Regional policy and the location of industry: an application of attraction theory

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ABSTRACT

Regional Policy and the Location of Industry: An Application of Attraction Theory

An active regional policy is exercised both by the British Government and by most other industrial countries. A major part of this policy in the United Kingdom has been and still is the relocation of industries from the prosperous regions. It would therefore seem as though the theories of the location of economic activity would have some relevance to this problem.

In reviewing the empirical work on location and industrial mobility, it is concluded that they lack a rigorous theoretical and methodological base, such that little reliance can be placed on their results. These studies do however suggest certain factors that need further examination in explaining location.

The theoretical work on location is found to be unable to generate many general results or suggest empirically testable models. These studies do however suggest certain analytical tools that are found useful to attraction theory.

Attraction theory is examined and modified. The limitations and assumption underlying this theory are made explicit. It is concluded that this model may be a useful took in evaluating government policy on the relocation of industry. The results of the application to the United Kingdom data are presented. These results seem to explain why certain regions have had higher unemployment than the national average and also suggest certain policy prescriptions.

In implementing the attraction model data from regional input-output tables are necessary. Therefore there is a discussion of

various methods of constructing these tables. The results of the method considered most appropriate is given for purposes of comparison with the attraction results.

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INTRODUCTION

Location theory - its relevance to British Regional Policy

British regional policy has been applied with various degrees of strength since the last war. One of the major components of this policy has been the movement of firms (whole or as branches) to the Development Areas from the South-East and West Midlands. The proportion of employment in manufacturing originating from moves from outside each region and moves originating in each region in the U.K. during the period 1945 - 1965 is shown in Table I.

If the size of the multiplier from these firms that moved to the Development Areas is taken into account it can be appreciated how dependent their economies have become on mobile firms. Because of the magnitude of these moves it is easy to see why, as Lever (127) claims, the supply of these moves is vital to the success of British regional policy. However it is not easy to say how many of these moves are due specifically to government policy, or how many would have taken place without policy[1]. Again it is not easy to see how many of these moves were due to the push factors (such as shortage of labour or refusal of I.D.Cs) or to pull factors (such as surplus labour or the availability of investment grants) in the Development Areas[2].

It would seem that location theory has some bearing on the supply of mobile firms. In reply to the question why manufacturing investment was eligible for automatic grants, but services got only selective help, Mr. Chataway (the then Minister of Industrial Development) is reported

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1 In this context see Moore and Thodes (144) who attempt to quantify the success of regional policy in job creation.

2 Various authors have tried to explain the movements described in the Howard (85) report. See for example Howard (86), Beacham and Osborne (9), Keeble (109), (111), Townroe (190) and Sant (162). The methodology and results of these studies will be discussed later.

Table I

Proportion of employment in manufacturing industry coming from industrial movers by region for the U.K. 1945 - 65.

	a	b	c
	Moves from the region to other region	Moves into the region from outside[1]	Total employment in manufacturing mid-1966 (000's)
Northern Ireland	-	21.3	187
Scotland	1.8	12.8	740
Wales	2.6	28.7	326
Northern	1.3	19.6	458
North West	3.6	7.7	1,364
South West	4.7	9.1	408
Yorkshire and Humberside	6.2	3.5	897
East Midlands	6.3	4.3	623
West Midlands	9.7	0.7	1,259
East Anglia	6.6	8.9	188
South East	16.4	1.2	2,603
U.K.	8.4	6.3	9,054

Source: Howard (85) Table 1, page 9.

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1 This includes moves from abroad as well as from other U.K. regions.

by the Guardian (68) as "a manufacturer can put his plant anywhere, and ship the goods to his customers; so manufacturing is automatically helpful regionally and deserves an automatic grant. Services on the other hand are mainly consumed locally; no amount of subsidy would persuade a barber or an accountant or any other service to set up in a place without customers, and any help would be so much wasted money. He would be looking for 'mobile' services - headquarters staff, computing bureaus and the like which might go elsewhere".[1] A similar view has been expressed more formally by Hill (83) and EFTA (49). The exact conditions for this hypothesis to be true will be examined later.

If this hypothesis were true, we would expect no area to have any locational advantage or disadvantage, and consequently the rate of growth of each industry would show no statistically significant variations over the regions. In a recent study, Brown (19) has shown that some of the problems of the Development Areas have been caused by the failure of 'growth' industries in these regions to grow as fast as the national average. This is by no means universally accepted and McCrone (137) reports some conflicting studies - some of which find no discrepancy in the growth rates. To some extent the results seem to depend on the time period over which the study is made and on how finely the industrial data is disaggregated[2].

The studies that find no difference in industrial growth rates between areas, claim that there are no locational disadvantages in

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1 The grants concerning selectivity against services have been recently changed, but this does not affect the principle of the location theory expressed about manufacturing industry.

2 For a discussion of these and further methodological problems present in this type of analysis see Brown (19), Stillwell (177), (178), Bishop and Simpson (12) or Buck (20).

the Development Areas, and that the problem is largely one of the region's industrial structure. The conflicting studies claim that their results show some locational disadvantages of the regions concerned, or that the 'growth' industries are linked in some way to the declining industries, either by final demand generated from local factor payments or through interindustry relations[1]. For example the shipbuilding industry in Scotland has been declining rapidly, and it may be possible that a growth industry, such as electronics, manufacturing radar equipment in Scotland, sells a large proportion to the local shipbuilding industry[2]. If the electronics industry finds it hard to expand in other markets then the fortunes of the two are linked. Similarly the factor incomes of shipyard workers will not be expanding rapidly, and so the local service sector will not expand as fast. Both these arguments imply that certain industries are orientated towards demand[3], thus giving an area with a large share of declining industries a locational disadvantage to certain demand orientated industries.

Even if we reject those components of growth studies that show areas to have locational disadvantages, we are still faced with the fact that all studies show the Development Areas to have an unfavourable industrial structure, from the point of view of increasing employment, over time. But this fact still has some relation to location theory, in that we have to explain why the majority of the 'growth' industries are to be found in the prosperous regions and why they have shunned the Development Areas. If their location is purely arbitary,

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1 This view has been expressed by Mackay (138) and in a more recent paper by McCrone (135)

2 This is purely a hypothetical example.

3 If the industry in question could switch its regional markets costlessly then it would not be described as being orientated towards demand.

Mr. Chataway may be correct in his statement and we may conclude that industry is footloose over the U.K. However if we can discover some rational explanation for this distribution of industry, there are important implications for the government's attitude towards industrial movement, which plays such a large part in regional policy.

McCrone (136) has recently suggested that location theory has little to offer as an explanation for the regional problems, since it is the national growth of demand for products and the region's structure that determines its rate of growth. This takes the extreme view[1] in the components of growth analysis, but even if it were true I hope to show that location theory is relevant to regional policy. However in the same paper McCrone puts forward a crude location theory, that much of industry is footloose, since transport costs are negligible in comparison with the gains from economies of scale. He suggests that location is determined more by access to a major market and the availability of labour than by any other factor. Rather than boldly assert these results, it is hoped in what follows to use economic analysis and a quantitative model in order to determine how mibile an industry really is. These results may then have implications on the government's location of industry policy, since different degrees of mobility in industry may be more usefully approached by a selective policy. This may replace the blanket I.D.C. controls in the generating regions and blanket grants in the receiving location[2].

Chapter I will examine in a qualitative way some of the factors that may be important in determining the location of industry, making the distinction between primary and secondary factors as suggested by

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1 Which is by no means universally accepted

2 For the latest regional policy proposals see H.M.S.O. (80). For the wider historical perspective of regional policy see McCrone (137).

Klaassen (114). Some of the less analytical approaches to the subject will then be examined, along with specific industry studies. In these studies it is hoped to bring out further factors, that, although less seemingly obvious, may influence location, but at the same time point out the qualitative nature of the results and certain methodological weaknesses of these approaches.

In Chapter II a review will be made of the analytical studies of location. The problems of deriving general results, even from simple hypotheses, and how these theoretical studies may ignore or be incapable of incorporating certain factors alluded to in Chapter I, will be discussed. The inability to make these models operational in a practical manner may be seen as one of the major drawbacks to this type of work.

Chapter III will present the attraction theory[1], pointing out some of the assumptions that are necessary in order to obtain an operational model. The usefulness of this system as a tool in regional policy will be discussed and finally its relationship to other theories of location, particularly those in Chapter I and II, will be examined.

Chapter IV will present a method and some results of constructing regional input-output tables for the U.K. Whilst at first this may not seem directly related to the remainder of the study it is included for a number of reasons. In the first place the construction of these tables is necessary to obtain data to test the attraction theory. Secondly they have played a large part in regional analysis in the past and as such the results for the U.K. may have some intrinsic appeal, and finally it is hoped to show that attraction theory attempts to combine supply and demand influences as opposed to the orientation of industry implicit in regional input-output analysis.

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1 First developed by Klaassen (113) and extended by Klaassen and Van Wickeren (115) and Van Wickeren (197).

Finally Chapter V will present the results of the application of the attraction theory to U.K. data. The various multipliers and results of a method of determining footloose industries will be presented. The possible policy implications of these results will then be examined.

This study may be seen not only as an investigation into the possibilities of using attraction theory as an aid to decision making in industrial location policy, but also as an attempt to investigate whether attraction theory can be tested by the use of estimated, rather than actual, regional input-output tables. If this is possible, then there will be an additional[1] way in which attraction models can be implemented, so giving a larger number of situations where it can be of use in planning industrial location decisions. This may be particularly important at the present time since most industrial countries are pursuing an active regional policy, and regional policy is also to be given an important part in the programme of the enlarged European Economic Community[2].

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1 For the other possible methods of implementing the attraction model see Van Wickeren (197).

2 See E.E.C. (44).

CHAPTER 1

Qualitative and Quantitative Methods of Determining the Location Characteristics of Industry

I.A. Introduction

Most theories of location attempt to set the traditional profit maximising producer in a spatial context. For an individual firm or industry we would wish to know the cost of inputs (such as labour, capital and intermediate products) over space and the costs of transporting these between various possible producing locations. We should also wish to know the spatial distribution of demand and the costs of transporting the final product. If we were then given information on the technologies available to the firm we hopefully should be able to obtain the maximum profit location. Transport costs would thus become another factor to take into account; for example, a firm may be willing to increase its transport costs by concentrating production at one location and thus sending the final product over a greater distance, if say there are economies of scale or labour were cheaper at a certain site[1].

However the location decisions of an individual firm cannot be looked at in isolation, since location decisions are interdependent. For example, rents may increase when firms crowd into an area, or the cost of an intermediate input at a location depends on the location of the supplier of that input.

In discussing the location decisions of a firm in a spatial context, a useful distinction between primary and secondary factors has been made by Klaassen (114). Primary factors include those mentioned

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1 For some studies of traditional substitution analysis involving the spatial dimension see Isard (89) and Moses (146).

above and are the ones generally analysed in location studies. However even when all these factors are taken into account and they indicate that a certain area shows the maximum profit, this is not necessarily the area where the activity will locate. We must take into account secondary factors of location. These consist not only of non-economic factors that may be classified under the heading of environmental, but also economic factors which may be called communication channels, either with firms below or above in the production process. "This group also includes contacts with service firms, with subcontractors and with consumers..... Good communicative channels are a necessity for many industries. Many other industries will accept unfavourable cost differentials in the primary factors of production, on the condition that adequate communication facilities be available"[1]. Consequently studies that are to be useful in government policy evaluation must not only take into account the primary factors of location, but also secondary factors and the interdependent nature of location decisions.

In this chapter, I wish to examine individually some of the factors, both primary and secondary, that will influence a firm facing location decisions. Not all relevant factors will be discussed but only those which also must be taken into account because of the added spatial dimension to the profit maximisation problem. A study of some of the techniques that are often used in empirical work on location will follow, bearing in mind both their relevance to government locational policy decisions and their ability to handle the

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1 Klaassen (114) page 81 - 2. A more detailed discussion of secondary factors and the interdependence of location decisions will be found later.

factors added to the profit maximisation problem by the introduction of the spatial dimension. However it will be shown that in many cases these applied studies lack a rigorous methodological base, and consequently the results that they generate may be open to doubt. The more rigorously formulated studies of location theory (both of the individual firm and optimal patterns of location) are often incapable of generating testable hypotheses and thus the literature has tended to keep them separate[1]. I shall maintain this distinction and discuss the theoretical work in a later chapter.

I.B. Additional variables encountered by a profit maximising firm in a spatial context

(i) Transport Costs

Transport costs have been the major factor in classical location theory in determining either:

(a) the location of the individual firm given market demand and supply points, or

(b) the market boundaries, given the location of the firm[2].

In the latter studies transport costs and the density of demand determine the market areas, and in the former studies transport costs alone are used to determine the optimal location of an activity. Only once this location is found are other factors (such as cheap labour or energy sites) allowed to cause the location to deviate from the site with the ~~cheapest~~ lowest transport costs.

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1 A similar view has been expressed by Brackett and Stevens (17)

2 Both these approaches have been reviewed by Been (11).

However, recently this approach has come under attack because of the conjunction of two types of studies. The first of these has been the many comparative cost studies[1], where it has been claimed that in the UK the costs of operating a plant in many industries are not significantly different between the various regions, and that, given a settling down period, plants can move successfully from say the South-East to any Development Area and have the same operating costs. Secondly it has been suggested that transport costs in many industries are now such a small proportion of total costs, that variations in transport costs between locations can be ignored.

These two views are implicitly embodied in Chataway's statement (see above) since they are necessary for a manufacturer to be able "to put his plant anywhere". McCrone (135 p. 370) expresses these views more formally as "Modern industry which has a high value added in relation to its bulk is much less closely tied to particular locations by accessibility to raw materials or transport costs. It is commonly thought that this makes modern industry more footloose than in the past."[2]

The demise of transport costs has been based on some factual evidence. Townroe (191) quotes a number of sources for this. Woodward (201) states that according to input-output

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1 These studies will be reviewed and evaluated in Section I.C.ii.

2 The quote continues "In fact it probably means that location becomes less governed by transport costs in the simple sense, and more by ease of contact with related activities, supplies, distribution, sources of finance etc." The view that industry may be tied to related activities and the other factors mentioned by McCrone is part of the basic hypothesis of attraction theory and will be discussed later.

tables for manufacturing industry in the UK, the direct cost of purchased transport averages only 3% of total costs. Hague and Newman (71) in comparing alternative locations for the clothing industry, concluded that in moving to Development Areas, transport costs increased by less than 1% of total costs. Similar studies to those mentioned above also conclude from comparing transport costs of plants located in London and the Development Areas, that actual transport cost differences are negligible[1].

It is possible that these results concerning transport costs were obtained because the types of industry studied above are the ones that have moved to the Development Areas and so are the ones where transport costs are necessarily less important. This would not allow us to draw any conclusions concerning the remainder of industry. However by using a different technique Tornqvist (187) came to the similar conclusion that transport costs are unimportant. Instead of observing actual transport costs, he calculated what the theoretical increase in transport costs for various sites would be over the minimum cost site for assembly and distribution of materials. He concluded (page 8) "the regional variations in transport costs that are demonstrated...... will not to any decisive extent be able to influence the choice of localisation for the production unit studied."

Despite the results of the studies reported above, there

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1 However it is interesting to note that Hague and Dunning (70) state that, although actual transport cost increases may be insignificant, it may be a serious handicap to the industry (the radio assembly industry in this case) to be out of touch with the market and away from Research and Development centres; and it is this type of cost that is difficult to quantify in comparative cost studies - this point will be raised later.

is still much evidence to suggest that transport costs are an important locational factor, or even if transport costs are small, there are still significant costs incurred in communications between two sectors that are some distance apart. We shall call this wider concept separation or communication cost and specific examples will be given below.

Lever (127) argues that transport costs may be more important than has been recently suggested. He quotes Edwards (42) "it is probable that transport accounts for at least 9% of total costs of producing and distributing. In addition to this 9% ... it is possible to define a set of distance costs. These include the cost of information transfer (post, telephone etc.) and personal travel as, for example, between a firm's main works and a branch plant, the higher levels of stock holding necessitated by remoteness from suppliers and costs incurred by the loss of face-to-face contact between suppliers and customers." Thus the view is expressed that not only have the extra actual transport costs involved in a distant location been underestimated, but that even these underestimate the real costs involved in separation.

A similar conclusion can be reached by tackling the problem with a different approach. Two studies both using linear programming techniques show that transport costs are a significant variable. Hopkins (84) studied the Household Furniture Industry (H.F.I.) where traditionally transport costs may be thought to be unimportant since they form a small part of total costs. A linear programming problem was set up in the conventional way, with a matrix of transport costs between all supply and demand given at each point. The cost of assembly of each input and the optimal distribution of output was calculated, along with the dual or shadow price. These costs and shadow prices of supplying an additional unit of H.F.I. production, along with various other factors thought to be important were regressed on employment and changes in employment in the H.F.I. Whilst the explanation is not very good (R^2 between 0.32 and 0.43) the transport cost variables and shadow prices were always highly significant and of the correct sign.

In a slightly different context, O'Sullivan (179), using linear programming, concluded that transport costs were important in determining the flows of goods. The flows of goods between regions of the U.K. generated from the Min. of Transport road survey data (141) were compared with flows generated from a linear programming model (where the objective function was to minimise transport costs and the constraints that demand should be met in each area, and that demand for a good should not exceed the supply in each area). The results were quite encouraging, in that the actual flows closely resembled the hypothetical optimum flows, especially with more homogeneous areas and homogeneous categories of goods.

From the two above linear programming studies it is difficult to tell whether it is pure transport costs that are a significant determinant of location and flows of goods, or if transport costs are a good proxy for additional separation or communication costs. If these two types of costs were highly correlated it would be impossible to distinguish between the effects of the two, but only to establish their joint importance.

In trying to explain the number of moves by industry between regions in the UK[1], Townroe (190) uses multiple regression analysis with the number of moves as the dependent variable and various independent variables that he considers as important in this movement process[2]. A number of variables are found to be significant and of the expected sign - one of these is the proportion of transport costs in total costs, which is negatively related to the degree of mobility. For this to be so it is argued that transport costs (or this as a proxy variable for the wider concept of distance costs) must be a significant factor in determining the location of an industry.

Physical distance as a proxy for transport costs and/or separation costs is often used in various studies to help explain the interaction of economic variables. A well-known technique is that of gravity models[3]. A recent study was undertaken by Keeble (109) to explain the number of jobs moved. The actual formulation used by Keeble is:

$$_{rs}M = \frac{_{s}A}{_{rs}d^{b}}$$

where

$_{rs}M$ = an index of predicted volume of industrial movement between origin r and destination s.

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1 That is the data described by Howard (85).

2 This technique can be criticised on methodological grounds. This will be done in Section I.C.v, where a more comprehensive discussion of this study will be found.

3 For a survey see Isard (90).

$_{s}A$ = attractiveness of receiving location s, measured in terms of labour availability.

$_{rs}d$ = distance between r and s, raised to some exponent b.

Although the above formulated can be critisised[1] it did show that with the UK distance may be a significant variable and improved the "explanation" of job movements.[2]

Evidence on the importance of physical distance (again as a proxy for transport costs and/or separation costs) can be gathered from some of the road surveys undertaken in the UK. Edwards (41) completed a study of the West Cumberland transport patterns, that showed that West Cumberland traded much more frequently with adjacent areas than with distant ones. The Ministry of Transport Road Survey (141) for Great Britain shows a very similar trading pattern, of heavy internal flows within a region, lighter flows to adjacent regions and even lighter

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1 Human behaviour does not have to conform to laws drawn from the physical sciences, but the study may be useful in bringing to light variables that should be investigated further by more traditional economic analysis. The problem of arbitarily specifying reduced form equations will be discussed in Section I.C.v.

2 This is important since many of the studies reported by other authors concern the USA, where transport is, other things being equal, more important due to the larger distances involved. However Swann (180, page 101) reports that even in Germany railway freight added 50% to the producer price of building materials, and between 9% and 26% to the prices of bulk goods. Surprisingly in the U.K., he reports that a 400-mile journey by rail in 1955 added 28% to the wholesale price of wheat and 22% to the price of barley and oats.

Estimated number of tons carried by road transport in 1962 analysed by region of origin and destination

Thousands

Destination / Origin	Northern	E. & W. Ridings	North Midland	Eastern	London & S. Eastern	Southern	South Western	Wales	Midland	North Western	Scotland	Total
Northern	70,600	5,640	770	420	630	100	220	270	920	1,900	2,080	83,550
E. & W. Ridings	3,860	101,480	5,190	610	1,190	350	250	210	2,050	6,600	450	122,240
North Midland	1,000	[illegible]	[illegible]	3,550	7,390	1,090	700	480	7,050	5,050	340	[illegible]
Eastern	210	620	3,150	71,900	14,180	[illegible]	600	130	1,730	830	140	98,330
London & S. Eastern	460	1,150	1,620	12,170	151,410	7,630	2,160	730	2,690	1,780	400	182,200
Southern	50	300	600	1,670	6,630	38,420	3,790	290	1,320	420	20	53,510
South Western	100	200	500	480	2,250	2,650	63,700	1,240	2,740	530	50	[illegible]
Wales	180	200	420	240	1,220	390	1,180	61,320	3,060	4,170	80	72,540
Midland	620	1,430	4,180	1,350	3,040	1,280	2,430	2,760	106,510	7,840	280	131,730
North Western	2,250	6,190	3,120	730	2,510	10	530	2,770	6,910	154,030	1,560	181,300
Scotland	1,410	460	280	80	370	60	70	40	280	1,320	169,190	[illegible]
Total	80,720	122,300	94,380	93,500	[illegible]	[illegible]	80,030	70,340	135,290	184,510	[illegible]	[illegible]

Source: Ministry of Transport Statistical Paper No. 6 Survey of Road Goods Transport 1962, Final Report H.M.S.O. 1966.

Fig. I.1 Northern Region trade by road transport 1962.

Main flows by road within the region and between the region and other parts of Great Britain in 1962

Source: Min. of Transport Statistical Paper No. 6
Survey of Road Goods Transport 1962

Final Report H.M.S.O. 1966
Page 20.

Key

Tonnage moved aggregate of flow in both directions

Over	Not Over
¼ million	½ million
½ million	1 million
1 million	

flows to distant regions[1] - see for example the map of trading patterns of the Northern Region (Fig. I.1 and Table I.1).

Despite the regularity in appearance of these interregional flows, I am very sceptical about the use of gravity models[2], since as Spiegelman (170) points out, in order for the gravity model to give a "good fit" there must be a considerable degree of aggregation. "When the total volume of traffic is disaggregated .... the extent to which the gravity model describes or explains any regular falling-off effect tends to decrease. In addition, there is admittedly a lack of any theory to explain values or functions which we assign to weights and exponents." (page 20) As terminal costs increase relative to the unit distance cost in the transport cost function[3], the ability of gravity models to predict will be further weakened. Rather than place much reliance on the results of gravity models, their use is, perhaps, to suggest that there are certain locational forces that result in the majority of trade taking place within a region, and that these forces need greater investigation by more systematic analysis.

Thus there seem to be two conflicting views - one that transport costs are so insignificant and can thus be ignored and the other that transport and/or separation costs are vital to the location decision. However, even if the former view were true, this does not make industry locationally mobile, since transport

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1 Isard (90) has some charts showing a similar inverse relationship between distance and volume of goods.

2 The form of the transport (communication)cost function will be shown to be crutial to attraction theory - see Ch. III.

3 To my knowledge the only systematic test of gravity models versus models based on optimising behaviour has been carried out by Hartwick (74) where the optimising models performed better - see Section IV.D for details of this study and further problems of using gravity models to predict interregional flows of goods.

costs are only one of many variables that would affect a location decision. If transport and/or separation costs are significant they must be analysed along with these other factors and not treated in isolation. These qualifications have important bearings on Chataway's statement quoted above.

(ii) Inter-Industry contacts and related factors

Whilst interindustry relations are important in the problem of profit maximisation in a non-spatial context, they must be regarded in a different light when spatial analysis is explicitly employed. There is a great deal of evidence to suggest that industries which are tied together through input-output relationships tend also to be found in the same geographical area. These associations are usually analysed with correlation coefficients. Richter (159) concluded "the data and analysis used ... lead to the conclusion that industrial linkages are agglomerative forces. Geographical associations are more common among linked than non-linked sectors."

Karaska (106) studied an urban input-output table, constructed from actual survey data, and from his analysis he said "the intent .... was to provide some insight into those complex forces which attract industry to a large metropolitan area. While not totally accurate, this mutual linkage partly described the external agglomeration forces."

Lever (127) carried out a similar analysis with UK data, only he distinguished between the older declining industries (such as textiles) which might be thought to be more linked, and the newer expanding ones, which are often regarded as the source of mobile industry for regional policy. In fact, quite surprisingly, the latter group was shown to be just as strongly spatially linked, as the older group, with its suppliers and customers. Within the newer group there were identified two sub-groups "the first comprising most branches of engineering and metal working, is shown to be strongly influenced by access to suppliers and customers, whereas the second type comprising most of the science based industries, such as plastics and pharmaceuticals are little influenced by functional linkages, but which seem to need the type of external economies most commonly available in the greater London Metropolitan area." Many other similar studies can be found, such as Streit (175), who tries to identify complexes of industries and one of the strongest appears to be the metal working and metal using sectors[1]. All these studies basically use correlation analysis and reach the same conclusion that industries that are linked through input-output relationships tend to be geographically associated.

A slightly different approach is adopted by Brown (19) in trying to explain output per head in various industries between regions in the U.K. Using regression analysis, Brown shows that output per head is positively related to the total amount of trade in the region - this is an indicator of the strength of general external economies as well as inter-industry relationships - and related to the size of the industry in that region - this is an indicator of the strength of intra-industry relationships or as Brown calls it economies of aggregation.

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[1] A similar conclusion about metal using sectors can be found in many diverse studies and examples will be given when they appear.

It is unlikely that these geographical associations and intra-industry relations could be explained by reference to transport costs alone, since many of the flows will be of small physical size, high value products. But there are, nonetheless, important economic reasons why these activities should be located close together. The explanation can perhaps best be presented by using Tosco's (189) classification[1];[2] he puts great emphasis on the importance of intra- and inter-industry relations in generating external economies and on the importance of general agglomeration effects, thus providing a strong locational tie for industry. These can be classified as:

(a) general external economies or agglomeration effect - these "refer not only to the advantage of infrastructure and site (standard requirements) and of labour supply (including skilled labour, but limited to skills that are widely used), but also to maintenance units common to the majority of manufacturing industries, to those depots of standard and catalogued products for which there is extensive demand and to industrial services of a general nature."[3] From this brief definition it may be judged that this type of linkage is difficult to quantify, but does provide a positive force for industry to agglomerate. Specific examples of this type of external economy will be mentioned later.

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1 The wider aspects of Tosco's study will be examined in Section I.C.iii.

2 For a practical application of Tosco's work see the EEC study (45)

3 Tosco (189) page 164.

(b) <u>sectoral external economies</u> -

these "refer not only to specialised infrastructure, special site requirements in some cases, and skilled labour supply, but also to the whole system of inter-industry relations which an industry in a given sector requires (sectoral services)"[1]. Again these may be difficult to quantify, but do provide a positive force for similar linked industries to agglomerate.

Tosco gives example of the type of relations that may influence the location of industry. He cites:

(i) <u>specialist units for maintenance and servicing of machinery and equipment, and related facilities</u>.

These services are necessary to keep the industry they serve in good working condition. Tosco (op. cit. page 162) suggests that "maintenance units must be located near customer plants so that workmen and technical staff can reach them within a short time and materials delivered quickly. Frequent contacts between customer units and maintenance units are also necessary, because in the intervals between overhauls, a number of operations have to be arranged, such as the preparation of materials." Because of these factors and the necessity for speed in these operations the maximum feasible distance between contacting units may be as little as 50 km. (approx. 30 miles). These specialist maintenance units require themselves other services of supplementary specialists, as well as depots of spare parts. This type of maintenance

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[1] Tosco (189) page 164.

units is required by most industries and they tend to serve a number of different industries, although in some cases they may serve only one industry and thus be classified as sectoral external economies.

(ii) Specialist units for repair of machinery and equipment and related facilities.

Much the same comments apply to this service as the one described above. It is, however, worth pointing out that this facility must be close at hand to the industry it serves because stoppages of work due to idle machinery can be very costly in modern integrated plants, and therefore, the speed with which the repairs can be carried out is an essential part of the service.

(iii) Subcontractors

These "specialised processing units produce materials or components or supply processes for the article produced by the customer firms. These processes are made-to-order according to required specifications." Tosco (op. cit. pg. 162). There are many techno-economic reasons why subcontractors are necessary in certain industries - for example a firm can use them to meet rapid changes in demand and if many firms are using the subcontractor in this manner, the subcontractor will experience a relatively even demand, or a firm may require certain specialist components so infrequently that it is not worth while for the firm to produce them itself, but a subcontractor supplying many firms in such a condition will find the production relatively cheap. It is also necessary that these subcontractors should be in close proximity to the industry they serve because of the number of face-to-face contacts, the need for quality control and the control of design. Tosco (op. cit.) suggests 100 km. (approx. 60 miles) as the limit within which the service can operate. Keeble (110), suggests a much lower limit for the maximum feasible separation for the subcontractors of the engineering sector in N-W London[1].

(iv) <u>Intermediate industries or local depots supplying standard and catalogued products</u>

These are the inputs that are used everyday in the production process and can be readily identified through input-output tables. Transport costs are not normally significant on these types of inputs and little face-to-face contact is necessary because of their standardised nature, which means that they can describe their products adequately in catalogues and brochures. If transport costs are important then there will be a tendency for the industries to locate together, but normally this is not the case. However a local depot of these inputs can be helpful in that it allows the user factory to hold lower stocks, mistakes in supplies can be corrected immediately, and production stoppages or imbalances can be avoided when infrequently used products are needed urgently.

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1 He also discusses some of the reasons for the necessity of subcontractors and why they should be located so close together.

(v) Raw material inputs

Similar comments apply to these when they are used as a standardised inputs, except that transport costs often tend to be more important and so may exert an influence. However, as mentioned above, less industry is tied in this way with improvements in transport and the declining importance of primary products in the production process.

(vi) Various technical, commercial and financial services

Again the frequency face-to-face contact makes location in the same proximity a necessity. Most of these services are required by all industries and so they can be classed as general external economies rather than sectoral economies.

Whilst not explicitly mentioned, embodied in most of the flows of goods described above is some notion of the constraining influence of information flows that are necessary for the goods flows to take place[1]. A specific example was found above in Hague and Dunning (70) concerning the information flows about research and development and of market changes in the radio assembly industry. The disruption caused by the lack of this information is very difficulty to quantify, and surveys, which give some idea of the frequency of contact between two parties, have often been used instead.

In the sphere of offic location, the importance of information flows has long been recognised. A government White Paper (H.M.S.O.(79)) attempts to quantify the costs of maintaining and/or substituting the

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1 The place of information flows in the location of all economic activity can be seen in Tornqvist (188) and Wood (200).

mode of communication, when previously closely located activities are separated. A similar study of Swedish government relocation is reported by Thorngren (184) who also discusses its wider implications - the stimulus to regional development of a relocated information source[1]. Thus physical distance separation not only includes transport costs, but also information cost, time costs and the other costs mentioned above. We shall use the term communication costs to cover all these costs.

This discussion of intra- and inter-industry relations in a spatial content adds to the factors already mentioned that are encountered in discussing the location of an industry or of a firm. It can be seen that for Chataway's statement (see above) to be true, as regards the mobility of industry, that many restrictive conditions must hold for profitability to be similar in a number of locations. In an attempt to discover just how mobile industry is I shall examine some of the methods that have been used in location studies, that attempt some quantitative or qualitative assessment of the problem.

Before discussing this however, it may be worth suggesting some lines for further research in trying to explain the results presented in the Howard report (85), by bringing together Keeble's (111) study involving distance and economic attractiveness of an area[2], and the findings of the studies reported above concerning industrial linkage and geographical association. This last factor, as far as the author knows, has never been introduced into the studies that try to explain why firms chose particular locations in which to locate.

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1 The importance of information in the private sector office location can be seen in Goddard (58, 59).

2 Measured in terms of surplus labour.

I.C. Some Qualitative and Quantitative Methods of Studying Industrial Location Forces

(i) Surveys

A large number of surveys have been carried out with an attempt to identify the factors that influence industrial location[1]. Most of these surveys deal with new movers in industry rather than long established firms, and ask such questions as why a move was made to a new location, what factors influenced the move, what factors are important locational determinants or to place in order of importance various factors influencing location[2].

The questions usually ask for subjective answers, that often involve ex post rationalisation of past decisions, or the respondent may give answers that he thinks the interviewer wants to hear[3]. Also the answers given can lead to contradictions. For example, in most studies, the availability of labour appears to be one of, if not the, most significant factor influencing location decisions. This was one of the conclusions that emerged from the studies by Cameron and Clark (23) and Cameron and Reid (24). Similarly Tosco (189) reports the results of a survey, interviewing Italian firms over their relocation policies in the Mezzogiorno. When asked generally what factors influenced their location of new plants, the most commonly cited factors were labour and site requirements. However when asked why they did not wish to set up new plants in the Mezzogiorno, the most commonly cited reply was the lack of auxiliary industries and industrial services.

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1 Brackett and Stevens (17) list some of these surveys

2 Examples of this type of study can be found in Cameron and Clark (23), Cameron and Reid (24), Howard (86) and N.E.D.C. (150).

3 Howard (86) page 9 lists some of the problems of interviewing techniques. Machlup (134) has discussed the problems of interviewing techniques in a different context to test marginalist theories.

Clearly there is a contradiction that has emerged in asking firms arbitarily to order criteria, and this lays open to doubt the usefulness of results obtained from such surveys[1].

Perhaps more value can be attached to the type of surveys that ask less subjective questions, and require, instead, more factual information[2]. These results may then be used tentatively to test hypotheses which previously existing data would be incapable of testing. Keeble (110) carries out such a study of manufacturing linkages in N-W London. Questions concerning the proportion of inputs and outputs bought and sold locally, and questions concerning the use of auxiliary industries and industrial services are asked in order to obtain some idea of the strength of local[3] economic linkages. From these replies it is possible to build up some kind of picture of the importance of such factors as the availability of skilled labour or the use of subcontractors. This may give a more objective clue as to the locational characteristics of an industry. Keeble presents a picture of the importance of local links in the engineering industry and particularly of the importance of subcontractors[4]. He describes the numerous advantages of subcontracting in this type of industry, and shows that such a relationship is most efficient when the two parties are geographically close. He states (page 173) "subcontracting

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1 Spiegelman (170) also doubts the usefulness of surveys in studying location decisions.

2 Even in this type of study there is probably a large reporting error in the answers.

3 Local in this study is restricted to N-W London.

4 This view was also expressed by Tosco (189).

engineering ..... demands close contact between supplier and purchaser, since the former's factory is generally operating largely as just another department of the latter's, and must conform very strictly to specification, materials and completion dates necessitated by the purchaser's overall production schedule. Such a relationship is undoubtedly facilitated by close geographical proximity." The study also comments on the value of other local industrial contacts, which again tend to agree with those expressed by Tosco (189).

Although this is a valuable piece of work, it would be difficult to go any further than the qualitative statements concerning the importance of linkages. Also it is not known whether the conditions described above are unique to the industries in N-W London, without further expensive surveys. But the evidence gathered in this paper does lend weight to the Tosco study (189) that interindustry relations are of crucial importance in the location decisions of some industries.

(ii) Comparative Cost Studies

As the title suggests, this technique involves comparing the total operating costs between two or more distinct sites. In these studies it is usually assumed that the output of the plant is sold in the same market at the same price, and the costs of transport to the market are included in total costs. Thus a comparison of cost is a comparison of profitability. If the differences in costs of various sites are large, then the industry will naturally go to the low cost sites and be locationally immobile, but if there is little or no difference between costs, then the industry is said to be footloose.

This technique has been carried out in two distinct ways. One way is to go and examine similar plants in different locations that are actually operating. This was the approach used by Luttrell (133), Hague and Newman (71), and Hague and Dunning (70). Comparisons were made between parent plants which tended to be in the S-E England and the W. Midlands, and branch plants operating in the Development Areas. Branch and parent companies were chosen as opposed to studying different firms, otherwise slight product differences and differences in management quality would probably have made the analysis extremely difficult.

Even between parent and branch companies there are conceptual as well as practical difficulties in gathering the data. In practice, labour productivity is extremely difficult to measure. Slightly different types of work were done by the branch plant. The branch plant may be used to absorb fluctuations in demand. The problem of apportioning shared head office staff is encountered. Unless the plants are of the same size, economies of scale may cause differences in average costs, also a 'settling-in' period is usually encountered by a branch plant at a new location before average costs reach their lowest level[1]. Table I.2 shows a typical example of the type of accounting done in comparative cost studies.

The type of industry that has been subjected to the above analysis, tends to be light or of an assembly work nature, such as textiles or radios. The major costs compared are labour and

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1 Details of further difficulties encountered can be found in Lutrell (132).

transport costs for inputs and outputs, although minor factors such as factory rents, heating and lighting etc. are usually considered. However these may not be the only costs involved in locating in a Development Area at some distance from traditional sites. The flows and costs of information transfer are not considered, or the importance of speed of delivery and necessity of frequent face-to-face contact cannot be quantified in this manner[1].

The second approach to comparative cost studies has been carried out on hypothetical plants in various sites, rather than with plants actually in operation. So rather than using actual data the costs are estimated - such as the local wage rate, transport costs of raw materials and finished products, power costs, local taxes, building costs etc. However, it is the wages and transport costs that tend to get the emphasis in this type of study. An additional problem is that the figures used are only hypothetical guesses which may turn out to be wrong in reality. Also if the project is a large one it may substantially alter local conditions, such as wage rates, once it has established itself.

Many of the industries subjected to the hypothetical plant comparative cost analysis almost fulfill the classical Weberian location triangle problem with two point sources of inputs and a point source of demand. An example of this is the iron and steel industry with the two major inputs of coal and iron ore. This heavy/basic type of industry is more amenable to comparative

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1 Hague and Dunning (70) comment on the radio assembly industry mentioned above is an example of those types of costs that cannot be easily quantified.

Table I.2

Actual cost of closing[1] at a branch compared with the actual cost at the main works

Cast Study A

1947

		Branch		Main Works		Branch to Main Works %
1.	Output in pairs		219,280	223,207		
		Total £	Price per pair	Total £	Price per pair	
2.	Wages	9,536	10.44	10,791	11.15	94
3.	Salaries	1,884	2.06	1,184	1.22	169
4.	Upkeep of premises	1,390	1.52	1,298	1.34	113
5.	Travel Communication	384	0.42	139	0.14	300
6.	Other	585	0.64	517	0.53	121
	Total	13,779	15.08	13,929	14.39	105

Source: Luttrell (133) Case A Table 26 page 37.

For the full details and description of actual case see Luttrell (op. cit.)

Notes (i) This is the first year of operation and so the firm was still suffering from settling in problems and by 1949 the branch was cheaper per unit of output than the main works

(ii) Notice how 5 (travel, communication) is the item that increased relatively the greatest.

(iii) The above categories are an aggregation of various categories since the data collected was very detailed. See Luttrell (op. cit.) pages 89 - 91 for details of the disaggregation.

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1 Closing - a technical term used in the shoe-making process.

cost studies, because the relatively easily measurable factors (such as transport costs on raw material) are probably fundamental in the location decisions of such industry. The less quantifiable costs such as the importance of the speed of delivery of spare parts and necessity for subcontractors are of little value to such industry, since it requires a steady flow of inputs to work at a predetermined capacity on long runs. Changes are not made quickly and hence the need for flexibility is obviated[1]. Most of these types of studies concluded that the industries were not footloose, but tied by the transport costs. Although changing technology which could alter the proportion of the various inputs used, usually changed the locational patterns of the industries in question.

To apply this analysis to the above industries seems reasonably safe, and a priori we would suspect that this type of industry is not footloose. But to apply the hypothetical plant sites to the lighter type of industries seems to have the same dangers, as discussed above, for actual plant sites. Smith (169) reports a number of studies of hypothetical sites of industries that were traditionally regarded as footloose such as clothing and electrical apparatus assembly. A study of an electrical apparatus firm searching for a new site is given in great detail because the data was available from a firm of economic consultants - Fantus Company (51). From this study Smith (op. cit. page 386) concludes "the location of the electric equipment industry is thus a case where the conventional cost approach is incapable of providing a sound explanation,

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1 Examples of this method applied to this type of industry are found for iron and steel in Isard and Capron (97) and Isard and Cumberland (98), for zinc smelting by Cotterill (33), for aluminium by Krutilla (121), and for oil refining by Lindsay (129).

despite the quality of the input cost data which are available. The external economies which appear to exercise the dominant influence on location choice are notoriously difficult to measure which made it impossible to incorporate them into the original variable cost model. If there were some way of accurately measuring areal differences in the cost advantages arising from access to an existing concentration of the electronics industry and related research activities, this could be added to the input cost data and could be expected to have a dramatic effect on the form of the total cost surface." Table I.3 shows the case study in question. A further problem with this analysis, is that it is usually restricted to one industry at a time, in trying to decide whether the industry may be footloose. Although one individual industry may not be footloose, it is possible that a group of closely related industries may be footloose when examined as a block. This further objection is overcome by industrial complex analysis and will be discussed next. However, apart from this there does seem to be serious objections to using comparative cost techniques to analyse and quantify the mobility of industry in the context of government regional policy.

(iii) Industrial Complex Analysis and Growth Poles

The concept of industrial complexes has been mentioned in connection with some of the studies reported above, however it is hoped to present in this section some of the more formal studies of industrial complexes. An industrial complex can be described as a group of industries, that, because of the strength of the inter-industry linkages, tend to be found together. These linkages may be so important, that whilst each individual member

Table I.3

Annual operating costs ($\not{}$1000) for an electrical appliance plant in selected locations

Item \ Location	1	2	3	4	5	6	7	8	9	10	11
Labour	1441	1179	1082	1092	1286	1249	1253	1057	1253	1085	1151
Materials (freight only)	180	355	352	360	270	261	271	361	328	366	315
Land and Buildings	64	179	204	188	243	233	242	170	237	189	243
Utilities	112	56	76	62	95	89	83	58	101	74	64
Taxes	101	44	50	48	92	99	25	46	99	96	56
Interplant Communication	8	24	24	24	18	15	18	24	21	24	21
Cost of relocation	0	153	153	153	153	153	153	153	153	153	153
Total Factory Cost	1905	1991	1940	1927	2156	2099	2046	1868	2192	1987	2013
Outbound Freight	259	226	228	231	230	229	224	225	228	243	224
Total Costs	2165	2217	2168	2158	2386	2327	2270	2093	2420	2231	2236
Difference from Location 1	(0)	+52	(+4)	(-7)	+221	+162	+106	(-71)	+256	+66	+72

Source: Smith (169) Table 18.8 page 364

The figures may not add up due to rounding

Notes (i) It will be noticed that the cheapest site (8) has very cheap labour but expensive freight costs. This is an example of transport costs being traded off for a cheap labour site.

(ii) It can also be seen from the 4 cheapest sites (ringed) that the difference between them is less than 3½% of average total costs. This figure is probably well within the margin of error of the data that has been collected, especially the labour cost figure, which will depend crucially on how the local labour force adapts to the factory in question and on its productivity. Because of the narrow range of costs, the factory would probably be regarded as mobile in the terms expressed above. Because of the very small differences shown above it can be appreciated how the introduction of external economies could substantially alter one's view of the least cost location.

of a complex may not be a viable candidate in a regional development programme (because of the extra costs imposed by being separated from other members of the complex) the complex taken as a whole may be viably moved to a depressed region if certain conditions are fulfilled. It seems, therefore, worthwhile to study interrelated industries as groups.

Kaprow and Rabinovick (105) suggest re-arranging the input-output tables into a block or block triangular form in an attempt to identify self contained groups of industries that might be described as complexes[1]. The problem arises when one attempts to quantify the costs involved in the inter-industry flows of goods, and when attempting to determine the mobility of the complex or to determine the effect of the stimulation of one or some members of the complex on the other related industries.

One of the earliest studies of complexes was undertaken by Isard and Kuenne (99) when they attempted to measure the impact of a new steel mill in the New York area. Studies for the least cost site in the area had been undertaken by Isard and Cumberland (98) using comparative cost techniques. From this initial steel plant there were assumed to be two major effects on the surrounding region:

(a) the supply effect - as a result of an increase in supply of steel in the area more metal using industries would be attracted. An estimate of this effect was obtained by examining other major steel producing areas and working out the ratio of steel workers to workers in the steel using industries. This measure of attractiveness was applied to the New York area in that it was assumed that the same ratios would be attracted,

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1 Yan (203) also describes this method of attempting to identify complexes.

(b) the demand effect - the new steel plant and the steel using plants attracted to the area will have a demand for other intermediate inputs some of which will be supplied from within the region. Also the wages of the people employed in new plants will have an income generating effect. These effects can be calculated through the conventional use of a regional input-output table[1].

Whilst this is an important study in that the effects of an industry can be seen on the supply side as well as the purely demand side[2], there are some criticisms that can be levelled against it. The calculation of the initial supply effect of the steel plant, i.e. the complex that would be attracted around the steel plant, seems somewhat arbitary. It assumes that the only locational influence on steel using industries is the potential supply of steel in the area, and after this the supply effect of all other industries can be ignored. To pre-determine the locational characteristics of the steel users in such a way seems an oversimplification. To pick out steel making as the basic industry in the whole complex again seems somewhat arbitary. It is possible that the location of the steel industry may be influenced to some extent by the demand for its products and so the industry cannot be looked at in isolation, but the location of the whole complex must be tackled in a simultaneous manner.

It may be that rather than a method of general application to the problem of examining other industrial complexes, the above analysis is just a practical expediency for the problem in hand. This may be bourne out by the fact that in examining the

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1 See Ch. IV for a discussion of the use of regional input-output models to calculate the effects of an exogeneous change in demand in a region.

2 The identification of both demand and supply effects will be seen to be of critical importance in attraction theory - see Ch. III.

possibilities of a chemical complex in Puerto Rico, a completely different approach is adopted by Isard, Schooler and Vietorisz (94)[1].

The study starts by identifying the main assets of Puerto Rico as access to the US mainland market, abundant supplies of cheap labour and possible use of Venezualan crude oil. Around these factors a chemical complex was planned.

From the basic process of refining oil many different types of by-products can be produced, that in turn are used in the manufacture of other goods. In order to reduce to manageable proportions the number of different types of complexes that are to be studied in greater detail, certain types are excluded or included, using a priori reasonable criteria. For example, processes that produced goods for which there was little market in Puerto Rico - such as anti-freeze - were excluded. Similarly, products where a very large plant was necessary in order to obtain economies of scale - such as rubber, or where vast amounts of capital and/or skilled labour per other employee were necessary - were both excluded. Thus the types of processes left, were the ones that were felt may be viable in Puerto Rico, because of the large market - such as for fertilisers that were previously imported - or because large amounts of labour were needed - such as for synthetic fibres. In all the complexes refined oil would have to be transported to the mainland US market.

Having cut down the number of complexes to a viable number, large amounts of engineering data were collected on the inputs required and the different outputs of each type of complex, and information concerning the minimum viable size of each process was obtained. The costs of production and the costs of transporting

1 An abridged version of this lengthy study can be found in Isard and Schooler (95), and Isard and Vietorisz (96).

the inputs to Puerto Rico and the outputs to the US markets were estimated. This was then compared with the best site for that complex in the USA - the best site rather than existing ones were used because it was felt that this is where the competition would be in the future. For certain complexes Puerto Rico seems to have a comparative cost advantage even when there is explicit allowance for economies of scale advantage that Puerto Rico may not be able to accommodate to as great an extent as the USA mainland.

This study is interesting in that it applies a modified form of comparative cost technique to a group of related industries, rather than considering each one separately. It is likely that one individual unit of the complex, e.g. the oil refining, may not be able to survive if only established on its own, but if established in a whole complex it becomes a viable proposition. It is also interesting to note that this is one of the few studies of complexes that has been carried out ex ante and some of the results actually implemented. Paelinck (153) reports that the oil refining and fertilisers have developed in accordance with the guideline of the study, but the synthetic fibre part has to some extent lagged behind.

Apart from some of the technical problems in the study, there are some further drawbacks that limit the applications to which the study can be put. The study requires large amounts of quantitative data on the engineering and technical aspects of production, that are generally not available, and are expensive to collect. Also the type of industry studied is particularly amenable to this type of analysis since the costs of the flows of goods between the various sectors of the complex can be relatively easily quantified. One doubts, as in the simple comparative cost studies, if this analysis could be meaningfully applied to complexes where the interrelations are less amenable to quantification in the traditional way and where the complexes are less strongly integrated in the quantity of goods that flow but the weaker relationships are nonetheless important[1].

The type of complex where these limitations are encountered most strongly is probably found in engineering and metal working sectors. These sectors have been the subject of two studies - one by the EEC (45)[2] which examined the possibility of establishing a viable complex in the under-industrialised South of Italy, and a second by Economic Consultants Ltd.[3] for the possibility "that the planned introduction of certain types of industry into one part of a region (in this case the Central Lancashire New Town) is more likely to create the conditions necessary for future regional economic growth than is the shepherding of a random group of mobile units to a wider area." (Livesey (130) page 225).

Despite the reception areas being completely different, the two reports are basically similar, both in the methodology used for the selection of complexes and in the final composition of the complex. Both studies stress the importance of inter-industry linkages in the widest sense (in that they are expected to be important locational determinants for various industries) and both stress the propulsive effect (in that they will create extra employment, stemming from the initial principal unit established in the region). Both studies use exclusion and inclusion criteria as a means of reducing the large number of potential complexes to a

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1 Isard and Schooler (95) admit these limitations.

2 Summarised in Tosco (189) and Allen and MacLennan (3).

3 My only source of reference to this study, which to my knowledge has not been published, has been through Livesey (130) who explains the methodology and summarises the results. My description of this study is, therefore, taken solely from Livesey.

final choice. These exclusion and inclusion criteria are constructed from what seem a priori reasonable premises. ~~I shall report~~ the Livesey article can be used as an illustration of this methodology. In the Livesey article various types of industrial complexes are defined and examined briefly in order to decide on which ought to be examined in greater detail. The one chosen is "industries which require the proximity of various support industries and services for which this is of major importance to the choice of a location. Such industries, subject to a complex system of technical interrelatedness, largely comprise these within the metal-fabricating sector; the majority of those require the proximity of numerous 'intermediate' activities which provide them with their production requirements of castings, forgings, tooling and so on. The intermediate activities in turn benefit from proximity to the users' plants." (Livesey op. cit. page 227).

Because the metal working sector has had a good record of growth in the past (and is expected to in the future) it is consequently chosen for further study. It is examined in detail as to the method of production (e.g. jobbing units, those producing one-off individual lots, or batch production) and the economic linkage of these types studied. The effect of geographical separation on these units is examined, and for many of these linkages 30 miles is deemed the feasible maximum separation distance, because the journey can be done there and back in a day leaving time for some work in between. Also contacts can still be maintained easily. There are also some general remarks on why these industries must stay close together, and some remarks concerning the suitability of Central Lancashire are made.

To narrow the field of enquiry certain exclusion criteria were applied:

(a) where the process was simple or indivisible so there would be little effect on ancillary services

(b) where a large minimum scale was necessary, e.g. cars

(c) where certain special sites requirements were necessary, e.g. shipbuilding

(d) where the activity was already present to a large extent in the area

(e) where poor market prospects were expected

(f) where the group was too small to give meaningful analysis.

After the exclusion criteria, preference criteria were applied to the final producers:

(a) **dividing** the remaining group into higher, average and/or lower growth potential

(b) **dividing** up into type of production, e.g. batch; and giving preference to those which were most likely to have the greatest effect on intermediate producers and jobs.

Similar exclusion and preference criteria were then applied to the intermediate producers, e.g. they were excluded if it was thought that they did not have to be in close proximity (i.e. not in Lancashire) to the units that they were supplying, and preference was given to those who would service a large number of customers rather than a few large specialised ones.

It is from these various exclusion and preference criteria that all the choices were made. To some extent these criteria seem to be based on intuition, subjective judgement and arbitary classifications. Extrapolation of growth trends is used frequently and no proper attempt is made to find the costs involved in creating the agglomerative forces. In fact, very little actual analysis, but more description, is used. However both studies, particularly the one reported by Tosco (189), do possess a wealth of detail that suggest that there are many costs in being separated from linked industries in the metals sector that cannot be adequately measured by the pure transport costs.

Some attempt has been made to try to put a more rigorous basis to these industrial complexes. These can be described as programming models. Paelinck (153) suggests an integer geometric programming technique, which through the use of the dual is able to select an optimal bunch of industries to form a viable complex. However in order to implement this technique it is necessary to know quantitatively the following data:

(a) investment and labour functions, i.e. production functions for each industry in the complex. He suggests that they could come from engineering data, although these may be expensive to collect

(b) prices and related data - these are probably available but again may be expensive to collect

(c) external economies - since this is an integral part of industrial complex analysis, these data will effect the final results to a great extent. However Paelinck does not suggest any way to quantify their important data, and as we have seen from the studies above there does not seem to be a widely accepted view of their quantification - the only concensus being that they are likely to be important.

This last point seems to make the model inapplicable despite the ingenious programming techniques demonstrated that would give optimal complexes.

Ghosh and Chakravarti (55) propose a method for the location of an industrial complex cast in linear programming terms, where the objective function can be the minimisation of production plus transport costs or minimisation of capital costs. The constraints on the model are the usual input-output accounting constraints[1]. Further restrictions can be placed on the transport sectors' and other sectors' capacities. Additional constraints such as a minimum viable size of output and various other refinements are presented. The problem again with this technique is that the objective function contains either simple transport costs (in which case these do not adequately measure separation costs), or if these were replaced by true distance separation costs they would be unquantifiable.[2]

The idea of industrial complexes is often associated with Hirschman's big push or the concept of growth poles, because these concepts encompass the dynamic effects of the backward and forward linkages. Hirschman's big push has been given little location significance, and so comments will be restricted to growth poles with their implicit determination of location. However in a review of the subject Darwent (36) concluded that the literature was not very fruitful because the concepts are loosely formulated in normative statements, with little rigorous analysis and relying on intuitive appeal. Darwent claims that the literature abounds in such statements as "it is better to concentrate investment in centres than scatter it around" or "bigger centres will be better than smaller ones in the amount of growth produced from a given level of investment." The basic idea stems from Perroux who says that a "propulsive industry" will stimulate others and cause economic growth to take place in growth poles. However, unless the theory is formulated more specifically and rigorously it seems little

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1 For a more detailed discussion of Ghosh and Chakravarki (55) see Appendix.I.

2 A further problem with this technique is that all the functions specified must be linear But this problem will be found again in connection with attraction theory (see Ch.III) where the functions must be assumed linear if any empirical results are to be obtained.

better than a tautology and can offer little as a development tool. Consequently this idea will not be developed any further.

From the discussion of industrial complex analysis there seems to be little to offer as a government policy aid, since most of the studies have certain methodological weakness and are often only appropriate for one specific case study, or often require expensive data collection or even data that are impossible to collect.

(iv) Studies suggesting various criteria by which to judge industrial location decisions

In some forms of analysis, various criteria are put forward by which to judge the locational characteristics of an industry. These can be classified as the ranking criteria and the cross-hauling criteria.

(a) Ranking Criteria

This criterion involves giving various ranks of scores to various locations and various industries, to reflect their characteristics and needs, and then to try to match them up.

Lowenstein (131) proposes a model for locating manufacturing industry in various parts of an urban area, although presumably the technique could be applied on an inter-regional scale. For each industry to be examined a score or weight for each possible locational factor is given. For example, if we were using a scale of 0 to 3, where 3 represents very strong preference and 0 very weak preference for a factor, then the textile industry may score 3 on the cheap female labour characteristic, but zero on the availability of raw materials. This would represent the surveyor's judgement that it was vitally important for the textile industry to be located near cheap labour, but that location of the raw material (say cotton) in the same area was of no consequence. Various scores or weights are then apportioned to the various possible receiving locations for each factor. Thus location A will obtain a high score for cheap female labour if that is present in location A. An iterative procedure is then undertaken so as to minimize the difference between what an industry requires of an area for all locational characteristics, and what that area possesses in terms of these characteristics. Thus industries can be allocated to various locations.

There seem to be two major objections to this method. In the first place the rankings are purely arbitary and depend very much on the subjective judgement of the person undertaking the study. If these scores are altered slightly it is possible that completely different results would be forthcoming. To obtain detailed knowledge of the scores in an objective way, i.e. to ascertain the relative costs of all the various factors would be a difficult task, and, as we have seen above, in comparative cost analysis there are numerous conceptual and practical difficulties of collecting these data - most of which would be extremely difficult to quantify. The second objection is that the model allocates industries individually, but as we have seen above, the locational decisions of one type of industry may be influenced by the locational decisions of another. Thus the system must be treated simultaneously and not individually. The theoretical problems found when locational decisions are interdependent will be discussed elsewhere.

A similar type of ranking weights and scores model is demonstrated by Guigou (69). It is an extension of a model by Ponsard[1]. Ponsard's model brought graph theory and associated algorithms to bear on the location problem. He does this by showing a graph of all possible locations, with the costs of movement where transportation is possible, along each branch of a graph. In each location the material and labour inputs available and the demand for final products is known. For each industry to be located in the model the requirements of inputs are obtained. The algorithm then tries to maximise the profits of the individual firm by assigning it to a particular location and by working out the cheapest (or shortest) route through the graph for the inputs and outputs from and to the nearest points at which they are supplied and demanded. This is repeated for all locations until a global optimum is found.

Guigou modifies this model, and in its new form is called ELECTRA (Elimination and Choice Translating Reality Model). Instead of being unicriterional (as

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1 For the details and reference to Ponsard's model see Guigou (op. cit.)

the Ponsard model is in terms of profits), this model can take into account multicriteria, i.e. it is able to handle many more important factors that may influence location but cannot be quantified in the traditional way. Examples of the factors that can be taken into account are the quality of the environment, labour relations etc. This is achieved in a similar manner to the Loewenstein model reported above, by assigning arbitary weights to locations concerning these characteristics and by assigning arbitary weights to the strength of requirements of these needs for each industry. The graph theory and assorted algorithms are then used to calculate the costs and profits of each criterion at each location. Unless the trivial case arises that one location is better on all criteria for one industry, the final choice of location for an industry can only be made by assigning further arbitary weights to reflect the strength of each criterion and so allowing the comparison to be made. The method then becomes implicitly a unicriterion one, so contradicting the author's claim. Despite being couched in terms of intricate mathematical terminology, this method does not live up to the author's claim of "a complete renewal of the theories of optimal location." (Guigou op. cit. page 314). The criticisms raised against the Lowenstein model can be made against this one, namely that the weights are arbitary and subjective, and that it ignores the importance of interindustry relations and the possibility of a simultaneous solution for all industries. As we have seen from earlier studies cited in the sections above, these may be of vital importance in the optimal location of industry.

(b) Cross-hauling Criteria

This is a criterm used in two separate studies both of which claim to be analysing the potential footlooseness of an industry - both studies start from the basis that if a good is both exported and imported from each region, i.e. there is considerable cross-hauling, then this would seem to be a good indication that the industry in question is footloose.

Farness (52) applies this technique to a large number of industries. The amount of cross-hauling is measured by comparing the proportion of national output produced in an area and the proportion of that area's output sold in the various regional markets. For example, if an area accounted for 20% of national output and if the pull of final demand were zero, then one would expect that area to have 20% of the market in each region. Although none will fulfill this ideal some will come close. The industries are deemed to be footloose as far as demand is concerned.

The inputs are then subjected to a criterion of the rank of importance of transport costs, to see if the industries are footloose on the supply side. Since most inputs are transportable, it is possible to see what would happen to the total transport cost bill if all inputs had to add 1,000 miles on to their journey. This cost is calculated for each industry for each input, and a ranking of penalty points is obtained - the ones with low ranking penalty points being deemed footloose as far as supply is concerned.

A very similar type of analysis was applied by Spiegelman (171) to the precision instrument industry. He showed that cross-hauling of final demand was very extensive, and firms regardless of their location, tend to sell all over the country. The inputs side was investigated and surveys concerning transport costs conducted. However these costs were found to be negligible, since most of the inputs were skilled labour and very small high value intermediate products. On both these criteria the precision instruments industry was consistently ranked as footloose, but there was significant clustering in this industry, which lead the author to suspect that other factors are working. A further survey was carried out asking firms to rank important locational factors - apart from personal reasons (a factor reported in most surveys) the availability of skilled and professional staff seemed to be consistently ranked high. In an attempt to test the results of the survey some bivariate regressions were carried out with the size of the industry against factors such as the distribution of engineering students or the distribution of skilled labour. However none of these appeared to be significant. So it was concluded that the clustering was due to a combination of factors at the site, and historical and inertia forces.

