DYNAMIC EQUILIBRIUM : GAME THEORY, CONTRACTS, AND SEARCH

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Preface

Much of the research that resulted in this thesis was begun during the tenure of an ESRC Research-Linked Studentship at the University of Warwick. I would like to thank first of all Avinash Dixit and Paul Weller, who, although they have not had a chance to see the thesis in a finished state, gave me much time and advice during my stay at Warwick. I would also like to acknowledge the many valuable conversations with, and comments from, both Alan Manning and Jonathan Thomas throughout the writing of the thesis, especially on the topic of long-term contracts. I am especially indebted to Jonathan for the material in section 2.6 which is joint work. I would like to thank my parents and Jill Hartley for all their support. Most of all, though, I would like to thank my supervisor, Nick Stern, for his continuous help, advice, and - especially - patience, over more years than I care to think about. This thesis would not have been written without him.

Declaration

I declare that the material contained in this thesis is solely my own work, with the exception of 2.6, which is joint work with Jonathan Thomas.

Summary

This thesis comprises three chapters centered on two common themes. The first theme is the application of non-cooperative game theory to economic questions; the second is the study of the kind of arrangements that can arise in the labour market as a response to asymmetric information.

The first chapter surveys recent developments in non-cooperative game theory, and then attempts an extension of the recent results characterising perfect equilibrium payoffs in repeated games without discounting to more general games. We choose the dynamic game framework for the generalisation, and shows that there are two jointly, but not individually sufficient conditions for the generalisation to go through.

We then turn to an application of these ideas to the theory of long-term contracts. The main motivation for this is that the view that wages and employment are determined by risk-sharing implicit contracts is now a well established alternative to fixed-price and market-clearing theories. In general, long-term arrangements may mitigate inefficiencies in the short-term contract that arise from various sorts of asymmetric information which are likely to be prevalent in worker-firm relationships. In this chapter two things are attempted; first, we try to integrate the game-theoretic approach to contracting of Radner with the work of Townshend, Rogerson, Roberts and Manning among others, and second, we characterise the optimal contract, and obtain some new results.

The labour market is also the topic of the third chapter. Here, we

attempt to extend a well-known model of "frictional" labour market equilibrium to the case where onr or both sides of the market differ in inherent characteristics (e.g. skills) which may be observable or unobservable. We show first that the equilibrium may be inefficient even in the absence of externalities which work through the matching technology. Also, the model with unobservable characteristics provides a framework for a theoretical analysis of the practice of firms of screening workers by unemployment duration. We show that in our model, there are screening equilibria, and also investigate in some detail the impact of exogenous variables on the equilibrium.

Introduction

This thesis is comprised of three essays grouped round two common themes. The first of these themes is the theory of repeated games; the "pure" theory is surveyed in Chapter 1, and many of the ideas discussed there are applied to the study of long-term contracts in Chapter 2. The major application of, and motivation for, analysing long-term contracts is the labour market, and the presentation and discussion of the results in Chapter 2 is in terms of risk-sharing contracts between firm and worker. The labour market is the second common theme, as the third chapter is concerned with questions of both the nature and efficiency of labour market equilibrium in a search framework where workers are heterogenous - in particular, we are concerned to explain, within a rational choice framework, the fact that worker's unemployment histories appear to affect their reemployment probabilities.

Nevertheless, the three chapters are quite self-contained, and it would be misleading and uninformative to introduce them by focussing on the central themes. Rather, we will discuss them individually, beginning with Chapter 1.

This begins with an overview of recent developments in the theory of non-cooperative games, concentrating in particular on recent results which attempt to characterise the degree of cooperation that can credibly be achieved between players in an infinite-horizon context - or, more technically, the perfect, (or sequential) equilibrium payoffs of repeated games. Many general results are now available for repeated games with complete information, and some more specific results for repeated games with asymmetric information, which suggest that the nature of the set of equilibrium payoffs depends crucially on whether the informational asymmetry is one or two sided.

By contrast, the extent to which the powerful results of the complete information case (i.e. the "folk" theorems surveyed in 1.1) generalise beyond the repeated game case is hardly known. The motivation for such an extension is self-evident, as virtually all realistic models of strategic economic interaction have intertemporal linkages. However, the question is one of method; is it better to analyse models on a case-by-case basis, or to search for general results?

In Chapter 1, we argue that if one is willing to consider the no discounting case, (which clearly imposes strong restrictions on preferences) then otherwise very general and satisfactory results can be obtained. In particular, it is shown that there are two conditions that are jointly, but not individually, sufficient for a "folk " theorem (namely, that all individually rational payoffs are perfect equilibrium payoffs) to be valid. A precise statement of these conditions reqiures some sort of structure, or model, and we choose the dynamic game framework, where intertemporal linkages are summarised by a state variable or variables. The major restriction of this framework is that it imposes additive separability of preferneces over time, but in applications, this is often not restictive. We then show that these two conditions can easily be checked in specific models, and we use Spence's model of strategic investment as an illustration. The general conclusion is that the two conditions will be satisfied in "most" economic models, and so for subgame perfection to have real force (e.g. in eliminating Pareto-efficient Nash equilibria) there must be discounting.

The second chapter brings some of these game-theoretic techniques to bear on the problem of long-term contracting with asymmetric information. Before going into details, it is worth recalling the origins of the current interest in long-term contracting.

In the mid 70's Azariadis and Baily , among others, suggested that the observed fact that firms responded to contractions in demand by laying off workers rather than lowering wages could be due to the fact that there "implicit contracts" between firm and workers (or union) which insured workers from wage fluctuations. Unfortunately, it was soon realised that two of the most realistic features of the Azariadis/Baily model (i.e. that it predicted more layoffs, or greater employment variation, than in a spot market economy, and that these layoffs were involuntary) depended crucially on the assumption that the firm could not make layoff payments to the workers.

If this assumption was dropped, one had a theory of real wage rigidity, but no theory of involuntary (or even inefficient) layoffs. One way of reintroducing these elements was to introduce asymmetric information into the risk-sharing contract, as Grossman and Hart(1981) observed. This too, had its limitations, however - for example, a very special information structure, and special conditions on preferences, are needed to generate involuntary layoffs as a feature of the optimal contract (see Moore(1985) or Arvan(1985)). More fundamentally, contract theory as a whole offers no explanation of why some workers do not get jobs - or contracts- in the first place, that is invluntary unemployment, as opposed to involuntary layoffs.

In spite of these drawbacks, implicit contract theory has had some recent sucesses - in particular, Farmer((1985), (1985a)) has argued convincingly that the inefficiencies in contracts with asymmetric information may provide a transmission mechanism for monetary policy to affect the real economy. To summarise then, the implicit contracts approach has established itself as an alternative paradigm to the spot-market over the last ten years, and this provides quite a motivation to investigate the nature of long-term labour contracts.

But our actual concerns in Chapter 2 are wider than this; our formal model includes those contracting problems where the rationale is not risk-bearing, but whose interest rather derives from the tension between the agent having private information and the principal wishing to extract all the surplus from him i.e. contracting problems with ex post participation constraints. In this case, allocative inefficiency can still arise, even with risk-neutrality.

The main interest of studying contracting with asymmetric information in a dynamic context is, of course, that one would expect that in a repeated setting, one could construct the contract so that the inefficiencies of the one-shot contract would be mitigated. This observation is very close to the idea that there are Pareto-superior equilibria in the repeated game to the Nash equilibrium of the one-shot game.

There have been two very diverse approaches to investigating the degree and nature of the improvement that long-term contracting can bring. The first is predicated on the assumption that the principal can precommit to particular contracts, and consists in showing that introducing history-dependence into the long-term contract makes the contract more efficient than a series of one-shot contracts.

There is not yet agreement about what aspects of the inefficiency history-dependence improves; the models presented differ, and the whole issue is greatly complicated by the fact that the long-term contract is also a savings contract for the agent, as Townshend(1982) observed, and this in itself introduces history-dependence. However, there seem to be two main answers; Townshend has shown that in a "pure" risk-sharing contract it improves first-period risk-sharing, and Roberts(1982) presents an argument which suggests that it allows the principal improved control over the agent's actions at zero cost.

A very different approach to the problem is taken by Radner(1981), (1985)) who observed that without precommitment by the principal, the repeated contracting problem could be considered as a repeated game, and therefore the question of improvement on the one-shot contract is formally equivalent to being able to find an equilibrium of the repeated contracting game which dominates the Nash equilibrium of the one-shot contracting game. In his 1985 paper, Radner was able to construct such an equilibrium, but also noted that it was not efficient i.e. there were other equilibria that Pareto-dominated it.

Three things emerge from this discussion. First, the conditions under which history-dependence occurs and the reasons for it are not yet well-understood. Second, Radner's game-theoretic approach is not well-integrated with the precommitment approach, where it is possible, in principle, to write down "the" contracting problem and solve it. Third, with the exception of Rogerson(1985) and Townshend(1982), nobody has actually attempted to do the latter, and as argued below, many questions concerning the structure of the optimal contract remain unanswered.

In Chapter 2, all three of these outstanding problems are tackled in an integrated way. The introduction to that chapter outlines in much greater detail what is actually invoved and what is achieved, but we mention two of the major findings. First, Robert's marginal improvement argument is shown to apply to only a special class of contracting problems,and the reasons for this are discussed in detail. Second, we are able to show that a general feature of the optimal long-term contract is that the agent's marginal utility of income is positively serially correlated, and this yields more operational predictions e.g. positively serially correlated wages in more specific contexts.

As already remarked, the model analysed in Chapter 2 has provides the theoretical basis for a particular view of the labour market. The last chapter, Chapter 3, is also concerned with the labour market, but from a search-theoretic viewpoint. We make little use there of the game-theoretic techniques developed in Chapter 1, apart from the notion of Nash equilibrium. The purpose of the chapter, at a general level, is to explore the consequences of the fact that workers (or firms) are heterogenous i.e. differ in productivity for equilibrium in a search model of the labour market.

In order to fully explain the motivation for this, it would help to give a brief review of the objectives and methods of the literature on "frictional" labour markets (a more detailed discussion is in 3.1 and 3.2 below).

It is a commonplace observation that a certain amount of unemployment is frictional, and this component of unemployment took on a greatly added significance following the introduction of the idea of the "natural" rate of unemployment by Friedman(1968) which has since been closely associated with firctional unemployment. Not surprisingly, this gave an impetus to the development of rigorous models of equilibrium search in the labour market, such as that of

Lucas and Rapping(1974). However, for reasons more fully discussed in 3.1 below, the most satisfactory search model, (and the one that has achieved prominence in the literature) is the one developed by Mortensen, Diamond, and Pissarides; this dispenses entirely with the idea of markets, trades taking place instead between firms and workers who are stochastically matched.

One interesting feature of this model is that it exhibits a rich array of inefficiencies of the type stressed by Tobin(1972), where the decisions by individual agents at several margins of decision-making (entry into the market, match-formation, search intensity) have external impacts on the matching probabilities of the other agents in the market. These "congestion" externalities have been extensively investigated by Diamond and Pissarides, among others, and the details are surveyed in section 3.2.

The first objective of Chapter 3 is to show that if workers are assumed to be heterogenous, a possibility that has not been investigated in the literature, then search equilibrium will be inefficient, even in the <u>absence</u> of congestion externalities. Furthermore, the direction of this inefficiency is unambigious - output and employment are too low in equilibrium.

The second objective of Chapter 3 is to use the matching model to investigate the idea that elements of a worker's employment or unemployment history can convey information about (unobservable) differences in worker attributes. This is motivated by the widespread belief that firms "screen" workers on the basis of e.g the length of the last unemployment spell. There is certainly some direct evidence that such practices go on, and this is also consistent with much of what is known about re-employment probabilities (see 3.3 for a full discussion).

What we try to do is build a model where such screening can be the outcome of fully rational behaviour on the part of firms. Our main focus of interest is how the level of screening is determined, and how it is affected by changes in exogenous variables. Among the more interesting results is that the level of screening falls with an aggregate demand shock. As the unemployment rises with the level of screening, screening provides a mechanism by which demand or productivity shocks can impinge upon employment, in addition to the more usual channel of a lowering of the marginal productivity at which a match is formed. Also, we find that an increase in the supply of workers increases the level of screening, and an increase in the level of vacancies decreases it. This is not the only issue discussed - there are also questions of multiple equilibria, and of reconciling the predictions of the model with the evidence, and these are all discussed at some length in Chapter 3.

As already emphasised, the differences between the chapters in subject matter, techniques, and motivation are at least as great as the similarities, especially between Chapter 3 and the rest. For this reason, we do not number equations and theorems consecutively throughout the thesis, but start the numbering afresh at the beginning of every chapter. For the same reason, there is no overall conclusion to mirror the introduction - instead, there are short conclusions to each chapter. However, the chapters have a common bibliography, and this, along with Appendices and footnotes, is to be found at the end of the thesis.

1. Perfect Equilibrium in Dynamic Games

Introduction

In this chapter we do two things. First, we provide a survey and overview of recent developments in the theory of repeated games, both with complete and incomplete information; in chapter 2 we will draw some of the results surveyed. Second, we attempt to analyse the extent to which the Aumann-Shapley -Rubenstein characterisation of perfect equilibrium in repeated games without discounting extends to more general settings where structural dependence between periods is allowed for, but retaining the no discounting assumption. We are able to derive two conditions that are individually necessary and jointly sufficient for Nash equilibrium paths to be perfect in a very wide class of games viz. all those that can be put in state-space form.

This is clearly interesting in its own right as the repeated game framework is extremely restrictive. Also, it gives some indication of the problems likely to be encountered in attempting to extend Abreu's(1982) characterisation of perfect equilibrium paths in repeated games with discounting to more general settings.

1.1 A Survey of Recent Developments in the Theory of Repeated Games

A repeated game with complete information is a constituent game in normal form played repeatedly, possibly an infinite number of times. Within this framework it is possible to have different information structures, in that the players may only be able to observe certain elements of the past history of play e.g. only last period's moves. What is essential is that all players should possess the same information at each date, even though this information may be limited. Games where in the constituent game some player can only observe the actions of another up to a random errror term, such as oligopoly with demand uncertainty (such as Green and Porter(1984)), or Radner's

(1985a) partnership games, do not fall into this category. Such repeated games are technically games with incomplete information. We make the distinction equilibria as the two sorts of repeated games have rather different properties.

Nash equilibria in repeated games are easy to characterise - moreover, it is well-known that these equilibria are extremely indeterminate. Much of the recent work in the area has centered round the attempt to characterise perfect Nash equilibria in such games. It is also well- known that the imposition of subgame perfection cannot completely resolve Nash indeterminacy (in contrast to what happens in other kinds of infinite-horizon games - e.g Rubenstein(1982)), so this is not a major motive. Rather, the motive is that at first sight, the infinite horizon makes it extremely difficult to characterise credible Nash strategies; as Abreu(1982) argues, one faces an infinite regress problem.

The picture is now quite well-developed for the complete information case, and we survey the most important of these results in what follows.

(a) A Formal Definition of a Repeated Game

The constituent game of a repeated game is a game in normal form, played T times (where possibly $T = \infty$) by n players, i = =1,2...n. Each player chooses an action, a_{it} at date t from a set A_i. This choice may be conditioned on the past history of play, $h_t = (a_1, a_2 \ldots\ldots a_{t-1})$, where $a_t = (a_{1t}, a_{2t}, \ldots a_{nt})$. All the results surveyed below assume that this choice can depend in an unrestricted way on h_t. Let the mapping from h_t into A_i be denoted f_{it} and its infinite Cartesian product over dates be f_i. Then f_i is a strategy for i in the repeated game.

Next, let A be the Cartesian product of the A_i, and A^T be the T-fold product of A, and let $a = (a_1, a_2 \ldots\ldots a_T)$ be a typical element of A^T. Then a is an outcome path of the repeated game. Clearly, any n-tuple of strategies $f = (f_1, f_2 \ldots f_n)$ will determine a particular outcome path.

Finally, payoffs over outcome paths for each player are given by $\sum_{t=1}^{T} \delta^t . u_i(a_t)$, with $0 < \delta \leq 1$. If T is infinite, the case of no discounting (i.e. $\delta = 1$) these payoffs are not well-defined, and we replace them with $\limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} u_i(a_t)$. If the limit exists in the latter expression, the discounted payoffs converge to the limsup as $\delta \to 1$.

Given the mapping from strategies to outcomes, there is clearly an induced preference relation over strategies for each player (see e.g.Rubenstein (1979) for details.) Then a Nash equilibrium in the repeated game is defined in the normal way viz. as an n-tuple of strategies that are mutual best responses. Subgame perfection requires that these strategies also be mutual best replies at each date conditional on every possible history of actions having occurred , <u>not</u> just the history generated by the equilibrium strategies.

This quite formal development of the repeated game model is necessary to a detailed exposition of some recent work, particularly Abreu's, where the distinction between strategies and outcome paths is crucial.

(b) Characterising Equilibria in Repeated Games

Our starting point is the observation that infinitely repeated games[1] admit a large number of Nash equilibrium outcomes. Furthermore, the set of equilibrium outcomes can easily be described by considering Nash equilibria of a special sort, those involving so-called trigger strategies. These strategies prescribe that player i play a_i^* unless and until precisely one player (possibly himself) deviates and plays $a_j' \neq a_j^*$. If $j \neq i$, then i switches to an action that punishes j, a_i^j and plays this for ever; if on the other hand, it is himself that has deviated, he switches from then on to playing his best reply to the punishment a_{-i}^i which he expects to ensue.

We might as well consider punishment actions that punish players most severely, i.e. punishments, that yield players only their security levels v_i in the constituent game, where v_i is defined as

$$(1) \qquad v_i = \min_{a_{-i}} \max_{a_i} u_i(a_i, a_{-i})$$

Let any vector of actions that achieves v_i be denoted $m^i = (m_1^i, m_2^i \dots m_n^i)$.

Now suppose that all players $j \neq i$ announce that they will play trigger strategies. What is the best that i can do in reply? If i is considering deviating, the best he can get with discounting is a payoff of $(1 - \delta) \max_{a_i \varepsilon A_i} u_i(a_{-i}^*, a_i) + \delta . v_i$,

because once he has deviated from a^* once, he can get at most v_i from then on. Hence a^* is the equilibrium outcome of a Nash equilibrium (in trigger strategies) if and only if a^* satisfies

$$(2) \quad u_i(a^*) \geq (1-\delta).\ u_i^*(a_{-i},\ a_i') + \delta.v_i,\ \text{all } a_i' \in A_i,\ \text{all } i.$$

Note in particular that as $\delta \to 1$, we find that any a^* that yields each player more than his security level v_i (i.e. is strictly individually rational for that player) is a Nash equilibrium outcome. This is one version of a result known as the "folk" theorem for repeated games. Without discounting (i.e. with long-run averaging) we get the essentially identical result that any weakly individually rational outcome is an equilibrium outcome.

We illustrate all this with an example from Fudenberg and Maskin (1983)) of a matrix constituent game;

	U	D
U	1,1	0,-2
D	-2,0	-1,-1

In this game, player i minimaxes player j by playing D with probability 1, and j's best response is to play U with probability 1, so $v_j = 0$ forj = 1,2. Then without discounting, any mixed strategies $(a^*_1,\ a^*_2)$ (where a_i denotes the probability that i plays U) which yield each player at least 0 can be sustained as an equilibrium outcome, using strategies where any defection by i is punished by j switching to D for ever.

Is such an equilibrium perfect, however? The answer is that for almost all games[2] it is not, the reason being that in the event that some player i deviates (a subgame never reached in Nash equilibrium, of course) the players who must then minimax i for ever, will in general not wish to do so. For example, in our matrix game 1 minimaxes 2 by playing D, but given that 2 responds by playing U, 1 can do better by switching to U himself. To put it another way, 1's threat to minimax 2 is not credible.

In a series of papers, Aumann,Shapley , and Rubenstein tackled this problem for the case without discounting. Their insight was to point out that in order to make trigger strategies credible (i.e. perfect) players must _also_ be punished for refusing to cooperate in the punishment of another player. Moreover, they showed exactly how this could be done.

A natural first attempt at modifying the trigger strategies would be to declare that players who fail to carry out minimax punishments should themselves be minimaxed for ever. However, this will not work in our example, as punishing one's opponent for ever is extremely costly, yielding a payoff of -2, so that each player would rather _be punished_ for ever than punish for ever.

Rather, the key to constructing credible punishment strategies was to make punishment periods _finite_ (play always returning to the original a^* at the end) and depend on the _number_ of previous deviations by any particular player from whatever is prescribed by his equilibrium strategy. Suppose that player i is punished for τ_n periods after his _nth_ deviation. Then, as punishment periods are finite, with limit averaging it costs nothing for a player to carry out a punishment. Similarly, no player i can gain or lose by a finite number of deviations from playing a_i^* the original cooperative agreement. Hence, to establish that any individually rational a^* can be sustained as a perfect equilibrium it suffices to show that no player i can do strictly better than $u_i(a^*)$ by deviating an infinite number of times.Now, the average gain from any sequence of deviations over T periods is bounded above by

$$(3) \qquad \frac{n(T).b_i}{T} + \frac{T - n(T)}{T}.(v_i - u_i(a^*))$$

where b_i is an upper bound on $u_i(a)$, and n(T) is the largest number of complete periods of punishment that can be fitted into T periods (i.e. n(T) is the largest integer n such that $\sum_{i=1}^{n} \tau_i < T$).

It is clear that as long as the τ_n increase fast enough with n so that $n(T)/T \to 0$ as $T \to \infty$, the limit average gain from deviation as $T \to \infty$ will be $v_i - (a_i^*)$ which is non-positive as long as a_i^* is individually rational. This can be achieved, for example, by making τ_n a geometric series i.e. $\tau_n = a^n$, $a > 1$.

We have therefore proved;

<u>Proposition (Aumann-Shapley-Rubenstein)</u>

Without discounting, any individually rational payoff of the constituent game is the payoff of some perfect equilibrium of the repeated game.

The validity of this result depends both on no discounting of the future and the repeated game structure. Discounting raises subtle and important problems. First, with discounting, the device of geometrically increasing punishments fails, as arbitrarily long punishments are not arbitrarily severe, because far-off punishments are relatively unimportant, so one would expect the imposition of perfection to have some real force.

In fact, we require, roughly speaking, any subgame-perfect equilibrium outcome a^* to satisfy (2) where v_i is replaced by w_i, the worst payoff that all players can credibly impose on i (and we expect that $w_i > v_i$). However, this raises a thorny problem; unlike the Nash case, where v_i is exogenously defined, w_i is endogenous. In fact, w_i must be the "worst" payoff for i associated with a perfect equilibrium (if such a worst payoff exists).

The reason for this is as follows. Let A^P be the set of outcome paths associated with perfect equilibria of the repeated game, and let f^* be any perfect equilibrium whose outcome path is a^*. Then w_i is simply the payoff to i from the continuation of f^* from t + 1 onwards contingent upon some (otherwise arbitrary) history where i deviates from a^* at t. By the perfectness of f^*, this continuation must also be a perfect equilibrium and w_i must therefore satisfy only

(4) $$w_i > \inf_{a \in A^P} \sum_{t=0}^{\infty} \delta^t . u_i(a_t)$$

Furthermore, it seems impossible to break into this circular definition of the w_i. For example, we do not know whether A^P is a closed set - i.e. whether worst credible punishments exist. Even if they do, we have no constructive means of finding them.

Abreu (1982) showed how this circularity could be overcome by restricting attention to what he calls simple strategy n-tuples. These are defined as follows. Let a^i be an outcome path. Then a simple strategy n-tuple $f(a^*; a^1, \dots a^n)$ prescribes that each player i play a^*_{it} at each date until one player (say i) deviates at τ in which case all players start playing $a^i = (a^i_0, a^i_1 \dots)$ at $\tau + 1$, $\tau + 2$ and so on. If $j \neq i$ deviates from this punishment, then a^j is started from the beginning, and most importantly, if i deviates from his own punishment, a^i is started over again from the beginning.[3] Note that unlike the trigger punishments, a^i_{it} need not be a best reply to $a^i_{-i,t}$ or in other words, the possibility that players cooperate in their own punishments is allowed for. Note also the dimensional simplification - a simple strategy is entirely characterised by n + 1 outcome paths, $(a^*, a^1 \ldots\ldots a^n)$.

The key property of a simple strategy n-tuple is that is is perfect if and only if one-shot deviations from it, contingent upon all histories, are not profitable for any player. The conditions which ensure this are, assuming a^* is stationary (i.e. $a^*_t = a^*$, all t);

(5) $$u_i(a^*) \geq (1 - \delta)\, u_i(a^*_{-i}, a'_i) + \delta \sum_{t=0}^{\infty} \delta^t u_i(a^i_t)$$

$$\text{all } a'_i \in A_i, \text{ all } i \; ;$$

(6) $$\sum_{t=\tau}^{\infty} \delta^{t-\tau} . u_i(a^j_t) \geq (1 - \delta)\, u_i(a^j_{-i,\tau}\; a'_i) + \delta \sum_{t=0}^{\infty} \delta^t . u_i(a^i_t)$$

$$\text{all } a'_i \in A_i, \text{ all } i, j, \text{ all } \tau.$$

(5) ensures that deviations from the original outcome path are not profitable for my player, whereas (6) ensures that deviations from any ongoing punishment $(a^j_\tau, a^j_{\tau+1} \ldots)$ are not profitable for any player.

The second fundamental property of a simple strategy n-tuple is that $f(a^*; a^1, \ldots . a^n)$ is perfect if and only if the "punishment" strategy n-tuple $f(a^i: a^1, \ldots . a^n)$ is also perfect, i=1,2...n. To see this, note that (6) is precisely the condition for $f(a^i; a^1 \ldots . a^n)$ to be perfect, all i as (5) is implied by (6) if $a^* = a^i$.(Abreu calls the n-tuple of "punishment" simple strategy n-tuples $\{f(a^i, a^1, \ldots a^n)\}_{i=1,\ldots n}$ a <u>simple penal code</u>.) Therefore, (6) is satisfied if and only if a^i is a perfect outcome path, all i. This fact enables us to prove that there exist "worst" punishments, or equivalently, that A^P is closed.

Let $(a_v^1 \ldots\ldots a_v^n)_{v=1}^{\infty}$ be a convergent sequence of n-tuples of outcome paths such that $a_v^i \in A^P$ all i, all v and such that the payoff from a_v^i converges[4] to $\inf\limits_{a \in A^P} \sum_{t=0}^{\infty} \delta^t . u_i(a_t)$. We know that (6) is satisfied for all v as the a_v^i are perfect equilibrium outcomes, so (6) is also satisfied in the limit as $v \rightarrow \infty$ by the continuity of payoffs. Let the limit of a_v^i be $\underline{a}^i$; then the strategy n-tuple $f(\underline{a}^i; \underline{a}^1, \ldots\ldots \underline{a}^n)$ is also perfect all i, and yields i the "worst" credible punishment- i.e. the infimum on the RHS of equation (4) above. Abreu calls $\{ f(\underline{a}^i; \underline{a}^1 \ldots\ldots \underline{a}^n)\}_{i=1..n}$ an <u>optimal</u> simple penal code.

However, what is most interesting about (6) is that it provides a set of necessary conditions that optimal penal codes must satisfy. First, it is apparent from (6) that (as already remarked) optimal penal codes may not have the "best reply" property, but players can <u>credibly</u> be persuaded to cooperate in their own punishments only if punishments are more severe at the beginning of the sequence - they must have something to lose by going back to the beginning.

Second, subject to this "stick and carrot" requirement, optimal penal codes may have a very simple form, as Abreu himself has shown for an oligopoly game. In fact, Fudenberg and Maskin (1983) have constructed a class of "simple" simple penal codes which are "approximately" optimal as the discount factor tends to unity in the sense that the payoff to i from a^i tends to his security level v_i as δ tends to 1. This enables them to prove a "folk" theorem with discounting[13], which states that (subject to a regularity condition in the n-person case) for any strictly individually rational payoff vector, there exists a δ^* such that this vector is the payoff of a perfect equilibrium for all $\delta^* < \delta < 1$. (see Fudenberg and Maskin (1983) Theorems 1 and 2).

The simple penal code involved is particularly "simple" with two players, and the algebra is simplified if we consider stationary oucome paths a^* - i.e. $a^*_t = a^*$, all t. Then choose a^* such that $u_i(a^*) > v_i$. Then the F/M simple penal code prescribes that in the event of any deviation <u>both</u> players should play the strategy m^j_i that minimaxes the other for k periods, and then revert to a^*, so that

$$(7) \qquad a^i = (\underbrace{m, m \ldots m}_{k \text{ times}};\ a^*, a^* \ldots\ldots) \text{ , with } m = (m^2_1 , m^1_2).$$

To prove the F/M "folk" theorem, (for the 2-person case and stationary outcome paths) we simply show that for any such a^*, there exists an integer k^* and number δ^* such that (5) and (6) are satisfied given (7) for all $1 > \delta > \delta^*$.

For convenience, normalise payoffs so that $v_i = 0$, i = 1,2. Then it is easily verified that (5) and (6) become

$$(8) \quad u_i(a^*) > (1 - \delta)\, u_i\, (a^*_{-i}, a'_i) + \delta . w_i \text{ , all } a'_i \in A_i, \text{ all } i,$$

and

$$(9) \qquad w_i \equiv (1 - \delta^{k^*+1}).u_i(m) + \delta^{k^*+1}.u_i(a^*) > 0, \text{ all } i.$$

As $u_i(a^*) > 0 \geq u_i(m)$, a k^* and δ^* satisfying (8) and (9) can certainly be found.

Next, we turn to the question of indeterminacy in finitely repeated games. First, it is well-known that if the constituent game has a unique Nash equilibrium, the only perfect equilibrium of the constituent game is repeatedly played. The argument is a simple backward induction one; in the final period, the only possible outcome in Nash equilibrium. Knowing this, no player can be induced to play anything other than his Nash equilibrium strategy in the penultimate period, and so on.

With multiple equilibria, this no longer holds true. Players can be threatened by the imposition of "bad" Nash equilibria in later periods to play strategies in earlier periods that are not mutual best replies - in other words, there may be some perfect equilibrium outcomes of the repeated game that are not entirely composed of Nash equilibria of the constituent game. This point has been made recently and independently by Benoit and Krishna (1985), Friedman (1985), and Moreaux (1985).

For example, consider the following matrix constituent game, due to Benoit and Krishna.

	B	C	D
B	5,3	0,0	2,0
C	0,0	2,2	0,0
D	0,0	0,0	0,0

This has <u>two</u> Nash equilibria - a "good" one (B,B) which Pareto-dominates the "bad" one, (C,C). Suppose now the constituent game is played twice, and players do not discount. Then the trigger strategies which prescribe that both players play D in period 0 and B in period 1, unless precisely one player deviates in period 0, in which case C is played by both players, are in perfect equilibrium even though (D,D) is <u>not</u> a Nash equilibrium of the constituent game.

Benoit and Krishna have exploited the potential of this simple idea remarkably fully. Under the assumptions of no discounting and that there are "enough" equilibria in the constituent game so that each player can be punished by switching from one equilibrium to another, they show that given any game repeated T times with associated "worst" perfect equilibrium payoffs, w_i^T , for each player , one can find a longer game T^*, with strictly lower "worst" perfect equilibrium payoffs for each player. Hence, in the limit, w_i^T tend to v_i for each i, although, surprisingly, they may not do so monotonically (see Benoit and Krishna, p.913). It follows immediately from this that under the stated assumptions, any strictly individually rational payoff can be sustained as the perfect equilibrium payoff of a game repeated often enough.

The argument is an ingenious one, and involves constructing three-stage punishments for each player i, <u>using</u> the w_i^T. The outcome path of the three- stage punishment involves a first phase where all other players minimax player i, a second reward phase where $j \neq i$ are rewarded for doing this (which is essential if the original T-period punishment w_i^T is preferred by any j to minimaxing i) and a third phase where play cycles through a finite sequence of constituent game Nash equilibria. Deviations by any player from the first two phases of this outcome path (say at time t) are punished by as many repetitions of the original T-period punishment as can be fitted into the remaining $T^{*} - t$ periods, "topped up" with playing j's "worst" constituent game Nash equilibrium in the remaining $T^{*} - t - T + 1$ periods at the end of the game. By constuction, these punishments are themselves credible - i.e. perfect equilibrium outcomes. Finally, by appropriate choice of the lengths of the various phases, the new punishments can be made more severe than the old.

Note that subject to the constraint imposed by the finite horizon, these punishment strategy n-tuples are simple in Abreu's sense. Therefore, one can view Benoit and Krishna's results as complementary to the work of Abreu and Fudenberg and Maskin - they exhibit simple penal codes which are "approximately" optimal in finite horizon games, if the horizon is long enough. Apart from these limiting results, however, the central question raised by Abreu's work - the characterisation of optimal penal codes - remains largely unanswered, and it is probably unreasonable to expect any general results, although a recent paper by Abreu, Pearce, and Stachetti does suggest a general method for finding them.

(c) Repeated Games with Incomplete Information

We have already mentioned repeated games with imperfect monitoring of actions. No general characterisation of "perfect" equilibria in such games is available, and it is probably fruitless to look for one. In fact, the results so far, especially Radner's work, suggest that the behavoiur of the equilibrium set depend on the information structure of the game ((Radner(1981), (1985), Radner, Myerson, and Maskin(1984)). In the first two papers, Radner considers a repeated principal agent relationship modelled as a game. The informational asymmetry is therefore one-sided; the agent can observe the contract that the principal offers (a contract offer is the principal's srtategy in the constituent game) but the principal cannot observe the agent's effort. Radner showed that without discounting, all fully Pareto-efficient payoffs that both players prefer to the one-shot equilibrium are perfect (or more precisely, sequential in the Kreps-Wilson sense) equilibrium payoffs in the repeated game. As stated, it is a weaker result than the A-S/R theorem, but it is in a sense much more striking, in that it asserts that the principal can completely overcome his informational disadvantage by imperfectly monitoring the agent's behaviour over long periods. Radner (1985) also established the "continuity" result that payoffs arbitrarily close to these same efficient payoffs can be attained as equilibrium payoffs if the discount rate is low enough.

By contrast, Radner et al. show that in an otherwise rather similar setting with two-sided imperfect monitoring, a "partnership" game where neither partner can observe the other's effort, there is a discontinuity in the equilibrium set of payoffs as the discount rate goes to zero. It is not yet clear that the one-sided / two-sided distinction is the crucial one for these results, but certainly the continuity result seems to generalise to other contracting problems. For example, we show in chapter 2 below that this continuity result goes through for principal-agent contracts with hidden information, rather than hidden actions.

1.2 The Folk Theorem for Dynamic Games without Discounting

The repeated game model is a very special one, and does not adequately capture many features of the economic environment where strategic interaction typically takes place. For example, in oligopolised industries there are structural linkages between periods arising from durable capital, inventories, intertemporal substitution on demand, and the like. Again, in macroeconomic applications, such as policy games there are a number of stock-flow conditions such as balance-of-payments equations, or intertemporal arbitrage conditions, whch impose linkages between periods.

