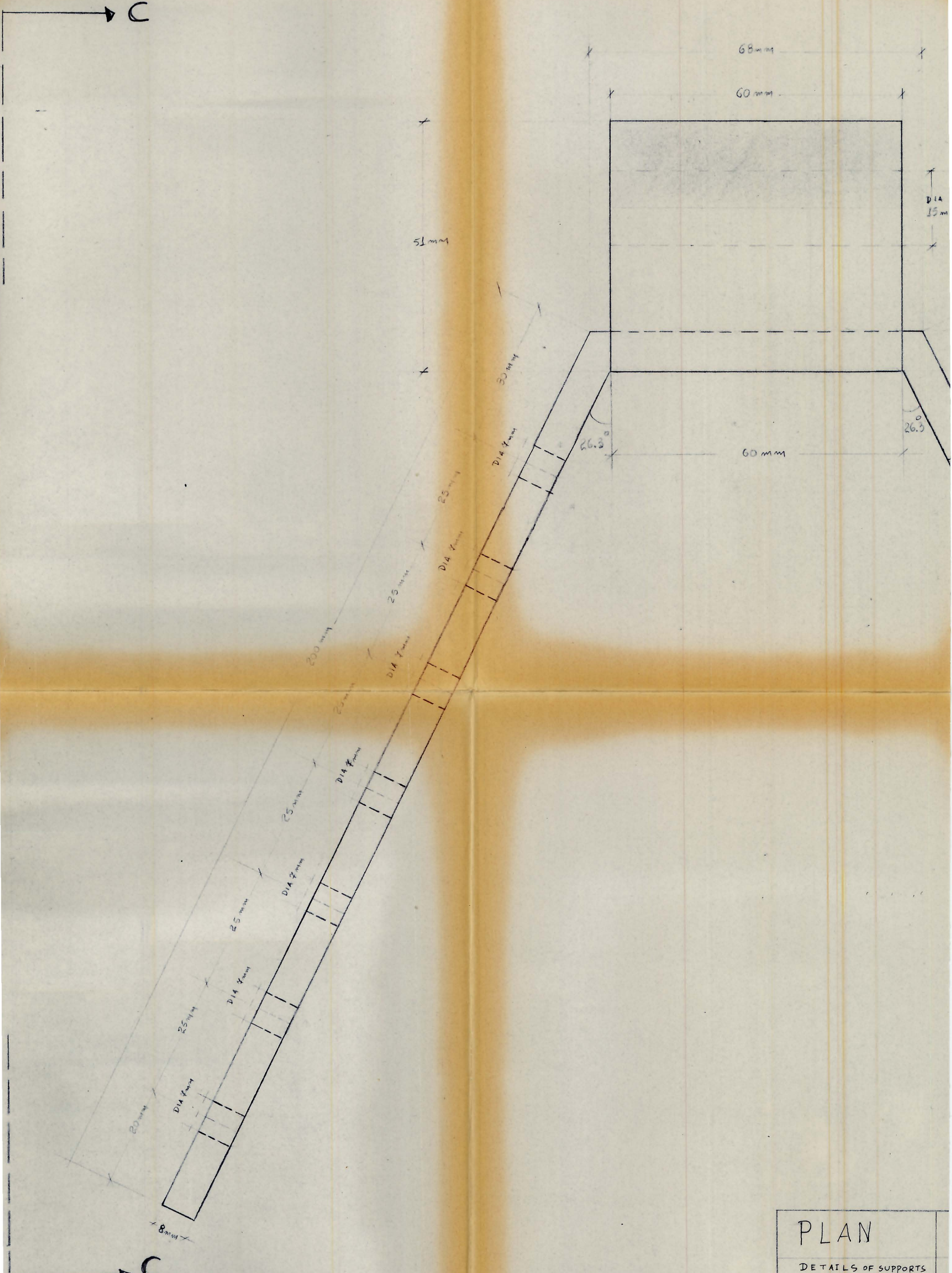
SCALE: 1cm = 5mm

G. MANOS

VIEW C – C

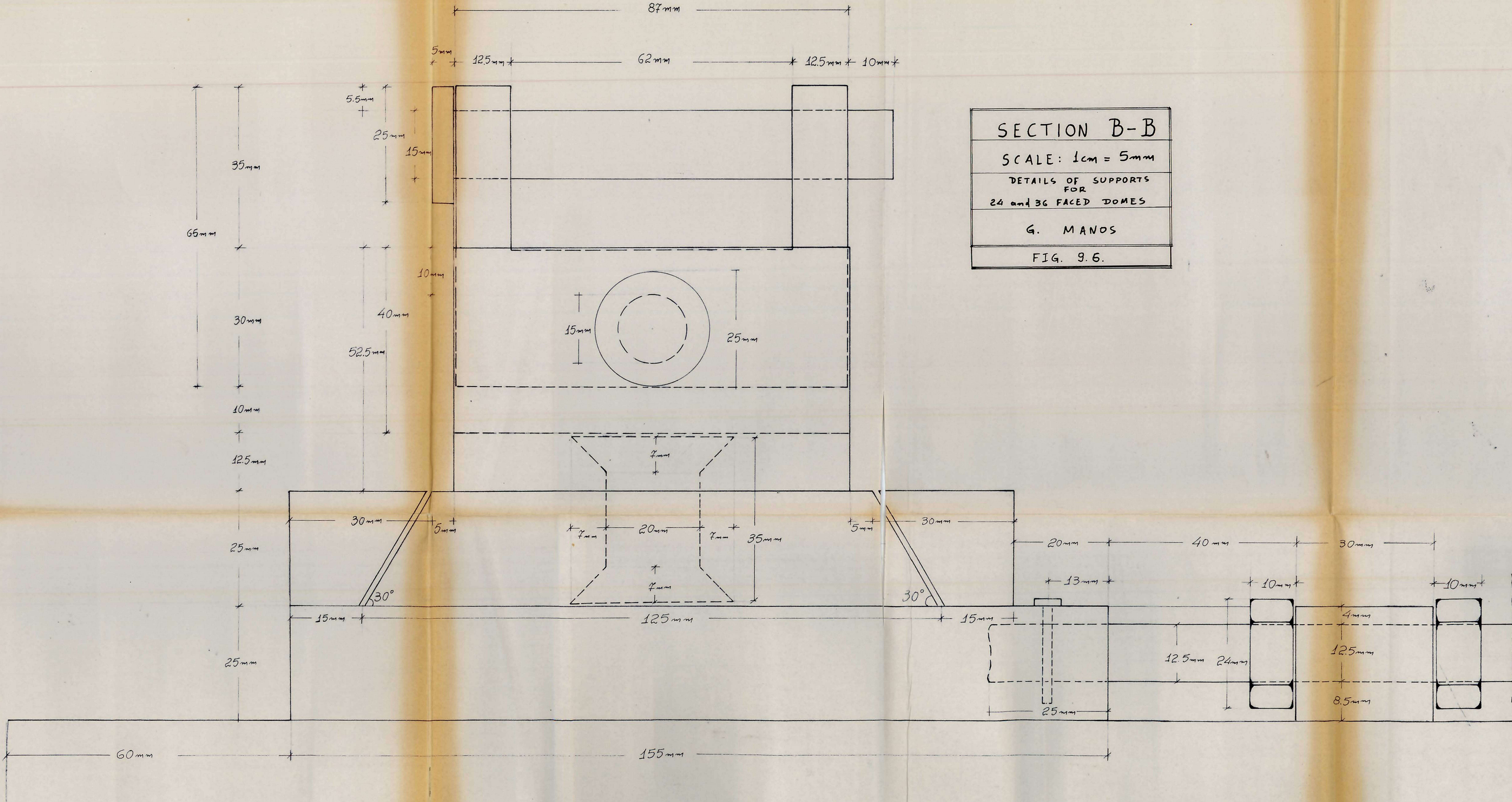
SCALE 1cm = 5mm

DETAILS OF SUPPORTS FOR 24 and 36 FACED DOMES	G. MANOS
FIG. 9.7.	



PLAN

DETAILS OF SUPPORTS FOR
24 and 36 FACED DOMES



SECTION B-B

SCALE: 1cm = 5mm

DETAILS OF SUPPORTS FOR 24 and 36 FACED DOMES

G. MANOS

FIG. 9.6.