Similar problems arise in both the above reported studies, in that transport costs, when they are used, do not reflect the true cost of distance separation. There may also be significant locational ties of immobile resources that are impossible to pick up by analysing cross-hauling, as was encountered in the case of precision instruments. Again, neither study is capable of handling industries simultaneously, and without using arbitary weighting there would be no way of determining the relative mobility of an industry that ranked high on the cross-hauling criteria as far as demand was concerned, but low as far as supply is concerned. This last problem will occur in all cases except where one industry ranks better on both tests than the ones with which it is being compared. Because of the many objections raised to both the ranking criteria and the cross-hauling criteria, there seems little application of these methods to aid government policy making.

(v) Econometric Studies

The term econometric is here taken to mean any statistical analysis that has been used to test the locational characteristics of an industry. A discussion of the validity of the methodology used will be left until after the studies have been presented. For ease of presentation these studies can be subdivided into single equation models and simultaneous systems.

(a) Single equation models

Single equation models have been a very popular technique in trying to explain the growth, the output or the number of moves in an industry or group of industries, by reference to a number of independent variables.

The model is expressed in the classical regression form:

$$_iY = \beta_0 + \beta_1 \, _iX_1 + \beta_2 \, _iX_2 + \dots + \beta_n \, _iX_n + \, _iU$$

where $_iY$ = the ith observation on the dependent variable to be explained

$_iX_1 \dots {_iX_n}$ = the ith observation of the independent variables used to explain the dependent variable

$\beta_0 \dots \beta_n$ = parameters to be estimated

$_iU$ = error term,

and where observations are taken for each i, which may be from time-series and/or cross section data depending on the specific problem in hand.

The independent variables that are chosen vary with the problem in hand, but seem to depend very much on the ingenuity and resources of the author. Some of the studies such as Townroe (190) try a priori to rationalise the choice of the independent variables and to predict the signs of the coefficients. For example, Townroe in trying to explain the total number of moves[1] by industry as described in the Howard report (85), uses the following independent variables:-

(i) rate of growth of the industry in terms of output and employment

(ii) average size of establishment in the industry

(iii) percentage of female labour employed

(iv) percentage of transport costs in total costs

(v) capital expenditure

(vi) labour intensity of production

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1 Measured both in terms of number of moves and in terms of total number of jobs involved.

(vii) floor space availability variables

(viii) a proxy for ties and linkages

(ix) rates

(x) degree of concentration.

It can be seen that all the above variables can be rationalised either by showing that a priori there should intuitively be a causal mechanism between the independent variable and the dependent variable or that the independent variables are proxies for other variables, which are not available, but if they were, would intuitively have a causal mechanism. For example, referring to Luttrell's findings (133), much mobile industry seems to be searching for pockets of female labour, and thus one would expect independent variable (iii) to be positively correlated with the number of moves. In the analysis, only the first four independent variables were of the correct sign and statistically significant using a t-test. Independent variable (i) had a positive coefficient, since one would expect faster growing industries to generate more moves. Independent variable (ii) had a negative coefficient since large plants are more able to absorb changes within the existing plant than smaller plants. Independent variable (iii) was positive, as explained, and independent variable (iv) had a negative coefficient since industries where transport costs are high are likely to be less mobile[1].

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1 For other multiple regression studies trying to explain Howard's results see Keeble (109) (111), Sant (162) or Beacham and Osborn (9).

A similar study was carried out by Fuchs (53) where he tries to explain the "comparative gain (by region) adjusted for industrial structure" for manufacturing industry in the U.S.A. He uses the following independent variables:-

(i) consumer demand
(ii) raw materials
(iii) taxes
(iv) foreign trade shifts
(v) federal government dispersal policy
(vi) wage levels
(vii) extent of unionisation
(viii) space (land)
(ix) climate
(x) initial economic activity.

A summary and criticism of Fuchs can be found in Burrows et. al. (22), Ch. 2.

Two other studies in a similar vein are one by Keeble and Hauser (112) which tries to explain the movement of industry in S.E. England measured in 18 different series by reference to 45 independent variables. The second is by Thompson and Mattila (183) who try to explain the absolute growth of employment in each industry across states in the U.S.A. by reference to 60 independent variables. These variables are first reduced in number by undertaking some correlation analysis, and discarding all the uncorrelated ones before undertaking multiple regression.

A discussion of the general problems involved in the above methodology will be found later, but one remark must be made here regarding some of the above reported studies. The single equation system of estimation has been heavily criticised by Burrows et. al (22) and originally by Heitman (77), and is especially relevant to Fuchs and Thompson and Mattila. Both Burrows et. al. and Heitman claim that the independent variables are not independent at all, but are really endogeneous and form part of a simultaneous sytem. For example, both the above criticised studies found that high wage levels were an unattractive factor to industry, and growth took place more rapidly in low wage districts. However, the wage rate in a state (or region) will be partly determined by the industrial expansion in that area, since this will represent an increase in demand for labour. Thus the wage rate is not exogenous but should be endogenously formed within a simultaneous sytem. If a simultaneous system is assumed to be a single equation model and estimated by ordinary least squares there will be bias in the estimated coefficients[1].

b. Simultaneous Systems

Because of the objection to single equation models cited above, both Heitman (77) and Burrows et. al. (22) propose simultaneous models. Heitman also criticises Thompson and Mattila (183) for failure to give any theoretical base to their model. He therefore suggests the following form as a model of the footwear industry:-

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[1] For a discussion of this problem see, for example, Christ (32).

$$Y_1 = F_1(Y_2, Y_3, Z_2, Z_3, Z_5, Z_6, U_1) \qquad \text{I.C.1}$$

$$Y_2 = F_2(Y_1, Y_3, Z_2, Z_4, U_2) \qquad \text{I.C.2}$$

$$Y_3 = F_3(Y_1, Y_2, Z_1, Z_2, Z_4, U_3) \qquad \text{I.C.3}$$

where

Y_1 = percentage change in output in footwear

Y_2 = average wage rate in footwear

Y_3 = percentage of footwear workers unionised

Z_1 = percentage of total labour force unionised

Z_2 = number of footwear establishments

Z_3 = percentage change in income in the region

Z_4 = percentage of total labour force unemployed

Z_5 = transportation costs

Z_6 = percentage change in population

and where as with conventional notation the Y's are endogeneous variables and the Z's are exogeneous. Inclusion of each of the variables was justified on a priori grounds. Unfortunately no data was available for Y_3, so equation I.C.3 was eliminated, and Z_1 substituted for Y_3 in equation I.C.1 and I.C.2.

The model was then cast in a linear form (since it seemed intuitively appealing and was convenient for estimation purposes), and the parameters of the simultaneous model estimated in the usual way. In the process one of the equations was overidentified, but fortunately two of the exogeneous variables were found to be insignificant and so dropped out, leaving the equation exactly identified.

The final results were:-

$$Y_1 = -1.37Y_2 - 0.53Z_1 + 0.03Z_2 + 2.38Z_3 \qquad \text{I.C.4}$$

$$Y_2 = 0.70Y_1 + 0.68Z_1 + 0.06Z_2 - 3.60Z_4 \qquad \text{I.C.5}$$

where all the insignificant variables have been dropped. All the above variables were significant and of the correct sign and can be rationalised quite easily. Z_2 and Z_3 are perhaps not so obvious. Z_2 picks up any external economies present in the footwear industry, and Heitman explains Z_3 as a proxy for the general performance of the regional economy and postulated a priori that the better the region performs the more the shoe industry will grow. However, this, ~~to my mind,~~ could be due to three reasons:

(i) growing strength of local demand

(ii) the local economy grows because of the increase in labour productivity and the shoe industry picks up some of this extra efficiency of labour

(iii) growth of agglomeration economies.

The fact that Heitman found transportation costs to be insignificant may seem to contradict reason (i) above, but it is possible that although pure transport costs are not a factor, it may be necessary to have close contacts with customers, particularly if fashions are changing rapidly.

A much more ambitious project was undertaken by Burrows et. al. (22) who used 1950 socioeconomic data to explain 1960 output for each industry at county level (in the U.S.A.), then using the parameters so estimated, they had to predict 1970 county output for each industry using 1960 socioeconomic data.

The model was postulated as follows:

$$E^{i\,j}_{1,t} = \phi^{i\,j}_{1}\left(\underline{Q}^{i}_{t}, P^{i}_{t}, V^{i}_{t-1}\right) \text{ industry } j \text{ county } i \qquad \text{I.C.6}$$

$$E^{i\,j}_{2,t} = \phi^{i\,j}_{2}\left(\underline{Q}^{i}_{t}, P^{i}_{t}, E^{i\,j}_{2\,t-1}, V^{i}_{t-1}\right) \qquad \text{I.C.7}$$

$$E^{i\,j}_{t} = E^{i\,j}_{1\,t} + E^{i\,j}_{2\,t} \qquad \text{I.C.8}$$

$$P^{i}_{t} = \phi_{1}\left(\underline{E}^{i}_{t-1}, \underline{Q}^{i}_{t-1} \ldots \underline{E}^{i}_{t-s}, \underline{Q}^{i}_{t-s}\right) \qquad \text{I.C.9}$$

$$V^{i}_{t-1} = \phi_{2}\left(\underline{E}^{i}_{t-1}, \ldots, \underline{E}^{i}_{t-n}\right) \qquad \text{I.C.10}$$

$$Q^{i}_{1,t} = \phi_{3}\left(\underline{Q}^{i}_{t-1}, V^{i}_{t-1}\right) \qquad \text{I.C.11}$$

$$Q^{i}_{m,t} = \phi_{4}\left(\underline{Q}^{i}_{t-1}, V^{i}_{t-1}\right) \qquad \text{I.C.12}$$

where $E^{i\,j}_{1,t}$ = employment in period t for county i industry j in new establishments not present in period t-1

$E^{i\,j}_{2,t}$ = employment in period t for county i industry j in previously established firms

P^{i}_{t} = labour force (or 'size') variable for county i in period t

V^{i}_{t} = industrial structure indicator for county i in period t

$\underline{E}^{i}_{t}$ $E^{i\,1}_{t}, \ldots\ldots E^{i\,f}_{t}$ = an F dimensional vector of employment rates by industry

$\underline{Q}^{i}_{t}$ = $Q^{i}_{1,t} \ldots\ldots Q^{i}_{m,t}$ = an m-dimensional vector of socioeconomic characteristics of county i in period t.

This exact specification was used so that the system would be recursive and consequently avoid simultaneity problems. The authors admit that by this specification they probably ignore some of the simultaneous relationships that should undoubtedly be present. Because of problems such as lack of data[1] they simplify the approach by employing the following "partially reduced-form" equation

$$E^{ij}_{t} = h^{j}\ Q^{i}_{t-1},\ P^{i}_{t},\ E^{ij}_{t-1} \quad j = 1 \ldots F \qquad \text{I.C.13}$$

In this procedure there are a number of statistical problems, such as autocorrelation of the error terms, and a number of theoretical problems, such as simultaneity, and these are discussed by the authors in Chs. 4 and 6. In the actual estimation procedure, various changes are made for resource based industries, for those dependent on local demand and other special caaes. Altogether 168 independent variables were tried in an attempt to explain the dependent variable! The criterion by which the socioeconomic variables were included was "solely whether the inclusion of a variable significantly reduced the standard error of the estimate. The reason for adopting this procedure was that the purpose of our equation was primarily that of prediction and for prediction one needs only worry about measuring the correlations of the independent variables with the dependent variables." (op. cit. page 58). The total estimates of employment for 1970 for each industry were made consistent with Almon's (4) input-output estimates for the national economy. Despite this care, some results were nonsensical, such as the estimate for Buena Vista City Virginia which had 0 employment in industry 17 (other transport) in 1960, but was predicted as having 481,012 in 1970!

In addition to the problems mentioned in each individual study, there are some serious methodological problems, both with the single equation models and the simultaneous systems. One objection can be made on purely statistical

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1 For other problems see Burrows et. al. (22) page 24.

grounds, that if 100 different independent variables are used to explain a dependent variable, one would expect 5 of the independent variables to be significant at the 5% level, but since most economic series are, to some extent correlated[1], it is likely that this figure will be greater. When one considers that Burrows et. al. used 168 independent variables and on average retained less than 10 of these in the final equations, it is not surprising that they were significant statistically.

A further objection is that correlation does not imply causation, and as Burrows et. al. states (op. cit. pages 58 - 9) "many of the variables retained had coefficients whose signs were contrary to expectations. As all retained coefficients were statistically significant, the explanation for incorrect signs is probably that the variables in question are proxies for other variables not included as a result of lack of data. While this fact does not affect the accuracy of the model's prediction if the relationships among the variables remain the same in future time periods, it does impair our ability to make inferences about the true structure in general and about causal relationships in particular. For example, the variable 'proportion of labour force employed in mining' in the equations in which it appeared was almost invariably negatively correlated with employment in non-mining activities. One cannot however, jump to the policy conclusion that the mining activity should be discouraged,

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1 Even to some extent in cross-section data.

as the reason for the negative correlation is undoubtedly that mining-intensive counties in general have other characteristics which are detrimental to economic activity. Reducing the level of mining employment would probably harm these counties as such a reduction would probably not have an impact on the socioeconomic characteristics which are the real cause of economic inactivity."

I have quoted this at length because it highlights the grave problems that exist in the use of this methodology. Estimation without knowledge of the underlying structures of the model, but merely estimating a hypothesised reduced form, has certain dangers. The negative correlation in mining activity with other activities can be seen to be a result of other mechanisms, which are relatively obvious. One could suggest, guessing at the underlying structure, that if mining activity declined, other activities, particularly local service activities, would decline too as a result of a reduction in factor incomes, and vice versa. This would give a positive correlation between the activities. There is no reason to suggest that mining activity will always be a perfect proxy for the other 'socioeconomic characteristics' that are negatively related to other activities, and without knowledge of the underlying structures, prediction can be difficult, as the above example concerning factor incomes shows. The mining example is relatively easy to understand and so there is little chance of making a wrong policy decision to discourage mining so as to increase other activity. But this is not to say that a similar mistake could not happen with other variables where the structure is not so obvious.

This type of problem is discussed in more general terms by Streissler (174) and Koopmans (116). Besides a wide discussion on the general problems in econometrics, Streissler stresses the dangers of the above methodology. He uses the analogy that the birth-rate and the number of storks in Austria were highly correlated, so providing supporting evidence for one particular theory of reproduction. However, recently the number of storks has continued to decline, but the birth-rate is now increasing. Could this be explained by the increasing productivity of storks, and a dummy variable or proxy series be introduced into the estimation to allow for this! The point he makes is that if one searches long enough one could find very good correlations. This is especially dangerous when reduced form equations are estimated directly without any knowledge of the underlying structures, and a computer is used to pick the statistically significant variables (Streissler op. cit. page 34 - 5). He also takes exception to what he describes as the Friedman view that "it is enough to find a good equation; don't bother what it means. Once an equation has proved itself it can be used without hesitation; and the proof of the equation is in its fit." (op. cit. page 39). However he contends "this position would, however, only be correct if economic and social systems were basically stationary stochastic processes" but this is certainly not true. Exception was also made to the tendency to linearise economic models just because it is convenient (op. cit. page 47). But he particularly draws attention to the dangers inherent in simultaneous models (op. cit. page 71 - 2) which must be specified correctly since, as he says "I like to put this problem in a nutshell.simultaneous econometric models do not only require precision statistics - these are already required for single equation regression models - they require precision theories as well. The errors due to misspecification again become more troublesome in forecast of continuance. For here the wrong parameter is likely to be magnified as it is multiplied by an ever-larger value of the variable."

A similar type of discussion can be found in Koopmans (119), and although this is specifically concerned with business cycles, Koopmans does stress the need for a proper specification of the structure before statistical testing can take place.

The criticism made against the above methods of research into locational determinants, do lay the usefulness of the results open to serious doubt. This is not to say that econometric tools are not useful in this location field, but that the theories need to be determined, and the structures rigorously formed. Without this, it is doubtful if much reliance can be placed on the results, that claim to explain the locational determinants and mobility of an industry. However, despite these serious reservations, it is interesting to note that in nearly all the studies reported above all the industries studied were adequately[1] 'explained' by the "independent" variables that were employed, implying that none of the industries are footloose. In the studi that dealt with the movement of manufacturing industry in

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1 Measured in terms of R^2

the U.K., again economic variables and government controls seemed to 'explain' the dependent variable. This would suggest that industry is not as mobile as implied in the Chataway statement.

Postscript to econometric studies

It may seem surprising that no mention has been made of any of the larger number of regional models or simulations that have been carried out recently.[1] Although these studies may be admirable for the purposes for which they are designed, they are of little use for the problem we are trying to tackle. This can be seen from the methods that they use to handle the production sectors of the model. These approaches have the following characteristics:

(i) they are very highly aggregated in the traditional Keynesian manner

(ii) they adopt a very simple view of location, such as the economic base theory, which hypothesises that certain industries produce solely for the local market and their size is determined by the basic sector, which depends solely on the stimulation of demand from outside the local market area (i.e. exports)[2]. It is hoped to show that economic base studies are a very special case of attraction theory. Also in many of the econometric studies carried out, using base theory for the productive sectors of the model, they often make further simplifying assumptions that the basic sector will maintain the same proportion of national output over time

(iii) where high levels of aggregation and base theory are not used, a regional input-output table is often plugged into the econometric model in order to obtain estimates for the individual industries. Again it is hoped to show that this is just another special case of attraction theory, in that it assumes that

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1 For a survey and review of seven large-scale regional forecasting models see Milliman (140). For a very lucidly presented econometric model of Ontario and review of six other regional econometric models see Haronitis (73).

2 There are many practical and theoretical difficulties in conducting economic base studies. For a discussion see Richardson (158).

industrial growth and therefore location is dependent on external demand[1]. Whilst this may be true of a very small open region, the larger the region becomes, the less likely is this assumption to hold, since the region will start to develop internal dynamics of growth[2].

This section is not intended to be a criticism of regional econometric models as such but merely to suggest that the sub-sections dealing with location decisions have been too simplified to be of use to government policy in the context that we are discussing. To look forward somewhat, it may be suggested that perhaps attraction theory could be plugged into regional econometric models in a similar way that regional input-output tables have been in the past.

(vi) Other Methods

(a) Area Studies

The locational characteristics of certain areas are often studied in an attempt to gain some insight into the determination of industrial location. (Surveys such as those done by Keeble on N-W London and Cameron and Clark on Scotland are examples of one type of this study - these have been reviewed above.) Usually the views expressed tend to be very subjective, and built up as a result of first hand information and long association with the area in question.

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1 Thus ignoring any supply effects - these have been postulated by various authors (see above) to be of critical importance to certain industries.

2 Similarly Richardson (158) has also criticised regional econometric models for relying solely on external demand to generate growth, and thus implying an over simple view of location.

Lichtenberg (128) has completed such a study of the New York area. The method of analysis is highly eclectic "we have placed some weight on our reading of history, some on our more precise measurement of recent trends in location, some on the analysis of colleagues in companion volumes dealing with such factors as labour and transportation, and some on the industry case studies made by still other participants in the New York Metropolitan Region Study". (op. cit. page 37) For most of the industries found in New York, Lichtenberg places great emphasis on the external economies available in the area, both of a general and sectoral nature to use Tosco's terminology. The apparel industry is given as a good example because of the flexibility needed in the production process. Lichtenberg states "the technical requirements of this production are such that producers must be able to cater to an ever-changing demand [in the sense of fashion changes as well as volume] by speedily tooling up (or rapidly drawing upon an established network of subcontractors and suppliers) and turning out their product in a short period." (op. cit. page 58). The need for rapid change in this industry means that stocks have to be kept low, labour hired and fired rapidly, extra rented space taken on and dropped at short notice etc. There needs to be access to other industrial services such as finance, designers and advertisers. These facilities can only be provided when the industry is highly concentrated so that the fluctuation in demand of all the various individual firms means that on average the suppliers of these ancillary products and services face a relatively steady demand. Also, many of these services can only be provided where the market is large - for example, an individual firm may be a poor risk for a capital loan, but if the financial source lends to many in the industry the risk will be minimised. Again, for many of the transactions described above, it is necessary to have face-to-face contact, which again requires the industries to be conglomerated. It is probably the need for flexibility to meet rapid change that requires industrial conglomeration, because if the product became standardised and long production runs could be undertaken, then the apparel industry would probably move out of New York in the search for cheaper labour sites, since the necessity for external economies would be gone.

A similar conclusion about the importance of external economies in the New York economy is reached by Chinitz (31) when he compares it with Pittsburgh. The latter lacks the industrial structure to encourage the growth of ancillary products and industrial services, so there is little incentive for new industry to establish in Pittsburgh. Townroe (191) suggests that similar advantages can be found in London for the furniture trade and in the W. Midlands conurbation for the metal trades, both of which "benefit from access to a common pool of labour with specialised skills from the availability of specialised services, or from the concentration of buyers" (sectoral external economies) - and which also benefit from using "common facilities of commerce and banking, of technical services, of education, of subcontracting and a wide range of adaptable skilled labour" (general external economies)[1].

The weakness of these studies is that they lack any analytical base and tend to rely on judgement and qualitative statements. They do, however, confirm our suspicions that industry may not be as footloose as once regarded.

(b) Individual Industry Studies

In discussing techniques to evaluate the mobility of industry, many individual industry studies have been mentioned. Some of these studies have no discernable methodology. Often they present a description of the location of the industry under study in map form, or give views on what are thought to be important locational requirements of the industry. Although these views may have an intuitive appeal, they are not backed up by any analysis or rigorous testing with hard data.

The industries studied in these loose descriptive terms cover a wide field - for example cars by Hurley (88), glass by Bain (7), pulp and paper by Hunter (87) and iron and steel by Alexandersson (2). These studies may have some value in that they may point out factors that should be incorporated into more rigorous theoretical developments - foe example Hurley mentions the importance of external economies of supplies for cars, Bain stresses the importance

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1 Further references to area studies can be found in Brackett and Stevens (17).

of labour without any traditional union prejudices of the older glass-making areas, and Hunter stresses the length of time that it takes an industry to adjust in the face of changing technology, which alters the source of supply of raw materials in the pulp and paper industry. However, Stevens and Brackett (17) in discussing individual industry studies in general state "despite the number of industry studies, there is a distinct lack of definitive research. Many of the conclusions about cause and effect are intuitive and go well beyond the evidence presented by existing data" (op. cit. page 13). They therefore seem to be of little use to the problem in hand.

I.D. Conclusions

From the studies that have been discussed there seems to be a lack of any adequately formulated practical models, that can be used by a government in a location of industry policy[1]. They tend to be either impractical or of very limited use or have serious theoretical drawbacks in their formulation. Therefore it seems necessary to look at the more general and more rigorously formulated theoretical work that has been done on industrial plant location and optimal spatial patterns, in an attempt to obtain some guidance for a government location policy. However in all the studies reported, some explanation of the locational characteristics of each industry has been put forward, and in almost all cases the industry was found to be explained by these characteristics. In very few cases was it concluded that industry was as mobile as Chataways's statement implies.

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[1] Consider also Spiegelman's (170) conclusion after reviewing location analysis "In general, existing industry location studies fail to provide meaningful statistical testing of hypotheses on location-determining factors.... Mostly, locational conclusions were based on examples and assertions....There is a great need for more profound work in the establishment and testing of hypotheses as to the causes of locational patterns and the reasons for changes in this pattern." (page 25)

CHAPTER II

A Review of Some Theoretical Location Studies

II.A Introduction

In the previous chapter an attempt was made to review some pragmatic approaches to location problem. It was seen that many of these works lacked a rigorous theoretical base and that little reliance could be put on the results. It was suggested that their use lay in an ad hoc description of factors of location that may influence the location decisions of economic activities, and therefore these factors may be worth studying in a more rigorous way.

It is necessary to examine the theoretical works on location for a number of reasons:

(i) ideally we should like a general theory with general functional relationships to be empirically testable and we must see if any work has been developed so far that will fulfill these conditions. It will be shown that there are no such theories in a spatial context,

(ii) the study will hopefully highlight the problems of deriving general results and show that to obtain any results it is often necessary to make specific assumptions about the functional relationships, or to make exogeneous, certain variables that ideally should be formed endogeneously in the model. This may help to explain why certain assumptions are necessary in attraction theory in order to obtain a consistent and testable model.

(iii) It will be shown in Chapter III that one of the basic assumptions necessary for attraction to be consistent is that industry is optimally distributed over the country and that this, in the context of the United Kingdom, must be done largely by a decentralised decision-making process based upon profit maximisation. This problem has been the subject of many studies and it will be shown what conditions are necessary to arrive at such an optimum, thus bringing more to light the necessary conditions and assumptions of attraction theory.

To review the whole of location theory will be a monumental task and most of it would not be very rewarding for the purposes of evaluating attraction theory. Some drastic pruning of the subject is therefore necessary. The general spatial equilibrium models[1] will be ignored since they have little hope of empirical application. The study of equilibrium in spatially separated markets - as originally formulated by Cournot[2] - will not be discussed since they require information that will not be generally available and so have little to contribute to attraction theory. The classical works of Von Thünen, Weber and Losch, do have bearing on attraction theory but they will not be studied here because they have been extensively studied elsewhere[3]

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1 That is the general equilibrium models of Walras, and Arrow and Debreu (6) placed in a spatial context - see for example Kuenne (122) or Takayama and Judge (181) Chapter 15 and 16.

2 These problems have been extended and solved by Samuelson (161) and Takayama and Judge (181) Chapter 12 - 14.

3 See in particular Been (11), Isard (89) or Von Boventer (16) or for Von Thünen in particular see Hartwick (75) or Alonso (5)

and some of the more recent studies reviewed here take many of their basic approaches from the classics and apply more sophisticated techniques.

The type of works that I shall discuss can be divided into two distinct categories:

(i) the first approach is to treat space as a continuous variable in either one (a straight line) or two (a flat plain) dimensional space. This is in the tradition of Von Thunen, Losch and Christaller, and the approach has been adopted by Bos (13), Serck-Hanssen (165) and Tinbergen (185) whose work will be examined. In all these studies there are common underlying features -

(a) there are usually economies of scale in production that give rise to a tendency for production to take place in the same spot. This may be replaced by or added to the assumption that industries need the output of other industries in order to produce, so again there is a tendency to conglomerate because of (b),

(b) resources are consumed in transferring products over space - these are usually termed transport costs[1],

(c) certain activities consume space as an input - such as agriculture and forestry - and these supply inputs to and/or demand output from the

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[1] Although of course the costs may include much more than transport costs (e.g. communication costs)

non-space consuming activities. This force tends to decentralise production.

Some of these models and the results obtained will be discussed but there are some points that I consider to be germane to this type of study:

(a) general results are very difficult to obtain and in order to obtain any results very specific assumptions must be made for example concerning the production function or the hierarchy of trading patterns,

(b) because of the simplifying assumptions used, the results would suggest that as transport costs decrease and the agricultural sector becomes of only minor importance as a supplier of inputs and demander of products, then all non-space consuming activity should be concentrated on one spot[1]. However there does seem to be evidence to suggest that urban diseconomies from both the consumer and producers point of view will become a major force despite perhaps continued economies of scale in production and the external advantages of being close to other producers[2]. To allow for this force would probably mean that no general results would be possible despite many other simplifying assumptions.

(ii) The second approach is to treat space as a series of discontinuous points between which commodities are transported but where they can only locate at a number of pre-determined

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1 Because of the economies of scale in production and the dependence on the output of other industries as an input.

2 For a short bibliography to some of the literature on urban diseconomies see Spiegelman (170).

points. Again a number of points are common to most of these studies:

(a) transport costs are incurred in transferring resources over space,

(b) economies of scale or indivisibilities are present

(c) point (b) is either replaced by or added to the fact that there are constraints on the system such as constraints on the capacity of production at each location, constraints on primary inputs (e.g. labour) or intermediate input (e.g. mineral deposit) at each location, constraints of a certain minimum final demand to be met at each location.

Thus there are two conflicting forces - points (a) and (c) giving a tendency towards dispersion of economic activity, and point (b) giving a tendency towards concentration of economic activity. When point (b) is dropped and output at each location is given, the problem becomes one of simply determining the optimum flows of goods between the producing and receiving locations subject to the particular constraints of the problem.

The framework of discontinuous space described above readily lends itself to programming analysis and it was in this context that linear programming was developed. However with programming no general results are possible in that the solution depends on the parameters fed into the model, which of course implies that we know the parameters. However this type of approach is often used to determine whether the market will give an optimum solution, or give any stable solution at all. Some further interesting results can be derived from this approach. These will be discussed. The integration of attraction theory and the studies to be discussed in this chapter will be attempted after the attraction model has been discussed in Chapter III. However, as mentioned above in the reasons for studying theoretical works on location, the main areas of comparison between attraction theory and other location theory will be:

(i) the ability of a decentralised decision-making process to sustain an optimal distribution of economic activity

(ii) the generality of the assumptions and functional relationships adopted

(iii) the degree to which the model is a partial model in taking endogeneous variables as exogeneous.

II.B Discontinuous Space

One of the earliest applications of activity analysis was to transportation problems involving discontinuous space[1]. In these problems an efficient transportation system could be devised. This type of analysis was extended[2] to include both the producing and transportation activities. When the dual of this programming analysis and the Kuhn-Tucker conditions for an optimal solution are employed it is possible to see implicitly marginal analysis in the interpretation of the results, which can be given concrete economic meanings[3]. This type of approach

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1 See for example Koopmans (117)

2 Notable by Beckmann and Marshak (16)

3 For an introduction to the dual and Kuhn-Tucker conditions see Baumol (8)

also shows what conditions are necessary for a decentralised decision-making process (e.g. profit maximizers guided by a price system) to bring about this optimum solution.

One of Takayama and Judge's (181) (pages 65 to 73) simpler examples of activity analysis will be given to illustrate both the approach of this type of analysis and the economic interpretation of the dual and the Kuhn-Tucker conditions[1]. This example will also form the basis of the application of the Kuhn-Tucker conditions to attraction theory[2].

The problem is as follows: we are given in each of n regions a known quantity of a mobile primary or intermediate product, which by passing through a production process is converted into a final commodity. The technology is known and fixed for each region. The regional demand for this final commodity is assumed to be fixed and known and this demand must be met. Each region has some capacity for processing each type of primary or intermediate product. Each region may have a unit plant cost for converting each type of primary or intermediate commodity into a final commodity and these costs or regional cost differentials are known. The production processes are in constant proportion for all output levels and these processes may vary from region to region.

Regional producing plants are considered in the category of an immobile primary commodity. All other commodities are assumed mobile and all possible pairs of regions are separated by a transport cost which is known. It is further assumed that when the total regional supplies of primary or intermediate products are converted into a final commodity, the resulting total potential supply of the final commodity is equal to or greater than total demand for production and for consumption purposes. Each type of product is assumed homogeneous and

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1 See also Stevens (176) and Kuenne (122) for a more limited discussion.

2 See Section III.K.

thus producing firms and consumers are indifferent as to their sources of supply.

We now wish to formulate a model which can be used to ascertain the level and location of processing for primary or intermediate commodities and the volume and direction of all primary intermediate and final products flows, that will minimise the aggregate transport and production costs. We are thus seeking the allocation and pricing system which will maximise the returns to each producer and/or resource holder and minimise the cost to consumers and resource users, subject to the constraints postulated.

We use the following notation:-

r,s denote regions $r,s = 1, 2 \ldots\ldots n$

${}_rF_k$ denote the fixed minimum demand requirements for final demand commodity k in region $r, k = 1,2 \ldots\ldots k$

${}_rX_p$ denote the given quantity of a mobile primary or intermediate product p available in region r (quantity available before in shipments, outshipments and use for production of final commodity) $p = k + 1, k + 2 \ldots\ldots R$.

${}_rX_L$ denotes the given quantity of an immobile primary commodity L (for example capacity of a producing plant) in region r. $L = R + 1, R + 2 \ldots\ldots M$.

${}_{rs}x_p$ denotes the variable quantity of the mobile primary or intermediate product p, shipped from region r to s to be used in producing a final commodity.

${}_{rs}f_k$ denotes the variable quantity of a mobile final product k shipped between regions r and s.

${}_{rs}t_p$ denotes the unit transport cost for a shipment of a primary or intermediate commodity p from region r to s which is independent of volume and direction.

$_{rs}t_k$ denotes the unit transport cost for a shipment of a final commodity k from region r to s which is independent of volume and direction.

$_r\beta_{pk}$ denotes the rate (constant proportion at all output levels) at which a mobile primary or intermediate product p is converted per unit of the process into a final product k in region r.

$_r\beta_{Lk}$ denotes the rate (constant proportion at all output levels) at which immobile primary product L is converted per unit of the process into a final product k in region r.

$_rg_k$ denotes the level of production of a final product k in region r.

$_rc_k$ denotes the unit plant cost of producing a final product k in region r. These unit costs involve outlays not otherwise included and they are assumed to be independent of the scale of plant operation.

$_re_k$, $_re_L$ denote the net availability (amount remaining after production, imports, exports and use) of the commodity at each stage of production or use in region r;

Consider the case of one final good, one mobile primary or intermediate good and one immobile primary product. The problem is to maximise -

$$f(X) = -\sum_r \sum_s {}_{rs}t_k \cdot {}_{rs}f_k - \sum_r \sum_s {}_{rs}t_p \cdot {}_{rs}x_p - \sum_r {}_rc_k \cdot {}_rg_k \qquad \text{II.B.1}$$

(i.e. minimise total transport and plant costs) subject to

$${}_r e_k \equiv {}_r g_k - \sum_s {}_{rs} f_k \geqslant 0 \text{ for all } r \qquad \text{II.B.2}$$

(i.e. the amount of the final product k, region r ships to itself and to other regions is equal to or less than the amount produced by the plant in region r)

$${}_r e_p \equiv {}_r X_p - \sum_s \left({}_{rs} x_p - {}_{sr} x_p\right) - {}_r \beta_{pk} \cdot {}_r g_k \geqslant 0 \text{ for all } r \qquad \text{II.B.3}$$

(i.e. the quantity of the primary or intermediate product p used in producing the final commodity k in region r minus inshipments of the product into region r, plus outshipments of the products from region r must be equal to or less than the native availability of the product in region r)

$${}_r e_L \equiv {}_r L_j - {}_r \beta_{Lk} \cdot {}_r g_k \geqslant 0 \text{ for all } r \qquad \text{II.B.4}$$

(i.e. the quantity of the immobile primary commodity used in producing the final commodity by the plant in region r is equal to or less than capacity or native availability)

$${}_r e_k \equiv \sum_s {}_{sr} f_k - {}_r F_k \geqslant 0 \text{ for all } r \qquad \text{II.B.5}$$

(i.e. the shipment of the final commodity to region r from itself and from other regions is equal to or greater than the demand in region r.)

$${}_r g_k,\ {}_{rs} f_k,\ {}_{rs} x_p \geqslant 0 \text{ for all } r \text{ and } s \qquad \text{II.B.6}$$

(i.e. all the decision variables relating to the producing and flow activities must be non-negative)

Following the Kuhn-Tucker conditions we will look at the necessary conditions for II.B.1 to be a maximum subject to constraints II.B.2 to II.B.6.

Form the Lagrangean

$$\phi(g,x,\lambda) = -\sum_r\sum_s {}_{rs}t_k \cdot {}_{rs}f_k - \sum_r\sum_s {}_{rs}t_p \cdot {}_{rs}x_p - \sum_r {}_rc_k \cdot {}_rg_k$$

$$+ \sum_r {}_r\lambda_1 \left({}_rg_k - \sum_s {}_{rs}f_k\right) + \sum_r {}_r\lambda_2 \left({}_rX_p - {}_r\beta_{pk} \cdot {}_rg_k - \sum_s ({}_{rs}x_p - {}_{sr}x_p)\right)$$

$$+ \sum_r {}_r\lambda_3 \left({}_rL_J - {}_r\beta_{Lk} \cdot {}_rg_k\right) + \sum_r {}_r\lambda_4 \left(\sum_s {}_{sr}f - {}_rk\right)$$

$$+ \left[\left(\sum_r\sum_s {}_r\lambda_5 \cdot {}_{sr}f_k + \sum_r {}_r\lambda_6 \cdot {}_rg_k + \sum_r\sum_s {}_r\lambda_7 \cdot {}_{rs}x_p\right)\right] \qquad \text{II.B.7}$$

and ${}_r\lambda_1, {}_r\lambda_2, {}_r\lambda_3, {}_r\lambda_4, {}_r\lambda_5, {}_r\lambda_6, {}_r\lambda_7 \geq 0$ for all rs II.B.8

For an optimum solution $(\bar{X}\ \bar{\lambda})$[1] the following necessary Kuhn-Tucker conditions must hold

a) $$\frac{\partial\bar{\phi}}{\partial_{rs}f_k} = {}_s\bar{\lambda}_4 - {}_r\bar{\lambda}_1 - {}_{rs}t_k + {}_{rs}\bar{\lambda}_5 \leq 0$$

$$\left(\frac{\partial\bar{\phi}}{\partial_{rs}f_k}\right){}_{rs}\bar{f}_k = 0 \quad \text{and} \quad {}_{rs}\bar{f}_k \geq 0$$

II.B.9

b) $$\frac{\partial\bar{\phi}}{\partial_r g_k} = {}_r\bar{\lambda}_1 - {}_r\beta_{pk}\,{}_r\bar{\lambda}_2 - {}_r\beta_{Lk}\,{}_r\bar{\lambda}_3 - {}_rc_k + {}_r\bar{\lambda}_6 \leq 0$$

$$\left(\frac{\partial\phi}{\partial_r g_k}\right){}_r\bar{g}_k = 0 \quad \text{and} \quad {}_r\bar{g}_k \geq 0$$

c) $$\frac{\partial\bar{\phi}}{\partial_{rs}x_p} = {}_s\bar{\lambda}_2 - {}_r\bar{\lambda}_2 - {}_{rs}t_p + {}_{rs}\bar{\lambda}_7 \leq 0$$

$$\left(\frac{\partial\phi}{\partial_{rs}x_p}\right){}_{rs}\bar{x}_p = 0 \quad \text{and} \quad {}_{rs}\bar{x}_p \geq 0$$

d) $$\frac{\partial\phi}{\partial_r\lambda_1} = {}_r\bar{e}_k = {}_r\bar{g}_k - \sum_s {}_{rs}f_k \geq 0$$

$$\left(\frac{\partial\phi}{\partial_r\lambda_1}\right){}_r\bar{\lambda}_1 = 0 \quad \text{and} \quad {}_r\bar{\lambda}_1 \geq 0$$

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1 A bar ($^{-}$) over a variable indicates its optimum level

II.B.9 /contd

e) $$\frac{\partial \phi}{\partial_r \lambda_2} = {}_r\bar{e}_p = {}_rX_p - {}_r\beta_{pK}\,{}_r\bar{g}_K - \sum_s \left({}_{rs}\bar{x}_p - {}_{sr}\bar{x}_p\right) \geq 0$$

$$\left(\frac{\partial \phi}{\partial_r \lambda_2}\right) {}_r\bar{\lambda}_2 = 0 \quad \text{and} \quad {}_r\bar{\lambda}_2 \geq 0$$

f) $$\frac{\partial \phi}{\partial_r \lambda_3} = {}_r\bar{e}_L = {}_rX_L - {}_r\beta_{LK}\,{}_rg_K \geq 0$$

$$\left(\frac{\partial \phi}{\partial_r \lambda_3}\right) {}_r\bar{\lambda}_3 = 0 \quad \text{and} \quad {}_r\bar{\lambda}_3 \geq 0$$

g) $$\frac{\partial \phi}{\partial_r \lambda_4} = {}_r\bar{e}_K = \sum_s {}_{sr}\bar{\xi}_K - {}_rF_K \geq 0$$

$$\left(\frac{\partial \phi}{\partial_r \lambda_4}\right) {}_r\bar{\lambda}_4 = 0 \quad \text{and} \quad {}_r\bar{\lambda}_4 \geq 0$$

h) $$\frac{\partial \phi}{\partial_{rs} \lambda_5} = {}_{rs}\bar{\xi}_K \geq 0$$

$$\left(\frac{\partial \phi}{\partial_{rs} \lambda_5}\right) {}_{rs}\bar{\lambda}_5 = 0 \quad \text{and} \quad {}_{rs}\bar{\lambda}_5 \geq 0$$

i) $$\frac{\partial \phi}{\partial_r \lambda_6} = {}_r\bar{g}_K \geq 0$$

$$\left(\frac{\partial \phi}{\partial_r \lambda_6}\right) {}_r\bar{\lambda}_6 = 0 \quad \text{and} \quad {}_r\bar{\lambda}_6 \geq 0$$

j) $$\frac{\partial \phi}{\partial_{rs} \lambda_7} = {}_{rs}\bar{x}_p \geq 0$$

$$\left(\frac{\partial \phi}{\partial_{rs} \lambda_7}\right) {}_{rs}\bar{\lambda}_7 = 0 \quad \text{and} \quad {}_{rs}\bar{\lambda}_7 \geq 0$$

For all r and s

The above results can be given a meaningful economic interpretation. We can define the λ's in the Lagrangean II.B.7 as efficiency or market prices and rents. Then conditions II.B.9a to II.B.9 j spell all the characteristics of the internal price and rent system that are consistent with an efficient production and allocation programme. In general these conditions state that in order to derive an efficient production and allocation programme, regional market prices and rents must be such that

(i) profits are zero on all production and flow processes and no process may show a positive profit (conditions II.B.9a, 9b, 9c). (This refers to the actions of arbitragers rather than rent that is tied to a location.)

(ii) Market prices of the mobile primary or intermediate and final commodity may exceed zero only if their regional net availability (${}_re_k$, ${}_re_p$ and ${}_re_L$) are equal to zero (conditions II.B.9d, 9e, 9g).

(iii) Rents or processing plants or immobile primary commodities may exceed zero only if the capacities in each case are fully utilised (condition II.B.9f).

The optimality conditions specified by II.B.9a to II.B.9j are thus seen to yield solutions for the pricing and allocation problem that are consistent with zero profit conditions of a perfectly competitive equilibrium[1].

Looking at the conditions in particular II 9a states that if any flow of the final product takes place between region r and s (that is ${}_{rs}\bar{f}_k > 0$) then because of II.B 9h ${}_{rs}\bar{\lambda}_5$ (the lagrangean counterpart of ${}_{rs}\bar{f}_k$) = 0 and $\frac{\partial \phi}{\partial_{rs}f_k} = {}_s\bar{\lambda}_4 - {}_r\bar{\lambda}_1 - {}_{rs}t_k = 0$. If we interpret ${}_s\bar{\lambda}_4$ and ${}_r\bar{\lambda}_1$ as the market prices of the final product of the demand and supply (producing) points respectively and ${}_{rs}f_k > 0$ then ${}_s\bar{\lambda}_4 - {}_r\bar{\lambda}_1 = {}_{rs}t_k$. Thus the difference in market price between demand and supply points s and r is the transport costs. Likewise when no flows take place

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[1] How this is defined can be found in Takayama and Judge (181) page 34 - 5.

$_{rs}\bar{f}_k = 0$ then $_s\bar{\lambda}_4 - {}_r\bar{\lambda}_1 \leq {}_{rs}t_k$. Because of condition II.B.9g the market price of the final product at demand location r ($_r\bar{\lambda}_4$) can only be positive when the amount of the final product shipped to region r is equal to the demand in region r. Likewise because of condition II.B.9d the market price of the final product of the producing point r ($_r\bar{\lambda}_1$) can only be positive when the sum of the amount shipped from region r to region s, for all s, is equal to the amount produced. Thus if excess supply exists of any producing point, the market supply price is zero.

Conditions II.B.9b states that of the primary or intermediate product transformed into the final product in region r, that is $_r\bar{g}_k > 0$ then because of II.B.9i $_r\bar{\lambda}_6$ (the Lagrangean counterpart of $_r\bar{g}_k$) = 0 and the market price of the product at the plant (producing) point $_r\bar{\lambda}_1$ must be equal to the value of the primary or intermediate product of the plant $_r\beta_{pk} \cdot {}_r\bar{\lambda}_2$ plus any internal rent (measure of profitability) that may accrue to the plant at that location $_r\beta_{Lk} \cdot {}_r\bar{\lambda}_3$ plus external unit producing costs $_rc_k$. Because of condition II.B.9e $_r\bar{\lambda}_2$ the opportunity cost of the primary or intermediate commodity may be positive only when the supply of the primary or intermediate product in region r is fully used. Likewise because of conditions II.B.9f plant rent $_r\bar{\lambda}_3$ may be positive only when the capacity of the plant or the available immobile primary commodity is exhausted. Thus, as before, the plant location earns no rent and the supply point is imputed no return unless capacity or availability is fully used.

Condition II.B.9c states that if the primary or intermediate product flows from region r to region s, that is $_{rs}\bar{x}_p > 0$ then because of II.B.9j $_{rs}\bar{\lambda}_7 = 0$ and the difference between the market price of the mobile primary or intermediate product in the two regions is equal to the unit transportation cost, that is ${}_s\bar{\lambda}_2 - {}_r\bar{\lambda}_2 = {}_{rs}t_p$. When no flows take place ${}_{rs}\bar{x}_p = 0$ then ${}_s\bar{\lambda}_2 - {}_r\bar{\lambda}_2 \leq {}_{rs}t_p$, as a condition parelling that for the final commodity.

For the total conditions, take the sum of $\left(\frac{\partial\phi}{\partial_{rs}f_k}\right){}_{rs}\bar{f}_k$, $\left(\frac{\partial\bar{\phi}}{\partial_r g_k}\right){}_r\bar{x}_k$

and $\left(\frac{\partial\phi}{\partial_{rs}x_p}\right){}_{rs}\bar{x}_p$ over all r and s while remembering conditions

II.B.9d to II.B.9g. This summation gives the following outlay and market price conditions:-

$$\sum_r\sum_s\left(\frac{\partial\phi}{\partial_{rs}f_k}\right){}_{rs}\bar{f}_k = \sum_r\sum_s {}_s\bar{\lambda}_4 . {}_{rs}\bar{f}_k - \sum_r\sum_s {}_r\bar{\lambda}_1 \cdot {}_{rs}\bar{f}_k - \sum_r\sum_s {}_{rs}t_k \cdot {}_{rs}\bar{f}_k = 0$$

$$\text{or} \sum_r\sum_s {}_{rs}t_k \cdot {}_{rs}\bar{f}_k = \sum_r\sum_s {}_s\bar{\lambda}_4 \cdot {}_{rs}\bar{f}_k - \sum_r\sum_s {}_r\bar{\lambda}_1 \cdot {}_{rs}\bar{f}_k \qquad \text{II.B.10}$$

$$\sum_r\left(\frac{\partial\phi}{\partial_r g_k}\right){}_r\bar{g}_k = \sum_r {}_r\bar{\lambda}_1 \cdot {}_r\bar{g}_k - \sum_r {}_r\beta_{pk} \cdot {}_r\bar{\lambda}_2 \cdot {}_r\bar{g}_k - \sum_r {}_r\beta_{Ik} \cdot {}_r\bar{\lambda}_3 \cdot {}_r\bar{g}_k$$

$$- \sum_r {}_rc_k \cdot {}_r g_k = 0$$

$$\text{or} \sum_r {}_r\bar{\lambda}_1 \cdot {}_r\bar{g}_k = \sum_r {}_r\beta_{pk} \cdot {}_r\bar{\lambda}_2 \cdot {}_r\bar{g}_k + \sum_r {}_r\beta_{Lk} \cdot {}_r\lambda_3 \cdot {}_r\bar{g}_k + \sum_r {}_rc_k \cdot {}_r\bar{g}_k \qquad \text{II.B.11}$$

$$\text{and} \sum_r\sum_s\left(\frac{\partial\phi}{\partial_{rs}x_p}\right){}_{rs}\bar{x}_p = \sum_r\sum_s {}_s\bar{\lambda}_2 \cdot {}_{rs}\bar{x}_p - \sum_r\sum_s {}_r\bar{\lambda}_2 \cdot {}_{rs}\bar{x}_p - \sum_r\sum_s {}_{rs}t_p \cdot {}_{rs}\bar{x}_p = 0$$

$$\text{or} \sum_r\sum_s {}_{rs}t_p \cdot {}_{rs}\bar{x}_p = \sum_r\sum_s {}_s\bar{\lambda}_2 \cdot {}_{rs}\bar{x}_p - \sum_r\sum_s {}_r\bar{\lambda}_2 \cdot {}_{rs}\bar{x}_p \qquad \text{II.B.12}$$

For the optimum $\bar{g}$, $\bar{x}$, $\bar{\lambda}$. At the optimum, total revenue from each of the producing and flow activities is exactly equal to the total transport and product cost outlays. For example, the left-hand side of II.B.10 is the total transport outlay on the final product. The right-hand side is the total excess value of the shipments of the final product in the demand locations. Thus for an efficient or optimum shipment pattern, the excess of delivered value over surplus point value is equal to the total transportation outlay. Therefore, there is no surplus remaining that can accrue to an arbitrager seeking to make a profit by rearranging the shipment pattern; the surpliers maximise the returns to each supply source and the commodity is distributed to the consumers at a minimum cost.

Similarly II.B.11 can be interpretted and the total value of the final product equals the cost of mobile inputs plus the cost of the immobile inputs plus fixed costs. II.B.12 is interpretted similarly to II.B.10 only this is shipments of mobile inputs instead of final goods. Thus the solution can be defined as an efficient one.

The dual of this problem can be given an economic meaning. Rewrite the Lagrangean II.B.7 in the following form:-

$$\phi(g,x,\lambda) = -\Big(\sum_r {}_rF_k \cdot {}_r\lambda_4 - \sum_r {}_rX_L \cdot {}_r\lambda_3 - \sum_r {}_rX_p \cdot {}_r\lambda_2 - \sum_r {}_r o_k \cdot {}_r\lambda_1\Big)$$

$$+ \sum_r \sum_s {}_{rs}f_k \left({}_s\lambda_4 - {}_r\lambda_1 - {}_{rs}t_k\right) + \sum_r {}_rg_k \left({}_r\lambda_1 - {}_r\beta_{pk} \cdot {}_r\lambda_2 - {}_r\beta_{Lk} \cdot {}_r\lambda_3 - {}_rc_k\right)$$

$$+ \sum_r \sum_s {}_{rs}x_p \left({}_s\lambda_2 - {}_r\lambda_2 - {}_{rs}t_p\right) \qquad \text{II.B.13}$$

where the non-negatively constraints will be ignored for simplicity and where ${}_r o_k$ is the real number zero, which means that all production levels in II.B.2 are variable for all r.

The Lagrangean II.B.13 forms the basis for the specification of the corresponding dual expression for our primal programming problem and it is to minimise

$$g(\lambda) = -\Big(\sum_r {}_rF_k \cdot {}_r\lambda_4 - \sum_r {}_rX_L \cdot {}_r\lambda_3 - \sum_r {}_rX_p \cdot {}_r\lambda_2 - \sum_r {}_r o_k \cdot {}_r\lambda_1 \qquad \text{II.B.14}$$

subject to

$${}_s\lambda_4 - {}_r\lambda_1 - {}_{rs}t_k \leq 0 \qquad \text{II.B.15}$$

$${}_r\lambda_1 - {}_r\beta_{pk} \cdot {}_r\lambda_2 - {}_r\beta_{Lk} \cdot {}_r\lambda_3 - {}_rc_k \leq 0 \qquad \text{II.B.16}$$

$_s\lambda_2 - {}_r\lambda_2 - {}_{rs}t_p \leq 0$ for all and s II.B.17

$({}_r\lambda_1, {}_r\lambda_2, {}_r\lambda_3, {}_r\lambda_4) \geq 0$ for all r II.B.18

Under the economic interpretation given for the Lagrangean multipliers, the dual programming problem is to find the set of imputed prices and rents $(\bar{\lambda})$ which will maximise the total revenue and the demand points for the final product over the total costs at the plants or production points[1]. The pricing and rent restriction of the dual programming problem are reflected in conditions II.B.9a to II.B.9c which state the condition that profits must be zero on all production and flow processes used and that no process may permit a positive profit. As noted previously, rent accrues to a plant location only if the plant capacity is used. In summary then the λ variables of the dual programming problem are interpretted as imputed prices at each demand and supply point for the primary or intermediate and final products and rent on plant locations, regional market supply prices for mobile primary or intermediate products may contain an element of rent reflecting the locational value of deposits, and the fact that profits after paying costs are zero for each firm in the industry. The rents to immobile factors (or factories) would give an indication of where further expansion of capacity would give the greatest returns.

This relatively simple example was given in order to illustrate the underlying economic concepts. The model can be extended to include multi-products and production functions resembling Leontief's and conceptually the problem does not change. Implicit in the analysis is that final demand is perfectly inelastic, i.e. the final demand must be met. A more general approach would be to have a more general demand function, thus recreating the Cournot problem, recently reformulated by Samuelson (161). Takayama and Judge (181) do go on to include this in their analysis - however since this is not relevant to attraction theory

[1] That is to say maximise profits.

it will not be discussed here[1].

The above type of analysis has been used in certain applied studies to work out optimal patterns of goods flows. The data required for the implementation of these models varies depending on the specific problem in hand, but as described in the above example, it generally requires

(i) transport costs between regions

(ii) capacities and/or costs of production in each region and/or inputs needed per unit of output

(iii) demand requirements to be met at each location.

The problem then involves minimising transport costs (usually plus costs of production). Studies of this type can be found by Henderson (78) who formulates the optimum flows of coal between regions of the United States, or by Ghosh (54) who similarly formulates the optimum flows of cement between regions of India. The latter also uses the dual formulation to calculate 'royalties' which are the rents on the immobile resources, to show where extension of capacity would be most rewarding. In the previous chapter however serious doubt was cast on the assumption that transport costs could effectively measure the cost of separating two functionally linked activities. While these remarks carry less weight when applied to the standardised regular flowing goods such as cement and coal, it is interesting to note that Ghosh (54) when comparing the 'optimum' flow of cements (calculated on production plus transport costs) with the flows that actually took place, fairly large discrepancies are found to exist. This may be due, as Ghosh suggests, to the imperfectly competitive structure of the cement industry, but a possible explanation is that transport costs are not an adequate explanation of separation costs - it would be expected that this problem would become more serious

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1 A more general approach would be to have the supply and demand schedules formed endogeneously in the system, e.g. the demand for a good is a function of all consumer activity and demand for all other goods. This is taking the problem into the realms of general equilibrium analysis first developed for a spaceless economy by Walras and shown in Arrow and Debreu's (6) work. This has been extended to a spatial economy by Isard and Ostroff (100) and Takayama and Judge (181) Ch. 16. The practical applications of this type of work are limited

in the type of industry discussed by Tosco (189).

When the output at each location is a variable in these activity analysis location studies, they become the rigorous counterpart of comparative cost studies. Both compare costs at each location which include transport costs. However because the activity analysis studies are formulated as a linear programming problem they are able to handle a greater number of variables and greater generalities - such as the interdependence of location, in that producing at one output will cause demand for intermediate inputs at that location and also cause competition in the final demand market, thus effecting the price of final intermediate and primary products.

A more ambitious study along the same lines is given by Moses (147) where he attempts to determine the optimum flows of all goods between all regions of the U.S.A. The aim was to minimise labour inputs (the only primary good which was needed to produce both transport and all other goods) subject to the constraints of regional capacities, regional technologies and requirements of regional final demand[1]. However again we face the problem when trying to apply the linear programming model, that transport costs probably do not adequately measure distance separation.

In discussing the example of Takayama and Judge (see above) it was shown how the price system under certain assumptions could bring about an efficient allocation of resources[2]. However under a different set of basic assumptions it is possible to show that the price system will not even bring about a stable solution, let alone an efficient one. This problem was first raised by Koopmans and Beckmann (119) when they

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1 This is very similar to the attempt to formulate the solution to the interregional input-output model as a linear programming problem that will be found in Section IV.F.

2 The importance of this result to attraction theory will be discussed in Ch. III.

set up the model as follows:

(i) the economies of scale or indivisibilities are such that in the competition for space only one plant can occupy each location and that plants cannot be divided between two different locations (there are the same number of plants as locations)

(ii) the profits of each plant depend on the location of other plants - this may be nothing more than the cost of buying inputs from other plants where the transport costs paid on these inputs is a variable, depending on how far the two locations are apart.

The model is cast formally as

$$\max \sum_{kr} {}_rR_k \cdot {}_rg_k - \sum_{kh} \sum_{rs} {}_{rs}t \cdot {}_{rs}x_{kh} \quad ^{1} \qquad \text{II.B.19}$$

where ${}_{rs}x_{kh}$ represents the flow of goods from location r to location s of the commodity which is supplied by plant k to plant h

${}_{rs}t$ represents the cost of transport per unit from region r to region s

${}_rR_k$ is the Revenue (before paying transport costs)of product k at location i

${}_rg_k$ is the size of plant k at location r.

Thus we are maximising the revenue of firms in all locations minus transport costs of flows of goods between them. The restrictions on the problem are

$${}_rg_k \cdot b_{kh} + \sum_s {}_{sr}x_{kh} = {}_rg_h \cdot b_{kh} + \sum_s {}_{kh}x_{rs} \qquad \text{II.B.20}$$

where k, h, i = 1,......n[2]

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[1] It is assumed that the R (revenues) and t (transport costs) are independent of the location of plants.

[2] It is assumed that the input and output flows of intermediate goods to and from a plant at a particular location are proportional to the size (${}_rg_k$) of the plant at that location.

where b_{kh} = required commodity flows from plant k to plant h in weight units

which states that the total inflow of the intermediate commodity (k, h) at the location r added to its production of that location equals the total outflow from r plus its consumption at that same location.

$$\sum_r {}_r g_k = 1 \qquad k=1,\ldots\ldots n \qquad \text{II.B.21}$$

$$\sum_k {}_r g_k = 1 \qquad r=1,\ldots\ldots n \qquad \text{II.B.22}$$

where II.B.21 expresses that precisely one plant of each kind is to be assigned and II.B.22 that each location can only accommodate one plant

$${}_r g_k \geq 0 \quad {}_{rs}x_{kh} \geq 0 \quad {}_{rr}x_{kh} = 0 \qquad \text{II.B.23}$$

$$k,h,r,s=1,2\ldots.n$$

which are the usual non-negatively constraints.

Both ${}_r g_k$ and ${}_{rs}x_{kh}$ are variables and the R, b and t are datum.

Without further restrictions the above problem allows divisible plants and is a normal linear programming problem for which a decentralised price mechanism would give an efficient solution. However if a further constraint is added

$${}_r g_k = 0 \text{ or } 1 \qquad k,r=1,\ldots\ldots.n \qquad \text{II.B.24}$$

which says that plants are not divisible then the problem is converted into a quadratic assignment problem to which there is no solution by market mechanism, as Koopmans and Beckmann (119) state (page 69) "No price system on plants on locations and on commodities in all locations that is regarded as given by plant owners, say, will sustain any assignment. There will always be an incentive for someone to seek a location other than the one he holds". For the actual argument of why there can be not even a stable solution see Koopmans and Beckmann (119), pages 68 - 70.

This non-operation of the price mechanism where indivisibilities (economies of scale) are present, is a rather worrying feature in location theory[1]. Vietorisz (194) discusses some of the indivisilibites and economies of scale that are particularly important in spatial analysis and will result in the non-convexities that cause the breakdown of the price system, since marginal adjustments can no longer be guaranteed to bring about a global optimum. An algorithm to solve the Koopmans and Beckmann problem has been formulated by Reiter and Sherman (156), where the algorithm consists of an iterative procedure, when each step of this consists of an iterative procedure itself. This can be regarded as finding a local optimum condition by marginal adjustments (the sub-iterative procedure) and then disturbing the equilibrium (the prime-iterative procedure) and allowing the sub-iterations to provide another local optimum. This is done a number of times to obtain some idea of the values of distribution of local optimum. This algorithm allows feasible calculation of practical sized problems, whereas enumerating all possible solutions even to moderate sized problems would be infeasible. However if analogy with the price mechanism is made to this algorithm, it is that in calculating the sub-iterations each mover (factory) must not only know his own benefits from the move, but he must know the social benefits/costs(i.e. the effects on all other factories together) and take these into account in his decision. Whilst the algorithm is ingeneous, this type of decentralised decision-making process seems unlikely to operate in a free market.

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1 See also Koopmans (118) pages 150 - 154.