It is therefore of interest to ask whether there is a simple characterisation of perfect equilibria in a more general class of infinite horizon games, and this is the objective of this chapter. We show that general results can be obtained for the case of no discounting, within the framework of dynamic games. (Basar and Oldser is the definitive survey of the literature on such games, and we give a formal definition below in 1.3.) While not all extensive-form games can be put in dynamic form, they cover an extremely wide class of games, essentially because the the state-space can be chosen arbitrarily. The main restriction of the dynamic game framework is that payoffs must be additively separable over time.

What we do is to present two conditions which are jointly sufficient for all indvidually rational payoffs to be perfect equilibrium payoffs in a very wide class of dynamic games, and also show by means of counterexamples, that neither of these two conditions alone is sufficient. In fact, neither of these two conditions alone is even sufficient to ensure that all Nash payoffs are perfect. These results can be taken as showing the extent to which the Aumann-Shapley-Rubenstein result generalises. In the course of this analysis, we also show that all individually rational payoffs are also Nash equilibrium payoffs if the first condition holds, but need not be otherwise. This is a generalisation of the original "folk" theorem for Nash equilibrium in repeated games.

These conditions are; first, that each player's security level - i.e. the worst payoff that can be imposed upon him over the infinite horizon by the other players - is independent of the initial state of the game, and second, that any individually rational, but otherwise arbitrary outcome path must also possess this independence property.

That these conditions are jointly sufficient is not entirely surprising, because if the long-run payoffs are independent of the initial state then timing and commitment are unimportant, and therefore the fact that perfect equilibria must involve credible punishments also becomes unimportant. The fact that neither of these conditions is individually sufficient is somewhat more surprising. The reason is that if we allow either payoffs from sequences of actions, or security levels, to be state-dependent it is possible to construct examples of games where players are able, by taking certain actions, to move the game to particular absorbing states which are especially advantageous for them. They may not be able to credibly threaten not to make such moves, even though such threats may be necessary (e.g. as parts of punishments) to sustain certain paths as Nash equilibrium paths. But then these Nash paths cannot be perfect.

To get these results, we need make two structural assumptions. The first is that the state-space is finite. The second is that the security levels are continuous at infinity i.e. the worst payoff all the other players can impose on any given player over a finite horizon converges to the worst payoff over the infinite horizon as the horizon length goes to infinity. It is not clear whether these conditions are necessary for the result, but it seems unlikely - as the counter-examples presented below make clear, without discounting, perfection seems only to have force in games where payoffs are not independent of inital states. (In addition, it seems likely that the "continuity at infinity" property follows from the finiteness of the state-space, but I have not been able to prove this[5].)

The layout of the rest of the chapter is as follows. In 1.3 we present the general dynamic game model and define security levels. In Section 1.4 we discuss the relationship between Nash equilibrium and individual rationality. In Section 1.5, we do the same for perfect equilibrium; it is here where we present the two counter-examples and the sufficiency theorem. In 1.6, we give an alternative statement of the state-independence condition on outcome paths, in terms of closed-loop strategies, or <u>rules</u>. Apart from its' general interset, this anticipates 1.7, where a version of Spence's (1979) investment model is analysed, as an application of the sufficiency theorem, and we contrast our findings with Fudenberg and Tirole's analysis of the same model. We show that state-independece of paths holds only if the rate of capital depreciation is positive. We conclude by briefly discussing the case with discounting.

1.3 Dynamic Games

(a) Notation and Equilibrium Concepts

In this section we give a brief description of the quite general class of dynamic games for which we establish results. For a thorough discussion of such games, see Basar and Oldser (1982).

We suppose n players and an infinite sequence of dates, $t = 0,1 \ldots$ At each date each player $i \in N$ chooses an action a_{it} lying in a compact subset A_i of some Euclidean space. The state at any time, x_t, lies in a finite subset X of another Euclidean space. Also, the state at $t + 1$ is entirely determined by x_t and $a_t = (a_{it} \ldots a_{nt})$ according to the state equation;

$$x_{t+1} = g(a_t, x_t)$$

where $g : X \times A \to X$ with $A = \underset{i \in n}{X} A_i$, and the initial state, x_o, is predetermined. Finally each $i \in N$ has a continuous per period payoff u_i defined on $X \times A$. Note that the repeated game is a special case of a dynamic game with a degenerate state space.

Next, define a history of length t, $h_t = (x_0, a_0 \ldots a_{t-1})$ as a complete description of the initial condition and actions taken up to and including time $t - 1$. It is assumed that this is the information available to each player ay time t. Therefore, a strategy for each player, f_i, is a sequence of maps, f_{it}, $t = 0,1\ldots$ from the space of all possible histories into actions, A_i.

We can now define a map, σ, from n-tuples of strategies, $f = (f_1 \ldots f_n)$ and an initial condition, x_0, into infinite sequences of actions, or outcome paths $a = (a_0, a_1 \ldots)$ in A^∞. If $a = \sigma(x_0, f)$ then the elements of a are defined inductively by

$$a_{ti} = f_t^i\,(x_0,\ a_0\ \ldots.\ a_{t-1})\ \text{,all } t,\ \text{all } i\epsilon N.$$

We are now in a position to define payoffs over outcome paths and strategies. First, define for each player a T-period sum of per period payoffs defined on A^∞ x X as follows;

$$U_i^T\,(a,\ x_0) = \sum_{k=0}^{T} u_i(a_k,\ x_k)$$

with $x_k = g(a_{k-1},\ x_{k-1})$ all $T > k > 1$.

Any initial condition/outcome pair $(a\ ,\ x_0)$, is now evaluated by i according to the long-run average payoff associated with it i.e. $\limsup_{T \to \infty} \frac{1}{T}\ U_i^T\,(a,\ x_o)$.

This is not the only concievable evaluation criterion which does not discount future payoffs- for example, the overtaking criterion also has this property (see e.g. Rubenstein(1979) . However, the "folk" thorem and counterexamples hold in an almost unchanged form for this latter criterion so the choice of the long-run average is not restrictive.

Strategy n-tuples f are evaluated by i according to

$$\limsup_{T \to \infty} \frac{1}{T}\ U_i^T\,(\sigma(f,\ x_0),\ x_0).$$

We are now in a position to define perfect equilibrium for the game. The strategies f_i are in perfect equilibrium if, conditional on <u>any</u> history h_t, the f_i are in Nash equilibrium. A formalisation of this in the repeated game case can be found in Rubenstein (1979), and it is easily adaptable to the dynamic game case. As we do not make use of the formalisation directly in what follows, we leave the details of the adaption to the reader.

It is worth remarking, however, that as presented the game described above is not necessarily one of perfect information, as players move simultaneously at each date (i.e. choose a_{it} in ignorance of a_{-it}). Therefore, perfect equilibrium in a finite-horizon version of the game (i.e. $T < \infty$) may not exist for the usual reasons that give rise to non-existence of Nash equilibrium in simultaneous-move games - discontinous payoffs, and so on. Furthermore, if the equilibrium exists and is unique, the equilibrium strategies must have the "closed-loop" property that the f_{it} only depend on h_t through the current state, x_t. Thus, except in very special circumstances (e.g. when the game is a repeated one) open-loop strategies cannot be prefect equilibrium strategies. Neither can more complex strategies, which depend on h_t in a richer way. Without uniqueness, of course, the latter is no longer true; multiple perfect equilibria in the finite-horizon game can be used to construct other perfect equilibria, as Benoit and Krishna (1985) have shown.

One other implication of the apparent simultaneity of moves is that it does not seem to easily accommodate dominant-player games, i.e. games where the dominant player moves first in each time period. (A common example is interaction between government and private agents described, for example, in Kydland and Prescott(1977), where the government is assumed to be able to move first in any time-period, not necessarily in a literal sense, but possibly by precommitment. See also Sargent(1981) and Gale(1982)). This can in fact be modelled in our framework, by means of either putting the within-period game in normal form, or by altering the information structure. The former works by redefining the action space of the follower to be the set of responses to the leader's action, or formally the set of all maps from A_l to A_f, where A_l is the leader's action set, and A_f the follower's. If both these sets are finite, then the set of all maps will also be finite, and there is no problem. If this is not the case, then technical problems could arise e.g. it may be difficult to guarantee the compactness of the follower's strategy set. The other alternative is to alter the information structure, so that the leader chooses at each t an action a_{tl} on the basis of h_t only, whereas the follower chooses an action on the basis of information (h_t, a_{tl}).

(b) Individual Rationality

In this section, we give an exact definition of the players' security levels in the game described above, and armed with these, define individually rational outcome paths. First, the worst average payoff over any finite number of periods, $T < \infty$ that the coalition of all other players can impose on player i is easily defined. It is simply i's minimax payoff evaluated over a T-period horizon, and will in general, be a function of x_0;

$$(10) \quad V_i^T (x_0) = \inf_{f^{-i}} \sup_{f^i} \left[\frac{1}{T} U_i^T (\sigma(f^i, f^{-i}, x_0), x_0) \right] .$$

Over the infinite horizon, the security level for i, $V_i^{\infty}(x_0)$, is defined as in (10) with the limsup operator before 1/T. It is now posible to state precisely the "continuity at infinity" assumption that we referred to in section 1.2. This is the condition that $\limsup_{T\to\infty} V_i^T(x_0) = V_i^{\infty}(x_0)$, and we assume this in what follows.

While the definition of the security levels is clear enough, they may be very difficult to compute from (10). It is possible to show, however, that if the u_i are differentiable, then the security levels over T-t periods, $V_i^{T-t}(x_t)$ must satisfy the Bellman-type equation

$$V_i (x_t)^{T-t} = \min_{a_{-i}\epsilon A_{-i}} \max_{a_i \epsilon A_i} \{ \frac{1}{T-t} . [u_i(a_i, a_{-i}, x_t) + (T-t-1) . V_i^{T-t-1}(x_{t+1})]\}$$

$$(11) \qquad \text{s.t.} \quad x_{t+1} = g(a_i, a_{-i}, x_t)$$

with terminal condition $V_i^0 (x_T) \equiv 0$. (The min and max operators are valid because of the continuity and compactness assumptions made).

Therefore the minimax payoff to i over a T-period horizon is given by the initial valuation function in (11) i.e. with t set equal to 0, and the security level can be found by takng the limit as $T \to \infty$. In particular examples, it should be therefore be quite easy to compute security levels, at least numerically.

We now turn to individual rationality. The natural definition of an individually rational outcome path is one for which at each date τ, the continuation path $a^\tau = (a_\tau, a_{\tau+1}, \ldots)$ yields each player at least his security level, evaluated in state x_τ, where $(x_1, x_2 \ldots)$ is the path of the state variable actually generated by a, for a given initial condition x_0. More formally, say that a is individually rational if

$$(12) \quad \limsup \frac{1}{T} \cdot U_i(a^\tau, x_\tau) \geq V_i(x_\tau), \text{ all } i, \text{ all } \tau, \text{ with}$$

$$x_\tau = g(a_{\tau-1}, x_{\tau-1}) \text{ and } x_0 \text{ given.}$$

Note that it is possible for a path to be individually rational for the game starting in one state, but not in another; this fact is important in interpreting some of the counterexamples which follow.

1.4 Nash Equilibrium and Individual Rationality.

It is well-known that in infinitely repeated games with no discounting, all individually rational paths are Nash equilibrium outcome paths - this is just the original "folk" theorem. We show here that in dynamic games the latter are always a subset of the former, and this subset may be strict. Also, we give a condition sufficient for them to coincide.

We begin with a definition of a Nash equilibrium path - a is such a path simply if no player prefers to deviate at any date t from a_t, given that he will be minimaxed from t+1 onwards. As finite sequences of payoffs are of no interest to the players, the object of deviation from a by player i is to switch the game into a state where his security level, which he knows will ensue after deviation, is relatively high. The condition which states that all such deviations are unprofitable is the following;

$$(13)\ \limsup_{T\to\infty} \frac{1}{T}.U_i^T(a_\tau,x_\tau) > V_i^\infty(x_{\tau+1})\ , \quad \text{all } i, \text{ all } \tau, \text{ with}$$

$$x_\tau = g(a_{\tau-1},x_{\tau-1}) \text{ and } x_{\tau+1} = g(a_{-i,\tau},\ b_i,\ x_\tau), \text{ all } b_i \text{ in } A_i\ .$$

Comparing (12) and (13), we can see that the latter condition is stronger as Nash paths must also be immune to manipulation by players trying to change the state. In fact, it is easy to find examples with individually rational outcome paths that are not Nash.

Consider the following example. There are two players, I and II and two states, A and B. The game starts in state A, where the following matrix game is played.

		II		
		U	M	D
	U	1,1	0,-2	1,0
I	M	-2,0	-1,-1	0,0
	D	0,1	0,0	$\frac{1}{2},\frac{1}{2}$

If U is played by either player <u>except</u> when M is being played by either player, the game moves into state B, and stays there. In state B, the same matrix game is played, except that 1 is added to all the payoffs. This means that starting in state A, either player can impose 0 on the other by playing M repeatedly, so security levels starting in A are 0. In B, of course, they are equal to the security levels in the constituent game i.e. 1. Therefore, the actions (D,D) played repeatedly constitute an individuallly rational path starting in state A, but are <u>not</u> a Nash equilibrium, as either player can guarantee himself 1 by switching to U at any time, thus moving the game into state B.

It is clear from this example and (12) and (13), however, that if if security levels were independent of the state, then Nash and individually rational paths would coincide. More formally, assume;

(A1) $V_i^\infty(x_0)$ is independent of x_0, all i.

Then we have;

<u>Theorem 1</u>

If (A1) holds, then all individually rational outcome paths are Nash equilibrium outcome paths.

As already noted, this result can be thought of as a generalisation of the classical "folk" theorem. We now turn to the main topic of the paper, the characterisation of perfect equilibrium.

1.5 Perfect Equilibrium and Individual Rationality

We begin by showing that (A1) on its' own is not sufficient for all individually rational paths to be perfect. What we do is present an example where (A1) holds, but where there are Nash equilibrium paths that are not perfect. Consider the following example. As in the earlier example, there are two states, A and B, and the game starts in state A, where the left-hand matrix game below is played. If either player chooses action D, the state switches to B, where it stays, and the right-hand matrix game is played. There are no mixed strategies.

State A (columns: player II)

	U	D
U	1,0	0,1
D	-1,0	0,1

State B (columns: player II)

	U	D
U	2,0	0,-1
D	2,0	0,-1

Note that in each constituent game, each player's security level is zero, so overall security levels in each state are also equal to zero, and so (A1) is satisfied. Now consider the outcome path consisting of repeated play of (U,U). This is certainly a Nash equilibrium, as it yields each player at least zero. However, it is not perfect, for the reason that once in state B, there is no strategy pair that yields indivisdually rational payoffs for both players, and gives player I less than 2. Therefore, by the Aumann-Shapley-Rubenstein theorem for repeated games, the worst payoff that II can credibly impose upon I in this state is 2, and so forseeing this, player I will always have an incentive to switch from U to D. This completes the example.

The distinctive feature of this example, however, is that there are individually rational outcome paths, such as (U,U) played repeatedly, whose payoffs to the players depends on the state. The question then arises; if we rule this out, do Nash and perfect paths coincide, and if this is so, can we in fact dispense with (A1)? To approach this question, the first step is to define state-independence of payoffs formally.

(A2) If a is an individually rational outcome path in the game starting in state x_0, then

$$\limsup_{T\to\infty} \frac{1}{T} \cdot U_i^T(a\ ,\ x) \text{ is independent of } x, \text{ all } i.$$

The first step is to show that (A2) on its own is also not sufficient to ensure that all individually rational (or Nash) paths are perfect i.e. (A1) is not redundant. Consider the following example. There are now three states, A, B, and C, and play starts in A, where the left-hand game is played. Again, there are no mixed strategies.

		II		
		U	M	D
	U	1,1	0,-2	1,-1
I	M	-2,0	-1,-1	-1,0
	D	-1,1	0,-1	$\frac{1}{2},\frac{1}{2}$

States A and B

	II		
	U	M	D
U	1,1	1,-2	1,-1
M	-2,1	0,0	-1,0
D	-1,1	0,-1	$\frac{1}{2},\frac{1}{2}$

State C

The transition between states occurs as follows. As soon as either player plays M, the game switches into state B; otherwise, the game remains in A. The game moves into C from B as soon as, and only if, both players stop playing M. State C is absorbing. In states A and B, security levels are equal to 0, and in C, are equal to 1, so that (A1) is violated.

On the other hand, (A2) is satisfied. To see this we consider three mutually exclusive and exhaustive subsets of all possible individually rational outcome paths for this game in turn. The first is the set of paths where only U or D is played at each date. Then it is clear by inspection of the payoff matrices above that the payoffs from such paths are independent of the initial state. The second possible subset is those paths where from some date τ onwards, at least one player plays M at each date. However, it is clear from inspection of the left-hand payoff matrix that such outcome paths can never be individually rational starting in state A. Hence, this subset is empty. Finally, the third subset consist of paths where up to some date τ no-one plays M, at τ at least one player plays M, whereas at $\tau + 1$, no-one plays M. Starting in state A and following such an outcome path, the game moves into state C at $\tau + 1$, and stays there. For such paths to be individually rational, then, after $\tau + 1$ the pair (U,U) must be played all but a finite number of times. Therefore, the payoffs from any element of this third set are 1 to each player, starting in any state, and so this third subset is indeed state-independent. As these three subsets exhaust all the possibilities, we have proved that (A2) is satisfied.

Now, the pair (D,D) played repeatedly is in fact a Nash equilibrium path, as any devation by one player can at best move the game into B, where the other player can impose a zero payoff on him by playing M for ever. In the event that one player deviates from D by playing M once and then switching to either U or D, however, it is not rational for the punisher to continue to play M, for by stopping, he can switch the game into C where he gets 1 rater than (at most) -1. Hence the Nash equilibrium is not perfect.

Thus, we have shown that neither (A1) nor (A2) are individually sufficient for Nash paths to be perfect. Our major positive result, however, is that jointly, they <u>are</u> sufficient, and it is to this that we now turn.

<u>Theorem 2</u>

If (A1) and (A2) hold, then all individually rational outcome paths are perfect equilibrium outcome paths.

This can be regarded as a generalisation of the Aumann-Shapley -Rubenstein result for repeated games, and the proof uses their device of rapidly increasing punishment periods for successive deviations. The main difference from the repeated game setting is that in a punishment phase, the behaviour of the player being punished is more complex, and in particular his best response to a T-period punishment over the T-period horizon may not be his best response over the infinite horizon. We do not need to characterise these punishments directly in the proof, but merely prescribe "lenient" punishments that not punish the the victim for any deviations from his own "best" response to punishment over a given number of periods.

Proof

The proof is constructive. That is, for any individually rational outcome path a^*, we find strategies $(f_1 ... f_n) = f$ which constitute a perfect equilibrium, and which generate the outcome path a^*, given the initial condition x_0; that is, $a^* = \sigma(f, x_0)$.

(i) The first step is to specify the strategies, which are as follows. First, we say that all $j \neq i$ punish i for t periods if they minimax him over the t-period horizon, as in (10) above, and say i responds to a t-period punishment if he plays his optimal response to this over t periods. (the inf and sup in (10) are always attained under the assumptions made in Section 1.3 so minimax strategies exist. Then, f prescribes;

(a) On subgames where no-one has deviated, play a^*;

(b) On subgames where i deviates from prescribed play for the nth time, all $j \neq i$ punish i for t_n periods, and i responds to the t_n period punishment, after which play reverts to a^*. By revert, we mean that if a^*_t was the last element of a^* to be played before the latest punishment phase started, a^*_0 is played as soon as it ends, and so on;

(c) The one exception to (b) is that if i fails to respond to his own punishment, in which case, his punishment continues unaltered;

(d) If more than one player deviates simultaneously, then no-one is punished.

(a) - (d) give a complete description of $(f_1 .. f_n)$ - i.e. describe actions for every player contingent upon every history, h_t - given a particular sequence of punishment durations.

(ii) We now specify this sequence; let it be a sequence $\{t_n^\infty\}$

satisfying

$$(\alpha) \quad \lim_{n\to\infty} \sum_{i=1}^{n-1} t_i/t_n = 0,$$

and if N(T) is the largest integer N such that $\sum_{i=1}^{N} t_i < T$, T = 1, 2 ...

$$(\beta) \quad \lim_{T\to\infty} N(T)/T = 0.$$

There certainly exist sequences satisfying (α) and (β); for example, the sequence defined inductively by $t_n = (\sum_{i=1}^{n-1} t_i)^2$, $t_i = 1$.

(iii) In view of the construction of these strategies (i.e. that along any equilibrium path of $f^1 \ldots f^n$, starting in any history, play eventually returns to a^*) and the fact that the limsup criterion ignores payoffs over finite periods, all we need to do to check that $f^1 .. f^n$ are perfect is that there exists no sequence of deviations, $(\tau_n)_{n=1}^{\infty}$ starting at any t, which yields some $i \in N$ a payoff strictly higher than from the equilibrium path. (By assumption (A2) furthermore, the latter is a constant independent of the state e.g. $U_i(a^*)$.) We can assume without loss of generality that this sequence is infinite,for if it were finite, so would be the period of punishment of i, and hence the gain to deviation would be precisely zero.

The first step in doing this is to note that the sequence of deviations by i, $(\tau_n^{\infty})_{n=1}$ generates both a sequence of intervals $(t_n)_{n=1}^{\infty}$ when i is being punished, and a further sequence , $(t_n^*)_{n=1}^{\infty}$ which is the the sequence of intervals when i is being not being punished. We can now establish some properties of $(t_n^*)_{n=1}^{\infty}$. First, suppose without loss of generality that t_n^* is increasing in n. Then define N(T) to be the largest integer N such that

$$(14) \quad \sum_{n=1}^{N} (t_n + t_n^*) < T .$$

It now follows from the fact that t_n increases so rapidly that either $\lim_{T\to\infty} \sum^{N(T)} t_n^* /T = 0$, or $t^*_{N(T)} \to \infty$. To see this, note that as the t_n^* are non-negative, the N(T) as defined in (14) is bounded above by the N(T) as defined in (a), so for the former, $\lim_{T\to\infty} N(T)/T = 0$. Now suppose that the t_n^* are bounded above by b. Then

$$0 \leq \lim_{T\to\infty} \sum_{i=1}^{N(T)} t_n^* /T \leq \lim_{T\to\infty} N(T).b/T = 0.$$

If on the other hand, the t_n^* are unbounded above, as N(T) goes to ∞ with T, then $t^*_{N(T)}$ goes to infinity with T also. There are therefore two cases to consider.

(a) ; $\underline{\lim_{T\to\infty} \sum_{i=1}^{N(T)} t_n^* /T = 0}$

In this case, from (14) , we conclude that $\lim_{T\to\infty} \sum_{n=1}^{N(T)} t_n /T = 1$, which in turn implies from property (α) that

$$(15) \lim_{T\to\infty} \frac{t_{N(T)}}{T} = 1,$$

or in other words, that the last punishment period eventually dominates. Then the average payoff over T periods for i is bounded above by

$$(16) \quad \max_{x\in X} \{ V_i^t N(T)(x) \} + \frac{(T-t_{N(T)}).b}{T}$$

where b is an upper bound on per-period utility, which exists under the assumptions made in Section 1.3 Then, as $t_{N(T)}$ goes to infinity with T, and x is finite, we can conclude from the "continuity at infinity" assumption that the first term in (16) goes to V_i^∞ with T. On the other hand, the second term goes to zero by (15).

(b); $\underline{t^*_{N(T)} \to +\infty}$

Here we have the case where $\limsup_{T\to\infty} \sum_{n=1}^{N(T)} t_n^* /T = k$ for some k with $0 < k < 1$, and where $\limsup_{T \to \infty} t_{N(T)}/T = (1-k)$.

We will now show that for any $\varepsilon > 0$, the payoff to deviation is bounded above by $k.U_i(a^*) + (1-k)V_i^\infty + \varepsilon$, so that deviation is not profitable if $U_i(a^*) > V_i^\infty$. First, choose any $\varepsilon > 0$. Then, it follows from (A2) and the finiteness of x that there exists a $T(\varepsilon)$ such that the following holds;

(17) $U_i^T(a^*, x) < T.(U_i(a^*) + \varepsilon)$ all $T > T(\varepsilon)$, all $x \in X$.

Then define

(18) $N(\varepsilon) = \{\min_n ;\ t_n^* > T(\varepsilon)\}$

(As $t_n^* \to +\infty$, such a $N(\varepsilon)$ exists). Now, at the end of each period of punishment, play reverts to a^*, so that the payoff to i over the interval of length t_n^* when he is not being punished is $1/t_n^* .U_i^{t_n}(a^*,x)$ where x is determined by the past history of play. Thus, from (17) and (18), it follows that the payoff to i over that proportion of T periods when he is not being punished is bounded above by

$$(19) \qquad \frac{1}{T}\cdot\sum_{n=1}^{N(\varepsilon)-1} t_n^*.b \;+\; \frac{1}{T}\cdot\sum_{n=N(\varepsilon)}^{N(T)} t_n^*.(U_i(a^*)+\varepsilon)$$

Taking the limsup of both sides of (19), we find that the payoff to i over that proportion of the time when he is not being punished is bounded above by $k(U_i(a^*)+\varepsilon)$. Now an argument as in case (a) establishes that over that proportion of the time when he is being punished, he gets at most $(1-k).V_i^\infty$. Therefore, his total payoff to deviation in this case is indeed bounded above by $k.U_i(a^*)+(1-k).V_i^\infty+\varepsilon$.

(iv) Taking both cases together, we see that deviation from f^i cannot be profitable for i as long as $U_i(a^*) > V_i^\infty$. But as a^* is individually rational by definition, this inequality always holds. This completes the proof.

Remarks

The theorem can be extended to games where A_i depends on the current state and actions of the other players in a stationary way i.e. $A_i = A_i(a_{-i,t}, x_t)$. Such dependence may arise in even simple economic examples (e.g dynamic duopoly with capacity constraints - see e.g. Fudenberg and Tirole(1983)). Also, if the inequality in the definition of individual rationality is changed from a weak to a strict one, the theorem should also go through in the case where infinite sequences of payoffs are evaluated according to the overtaking criterion.

A more pressing problem is to relax the assumption that X is finite. Unfortunately, it seems impossible to do this without imposing a stronger version of the state-independence condition (A2), which would require that the payoff from any individually rational path converge to a limit value uniformly in $\bar{X}$. When $\bar{X}$ is finite, of course, pointwise convergence implies uniform convergence, and so we avoid the problem of having to make this additional assumption. It is hard to imagine that uniform state-independence would be a necessary condition for Nash paths to be perfect, so relaxing the finiteness condition in this way would make the possible results much less sharp. A more appealing approach would perhaps be to approximate any game with a compact state-space by a sequnce of games with finite state-spaces, and try and show that the set of equilibrium paths in the latter converged to that of the former. A third possiblity is simply to try and verify inequality (17) in the proof above for particular examples, for it is this that is essential, not compactness per se. This is in fact what we do below when applying Theorem (2) to the Spence model.

1.6 Rules and Outcomes

The natural focus of interest in many dynamic games is not so much on sequences of actions as on sequences of rules which prescribe actions to be taken in any particular state e.g. industry growth paths in Spence's duopoly game. While there is, of course, a mapping from these rules to outcomes, the outcome path associated with any particular rule will, in general, depend on the initial state, x_o. For example, in the duopoly game if the rule for each firm is to accumulate capacity up to a "target" and produce up to capacity in each period, then the actual sequence of outputs and investment levels will depend on the initial capital stocks. Hence, the question - are the outcomes associated with any rule for the outcomes of perfect equilibria in the dynamic game ? - can only be answered using Theorem 2 above by "testing" each outcome path of the rule for each x_0 to see whether it satisfies (A2).

Therefore, it seems natural to reformulate (A2) so that it applies to the rule directly. It turns out that by doing this, we can we can prove a theorem which is not a special case of Theorem 2. The intiution for this is as follows. Inspection of the proof of Theorem 2 reveals that for it to go through, at the end of a punishment period, play need not return to a^* itself, but just something that always yields approximately limsup $\frac{1}{T} \cdot U_i^T(a^*, x_0)$ to every player over a long enough time horizon - for example, a closed loop rule that has this property.

To make this more formal, consider the following condition.

(A2') Let a be individually rational in the game starting in x_0. Then there exists a f^c such that

$$\limsup \frac{1}{T} \cdot U_i^T(\sigma(f^c, x), \quad x) = \limsup \frac{1}{T} \cdot U_i^T(a \ , x_0)$$

for all x, all i.

We now show that (A2') is a weaker requirement than (A2). To begin with, suppose (A2) holds. Then the (open-loop) rules defined implicitly by $a = \sigma(f^c, x)$ all x certainly satisfy (A2'). Note also that in the case of a repeated game, a thus defined trivially satisfies (A2'). We now show that for some games, (A2') may be satisfied while (A2) is not.

Consider the following very simple example. There are two states, A and B, and play starts in state A, where the left-hand matrix game is played. There are no mixed strategies and both states are absorbing. Security levels are 0 for both players. However, this game does not satisfy (A2) as any outcome path where (U,U) is played often enough is not state-independent.

State A

		II U	II D
I	U	1,0	0,0
I	D	0,0	1,0

State B

		II U	II D
I	U	2,0	0,0
I	D	0,0	1,0

Suppose, without loss of generality, that the outcome path is such that (U,U) is played a fraction k of the time and (D,D) the rest.

Then this yields player I a payoff of k in state A and 2k in state B. (Note that II always gets 0). Now it is always possible to choose a closed-loop rule such that (U,U) is played a fraction k of the time in state A and 1/2.k of the time in B - hence (A2') is satisfied in this case.

However, it is important to note that (A2') is not satisfied in cases where the folk theorem fails, but (A1) holds e.g. the first example presented in 1.5. There, in state B, there is no outcome path that yields the players the pair (1,0).

Now we can state the generalisation of Theorem 2;

Theorem 3

If (A1) and (A2') hold, then all individually rational outcome paths are perfect equilibrium paths.

We just give a sketch of the proof here. The proof follows that of Theorem 2, except that when a period of minimaxing a deviator is over, and the game is in (say) state x, play reverts to $\sigma(f^c,x)$ rather than a^*. As $\sigma(f^c, x)$ yields individually rational payoffs to all players, this is acceptable to the punishers, and part (iii) of the proof goes through virtually unchanged.

1.7 An Application-Duopoly with Durable Capital

In this section, we discuss the implications of the "folk" theorem, theorem 2, to a game of duopoly with durable capital, due originally to Spence (1979), and further analysed by Fudenberg and Tirole (1983).

We begin by checking the hypotheses of the theorem. We show that as long as there is a strictly positive rate of depeciation, outcome paths in this game (which are sequences of pairs of output and investment levels) are state-independent in the sense of the definition given in 1.4. (Without a positive rate of depreciation, the limsup of average payoffs can depend on the initial capital stock, and so the state-independence condition (A2) is violated.)

If we also assume that the state-space is compact, then it is also true that along any convergent subsequence, payoffs starting in any state converge to the limit uniformly in the initial state. This uniformity of convergence is important if we wish to apply Theorem 2 to the model, as in the Spence model, the state-space is the space of pairs of non-negative capital stocks, and hence not finite, so one cannot obtain equation (1.7) in the proof of theorem 2 directly.

It is also simple to show that security levels are both continuous at infinity and state-independent, so we can conclude that with positive depreciation, any individually rational outcome path is a perfect equilibrium path in the Spence model.

What are these individually rational paths? If the rate at which firms can invest is large enough, or if there is zero capital depreciation, then the security levels of the two firms will be zero, and so any outcome path which yields firms non-negative average profits will be perfect. Otherwise, the security levels may be strictly greater than zero, which is perhaps the more interesting case.

One can restate this result in terms of Spence's industry growth paths. An industry growth path (IGP) , in game theoretic terms, is a closed loop strategy n-tuple involving an initial expansion (or contraction, if there is a positive rate of depreciation) of the capital stocks to attain target levels, followed by constant output and replacement investment only. We have therefore shown that it may be the case that any IGP that generates outcome paths

which yield non-negative profits is "credible"; any "target" capacities that give each firm individually rational payoffs can be reached by credible threats of rapid expansion of capacity if the target is not achieved. In particular, all Pareto-efficient pairs of "target" capacities can be so attained.

These results contrast sharply with the findings of Fudenberg and Tirole(1983) who analyse a continuous-time version of this model with zero depreciation, and restrict their attention to closed-loop strategies (they call the resulting game a "state-space game"). They show, among other things that the <u>only</u> perfect equilibrium closed-loop strategies are "investment races" where both firms invest as fast as possible until a "finishing line" (F/T more accurately call it a "terminal surface") is reached. The point reached on this surface, and hence the long-run payoffs associated with these equilibria, depend <u>crucially</u> on the initial capital stocks.

However, this is not a consequence of zero depreciation per se, but rather the restricted strategy spaces which F/T use - although, as remarked above, state-independence fails in this case so that we cannot apply the folk theorem directly, it is possible to prove a similar folk theorem specifically for the Spence model, exploiting its special structure.

Finally, it is possible to say more about the structure of the perrfect equilibrium strategies. It turns out that the game may be "unstable" in the sense that imposing the security level payoff on player j can make player i worse off than he would be if he recieved his own security level. This means that outcome paths that yield each player payoffs below a certain level can only be supported by "finite" punishments of the A-S/R type discussed in 1.1.

(a) The Model

There are two firms, $i = 1, 2$. The state vector, $k = (k_1, k_2)$ is a pair of non-negative capital stocks. The action for the ith firm at time t, a_{it}, is a pair (I_{it}, y_{it}) of an investment level and output respectively. The revenue function for firm i, $R_i(y_i, y_j)$ is bounded above and non-negative, decreasing in y_j (strictly so if $y_i > 0$) and satisfies

$$(20)\quad \lim_{y_i \to 0} R_i = \lim_{y_i \to \infty} R_i = \lim_{y_j \to \infty} R_i = 0$$

The state equations are

$$(21)\quad k_{it} = k_{i,t-1}(1 - \delta) + I_{i,t-1} \qquad i = 1,2$$

where $0 < \delta < 1$ is the rate of depreciation. We assume that the initial capital stocks must be bounded above, which implies with a positive rate of depreciation, that all the k_{it} must be bounded above. Output is produced from capital and labour with $y_i < \min(k_i, l_i)$. Payoffs over actions are given by revenue minus short-run costs and investment costs;

(22) $R_i(y_i, y_j) - w.y_i - \rho I_i$

where w is the wage, and ρ the cost of investment goods.

Then the action space for i is

(23) $A_i(k_i) = [0, \bar{I}] \times [0, k_i]$

where $\bar{I}$ is an upper bound on the rate of investment. Note that the action space depends on the current capital stock, but in a stationary way, so (as explained in the remark following the proof of theorem 2) this theorem can be applied to this case.