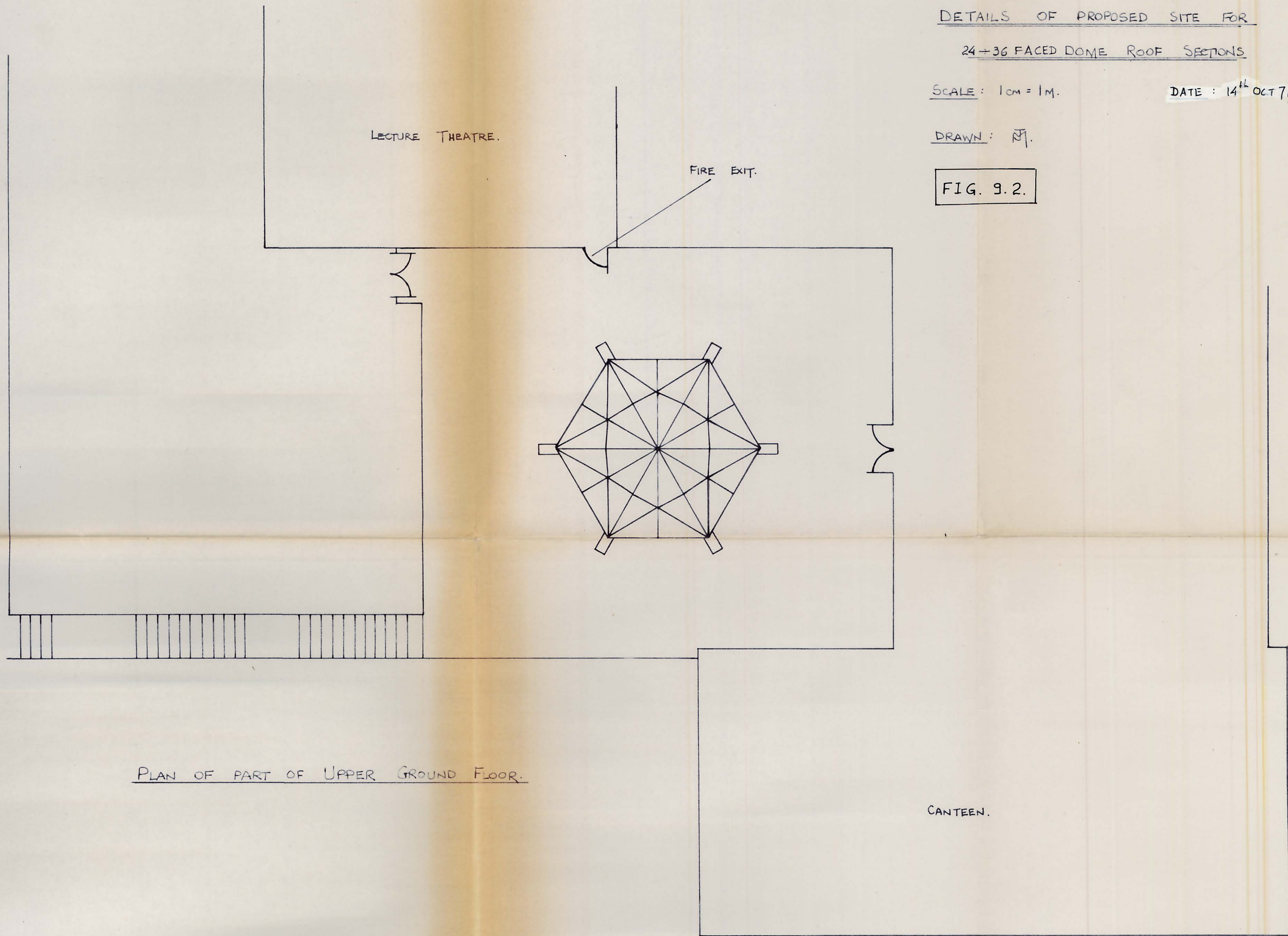
DETAILS OF PROPOSED SITE FOR
24 + 36 FACED DOME ROOF SECTIONS

SCALE: 1 CM = 1 M.