Some more optimistic results are reported by Manne (139) where a decentralised price mechanism can be shown to operate even where the producers are facing economies of scale. However in this example the economies of scale are represented by a fixed cost of production and then a constant marginal cost. Of course the fixed cost is not incurred if production is zero at any location. As the problem is set up, it is quite simple, in that certain final demands are to be met in certain locations. Transport costs, fixed costs, and marginal costs are all known in advance, and the aim is to meet final demand minimising transport costs, fixed costs and marginal costs (in the example given, marginal costs are ignored since they are assumed the same at each location and so they will not alter the result - dropping this assumption would not substantially alter the problem). A one move at a time algorithm is then presented for which there is an analogy with a decentralised price mechanism - see Manne (139) pages 219 - 220 for the details. Because of the fixed costs of production, marginal costs pricing cannot be pursued in each market otherwise every factory would operate at a loss and optimum output would be zero. Thus the algorithm has some elements of discriminatory pricing in it and in this sense the solution may not be regarded as an efficient solution. However, even a centralised decision process would have the problems of paying for the fixed costs of production by some means, and the mechanism suggested by Manne does have the advantage of minimising total costs.

A similar type of problem to the one posed by Manne can be found in Bos (13) Ch. 5, but a longer discussion of the same problem can be seen in Frock-Hanssen (105) Ch. 7 where an attempt is made to devise a system of incentives given by central authorities such that total costs are minimised and the solution is efficient - one of the conditions being that marginal cost is charged in each location[1].

Summary to discontinuous space

(i) Purely theoretical work - it has shown that where there are constant returns to scale (and certain other simplifying assumptions) that a decentralised price system will lead to an efficient use of resources, but that economies of scale or indivisibilities may prevent this - although under certain circumstances (see Manne (139)) this may not be so.

(ii) Application of theoretical work - this has been limited to working out optimal flows and optimal output under simplifying assumptions - not the least of which is that transport costs are an adequate description of the costs of overcoming distance. This raises the whole question of Klaassen's distinction between primary factors and secondary factors as an influence on location decisions[2]. The type of works we have discussed can only really measure the primary factors, such as marginal and fixed cost of production and transport costs, and necessarily secondary factors are ignored because if we assume, for example, that transport costs are not an adequate measure of separation costs, then we face the problem of how to measure these intangible costs of separation.

It should be pointed out that all the studies that have been described are partial short-run models, since some variables are taken as given that ideally should be formed endogeneously.

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1 For a different approach to decentralised decision-making in spatial analysis see Tung (192) where the costs and benefits of decentralised versus centralised decisions and the costs and benefits of participation and non-participation are explicitly taken into account in a spatial framework. However the scope of our study does not encompass this type of analysis.

2 See section I.B, above for a discussion of this distinction.

However full equilibrium or even dynamic models have not been discussed because they have little practical application or relevance to attraction theory.

II.C Continuous Space

The history of continuous space analysis can be traced back as far as Von Thunen, through Weber to Losch and Christaller[1]. However these will not be discussed here since they are not directly relevant to attraction theory and most of the essential elements of these classical studies can be found in the three studies that will be discussed below - Tinbergen (185), Bos (13) and Gerck-Hanssen (105). Not the whole of the latter two's studies will be discussed, but it is hoped to bring out some of the difficulties encountered in this type of work and where possible point out similarities/differences to attraction theory.

Bos (13) starts off with a simple problem of allocating factories of one industry along a one dimensional line, along which agricultural output is evenly spread. The factories are to supply goods to the agricultural population at a density of μ or receive raw materials from agriculture at a density of μ. Transport costs are incurred in moving goods and the individual factories exhibit economies of scale. The aim is to supply the demand whilst minimising production and transport costs. The cost function is given as

$$C = y^{1} + yv + T \qquad \text{II.C.1}$$

where C = total costs

v = variable production volume

T = total variable transport costs

y^{1} = total fixed costs

y = constant variable production costs per unit of product.

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[1] For a discussion of these classical works, see, for example, Been (11), Von Boventer (16) or Isard (89).

Now let t = variable transportation cost per unit of product and per unit of distance

variable r_1 = maximum supply distance on one side of the industry

r_2 = " " " " other " " " "

Therefore

$$T = \int_0^{r_1} \mu t r_1 \, dr_1 + \int_0^{r_2} \mu t r_2 \, dr_2 = \tfrac{1}{2} \mu t (r_1^2 + r_2^2) \qquad \text{II.C.2}$$

Assume all demand is met

$$V = \mu (r_1 + r_2) \qquad \text{II.C.3}$$

Therefore cost per unit of product k equals

$$k = \frac{C}{v} = \frac{y^1}{v} + y + \frac{T}{v} \qquad \text{II.C.4}$$

This total cost has to be minimised with respect to variations in r_1 and r_2 subject to II.C.2 and II.C.3. Substituting these two equations into II.C.4 and setting

$$\frac{\partial C}{\partial r_1} = \frac{\partial C}{\partial r_2} = 0 \qquad \text{II.C.5}$$

we find $\bar{r}_1 = \bar{r}_2$ where a bar $^-$ is optimum value, i.e. the optimum supply distances are equal on both sides of the production centre (which is expected) and it follows that the production units are located on the market line at equal distances of $2\bar{r}$ to also find that

$$\bar{r} = \left(\frac{v^1}{\mu . t}\right)^{\frac{1}{2}} \qquad \text{II.C.6}$$

which means that the market area is related to the size of the fixed costs i.e. the bigger the economies of scale the larger the market area. Also the size of the market area is inversely related to transport costs and the density of demand. From these results we can find the optimum size of production unit $(\bar{v})$ total transport costs in the optimum solution $(\bar{T})$ and optimum average costs $(\bar{k})$. Bos extends this analysis to cover circular market areas, non-circular market areas (e.g. hexagons), where agriculture not only demands the products of industry but supplies inputs as well in terms of raw materials and also to allow for discontinuities in demand, since we can no longer assume demand to be the same all over because the factory workers will demand some of the product that is being produced - this overcomes one of the weaknesses of Losch's analysis.

An attempt is then made by Bos to extend the above type of analysis to include more than one type of industry. Since industries are allowed to trade with each other, the dimensions of the problem are increased substantially and Bos shows in Ch. 4 that no general conclusion can be drawn from this type of analysis despite some further simplifying assumptions. In fact the results show that each industry's market area ($\bar{r}_h$ h=1,2....H) is dependent not only on the values of the y_h, μ_h, t_h, but also on all the other $\bar{r}_h$ (h=1,2....H), and only a trial and error solution to the problem exists.

One of the problems of generalising the above analysis was that as the number of industries increased, the number of possible types of centres (collections of different types of industries) increased rapidly For example, consider a system where only two industries (1 and 2) are found, then it is possible to have three types of centres. those containing industry 1 above. those containing industry 2 above and those containing both industries 1 and 2. With these three types of centre, five systems of centres are possible - 1, 2, 12-1, 12-2, 12-1. 2-12 - where a dash separates systems of centres and a comma separates the types of centres in a system. In an attempt to reduce the dimensions of this problem, Bos looks at the Tinbergen (189) hypothesis of spatial dispersion and attempts to see in what circumstances this proposed arrangement of economic activity would be optimal. I shall describe the Tinbergen hypothesis (taken from Bos (13) pages 20 - 22) in detail as it is not dissimilar to the Loschian, Christaller patterns of hierarchial centres.

Consider a plane over whose surface agricultural production (i.e. supplies of industrial inputs) and population (i.e. demand for industrial products) is spread evenly. All other production is organized in enterprises and each enterprise produces only 1 commodity. All enterprises of the same industry are assumed to have the same minimum size at which the production costs per unit of product are at a minimum and do not increase at higher production levels. The price of all commodities are uniform and made equal to 1. through the choice of the units of quantity of each product and there is the assumption that transportation costs are paid by the producer. Quantities therefore represent value. All commodities are consumer goods and fixed proportions α_0 to α_k of income are assumed to be spent on agricultural product 0 and the other products k (k=1, 2....k). With a given income y for the economy as a whole, the total demand for product h and consequently the total number n_h of production units of minimum size needed to serve the economy are given. The industries can be ranked according to their number of production units n_h in such a way that

$$n_1 > n_2 > n_3 \ldots\ldots \quad n_K = 1$$

If $n_K > 1$ the area of the economy can be split up into n_h smaller areas. An industry hc is said to be of higher ranking that industry k if $n_h < n_k$. Industry 1 is of the lowest rank and industry h of the highest.

The problem is to combine the production units of various industries into centres so as to minimise the production and transportation costs. Tinbergen's hypothesis is:

(a) In each centre with an industry of given rank k, all industries of lower rank are also located.

(b) There is only one production unit of the highest ranking industry in a centre. This industry is the only one in the centres exporting to other centres and the agricultural area. The production of all other industries in the centres is consumed within the centre.

Consequently there is a hierarchy of groups of centres. There are as many groups of centres as there are industries. Each group can be distinguished according to the highest-ranking industry present in it and is given a corresponding rank. Centres of the lowest rank consist of one production unit of the lowest-ranking industry only. They export part of their production to the agricultural area and have to import all other products from centres of the higher ranks and from agricultural areas. The centre of the next-lowest rank 2 consists each of one enterprise of industry 2 (exporting part of its production to centres of rank 1 and to the agricultural area) and of production units of industry 1, which supply only the population of centres 2. The number of centres in each group of a given rank diminishes as the rank of the centres increases. There is only one centre of the highest rank in which are located production of all industries. The production units of the highest ranking industry exports to all lower centres and to the agricultural area. This centre's only imports are agricultural products. With two industries (1 and 2) under Tinbergen's hypothesis there would be two types of centres - those containing industries 1 and 2 and those containing only industry 2[1]. Contrast this with the total possible number of centres listed above.

However even with various different types of transport cost functions Bos is unable to prove generally that the Tinbergen hypothesis gives

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1 Where industry 2 is of the lowest rank

the optimum location of economic activity: it is only with very specific assumptions not only about the transport cost function but also assumptions about the relative transport costs of the various goods that Tinbergen's system is an optimum solution. For example, when $t_o = t_2 = 0$ and $t_1 \neq 0$ a Tinbergen hypothesis is better than all other combinations of systems when certain conditions are fulfilled concerning the values of the α's (see Ch. 5 of Bos).

In order to investigate the values of the variables where the parameters of the model are changed and how the systems of centres change in order to minimise total cost, Bos (pages 67 - 70) casts it in a programming format. In order to do this the assumption about continuous space has to be abandoned and the problem becomes similar to the example given from Takayama and Judge above. The formation of the problem in such terms does indicate difficulties of obtaining general results and that the solution depends on an iterative procedure where the actual parameters must be fed into the model in order to obtain results. The economic environment of the model is similar to that in Tinbergen's hypothesis except the transport costs within a centre are assumed zero.

<u>Data of model</u>

Y = total national income

Y_k total national production of industry K ($k=0, 1, 2....K$)

Y^*_k minimum production of a production unit of industry k($k=1,2....K$), i.e. economies of scale effect

${}_rY_o$ agricultural production in centre r($r=1,2....M$)

<u>Unknowns of model</u>

${}_rY$ income of centre r

${}_rY_k$ production of k in r

${}_{rs}E_k$ exports of k from centre r to s ($r,s=1,2....M$) for $r \neq s$

Coefficients of Model

α_k propensity to spend on k ($\sum_k \alpha_k = 1$)

${}_{rs}t_k$ cost of transporting one unit of product k from centre r to s

The following are the equations

$${}_rY_k = \alpha_k \cdot {}_rY + \sum_s {}_{rs}E_k - \sum_s {}_{sr}E_k \qquad (r \neq s) \qquad \text{II.C.7}$$

which says production of k in r equals demand for k in r plus exports to centres s minus imports from other centres s

$${}_rY_o = \alpha_o \cdot {}_rY + \sum_s {}_{rs}E_o - \sum_s {}_{sr}E_o \qquad (r \neq s) \qquad \text{II.C.8}$$

which is interpretted as the same as above only for agriculture

$${}_rY = {}_rY_o + \sum_k {}_rY_k \qquad \text{II.C.9}$$

which says income of centre r equals agricultural plus non-agricultural production

$$\sum_r {}_rY = Y \qquad \text{II.C.10}$$

The sum of the incomes of all centres equals total national income

$$\sum_r {}_rY_k = Y_k \qquad \text{II.C.11}$$

The total production of commodity k in all centres equals the total national production of k

$$Y^*_k \leq {}_rY_k \leq Y_k \quad \text{or} \quad {}_rY_k = 0 \qquad \text{II.C.12}$$

which says that the production of h in r must be above the fixed minimum and below total national production of h or must not be produced at all.

All variables must be non-negative and the aim is to minimise total transport costs

$$T = \sum_r \sum_s \sum_k {}_{rs}t_k \cdot {}_{rs}E_k \qquad \text{II.C.13}$$

However, not all the equations in the model are independent. II.C.10 can be derived by substitution of II.C.7 into II.C.9 using the equality between total exports and total imports for all centres together for any product h. So -

$$\sum_r {}_rY = \sum_r {}_rY_o + \sum_r \sum_k {}_rY_k = \sum_r {}_rY_o + \sum_r {}_rY_o + \sum_r \sum_k \alpha_k . {}_rY = \sum_r {}_rY_o + \sum_k \alpha_k . \sum_r {}_rY$$

or

$$\sum_r {}_rY = \frac{1}{1 - \sum_k \alpha_k} \sum_r {}_rY_o \qquad \text{II.C.14}$$

C.11 can be derived from II.C.7 and II.C.14 since

$$\sum_r {}_rY_k = \alpha_k . \sum_r {}_rY = \frac{\alpha_k}{1 - \sum_k \alpha_k} . \sum_r {}_rY_o \qquad \text{II.C.14}$$

Since $\alpha_o + \sum_k \alpha_k = 1$ II.C.7 and II.C.8 can be substituted into II.C.9 so that

$$\sum_k \sum_s {}_{rs}E_k = \sum_k \sum_r {}_{rs}E_k \qquad \text{II.C.16}$$

or exports equal imports for each centre.

So we can now omit II.C.10 and II.C.11 from the model. If the inequalities are made equalities then the number of equations is rKM + 2M while the number of variables is M(M-1) (K+1) + 6KM or $KM^2 + M^2 - 3M$ degrees of freedom and as long as K > 1 and M > 1 it is positive, and so no general solution is possible but one to be solved by iterative methods. The model can be altered so that the economies of scale resemble those postulated by Manne (139), i.e. a fixed cost and constant marginal cost. Presumably the assumption could also be relaxed that all factories are vertically integrated producing only final goods, and intermediate (or Leontief) type goods allowed. This again would reinforce the non-solubility of general cases[1].

In a similar straight line economic environment to Bos', Serck-Hanssen derives optimality conditions for a series of factories whose output is fixed and whose only variable is its location along a straight line - these are called Weber models. The approach is slightly different from Bos' in that much concern is given to the problem of whether a decentralised market mechanism can sustain the optimality conditions or if postulated mechanism of market growth will lead to a non-optimal distribution of industry - for example he concludes that there is a tendency for industrial centres to become too big, in that no one individual factory-owner will have any incentive to move from the industrial centre because his transport costs would be increased since his trade partners will still remain behind in the centre - but it may well be optimal for a group of industries to move out simultaneously since total costs would thereby be reduced.

The study then turns to the problem of allowing the locations to be fixed, but the outputs of factories to be variable. It is interesting to note that Serck-Hanssen (op.cit) drops the assumption of continuous space and adopts the expediency of discontinuous space, so that the problem can be subject to programming analysis. Of course no general results are possible with this type of approach, although he does discuss the necessary conditions for an optimal solution (in a similar manner to that described above by the Takayama and Judge example), and

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1 It is interesting also to note that when the assumption concerning all goods being final goods under the Tinbergen hypothesis is dropped and intermediate goods are allowed; even when a strict hierarchy of centres is imposed as assumed under the hypothesis: the strict exporting and importing patterns of goods hypothesised breaks down. Again showing that the hypothesis may not hold in more generally realistic situations. For a discussion of this see Bos (13) Ch. 7.

also discusses when a market solution to this optimisation problem may break down. From the attraction theory point of view, the major interest of this work is that with output variable, no general solution was possible, but programming models must be used.

In the review of theory given so far it may be thought surprising that no mention has been made to the traditional theory of the firm where substitution between various factor inputs is allowed in determining the optimum location. This is due to two reasons:-

(i) little work has been done on substitution in the theoretical sense. Isard (89) is an exception when he applied the principle of substitution to the Weberian location problem, but the substitution principle was not allowed to extend to the production function, where a rigid input structure was still assumed. It was Moses (149) who first allowed substitution between inputs in the traditional way, integrating these into a continuous spatial analysis. However these theories have not been developed[1].

(ii) On the empirical side there seems little hope of implementing the more general Moses model mentioned above. Isard, Schooler and Vietorisz (94) have used the substitution principle in their modern Weberian analysis, however this approach has been criticised above in Section I.C.

Summary and conclusion to continuous space

The same problems that were encountered in the analysis of discontinuous space can be found with continuous spatial analysis - namely that Klaassen's primary location factors can be handled, but the secondary factors tend to be ignored when the work is empirically

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[1] For a recent extension of Moses' work see Nijkamp and Paelinck (152).

implemented. Further, we have seen that general results are notoriously difficult to obtain, and those that can be, are made under very specific simplifying assumptions - for example it was shown that the Tinbergen hypothesis of di[illegible] does not give a system of general application of the structure of economic centres. Also we have seen that in the studies when the output of factories was allowed to be a variable, programming analysis was resorted to, so that no general results were possible. Finally, we have seen that all the problems are subject only to partial analysis, in that many potential variables are taken as given.

CHAPTER III

The Theory of Attraction Models

III.A. Introduction

In previous chapters we have examined empirical and theoretical models of location as a tool for the government's location of industry policy. In this chapter attraction theory[1] will be explained. It is hoped to show how attraction theory can be used to identify potentially mobile industries and can thereby contribute to location policy. It is also hoped to make explicit the assumptions that are necessary for attraction theory to be consistent. An attempt will be made to integrate attraction theory with interregional input-output analysis, and attraction theory with the main body of location theory. Finally it will be shown that some of the theories used in regional analysis are just special cases of attraction theory.

Although this chapter will be concerned solely with the theoretical developments of attraction theory, the availability of data will always be borne in mind so that the structural equations of the model can be estimated. Consequently, theoretical considerations such as dynamic attraction models[2] will not be considered. The empirical implementation of attraction theory will be presented in Ch. V, and the estimation of input-output data for each region (the basic observations of attraction theory) in the U.K. will be given in Ch. IV.

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1 First developed by Klaassen (113) and Klaassen and Van Wickeren (115), but the latest and most comprehensive study of attraction theory is found in Van Wickeren (197) and consequently most of the references will be made to this work.

2 For a discussion of dynamic attraction models see Van Wickeren (197) Ch. 6, published separately by Van Wickeren and Smit (199).

III.B. Definition of total communication costs - the basis of attraction theory

Let us start by defining a typical 3-industry (1, 2, 3), 2-region (r, s) input-output system that trades with the remainder of the world[1]. This is shown in Table III.1.

Let the term communication costs represent all the cost of contact per unit of flow between two sectors[2], and assume

(i) communication costs for trade within a region are zero

(ii) communication costs for trade outside a region are positive.

Now let us define total communication costs for industry 1 in region r (${}_rT_1$).

$$
{}_rT_1 = {}_{rs}t^d_{11}\cdot{}_{rs}X_{11} + {}_{rs}t^d_{12}\cdot{}_{rs}X_{12} + {}_{rs}t^d_{13}\cdot{}_{rs}X_{13} + {}_rt^d_{1e}\cdot{}_re_1 + {}_{rs}t^d_{1f}\cdot{}_{rs}f_1 + {}_{sr}t^s_{11}\cdot{}_{sr}X_{11}\cdot{}_{sr}t^s_{21}\cdot{}_{sr}X_{21} + {}_{sr}t^s_{31}\cdot{}_{sr}X_{31} \qquad \text{III.B.1}
$$

where ${}_{rs}t^d_{ij}$ = unit cost of communication for industry i in region r to export one unit of product i to industry j in region s. The superscript d is to emphasize that this flow is being demanded from industry i and ${}_{sr}t^s_{ij}$ = unit cost for industry j in region r to import one unit of product i from region s. The superscript s is to emphasize that the flow is being supplied from industry i.

The other t's are the unit communication cost of the variable associated with the flow of goods to which the particular t is attached.

It must be noted that:

(i) for simplicity labour is assumed to be a non-transportable good and will therefore not generate

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[1] For a fuller discussion of interregional input-output analysis see Ch. IV.

[2] This term will include not only transport costs but also any information or separation costs. For a discussion of the type of costs included, see Ch. I, or Van Wickeren (197) for some examples. Some further specific examples of communication costs will be given below in section III.M.

Table III.1

A two-region, three-good input-output table

		Intermediate goods						exports abroad	Final demand in r	Final demand in s	
		1	2	3	1	2	3				
Inter-	1	$_{rr}X_{11}$	$_{rr}X_{12}$	$_{rr}X_{13}$	$_{rs}X_{11}$	$_{rs}X_{12}$	$_{rs}X_{13}$	$_{r}e_{1}$	$_{rr}f_{1}$	$_{rs}f_{1}$	$_{r}g_{1}$
mediate	2	$_{rr}X_{21}$	$_{rr}X_{22}$	$_{rr}X_{23}$	$_{rs}X_{21}$	$_{rs}X_{22}$	$_{rs}X_{23}$	$_{r}e_{2}$	$_{rr}f_{2}$	$_{rs}f_{2}$	$_{r}g_{2}$
	3	$_{rr}X_{31}$	$_{rr}X_{32}$	$_{rr}X_{33}$	$_{rs}X_{31}$	$_{rs}X_{32}$	$_{rs}X_{33}$	$_{r}e_{3}$	$_{rr}f_{3}$	$_{rs}f_{2}$	$_{r}g_{3}$
	1	$_{sr}X_{11}$	$_{sr}X_{12}$	$_{sr}x_{13}$	$_{ss}X_{11}$	$_{ss}X_{12}$	$_{ss}X_{13}$	$_{s}e_{1}$	$_{sr}f_{1}$	$_{ss}f_{1}$	$_{s}g_{1}$
Goods	2	$_{sr}X_{21}$	$_{sr}X_{22}$	$_{sr}X_{23}$	$_{ss}X_{21}$	$_{ss}X_{22}$	$_{ss}X_{23}$	$_{s}e_{2}$	$_{sr}f_{2}$	$_{ss}f_{2}$	$_{s}g_{2}$
	3	$_{sr}X_{31}$	$_{sr}X_{32}$	$_{sr}X_{33}$	$_{ss}X_{31}$	$_{ss}X_{32}$	$_{ss}X_{33}$	$_{s}e_{3}$	$_{sr}f_{3}$	$_{ss}f_{3}$	$_{s}g_{3}$
Imports from abroad		$_{r}m_{1}$	$_{r}m_{2}$	$_{r}m_{3}$	$_{s}m_{1}$	$_{s}m_{2}$	$_{s}m_{3}$				
Primary (from r		$_{r}p_{1}$	$_{r}p_{2}$	$_{r}p_{3}$	0	0	0				
Goods (from s		0	0	0	$_{s}p_{1}$	$_{s}p_{2}$	$_{s}p_{3}$				
		$_{r}g_{1}$	$_{r}g_{2}$	$_{r}g_{3}$	$_{s}g_{1}$	$_{s}g_{2}$	$_{s}g_{3}$				

Notation (i) x = intermediate goods

m = imports from abroad

p = primary inputs

g = total output

e = exports abroad

f = final demand

(ii) subscripts before a variable indicate regions, and subscripts after indicate industries. Where more than one subscript is found together the direction of flow is found by reading left to right, e.g. $_{rs}X_{23}$ means the flow of intermediate good from industry 2 in region r to industry 3 in region s.

any communication costs. Thus we will be dealing with inter-industry relations only. This assumption will be relaxed later.

(ii) It is assumed that imports from abroad are of a non-competing nature with home production and the cost of communication is fixed per unit of import regardless of the location of the importing industry. So they can be ignored since they will not influence location decisions. This will be discussed further in Ch. V.

Definition III.B.1 applies to the case where there are only 2 regions (r,s). Practical problems would arise had we included more than 2 regions because separate t's would be needed for each region depending where supplies came from or output was sold to, e.g. ${}_{r1}t^{d}_{ij}$, ${}_{r2}t^{d}_{ij}$, ${}_{r3}t^{d}_{ij}$ etc. [1]. This would be impracticable when it came to the empirical testing of the model because:

(i) when it comes to testing the equations it will be seen that there would not be sufficient degrees of freedom to permit this

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[1] ${}_{rs}t^{d}_{ij}$ can stand as a definition even when there are more than two regions and would be the aggregation of the individual regional t's. However if we are to estimate the coefficients of the model and use it for prediction, there must be certain restrictive assumptions made about the t's and the communication cost function. It is with this estimation and prediction in mind that the assumptions laid out above are discussed. See also Section III.D. for an explicit discussion of the assumption necessary for the coefficients to be estimated from cross-sectional data.

(ii) the data to construct interregional input-output flows to and from a system of n regions (where $n > 2$) is not available. We are constrained to a system where $n = 2$.[1] [2]

In this framework with only two regions, region r represents the region under study and region s represents the whole of the nation minus the region under study - henceforth called the Rest of the United Kingdom (RUK). Since each t will only be subscripted with rs and sr this seems to imply that the communication cost function is composed purely of a fixed cost per unit of output, that is invariant with physical distance.[3] It seems as though it is irrelevant with which region the trade is being conducted. However if one assumes that the interregional trading coefficients (and therefore patterns) are constant[4], between region r and all the other regions that make up region s (i.e. R.U.K.)[5], then, for example, the ${}_{rs}t^{d}_{11}$ represents the average of all the t's with all the regions that compose R.U.K.[6], and so will

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[1] For the details of this data and why it is only possible to have a model for each region (where the only region, apart from the region in question, is one composed of all the Rest of the UK aggregated together) see Ch. IV.

[2] This is an example of the generality of our theory being constrainted by the desire to obtain an empirically testable model.

[3] Bos (13) Ch. 5 uses such a function in one of his models.

[4] See Ch. IV for a discussion of the concept and problems involved in assuming the constancy of interregional trading coefficients.

[5] It will be shown later in another context that a necessary assumption of attraction theory is that the trading coefficients between region r and s are constant. This is a weaker assumption than the one made here that the coefficients are constant between r and all the regions individually that compose s.

[6] That is ${}_{rs}t^{d}_{11} = \frac{\sum_{k=1}^{k} {}_{rk}t^{d}_{11}}{K}$ where there are K + 1 regions and the (K + 1) th region = region r (i.e. the one under consideration).

not change. Thus in the definition we have used it seems necessary to assume either:

(i) a communication costs zero inside the region but for trade outside the region they are positive and invariant with distance, or

(ii) constant trading coefficients between all regions of the system[1].

Now let us make one further simplifying assumption, that the cost of exporting one unit of output is the same regardless of the purchasing sector (i.e. intermediate industries, exports, final demand). Thus all the t^d's for one industry will not be the same, but we leave the t^s's to vary depending upon the sector with which trade is carried out. This assumption is made for the following reasons:

(i) the number of degrees of freedom in the equations to be tested would not permit us to estimate a separate demand effect for each sector[2],

(ii) it seems intuitively reasonable that demand costs can be described as an homogeneous term, and we are interested in its aggregate effect on location[3].

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1 Van Wickeren (197) page 4 states that he assumes that communication costs vary with distance, therefore, he must implicitly assume that trading patterns between all regions examined are constant, if a definition involving specific regional terms is used. If national coefficients are used this problem does not arise. (For a discussion of specific regional and national coefficients see Section III.F.)

2 It will be seen from Ch. IV. that we have only 11 observations - one for each region of the UK. This is because we can only estimate one interregional input-output table for one year for each region, with the present data availability.

3 For example, from an increase in demand of a similar magnitude by either final consumers or an intermediate consumer in the region, we would expect to see a similar effect on the industry under study.

However we are interested in each supply industry individually as a locational influence[1], and since a priori we suspect for each industry that only a few of the larger suppliers will be important, the degrees of freedom problem will not arise.

Using the above assumptions, equation III.B.1 can be re-written:

$$
\begin{aligned}
{}_rT_1 = {}_rt_{1d}\left({}_rg_1 - \left({}_{rr}x_{11} + {}_{rr}x_{12} + {}_{rr}x_{13} + {}_{rr}f_1\right)\right) \\
+ {}_rt_{11}\left({}_r\beta_{11}\cdot{}_rg_1 - {}_{rr}x_{11}\right) + {}_rt_{21}\left({}_r\beta_{21}\cdot{}_rg_1 - {}_{rr}x_{21}\right) \\
+ {}_rt_{31}\left({}_r\beta_{31}\cdot{}_rg_1 - {}_{rr}x_{31}\right)^2 \qquad \text{III.B.2}
\end{aligned}
$$

where the ${}_r\beta$'s are the regional specific constant Leontief input coefficients and

$${}_r\beta_{ij}\cdot{}_rg_j \equiv {}_{rr}x_{ij} + {}_{sr}x_{ij}$$

which says that to produce a given output of industry ${}_j({}_rg_j)$ a certain amount of industry i's production is required, regardless of the region of origin of this input. The amount of input is determined by the

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1 For example, the industry that processes raw sugar beet uses as inputs both sugar beet and paper bags. However a £1m. increase in supply of each of these supplies would not have the same influence on the processing industry; since raw sugar beet cannot be moved very far because of the high communication costs (in this case due largely to high transport costs) the processing industry will tend to locate near raw beet, but paper bags can be moved interregionally to the processing industry at relatively low communication costs and so will not influence the location of the processing industry. These ideas will be discussed in more detail in Section III.M.

2 Where ${}_{rs}t^d_{11} = {}_{rs}t^d_{12} = {}_{rs}t^d_{13} = {}_rt^d_{1e} = {}_{rs}t^d_{1f}$ by assumption and now called ${}_rt_{1d}$, and both the superscript and the subscript s are dropped from the ${}_{sr}t^s_{ij}$ since no confusion can be caused by re-naming this ${}_rt_{ij}$.

technological requirements as shown by ${}_r\beta_{ij}$. Thus equation III.B.2 says exactly the same as equation III.B.1 except that equation III.B.1 sums all the costs of communication by exports of demand and imports of inputs, whereas III.B.2 says that total communication costs consist of exporting what is not sold within the region of production plus the costs of importing what is not bought within the region of production. By definition of total output these two formulations are necessarily equal.

It will be noticed from this definition of communication costs, that

$$ {}_{rr}x_{11} + {}_{rr}x_{12} + {}_{rr}x_{13} + {}_{rr}f_1 \qquad \text{III.B.3} $$

does not equal the total demand in region r for product 1, but is the demand in region r for product 1 that is produced in region r. The total demand in region r for product 1 includes this, plus that part of total demand which is imported from region s. Thus total demand for 1 in r is

$$ {}_{rr}x_{11} + {}_{rr}x_{12} + {}_{rr}x_{13} + {}_{sr}x_{11} + {}_{sr}x_{12} + {}_{sr}x_{13} + {}_{rr}f_1 + {}_{sr}f_1 \qquad \text{III.B.4} $$

Now contrast equation III.B.2 definition of communication costs with the definition of communication costs given by Van Wickeren (197) page 7:

$$ {}_jT_k = \underline{t_{kd}({}_jg_k - {}_jd_k)} + \sum_h t_{hk} (\beta_{hk} \cdot {}_jg_k - \alpha_{hk} \cdot {}_jg_h) \qquad \text{III.B.5} $$

(Only the part of the equation underlined is relevant to the argument.)

In this equation:

$${}_j d_k = \sum_{h=1}^{n} {}_j r_{kh} + {}_j f_k = \underline{\text{total}} \text{ of intermediate and final demand}$$

for product k in region j. III.B.6

Thus the ${}_j d_k$ in III.B.5 and III.B.6 is identical to the definition of total demand for the product as used in III.B.4.[1] However this definition of total demand is not appropriate to use in the definition of total communication costs. Subtracting it from total output underestimates communication costs since some costs have to be paid on that part of intermediate and final demand that is imported from outside the region. Thus the costs involved in transferring the goods shown in the bottom left hand quadrant of Table III.1 are ignored.

Using total regional demand in the definition of communication costs leads to certain problems that can only be solved by imposing arbitary constraints. Consider the example shown in Van Wickeren (197) page 28:

$${}_j T_k = t_{kd}({}_j g_k - {}_j d_k) + \sum_{h=1}^{n} t_{hk}(\beta_{hk} \cdot {}_j g_k - \alpha_{hk} \cdot {}_j g_h) \qquad \text{III.B.7}$$

which is constrained

(i) $h \neq t$

(ii) ${}_j d_k \leqslant {}_j g_k$

(iii) $\alpha_{hk} \cdot {}_j g_h \leqslant \beta_{hk} \cdot {}_j g_k$

(iv) t_{kd} and t_{hk} equal for all regions.

Only constraint (ii) interests us at the moment, and this says that the total demand for product k in region ${}_j({}_j d_k)$ cannot exceed total

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[1] This can be seen again more explicitly in Van Wickeren (197) pages 11 - 12 where each term of $\sum_{h=1}^{n} {}_j r_{kh}$ is written separately as $\beta_{11} \cdot {}_j g_1 + \beta_{12} \cdot {}_j g_2 + \beta_{13} \cdot {}_j g_3$, which is total intermediate demand for industry 1 in region j in a three-industry model.

production of product k in region j. However there are many cases where the demand in a region exceeds the production in that region. This constraint has to be introduced if we are to avoid absurd results when the equations are finally estimated[1]. Without this constraint (using total demand), it would be possible to find that industry had a negative attraction to demand. But by defining communication costs as above in equations III.B.1 and III.B.2 we know that ${}_r dd_1 \leq {}_r g_1$ in all cases where ${}_r dd_1 \equiv {}_{rr}x_{11} + {}_{rr}x_{12} + {}_{rr}x_{13} + {}_{rr}f_1$. The double dd is used to distinguish demand in a region for products made in the region from total regional demand. Thus by using a more defensible definition of communication costs, we avoid having to put arbitary constraints on the equations. It thus seems preferable to use ${}_r dd_1$ rather than Van Wickeren's ${}_r d_1$.

III.C. <u>Derivation of the equations to be estimated</u>[2]

Let us re-write equation III.B.2 as

$$ {}_r t_1 \cdot {}_r g_1 = {}_r t_{1d}({}_r g_1 - {}_r dd_1) + \sum_{i=1}^{n} {}_r t_{i1}({}_r \beta_{i1} \cdot {}_r g_1 - {}_r \alpha_{i1} \cdot {}_r g_i) $$

III.C.1

where n = number of industries in the system

${}_r t_1$ = average communication cost for one unit of output of industry 1 in region r

${}_r \alpha_{i1}$ = the proportion of output of industry i in region r that is sent to industry 1 in region r. That is to say ${}_r \alpha_{i1} \cdot {}_r g_i = {}_{rr}x_{i1}$ for each i.

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[1] The equations to be estimated will be discussed in Section III.C.

[2] What is termed 'equations to be estimated' in my model are called 'reduced form' by Van Wickeren (197). However a reduced form equation implies that in each equation there is one endogeneous variable explained by one/several exogeneous variables. However I will show that attraction theory is a simultaneous system (see Section III.D.) and that there are endogeneous variables on the right hand side of the equation, so that reduced form is an inappropriate description.

Multiplying out equation III.C.1

$$ {}_r t_l \cdot {}_r g_l = {}_r t_{ld} \cdot {}_r g_l - {}_r t_{ld} \cdot {}_r dd_l + \sum_{i=1}^{n} {}_r t_{il} \cdot {}_r \beta_{il} \cdot {}_r g_l - \sum_{i=1}^{n} {}_r t_{il} \cdot {}_r \alpha_{il} \cdot {}_r g_l \qquad \text{III.C.2} $$

Collecting terms with ${}_r g_l$ to one side and dividing through by $\left({}_r t_{ld} + \sum_{i=1}^{n} {}_r t_{il} \cdot {}_r \beta_{il} - {}_r t_l\right)$ gives

$$ {}_r g_l = \frac{{}_r t_{ld}}{{}_r t_{ld} + \sum_{i=1}^{n} {}_r t_{il} \cdot {}_r \beta_{il} - {}_r t_l} \; {}_r dd_l \; + $$

$$ \sum_{i=1}^{n} \frac{{}_r t_{il}}{{}_r t_{ld} + \sum_{i=1}^{n} {}_r t_{il} \cdot {}_r \beta_{il} - {}_r t_l} \; {}_r \alpha_{il} \cdot {}_r g_i \qquad \text{III.C.3} $$

Now let

$$ \left(\frac{{}_r t_{ld}}{{}_r t_{ld} + \sum_{i=1}^{n} {}_r t_{il} \cdot {}_r \beta_{il} - {}_r t_l} \right) \equiv \lambda_{ld} $$

to be called the demand attraction coefficient

and

$$ \left(\frac{{}_r t_{il}}{{}_r t_{ld} + \sum_{i=1}^{n} {}_r t_{il} \cdot {}_r \beta_{il} - {}_r t_l} \right) \equiv \lambda_{il} $$

to be called the supply attraction coefficient

Also let us define a new coefficient to partially replace ${}_r dd_l$

Define ${}_{rr} x_{il} = {}_r \delta_{il} \cdot {}_r g_l$, where ${}_r \delta_{il}$ represents the unit input of good i produced inside region r, per unit of output of good l in region r.[1]

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[1] Contrast this with the Leontief coefficient where ${}_{rr} x_{il} + {}_{sr} x_{il} = {}_r \beta_{il} \cdot {}_r g_l$. The difference between the Leontief and the above is that part of intermediate demand in a region that is imported from another region. When imports from another region are zero (i.e. ${}_{sr} x_{il} = 0$) then and only then does ${}_r \beta_{il} = {}_r \delta_{il}$.

Therefore equation III.C.3 can be re-written for 3 industries

$$_rg_1 = \lambda_{1d} \cdot {}_{rr}f_1 + \lambda_{1d}({}_r\delta_{11} \cdot {}_rg_1 + {}_r\delta_{12} \cdot {}_rg_2 + {}_r\delta_{13} \cdot {}_rg_3) +$$

$$\lambda_{11} \cdot {}_r\alpha_{11} \cdot {}_rg_1 + \lambda_{21} \cdot {}_r\alpha_{21} \cdot {}_rg_2 + \lambda_{31} \cdot {}_r\alpha_{31} \cdot {}_rg_3 \qquad \text{III.C.4}$$

Let us assume that the λ's can be estimated empirically[1] and then re-write equation III.C.4 which is for a single industry, for a whole system of industries (1, 2 .... n)

$$_r\underline{g} = \hat{1}\,{}_r\Delta\,{}_r\underline{g} + (LA)'\,{}_r\underline{g} + \hat{1}\,{}_{rr}\underline{f} \quad ^2 \qquad \text{III.C.5}$$

where $_r\underline{g}$ = n x 1 vector of gross outputs in region r

$_{rr}\underline{f}$ = n x 1 vector of final demands in region r for products of region r

$\hat{1}$ = n x n null matrix, apart from the main diagonal which consists of the demand attraction coefficients λ_{1d} i=1, 2....n.

$_r\Delta$ = n x n matrix of the δ elements for region r

$(LA)'$ = n x n matrix composed of the transpose of the supply attraction coefficients (λ_{ij} i, j=1, 2....n) and its associated coefficient of internal regional supply (${}_r\alpha_{ij}$ i, j=1, 2....n),

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[1] This part will be raised later in Section III.D.

[2] A bar _ under a symbol indicates a vector.

where for a three-industry system

$$LA \equiv \begin{pmatrix} \lambda_{11} \cdot r^{\alpha_{11}} & \lambda_{12} \cdot r^{\alpha_{12}} & \lambda_{13} \cdot r^{\alpha_{13}} \\ \lambda_{21} \cdot r^{\alpha_{21}} & \lambda_{22} \cdot r^{\alpha_{22}} & \lambda_{23} \cdot r^{\alpha_{23}} \\ \lambda_{31} \cdot r^{\alpha_{31}} & \lambda_{32} \cdot r^{\alpha_{32}} & \lambda_{33} \cdot r^{\alpha_{33}} \end{pmatrix}^{1}$$

From III.C.5

$$[I - \hat{1}\, {}_r\Delta - (LA)']\, {}_r g = \hat{1}\, {}_{rr} f \qquad \text{III.C.6}$$

where I = n x n unit matrix. Therefore

$${}_r g = [I - \hat{1}\, {}_r\Delta - (LA)']^{-1}\, \hat{1}\, {}_{rr} f \qquad \text{III.C.7}$$

where $[I - \hat{1}\, {}_r\Delta - (LA)']^{-1}$ will be termed the attraction matrix[2]. This attraction matrix contrasts with Van Wickeren's (197) attraction matrix[3], which is shown to be $[I - \hat{1} B - LA]^{-1}$. These two attraction matrices will only be the same when $B = {}_r\Delta$, which is only true in the limiting case where there are no imports of intermediate inputs. That is, the bottom left-hand quadrant of

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[1] Van Wickeren (197) defines LA as I have defined (LA)′ , but I think the transpose notation causes less confusion because it adheres to the conventional subscripting of matrices, and reading left to right on the subscripts maintains the direction of flow of goods.

[2] The attraction matrix shows the composition of the multiplier in a similar way to the Leontief $(I-B)^{-1}$. However this point will be discussed later in Section III.G.

[3] See for example page 87 Van Wickeren (197).

the interregional input-output Table III.1 will be empty[1]. This will generally not be the case. The discrepancy between the two attraction tables comes about because of the basic differences in defining total communication costs[2], and I have argued that the definition involving ${}_r dd_i$ rather than Van Wickeren's ${}_r d_i$ is a more accurate description of communication costs, and therefore more likely to lead to accurate results.

III.D. Some theoretical problems involved in the estimation of the parameters of the attraction model

In order to use the attraction model, it is necessary to determine statistically the λ's of the system. It is only through estimation of the λ's that we can obtain any knowledge about the various t's in the system[3].

It is proposed to use regression analysis to estimate these λ's, and this must be done from regional cross-section data[4]. Thus for the estimates to be consistent, the parameters must contain only elements that are constant across all regions. We defined

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[1] For this to be true of all regions, then the top right-hand quadrant of Table I must also be empty, so making each region completely independent of all other regions.

[2] See Section III.B. above.

[3] We showed in Chs. I and II that knowledge of these t's is vital if we are to obtain any knowledge about the mobility of an industry.

[4] Cross-section data must be used because, to date, there is only one input-output table from which estimates of regional tables can be made. It is unlikely that we shall ever be able to use time series data to estimate an equation for each region separately, because the time intervals of which the Census of Production(the basis of the input-output tables) are produced is so great, that many things that we can assume to be constant in the short run, will become variables in the long run.

$$\lambda_{1d} \text{ as } \frac{{}_r t_{1d}}{{}_r t_{1d} + \sum_{i=1}^{n} {}_r t_{i1} \cdot {}_r \beta_{i1} - {}_r t_1}$$

and

$$\lambda_{i1} \text{ as } \frac{{}_r t_{i1}}{{}_r t_{1d} + \sum_{i=1}^{n} {}_r t_{i1} \cdot {}_r \beta_{i1} - {}_r t_1}$$

Therefore to estimate these parameters we must assume:

(i) that the β's are constant across all regions, i.e. we must assume that each region uses the same technology[1]. Therefore from now on, we will drop the regional specific r in ${}_r\beta_{ij}$ because we assume them to be all the same.

(ii) That each t is constant across all the regions, i.e. we must assume that regardless of the region of origin or destination of goods, there is a constant unit communication cost for each good (excluding of course the intra-regional flows which were assumed to be zero). Thus the t's must be invariant with physical distance[2] if cross section data is used. This assumption may be relaxed if time series data were available[3], but this theoretical possibility will not be discussed.

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[1] As will be seen in Ch. IV, this assumption was necessary in order to construct the interregional input-output tables. Consequently we are introducing nothing new by making this assumption here.

[2] We saw in Section III.B. that this was one of two alernative assumptions that we found necessary to define total communication costs for an individual region when using specific regional coefficients. It is now no longer an alternative, but a necessary assumption. Thus Van Wickeren's (197) claim, page 4, that communication costs vary with distance is inconsistent with the use of cross-section data for specific regional coefficients, and can only be used when national coefficients are used. For a discussion of national vs. regional coefficients see Section III.F.

[3] We have argued above that this is unlikely.

We are also forced to make one further assumption. Consider Table III.2, where the normal interregional input-output table is shown, with the t in the left of the cell representing the communication cost per unit of good that is paid by the buyer of that good, and the right of the cell representing the cost paid by the seller.

Now in the actual estimation of the results it will be shown that most of the t_{id}'s are positive[1], but only a small number of the t_{ij}'s will be positive. Thus, for example, $t_{2d} > 0$ but $t_{21} = 0$[2]. For this to be consistent we must assume that in selling a good, the seller pays a fixed unit cost (t_{id}), regardless of which sector is buying that good. But for some of the sectors to which the sales are made, the cost of the transaction per unit of good will exceed this cost (i.e. t_{id}) and consequently the buyer must pay the excess (i.e. t_{ij}). For these sectors where there is no excess to be paid by the buyer then $t_{ij} = 0$[3].

Having made the above assumptions, the attraction model can then be estimated from cross-section data, since the coefficients to be estimated are constant across all regions. However, the Ordinary Least Squares estimating process cannot be used, without the results being biased, because the attraction system is of a simultaneous nature[4 5].

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1 See Ch. V for the results of the attraction analysis and the demand coefficients. See also Van Wickeren's (197) results page 80 - 1 for a similar conclusion that most demand attraction coefficients are positive.

2 This will be shown by $\lambda_{2d} > 0$ and $\lambda_{21} = 0$, which means that good 2 is influenced to some extent by demand and so the flow must have a cost, but good 1 is not influenced by the supply of good 2 since this is costless - see also Appendix III.

3 The problem of different costs accruing to difference sectors when buying the same good will be examined theoretically in greater detail in Section III.K when the Kuhn-Tucker conditions are applied to the attraction model.

4 Van Wickeren (197) uses Ordinary Least Squares - see pages 71 - 78.

5 See for example Christ (32) Ch. IX section 11 for a discussion of the bias when Ordinary Least Squares is used to estimate a simultaneous system.

Table III.2

Communication costs incurred by buyers and sellers in interregional trade

			t_{11} / t_{1d}	t_{12} / t_{1d}	t_{13} / t_{1d}
			t_{21} / t_{2d}	t_{22} / t_{2d}	t_{23} / t_{2d}
			t_{31} / t_{3d}	t_{32} / t_{3d}	t_{33} / t_{3d}
t_{11} / t_{1d}	t_{12} / t_{1d}	t_{13} / t_{1d}			
t_{21} / t_{2d}	t_{22} / t_{2d}	t_{23} / t_{2d}			
t_{31} / t_{3d}	t_{32} / t_{3d}	t_{33} / t_{3d}			

Notes (i) The figure in the top left of each cell represents the unit cost incurred by the buyers.

(ii) The figure in the bottom right of each cell represents the unit cost incurred by the seller.

(iii) The "t"'s no longer have any specific regional subscripts since they are assumed to be constant over all regions.

(iv) The top left and bottom right hand quadrants are empty because the communication costs associated with intra-regional trade flows are zero by assumption.

The simultaneity in the system can be seen intuitively from a small example. Suppose the output (or location) of industry i is influenced both by demand for its output and by the supply of product j. Suppose product j is partly influenced by demand. Now assume an exogeneous increase in demand for product i, which will stimulate its output in the region. This will cause either directly or indirectly an increase in the intermediate demand for product j, which in turn will stimulate product i through the supply effect. If the system is stable, these rounds of interaction will converge[1], with the two industries operating simultaneously on each other[2].

III.E. <u>The validity of the attraction model as a predictive system</u>

It is necessary to consider if we can legitimately manipulate a definition such as III.C.1 into a predictive equation such as III.C.7, without, so far, having made any behavioural hypothesis. This is done by Van Wickeren (197) by the "normalisation hypothesis" which says "in reality a sample of activities spread over a selected area has such a spatial structure that deviations from the optimal structure do not have statistical significance" (page 63). Thus an optimum pattern of location is assumed[3], and there is an implicit minimisation of communication costs in the system, and consequently we can turn a definition into an equation expressing output in a region in terms of fixed

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[1] This point will be discussed in greater detail later in Section III.G.

[2] See Appendix II for a more rigorous formulation of the attraction model as a simultaneous system. A proof that each equation in the system is general overidentified by the order conditions will be given, and a discussion of the practical and theoretical problems involved in estimating the attraction model as a simultaneous system will be found.

[3] For the conditions necessary for an optimum pattern of location and trade to exist, see the application of the Kuhn-Tucker conditions, to the attraction model, below in Section III.K.

coefficients and supply and demand factors. If the normalisation hypothesis holds true, then the system, in order to keep communication costs to a minimum, will be organised such that where the t's are very large between sectors these sectors will be attractive to each other and found close together[1 2]. In the short run the t's will probably hold constant, but in the longer run, especially with changing technological relationships, it is likely that the unit communication costs will alter relatively, and so the λ's will no longer be stable.

However for predicting in the short run it is necessary to manipulate an equation such as

$$ {}_r g = \hat{1}_{\cdot rr} f + \hat{1}_{\cdot r} \Delta \, {}_r g + (LA)' \, {}_r g $$

into

$$ {}_r g = \left[I - \hat{1}_r \Delta - (LA)' \right]^{-1} \hat{1}_{rr} f $$

This is similar to manipulating the Leontief definition

$$ BX + F = X $$

into

$$ X = (I - B)^{-1} $$

which is predictive.

This can be legitimately done because the elements of the B matrix are constants - they are constant because that is the only way that a bill of goods can be produced, so there is an implicit minimisation of

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1 This concept will be expanded when the relevant region is discussed in Section III.M.

2 It is shown in Appendix III that the larger the communication costs between two sectors, the larger the λ coefficients. A discussion of the range of values and interpretation of the λ's will also be found there.

the cost of inputs expressed in the B matrix. If we are to manipulate the attraction system in a similar way we must assume that the ${}_{r}\Delta$ and A matrices (composed of δ and α coefficients respectively) are constant[3], in the short run.

This is perhaps the most restrictive assumption made so far in attraction theory, but can be rationalised by claiming that because of the costs[2] in the system of changing geographically either the sources of inputs or the destination of sales, industries will tend to adhere to the same patterns of trade, as described by the δ and α coefficients.

III.F. National vs. regional coefficients[3]

In the above arguments we have defined total communication costs for each region in terms that are specific to that region, and then shown what assumptions are necessary if these specific regional definitions are to be used in cross-section analysis. We have also remarked that certain of these assumptions are not necessary if national coefficients are used. It is to the problem of whether to use national or specific regional coefficients that we now turn.

The problem stems from the fact that the λ's have to be estimated from cross-section data, and consequently the λ's must be composed entirely of terms that are constant across regions.

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1 We have already assumed that the λ's in the Υ and L matrices are constant in the short run by the normalisation hypothesis.

2 Such as information costs

3 Van Wickeren's (197) discussion of national vs. regional coefficients can be found on pages 8, 15, 26 - 31.

Proof that $\lambda_{kd} + \sum \lambda_{hk} = 1$

Using specific regional coefficients the equations to be estimated can be written[1]

$$ {}_j g_k = {}_j\lambda_{kd} \cdot {}_j d_k + \sum_{h=1}^{n} {}_j\lambda_{hk} \cdot \frac{{}_j\alpha_{hk}}{{}_j\beta_{hk}} \; {}_j g_h $$

where

$$ {}_j\lambda_{kd} = \frac{{}_j t_{kd}}{{}_j t_{kd} + \sum_{h=1}^{n} {}_j t_{hk} \; {}_j\beta_{hk}} $$

and

$$ {}_j\lambda_{hk} = \frac{{}_j t_{hk} \; {}_j\beta_{hk}}{{}_j t_{kd} + \sum_{h=1}^{n} {}_j t_{hk} \; {}_j\beta_{hk}} $$

Using national coefficients the reduced form can be written

$$ {}_j g_k = \lambda_{kd} \; {}_j d_k + \sum_{h=1}^{n} \lambda_{hk} \; \frac{\alpha_{hk}}{\beta_{hk}} \; {}_j g_h $$

where

$$ \lambda_{kd} = \frac{t_{kd}}{t_{kd} + \sum_{h=1}^{n} t_{hk} \, \beta_{hk}} $$

and

$$ \lambda_{hk} = \frac{t_{hk} \, \beta_{hk}}{t_{kd} + \sum_{h=1}^{n} t_{hk} \, \beta_{hk}} $$

---

[1] The equation presented here is the "reduced form" found in Van Wickeren (197) page 15. This is done merely to permit easy cross-references. The extra β term is included by Van Wickeren merely to constrain the coefficients to sum to unity. Neither the inclusion of this β term nor the use of Van Wickeren's reduced form will affect the relevance of the argument (to be presented in this section) to the attraction model derived in this chapter.

where the α's and β's are estimated directly from national input-output tables and the t's are national averages.

There are three basic arguments used by Van Wickeren to favour national rather than regional coefficients:

(i) since we are using cross-section data to estimate the coefficients, we would ideally like them to contain terms that were constant across all regions, rather than variables. When national coefficients are used, this problem is solved. However, we have shown that by making certain assumptions concerning the regional technologies (β) and the communication cost functions (t), specific regional coefficients can be used since the parameters to be estimated will be constant across regions.

(ii) A second argument, at the practical level, is that regional data is often lacking. However in our case this does not apply, since one of the basic aims of the study is to investigate whether the attraction model can be implemented with estimated regional data.

(iii) A third theoretical argument is "economic activities are supposed to be distributed over the ...... country in a national way ,..... From this it follows that differences between ${}_j\alpha_{hk}$ and α_{hk} could be caused only by a non-normal distribution over regions"[1].

---

[1] Van Wickeren (197), pages 8 - 9.

This last argument seems dubious, since ${}_j\alpha_{hk}$ will depend on the structure of industry in that region and on the technology used in that region[1]. Now the structure can vary from region to region because of rational economic causes[2].

(a) Certain industries will be footloose and their distribution between regions will appear random. This will exogeneously create different supply and demand factors between regions depending on where they arbitarily[3] choose to locate.

(b) Certain industries will be attracted towards certain natural resources, which are randomly distributed. This again will create different supply and demand factors between regions.

(c) Exogeneous final demand will vary quite markedly from region to region, since it is composed of investment expenditure, government expenditure, exports abroad and consumer expenditure. This last factor will vary considerably since it will be affected not only by consumer incomes and transfer payments, but also by

---

1 We have already assumed that technology does not vary between regions and so the ${}_j\alpha_{hk}$ will depend on the structure of industry in each region.

2 Van Wickeren (197) admits this (page 25) and suggests some reasons such as comparative advantage and economies of scale. Even without recourse to these "non-linear" arguments there are still rational economic reasons for the variation in structure. These reasons are given above in the main text.

3 If their location decisions were not arbitary, then they would not be completely footloose.

different regional tastes[1]. Consequently, using the national $_{hk}$ across all regions will give a false impression because this is an average of all the assumed rational regional distribution of goods[2], rather than the actual regional ones under consideration, and so will give a false definition of total communication costs[3].

It thus seems possible to defend regional coefficients in favour of national ones from the attacks made on them. It also seems possible to fault the use of national coefficients and these faults do not seem to apply when regional coefficients are used.

(i) Van Wickeren (197) states "the argument in favour of specific (regional) coefficients is connected with the arguments already mentioned [this concerned regions having different structures due to rational reasons]. If estimates based on (internationally)

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[1] Consider a simplified numerical example. Imagine that 100 units of good 1 are produced in each of two regions. This good is used only as an intermediate product in the goods 2 and 3 - both of which are purely demand orientated, i.e. they locate where demand is. Now in the first region consumers purchase good 2 and no good 3, and vice versa in the second region; this may be due to difference in tastes. Assume good 1 needs also some intra-industry inputs in order to produce. The system for the 2 regions and the composite national system can be shown as:

	Region I			Region II			National Average		
Good no.	1	2	3	1	2	3	1	2	3
Units of output received from Good 1	20	80	0	20	0	80	40	80	80
Coefficient	0.2	0.8	0	0.2	0	0.8	0.2	0.4	0.4

[2] $\alpha_{hk} = \underline{\sum_{j=1}^{m} {}_j\alpha_{hk}}$ where there are m regions.

[3] It is surprising that in Van Wickeren's discussion of regional vs. national coefficients, that even in the definition of communication costs using national coefficients, the regional definition of demand $({}_jd_k)$ is used. If the above argument concerning the use of national coefficients is valid, then the intermediate demand of each region should be the same as the national average since the structures are identical. However, we have tried to show that this argument is not valid.

false observations are carried out, their results will not be very reliable."[1]

(ii) A further problem arises from the fact that the industries classified in input-output tables are an aggregation of non-homogeneous industries. This means that the central diagonal elements of an input-output table are relatively large figures, and cannot be ignored a priori in the attraction analysis[2]. However if we use national coefficients, the "reduced form" to be estimated for industry 1 over the k various regional observations is:

$$ {}_1g_1 = \lambda_{1d}\ {}_1d_1 + \lambda_{11}\frac{\alpha_{11}}{\beta_{11}}\ {}_1g_1 + \lambda_{21}\frac{\alpha_{21}}{\beta_{21}}\ {}_1g_2 + \quad \lambda_{n1}\cdot\frac{\alpha_{n1}}{\beta_{n1}}\ {}_1g_n $$

$$ \vdots $$

$$ {}_kg_1 = \lambda_{1d}\ {}_kd_1 + \lambda_{11}\frac{\alpha_{11}}{\beta_{11}}\ {}_kg_1 + \lambda_{21}\frac{\alpha_{21}}{\beta_{21}}\ {}_kg_2 + \ \cdot\lambda_{n1}\frac{\alpha_{n1}}{\beta_{n1}}\ {}_kg_n $$ [3]

Consider the coefficient λ_{11} and its associated variable. There is a perfect correlation between ${}_ig_1$ and $\frac{\alpha_{11}}{\beta_{11}}\ {}_ig_1$ over all i, i = 1, 2....k, since α_{11} and β_{11} are national coefficients and do not vary over regions. Consequently in the estimation procedure, the λ_{11}

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[1] Van Wickeren (197) page 32.

[2] For example, to use Van Wickeren (197) page 36, the textile sector is an aggregation of weaving, spinning and finishing sectors, where the inter-industry flows between these sectors may be important in determining the location of one or more of these sectors. When they are aggregated these inter-industry flows become intra-industry flows.

[3] This is the reduced form used by Van Wickeren (197) page 37. Using the one derived in this study would not materially affect the arguments presented.

would be set to $\frac{\beta_{11}}{\alpha_{11}}$ [1] and all the other coefficients zero. If the regional coefficients ${}_{j}\alpha_{hk}$ were used the problem would not arise - unless of course an industry is attracted to nothing else but itself (which is a priori unlikely), in which case we would expect this peculiar result to arise from the analysis.

Because of the inconclusive nature of the arguments against regional coefficients and the fact that there are both theoretical and practical problems involved in using national coefficients, I propose to use regional coefficients (as shown so far) in the rest of this study.

III.G. Interpreting the multipliers

We have shown above how to estimate and interpret the λ's, and how the system can be used for predictive purposes. The assumptions that were necessary to make these steps have been discussed. We now turn to the attraction matrix $[I - \hat{\lambda}\,{}_{r}\Delta - (LA)']^{-1}$ and show how this can be interpreted in a similar way to the Leontief $(I - B)^{-1}$ matrix, which shows, as a result of an exogeneous change in demand, how the multiplier is composed. Thus for example, the cell row 2 column 1 of the attraction matrix shows the direct and indirect effects on industry 2 as a result of expanding the demand of industry 1 by 1 unit. This effect is composed of two parts:

(i) the demand effect - this is similar to the Leontief demand effect, except that the $\lambda_{kd} \leqslant 1$ [2] and ${}_{j}\delta_{hk} \leqslant {}_{j}\beta_{hk}$ -

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[1] $\frac{\beta_{11}}{\alpha_{11}} = 1$ in the case discussed here.

[2] See Appendix III for proof of this.

so consequently the demand effect is generally less than the traditional Leontief demand effect[1],

(ii) the supply effect - this shows that when the output of say industry 1 expands, some of its production (depending on ${}_{j}\alpha_{12}$) will be available as an intermediate input to industry 2, and this will have a stimulating effect on industry 2 providing $\lambda_{12} > 0$.