(b) Analysis

We begin with the issue of long-run security levels in this model. As R_i is decreasing in y_j (strictly so as long as $y_i > 0$) then j's unique minimax strategy is to expand capacity as fast as possible (i.e. gross investment at rate $\bar{I}$) and produce up to capacity as long as $y_i > 0$.

Assume to begin with that there is a strictly positive rate of depreciation, i.e. $0 < \delta < 1$. Then from (21), j's capacity will tend to $k_j = \bar{I}/\delta$ from any initial position. Hence, in the long run, the worst j can do to i is to produce at $y_j = \bar{I}/\delta$ in each period. As the maximum sustainable capacity for i is also $\bar{I}/\delta$, then the payoff to i from i's best response to $y_j = \bar{I}/\delta$, is in the long run, the maximum of 0 (as i always has the option of closing down) and w_i, where

$$(2\ ) \quad w_i = \max_{0 < y_i < \bar{I}/\delta} \left\{ R_i(y_i, \bar{I}/\delta) - (w + \delta\rho).y_i \right\}$$

Therefore, with depreciation,

$$V_i^\infty = \max\,(w_i,\ 0). \tag{25}$$

The payoff to j from minimaxing i is

$$z_j^\infty = R_j(\overline{I}/\delta,\ y_i^*) - (w + \rho\delta).\overline{I}/\delta \tag{26}$$

where y_i^* is a maximiser in (2_4) . If $\overline{I}$ is large enough, then it is possible that $z_j^\infty < V_j^\infty$, and so the game is"unstable", as claimed.

In the case of no depreciation, any finite level of capacity can be sustained at zero average cost over the long run, so that the equation determining w_i now is (24) with $\rho = 0$ and $\overline{I} = +\infty$. Therefore, w_i and hence V_i^∞ are both 0.

Next, outcome paths. We must show first that with positive depreciation, the payoffs to each firm from any outcome path $a = (a_0, a_1 \ldots)$ with $a_t = (I_t, y_t)$ do not depend on the initial state. This is easy to do. Payoffs depend on the limsup of $\xi_{it} = \min(y_{it}, k_{it})$ and I_{it}, and as, in turn, the limsup of k_{it} is independent of initial stocks, from equation (21), the result follows.

Next, we must show that payoffs "converge" uniformly in the inital state, or more precisely that any subsequential limit of the average payoff converges to its limit uniformly in the inital capital stocks. For this, all we need show is that any convergent subsequence of ξ_{it} converges to its limit uniformly in the initial capital stock, k_{i0}. This in turn requires that any convergent subsequence of k_{it} converges uniformly in the initial capital stock. Without loss of generality, suppose that k_{it} itself converges to a limit, k. Then for any $\epsilon > 0$, there exists a T such that $|k_{it}(0) - k| < \varepsilon/2$ for all $t > T$. But as $|k_{it}(0) - k_{it}(k_{i0}))| < (1-\delta)^t.b$, where b is the upper bound on initial capital stocks, one can choose a T′ such that $|k_{it}(0) - k_{it}(k_{i0}))| < \varepsilon/2$ for all $t > T'$. Therefore,

$$|k_{it}(k_{i0}) - k| < \epsilon \quad \text{for all } t > \max(T, T') \text{ , all } k_{i0} \tag{27}$$

so that k_{it} converges uniformly in the initial stocks, as required. As (27) holds along any convergent subsequence of k_{it}, the analogue of (27) will hold both for any convergent subsequence of ξ_{it} and hence also for any convergent subsequence of per period and average payoffs. Furthermore, we already know that the limsup of the average payoff is independent of the initial state. We can put these facts together to deduce that equation (17) in the proof of theorem 2 is valid for this model.

Conclusions

The major limitation of these results is that they only apply to games without discounting. Also, we know from 1.1. that a completely different approach is required to games with discounting - i.e. Abreu's approach. However, the main conclusion of this chapter is that the A-S/R approach to repeated games without discounting carries over to a more general setting, in the sense that it can be used to identify necessary and sufficient conditions for individually rational paths to be perfect.

In the same way, one might fruitfully search for necessary and sufficient conditions for the existence of optimal penal codes in dynamic games. This might be quite difficult, as the natural description of a penal code in a dynamic game would seem to be in terms of closed-loop strategies, rather than outcome paths, and one would have to show that the space of such strategies was compact if one wished to generalise Abreu's existence argument directly. However, this seems a line of thought worth pursuing.

2. Long-Term Contracts

Introduction

This chapter draws on the methods and results covered in the first chapter in investigating some aspects of the theory of long-term contracts.

As Hart and Holmstrom (1985) point out, the theory of contracts essentially deals with economic transactions which have the character of a relationship-specific investment - once the investment is made, it is worth more to the two parties than it would be outside. For example (and this is the kind of example we have in mind in what follows) the "investment" may be the acquisition of firm-specific human capital by a workforce, or the accumulation of mobility costs, so that after a certain point, the firm and worker are "locked in" together.

In these circumstances[1] (and only with such a lock-in effect) there is a rationale for signing explicit contracts, for two reasons. This first is risk-sharing i.e. the locked-in partners may be able to achieve a reallocation of risk bearing that cannot be achieved by the market, and such risk-sharing contracts have overwhelmingly been the preoccupation of the literature, as explained in the introduction to the thesis. The second, which has only recently been investigated see e.g. Grout (1984) is that if the relationship involves investment, in the absence of binding contracts concerning the level of investment, (more exactly, if the level of investment is chosen prior to the decision on the division of the surplus), the level of investment will be inefficient.

This chapter is concerned with risk-sharing contracts. The interest of such contracts, both from a theoretical point of view and from the point of view of applications, is in the conflict between risk-sharing and asymmetric information between the contracting parties. Two alternative types of asymmetric information structure have been extensively analysed, and have recently been dubbed "hidden action" and "hidden information" problems respectively by Arrow.

The former covers cases where the agent takes an action which is unobservable by the principal, although to make the problem interesting he must be able to observe some stochastic consequences of the agent's action. Such unobservability is usually referred to as moral hazard. The problem of risk-sharing in a static setting with moral hazard has been intensively investigated, notably by Mirrlees, Holmstrom and Grossman and Hart, among others and most general features of the optimal contract are now known (see Hart and Holmstrom (1985)).

The second kind of informational asymmetry, hidden information, is where the agent observes the value of a random variable which is relevant to the risk-sharing problem, and which is unobservable to the principal - hence hidden information. Starting with the work of Grossman and Hart(1981), most of the attempts to modify the Azariadis-Baily implicit contract model have made use of this type informational asymmetry - see for example, the 1983 Quarterly Journal of Economics Supplement. For example, the firm may observe a productivity shock that is unobservable to the worker. In this case, the first-best risk-sharing contract is no longer incentive-compatible, as the same wage were paid in all states, the firm would always wish to announce the highest realisation of the shock, as this wuold elicit the largest labour supply under the terms of the first-best contract. (see also Azariadis(1983)).

Alternatively, the firm may not be able to observe the opportunity wage available to the worker if he leaves the contract, or perhaps the worker's valuation of leisure. Moore(1985) argues that the former is the relevant opportunity cost if layoffs are permanent, and the latter if layoffs are temporary, and shows that in the former case, there is overemployment relative to the first-best, and in the latter , underemployment, and in addition, layoffs are involuntary.

To conclude, we see that with hidden information, there will be inefficiencies relative to the first-best, but the direction of these inefficiencies depends on the structure of the particular problem at hand.

Attention has naturally turned more recently to the risk-sharing/asymmetric information tradeoff where the relationship is repeated over a sequence of dates. Four questions naturally arise here:

(a) is it possible to improve on a series of short-term contracts i.e. introduce history-dependence into the contract;

(b) what is the structure of the optimal contract;

(c) what is the asymtotic behaviour of the optimal contract e.g. as the time horizon goes to infinity or the discount rate goes to 1;

(d) what are the observable consequences of the long-term contract (over and above the predictions of the short-term contract?

Perhaps the most fundamental of these questions is part (a), but there is, as yet, no complete answer, and several contributions seem to be in conflict with one-another. First, in an important paper, Townshend (1982) argued that one motivation for history-dependence was improving risk-sharing. What his paper also made clear however, was that in an intertemporal setting one role of the contract was to act as a vehicle for the agent to save or dissave, and a "pure savings contract" where the agent just saved at a given rate of interest was also "history dependent" i.e. the repayment in period 2 depends on the amount saved in period 1. He was also able to show that in his paticular model, which was quite special, the optimal contract was not simply a savings contract One can interpret this as showing that the optimal contract history-dependence over and above that induced by savings.

A recent paper by Manning (1985) sheds some light on this issue. He assumes that the payments of principal and agent are quasi-linear in income, and shows that in this case, history-dependence is desirable if and only if the participation constraints take a certain form. (With quasi-linearity of course, both the savings and insurance motives for history-dependence are absent.)

Finally, Roberts (1982) presents an alternative argument that asserts that history-dependence is always desirable, but he specifies the agent's participation constraint in a rather different way to the rest of the literature.

What we do in Sections 2.2 and 2.3 is integrate and reconcile these contributions (especially Roberts and Manning) and also present some new arguments. The central conclusion is that there are two conditions, which without savings by the agent are individually <u>necessary and sufficient</u> for history dependence.

The first is that the utility of income of either principal or agent be non-linear; and the second, roughly speaking, is that the decision to "leave" the contract, or choose a status quo payoff, be reversible.

We now turn to the question of the structure of the optimal contract. A fundamental problem of definition arises here; optimal relative to what? In general terms, optimal relative to the behaviour of the agent, so that the optimal contract, in a static context, can be thought of as a Stackleberg equilibrium in the game where the principal moves first and chooses the contract, and the agent then responds. The game-theoretic viewpoint is not very illuminationg in the static context, precisely for the reason that there is effectively "one" Stackleberg equilibrium i.e. all Stackleberg equilibria give the principal the same payoff.

Whether this simple picture generalises to the many-period case depends on whether it is assumed that the principal can precommit to carry out the terms of the contract in future periods or not.

If he can, then there is no ambiguity in the notion of an "optimal" contract - it is, again, one of the Stackleberg equilibrium contracts in the normal form of the contracting game - all of which give him the same payoff.

If he cannot, then one must view the contracting game as a repeated game. With a finite horizon, the only equilibrium is a sequence of (static) Stackleberg equilibria, but in the more interesting case of an infinite horizon, there may be many equilibria even if a perfectness (or similar) condition is imposed, as we have seen in 1.1.

The implication is that the "optimal" contract may not be well defined, as Radner (1985) points out; there may be many different Stackleberg equilibrium contracts, which yield the principal different payoffs.

It was Radner's considerable achievement in the same paper to explicitly construct such an equilibrium, involving quite a simple and plausible strategy for the principal, called a review strategy, which is discussed further below in 2.6. Radner recognised, furthermore, that his review strategy equilibrium was in general, not Pareto efficient (see Radner (1985) Section 8.2).

To find the all the Pareto-efficicent equilibria of the contracting game without precommitment is, of course, a very difficult task, and it is not attempted here. What we do is much more modest; we suggest a way in which one of these equilibria - the one which gives the principal the highest payoff - may be characterised. The argument is based on the fairly obvious fact that if the principal does not wish to renage on the precommitment equilibrium contract (or one of them, if it is not unique) then this contract is also an equilibrium contract in the non-precommitment game and is furthermore, Pareto-efficient. (see 2.1 below). Then, assuming that we can find such a precommitment contract, we will have also found an efficient non-precommitment contract. (Whether we can do so is discussed in greater detail in 2.6(b)). This provides a link between Radner's approach to the problem and the more orthodox (i.e. non-game theoretic approach.

The properties of the optimal precommitment contract are the main focus of the second half of the chapter. The contract has an extremely simple structure; it decomposes into a series of contracts at each date, linked only by a state variable, w_t, which is the expected discounted utility of the future contract seen from time t onwards. At each date, the principal chooses, in addition to income/action pairs, w_{it+1}, which is the expected discounted utility of the agent from t+1 onwards, given an "announcement" of type i.

Thus, history dependence works entirely through the w_t; if $w_{ti} \neq w_{tj}$, then the payments and actions on offer at t will depend on the past history of types. We find also that in each period, the same incentive constraints bind as in the static problem, and that the history-dependence serves precisely to relax these constraints (this is the important insight of Roberts (1982)). This is not terribly surprising. What we do find is several interesting intertemporal features of the optimal contract.

First, we find that Rogerson's (1985) intertemporal marginal utility of income condition also holds in this model - this is just an expression of the condition that the principal is "saving" optimally on behalf of the agent.

Second, and more importantly, we obtain a new and general result; that the marginal utility of income of the agent is positively correlated over time. (This is true whether it is the upward or downward incentive constraints that bind in the optimal contract.) Under some simplifying assumptions, we show that this result implies positive serial correlation of wages in labour contracts.

Finally, we deal with two other issues. The first is the asymtotic behaviour of the infinite horizon contract as the discount factor goes to unity. We show that in the limit, the first-best payoffs can be attained as equilibrium payoffs. The reason for this has already been established by Radner (1981); without discounting, the principal can, by monitoring all the announcements of the agent can determine whether he is lying a positive fraction of the time to any desired degree of accuracy. As long as the monitoring period is finite, it has zero cost to the principal, so that the principal can costlessly detect deviations, and so can enforce the first-best.

However, the argument we use to establish this is quite different - we show that the cost, in terms of the adjustment of the w_{it}, of making the first-best contract incentive-compatible in each period becomes negligible as the discount factor goes to unity.

The second issue is the use of an additional instrument, the threat of terminations, which is dealt with in 2.7. We show that terminations are of no help in relaxing incentive-compatibility constraints, even without history-dependence in the rest of the contract. We relate this to the recent literature which explains involuntary terminations, and hence involuntary unemployment, as an incentive device (Shapiro and Stiglitz(1984)).

2.1 The Model and a Characterisation of Efficient Equilibria of the Contracting Game

This section is divided into four parts. In the first, we present a general formulation of the risk-sharing problem in a static context. In the second, following Radner(1981), (1985), we formulate a model of the dynamic contracting problem as a game, where the principal's strategy is to choose a contract, and the agent's strategy is to decide on announcement of a type from a given set at every date and contingency. The reason for approaching the problem in this way, is because(as we argued in the introduction), if the principal cannot precommit it is the only way of formulating the problem. Having done this, we obtain a simple but extremely useful characterisation of (some of the) efficient equilibria, which enables us to analyse the structure of these efficient equilibria only by considering the equilibria of the contracting game where the principal can precommit; the latter have a much simpler structure.

In the third part, we analyse precommitment equilibria. The first step is to prove a revelation principle; for the class of precommitment equilibria that we are interested in, it is possible to assume that the agent is using truth-telling strategies. Using this fact we show that the conditions for a (Nash or perfect) precommitment equilibrium are in fact equivalent to a sequence of easily understood dynamic incentive-compatibility constraints. We can then express the principal's choice of optimal strategy as a dynamic programming problem, and the details of this are covered in the latter half of the third section. The fourth section presents some sufficient conditions for existence of a solution to the optimal contracting problem.

(a) The Static Contracting Problem

We start by describing a conventional static (possibly risk-sharing) problem, the elements of which are as follows. The utility of the principal, $v(x, y)$ depends on the "action", x, taken by the agent and on the income transfer, y, between agent and principal. Here, the word "action" denotes any variable apart from income, which affects the income of both parties and is part of the contractual agreement. The utility of the agent, $u(x, y, \theta)$ depends on x, the action taken, on the income transfer y, and also his type θ_i drawn from a finite set $\Theta = \{\theta_1 \ldots \theta_n\}$. We order the θ_i so that $\theta_1 < \theta_2 \ldots < \theta_n$. A (static) contract, c, is a 2n-tuple of actions and payments conditional on types $\{x_i, y_i\}_{i=1}^n$.

Before choosing an action/income pair (x_i, y_i) the agent will observe his type, some θ_i in Θ. The principal, by contrast, does not observe the agent's type, but knows that he is drawn from a distribution over the types which is common knowledge. Suppose that the probability that the agent is of type i is π_i. Given this asymmetry, it is well-known that attention can be restricted to contracts that satisfy the self-selection constraints

$$(1) \qquad u(x_i, y_i, \theta_i) \geq u(x_j, y_j, \theta_i) \qquad \text{all } \theta_j \neq \theta_i.$$

Furthermore, we require feasibility (or participation) constraints to be satisfied to ensure that the principal and agent are both willing to take part in the contract.

These latter constraints can have several different specifications, depending on whether or not the agent is assumed to observe his type before or after a contract is made, and depending on whether the payoff to the agent in the event of no contract being signed depends upon his type or not. Suppose first that the "no contract" payoff to the agent is independent of his type. Then, in the case where the agent observes his type before the contract is signed, feasibility for the agent requires

(2) $u(x_i, y_i, \theta_i) \geqq \bar{u}$, all θ_i.

If it is assumed that that $u(x, y, \theta)$ is increasing in θ, then (2) can be simplified to

(3) $u(x_1, y_1, \theta_1) \geqq \bar{u}$.

If, the other hand, the agent observes his type only after the contract is signed, then feasibility for the agent requires simply that

(4) $\sum \pi_i u(x_i, y_i, \theta_i) > \bar{u}$.

In the event that the "outside" payoffs of the agent do depend on his types, then (2) must be modified to

(5) $u(x_i, y_i, \theta_i) \geqq \bar{u}_i$, all θ_i,

where $\bar{u}_i$ is the outside opportunity of a type i, and cannot necessarily be simplified to $u(x_1, y_1, \theta_1) > u_1$. By contrast, constraints of the <u>ex ante</u> type still take the form (4) with the modification that $\bar{u} = \Sigma\ \pi_i . \bar{u}_i$.

The choice of participation constraint depends, of course, upon the problem analysed. For example, in the theory of non-linear pricing, it may reasonably be supposed that the consumer has knowledge of his "type' (i.e. his income, or preference for the good) before buying and so the ex post constraints of type (2) are appropriate. In this case, however, the assumption that the no contract option yields all types an equal payoff may be unreasonable - for example, if the consumers who differ in initial incomes. In contracts whose rationale is risk-sharing, however, such as the Azariadis-Grossman-Hart model of labour contracting, or Townshend's (1982) model, the ex ante constraint is obviously appropriate.

It is important to note that the problem with ex post constraints is _not_ a risk-sharing problem - what makes it non-trivial is the conflict between asymmetric information and the desire by the principal to extract all the surplus from the agent. (Nevertheless, as we show in 2.2. and 2.3, history-dependence is desirable under exactly the same conditions as with the ex ante constraints.)

For completeness, in what follows we consider multiperiod contracts with both ex ante and ex post participation constraints. Our results apply _a fortiori_ to the case where both sorts of constraints are in operation (as in for example Manning(1986),Manning and Lockwood (1985)). For analytical tractibility, we suppose that if the constraints are ex post that the agent's outside payoff is type-independent.

Finally, we do not bother with an explicit specification of a participation constraint for the principal, as none of the results below are substantially affected by its inclusion.

The most general form of optimal contract is any feasible contract (i.e. satisfying (2), (3) and/or (4)) which maximises the weighted sum of utilities of both principal and agent. However, we restrict our analysis to contracts which maximise the utility of the principal,

$\sum \pi_i . v(x_i, y_i)$. This rules out application of our results to multiperiod optimal taxation problems, but covers the non-linear pricing problem for a discriminating monopolist, and labour contracts with asymmetric information as should be clear to those acquainted with the literature (see for example, Spence (1980) on monopoly pricing, or Hart (1983) on labour contracts).

(b) The Dynamic Contracting Problem

Here the principal and agent contract over a sequence of time-periods $t = 0, 1...T$. This sequence may possibly be infinite, i.e. $T = \infty$. The per-period payoffs are as before in static case, and the sequence of random variables $\{\theta_t\}_{t=0}^{T}$ follows a specified stochastic process. We shall be exclusively concerned with the case where the θ_t are independent over time, and this assumption is in fact crucial for our approach to the problem.

At this point, we will formalise the contracting problem as a non-cooperative game, following Radner (1981), (1985), for reasons already discussed. In this context, a strategy for the principal is just a sequence of per period contracts $\{c_t\}_{t=0}^{T}$, where $c_t = (x_{it}, y_{it})_{i=1...n}$, where the choice of c_t is of course conditional on the information available to the principal at the beginning of period t.

What is this information? The principal has, at this point, observed the past choices of strategy of the agent, so his information at this point will be a history of these past choices. What these choices are depends on the strategy set of the agent. We have some discretion in specifying this strategy set. One possibility is that at each t, the agent simply announces a type $\theta_t \in \Theta$. Another is that he reveals his type indirectly by a choice of income/action pair. In the event that for some t, $x_{it} = x_{jt}$, $y_{it} = y_{jt}$ $i \neq j$ (i.e. "bunching") then the second type of "indirect" strategy reveals less information that the first to the principal. It turns out that as the θ_t are independent, it makes no difference[2] to the form of the equilibrium which strategy is available to the agent and so we will assume the former specification. Then the principal's information at t when he chooses c_t is simply a history of announced types $a^{t-1} = (a_0, a_1 \ldots\ldots a_{t-1})$, with $a_t \in \Theta$, and a history of per period contracts, $c^{t-1} = (c_0, c_1 \ldots c_{t-1})$ that he himself has previously offered.

We are now ready to give a slightly more formal description of the principal's strategies. At any $t = 0, 1 \ldots T$, contingent upon (a^{t-1}, c^{t-1}) the principal chooses a c_t ; i.e. $c_t = g_t(a^{t-1}, c^{t-1})$. A strategy for the principal is then a sequence of functions $g = (g_0, g_1 \ldots g_T)$.

The agent's strategies are similarly defined. First, let $\theta^t = (\theta_0, \theta_1 \ldots \theta_t)$ be any history of realised types. At each date t, Then, contingent upon his information at this point, which may be (a^{t-1}, θ^t, c^t) or $(a^{t-1}, \theta^{t-1}, c^t)$ the agent can decide either to leave (an option which we denote by ∞) or stay in the contract. If he stays in he makes an announcement a_t, once he has observed his current type, θ_t. Hence, this announcement will be a function of (a^{t-1}, θ^t, c^t) ; i.e. $a_t = f_t(a^{t-1}, \theta^t, c^t)$. By a slight abuse of notation - because we are not writing down the participation decision formally - let the sequence of f_t be f, the agent's strategy.

Outcome paths in this game are sequences of income/action pairs $\{x_t, y_t\}$. One can, in the usual way, construct mappings from strategies (f, g) to outcomes. Given the stochastic nature of the problem, these mappings are random functions. Finally, payoffs over outcomes are discounted sums of expected utilities for each player

i.e. $E \sum_{t=0}^{\infty} \gamma^t \cdot u(x_t, y_t, \theta_t)$ and $E \sum_{t=0}^{\infty} .\delta^t.v(x_t, y_t)$,

unless the agent terminates the contract, in which case he gets $\underline{u}_t$ and the principal gets $\underline{v}_t$. Note that γ and δ are the discount factors of the agent and principal. In what follows, we assume $\delta = \gamma$ unless something explicitly to the contrary is said, and denote the common discount factor by δ.

The game is technically one of incomplete information, because the principal cannot directly observe the past history of types. Hence, it is essential to introduce the idea of a probability assessment, m_t, which is a probability measure over all possible histories up to that point viz. Θ^{t-1}, and represents the principal's probability assessment at the beginning of t that a particular history has occurred. Where applicable, (i.e. on the equilibrium path) it should be consistent with the principal's own information and knowledge of f - off the equilibrium path it can be arbitrarily defined.

Given such an assessment, it is now possible to define a sequential equilibrium in the Kreps-Wilson (1982) sense for this game. We call such an equilibrium simply an equilibrium.

It is worth pausing at this point to observe that there is no way in which one could hope to characterise the equilibria of the contracting game in general. Radner managed to construct one type of equilibrium, an equilibrium in review strategies, only with a good deal of ingenious argument and by drawing heavily on the notion of a trigger strategy equilibrium in the theory of repeated games. Unfortunately, as there are many types of equilibria in repeated games with complete information, there will be many types of equilibria in this game. However, of particular interest are those equilibria whch are efficient in the sense that there exists no other sequential equilibrium which gives both players a strictly higher payoff. These are interesting not least because as Radner points out, his equilibria in review strategies are not efficient, so the structure of efficient equilibria is an open question.

What we argue now is that it is possible to characterise one of the efficient equilibria of the contracting game. First, formally define a precommitment equilibrium to be a sequential equilibrium of the contracting game where the principal can precommit to g at time 0. The connection between precommitment equilibria and efficient equilibria is the following.

Theorem 1

If (f^*, g^*) is a precommitment equilibrium and generates an outcome path that is also the outcome path of a (sequential) equilibrium, then this outcome path is also the outcome path of an efficient sequential equilibrium.

Proof

Suppose to the contrary that the outcome path of (f^*, g^*) is not the outcome path of an efficient sequential equilibrium. Then there exists an equilibrium (f^{**},g^{**}) which gives both players strictly more than (f^*, g^*). But then as the principal could precommit to the contract implicit in g^{**} and do better than g^*, this contradicts the definition of (f^*, g^*).

Although the proof is simple, this is a result of great importance, as it means one can hope to characterise one of the efficient equilibria of the contracting game by doing something very much simpler - i.e. characterising "the" precommitment equilibrium[3]. If one can then demonstrate that the principal, if threatened with a credible punishment - such as reversion to the static contract - would not wish to renage on this contract, then one has, by Theorem 1, a contract which is part of sequential equilibrium of the game.

In the next section, we investigate precommitment equilibria.

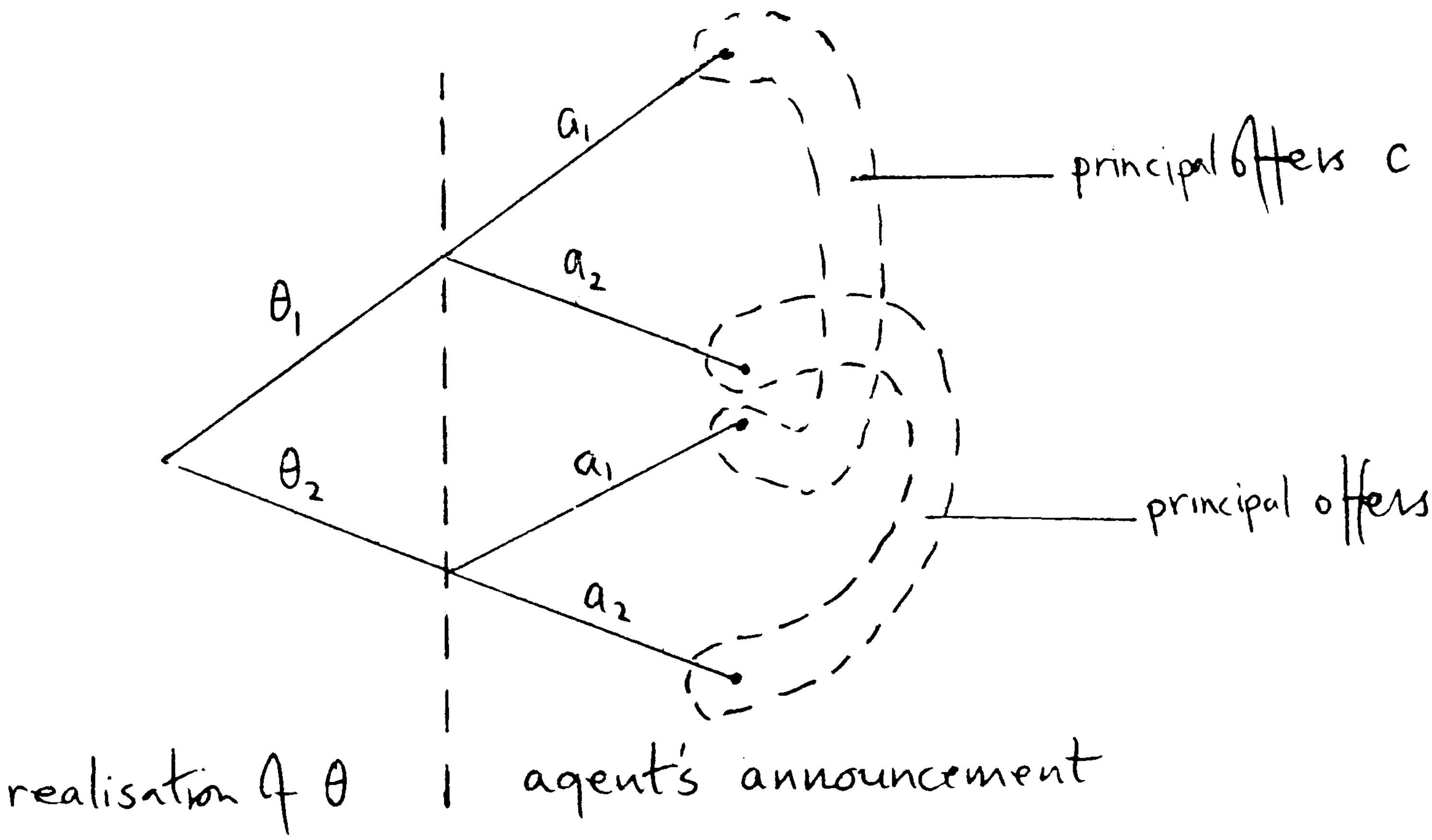
(c) Precommitment Equilibria

It is useful to start by showing that in any equilibrium of the contracting game, neither player will wish to condition their actions at each date on θ^{t-1}. First, the principal cannot condition the contract at any date t on θ^{t-1}; and one would then conjecture that as preferences are separable over time and the θ_t are independent, then the agent cannot do any better by such conditioning either. More formally, let $F^{t+}(c^{t-1}, \theta^{t-1}, a^{t-1}; g)$ be the set of strategies from t onwards for the agent that are best replies to g, conditional upon $(c^{t-1}, \theta^{t-1}, a^{t-1})$, then there is an element of F^{t+} that is independent of θ^{t-1} at every date. This is in fact easily proved in the finite-horizon case by backwards induction.

To make the point, consider a two period example pictured in partial game-tree form below. There are two possible types in the first period, and the principal expects the agent to tell the truth.

Hence, at the end of the first period, the principal's information sets are as shown, and he will offer the same contract in the second period contingent upon either node in any given information set. In the second period, it is clear that as the game is now one-shot, there is a best reply by the agent to either c or c′ that does <u>not</u> depend on the particular node in either of the two information sets.

Figure 2.1.1



This property can be extended to the infinite-horizon case by arguing by contradiction, if we suppose that the agent's per period payoff is bounded. Suppose to the contrary that given g, there was at some date t, continuation stategies $f^{t+} \neq f^{t+*}$ such that f^{t+} was strictly best against g on history θ^{t-1} and f^{t+*} was strictly best on history θ^{t-1*}, other components of the histories being the same.

Then, as discounted payoffs go (uniformly, by boundedness of the per period payoffs) to zero, there exists a long enough finite horizon such that the same is true over this finite horizon, which contradicts the earlier result.

We can therefore restrict ourselves, without loss of generality, to stategies where neither principal nor agent condition upon θ^{t-1}. Note that this does not rule out Radner-type review strategies - for example, the principal could treat the agent if he were truth-telling, and punish him if the average of the a_t fell too low below the population mean.

While not of great interest in itself, this fact enables us to prove the following;

Theorem 2; The Dynamic Revelation Principle

Suppose that f^* is a best reply to g^* of the contracting game. Then there exists a pair (f^{**}, g^{**}), where f^{**} is a best reply to g^{**} and which yield each player the same payoff, where f^{**} is truthtelling i.e. $f_t^{**}(c^t,\theta^t,a^{t-1}) = \theta_t$, all t and on all histories.

Proof

Consider the initial equilibrium. One can think of the strategy of the agent at each date as choosing, contingent upon (c^t , a^{t-1}), a permutation function $\sigma : \Theta \to \Theta$ which may not be the identity function, and need not be even one-to-one.

Let $c_t^* = g_t^*(c^{t-1},a^{t-1})$. Now construct at each date a new contract c_t^{**} from c_t^* by

$x_{it}^{**} = x_{\sigma(i)t}^*$, $y_{it}^{**} = y_{\sigma(i)t}^*$, all i, t.

Then it is easily established that if f^* is a best reply to g^*

contingent on any history observable by the agent, then f^{**} is a best reply to g^{**} on any such history. By construction, both pairs of strategies yield both players the same payoffs. This completes the proof.

Unfortunately, the revelation principle does not go the other way i.e. if (f^*,g^*) are an equilibrium, it does not follow that g^{**} as constructed above is a best response to truthtelling. The reason for this is that the initial equilibrium may have the agent behaving in a "hostile" way to the principal, and switching to truthtelling may create an incentive to the principal to redesign the contract (rather than just "garble" it, to get more out of the agent. For example, consider a one-shot game with two states, and suppose the initial "equilibrium" involves the agent picking the worst element for the principal from c, the contract, whatever θ. Then it is best for the principal to offer a contract (x,y) that is constant across both states. Now suppose the agent switches to the truthtelling strategy - then the principal can do better by not offering the same contract, but the first-best. This example is contrived, for neither the first nor the second strategy ascribed to the agent can ever be an equilibrium strategy for him in the one-shot game - the only equilibrium strategy is myopic optimisation against c i.e. choosing the best element of c for any realisation of θ. However, we cannot rule such strategies out in the repeated game.

This asymmetry in the revelation principle is important - if the argument went the other way as well, we could conclude that all sequential equilibria were truth-telling which (as we shall see below) would enforce a particular structure on them.

We now have a very useful corollary of Theorem 2.

Corollary

If there exists a precommitment equilibrium of the contracting game, (f^*, g^*), then there exists another precommitment equilibrium (f^{**}, g^{**}) where f^{**} is truth-telling.

This follows immediately from Theorem 2, and is extremely useful; it means that in analysing the precommitment equilibrium, we can replace the standard "best reply correspondence" of the second mover (i.e. the agent) on each subgame with a a series of conditions on g (one on each subgame (a^{t-1}, c^{t-1})) which guarantee that truthtelling is a best response to g from then on.

We are not yet home and dry, however, as these conditions are potentially very complex; they should ensure that all (possibly very complex) sequences of deviations from truthtelling do not pay. The following Lemma, proved in the Appendix, tells us that it is sufficient to consider "one-shot" deviations, if (i) the time horizon is finite, or (ii) the agent has a positive discount factor strictly less than 1. Note that by precommitment, we do not have to index the subgames by c^{t-1} , as it is not neceesary to specify the behaviour of principal or agent in the event that the principal offers the "wrong" contract at some date.

Lemma 1

Assume either (i) $T < \infty$, or (ii) $T = \infty$ and $0 < \delta < 1$,and $u(x, y,\theta)$ is bounded.