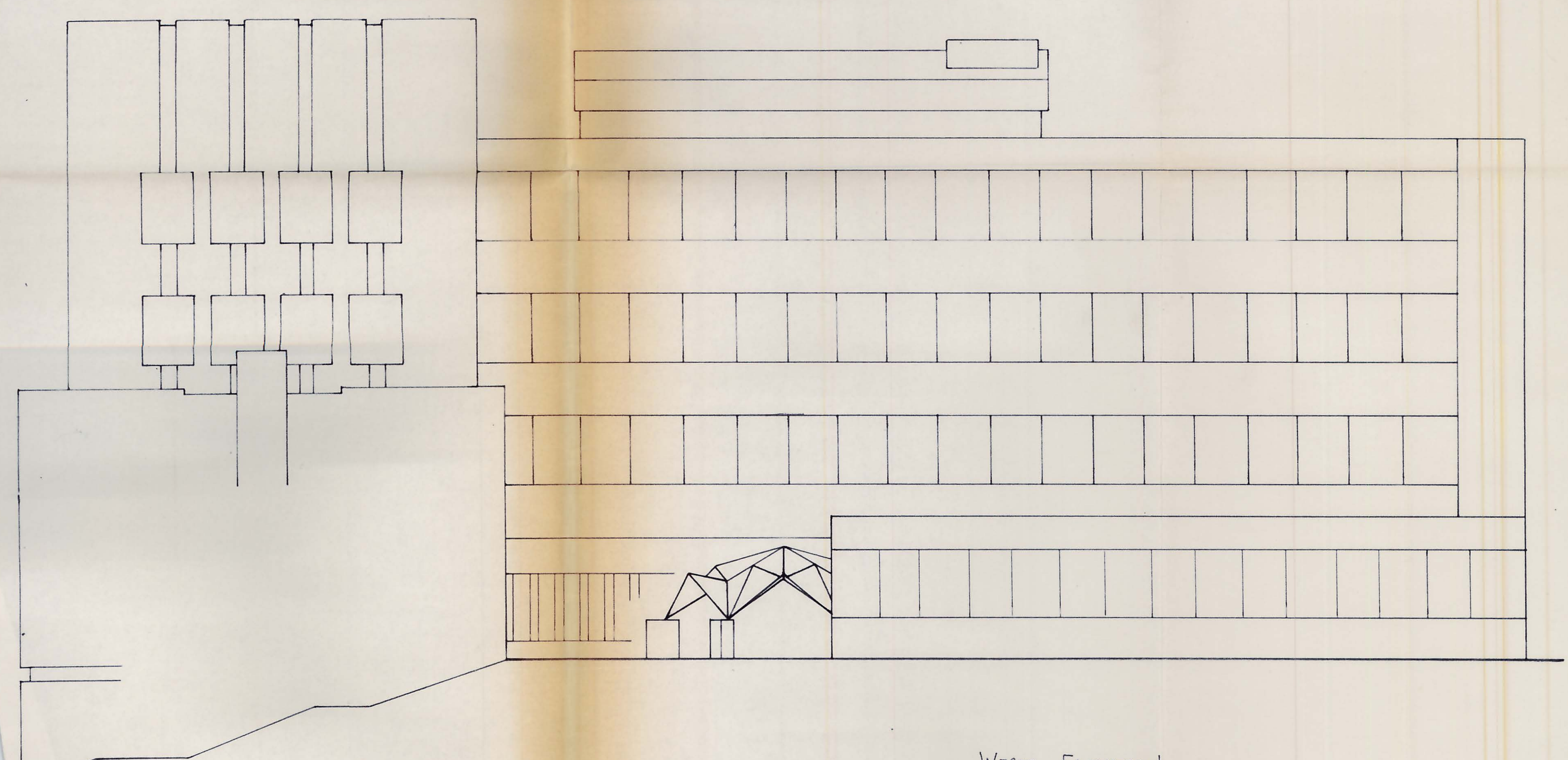
DATE: 14th OCT 71.

DRAWN: RM.

FIG. 9.2.



PLAN OF PART OF UPPER GROUND FLOOR.



WEST ELEVATION.