Both these forces then interact and have a direct and indirect effect. This can be demonstrated as follows. It is well known that the effects of the Leontief multipliers can be shown round by round by a power series[2].

$$I + B + B^2 + B^3 + \ldots\ldots\ldots\ldots + B^n$$

This can be done since

$$(I - B)\ (I + B + B^2 + \ldots\ldots B^n) = I - B^n$$

Now because all the column totals of B are less than 1 and non-negative, each element in B^n gets smaller as n becomes larger, and so the error

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1 This can be demonstrated by a small example. Assume that good 1 needs some inputs from good 2 of say 50 units in order to meet a final demand requirement. Generally good 2 will not expand its production by 50 units in the region, but the expansion will depend on how much it is attracted by demand (i.e. λ_{2d}). If industry 2 is not influenced by demand (i.e. λ_{2d} approaches zero) then most of the expansion of industry 2 will take place in another region where the factors that influence its location (in this case supply factors) are available. This is because if $\lambda_{2d} = 0$, then $t_{2d} = 0$ and demand can be moved between sectors costlessly and so no locational influence will be felt.

2 See for example Waugh (196) or Yan (203).

in using the power series as a proxy for $(I - B)^{-1}$ gets smaller, since $I - B^n$ approaches I, which is the result one obtains by post multiplying a matrix by its inverse[1].

If the attraction model is to be stable and not to explode, it seems possible to estimate the inverse by the expansion of a power series. This round by round effect of the multiplier will be useful for exposition purposes of how the multiplier is composed[2].

Let $\hat{\imath}_{\cdot r}\Delta \equiv Z$

$(LA)' \equiv W$

Therefore

$$[I - \hat{\imath}_{\cdot r}\Delta - (LA)'] \equiv (I - Z - W) \equiv [I - (Z + W)]$$

We can estimate the inverse as:

$$[I - (Z + W)]^{-1} \quad I + (Z + W) + (Z + W)^2 + \dots + (Z + W)^n$$

as n

$= I + Z + W$ round 1

$+ Z^2 + W^2 + ZW + WZ$ round 2

$+ Z^3 + Z^2W + ZWZ + ZW^2 + WZW + WZ^2 + W^3 + W^2Z$ (3) round 3

etc.

Any term beginning with Z can be interpretted as demand effect, and any term beginning with W can be interpretted as the supply effect. This can be best illustrated by a small numerical example.

---

1 The exact conditions for the power series to converge are known as the Hawkins-Simon (76) conditions. For an introduction see Chiang (30).

2 It will also prove useful in the analysis in later sections.

3 Note that generally $WZ \neq ZW$ since W and Z are matrices not scalers.

Assume Z =

1, 1	0.2	1, 2	0.1	1, 3	0.0
2, 1	0.1	2, 2	0.0	2, 3	0.2
3, 1	0.2	3, 2	0.1	3, 3	0.0

where, for example, row 1 column 2 (1, 2) shows that for each unit of product 2 produced, the direct effect on product 1 in the region is 0.1. This is because product 2 requires (demands) some of product 1 (in proportion to ${}_j\delta_{12}$) and product 1 is attracted by this demand (in proportion to its demand attraction coefficient λ_{1d}).

Assume W =

1, 1	0.1	2, 1	0.1	3, 1	0.2
1, 2	0.2	2, 2	0.1	3, 2	0.0
1, 3	0.0	2, 3	0.2	3, 3	0.1 [1]

Where, for example, 2, 1 means that when product 2 produces some output it sends some of its output to product 1 (in proportion to the supply coefficient ${}_j\alpha_{21}$) and product 1 is attracted to this supply (in proportion to its supply attraction coefficient λ_{21}).

Now consider WZ matrix multiplication

$1,1 = [(0.1)(0.2) + (0.1)(0.1) + (0.2)(0.2)]$ $1,2 = [(0.1)(0.1) + (0.1)(0.0)$ $(0.2)(0.1)]$ etc.

$2,1 = [(0.2)(0.2) + (0.1)(0.1) + (0.0)(0.2)]$ etc.

This shows because from an initial demand effect shown by Z there will be a feedback stimulus through the supply effect. For example, the cell 1,1 of the WZ matrix shows that from the demand for inputs from product 1 for products 1, 2 and 3, these later will have a feedback

---

[1] Note that $W \equiv (LA)'$, so that the ordering of the rows and columns is transposed in order to be consistent with the notation.

supply influence on the location of product 1. This works in the following way: product 1 demands as intermediate inputs some units of product 2 (shown by ${}_j\delta_{21}$) and attracts a proportion of this demand (shown by λ_{2d}). The result is that product 2 expands production in the region by 0.1 units[1]. This is what the Z matrix shows. However once product 2 has been attracted to the region, it will create supplies of product 2 for product 1 (shown by ${}_j\alpha_{21}$) which will be attractive to product 1 (shown by λ_{21}) and this results per unit of output of product 2 of an increase in 0.1 units of product 1[2]. Thus from the 0.1 units of product 2 attracted by demand the supply effect on product 1 is (0.1)(0.1). This is shown as the second term in all 1,1 of the WZ matrix. The other two terms in this cell (0.1)(0.2) and (0.2)(0.2) can be explained in the same way as above only using products 1 and 3 respectively instead of product 2. They all have an influence on product 1 which can be found by adding the terms together. Thus the WZ matrix shows the supply effects generated in that round.

Now consider ZW matrix multiplication:

$$1,1 = \left[(0.2)(0.1) + (0.1)(0.2) + (0.0)(0.0)\right] \quad 1,2 = \left[(0.2)(0.1) + (0.1)(0.1) \; (0.0)(0.2)\right] \quad \text{etc.}$$

$$2,1 = \left[(0.1)(0.1) + (0.0)(0.2) + (0.2)(0.0)\right] \quad \text{etc.}$$

This shows from an initial supply effect shown by W there will be a feedback stimulus through the demand effect for example, cell 1,1 means that from the supply of 1 to other products and the stimulation to these products (namely 1, 2 and 3 with effect 0.1, 0.2, 0.0) through

---

1 See Z matrix row 2 column 1 (2,1)

2 See W matrix row 1 column 2 (2,1)

supply, these products are going W demand per unit of output some output of product 1 (shown by 0.2, 0.1, 0.0). Adding these terms together will give the total demand effect on product 1 resulting from row 1 of the W matrix. Thus the ZW matrix shows the demand effects generated in that round.

By similar arguments any term beginning with W can be called the supply effect and any term beginning with Z the demand effect.

We are now in a position to interpret the whole power series. The I represents the initial effect if the product is bought directly from a firm in the region or if the firm is placed there by government policy. If there is just a general increase in final demand (perhaps as a result of an increase in consumer incomes) the I will have to be scaled down by the appropriate λ_{kd}'s, because not all the expansion of the industry will take place in the region, but will depend on how much influenced by demand that industry is. This is in fact the general case arrived at in equation III.C.7

$$ {}_{r}g = [I - \hat{\lambda}\,{}_{r}\Delta - (LA)']^{-1}\,\hat{\lambda}\,{}_{rr}f $$

So the initial effect will depend on the type of policy that is actually implemented[1].

After the initial effect we can proceed through the power series round by round:

---

[1] If λ_{kd}'s are very low and a government merely increases final demand say through transfer payments in a region, then most of the effect will leak outside the region, and the employment created even after the multiplier will therefore be very small. However, if a government buys directly from a firm in the region, all of the initial effect will stay in the region and employment created will necessarily be larger.

Z = direct demand effects as a result of initial expansion

W = direct supply effects as a result of initial expansion

Z^2 = the indirect effect on demand of the above Z effects

W^2 = the indirect effect on supply of the above W effects

ZW = the indirect effect on demand of the above W effects

WZ = the indirect effect on supply of the above Z effects

etc.

Tracing through the multiplier round by round may be useful in estimating where bottlenecks in excluded factors, such as labour, may be encountered. From a practical point of view, expansion of a power series to estimate the multiplier may be more economical on computer time than inverting the matrix[1].

III.H. Interregional feedback effects of the attraction multipliers - an integration with interregional input-output tables

In interregional input-output analysis, the feedback effects between regions can form a significant part of the total multiplier, since feedback effects can alter not only the size of the multiplier for each sector but also perhaps the rankings of the effects of different types of investment programmes, if some industries have a larger feedback than others[2]. It would thus seem to be important to try to integrate them into the attraction model.

As shown above we can obtain an attraction multiplier without feedback effects of the effect on production in one region, by:

---

[1] The results of this, and other experiments on the attraction tables will be reported in Ch. V.

[2] See Chapter IV for a discussion of interregional input-output systems and feedback effects.

$[I - \hat{T}_r \Delta - (LA)']^{-1} . \hat{T} f$

Now let us consider the leakages from this system to RUK[1]. This can best be analysed by examining the leakages as originating from two sources:

(i) During the initial stimulation of the regional economy by the exogeneous f, not all of that which is demanded is produced in the region[2], but only a certain proportion[3], as indicated by $\hat{T}$, the remainder, $I - \hat{T}$, being produced elsewhere in the nation. Now at each round of feedbacks from RUK some products are being demanded as intermediate inputs, but again, not all these will be produced in the region[3], only that proportion indicated by $\hat{T}$, the remainder $I - \hat{T}$, being produced elsewhere in the nation. Let this leakage be called leakage type I.

(ii) Of that product that is produced in the region as a result of the impact from an exogeneous f (and after the leakage type I) there will still be a further leakage into RUK. This comes from that proportion of intermediate input that has to be imported into the region from RUK(i.e. the bottom left hand quadrant of Table IV.I). Now in constructing

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[1] RUK stands for the Rest of the United Kingdom and means the whole of the United Kingdom minus the region in question.

[2] As would be the case in normal interregional input-output models.

[3] Since industries are not generally wholly demand orientated.

attraction theory we assumed that the $_r\Delta$ matrix (i.e. the top right-hand quadrant of Table IV.I) is composed of constant coefficients. We also assume the technology used is constant (i.e. the Leontief B matrix). Therefore the $(B - {}_r\Delta)$ matrix is constant - this is the bottom left-hand quadrant of Table IV.I. Thus we can estimate this further leakage - let it be called leakage type II.

From these two components of demand on the RUK, we could estimate the effects, not only on the RUK itself, but also on how much will be demanded back from the region. This last effect is shown by the top right-hand quadrant[1] of the matrix in Table III.1. The process could then be repeated iteratively, until the system converged.

It may seem at first as though we are ignoring the supply effects in RUK because the coefficients in the top right and bottom right-hand quadrants of Table III.1 also show supplies from the region and from RUK to RUK economy in a similar way that demand did. But this is not so, because we only wish to know how many goods are going to be produced in RUK in order to meet the demand that has leaked from the region. This is purely a technological question[2] of how many intermediate goods are necessary physically to provide for the given demand. Introduction of the supply effect would only be necessary if we were to subdivide the RUK into regions, and wanted to know how the leakage from the initially stimulated region was allocated amongst the other regions. Then we would have to know the coefficients of supply to each of these other regions, and then each of these regions would have a demand

---

1 This quadrant can be shown to be composed of constant coefficients from the assumptions already made, in a similar way that the $(B - {}_r\Delta)$ matrix was shown to be constant.

2 That is once the trading patterns are fixed. They are by assumption in the short run - see Ch. IV for a discussion of fixed trading patterns in input-output.

leakage in a similar manner to the original region. However, this would be beyond our data capacity and so will not be developed.

Rather than estimate the feedback process round by round as in the expansion of the power series, it was found easier to tackle this problem in the following way. Using Table III.1 as the basis for the elements and matrices of the interregional input-output system:

(i) let the coefficient of the intra-regional flows in the top left-hand quadrant be ${}_{r}\Delta$ and each element ${}_{r}\delta_{ij}$

(ii) let the coefficients of the flows from the region to RUK, i.e. the top right-hand quadrant be called σ and each element σ_{ij}

(iii) let the coefficients of the flows from RUK to the region, i.e. the bottom left-hand quadrant be called ${}_{r}\sigma$, and each element ${}_{r}\sigma_{ij}$

(iv) the coefficients of the intra-RUK flows, i.e. the bottom right-hand quadrant be called Δ and each element δ_{ij}.

So that $B - \Delta = \sigma$ and $B - {}_{r}\Delta = {}_{r}\sigma$

Summarizing the notation so far:

${}_{r}\Delta$ $({}_{r}\ {}_{ij})$	σ (δ_{ij})
${}_{r}\sigma$ $({}_{r}\sigma_{ij})$	Δ (δ_{ij})

Let (i) $f_{1\,rex}$ = an exogeneous (ex) increase in demand in the region (r) for product 1. ($\underline{F}_{rex}$ = a vector of such demands)

(ii) $f_{1\,ex}$ = an exogeneous (ex) increase in demand in the RUK for product 1. ($\underline{F}_{ex}$ = a vector of such demands)

(iii) $f_{1\,r}$ = the increase in demand for product 1 in the region (r) as a result of the feedback effects from RUK ($\underline{F}_r$ = a vector of such demands)

(iv) $f_{1\sigma}$ = increase in demand in RUK for product 1, as a result of the demand for intermediate inputs in the region ($\underline{F}_\sigma$ = a vector of such demands) i.e. Leakage type II

(v) $f_{1\lambda}$ = increase in demand in RUK for product 1 as a result of the direct demand leakage from the region ($\underline{F}_\lambda$ = a vector of such demands) i.e. Leakage type I .

Also let ${}_rg$ = vector of gross outputs in the region and

g = vector of gross output in RUK.

The feedback system will be shown for industry 1, before writing in general matrix notation. All the variables will be in terms of increases, so this is omitted from the notation for the sake of clarity. Assume a three-good economy.

(i) The direct effect on product 1 in the region (${}_rg_1$) as a result of an exogeneous change in demand in the region ($\lambda_{1d} \cdot f_{1rex}$) and/or as a result of feedback from RUK ($\lambda_{1d} \cdot f_{1r}$) and/or as a result of the effect of intermediate demand and supply of the output of other industries in the region, can be written

$$_r g_1 = \lambda_{1d}\ _r\delta_{11} + \lambda_{1d}\ _r\delta_{12}\ _r g_2 + \lambda_{1d}\ _r\delta_{13}\ _r g_3 + \lambda_{11}\ _r\alpha_{11}\ _r g_1 + \lambda_{21}\ _r\alpha_{21}\ _r g_2 + \lambda_{31}\ _r\alpha_{31}\ _r g_3 + \lambda_{1d} f_{1rex} + \lambda_{1d} f_{1r}$$

(ii) The direct effects on RUK (g_1) as a result of Type II ($f_{1\sigma}$) and/or Type I ($f_{1\lambda}$) leakages from the region and/or as a result of intermediate demand in the RUK, and/or as a result of a change in final demand in RUK (f_{1ex})

$$g_1 = \delta_{11} g_1 + \delta_{12} g_2 + \delta_{13} g_3 + f_{1\sigma} + f_{1\lambda} + f_{1ex}$$

(iii) The direct effects on product 1 in RUK as a result of the leakages of intermediate demand from the region, i.e. leakage Type II

$$f_{1\sigma} = {}_r\sigma_{11}\ _r g_1 + {}_r\sigma_{12}\ _r g_2 + {}_r\sigma_{13}\ _r g_3$$

(iv) The direct increase in demand on product 1 in the region as a result of the leakage of intermediate demand from RUK

$$f_{1r} = \sigma_{11} g_1 + \sigma_{12} g_2 + \sigma_{13} g_3$$

(v) The direct increase in demand on product 1 in RUK as a result of the demand leakage from the region i.e. leakage Type I

$$f_{1\lambda} = (1-\lambda_{1d})\ _r\delta_{11}\ _r g_1 + (1-\lambda_{1d})\ _r\delta_{12}\ _r g_2 + (1-\lambda_{1d})\ _r\delta_{13}\ _r g_3 + (1-\lambda_{1d}) f_{1rex} + (1-\lambda_{1d}) f_{1r}$$

Re-writing these five equations respectively in general matrix notation:

$$ {}_r g = \hat{I}\,{}_r\Delta\,{}_r g + (LA)'\,{}_r g + \hat{I}\,\underline{F}_{rex} + \hat{I}\,\underline{F}_r \quad [1] \qquad \text{III.H.1} $$

$$ g = \Delta g + \underline{F}_\sigma + \underline{F}_\lambda + \underline{F}_{e\lambda} \qquad \text{III.H.2} $$

$$ \underline{F}_\sigma = {}_r\sigma\,{}_r g \qquad \text{III.H.3} $$

$$ \underline{F}_r = \sigma g \qquad \text{III.H.4} $$

$$ \underline{F}_\lambda = (I-\hat{I})\,{}_r\Delta\,{}_r g + (I-\hat{I})\,\underline{F}_{rex} + (I-\hat{I})\,\underline{F}_r \qquad \text{III.H.5} $$

Substitute III.H.3, III.H.5 and III.H.4 into III.H.2

$$ g = \Delta g + {}_r\sigma\,{}_r g + (I-\hat{I})\,{}_r\Delta\,{}_r g + (I-\hat{I})\,\underline{F}_{rex} + (I-\hat{I})\,\sigma g + \underline{F}_{e\lambda} \qquad \text{III.H.6} $$

and substitute III.H.4 into III.H.1

$$ {}_r g = \hat{I}\,{}_r\Delta\,{}_r g + (LA)'\,{}_r g + \hat{I}\sigma g + \hat{I}\,\underline{F}_{rex} \qquad \text{III.H.7} $$

From III.H.6 we can obtain an expression for without a g on the R.H.S.

$$ (I-\Delta-\sigma+\hat{I}\sigma)\,g = {}_r\sigma\,{}_r g + (I-\hat{I})\,{}_r\Delta\,{}_r g + (I-\hat{I})\,\underline{F}_{rex} + \underline{F}_{e\lambda} \qquad \text{III.H.8} $$

---

[1] Remember $(LA)'$ is a matrix where each element is $\lambda_{ij}\cdot{}_r\alpha_{ij}$ rather than the result of matrix multiplication.

Therefore

$$g = [I - \Delta - \sigma - \hat{i}\,\sigma]^{-1} [\,{}_r\sigma\,{}_rg + (I - \hat{i})\,{}_r\Delta\,{}_rg + (I - \hat{i})\,\underline{F}_{rex} + \underline{F}_{ex}] \qquad \text{III.H.9}$$

Define $(I - \hat{i})$ as ϕ

and $[I - \Delta - \sigma + \hat{i}\,\sigma]^{-1}$ as ψ

then substitute III.H.9 into III.H.7

$${}_rg = \hat{i}\,{}_r\Delta\,{}_rg + (LA)'\,{}_rg + \hat{i}\,\underline{F}_{rex} + \hat{i}\,\sigma\,\psi[\,{}_r\sigma\,{}_rg + \phi\,{}_r\Delta\,{}_rg + \phi\,\underline{F}_{rex} + \underline{F}_{ex}] \qquad \text{III.H.10}$$

Multiplying out and collecting terms with ${}_rg$ to the L.H.S. gives:

$$[I - \hat{i}\,{}_r\Delta - (LA)' - \hat{i}\,\sigma\,\psi\,{}_r\sigma - \hat{i}\,\sigma.\psi\,\phi\,{}_r\Delta]\,{}_rg = \hat{i}\,\sigma\,\psi.\phi\,\underline{F}_{rex} + \hat{i}\,\sigma.\psi\,\underline{F}_{ex} + \hat{i}\,\underline{F}_{rex} \qquad \text{III.H.11}$$

Therefore

$${}_rg = [I - \hat{i}\,{}_r\Delta - (LA)' - \hat{i}\,\sigma\,\psi\,{}_r\sigma - \hat{i}\,\sigma\,\psi.\phi\,{}_r\Delta]^{-1} [\hat{i}.\sigma\,\psi\,\phi.\underline{F}_{rex} + \hat{i}\,\sigma.\psi\,\underline{F}_{ex} + \hat{i}\,\underline{F}_{rex}] \qquad \text{III.H.12}$$

This can be seen to be an equation for ${}_rg$, expressed in terms of an initial matrix composed entirely of constant coefficients post multiplied by three final demand vectors (which are themselves pre-multiplied by constant coefficients). Multiplication by the $\hat{i}\,\sigma.\psi\,\phi\,\underline{F}_{rex}$

and $\hat{1} \underline{F}_{rex}$ vectors will tell us the direct and indirect feedback effects on output in the region as a result of a change in final demand in the region. The $\hat{1} \underline{F}_{rex}$ vector shows the effect from the initial increase in final demand in the region, and the $\hat{1} \sigma \psi \phi \underline{F}_{rex}$ vector shows the effect from the initial increase in demand in the region, some will leak out to R.U.K. straight away but this will have a feedback effect on the region.

The $\hat{1} \sigma \psi \underline{F}_{ex}$ vector will tell us the direct and indirect feedback effects on output in the region as a result of a change in final demand in RUK.

Alternatively we could obtain an expression for g instead of $_r g$. This can be done by collecting all the terms containing $_r g$ to the L.H.S. of equation III.H.7

$$[I - \hat{1}\ {}_r\Delta - (LA)'\ ]\ {}_r g = \hat{1}.\sigma\ g + \hat{1}.\underline{F}_{rex} \qquad \text{III.H.13}$$

Therefore

$${}_r g = [I - \hat{1}\ {}_r\Delta - (LA)'\ ]^{-1}[\hat{1}.\sigma.\ g + \hat{1}.\underline{F}_{rex}]$$

III.H.14

Define $[I - \hat{1}\ {}_r\Delta - (LA)'\ ]^{-1}$ as Θ

and substitute III.H.14 into III.H.6

$$g = \Delta\ g + {}_r\sigma[\Theta\ \hat{1}\ \sigma\ g + \Theta\ \hat{1}\ \underline{F}_{rex}]$$
$$+ \phi\ {}_r\Delta[\Theta\ \hat{1}\ \sigma\ g + \Theta\ \hat{1}.\underline{F}_{rex}] + \underline{F}_{ex} + \phi\ \underline{F}_{rex} + \phi.\sigma\ g$$

III.H.15

Collecting all terms with g to the L.H.S.

$$[I - \Delta_{-r}\, \sigma\, \theta\, \hat{i}_r\, \sigma - \phi_r\, \Delta\, \theta\, \hat{i}\, \sigma - \phi_r\, \sigma]\, g =$$

$$_r\sigma_r\, \theta\, \hat{i}_r\, \underline{f}_{rex} + \phi_r\, \Delta\, \theta_r\, \hat{i}\, \underline{f}_{rex} + \underline{f}_{\ell x} + \phi\, \underline{f}_{rex}$$

III.H.16

Therefore

$$g = [I - \Delta_{-r}\, \sigma\, \theta\, \hat{i}_r\, \sigma - \phi_r\, \Delta_r\, \theta\, \hat{i}\, \sigma - \phi\, \sigma]^{-1}$$

$$[_r\sigma\, \theta\, \hat{i}\, \underline{f}_{rex} + \phi_r\, \Delta\, \theta_r\, \hat{i}\, \underline{f}_{rex} + \underline{f}_{\ell x} + \phi\, \underline{f}_{rex}]$$

III.H.17

Which shows output in the RUK in terms of constants and changes in final demand in the region and RUK.

III.I. One practical method of using the attraction model to identify footloose industries[1]

It is well known that the ordering of industries in input-output tables is purely arbitary, and by exchanging the relative position of any two rows and similarly exchanging their corresponding columns, the information in the table is unaltered. Now it may be possible to exchange the rows and columns of a national input-output table, such that the resulting form is triangular, where all (or most) of the elements above the leading diagonal are zero. Such a scheme is shown in Fig. III.1.

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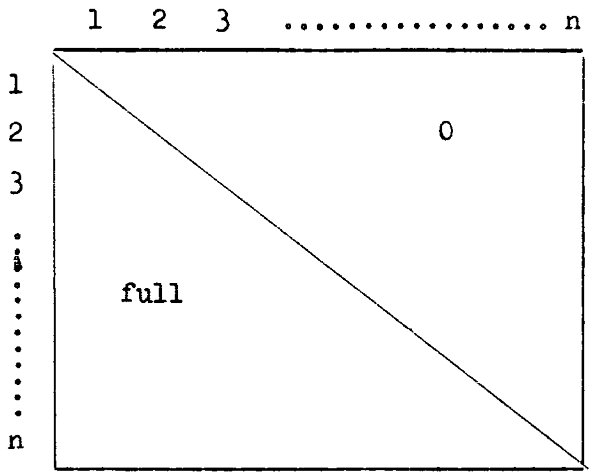
[1] The concept of footloose industries in the attraction model will be discussed in Section III.M. See also Van Wickeren (197) Ch. 4.

Fig. III.1 A triangularised input-output table



This is often done at the national level in order to identify 'basic' industries[1]. Let us apply this type of analysis to the regional attraction model.

Consider the case where demand was the only factor that influenced location, and examine the $\left[I - T_{\cdot r} \Delta - (LA)' \right]^{-1}$ where $(LA)'$ is a null matrix. Now if this matrix could be triangularised in the manner described above, we would be able to identify the industries near the beginning of the ordering (e.g. 1 and 2) as relatively footloose, and those near the bottom (e.g. n and n-1) as relatively non=footloose. 1 and 2 are relatively footloose because when stimulated exogeneously, they will influence the location of others through the demand effect (which is shown by the relatively full column 1) since other industries have directly or indirectly expensive (high) communication costs (t's) with sectors 1 and 2; but when other industries are stimulated exogeneously, there is no effect on industries 1 and 2 (which is shown by the relatively empty row) since industries 1 and 2 have no expensive communication costs with other sectors. Consequently, the location

[1] This is discussed in Yan (203) and examples are given in Simpson and Tsuki (168). A specific computer algorithm for minimising the sume of the above diagonal elements is given in Ramsey et. al. (155).

of industries 1 and 2 is not dependent on the demand from other industries and may be described as footloose with respect to inter-industry relations[1]. These industries would be good candidates for a regional development policy because not only are they relatively footloose, and so can be moved cheaply, but also their columns tend to be full, so giving them a large multiplier effect on the region.

A small numerical example will help to show the point about the industries at the top of the ordering after triangularisation being relatively footloose. Suppose the following are the ${}_r\delta$ coefficients for a 3-industry model:

	1	2	3
1	0.2	0.2	0.2
2	0.2	0.2	0.2
3	0.2	0.2	0.2

These coefficients have all been made the same, so that when the final result is produced it will be easier to see to which factors the ordering of the industry is attributable. Now assume the following demand coefficients have been estimated:

industry	1	2	3
demand coefficient	0	0.5	1.0

Now a priori we know industry 1 to be footloose as far as demand is concerned, industry 3 to be totally locationally tied to demand and industry 2 is an intermediate case[2]. The $\hat{\Lambda} \cdot {}_r\Delta$ matrix is:

---

1 Remember only demand is being considered in this case.

2 See Appendix III for a proof of this and how the various λ_{kd}'s are related to their respective t_{kd}'s.

	1	2	3
1	0	0	0
2	0.1	0.1	0.1
3	0.2	0.2	0.2

Where, for example, cell 1, 2 = 0 because when expanded industry 2 will demand some of industry 1, but there will be no effect on industry 1 in the region, since its attraction to demand is zero (i.e. $\lambda_{1d} = 0$) since t_{1d} must = 0.

Now the $[I - \hat{\lambda}_d A]^{-1}$, showing the direct and indirect requirements is[1]

	1	2	3
1	1	0	0
2	0.143	1.143	0.143
3	0.286	0.286	1.286

This matrix is already on its most triangular form (i.e. ordered industries 1, 2, 3) and we know a priori in this simple example industry 1 is footloose and industry 3 to be locationally tied.

We can apply a similar analysis to the supply side as we did with demand. Consider the case where supply is the only influence where $\hat{\lambda}_s A$ is a null matrix. If the attraction matrix could be triangularised, then industries near the top of the ordering can be described as footloose, since the direct and indirect effects of all other industries on these is small, i.e. industry 1 is not attracted by the supply of others. Conversely industries near the bottom of

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[1] The inversion was estimated on a small programmed calculator and so may be subject to small errors. In any case the figures are rounded to three places of decimals.

the ordering are not footloose since they depend on supplies of all the other industries. Again industry 1 would be a good candidate for regional development since, not only is it footloose, but the column tends to be relatively full, so making the multiplier effect larger.

A small numerical example will help to make the point. Suppose the following are the coefficients for a 3-industry model are:

	1	2	3
1	0.2	0.2	0.2
2	0.2	0.2	0.2
3	0.2	0.2	0.2

Again, these coefficients have all been made the same, so that when the final result is produced it will be easier to see to which factors the ordering of the industry is attributable. Now assume the following have been estimated:

$$ {}_r g_1 = 0.\,{}_r\alpha_{11} + 0.\,{}_r\alpha_{21} + 0\,{}_r\alpha_{31} $$

$$ {}_r g_2 = 0.5\,{}_r\alpha_{12} + 0.5\,{}_r\alpha_{22} + 0.5\,{}_r\alpha_{32} $$

$$ {}_r g_3 = 1.0\,{}_r\alpha_{13} + 1.0\,{}_r\alpha_{23} + 1.0\,{}_r\alpha_{33} $$

A priori we know industry 1 to be footloose as far as supply is concerned and industry 3 to be locationally tied to supply[1].

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[1] See Appendix III for a proof of this and how the various λ_{ij}'s are related to their respective t_{ij}'s.

The $(LA)'$ matrix is:

	1	2	3
1	0	0	0
2	0.1	0.1	0.1
3	0.2	0.2	0.2

This is identical to the $\hat{l}_r \Delta$ matrix shown above and so it is not necessary to repeat the inversion. We also know that in the triangularisation process, the optimum ordering is 1, 2, 3 where a priori we know industry 1 to be footloose.

Now these arguments can be combined and the full $[I - \hat{l}_r \Delta \cdot (LA)']^{-1}$ triangularised, since we know that this is composed of a series of supply and demand effects[1]. If this matrix approaches a triangular form, we know that the industries labelled 1, 2 etc. will be good candidates for a regional development programme for both the reasons mentioned above (i.e. they are both relatively footloose and have larger multiplier effects). However it must be stressed that they are only footloose as far as the influence of inter-industry relations are concerned[2].

A small numberical example may help to make the point clearer. Assume the following have been estimated:

$$ {}_r g_1 = 0\ \lambda d_1 + 0\ {}_r d_{11} + 0\ {}_r d_{21} + 0\ {}_r d_{31} $$

$$ {}_r g_2 = 0.5\ \lambda d_2 + 0.5\ {}_r d_{12} + 0.5\ {}_r d_{22} + 0.5\ {}_r d_{32} $$

$$ {}_r g_3 = 0.9\ \lambda d_3 + 1.0\ {}_r d_{13} + 1.0\ {}_r d_{23} + 1.0\ {}_r d_{33} $$

---

[1] See Section III.G. above.

[2] The introduction of other factors into the attraction model will be shown below in Section III.J.

Where we know a priori that industry 1 is footloose and industry 3 locationally tied[1]. Assuming the ${}_r\Delta$ and A matrices as above, then $[I - \Gamma\, {}_r\Delta - (LA)']^{-1}$ is:

1	0	0
0.476	1.476	0.476
0.905	0.905	1.905

Which is again in its most triangular form when ordered 1, 2, 3. Thus triangularisation may be seen as a useful tool to select footloose industries[2], and so play a part in regional development programmes.

III.J. Introduction of addition factors into attraction analysis - a study of the influence of labour on industrial location

To date, we have been concerned with the mobility of an industry relative to inter-industry relations. However, as we have seen, other factors have been suggested as important in determining the location of industry[3]. One of the principal factors singled out has been the availability (either quality or quantity) and price of labour. It would

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[1] Again see Appendix III for a proof of this.

[2] It must be stressed concerning the above analysis that the triangular ordering of industries shown above may not give the ordering of industries with the least communication costs. This is because we cannot solve absolutely for all the t_{kd}'s and t_{ij}'s but they can only be solved relatively for the individual industries' t_k - see Appendix III for this. Thus although an industry may only have a small element above the main diagonal and so have a high position in the ordering, it is possible that the t's may actually be quite large and the industry may in fact not be footloose. But, ceteris paribus, it seems that industries high in the ranking will more likely be footloose because they are influenced much less by other industries.

[3] See Ch. 1 for example.

thus seem possible to increase the generality of the attraction model, by introducing labour into the analysis. However, in doing this I shall take a slightly different course from the one conventionally adopted[1], so that it will be possible to see how footloose an industry is with regard to inter-industry relations and labour together, and then to introduce the effects of consumption into the attraction multipliers.

Labour can be introduced into the normal Leontief input-output model[2], and provided that certain restrictive assumptions are made concerning the consumption function, labour can be treated as a normal intermediate good. However, labour cannot be introduced into our formulation of the attraction model quite so easily. This is because of certain restrictive assumptions concerning labour were necessary to construct the interregional input-output data. These were:

(i) labour is a non-transportable good, i.e. in the short run there is no interregional migration, thus in an interregional input-output table, we need two matrices of inputs of intermediate goods, since we need to know the origin of the goods, but we only need a vector of primary inputs, since we assume all labour originates in the region;

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1 The conventionally adopted approach is taken to be Van Wickeren (197) pages 47 - 51 for a static model and Ch. 6 for a dynamic model. This approach is slightly different because Van Wickeren's aim was to predict the amount of labour that would be required, rather than the mobility of industry.

2 See Ch. IV for a discussion of the introduction of labour and consumption into input-output.

(ii) there is no substitution of inputs in the Leontief production function, although with intermediate goods the region of origin may be varied, the type of good may not.

Now let us introduce labour into the attraction model (using the two assumptions mentioned above) treating labour as a normal intermediate good.

Define:

$$t_k \cdot {}_r g_k = t_{kd}({}_r g_k - {}_r dd_k) + \sum_{h=1}^{n} t_{hk} (\beta_{hk} \cdot {}_r g_k - {}_r\alpha_{hk} \cdot {}_r g_h)$$

$$+ t_{Lk}(\beta_{Lk} \cdot {}_r g_k - {}_r\alpha_{Lk} \cdot {}_r g_L)$$

where all the symbols have their usual meaning, except: ${}_r g_k$ runs from 1, 2.....n, L where there are n industries and L = labour sector.

t_k = unit communication costs (or extra production costs that now includes a measure of labour cost shortage in the region rather than just referring to interindustry relations)

t_{Lk} = unit communication cost per unit of labour into industry k, when that labour is not at hand in the region.

$_{Lk}$ = Leontief coefficient for labour demand from k in region r.

${}_r\alpha_{Lk}$ = supply coefficient from L to k in region r.

Using the usual methods the equation to be estimated is:

$${}_r g_k = \frac{t_{kd}}{M} \cdot {}_r dd_k + \sum_{h=1}^{n} \frac{t_{hk}}{M} \cdot {}_r\alpha_{hk} \cdot {}_r g_h + \frac{t_{Lk}}{M} \cdot {}_r\alpha_{Lk} \cdot {}_r g_L$$

where $M \equiv t_{kd} + \sum_{h=1}^{n} t_{hk} \cdot \beta_{hk} + t_{Lk} \cdot \beta_{Lk} - t_k$

In estimation, ${}_r g_k$ is regressed on

$$\lambda_{kd} \cdot {}_r dd_k + \sum_{h=1}^{n} \lambda_{hk} \cdot {}_r\,{}_{hk} \cdot {}_r g_h + \lambda_{Lk} \cdot {}_r\,{}_{Lk} \cdot {}_r g_L$$

in order to estimate the λ's.

However, using assumptions (i) and (ii) above, i.e. that there is no interregional trade and no substitution for labour then

${}_r\alpha_{Lk} \cdot {}_r g_L = \beta_{Lk} \cdot {}_r g_k$ always,

i.e. an industry always gets the labour it needs in the region.

But β_{Lk} is a constant across all regions, by assumption. Thus the regression will always estimate ${}_r\alpha_{Lk} \cdot {}_r g_L$ as the important factor, ignoring all others and set $\lambda_{Lk} = \frac{1}{\beta_{Lk}}$[1]. This result is consistent with the assumptions (i) and (ii) made concerning labour - that labour is immobile between regions (i.e. that it has infinitely high communication costs) and is needed in fixed proportions (i.e. it cannot be substituted). All other inputs can be brought into the region at a cost, that although positive, is much cheaper than that of moving labour. So industry always locates near labour.

In order to obtain meaningful results concerning labour, it is necessary to relax one or both of our assumptions concerning labour. The immobility of labour between regions seems a reasonable assumption to make in the short run since it seems unlikely to migrate between regions instantaneously in response to local changes in demand[2]. If the immobility assumption holds true, then it is necessary to relax the substitution assumption and it seems possible that we can do this

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[1] This is similar to one of the problems encountered when national, rather than regional, coefficients are used - see Section III.F.

[2] In a long run model this assumption would have to be relaxed. See for example Creedy (34) as an explanation of UK interregional migration.

meaningfully. We can assume that if there is not enough of the correct type of labour available in a region it can be substituted for capital, or other types of less skilled, wrong aged/sexed or overtime labour, which costs more (t_{Lk}) per unit of output. So t_{Lk} can be defined as the extra cost per unit of output involved in having less labour of the correct type than the optimum Leontief coefficient, such that:

$$\beta_{Lk} \cdot r^{g_k} \lesseqgtr r^{\alpha_{Lk}} \cdot r^{g_L}$$

rather than:

$$\beta_{Lk} \cdot r^{g_k} = r^{\alpha_{Lk}} \cdot r^{g_L} \text{ as was assumed above.}$$

Thus, substitution of labour seems reasonable, if it cannot be bought from inside the region or imported. Labour is not, however, an homogeneous good as is, say, iron-ore, but can be subdivided by quality into perhaps three meaningful categories on which data ia available - skilled, semi-skilled and unskilled. If labour can be subdivided into these categories, it makes the abandonment of the assumption of no substitution even more plausible, since there is more flexibility introduced into the system.

If the above analysis is used with labour, then the equation to be estimated is

$$r^{g_k} = \lambda_{kd} \cdot r^{dd_k} + \sum_{h=1}^{n} \lambda_{hk} \cdot r^{\alpha_{hk}} \cdot r^{g_h} + \sum_{L=1}^{3} \lambda_{Lk} \cdot r^{\alpha_{Lk}} \cdot r^{g_L}$$

where L = 1, 2, 3 if there are three categories of labour. Unfortunately

${}_r\alpha_{Lk} \cdot {}_r g_L$ cannot be measured directly, and a proxy measure must be taken. Various authors have suggested proxy measures for the tightness of the labour market and these have been related to vacancies and unemployment rates and/or the absolute numbers of employed, unemployed and vacancies[1]. If a proxy can be found that is highly correlated with the unmeasurable variable, then there will be no problem in the estimation.

Placing labour in a whole system of equations can give two alternative forms:

(i) where labour previously was the only exogeneous demand so the system is now closed

$$[I - \hat{\iota}\,{}_r\Delta - (LA)']\,{}_r\underline{g} = \underline{0}$$

where a unique non-trivial solution can be obtained by arbitarily predetermining one of the g's (say the total labour supply) and solving everything in relative terms[2];

(ii) where there are other exogeneous demands, such as government spending then

$${}_r\underline{g} = [I - \hat{\iota}\,{}_r\Delta - (LA)']^{-1}\,\hat{\iota}\,{}_r\underline{F}$$

In both systems labour is included as an intermediate good. For a 2-good, 1-labour type system the $(LA)'$ matrix would look as follows:

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[1] See for example Dow and Dicks-Mireaux (39), or Davies (37) for an argument for the use of absolute numbers rather than rates.

[2] See Appendix IV for the technique of doing this, in the context of a conventional input-output framework. But the analysis is easily generalised to include attraction models.

$$\begin{bmatrix} \lambda_{11} \cdot {}_r\alpha_{11} & \lambda_{21} \cdot {}_r\alpha_{21} & \lambda_{L1} \cdot {}_r\alpha_{L1} \\ \lambda_{12} \cdot {}_r\alpha_{12} & \lambda_{22} \cdot {}_r\alpha_{22} & \lambda_{L2} \cdot {}_r\alpha_{L2} \\ 0_{1L} & 0_{2L} & 0_{LL} \end{bmatrix}$$

where the bottom row consists entirely of zeros, since labour is immobile in the short run and so is not attracted by the supply of other goods. The last column will have some positive figures, since, a priori, labour will be attractive to some other industries.

The $\hat{\Gamma}$ matrix presents some problems, since λ_{Ld} will represent how much the labour supply will expand as a result of an increase in demand for labour. Thus if it is assumed that the supply (from say the unemployed and non-employed) will increase by half of what is demanded then $\lambda_{Ld} = 0.5$. However, attraction theory, as presented here, does not purport to explain labour availability, but the figure will have to be estimated from micro-labour market studies. This figure is likely to vary from skill to skill since, say, unskilled labour is more likely to be forthcoming to meet demand because of a pool available. It will also vary from region to region because it will depend on local labour market conditions. This figure will necessarily be subjective and therefore it seems worthwhile to conduct some sensitivity analysis to see how it will affect the various multipliers.

We have assumed already that there is no interregional trade in labour therefore the ${}_r\Delta$ matrix will be as follows:-

$$\begin{bmatrix} {}_r\delta_{11} & {}_r\delta_{12} & {}_r\delta_{1L} \\ {}_r\delta_{21} & {}_r\delta_{22} & {}_r\delta_{21} \\ \beta_{L1} & \beta_{L2} & 0 \end{bmatrix}$$

The last row is the same technology as the nation since all labour is demand inside the region. The last cell of the last row is empty because labour does not consume labour directly.

In many ways the introduction of labour into attraction analysis is unsatisfactory because:

(i) we cannot estimate an equation directly for the labour industry since its equation would be

$$ {}_rt_L \cdot {}_rg_L = t_{Ld}({}_rg_L - {}_rdd_L) + \sum_{h=1}^{n} t_{hL}(\beta_{hL} \cdot {}_rg_L - {}_r\alpha_{hL} \cdot {}_rg_h) $$

which given our assumptions concerning labour is meaningless. We have therefore to resort to a pragmatic solution to the problem which leads to (ii).

(ii) A subjective value is placed upon λ_{Ld}, and if the results prove to be sensitive to this figure we will not be able to predict accurately.

(iii) The $(LA)'$ matrix has certain unsatisfactory implicit assumptions concerning the internal regional labour market, since the last column assumes that the labour sector sends a fixed proportion to each other sector. This is extremely unlikely to hold, even in the very short run.

However, despite these serious objections, it is probably worth including labour:

(a) in order to obtain some idea of the importance of labour in determining the mobility of industry - for example, using only interindustry relations, when the equation is tested for an industry it may be that there is no explanation of the industry and so one may be tempted to think the industry is mobile[1]. However, the inclusion of labour may substantially alter these results[2]. In the estimation of the equations for each industry when labour is included to see how footloose an industry is, it is not necessary to employ the most restrictive assumptions concerning labour market behaviour embodied in the constant α's. This assumption is only necessary for predictive purposes, so an important part of the labour analysis can be carried out without using the most restrictive assumptions.

(b) The feedback effects of consumption may substantially alter the multiplier effect. If however the values of the multipliers are not much affected by λ_{Ld} and $\lambda_{Lk} \cdot r^{\alpha}{}_{Lk}$ (k=1, 2, ...n) then subjective estimates of λ_{Ld} and unreal assumptions about the local labour market (embodied in $\lambda_{Lk} \cdot r^{\alpha}{}_{Lk}$) will not greatly affect the accuracy of the results, but including them (even if only based on informed guesses) may be more accurate than excluding them.

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1 For a more detailed discussion see Section III.M. on the relevant region.

2 See Appendix VIII for possible misspecification problems when labour is omitted from the analysis.

A comparison of the results of including and excluding labour will be given in Ch. V, where all the other results will be presented.

If the attraction model was to be used as a subset of a larger econometric model, as regional input-output tables sometimes are[1], then it would be preferable (because of the problems discussed above) to exclude the labour sector from the attraction analysis altogether and to include it in another subset of the model. Feedback effects between labour and industry could still be preserved without making use of the assumptions mentioned above[2].

Recently input-output techniques have been used to encompass 'goods' not traditionally regarded as being within the system - such as environmental factors. It may be possible to do this with attraction theory, since certain sectors may be largely attracted by the environment (e.g. tourism) whilst others may make large negative effects on the environment (e.g. chemicals). Thus negative attraction may develop due to these links - so introducing a chemical plant into an area may have a positive effect on other industries, but from this must be subtracted the hindrance to the tourist trade. Taking environment in its widest sense to include urban amenities, we may be able to explain the location of labour by reference to the supply of goods. The London area is attractive to certain types of labout not only because of the demand for labour but also because of the unique cultural facilities available there. The South Coast is attractive to the elderly because of the climate etc. However, we will go no further than mention the possibilities because of lack of data, not least of which is the high aggregation of the UK input-output data in the

---

[1] See Milliman (140) for a discussion of models of this kind

[2] This is mentioned as a suggestion for future research, since I do not wish to discuss regional econometric studies in this work.

service sectors. Although with the growth in real incomes, these types of factors may become more important in future research.

III.K. Cost minimisation, profit maximisation and the assumption of optimally located industry

In attraction theory we have assumed that firms try to minimise communication costs, and that this leads to an optimal pattern of industrial location and production. It is normal practice to assume that firms try to maximise profits, rather than minimise costs. However it will be shown that these two objectives are exactly the same when it is assumed that:

(i) there is a perfectly competitive market structure,

(ii) unit production costs (excluding communication costs) are the same in all regions,

(iii) there is a certain fixed final demand to be met in each region and this demand will be constant regardless of price, i.e. demand is perfectly inelastic[1].

Concerning the assumption of an optimal pattern of industrial location, we showed in Ch. II that given certain conditions a decentralised decision-making system based on profit maximisation will lead to an optimal solution. However, it was shown that this was not generally the case, and plausible situations could be hypothesied where a decentralised system would not lead to an optimal solution. It will be shown below that the attraction model will lead to an optimal solution with firms making decentralised decisions based on profit maximisation (or communication cost minimisation which will be shown to be

[1] The problem of more general price formation models will be discussed in Section III.L, although it may be noted here that in its present formulation the attraction model is probably incapable of incorporating more general price formulation models.

the same thing within the framework of the constraints mentioned above). The problem can be handled by casting it as a linear programming problem and applying the Kuhn-Tucker conditions[1]. In this way it is hoped to show the connection between attraction theory and traditional location theory and also point out a further assumption that is implicit in the formulation of attraction theory.

Assume a 2-region (i, j) 2-industry (1,2) model - altering the dimensions of the problem would not involve any conceptual difficulties, but the problem is cast in such a small dimension so that the results will readily be interpretable and even with such a small dimension to the problem the full formulation is rather long.

The notation is the usual input-output/attraction theory notation with the addition of:

(i) $_rP$ and $_sP$ are the total supplies of immobile primary input (e.g. labour in the short run) in a region r and s respectively.

(ii) β_{p1} is the Leontief coefficient for primary inputs per unit of output of product 1.

(iii) $_sF_1$ is total given final demand for product in region s.

(iv) $_{sr}f_1$ is the variable flow of product 1 from region s to region r for use in final demand.

(v) t_{f1} is the unit communication cost incurred by consumers importing a unit of product 1 for use in final demand. This corresponds to the cost of

---

[1] This approach will be seen to be very similar to the location problem formulated by Takayama and Judge (181) and described in Section II.B above.

importing intermediate good - t_{hk} - the use of this will become clearer later.

Givens	P	β	F	t
Variables	g	X	f	

The aim is to minimise total communication costs subject to a number of constraints. This can be formulated as a maximisation problem by multiplying through the objective function by -1.[1]

$$\text{Max } Z(g, X, f) = -\big[\, {}_{rs}x_{11}(t_{1d} + t_{11}) + {}_{rs}x(t_{1d} + t_{12}) + {}_{rs}f_1(t_{1d} + t_{f1}) + {}_{rs}x_{21}(t_{2d} + t_{21}) + {}_{rs}x_{22}(t_{2d} + t_{22}) + {}_{rs}f_2(t_{2d} + t_{f2}) + {}_{sr}x_{11}(t_{1d} + t_{11}) + {}_{sr}x_{12}(t_{1d} + t_{12}) + {}_{sr}f_1(t_{1d} + t_{f1}) + {}_{sr}x_{21}(t_{2d} + t_{21}) + {}_{sr}x_{22}(t_{2d} + t_{22}) + {}_{sr}f_2(t_{2d} + t_{f2})$$

Subject to

1. $${}_{rr}f_1 + {}_{sr}f_1 \geq {}_rF_1 \qquad {}_{rr}f_2 + {}_{sr}f_2 \geq {}_rF_2$$
$${}_{ss}f_1 + {}_{rs}f_1 \geq {}_sF_1 \qquad {}_{ss}f_2 + {}_{rs}f_2 \geq {}_sF_2$$

i.e. all final demand is met

2. $${}_{rr}x_{11} + {}_{sr}x_{11} \geq \beta_{11}\, {}_rg_1 \qquad {}_{ss}x_{11} + {}_{rs}x_{11} \geq \beta_{11}\, {}_sg_1$$
$${}_{rr}x_{21} + {}_{sr}x_{21} \geq \beta_{21}\, {}_rg_1 \qquad {}_{ss}x_{21} + {}_{rs}x_{21} \geq \beta_{21}\, {}_sg_1$$
$${}_{rr}x_{12} + {}_{sr}x_{12} \geq \beta_{12}\, {}_rg_2 \qquad {}_{ss}x_{12} + {}_{rs}x_{12} \geq \beta_{12}\, {}_sg_2$$
$${}_{rr}x_{22} + {}_{sr}x_{22} \geq \beta_{22}\, {}_rg_2 \qquad {}_{ss}x_{22} + {}_{rs}x_{22} \geq \beta_{22}\, {}_sg_2$$

i.e. the technological constraints on intermediate inputs

---

[1] Since unit costs (excluding communication costs) are assumed the same in each region then these can be ignored in cost minimisation.

3. $\beta_{P1}\,{}_{r}g_1 + \beta_{P2}\,{}_{r}g_2 \le {}_{r}P \qquad \beta_{P1}\,{}_{s}g_1 + \beta_{P2}\,{}_{s}g_2 \le {}_{s}P$

i.e. cannot exceed the stock of primary inputs

4.
$$
\begin{aligned}
&{}_{rr}x_{11} + {}_{rr}x_{12} + {}_{rs}x_{11} + {}_{rs}x_{12} + {}_{rr}f_1 + {}_{rs}f_1 \le {}_{r}g_1 \\
&{}_{rr}x_{21} + {}_{rr}x_{22} + {}_{rs}x_{21} + {}_{rs}x_{22} + {}_{rr}f_2 + {}_{rs}f_2 \le {}_{r}g_2 \\
&{}_{ss}x_{11} + {}_{ss}x_{12} + {}_{sr}x_{11} + {}_{sr}x_{12} + {}_{ss}f_1 + {}_{sr}f_1 \le {}_{s}g_1 \\
&{}_{ss}x_{21} + {}_{ss}x_{22} + {}_{sr}x_{21} + {}_{sr}x_{22} + {}_{ss}f_2 + {}_{sr}f_2 \le {}_{s}g_2
\end{aligned}
$$

i.e. an industry cannot supply to other sectors more than it produces itself

5. Non-negativity of all variables

$g \geq 0$ for all g

$x \geq 0$ for all x

$f \geq 0$ for all f

Now form the lagrangean in the usual way - see Baumol (8) or Takayama and Judge (181).

$$
\begin{aligned}
Z(g, X, f, \lambda) = {} & Z(g, X, f) + \lambda_1({}_{rr}f_1 + {}_{sr}f_1 - {}_{r}F_1) + \\
& \lambda_2({}_{rr}f_2 + {}_{sr}f_2 - {}_{r}F_2) + \lambda_3({}_{ss}f_1 + {}_{rs}f_1 - {}_{s}F_1) + \lambda_4({}_{ss}f_2 + {}_{rs}f_2 - {}_{s}F_2) \\
& + \lambda_5({}_{rr}x_{11} + {}_{sr}x_{11} - \beta_{11}\,{}_{r}g_1) + \lambda_6({}_{ss}x_{11} + {}_{rs}x_{11} - \beta_{11}\,{}_{s}g_1) \\
& + \lambda_7({}_{rr}x_{21} + {}_{sr}x_{21} - \beta_{21}\,{}_{r}g_1) + \lambda_8({}_{ss}x_{21} + {}_{rs}x_{21} - \beta_{21}\,{}_{s}g_1) \\
& + \lambda_9({}_{rr}x_{12} + {}_{sr}x_{12} - \beta_{12}\,{}_{r}g_2) + \lambda_{10}({}_{ss}x_{12} + {}_{rs}x_{12} - \beta_{12}\,{}_{s}g_2) \\
& + \lambda_{11}({}_{rr}x_{22} + {}_{sr}x_{22} - \beta_{22}\,{}_{r}g_2) + \lambda_{12}({}_{ss}x_{22} + {}_{rs}x_{22} - \beta_{22}\,{}_{s}g_2) \\
& + \lambda_{13}({}_{r}P - \beta_{P1}\,{}_{r}g_1 - \beta_{P2}\,{}_{r}g_2) + \lambda_{14}({}_{s}P - \beta_{P1}\,{}_{s}g_1 - \beta_{P2}\,{}_{s}g_2)
\end{aligned}
$$

$$+ \lambda_{15}\left({}_{r}g_1 - {}_{rr}x_{11} - {}_{rr}x_{12} - {}_{rs}x_{11} - {}_{rs}x_{12} - {}_{rr}f_1 - {}_{rs}f_1\right)$$
$$+ \lambda_{16}\left({}_{r}g_2 - {}_{rr}x_{21} - {}_{rr}x_{22} - {}_{rs}x_{21} - {}_{rs}x_{22} - {}_{rr}f_2 - {}_{rs}f_2\right)$$
$$+ \lambda_{17}\left({}_{s}g_1 - {}_{ss}x_{11} - {}_{ss}x_{12} - {}_{sr}x_{11} - {}_{sr}x_{12} - {}_{ss}f_1 - {}_{sr}f_1\right)$$
$$+ \lambda_{18}\left({}_{s}g_2 - {}_{ss}x_{21} - {}_{ss}x_{22} - {}_{sr}x_{21} - {}_{sr}x_{22} - {}_{ss}f_2 - {}_{sr}f_2\right)$$
$$+ \lambda_{19}\,{}_{r}g_1 + \lambda_{20}\,{}_{r}g_2 + \lambda_{21}\,{}_{s}g_1 + \lambda_{22}\,{}_{s}g_2 + \lambda_{23}\,{}_{rr}f_1 + \lambda_{24}\,{}_{rs}f_1$$
$$+ \lambda_{25}\,{}_{rr}f_2 + \lambda_{26}\,{}_{rs}f_2 + \lambda_{27}\,{}_{ss}f_1 + \lambda_{28}\,{}_{sr}f_1 + \lambda_{29}\,{}_{ss}f_2 + \lambda_{30}\,{}_{sr}f_2$$
$$+ \lambda_{31}\,{}_{rr}x_{11} + \lambda_{32}\,{}_{rr}x_{12} + \lambda_{33}\,{}_{rs}x_{11} + \lambda_{34}\,{}_{rs}x_{12}$$
$$+ \lambda_{35}\,{}_{rr}x_{12} + \lambda_{36}\,{}_{rr}x_{22} + \lambda_{37}\,{}_{rs}x_{12} + \lambda_{38}\,{}_{rs}x_{22}$$
$$+ \lambda_{39}\,{}_{ss}x_{11} + \lambda_{40}\,{}_{ss}x_{12} + \lambda_{41}\,{}_{sr}x_{11} + \lambda_{42}\,{}_{sr}x_{12}$$
$$+ \lambda_{43}\,{}_{ss}x_{21} + \lambda_{44}\,{}_{ss}x_{22} + \lambda_{45}\,{}_{sr}x_{21} + \lambda_{46}\,{}_{sr}x_{22}$$

The Kuhn-Tucker conditions for $Z(g, X, f, \lambda)$ to be a non-negative saddle point are [1 2].

$$\frac{\partial z}{\partial\,{}_{r}g_1} = -\lambda_5 \beta_{11} - \lambda_7 \beta_{21} - \lambda_{13} \beta_{11} + \lambda_{15} + \lambda_{19} \leq 0$$
$${}_{r}g_1\left(\frac{\partial z}{\partial\,{}_{r}g_1}\right) = {}_{r}g_1\left(\quad " \quad " \quad " \quad \right) = 0$$
$$\frac{\partial z}{\partial \lambda_{19}} = {}_{r}g_1 \geq 0$$
$$\lambda_{19}\left(\frac{\partial z}{\partial \lambda_{19}}\right) = \lambda_{19}\left({}_{r}g_1\right) = 0 \qquad \lambda_{19} \geq 0$$

} conditions for ${}_{r}g_1$ and its associated lagrangean

---

1 Not all the partial derivatives will be given, only those relating to good 1 in region r and s. The results thus obtained will be sufficient to serve as illustrative purposes.