Then truthtelling is a best response to g in all subgames a^{t-1} if and only if

$$(6) \quad u(g_t(a^{t-1}, \theta_t), \theta_t)$$

$$+ \quad E \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}.u(g_\tau(a^{t-1}, \theta_t, \theta_{t+1} \ldots\theta_\tau), \theta_\tau)$$

$$\geq u(g_t \ (a^{t-1}, a_t), \theta_t)$$

$$+ E \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}. u(g_\tau(a^{t-1}, a_t , \theta_{t+1}\ldots\theta_\tau), \theta_\tau)$$

all $a_t \neq \theta_t$, and also at each date and contingent upon each a^{t-1}, the agent does not wish to choose ∞ i.e. leave the contract.

The constraints (6) are clearly generalisations of the incentive constraints in the static problem. The additional term on either side reflects the "long run" effect of announcing a type a_t. As they stand, constraints (6) are still very complicated, but they can be considerably simplified by defining the following expected discounted utility variable;

$$(7) \qquad w_{t+1}(a^{t-1}, a_t) \quad = E \sum_{\tau=t+1}^{\infty} \delta^{\tau-t-1}. u(g_\tau(a^{t-1}, a_t, \theta_{t+1}\ldots\theta_\tau) \theta_\tau)$$

This is the expected future discounted utility of the agent from t+1 onwards if he announces a type a_t at t, conditional upon a history of announcements a^{t-1} and if he tells the truth from then on. Note that the w_t satisfy the recursion relationship

$$(8) \quad w_{t+1}(a^{t-1}, a_t) = E\,.u(g_{t+1}(a^{t-1}, a_t, \theta_{t+1}), \theta_{t+1}) + \delta.w_{t+2}(a^{t-1}, a_t, \theta_{t+1})$$

Notice also that this is the <u>only</u> definitional constraint that the w_t must satisfy. Therefore, we can think of the w_{t+1} functions as chosen by the principal, along with g_t at each t subject to (8) lagged one period and (6), and the participation constraints. The exact form of the participation constraints depends on whether the decision to leave the contract is irreversible or not. Suppose that it is for the moment. Then the decsion to stay before observing θ_t requires the following ex ante constraint to be satisfied;

$$(EA) \quad E\ u(g_t(a^{t-1}, a_t), \theta_{it}) + \delta.w_t(a^{t-1}, a_t) \geqq \underline{u}_t$$

and similarly the ex post constraint is

$$(EP) \quad u(g_t(a^{t-1}, a_t), \theta_t) + \delta.w_t(a^{t-1}, a_t) \geq \underline{u}_t, \text{ all } \theta_t.$$

Finally, the maximand of the principal is of course,

$$E \sum_{\tau=t}^{\infty} \delta^{\tau-t}.v(g_\tau(\theta^t)).$$

Now by the principle of irrelevant information, or Bellman's principle of optimality, (see e.g Whittle(1983)) the optimal g_t and w_{t+1} are not <u>directly</u> conditioned upon a^{t-1}; they only depend upon a_t and the state variable w_t. Thus, the optimal contract $(x_{it}, y_{it})_{i=1..n}$ at any date t must solve the problem

$$V_t(w_t) = \sup_{(x_{it}, y_{it}, w_{it+1})} E \left[v(x_{it}, y_{it}) + \delta . V_{t+1}(w_{it+1}) \right]$$

subject to;

$$(IC_{ij}) \quad u(x_{it}, y_{it} \; \theta_i) + \delta . w_{it+1} \geq u(x_{jt}, y_{jt}, \theta_i) + \delta . w_{jt+1} \text{ all } j \neq i$$

$$(F) \quad w_t = E \left[u(x_{it}, y_{it}, \theta_i) + \delta . w_{it+1} \right]$$

and either

$$(EA) \quad E \left[u(x_{it}, y_{it}, \theta_i) + \delta . w_{it+1} \right] \geq \underline{u}_t$$

or

$$(EP) \quad u(x_{it}, y_{it}, \theta_i) + \delta . w_{it+1} \geq \underline{u}_t, \text{ all } i$$

if the constraints are irreversible, and

$$(EA') \quad E\, u(x_{it}, y_{it}, \theta_i) \geq \underline{u}_t$$

or

$$(EP') \quad u(x_{it}, y_{it}, \theta_i) \gtreqqless \underline{u}_t, \text{ all } i$$

if the constraints are reversible, and finally

$$(\alpha) \quad w_{it+1} \geq \underline{u}_{t+1}, \text{ all } i.$$

Thelast constraint, which we have not met before, embodies the consistency requirement that the principal cannot commit himself to offer a contract which offers the agent less than his participation level of utility in the next period.

We know that such a valuation function exists if either the time horizon is finite or v, the principal's utility is bounded, but we do not yet know whether we can replace the sup by the max operator i.e.

whether there exists an optimal contract. We defer discussion of this question to the next section.

The dynamic programming formulation makes clear the nature of the history dependence in the contract - it works only through the (F) constraint, and so the structure of the optimal contract, as already remarked, is much simpler than Radner's review contracts. In particular, there are no separate punishment phases, and the evolution of the whole contract over time is as a series of static contracts linked by the state variable w_t which follows a Markov process.

Finally, we note that the participation constraints can be simplified somewhat in the irreversibility case. First, (F), in conjunction with the (α) constraint $w_t \geq \underline{u}_t$ in fact implies (EA) at time t, so that (EA) can in fact be dropped. Second, if $u_\theta \geq 0$ then (EP) can be replaced by $u(x_{1t}, y_{1t}, \theta_1) + w_{1t} \geq \underline{u}_t$. We shall use these simplifications in what follows.

(d) Existence of the Optimal Contract

It is quite possible for an optimal contract not to exist, even in the finite-horizon case. For example, Manning(1985) has shown that when the preferences of both principal and agent are quasi-linear and the income transfer y_{it} is unbounded, then no solution exists when the discount factor of the agent exceeds that of the principal, essentially because the principal can borrow an unlimited amount from the agent at favourable terms. This situation is not consistent with our assumption that u and v are bounded. However, boundedness itself is not sufficient to guarantee a solution. Sufficient conditions are;

(A1) c_t lies in a compact set, C.

(A2) both u and v are continuous in x and y.

In addition, we need a condition which says that the $\underline{u}_t$ are not so high that the feasible set at any date is empty. Such conditions are fairly straightforward but tedious to formulate in the finite-horizon case. In the infinite horizon case, where it is necessary to impose stationarity on the $\underline{u}_t$ (i.e. $\underline{u}_t = \underline{u}$ all t), a sufficient condition would be for example, that there exists a c in C such that $u(x_i, y_i\ \theta_i) > \underline{u}$ all i.

Given these two assumptions, and a non-emptiness condition, it is possible to show that a solution exists in both the finite and infinite horizon cases. In the finite horizion case, using the terminal condition $V_{T+1} = 0$, it is possible to show by induction and the Theorem of the Maximum that each V_t is continuous, and that each w_t must lie in a compact set, and so existence follows immedately. In the infinite-horizon case, it is easy to see that the w_{it+1} lie in the compact set $[\underline{u}/(1-\delta),\ b/(1-\delta)]$, where b is an upper bound on u which exists by (A1) and (A2), and then one can show that V is continuous , and the existence of a solution follows.

Therfore, (A1) and (A2) are jointly sufficient for existence.

2.2 Quasi-Linearity, Participation Constraints, and History-Independence; a Reconciliation of Some Recent Results.

In this section, we present and reconcile the results of Manning and Roberts on the question of history-dependence. In particular, while Manning shows that with quasi-linearity of both principal and agent's preferences, the contract is history-independent, Roberts presents a labour contracting example with quasi-linear preferences, where there is history-dependence.

Our argument is that Robert's model is only consistent with a version of our model where the participation constraints are ex post and at least some of them are also reversible. We then show that in the context of his example, if they are replaced by irreversible constraints, the resulting contract is history-independent.

This means that Robert's general, "marginal improvement" argument establishing history-dependence is not valid unless at least some of the participation constraints are reversible, and we explain why this is so. Thus, the current literature leaves one important question open; is non-linearity in the utility of income sufficient for history-dependence? We deal with this question in the next section, 2.3.

We begin by reviewing the result that if the payoffs of both principal and agent are linear in income, and the participation constraints are of the irreversible type, then the optimal contract does not exhibit any history-dependence. By "linear in income", we mean that $u(x, y, \theta) = u(x, \theta) + y$ and $v(x, y) = v(x)-y$. Given our characterisation of the optimal contract in 2.1.(c) above, we know that the past history of θ only affects the current contract through the state variable, w^t. Hence, a necessary and sufficient condition for history-dependence is that the w_{it+1} be independent of i, for all i and all t.

In the case of an ex ante participation constraint, the argument is straightforward - with quasi-linearity, the efficient, or first-best, contract is incentive-compatible in the static context[3]. Therefore, the optimal multiperiod contract will simply be a sequence of static contracts. In fact, this is also true if the constraint is ex ante and reversible.

With the ex post participation constraint, of course, the optimal static contract is no longer efficient. However, Manning (1984) has shown for the finite-horizon case that the history-independence property still obtains. One advantage of the dynamic programming approach developed in Section 2.1(c) is that it allows a very simple proof of this result, and indeed its generalisation to the infinite horizon case.

Consider the constituent problem at time t in the infinite horizon case. Assuming that the valuation function V(w) is differentiable (we prove this in 2.4 below) the first-order conditions for y_{it} and w_{it+1} are , with quasi-linearity;

$$(9) \quad -\pi_i + \pi_i \cdot \mu + \phi_i + \sum_{i \neq j} \lambda_{ij} - \sum_{j \neq i} \lambda_{ji} = 0 \text{ , all i ,}$$

and

$$(10) \quad \pi_i \cdot V'(w_{it+1}) + \pi_i \cdot \mu + \phi_i + r_i + \sum_{i \neq j} \lambda_{ij} - \sum_{j \neq i} \lambda_{ji} = 0 \quad \text{all i}$$

where μ is the multiplier on (F), λ_{ij} the multiplier on (IC_{ij}), ϕ_i the multiplier on (EP), and r_i the multiplier on (σ). It follows from (9) and (10) that if $r_i = r_j = 0$, $V'(w_{it+1}) = V'(w_{jt+1})$. Now it is possible to show that V(w) is concave (see Lemma 6 below) so this implies $w_{it+1}=w_{jt+1}$. If on the other hand both (σ) constraints are binding, we have again $w_{it+1}=w_{jt+1}$. The final case is where r_i is binding but r_j is not. Then from (9) and (10) $V'(w_{jt+1}) = -1 > V'(w_{it+1}) = -(1+r_i)$, so by concavity of V, $w_{jt+1} < w_{it+1}$, which is a contradiction , so $w_{it+1}=w_{jt+1}$ after all. Thus, we have demonstrated, conditionally on some facts about V, that we get history-independence even with ex post constraints.

Manning (1985) also examines in some detail the structure of the contract in the quasi-linear case, and shows that while it is history independent, it is non-stationary, or time-dependent. In particular, all per period contracts except the first period's are efficient. This is achieved by offering a high enough w_t or level of expected discounted utility to the agent in t = 1, 2 etc. to achieve the first-best, and recouping this excess of w_t over the participation level, u, by an income transfer in the first period, period 0. (For this to be optimal, the principal must discount the future (weakly) less than the agent - if the discount factors go the other way, then there is no solution.) Therefore, we know that although history-independent, long-term contracts with quasi-linearity exhibit a front-end loading property reminiscent of Holmstrom's although it arises for different reasons.

Clearly, we must reconcile the history-independence result with Roberts (1982) who presents a general argument to show that history-dependence pays. The difference is in the specification of the participation constraints in the two models. Roberts does not have participation constraints explicitly as such, but rather a status quo choice of action by the agent, which he may choose for some realisations of θ.

However, it will be argued here that the example of labour contracting Roberts presents in the paper makes it fairly clear that the only "participation" constraint consistent with his model is a period by period or reversible , one,and this accounts for the difference in results - Manning (1985) has shown that even with quasi-linearity, one has history-dependence with period-by-period participation constraints. We begin by presenting Roberts example, and arguing that it can only be interpreted as a case with period by period participation constraints. We then show that if one changes the participation constraint in the example to an irreversible one, history-dependence is no longer desirable, and in fact the optimal contract takes on the form predicted by Manning (1985) - the static (inefficient) contract in the first period, followed by an efficient contract in the second, with an income transfer to the principal in the first period recouping the higher payoff he must allow the agent in the second, in order to achieve efficiency.

Roberts example concerns a firm and a worker contracting over two periods. The firm observes a random productivity shock θ (distributed uniformly on the unit interval) which is unobservable to the worker. Output is the product of θ and labour input. Roberts assumes that the labour input can only take on two values 0 and 1 (unemployed and employed). Given the informational asymmetry the agent is consequently the principal.

Finally, payoffs are linear in income i.e. the worker gets $w(\theta) - R{\cdot}L(\theta)$ where w is the wage, $R < 1$, and the firm gets $\theta{\cdot}L(\theta) - w(\theta)$. The ex post constraints require the latter to be at least zero in every contingency. Now consider the static contract. Now the wage can only be conditioned upon employment - w(e) - or unemployment - w(u). The <u>ex post</u> constraints require $w(u) \geq 0$ and in any optimal contract $w(u) = 0$. Hence, the firm will choose to employ the worker only in those states where $\theta > w(e)$. Consequently, the payoff to the worker is $(1 - w(1))(w(1) - R)$ which is maximised by $w(e) = \frac{1 + R}{2} > R$ so the contract is inefficient.

Roberts considers this one-shot contract situation repeated twice, and allows second-period wages to depend on employment or unemployment in the first period. Again from the <u>ex post</u> constraints, second-period transfers in unemployment states will be zero. Hence, let $w_1(u)$, $w_1(e)$ be second period wages in second-period <u>employment</u> states, given unemployment and employment respectively in the first period.

Then second period expected profits to the firm will be either

$$(11) \quad \int_{w_1(u)}^{1} (s - w_1(u))ds \quad \text{or} \quad \int_{w_1(e)}^{1} (s - w_1(e))ds$$

depending on which first-period contingency has occurred.

Now let $w_0(e)$, $w_0(u)$ be <u>first-period</u> income transfers to the worker in employment and unemployment states respectively. Then from (11) the marginal first period θ above which the firm will employ the worker, θ_0, is determined by

$$(12) \quad \theta_0 = w_0(e) - w_0(u) - \int_{w_1(e)}^{1} (s - w_1e))ds + \int_{w_1(u)}^{1} (s - w_1(u))ds$$

If the participation constraints are of the irreversible type, it requires

$$(13) \quad w_0(u) + \int_{w_1(u)}^{1} (s - w_1(u))\, ds \geq 0$$

$$(14) \quad \theta - w_0(e) + \int_{w_1(e)}^{1} (s - w_1(e))ds \geq 0 \quad , \theta \geq \theta_0,$$

or that the expected (discounted) utility over both periods always be non-negative. In fact, (12) and (13) imply (14).

If on the other hand, the participation constraints are of the period-by-period type, they require

$$(15) \quad - w_0(u) \geq 0$$

$$(16) \quad \theta - w_0(e) \geq 0$$

The problem that Roberts analyses is in fact "mixed"; that is, he implicitly assumes (15) and (14) . He in fact sets $w_0(u) = 0$, which yields the same outcome as (15). To derive the solution in this case, note that the maximand, or expected payoff to the worker over two periods, is

$$(17) \quad (1 - \theta_0)(w_0(e) - R) + \theta_0 \cdot w_0(u) + (1 - \theta_0)(1 - w_1(e))(w_1(e) - R)) + \theta_0(1 - w_1(u)).(w_1(u) - R)).$$

Maximising this with respect to first and second-period wages, and θ_0 , subject to (12), and (15) yields

$$(18) \quad w_1(u) = \frac{1 + \theta_0 \cdot R}{1 + \theta_0}, \quad w_1(e) = R, \; w_0(u) = 0, \quad w_0(e) \leq \frac{1 + R}{2}.$$

As $w_1(u) \neq w_1(e)$, there <u>is</u> history dependence in the optimal contract. It serves to improve first-period efficiency, as one would expect; as $w_1(u) > w_1(e)$, and $w_0(e) \leq (1 + R)/2$, from (12) it follows that θ_0 is less than $(1 + R)/2$, its value in the static contract.

If one examines the constraints, the cause of history-dependence of the solution (18) with $w_1(u) \neq w_1(e)$ becomes more apparent. The worker would <u>like</u> to set $w_1(u) = w_1(e) = R$, and recoup the surplus this gives the firm in the first period, as Manning has shown. By choosing $w_0(e)$ appropriately, he can do this following an employment state, so $w_1(e) = R$; but as $w_0(u)$ is effectively constrained to be zero he <u>cannot</u> do this following an unemployment state.

One can confirm that this line of argument is correct by solving the problem with the irreversible participation constraint (13) replacing (15). Then, as $w_0(u)$ can now be freely varied, we find that the solution has second-period efficiency in <u>both</u> contingencies i.e. $w_1(u) = w_1(e) = R$, and in the first period, θ_0 is at the same value as in the static contract, viz $\theta_0 = (1 + R)/2$.

This argument suggests that Roberts general argument in the first part of the paper is crucially dependent on the period-by-period nature of the participation constraints.

His argument, in the context of this example, is as follows. Suppose the long-term contract consists of static contracts, i.e. with $w_0(e) = w_1(e) = w_1(e) = (1 + R)/2$, $w_0(u) = 0$. Then there is a gain to first-order from lowering θ_0 of $(w_0(e) - w_0(u) - R)$, which, calculated at the static contract values, is strictly positive. Now, by definition, this gain cannot be realised by lowering $w_0(e)$, as it is already chosen optimally.) However, it <u>can</u> be achieved, for example, by lowering $w_1(e)$ slightly, and as $w_1(e)$ is optimally chosen this will have zero cost to first order.

In the case with irreversible participation constraints, however, this argument does <u>not</u> conclusively establish the desirability of history-dependence, for the simple reason that this same gain can also be realised by raising $w_0(u)$, the first-period layoff payment from zero (this is possible from (13)); in addition, this change will give the worker a direct gain to first order of θ_0, and so is actually a preferable method of lowering θ_0 than introducing history-dependence.

We can conclude then, that Proposition 2 of Roberts(1982) is limited to contracts with period-by-period participation constraints. This raises two issues. First, how reasonable is it to formulate contracting problems in this way, and second in contracts with irreversible participation constraints, what are the (sufficient) conditions for history-dependence in the optimal contract?

It is a major objective of this chapter to deal with the second question, and we discuss this at some length in 2.3. To answer the first question, it is helpful to consider the intermediate case in between full irreversibility and full reversibility - partial irreversibility. That is, the agent, by leaving the contract must pay a fixed cost, c, and forfeit some fraction γ of expected discounted payoffs, these may represent legal costs, cost of strikes, scrapping costs etc., which gives rise to the constraints;

$$(19) \qquad u(x_i, y_i, \theta_i) + \delta . \gamma . w_i + c \geq \underline{u}$$

Now consider the FOC for w_{it+1} and y_{it} with (EP) replaced by (19), retaining the assumption of quasi-linearity of payoffs. One then gets after some manipulation;

$$(20) \qquad V'(w_{it+1}) + 1 + \lambda_i . (\delta - 1)/\pi_i = 0,$$

so that unless $\delta = 1$ there will be history dependence, in general. Therefore, the history-independence is not robust to changes in the participation constraints, which to some extent vindicates Roberts' approach.

To summarise, the results of Manning and Roberts are displayed in the following table;

	irreversible		reversible	
	EA	EP	EA	EP
Both parties linear utility of income	I	I	I	D
At least one party non-linear utility of income	?	?	D	D

Here, I and D denote that under the stated conditions, the contract is history-independent or dependent resectively. Note that it has not been established whether non-linear utility is sufficient for history-dependence under irreversible constraints, and this is what we turn to next.

2.3 Non-Linearity as a Sufficient Condition for History-Dependence

In this section, we show that is if that the utility of income of either the principal or agent or both be everywhere non-linear, then this sufficient for long-term contracts to exhibit history-dependence, if the reservation utilities are low enough.

In fact, this condition alone is sufficient for the Pareto-frontier associated with the long-run contract to lie outside the Pareto-frontier associated with a sequence of static contracts - and this is really the essence of history-dependence.

The argument that establishes this is a two-stage one. The first stage uses Robert's insight that a small degree of history-dependence can be introduced into the contract at zero cost, to first order, (although in our framework this is done by changing the w_i slightly from a position where w_i=w all i), and can be used to relax the incentive-constraints.

The second step is to exploit this relaxation to make one or both of the principal and agent strictly better off, by improving the insurance offered bythe contract. This can only be done if the marginal rate of substitution between incomes in some pair of states differs between principal and agent. This condition in turn, will hold at the second-best static contract if and only if at least one of the parties has a non-linear utility of income.

To make the point as clearly as possible, we specialise the general model to a special case that can be interpreted as modelling a risk-sharing contract between firms and workers (see e.g. Grossman and Hart (1981), Azariadis (1983)) and which we call from now on the GHA model. Also, we assume two states and two periods, and no discounting. The participation constraint is *ex ante*.

We suppose that the payoff to the firm is just profit, $f(\theta,l)-y$ where l is the labour input and w the real wage, with $f_l > 0$, $f_{ll} < 0$, $f_{l\theta} > 0$, and that the payoff to the worker is just $u(y_i) - l$. We assume u is increasing and strictly concave.

The first-best risk-sharing contract involves a constant real wage (i.e. 100% risk-bearing by the firm) and employment strictly increasing in θ. As θ is only observable by the firm, therefore, this constract is not incentive compatible - the firm would always wish to announce the highest θ.

Let the static second-best Pareto-frontier be described by $V=V^s(w)$, where V is the principal's (here the workers) expected payoff, and w is the agent's here the firm's payoff). Then we know from 2.1 that the Pareto-frontier for the two-period contract is described by

$$V(w) = \max_{w_i, l_i, y_i} \sum_{i=1}^{2} \pi_i (u(y_i) - l_i + V^s(w_i))$$

$$\text{s.t.} \qquad (IC_{ij}) \quad f(\theta_i, l_i) - y_i + w_i \geq f(\theta_i, l_j) - y_j + w_j \qquad (21)$$

$$i=1,2$$

$$(F) \quad \sum_{i=1}^{2} \pi_i (f(\theta_i, l_i) - y_i + w_i)) = w$$

$$(\sigma) \qquad w_i \geq \underline{u}$$

Recall that the $\underline{u}$ in the (σ) constraints is next period's reservation level of utility, and also that constraint (EA) can be dropped from the problem (under the assumption that w is greater than the agent's reservation utility over two periods.) Finally, we note that the equality constraint (F) can be replaced by an inequality constraint, as the constraint will always bind at the solution to (21).

Our argument is now as follows. First, and most importantly, we show that in the solution to (21) without the $w_i > \underline{u}$ constraints, we will have history dependence i.e. $w_1 \neq w_2$. This shows that the Pareto frontier for the entire contract lies above the sum of the static Pareto frontiers for the two periods i.e. $v(w) > 2v^s(w)$ and is really the essence of history dependence. Whether we have history dependence in the actual contract depends of course, on whether or not the $w_i > u$ constraints are violated at the solution to (20) without these constraints. It seems likely that if $\underline{u}$ is low enough, they will not be violated, and history-dependence will occur. However, this is essentially a technical matter and is related to other intertemporal features of the contract, such as the evolution of wages over time and so we defer discussion of this to the next section, which is more technical.

Assume then to the contrary that $w_1 = w_2$. Then the (l_i, y_i) i=1,2 that solve (21) is the second-best static contract, and it is easy to show that in this case, only the upward constraint, (IC_{12}) is binding and also that $y_2 > y_1$, so that there is incomplete insurance. By raising w_1 by ε/π_1 and lowering w_2 by ε/π_2 the principal can relax this constraint, without affecting the (F) constraint as an inequality constraint. Assuming $V^S(w)$ is differentiable (which follows from differentiability of f and u) the cost of doing this, to first order, is zero.

Now the principal is in a position to exploit the slight relaxation in the incentive constraint, by using the fact that his own marginal rate of substitution between y_1 and y_2 is $-\pi_1 u'(y_1)/\pi_2 u'(y_2)$ which is less than $-\pi_1/\pi_2$, whereas the agent's is exactly $-^{\pi 1}/_{\pi 2}$. The principal can do this by raising y^1 by ε/π^1 and lowering

y_2 by ϵ/π_2 leaving the expected utility of the agent unchanged and without violating the upward incentive constraint. This change will, to first-order, make the principal better off by an amount $u'(y_1)-u'(y_2) > 0$ thus contradicting the hypothesis that $w_1=w_2$ can be optimal.

This argument can be compared to Townshend's (1982) analysis of a pure insurance contract. He argues that introducing history dependence (over and above optimal saving) can lead to an improvement as the agent can be insured against an income fluctuation in the first period. This is similar to what is going on here; however, here it is the principal who can be better insured against profit fluctuations by introducing history-dependence. Also observe that the argument is symmetric; that is, history dependence can also be shown to be desirable if the agent has non-linear preferences over income.

More importantly, exactly the same arguments apply if the participation constraint is ex post; again, the principal improve his own insurance slightly. This seems quite surprising, as in the ex post case, the contract is not a risk-sharing one. What is giong on, in fact, is that he is risk-sharing with himself - when $w_1 = w_2$, he is fully insured with respect to the w_i, but faces considerable income risk . As the w_i and y_i can be traded one for one without affecting the constraints, there is clearly scope for intertemporal allocation of risk.

2.4 A Characterisation of the Optimal Contract

The "local improvement" argument of the last section suggests that in the optimal long-run contract, the upward incentive constraints will still be binding, and the choice of the w_i will be such as to relax these constraints i.e. $w_i > w_{i+1}$, all i. This is in fact the case under certain conditions, but the argument is not entirely straightforward, mainly because it requires that V_t be concave. This in turn will be the case if one third cross-derivative of f, $f_{11\theta}$, is non-positive.

Nevertheless, the main message is that under certain conditions, the structure of the static contract largely carries over to the long-run contract. This is, of course, also true of the class of problems where the <u>downward</u> constraints bind in the static second-best contract (i.e. the downward constraints continue to bind in the long-run contract, and $w_i > w_{i-1}$), and this can be proved using the arguments of this section.

The class of problems we prove results for are those problems where by a suitable redefinition of variables, the principal's payoff can be written as a linear function of both action and income variables, and where the agent's payoff can be written as additively separable function of both variables, satisfying certain conditions, viz.,

$$\text{(A3)}\qquad \begin{aligned} &v(x,\ y) = -(x+y) \\ &u(x,\ y\ \theta) = f(x,\ \theta) + g(y),\ \text{with } f_x,\ f_\theta,\ f_{x\theta},\ g_y > 0\ , \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad f_{xx},\ g_{yy} < 0. \end{aligned}$$

Such a specification is in fact consistent with a wide class of problems - the main restriction implicit in (A3) is that of additive separability. For example, the GHA model of labour contracts presented in 2.3 above can clearly be put into this framework, by defining l = x, and redefining variables with y = $-u(z)$, and $g(y) = -z$, so $g(y) = -u^{-1}(-y)$. (It is easy to check that g is increasing and concave if u is.)

In addition, we will require that the marginal utility of income be unbounded, as y approaches its lower bound, (which, by (A1) above, is in the consumption set) and which, without loss of generality, we can take to be 0;

(A4) $\lim\limits_{y \to 0} u'(y) = +\infty$.

This is, of course, to ensure an interior solution for all the y_{it}, which simplifies the proofs, although it is not essential.

We begin by restating the general per-period contracting problem faced by the principal which was derived in 2.1(b) above, assuming now that (A3) holds. We consider only the case with ex ante constraints, but stress that the results obtained in this section apply equally to the ex post case.

$$V_{t-1}(w) = \max_{(x_i, y_i, w_i)} \sum \pi_i (\delta . V_t(w_i) - (x_i + y_i)) \quad \text{s.t.} \qquad (P_t)$$

$$(IC_{ij}) \quad f(x_i, \theta_i) + g(y_i) + w_i \gtrless f(x_j, \theta_i) + g(y_j) + w_j \qquad j \neq i ,$$

$$(F) \quad \sum \pi_i (f(x_i, \theta_i) + g(y_i) + w_i) \geq w ,$$

$$(\alpha) \quad w_i \geq \underline{u} .$$

Note that for simplicity, we drop the time subscripts on the choice variables, but in what follows it is important to remember that w is the expected discounted utility that the principal promised the agent in period t onwards, and is therefore not a choice variable of the principal, whereas the w_i are the expected discounted utilities promised to the agent from t+1 onwards.

We can now state the main result of this section;

Theorem 3

Suppose that (A3) and (A4) hold, and in addition that $f_{xx\theta} < 0$. Suppose also that $(x_i, y_i, w_i)_{i=1...n}$ solves (P_t). Then

(a) $w_i \geq w_{i+1}$ all i= 1,2..n-1;

(b) $x_{i+i} \geq x_i$, $y_i \geq y_{i+1}$ all i= 1,2..n-1 with at least one strict inequality;

(c) only the upward incentive constraints bind at the solution.

Parts (b) and (c) state precisely the extent to which the long-run contract inherits the features of the static contract. Part (a), by contrast, is the main result concerning the direction of history-dependence. Unfortunately, for a fixed discount rate, it is impossible to give conditions under which any of the inequalities in (a) are strict. The reason why one cannot say anything in general is that if one of the inequalities were strict, then the principal prefers to give the agent some rent from the contract, (i.e. set $w_i > \underline{u}$, some i) even though this is costly to him, because of the benefit of being able to meet the incentive constraints at lower cost in terms of distortions in the per period contract. One cannot in general say anything about the relative magnitudes of these costs and benefits.

What can be shown (and is shown in 2.6 below) is the discount rate is high enough, and $w > \underline{u}$, then at least one of these inequalities will be strict i.e. there will be history-dependence in the infinite-horizon case, so that for δ high enough, the benefits of history dependence outweigh the costs. A necessary and sufficient condition for $w > \underline{u}$, furthermore, is that there be a 'lock-in' effect, i.e. that by entering the contract for one period, the agent lowers the expected discounted utility of the alternative occupation etc. were he to leave. There are many different assumptions, in the context of specific models, that would generate this effect.

We now turn to the proof of Theorem 3 , which proceeds by a series of Lemmas.

Lemma 2

Assume (A3) and (A4) hold. If g is differentiable , V_t is differentiable, and $V_t' = -\mu$, where μ is the multiplier on (F).

Proof

To prove differentiability of V_t , let $y_i(w)$ be the optimal choice of y_i at w. Then define, for any $h \neq 0$,

$$\Delta y_i(h, w) \equiv g^{-1}(g(y_i(w)) + h) - g_i(w).$$

Then if $(x_i(w), y_i(w), w_i(w))_{i=1...n}$ is a solution to (P_t) at w, then $(x_i(w), y_i(w) + \Delta y_i(h, w), w_i(w))$ is feasible at w + h. Therefore, by revealed preference,

$$(22) \quad -\sum \pi_i . \Delta y_i(-h, w + h) \geq V_t(w + h) - V_t(w) \geq -\sum \pi_i . \Delta y_i(h, w)$$

Dividing (22) through by h and using the fact that

$$\lim_{h \to 0} \frac{\Delta g_i(h, w)}{h} = \lim_{h \to 0} \frac{\Delta g_i(-h, w+h)}{h} = \frac{1}{g'(y_i(w))}$$

(by L'Hopital's rule and the continuity of $y_i(w)$) we infer that

$V_t'(w) = -\sum\pi_i \cdot \frac{1}{g'(y_i(w))}$. Now by (A4) the y_i always have an interior solution, so that $\frac{1}{g'(y_i)} = \mu + \frac{1}{\pi_i} \cdot \sum_{j\neq i}(\lambda_{ij} - \lambda_{ji})$, where $\lambda_{ij} > 0$ is the multiplier on (IC_{ij}).

Substituting in the expression for $V_t'(w)$ and cancelling terms ,we find that $V_t'(w) = -\mu$, as required.

The second step, and a standard one in analysing this kind of problem, is to note that the "global" incentive constraints (IC_{ij}) $j\neq i$, can be replaced by the "local" incentive constraints $(IC_{i,i-1})$ and $(IC_{i,i+1})$ and an appropriate monotonicity condition. For completeness, we give a proof of this result, although similar results are available elsewhere (e.g. Hart(1983)). (Because of the dimensionality of the problem, with three variables instead of the usual two, such a property only holds if the payoff of the agent can be written in additively separable form.)

Lemma 3

In problem (P_t) above, (IC_{ij}) $j\neq i$, $i = 1..n$ hold if and only if $(IC_{i,i-1})$,$(IC_{i,i+1})$ and $x_i > x_{i-1}$ $i = 1,..n$ hold.

Proof

(a) We show that if the (IC_{ij}) hold, then $x_i > x_{i-1}$. For suppose to the contrary that $x_i < x_{i-1}$. Then $(IC_{i-1,i})$ implies

(23) $f(x_{i-1}, \theta_{i-1}) + g(y_{i-1}) + \delta.w_{i-1} > f(x_i , \theta_{i-1}) + g(y_i) + \delta.w_i$

But $x_i < x_{i-1}$ and $f_{x\theta} > 0$ imply that

(24) $f(x_{i-1}, \theta_i) - f(x_i, \theta_i) > f(x_{i-1}, \theta_{i-1}) - f(x_i, \theta_{i-1}) > 0$

and (23) and (24) together imply that $(IC_{i,i-1})$ is violated, a contradiction.

(b) We show that if the $(IC_{i,i+1})$ constraints and the monotonicity conditions hold, then the $(IC_{i,i+2})$ constraints hold. The lemma then follows by repeated application of this argument.

From $(IC_{i,i+1})$ we know that

$$(25)\quad f(x_i,\ \theta_i) + g(y_i) + \delta.w_i \geq f(x_{i+1},\ \theta_i) + g(y_{i+1}) + \delta.w_{i+1}$$

$$(26)\quad f(x_{i+1},\ \theta_{i+1}) + g(y_{i+1}) + \delta.w_{i+1} \geq f(x_{i+2}, \theta_{i+2}) + g(y_{i+2}) + \delta.w_{i+2}$$

Then, as $x_{i+2} \geq x_{i+1}$, and $f_{x\theta} > 0$, it follows that

$$(27)\quad f(x_{i+1},\ \theta_i) - f(x_{i+2},\ \theta_i) > f(x_{i+1},\ \theta_{i+1}) - f(x_{i+2},\ \theta_{i+1}) \ .$$

Now from (27) and (26), it follows that

$$(28)\quad f(x_{i+1},\ \theta_i) + g(y_{i+1}) + \delta.w_{i+1} \geq f(x_{i+2},\ \theta_i) + g(y_{i+2}) + \delta.w_{i+2} \ .$$

Finally, from (28) and (25), it follows immediately that $(IC_{i,i+2})$ holds. This completes the proof.

This means that we can replace the "global" incentive constraints in P_t with the "local" upward and downward constraints. The next step is to show that only the upward incentive constraints are binding at the solution to (P_t) as long as V_t is concave.