2 All the variables g, x, f and λ should have a bar ($^{-}$) over them to indicate an optimum value, however these are left out for sake of clarity.

$$\frac{\partial L}{\partial {}_s g_1} = -\lambda_6 \beta_{11} - \lambda_8 \beta_{21} - \lambda_{14} \beta_{p1} + \lambda_{17} + \lambda_{21} \leq 0$$

$$\left(\frac{\partial L}{\partial {}_s g_1}\right) \cdot {}_s g_1 (\quad " \quad " \quad " \quad) = 0$$

$$\frac{\partial L}{\partial \lambda_{21}} = {}_s g_1 - 0$$

$$\lambda_{21} \frac{\partial L}{\partial \lambda_{21}} = \lambda_{21} ({}_s g_1) = 0 \qquad \lambda_{21} \geq 0$$

conditions for ${}_s g_1$ and its associated lagrangean

$$\frac{\partial z}{\partial {}_{rr} f_1} = \lambda_1 - \lambda_{15} + \lambda_{23} \leq 0$$

$$f_1 \left(\frac{\partial z}{\partial {}_{rr} f_1}\right) = {}_{rr} f_1 (\quad " \quad " \quad) = 0$$

$$\frac{\partial z}{\partial \lambda_{23}} = {}_{rr} f_1 \geq 0$$

$$\lambda_{23} \left(\frac{\partial z}{\partial \lambda_{23}}\right) = \lambda_{23} ({}_{rr} f_1) = 0 \qquad \lambda_{23} \geq 0$$

conditions for ${}_{rr} f_1$ and its associated lagrangean

$$\frac{\partial z}{\partial {}_{rs} f_1} = -t_{1d} - t_{s1} - \lambda_{15} + \lambda_3 + \lambda_{24} \leq 0$$

$$f_1 \left(\frac{\partial z}{\partial {}_{rs} f_1}\right) = {}_{rs} f_1 (\quad " \quad " \quad " \quad) = 0$$

$$\frac{\partial z}{\partial \lambda_{24}} = {}_{rs} f_1 \geq 0$$

$$\lambda_{24} \left(\frac{\partial z}{\partial \lambda_{24}}\right) = \lambda_{24} ({}_{rs} f_1) = 0 \qquad \lambda_{24} \geq 0$$

conditions for ${}_{rs} f_1$ and its associated lagrangean

$$\frac{\partial z}{\partial {}_{ss} f_1} = \lambda_3 - \lambda_{17} + \lambda_{27} \leq 0$$

$$f_1 \left(\frac{\partial z}{\partial {}_{ss} f_1}\right) = {}_{ss} f_1 (\quad " \quad " \quad) = 0$$

$$\frac{\partial z}{\partial \lambda_{27}} = {}_{ss} f_1 \geq 0$$

$$\lambda_{27} \left(\frac{\partial z}{\partial \lambda_{27}}\right) = \lambda_{27} ({}_{ss} f_1) = 0 \qquad \lambda_{27} \geq 0$$

conditions for ${}_{ss} f_1$ and its associated lagrangean

$$\frac{\partial z}{\partial {}_{sr} f_1} = -t_{1d} - t_{s1} - \lambda_{17} + \lambda_1 + \lambda_{28} \leq 0$$

$$f_1 \left(\frac{\partial z}{\partial {}_{sr} f_1}\right) = {}_{sr} f_1 (\quad " \quad " \quad " \quad) = 0$$

$$\frac{\partial z}{\partial \lambda_{28}} = {}_{sr} f_1 \geq 0$$

$$\lambda_{28} \left(\frac{\partial z}{\partial \lambda_{28}}\right) = \lambda_{28} ({}_{sr} f_1) = 0 \qquad \lambda_{28} \geq 0$$

conditions for ${}_{sr} f_1$ and its associated lagrangean

$$\frac{\partial z}{\partial_{rr}x_{11}} = \lambda_5 - \lambda_{15} + \lambda_{31} \leq 0$$

$$_{rr}x_{11}\left(\frac{\partial z}{\partial_{rr}x_{11}}\right) = {_{rr}x_{11}}(\lambda_5 - \lambda_{15} + \lambda_{31}) = 0$$

$$\frac{\partial z}{\partial \lambda_{31}} = {_{rr}x_{11}} \geq 0$$

$$\lambda_{31}\left(\frac{\partial z}{\partial \lambda_{31}}\right) = \lambda_{31}({_{rr}x_{11}}) = 0 \qquad \lambda_{31} \geq 0$$

conditions for $_{rr}x_{11}$ and its associated lagrangean

$$\frac{\partial z}{\partial_{rr}x_{12}} = \lambda_9 - \lambda_{15} + \lambda_{32} \leq 0$$

$$_{rr}x_{12}\left(\frac{\partial z}{\partial_{rr}x_{12}}\right) = {_{rr}x_{12}}(\lambda_9 - \lambda_{15} + \lambda_{32}) = 0$$

$$\frac{\partial z}{\partial \lambda_{32}} = {_{rr}x_{12}} \geq 0$$

$$\lambda_{32}\left(\frac{\partial z}{\partial \lambda_{32}}\right) = \lambda_{32}({_{rr}x_{12}}) = 0 \qquad \lambda_{32} \geq 0$$

conditions for $_{rr}x_{12}$ and its associated lagrangean

$$\frac{\partial z}{\partial_{ss}x_{11}} = \lambda_6 - \lambda_{17} + \lambda_{39} \leq 0$$

$$_{ss}x_{11}\left(\frac{\partial z}{\partial_{ss}x_{11}}\right) = {_{ss}x_{11}}(\lambda_6 - \lambda_{17} + \lambda_{39}) = 0$$

$$\frac{\partial z}{\partial \lambda_{39}} = {_{ss}x_{11}} \geq 0$$

$$\lambda_{39}\left(\frac{\partial z}{\partial \lambda_{39}}\right) = \lambda_{39}({_{ss}x_{11}}) = 0 \qquad \lambda_{39} \geq 0$$

conditions for $_{ss}x_{11}$ and its associated lagrangean

$$\frac{\partial z}{\partial_{ss}x_{12}} = \lambda_{10} - \lambda_{10} + \lambda_{40} \leq 0$$

$$_{ss}x_{12}\left(\frac{\partial z}{\partial_{ss}x_{12}}\right) = {_{ss}x_{12}}(\lambda_{10} - \lambda_{10} + \lambda_{40}) = 0$$

$$\frac{\partial z}{\partial \lambda_{40}} = {_{ss}x_{12}} \geq 0$$

$$\lambda_{40}\left(\frac{\partial z}{\partial \lambda_{40}}\right) = \lambda_{40}({_{ss}x_{12}}) = 0 \qquad \lambda_{40} \geq 0$$

conditions for $_{ss}x_{12}$ and its associated lagrangean

$$\frac{\partial z}{\partial\, {}_{rs}x_{11}} = -t_1 d - t_{11} + \lambda_6 - \lambda_{15} + \lambda_{33} \leq 0$$

$${}_{rs}x_{11}\left(\frac{\partial z}{\partial\, {}_{rs}x_{11}}\right) = {}_{rs}x_{11}\left(\quad '' \quad '' \quad '' \quad \right) = 0$$

$$\frac{\partial z}{\partial \lambda_{33}} = {}_{rs}x_{11} \geq 0$$

$$\lambda_{33}\left(\frac{\partial z}{\partial \lambda_{33}}\right) = \lambda_{33}\left({}_{rs}x_{11}\right) = 0 \qquad \lambda_{33} \geq 0$$

conditions for ${}_{rs}x_{11}$ and its associated lagrangean

$$\frac{\partial z}{\partial\, {}_{rs}x_{12}} = -t_1 d - t_{12} + \lambda_{10} - \lambda_{15} + \lambda_{34} \leq 0$$

$${}_{rs}x_{12}\left(\frac{\partial z}{\partial\, {}_{rs}x_{12}}\right) = {}_{rs}x_{12}\left(\quad '' \quad '' \quad \right) = 0$$

$$\frac{\partial z}{\partial \lambda_{34}} = {}_{rs}x_{12} \geq 0$$

$$\lambda_{34}\left(\frac{\partial z}{\partial \lambda_{34}}\right) = \lambda_{34}\left({}_{rs}x_{12}\right) = 0 \qquad \lambda_{34} \geq 0$$

conditions for ${}_{rs}x_{12}$ and its associated lagrangean

$$\frac{\partial z}{\partial\, {}_{sr}x_{11}} = -t_1 d - t_{11} + \lambda_5 - \lambda_{17} + \lambda_{41} \leq 0$$

$${}_{sr}x_{11}\left(\frac{\partial z}{\partial\, {}_{sr}x_{11}}\right) = {}_{sr}x_{11}\left(\quad '' \quad '' \quad '' \quad \right) = 0$$

$$\frac{\partial z}{\partial \lambda_{41}} = {}_{sr}x_{11} \geq 0$$

$$\lambda_{41}\left(\frac{\partial z}{\partial \lambda_{41}}\right) = \lambda_{41}\left({}_{sr}x_{11}\right) = 0 \qquad \lambda_{41} \geq 0$$

conditions for ${}_{sr}x_{11}$ and its associated lagrangean

$$\frac{\partial z}{\partial\, {}_{sr}x_{12}} = -t_1 d - t_{12} + \lambda_9 - \lambda_{17} + \lambda_{42} \leq 0$$

$${}_{sr}x_{12}\left(\frac{\partial z}{\partial\, {}_{sr}x_{12}}\right) = {}_{sr}x_{12}\left(\quad '' \quad '' \quad '' \quad \right) = 0$$

$$\frac{\partial z}{\partial \lambda_{42}} = {}_{sr}x_{12} \geq 0$$

$$\lambda_{42}\left(\frac{\partial z}{\partial \lambda_{42}}\right) = \lambda_{42}\left({}_{sr}x_{12}\right) = 0 \qquad \lambda_{42} \geq 0$$

conditions for ${}_{sr}x_{12}$ and its associated lagrangean

$$\frac{\partial z}{\partial \lambda_{13}} = {}_rP - \beta_{p1}\, {}_r g_1 - \beta_{p2}\, {}_r g_2 \geq 0$$

$$\lambda_{13}\left(\frac{\partial z}{\partial \lambda_{13}}\right) = \lambda_{13}\left(\quad '' \quad '' \quad '' \quad \right) = 0 \qquad \lambda_{13} \geq 0$$

$$\frac{\partial z}{\partial \lambda_5} = {}_{rr}x_{11} + {}_{sr}x_{11} - \beta_{11}\, {}_r g_1 \geq 0$$

$$\lambda_5 \left(\frac{\partial z}{\partial \lambda_5}\right) = \lambda_5 \left(\quad " \quad \quad " \quad \quad " \quad \right) = 0 \qquad \lambda_5 \geq 0$$

$$\frac{\partial z}{\partial \lambda_1} = {}_{rr}f_1 + {}_{sr}f_1 - {}_r F_1 \geq 0$$

$$\lambda_1 \left(\frac{\partial z}{\partial \lambda_1}\right) = \lambda_1 \left(\quad " \quad \quad " \quad \right) = 0 \qquad \lambda_1 \geq 0$$

$$\frac{\partial z}{\partial \lambda_{15}} = {}_r g_1 - {}_{rr}x_{11} - {}_{rr}x_{12} - {}_{rs}x_{11} - {}_{rs}x_{12} - {}_{rr}f_1 - {}_{rs}f_1 \geq 0$$

$$\lambda_{15} \left(\frac{\partial z}{\partial \lambda_{15}}\right) = \lambda_{15} \left(\quad " \quad \quad " \quad \quad " \quad \quad " \quad \right) = 0$$

$$\lambda_{15} \geq 0$$

<u>Interpretation</u>

Let us interpret the following λ's as:

λ_{15} = marginal unit cost of product 1 in region r

λ_{17} = " " " " " 1 " " s

λ_{13} = unit price of primary input in region r

λ_{14} = " " " " " " s

λ_1 = price of product 1 in region r for final demand

λ_3 = " " 1 " " s " " "

λ_5 = " " 1 " " r as an input in product 1

λ_9 = " " 1 " " r " " " " 2

λ_7 = " " 2 " " r " " " " 1

λ_6 = " " 1 " " s " " " " 1

λ_8 = " " 2 " " s " " " " 2

λ_{10} = " " 1 " " s " " " " 2

(a) If ${}_r g_1 > 0$ (i.e. strictly positive) then from $\lambda_{19}\left(\frac{\partial z}{\partial \lambda_{19}}\right) = 0$, $\lambda_{19} = 0$.

This result holds for any variable that is strictly positive, its associated lagrangean = 0 (i.e. a strict equality). When the variable = 0, the associated lagrangean > 0. Now if $_{r}g_{1} > 0$ and $\lambda_{19} = 0$, then from $\frac{\partial z}{\partial_r g_1} \leq 0$ and $_{r}g_{1}\left(\frac{\partial z}{\partial_r g_1}\right) = 0$, it follows:

$$\lambda_{15} = \beta_{11}\lambda_5 + \beta_{21}\lambda_7 + \beta_{p1}\lambda_{13}$$

which using the interpretation of the λ's above says that if any output of product 1 is produced in region i ($_{r}g_{1} > 0$) then the marginal cost of product 1 in that region (λ_{15}) is equal to the cost in that region of all the inputs ($\beta_{11}\cdot\lambda_5 + \beta_{21}\cdot\lambda_7 + \beta_{p1}\cdot\lambda_{13}$).

(b) From $\left(\frac{\partial z}{\partial\lambda_{13}}\right) \geq 0$ and $\lambda_{13}\left(\frac{\partial z}{\partial\lambda_{13}}\right) = 0$, λ_{13} can only > 0 when $_{r}p - \beta_{p1}\cdot{}_{r}g_{1} - \beta_{p2}\cdot{}_{r}g_{2} = 0$ and $\lambda_{13} = 0$, when $_{r}P - \beta_{p1}\cdot{}_{r}g_{1} - \beta_{p2}\cdot{}_{r}g_{2} > 0$. Which says that the cost of the primary input can only be positive when that input is fully utilised, or conversely when that input is not fully utilised the cost (or marginal product) of that input = 0.

(c) From $\frac{\partial z}{\partial\lambda_1} \geq 0$ and $\lambda_1\left(\frac{\partial z}{\partial\lambda_1}\right) = 0$, if $\lambda_1 > 0$, then $_{rr}f_{1} + {}_{sr}f_{1} = {}_{r}F_{1}$ which says that when final demand of product 1 in region i has a cost then there will be no free dispersal of that product, i.e. the original constraint holds as a strict equality. This result can be shown for any variable and its associated lagrangean (price). For example, from

$\frac{\partial z}{\partial\lambda_{15}} \geq 0$ and $\lambda_{15}\left(\frac{\partial z}{\partial\lambda_{15}}\right) = 0$ when $\lambda_{15} > 0$ then $_{r}g_{1} = {}_{rr}x_{11}$ $_{rr}x_{12} + {}_{rs}x_{11} + {}_{rs}x_{12} + {}_{rr}f_{1} + {}_{rs}f_{1}$, which says that where the marginal cost of product 1 in region r >0 then there is no free dispersal or surplus product.

(d) Assume ${}_{rr}x_{11} > 0$, ${}_{rr}x_{12} > 0$ and ${}_{rr}f_1 > 0$, then from $\frac{\partial z}{\partial_{rr} f_1}$, $\frac{\partial z}{\partial_{rr} x_{12}}$, $\frac{\partial z}{\partial_{rr} x_{11}}$ and their respective associated lagrangeans we can obtain the following results:

$$\lambda_1 = \lambda_{15}$$

$$\lambda_9 = \lambda_{15}$$

$$\lambda_5 = \lambda_{15}$$

$\lambda_1 = \lambda_9 = \lambda_5 = \lambda_{15}$ which says that as long as good 1 flows from region r to all sectors in region r the price is equalised between those sectors, and equal to the marginal cost in that region. This is to be expected since the cost of moving goods inside a region is assumed to be zero.

Similarly when ${}_{ss}x_{11} > 0$, ${}_{ss}x_{12} > 0$ and ${}_{ss}f_1 > 0$ then $\lambda_{17} = \lambda_3 = \lambda_6 = \lambda_{10}$.

(e) Problems arise however when goods flow interregionally. Let us assume that ${}_{r}g_1$ and ${}_{s}g_1$ both > 0 (i.e. good 1 is produced in both regions) but region r produces more than it consumes and region s does not produce enough, thus region r will export to region s and no cross-hauling will take place as this would be inefficient. Assume further that ${}_{rs}x_{11} > 0$, ${}_{rs}x_{12} > 0$ and ${}_{rs}f_1 > 0$, i.e. region r ships some of product 1 to each sector in region s, remembering that ${}_{sr}x_{11} = 0$, ${}_{sr}x_{12} = 0$ and ${}_{sr}f_1 = 0$ by the no cross-hauling assumption. From

$\frac{\partial z}{\partial_{rs} f_1}$, $\frac{\partial z}{\partial_{rs} x_{11}}$, $\frac{\partial z}{\partial_{rs} x_{12}}$, $\frac{\partial z}{\partial_{sr} x_{11}}$, $\frac{\partial z}{\partial_{sr} x_{12}}$, $\frac{\partial z}{\partial_{sr} f_1}$,

and their associated lagrangeans, we can obtain the following results:

(i) $\lambda_3 = \lambda_{15} + t_{1d} + t_{f1}$

(ii) $\lambda_1 + \lambda_{28} < {}_{17} + t_{1d} + t_{f1}$ and $\lambda_{28} > 0$

(iii) $\lambda_6 = \lambda_{15} + t_{1d} + t_{11}$

(iv) $\lambda_5 + \lambda_{41} < t_{1d} + t_{11}$

(v) $\lambda_{10} = t_{1d} + t_{12} + \lambda_{15}$

(vi) $\lambda_9 + \lambda_{42} < t_{1d} + t_{12} + \lambda_{17}$ and $\lambda_{42} > 0$

and from previous results obtained in (d) above we know that (vii) $\lambda_1 = \lambda_9 = \lambda_5 = \lambda_{15}$ and (viii) $\lambda_3 = \lambda_6 = \lambda_{10} = \lambda_{17}$. Let us first look at conditions (i) to (vi) individually. Condition (i) says that the price of final demand for good 1 in region s is equal to the marginal cost of production in region r plus the communication costs involved. Similarly condition (iii) says the price of good 1 as an input to good 1 in region s is equal to the marginal cost of production in region r plus the communication costs involved. A similar argument can be repeated for λ_{10} with condition (v). Conditions (ii), (iv) and (vi) show that back-hauling would be inefficient because the marginal cost of production in region r plus the communication costs exceed the price in region s. However with regard to conditions (i), (iii), (v) and (viii) taken together, we find that for all these conditions to hold $t_{f1} = t_{11} = t_{12}$. So the pattern of trade that we have described can only be optimal when these conditions hold. This may at first seem surprising since a basic postulate of attraction theory is that generally

$t_{11} = t_{12}$. Now let us assume that in this case $t_{f1} = 0$ (generally assumed in attraction theory since t_{f1} is not mentioned) and for the sake of this example let $t_{12} > t_{11} > 0$. Now a priori for an optimum solution (in the conditions we have described with region s having an import deficit in good 1) we would expect the sector that is most expensive (sector 2) to supply by imports (i.e. where the communication costs were highest) to be supplied by region s's production. Then the next most expensive sector (sector 1) supplied and so on until all of region s production of good 1 is exhausted. The remaining unfulfilled demand would be satisfied by imports from region r, since with these remaining sectors the communication costs are relatively low. Only if $t_{f1} = t_{11} = t_{12}$ could region r supply some of product 1 to each sector in region s and this series of flows still be optimal.

(f) Let us examine a system of flows where $t_{12} > t_{11} > t_{f1} = 0$ and assume the flows of goods to be as follows:

(i)	${}_{rr}x_{11} > 0$	(ii)	${}_{rr}x_{12} > 0$
(iii)	${}_{rs}f_{1} > 0$	(iv)	${}_{rs}x_{11} = 0$
(v)	${}_{rs}x_{12} = 0$	(vi)	${}_{rs}f_{1} > 0$
(vii)	${}_{ss}x_{11} > 0$	(viii)	${}_{ss}x_{12} > 0$
(ix)	${}_{ss}f_{1} = 0$	(x)	${}_{sr}f_{1} = 0$
(xi)	${}_{sr}x_{11} = 0$	(xii)	${}_{sr}x_{12} = 0$

This system shows that region r supplies all its own requirements of good 1 (i.e. it imports none) and that it only exports to final demand in region s, sectors 1 and 2 in region s being supplied by internal production. This is a priori reasonable since the communication costs associated with final demand are smallest and consequently it is expected that any exports from region r to region s would automatically go to final demand first of all. From conditions (i) to (xii) we can obtain the following (in) equalities respectively:

(i) $\lambda_1 = \lambda_{15}$

(ii) $\lambda_9 = \lambda_{15}$

(iii) $\lambda_5 = \lambda_{15}$

(iv) $\lambda_6 + \lambda_{33} < \lambda_{15} + t_{1d} + t_{11}$ and $\lambda_{33} > 0$

(v) $\lambda_{10} + \lambda_{34} < t_{1d} + t_{12} + \lambda_{15}$ and $\lambda_{34} > 0$

(vi) $\lambda_3 + \lambda_{15} + t_{1d} + t_{f1}$

(vii) $\lambda_6 = \lambda_{17}$

(viii) $\lambda_{10} = \lambda_{17}$

(ix) $\lambda_3 + \lambda_{27} < \lambda_{17}$ and $\lambda_{27} > 0$

(x) $\lambda_1 + \lambda_{28} < \lambda_{17} + t_{1d} + t^{F}_{1}$ and $\lambda_{28} > 0$

(xi) $\lambda_5 + \lambda_{41} < t_{1d} + t_{11} = \lambda_{17}$ and $\lambda_{41} > 0$

(xii) $\lambda_9 + \lambda_{42} < t_{1d} + t_{12} + \lambda_{17}$ and $\lambda_{42} > C$

The prices in the exporting region r are all equal:

$$\lambda_1 = \lambda_5 = \lambda_9 = \lambda_{15}$$

The prices of the non-interregionally traded good in region s are equal and both equal to the marginal cost of production:

$$\lambda_6 = \lambda_{10} = \lambda_{17}$$

Now the price of product 1 as an input to final demand in region s:

$$\lambda_3 = \lambda_{15} + t_{1d} + t_{f1}$$

but from condition (ix)

$$\lambda_3 < \lambda_{17} - \lambda_{27} \quad \text{where } \lambda_{27} > 0$$

Therefore

$$\lambda_3 < \lambda_{17} = \lambda_6 = \lambda_{10} < \lambda_{15} + t_{1d} + t_{f1}$$

And from conditions (iv) and (v) we know

$$\lambda_6 < \lambda_{15} + t_{1d} + t_{11} \text{ and } \lambda_{10} < \lambda_{15} + t_{1d} + t_{12}$$

So the price of good 1 for final demand in region s is less than good 1 as an intermediate input in region s's where the final demand is imported from outside the region. This may seem odd at first, since intra-regional trade flows are costless, one would expect the price of a good to be equal in all sectors in a region. In the above situation, without further restrictions, an arbitrager would buy as final demand good 1, and re-sell it to the other demanding sectors 1 and 2. Since there are no communication costs internally and the price is cheaper in the final goods sector than the intermediate goods sector the arbitrager would make a profit, and this would result in increased efficiency since sectors 1 and 2 would obtain the good cheaper and the price equalised in all sectors in the importing region. However, we saw in (e) above that when the prices in each sector in the importing region are equal and some are imported into every sector, then $t_{f1} = t_{11} = t_{12}$. However our assumptions in this section are that t_{12} $t_{11} > t_{f1} = 0$. But by allowing an arbitrager to re-sell goods that were imported as final demand to intermediate sectors and letting this re-sale be costless, we are implying that the cost of importing a good is equal for all sectors since it can be most efficiently done by importing to the sector with the cheapest communication costs and re-selling intra-regionally and costlessly from this cheapest sector. Thus in order to have a condition that communication costs can vary between sectors, we must also assume that once a good is imported by a sector it cannot be re-sold by that sector to another sector in the same region. This condition is necessary for attraction theory to be formulated.

(g) In section (f) we gave an example where in the importing region, the imported good was supplied only to a sector that was not supplied by internal production. Let us consider a case where imports are supplied, not to all sectors, but (in this case) to two sectors, one of which is supplied solely by imports (final demand) and one of which is supplied partially by imports and partially by internal production (sector 1). This would seem to imply as in the case above $t_{12} > t_{11} > t_{f1} \geqslant 0$. The system of flows described here is identical to the ones given in (f) above, apart from (iv) where ${}_{rs}x_{11} > 0$ rather than ${}_{rs}x_{11} = 0$. This changes the associated inequality (iv) into an equality (iv) $\lambda_6 = \lambda_{15} + t_{1d} + t_{11}$. From this system of flows we obtain:

(i) $\lambda_1 = \lambda_9 = \lambda_5 = \lambda_{15}$, the usual conditions of price equality in all sectors in the exporting region,

(ii) $\lambda_{17} = \lambda_{10} = \lambda_6 = \lambda_{15} + t_{1d} + t_{11} > \lambda_3$

and

$\lambda_{13} = \lambda_{15} + t_{1d} + t_{f1}$

and

$\lambda_{10} < \lambda_{15} + t_{1d} + t_{12} - \lambda_{34}$ and $\lambda_{34} > 0$

therefore

$\lambda_{10} < \lambda_{15} + t_{1d} + t_{12}$

Thus the sectors to which product 1 (produced in region s) is sold, the price in region s is equal to the marginal cost of production, i.e. $\lambda_{17} = \lambda_{10} = \lambda_6$. Now product 1 is also imported as an intermediate product into sector 1 and the price of this is λ_6, and $\lambda_6 = \lambda_{15} + t_{1d} + t_{11}$ which is the cost in region i plus the communication costs. Now all these prices $(\lambda_{17}, \lambda_{10}, \lambda_6) > \lambda_3 = \lambda_{15} + t_{1d} + t_{f1}$, and from $\lambda_6 > \lambda_3$ it follows that $t_{11} > t_{f1}$. Also since $\lambda_6 = \lambda_{10}$ and $\lambda_{10} < \lambda_{15} + t_{1d} + t_{12}$, it follows that $t_{12} > t_{11}$.

It has thus been demonstrated by postulating a set of a priori reasonable interregional flows, then the conditions that follow from the interregional flows concerning the relative magnitude of the communication costs are consistent (i.e. $t_{12} > t_{11} > t_{f1}$)[1] Conversely it could have been shown that given the communication costs, what the interregional flows must be for an optimal solution. The problem was only tackled in the former way for ease of presentation and this does not affect the result.

(h) We can also show that when the output of a product in a region is positive then there are no profits to be made, since the price can be totally accounted for by the costs. Let us examine $\frac{\partial z}{\partial_1 g_1}$ and ${}_r g_1 \left(\frac{\partial z}{\partial_r g_1} \right)$ conditions.

When ${}_r g_1 > 0$ then

$$\lambda_{15} = \beta_{11} \lambda_5 - \beta_{21} \cdot \lambda_7 - \beta_{p1} \cdot \lambda_{13}$$

Now we know that when any sector in region r buys from industry 1 in region s that the price is equal to λ_{15}, i.e. $\lambda_{15} = \lambda_1 = \lambda_5 = \lambda_9$. Now λ_{15} is totally accounted for by costs of production since λ_5, λ_7 and λ_{13} were interpretted as the cost of inputs. Similarly when industry 1 in region r sells to region s the

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1 Showing that the most expensive goods to transport are less transported than the cheaper to transport goods and vice versa, which is what one would expect ceteris paribus.

price of the product is equal to λ_{15} plus the costs of communication. Consequently there is no market - either in region r or s - where profits are being earned. A similar analysis can be repeated for production of the other goods.

(i) The dual of the primal problem could be formulated[1], where the objective function would be to find a set of prices and rents which will maximise profits (revenue minus costs) subject to the constraint that profits that are arbitrager can make must be zero. Thus as long as the final demand is perfectly inelastic (i.e. will buy a fixed amount regardless of the price) and unit costs (excluding communication costs) are the same in each region[2], the minimisation of transport costs and the maximisation of profits can be shown to be equivalent problems, since the primal and dual are essentially identical.

The possible existence of this optimal solution lends weight to our assumption that industry will distribute itself (or through the actions of arbitragers) optimally over the regions. Thus it is possible to turn a definition such as III.C.1 into a predictive equation such as III.C.7. Since we assume the costs to be at a minimum, and that firms behave so as to make the α and ${}_{r}\delta$ coefficients stable in

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1 In a similar way to the example given from Takayama and Judge (181) in Section II.B.

2 Assuming of course the market structure is perfectly competitive in all sectors.

the short run, and so prediction is possible. If industry were not optimally distributed we could turn the definition round into a form that could be estimated, but we could not be sure that these estimated relationships would not be based on any consistent behaviour. Therefore the assumption of optimally distributed industry is vital to attraction theory, and we have shown in this section what assumptions are necessary for this distribution, given the present formulation of the attraction model.

III.L. Attraction theory and more general price formulation models

The system by which price has been implicitly formulated in attraction theory, as so far discussed, has been very specific[1]. This section will examine the possibilities of using one other simple price formulation model, and go on to mention the extensions in this field that although may be theoretically desirable (since they would increase the generality of the model) may be practically impossible.

Let us consider in detail the case where the prices of all products and primary inputs are fixed exogeneously in each region. This may be due to government intervention, price leadership, the influence of unions etc. Now the aim of the firm is to maximise profits[2], which in attraction theory notation can be defined as:

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[1] That is that demand perfectly inelastic and market structure pefectly competitive, and so the price is determined by the marginal cost of production (imports) in that market at the given volume.

[2] Communication costs minimisation and profit maximisation no longer being identical.

$$ {}_r\pi_k \cdot {}_r g_k = {}_r p_k \, {}_r dd_k + {}_s p_k ({}_r g_k - {}_r dd_k) - t_{kd}({}_r g_k - {}_r dd_k) - \left[\sum_h ({}_r p_h \, {}_r\alpha_{hk} \, {}_r g_h + {}_s p_h \, {}_{sr}x_{hk}) + \sum_h t_{hk}(\beta_{hk} \, {}_r g_k - {}_r\alpha_{hk} \, {}_r g_k) \right] - \beta_{Lk} \, {}_r g_k \cdot {}_r p_L \qquad \text{III.L} $$ [1]

where the notation has its usual meaning and in addition

$_r\pi_k$ = unit profit of industry k in region r

$_rp_k$ = unit price of " " " "

$_sp_k$ = " " " " " " s (where s 2 all other regions)

$_rp_h$ = " " " " h " r

$_sp_h$ = " " " " " " s

$_rp_l$ = " " " labour " " r

A firm will try to maximise its profits but is subject to certain constraints such as:

(i) $_{sr}X_{hk} + {}_r\alpha_{hk} \cdot {}_r g_h \geqslant \beta_{hk} \cdot {}_r g_h$

(ii) that it cannot sell in any one market more than is demanded by final plus intermediate users in that market. This problem with inequality constraints cannot be solved by classical methods but only handled in a linear programming format, which unfortunately necessitates knowledge of the parameters p's and t's. Even if the inequalities of the constraints were transformed into equalities, lagrangean methods would not give a solution either, without knowledge of the p's and t's. We can however take the following

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[1] This assumes that primary inputs are non-transportable factors.

approaches:

a. The first approach is to attempt to handle equation III.L.1 in the usual attraction manner by assuming that industry is optimally distributed with regard to the need prices and rearranging equation III.L.1 we obtain the following:

$$-{}_r\Pi_k\,{}_rg_k - t_{kd}\,{}_rg_k - \sum_h t_{hk}\,\beta_{hk}\,{}_rg_k = -{}_rp_k\,{}_rdd_k - {}_sp_k\left({}_rg_k - {}_rdd_k\right) - t_{kd}\,{}_rdd_k + \sum_h {}_rp_h\,{}_r\alpha_{hk}\,{}_rg_h + \sum_h {}_sp_h\,{}_{sr}x_{hk} - \sum_h t_{hk}\,{}_r\alpha_{hk}\,{}_rg_h + \beta_{hk}\,{}_rg_k\,{}_rp_L \qquad \text{III.L.2}$$

Let $Z = \left(-{}_r\Pi_k - t_{kd} - \sum_h t_{hk}\,\beta_{hk}\,{}_rg_k\right)$ III.L.3

$${}_rg_k = \frac{-{}_rp_k\,{}_rdd_k}{Z} - \frac{{}_sp_k\left({}_rg_k - {}_rdd_k\right)}{Z} - \frac{t_{kd}}{Z}\,{}_rdd_k + \frac{\sum_h {}_rp_h\,{}_r\alpha_{hk}\,{}_rg_h}{Z} + \frac{\sum_h {}_sp_h\,{}_{sr}x_{hk}}{Z} - \frac{\sum_h t_{hk}\,{}_r\alpha_{hk}\,{}_rg_h}{Z} + \frac{\beta_{hk}\,{}_rg_k\,{}_rp_L}{Z} \qquad \text{III.L.4}$$

Now ${}_r\Pi_k,\ t_{kd},\ \sum_h t_{hk},\ \beta_{hk} \geq 0$ III.L.5

$$-{}_r\Pi_k - t_{kd} - \sum t_{hk}\,\beta_{hk} = Z \leq 0 \qquad \text{III.L.6}$$

In theory it is possible to obtain information on

i. ${}_rp_k$

ii. ${}_sp_k$

iii. ${}_rp_h$

iv. ${}_sp_h$

v. ${}_rp_h$

if regional prices were available. Now if supernormal profits were taken away by competition such that ${}_r\pi_k$ was a constant across all regions, then equation III.L.4 could be estimated by regressing

$$
{}_r g_k = \lambda_1 \, {}_r p_k \, {}_r dd_k + \lambda_2 \, {}_s p_k ({}_r g_k - {}_r dd_k) + \lambda_3 \, {}_r dd_k + \lambda_4 \, {}_r p_h \, {}_r \alpha_{hk} \, {}_r g_h + \lambda_5 \, {}_s p_h \cdot {}_{sr} x_{hk} + \lambda_6 \, {}_r \alpha_{hk} \, {}_r g_h + \lambda_7 \beta_{hk} \, {}_r g_k \, {}_r p_h
$$

III.L.7

where

$$\lambda_1 = \lambda_2 = \frac{1}{{}_r\pi_k + t_{kd} + \sum_h t_{hk}\beta_{hk}}$$

$$\lambda_3 = \frac{t_{kd}}{{}_r\pi_k + t_{kd} + \sum_h t_{hk}\beta_{hk}}$$

$$\lambda_4 = \lambda_5 = \lambda_7 = \frac{1}{-({}_r\pi_k + t_{kd} + \sum_h t_{hk}\beta_{hk})}$$

$$\lambda_6 = \frac{t_{hk}}{{}_r\pi_k + t_{kd} + \sum_h t_{hk}\beta_{hk}}$$

Now t_{kd} and $t_{hk} > 0$

Therefore assuming all the t's stay constant and all the other restrictions assumed in attraction analysis hold then the various coefficients can be interpretted as

i. λ_1 says that ${}_r g_k$ is positively related to ${}_r p_k$

ii. λ_2 says that ${}_r g_k$ is positively related to ${}_s p_k$

iii. λ_3 says that ${}_r g_k$ is positively related to demand for k in r.

iv. λ_4 says that ${}_r g_k$ is negatively related to price of input h in r

v. λ_5 says that ${}_r g_k$ is negatively related to price of input h in s

vi. λ_6 says that ${}_r g_k$ is positively related to supply of h in r

vii. λ_7 says that ${}_r g_k$ is negatively related to price of labour in r

It is legitimate to estimate $\lambda_1 \ldots\ldots {}_7$ by regression since all terms are seen to be constant across regions. However because of lack of regional price data, this would be impractical.

b. A second approach is to examine the consequence of erroneously implementing the usual attraction model when prices are given exogeneously. Let us assume that the real price of primary inputs (i.e. productivity x money wage) and/or the price at which the goods can be sold vary between regions, such that the profits of manufacturing products k in region r are less than the average, ceteris paribus. Now competition will mean that sub-normal profits are impossible - how will the regional market rectify itself. Because of sub-normal profits, firms will be forced out of production and this will have two effects that will both force profits towards an equilibrium:

i. because of a reduction in supply of good k in region r, the price will be forced up[1], and the cost of primary inputs fall as a result of a decrease in demand. These possibilities may, however, be ruled out as they violate the institutional restraints.

ii. At the same time as competitors are falling out, those who remain in the market will find that they can sell a greater proportion of their output on the regional market, thus avoiding the cost of exporting shown by t_{kd}. And also, because of the smaller number of firms in the regional market the remaining ones will find that they can buy a greater proportion of their inputs on the regional market, thus avoiding the cost of importing shown by t_{hk} for each h. These factors will boost profits towards the average.

Thus for a region at a competitive disadvantage because of either price or costs, we will find that for the demand in that region or for the supply of inputs in that region, that output will be less than the expected (predicted) output. This is a priori reasonable. For example

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[1] Note that the prices of traded goods can never vary in equilibrium between regions by more than t_{kd} and t_{hk} - see the Takayama and Judge (181) example, Section II.B., or the example given in Section III.K.

suppose we estimate

$$_{r}g_{k} = \lambda_{1} \cdot {}_{r}dd_{k} + \lambda_{2} \cdot {}_{rr}X_{hk} \quad \text{over all } j. \qquad \text{III.L.8}$$

Now by assumption for a region at a competitive disadvantage $_{r}dd_{k}$ and/or $_{rr}X_{hk}$ will be higher than average for the other non-disadvantaged regions. Thus the predicted $_{r}g_{k}$ will exceed the actual $_{r}g_{k}$ for the disadvantaged region r.

A more serious problem arises if say the real wages varied considerably across all regions, such that the relationship between $_{r}g_{k}$ and $_{r}dd_{k}$ and $_{rr}X_{hk}$, varied considerably such that we reject the relationship between them since it may appear statistically insignificant[1].

However, ignoring this possibility, if prices were given exogeneously and are assumed that they stay constant relative to one another then we may be able to test that the disadvantaged regions (shown by an underestimation of output from the predicted output) grow more slowly than the advantaged region.

Approach a. above can be adopted (if regional prices are available) without violating the assumption that industry is optimally distributed through a decentralised decision-making system[2], and thus it is legitimate to apply the attraction analysis approach to this simple price formulation system. In order to give an attraction theory a wider application it would be desirable to have prices formulated through more general market systems. This could be done by using the more general exogeneously given demand functions as used in the Cournot problem, or the endogeneously formed demand functions of general equilibrium[3].

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[1] The problem of misspecification when using III.L.8 when the true equation is III.L.7 will be ignored in this context since the approach is not going to be developed. This does not imply that it is unimportant.

[2] This assertion will not be proved here, but similar and more general price formulation models with efficient solutions from decentralised decision-making can be found in Takayama and Judge (181).

[3] These have been mentioned in Section II.B.

However, it is unlikely that these could ever be empirically implemented, and so will not be discussed.

III.M. Some general remarks concerning the attraction model and an overview of the theory - the relevant region

(i) Some General Remarks

In the discussion of attraction theory so far, I have tried to point out the msin assumptions implicitly and explicitly embodied in model. I have also tried to point out the relationship of the attraction theory of location to other theories of location. However I should like to make a few further points.

(a) As in many of the theoretical works on location described in Chapter II, attraction theory deals with discontinuous space, between which distance costs are incurred, but within which no costs are found to transfer goods between sectors. However even some of the continuous spatial studied were forced to adopt this expedient when solving more general location problems.

(b) One of the advantages of attraction theory is that it is an empirically implementable model, and this is only possible by making certain, often heroic, assumptions. Attraction theory would probably be incapable of incorporating more general functions, which would give it a wider application, since for example, we have to show that a decentralised decision-making system can lead to an optimal solution. The communication cost function is another example of a particularly simple function that is used. The t's are proportional to the volume of goods shipped and not dependent on distance - this is necessary in order to estimate the λ's. This function must also not exhibit any economies of scale. For example, if there was a fixed cost of communication as well as the proportional cost when output was positive the definition of total communication costs would be:

$$_r g_k \cdot t_k = \Omega_k + t_{kd}(_r g_k - {_r dd_k}) + \sum_{h=1}^{n} t_{hk}(\beta_{hk} \cdot {_r g_k} - {_r \alpha_{hk}} \cdot {_r g_h})$$

III.M.1

where the notation has its usual meaning and Ω_k = the fixed costs of communication for industry k as long as output is positive.

Therefore

$$_r g_k = \frac{-\Omega}{t_{kd} + \sum_h t_{hk}\beta_{hk} - t_k} + \frac{t_{kd}}{t_{kd} + \sum_h t_{hk}\beta_{hk} - t_k} {_r dd_k}$$

$$+ \sum_h \frac{t_{hk}}{t_{kd} + \sum_h t_{hk}\beta_{hk} - t_k} {_r \alpha_{hk}} \, {_r g_h}$$

III.M.2

where the term $\frac{-\Omega}{t_{kd} + \sum_h t_{hk}\beta_{hk} - t_k}$ can be interpretted as the intercept. Therefore in the empirical section of the study a test will be made to see if there is a significant intercept. If the intercept is insignificant we can reject the hypothesis that there are any fixed costs of communication. However if it is significant that the function exhibits non-constant returns to scale, then the assumption that a decentralised market mechanism will lead to an optimal solution may be to doubt[1].

(c) Attraction theory is only a partial theory of location in that many variables are given exogeneously that ideally we should like the model to form endogeneously. For example, we take as given the perfectly inelastic demand (composed of consumers, government and investment), the equal production costs (excluding communication costs) at each location, the given labour supply at each location, the location and output of certain industries (either footloose industries or industries where no was made to explain their location by attraction theory) and the communications sector. However, most other theories of location described in Chapter 1 and Chapter II are partial theories and so in this sense attraction theory is not unique.

(d) Attraction theory is based upon communication costs, which are hypothesised to be significantly different from transport costs. It is with this that attraction theory attempts to incorporate some of the secondary factors of location as well as the primary factors (to use Klaassen's (113) distinction again) and to incorporate the advantages of being in a complex with other industries. These factors were discussed in Chapter 1. Some examples of communication costs have been given in Chapter 1[2] in relation to other studies; however, it seems worthwhile

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[1] Situations where a decentralised market mechanism will not operate has been described above in Chapter II.

[2] Van Wickeren (197) also mentions some examples of communication costs.

to list a few examples of communication costs that cause costs to be incurred when a flow of goods takes place between two spatially separated sectors:

i. traditional transport costs

ii. sales lost by not being in close contact with customers - not only final demand but also intermediate goods - or conversely the cost of establishing a sales office in an area you wish to supply at some distance from your factory,

iii. the extra cost of providing storage facilities - when an industry is located near suppliers it is able to keep small stores as any shortage for production can be made good quickly, but location at a distance precludes this,

iv. when a machine breaks down, production is lost while the machine is out of order - this can be quickly mended if specialist engineering firms and/or suppliers of the machine are near at hand, but there may be long delays in a more remote location,

v. transfer of information of the latest techniques, fashions and marketing and buying opportunities may be difficult (and therefore expensive) at long distances,

vi. where sub-contractors and flexibility in production are important,

vii. where personal contacts in business are important.

In attraction theory we have been forced to assume that these costs are related purely to the flow of goods and not distance.

This may seem heroic especially for such a wide range of costs that could possibly have a wide range of functions involving distance and non-constant returns to scale. However, it is only by specifying certain functional forms that results can be obtained for empirical testing - and this is the price that we must pay for this advantage.

(ii) <u>An overview of attraction theory - the relevant region and its relation to some other location theories</u>[1]

The relevant region of an industry can be defined as the area within which the industry is footloose - thus each industry can potentially have a different relevant region. For example, assume that no supply and/or demand and/or labour factors constrain the location of an industry - then the industry is free to locate anywhere within the nation, and the nation is said to be its relevant region. Now consider the example of a corner shop - this is obviously purely demand orientated and the relevant region may be as small as a few streets because if the shop moves outside this area it will lose all its customers. Thus the corner shop may be footloose within a few streets and this is its relevant region.

For location decisions it is hypothesised that an industry will attempt to minimise the communication costs[2] with other sectors and the further from these sectors that it locates the higher these costs. Remembering this hypothesis; ideally we should like to have the country divided into incredibly small

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[1] For a fuller discussion of the relevant region see Van Wickeren (197) Chapter IV.

[2] Communication cost minimisation and profit maximisation having already been shown to be identical under the set of conditions and assumptions imposed on the model.

regions (perhaps even the few streets of the corner shop) and test for each industry for supply and demand attraction within this definition of a region. Obviously we would not get a very good explanation for most industries and so we could consider them to be footloose in some area bigger than the one tested.

These regions should then be aggregated into larger regions and the process repeated. However, the aggregation of regions should not be purely arbitary but should consist of adjacent regions where the newly formed region has less external trade than the two regions that formed it did. With this criterion we can assume that the aggregated regions are complementary because they create a more homogeneous system of regions. Within this new region some more industries will be explained adequately by supply and/or demand and this is then their relevant region. This process is continued until the only region left is the nation, and the industries that are left unexplained are then considered footloose as far as the nation is concerned.

Each industry will have a different relevant region because the costs of communication for each industry will not have the same threshold level, before which it is zero and after which it is positive for contact between buyers and sellers. For example, the costs of buyers communicating with a corner shop will be zero for people very near the shop but for people, say ¼-mile away they will be positive and so the customers will not shop there and consequently the shop cannot move more than a few hundred yards without losing all its customers.

The costs of communicating with a hypermarket will not, however, increase at the same threshold level as did the corner shop and people may be willing to travel 20 - 30 miles to pay visits to such shopping centres, but after, say 40 miles, people would not travel. Thus a hypermarket is footloose within a wider range of area because the threshold cost is less than with the corner shop, even though both are demand orientated.

Similar types of argument can be supplied to industries that are supply orientated. It is possible that certain types of engineering firms need to be near an engineering complex for the supply of sub-contractors' services, and Keeble (110) has suggested that the possible critical distance for this type of supply may be as small as 5 miles. Tosco (189) suggests that 20 - 50 kms. (about 15 - 40 miles) may be the maximum distance over which ancillary services to a metal working complex can possibly communicate with any efficiency[1].

For some types of industries the communication cost will hardly vary with distance, and so regardless of where they locate they will not increase their communication costs. This type of industry is then footloose to locate anywhere within the nation and is the type of industry we are hoping to identify in attraction analysis, if it is to be used as a government regional policy aid.

Unfortunately we cannot carry out the estimation of the attraction theory as outlined above, because we only have input-output tables for one set of regions and consequently we are only

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[1] For more details of these studies see Chapter I above.

testing one area. Thus if we explain the location of our industry with our analysis, we do not know whether the relevant region of that industry is the area we have tested, or if that area is an aggregation of a number of relevant regions. Similarly if we do not explain an industry we do not know if that industry is foot-loose in the nation as a whole or if the relevant region is bigger than the region we have tested, but smaller than the nation. However, as a general rule, we will assume that if we cannot explain an industry then the relevant region is the nation.

The concept of the relevant region and supply and/or demand attraction can be used to analyse some other well known regional techniques and show that they are only special cases of attraction theory.

(a) In attempting to estimate regional input-output tables, Leontief (123) and Isard (91) attempt to classify industries into local, regional, national and international industries, or into as many sub-divisions as are thought appropriate[1]. A local industry is defined as one where consumption and production balance at the local level (whatever local is defined as), and similarly with the other classifications. This assumes that no products of local industries cross the boundaries of the local markets. In attraction terminology a local industry is purely demand orientated as its relevant region is the local region. This implies that demand attraction is the only significant factor and ignores supply attraction.

[1] See Section IV.E for a further discussion of this type of input-output model.

(b) Many comparative cost studies have been done with the aim of establishing whether any industries incur any extra cost in going to a new location (usually a development area). This is an attempt to identify whether the industries studied are footloose or not, i.e. can the industries be successfully transplanted to new areas without suffering any cost disadvantage. If the answer is affirmative then the industry is said to be footloose in that they are not locationally tied to certain areas. In terms of attraction theory the relevant region is the nation. We have argued in Chapter I that these studies are expensive to carry out and it is extremely difficult to identify and quantify all the types of flows such as information, whereas the attraction model may be able to do these more successfully.

(c) In many local multipliers studies a distinction is made between basic and non-basic industries[1]. Basic industries are those that expand as a result of external stimuli from export demand, whereas non-basic industries are purely dependent on the demand generated by the workers in the basic sector. Thus non-basic is assumed to be completely demand orientated and the relevant region is the area under study from which they export none of their products. Basic industries are not demand orientated since they export most of their production, but no reason is given for their location in the region under study.

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[1] Basic industry here is not meant in the sense of a heavy industry that the term is sometimes used in development economics. For a study of basic industry used in this context see, for example, Thomas (182).

(d) Interregional input-output studies have been mentioned already[1], and in them all industries are assumed to be orientated towards demand, with supply having no influence. This is merely a special case of the attraction model where all $\lambda_{kd} = 1$ and $\lambda_{hk} = 0$. However it seems preferable to test for these parameters rather than predetermine them[2].

All these cases above are just special cases of attraction theory, but it seems a better approach to test for demand and/or supply attraction and so calculate the relevant region rather than assume these beforehand.

To conclude the Chapter on attraction theory it seems possible to say from the above discussion that whilst attraction theory may be a more general case of some other location theories, it is nevertheless only a partial short-run model that specifies in a non-general manner many of the relationships between the variables in order to produce a testable form. However, it has the advantage of handling location decisions simultaneously and of having a more rigorous theoretical base than many of the empirical studies discussed in Chapter 1, but is perhaps less general than many of the purely theoretical works discussed in Chapter II. It is this compromise between the two that makes attraction theory a tool that can possibly be used to aid government decision-making.

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[1] See Chapter IV for a full discussion of regional input-output.

[2] See also Section III.H. for a connection between attraction theory and interregional input-output.

CHAPTER IV

Methods of Construction Interregional Input-Output Tables - a Survey and Results of One Method[1]

IV.A Introduction

Interregional input-output tables have been discussed above in relation to the attraction model. We discussed how they are a special case of the attraction model[2]. We have also seen how the data of a regional input-output model is needed as basic data to implement the attraction analysis. However, input-output models have been used extensively in their own right in regional analysis[3]. Their popularity is due partly to their simple consistent structure and partly to their detailed analysis of exogenous changes in demand for the region's product: the multiplier is shown working through all the industries in a region[4]. Therefore the tables are worth constructing not only to obtain the basic data for the attraction analysis but also for purposes

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1 This chapter was written at the early stages of the Ph.d. work. Subsequent to this there has been published a whole text on regional input-output analysis by Richardson (157). There is inevitably some overlap, but the original study is presented in full so that the reader can understand why one particular method of constructing the tables was followed. The method proposed is not totally new and has been used by other authors in the past (these will be mentioned at the appropriate place in the text) although certain minor details do differ.

2 See Section III.H and III.M.

3 For a bibliography of some regional input-output tables see Borque and Cox (15).

4 For an example of this type of impact study see Leontief et. al. (126).

of comparison with both the attraction results and with other input-output tables.

There are different ways of deriving the regional input-output model, each with different data problems and techniques of estimation. This brief survey shows the assumptions embodied in each method, along with the advantages and disadvantages. Finally one method used to estimate regional input-output tables for the 11 standard regions of the U.K. is given, and the Northern region's results are summarised. These tables will form the basic data to implement the attraction model.

IV.B Solution of a 2-region 3-commodity interregional input-output model

The form of a two-region, three-commodity interregional input-output table is shown in Table IV.I. (which is exactly the same as Table III.1, but is reproduced here for ease of reference). Using only two regions is not as restrictive as it may first appear, since region r can be the region that is being studied, and s can be the rest of the country[1]. Theoretically the model can be adapted to encompass any number of regions, but the size of the table increases with the square of the number of regions. Such adaptions are unwieldy and costly to construct, and in the case of some techniques of estimation impossible to construct. Consequently most of the comments will be restricted to a two-region model[2], although mention will be made of methods that are capable of generating larger tables.

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[1] This is the scheme adopted by the Welsh (151) study.

[2] We saw in Ch. III that it was possible to implement the attraction model with a two-region model of this type.

Table IV.1

An interregional input-output system for 2 regions and 3 goods

		region r 1	region r 2	region r 3	region s 1	region s 2	region s 3	exports abroad	To final demand in r	To final demand in s
region r	1	${}_{rr}x_{11}$	${}_{rr}x_{12}$	${}_{rr}x_{13}$	${}_{rs}x_{11}$	${}_{rs}x_{12}$	${}_{rs}x_{13}$	${}_{r}e_{1}$	${}_{rr}f_{1}$	${}_{rs}f_{1}$
	2	${}_{rr}x_{21}$	${}_{rr}x_{22}$	${}_{rr}x_{23}$	${}_{rs}x_{21}$	${}_{rs}x_{22}$	${}_{rs}x_{23}$	${}_{r}e_{2}$	${}_{rr}f_{2}$	${}_{rs}f_{2}$
	3	${}_{rr}x_{31}$	${}_{rr}x_{32}$	${}_{rr}x_{33}$	${}_{rs}x_{31}$	${}_{rs}x_{32}$	${}_{rs}x_{33}$	${}_{r}e_{3}$	${}_{rr}f_{3}$	${}_{rs}f_{3}$
region s	1	${}_{sr}x_{11}$	${}_{sr}x_{12}$	${}_{sr}x_{13}$	${}_{ss}x_{11}$	${}_{ss}x_{12}$	${}_{ss}x_{13}$	${}_{s}e_{1}$	${}_{sr}f_{1}$	${}_{ss}f_{1}$
	2	${}_{sr}x_{21}$	${}_{sr}x_{22}$	${}_{sr}x_{23}$	${}_{ss}x_{21}$	${}_{ss}x_{22}$	${}_{ss}x_{23}$	${}_{s}e_{2}$	${}_{sr}f_{2}$	${}_{ss}f_{2}$
	3	${}_{sr}x_{31}$	${}_{sr}x_{32}$	${}_{sr}x_{33}$	${}_{ss}x_{31}$	${}_{ss}x_{32}$	${}_{ss}x_{33}$	${}_{s}e_{3}$	${}_{sr}f_{3}$	${}_{ss}f_{3}$
imports from abroad		${}_{r}m_{1}$	${}_{r}m_{2}$	${}_{r}m_{3}$	${}_{s}m_{1}$	${}_{s}m_{2}$	${}_{s}m_{3}$			
primary inputs	from r	${}_{r}p_{1}$	${}_{r}p_{2}$	${}_{r}p_{3}$	0	0	0			
	from s	0	0	0	${}_{s}p_{1}$	${}_{s}p_{2}$	${}_{s}p_{3}$			

<u>Assume the following are known</u>[1]:-

(i) total production of each industry in each region ($_rg_i$)

(ii) the technology used in each region (the Leontief β's). Although these may not be known it is reasonable to assume that both regions use the same Leontief technology as the nation, especially in a fairly homogeneous country such as the U.K.[2]

(iii) Final demands in each region[3]

(iv) a national input-output table.

<u>The following are unknowns</u>:-

(i)	$3^2 \cdot 2^2$	intermediate flows	=	36
	or generally with n regions, h commodities			$n^2.h^2$
(ii)	$2 \cdot 3$	import flows	=	6
	or generally			n.h
(iv)	$2 \cdot 3$	export flows		
	or generally			n.h.
(v)	$2^2 \cdot 3$	final demand flows	=	12
	or generally			$n^2.h$

Total unknowns = 36 + 6 + 6 + 6 + 12 = 66

or generally $n^2.h^2 + n^2.h + 3.n.h$

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1 These assumptions of what is known are made on the basis of the data for the U.K. and its regions, that is either known or can be readily estimated. For different countries the availability of data may be different, and therefore there may be better methods for constructing tables than the ones presented here.

2 This assumption will be examined later.

3 These have to be estimated in the case of the U.K. - see Section IV.G.

Equations

(i) How output is allocated.

The output of i from r is used partly as an input into industry i

in r and partly as an input into other industries in r $\sum_{j=1}^{3} {}_{rr}x_{ij}$

and partly as inputs into industries in region s $\sum_{j=1}^{3} {}_{rs}x_{ij}$

and partly as exports abroad from region r ${}_{r}e_{i}$

and partly as final demand in region r ${}_{rr}f_{i}$

and partly as final demand in region s ${}_{rs}f_{i}$

Hence

$$\sum_{j=1}^{3} {}_{rr}x_{ij} + \sum_{j=1}^{3} {}_{rs}x_{ij} + {}_{r}e_{i} + {}_{rr}f_{i} + {}_{rs}f_{i} = {}_{r}g_{i} \qquad \text{IV.B.1}$$

A similar equation applies for that part of commodity i which is output in region s: ${}_{s}g_{i}$

Since i = 1, 2, 3 and there are two regions there are 2.3 = 6 such equations

or generally n.h

(ii) Where inputs originate

By accounting definition, total inputs = total output. The input of i in r is partly an input from industry i in r and

partly an input from other industries in r $\sum_{j=1}^{3} {}_{rr}x_{ji}$

and partly as inputs from industries in region s $\sum_{j=1}^{3} {}_{sr}x_{ji}$

and partly as imports from abroad into r ${}_{r}m_{i}$

and partly as primary inputs within r ${}_{r}p_{i}$

Hence

$$\sum_{j=1}^{3} {}_{rr}x_{ji} + \sum_{j=1}^{3} {}_{sr}x_{ji} + {}_{r}m_{i} + {}_{r}p_{i} = {}_{r}g_{i} \qquad \text{IV.B.2}$$

A similar equation applies for that part of commodity i which is an input in region s $\quad {}_{s}g_{i}$

Since i = 1, 2, 3 and there are two regions, there are 2.3 = 6 such equations or generally n.h.

(iii) The technology

$${}_{r}\beta_{ij} = \frac{{}_{rr}x_{ij} + {}_{sr}x_{ij}}{{}_{r}g_{j}} = \text{technology in region r} \qquad \text{IV.B.3}$$

Since there are 3 industries there are $3^2 = 9$ such equations for each region. For two regions there are 18 equations or generally $h^2.n$.

$${}_{r}\beta_{mi} = \frac{{}_{r}m_{i}}{{}_{r}g_{i}} = \text{import coefficient} \qquad \text{IV.B.4}$$

There are 3.2 such equations = 6 or generally n.h.

$${}_{r}\beta_{pi} = \frac{{}_{r}p_{i}}{{}_{r}g_{i}} = \text{primary input coefficient} \qquad \text{IV.B.5}$$

There are 3.2 such equations = 6 or generally n.h.

(iv) Others

$$ {}_r e_i + {}_s e_i = E_i \qquad \text{IV.B.6} $$

Where E = total exports of industry i abroad.

There are 3 such equations

or generally h.

Alternatively, exports may be known directly from each

region in certain cases. In which case

6 equations

or generally n.h

$$ {}_{rr} f_i + {}_{sr} f_i = {}_r F_i \qquad \text{IV.B.7} $$

There are 2.3 = 6 such equations

or generally n.h.

However not all these equations are independent since by

definition

$$ \sum_{j=1}^{3} {}_r \beta_{ji} + {}_r \beta_{mi} + {}_r \beta_{pi} = 1 \qquad \text{IV.B.8} $$

Therefore IV.B.2 can be formed from IV.B.3, IV.B.4, IV.B.5

and IV.B.8.

Therefore the total number of equations is

6 + 18 + 6 + 6 + 3(6) + 6 = 45(48)

and the total number of unknowns is 66.

Let us assume that we know the exports directly and so can fill in these cells. This leaves us 60 unknowns and 42 equations. However we can also fill in directly the imports from abroad (m) and the primary inputs sector (p) from equations IV.B.4 and IV.B.5. This leaves 30 equations and 48 unknowns. Or in the general case

(i) $n^2.h^2 + n^2.h$ unknowns

(ii) $2.n.h. + h^2.n$ equations

and since n and h are both integers greater than 1 the system is generally insoluble.

Despite this basic insolubility described above, many regional and interregional input-output tables have been constructed. Various methods have been used in order to fill in the missing cells. These can be classified as

(i) surveys

(ii) gravity type models

(iii) Leontief-Isard balance models

(iv) location quotients and optimising behaviour models.

These four types of methods will be examined in turn.

IV.C. <u>The Survey method</u>

If these were conducted in the same way as the national input-output table the expense would be enormous. However Hansen and Tiebout (72) have a cheaper method based on the assumption that firms know better to where they sell (by industry and region) than from where they buy. A questionnaire was sent to firms concerning only their sales. This allows the rows of the input-output table to be filled in. However even this method is not cheap, and is risky in the sense that the questionnaire may elicit a poor response rate[1]. Consequently this method

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[1] Because of the expense involved this method is not practical for more than 1 region, and to implement the attraction model we would need all 11 regions.

will not be considered further[1].

IV.D. Gravity type models[2]

The hypothesis behind most gravity models is that the flows of commodities between two regions can be expressed in the form

$$ {}_{rs}Z_i = \phi\left(\frac{{}_rg_i \cdot {}_sD_i}{{}_{rs}t_i}\right) \qquad \text{IV.D.1} $$

${}_{rs}Z_i$ = total flow of good i from region r to region s

$$ \sum_{j=1}^{h} {}_{rs}x_{ij} + {}_{rs}f_i \qquad \text{IV.D.2} $$

${}_sD_i$ = total demand in region s for products of industry i (intermediate and final)

$$ \sum_{j=1}^{h} {}_{ss}x_{ij} + \sum_{j=1}^{h} {}_{rs}x_{ij} + {}_{rs}f_i + {}_{ss}f_i \qquad \text{IV.D.3} $$

${}_{rs}t_i$ = distance between region s and r (or the cost of transport of good i between s and r). This term is often squared to make it similar to the Newtonian physical gravity model, or can be raised by any exponent power to give a better fit.

This basic expression can take many different forms[3] -

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1 See also Schaffer (164) for other possible types of surveys that involve less work than would be involved in a full survey.

2 For a further discussion of the gravity hypothesis see Section I.B.

3 Isard (90) has a discussion on the use of gravity models. For a recent example see Dramais and Glejser (40).

However I shall restrict the discussion to the Leontief and Strout (124) model. As Polenske (154) shows, the data requirements are:

(i) set of final demands by region $({}_sF_i)$

(ii) technical coefficients by region $({}_s\beta_{ij})$

(iii) trade coefficients $({}_{rs}q_i)$.

The following equations are then derived:

$$ {}_sD_i = \sum_{j=1}^{h} {}_s\beta_{ij} \cdot {}_sg_j + {}_sF_i \qquad \text{IV.D.4} $$

$$ {}_sg_i = \sum_{r=1}^{n} {}_{sr}Z_i \qquad \text{IV.D.5} $$

$$ {}_sD_i = \sum_{r=1}^{n} {}_{rs}Z_i \qquad \text{IV.D.6} $$

$$ {}_{rs}Z_i = \frac{{}_rg_i \cdot {}_sD_i}{{}_og_i} \cdot {}_{rs}q_i \qquad \text{IV.D.7} $$

where $r \neq s$

Where ${}_og_i$ = total amount of commodity i produced (and consumed) in all regions.

${}_{rs}q_i$ = a trade parameter which is a function of transferring commodity i from region r to region s (where the transfer costs reflect the various factors, including transport costs, which determine interregional trade).

Equation IV.D.4 states:

Total demand in region s = the total of intermediate and final demand in region s.

Equation IV.D.5 states:

Total output of region s = total of all flows to all other regions and to itself (i.e. inter-and intra-regional flows) from s.

Equation IV.D.6 states:

Total demand in region s = the total of flows from all other regions and itself (i.e. inter- and intra-regional flows) to s.

Equation IV.D.7 states:

The flow of i from region r to s is a function of output of i in s, demand for i in s, the trade parameter and total national production of i.

Substitute (IV.D.7) into (IV.D.5) and then (IV.D.6)

$$ {}_{s}g_{i} = \sum_{r=1}^{n} {}_{sr}Z_{i} + \frac{{}_{s}g_{i} \sum_{r=1}^{n(r \neq s)} ({}_{r}D_{i} \cdot {}_{sr}q_{i})}{{}_{o}g_{i}} + {}_{ss}Z_{i} \qquad \text{IV.D.8} $$

Which says production of commodity i in region s is equal to total amount of good i produced and sold in region s plus the production sold to all other regions

$$ {}_{s}D_{i} = \sum_{r=1}^{n} {}_{rs}Z_{i} = \frac{{}_{s}D_{i} \sum_{r=1}^{n(r \neq s)} ({}_{r}g_{i} \cdot {}_{rs}q_{i})}{{}_{o}g_{i}} + {}_{ss}Z_{i} \qquad \text{IV.D.9} $$

Which says total consumption of good i in region s equals the total amount of commodity i produced and used in that region plus the amount imported from all other regions.