<u>Lemma 4</u>

Suppose V_t is concave. Then at the solution to (P_t), if $x_i > x_{i-1}$ then $(IC_{i,i+1})$ is binding and $(IC_{i,i-1})$ is slack.

Proof

(a) Suppose to the contrary that $(IC_{n,n-1})$ is binding and $x_n > x_{n-1}$. Then $(IC_{n-1,n})$ must be slack, as both upward and downward constraints cannot bind when $x_i > x_{i-1}$. We argue that this implies that this implies that $(IC_{k,k-1})$ is binding, all $k < n$. To see this, suppose by contrast that $(IC_{n-1,n-2})$ is slack. Then from the FOC for the problem P_t, we find

$$(29) \quad g_y(y_{n-1}) = (\mu - (\lambda_{n-2,n-1}+\lambda_{n,n-1})/\pi_{n-1})^{-1}$$

$$> (\mu + \lambda_{n,n-1}/\pi_n)^{-1} = g_y(y_n)$$

$$(30) \quad f_x(x_{n-1},\theta_{n-1}) = (\mu - (b_{n-1,n-2}.\lambda_{n-2,n-1}+b_{n-1,n}\cdot\lambda_{n,n-1})/\pi_{n-1})^{-1}$$

$$> (\mu + b_{n,n-1}\cdot\lambda_{n,n-1}/\pi_n)^{-1} = f_x(x_n,\theta_n)$$

where

$b_{ij} = f_x(x_i, \theta_j)/f_x(x_i,\theta_i) > 0$, so from $g_{yy} < 0$ and $f_{xx} < 0$ it follows that $y_{n-1} < y_n$ and $x_{n-1} < x_n$. Next,

$$(31) \quad V'_t(w_{n-1}) = -\mu + (\lambda_{n-2,n-1}+\lambda_{n,n-1} - \zeta_{n-1})/\pi_{n-1}$$

$$V'_t(w_n) = -\mu + (\lambda_{n,n-1}- \zeta_n)/\pi_n.$$

It can be shown from that as V_t is concave, either both (α) constraints are binding, or $w_{n-1} < w_n$. This implies,as $x_{n-2} < x_n$ and $y_{n-1} < y_n$, that $(IC_{n-1,n})$ is violated, a contradiction. Thus, $(IC_{n-1,n-2})$ must be binding after all. Hence, repetition of the argument implies that all the downward constraints must be binding if the topmost one is.

But then an argument similar to the one above implies that $x_1 < x_n$, $y_1 < y_n$, and $w_1 \leq w_n$, so that (IC_{1n}) is violated. Hence, we infer that

$(IC_{n,n-1})$ must be slack at the solution.

(b) There are now two possibilities; either $(IC_{n-1,n})$ is slack at the solution or it is binding . If it is the former, the entire above argument is repeated, with n-1 replacing n, to infer that $(IC_{n-1,n-2})$ must be slack. One can then infer from the FOC that $x_{n-1} < x_n$, $y_{n-1} < y_n$, $w_{n-1} < w_n$, so that $(IC_{n-1,n})$ is violated, a contradiction. Hence $(IC_{n-1,n})$ must be strictly binding (i.e. $\chi_{n-1,n} > 0$) at the solution.

(c) The final step is to show that if $(IC_{i-1,i})$ is strictly binding, then $(IC_{i-1,i-2})$ is also strictly binding. The argument is along the lines of part (a) above and is consequently omitted.

The proof of this Lemma should make it clear that concavity of V_t is vital to ensure that the structure of the static problem is retained in the dynamic case. Next, we show that concavity of V_t also allows us to establish the relationship between the w_i at the solution to (P_t).

Lemma 5

Suppose that V_t is concave. Then at a solution to (P_t), $w_i < w_{i-1}$ all $i = 2...n$.

Proof

Suppose to the contrary that $w_{i-1} < w_i$ at the solution. From the FOC to (P_t) and the concavity of V_t, a necessary condition for this is that

$$V_t'(w_{i-1}) = -\mu - (\lambda_{i-1,i} + \zeta_{i-1} - \lambda_{i-2,i-1})/\pi_{i-1} >$$

$$\mu - (\lambda_{i,i+1} + \zeta_i - \lambda_{i-1,i})/\pi_{i-1} = V_t'(w_i)$$

As $\zeta_{i-1} > 0$, from the FOC for y_i and y_{i-1} and the concavity of g, we infer that $y_{i-1} < y_i$. As $x_{i-1} \leq x_i$ by Lemma 2, , $(IC_{i-1,i})$ is violated, a contradiction.

In view of these intermediate results, Theorem 3 can now be established simply by showing that V_t is concave if V_{t+1} is (if the time-horizon is finite) or if the horizion is infinite, showing that the functional operator implicit in P_t preserves concavity of V. This is easy enough to do. the first step is to define a <u>less-constrained problem</u> which is P_t with the downward constraints omitted i.e. simply retaining the monotonicity constraints $x_i \geq x_{i-1}$ and the $(IC_{i,i+1})$. Let the valuation function for the less constrained problem be V_t^* .

Then we have the crucial result;

<u>Lemma 6</u>

Suppose that (A3) and (A4) hold, and in addition $f_{xx\theta} \leq 0$. Then, in the finite-horizon case, V_t^* is concave, all t, and in the infinite-horizon case, V^* is concave.

<u>Proof</u>

We only give the proof for the infinite-horizon case; the proof for the finite-horizon case is almost identical, except it proceeds by induction. Say that a contract is feasible at w if it satisifes all the incentive-constraints, and the (F) and (α) constraints, and gives the agent at least w in expected utility. Then the lemma will be proved if for any w, w′, we can find a contract feasible at $p.w +(1-p).w'$ which yields the principal at least $p.V^*(w) + (1-p).V^*(w')$, under the hypothesis that V^* is concave.

We will in fact construct such a contract. Let $(x_i, y_i, w_i)_{i=2\ldots n}$ and $(x_i', y_i', w_i')_{i=1\ldots n}$ be the optimal contracts at w and w′ respectively.

Now define a new contract as follows;

$$(32)\quad x_i'' = p.x_i + (1-p).x_i'$$

$$(33)\quad w_i'' = p.w_i + (1-p).w_i'$$

and with the y_i'' defined implicitly by the condition

$$(34)\quad p(f(x_i, \theta_i) + g(y_i) + \delta.w_i) + (1-p).(f(x_i', \theta_i) + g(y_i') + \delta.w_i')$$
$$= (f(x_i'', \theta_i) + g(y_i'') + \delta.w_i'')$$

Now from the concavity of f and g and (32) and (33), we can infer that y_i'' $p.y_i + (1-p).y_i'$, so that if V^* is concave, the entire contract will yield a higher payoff to the principal than $p.V^*(w)$ + $(1-p).V^*(w')$. Furthermore, by (34) the new contract yields the agent expected utility of precisely $p.w + (1-p).w'$ and by construction satisfies the monotonicity and (α) constraints.

Therefore, the proof will be complete if we can also show that the new contract also satisfies the upward incentive constraints $(IC_{i,i+1})$. Let $u_{ij} = f(x_j, \theta_i) + g(y_j) + \delta.w_j$. Then we need to show that $u''_{i,i} > u''_{i,i+1}$. But, from (34),

$$(35)\quad u''_{i,i} - u''_{i,i+1} = \left[p(u_{i,i} - u_{i,i+1}) + (1-p).(u'_{i,i} - u'_{i,i+1})\right]$$
$$+ \left[p.u_{i,i+1} + (1-p).u'_{i,i+1} - u''_{i,i+1}\right]$$

Now the first bracketed term in (35) is non-negative by the incentive-compatibility of the original contracts, so we only need to show that the second term is non-negative. Now from (34),

$$(36) \quad \left[p.u_{i,i+1} + (1-p).u'_{i,i+1} - u''_{i,i+1}\right] =$$

$$\left[p.f(x_{i+1},\theta_i) + (1-p).f(x'_{i+1},\theta_i) - f(x''_{i+1},\theta_i)\right] -$$

$$\left[p.f(x_{i+1},\theta_{i+1}) + (1-p).f(x'_{i+1},\theta_{i+1}) - f(x''_{i+1},\theta_{i+1})\right]$$

Now from (33) in turn, (36) will be non-negative if for any random variable x, the term $Ef(x, \theta) - f(Ex, \theta)$ is decreasing in θ, or equivalently, $Ef_\theta(x, \theta) - f_\theta(Ex, \theta) < 0$. But this in turn states that f_θ is a concave function of x, or $f_{xx\theta} < 0$. This completes the proof.

The proof of Theorem 3 now follows directly from Lemmas 2-6 and the fact that the valuation function for the less-constrained problem is identical to the valuation function for the ordinary problem viz. $V(w) \equiv V^*(w)$. This last fact follows in turn from the fact that at any solution to the less-constrained problem, the downward incentive constraints are also satisfied, so the solution to the less-constrained problem is feasible in the original problem.

2.5 The Intertemporal Structure of the Optimal Contract

The previous section established, among other things, that the within period distortions of the long-term contract mirror those of the static contract i.e. there will be too much employment in all but the lowest productivity state, θ_1, and also incomplete insurance, with the worker's income being lower in low-productivity states.

While it is certainly of some interest to know the nature of within-period inefficiencies of the optimal dynamic contract, it is perhaps more important to analyse the time-series behaviour of payments, actions, and payoffs in the optimal contract (Holmstrom and Hart (1985) identify this as a major rationale for investing long-term contracts). At a general level, the only result to date is Rogerson's (1985) characterisation of the time series behaviour of payments in the repeated-principal-agent model with moral hazard: he shows that the agent's average payment is either increasing or decreasing over time depending on whether the inverse of his marginal utility function is concave or convex. We are able to show that exactly the same result is true in our model. Furthermore, this is not surprising, it is a reflection of the fact that the optimal contract is a savings, as well as an insurance contract, and characterises the degree of equalisation of the agent's incomes over time, that is consistent with incentive-compatibility. Interestingly enough, the Rogerson condition is exactly the opposite of the one that would arise if the agent could independently save or dissave at a rate of interest equal to the discount rate, $1/\delta - 1$. That is, standaed savings condition is that the marginal utility of income follows a martingale, whereas Rogerson's condition is that the inverse of the marginal utility of income follows a martingale.

Because we have characterised the within-period distortion and the direction of history-dependence, however, there is much more that we can say about the optimal contract. In particular, we can establish the signs of the serial correlations of some key variables in the contract. Our fundamental result is that the marginal utilities of income of the agent are positively correlated over time. It is important to note that this result holds true also in the case where the downward incentive constraints bind i.e. the case where $f_{x\theta} < 0$. Using this basic result, we can show that under certain assumptions,this implies wages are positively correlated over time.

Thus, history dependence generates "persistence" of exogenous shocks in the sense that even if the θ_t which represents exogenous demand or supply shocks in the GHA model are independently distributed, the endogenous variables (e.g. wages) are serially correlated. While there are other perhaps more plausible, mechanisms which generate persistence, such as inventory accumulation, labour adjustment costs, or overlapping contracts, (see Sheffrin (1983) or Taylor (1980)), this work adds another possibility.

We begin by establishing Rogerson's result on wage payments for this model. In the context of our multiperiod model, his result can be proved first by showing that the inverse of the marginal utility of income of the agent follows a martingale process i.e.

$$(37)\quad 1/u'(y_t) = E[1/u'(y_{t+1})|y_t]$$

Then the result follows by observing that from (37) , y_t is greater or less than $F(y_{t+1})$ as u' is convex or concave respectively.

We can, in fact, derive (37) under more general conditions than those imposed in (A3) and (A4). All that is required is an interior solution for the y_t, for which (A4) is sufficient, and for the w_{t+1}. For then the first order conditions for the y_{it} give

$$(38)\quad -\pi_i + u'(y_{it})\,[\sum_{j\neq i} \lambda_{ijt} - \sum_{j\neq i} \lambda_{jit} + \pi_i \cdot \mu_t] = 0$$

Dividing (38) through by $u'(y_{it})$ and summing across all i gives

$$(39)\quad \sum_{i=1}^{n} \pi_i/u'(y_{it}) = 1/\mu_t.$$

Next, assuming also an interior solution for the w_{it+1}, we have from the first-order conditions,

$$(40)\quad u'(y_{it}) = 1/V'(w_{it+1})$$

But from Lemma 2 , we have that

$$(41)\quad V'_t(w_{it+1}) = -\mu_{t+1}$$

so combining (40) and (41) with (39) advanced forward one period, we get

$$(42)\quad 1/u'(y_{it}) = \sum_{i=1}^{n} \pi_i/\, u'(y_{it+1})$$

which is (37) This completes the argument.

We now turn to the question of intertemporal correlations. Let u_t be the utility of the agent at time t generated by the optimal contract.

Theorem 4

Assume that $E[u'(y_t)^2] < \infty$. Under assmptions (A3) and (A4), $u'(y_t)$ and $u'(y_{t-1})$ are positively correlated.

Proof

We already know from Lemma 2 and (38) and (39) above that if the y_{it} have an interior solution, then

$$(43) \quad E[u'(y_t) \mid w_t = w] = -1/V'_t(w)$$

But as V_t is concave from lemma 6, we can conclude that the left-hand side is decreasing in w. Now as w_t and y_{t-1} are both monotonically decreasing in θ_{t-1} by Theorem 3, we can conclude that $u'(y_{t-1})$ is monotonically increasing in θ_{t-1}. Therefore,

$$(44) \quad E[u'(y_t) \mid u'(y_{t-1}) = \Omega]$$

is increasing in Ω. Then as $u'(y_t)$ has finite variance by assumption, by the result in Parthasarathy (1977) p 206 we have

$$(45) \quad E[u'_t \mid u'_{t-1} = \Omega] = Eu'_t + \frac{\operatorname{cov}(u'_t, u'_{t-1})}{\operatorname{var}(u'_t)} \cdot [\Omega - Eu'_{t-1}] .$$

As the left-hand side of (45) is increasing in Ω, we conclude that $\operatorname{cov}(u'_t, u'_{t-1}) \geq 0$.

As the sign of the covariance is invariant to linear transformations of the variables, we can conclude that if marginal utility is proportional to total utility, then <u>total</u> utilities will be positively correlated. This will only be the case where utility is exponential. Hence, we have the corollary;

Corollary

Suppose u(y) is exponential. Then $\operatorname{cov}(u_t, u_{t-1}) > 0$.

Now we can apply these results to the GHA model. Recall that when we transformed the GHA model into our canonical form, we defined the utility of income of the agent to be minus the real wage. Hence, we can conclude from the corollary that if the worker's utility is exponential, then wages will be positively serially correlated.

2.6 Asymtotic Results for Low Discount Rates

We now turn to discuss the nature of the contract as the discount rate goes to zero. Although it is not our major purpose, a by-product of this analysis is a sufficient condition for history dependence which is easily interpretable without a particular model in mind - viz. that the discount rate be low enough. Our main purposes, however, are two; (i) to attempt an analysis of the asymtotic behaviour of the optimal long-run contract and (ii) to show that if the discount rate is low enough, the principal will not wish renage from the optimal long-run contract - so that this contract is actually an equilibrium outcome of the contracting game without precommitment. (Then, from the arguments presented in 2.1 above, it is an efficient equilibrium outcome.)

The argument is in several stages, and in the course of it we prove that under certain assumptions, the first-best can be approximately attained by a feasible contract if the discount rate is low enough, a result similar to Radner's results (Radner (1981), (1985)) although the argument is quite different.

Our starting point is Radner's 1985 paper. There, he shows that for a principal-agent problem involving moral hazard, the payoffs from any optimal second-best contract actually tend to first-best payoffs for both principal and agent as discounting goes to zero. More precisely, Radner is able to show that for any given $\varepsilon > 0$, there exists a contract which pays both parties within ε of any point on the first-best Pareto frontier which is an equilibrium in the contracting game for δ high enough.

As we have already said, Radner characterises this contract as a particular kind of equilibrium in the contracting game, where the agent is assessed for a certain number of periods, and then his cumulative performance (sum of outputs) is reviewed. If it falls below a certain level, the agent is punished for a given number of periods by a reversion to the second-best static contract, and then the assessment starts again. If he passes the review, the assessment phase is repeated from scratch.

Radner calls this an equilibrium in review strategies, but we can equally well call the principal's strategy in this equilibrium a review contract.

We can make two observations at this point. The first is that using Radner's arguments, it is possible to demonstrate that in our asymmetric information model also a review contract can approximately attain the first-best as the discount rate goes to zero. This in itself is quite remarkable, considering the very different structure of the moral hazard and asymmetric information problems, and is explained by the fact that Radner avoids having to analyse the optimal response of the agent to a review contract. Rather, his method relies on finding upper and lower bounds to payoffs of the principal and agent from the review contract, and the arguments used to establish these bounds are extremely general.

The second point is that (as we have noted in 2.1, and as Radner himself observes) the review contract is not the second-best optimal contract at any given discount rate. Hence, although Radner's results prove convergence of <u>payoffs</u> to the first-best, we cannot make any inferences from this about the structure of the optimal contract itself when δ is close to 1.

With our dynamic programming approach, it is possible to improve substantially on both these limitations. First, we can provide an extremely simple and intuitive argument to show that the second-best Pareto-frontier converges to the first-best as the discount factor tends to unity. Second, we can deduce two important features of the limiting contract as $\delta \to 1$. First, the sequence of expected discounted utilities generated by the contract, (w_t, v_t) , $t = 0,1\ldots$ tend to common limits w, v as $\delta \to 1$, although they may not do so uniformly in t. This implies that as the contract in each period, t, only depends on past history through w_{t-1}, the contract becomes "approximately" history independent as δ goes to 1. It also implies that the per period contracts, $(x_{it}, y_{it})_{i=1..n}$ all tend to the first-best contract $(x_i^*, y_i^*)_{i=1..n}$ although again, they may not do so uniformly in t.

Third, we can show that for a high enough discount rate, this contract may be an (efficient) equilibrium contract in the contracting game without precommitment by the principal.

We can conclude that at least one efficient contracting equilibrium is not of the "review" type for a high enough discount rate; rather than being backward looking (i.e. rewarding the agent on the basis of the average of past performances) it is forward-looking, in that the optimal contract, as $\delta \to 1$, exploits the fact that the "cost" in terms of the w_i, of implementing any per period contract $(x_i, y_i)_{i=1..n}$ goes to zero as $\delta \to 1$.

We now turn to the analysis of the model as δ goes to 1, and begin with the precommitment case.

(a) Convergence of the Pareto Frontier and the Optimal Contract to the First-Best

In this section, of course, we are only concerned with the the infinte-horizon case. Let $V(w, \delta)$ be the principal's infinite horizon valuation function, where we note now the explicit dependence of the function upon the discount factor. In order to investigate the asymtotic behaviour of this function, it is necessary to normalise by premultiplying all per period payoffs by $(1 - \delta)$. Then it follows immediately that $V(w, \delta) \leq V^f(w)$, where $V^f(w)$ is the first-best Pareto frontier.

Next, in order to ensure stationarity of the problem, we need to assume that the (α) constraints are stationary. However, to get the result, we also need to assume some "lock-in" effect i.e. that in each period but the first, the payoff from the alternative occupation to the agent is lower than the precontracting alternative. Let $\underline{u}$ be this alternative payoff to the agent. It seems that to obtain the convergence results, we need to assume that the lock-in effect is extreme i.e.

(A5) $\underline{u} = -\infty$.

Of course, (A5) is only consistent with certain specifications of preferences, such as g being logarithmic.

Finally, we shall not assume (A1) and (A2), but make the less satisfactory assumption that an optimal contract exists. The reason for this is to make the following crucial assumption;

(A6) u_x, $u_y > 0$, and x and y are unbounded above.

The implication of (A5) and (A6) is that the w_{it+1} are defined on the whole real line, and this is crucial to what follows. given this, however, we can show that $\lim_{\delta \to 1} V(w, \delta) = V^f(w)$. In other words, for low enough discount rates, payoffs arbitrarily close to the first best can be attained in the optimal contract.

The argument is simple. Consider a contract which yields the agent w in expected utility, and where the actions and income transfers in the first period are those of the first-best static contract i.e. $(x_i^*, y_i^*)_{i=1\ldots n}$, and in subsequent periods, contingent upon the w_i, the contract is the second-best optimal one. We choose for any w,the particular first-best contract which gives the agent w i.e.

(46) $Eu((x_i^*, y_i^*, \theta_i) = w$

Now, given (A5) and (A6), there are <u>no</u> restrictions on the choice of the w_i, so it is possible to choose the w_i to make this contract incentive-compatible. The argument is exactly the same as the one that establishes that with an ex ante constraint, and linearity of the agent's payoffs in income, the first-best actions can be made incentive-compatible by unrestricted choice of the income variables - the w_i here just play the role of the y_i. Also, from (46) and the fact that the whole contract gives the agent w, it follows that Ew_i = w.

Now this contract just described satisfies the participation and incentive-compatibility constraints. By (A5), it also satisfies the (α) costraints (or more accurately, (A5) makes the (α) constraints irrelevant) the so that it must be (weakly) dominated by the optimal contract i.e.

(47) $V(w, \delta) \geq (1-\delta).E\ v(x_i^*, y_i^*) + \delta.EV(w_i, \delta)$.

We can now prove the key result, which says that the cost of making the first-best static contract incentive-compatible, $V(w, \delta) - EV(w_i, \delta)$, divided by the normalising factor, $(1-\delta)$, goes to zero as $\delta \to 1$.

Lemma 7

If $V(w,\delta)$ is continuously differentiable, and the w_i are described above then for any $\varepsilon > 0$ there exists $\delta^0 < 1$ (possibly depending on w) such that

$$EV(w_i, \delta) > V(w, \delta) - (1-\delta).\varepsilon/\delta \quad \text{for } \delta \geq \delta^0.$$

Note that we know that V is continuously differentiable from Lemma 2 .

Proof

First, note that for a fixed w, the variation in the w_i needed to make (x_i^*, y_i^*) incentive-compatible is bounded - in fact,

(48) $|w_i - w_j| < B.(1-\delta)/\delta$.

Now by the mean value theorem,

$$\frac{EV(w_i, \delta) - V(w,\delta)}{(1-\delta)} = \frac{E[V'(\tilde{w}_i, \delta).(w_i-w)]}{(1-\delta)}$$

where $\tilde{w}_i$ is between w_i and w, so

(49) $$\frac{EV(w_i, \delta) - V(w,\delta)}{(1-\delta)} = \frac{E[(V'(\tilde{w}_i, \delta)-V'(w,\delta)).(w_i-w)]}{(1-\delta)}$$

using $Ew_i = w$.

Then using (48) , and the fact that V has continuous derivatives (with repect to w) we have for any $\varepsilon > 0$

$$E\left[(V'(\tilde{w}_i, \delta)-V'(w,\delta)).(w_i-w)\right] > -\left|E \quad V'(\tilde{w}_i, \delta) - V'(w, \delta)\right| .B.(1-\delta)/\delta$$

(50)

$$> - \varepsilon.(1-\delta)/\delta$$

for δ greater than some δ^0, as the w_i tend to w as δ goes to 1. It follows immediately from (49) and (50) that

$$(51) \quad \frac{EV(w_i, \delta) - V(w,\delta)}{(1-\delta)} < \varepsilon/\delta \text{ for } \delta > \delta^0,$$

and the lemma follows immediately from (51) .

Applying Lemma 7 to equation (47) , we find that for $\delta > \delta^0$, $V(w,\delta) > Ev(x_i^*, y_i^*) - \varepsilon = V^f(w) - \varepsilon$. We have therefore proved;

Theorem 5

Assume (A5) and (A6). For any $\varepsilon > 0$, there exists a $\delta^0 < 1$ such that $V(w,\delta) > V^f(w) - \varepsilon$ for all $\delta \geq \delta^0$.

This is analagous to theorems 7.1. and 7.2 of Radner(1985); in fact, it is possible to show that the above arguments also apply to Radner's model, which is a "moral hazard" one i.e. a contracting problem with unobservable actions. We now turn to the behaviour of the optimal contract as $\delta \to 1$. Recall that w is the expected utility that the contract must provide in the first period. Then we have;

Theorem 6

Assume (A5) and (A6). Then as δ goes to 1;

(a) w_t goes to w, all t.

(b) the per-period contracts, $(x_{it}, y_{it})_{i=1..n}$, converge to the first- best static contract which pays the agent exactly w.

Proof

(a) First fix w_t. Then $w_{it+1} \equiv w_i(w_t)$ goes to w_t itself as δ goes to 1. To see this, note that all the w_i must converge to the same limit as δ goes to 1. For suppose not. Then as the w_i lie in a compact set for all δ - this follows from the boundedness of u and the fact that the consumption set is closed - along some subsequence of δ, there exists a pair w_i, w_j which tend to different limits $w_i < w_j$. But then for δ high enough, it follows that incentive constraint (IC_{ij}) will be violated, contradicting the feasibility of the contract. Therefore, all the w_i tend to a common limit. from the participation constraint (EA), this limit must be w_t itself.

It now follows immediately that for all possible histories θ^t, w_1 tends to w, w_2 tends to w_1, and so on, as δ goes to 1. Therefore, w_t tends to w for all t, although it may not do so uniformly in t. This proves part (a).

(b) Suppose without loss of generality that the optimal contract at each dae is unique. By the stationarity of the problem, the optimal contract at t, $(x_{it}, y_{it})_{i=1...n}$ is a stationary function of w_t. Furthermore, it is a continuous function, by the Theorem of the Maximum. Hence, as all the w_t tend to a common limit, so do all the per period contracts $(x_{it}, y_{it})_{i=1...n}$. By Theorem 5, this limiting contract must be a first-best contract that pays the agent an expected utility of w. This completes the proof.

(b) Renaging by the Principal when the Discount Rate is Low

We show here that if the discount rate is low enough, the optimal contract may be (part of) an equilibrium outcome in the contracting game without precommitment. By the results in 2.1, then, it must be an efficient equilibrium outcome, and hence the question raised by Radner concerning the nature of efficient contracting equilibria has been partially answered.

It is very simple to show this. Consider a strategy for the principal where in the event of any deviation from the prescribed contract by himself, the principal plays the optimal static contract from then on, and where the agent optimises against the principal's strategy at all times and in all contingencies. This will be and equyilibrium in the contracting game without precommitment if and only if (a) the contract is optimal i.e. solves the dynamic programming problem in 2.1.(b) and (b) the principal does not wish to renage at any date. Given that the agent is himself always optimising, the best the principal can get from the latter course of action is $V^s(\underline{u})$. Then the no-renaging condition simply requires that

(52) $V(w_{it+1}, \delta) \geq V^s(\underline{u})$, all t, all i .

Now if all the w_{it+1} are bounded above by B, it follows immediately from (A5) and Theorem 5 that if $V^f(B) > V^s(-\infty)$, there exists a δ^0 such that (52) will be satisfied for all $\delta \geq \delta^0$.

Unfortunately, it is not clear that $V^f(B) > V^s(-\infty)$ can be satisfied under the same assumptions as those which make the preceeding analysis valid - in particular, (A6).

2.7 The Role of Terminations as an Incentive Device

In the real world, it is common enough to see the threat of termination of a contract used as an incentive device, for example, in the labour market. Although fires constitute a small proportion of total job separations (Johnson and Layard (1984)), it is well known that this understates the number of separations that take place because of unsatisfactory performance. This is for two reasons; first, many workers quit because they anticipate being fired, and the latter would do more damage to their work record - it is notoriously difficult to separate fires and quits. Second, in the event of redundancies, employers have some discretion in deciding which workers to make redundant, even if most adhere to the "last-in, first-out" principle, in general terms, (see Oswald and Turnbull (1985)), and will use this discretion to lose unsatisfactory workers.

Recently several papers (e.g. Malcolmson (1981), Shapiro and Stiglitz (1982), McLeod and Malcolmson (1985)) have attempted to model the connection between the role of involuntary terminations, or fires, as an incentive device, and involuntary unemployment. At the heart of the first two papers is a moral hazard problem where worker effort is unobservable. These papers suppose that firms offer "boundedly rational" or simple contracts, as an attempt to deal with this problem.

They are simple in that wage payments cannot be contingent upon output; the only incentive device to induce effort is a probability of job termination (or more precisely, a decision to terminate if the worker's effort is found to be low in the Shapiro-Stiglitz model). Termination can only be effective if firms do not fully insure their workers against fires; in fact, Shapiro and Stiglitz show that in equilibrium these firms will offer no unemployment insurance. Thus, in equilibrium, unemployment must be involuntary.

While the advantage of this work is that it provides a theory of truly involuntary unemployment, it clearly rests on the "assumption" of simple nature of the contracts. This assumption will be critical if in more complicated unrestricted contracts, termination is not necessary as an incentive device. What we do in this section is show that in the context of our model, termination is of no use as an incentive device unless the contract is extremely restricted. In fact, we show that even if the per period contracts (x_{it}, y_{it}) $i=1\ldots n$ are restricted to be history-independent, so that the only possible intertemporal linkages in the contract are through probabilities of terminations being contingent upon last period's announcements, the optimal contract will not make use of these - the optimal contract will have all termination probabilities equal to zero.

In fact, the only time when termination is a useful instrument is when income-transfers between principal and agent cannot be made contingent upon announcements, which is analogous to the Malcolmson-Shapiro-Stiglitz assumption that wages cannot be based on output/performance. This suggests that a flat wage schedule is really a very strong assumption.

The second possible role for terminations is in the case where the principal cannot precommit to the contract. (This is the case considered by Radner (1985)). Here, it is on the face of it possible that certain long-term contracts can be sustained by the threat of principal (or agent) terminating if the principal reneges on a per period contract. What we show here is that this threat is <u>only</u> useful if the principal and agent must make the termination decision independently of one another i.e. move simultaneously, or if the agent decides to terminate first in any period. Neither of these correspond to the natural sequence of decision-making in our model, where the principal is first mover, as explained below.

We begin with analysing the role of terminations in restricted contracts and for simplicity we work with the two period contracting problem presented in 2.3. Let ρ_i be the probability that the contract is <u>not</u> terminated in the second period, following an announcement that the agent is of type i in the preceding period, and let $\underline{v}$ be the payoff to the principal from the alternative occupation in the second period. With this additional instrument the two-period labour contracting problem becomes

$$\max_{w_i, \ell_i, y_i, \rho_i} \sum_{i=1}^{n} \pi_i(u(y_i) - l_i + \rho_i . V^{s}(w_i) + (1-\rho_i).\underline{v})$$

$$\text{s.t.} \quad f(\theta_i\ l_i) - y_i + \rho_i(w_i - \underline{u}) \geqq f(\theta_i, l_j) - y_i + \rho_j(u_j - \underline{u})$$

$$\text{and} \quad \sum_{i=1}^{n} \pi_i\left[f(\theta_i, l_i) - y_i + \rho_i . w_i + (1-\rho_i)\underline{u}\right] \geqq w$$

$$w_i > \underline{u}$$

The first order conditions with respect to the y_i and ρ_i are, assuming an interior solution for the y_i,

$$(53)\ \pi_i . u'(y_i) - \lambda_i, {}_{i+1} + \lambda_{i-1,i} - \pi_i . \mu = 0$$

$$(54)\ \pi_i(V^s(w_i) - \underline{v}) + (w_i - \underline{u})(\lambda_{i,i+1} - \lambda_{i-1,i} + \pi_i . \mu) \lessgtr 0$$

with complementary slackness for ρ_i in (54) . Substituting (53) in

(54) we find that it becomes

$$(55)\ \pi_i\left[(V^s(w_i)-\underline{v}) + (w_i - \underline{u}).u'(y_i)\right].$$

If there is any surplus to be had in the contract in the second period i.e. $V^s(\underline{u}) > \underline{v}$, we know that at least one of $V^s(w_i)-\underline{v}$ and $w_i-\underline{u}$ is strictly positive, and $u'(y_i)$ is always strictly positive so (55) will also be strictly positive, implying $\rho_i=1$, all i, or in other words, there are no terminations in the optimal contract. Note that as the above argument does not rely on the first-order conditions for the w_i; it also applies to the case when the contract is constrained to be history-dependent, i.e. $w_i=w_j$ all i, j. (In this case, of course, the principal will set $w_i = \underline{u}$, all i). This establishes the claim that terminations are redundant as an incentive device, even if contracts are restricted to be short-term.

The picture is different if income transfers also cannot be contingent upon announcements. Then if $V^s(w) - \underline{v}$ is sufficiently small and $w - \underline{u}$ is positive, then by varying the ρ_i, the principal can at minimal cost to himself, introduce self-selection into the contract. To ensure $w > \underline{u}$, however, in the optimal contract, requires that the principal discount the future more than the agent, and some condition like this may be sufficient to prove formally that terminations i.e., $\rho_i<1$ are used in the optimal contract where $w_i=w$, $y_i=y$.

We now turn to the use of terminations as a device to prevent renaging by the principal. The kind of renaging we are interested in is where the principal renages on an agreement to give the worker w_{it+1} in the next period, but tries to reduce the agent's surplus. We assume that the principal is trustworthy <u>within</u> the time period i.e.

if he offers (x_{it}, y_{it}) i=1...n he can be relied upon to stick to the payments y_{it} at t.

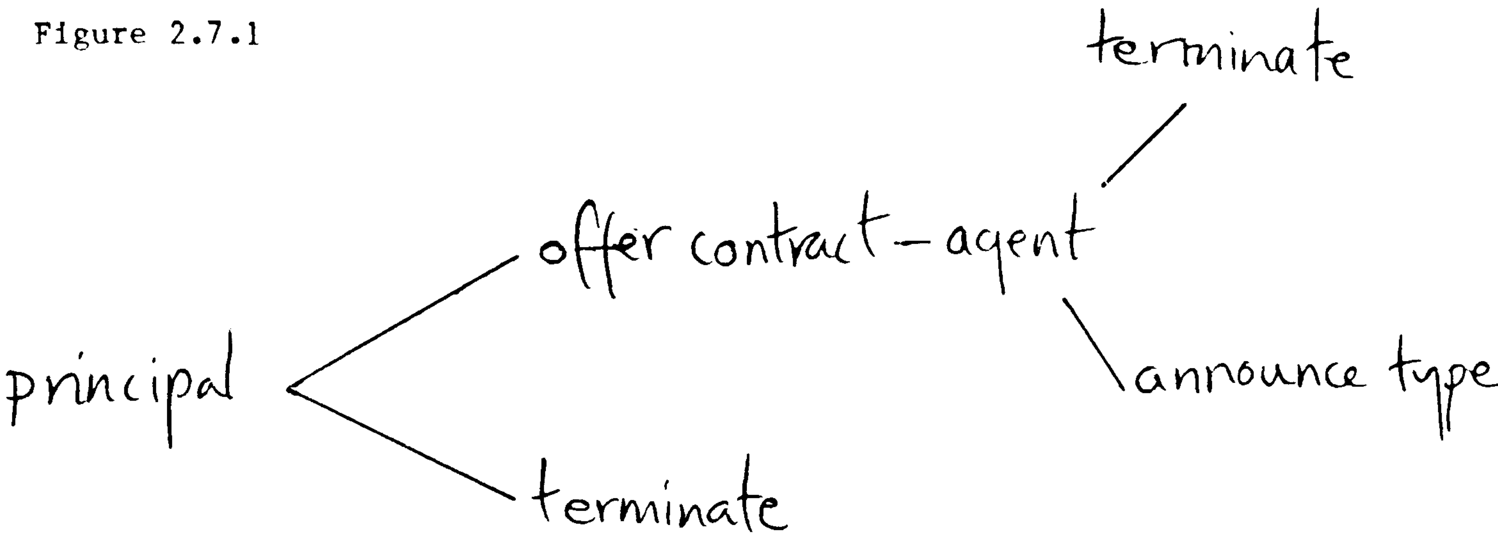
The issue is straightforward enough in broad terms; the principal will not wish to renage on the current contract (x_{it}, y_{it})i=1...n if and only if the gains from doing so do not outweigh the punishment that ensues. Of particular interest are the worst punishments that can credibly be imposed on the principal, or what Abreu (1982) calls optimal punishments which are simply the (sequential) equilibria of the contracting game which yield the principal the lowest payoff.