The system treats as unknowns

(i) total production in each region $({}_{s}g_{i})$

(ii) total demand in each region $({}_{s}D_{i})$

(iii) amount of commodity produced and used in each region $({}_{ss}Z_{i})$

which is 3 n.h unknowns.

Using equations IV.D.4, IV.D.8, IV.D.9 gives 3 n.h equations.

The system can be solved and then the interregional flows $({}_{sr}Z_i)$ found by substituting into IV.D.7.

The problem is how to estimate the trade parameters $({}_{rs}q_i)$. Two methods are given by Polenske (154) depending on the data available.

(i) when all the data in base year estimates are known, as with Polenske's study of Japan. However this data is not available for the U.K. and so I will not discuss the method further.

(ii) When base year estimates are lacking let

$$ {}_{rs}q_i = ({}_r u_i + {}_s v_i) \cdot {}_{rs}t_i \cdot {}_{rs}c_i \qquad \text{IV.D.10} $$

Therefore for good i where $r \neq s$, IV.D.5 becomes

$$ {}_{rs}Z_i = \frac{{}_r g_i \; {}_s D_i}{{}_o g_i} ({}_r u_i + {}_s v_i) \cdot {}_{rs}t_i \cdot {}_{rs}c_i \qquad \text{IV.D.11} $$

where

(i) ${}_{rs}t_i$ is intended to be a measure of the inverse of the per unit transport cost of moving good i from region r to region s. Lack of information may necessitate the measure being estimated by the inverse of the distance involved (making a familiar gravity model)

(ii) ${}_r u_i$ and ${}_s v_i$ are parameters characterising in a summary way the relative positions of region r vis-a-vis all other regions as a supplier, and of region s as a user of good i.

(iii) ${}_{rs}c_i$ can only take two possible values: zero and one. ${}_{rs}c_i = 0$ when $r = s$, so this equation does not estimate the intra-regional flow. In all other cases ${}_{rs}c_i = 1$ except when the constructor of the table feels that the flows of any goods are unlikely (using intuition or casual empiricism). For example, bricks are unlikely to flow from Scotland to the South-Western region because it is too far for bulky, low-value goods to be transported.

${}_{r}u_i$ and ${}_{s}v_i$ can be computed indirectly if we know

(i) total product ${}_{r}g_i$

(ii) total demand ${}_{s}D_i$

(iii) internal use of domestic product ${}_{ss}Z^{\mu}_i$

where the superscript μ stands for base year estimate.

If ${}_{r}u_i$ and ${}_{s}v_i$ are substituted into IV.D.8 and IV.D.9

$$ {}_{r}g^{\mu}{}_{i} \sum_{s=1}^{n} \left[{}_{s}D^{\mu}{}_{i} ({}_{r}u_i + {}_{s}v_i) \cdot {}_{rs}t_i \cdot {}_{rs}c_i \right] \qquad \text{IV.D.8a} $$

$$ = ({}_{s}g^{\mu}{}_{i} - {}_{ss}Z^{\mu}{}_{i}) \cdot {}_{o}g^{\mu}{}_{i} $$

$$ {}_{s}D^{\mu}{}_{i} \sum_{r=1}^{n} \left[{}_{r}g^{\mu}{}_{i} ({}_{r}u_i + {}_{x}v_i) \cdot {}_{rs}t_i \cdot {}_{rs}c_i \right] $$

$$ = ({}_{s}g^{\mu}{}_{i} - {}_{ss}Z^{\mu}{}_{i}) \; {}_{o}g^{\mu}{}_{i} $$

where ${}_{rr}c_i = 0$ corresponding to ${}_{rr}q_i = 0$.

For 1 good 2.h. equations IV.D.8a and IV.D.9a

and 2.h. unknowns (h and k)

Having obtained all this information we still cannot fill in all the cells of the interregional input-output table as shown in Table IV.1. Unknowns in system of Table IV.1 = 48.

Equations in system of Table IV.1 = 30 (that is taking the most optimistic view of known data.
Additional equations available from the solution of the Leontief - Strout gravity model are

$$ {}_{rr}f_i + {}_{rr}x_{i1} + {}_{rr}x_{i2} + {}_{rr}x_{i3} = {}_{rr}Z_i \qquad \text{IV.D.12} $$

where ${}_{rr}Z_i$ = internal use of domestic product i = $\sum_{j=1}^{3} {}_{rr}x_{ij}$

There are 6 such equations. Also:

$$ {}_{rs}f_i + {}_{rs}x_{i1} + {}_{rs}x_{i2} + {}_{rs}x_{i3} = {}_{rs}Z_i = \text{total flow of good i from} $$

$$ \text{region r to s} = \sum_{j=1}^{3} {}_{rs}x_{ij} \qquad \text{IV.D.13} $$

yielding a further 6 equations.

However in a two-region system IV.D.13 can be found directly from IV.D.12 and IV.B.1, once we have allocated the exports abroad as assumed. So in fact we only obtain an additional 6 equations. In the general case there will not be $n.h^2$ additional equations but only $(n.h^2 - n^h)$, since the flow from one region to any one other region can be deduced from total output of the former region and its flows to all other regions. Thus we cannot solve the system of Table I solely by the Leontief - Strout (124) method.

This is to be expected since the Leontief - Strout (124) model only tells us the total flow of one commodity between regions, but not how the flow is allocated between each industry in the region of destination (similarly with intra-regional total flows.) The system can be uniquely determined by making an additional assumption that if say 20%

of the total demand for good i is satisfied by imports from another region, then 20% of each individual sector demanding good i is satisfied from imports from that region.

$$\frac{{}_{rs}Z_i}{{}_{s}D_i} = \frac{{}_{rs}x_{ji}}{{}_{s}\beta_{ji} \cdot {}_{s}g_i} \qquad j = 1 \ldots\ldots\ldots h \qquad \text{IV.D.14}$$

where

(i) ${}_{rs}Z_i$, ${}_{s}g_i$, ${}_{s}D_i$ are the total flow between region r and s of good i, total output and total demand respectively, and all are known,

(ii) ${}_{s}\beta_{ij}$ is the known technology

(iii) ${}_{rs}x_{ij}$ is the unknown import for each individual cell

The Leontief - Strout (124) model has several disadvantages when used in the United Kingdon.

(i) Since we do not know the q's we will have to use the method of equations IV.D.8a and IV.D.9a which needs to know ${}_{rr}Z_i$. These are not readily available for the U.K., although various sample surveys are available that could be utilized if such a study were implemented. Steele (172) has a list of such surveys but they are not very complete and are disaggregated into only a few sectors, rather than the 70 used in the U.K. input-output tables.

(ii) The model assumes economic behaviour can be described and predicted by analogy with physical interaction theories. It is preferable to build up a model based on assumptions concerning economic behaviour.

(iii) Hartwich (74) generated transportation problems by Monte Caro methods and solved by linear programming. The transportation cost minimizing flows were compared with the flows generated by gravity methods. This was done because it is possible to derive gravity type functions from cost minimization hypothises. However he concluded "that simple gravity values will in general not be transportation cost minimizing values".[1]

(iv) Gravity models only seem to perform well when the level of aggregation is high - see Spiegelman (170) and Section I.B - so that the no cross-hauling criteria of models based on optimising behaviour is no longer appropriate[2]. In our case of 70 industries the level of aggregation is quite low.

(v) The model does not give a complete interregional input-output table without additional assumptions, but this problem is not unique to gravity models.

(vi) It must be stated that an advantage of gravity models is that they can be adapted to encompass more than two regions, and they can be implemented for the U.K. situation. Gordon (60, 61, 62) has a lengthy discussion of gravity models in this context, that is more sophisticated than the ones discussed here. However this work is still in progress and so his results cannot be compared with the ones that will be presented here in Section IV.G.

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1 O'Sullivan (179) also showed that linear programming gives a good predictor of flows of goods, see Section I.B.

2 Models based on optimising behaviour will be discussed in Section IV.F.

IV.E. Isard - Leontief balanced regional model

Following Leontief (123) the Isard - Leontief balanced model combines:

(i) input-output analysis

(ii) the fact that some goods are produced in the location where they are consumed and no interregional trade takes place in them,

(iii) some goods travel a long way from the production centre before being consumed, i.e. the supply and demand balance only of the national level.

The type of argument used in points (ii) and (iii) could be repated at all levels, so we could distinguish international, national, regional, sub-regional, local etc. goods. The system is presented below, using only two types of good but the argument could be extended to include many.

There are M commodities:

$M = 1, 2 \ldots\ldots h, h + 1, h + 2 \ldots\ldots\ldots\ldots m$

The first h are regional goods in the sense that production and consumption balance in each region. The last m - h are national goods: production and consumption balance only at the national level.

Let $p = 1, 2 \ldots\ldots h$ (regional)

$g = h + 1, h + 2 \ldots\ldots m$ (national)

(i) ${}_o\underline{g}_i$, ${}_o\underline{g}_p$, ${}_o\underline{g}_k$ represent total national output of all, regional and national goods respectively[1].

(ii) ${}_r\underline{g}_i$, ${}_r\underline{g}_p$, ${}_r\underline{g}_k$ are the corresponding regional outputs.

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[1] A bar under a symbol indicates a matrix or vector.

(iii) $_oF_i$, $_oF_p$, $_oF_k$ are the corresponding total final demand

(iv) $_rF_i$, $_rF_p$, $_rF_k$ are the corresponding total regional final demand.

There are two sets of constraints

$$b_{ij} = \frac{x_{ij}}{g_j} \qquad \text{IV.E.1}$$

The usual Leontief coefficient, which is assumed to hold for all regions

$$_rH_k = \frac{_rg_k}{_og_k} \qquad \text{IV.E.2}$$

where $_rH_k$ = the proportion of total national output of the national commodity k produced in region r.

Now

$$_og_i - \sum_{j=1}^{M} b_{ij} \; _og_j = {_oF_i} \qquad \text{IV.E.3}$$

The usual Leontief system

$$_og_i = \sum_{j=1}^{M} A_{ij} \; _oF_i \qquad \text{IV.E.4}$$

where A_{ij} = elements of the inverse of the matrix of input coefficients[1].

Regional output of any commodity can be determined by multiplying the previous derived national output by the appropriate regional percentage

$$_rg_k = {_rH_k} \cdot {_rg_k} \qquad \text{IV.E.5}$$

To derive the required output of regional commodities one has to separate from system IV.E.3 the first h equations describing the

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1 More commonly written $(I-B)^{-1}$

overall production - consumption balance of these particular goods, and then split each one of them into n regional balance equations. For every region for each given p a separate set of input-output equations is obtained for all regionally balanced goods and services

$$ {}_r g_p - \sum_{j=1}^{m} \beta_{pj} \cdot {}_r g_j = {}_r F_p \qquad \text{for all } p \qquad \text{IV.E.6} $$

Each such system contains h equations - one for every regionally balanced output - and M + 1 variables since the total regional consumption of any required good or service depends on direct final demand and upon the input requirements of all industries (regional as well as national) operating in that region. But the final demand located in a particular region ${}_r F_p$ is given, whilst m - h outputs of national commodities produced in the region have already been determined through IV.E.4 and IV.E.5. Solving IV.E.6 for the remaining h outputs of regional goods allows us to write:

$$ {}_r g_p = \sum_{k=h+1}^{m} B_{pk} \cdot {}_r g_k + \sum_{j=1}^{h} C_{pj} \cdot {}_r F_j \qquad \text{IV.E.7} $$

The two sets of constants B_{pk} and C_{pj} are computed from the basic input coefficients β_{ij}[1]. The ${}_r g_g$'s in the first right-hand term can be obtained from IV.E.4 and IV.E.5.

Thus every regional output of each regional commodity can finally be derived from a given bill of goods. That bill must be described in terms of the total outside demand for each nationally balanced group of commodities and separate regional final demand for all regional goods and services. From the output of regionally and

---

[1] See Appendix VI.

nationally balanced goods in each region the requirements of each good can be calculated. We know that no interregional trade takes place in regional goods and the balance of national goods can be computed.

The input-output system is not fully determined since we know only net flows of national goods not gross flows, unless additional assumptions are made. Also we do not know how to allocate the imports from other regions among the industries demanding this import unless we make the additional assumption that was made in the Leontief - Strout (124) model that each was in the same proportion.

The major drawback with this system is that it is very difficult to identify national and regional industries in practice. Isard (91) has attempted this but had very little success as the industries were not easily classifiable. To attempt to split the industries into two categories is similar to the basic/non-basic industry argument, when the latter are the regionally balanced goods that are deemed to be purely demand orientated, since no flows leave the region. This is a gross oversimplification according to attraction theory.

IV.F. Programming and location quotients

(i) Crude location quotients - an implicit optimising behaviour model

These models can be used to give estimates of inter-regional flows, a typical model would be:

(a) $\dfrac{{}_r g_n}{\sum_{i=1}^{h} {}_r g_i}$ gives the output of industry n in region r as a proportion of total regional output.

(b) $\dfrac{{}_o g_n}{\sum_{i=1}^{h} {}_o g_i}$ gives the output of industry n in the nation as a proportion of total national output.

Three possible situations can arise:

1. (i) > (ii) when the region is assumed to be a net exporter of product n because it has more than its requirements.
2. (i) < (ii) where the region is assumed to be a net importer
3. (i) = (ii) where the region neither imports nor exports in terms of net flows.

Employment data is often used instead of output - since the former is more readily available - but this implies that the output per head if the same in each region. Whether employment or output data are used the following assumptions are implied in the above system:

(a) The technology of the region and the nation are similar - in most regional studies this assumption is made because of lack of any other data.

(b) Consumption patterns of final demand users are the same in each region, and intermediate demand and industrial structure is the same in each region[1]. These assumptions imply that the regional economy has the same proportion as the national economy;

$$\frac{{}_r g_k}{\sum_{i=1}^{h} {}_r g_i} = \frac{{}_o g_k}{\sum_{i=1}^{h} {}_o g_i}$$

for all n. But this is inconsistent with flows to and from the region.

---

[1] This must be so because if it was not, it is possible that a region may have a larger proportion of output than the nation, but will not export any to other regions because its structure means that there is an extra intermediate demand for the product inside the region.

(c) For the net flows calculated to be regarded as gross flows it must be assumed that no cross-hauling of goods takes place: similar products are not shipped both to and from a region. At a high level of aggregation this is not true since this will conceal in one classification many types of products. However the more disaggregated the table the more likely this assumption is to hold, because transport costs can be saved by buying in the region an identical product that was previously imported from another region. Thus the no cross-hauling rule implies some optimisation behaviour. The level of disaggregation of an industrial classification can be roughly judged by how empty the principal diagonal is in the input-output matrix. The emptier the $_{ij}$ cells (when i = j) the more disaggregated the table tends to be.

For an explicit description and computer algorithm of a location quotient method of constructing regional input-output tables, see Jones and Golam (102) which is based on Shaffer and Chu (163). Many other regional input-output studies using modified forms of location quotients (which overcome some of the problems mentioned above) have been carried out, using national tables as a basis - see for example Hewings (81, 82).

(ii) <u>The Welsh study - an explicit optimising behaviour model</u>

The Welsh model (151) as presented by Round (160) is given here to demonstrate the use of location quotients and the problems involved. The following assumptions are made:

Assumption I: regional technological coefficients are the same as the national ones. In a country as homogeneous as the U.K. this assumption should not lead to too great discrepancies. In the Welsh model, certain industries were modified in the light of locally available knowledge, but did not appear to be too different from the U.K. estimates[1].

Assumption II: final demand in the region, which is given or can be estimated, is satisfied by regional production, which also is given, as the first allocation flow. One can assume that the costs of selling to final demand must be high to sectors outside the region so that final demand has to be satisfied from within the region, as long as there is enough production in the region to satisfy this final demand.

$$rs\underline{f} = sr\underline{f} = 0 \qquad \text{IV.F.1}$$

where the $\underline{f}$ are vectors. If there is not sufficient production within the region, then some interregional flows of final demand would take place.

---

1 This assumption has been tested to some extent with USA data by Czamanski (35) and Walderhaug (195). Both compared an actual survey table based on Washington State, with tables derived from second-hand methods. Czamanski's results were quite encouraging apart from what he called 'problem sectors'. These are basically the sectors in which the region has a high degree of specialisation. Since the USA is a more hetrogeneous country than the UK, the problem sectors may not give rise to as many discrepancies in the UK. Walderhaug, page 84, is more optimistic and concludes "the investigation suggests that technical coefficients for local input-output tables of acceptable quality can be developed from national input-output data."

Therefore:

$$_r\underline{g} - {}_r\underline{F} = {}_r\underline{U} \qquad \text{IV.F.2}$$

where $_r\underline{U}$ = vector of total intermediate outputs available from region r, to region r and the rest of the nation(s), after final demand has been satisfied.

<u>Assumption III</u>: total intermediate outputs of each industry in region r are distributed into national industries in the same relative proportion as the distribution of the total intermediate output of the corresponding U.K. industries amongst U.K. industries[1].

<u>Assumption IV</u>: as far as possible the industries in region r obtain their inputs from supplying industries located within region r and likewise industries in region r sell their outputs as far as possible to industries within the region[2].

From assumption III we can define a h x h matrix $_r\underline{X}$ of intermediate flows of goods and services from region r to U.K. industries (region o) where r + s = 0.

$$_r\underline{X} = {}_r\hat{U} \cdot {}_o\hat{U}^{-1} \cdot {}_o\underline{X} \qquad \text{IV.F.3}$$

where:

$_o\underline{X}$ ≡ national input-output matrix[3]

---

1 That is the coefficients found by dividing each element of a row by the row total are constant. Whereas the coefficient in the attraction model is found by dividing each element along a row in the top left-hand quadrant of Table IV.1 by the output of each industry, the coefficient in the Welsh study is found directly from a national input-output table and these coefficients across the rows are assumed to hold for the region (r).

2 Thus there is no cross-hauling so as to minimise transport costs (although this is not explicitly stated).

3 This is a matrix of actual flows of goods, not the coefficients.

$_r\hat{U}$ ≡ the diagonal matrix with $_r\underline{U}$ as the principal diagonal

$_o\hat{U}^{-1}$ = inverse of the matrix with $_o\underline{U}$ as the principal diagonal

$_o\underline{U}$ = vector of total national intermediate outputs.

The meaning of $_r\underline{W}$ is

$_r\hat{U}\ _o\hat{U}^{-1}$ gives the proportion of total intermediate output that the region has of each product.

$_r\hat{U}.\ _o\hat{U}^{-1}.\ _o\underline{X}$ gives the availability to each industry in the U.K. (r + s) of products from industries in region r.

For example, if $_o\underline{X}$ were from a 3 x 3 model

1	6	12	24
2	12	18	0
3	12	12	16

and region r has: half of intermediate output

of industry 1; one third of intermediate output

of industry 2: quarter of intermediate output

of industry 3[1]

i.e. the vector half, third, quarter is the principal diagonal of $_r\hat{U}.\ _o\hat{U}^{-1}$,

then if Assumption III holds, region r would be expected to have available:

for industry 1: half.6 = 3 from industry 1

for industry 2: half.12 = 6 from industry 1

for industry 3: half.24 = 12 from industry 1

for industry 1: third.12 = 4 from industry 2 etc.

---

[1] Note that the vector of proportions of intermediate output can differ from the demand for intermediate inputs because of the different structure of final demands in Wales and R.U.K.

Premultiplying ${}_{o}\underline{X}$ by ${}_{r}\hat{U}.{}_{o}\hat{U}^{-1}$ gives

1	3	6	12
2	4	6	0
3	3	3	4

$= {}_{r}\underline{X}$

From Assumption I we can define a h x h matrix ${}_{r}*\underline{X} = {}_{o}\underline{X}.{}_{o}\hat{V}^{-1}.{}_{r}\hat{V}$

IV.F.4

which gives the intermediate flows of goods and services to industries in region r from industries in the nation as a whole, i.e. its total demands for intermediate products.

Where:

${}_{r}\underline{V}$ vector of region r total intermediate inputs, i.e. its total demand from each sector in region r.

${}_{o}\hat{V}^{-1}.{}_{r}\hat{V}$ is the proportion of total intermediate inputs that the region requires of each product.

${}_{o}\underline{X}.{}_{o}\hat{V}^{-1}.{}_{r}V$ will give the demand by each sector for intermediate products of all other sectors regardless of origin.

If ${}_{o}\underline{X}$ is defined as before and region r needs as total intermediate inputs

(a) tenth of 1

(b) half of 2

(c) three quarters of 3,

i.e. the vector, tenth, half, three quarters is the principal diagonal of ${}_{r}\hat{V}\,{}_{o}\hat{V}^{-1}$.

Post multiply ${}_{o}\underline{X}$ by ${}_{r}\hat{V}{}_{o}\hat{V}^{-1}$ and get the requirements

1	0.6	6	18
2	1.2	9	0
3	1.2	6	12

$\equiv \; {}_{r*}\underline{X}$

From Assumption IV the intra-regional flow matrix can be obtained

$$ {}_{rr}x_{ij} = \min \quad {}_{r}x_{ij}, \; {}_{r*}x_{ij} \qquad \text{IV.F.5}$$

where

(a) ij signifies a typical element of the matrix

(b) ${}_{rr}\underline{X}$ is the matrix of intra-regional flows, i.e. made up of ${}_{rr}x_{ij}$ elements.

For the complete 2-region input-output model we define the following

${}_{rr}x_{ij}$	${}_{rs}x_{ij}$
${}_{sr}x_{ij}$	${}_{ss}x_{ij}$

After the ${}_{rr}\underline{X}$ matrix has been filled using IV.F.5, 3 cases are possible

(a) ${}_{r}x_{ij} > {}_{r*}x_{ij}$ - the region r is a net exporter to region s of intermediate output of industry i when supplying industry j (the output of i for j in region r is greater than the demands by j in region r for i, and once having satisfied this internal demand it exports the surplus available).

Therefore

1. $_{rs}x_{ij} = {_r}x_{ij} - {_{r*}}x_{ij}$, i.e. it exports the surplus available.

2. $_{sr}x_{ij} = 0$ no imports since cross-hauling is eliminated by Assumption IV.

3. $_{ss}x_{ij} = {_o}x_{ij} - {_r}x_{ij}$ the trade among the industries in the rest of the nation is the national input-output tables minus intra-regional trade in r and exports r to s.

(b) $_{r*}x_{ij} > {_r}x_{ij}$

Therefore

1. $_{sr}x_{ij} = {_{r*}}x_{ij} - {_r}x_{ij}$ (it imports what it cannot produce itself).

2. $_{rs}x_{ij} = 0$ (no exports as no cross-hauling)

3. $_{ss}x_{ij} = {_o}x_{ij} - {_{r*}}x_{ij}$

(c) $_{r}x_{ij} = {_{r*}}x_{ij}$ - then on balance region r neither exports not imports intermediate output of i to or from s.

Therefore

1. $_{rs}x_{ij} = 0$

2. $_{sr}x_{ij} = 0$

3. $_{ss}x_{ij} = {_o}x_{ij} - {_r}x_{ij}$

Continuing the numerical example we have

$$_{r}x_{ij} = \begin{matrix} 3 & 6 & 12 \\ 4 & 6 & 0 \\ 3 & 3 & 4 \end{matrix} \qquad {_{r*}}x_{ij} = \begin{matrix} 0.6 & 6 & 18 \\ 1.2 & 9 & 0 \\ 1.2 & 6 & 12 \end{matrix}$$

$$
{}_{o}x_{ij} = \begin{matrix} 6 & 12 & 24 \\ 12 & 18 & 0 \\ 12 & 12 & 16 \end{matrix}
$$

yielding

	0.6	6	12	2.4	0	0	
${}_{rr}x_{ij}$	1.2	6	0	2.8	0	0	${}_{rs}x_{ij}$
	1.2	3	4	1.8	0	0	
	0	0	6	3	6	6	
${}_{sr}x_{ij}$	0	3	0	8	9	0	${}_{ss}x_{ij}$
	0	3	8	9	6	4	

where final demands and primary inputs are determined beforehand. As can be seen this is a refined location quotient since it compares how much is needed for each cell, how much is available and then calculates exports and imports.

The Welsh model has an implication of minimizing transport costs in Assumption IV that eliminates cross-hauling. Taking this criterion we can reformulate the problem into a linear programming one, with the objective function aiming at minimizing transport costs as is implied in the location quotient studies. Using the notation of Table IV.1

Minimize

$$
\sum_{r=1}^{2} \sum_{s=1}^{2} \sum_{i=1}^{3} \sum_{j=1}^{3} {}_{rs}t_{ij} \cdot {}_{rs}x_{ij} + \sum_{r=1}^{2} \sum_{i=1}^{3} {}_{r}t_{i} \cdot {}_{r}e_{i}
$$

$$\sum_{r=1}^{2} \sum_{i=1}^{3} {}_{r}t_{mi} \cdot {}_{r}m_{i} + \sum_{r=1}^{2} \sum_{s=1}^{2} \sum_{i=1}^{3} {}_{rs}t_{i} \cdot {}_{rs}f_{i} \qquad \text{IV.F.6}$$

where t = unit transport[1] cost of its associated variable. Subject to ${}_{rs}x_{ij} \geqslant 0$ where r = 1, 2 s=1, 2, i=1, 2, 3, j=1, 2, 3 and the equations IV.B.1 through to IV.B.8 hold as linear (in)equality constraints[2].

Given a unique set of ${}_{rs}t_{ij}$ there will be a unique solution (excluding the dangers of degeneracy) for the interregional flow model. The Welsh model assumes that final demand is satisfied from within the region before any intermediate demand; this can be allowed for by making the transport costs of importing final demand so large that this flow will be forced to zero in the minimization procedure. Whether the assumption concerning final demand is true or not depends on the actual cost of importing final goods, and so is an empirical question.

In this ideal system it was assumed that each ${}_{rs}t_{ij}$ was known; however in practice this is unlikely to be so for all the categories of goods and even if the transport costs of say textiles is known from region r to s it is unlikely that we will know the differing costs of importing textiles to two different sectors in the same region.

Faced with a complete lack of information on transport costs the following assumptions can be made:

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[1] or communication cost. However we shall ignore this controversy here so as to keep the discussion simple.

[2] The formulation of this problem is similar to the ones shown in Ch. I, II and particularly Section III.K. and Moses (147). It will be demonstrated here that the problem cannot be solved with second-hand data and reasonable assumptions about the t's.

(a) ${}_{rs}t_{ij}$ (where r = s) = 0 (transport costs within the region are zero).

(b) ${}_{rs}t_{ij} > 0$ (where $r \neq s$, there are positive transport costs between regions).

It is impossible to make any assumption concerning the relationship between the cost of transport from regions r to s of good i to, say, the textiles sector or to the engineering sector, unless actual information is available. However if we assume that ${}_{rs}t_{ij} = c_i$ for all j that is the cost of importing good i from region r to sectors j ($j = 1, 2 \ldots n$) in region s is some positive constant, where the constant may be estimated from freight rates if they are available. However this will not give a unique answer to the linear programming problem.

Assume that the region in question has a net export surplus available for a certain product. It imports none of the product, since cross-hauling always leads to extra transport costs, and it exports the surplus to the other region. However there is no criterion by which to allocate this export surplus across the row of imports to the rest of the nation. Sending all the exports to a few sectors in the rest of the nation, with the remainder of the sectors in the rest of the nation getting the supply from themselves, will yield no more saving in transport costs than allocating a few exports to each of the sectors that have to import that good. For example if the region had a net export surplus of 30, both the situations shown in Tables IV.2 and IV.3 would lead to a minimization of transport costs (given our assumptions concerning transport costs) whilst fulfilling the constraints.

Table IV.2

One solution to the transport cost minimization problem

	r			s		
	1	2	3	1	2	3
	10	12	20	30	0	0
	0	0	0	70	70	150
total inputs	10	12	20	100	70	150

Table IV.3

Another solution to the transport cost minimization problem

	r			s		
	1	2	3	1	2	3
	10	12	20	10	10	10
	0	0	0	90	60	140
total inputs	10	12	20	100	70	150

If we have two such optimal basic solutions to a linear programming problem then there are an infinite number of solutions made up of linear combinations of these two solutions.

For the Welsh model a unique solution is obtained without resorting to different t's for the same product between different sectors. It is necessary to look closely at this aspect of the Welsh study because I believe that Assumption III is somewhat arbitary and inconsistent. The assumption is that total intermediate output of each industry in region r is distributed into national (total UK) industries in the same relative proportions as the distribution of total intermediate output of the corresponding national industries amongst national industries.

$$\frac{{}_{rr}x_{ij} + {}_{rs}x_{ij}}{{}_{r}g_{i}} = a_{ij} \quad \text{for all } j \qquad \text{IV.F.7}$$

Where a_{ij} stands for the Welsh allocation coefficient. On this assumption, flows across the rows in an input-output table must always equal this constant allocation coefficient. In contrast, the Leontief coefficient (Assumption I) is

$$\frac{{}_{rr}x_{ij} + {}_{sr}x_{ij}}{{}_{r}g_{j}} = \beta_{ij} \qquad \text{for all } i \qquad \text{IV.F.8}$$

This latter relationship is dictated by the technology of the production process, whereas the Welsh allocation coefficient has no reason for this to be a constant. Since there is no technological or economic compulsion[1] for industry i always to sell a

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[1] We will show that it is not an economic necessity since this situation <u>will not</u> minimize total transport costs.

fixed proportion of its output to industry j[1].

Table IV.4 was based on the example worked out above for the Welsh study, but with only industry 3 inserted. This simplifies for illustrative purposes, but the principle would hold for all industries. In this situation the region in the top left-hand quadrant exports 1.8 of good 3 and imports 11, i.e. a net import balance of 9.2. If the Welsh Assumption III is dropped then the region would have no exports because cross-hauling is eliminated. This would mean the ${}_{rs}x_{31}$ cell would be reduced to zero and the ${}_{ss}x_{31}$ cell increased to 10.8 at the expense of 1.8 exports to r. The problem is how to allocate the exports from s to r and the extra intra-regional use in r.

There are 6 unknowns

${}_{rr}x_{31}$ ${}_{rr}x_{32}$ ${}_{rr}x_{33}$

${}_{sr}x_{31}$ ${}_{sr}x_{32}$ ${}_{sr}x_{33}$

but only 5 equations

1. ${}_{rr}x_{31} + {}_{sr}x_{31} = 1.2$
2. ${}_{rr}x_{32} + {}_{sr}x_{32} = 6$
3. ${}_{rr}x_{33} + {}_{sr}x_{33} = 12$

(equations 1–3) must hold because of Leontief coefficients

---

[1] This is not inconsistent with attraction theory because attraction theory is based on an interregional input-output table where we are assuming flows are optional, i.e. they minimize transport (or communication) costs. Once these optional flows have been established then we have argued in Section III.E. that the ${}_{r}\alpha$ coefficients in the attraction model will be stable in the short run, and so allow us to implement the attraction model. But the constant 'a' coefficients of the Welsh model are <u>not</u> based on a consistent optimising behaviour, as will be shown.

Table IV.4

An inconsistency in the no-cross-hauling assumption

	1	2	3	1	2	3
1						
2						
3	${}_{rr}x_{31}$ 1.2	${}_{rr}x_{32}$ 3	${}_{rr}x_{33}$ 4	${}_{rs}x_{31}$ 1.8	${}_{rs}x_{32}$ 0	${}_{rs}x_{33}$ 0
1						
2						
3	${}_{sr}x_{31}$ 0	${}_{sr}x_{32}$ 3	${}_{sr}x_{33}$ 8	${}_{ss}x_{31}$ 9	${}_{ss}x_{32}$ 6	${}_{ss}x_{33}$ 4

4. $_{sr}x_{31} + _{sr}x_{32} + _{sr}x_{32} = 9.2$ - total exports

5. $_{rr}x_{31} + _{rr}x_{32} + _{rr}x_{33} = 10$ - total intra-regional use.

If there are h commodities and n regions, there are h.n. unknowns and h + n equations. So the system is insoluble if $h > 2$ and $n > 1$.

Some method has to be devised whereby the flows are allocated, since no assumptions concerning transport costs would give a unique solution. One method would be to say (as suggested in equation IV.D.14) from the above situation that because region r imports $\frac{9.2}{10 + 9.2}$ 100% of its total requirements of good 3, then each industry in region r that requires good 3 obtains the same percentage from imports

e.g. $_{sr}x_{31} = \frac{9.2}{10 + 9.2}(1.2)$ and consequently

$$_{rr}x_{31} = 1.2 - \frac{9.2}{10 + 0.2}(1.2)$$[1]

This could be rationalized by assuming that the sales efforts to each sector of exports are subject to diminishing returns[2], but if we assume that each industry is competing for products inside the region (or the nation) for products produced there, it does not seem unreasonable that they can procure the same

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1 This is suggested by Moore and Petersen (143) and Chenery et. al. (29) - see Section III.F.iii for a discussion of their work

2 but this non-linear argument would then be inconsistent with its use in the attraction model.

percentage each[1]. Although this assumption may be somewhat arbitary it does not lead to internally conflicting results as did Assumptions III and IV in the Welsh model. This allowed region r to export to region s products of industry 3 to industry 1, and consequently industry 3 in r would not be allowed to import any of industry 1 from region s, because these flows would be cross-hauling. However it allowed industry 2 in region r to import products of industry 3 from region s, as long as no products of industry 3 were exported from r to industry 2 in s. However since the products of industry 3 are homogeneous regardless of whether they are going to industry 1 or 2 this means that cross-hauling is in fact taking place. Because of this internal inconsistency the Welsh method will not be used.

---

1 We saw in Section III.K that for some of the good imports by the region to go to each sector in the region, then the t's (transport/communication costs) between each sector must be the same. But without knowledge of the t's nothing can be done apart from the assumption described above. Although this is to some extent inconsistent with the attraction model we are going to test, I do not think that it will be too serious since the assumption we are using determines the flows of goods to each sector by preference to total production in the region and total intermediate and final demand in region for that product, which is determined by incomes, tastes and industrial structure. It is these factors that one would expect to play a major role (in determining how much is allocated to each sector). Also other authors (see in particular Boster and Martin (14), Moses (149) and Chenery et. al. (29))do this so that accurate input-output tables can be obtained from second-hand data - see Sections IV.F.iii and IV.vii for a discussion of these works.

(iii) Some other optimising models

Moore and Petersen (143) have constructed an input-output table for Utah alone[1] using assumptions very similar to those suggested above. They first obtain the gross outputs of each sector in Utah and then calculate final demand. They then calculate intermediate demand by post multiplying the national input-output table by the proportion of gross output of each sector using a diagonalised matrix.

$$ {}_{o}\underline{X} \cdot \hat{V}^{-1} \cdot {}_{r}\hat{V} $$

Thus total demand in Utah is found by summing final demand and intermediate demand and comparing this with the known gross output. If output exceeds demand then the surplus is exported, but if demand exceeds output the deficit is imported and the imports allocated to the deficit sectors in the same proportion as imports are to total requirements. This approach follows Isard (92) who constructs a net export/import flow for New England using this balance technique. Isard does point out that to obtain gross flows from net flows by this method one must assume no cross-hauling and he is apprehensive about deriving regional input-output tables in this way. We have argued above that ability to call the net flows the gross flows depends on the disaggregation of the input-output table, and how near the regions in question come to being spacially separated points. If this held then transport costs within regions would be zero

---

1 This is a single region input-output table, rather than the two-region case we have been discussing.

and positive between regions. How far this holds depends on how well the regions have been dilineated, because it is at the regional level only that data is available. For example North Wales may have a surplus in a product and South Wales a deficit - giving zero on balance. However it is possible that North Wales finds it cheaper to sell to Merseyside and South Wales to import from Severnside because that is the way the major lines of communication run. Since we only have data for the administrative regions we have to bear in mind that these may not be the most economically integrated units into which the country can be divided and remember this as a weakness of the study[1].

Chenery et. al. (29) and Moses (149) in construction of their interregional input-output models, also assume that if say 10% of the total of region 1's steel is imported, then each sector that needs steel in region 1 is assumed to have 10% made up of imports. These two studies also bring up the stability of what Chenery calls 'supply coefficients' and Moses 'trade coefficients'. In the input-output matrix shown in Table IV.I, these refer to the bottom left and top right quadrants[2]. A static description of interregional trade can be built up where all the cells are filled in by making the proportionality assumption concerning imports. But if the model is to be used for prediction the assumption must be made that these coefficients

---

1 However casual empiricism from the Min. of Transport road surveys (141) would suggest that the regions are reasonably well defined from the economic stand point - see Fig. I.1 and Table I.1.

2 If these 'trade' or 'supply coefficients' are stable then so is the δ coefficient in the attraction analysis.

are stable, so if region 1 imports 10% of its steel now, and if there is an increase in demand for steel users in that region, they will have to increase their demand for steel (direct and indirect) and 10% of this additional demand will come from imports from other regions[1].

To some extent this may seem arbitary because there is no technical reason for doing so, but this problem was the subject of Moses' (149) study and his results give rise to some optimism concerning the stability of these coefficients. Isard (93) overcame the problem by assuming that imports of steel from one region are a technically different product from the steel produced within the region, thus the trade/supply coefficients become technically necessary Leontief coefficients and prediction is possible. However Isard's is a purely theoretical paper that would be impossible to construct from second-hand data, since the method outlined above relies on calculating the net flows of an homogeneous product from a region.

It must be noted that a disadvantage of all the implicit or explicit behaviour optimising models that have been presented here, is that they can only handle a system of two regions at once, where one must be the region under study and the other must be the nation minus the region under study.

IV.G. An example: the Northern region

It is for the reasons given in Section IV.F. that I propose to follow the methods used by various authors[2] mentioned above and:

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1 For a summary of these studies see Kuenne (122).

2 Namely Moses (149), Moon and Petersen (143) and Chenery et. al (29).

(i) calculate gross output of each region from 1963 Census of Production data (26)[1]

(ii) calculate final demand (excluding exports abroad and to other regions) from such sources as the Family Expenditure Survey (50)[2]

(iii) post-multiply the national input-output matrix for intermediate goods by the matrix which results by setting the proportions of gross output along the principle diagonal of an otherwise null matrix, and sum the rows of the resulting product matrix.

(iv) Add the row sums to the final demands and so calculate total demand from within the region, and compare this with available supply (i.e. gross output).

(v) If the region has a net surplus then all the regional demands are filled by regional output and the surplus is allocated to exports as in the studies described above.

(vi) If the region has a net deficit, the deficit is imported and imports to each sector are allocated as shown above.

It thus seems possible to construct cheaply and quickly interregional input-output tables (of the type shown in Table IV.1) for each region of the U.K. This can be done by employing relatively well

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1 At the time of implementing the analysis the Census of Production for 1968 was not available.

2 A heavy reliance was placed on Woodward (202) in the construction of these estimates. However since completing this work there has come to my notice other work on regional social accounts that go much deeper into the problems involved - namely Gordon (63, 64, 65, 66) and Tompkins (186). These studies treat the problem more rigorously but because of time constraints it was decided in this study to employ existing results rather than going to many primary sources.

established techniques of construction that seem to be internally consistent and by employing data that is relatively accessable. This seems reasonable in the light of the indeterminacy of the model described by equations IV.B.1 to IV.B.8.

Although this method may give, as we have argued, a reasonable static model the main use of input-output tables is for predictive purposes. Moses (198) has pointed out an inconsistency in this type of input-output table when used for prediction. He argues that imports of products are only made when the region cannot supply any more of the product itself, so it must be working at full capacity, otherwise transport costs would be saved by not importing. But use of the inverse matrix for prediction assumes that sectors that import will continue to import the same proportion and produce the same proportion of the product, before and after the increase in demand[1]. Although this is a valid criticism, the Moses (149) study and that of Chenery et. al. (29) give rise to some optimism concerning the stability of these coefficients.

An aggregated version of one of the 11 standard regions input-output tables is presented here. The original had 73 intermediate goods, 5 primary inputs and 4 final demands; so giving a 144 row x 146 column table which is too big for illustrative purposes. To aggregate 73 industries into 6 involves somewhat arbitary aggregation but the table serves for illustration. The table for the Northern Region is as Table IV.5.

---

1 Consequently the implication is that there is some spare capacity remaining in the region, otherwise it would not be able to expand production. This difficulty is not encountered in attraction theory since industries are no longer purely demand orientated, but are limited by the supply constraints as well. Expansion of output of some sectors will lift some of these constraints and so allow some other sectors' output to expand.

Table IV.5

The Northern input-output table diagramatically

Intermediate flows: North to North	Intermediate flows: North to rest of U.K.	North to final demand	row sums
Intermediate flows: Rest of U.K. to North	Intermediate flows: Rest of U.K. to Rest of U.K.	Rest of U.K. to final demand	
Primary inputs in North	Primary inputs in Rest of U.K.		
column sums			

The numerical values are shown in Table IV.6.

The actual table looks as though there is cross-hauling of products, but it must be remembered that each 'industry' in the table represents 12 industries for which there was no cross-hauling in the original interregional flow table - although on aggregation it appears that cross-hauling is present. Cross-hauling could have been eliminated from the demonstration table by aggregating the industries before the interregional flows had been calculated; but this would have underestimated the interregional flows, since the surpluses and deficits of the aggregated industries would have to some extent cancelled each other out.

Remembering the limitations of the multiplier, the $(1 - B)^{-1}$ matrix is shown in Table 7 where labour has been made endogeneous to the system. The matrix that was inverted was slightly different from the one shown in Table 6, because the system shown in Table 6 is a closed system[1]. However in obtaining multipliers for the region the main interest is in the amount of employment that is created directly and indirectly as a result of a change in final demand. Therefore the labour inputs were separated from the primary inputs vector shown in Table IV.6, and final demand to consumers separated from total final demand. The invested matrix was of the form of Table 8[2] with labour included as a normal intermediate.good.

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1 See Yan (203) for a discussion of open and closed input-output systems. This closed system could be solved in relative terms of one variable - see Appendix IV.

2 In showing how to calculate interregional feedback effects from input-output tables Richardson (157) page 79 partitions the matrix and solves for each region separately. However this method involves many more small inversions that inverting the whole 140 x 140 at once. By partition we would need to invert 8 70 x 70 matrices in order to obtain the same information.

Interregional input-output table for Northern region and the Rest of the U.K.

Ind. No.	1	2	3	4	5	6	1	2	3	4	5	6		
1	57.2	34.4	0.5	1.2	4.2	27.7	4.9	19.8	1.0	3.5	7.2	52.0	205.1	418.5
2	19.3	120.7	26.4	25.1	4.7	21.8	10.0	43.1	17.5	27.9	9.5	18.5	179.6	524.1
3	7.8	9.8	26.3	8.2	1.5	16.6	0.3	0.8	2.2	1.2	0.2	2.4	126.3	203.6
4	5.7	5.9	4.6	26.2	10.1	14.3	2.4	1.2	0.8	11.4	0.7	8.4	98.9	190.6
5	8.9	8.9	4.9	2.6	14.3	45.2	0.2	0.1	0.2	0.1	0.5	0.7	69.3	156.0
6	51.9	78.9	23.2	18.9	20.0	161.4	2.7	2.2	0.9	1.5	1.7	9.0	845.4	1217.6
1	7.8	6.5	0.5	0.4	0.8	8.1	1471.1	190.3	17.2	53.7	124.3	555.0	3988.4	6424.2
2	2.0	9.2	3.7	2.3	0.5	2.5	277.0	973.9	496.2	778.1	160.9	392.7	1228.0	4327.0
3	2.1	1.0	2.9	0.9	0.3	2.7	68.3	112.5	514.3	258.3	51.1	323.2	2447.9	3785.6
4	3.6	5.0	5.8	12.0	8.9	14.0	134.7	115.1	207.2	1548.0	342.4	485.0	3445.3	6326.9
5	4.1	4.1	2.1	1.4	6.8	25.0	198.2	116.5	146.8	181.1	635.6	1199.5	1713.2	4234.3
6	1.5	2.2	0.8	0.5	0.6	5.8	854.6	630.8	440.3	612.9	577.0	2800.6	15646.5	21574.0
	246.2	237.7	102.1	90.9	83.1	872.1	3399.8	2121.3	1941.0	2849.1	2323.2	15727.1	0.0	29993.6
	418.3	524.2	203.8	190.6	155.8	1217.2	6424.1	4327.4	3785.6	6326.7	4234.2	21574.3	29993.9	

Note: The row and column totals do not match exactly since they are an aggregation of 12 sectors which were themselves subject to rounding errors involved in the computer calculations.

Inversion of input-output table for Northern region and the rest of the U.K. with labour endogeneous

	1	2	3	4	5	6	1	2	3	4	5	6	Consumer Demand in North	Consumer Demand in Rest of U.K.
1	1.35	0.25	0.20	0.20	0.20	0.23	0.01	0.02	0.01	0.01	0.01	0.01	0.31	0.01
2	0.14	1.37	0.26	0.27	0.12	0.09	0.01	0.02	0.02	0.02	0.01	0.01	0.09	0.01
3	0.06	0.06	1.18	0.09	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
4	0.06	0.05	0.07	1.20	0.12	0.06	0.00	0.00	0.00	0.01	0.00	0.00	0.06	0.00
5	0.11	0.10	0.12	0.10	1.18	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
6	0.62	0.60	0.63	0.60	0.57	0.64	0.01	0.02	0.02	0.02	0.01	0.01	0.75	0.01
1	0.14	0.13	0.13	0.14	0.13	0.13	1.51	0.29	0.29	0.28	0.29	0.31	0.16	0.41
2	0.06	0.08	0.10	0.10	0.08	0.06	0.16	1.39	0.30	0.31	0.17	0.13	0.07	0.13
3	0.03	0.02	0.04	0.04	0.03	0.02	0.06	0.08	1.21	0.11	0.07	0.07	0.03	0.06
4	0.11	0.10	0.15	0.20	0.19	0.12	0.14	0.16	0.22	1.46	0.25	0.17	0.13	0.18
5	0.09	0.08	0.10	0.09	0.13	0.10	0.17	0.17	0.21	0.19	1.32	0.22	0.09	0.21
6	0.17	0.16	0.20	0.21	0.21	0.17	0.66	0.70	0.78	0.75	0.72	1.78	0.18	0.92
Labour in North	0.88	0.71	0.88	0.86	0.78	0.97	0.01	0.02	0.02	0.02	0.01	0.01	1.56	0.01
Labour in Rest of U.K.	0.19	0.18	0.23	0.24	0.25	0.19	0.78	0.82	1.04	0.97	0.95	1.10	0.21	1.70

Table IV.8

The Northern input-output table diagramatically - with labour as an endogeneous intermediate good

Intermediate flows North to North	Intermediate flows North to Rest of U.K.	From North to consumers in North	From North to consumers in R.U.K.	Other final demand - exogeneous
Intermediate flows Rest of U.K. to North	Intermediate flows Rest of U.K. to Rest of U.K.	From R.U.K. to final consumers in North	From R.U.K. to final consumers in R.U.K.	
Labour inputs from North to North	Labour inputs from North to Rest of U.K. = 0	0		
Labour inputs from Rest of U.K. to North = 0	Labour inputs from Rest of U.K. to Rest of U.K.			
Other primary inputs - taken as exogeneous				

The bottom right-hand square of Table IV.8 is zero since final demand does not consume labour directly, and the primary inputs from R.U.K. to North and from North to R.U.K. are both zero since labour is assumed non-transportable in the short run. Each column was then divided by the column totals of the matrix in Table IV.6 (so the coefficients will not sum to unity); this matrix of coefficients was taken from a unit matrix and inverted. The result is shown in Table IV.7. Apart from the assumptions mentioned earlier concerning the stability of the trading coefficients, which must also apply to final consumers as well as intermediate users; it is also assumed that the marginal propensity to consume is equal to the average propensity to consume. This assumption was made due to the lack of other available information on the consumption patterns by region of the 73 industries. The results of Table IV.7 are interpreted in the following way. Assume that there was an increase in demand of 1 unit for products of industry 1 in the Northern region; this would require directly and indirectly 1.35 units of gross output of industry 1 in the Northern region, 0.14 of industry 2 in the North etc., 0.14 of industry 1 in the Rest of the U.K., 0.06 of industry 2 in the Rest of the U.K. etc., 0.88 units of labour in the North and 0.19 units of labour in the Rest of the U.K. It should be remembered that this is the maximum that the multiplier allows since in this system it is assumed:

(i) the marginal propensity to consume = the average

(ii) all available inputs are bought within the region before looking outside. This minimises the 'leakage' effects of the multiplier[1]

---

1 For a rigorous proof of the upward bias on multipliers involved by assuming no cross-hauling see Jones, etl. al. (103).

(iii) The Moses (148) criticism mentioned above.

In spite of the assumption of minimum leakages, one unit of expenditure in the Northern region creates less than 1 unit of employment in that region in all cases. This seems to be considerably less than the multiplier obtained in the many non-input-output multiplier studies[1 2]. A possible explanation is that no account has been taken of feedbacks in government expenditure, since this sector has been treated as exogenous. However it is felt that this would not increase the multiplier effect considerably.

The actual labour requirements (direct and indirect) of each sector for the increases in final demand can be found with reference to Table IV.9 which shows the labour coefficients of each industry. For example, 1 unit of output of industry 1 in the North requires directly 0.36 units of labour. Thus if demand for industry 1 in the North were increased by 1 then the direct and indirect labour requirements obtained in Table IV.7 with the labour coefficients of Table IV.9:

(i) 1.35 (0.36) from industry 1 in the North

(ii) 0.14 (0.19) from industry 2 in the North

etc.

---

[1] See for an example Greig (67)

[2] It is interesting to note that Boster and Martin (14) investigated the results obtained from survey based input-output tables and non-survey based tables, and did not find great discrepancies. They concluded that the little extra accuracy gained in constructing survey based tables (assuming these to be true and free from reporting errors) was not worth the extra enormous expense involved.

Table IV.9

Labour coefficients for Northern Region and Rest of U.K.

Northern Region						Rest of United Kingdom						Labour in North	Labour in Rest of U.K.
1	2	3	4	5	6	1	2	3	4	5	6		
0.36	0.19	0.33	0.32	0.30	0.48	0.21	0.21	0.35	0.27	0.30	0.49	0.0	0.0

(vi) 0.62 (0.48) from industry 6 in the North

The total of which = 0.88

(vii) 0.14 (0.21) for industry 1 in the Rest of the U.K.

(viii) 0.06 (0.21) for industry 2 in the Rest of the U.K.

etc.

(xii) 0.17 (0.49) for industry 6 in the Rest of the U.K.

The total of which = 0.19

This can be done for each industry for which demand increases and shows where the resulting increase in demand for labour will occur.

It was felt that the multiplier may be sensitive to changes in the propensity to consume and so this was tested in the following way. The technical coefficients in the final demand columns were multiplied by a number less than one, then the whole matrix was subtracted from the unit matrix and inverted. The number less than one is how much the marginal propensity to consume is as a proportion of the average. Each industry was multiplied by the same number because it was not known which industry would have the highest marginal propensity to consume since the regional data is lacking. (In any case the industries in this 14 x 14 study are arbitary aggregations from the 140 x 140 table.) This operation was repeated with numbers ranging from 0.10 to 1.00 in multiplies of 0.05. On average the direct and indirect labour requirements seemed to have been reduced by about 40% by taking this range. The whole table for a propensity to consume 0.75 of the average (perhaps a reasonable estimate) is reproduced in Table IV.10. The whole range of values of these marginal propensity to consume, together with direct and indirect labour requirements in the North and in the rest of the U.K. is given in Table IV.11 for industry 1.

Table IV.10

Inverted input-output table for Northern region with the marginal propensity to consume less than the average propensity to consume

	1	2	3	4	5	6	1	2	3	4	5	6	Consumers demand in North	Consumers demand in Rest of U.K.
1	1.29	0.20	0.14	0.14	0.15	0.16	0.01	0.02	0.01	0.01	0.01	0.01	0.20	0.01
2	0.12	0.35	0.24	0.25	0.10	0.08	0.01	0.02	0.01	0.02	0.01	0.01	0.06	0.01
3	0.05	0.05	1.18	0.08	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
4	0.05	0.04	0.06	1.19	0.11	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
5	0.47	0.48	0.48	0.46	0.44	1.48	0.01	0.01	0.01	0.01	0.01	0.01	0.49	0.01
1	0.10	0.09	0.09	0.09	0.09	0.08	1.44	0.22	0.19	0.19	0.21	0.22	0.09	0.26
2	0.05	0.07	0.08	0.08	0.06	0.04	0.13	1.37	0.27	0.29	0.14	0.10	0.04	0.08
3	0.02	0.02	0.04	0.03	0.02	0.02	0.04	0.07	1.19	0.10	0.05	0.05	0.01	0.04
4	0.08	0.08	0.12	0.17	0.16	0.09	0.11	0.13	0.18	1.42	0.21	0.12	0.08	0.12
5	0.06	0.06	0.07	0.07	0.11	0.07	0.13	0.13	0.16	0.15	1.28	0.17	0.05	0.13
6	0.10	0.10	0.12	0.13	0.14	0.10	0.51	0.54	0.58	0.55	0.56	1.56	0.09	0.59
Labour in North	0.77	0.62	0.77	0.76	0.68	0.85	0.01	0.02	0.01	0.01	0.01	0.01	1.37	0.01
Labour in Rest of U.K.	0.13	0.13	0.16	0.17	0.18	0.13	0.66	0.70	0.89	0.83	0.81	0.93	0.12	1.44

0.75 = marginal propensity to consume as a proportion of the average

Table IV.11

Direct and Indirect labour requirements of industry I in Northern region with various marginal propensities to consume

Marginal propensity to consume as a proportion of average	Direct and indirect requirements of labour (a) in the North	(b) in Rest of U.K.
1.00	0.88	0.19
0.95	0.86	0.18
0.90	0.83	0.16
0.85	0.81	0.15
0.80	0.79	0.14
0.75	0.77	0.13
0.70	0.75	0.12
0.65	0.74	0.11
0.60	0.72	0.10
0.55	0.70	0.10
0.50	0.69	0.09
0.45	0.67	0.08
0.40	0.66	0.08
0.35	0.64	0.07
0.30	0.63	0.07
0.25	0.62	0.06
0.20	0.61	0.06
0.15	0.60	0.05
0.10	0.58	0.05

Figures rounded to 2 places of decimals

The operation described above would not be valid for any 'industry' except labour, since the assumptions concerning the technology of an input-output system necessitate fixed coefficients which are stable so that production can be made. However with the labour 'industry' this is not obligatory since the amount of goods necessary to produce one unit of labour diminishes the more labour is produced, i.e. the marginal propensity to consume is less than the average. For example, initially to produce 1 unit of labour it may have been necessary to have 0.5 units of food industry and 0.5 of services. However to produce a further unit of labour only 0.4 food and 0.4 services may be necessary. This reduction in the coefficients is some measure of the marginal propensity to consume and was tested for the wide range of values down to 0.10 which nearly excludes labour from the system. Thus making it exogeneous and ignoring its feedback effects[1]. A summary of some of the results obtained from manipulation of the full 140 x 140 interregional table for the Northern region can be found in Appendix VII.

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[1] It should be noted that when the input-output table is inverted to obtain multipliers, the elements along the central diagonal are retained. This contrasts with the U.K. tables for 1963 (27) but is in line with the tables for the U.S.A. (143). However, as shown in Appendix V, the only difference between a consolidated system (where the central diagonal is zero) and a non-consolidated system (where it is non-zero) when the matrix is inverted is the actual value on the central diagonal. Thus the multiplier is not affected by feed-back effects on this element and so to some extent its inclusion is arbitary and will not affect the direct and indirect labour requirements. It was included in the input-output analysis so direct comparison could be made with the attraction tables.

Chapter V

Methods of implementing the attraction model with UK data, and some of the results obtained

V.A. Basic Equations Estimated

The interregional input-output tables that were estimated as shown in Section IV.G were used as the basic data in estimating an equation for each industry of the attraction model. As described in Appendix II the attraction model is a simultaneous system, and so Two Stage Least Squares (2.S.L.S.) was used to estimate the parameters. This method involves the use of all the exogeneous variables (even those excluded from each equation) being used to obtain new estimates of the included endogeneous variables, which are then used to obtain estimates of the parameters in the original equation[1]. However the number of exogeneous exceeded the number of observations, so a principal components analysis was carried out on these exogeneous variables[2]. These principal components were then used in place of the exogeneous variable in the first stage of estimation, while the second stage was carried out in the usual way.

In all industries there were a large number of potential explanatory variables, since most sectors trade to some extent with a large number of other sectors. Therefore a rule of thumb was adopted - if in the original U.K. input-output table (27) an industry obtained less than 2% of its inputs from another industry then this latter industry would not be considered as a potential supply influence[3]. Of the remaining potential explanatory variables only those whose coefficients

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1 For a complete discussion of 2.S.L.S. see for example Johnstone (101)

2 See Appendix II for the full details

3 This expediency was also adopted by Van Wickeren (197) Ch. 5.

were significantly different from zero at the 10% level were included.

Imports caused some problems. It was felt that they could influence the location of industry through their point of entry into the country. In the U.K. these points of entry were taken to be sea ports[1] since data for these were readily available from the Digest of Port Statistics (38). Table C of this U.K. input-output table (27) gives a breakdown of the foreign industry of origin of imports, for each importing industry in the U.K. Only those imports by a sector that were supplied largely by the products of a single foreign industry were taken as a possible explanatory variable[2]. There were then two possible cases:-

(i) Where a commodity was imported by a sector and none of that commodity was purchased (or produced) in the U.K. (i.e. non-competitive imports) the imported commodity was taken as a separate attraction force. For example, industry 11 (Tobacco) imported a large proportion of its inputs (38% of gross output was imported) and of this almost 99% consisted of products of the foreign agricultural industry. The U.K. agricultural sector supplied no inputs to industry 11. Thus the ports that imported tobacco were regarded as a separate possible attraction force to industry 11.

(ii) Where a commodity was imported by a sector as well as that sector obtaining some of that

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1 Airports may become significant in the future, but were excluded from the present analysis.

2 Where imports into an industry were composed of small flows from many different foreign industries, these were ignored since it was felt that they would not significantly affect the location of the importing industry.

commodity inside the U.K. from other sectors (i.e. competitive imports). The imported commodity was added on to the regionally domestically produced commodity since the attraction effect was felt to be similar. For example, industry 5 (grain milling) used the products of agriculture both from domestic and external sources. The imports by each port were added onto that region's internal domestic supply of agricultural products in order to obtain a composite agricultural supply attraction.

This procedure with imports was justified on the grounds that in order to import some goods into the U.K. a fixed cost per unit is incurred in buying the good at the point of entry, since the imports must be landed in the U.K. from abroad - the industry has no influence over this[1]. There is, however, a cost over which the industry has some influence in that costs of communication will be saved if the industry locates near the port where the commodity is landed[2].

The results of the 2.S.L.S. analysis for each industry are presented in Table V.1. Multicollinearity made it necessary to aggregate certain industries but even in some of the remaining industries there was evidence of some multicollinearity that would reduce the efficiency of the estimators[3], and may cause us to reject certain variables

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1 Unless of course it locates at the supply source outside the U.K.

2 See Van Wickeren (197) Ch. 5 for the method he uses to deal with imports. The method used here implies that in the case of competitive imports the costs of imports in the region where the imports are landed are identical with the region's domestically produced supply.

3 That is to say multicollinearity exaggerates the size of the standard errors of the estimated coefficients.

Table V.1

Results of 2.S.L.S. analysis for each industry in the attraction analysis

Notes

(i) The figures after the coefficients show which industry is the attracting force - d = demand

a = supply (for α coefficient)

m = imports

(ii) The figures in brackets under the coefficients are the t-values.

(iii) Industry 66 and 67, and 69 and 70 have been aggregated because of multicollinearity.

(iv) Industry 20 and 21 have been aggregated because data for each industry separately was not available in the 1963 Census of Production.

(v) No tests for industry 1, 2, 3 or 4 were undertaken because they were thought to be influenced by factors not within the scope of the attraction model.

Industry Number	Name	Regression Coefficients	R^2
1	Agriculture	not included in the attraction model	
2	Forestry and Fishing		
3	Coal Mining		
4	Other mining and quarrying		
5	Grain milling	= 0.80(d) + 0.67(al + m) (4.37) (2.04)	0.92

Industry Number	Industry Name	Regression Coefficients	R^2
6	Other cereal food	= 0.30(d) + 0.63(a5) + 11.36(a9) (2.30) (2.42) (2.62) + 3.60(a16) + 3.74(a58) (2.54) (2.47)	0.96
7	Sugar	= 1.32(a1 + m) (9.80)	0.86
8	Cocoa, chocolate and sugar confectionary	Not significant at the 10% level	
9	Other food	= 0.19(d) + 8.50(a9) + 3.89(a37) (2.42) (3.42) (2.72) + 9.95(a58) (2.98)	0.93
10	Drink	Not significant at the 10% level	
11	Tobacco	Not significant at the 10% level	
12	Mineral oil refining	= 0.87(d) + 0.96(a4 + m) (1.98) (2.44)	0.95
13	Paint and printing ink	= 0.43(d) + 9.27(a17) + 1.11(a18) (2.67) (3.22) (2.76)	0.96
14	Coke ovens	= 1.60(a3) (63.05)	0.99
15	Pharmaceutical and toilet preparations	= Not significant at the 10% level	
16	Soaps, oils and fats	= 0.27(d) + 7.42(a18) + 10.70(a58) (7.11) (6.75) (3.63)	0.93
17	Synthetic resins and plastic materials	= 0.35(d) + 3.28(a18) (3.16) (8.04)	0.98
18	Other chemicals and allied industries	= 0.49(d) + 7.43(a18) (5.89) (12.08)	0.99
19	Iron and steel	= 0.68(d) + 0.73(a4 + m) + 4.75(a14) (3.30) (2.16) (2.35) + 1.16(a1a) (2.21)	0.98

Industry Number	Industry Name	Regression Coefficients	R^2
20/21	Light metals and other non-ferrous metals	= 0.41(d) + 16.53(a4+m) + 0.71(a20/21)	
		(12.97) (9.86) (3.29)	
		+ 13.91(a38)	0.97
		(5.02)	
22	Agricultural machinery	= 19.04(a31)	0.98
		(3.71)	
23	Machine tools	= 0.19(d) + 1.43(a19) + 1748(a31)	0.94
		(2.64) (2.23) (4.95)	
24	Engineer's small tools	= 0.62(d) + 1.90(a19) + 19.24(a24)	0.92
		(4.40) (3.88) (4.97)	
25	Industrial engines	= 21.02(a31)	0.97
		(33.29)	
26	Textile machinery	= 15.10(a31)	0.96
		(7.21)	
27	Contractors' Plant and Mechanical handling equipment	= 15.51(a31)	0.95
		(46.01)	
28	Office machinery	= 14.80(a31)	0.90
		(19.09)	
29	Other non-electrical machinery	= 0.16(d) + 16.33(a31)	0.96
		(2.60) (5.56)	
30	Industrial plant and Steel work	= 0.68(d) + 14.49(a29) + 7.26(a31)	0.97
		(3.63) (5.14) (3.34)	
31	Other mechanical engineering	= 0.21(d) + 0.16(a19) + 22.53(a31)	0.95
		(2.09) (2.53) (5.70)	
32	Scientific instruments etc.	= 0.42(d) + 10.28(a32)	0.93
		(3.86) (19.08)	
33	Electrical machinery	= 0.23(d) + 2.81(a19) + 17.76(a33)	
		(3.70) (2.25) (2.44)	
		+ 6.08(a34)	0.96
		(1.98)	

Number	Industry Name	Regression Coefficients	R^2
34	Insulated wires and cables	= 0.27(d) + 8.32(a38) (7.07) (47.71)	0.99
35	Radio and Telecommunications	= 0.31(d) + 8.49(a20/21) + 4.41(a31) (4.93) (2.64) (2.28) + 5.64(a35) + 2.04(a38) (3.11) (2.30)	0.94
36	Other electrical goods	= 0.48(d) + 9.23(a20/21) + 14.00(a35) (3.00) (8.77) (2.52)	0.91
37	Cans and metals boxes	= 0.85(d) + 1.03(a19) (8.58) (3.21)	0.91
38	Other metal goods	= 0.45(d) + 0.88(a19) + 3.68(a20/21) (4.15) (2.67) (2.94) + 3.27(a38) (2.22)	0.93
39	Shipbuilding and marine engineering	= 0.61(d) + 3.96(a19) + 4.47(a39) (8.33) (3.55) (3.68)	0.98
40	Motor vehicles	Not significant at the 10% level	
41	Aircraft	= 0.16(d) + 0.83(a38) + 5.10(a41) (7.05) (2.33) (22.20)	0.94
42	Other vehicles	= 0.19(d) + 0.40(a19) + 26.49(a31) (2.26) (2.35) (5.12)	0.95
43	Production of man-made fibres	= 3.71(a17) + 6.38(a18) (2.34) (3.33)	0.91
44	Cotton etc. spinning and weaving	= 0.58(d) + 3.94(a44) (2.12) (5.64)	0.97
45	Wool	= 0.42(d) + 15.75(al+m) + 5.17(a43) (6.45) (2.36) (3.12) + 0.45(a45) + 34.79(a47) (2.77) (2.47)	0.96
46	Hosiery and Lace	= 0.28(d) + 1.48(a43) + 15.41(a46) (4.05) (2.53) (18.21)	0.99

Industry Number	Industry Name	Regression Coefficients	R^2
47	Textile finishing	= 0.52(d) + 11.59(a18) (2.66) (4.58)	0.93
48	Other textiles	= 0.80(d) + 2.45(a45) + 3.19(a48) (6.25) (2.72) (2.17)	0.98
49	Leather, leather goods and fur	Not significant at the 10% level	
50	Clothing	= 0.59(d) + 1.22(a44) + 11.08(a50) (5.08) (5.02) (4.06)	0.98
51	Footwear	= 3.26(a49) + 10.92(a51) (3.96) (2.20)	0.92
52	Cement	= 0.96(d) + 11.64(a12) (12.80) (9.50)	0.97
53	Other building materials etc.	= 0.88(d) + 6.23(a04) (9.55) (2.21)	0.96
54	Pottery and glass	Not significant of the 10% level	
55	Furniture etc.	= 0.47(d) + 16.98(a55) (4.77) (9.57)	0.96
56	Timber and miscellaneous wood manufactures	= 0.47(d) + 1.89(a38) + 9.29(a56) (5.22) (2.38) (6.48)	0.97
57	Paper and Board	= 0.52(d) + 4.79(a3) + 13.37(a57) (4.49) (5.00) (4.21)	0.98
58	Paper products	= 0.79(d) + 1.82(a57) (7.98) (5.55)	0.99
59	Printing and publishing	= 0.36(d) + 7.36(a59) (5.20) (16.03)	0.98
60	Rubber	= 0.24(d) + 1.95(a18) + 27.71(a38) (3.74) (2.71) (9.43)	0.98
61	Other manufacturing	= 0.44(d) + 10.48(a18) + 2.70(a38)	0.95
62	Construction	= 1.03(d) (60.54)	0.98

Industry Number	Industry Name	Regression Coefficients	R^2
63	Gas	= 1.17(d) (37.52)	0.91
64	Electricity	= 1.02(d) (50.45)	0.98
65	Walter supply	= 1.03(d) (100.65)	0.99
66/67	Other transport and road and rail transport	= 1.60(d) (23.34)	0.92
68	Communication	= 0.80(d) + 18.09(a35) (15.81) (7.14)	0.96
69/70	Distributive Trades and miscellaneous services	= 1.09(d) (90.35)	0.99

as insignificant because of the resulting low t-values. Thus there is a danger we may have rejected a true relationship so misspecifying the equation. Since the rejected variable is correlated with at least one of the remaining variables in the equation by virtue of the multicollinearity then the results will be biased[1].

Attraction analysis was thought to be inappropriate in explaining the first four industries (agriculture, forestry and fishing, coal mining and other mining and quarrying) since it was felt that other factors, such as the location of deposits of coal, influenced these industries rather than interindustry relations[2]. For certain industries the regression results proved inconclusive and these are designated 'not significant' in Table V.1.

In all the industries reported no intercept term was used in the regression analysis. However, as explained in Section III.M, an intercept term may be interpreted as a fixed cost of communication which if present will cast doubt on the necessary assumption that industry is optimally distributed. All the regressions were, therefore, re-run with an intercept term. It proved insignificantly different from zero in all cases except industry 63 (gas) where it was just significant, and industry 66/67 (transport) where it was highly significant. In the case of gas the intercept was negative, which is not inconsistent with the theory in Section III.M. The intercept in the equations to

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1 See Appendix VIII for a discussion of the effects of multicollinearity in attraction analysis.

2 This is not to say that some other industries such as chemicals and shipbuilding do not have strong site requirements, but it was felt that these industries would be [illegible] influenced by inter-industry relations, and so meaningful results may be obtained from the regressions.

to be estimated is

$$-\left(\frac{\Omega_k}{t_{kd} + \sum_h t_{hk}.\beta_{hk} - t_k}\right)$$ [1]

where all the terms are positive and $(t_{kd} + \sum t_{hk}.\beta_{hk}) > t_k$ unless Ω_k is very large in relation to t_{kd} and $\sum t_{hk}.\beta_{hk}$. This is difficult to explain although two alternative explanations present themselves.

(i) Since we are estimating nearly 70 equations, and even though the 'true' equation for each industry may not have an intercept, it is possible that we could identify a significant intercept by chance in the case of industry 63.

(ii) The gas industry is an aggregation of two industries - gas production and gas distribution - and it is possible that each may be influenced by different characteristics. It is possible that gas distribution is wholely influenced by demand as that gas production by demand and/or coal inputs. However aggregating the two leads to difficulties.

The explanation for a significant positive intercept for industry 66/67 may be that a large proportion of its output is exported (over 25%) and in this particular industry it is possible that the communication costs with overseas buyers for such places as London, is the same as with buyers in the South-East. If this is so, and the industry is demand orientated, then the use of internal regional demand in the regression will underestimate the actual demand, and cause a positive intercept and explain why the demand attraction coefficient is greater than one, when the theory predicts that it should not exceed one.[2]

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1 Remember Ω_k is the fixed cost of communication

2 See Appendix III

However in the estimation of the attraction table[1] the estimates for industry 63 and 66/67 will be based on the parameters without the intercept[2], but the results should be regarded with suspicion[3].

In Section III.1 some alternative models of price formulations were discussed and it was shown that if our assumptions about pricing behaviour were invalid, it was possible that what we are trying to estimate across regions, could be variables rather than constants. Let us assume that the prices of inputs and/or outputs and/or labour productivity are random variables across regions. This will give the coefficients λ_{kd} and λ_{hk} a random component - $\lambda_{kd} + u_{kd}$ and $\lambda_{hk} + u_{hk}$. For example if the price of the final product is high in a region then, ceteris paribus, for a good attracted by, say, demand, for a given level of demand, in order that normal profits shall still be earned, output will be higher. This will be reflected in $u_{kd} > 0$, and vice versa when the price is low. The assumption of the randomness of these factors may be valid for, say, labour productivity, but it may be that the price of the good in a region is an interaction of general supply functions (regional output) and general demand functions (which is one of the variables on the right-hand side of the equation) and so the u's will not be random. However there seems to be no way to test this last proposition within the present framework. If we take the simple model where the coefficients are for, say, demand $\lambda_{kd} + u_{kd}$ and the expected value of $u_{kd} \neq 0$, we can adopt a method developed by

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1 See Section V.B.

2 Although the other parameters were changed slightly by excluding the intercept, the change was not very great.

3 Making the demand attraction coefficients greater than one will not cause the attraction table to 'explode' (i.e. it will converge) in this case, because of the large proportion of total output of both industries sent to final demand.

Burns (21) to make efficient estimates of the random coefficients[1]. It is interesting to note that even if the coefficients have a random element, the ordinary estimates of these coefficients will still be unbiased, but inefficient. Some industries were tested using this method but the results were not very exciting and there was little change in the standard errors. Because of this we can tentatively conclude that there is probably not a random element in the coefficient[2].

From the results in table V.1 it is interesting to note the importance of industry 31 (other mechanical engineering) in explaining the location of many metal and machinery sectors. Industry 31 is made up of Minimum List Heading (M.L.H.) 342 and 349 from the 1968 Standard Industrial Classification (172) where part of 349 is defined as 'Establishments undertaking general sub-contract and repair work' (page 16). This is consistent with the conclusions of both Tosco (189) and Keeble (110)[3], in that with metal working and engineering it was of fundamental importance that subcontractors and repair work were located close by and that the absence of this type of industry in an area may be a major constraint on regional development. Many of the results are what one might expect on a priori grounds, in that certain industries such as services (69/70) are orientated towards demand and certain industries such as sugar (7) and coke ovens (14) are orientated towards their bulky raw material - raw sugar and coal respectively. Also there seems some evidence to suggest that the metals and mechanical engineering sectors are all linked to form a complex. Similarly there seems a complex, although weaker, in the textiles sectors.

Some of the results are quite surprising - particularly our inability to explain motor vehicles (40). There are two possible

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1 See Burns (op. cit) for the exact assumptions concerning the error terms for the method to be valid. Although the method shown by Burns is for O.L.S. this can be generalised for 2.S.L.S. - private communication with M. Burns.

2 This suggests that price is not a random exogeneous variable across regions, and is consistent with the pricing framework of the attraction model discussed in Section III.K.

3 See Ch. 1, particularly Section I.B.

reasons for this:-

(i) The Government has recently had a major influence on the location decisions of this industry - particularly with respect to new plants in Merseyside and Central Scotland - and it is possible that these new plants are inefficiently located with respect to interindustry relations in that the linkages are not available in these reception areas. Therefore we cannot explain their location by attraction theory.

(ii) The variables of the R.H.S. of the equation for motor vehicles had a very high degree of multicollinearity, which, as discussed above, would reduce the efficiency of the estimates, causing us to reject the explanatory variables as insignificant, when in fact they could have been significant.

It is interesting to note that in all the possible combinations of explanatory variables tried on industry 40, one did come out significant all the time - this was industry 31. However the degree of explanation (measured by R^2) was not sufficient to justify a total explanation of industry 31 above. The multicollinearity problem could have been overcome by aggregating all the explanatory variables of industry 40. However this would have meant aggregating these variables everywhere else, if the results were to be used in estimating the attraction table (see below). The loss of detail was thought not to be a whorthwhile price to pay for explanation of industry 40, and consequently it was left unexplained, although this does not imply that the industry has no locational ties.

V.B. Construction and inversion of the attraction tables

As discussed in Section III.C. and III.F. the effects of exogeneous changes in demand or public policy can be obtained by calculating the attraction matrix[1].

$$\text{(i)} \quad {}_r y = [I - \hat{\lambda}_r \Delta - (LA)']^{-1} \hat{\lambda} \cdot {}_r f$$

which will show the effects of an increase in demand by final users in the region - some of the original stimulus will leak out of the region immediately (shown by the $\hat{\lambda}$ matrix).

$$\text{(ii)} \quad {}_r y = [I - \hat{\lambda}_r \Delta - (LA)']^{-1} \cdot {}_r f$$

which will show the effects of an increase in demand by final users for products produced in the region. This would be, say, the C.E.G.B. buying its equipment from a firm in the Northern region.

$$\text{(iii)} \quad {}_r y \, [I - \hat{\lambda}_r \Delta - (LA)'] = 0$$

The method for a solution to such a system can be found in Appendix IV. We would need such a solution if we were to calculate the effects of, say, the government establishing or subsidising a new factory in a region, which would stimulate the local economy through supply and demand effect.

The solution to this last system requires a different matrix to be inverted for each industry's result. However the solution of systems (i) and (ii) only require one matrix to be inverted and the effects on all industries can be obtained from this. However, since

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[1] In all these cases the interregional feedback effects are ignored.

matrix inversion of the order 70 x 70 is rather expensive in terms of computer time, it was felt that it would not be worthwhile solving the last system unless one had some specific projects in mind[1]. The other two systems were solved for all regions using a standard inversion routine. Some experiments were carried out concerning the expansion of the series $I + [\uparrow_r \Delta + (LA)'] + [\uparrow_r \Delta + (LA)']^2$ + etc.[2] However it took 18 iterations before the results to 2 decimal places were the same as those obtained by inversion, and the time taken to do this number of iterations was significantly longer than that used to the inversion routine. Even the time taken to get the results correct to one decimal place presented no significant saving over inversion. Therefore all the results were obtained by inversion. The results for the Northern and South-East regions are presented in Table V.2[3]. The full attraction table (70 x 70) will not be shown because of constraints on space and the individual elements have been interpreted theoretically already. It is thus only the column totals or multipliers that we are now interested in. In Table V.2

(i) column a shows the column sums of $[I - \uparrow_r \Delta - (LA)']^{-1}$ which may be interpreted the value of the multiplier when products are bought directly in the region,

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1 A few experimental results are reported in Table V.2 for comparison with the other attraction multipliers. These show that when output of the industry in question is set equal to 1.0 what the output of all the others must be (in labour terms) for the system to remain in equilibrium.

2 See Section III.G.

3 The results for all regions are shown in Appendix IX - by presenting the results from only two contrasting regions it is hoped to demonstrate the interpretation more clearly.

Table V.2

Attraction multipliers for two regions

Notes

(i) All figures are rounded to 2 decimal places

(ii) All figures are in value of labour employed

(iii) (a) shows the column sums of $\left[I - \hat{\imath}_r\Delta - (kA)'\right]^{-1} {}_r f$

(iv) (b) shows the column sums of $\left[I - \hat{\imath}_r\Delta - (kA)'\right]^{-1} \hat{\imath}_r f$

(v) in Notes (iii) and (iv) ${}_r f$ is taken on a vector with 1 in each element.

(vi) Figures in brackets after industries 14, 18, 19, 31 and 41 in column (a) of the Northern region are the result of the system ${}_r g\left[I - \hat{\imath}_r\Delta - (kA)'\right] = 0$ when the industry in question has its output set equal to 1.0 and all other industries solved relative to this.

(vii) For the multipliers for the other nine regions see Appendix IX.

Industry	Northern Region		South-East	
	a	b	a	b
1	0.39	0.0	0.34	0.0
2	0.54	0.0	0.53	0.0
3	1.13	0.0	0.92	0.0
4	1.08	0.0	1.44	0.0
5	0.30	0.24	0.29	0.24
6	0.36	0.11	0.37	0.11
7	0.16	0.0	0.16	0.0
8	0.40	0.0	0.41	0.0
9	0.67	0.13	0.78	0.15
10	0.31	0.0	0.32	0.0

Industry	Northern Region		South-East	
	a	b	a	b
11	0.27	0.0	0.31	0.0
12	0.22	0.19	0.21	0.19
13	0.67	0.29	0.55	0.24
14	2.28 (2.19)	0.0	0.08	0.0
15	0.59	0.0	0.51	0.0
16	0.52	0.14	0.48	0.12
17	0.77	0.27	1.02	0.36
18	3.12 (0.52)	1.53	1.86	0.91
19	1.22 (0.64)	0.83	0.88	0.60
20/21	0.88	0.36	1.22	0.50
22	0.55	0.0	0.48	0.0
23	0.61	0.12	0.53	0.10
24	0.69	0.43	1.25	0.78
25	0.61	0.0	0.56	0.0
26	0.59	0.0	0.57	0.0
27	0.56	0.0	0.48	0.0
28	0.57	0.0	0.53	0.0
29	1.12	0.18	0.63	0.10
30	0.65	0.44	0.53	0.36
31	5.02 (5.59)	1.05	3.35	0.70
32	0.79	0.33	4.18	1.75
33	1.56	0.36	1.24	0.29
34	0.94	0.25	0.61	0.16
35	1.84	0.57	1.62	0.50
36	0.56	0.27	0.54	0.26
37	0.99	0.84	1.39	1.18
38	0.94	0.43	1.44	0.65

Industry	Northern Region		South-East	
	a	b	a	b
39	1.16	0.71	0.89	0.54
40	0.44	0.0	0.40	0.0
41	0.63 (0.21)	0.10	3.00	0.48
42	0.65	0.12	0.56	0.11
43	0.54	0.0	0.38	0.0
44	0.89	0.52	0.65	0.38
45	0.48	0.20	0.33	0.14
46	0.49	0.31	0.43	0.27
47	1.16	0.60	0.61	0.32
48	0.53	0.42	0.42	0.33
49	1.05	0.0	0.52	0.0
50	0.80	0.47	0.74	0.44
51	0.67	0.0	0.53	0.0
52	0.37	0.36	0.38	0.37
53	0.51	0.45	0.52	0.46
54	0.58	0.0	0.57	0.0
55	1.27	0.60	1.72	0.81
56	0.71	0.33	1.01	0.48
57	1.07	0.55	1.66	0.86
58	0.86	0.48	0.92	0.73
59	0.82	0.29	3.05	1.10
60	0.54	0.13	0.48	0.11
61	0.54	0.24	0.52	0.23
62	0.56	0.58	0.57	0.59
63	0.42	0.49	0.40	0.47
64	0.29	0.30	0.30	0.30
65	0.41	0.42	0.40	0.42

Industry	Northern Region		South East	
	a	b	a	b
66/67	0.48	0.75	0.49	0.75
68	0.68	0.55	0.69	0.55
69/70	0.57	0.62	0.61	0.66

(ii) column b shows the column sums of $[I - \hat{\tau}_r \Delta - (LA)']^{-1} \hat{\tau}$ which may be interpreted as the value of the multiplier when final demand inside the region increases.

Thus the difference between the two multipliers is that with the former all the original expenditure is spent directly inside the region, but with the latter a certain proportion $(1 - \lambda_{\cdot kd})$ leaks out of the region immediately since industries are not totally demand orientated. Only that proportion that does not leak out can have any effect on the region.

In both cases not the gross output, but labour requirements are shown[1].

Some of the multiplier effects in the two regions are not significantly different from each other. However certain industries have significantly different effects in each region because of the different attractiveness of each region as expressed by its industrial structure.

(a) Industries that have got a significantly greater effect in the Northern region than in the South-East[2].

(i) Coke (14) - this is because of the effects on the coal mining sector, where the links are particularly

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[1] This is not, of course, identical to the number of jobs since wage rates differ between occupations.

[2] It must be appreciated that these results are based on the 1963 Census of Production (26) and that year's industrial structure, and as we have argued in Ch. III, attraction theory is only a short run model. Since 1963 the industrial structure of the Northern region has been changed substantially, not a little by government regional policy. Thus since 1963 not only will the technology of the Northern region have changed substantially, but also its internal regional trading structure, and therefore it may be dangerous to apply these results to the Northern region at present without up-dating the results.

strong in the Northern region, and non-existent in the South-East. However, it must be noted that because the demand attraction coefficient (λ_{141}) is zero this effect will only be produced when coke is bought directly in the region and the coal mining industry is able to expand the supply of coal, which is the determinant of the location of the coke industry.

(ii) Chemicals (18) - although the multiplier effects in both regions are quite high (due to the general attractiveness of chemicals) the effect in the Northern region is significantly greater, due to the higher internal multiplier effect of chemicals, which already forms a tight complex in the Northern region where the structural links are much stronger developed. Because of the high internal multiplier effects of chemicals, industries that are attracted by chemicals - such as 13, 15, and 17 - are stimulated much more.

(iii) Iron and steel (19) - again this is due to the Northern region and have a complex of iron and steel making and using, where the structural links are strongly developed. The large dependence on heavy iron and steel using industries has been a feature of the Northern region for a long time and has caused the links to become stronger than in the South-East.

(iv) Other non-electrical machinery (29) - this is larger in the Northern region because of the existence of the structural links with heavy iron and steel working sectors which are stronger in the Northern region. Therefore they are stimulated by demand when industry 29 expands.

(v) Other mechanical engineering (31) - in both regions the effect of these industries is very large. However, the larger effect in the North is again partly due to the links with the heavy metal-making and using sectors. The major difference in the multipliers being due to the larger effect on 19 (iron and steel) 42 (other vehicles) and 30 (industrial plant and steel work). It is also partly due to the fact that this industry must have previously been more of a bottleneck in the Northern region on the supply side. This will be explained more fully below.

(vi) Shipbuilding and marine engineering (39) - here the larger multiplier is due to the stronger structural links in the Northern region with the iron and steel making (19) and with 39 itself.

(vii) Textile finishing (47) - this has a larger effect in the Northern region because of the stronger links with the chemical complex in that region.

(b) Industries that have a significantly greater effect in the South-East than in the Northern region

(i) Plastics (17) - the greater influence in the South-East cannot be ascribed to any specific sectors, but stems from the fact that industry 17 demands from and supplies to the more modern lighter industries with which the structural links are much better developed in the South-East than the Northern region. This is despite the fact that in the Northern region the chemical industry (18) is stimulated much more through the demand effect from plastics.

(ii) Light metals (20/21) - the product of this sector is an important supply determinant of certain electrical machinery sectors, and again these links are more strongly represented in the South-East. These links are not as well developed in the Northern region.

(iii) Engineers' small tools (24) - the difference is due to the attraction of 24 to itself and the resulting demand effect for intermediate inputs that are more likely to be made in the South-East because of the stronger links there.

(iv) Scientific instruments (32) - this is caused mostly by the attraction of 32 to itself to form an industrial complex that manufactures this type of product[1].

(v) Cans and metal boxes (37) - the difference is due mainly to the supply influence on industry 9 (which includes food canning) which then influences

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1 There are many sound reasons for industry 32 to form a complex - these have been described for Boston U.S.A. - see for example Spiegelman (171) or Simshoni (166)

industry o. These links are not strongly developed in the Northern region. This is despite the greater influence in the Northern region on the iron and steel sector, which is a large intermediate input to industry 37.

(vi) Other metal goods (38) - the supply effects of this good is again more on the typical goods produced in the South-East because of the stronger linkages. This type of industry is not integrated well into the Northern region's economy.

(vii) Aircraft (41) - this again forms a complex that is attractive to itself in a similar way that 32 did.

(viii) 55, 56 and 57 - which may be loosely described as a wood-working complex. This complex has higher internal linkages in the South-East, and some of the reasons for this type of agglomeration to develop have been suggested elsewhere[1].

(ix) Printing and Publishing (59) - this again forms a sector that is attractive to itself (i.e. high internal multiplier effects) and has the linkage much more highly developed in the South-East.

The conclusion that we may derive from this is that where the effect of supply of a product is a large influence in the multiplier effect, then this product must have been a bottleneck to expansion in that region. For example, suppose that all the previous output

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[1] See for example Towrroe (191).

of a product X was used in the region (and none was exported) then the ${}_r\alpha$ coefficient (i.e. the coefficients in the A matrix) would be higher than if some had been exported. Now when good X has an increase in output there will be a greater effect in the region when the α's are higher[1], because this must have previously been a constraint on production. However in the region where the α's are low (because much is exported) there can have been little constraint on the output of other goods that used good X as an intermediate input, otherwise good X would not have been exported. Consequently the multiplier effects of good X in a region where a lot was previously exported will not be great, because no production constraint or bottleneck is being overcome.

Similarly where the intermediate demand effect is larger in a region, this is due to the ${}_r\delta$ coefficient (i.e. the coefficient in the ${}_r\Delta$ matrix) being higher[2], which means less of the good was imported as an intermediate product. Now if much of the product is already being imported (smaller ${}_r\delta$) then demand in the region can have been no constraint. So increasing the demand will have little effect on output since no bottle-neck is being overcome. But if none was imported (higher ${}_r\delta$) then demand may have been a constraint, and so increasing demand will have a greater effect.

So for both supply and demand the attractiveness of a region depends not only on the λ_{kj}'s and λ_{hk}'s (which are constant across

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1 Assuming that some $\lambda_{xk} > 0$ $(k = 1,2....n)$, and the λ's are constant across all regions by assumption, so it is only the ${}_r\alpha$'s that can make the difference.

2 Assuming that $\lambda_{kd} > 0$, and the λ's are constants across all regions.

regions)[1], but also on its existing structure and trading patterns. Thus different regions may be described as having different attractiveness to certain industries and different multiplier effects.

We can see from Table V.2 of the multipliers that the Northern and South-East regions have different attractiveness to various industries because of their different industrial structure. The type of industry that the Northern region is attractive to tends to be heavy basic industries, such as iron and steel, chemicals, coke and heavy steel fabricating sectors. However there is one significant exception - industry 31 - if this industry could be stimulated then there would be a significant effect on the region, and so this industry must have been a major bottleneck to the expansion of existing industry. The type of industry that has a large effect in the South-East tends to be the more modern lighter type of industry.

As can be seen from Table V.3 the national employment of industries having a large effect in the Northern region has generally been declining, whilst national employment of those having a large effect in the South-East has generally been expanding. So employment expanding industries have a bigger effect on the South-East and declining ones have a bigger effect in the Northern region. Thus these attraction multipliers may give some indication of why the Northern region is depressed and why the South-East has a buoyant economy.

Table V.4 shows the proportion of output in various industries in the Northern and South-East regions. To obtain some idea of the

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1 It will be noted from Section III.C that we had to assume the t_{kd} and t_{hk}'s constant across all regions, because only cross-section data was available - so λ_{kd} and λ_{hk} are constant across regions. If time series data were available the t_{kd} and t_{hk} (and therefore λ_{kd} and λ_{hk} could vary across regions, showing that certain regions had better communications access than other regions, and so the multiplier effects would be greater.

Table V.3

Total percentage increase in employment of certain manufacturing industries in the whole U.K. over the period 1959-1968

Notes:

(i) The industry number refers to the classification as given in input-output tables for the U.K. (27)

(ii) The employment figures are aggregated from the subdivision of the industries shown here, i.e. they are based on M.L.H. S.I.C. 1958.

(iii) Only the years 1959-1968 are considered because up to 1959 the employment was classified by M.L.H. S.I.C. 1948 which is not directly comparable with 1958 S.I.C.

All manufacturing	3.6%
All industries (services and manufacturing)	4.8%

(a) Industries that have a large attraction multiplier in the Northern Region

(i)	14 - coke over	- 19.1%
(ii)	18 - chemicals	- 5.7%
(iii)	19 - iron and steel	- 1.3%
(iv)	29 - other non-electrical machinery	10.7%
(v)	31 - other mechanical engineering	18.2%
(vi)	39 - shipbuilding	- 24.3%
(vii)	47 - textile finishing	- 18.2%

(b) Industries that have a large attraction multiplier in the South-East Region

(i)	17 - plastics	43.3%
(ii)	20/21 - light metals	8.8%
(iii)	24 - engineers small tools	37.4%
(iv)	32 - scientific instruments	11.8%
(v)	37 - cans and metal boxes	- 0.5%
(vi)	38 - other metal goods	13.2%
(vii)	41 - aircraft	- 15.2%
(viii)	56 - timber	27.7%
(ix)	55 - furniture	- 2.1%
(x)	57 - paper	2.1%
(xi)	59 - printing and publishing	14.3%

Source of figures - Department of Employment and Productivity - British Labour Statistics Historical Abstract 1886 - 1968 (18) Table 138 and 135.

Table V.4

Proportion of national output in various industries for the Northern and South-East regions

Note: industry 31 is included in both the Northern and South-East Regions, because, even though the effect is greater in the Northern region, it is still very large in the South-East.

(a) Industries having a large multiplier effect in Northern Region

(i)	14 - coke	17.2%
(ii)	18 - chemicals	17.3%
(iii)	19 - iron and steel	10.3%
(iv)	29 - other non-electrical machinery	3.0%
(v)	31 - other mechanical engineering	4.6%
(vi)	39 - shipbuilding	19.5%
(viii)	47 - textile finishing	0.7%

(b) Industries having a large multiplier effect in the South-East Region

(i)	17 - plastics	22.6%
(ii)	20/21 - light metals	20.8%
(iii)	24 - engineers' small tools	34.1%
(iv)	31 - other mechanical engineering	36.3%
(v)	32 - scientific instruments	71.1%
(vi)	37 - cans and metal boxes	25.7%
(vii)	38 - other metal goods	24.2%
(viii)	41 - aircraft	26.6%
(ix)	55 - furniture	55.5%

(x)	56 - timber	40.5%
(xi)	57 - paper	26.2%
(xii)	59 - printing and publishing	59.1%

Source: 1963 Census of Production (26)

relative sizes of the two economies the proportion of the U.K. population living in

(a) the Northern Region is 6.06%

(b) the South-East Region is 31.14% [1]

Now it can be seen that the few industries that have both a high growth of employment at the national level, and a high multiplier effect in the Northern region (industry 29 and 31) are only a small proportion of the regional economy (5.0% and 4.6% respectively as a percentage of national output) So they are unlikely to cause large increases in absolute numbers employed. However, these industries that have had both a declining employment at the national level and a high multiplier effect in the Northern region (industries 11, 18, 19 and 9) are a large proportion of the regional economy (17.2%, 17.3%, 10.3% and 19.5% respectively as a percentage of national output). So their decline will have caused a large absolute reduction in the number of jobs. The situation in the South-East is reversed to some extent (although the pattern is not quite as clear as in the Northern region). One would not however expect the pattern to be as clear in such a diversified region such as the South-East as in a relatively specialised one such as the Northern region.

It is interesting to compare [illegible] of the attraction multipliers for the Northern region with its input-output multipliers[2]. It will be noticed that the input-output multipliers for each industry are relatively close in magnitude to each other, whereas the attraction multipliers have a much larger range. Also the average multipliers

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[1] Source - Abstract of Regional Statistics 1970 (1) Table 5.

[2] Given in Appendix VII.

Table V.5

Average and standard deviations of input-output multipliers and attraction multipliers for Northern Region

	Name	Average	Standard Deviation
(i)	Column a of Appendix VI	0.689	0.164
(ii)	Column a of Appendix VI (multiplier in brackets)	0.506	0.120
(iii)	Column a of Table V.II for Northern Region	0.807	0.687
(iv)	Column b of Table V.II for Northern Region	0.320	0.306

given in column (a) of Table V.2 are larger on average than the input-output multipliers[1], but those in column (b) of Table V.2 are smaller on average (see Table V.5)[2]. If the input-output multipliers were correct, then different policies would have similar effects. Howevever, if, as we have argued, regional input-output is just a special case of the attraction model and the latter's results are more reliable, then it is very important which type of industry is stimulated and by which policy, i.e. buying directly from an industry or expanding final demand in the region. This would suggest a more discriminatory policy rather than blanket controls and incentives[3].

V.C. The influence of labour as an attractive force in location[4]

As was discussed in Section III.J. the structural attraction equation for industry k where three different types of labour are included is:

$$_{r}g_{k} = \lambda_{kd} \cdot {}_{r}dd_{k} + \sum_{h=1}^{n} \lambda_{hk} \cdot {}_{r}\alpha_{hk} \cdot {}_{r}g_{h} + \sum_{L=1}^{3} \lambda_{Lk} \cdot {}_{r}\alpha_{Lk} \cdot {}_{r}g_{L}$$

As stated in that section ${}_{r}\alpha_{Lk} \cdot {}_{r}g_{L}$ cannot be measured directly and various proxy measures have been suggested. Data on various

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1 This is even without interregional feedback effects and without labour in the attraction multipliers that are present in the input-output multipliers.

2 It should also be noted that the rankings of the values of the multipliers obtained from attraction analysis and from input-output studies were significantly different. Whereas the input-output multipliers from all regions had a similar ranking to each other and the attraction multipliers also had a similar ranking to each other.

3 For further experiments with the attraction matrix are presented in Appendix X where the results of interregional feedback (see Section III.H) and of separate supply and demand influences (see Section III.G) are shown.

4 For a discussion of the problem of misspecifying an equation with regard to labour (i.e. of excluding labour from the analysis) when labour is correlated to some extent with the other included variables, see Appendix VIII.

labour market measures by region can be obtained from the Ministry of Labour Gazette, and Census of Population, which allows a breakdown by three skill categories for males and females, employed and unemployed; also available are such measures as activity rates and vacancies. In trying to estimate a measure for ${}_{r}\alpha_{Lk} \cdot {}_{r}g_{L}$ some practical difficulties were encountered.

(i) If too many different series were used the degrees of freedom in each equation would have been very small.

(ii) Many of the series suggested, of skill and sex breakdown, were highly correlated, and therefore it was impossible to sort out the separate effects of each series.

(iii) Even if we had a measure of the tightness of the labour market in each region, was this figure to apply to all industries in the region, i.e. did each industry in the region face the same labour market?

(iv) Labour market areas are generally much smaller than the region - for example, in the North-west region the Liverpool and Manchester conurbations each will have separate labour markets, because the maximum distance that people are willing to commute is less than the distance between the two areas. Thus any measure of labour market tightness for any region will be an average of some separate (and to some extent independent) labour markets.

Because of these problems (the last two of which are probably insuperable in the present context) it was decided to use principal component analysis to obtain a measure of the tightness of the regional labour markets. This was carried out as follows. A breakdown of absolute[1] numbers of male employed and unemployed by skill (skilled, semi-skilled and unskilled) were obtained. A breakdown of female employed to skill was obtained - no analysis was based on female unemployment figures because this does not measure accurately the number of females seeking work, whereas male unemployment is probably an accurate reflection of the number of males seeking work. To obtain some idea of the extent of female hidden labour, it was assumed that the South-East region (i.e. the one with the highest female activity rate) plus 5% of the women of working age, was the maximum female activity rate that any region could achieve. For this maximum figure and the number already working, the difference represents those women unemployed but willing to work. Series on vacancies were obtained for males and females.

With these various series, each of which will have some effect on the tightness of the labour market, a principal components analysis was carried out. The correlation between all the variables was surprisingly high, and in fact the first principal component accounted for 92% of the total variation amongst all the measures used. The second component only added a further 3% 'explanation'. Because this second component added so little 'explanation' and the first was so all embracing, it was decided to use only the first principal component and so retain more degrees of freedom. Table V.o gives the industries for which the labour coefficients turned out to be significant. However it is impossible to report the equations in a similar manner to those given for intermediate industries in Table V.1 because we only have:

(i) $n_k \cdot {}_r\alpha_{Lk} \cdot {}_r g_L = {}_r C_1$

(ii) $\dfrac{\lambda_{Lk}}{n_k} = C_2$

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[1] In all the analysis of labour markets absolute numbers rather than percentages were used - for the arguments for why absolute numbers should be used see Davies (37).

Table V.6

Results of including labour in attraction analysis as a normal intermediate good

Industry No.	The labour coefficient was (a) significant	(b) insignificant
1, 2, 3 and 4	not tested	
5		x
6		x
7		x
8		x
9	x	
10	x	
11		x
12		x
13		x
14		x
15	x	
16		x
17		x
18		x
19		x
20/21		x
22		x
23		x
24	x	
25		x

Industry No.	The labour coefficient was (a) significant	(b) insignificant
26		x
27		x
28	x	
29		x
30		x
31	x	
32	x	
33		x
34		x
35		x
36	x	
37		x
38	x	
39		x
40		x
41	x	
42		x
43		x
44	x	
45	x	
46		x
47		x
48		x
49	x	
50		x
51		x
52		x

Industry No.	The labour coefficient was (a) significant	(b) insignificant
53		x
54		x
55		x
56		x
57		x
58		x
59		x
60		x
61		x
62		x
63		x
64		x
65		x
66/67		x
68		x
69/70		x

where n_k . ${}_r\alpha_{Lk}$. λ_{Lk} are unknowns

${}_rC_1$, and C_2 are known estimated coefficients.

The first expression is obtained from the fact that we only know some unknown multiple (n_k) of the value ${}_r\alpha_{Lk}$. ${}_rg_L$. We are assuming that the proxy measure we are using for the labour supply (i.e. the principal component) is perfectly (very well) correlated with actual labour supply, but we do not know if each value of the proxy is say half or three times the value of the true variable. Therefore in the second expression we do not know what multiple value of the real λ_{Lk} is that we are estimating.

Theoretically ${}_rg_L$ (total labour supply) should be an easily measurable quantity in value terms, and if we know it, then from the first expression:

$$n_k \cdot {}_r\alpha_{Lk} = \frac{{}_rC_1}{{}_rg_L}$$

and multiplying this by the second expression

$$\frac{\lambda_{Lk}}{n_k} \cdot n_k \cdot {}_r\alpha_{Lk} = \lambda_{Lk} \cdot {}_r\alpha_{Lk} = \frac{{}_rC_1 \cdot C_2}{{}_rg_L}$$

The composite figure $\lambda_{Lk} \cdot {}_r\alpha_{Lk}$ can then be calculated for each region, and this is the figure that is needed for the labour sector in the $(LA)'$ matrix. But only the composite value is known and not the individual effect, i.e. we cannot report λ_{Lk} in a similar way that we reported λ_{kd} and λ_{hk} in Table V.1.

Thus if we have an estimate of the attraction multiplier with labour feedbacks we must have an estimate of ${}_rg_L$. This can be defined as the total supply, in value terms, of labour from the working population where all reserves of labour are used, i.e. when the maximum activity rates for that individual region are utilised. This figure is not directly available and was estimated in the following way:

(i) The absolute number of males employed and unemployed for each region is known but not all these would be available for work since some are unemployable and some will always be changing jobs. These people will never be available for work and should therefore be discounted from the total supply of males. A conventional figure of 1% unemployables and ½% changing jobs was adopted and subtracted from the total males employed and unemployed.

(ii) The absolute number of females employed and unemployed and activity rates is known for each region. Ideally from local micro-labour market studies we should obtain estimates of the maximum females' activity rate that each region would be willing to sustain. However a rule of thumb was adopted that the region with the highest activity rate plus 5% was the maximum that all regions could sustain.

(iii) Income per head for total regional population is available[1] for each region, therefore knowing total regional working population and total regional population it is possible to work out the average employment income (or product) per worker.

(iv) Knowing the average employment income per worker and knowing the total potential supply of workers available

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[1] See Woodward (202) for a summary of some of the results in the Regional Problems of the Enlarged Community (44).

in a region, and assuming the additional workers earn (produce) the same as the average in the region, a figure in value terms can be obtained for the total potential supply of labour for each region, i.e. for $_rg_L$.

Thus estimates of $\lambda_{Lk} \cdot {}_i\alpha_{Lk}$ for each k and each region can be obtained from estimates of $_rg_L$, the principal components analysis and the regression results when labour is included. To introduce labour into the attraction table we assume that intermediate goods do not attract labour, so $\lambda_{Ll} \cdot {}_r\alpha_{kL} = 0$ for all k[1]. In the Υ matrix when labour is included λ_{Ld} would be how responsive the regional labour market is to changes in demand for labour. Since this will be a subjective judgement (unless detailed micro-studies are undertaken) a simple sensitivity analysis was carried out with λ_{Ld} set at values of 1.0 and 0.5. In the $_r\Delta$ matrix $_r\delta_{Lk} = \beta_{Lk}$ for all k by assumption and:

$$_r\delta_{kL} = \frac{{}_rp_k \cdot {}_{rr}f_{kL}}{{}_rc_L} \quad \text{for all } k \qquad \text{V.C.1}$$

where the notation has its usual meaning except

(i) $_rp_k$ which can vary between 0 and 1 and is some measure of the marginal propensity to consume as a proportion of the average[2]. See Section IV.G above for a discussion of this concept when applied to traditional regional input-output tables.

(ii) $_rc_L$ is the total value of labour actually employed in region r, as opposed to $_rg_L$ which is the total potential value.

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1 See Section III.J. for a fuller discussion of these points

2 In circumstances where we do not know $_rp_k$ for each different k it is taken as a constant.

Table V.7

Results of including labour in attraction analysis - the effects on the multiplier for the Northern Region

Notes

(i) λ_{LD} = a measure of the responsiveness of local labour markets to changes in demand for labour.

(ii) p = the marginal propensity to consume as a proportion of the average propensity to consume (see Section V.C).

(iii) The results are all in terms of value of labour employed from the matrix $[I - \hat{1}_r \Delta - (LA)']^{-1}$

Industry	$\lambda_{LD} = 0.5$			$\lambda_{LD} = 1.0$		
	p=0.0	p=0.5	p=1.0	p=0.0	p=0.5	p=1.0
1	0.61	0.67	0.75	0.85	1.06	1.42
2	0.85	0.94	1.05	1.18	1.48	1.97
3	1.77	1.96	2.19	2.46	3.09	4.11
4	1.69	1.87	2.09	2.37	2.97	3.96
5	0.47	0.52	0.59	0.66	0.83	1.10
6	0.57	0.63	0.71	0.79	1.00	1.32
7	0.25	0.28	0.31	0.35	0.44	0.59
8	0.63	0.70	0.78	0.88	1.10	1.47
9	1.04	1.15	1.29	1.46	1.82	2.42
10	0.49	0.54	0.60	0.68	0.85	1.13
11	0.42	0.47	0.52	0.59	0.74	0.98
12	0.35	0.39	0.43	0.49	0.61	0.81
13	1.07	1.18	1.32	1.49	1.86	2.48
14	3.57	3.95	4.41	4.98	6.23	8.31
15	0.92	1.02	1.14	1.29	1.61	2.15

Industry	$\lambda_{LD} = 0.5$			$\lambda_{LD} = 1.0$		
	p=0.0	p=0.5	p=1.0	p=0.0	p=0.5	p=1.0
16	0.82	0.91	1.01	1.14	1.43	1.91
17	1.29	1.42	1.59	1.79	2.25	3.00
18	5.02	5.55	6.21	7.01	8.76	11.68
19	1.91	2.12	2.37	2.67	3.34	4.45
20/21	1.29	1.42	1.58	1.79	2.23	2.97
22	0.86	0.95	1.06	1.20	1.50	2.00
23	0.96	1.06	1.18	1.34	1.67	2.23
24	1.09	1.21	1.36	1.54	1.93	2.58
25	0.96	1.06	1.18	1.34	1.67	2.23
26	0.93	1.02	1.15	1.29	1.62	2.15
27	0.87	0.96	1.08	1.21	1.52	2.02
28	0.90	1.00	1.11	1.26	1.57	2.10
29	1.74	1.93	2.15	2.43	3.04	4.05
30	1.01	1.12	1.25	1.42	1.77	2.36
31	7.73	8.55	9.55	10.79	13.49	17.99
32	1.23	1.36	1.52	1.71	2.14	2.85
33	2.43	2.69	3.00	3.39	4.24	5.65
34	1.47	1.62	1.81	2.05	2.56	3.42
35	2.26	2.50	2.79	3.16	3.94	5.26
36	0.88	0.97	1.09	1.23	1.54	2.05
37	1.55	1.71	1.92	2.16	2.70	3.61
38	1.46	1.61	1.80	2.04	2.55	3.40
39	1.82	2.01	2.25	2.54	3.17	4.23
40	0.69	0.76	0.85	0.96	1.20	1.60
41	0.99	1.09	1.22	1.37	1.72	2.29
42	1.01	1.12	1.25	1.41	1.76	2.35
43	0.86	0.95	1.06	1.20	1.50	2.00
44	1.39	1.54	1.72	1.95	2.43	3.24

Industry	λ_{LD} = 0.5			λ_{LD} = 1.0		
	p=0.0	p=0.5	p=1.0	p=0.0	p=0.5	p=1.0
45	0.75	0.83	0.93	1.05	1.32	1.75
46	0.78	0.86	0.96	1.09	1.36	1.81
47	1.80	1.99	2.23	2.51	3.13	4.17
48	0.83	0.92	1.02	1.16	1.45	1.93
49	1.66	1.83	2.05	2.31	2.89	3.85
50	1.23	1.38	1.54	1.74	2.17	2.90
51	1.04	1.16	1.29	1.46	1.82	2.43
52	0.58	0.64	0.72	0.81	1.01	1.35
53	0.81	0.89	0.99	1.12	1.40	1.86
54	0.92	1.01	1.14	1.28	1.60	2.14
55	1.99	2.20	2.40	2.78	3.48	4.63
56	1.12	1.23	1.38	1.56	1.95	2.59
57	1.68	1.85	2.07	2.34	2.92	3.90
58	1.34	1.49	1.66	1.88	2.35	3.13
59	1.28	1.42	1.58	1.79	2.23	2.98
60	0.85	0.94	1.05	1.19	1.49	1.98
61	0.86	0.95	1.06	1.20	1.50	1.99
62	0.88	0.97	1.08	1.22	1.52	2.04
63	0.65	0.72	0.81	0.91	1.14	1.52
64	0.46	0.51	0.57	0.64	0.80	1.07
65	0.63	0.70	0.77	0.87	1.09	1.44
66/67	0.76	0.84	0.94	1.06	1.32	1.76
68	1.07	1.18	1.32	1.49	1.86	2.48
69/70	0.89	0.98	1.11	1.25	1.56	2.08

Thus V.C.1 says that $_{r}\delta_{kL}$ (the coefficient showing how much final consumption as a proportion of total income is bought inside the region) depends on $\frac{_{rr}f_{kL}}{_{r}q_{L}}$ (that proportion that was previously bought in the region) and $_{r}\nu_{k}$ (a measure of the marginal propensity to consume). In order to test the sensitivity of the results to $_{r}\nu_{k}$, the inversion was carried out setting $_{r}\nu_{k}$ = 1, 0.5 and 0.0. All the results where labour is included are set out in Table V.7 [1]. This is only for the Northern region since matrix inversion is relatively expensive in terms of computer time, and only one region was necessary for purposes of comparison. However, it must be remembered that when using the results with labour included there are many restrictive assumptions that must be made in order for labour to be introduced[2], and this may make the estimates with labour subject to a greater degree of error. Table V.7 can be compared with column (a) of Table V.2 for the Northern region which shows the multipliers without any influence of labour. The results do seem to be particularly sensitive to the values of λ_{Ld} (responsiveness of the local labour market to changes in demand for labour) and the value of the marginal propensity to consume. [illegible] λ_{Ld} the value of the m.p.c. becomes more critical. Thus if the local labour market is tight or slow to react, including labour is probably not of great importance but in a labour surplus region the labour effect can be quite significant.

V.D. Footloose industries and triangularisation of the attraction matrices

It was shown in Section III.I that triangularisation of the attraction matrix would help us to identify footloose industries, - i.e. industries that had empty rows and full columns could be regarded as footloose. However it must be stressed again that this is not

1 The industries for which labour was found to be significant are reported in Table V.6

2 For details see Section III.J.

necessarily a way of discovering the industries with the smallest communication cost, since the t's can only be solved relatively for each industry, no interindustry comparisons of the t's cannot be made[1]. However, if an industry has a (nearly) empty it is (almost) certain that it has no communication costs with other industries and can therefore be regarded as footloose.

The method used to triangularise the matrix was based on Ramsey et. al. (155). They suggest a computer algorithm that involves exploring the various branches or possible permutations of orderings of industries. Although the algorithm does not terminate until all possible enumerations have been explored[2], it is possible to stop the algorithm at any stage and read the best solution so far obtained. The authors report some simulation studies, that although the algorithm took a long time to terminate, the optimal solution is often obtained very early in the computation. This process of finding an early optimal solution is especially speeded up if:

(i) many of the elements in the matrix are zero, such that the number of branches to explore is reduced,

(ii) a 'good' starting bound is entered before the algorithm is run.

The authors suggest that although 20 x 20 matrices may be the maximum size to solve general problems using the algorithm alone, when the above conditions are fulfilled the algorithm can be used on large matrices to give a greatly improved ordering.

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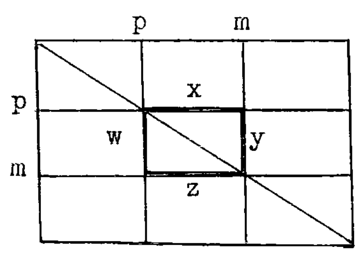
1 See Appendix III

2 Which on a large matrix such as the ones we shall be operating on would be impractical in terms of computer time, since there are n! ways of ordering an n x n matrix.

An initial 'good' starting bound was obtained by using a method devised by Simpson and Tsuki (168)[1]. This involves making only pairwise changes of industries if the change will results in a decrease in the sum of the above diagonal elements. Thus if one is considering changing the position in a matrix **A** of row/column p with row/column m (where p < m.) the change should only be made if.

$$\sum_{i=0}^{m-p} A_{p,\,p+i} + \sum_{i=0}^{m-p} A_{p+i,\,m} < \sum_{i=0}^{m-p} A_{m,\,p+i} + \sum_{i=0}^{m-p} A_{p+i,\,p}$$

or in diagramatic form:



the change should be made if x + y > w + z, where x, y, w and z are the sums of the elements in the part of the rows and columns of p and m shown in the diagram.

The pairwise changes were made first and then the algorithm run. Two properties of matrices make the algorithm easy to run[2].

(i) For any permutation of the matrix A (elements a_{ij}), then either a_{ij} or a_{ji} ($i \neq j$) must appear above the diagonal. Thus one can construct of transformed matrix B, such that $b_{ij} = b_{ji} = \min(a_{ij}, a_{ji})$, $j \neq i$. The new matrix B can be interpretted as the amount that is independent of the ordering of the rows and column. Thus one needs only to work on a matrix **C ≡ A − B**.

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1 Ramsey et. al. (155) suggests the use of this method to obtain a starting bound.

2 [illegible] Ramsey [illegible] et al. (op. cit.) page 3 for full details.

(ii) Finding the ordering that will minimize the sums of the above diagonal of C is equivalent to finding the optimal ordering of A.

The attraction matrix was thus transformed and all calculations carried out on this transformation. Now, although the algorithm did not terminate in the allocated time period for any of the attraction tables, we can be reasonably sure that the solution was quite near an optimal solution since in all cases the sum of the above diagonal elements of the transformed matrix was less than 1 and in some cases less than 0.5. So there would have been little chance of improvement if the algorithm had been allowed to run longer.

To test to see if there was a significant measure of relationship among the various rankings, a Kendall coefficient of concordance ( ) was computed for all the observations[1]. The Spearman rank correlation coefficient (r h o)[2] was then calculated for each pair of observations and the results for both are given in Table V.8. This shows a remarkable degree of similarity between regions on what are footloose industries. For all the ranks the best estimate of the true ordering is provided by the rank of the summed ranks. This is given in Table V.9 and shows the ranked footlooseness of industry. Obviously the industries that we did not try to explain and the ones that we could not explain came out as footloose in this analysis, since all their elements are zero and so their initial ordering is arbitrary. Also a region that had none of an industry before would show the industry as footloose, since all the elements would be zero but this should not be taken to mean that the industry is footloose.

In comparing the rankings of footloose industries between regions

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1 See Siegel (167) pages 229 - 239

2 See Siegel (op. cit.) pages 202 - 213

Table V.8

Spearman rank correlation coefficients of rankings from triangularisation of attraction tables

Notes

(i) The Spearman rank correlation coefficients are based on the rankings of the triangularisation of attraction tables given in Table V.9

(ii) All the coefficients are significantly different from zero on a 1 rail t-test at the 0.0005 level (see Siegel (167) page 212).

(iii) The Kendall coefficient of concordances (w) for all the severn regions was 0.88 and on a chi-squared test this was also highly significant (see Siegel (167) page 236).

		(2)	(3)	(4)	(5)	(6)	(7)
Northern	(1)	0.88	0.79	0.81	0.83	0.85	0.88
Yorkshire and Humberside	(2)	\	0.90	0.81	0.93	0.90	0.93
East Midlands	(3)		\	0.73	0.86	0.85	0.88
South East	(4)			\	0.80	0.86	0.83
West Midlands	(5)				\	0.84	0.86
North West	(6)					\	0.94
Scotland	(7)						\

Table V.9

Rankings of industries from triangularisation of attraction matrices - 7 regions

Notes: where industries tie for a rank, they are allocated the average of the range of ranks occupied by those industries - for example 12 industries tied for rank 1 (either because we did not try to or could not explain them) and so they are all allocated an equal rank of 6, and then the next industry is ranked 13 and so on.

	(1) Northern	(2) Yorks and Humberside	(3) East Midlands	(5) South East	(7) West Midlands	(8) North West	(10) Scotland	Rank of Summed Ranks
1	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
2	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
3	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
4	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
5	47.5	60.0	50.0	63.0	39.0	63.0	58.0	59.0
6	63.5	62.0	62.0	65.0	64.0	65.0	62.0	65.0
7	13.0	12.0	12.0	13.0	12.0	12.0	12.0	13.0
8	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
9	46.0	57.0	49.0	61.0	60.0	60.0	56.0	60.0
10	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
11	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
12	27.0	35.0	31.0	59.0	32.0	61.0	60.0	44.0
13	39.0	32.0	23.0	51.0	36.0	42.0	37.0	37.0
14	12.0	13.0	13.0	6.5	14.0	13.0	13.0	12.0
15	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
16	45.0	53.0	54.0	60.0	48.0	62.0	48.0	56.0
17	56.0	25.0	22.0	35.0	35.0	49.5	32.0	35.0

	(1) Northern	(2) Yorks and Humberside	(3) East Midlands	(5) South East	(7) West Midlands	(8) North West	(10) Scotland	Rank of Summed Ranks
18	52.0	46.0	53.0	47.0	34.0	47.0	47.0	48.0
19	62.0	59.0	51.0	37.0	55.0	37.0	52.0	53.0
20/21	30.5	39.5	28.5	32.5	46.5	34.5	38.5	33.5
22	19.0	20.0	17.0	18.0	20.0	18.0	19.0	16.0
23	23.0	28.0	24.0	23.0	41.0	23.0	21.0	23.0
24	22.0	42.0	34.0	39.0	40.0	26.0	24.0	29.0
25	24.0	16.0	25.0	20.0	23.0	19.0	18.0	19.0
26	20.0	22.0	20.0	17.0	19.0	24.0	17.0	18.0
27	33.0	26.0	38.0	22.0	21.0	20.0	25.0	24.0
28	21.0	18.0	18.0	19.0	18.0	17.0	27.0	17.0
29	26.0	37.0	36.0	45.0	29.0	32.0	36.0	31.0
30	43.0	38.0	37.0	46.0	42.0	36.0	41.0	41.0
31	18.0	17.0	16.0	16.0	17.0	16.0	16.0	15.0
32	125.0	21.0	19.0	30.0	25.0	27.0	28.0	21.0
33	42.0	33.0	41.0	42.0	44.0	44.0	30.0	39.0
34	36.0	15.0	33.0	41.0	16.0	43.0	29.0	27.0
35	38.0	29.0	42.0	43.0	26.0	31.0	34.0	32.0
36	41.0	30.0	43.0	44.0	31.0	33.0	35.0	36.0
37	50.0	34.0	32.0	36.0	38.0	25.0	20.0	28.0
38	34.0	50.0	39.0	40.0	62.0	39.0	40.0	43.0
39	65.0	31.0	15.0	38.0	22.0	38.0	55.0	38.0
40	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
41	15.0	19.0	40.0	29.0	24.0	30.0	23.0	22.0
42	37.0	27.0	26.0	21.0	30.0	22.0	26.0	25.0
43	53.0	52.0	58.0	15.0	51.0	49.5	33.0	45.0
44	29.0	55.0	57.0	25.0	57.0	56.0	57.0	50.0
45	57.0	65.0	65.0	28.0	52.0	52.0	61.0	58.0
46	16.0	23.0	61.0	27.0	27.0	21.0	31.0	26.0

	Northern	Yorks and Humberside	East Midlands	South East	West Midlands	North West	Scotland	Rank of Summed Ranks
47	17.0	48.0	63.0	24.0	28.0	51.0	49.0	40.0
48	58.0	54.0	66.0	34.0	61.0	55.0	63.0	61.5
49	16.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
50	63.5	63.0	59.0	26.0	49.0	29.0	59.0	54.0
51	14.0	14.0	14.0	14.0	13.0	14.0	14.0	14.0
52	28.0	36.0	27.0	47.0	58.0	15.0	15.0	30.0
53	61.0	61.0	46.0	55.0	56.0	58.0	54.0	61.5
54	6.0	6.0	6.0	6.5	6.0	6.0	6.0	6.0
55	32.0	24.0	21.0	31.0	15.0	28.0	22.0	20.0
56	51.0	51.0	44.0	54.0	54.0	46.0	43.0	51.5
57	49.5	44.0	30.0	56.0	33.0	41.0	45.0	42.0
58	44.0	45.0	48.0	58.0	45.0	45.0	46.0	49.0
59	47.5	43.0	47.0	49 0	43.0	40.0	44.0	46.0
60	35.0	41.0	55.0	48.0	63.0	48.0	53.0	51.5
61	54.0	47.0	50.0	62.0	49.0	54.0	50.0	57.0
62	59.5	58.0	60.0	53.0	53.0	57.0	66.0	63.0
63	55.0	56.0	45.0	52.0	50.0	59.0	51.0	55.0
64	68.0	66.0	64.0	66.0	66.0	66.0	64.0	66.0
65	49.5	49.0	35.0	50.0	37.0	53.0	42.0	47.0
66/67	66.5	67.5	67.50	67.5	67.5	67.5	67.5	68.0
68	59.5	64.0	52.0	64.0	65.0	64.0	65.0	64.0
69/70	69.5	69.5	69.5	69.5	69.50	69.5	69.5	69.50

Table V.10

Rankings of industries from triangularisation of attraction matrices - 4 regions

Notes: industry market * are not present in the region in question and thus appear to be ranked low.