We shall not attempt to derive optimal punishments in our model, but merely show that optimal punishments do not involve terminations of the contract be either party. This raises the question of what the optimal punishment payoffs for the principal are - we argue, rather in formally, that they are the payoffs from the short-term contract, $V^s(\underline{u})$.

Consider first the timing of moves by the two parties in a typical time period when the principal cannot precommit to an entire contract in advance. First, the principal offers a contract $(x_{it}, y_{it}, w_{i,t+1})$ i=1...n. The worker decides whether to accept or reject it, (possibly before or after having observed θ_f). Then if he accepts, he makes a type announcement.

We have not yet specified at what point the principal makes his termination decision, because if he can precommit, he will never wish to use it, as we have just shown. However, the only sensible timing is if he decides to terminate <u>prior</u> to the agent, so that the sequence of events is as follows;

Figure 2.7.1



Because the principal is a Stackleberg leader, the threat of termination cannot credibly be used against him. For example, suppose the agent threatens to terminate the contract if the principal renages on a promise to offer a contract at t that gives the agent $w > \underline{u}$. Would the agent actually carry this out? The answer is not, because the most he can get from this strategy is $\underline{u}$, whereas if the principal renages by offering a contract that yields him strictly more than $\underline{u}$ *in the current period*, then he can always do better by accepting it, no matter what happens to him for not carrying out his own pubishment threat in t+1, t+2, and so on.

This problem would of course, not arise if principal and agent made the termination decision simultaneously - then termination is a credible threat, as termination decisions are mutual best responses, or in Nash equilibrium, on *any* game histories where termination has not yet taken place (see McLeod and Malcolmson (1985)) - or indeed if the agent made the termination decision first in each period.

The question remains as to what is the worst punishment that can credibly be inflicted on the principal in the absence of termination. (We have already discussed Nash punishments in 2.6(b)). In the complete information version of this game, i.e. where θ_t is observable by both parties; we know from the results of Fudenberg and Maskin (1983) that the principal can "nearly" be driven down to his security level, if the discount factor is high enough. His security level attained by the agent choosing the worst (for the principal) of the (x_i, y_i) pairs in the static contracts, so the principal's security level is in fact

$$v_{min} = \max_{x,y} v(x,y)$$

$$\text{s.t. either} \quad Eu(x, y, \theta_i) \geqq \underline{u} \quad \text{or} \quad u(x, y, \theta_1) \geqq \underline{u}$$

depending on whether the participation constraint is either <u>ex ante</u> or <u>ex post</u>. The punishment which achieves the principal's security level is however, (a) not a best response to any possible contract offer (b) only yields the agent $\underline{u}$. This raises difficulties. First, (a) implies that the agent will have an incentive to deviate from the punishment; these deviations can be detected to any degree of accuracy over a long enough time period, but once detected, the agent can only be "punished" by having $\underline{u}$ imposed upon him, which was what he was getting anyway. The conclusion seems to be that v_{min} cannot be credibly imposed on the principal, but whether a punishment arbitrarily close to v_{min} can be so imposed is an open question.

Conclusions

In this chapter we have attempted a fairly thoroughgoing analysis of the nature of long-term contracts, with the exception that we have not made a systematic attempt to isolate the savings element of the contract from the risk-sharing element. This is important because one of the main conceptual issues in long-term contracts is whether there is history-dependence in the long-term contract over and above that implied by the savings component.

There are two ways in which savings could be introduced explicitly. The first is to suppose that in each period, the principal offers a level of savings - at a given rate of interest - to the agent as part of the contract. The second is to suppose that the principal chooses the contract as above, subject to the condition that the agent is saving optimally with a third party. The first approach seems most appropriate for analysing the history-dependence question, as at allows a decomposition of the contract into a savings part and a within period transfer. This is the approach taken by Malcolmson and Spinnewyn(1985), who show for the case of contracting with hidden actions, there is no residual history dependence over and above that implied by the savings part.

It is an obvious topic for future research to see whether this result extends to hidden information problems of the type studied here; in view of Townshend's result that the optimal contract is <u>not</u> a savings contract, it seems likely that it does not.

In addition, it is possible to show that with savings contracts, the kind of history dependence studied by Roberts(1982) does not disappear even with quasi-linearity. The reason for this is as follows. The source of history dependence is equation (15) - comparing it with equation (14), we see that the problem is that second-period profit cannot be transferred into the first period. This could of course be achieved by a zero interest rate savings contract which allowed, contingent upon unemployment, the agent to <u>dissave</u> by an amount $\int_{w_1(u)}^{1} (\theta - w_1(u))d\theta = k$.

But then the agent would, in a period 1 unemployment state, have a payoff of $-k < 0$ i.e. would have to repay the loan, and so would prefer to leave the contract. In fact,one can show that allowing for a savings instrument in the contract changes nothing - the optimal contract is still as ddescribed in 2.2. Hence, the Malcomson-Spinnewyn arguments do not apply to this sort of history dependence.

3 Search Equilibrium with ex ante Heterogeneity: Efficiency and Screening via Employment Histories

Introduction

This chapter is concerned with the consequences of differences in skills between workers (or the productivities of firms), both observable and unobservable to the other side of the market, for labour markets where search for trading opportunities is costly.

Our major aims are two : first, to demonstrate that the presence of such ex ante differences, search equilibrium displays a kind of inefficiency which has not been noted in the literature before. Furthermore, the direction of this inefficiency is unambiguous - in equilibrium, both output and employment, are lower than is socially optimal. Our second aim is to analyse the nature of equilibrium in the case where worker skills are unobservable by prospective employers. In particular, we show that equilibrium in this case must often be characterised by informational externalities, in that certain elements of an individual worker's employment history, (e.g. duration of last spell of employment or unemployment, number of different jobs), convey valuable information about individual worker's skills. In this case, we show that, in equilibrium, firms may want to make their hiring decisions contingent upon these employment histories (e.g. reject applicants who have had too many jobs, or who have been unemployed too long).

There is some direct evidence that such hiring policies are pursued, and it is certainly a widely held belief among the unemployed that such practices are in operation (see the discussion in 3.3. below). Also, there is some statistical evidence consistent with such policies, especially for the duration of unemployment, where it is very clear that at an aggregate level, re-employment probabilities decine with the duration of unemployment. However, there are a number of competing explanations for this latter phenomenon, and so we defer detailed discussion of the evidence to a separate section.

The models presented to discuss these issues are of the stochastic search, or matching type, pioneered in economics by Mortensen, and since much developed by Diamond and Pissarides, among others. This type of search model is not the only one available, but seems much superior to the alternatives, both because it provides a coherent and well-understood framework in which to discuss questions of efficiency, and because it arguably provides a better description of the kind of search behaviour that actually goes on, at least in unskilled labour markets, than other available models.

We begin with a more detailed discussion of the relative merits of the various types of search models in Section 3.1. In Section 3.2 we survey the Mortensen-Diamond-Pissarides (MDP) model, and also the results on the inefficiency of equilibrium obtained by these authors and others. The ex ante heterogeneity model with symmetric information is presented in 3.3 and the basic inefficiency result in 3.4. In Section 3.5 we discuss evidence suggesting that screening via employment histories may be common. Equilibrium concepts for the model with asymmetric information are presented in 3.6, and pooling and separating equilibria are analysed in Sections 3.7 - 3.10. In 3.11 we attempt to reconcile the predictions of the model with the evidence, and we conclude by discussing possible shortcomings and extensions of the analysis.

3.1 Sampling vs. Matching Models of Search Equilibrium

There is a vast and diverse literature on equilibrium in markets that are not "frictionless". Even the notion of "friction" is not unambiguous. It can mean either that information about agents on the other side of the market can only be acquired at some cost, either in money or time, or that the action of pairing, or matching, with a particular agent on the other side of the market is itself costly, in the sense that if the match is rejected, it takes time or money to consummate another match. Any information that is then exchanged is between matched partners. These two kinds of friction are not identical; in the former, information flows are independent of the process by which e.g. buyer and seller are eventually paired, whereas in the latter they are inherently constrained by the matching process - any information exchanges take place between matched partners.

Consider for example, the Salop-Stiglitz (1977) model of bargains and ripoffs. Here, consumers simply choose whether or not to become informed of the location of low-price shops at a cost, (e.g. by buying a local newspaper)and then go to such a shop(e.g. by choosing a shop at random). Hence, the information is obtained before (and independently of) the match being made.

On the other hand, models such as Diamond (1982) or Pissarides (1984) have firms and workers, (or sellers and buyers) first being stochastically matched, and then if the match is accepted the wage (price) being determined as a function of the productivity or value of the match . The probability that a match takes place over a unit time period may be either exogenous or under the control of the agent.

Now from the point of view of the agent, a match is a sample from the equilibrium distribution of wages and prices. However, his sampling problem differs from the standard one (as exposited for example, in Morgan and Manning(1985)), as the agent (e.g. the worker) cannot choose the number of firms to sample in any given time period - he is effectively constrained to follow a sequential search strategy. (This is inherent in the assumption that matches follow a Poisson process, and time periods are very short, so that the probability of several matches in a unit time period is negligible.)

Hence, in sharp contrast to the Salop-Stiglitz type of model, sampling and matching are constrained to be identical acts.

One important consequence of this is that if the sample/match is rejected, the whole process starts again, so that this type of model is inherently dynamic, (although assuming stationarity makes it analytically quite tractable). This is an additional difference from Salop/Stiglitz type models, and in fact points to an inconsistency in the latter - although their consumers follow a fixed-sample-size search rule, this is not the optimal rule under their assumptions: as Reinganum(1979) points out, the optimal rule is seqential sampling. But then an explicit time-scale must be introduced into the model.

It is convenient then , and not too unreasonable, to call models such as the Salop-Stiglitz one sampling models, where the timing of events is

buyers - acquire information
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad >$ match
sellers - set price

By contrast, it is natural to call models of the other kind *matching* models, where the sequence of events is

$$\left.\begin{array}{l}\text{buyer}\\ \text{seller}\end{array}\right\rangle \text{ matched - exchange information } \left\langle\begin{array}{l}\text{accept - set price}\\ \text{reject}\end{array}\right.$$

Models where the matching process is not stochastic, but where agents sample sequentially, such as Reinganum's model, occupy an intermediate position in this schema. For example, in her model, sellers set price *before* being matched.

One important implication of the differing structure of the two models is that the price and wage setting rules must often differ. In the matching model, matched partners are in a situation of bilateral monopoly, so that unless binding price or wage contracts can be signed, the price or wage must be determined by bilateral bargaining (for an explicit model of such a process, see Rubenstein and Wolinsky(1985)). If one side or the other had the ability to set prices in advance (as is commonly assumed in sampling models), this would make little difference, as this agent could not use the price/wage to affect the flow of matches that he encounters - all that would happen is that he could appropriate all the surplus from the match. By contrast, the trade-off between price set and the flow of customers is fundamental to sampling models - this trade-off will depend on the distribution of search costs in the population, and depending on this distibution, various equilibrium price configurations can arise (single-price equilibria, price-dispersion, etc) as Braverman(1980) shows.

As well as differing in these essentials, these models differ in the emphasis they place on various different features. For example, sampling models stress how equilibrium differs with different assumptions about buyer sampling behaviour (e.g. Burdett and Judd, (1983)) whereas in the matching model, agents are compelled to "sample" sequentially.

For our purposes, the choice made here between matching and sampling models is on both grounds of realism, and analytical richness and tractability.

On the first, the choice between them depends largely on what control workers actually have over the search process. Sinfield (1981) argues that this differs considerably between skilled and unskilled workers. The former are more in demand, and so skilled jobs are much more frequently advertised; in addition to which, as Sinfield says; "there are certain known places to look for notification of jobs besides the job shop and an efficient job seeker who inteds to stay within a certian range of jobs may save himself the tedious and depressing trudge from factory to factory that generally still has to be made by those without a skill." This suggests that skilled wokers have considerable control over the sampling process - in particular, they may be easily able to vary the intensity of search. By contrast, Sinfield arues that for the unskilled, job search is highly random; vacancies are rarely advertised, and they rely to a great extent on word of mouth and the reccommendation of friends and relatives.

Hence, one could argue that the MDP model is more applicable to unskilled rather than skilled labour markets.

On the second, the matching model is vastly preferable. It has been developed, principally by Mortensen, Diamond and Pissarides, to the point where there is a single commonly accepted, plausible, and easily understood notion of equilibrium. In addition, it has been shown that the equilibrium is inefficient on several important margins of decision-making, worker mobility, search intensity, and job acceptance, and the reasons for this inefficiency are well understood (see especially Pissarides (1983)). In addition, it is also on the way to becoming an alternative "microfoundations" which can account for some macroeconomic phenomena which are beyond the market-clearing or fixed-price approaches (see especially Pissarides(1985)). By contrast, there are few common themes in the sampling model literature, and even conditions under which wage (or price) dispersion occurs are not fully understood.

For these reasons, we choose to work with the MDP model in examining issues raised by transferable (and possibly unobservable) skills. While it is probably possible to construct a sampling type model where unemployment histories convey useful information, it would no doubt be extremely difficult.

3.2 A survey of the literature

The various inefficiencies that can arise in the MDP model have been intensively investigated over the past few years. The current state of knowledge is thoroughly exposited and surveyed by Pissarides (1983). It seems that there is no version of the model where firms or workers have inherently different skills or productivities which they bring to a match, although one version of the MDP model allows the productivity of the matches to be stochastic, or *ex post* heterogeneity. Similarly, there is no analysis of the implications of asymmetric information about match productivities - although Jovanovic (1979), (1984) has analysed the case where both firm and worker have common, but incomplete, information about match productivities, and learn about these by observing the cumulative flow of output over time. This yields some interesting predictions about the relationship of wages and separation probabilities to job tenure. In particular, he is able to show that job separation hazard rates (the conditional probabilities of moving from one job to another or from a job to unemployment), depend on the length of job tenure as a result of optimising behaviour on the part of workers and firms. Furthermore, they do so in a way that is consistent with the empirical evidence, first rising and then declining (e.g. Jovanovic (1979), p. 981).

Therefore, Jovanovic's work is quite close in spirit to my attempt to model duration dependence, although our approaches differ substantively both in the assumptions made and in the issues addressed.

I conclude this section by describing the inefficiencies arising in the MDP model in some detail, in order to set the stage for my inefficiency results with *ex ante* heterogeneity.

To do this, I begin by sketching a few features of the MDP model. Suppose that the labour market is composed of L workers and J firms, which are identified with jobs. U of these workers are unemployed and V of these jobs are vacant. The number of matches per unit time period is $x(u, v)$ with $\partial x/\partial u, \partial x/\partial v > 0$ (and $u = U/L$, $v = V/L$). When a firm and worker are matched, they must decide to accept or reject the match. If the productivity of the match is stochastic, this is a non-trivial decision - both parties must decide on a reservation productivity, below which a match is rejected. If, on the other hand, all matches have the same productivity, all matches will be accepted in equilibrium. The wage is then set to share out the surplus from the match in proportion to the bargaining strengths of the two parties.

Apart from the choice of the reservation productivity, there are two other margins of decision-making in the model which may give rise to inefficiencies - mobility, (or freedom of entry and exit of firms and workers) and choice of search intensity (advertising) by workers (firms).

What Pissarides shows is that in successive versions of the model where only one of these three margins of decision-making is operative, the equilibrium will be inefficient, although the direction of the inefficiency - i.e. whether output and employment are too high or too low - is ambiguous, unless additional assumptions are introduced.

In the case of mobility, for example, by entering the market both firms and workers incur costs (i.e. the cost of opening up a vacancy for firms, the cost of returns from alternative activities for workers) and obtain the potential benefit of a flow of output from a match, and in addition raise the numbers of job matchings by $\partial x/\partial v$ and $\partial x/\partial u$ respectively.

To put it another way, a small increase in v will increase the matching probabilities of workers x/u slightly, and reduce the matching probabilities of other firms x/v. A small increase in u will have the opposite effect. Therefore, the direction of the externality - whether it leads to under- or over-employment - is not clear. Furthermore, calculation of the marginal external impacts is complicated by the frictional nature of the market; agents do not simply equate flow costs and benefits (typically, the costs and benefits are discounted by different factors, as in Pissarides (1983) equations 5 and 9), whereas in determining socially efficient levels of the labour force, unemployment, and vacancies, flow costs and benefits are simply added together (c.f. Pissarides (1983) equation 25).

For these reasons, it can be shown that the inefficiency centres round the relationship between the marginal impacts $\partial x/\partial u$, $\partial x/\partial v$ and the firm and worker matching probabilities, x/v, x/u. In fact for efficiency, $\frac{\partial x}{\partial v} = (1-\beta)\ x/v$, $\frac{\partial x}{\partial u} = \beta . x/u$ is required. At best, with free entry and exit for one side of the market, these conditions can only hold by accident; at worst, with free entry for both sides, they are incompatible with equilibrium, as a necessary condition for them to hold is that x(u,v) exhibit constant returns to scale, and the latter leads to non-existence of equilibrium (Pissarides, op. cit p. 16).

A very similar kind of inefficiency arises when match productivities are stochastic. A decision to break up a match increases the number of matches in the economy by $\frac{\partial x}{\partial u} + \frac{\partial x}{\partial v}$, an effect which is external to the matched parties. Efficiency requires $\beta.x/u + (1-\beta).x/v = \partial x/\partial u + \partial x/\partial v$, a less stringent requirement than for the mobility case. The only case where it will be satisfied (except by accident) is where $x(u,v)$ has constant returns to scale and $u = v$. In this case, x/u and x/v are independent of small changes in u or v so match break-ups can have no external effects.

Finally, decisions about search and advertising may also lead to inefficiencies for much the same reason; for example,increasing search intensity may decrease other workers' matching probabilities, and raise those of all firms, effects which are not internalised.

More recently, McKenna (1985) has presented a model which essentially extends the Diamond (1982) and Pissarides (1983) analysis of mobility to allow for mobility costs that differ between individuals; as one would expect, the inefficiency results are very similar. McKenna calls these costs search costs which seems a little misleading; as they are presented, they are not the flow costs of search, but the discounted present value of these search costs. This DPV then is the "fee" a potential entrant must pay to make a once and for all entry into the labour force.

What is common to all these external effects is that they work through the matching probabilities; all the decisions have marginal impacts on these probabilities that are not internalised. This is an important point to note, as the inefficiencies that arise as a consequence of ex ante heterogeneity arise in an entirely different way.

3.3 A MDP Model with Transferable Skills

It has been argued in 3.2 that a major limitation of the matching type of search model is that the implications of *ex ante* heterogeneity have not been explored. In this section we present a simple MDP type model where workers have transferable skills, and derive socially optimal rules for match formation. The model is in fact a generalisation of the one set out in Pissarides (1984), and so allows for total match productivity to be composed of the sum of a match-specific component, θ, and the transferable ability of skill of the worker, a_i, where i indexes the worker's type.

We allow a match-specific component for two reasons; first, it enables us to compare our results with the literature on matching with *ex post* heterogeneity, especially Pissarides(1984), and second, it provides a certain amount of "noise" in the model, which mitigates a "discouraged worker" effect which arises if θ is constant.

We now set out the model formally.

The Model

The economy is composed of workers and jobs. Firms are identified with jobs. There are L workers who are independently drawn from a discrete distribution of abilities $a_1,..a_n$, $a_i > 0$, with probabilities $\pi_1...\pi_n$, and J identical jobs. A worker of ability a_i when matched with a job can produce $y = \theta+a_i$. Both θ and a_i are common knowledge. The match specific productivity element, θ, is distributed continuously on $[o,\infty)$ with density function $f(\theta)$. Thus output to depend both on a match-specific component, θ, and the transferable skill of the worker, a, and is the distinctive feature of our analysis.

Jobs and workers are brought together by a matching technology in which the number of matches in a given unit time period, $X(U, V)$ is a function of unemployment and vacancy numbers. We follow the rest of the literature in not trying to model the matching technology explicitly. Rather we just assume that the number of matches is increasing in the numbers of both vacancies and unemployed. To abstract from the externalities arising from job and worker rejection investigated by Pissarides(1983),(1984) we assume that $L=J$ (i.e. there is no excess demand or supply in the labour market), and that X exhibits constant returns to scale. From the previous discussion in 3.2, we know that this implies that the matching probabilities X/U and X/V are constant and independent of U and V..

Finally, jobs break up at an exogenous rate s per period. We shall only be concerned with steady states in the model i.e. where flows into and out of unemployment are equal for each ability type. The expected flow into unemployment for each type is $(L_i-U_i)s$, or in per capita terms, $(1-u_i)s$. The flow out of unemployment is calculated as follows. The probability of any unemployed worker making a job contact is $X(U,V)/U = X(1,1) \equiv x$ by constant returns to scale and $U=V$. The probability of such a match being accepted is q_i. There are U_i such workers, so the total expected flow is $q_i.x.U_i$, or in per capita terms $q_i x.u_i$, with $u_i = U_i/L_i$. By the law of large numbers, actual and expected flows per capita can be taken to be equal, and so we can write the flow equilibrium conditions

(1) $(1 - u_i).s = q_i.x.u_i, \qquad i = 1,2,....n.$

Solving for the equilibrium unemployment levels, we have;

(2) $u_i = \dfrac{s}{s+ q_i.x}$.

Efficiency

We are now in a position to derive the efficent, or socially optimal, rules for match formation. As both firms and workers are assumed to be risk-neutral and there are no vacancy costs or unemployment costs and benefits, the maximand is the discounted flow of output per capita, which by the law of large numbers is non-random. There is no real loss of generality in taking the maximand to be the steady-state flow of output, as in what follows steady-state optimal rules are either identical to the rules which maximise discounted flows, or correspond to the special case where the interest rate is set to zero. This can either be checked directly, or by using the formulae of Diamond(1980).

First, observe that from the law of large numbers, output per capita can be written

$$(3) \qquad Y = \sum_{i=k}^{n} \pi_i(1-u_i)Y_i^e$$

where u_i is as in (2), and

$$(4) \qquad Y_i^e = a_i + \int_{\theta_i}^{\overline{\theta}} \theta f(\theta)d\theta/(1-F(\theta_i)).$$

Hence, form (2) u_i depends on q_i. In turn, q_i is determined by a critical reservation index of ability k, and job specific productivity, θ_i, such that the match is rejected if $i < k$ or $\theta <$ θ_i^*. In other words, $q_i = x.(1-F(\theta_i))$ if $i \geqq k$, and 0 otherwise.

We show in this section that the efficient reservation values (i.e. those that maximise the steady-state flow of output) have two properties. First, matches should be formed _whatever_ the ability of the worker (i.e. k=1) and second, the reservation job-specific productivity, θ_i depends non-trivially on i.

It is immediately obvious from (3) that k=1 maximises the flow of output as long as $Y_i^e > 0$, and the latter holds by assumption. In other words, no matches should be rejected solely because the worker's ability is too low.

Next recalling the definition of u_i from (2), and using the fact that $q_i = x.(1-F(\theta_i))$, if the θ_i maximise Y, then it follows that each θ_i maximises

$$(5) \quad \frac{\left[\int_{\theta_i}^{\overline{\theta}} \theta f(\theta)d\theta + a(1-F(\theta_i))\right]}{s/x \quad + (1-F(\theta_i))}$$

This implies, from the first order conditions;

$$(6) \quad \theta_i = \frac{(1-F(\theta_i).x.\theta_i^e - s.a_i}{s + (1-F(\theta_i)).x}$$

where $\theta_i^e = \int_{\theta_i}^{\overline{\theta}} \theta f(\theta)d\theta/(1-F(\theta_i))$.

When a=0, (6) reduces to formula (29) in Pissarides with b=k=0, using the fact that $X(1,1) = \frac{\partial X}{\partial u} + \frac{\partial X}{\partial v}$ from Euler's theorem.

In addition, from (6),

$$(7) \quad \frac{d\theta_i}{da^i} = \frac{-s}{s + (1-F(\theta_i)).x}$$

so that θ_i is strictly decreasing with a as long as $\theta_i > 0$. Hence, for high enough ability types, the match should be accepted for <u>any</u> realisation of the match-specific shock.

3.4 Equilibrium and Inefficiency

In this section we describe the equilibrium of the transferable skills model and explain how and why it is inefficient.

Any equilibrium is fully described by a wage function $w(\theta,i)$ which specifies the wage to be paid to a type i worker in a match with match-specific productivity θ, and reservation job-specific productivities θ_i , and an ability k such that a job match is rejected if and only if $\theta < \theta_i$ or $i < k$. Of course, these reservation values will in general be different to the socially optimal ones - that is the whole point of this section - but the context should make it sufficiently clear which is which, without having to index them differently.

What we will show, in fact, for the case where θ is not "very" random is that if the firm has some bargaining power over wages - i.e. appropriates some of the surplus - then the equilibrium marginal, or reservation type, k, will be strictly greater than 1. In other words, the equilibrium will be inefficient, and output and employment will be too low relative to the optimum.

Furthermore, (unlike the congestion inefficiencies surveyed in 3.2?) the cause of this inefficiency is transparently simple. From the point of view of the firm, worker ability is a random variable, so that if a worker's ability is too low relative to the mean, the firm will reject the match in the hope of being matched with a better worker. However, from the point of view of the whole economy, worker abilities are <u>fixed</u>; - per capita, there are π_i workers with productivity a_i - so there is nothing to be gained by breaking up a match with a low productivity worker.

We turn now to a description of the equilibrium. The equilibrium wage and reservation values are determined by asset equations implicitly defining net worths, or asset values, for both firms and workers, and a rule for wage determination.

Let $W_e(\theta,i)$ be the net worth of an employed worker of ability a_i in a match with productivity θ, and $W_u(i)$ be the net worth of an unemployed worker of type i. Similarly, let $W_f(\theta,i)$ be the net worth of a job with a match of productivity $a_i + \theta$, and W_v the net worth of a job vacancy.

These values must satisfy the standard asset equations that the rate of return times net worth equals flow income plus expected capital gain. For the workers, these asset equations are;

(8) $$r.W_e(\theta,i) = w(\theta,i) + s(W_u(i) - W_e(\theta,i))$$

(9) $$r.W_u(i) = q_{wi}(W_e^e(i) - W_u(i))$$

with

(10) $$q_{wk} = x.(1-F(\theta_k))$$

and

(11) $$W_e^e(i) = \int_{\theta_i}^{\overline{\theta}} W_e(\theta,i)f(\theta)d\theta/1-F(\theta_i)$$

Analogous equations for jobs as assets are

(12) $$r.W_f(\theta,i) = (a_i + \theta - w(\theta,i)) + s(W_v - W_f(\theta,i))$$

(13) $$r.\ W_v = \sum_{i=1}^{n} \pi_i^*.q_{wi}(W_{fi}^e - W_v)$$

$$(14) \qquad W^e_{fi} = \left[\int_{\theta_i}^{\overline{\theta}} W_f(\theta,i) f(\theta) d\theta \right] / (1-F(\theta_i))$$

where π^*_i is the probability that the worker matched with the firm is of type i, and is in fact an endogenous variable, being π_i times the unemployment rate of type i workers, apprpriately normalised;

$$(15) \qquad \pi^*_i = u_i . \pi_i / \sum_{i=1}^{n} u_i . \pi_i .$$

The reservation θ_i are now determined by the condition that each side of the market must be indifferent between rejecting and accepting the match, given that the worker is of type i i.e.

$$(16) \qquad W_f(\theta_i,i) = W_v$$

$$(17) \qquad W_e(\theta_i,i) = W_u(i).$$

The precise definition of the reservation ability index k is given below.

Although at first sight it is possible that (16) and (17) give different answers, the rule we adopt for wage determination ensures that there is no inconsistency. Following Diamond and Pissarides, we suppose that the total surplus from the match, $W_e(\theta,i) + W_f(\theta,i) - (W_v + W_u(i)$ is divided up according to a generalised Nash bargain, so that the worker receives a fraction β of it. Then the wage is implicitly determined by

$$(18) \qquad W_e(\theta,i) - W_u(i) = \frac{\beta}{1-\beta} \left[W_f(\theta,i) - W_v \right]$$

It should be clear from (18) that (16) and (17) are in fact identical equations.

To bring out the issues clearly, we first examine the special case where there is <u>no</u> match-specific element to productivity. We show that for this case, matches involving low ability types may be rejected, implying that the equilibrium is inefficient, and output and employment are too low relative to the social optimum.

<u>Case 1; $\theta \equiv 0$</u>

In this case, $q_{wi}=x$ if i is an ability type that is employed in equilibrium, and $q_{wi}=0$ otherwise. In this case, if k is the critical index, simple manipulation of (8)-(15) and (18) imply that the equilibrium wage, as a function of i, is

$$(19) \quad w(i) = \left[a_i - \frac{q_f.(1-\beta^*).a^e}{r+s+q_f}\right] . \frac{\beta(r+s+x)}{r+s+\beta.x}$$

where $a^e = \sum_{i=k}^{n} \pi_i^* . a_i \Big/ \sum_{i=k}^{n} \pi_i^*$ is the (conditional) average productivity of a worker drawn from the pool, $q_f = x. \sum_{i=k}^{n} \pi_i^*$ is the probability of the firm filling the vacancy in a unit time period, and finally $\beta^* = \beta(r+s+x)/(r+s+(1-\beta).q_f+\beta.x)$. Note that as long as $\beta < 1$, the wage of a type i worker depends on the productivities of other types in the economy, a phenomenon not apparent in the Diamond-Pissarides model.

As $W_e(i) - W_u(i) = w(i)/(r+s+x)$ from (8) and (9), it follows immediately from (16) that the critical index k is the smallest index such that $w(k) \geq 0$, or, from (19), the <u>smallest</u> k such that

$$(21) \quad a_k \gtreqless \frac{q_f.(1-\beta^*).a^e}{r+s+q_f} .$$

The first thing to note is that if β is small (i.e. worker bargaining power is low) $k > 1$, so that the equilibrium will be efficient, as claimed. It is also apparent that as $\beta \to 1$, eventually $k=1$, so that if worker bargaining power is high enough, the equilibrium will be efficient.

To understand the cause of the inefficiency, consider (21) with $\beta < 1$. Here the firm compares the productivity of a prospective match, a_i, with the average productivity of another "draw" from the worker population a^e. If the former is low enough relative to the latter, the match is rejected.

However, from the point of view of the economy as a whole, worker ability is not a random variable. Human capital per capita is fixed at $\sum_{i=1}^{n} a_i \pi_i$, and hence aggregate output cannot be increased by rejecting any job-worker match.

There is one problem with this equilibrium, however - workers of types below k, the critical index, will never be employed. Hence, there is no return to search, and so if there were even the slightest cost to search, they would drop out of the search process, which would have an impact on the matching probabilities of firms and the remaining workers. To model this "discouraged worker" effect, therefore, means endogenising the matching probabilities, which would be rather complicated, while the basic ineffficency result would not change i.e. the equilibrium would now have low-productivity types quitting the market, which would not be socially optimal, as long as search costs were low enough.

We now turn to the case where the match-specific productivity is genuinely random.

Case 2; θ random

To keep the algebra simple, we only deal with the polar cases where either firm or worker can obtain all the surplus i.e. either $\beta=0$ or $\beta=1$. We show that in the former case, the reservation values θ_i are not the efficient ones, whereas in the latter case, they are efficient. Consequently, one would expect that in any equilibrium where both parties obtain a strictly positive share of the surplus ($0<\beta<1$) will be inefficient also, and this is in fact the case, the demonstration is tedious and unrewarding, and is consequently omitted.

$\beta=0$

Here, the only possible equilibrium wage function is $W(\theta,i) = 0$ for all θ,i. In this case, from (12) and (13), we obtain;

$$(22) \qquad W_f^e - W_v = \frac{\sum_{i=1}^{n} \pi_i^* \cdot q_w \; (a_i + \theta_i^e)}{r + s + qf} \quad , \quad q_f = \sum_{i=1}^{n} \pi_i^* \cdot q_{wi}$$

with θ_i^e defined as in (6). Substituting this back into (13), solving for W_v and then rearranging (12), we can find an explicit expression for $Wf(\theta_k,k)-W_v$. Setting this equal to zero in accordance with (16), setting r=0 for purposes of comparison with the efficient case) we obtain n-k-1 simultaneous equations implicitly defining the equilibrium reservation values;

$$(23) \qquad a_i + \theta_i = \frac{\sum_{j=k}^{n} \pi_j^* \cdot (1-F(\theta_j) \; \cdot (a_j + \theta j^e)}{s + \sum_{j=k}^{n} \pi_j^* \cdot (1-F(\theta_j))}$$

The critical index k, is now determined as the smallest index such that the n-k-1 equations (23) have solutions $\theta_i > 0$, $i=k...n$. Note first that if $\theta_j = \theta_j^e = 0$, then (23) reduces to (21) with $\beta^* = 0$. Therefore, the underlying inefficiency discussed in the previous section is still present, and so output and employment will still be too low, as long as the variability of θ is low enough. However, for a given distribution for θ, it is impossible to say anything in general about the direction of the inefficiency i.e. whether the θ_i will be greater or smaller than their socially optimal counterparts defined by (6). Note also that if there is only one ability type, (23) reduces to (6), confirming Pissarides' result that the equilibrium will be efficient.

$\underline{\beta=1}$

Here, the only possible equilibrium wage function is $w(\theta,i) = a_i + \theta$ for all θ,i. In addition, as $W_f(\theta,i) = W_v = 0$, firms are indifferent as to which types they employ. Simple manipulation of the asset equations then implies that

$$(24)\quad W_e(\theta,i) - W_u(i) = \frac{1}{r+s} \cdot \left[a_i + \theta - \frac{x.(1-F(\theta_i)).(a_i + \theta_i)}{s + r + x.(1-F(\theta_i))} \right]$$

Setting (24) to zero in accordance with equation (17), we obtain an equation for the equilibrium θ_i which, for r= 0, is identical to the socially optimal equation (6).