	(4) East Anglia	(6) South West	(9) Wales	(11) Northern Ireland
1	10.0	7.0	8.0	9.0
2	10.0	7.0	8.0	9.0
3	10.0	7.0	8.0	9.0
4	10.0	7.0	8.0	9.0
5	61.0	62.0	55.0	63.0
6	64.0	66.0	63.0	66.0
7	20.0	15.0	8.0*	9.0*
8	10.0	7.0	8.0	9.0
9	62.0	61.0	54.0	59.0
10	10.0	7.0	8.0	9.0
11	10.0	7.0	8.0	9.0
12	10.0*	53.0	62.0	9.0*
13	53.0	28.0	34.0	41.0
14	10.0*	7.0*	17.0	9.0*
15	10.0	7.0	8.0	9.0
16	59.0	60.0	60.0	52.0
17	52.0	19.0	44.0	
18	51.0	45.0	59.0	49.0
19	10.0*	31.0	61.0	54.0
20/21	37.0	30.0	47.0	36.0

	(4) East Anglia	(6) South West	(9) Wales	(11) Northern Ireland
22	39.0	22.0	21.0	22.0
23	28.0	24.0	22.0	9.0*
24	22.0	38.0	24.0	30.0
25	10.0*	27.0	8.0*	9.0*
26	30.0	23.0	8.0*	27.0
27	33.0	24.0	23.0	21.0
28	10.0*	17.0	8.0*	23.0
29	49.0	41.0	32.0	42.0
30	26.0	42.0	33.0	26.0
31	24.0	20.0	20.0	20.0
32	31.0	26.0	27.0	37.0
33	25.0	34.0	39.0	38.0
34	10.0*	33.0	37.0	35.0
35	42.0	39.0	38.0	45.0
36	44.0	40.0	40.0	46.0
37	32.0	21.0	35.0	43.0
38	40.0	32.0	36.0	32.0
39	34.0	56.0	25.0	58.0
40	10.0	7.0	8.0	9.0
41	23.0	35.0	26.0	24.0
42	45.0	29.0	42.0	25.0
43	10.0*	47.0	66.0	60.0
44	50.0	49.0	50.0	64.0
45	36.0	51.0	31.0	61.0
46	27.0	18.0	19.0	39.0
47	10.0*	7.0*	28.0	55.0
48	38.0	52.0	49.0	62.0

	(4) East Anglia	(6) South West	(9) Wales	(11) Northern Ireland
49	10.0	7.0	8.0	9.0
50	55.0	50.0	52.0	50.0
51	21.0	16.0	18.0	19.0
52	47.0	43.0	57.0	31.0
53	58.0	o3.0	56.0	57.0
54	10.0	7.0	8.0	9.0
55	35.0	26.0	29.0	24.0
56	60.0	58.0	43.0	48.0
57	29.0	44.0	45.0	28.0
58	48.0	59.0	53.0	40.0
59	56.0	37.0	30.0	47.0
60	43.0	46.0	46.0	33.0
61	54.0	54.0	64.0	56.0
62	57.0	57.0	61.0	67.0
63	63.0	55.0	58.0	53.0
64	66.0	67.0	67.0	67.0
65	46.0	48.0	48.0	44.0
66/67	68.0	68.0	68.0	68.0
68	65.0	64.0	65.0	65.0
69/70	69.0	69.0	69.0	69.0

not all regions were used, since East Anglia, the South-West, Wales and Northern Ireland were excluded. This was done because these four regions lacked a number of different industries, such that in the triangularisation process they would be ranked as highly footloose since all coefficients were zero. This would have given distorted results. Thus table V.9 is based only on regions Northern (1), Yorks. and Humberside (2), East Midlands (3), South-East (5), West Midlands (7), North West (8) and Scotland (10).

The rankings of the remaining four regions (East Anglia (4), South West (6), Wales (9) and Northern Ireland (11) are given in Table V.10. The industries where there is zero output in these regions are marked with an asterisk and so these industries appear to be footloose from the triangularisation process, but in actual fact nothing can be said about them. The other industries however are directly comparable.

It should also be remembered that labour has been excluded from this analysis because it was felt that the restrictive assumptions that are necessary to introduce labour into the analysis made the results of labour less reliable. Therefore in interpreting the result one must regard industries where there was some influence of labour in the location (see Table V.6) as less footloose than the rankings might imply. It would have been interesting to carry out the triangularisation when labour was included but the expense in terms of computer time was thought not to be worthwhile.

V.E. Conclusion

One of the principal aims of this study was to see if the attraction model could be implemented using data derived from second-hand input-output tables. This has been done, although it is difficult to say how reliable the results have been. However, they do seem to explain to some extent why the Northern region is a depressed region and why the South-East is a relatively prosperous one. The findings also lend support to Keeble (110) and Tosco (189) that industry 21 (subcontractors and repairs) may be a key to regional development particularly with the metal, machine and engineering sectors, and because of the large multiplier effects of this industry, it seems to have been a major constraint on regional growth.

The model can also be seen to be a useful one in analysing the effects of exogeneous changes (such as changes in government policy) on the regional economy. It has been shown how the total multiplier effects can be calculated - and how this can be refined by the introduction of labour, and with interregional feedback effects. It has also been shown how the effects of exogeneous shocks can be traced round by round and by the separate supply and demand influences. Consequently, the bottlenecks (such as labour supply) to regional growth may be able to be identified.

Apart from the possible explanatory or predictive value of the model with regard to exogeneous changes, the results also suggest certain policy prescription. This comes about because of two related findings:

(i) For most industries it was possible to explain their location, which would seem to indicate that manufacturing industry is not as footloose in its location as was claimed by Chelaway.[1] A method was also shown how to rank ordinally the relative footlooseness of these industries. This has implications for the uniform blanket Industrial Development Certificate (I.D.C.) control of expansion of industry in the prosperous regions. If certain industries do vary

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[1] See Introduction

markedly in their degree of mobility, a much more selective application of I.D.C. would seem appropriate. Some industries would have their costs (and therefore efficiency) adversely affected by being unable to expand in certain regions, whereas for others the costs may be relatively small, and so can be moved relatively easily.

(ii) We have seen also how different industries have markedly different effects on employment income in the region. Now if the aim is to maximise (or increase quickly) employment income in the Development Areas, this suggests some alteration in the uniform investment grant and Regional Employment Premium (R.E.P.) policy. It may well be worth 'bribing' an industry, which has large external effects on the regional economy, more than one which has relatively limited multiplier effects. This policy may be better than trying to encourage industries that seem to have good long term growth prospects (but small multiplier effects). So although the growth prospects of the actual industry in question may be less, the total effects may be greater in the long run.

We have argued that attraction theory is only a short run model so it may be worth while up-dating it when more recent Census of Production data become available, particularly since the more recent ones have more data disaggregated at the regional level[1], and because more recent input-output tables are classified into even more

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1 For example, investment will be disaggregated to the regional level. This type of data will greatly aid in the construction of regional social accounts, and so facilitate more accurate estimates of exogeneous final demand.

homogeneous industries by virtue of further disaggregation from those classified in the 1963 Input-Output Table (27).

Besides being a short run model, attraction theory was also seen to be partial, in that many variables that are endogeneous to the system were taken as exogeneous. It would thus seem worthwhile to try to integrate the attraction model into regional econometric models. This would give attraction theory more feedbacks and would give the econometric models a more general view of location decisions and overcome some of the difficulties of using highly aggregated Keynesian relationships at regional level.

Despite its limitations and drawbacks the attraction model would thus seem to be a useful tool to be employed in analysis and prescription, since most industrialised countries and the enlarged E.E.C. are now pursuing an active regional policy.

Appendix I

Ghosh and Chakravarti's (55) input-output linear programming model

Ghosh presents various linear programming input/output models as an aid in determining the location of an industrial complex. This approach has the advantage of being able to handle the location and output of all industries simultaneously, but its weakness is that only transport costs are directly measurable and so Klaassen's (114) secondary factors of location are ignored. The model of Ghosh and Chakravarti (55 pages 167-8)[1] will be used as example of this type of approach, and a number of small errors will be pointed out, which will demonstrate the necessity for a strict input-output accounting framework when these models are used.

Model 1

Minimise $\sum_{r}\sum_{i} {}_{r}L_{i} \cdot {}_{r}g_{i} + \sum_{r}\sum_{s}\sum_{i} {}_{rs}X_{i} \cdot {}_{rs}t_{i}$

subject to

$$(1 - \beta_{ii})\ {}_{r}g_{i} - \sum_{j} {}_{r}\beta_{ij} \cdot {}_{r}g_{j} - \sum_{s} {}_{rs}X_{i} - \sum_{r} {}_{rs}X_{i} = {}_{r}F_{i} \quad (r \neq s)$$

For all i, j, r and s (sic)

(plus some constraints on the transport system's and production capacity. However these do not interest us at the moment, and so can be ignored).

where ${}_{r}L_{k}$ = = unit labour cost of industry k in region r

${}_{r}g_{k}$ = output of industry k in region r

$(r)\beta_{ij}$ = (regional) Leontief coefficient $\left(\sum_{s} \frac{{}_{sr}x_{ij}}{{}_{r}g_{j}}\ (r)\beta_{ij}\right)$

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[1] This model is also reproduced in Ghosh and Chakravarti (56) where some actual empirical results are shown.

$_{rs}x_{ij}$ = flow of good i in region r to industry j in region s

$_{rs}X_i$ = total flow of good i from region r to region s

$(\sum_j {}_{rs}x_{ij} + {}_{rs}f_i)$[1]

$_{rs}f_i$ = flow of good i in region r to final demand in region s

$_rF_i$ = total final demand in region r for good i

$_{rs}t_i$ = unit transport cost of good i from region r to region s.

In Ghosh and Chakravarti (56 Ch. 4) the last minus sign on the LHS is a plus $(+\sum_r {}_{rs}X_i)$. The equation above is a misprint and we shall read it as a plus sign.

In this equation regional and national technical coefficients are mixed (that is β_{ii} and $_r\beta_{ij}$ are both used). If the region uses the same technology as the nation then β_{ii} and β_{ij} can be used and if the regional uses its own technology then $_r\beta_{ii}$ and $_r\beta_{ij}$ can be used, but not a mixture. Also the term $\sum_j {}_r\beta_{ij} \cdot {}_rg_j$ should be constrained $j \neq i$ otherwise the intra-industry demand term is counted twice (that is $(r)\beta_{ii} \cdot {}_rg_i$). Finally exports and imports to and from abroad are ignored. If imports are regarded as non-competing goods with home produced goods and the transport costs are the same regardless of location, then they can be ignored, but exports abroad cannot[2].

Further mistakes can be found in some of the other models, for example Model 2 page 108 the accounting constraint is given as

$$\sum_r {}_rg_i - \sum_r {}_r\beta_{ii} \cdot {}_rg_i - \sum_r \sum_j {}_r\beta_{ij} \cdot {}_rg_j = \sum_r {}_rF_i$$

---

[1] This must hold if the accounting balance is to hold, although this formulation is not explicitly stated in the text and no distinction is made between final and intermediate goods in $_{rs}X_{ij}$.

[2] Exports abroad can be ignored as a cost of transport if it is assumed that this cost is the same regardless of which region is doing the exporting, but they cannot be ignored in the accounting balance.

where again $j \neq i$ although it is not specifically stated. If $j \neq i$ then this equation actually holds true (ignoring exports and imports) in that the output of good i over all regions minus all that is sent as intermediate goods to all regions equals final demand for good i over all regions. Although this total constraint must hold in a linear programming model, i.e. it is a necessary condition, it is not a sufficient condition, since a separate accounting balance constraint for final must hold for each good in each region.

The formulation of input-output linear programming models becomes much easier to make internally consistent if a table of the regional input-output relationships is set us, such as the ones shown in Table III.1 or Table IV.1. Although these are only for 2 regions they can easily be generalised for n regions. It is hoped that the models formed from these types of tables that are presented in Ch. III and IV are internally consistent.

Appendix II

The attraction model as a simultaneous system

In Section III.D. it was mentioned that the attraction system is a simultaneous system. This can be shown explicitly by writing the whole system of the attraction model:

1) $g_1 = \lambda_{1d} \cdot dd_1 + \lambda_{11} \cdot (\alpha_{11} \cdot g_1) + \lambda_{21} \cdot (\alpha_{21} \cdot g_2) + .. + \lambda_{n1} \cdot (\alpha_{n1} \cdot g_n) + U_1$

n) $g_n = \lambda_{nd} \cdot dd_n + \lambda_{1n} \cdot (\alpha_{1n} \cdot g_1) + \lambda_{2n} \cdot (\alpha_{2n} \cdot g_2) + .. + \lambda_{nn} \cdot (\alpha_{nn} \cdot g_n) + U_n$

n+1) $dd_1 \equiv \phi_1(F_1, g_1 \ldots g_n, \Delta)$

n+n) $dd_n \equiv \phi_n(F_n, g_1 \ldots g_n, \Delta)$

where there are: 2 n equations

2 n unknown (n of the g's and n of the dd's)

and where dd's are endogeneous variables

g's are endogeneous variables

F's are exogeneous variables

Δ is a matrix of coefficients made up of elements δ_{ij}

λ's, α's and δ's are constant coefficients.

Equations 1) to n) show that output of each industry is a function of internal regional demand and internal regional supply.

Equations n+1) to n+n) show that internal demand is a function of the regions structure, final demand and technology.

The order conditions for identification of each equation of a simultaneous system are[1]:

$$K - J \geqslant H - 1$$

where K = total number of exogeneous variables in the system

J = total number of exogeneous variables included in the particular equation.

[1] See Christ (32) Ch. VIII Section 3.

H = total number of endogeneous variables included in the particular equation.

Now in an attraction system of N industries:

(a) for equations 1) to n)

K = N since there is a final exogeneous demand (F) for each industry.

J = 0 since in equations 1) to n) there are no exogeneous variables included.

H theoretically could include all the g's, i.e. N in number, but in practice H numbers only about 4 at the most.

So $K - J > H - 1$ for equations 1) to n) and each equation is over-identified.

(b) for equations n+1) to n+n)

theses equations can be solved directly with the data that is used to implement the model, since they are taken from the regional input-output tables. For prediction purposes internal intermediate demand is formed endogeneously in the system from $g_1 \ldots g_n$ and Δ .

Using Ordinary Least Squares (O.L.S.) to estimate a simultaneous system will lead to biased results - two Stage Least Squares (2.S.L.S.) is therefore suggested by various authors[1]. In order to carry out 2.S.L.S. and estimate each equation separately it is necessary to have a matrix of exogeneous variables even those that are excluded from the equation - in our case this is the F of which there are N in number, with an observation for each region of the U.K.[2] In this situation

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[1] See for example Christ (32), Johnstone (101) or Haronitis (73) for a theoretical discussion of why 2.S.L.S. is preferable to O.L.S. Mosback and Wold (145) have systematically tested (by Monte Carlo methods) O.L.S. vs. 2.S.L.S. for small scale interdependent models. See Ch. 13 for a summary of their results.

[2] Strictly speaking the industries that we were unable to explain or decided not to explain (See Section V.A.) should be included as exogeneous variable since they are taken as given.

$N > k$ where k is the number of observations. In cases of this nature it has been suggested that principal components analysis be used[1]. If the exogeneous variables are highly correlated then only a very few principal components vectors would be needed to 'explain' the variations amongst the exogeneous variables. In fact the exogeneous variables were generally quite highly correlated such that the cumulative percentage of variation 'explained' by the values of the F's was:

No.	1	2	3	4	5	
Cumulative Percentage	0.82	0.90	0.94	0.96	0.97	etc.

The only binding restriction on the use of the principal components vectors was that we should be able to identify each equation and so we needed at least as many vectors as included endogeneous variables. After this there is a trade off problem if we include more principal components vectors a larger proportion of the variations amongst the exogeneous variables would be included[2], but with a larger number of vectors included we would be decreasing the degrees of freedom in the equation we are trying to estimate. Consequently the following rule of thumb was adopted in the estimation procedure[3]: 4 vectors were used when the number of endogeneous variables was 4 or less. This is because this still left us with a satisfactory number of degrees of freedom and

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1 See Johnson (101) Ch. 13 Section 5 for a discussion of principal components analysis in simultaneous systems.

2 Although it must be noted that the net increase in 'explanation' diminishes with each additional vector.

3 For more formal tests of how many to include see Koutsoyiannis (120) Ch. 17 Section 6.

'explained' 94% of the variation of the exogeneous variable. When there were more than 4 endogeneous variables, the same number of principal components were used in order to identify exactly the endogeneous variables.

Appendix III

Interpretation of the attraction coefficients (the λ's)

Intuitively we have interpretted the λ_{kd} as the demand attraction coefficient and this as the proportion explained by demand. Similarly λ_{hk}'s are the supply attraction coefficients and show the relatively explanatory power of the various supplying industries. It is attempted below to derive these in a more rigorous way.

We have the following coefficients.

(i) $$\lambda_{kd} \quad \frac{t_{kd}}{t_{kd} + \sum_{h=1}^{n} t_{hk} \cdot \beta_{hk} - t_k}$$

(ii) $$\lambda_{hk} \quad \frac{t_{hk}}{t_{kd} + \sum_{h=1}^{n} t_{hk} \cdot \beta_{hk} - t_k}$$ for each supply industry h.

We can identify 5 possible cases of supply and/or demand attraction.

A. When an industry is purely demand orientated and any supply that it imports has no costs,

we know (i) $t_{hk} = 0$ for each h by assumption

(ii) $t_k = 0$ since there are no actual costs incurred, since $t_{hk} = 0$ and no demand crosses the boundaries of the relevant region (see Section III.M for definition of this) so the average is zero,

(iii) $t_{kd} > 0$ by assumption

Therefore

(i) $\lambda_{kd} = 1$

(ii) $\Sigma \lambda_{hk} = 0$

(iii) $\lambda_{kd} + \Sigma\lambda_{hk} = 1$

B. When an industry is purely supply orientated and the demand that it exports has not cost

we know (i) $t_{kd} = 0$ by assumption

(ii) $t_k = 0$ see reason above

(iii) $t_{hk} > 0$ for at least one, h by assumption

(iv) $0 < \sum_h \beta_{hk} < 1$ Leontief coefficient

Therefore (i) $\lambda_{kd} = 0$

(ii) $\Sigma \lambda_{hk} = \Sigma \dfrac{t_{hk}}{\sum_h \beta_{hk} . t_{hk}} = \dfrac{1}{\beta_{hk}} > 1$

(iii) $\lambda_{kd} + \Sigma \lambda_{hk} > 1$

C. When an industry is purely demand orientated, but the supply production that it has to import is not costless

we know (i) $t_{kd} > \Sigma t_{hk}$ or else not demand orientated

(ii) $\Sigma t_{hk} > \Sigma t_{hk} . \beta_{hk}$ since $\beta_{hk} < 1$

(iii) $t_{kd} > \Sigma t_{hk} . \beta_{hk}$ by transitivity

(iv) $t_{kd} > t_k$ because t_{kd} is the most expensive unit communication cost but t_k is only the aaverage, which include many that are costless

(v) $\Sigma t_{hk} . \beta_{hk} > t_k$ since t_{hk} is a cost actually incurred but t_k an average of all flows of goods including some when no cost is incurred.

Therefore (i) $\lambda_{kd} < 1$

(ii) $\Sigma \lambda_{hk} < 1$

(iii) $\lambda_{kd} + \Sigma \lambda_{hk} > 1$

This case is quite unlikely in practice because if supply imports have some cost then it is unlikely that demand will be the sole influence on location, unless t_{kd} is very large in relation to t_{hk} in which case

(1) $\lambda_{kd} \rightarrow 1$

(2) $\Sigma\lambda_{hk} \rightarrow 0$

i.e. the limiting case of A. above.

D. When an industry is purely supply orientated but the demand it has to export is not costless

we know (i) $\Sigma t_{hk}.\beta_{hk} > t_{kd}$

(ii) $t_{kd} > t_k > 0$

therefore (i) $\lambda_{kd} < 1$

(ii) $\Sigma\lambda_{hk} \lesseqgtr 1$

depending on whether $\Sigma t_{hk} + t_k \lesseqgtr t_{kd} + \Sigma t_{hk}.\beta_{hk}$

This case is again unlikely in practice because if demand exports have some cost then it is unlikely that supply will be the sole influence on location, unless t_{hk} is very large in relation to t_{kd} then as $t_{kd} \rightarrow 0$, i.e. as the cost of demand gets less then $t_k \rightarrow 0$

and (1) $\lambda_{kd} \rightarrow 0$

(2) $\Sigma\lambda_{hk} \rightarrow \frac{1}{\beta_{hk}}$

i.e. the limiting case of B. above.

E. When an industry is partly supply and partly demand orientated

we know (i) t_{kd} and $\Sigma t_{hk}.\beta_{hk} > t_k$

but (ii) $\Sigma t_{hk} \lesseqgtr t_{kd}$ depending on which is most important

(iii) $t_k > 0$

therefore (i) $\lambda_{kd} < 1$

(ii) $\Sigma\lambda_{hk} \lesseqgtr 1$ depending on whether $\Sigma t_{hk} + t_k \lesseqgtr t_{kd} + \Sigma t_{hk}\,{}_{j}\beta_{hk}$.

The denominator of the two expressions (λ_{kd} and $\Sigma\lambda_{hk}$) is exactly alike, so it depends on whether $t_{kd} \lesseqgtr \Sigma t_{hk}$.

We know always $\lambda_{kd} + \Sigma\lambda_{hk} > 1$

As Σt_{hk} increases $\lambda_{kd} \rightarrow 0$

" " " " $\Sigma\lambda_{hk} \rightarrow \frac{1}{j^{\beta}hk}$ 1, i.e. the limiting case of B.

As Σt_{hk} decreases $\lambda_{kd} \rightarrow 1$

" " " " $\Sigma\lambda_{hk} \rightarrow 0$, i.e. the limiting case of A.

So we can see that λ_{kd} represents the proportion explained by demand, and therefore $1 - \lambda_{kd}$ the proportion explained by supply.

We can find out within each industry which is the highest unit communication cost since we can solve for each equation the <u>relative</u> values of each of the t's. However because they are only relative values we cannot compare across industries.

Consider a typical industry k which is partly explained by demand (λ_{kd}) and partly by two supply industries (λ_{1k} and λ_{2k}) and the following results were estimated:

$$\lambda_{kd} = \frac{t_{kd}}{t_{kd} + t_{1k}\beta_{1k} + t_{2k}\beta_{2k} - t_k} = C_1$$

$$\lambda_{1k} = \frac{t_{1k}}{t_{kd} + t_{1k}\beta_{1k} + t_{2k}\beta_{2k} - t_k} = C_2$$

$$\lambda_{2k} = \frac{t_{2k}}{t_{kd} + t_{1k}\beta_{1k} + t_{2k}\beta_{2k} - t_k} = C_3$$

In this system there are 3 equations and 4 unknowns (t_{kd}, t_{1k}, t_{2k}, t_k) with the β's being constant. Generally there will be 1 more unknown than equations.

Now let $t_k = 1$ so the system is to be solved in relative terms to t_k (which is the average unit communication cost for all output).

The system can be re-written

$$t_{kd} = C_1(t_{kd} + t_{1k} \quad {}_{1k} + t_{2k} \quad {}_{2k} - 1)$$

$$t_{1k} = C_2(\text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"})$$

$$t_{2k} = C_3(\text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"})$$

Re-arranging in matrix form

$$\underset{\text{I} \qquad \text{A}}{\begin{pmatrix} 1 - C_1 & - C_1 . \beta_{1k} & - C_1 . \beta_{2k} \\ - C_2 & 1 - C_2 . \beta_{1k} & - C_2 . \beta_{2k} \\ - C_3 & - C_3 . \beta_{1k} & 1 - C_3 . \beta_{2k} \end{pmatrix}} \underset{\text{Y}}{\begin{pmatrix} t_{kd} \\ t_{1k} \\ t_{2k} \end{pmatrix}} = \underset{\text{X}}{\begin{pmatrix} - C_1 \\ - C_2 \\ - C_3 \end{pmatrix}}$$

Therefore $Y = (I-A)^{-1} X$

where we can assume I-A is non-singular and we have an expression for the t's in terms of all the known values the C's and β's.

Appendix IV

Solution of a closed input-output system

I am indebted to John D. Hey for showing me this method.

Even though the matrix to be inverted may be singular, some meaningful results can be obtained from the closed system in that if one final output is predetermined then all the other outputs are determined since they are all linear combinations of each other.

In a closed system

$$\underline{\underline{B}}.\underline{g} = \underline{g}$$

where (i) $\underline{\ }$ denotes vector or matrix

(ii) $\underline{\underline{B}}$ the Leontief coefficient matrix

(iii) $\underline{g}$ vector of gross outputs

$$\underline{g} - \underline{\underline{B}}\underline{g} = \underline{0}$$

$$(\underline{I} - \underline{B})\underline{g} = \underline{0}$$

where $\underline{g} = \underline{0}$ or $(\underline{I} - \underline{B})$ is singular.

Let us assume $(\underline{I} - \underline{B})$ is singular and ignore the trivial solution $\underline{g} = 0$[1]

However partitioning for an nxn system

$$\left(\begin{array}{c|c} \underset{(n-1)\ x\ (n-1)}{\underline{E}} & \underset{n-1}{\underline{b}} \\ \hline \underset{n-1}{\underline{c}} & d \end{array}\right) \left(\begin{array}{c} \underset{n-1}{\underline{g}} \\ \hline g_n \end{array}\right) = \left(\begin{array}{c} \underset{n-1}{\underline{0}} \\ \hline 0 \end{array}\right)$$

---

1 In a closed system I - B is singular because any one row can be formed from a linear combination of all the other rows since $\sum_{i=1}^{n} \beta_{ij} = 1$ for all j in a closed system

where the notation under the vector or matrix denotes its size, and d, g_n and 0 are scalars.

Therefore $\underline{E}.\underline{g} + \underline{b}.g_n = \underline{0}$

$$\underline{E}.\underline{g} = - \underline{b}.g_n$$

$$\underline{g} = - \underline{E}^{-1}.\underline{b}.g_n$$

So the gross outputs of all other industries $\underset{n-1}{\underline{g}}$ can be obtained by predetermining the output of one industry $.g_n$.

This method can also be used for the solution of a closed attraction system.

Appendix V

Consolidated and non-consolidated input-output systems

I am indebted to John Creedy for this proof.

Define

	production sector	all other	Total
production sector	Z	f	q
all other	y	o	y
total	q	y	

Z = a matrix of intermediate flows

f = a vector of final demands

y = factor incomes

For a consolidated matrix (two industries)

$$Z = \begin{bmatrix} o & Z_{12} \\ Z_{21} & o \end{bmatrix}$$

$$A = Z\hat{q}^{-1} = \begin{bmatrix} o & \frac{Z_{12}}{q_2} \\ \frac{Z_{21}}{q_1} & o \end{bmatrix}$$

For non-consolidated

$$Z^1 = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Z'\hat{q}^{-1} = A^1 = \begin{bmatrix} \frac{Z_{11}}{q_1+Z_{11}} & \frac{Z_{12}}{q_2+Z_{22}} \\ \frac{Z_{21}}{q_1+Z_{11}} & \frac{Z_{22}}{q_2+Z_{22}} \end{bmatrix}$$

Then $Q = [I - A]^{-1} f = \frac{1}{1 - \frac{Z_{12}\ Z_{21}}{q_1\ q_2}} \begin{bmatrix} 1 & \frac{Z_{12}}{q_2} \\ \frac{Z_{21}}{q_1} & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$

$$= \frac{q_1\ q_2}{q_1\ q_2 - Z_{12}\ Z_{21}} \begin{bmatrix} 1 & \frac{Z_{21}}{q_2} \\ \frac{Z_{21}}{q_1} & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

and $Q^1 = \left[I - A^1\right] f = \frac{1}{\left(1 - \frac{Z_{11}}{q_1+Z_{11}}\right)\left(1 - \frac{Z_{22}}{q_2+Z_{22}}\right) - \frac{Z_{12}\ Z_{21}}{(q_2+Z_{22})\ (q_1+Z_{11})}}$.

$$\begin{bmatrix} 1 - \frac{Z_{22}}{q_2+Z_{22}} & \frac{Z_{12}}{q_2+Z_{22}} \\ \frac{Z_{21}}{q_1+Z_{11}} & 1 - \frac{Z_{11}}{q_1+Z_{11}} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$= \frac{(q_1 + Z_{11})(q_2 + Z_{22})}{q_1 q_2 - Z_{12}Z_{21}} \begin{bmatrix} \dfrac{q_2}{q_2 + Z_{22}} & \dfrac{Z_{12}}{q_2 + Z_{22}} \\ \dfrac{Z_{21}}{q_1 + Z_{11}} & \dfrac{q_1}{q_1 + Z_{11}} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Consider first elements

$$Q_1^1 = \frac{(q_1 + Z_{11})}{q_1 q_2 - Z_{12}Z_{21}} (q_2 f_1 + Z_{12} f_2)$$

$$Q_1 = \frac{q_1 q_2}{q_1 q_2 - Z_{12}Z_{21}} \left(f_1 + \frac{Z_{12} f_2}{q_2} \right)$$

$$Q_1^1 - Q_1 = \frac{Z_{11}(q_2 f_1 + f_2 Z_{12})}{q_1 q_2 - Z_{12}Z_{21}} \qquad (1)$$

Now from previous derivation Z + f = q

i.e. $$\begin{bmatrix} o & Z_{12} \\ Z_{21} & o \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Substitute for q_1 and q_2 in (1)

$$Q_1^1 - Q_1 = \frac{Z_{11}\left[(Z_{21} + f_2)\, f_1 + f_2 Z_{12}\right]}{(Z_{12} + f_1)(Z_{21} + f_2) - Z_{12}Z_{21}}$$

$$= Z_{11}$$

Similarly it can be shown that the second element differences is Z_{22}.

Consequently the only difference between the inversion of a consolidated and non-consolidated matrix is the absolute figure of the central diagonal of the original matrix and so will not affect the problem. We shall use the non-consolidated matrix since the intra-industry effects may be important in attraction theory.

Appendix VI

The derivation of the Leontief-Isard balance model

The derivation of these results of how to obtain B and C matrices used in equation IV.E.7 is taken from Leontief (123). A bar under a symbol indicates a matrix.

$$\underline{B} \equiv \begin{bmatrix} \underline{b}_{pp} & \underline{b}_{pk} \\ & \\ \underline{b}_{kp} & \underline{b}_{kk} \end{bmatrix} \equiv \text{input coefficients matrix}$$

where

$$\underline{b}_{pp} \equiv \begin{bmatrix} b_{11} & b_{12} & b_{1h} \\ b_{21} & & \\ b_{h1} & & b_{hh} \end{bmatrix} \quad \underline{b}_{pk} \equiv \begin{bmatrix} b_{1,h+1} & b_{1,h+2} & b_{1m} \\ & & \\ b_{h,h+1} & & b_{h,m} \end{bmatrix}$$

$$\underline{b}_{kp} \equiv \begin{bmatrix} b_{h+1,1} & b_{h+1,2} & b_{h+1,h} \\ b_{h+2,1} & & \\ & & \\ b_{m1} & & b_{mh} \end{bmatrix} \quad \underline{b}_{pk} \equiv \begin{bmatrix} b_{h+1,h+1} & b_{h+1,h+2} & b_{h+1,m} \\ b_{h+2,h+1} & & \\ & & \\ b_{m,h+1} & & b_{m,m} \end{bmatrix}$$

The balance equation for the regional outputs of region r is:

$$\left[\underline{I} - \underline{b}_{pp} - \underline{b}_{pk}\right] \begin{bmatrix} {}_r\underline{g}_p \\ {}_r\underline{g}_k \end{bmatrix} = {}_r\underline{F}_p$$

$$= {}_r\underline{g}_p - \underline{b}_{pp} \cdot {}_r\underline{g}_p - \underline{b}_{pk} \cdot {}_r\underline{g}_k = {}_r\underline{F}_p$$

Therefore $\left[\underline{I} - \underline{b}_{pp}\right] {}_r\underline{g}_p = {}_r\underline{F}_p + \underline{b}_{pg} \cdot {}_r\underline{g}_k$

$${}_r\underline{g}_p = \left[\underline{I} - \underline{b}_{pp}\right]^{-1} {}_r\underline{F}_p + \left[\underline{I} - \underline{b}_{pp}\right]^{-1} \underline{b}_{pk} \cdot {}_r\underline{g}_k$$

where $\left[\underline{I} - \underline{b}_{pp}\right]^{-1}$ $\underline{C}$)

) in equation IV.E.7

$\left[\underline{I} - \underline{b}_{pp}\right]^{-1} \underline{b}_k$ $\underline{B}$)

Industry name and number		Expenditure on products of North		Expenditure on products of Rest of UK		See Notes		
		Effects in North (a)	Effects in RUK (b)	Effects in North (c)	Effects in RUK (d)	(e)	(f)	(g)
1.	Agriculture	0.59 (0.44)	0.13	0.01	0.72	0.22	1.17	0.17
2.	Forestry and Fishing	0.75 (0.55)	0.14	0.02	0.88	0.37	0.10	0.01
3.	Coal Mining	1.02 (0.75)	0.17	0.02	1.20	0.61	2.32	1.22
4.	Other mining and quarry	0.73 (0.53)	0.15	0.02	0.87	0.26	0.11	0.07
5.	Grain milling	0.39 (0.29)	0.09	0.01	0.47	0.09	0.11	0.02
6.	Other cereal foodstuffs	0.58 (0.42)	0.15	0.01	0.72	0.21	0.63	0.09
7.	Sugar	0.30 (0.22)	0.06	0.01	0.36	0.07	0.03	0.01
8.	Cocoa, chocolate and sugar confectionary	0.57 (0.41)	0.16	0.01	0.71	0.23	0.12	0.02
9.	Other food	0.60 (0.44)	0.15	0.01	0.74	0.15	0.27	0.04
10.	Drink	0.55 (0.40)	0.14	0.01	0.68	0.23	0.34	0.05
11.	Tobacco	0.36 (0.26)	0.09	0.01	0.45	0.14	0.10	0.01
12.	Mineral Oil Refining	0.21 (0.15)	0.06	0.01	0.26	0.05	0.01	0.01
13.	Paint and Printing ink	0.68 (0.58)	0.15	0.03	0.81	0.22	0.15	0.11
14.	Coke Ovens	1.00 (0.70)	0.17	0.13	1.0	0.09	0.10	0.08
15.	Pharmaceuticals and toilet preparations	0.66 (0.48)	0.16	0.02	0.80	0.20	0.04	0.01

Industry name and number		Expenditure on products of North		Expenditure on products of Rest of UK				
		Effects in North (a)	Effects in RUK (b)	Effects in North (c)	Effects in RUK (d)	See notes (e)	(f)	(g)
16.	Soap oils and fats	0.47 (0.34)	0.13	0.01	0.58	0.12	0.12	0.06
17.	Synthetic resin and plastic material	0.55 (0.41)	0.13	0.02	0.64	0.18	0.23	0.20
18.	Other chemicals and allied industries	0.58 (0.43)	0.13	0.02	0.68	0.22	0.60	0.36
19.	Iron and Steel	0.73 (0.54)	0.16	0.04	0.86	0.30	3.33	3.13
20.	Light metals	0.57 (0.41)	0.12	0.01	0.68	0.29	0.33	0.30
21.	Other non-ferrous metals	0.41 (0.31)	0.18	0.01	0.60	0.15	0.18	0.18
22.	Agricultural machinery	0.74 (0.54)	0.20	0.02	0.93	0.14	0.01	0.01
23.	Machine Tools	0.82 (0.61)	0.18	0.02	0.99	0.27	0.07	0.06
24.	Engineers small tools	0.85 (0.63)	0.18	0.02	1.02	0.39	0.03	0.03
25.	Industrial Engines	0.84 (0.62)	0.22	0.02	1.06	0.44	0.40	0.36
26.	Textile machinery	0.80 (0.59)	0.21	0.02	1.01	0.37	0.01	0.01
27.	Contractors plant and mechanical handling equipment	0.76 (0.56)	0.21	0.02	0.95	0.29	0.07	0.05
28.	Office machinery	0.80 (0.58)	0.14	0.01	0.94	0.42	0.02	0.01
29.	Other non-electrical machinery	0.79 (0.58)	0.19	0.02	0.97	0.35	0.32	0.25
30.	Industrial plant and steel work	0.85 (0.63)	0.19	0.03	1.03	0.34	0.52	0.50
31.	Other mechanical engineering	0.83 (0.61)	0.19	0.02	1.01	0.41	1.53	1.45
32.	Scientific Instruments etc.	0.78 (0.57)	0.20	0.01	0.98	0.41	0.15	0.13

Industry name and number		Expenditure on products of North		Expenditure on products of Rest of UK		See notes		
		Effects in North (a)	Effects in RUK (b)	Effects in North (c)	Effects in RUK (d)	(e)	(f)	(g)
33.	Electrical machinery	0.86 (0.63)	0.19	0.02	1.05	0.41	41.32	41.24
34.	Insulated wires and cables	0.53 (0.40)	0.23	0.02	0.76	0.20	0.31	0.29
35.	Radio and telecommunications	0.82 (0.60)	0.17	0.01	0.99	0.38	1.00	0.88
36.	Other electrical goods	0.77 (0.56)	0.19	0.02	0.95	0.30	0.43	0.25
37.	Cans and metal boxes	0.77 (0.57)	0.16	0.04	0.90	0.20	0.04	0.01
38.	Other metal goods	0.70 (0.52)	0.20	0.02	0.89	0.29	0.64	0.50
39.	Shipbuilding and engineering	0.92 (0.68)	0.22	0.03	1.14	0.44	0.17	0.13
40.	Motor vehicles	0.61 (0.45)	0.33	0.02	0.93	0.23	0.06	0.02
41.	Aircraft	0.78 (0.57)	0.32	0.01	1.10	0.42	0.02	0.02
42.	Other vehicles	0.93 (0.68)	0.20	0.02	1.13	0.43	0.25	0.14
43.	Production of man-made fibres	0.48 (0.36)	0.11	0.03	0.58	0.18	0.04	0.01
44.	Cotton etc. spinners and weaving	0.53 (0.39)	0.21	0.02	0.73	0.23	0.07	0.02
45.	Wool	0.58 (0.43)	0.12	0.12	0.69	0.28	0.18	0.03
46.	Hosiery and Lace	0.65 (0.48)	0.24	0.06	0.88	0.25	0.06	0.01
47.	Textile finishing	0.84 (0.62)	0.16	0.02	1.00	0.41	0.02	0.01
48.	Other textiles	0.59 (0.43)	0.17	0.01	0.75	0.25	0.18	0.05
49.	Leather, leather goods and fur	0.55 (0.41)	0.14	0.01	0.68	0.23	0.08	0.01
50.	Clothing	0.67 (0.49)	0.25	0.01	0.91	0.32	0.70	0.11

Industry name and number	Expenditure on products of North		Expenditure on products of Rest of UK		See notes		
	Effects in North (a)	Effects in RUK (b)	Effects in North (c)	Effects in RUK (d)	(e)	(f)	(g)
51. Footwear	0.73 (0.53)	0.23	0.01	0.95	0.33	0.21	0.03
52. Cement	0.63 (0.46)	0.15	0.03	0.76	0.19	0.01	0.01
53. Other building materials etc.	0.76 (0.55)	0.17	0.02	0.92	0.32	0.43	0.35
54. Pottery and gloss	0.86 (0.63)	0.17	0.02	1.01	0.43	0.56	0.48
55. Furniture etc.	0.76 (0.56)	0.20	0.01	0.95	0.35	0.24	0.05
56. Timber and misc. wood manufacturers	0.58 (0.42)	0.12	0.01	0.69	0.28	0.22	0.14
57. Paper and Board	0.57 (0.41)	0.13	0.01	0.68	0.20	0.12	0.08
58. Paper Products	0.58 (0.42)	0.18	0.01	0.75	0.24	0.33	0.23
59. Printing and publishing	0.77 (0.57)	0.23	0.01	1.00	0.40	0.42	0.12
60. Rubber	0.65 (0.47)	0.19	0.02	0.82	0.28	0.08	0.04
61. Other manufacturing	0.70 (0.51)	0.18	0.02	0.86	0.30	0.41	0.28
62. Construction	0.84 (0.62)	0.18	0.02	1.02	0.42	1.11	0.41
63. Gas	0.84 (0.62)	0.16	0.05	0.95	0.28	0.47	0.22
64. Electricity	0.64 (0.47)	0.11	0.04	0.71	0.21	0.78	0.29
65. Water Supply	0.61 (0.45)	0.10	0.01	0.71	0.35	0.17	0.06
66. Road and Rail transport	0.93 (0.68)	0.18	0.02	1.10	0.56	3.29	1.59
67. Other transport	0.52 (0.38)	0.10	0.01	0.61	0.28	0.73	0.33
68. Communication	0.97 (0.71)	0.13	0.01	1.11	0.60	0.99	0.44
69. Distributive trades	0.77 (0.57)	0.14	0.01	0.91	0.46	7.22	2.33

Industry name and number	Expenditure on products of North		Expenditure on products of Rest of UK		See notes		
	Effects in North (a)	Effects in RUK (b)	Effects in North (c)	Effects in RUK (d)	(e)	(f)	(g)
70. Miscellaneous services	0.86 (0.63)	0.18	0.01	1.03	0.59	8.47	2.80
71. Public Admin., defence, health and education	1.46 (1.07)	0.17	0.01	1.62	1.0	0.0	0
72. Domestic services etc. to households	1.46 (1.07)	0.17	0.01	1.62	1.02	1.28	0.19
					TOTAL =	86.63	63.21

Appendix VII /contd

A summary of some of the results that can be obtained by manipulating the 146 x 146 North - Rest of the UK inter-regional input-output table.

Column (a)

This shows the effects on all industries in the North of an increase in final demand of 1 for the products of the Northern region. Here labour is treated as endogenous and the m.p.c. - a.p.c. The figure in brackets is where the m.p.c. = 0.2 of the a.p.c.

This figure can be interpretted as the multiplier

Column (b)

This shows the effects on all industries in the Rest of the U.K. of an increase in final demand of 1 for the products of the Northern region. Labour is again endogenous and the m.p.c. = a.p.c.

Column (c)

This shows the effects on all industries in the North of an increase in final demand of 1 for the products of the Rest of the U.K. Again the m.p.c. = a.p.c.

Column (d)

This shows the effects on all industries in the Rest of the U.K. of an increase in final demand of 1 for the products of the Rest of the U.K. Again the m.p.c. = a.p.c.

Column (e)

This shows the effects (direct and indirect) on the industry in question in the North as a result of an increase in final demand of 1 for the products of the Northern region. Again the m.p.c. = a.p.c. Thus the difference between column (a) and column (e) is the direct and indirect effects on all other industries in the North as a result of an increase in final demand of 1 of the industry in question in the North.

Column (f)

This shows the effects (direct and indirect) of £100 expenditure in Northern electrical machinery (industry 33 - minimum list heading 361 of the 1968 standard industrial classification) on all the other industries in the North. Again m.p.c. = a.p.c. This industry is a typical investment good producing industry in the Northern region.

Column (g)

Same as column (f) only m.p.c. = 0.2 a.p.c.

Note 1

All the figures in this table are in money units worth of employment. This may be taken as a rough proxy for the number of jobs but to convert money units of employment into actual job numbers it must be ascertained what the average wage is for each industry - for example coal mining may create more money units worth of jobs than say clothing, but clothing may actually be a larger number of jobs since the average wage is lower in the clothing industry. Whether this is worth doing will depend on what the aims of the policy are (i.e. maximise the number of jobs or maximise the value of labour employed).

Note 2

The multipliers in the first four columns are for an increase in expenditure of final demand for the products of the North (effects shown in columns (a) and (b)) or for the Rest of the UK (effects shown in columns (c) and (d)). It must be remembered that this is different from an increase in final demand in the North, say, where some would come directly from the Rest of the UK as well as directly from the North itself. This effect could be shown by post-multiplying the $(1 - B)^{-1}$ matrix by the vector of final demand ($\underline{F}$) where the form of $\underline{F}$ in a 3 good economy would be:

$$\underline{F} = \begin{pmatrix} 0 \\ x_1 \\ 0 \\ 0 \\ x_2 \\ 0 \end{pmatrix}$$

where $x_1 + x_2$ = total increase in final demand for the product no. 2 in the North

x_1 = the amount of the increase coming initially from the North

x_2 = the amount of the increase coming initially from the Rest of the UK, that leakage out of the North directly.

It could be assumed that a similar proportion will come from the North as in the past.

This method has not been shown in this appendix because of expense in computer time in manipulating such large matrices.

Appendix VIII

Some problems of multicollinearity and misspecification in attraction analysis

As stated in section V.1 multicollinearity made it necessary to aggregate certain industries (20 and 21, 66 and 67, and 69 and 70) and was so serious in the case of industry 40 that no reliable estimates were possible. Even in the other industries where estimates were obtained there was some evidence of collinearity between the variables on the R.H.S. of the equation[1]. As discussed in that section this may cause us to reject certain variables as insignificant (since the efficiency of the estimates is reduced), when in fact they are not, so mis-specifying the equation. It is also important to note that if labour is excluded from the equation, when in fact it should be included, and labour and the other included variables are correlated (i.e. they are not orthogonal), then we have another misspecification problem. When the misspecification problem involves excluding variables that are correlated with the remaining variables then there is a bias on the coefficients of the remaining variables[2]. Thus multicollinearity is probably one of the most serious problems encountered in attraction analysis. Little can be done to avoid multicollinearity in our case, since most of the conventional solutions involve the use of extra information to overcome the collinearity between variables. We have used cross-section data in our analysis, but if time series data ever does come available it will not be able to be used to overcome collinearity because the assumption that allows us to estimate our equations will not hold with time series data. Koutsoyiannis (120) suggests the use of principal components analysis to obtain some orthogonal variables

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1 It is interesting to note that Van Wickeren (197) page 76 encountered some multicollinearity problems.

2 See Koutsoyiannis (120) pages 245 - 248.

from correlated variables, but this method is not recommended to overcome multicollinearity since less information is being used. For a useful summary of the problems and possible solutions to multicollinearity and misspecification problems in applied econometric work see Giles (57), but it is unlikely if any of the methods suggested could be used to overcome multicollinearity in attraction theory.

Appendix IX

Results of inverting the attraction matrices for all regions

Results of $[I - \hat{1}._{r}\Delta - (LA)']^{-1}$ attraction matrix for all regions[1].- column sums. This is equivalent to column a in Table V.II. All results are in terms of value of labour employed.

Ind. No.	Yorkshire and Humberside	East Midlands	East Anglia	South West	West Midlands	North West	Wales	Scotland	Northern Ireland
1	0.89	0.37	0.36	0.32	0.31	0.48	0.46	0.47	0.47
2	0.58	0.48	0.49	0.53	0.48	0.61	0.55	0.60	0.58
3	1.55	1.02	0.59	0.92	1.25	1.35	1.56	1.34	0.59
4	1.89	0.88	1.25	0.69	3.24	1.76	1.48	1.45	0.84
5	0.34	0.30	0.30	0.33	0.26	0.35	0.32	0.33	0.31
6	0.45	0.33	0.41	0.37	0.32	0.46	0.39	0.40	0.42
7	0.19	0.15	0.17	0.17	0.14	0.19	0.07	0.18	0.07
8	0.52	0.39	0.47	0.42	0.37	0.53	0.48	0.45	0.46
9	1.19	0.85	0.90	0.99	0.80	1.28	0.58	1.05	1.25
10	0.37	0.31	0.34	0.31	0.30	0.37	0.34	0.34	0.34
11	0.33	0.25	0.34	0.33	0.13	0.41	0.32	0.32	0.30
12	0.37	0.18	0.06	0.18	0.17	0.27	0.30	0.25	0.05
13	1.35	0.64	0.80	0.44	0.51	0.95	1.08	0.83	1.02
14	2.63	1.61	0.08	0.08	6.23	2.75	2.06	4.58	0.08
15	0.99	0.55	0.68	0.46	0.45	0.78	0.84	0.70	0.79
16	0.78	0.50	0.56	0.51	0.39	0.59	0.67	0.65	0.83
17	4.42	1.17	1.16	0.83	0.77	1.82	2.78	1.50	0.17
18	8.86	3.31	3.68	1.47	1.65	4.68	6.77	4.41	6.26
19	1.09	1.19	0.21	0.87	1.51	1.89	0.82	1.44	1.51
20/21	0.74	0.95	0.73	0.71	0.69	1.01	0.59	1.04	1.08

1 Except Northern and South-East which are given in Table V.II.

Ind. No.	Yorkshire and Humberside	East Midlands	East Anglia	South West	West Midlands	North West	Wales	Scotland	Northern Ireland
22	0.56	0.56	0.43	0.43	0.59	0.64	0.53	0.60	0.54
23	0.61	0.62	0.50	0.50	0.63	0.66	0.58	0.64	0.39
24	1.39	0.85	0.57	1.01	1.42	0.97	0.71	0.81	0.95
25	0.70	0.63	0.37	0.52	0.65	0.70	0.37	0.66	0.37
26	0.64	0.62	0.53	0.51	0.62	0.71	0.34	0.65	0.60
27	0.57	0.58	0.43	0.43	0.59	0.65	0.53	0.60	0.55
28	0.57	0.57	0.40	0.49	0.56	0.61	0.40	0.58	0.55
29	0.73	0.70	0.50	0.50	0.91	0.84	0.69	0.95	0.60
30	0.65	0.64	0.48	0.48	0.67	0.68	0.59	0.70	0.63
31	3.61	5.29	4.68	3.47	4.39	5.15	2.01	6.12	3.88
32	1.31	0.66	3.65	3.87	0.83	1.23	1.08	4.43	1.69
33	1.61	1.17	0.69	0.69	1.58	1.70	0.82	1.05	0.89
34	1.10	0.71	0.20	0.44	1.43	0.97	0.62	0.69	0.56
35	0.85	1.24	1.06	1.11	0.98	1.29	1.08	1.11	1.37
36	0.65	0.55	0.54	0.49	0.54	0.67	0.61	0.63	0.60
37	2.75	1.04	2.76	2.32	1.92	2.93	0.68	2.29	3.14
38	1.19	1.57	0.64	1.44	1.28	2.93	1.73	1.37	1.29
39	0.90	0.66	0.69	1.03	0.80	1.41	0.85	1.26	1.19
40	0.48	0.46	0.34	0.39	0.48	0.55	0.45	0.50	0.46
41	2.22	6.22	0.59	5.87	6.22	6.94	2.05	3.02	6.34
42	0.69	0.64	0.55	0.53	0.65	0.69	0.63	0.69	0.68
43	1.96	0.45	0.17	0.36	0.33	0.70	0.75	1.11	0.71
44	6.31	0.79	3.71	0.90	0.58	3.30	0.99	5.08	4.13
45	0.50	0.38	0.41	0.51	0.68	0.73	0.58	0.75	0.81
46	1.25	3.89	0.88	0.45	0.53	1.15	0.61	7.51	6.31
47	3.92	0.67	0.41	0.41	0.69	0.90	0.80	2.27	1.00
48	1.04	0.61	0.56	0.50	0.58	0.87	0.65	0.97	0.89
49	0.57	2.24	2.13	1.59	0.56	1.14	0.55	0.57	0.74
50	1.79	1.07	0.98	0.62	0.53	1.39	0.65	1.76	1.88

Ind. No.	Yorkshire and Humberside	East Midlands	East Anglia	South West	West Midlands	North West	Wales	Scotland	Northern Ireland
51	0.59	0.74	0.82	0.76	0.50	0.97	0.51	0.57	0.61
52	0.46	0.37	0.42	0.38	0.34	0.46	0.44	0.41	0.40
53	0.61	0.51	0.52	0.51	0.49	0.59	0.57	0.55	0.53
54	0.74	0.59	0.62	0.55	0.54	0.68	0.68	0.64	0.65
55	1.26	1.48	1.11	0.89	0.82	2.09	0.81	0.92	1.38
56	1.11	0.96	0.89	0.94	0.76	1.03	0.66	0.93	0.73
57	2.07	0.83	1.82	2.24	0.78	3.14	1.94	1.47	1.25
58	1.77	0.87	2.19	1.43	0.90	2.05	1.06	1.38	1.83
59	1.72	1.47	2.20	1.37	1.62	2.77	0.85	2.40	0.96
60	1.06	0.54	0.71	0.46	0.45	0.80	0.77	0.82	0.84
61	0.91	0.53	0.62	0.49	0.47	0.76	0.76	0.70	0.68
62	0.61	0.57	0.54	0.54	0.57	0.62	0.56	0.59	0.58
63	0.43	0.41	0.37	0.38	0.41	0.45	0.02	0.44	0.42
64	0.30	0.29	0.28	0.28	0.28	0.31	0.30	0.30	0.29
65	0.44	0.41	0.41	0.40	0.40	0.43	0.43	0.42	0.42
66/67	0.50	0.47	0.47	0.48	0.52	0.52	0.50	0.50	0.45
68	0.69	0.67	0.67	0.70	0.65	0.70	0.68	0.69	0.68
69/70	0.63	0.57	0.58	0.58	0.56	0.63	0.59	0.62	0.59

Appendix X

Some experiments with interregional feedback effects and separating out the supply and demand effects in the Northern region attraction table

Notes

(i) Col a - effects on region as a result of a change in demand of 1 for each product inside the region, when all the original product is bought directly inside the region. That is to say $[I - \uparrow \cdot {}_r\Delta - (LA)' - \uparrow \cdot \sigma \cdot \psi \cdot {}_r\sigma - \uparrow \cdot \sigma \cdot \psi \, \phi \, {}_r\Delta]^{-1} [\uparrow \cdot \sigma \cdot \psi \, \phi \, F_{rex} + I \, F_{rex}]$

(ii) Col b - effects on region as a result of a change in demand of 1 for each product inside the region, when some of the original product leaks directly out of the region because of the demand leakage $(I - \uparrow)$. That s to say $[I - \uparrow \cdot {}_r\Delta - (LA)' - \uparrow \sigma \cdot \psi \, {}_r\sigma - \uparrow \cdot \sigma \cdot \psi \cdot \phi \, {}_r\Delta]^{-1} [\uparrow \cdot \sigma \, \psi \, \phi \, F_{rex} + \uparrow F_{rex}]$.

(iii) Col c - effects on region as a result of a change in demand of 1 for each product in the RUK. That is to say $[I - \uparrow \cdot {}_r\Delta - (LA)' - \uparrow \cdot \sigma \, \psi \, {}_r\sigma - \uparrow \, \sigma \, \psi \, {}_r\Delta]^{-1} [\uparrow \, \sigma \, \psi \, f_{ex}]$

(iv) Col d - effects on RUK as a result of a change in demand of 1 for each product in the Northern region when some of demand leakages out initially $(I - \uparrow)$. That is to say $[I - \Delta - {}_r\sigma \cdot \Theta \cdot \uparrow \cdot \sigma - \phi \cdot {}_r\Delta \cdot \Theta \cdot \uparrow \cdot \sigma - \phi \cdot \sigma]^{-1} [{}_r\sigma \cdot \Theta \cdot \uparrow \, F_{rex} + \phi \, {}_r\Delta \cdot \Theta \cdot \uparrow \cdot f_{rex} + \phi \, f_{rex}]$

(v) Col e - is the total direct and indirect effects of the multiplier resulting from demand influences divided by the total direct and indirect effects of the multiplier resulting from supply influences. Thus when the result is greater than 1 for a particular industry, the stimulation of that industry has more direct and indirect effects on other industries through the demand effect than through the supply effect, and vice versa. This result is obtained from the column sums of the expansion of the power series (see Section III.G.)

(vi) Col f - is the total direct and indirect effects of the demand effects of a change in final demand of 1 for all industries on the industry in question, divided by the total direct and indirect effects of the supply effect as a result of a change in final demand of 1 for all industries. Thus when the result is greater than 1 for a particular industry, then the demand effects on that industry from all other industries are greater than the supply effects on that industry from all other industries, and vice versa. Where certain industries are solely influenced by supply, these have been designated 'all supply', because they are not influenced by demand at all, and vice versa for 'all demand'. 'n.a.' signifies 'not applicable' since these industries are not influenced at all by either supply or demand.

Both col e and f relate to the effects in the Northern region.

	a	b	c	d	e	f
1	0.79	0.0	0.0	0.43	4.06	n.a.
2	0.86	0.0	0.0	0.53	9.27	n.a.
3	1.95	0.0	0.0	0.71	0.93	n.a.
4	1.91	0.0	0.0	0.52	1.39	n.a.
5	0.52	0.42	0.004	0.19	2.60	n.a.
6	0.74	0.22	0.003	0.36	7.32	n.a.
7	0.24	0.0	0.0	0.22	20.18	all supply
8	0.71	0.0	0.0	0.43	6.52	n.a.
9	1.47	0.28	0.002	0.44	1.09	0.02
10	0.54	0.0	0.0	0.41	10.30	n.a.
11	0.40	0.0	0.0	0.27	7.09	n.a.
12	0.35	0.30	0.009	0.09	4.88	125.25
13	1.30	0.58	0.050	0.39	3.08	0.43
14	5.08	0.0	0.0	0.55	0.60	all supply
15	1.10	0.0	0.0	0.49	5.05	n.a.

	a	b	c	d	e	f
16	0.96	0.26	0.020	0.32	2.04	0.25
17	1.54	0.54	0.050	0.34	1.42	0.19
18	5.93	2.98	0.022	0.86	0.62	0.48
19	2.55	1.73	0.029	0.52	0.79	0.99
20/21	1.51	0.62	0.008	0.39	2.43	0.63
22	1.09	0.0	0.019	0.56	2.65	all supply
23	1.06	0.20	0.015	0.53	2.07	0.07
24	1.13	0.69	0.013	0.34	1.75	0.28
25	1.21	0.0	0.017	0.64	2.27	all supply
26	1.12	0.0	0.0	0.61	2.44	all supply
27	1.13	0.0	0.0	0.56	1.95	all supply
28	0.89	0.0	0.008	0.57	1.44	all supply
29	2.21	0.35	0.015	0.57	0.68	0.07
30	1.36	0.93	0.023	0.37	2.57	0.12
31	9.59	2.01	0.015	0.89	0.29	0.72
32	1.31	0.55	0.012	0.49	0.54	0.06
33	2.87	0.66	0.015	0.61	0.34	0.06
34	1.79	0.48	0.015	0.46	0.48	0.25
35	2.64	0.82	0.009	0.57	0.50	0.09
36	1.10	0.53	0.015	0.41	3.25	0.14
37	2.32	1.97	0.053	0.51	1.32	1.97
38	1.80	0.81	0.021	0.48	0.90	0.78
39	2.33	1.42	0.028	0.48	0.62	0.24
40	1.03	0.0	0.0	0.57	2.60	n.a.
41	1.15	0.18	0.009	0.61	0.72	0.04
42	1.24	0.24	0.019	0.59	2.17	0.05
43	0.97	0.0	0.037	0.35	1.28	all supply
44	1.80	1.05	0.011	0.45	0.43	0.75
45	0.93	0.39	0.006	0.35	1.30	0.42
46	1.18	0.73	0.011	0.39	1.39	0.13
47	2.05	1.07	0.016	0.51	0.70	1.62

	a	b	c	d	e	f
48	0.96	0.77	0.007	0.26	1.56	0.33
49	2.08	0.0	0.012	0.42	0.28	n.a.
50	1.62	0.96	0.006	0.44	0.64	0.05
51	1.38	0.0	0.00	0.58	0.44	all supply
52	0.69	0.67	0.008	0.21	12.21	1.58
53	0.92	0.81	0.008	0.21	12.50	0.91
54	0.99	0.0	0.0	0.61	6.22	n.a.
55	2.37	1.12	0.008	0.51	0.50	0.05
56	1.00	0.50	0.006	0.30	0.73	0.31
57	1.91	0.99	0.009	0.44	0.80	0.68
58	1.05	1.31	0.002	0.42	1.10	0.82
59	1.40	0.51	0.003	0.48	0.78	0.23
60	0.97	0.23	0.004	0.42	4.13	0.37
61	1.03	0.45	0.03	0.38	4.66	1.10
62	1.10	1.10	0.009	0.15	12.61	all demand
63	0.88	0.92	0.009	0.28	8.80	"
64	0.49	0.49	0.004	0.21	11.00	"
65	0.54	0.54	0.003	0.06	8.36	"
66/67	0.67	0.85	0.008	0.08	7.95	"
68	0.83	0.67	0.002	0.17	20.19	"
69/70	0.78	0.83	0.004	0.07	23.27	"

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