To summarise; we have demonstrated that the presence of ex ante hetereogeneity in search models generates an inefficiency that is conceptually distinct from, and in some ways easier to understand, than inefficiencies of the "congestion" type which have been analysed before in the literature, and which arise from individuals ignoring the external impacts of their own actions on matching probabilities.

Furthermore, as we shall see in what follows, these inefficiencies can persist with asymmetric information. We will analyse a model of asymmetric information in some detail in 3.5 - 3.10. However, we can note one interesting fact now, which is that the introduction of asymmetric information can improve efficiency. Suppose for simplicity the firm has monopoly power, and cannot observe the type of the worker when the hiring i.e. matching decision is made. Then the only possible equilibrium is where the wage is identically zero and all types are hired. Thus, if θ is non-random, full efficiency is restored , and even if it is random, inefficiency may be alleviated.

3.5 Evidence for Discrimination on the Basis of Unemployment Histories

There are essentially three kinds of evidence that one can appeal to. The first kind is individual testimony from the unemployed that aspects of their employment history count against them in looking for work. The second is direct evidence that such discrimination is taking place. The third is statistical evidence concerning re-employment probabilities.

As for the first, the interviewees reported in Sinfield op. cit. and Harrison(1976) say that histories of short job tenure and (especially) a long current duration of unemployment count against them. In addition, unemployed young people clearly recognise a "Catch-22" of no job without experience. In spite of this, there seems to be little direct evidence that such discriminatory pratices are going on - Harrison cites only one NEDO study which found that "the long term unemployed may not even get as far as seeing any employers.Those on the register a long time were much less frequently submitted for jobs than the new arrivals." This lack of direct evidence is not really surprising, however; it is unlikely that firms would be explicit about a practice that appears as discriminatory.

The third type of evidence is far more indirect. First, at the aggregate level, the probability of re-employment falls with unemployment. The evidence is quite unambiguous on this; the average duration of completed spells of unemployment (as measured for example, by the ratio of the unemployment stock to flow) is much lower than the average duration of current uncompleted spells. The former has consistently been 2-5 times the latter for both Britain and the US over the last twenty-five years (see Johnson and Layard (1984)). This means as Salant (1977) shows, that duration data cannot be generated by a process whereby the probability of any individual unemployed worker leaving unemployment is constant both across time and across individuals - either re-employment probabilities must differ significantly across individuals (heterogeneity) or the probabilities must be genuinely duration-dependent, and decline with time unemployed.

Much recent work, e.g. McGregor (1978), Nickell (1979), Lancaster (1979), Lynch (1985), has examined the question of how much of this effect can be attributed to observed differences between workers (e.g. age, marital status, skills, education, place of residence), and find that conditional re-employment probabilities still decline, sometimes sharply over the duration of the unemployment spell. However, both Nickell (1979) and Lancaster (1979) pointed out that this may be due to unobserved (by the econometrician) heterogeneity among workers. Even with adjustments for such unobserved heterogeneity, however, both Nickell (1979) and Lynch (1985) find that such a duration effect still persists.

The problem with interpretating such results is that there are several other explanations for them, besides that of employer discrimination. We can divide them into three types;

(i) human capital; adults may lose valuable skills, and youths may never have a chance to acquire them, over lengthy spells of unemployment;

(ii) search intensity; loss of morale may cause the unemployed to search less intensively;

(iii) a rising reservation wage; the minimum acceptable wage may rise as unemployment duration increases.

There is evidence that the first two kinds of effects occur in practice, whereas both the evidence and theoretical reasoning are <u>against</u> the third; i.e. reservation wages should, and do, <u>fall</u> over time.

As regards (i) and (ii), much of the evidence on the psychological effects of long-term unemployment (Harrison(1976), Sinfield(1981)) shows a pattern of initial optimism about re-employment, followed by a more pessimistic re-evaluation when decisions e.g. to look for less skilled jobs are made, followed finally by fatalism. Interviewees say that they search less intensively as time goes on. In addition, they become apprehensive about taking on jobs that require them to use their former skills (and may of course have skills that are fast becoming obselete), although there is little evidence that long unemployment irreversibly destroys an individual's ability to work.

As for the reservation wage, one would expect it to fall over time on theoretical grounds because (a)unemployment benefits either decline, or certainly do not increase, over time, (b) finite search horizons, and (c) learning effects- those who do not leave unemployment are those who recieve relatively low wage offers, and these offers are likely to cause their beliefs about the wage distribution to become more pessimistic. On the empirical side, reservation wages are not directly observable. However, as Keifer and Neumann(1979) have shown, it is possible nevertheless to estimate the effect of various parameters, including the duration of unemployment on the reservation wage. For U.S. data they show that the reservation wage significantly declines with time. Narendrathan, Nickell, and Stern(1985) , using different methods, also conclude that the reservation wage for U.K males declines with duration.

To summarise, then, there seems to be considerable evidence that then duration of individual spells of unemployment adverely affects those individual's reemployment probabilities. There is also anecdotal evidence that other aspects of individual work histories affect reemployment, although thre is very little econometric work relating to this. Heckman and Borjas(1980) provide methods for estimating and testing for such relationships, but their own empirical results are inconclusive.

Therefore, we concentrate on modelling the duration effect, although the approach used could easily be adapted to model other aspects of discrimination via unemployment history.[1] In the next section, , we give a general description of the modelling strategy, and then lay out the model and discuss possible equilibria.

3.6 A Model of Screening via Unemployment Duration: Modelling Strategy,Equilibrium Concepts, and an Overview of the Main Results

The evidence suggests that employers, if they are rational, construe the duration of unemployment as conveying information that the worker posesses some undesirable characteristic (e.g. low ability, or reliability, high propensity to quit) that is costly in terms of time or money to observe directly.

Now, the duration can only convey interesting information if workers of different "types" (i.e. who differ in some payoff relevant way to the firm) have different re-employment probabilities. There are several different ways in which this could happen. Suppose, for example, more highly productive workers searched harder, and firms simply hired all workers they were matched with at the same wage. However, this (and other similar devices) are ad hoc in the sense that there are no differential incentives for productive workers to search harder - all workers get the same wage. It is fairly clear, in fact, that the only way that a relationship between productivity and re-employment can be generated which is not ad hoc in this sense is to suppose that firms get some "noisy" signal about worker productivities when they are matched - i.e. they test workers. Then if productive workers are more likely to do well at the test, any sensible hiring rule based on test results (only hiring workers who "pass" or do well on, the test) will automatically embody information in the duration. This is the reasoning behind the model.

This line of reasoning really establishes the first important result; viz. that a necessary pre-condition for screening via duration is the presence of informational externalities; (some of) the information gleaned by individual firms by testing is embodied in the duration.

Thus, duration conveys information about workers rather like the price in an asset market does about asset returns. However, there are interesting differences. In Walrasian rational expectations models, it is well-known that, generically, equilibria exist where the price effectively reveals all private information to each agent. This is not the case in this model - the equilibrium is not fully revealing. More precisely, the durations of individuals who have taken (and failed) one or more tests do not fully reveal this information.

This has the important consequence that firms may still find it in their interests to gather information about workers privately, even when the duration carries <u>some</u> information, and so the Grossman/Stiglitz existence problem need not arise. In fact, there may be multiple equilibria; in particular, pooling equilibria, where the duration conveys no information and screening equilibria may co-exist at the same parameter values.

Also, we are able to obtain several interesting comparative statics results. First, we show that the level of screening will fall following a positive shock to productivity, as long as the wage does not fully adjust to the shock. (Pissarides(1985) argues that in a search framework, this lack of full adjustment is the <u>only</u> way in which the business cycle can impinge on the labour market). In turn, a fall in the level of screening reduces unemployment. Thus screening provides an alternative channel through which aggregate demand shocks can affect unemployment, the more usual channel being through an increase in the proportion of job matches that are accepted (Pissarides(1985)). The reason for the fall in screening is that the benefit from screening, as measured by the ratio of net profits from employing high and low productivity workers, falls in a boom, as long as wages do not fully adjust.

Second, an increase in excess supply in the labour market will lead to a worsening of the screening effect. This works through the cost side; an increase in labour market slack makes it easier for employers to find job matches, and so one of the costs of screening (the waiting cost incurred by a job match) goes down.

We now set out the model rather more formally.

The Model

We suppose an economy with equal numbers of jobs, J and workers, L. Firms are comprised of large numbers of jobs. It is useful to think of jobs as machines which can only be operated by one worker. All jobs are identical, but workers may be of two types, 1 and 2, so that $L = L_1 + L_2$. A worker of type i and a job can produce y_i units of output with $y_1 > y_2 > 0$, so that type 1 workers are more productive in every job match. This is in contrast to the standard MDP model, where productivities pertain to matches, not workers.

The matching process is a Poisson one, with workers encountering a single job with probability x_w and a job "contacting" a worker with probability x_f in any "short" unit time period. In general, x_w and x_f will depend on the numbers of unemployed of each type, U_i and the number of vacancies, V (see e.g., Diamond (1982)) and will so be endogenous. However, we wish to abstract from this complication to begin with as we did in 3.3.and 3.4. We know this can be done by assuming that the matching function $\chi(U_1+U_2, V)$ has constant returns to scale and that there is no excess supply of, or demand for jobs on aggregate, i.e. $L_1 + L_2 = J$. Then $x_f = x_w = x$.

We depart from previous analyses of models of this type by assuming that firms only have partial information about the type, or productivity of any particular worker. In particular, we suppose that each firm can test at zero cost a prospective worker that it is matched with, but the test is an imperfect sorting device in that the more productive type 1 workers always pass the test, but type 2 workers also pass with probability ϕ. We also assume that the wage is exogenously given at a level b, equal perhaps to an opportunity wage or benefits. Given that in a search framework, matched partners are in a situation of bilateral monopoly it would be more satisfactory to adopt the Diamond-Pissarides wage bargaining approach, where the wage is determined by the condition that the surplus from the match is divided between the parties in proportions corresponding to their bargaining strengths. The problem with this arises in our "screening" equilibrium below, the wage must there depend on the duration of unemployment of the worker, and the analysis consequently becomes very complicated.

Finally, we suppose that workers are finitely lived - they have a transition probability of entering retirement of δ from either employment, or unemployment. Hence,we must suppose that there are δL_i "births" or flows of new workers of each type per period in order to offset retirement flows. We suppose new workers enter into the unemployment pool.

While it is possible (and desirable) to ring changes on the basic model, it seems unlikely that the qualitative conclusions (existence and multiplicity of equilibria and the main comparative statics results) would change a great deal. In spite of this, it would be very interesting to model the worker's decision-making explicitly, especially the reservation wage and search decisions, and we discuss these matters in the conclusion to this chapter.

It is apparent, upon reflection, that there are potentially three types of symmetric pure strategy[2] equilibrium in the model above, depending on firms hiring policies and the information generated by these policies. The first possibility is that all firms decide not to test workers, but simply hire every worker that they are matched with. It is usual to call such an equilibrium a pooling equilibrium. If all other firms are behaving in this way, then no particular firm can glean any information about the productivity of any particular worker from his duration in unemployment, and so the firm's hiring policy cannot be dependent on unemployment duration. Hence, the sole condition for equilibrium is that every firm finds it too costly (in terms of a probability of a match foregone) to test workers.

Suppose now, by contrast, that this is not the case - i.e. firms test all workers and hire only those who pass the test. By doing so, they create an informational externality; the duration of an individual worker's unemployment duration will now convey information about his type. In particular, if all firms have such a hiring policy, the longer a worker has been unemployed, the more likely he is to be of low productivity.

Therefore, if other firms choose to make use of this information, it can be shown that they will do so by operating a "cutoff" hiring rule i.e., only hiring workers with a duration less than some critical value, t*, <u>given</u> that all other firms are also operating cutoff rules[3]. We can show that in some circumstances, such an equilibrium in cutoff rules exists.

The third possibility is that firms do not wish to make use of this information, even when it is available, i.e. where all firms test workers, but their hiring rules do not depend on the duration of worker unemployment. Such an equilibrium musts, in fact, be a special sort of cutoff rule equilibrium where $t^* = +\infty$. We call such aan equilibrium a trivial cutoff rule equilibrium. The possibilities are depicted below:

	$t^*<\infty$	$t^*=\infty$
Testing	cutoff rule equilibrium	trivial cutoff rule equilibrium
No testing		pooling equilibrium

3.7 Pooling Equilibrium

As argued above, this equilibrium is characterised by the condition that individual firms do not find it profitable to test workers. We have assumed that the direct cost of testing is zero, but in a search framework there is, of course, an implicit waiting cost of testing, which is that the match may be rejected and the firm must therefore wait for another suitable match.

Let $\lambda_i = L_i/(L_1 + L_2)$ be the fraction of type i workers in the population. In a pooling equilibrium, the fraction of the employment pool which are type i is also λ_i. Therefore, the expected flow of profit from a match if the firm does not test is simply

$$\lambda_1(y_1 - b) + \lambda_2(y_2 - b). \tag{25}$$

If the firm does test, it is

$$\frac{\lambda_1(y_1 - b) + \phi\cdot\lambda_2(y_2 - b)}{\lambda_1 + \phi\lambda_2} \tag{26}$$

by Bayes' rule.

Now a simple dynamic programming argument establishes that the hiring policy is the one which gives the firm the greatest expected capital gain from hiring. In turn, as there are no flow costs to a vacancy, the expected capital gain is simply the expected flow of profit from a match, (multiplied by $\lambda_1 + \phi.\lambda_2$, the probability that a matched worker passes the test in the testing case) divided by the sum of the effective discount rate, $(r + \delta)$ (where r is the rate of interest) and the probability of a sucessful match: in the case without testing, this is simply x, and with testing, this is $x(\lambda_1 + \phi.\lambda_2)$.

Hence, the firm will wish to test a given worker if and only if

$$(27)\quad f(\lambda_2) = \frac{\lambda_1(y_1 - b) + \phi\lambda_2(y_2 - b)}{r + \delta + x(\lambda_1 + \phi\cdot\lambda_2)} - \frac{(y_1 - b) + \lambda_2(y_2 - b)}{r + \delta + x}$$

$$\gtrless 0$$

Note first that as $(r + \delta) \to 0$, it always becomes profitable to test as the implicit cost of waiting goes to zero, and the flow of profit under testing, (26) above, is always greater than the flow of testing without, (25) above.

Also, $\lim_{\lambda_2 \to 0} f(\lambda_2) = 0$, as when λ_2 is very low, there is very little to be gained or lost from testing, and $\lim_{\lambda_2 \to 1} f(\lambda_2) < 0$, as when $\lambda_2 \approx 1$, the gain from testing (i.e. (26) - (25)) is negligible, whereas the cost (i.e. a hiring probability of approximately $\phi\cdot x$ rather than x) is strictly positive. Finally, f can be shown to be quasi-concave[4] and $f'(0) > 0$.

Putting these facts together, we conclude that there is a unique value of λ_2, λ_2^0 , such that a pooling equilibrium is possible if and only if $1 \gtrless \lambda_2 \gtrless \lambda_2^0$.

3.8 Cut-off Rule Equilibrium

We will define here the conditions which define an equilibrium in cut-off rules, and also consider questions of uniqueness and stability. Recall that this equilibrium is described by (a) a cut-off time t^* such that only workers with unemployment duration $\tau < t^*$ are tested, and (b) the condition that only workers that pass the test are hired.

To see why a cut-off rule may be part of an equilibrium, suppose to the contrary that all firms are testing workers, no matter what their unemployment duration, and only hiring those who pass the test. Then, as the transition probability out of unemployment is lower for type 2 workers, the probability that any worker who has been unemployed for exactly time τ is of type 2 is increasing in τ, and in the limit, as $\tau \to \infty$, approaches 1.

Assume now that if a firm was sure that a worker was of type 2, it would not wish to employ him. If the worker's unemployment duration was high enough, the firm could be almost sure that he was of type 2. Therefore, the firm could do better by not bothering to test him i.e. practice a cutoff rule.

This argument, of course, is not sufficient to establish the existence of a cutoff rule equilibrium, but it gives an indication of how to proceed.

First, we know already that the firm encounters a worker with probability x per unit time period. If the worker has been unemployed for τ periods, there is a probability $P_i(\tau; t^*)$ that he is of type i, given that all firms (except possibly the firm under consideration) are following a cutoff rule with cutoff time t^*. Note that the viability of such a definition requires that firms are small relative to the market, or equivalently, that L and J are large. Note also that for $\tau > t^*$, both types exit with equal probability, δ, from unemployment, so then $P_i(\tau; t^*)$ is independent of t.

Then write

$$(28) \qquad P_i(\tau; t^*) = P_i^+(t^*; t^*) \qquad \tau > t^*.$$

At this stage it is not necessary to give an explicit formula for $P_i(\tau; t^*)$ - we can write down the equation characterising t^*, and also the testing condition, without it.

(a) Characterisation of Equilibrium

The first step is to write down equations for the asset values of vacant jobs, and also jobs filled with a worker of duration τ. Let the latter be $W_f(\tau; t, t^*)$ where t is the firms own cutoff time, and t^* that of the market, and similarly $W_v(t, t^*)$. (Note that the state space is $v \times (f, \tau)$, $\tau \in (0, \infty)$, and does not include t or t^*.) Then we know that the return on each asset must be equal to the flow of profit plus expected capital gains, or

$$(29) \quad r\, W_f(\tau; t, t^*) = \frac{P_1(\tau; t^*)(y_1 - b) + \phi \cdot P_2(\tau; t^*)(y_2 - b)}{P_1(\tau; t^*) + \phi\, P_2(\tau; t^*)}$$

$$+ \quad \delta(W_v(t, t^*) - W_f(\tau; t, t^*))$$

for a filled job, and for a vacancy;

$$(30) \qquad r \cdot W_v(t, t^*) =$$

$$x \cdot \int_0^t (W_f(\tau; t, t^*) - W_v(t, t^*))(P_1(\tau; t^*) + \phi \cdot P_2(\tau; t^*))f(\tau; t)d\tau$$

if $\quad t < t^*$, and.

$$x.\int_0^t (W_f(\tau;\ t,\ t^*) - W_v(t,\ t^*))(P_1(\tau;\ t^*) + \phi\cdot P_2(\tau;\ t^*))f(\tau;\ t)d\tau$$

$$+ (F(t;\ t^*) - F(t^*;\ t^*))\cdot(P_1^+(t^*;\ t^*) + \phi\cdot P_2^+(t^*;\ t^*)).$$

$$(W^+(t^*;\ t,\ t^*) - W_v(t,\ t^*)) \qquad \text{if } t > t^*,$$

where $F(\tau;\ t^*)$ $(f(\tau;\ t^*))$ is the distribution (density) function of exit times from unemployment averaged over all workers. Also, it follows from (28) and (29) that $W_f(\tau;\ t,\ t^*)$ is independent of τ for all $\tau > t^*$, so we set it equal to $W_f^+(t^*;\ t,\ t^*)$. The optimal cutoff rule for the firm, t, maximises (30), where the dependence of W_v on t is not taken into account.[5] Given this, the first-order necessary condition is that

$$(31) \qquad W_f(\tau;\ t,\ t^*) \geq W_v(t,\ t^*) \geq W_f^+(t^*;\ t,\ t^*)$$

at $\tau = t = t^*$, and the sufficient condition is that $W_f(\tau;\ t,\ t^*)$ is decreasing in τ. But by inspection of (29), it is clear that the latter is satisfied - recall that $P_2(\tau;\ t^*)$ is increasing in τ.

We now solve explicitly for the asset values. First, setting $t = t^*$ in (29) and (30), we find that

$$(32) \quad \int_0^t W_f(\tau;\ t,\ t^*) - W_v(t,\ t^*))(P_1(\tau;\ t^*) + \phi\, P_2(\tau;\ t^*))f(\tau;\ t^*)d\tau$$

$$= \frac{\Omega_1(t,\ t^*)(y_1 - b) + \phi\,\Omega_2(t,\ t^*)(y_2 - b)}{r + \delta + x(\Omega_1(t,\ t^*) + \phi.\Omega_2(t,\ t^*))}$$

where

$$\Omega_i\ (t,\ t') = \int_0^t P_i(\tau;\ t')\ f(\tau;\ t')d\tau.$$

(32) gives an equation for the expected capital gain from following cutoff rule t - not surprisingly, it is equal to the (discounted) expcted profits form doing so.

Then, setting $t = t^*$ in (32), and using (29) and (30), we find that capital gains are

$$(33) \qquad (r + \delta)(W_f(\tau;\ t^*,\ t^*) - W_v(t^*,\ t^*)) =$$

$$\frac{P_1(\tau;\ t^*)(y_1 - b) + \phi . P_2(\tau;\ t^*)(y_2 - b)}{P_1(\tau;\ t^*) + \phi\ P_2(\tau;\ t^*)} - a, \quad \tau < t^*$$

$$\frac{P_1^+\ (t^*;\ t^*)(y_1 - b) + \phi . P_2^+\ (t^*;\ t^*)(y_2 - b)}{P_1^+\ (t^*;\ t^*) + \phi\ P_2^+\ (t^*;\ t^*)} - a \quad \tau > t^*$$

$$\text{where} \quad a = \frac{x \cdot [\Omega_1(t^*,\ t^*)(y_1 - b) + \phi . \Omega_2(t^*,\ t^*)(y_2 - b)]}{r + \delta + x(\Omega_1(t^*,\ t^*) + \phi . \Omega_2(t^*,\ t^*))} .$$

Note that for $\tau > t^*$ (33) is the capital gain accruing to a firm which (accidently) tested a $\tau > t^*$ worker, given that it was following a t^* - cutoff rule - an "off the equilibrium" path event. It follows immediately from (31) and (33) that (31) can only be satisfied if $W_f(t^*;\ t^*,\ t^*) > W_f^+\ (t^*;\ t^*,\ t^*)$, or equivalently, $P_1(t^*;\ t^*,\ t^*) > P_1^+\ (t^*;\ t^*)$. We show in the appendix that the latter inequality holds automatically. In fact, as the conditional probabilities are discontinuous from the right at t^*, we have some latitude in our first equilibrium condition and (arbitrarily) take t^* to be defined by $W_f(t^*;\ t^*,\ t^*) = W_v(t^*,\ t^*)$. Let $\Omega_i^* = \Omega_i(t^*,\ t^*)$. Then condition (31) becomes

$$(34) \qquad \frac{P_1(t^*;\ t^*)(y_1 - b) + \phi . P_2(t^*;\ t^*)(y_2 - b)}{P_1(t^*;\ t^*) + \phi . P_2(t^*;\ t^*)}$$

$$= \frac{x(\Omega_1^* + \phi . \Omega_2^*)}{r + \delta + x(\Omega_1^* + \phi . \Omega_2^*)} \cdot \frac{\Omega_1^*(y_1 - b) + \phi . \Omega_2^*(y_1 - b)}{\Omega_1^* + \phi \cdot \Omega_2^*}$$

Put in this form, the equilibrium condition is somewhat more transparent. On the left-hand side is the expectd profit from a "type" t^* worker. On the right hand side is a constant between zero and one, which represents the costs of waiting, times the expected profit from a random draw from the unemployment pool i.e. an "average" worker.

(34) therefore states that the firm is indifferent between accepting a type t^* worker now and waiting to make another draw from the pool of unemployed. This is analagous to the condition determining the reservation productivity in 3.4, and arises for the same reason.

Finally, we must deal with the condition that ensures that firms wish to test workers that pass the cutoff rule. For simplicity we suppose that the decision to test does not depend on the particular duration of the worker. This could be justified in several ways, and in any case, no result of importance hinges upon it.

Suppose that all other firms are testing workers and have a cutoff rule, t^* satisfying (34). The alternative policy for the firm is not to test, and choose another cutoff rule, t^{**}.

By the preceeding arguments, t^{**} must in fact satisfy an analogue of (34) with $\phi = 1$. The testing condition then requires that the expected capital gain (taken over workers of all durations) from testing exceeds that from not testing. This is, using (30)

$$(35) \qquad \frac{\Omega_1(t^*, t^*)(y_1 - b) + \phi.\Omega_2(t^*, t^*)(y_2 - b)}{r + \delta + x(\Omega_1(t^*, t^*) + \phi.\Omega_2(t^*, t^*))} - \frac{\Omega_1(t^{**}, t^*)(y_1 - b) + \Omega_2(t^{**}, t^*)(y_2 - b)}{r + \delta + x(\Omega_1(t^{**}, t^*) + \Omega_2(t^{**}, t^*))} > 0$$

It is of particular interest to know if this condition can be saatisfied simultaneously with condition (27) which guarantees the existence of a pooling equilibrium. Because one cannot compare t^* and t^{**} in general, this cannot be done analytically, but it should be possible to check for particular numerical examples.

(b) Existence and Multiplicity of Equilibrium

At this stage, we call on some explicit formulae for the conditional probabilities which are derived in the appendix to chapter 3. First,

$$(36) \qquad P_i(\tau;\ t^*) = \frac{f_i(\tau;\ t^*)\ \pi_i(t^*)}{\sum_{i=1}^{2} f_i(\tau;\ t^*)\ \pi_i(t^*)}$$

by Bayes' rule, where $\pi_i(t^*)$ is the firm's prior belief that the worker is of type i, and so is the fraction of the unemployment pool made up of type i workers i.e.

$$(37) \qquad \pi_i(t^*) = \frac{u_i(t^*)\cdot\lambda_i}{\sum_{i=1}^{2} u_i(t^*)\cdot\lambda_i}$$

where $u_i(t^*)$ is the equilibrium unemployment rate for type i workers which can be computed as;

$$(38) \qquad u_i(t^*) = \frac{\delta}{\delta + h_i\cdot(1 - \exp(-(h_i + \delta)t^*)}$$

with $h_1 = x_1 \quad h_2 = \phi\cdot x$. Finally,

$$(39) \qquad f_i(\tau;\ t^*) = \begin{cases} (h_i + \delta).\exp(-(h_i + \delta)\tau\ ) & \tau < t^* \\ \delta.\exp(-\delta(\tau - t^*))\ .\ \exp(-(h_i + \delta)t^*) & \tau > t. \end{cases}$$

A number of properties of the conditional probabilities follow from this definition. First, $u_1(t^*) < u_2(t^*)$ all $t^* > 0$, so that $\pi_2 > \lambda_2$, so that type 2's suffer relatively more unemployment. It is also simple to establish[5] that $P_2(\tau;\, t^*)$ is strictly increasing in τ, as claimed, for $\tau < t^*$. This means, that viewed as a signal about productivity, τ is "bad news" in Milgrom's(1981) sense - that is, a higher τ implies that for <u>any</u> prior π_1, π_2, the posterior probability that the worker is highly productive is lower.

Using (36) - (39), (34) becomes, after some manipulation,

$$(34') \qquad g(t^*) = \frac{(y_2 - b) + \alpha(t^*)(y_2 - b)}{1 + \alpha(t^*)} - \frac{x.\beta(t^*)}{r + \delta + x.\beta(t^*)} \cdot \left[\frac{(y_1 - b) + \gamma(t^*)(y_3 - b)}{1 + \gamma(t^*)} \right] = 0$$

with

$$(40) \qquad \alpha = \phi.\frac{\pi_2(t)}{\pi_1(t)} \cdot \frac{(\delta + h_2)}{(\delta + h_1)} \cdot \exp((h_1 - h_2)t\,),$$

$$\beta(t) = \pi_1(t).(1 - \exp(-(\delta + h_1)t))\,) + \pi_2(t)\cdot\phi\cdot(1 - \exp(-(\delta + h_2)t)),$$

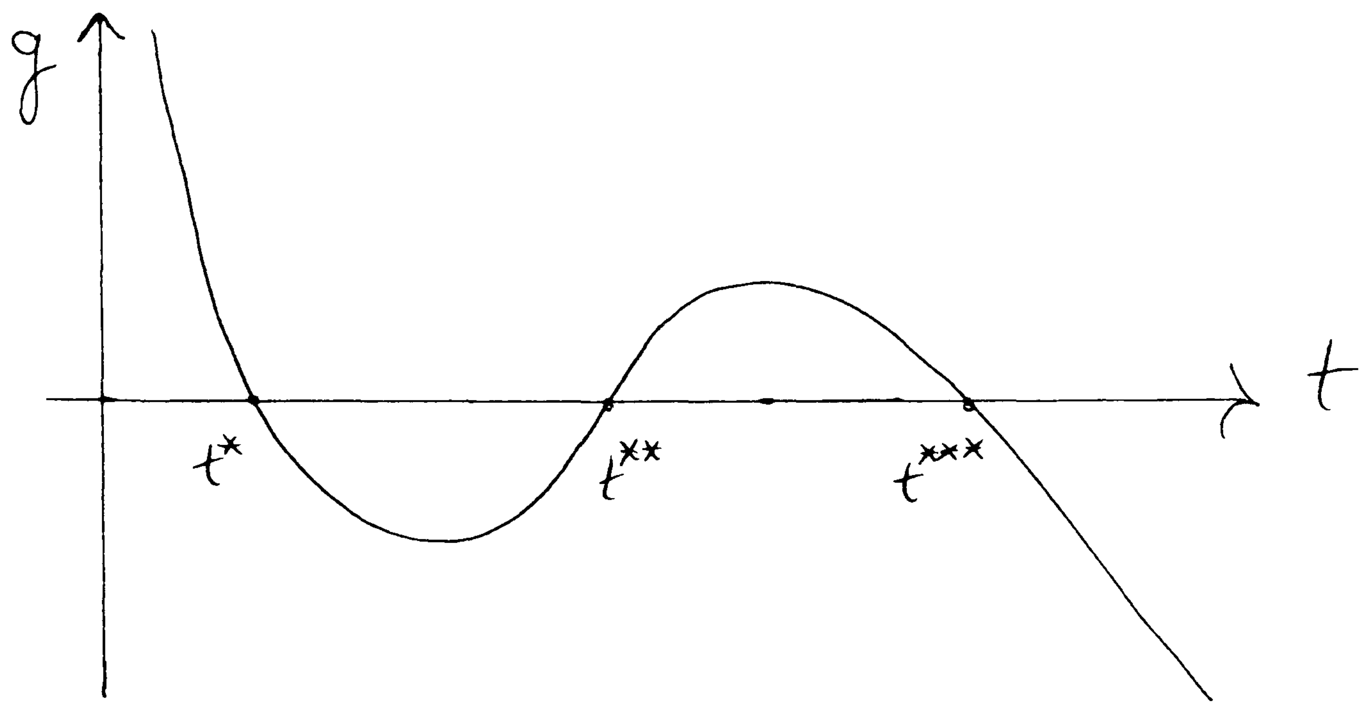
$$\gamma(t) = \beta.\frac{\pi_2(t)}{\pi_1(t)} \cdot \frac{(1 - \exp(-(\delta + h_2)t))}{(1 - \exp(-(\delta + h_1)t))}.$$

We can now establish rigorously the conditions under which (34) has a solution $0 < t^* < \infty$. First, obseve that as $t \to \infty$, $\alpha(t) \to \infty$ whereas $\beta(t)$ and $\gamma(t)$ tend to finite, strictly positive limits $\bar{\beta}$, $\bar{\gamma}$, so that if y_2 is small enough relative to y_1, then $\lim_{t \to \infty} g(t)$ is strictly negative (see condition (A) below for a precise statement).

On the other hand, as $t \to \infty$, $\alpha(t)$ and $\gamma(t)$ tend to the same limit (using L'Hopital's rule) so that as long as $r + \delta > 0$, $g(t)$ will become strictly positive, so by continuity of g, there will be at least one non-trivial cutoff rule equilibrium.

There may, of course, be more than one, as shown.

Figure 3.8.1



It is apparent that equilibrium like t^* and t^{***}, where $g'(t\ ) < 0$ are stable, whereas equilibria like t^{**} are unstable. The reason for this is as follows. Take any $t^* < t < t^{**}$. Then by definition of g(t) (see (33)) $W_f(\tau;\ t,\ t) - W_v(t,\ t) < 0$ at $\tau = t$. Hence, from (34), each firm could do better by reducing t slightly, so that the economy will eventually move to t^*. (Note that the same argument cannot apply for t at which $g(t) > 0$, as W_f is discontinuous in τ at that point).

Finally, it is fairly apparent that if $\lim_{t \to 0} g(t) > 0$, there exists a <u>trivial</u> cutoff rule equilibrium, with $t^* = \infty$. To see this note that $\lim_{t \to \infty} g(t) > 0$ implies $W_f(\infty; \infty, \infty) \geqq W_v(\infty, \infty)$. Then as $W_f(\tau; \infty, \infty) > W_f(\infty; \infty, \infty)$ for all $\tau < \infty$, it always pays any firm to test any worker of whatever duration, as long as all other firms are doing the same.

It turns out that this limiting value of g(t) has an interesting interpretation. In fact, consider the condition

(A) $\lim_{t \to \infty} g(t) =$

$$y_2 - b - \frac{x(\bar{\pi}_1 + \phi.\bar{\pi}_2)}{r + \delta + x.(\bar{\pi}_1 + \phi\bar{\pi}_2)} \cdot \frac{\bar{\pi}_1(y_1 - b) + \phi.\bar{\pi}_2(y_2 - b)}{\pi_1 + \phi.\bar{\pi}_2} < 0.$$

This is precisely the condition that says that the firm should <u>not</u> hire a type 2 worker, but should wait and draw another worker from the pool, if the proportion of type 2 workers in the population is $\bar{\pi}_2 = \lim_{t \to \infty} \pi_2(t)$.

We summarise;

<u>Theorem 1</u>

Assume that (11) holds. Then if condition (A) holds, a non-trivial cutoff rule exists. If condition (A) does not hold, a trivial cutoff rule equilibrium exists.

3.9 Equilibrium with an Endogenous Unemployment - Vacancy Ratio

So far we have assumed that the labour market is in equilibrium in the sense that the number of jobs equals the number of workers. This is highly restrictive, both because its lack of realism, and because it prevents us from ascertaining changes in the working population or jobs on the cutoff rule.

In this section we define equilibrium for the more general case with an arbitrary J/L ratio. Recall that the number of matches per unit time period is $x(U_1 + U_2, V)$. Hence, the probability of any unemployed worker matching with a firm is

$$(41) \qquad \frac{x(U_1 + U_2, V)}{U_1 + U_2} = x\left(1, \frac{v}{\lambda_1 u_1 + \lambda_2 u_2}\right)$$

where $v = V/L$, and assuming constant returns to scale for x. Let $u = \lambda_1 u_1 + \lambda_2 u_2$, the average unemployment rate. Then it is easily seen that $v = u - s$, where $s = (L - J)/L$ is a measure of excess supply in the market. Therefore, the probability for a worker of a match is $x(1, 1 - s/u)$. A similar argument establishes that the probability of a match for the firm can be written

$$(42) \qquad \frac{x(U_1 + U_2, V)}{V} = x\left(\frac{u}{u - s}, 1\right)$$

Now a cutoff rule equilibrium is characterised by the testing condition (35) as before, and two simultaneous equations in t and u, the aggregate rate of unemployment.

First, from (34), we have

$$(34') \qquad g\left(t, x\left(\frac{u}{u - s}, 1\right)\right) = 0$$

where the dependence of g on x is now explicit.

Second, we have from (38),

$$(43) \quad u - \frac{\delta\lambda_1}{\delta + x(1 - s/u)(1 - \exp(-(x(1 - s/u) + \delta).t))}$$

$$+ \frac{\delta\lambda_2}{\delta + \phi x(1 - s/u)(1 - \exp(-(\phi \cdot x(1 - s/u) + \delta).t))}$$

$$= \quad h(u, t) \quad = \quad 0.$$

We now have two equilibrium conditions, (34′) and (43). It is possible to show that this system has at least one solution (t^*, u^*) under a condition similar to condition (A) above.[6]

The stability condition is, however, slightly different; it requires the <u>total</u> derivative of g with respect to t to be less than zero i.e.

$$(44) \quad \underset{?}{\frac{\partial g}{\partial t}} + \underset{?}{\frac{\partial g}{\partial x}} \cdot \underset{+}{\frac{\partial x}{\partial u}} \quad \frac{s}{(u - s)^2} \quad \cdot \underset{+}{\frac{\partial h}{\partial t}} \quad \underset{+}{\frac{\partial h}{\partial u}}^{-1} < 0.$$

The signs of the partial derivatives are as indicated, so as long as there is excess supply in the labour market (i.e. $s > 0$) at least one of $\partial g/\partial t$ and $\partial g/\partial x$ must be negative for stability. We argue that it is reasonable for $\partial g/\partial t$ and $\partial g/\partial x$ to have the same sign in the next section, so that under this "reasonable" condition, $\partial g/\partial t$ is again negative for stability.

It is worth remarking at this point that our way of relaxing the condition that the number of jobs and workers are equal is somewhat <u>ad hoc</u>. In particular, it is possibly more desirable to suppose that the number of jobs is determined endogenously, by profit-maximising behaviour. In fact, Pissarides (1985) argues that if the capital stock is perfectly flexible i.e. there are no delivery lags then profit-maximising implies that the asset value of a vacancy should be zero. Imposing this condition on equilibrium clearly represents the opposite extreme to our approach.

We defend the option taken here on three grounds. First, by making the J/L ratio exogenous, we can evaluate directly its effect on the cutoff time and on unemployment. The second reason is technical. Imposing $W_v = 0$ as an equilibrium condition, as Pissarides suggests, would lead to entirely different equilibrium conditions to the case with $L = J$.

In particular, if $t^* < \infty$, t^* would be defined by the condition $P_1(t^*; t^*)(y_1 - b) + \phi . P_2(t^*; t^*)(y_2 - b) = 0$, which requires $y_2 < b$, or that less productive workers should actually have negative net products, if b is the shadow price of leisure.

Thirdly, instantaneous adjustment of capital is highly unrealistic, and so our approach can at least capture the short-run implications of changes in the labour force.

3.10 Comparative Statics

In this section, we analyse the effect of changes in the parameters on both the equilibria with symmetric and asymmetric information. The main focus of interest is on the screening equilibrium - in particular, how productivity shocks and the excess supply of labour affect screening - and so we start with this case.

(a) The Asymmetric Information Model

We shall concentrate exclusively on the screening equilibrium, and will analyse the effects of changes in the wage, b, excess supply, s, and aggreagate and relative productivity changes (equiproportionate changes in y_1 and y_2, and changes in their ratio) on the equilibrium. In addition, we examine the impact of changes in some parameters on the "informativeness" of unemployemnt duration as a signal.

We begin by reparameterising; set $y_2 = y$, $y_1 = \theta y$, $\theta > 1$. Then changes in y can be interpreted as aggregate output shocks, and changes in θ as changes in relative productivity. We begin with the effect of small changes in parameters on t^*, the cutoff time. Under the assumption that the equilibrium is stable,we obtain the following results.

		$\partial t^*/\partial y$	$\partial t^*/\partial \theta$	$\partial t^*/\partial b$	$\partial t^*/\partial x$	$\partial t^*/\partial s$
u/v ratio	constant	+	- ?	-	?	not defined
	variable	+	- ?	-	not defined	- ?

First, $\partial t^*/\partial y > 0$ indicates that an aggregate productivity increase decreases screening, and conversely, a wage increase increases it.

In fact, it is possible to show (c.f. equation (34)) that equiproportionate changes in y and b, leave t^* unchanged, so that as long as an aggregate productivity (or demand) shock is not fully matched by the wage, it will reduce screening and reduce unemployment. This provides an alternative channel by which aggregate demand shocks might affect the labour market to the one outlined in Pissarides (1985), where an increase in aggragate demand lowers the productivity of the marginally acceptable match.

One would expect that an increase in θ, increasing the relative value of more productive workers, would be a stimulus to screening, hence lowering the cutoff time, and this is in fact the case if the discount rate is low enough. In fact,

$$(45) \quad \frac{\partial t^*}{\partial \theta} = \frac{y/(1+\alpha) \quad - \quad x.\beta.y/(r+\delta+x.\beta).(1+\gamma)}{D}$$

where $D > 0$ by stability considerations. As $\alpha > \gamma$ in equilibrium, this will be negative if $r+\delta$ is low enough.

We now turn to effects of changes in the matching technology. The most straightforward case is when u = v , for then this is simply a question of the effects of changes in x, the per period matching probability. One might suppose that this model exhibits a homogeneity property in that a doubling of the expected waiting time to a match (i.e. a halving of x) would lead to a doubling of t^*. For this we require that $-\partial g/\partial \frac{1}{x} = \partial g/\partial x.x^2 = \partial g/\partial t$.

Inspection of (34′) indicates that, in general, this is unlikely to be true, although one cannot say whether the effect is greater or less than proportional i.e. whether $\partial t^*/\partial \frac{1}{x}$ is greater or less than 1. It does seem plausible, however, to assume that $\partial t^*/\partial \frac{1}{x}$ is greater than zero. This in turn requires $\partial g/\partial x$ and $\partial g/\partial t$ to have the same sign, so from the stability condition (44) they must both be negative.

If we make this assumption, it is possible to show that $dt^*/ds < 0$, i.e. that an increase in labour market slack makes screening more severe. The explanation for this result is as follows. First, an increase in s will have both a direct effect and an indirect effect, via u, on the probability that the firm gets a match , $x(u/(u-s), 1)$. In fact,

$$(46) \qquad \frac{dx}{ds} = \frac{\partial x}{\partial u}\left[\frac{u}{(u-s)^2} - \frac{s}{(u-s)^2}.\frac{du}{ds}\right].$$

As $\partial x/\partial u > 0$, an increase in s will have a positive (negative) effect on the firm's matching probability as the elasticity of u with respect to s is less than (greater than) unity. Now it is possible to show that[7] this elasticity is less than or greater than 1 as $\partial g/\partial t$ is less than or greater than 0. Hence, dx/ds has the opposite sign to $\partial g/\partial x$. Finally, from (34′)

$$(47) \qquad \frac{dt^*}{ds} = \frac{-\partial g/\partial x.dx/ds}{\partial g/\partial t} .$$

As we have just established that dx/ds has the opposite sign to $\partial g/\partial x$, and $\partial g/\partial t$ is less than zero by stability, dt^*/ds must be negative, as claimed.

This argument could be put less formally by saying that "normally" (i.e. when $\partial g/\partial t < 0$) an increase in slack improves the matching probability of the firm, thus reducing the waiting cost of rejecting any worker of any given duration of unemployment. Hence, as the expected profit for the firm from any worker employed is decreasing in his or her unemployment duration, it pays the firm to reduce the cutoff duration below which it is willing to test workers.

One important consequence of this effect is that it enhances the effects of labour market slack on unemployment i.e. the total effect of s on u is

$$(48) \qquad \frac{du}{ds} = \left.\frac{\partial u}{\partial s}\right|_{t^* \text{ const.}} + \frac{\partial u}{\partial t^*} \cdot \frac{dt^*}{ds}$$

where the indirect effect $\frac{\partial u}{\partial t^*} \cdot \frac{dt^*}{ds}$ is positive. One implication of this is that it is possible for the total elasticity $\frac{s}{u} \cdot \frac{du}{ds}$ to be greater than unity (see footnote 7) whereas the elasticity of u with respect to s for a fixed t^* is always less than unity[8]. In other words, the screening effect may mean that a 1% increase in labour market slack leads to a greater than 1% increase in unemployment.

We now turn to the question of the informativeness of duration as a signal of productivity. The natural measure of informativeness is the likelihood ratio $f_2(\tau;t^*)/f_1(\tau;t^*)$; the greater this is in absolute difference from 1, the greater is the difference between the firm's prior and posterior assessments of the worker's type. We shall be concerned with the impact of exogenous variables on this ratio. To begin with, any increase in t^* will inccrease this absolute difference for any $0 < \tau < \infty$, as long as $f_2(t^*;t^*)/f_1(t^*;t^*) > 1$. The reason is that f_2/f_1 non-decreasing in t^*; independent of t^* for $\tau < t^*$, and is proportional to $\exp((1-\phi).t^*)$ for $\tau > t^*$.

In other words, an increase in either the cutoff time can increase the informativeness of the signal. This shows that the fundamental positive informational externality on which this model is based- i.e. that testing creates information for other firms - may be mitigated by the fact that in trying to take advantage of it, firms reduce it's quality.

It is also possible to show[9] that f_2/f_1 is increasing (decreasing) in in x as long as $\tau > (<) \delta.[(x+\delta).(\phi.x+\delta)]^{-1}$. Armed with these facts, it is possible to deduce the effects of various parameters on the informativeness of τ, but the picture seems inconclusive. We consider only the example of excess labour supply. We know that this reduces both t^* and x (for the workers). Then the increase in s will reduce informativeness via the effect on t^*, but its effect via x will be indeterminate.

A final result worth noting is the effect of changes in parameters on the <u>ratio</u> of unemployment rates, $u_2(u,t^*)/u\ (u,t^*)$. We can show[10] that for this ratio will be increasing in t^* - for a fixed u - and x, as long as ϕ, the probability that type 2's pass the test, is low enough. Then the effects of a change in any of the parameters on the ratio can be deduced via their effects on u and t^*. We will only do this for the most important of these i.e. changes in labour excess supply. We already know that an increase in s decreases t^*; it will also decrease the worker matching probability $x(1, 1-s/u)$ as long as the unemployment elasticity with respect to s is less than unity. Therefore, an increase in s tends to equalise the rates of unemployment. As $u_2 > u_1$, it could be argued in the case that $\lambda_i = 1/2$ that this reduces the prior information available to firms concerning worker's types.

(b) The Symmetric Information Model

In this section, we consider the impact of tax parameters and other exogenous variables on the symmetric information equilibrium, and in particular determine how the inefficiency of the equilibrium may be alleviated by government intervention.

From the arguments of section 3.4, it is clear that for the case where θ is not very variable the degree of inefficiency of equilibrium turns on (a) the relationship between the profit to the firm of employing a given worker, $a_i - w_i$, and the expected profit it can get from another random draw, $a^e - w^e$, and (b) the degree of worker bargaining power, θ. We have already remarked that one way of achieving efficiency of equilibrium is to raise worker bargaining power so that workers appropriate all, or nearly all, of the surplus. Another way of achieving complete efficiency is to impose a 100% profit tax, combined with lump-sum redistribution of the proceeds, as this would equalise the firms' after-tax profits from each type at zero, and it would therefore be indifferent about whom it hired.

One might conjecture from this that introducing a profit tax of less than 100% will improve efficiency from the no-tax status quo. It is also of interest to know more generally how in equilibrium, taxes interact with worker bargaining power to affect efficiency. It turns out that these two questions are closely related. An increase in a profit tax is (partially) shifted to workers by a lowering of the workers' share of the average product, θ^* (recall that $w^e = \theta^*.a^e$), and the tax rate affects the reservation productivity only through θ^*, as we show below. Therefore, we have the striking result that the imposition of a less than 100% profit tax decreases efficiency, whereas a 100% tax will restore full efficiency - that is, there is a discontinuity of the equilibrium matching rules in the tax rate at this point.

The introduction of a wage tax will also have perverse effects, for the same reason; some of the tax will be shifted onto the firm by an increase in effective worker bargaining power, β^*, so the efficiency of equilibrium will rise. Finally, it turns out that if the proceeds from the tax are distributed to either firms or workers not in a lump-sum fashion, but in a way that varies with the state that they occupy, then this discontinuity may be eliminated. For example, if some positive fraction of the proceeds is redistributed only to firms who are currently employing a worker (i.e. a firm employment subsidy), or only to workers that are currently employed (a worker employment subsidy) then equilibrium becomes efficient as the profit or wage tax rate goes to unity.

It is worth noting that such discontinuity of equilibrium in the tax parameters does not arise in Walrasian models, except in pathological cases. Our results would seem to indicate that such discontinuities are the norm in matching models, and so one should be cautious in applying arguments derived in a Walrasian setting to these models.

We now turn to a formal analysis of the effects of profit and wage taxes. Let τ_f be the profit tax rate, so the firms' net profit from employing a type i worker is $(1-\tau_f).(a_i - w_i)$, and τ_w the wage tax rate, so that the after-tax wage is $(1-\tau_w).w_i$. Then it is simple to show that if the proceeds from the taxes are distributed in a lump-sum fashion, (21) becomes

$$a_k > \frac{x.q_f.(1-\beta^*(\tau_f, \tau_w)).a^e}{r + s + x.q_f} \tag{49}$$

where

$$\beta^*(\tau_f, \tau_w) = \frac{\beta.(1-\tau_f).(r+s+x)}{\beta.(1-\tau_f).(r+s+x) + (1-\beta).(1-\tau_w)(r+s+q_f.x)} \tag{50}$$

First, from (50), it is apparent that for β strictly between 0 and 1, $\beta^*(\tau_f, \tau_w)$ is decreasing in τ_f and increasing in τ_w. The reason for this is quite simple. For a given profit tax rate, $(1-\beta^*).(1-\tau_f)$ is the firms' equilibrium post-tax share of the average product of the worker, a^e. As $1-\tau_f$ falls with an increase in the profit tax rate, $(1-\beta^*)$ rises to compensate the firm for this, so β^* must fall. An exactly similar argument explains why β^* is increasing in τ_w. Next, it is possible to show that the index k is non-increasing in β (we have already argued intuitively that this is the case in the previous section). Therefore, by the previous argument, k will be non-increasing in a wage tax, and non-decreasing in a profit tax, so that the former will improve, and the latter worsen, efficiency.

Now, suppose that some of the tax revenue is redistributed as lump-sum payments to firms whose vacancies are filled, and also lump-sum payments to workers who are currently employed. Let these payments be b_f and b_w respectively. Then the net profit from a filled vacancy is $(1-\tau_f).(a_i-w_i) + b_f$, and the net income for an employed worker of type i is $(1-\tau_w).w_i+ b_w$. It is then easy to show that (49) becomes

$$a_k > \frac{x.q_f.(1-\beta^*(\tau_f,\ \tau_w)).a^e}{r + s + x.q_f} + \tag{51}$$

$$\left[\frac{x.q_f.(1-\beta^*(\tau_f,\ \tau_w))}{r + s + q_f.x} - 1\right].\left[\frac{b_f}{1 - \tau_f} + \frac{b_w}{1 - \tau_w}\right]$$

As the coefficient on $b_f/(1-\tau_f) + b_w/(1-\tau_w)$ is negative, it follows that an increase in b_f or b_w from zero will improve the efficiency of the eouilibrium. The relative effectiveness of these two instruments depends on the relationship between the tax rates - if $\tau_f > \tau_w$, then b_f is relatively more effective, and vice versa.

In fact, for any positive firm (worker) employment subsidy there exists a critical tax rate, τ_f^* (τ_w^*), such that for all $\tau_f > \tau_f^*$, $(\tau_w > \tau_w^*)$ the equilibrium is efficient. The reason for this is simply that as long as b_f is strictly positive, the right-hand side of (51) goes to minus infinity as τ_f goes to 1, and similarly for τ_w. This has the following policy implication; it is possible, by an appropriate choice of profit tax and firm employment subsidy ($b_f >$ 0),to run an unemployment benefit scheme ($b_w < 0$) which has almost no negative effect on output or employment. The intuitive explanation for this is simply that by making the profit tax arbitrarily large relative to the wage tax, the incentive effects of an firm employment subsidy relative to unemployment benefit can be made arbitrarily large.

Finally, it is worth comparing these results to those obtained by Pissarides(1985a) for a stochastic matching model with an endogenous unemployment vacancy ratio, but where workers and firms are homogenous, and where vacancies are determined by a zero profit condition. In this setting, employer subsidies and unemployment benefits have very similar effects to the ones noted above; the former increases output and employment, and the latter decreases it. The mechanism by which this occurs is however, completely different. In the case of unemployment benefit, for example, an increase in unemployment benefit raises the wage, so the profitability of a vacancy falls below zero, so vacancy numbers fall also to restore equilibrium. This lowers the matching probabilities of workers, and so raises unemployment.

3.11 Theory and Evidence

In this section, we try to relate the predictions of the model to the available evidence.

The data mentioned in 3.5 was the ratio of the average duration of current uncompleted spells to the average duration of completed spells for males in the U.K. over the period. Recall that the higher this is, the less likely it is that the data are generated by a process where unemployment exit times are both independent of duration and the same across individuals.

The behaviour of this ratio for males in the U.K. is intriguing - it has shown a secular decline over the period 1962-1983 from a level of about 4 to 5 in the early '60s to around $1\frac{1}{2}$ in the early eighties, (Jackson and Layard (1984)). Furthermore, large increases in the percentage rate of unemployment (e.g. in 1975, and 1979-81) cause sharp falls in the ratio, which are subsequently partially offset.

Unfortunately for our purposes, there are several possible explanations for these trends. First, and most importantly, it is possible that a rising rate of growth of unemployment produces this effect, in that compared to the case where the rate of growth of unemployment is constant, there will be proportionately more recent entrants into the unemployment pool as time proceeds. This will lower the average current uncompleted spell, even if the expected completed spell length remains constant Conversely, as the rate of growth of unemployment falls, the ratio should rise.(see Main (1981) for a more detailed discussion of this).

This argument implies that we should see "undershooting" of the ratio following particularly sharp rises in unemployment, and this is precisely what we do observe. In 1980, for example, with unemployment at 8.3%, the ratio was 1.74; in 1981, when unemployment had jumped to 13% the ratio declined sharply to 1.09, and recovered to 1.22 and 1.4 in the following two years as the rate of unemployment slowed.

Unfortunately, this effect is likely to swamp anything else that could have led to such a fall. For example, there may be changes over time in the distribution of characteristics across the workforce affecting re-employment probabilities. Or there may be changes in other factors truly affecting duration dependence e.g. search intensity declining less rapidly over the unemployment spell.

There seems no real way of assessing the relative importance of these other factors, and especially whether they provide a complete explanation of the movements in this ratio. It would be much more desirable to have cross-section data on this ratio, perhaps by occupation or region, but this does not seem to be available.

However, assuming (without much justification, it must be said) that these other factors do not fully explain the fall in the ratio then changes in screening must be such as to decrease it. Now, Salant(1977) shows that if τ is the random exit time from unemployment, , this ratio is in fact equal to

$$(54) \quad \frac{1}{2} \cdot \frac{\mathrm{var}(\tau)}{(E(\tau))^2} + 1$$

If we calculate $\mathrm{var}(\tau)$ and $E(\tau)$ explicitly from the aggregate density function, $f(\tau;t^*)$ and substitute into (54), we find that the ratio is a rather complicated, non-monotonic function of the cut-off time, t^* . However, at $t^* = \infty$, it is greater than unity, and at $t^* = 0$, it is equal to unity, and so must be at least somewhere decreasing in t^*, as one would expect - recall that increases in t^* correspond to a lessening in screening, which should decrease the ratio.

Thus the data is, very roughly speaking, consistent with an increasing t^* over time, which is in turn not really consistent with the prediction of the model, which is that increases in unemployment should reduce t^*. However, this inconsistency merely highlights the fact that the model is a steady-state one; it seems plausible that a large increase in the aggragate unemployment rate will reduce the informativeness of duration, and so lead to a reduction in screening. The investigation of the dynamics of the model is certainly a topic for future work.

Conclusions

As remarked in the previous section, the most serious limitation of the duration dependence model is that it is a steady-state model, and so it is very difficult to compare its predictions directly with the evidence. However, relaxing this assumption would be extremely difficult, as the conditional probabilities would then depend upon the entire time path of past unemployment levels for the two types, which would in turn depend upon the past values of the cutoff time (the latter would themselves depend on calendar time). However, even if analytical results are not possible without simplifying assumptions outside the steady state, the general picture would seem to be as follows. Suppose that initially, unemployment is below its steady-state level; then flows into unemployment would exceed flows out, which would tend to to lower the proportion of type 2's in the unemployment pool, therefore possibly making screening less attractive and thus raising the equilibrium cutoff time. It would be interesting to try and make this argument more rigorous.

The other possible extension would be to model screening on other elements of an individual's unemployment history, for example the duration of the last spell of employment. Whether this would be worht while is another question; this rather depends on whether employers actually make use of this piece of information. It seems more likely that they would be concerned with the reason why the worker lost the last job, rather than the length of the last employment spell.

Footnotes

Chapter 1

1 This is also true of finitely repeated games, and the demonstration of this involves similar sorts of "trigger"strategies. Of course, the equilibrium outcomes will be affected by "endgame" considerations, for example that the outcome in the final period must be a constituent game equilibrium.

2 The exceptions are games where the payoffs of some Nash equilibrium of the constituent game coincides with the minimax payoffs for all players e.g. the Prisoner's Dilemma.

3 To complete the description of the strategy, we suppose that if two or more players deviate simultaneously, play returns to a^* form the beginning.

4 By the compactness of the space of outcomes, and the continuity of payoffs (in the product topology) such a sequence exists. See Abreu(1982) for a more detailed exposition of this point.

5 In the theory of average-cost programming, which is the single-person analogue of a dynamic game without discounting, continuity of minimal average cost at infinity certainly holds if the state-space is finite, although this is no longer the case if it is infinite (see Whittle(1983), p.132 and 147).

Chapter 2

1 It is fairly easy to show in a one-period context that if there are no mobility costs incurred by workers moving from one firm to another ex post - i.e. after contracts have been "signed" and the exogenous shock is realised - then risk-sharing contracts cannot co-exist with a spot market for labour. The reason for this is that the contract must pay a wage at least equal to the market wage in every state to prevent workers leaving, and must also pay the worker an expected utility equal to the expected utility obtainable on the spot market, in order to attract workers to "sign" contracts in the first place. Hence, the contract must pay the spot-market wage in every contingency. With many periods, on the other hand, this is no longer true, as Holmstrom has shown; contracts may co-exist with spot markets by "front-end loading" i.e. paying a wage below the spot-market level in the first period, so that (by equality of lifetime expected utility) the average contract wage will be above the spot market average in the second, thus allowing for risk-sharing.

2 This follows from the result established in 2.1(c) that there is always a precommitment equilibrium where both principal and agent do not condition their actions on past values of θ, so that it does not matter to what extent this information is revealed.

3 Recall that all precommitment equilibria yield the same payoff to the principal in every subgame, so it does not matter which of the equilibria we analyse.

the previous section; what is required is simply condition (A) with x replaced by $x(\bar{u}/\bar{u}-s,\ 1)$.

7 By total differentiation of (34)′ and (43), it is possible to show that $\frac{s}{u}\cdot\frac{du}{ds} = \frac{\partial r/\partial t + D}{D}$

where $D > 0$ by stability, so the result follows.

8 This is proved as follows. From (43),

$$\frac{s}{h}\cdot\frac{\partial u}{\partial s} = -\frac{s}{u}\cdot\frac{\partial h}{\partial s}\Big/\frac{\partial h}{\partial u} = \left(\frac{\partial h}{\partial u} - 1\right)\Big/\frac{\partial h}{\partial u}.$$

As $\frac{\partial h}{\partial u} > 1$ again from (43) it follows that $0 < \frac{s}{n}\cdot\frac{\partial u}{\partial s} < 1$.

9 This is easy enough to establish, as from equation (39) we can deduce that $\partial(f_2/f_1)/\partial x$ is proportional to $(1-\phi)\left[(x+\delta)(\phi x+\delta)\tau-\delta\right]$.

10 First, let $u_2/u_1 = k$. Then $\partial k/\partial x$ has the sign of

$$\frac{(1-a_1)+x.t^*.a_1}{\phi(1-a_2)+\phi^2x.t^*.a_2} - \frac{\delta+x.(1-a_1)}{\delta+\phi x.(1-a_2)}$$

with $a_1 = \exp(-(x+\delta)t^*)$

$a_2 = \exp(-(\phi x+\delta)t^*)$

It is apparent that as $\phi \to 0$, this expression goes to $+\infty$, so that for ϕ low enough, $\partial k/\partial x > 0$.

Next, we turn to $\partial k/\partial t^*$. This has the same sign as

$$\frac{x.(x+\delta).a_1}{\phi x.(\phi x+\delta).a_2} - \frac{\delta+x.(1-a_1)}{\delta+\phi x.(1-a_2)}$$

so the same argument applies – for ϕ low enough, $\partial k/\partial t^* > 0$.

Appendix to Chapter 2

Here we give a formal definition of a subgame-perfect equilibrium of the contracting game with precommitment by the principal described in 2.1(b), and then prove Lemma 1.

It will simplify the exposition considerably, and does not alter the basic argument, if we ignore the fact that the principal and agent can terminate the contract. In this case, recall that a strategy in the game for the agent is, at each t, to announce a type a_t, conditional upon a past history of types and announcements; i.e. $a_t = f_t(a^{t-1}, \theta^t)$.

Now define the history of announced types a^t contingent upon a particular θ^t, a^τ, $(\tau < t)$ and f recursively as follows;

$$(A1)\quad a^t(\theta^t, a^\tau, f) \equiv ((a^{t-1}(\theta^{t-1}, a^\tau, f), f_t(a^{t-1}(\theta^{t-1}, a^\tau, f), \theta^t))$$

and $a^\tau(\theta^\tau, a^\tau, f) \equiv a^\tau$,

Next, define

$$(A2)\quad g_t(\theta^t, f|a^\tau) = g_t(a^t(\theta^t, a^\tau, f))$$

so that $g_t(\theta^t, f|a^\tau)$ is the contract on offer at t given that θ^t occurs, and that a^t follows a^τ up to $\tau < t$ and is generated by f from then on.

Now, (f, g) is a subgame-perfect equilibrium of the contracting game withprecommitment if (i) g is a best response to f for the principal at t=0, and (ii) f is best against g for the agent on all subgames a^{t-1}.

The condition for (ii) is that

$$\text{(A3)}\quad E\Big[\sum_{\tau=t}^{T}\delta^{\tau-t}.u(\ g_t(\theta^t\ ,\ f|a^{t-1}),\ \theta_\tau)\ \Big]\quad >$$

$$E\Big[\sum_{\tau=t}^{T}\delta^{\tau-t}.u(\ g_t(\theta^t\ ,\ f'|a^{t-1}),\ \theta_\tau)\ \Big]\ ,$$

all $f' \neq f$, all a^{t-1}, all $t = 0.....T$.

We do not bother to state the equilibrium condition for the principal, as it is not required, but it is along the lines of (A3) with t=0. The first point to be made is that from Theorem 2, we can restrict our attention to those equilibria where the agent always announces his correct type. Therefore, let f^0 be the truthtelling strategy in the sequel.

We are now in a position to prove Lemma 1.

Proof of Lemma 1

Clearly, condition (6) in the text is a necessary condition. We prove sufficiency of (6) in the two cases separately.

(i) $T < \infty$

Suppose to the contrary there exists a $f^n \neq f^0$ where the agent lies $n < T-t$ times at dates $t < t_1 < t_2 ... < t_n$, and this is profitable at t on some subgame a^{t-1}. By condition (6) in the text, the last deviation at t_n cannot be profitable in any subgame starting at t_n, so that (A3) above holds at $t = t_n$, $f = f^0$ and $f' = f^n$ i.e.

$$\text{(A4)}\quad E\Big[\sum_{\tau=t_n}^{T}\delta^{\tau-tn}.u(\ g_t(\theta^t\ ,\ f^0|a^{tn-1}),\ \theta_\tau)\ \Big]\quad >$$

$$E\Big[\sum_{\tau=t_n}^{T}\delta^{\tau-tn}.u(\ g_t(\theta^t\ ,\ f^n|a^{tn-1}),\ \theta_\tau)\ \Big]$$

Now let f^{n-1} be the strategy which follows f^n up to and including t_{n-1}, and follows f^0 thereafter. Then, note that for all $t_{n-1} < t < T$

, $g_t(\theta^t, f^k | a^{t_{n-1}-1})$ are the same for k = 0, n-1, because both strategies prescribe truthtelling between these dates, and that the same holds for k = 0, n-1, n for $t_{n-1} < t < t_n$. Using this fact and inequality (A4), we can then show that f^{n-1} yields at least as high a payoff as f^n in all subgames beginning at t_{n-1}. Sucessive repetitions of this argument establish that f^{n-k}(similarly defined to f^{n-1}) yields at least as high a payoff as f^n in all subgames beginning in t_{n-k}, so that we conclude for k = n that

(A5) $$E[\sum_{\tau=t}^{T} \delta^{\tau-t} . u(g_t(\theta^t, f^0 | a^{t-1}), \theta_\tau)] \geq$$

$$E[\sum_{\tau=t}^{T} \delta^{\tau-t} . u(g_t(\theta^t, f^n | a^{t-1}), \theta_\tau)]$$

all a^{t-1}, so that f^n cannot improve on f after all.

(ii) <u>$T = \infty$, $0 < \delta < 1$</u>

Suppose to the contrary there exists a f^∞ where the agent lies at a sequence of dates t_n, n = 1, 2 .....∞, and which yields a higher payoff than f^0 in some subgame beginning at t. By the boundedness of u, we know that there exists a $0 < b < \infty$ such that

(A6) $$E[\sum_{\tau=t}^{\infty} \delta^{\tau-t} . u(g_t(\theta^t, f^\infty | a^{t-1}), \theta_\tau)] <$$

$$E[\sum_{\tau=t}^{t_n} \delta^{\tau-t} . u(g_t(\theta^t, f^n | a^{t-1}), \theta_\tau)] + b. \frac{1-\delta^{t_n+1}}{1-\delta}$$

where f^n is the strategy which follows f^∞ up to and including t_n, and f^0 thereafter. As (A6) holds for all n, we can combine (A5) and (A6) and take limits as $n \to \infty$ to yield a contradiction.

Appendix to Chapter 3

In this appendix, we derive the formulae for the conditional probabilities given in equations (36) to (39) of the text.

We show first that the distribution of exit times for a given type i has the density function

$$f_i(\tau;t^*) = \begin{cases} (h_i+\delta).\exp(-(h_i+\delta).\tau) & \text{if } \tau \leq t^* \\ \delta.\exp(-\delta(\tau-t^*)).\exp(-(h_i+\delta)) & \text{if } \tau > t^* \end{cases} \qquad \text{(A1)}$$

where $h_1 = x$, $h_2 = \phi.x.$ so that f_i is left-continuous in τ, and has a downward jump at $\tau = t^*$.

This is easily proved by considering a discrete approximation to the model with time periods of length 1/n. In this case, we suppose that if a worker of type i has experienced less than or equal to $\lceil n.t^* \rceil$ periods of unemployment (where $\lceil a \rceil$ denotes the smallest integer greater than or equal to a) he will leave unemployment with probability $(h_i+\delta)/n$ per period, whereas if he has more than $\lceil n.t^* \rceil$ period's previous unemployment, he will leave with probability δ/n. Then the probability of exiting in period $\lceil n.\tau \rceil$ is

$$\pi_{i\tau} = \begin{cases} (1-(h_i+\delta)/n)^{\lceil n.\tau \rceil - 1}.(h_i+\delta)/n & \text{if } \lceil n.\tau \rceil \leq \lceil n.t^* \rceil \\ (1-(h_i+\delta)/n)^{\lceil n.t^* \rceil}.(1-\delta/n)^{\max\{0,\ \lceil n.\tau \rceil - \lceil n.t^* \rceil - 1\}}.\delta/n & \text{if } \lceil n.\tau \rceil > \lceil n.t^* \rceil \end{cases}$$

Now $\pi_{it}/(1/n) = n.\pi_{it}$ is the probability of exiting in $\lceil n.\tau \rceil$ normalised by the period length. By inspection, we can see that $\lim_{n \to \infty} n.\pi_{i\tau} = f_i(\tau;t^*)$ as defined in (A1), which completes the derivation of the formula.

We now derive the formulae given in equations (36) and (38) of the text. In each case, the derivation is best understood by considering a discrete approximation as above, but we dispense with this intermediate step, having already obtained an explicit formula for the exit density. We begin with equilibrium unemployment rates.

The equilibrium condition is that the expected flow of each type of worker into unemployment, $L_i.\delta$, must be equal to the expected flow out. The latter is composed of two elements. The first is the expected flow into employment, which is equal to the probability of exiting into employment, $h_i.\int_0^{t^*} f_i(\tau;t^*)d\tau$, times total unemployment of that type, U_i. The second element is the expected flow out of the labour force, which is simply $\delta.U_i$. Equating flows in to flows out, and dividing through by L_i to express them in per capita terms, we obtain

$$\delta = [\ h_i.\int_0^{t^*} f_i(\tau;t^*)d\tau + \delta].u_i \quad .$$

Rearrangement of this equation to solve for u_i yields equation (38) in the text.

It now follows that the firm's prior belief that a sampled (i.e. matched)worker is of type i is $u_i.\lambda_i/(u_1.\lambda_1+u_2.\lambda_2) = \pi_i$, as in equation (37). Therefore, if it is matched with a worker whose duration is τ it's posterior belief that the worker is of type i is

$$\frac{f_i(\tau;t^*).\pi_i}{\sum_{i=1}^{2} f_i(\tau;t^*).\pi i} \qquad \text{(A2)}$$

which is the formula for $P_i(\tau;t^*)$ in the text.

Finally, we use this formula to demonstate that $P_1(t^*;t^*) > P_1^+(t^*;t^*)$, as claimed in the text. Substituting (A1) into (A2), we find that this inequality is equivalent to;

$$\frac{(h_1+\delta).\exp(-(h_1+\delta)).\pi_1}{\sum_{i=1}^{2}(h_i+\delta).\exp(-(h_i+\delta)).\pi_i} > \frac{\exp(-(h_1+\delta)).\pi_1}{\sum_{i=1}^{2}\exp(-(h_i+\delta)).\pi_i} \tag{A3}$$

As $h_1 > h_2$, (A3) is always satisfied with a strict inequality.

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