Repurchasing Debt

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Abstract

In this paper we build a theoretical model of corporate debt repurchases. First, we find that the firm that buys back its own debt from a creditor must pay a premium over the price at which the same creditor is willing to trade with third parties. This is because the repurchase by a firm leads to a dollar-for-dollar reduction in the amount of cash or assets available to pay the remaining debt. Second, the repurchase price is lower when there are multiple bondholders because of cross-creditor externalities. Therefore, we challenge the view that restructuring more dispersed debt is always more costly to implement. Third, when bankruptcy costs are significant, there is a range of prices below face value at which debt can be repurchased. Fourth, we show that repurchases contribute to flexibility in firms' capital structure and increase ex-ante firm value, but have limited power to mitigate debt overhang.

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1. Introduction

The low price of corporate debt in the secondary securities market and recent tax incentives arising from the post-crisis American Tax Recovery and Reinvestment Act of 2009 presented an opportunity to many firms to restructure or reduce outstanding debt on more favorable terms. By repurchasing their debt with cash or assets, companies were able to reduce their existing indebtedness (which carries no tax advantage net of cash) at less than the original face value, reduce their interest costs, and remove restrictive covenants. In 2010, for example, companies initiated 190 cash repurchases of publicly traded bonds, with an aggregate amount of $36.3 billion, compared to just 49 transactions during the period 1986-1996 (see Figure 1).[1] Despite the increasing number of debt repurchases, the academic literature on this topic is virtually nonexistent. In this paper, we provide a formal theoretical framework for corporate debt buybacks, with the goal of understanding when a repurchase is optimal, and what the implications it has for shareholders and creditors. The framework lends itself to a number of applications and empirical predictions.

First, we show that creditors should not sell risky debt back to the company at the market price–i.e., the price at which they would be willing to trade with third parties. We provide an example of a firm with a sole lender or a group of coordinated lenders, under Modigliani-Miller conditions. In this frictionless setting, the minimum price at which the lender agrees to sell the marginal unit of risky debt back to the firm is equal to the face value of the debt, above the market value. All additional bonds are also repurchased at the face value. Note, however, that it is impossible to buy back all debt at the face value, since if there were enough cash to do this, the debt would not be risky. The basic idea is that using cash or any safe asset for repurchase adversely affects the value of the remaining debt claims and does not reduce the probability of bankruptcy since the firm's liabilities and assets are

[1] These estimates are conservative because many repurchases are not recorded in the FISD database. They are omitted if they were negotiated privately or structured as exchanges for cash, or bundled with assets, common shares, or senior debt. For example, Imax Corp. recently exchanged $90 million in notes at less than 24% of their face value; however that transaction does not appear in our data.

reduced by the same amount. In essence, the debt is secured by cash and assets inside the firm, so that repurchasing the debt amounts to paying the lender with his own money.

Second, debt that is held by many shareholders can be repurchased at a significantly lower price. Bonds can be repurchased on the open market or using a tender offer at prices close to the market price, as long as there are small investors willing to sell their entire stake. The important difference from the frictionless case is that the sellers do not internalize a decline in the value of the remaining debt because it is held by other investors. We show that the equilibrium outcome in this case depends on the price offered in the repurchase. The repurchase is guaranteed to be successful for any offer price above the market price, and may even be successful for prices below the market price if investors are optimistic about the repurchase completion. Intuitively, there is a strong incentive to participate in the repurchase because the price is expected to decrease. Those investors that do not tender or exchange their bonds are exposed to increased risk and lower value.[2]

Third, we discuss bankruptcy costs, tax, and transaction costs in the context of our model, and show that (1) costly bankruptcy encourages repurchases; and (2) taxation and transaction costs discourage repurchases. Intuitively, fixed and proportional bankruptcy costs decrease recovered value to a lender following firm default and therefore encourage bondholders to make concessions. The model gives a range of prices at which a repurchase is possible. Within this range, the negotiated price depends on the relative bargaining power of shareholders and bondholders. Expected bankruptcy costs in repurchases are reduced in two distinct ways. Cash or assets transferred to creditors before bankruptcy reduce the proportional bankruptcy costs. Additionally, the repurchase at a price below the face value reduces the probability of bankruptcy. Taxation is shown to affect the repurchase incentive primarily through cancellation of indebtedness (COD) tax, which results in an additional cost proportional to the size of the repurchase discount.

Fourth, we argue that, from the ex-ante perspective, the ability to repurchase debt is

[2]There are legal restrictions applicable to the tender offer repurchases of publicly traded debt, that prohibit changing the debt principal without the debtholders' unanimous consent. We discuss this later on.

beneficial to the firm. Repurchasing at prices below the face value—e.g., through the tender offer—increases firm value and the firm's debt capacity. Although bondholders may be exploited ex-post, the overall effect on firm value is positive because shareholders gain more than bondholders lose. We discuss features of creditor structure and debt contract design that decrease the firm's ability to repurchase debt because they will also decrease firm value. For example, it is easier to repurchase publicly traded debt because there are small investors who can sell their entire stake. At the same time, we find that convertible debt is harder to repurchase because of the additional regulatory requirements. Contrary to casual intuition, a call option has little effect on the value of the repurchase option since the former is in the money when the debt value is high, and the latter when the debt value is low.

Although a repurchase reduces firm indebtedness, we show that in most cases it cannot mitigate agency conflicts originating from debt, such as the underinvestment problem. At the root of the problem is the condition that, unless the repurchase price is lower than the face value, the bankruptcy risk and debt overhang will not be affected by buying back debt. We further show that it may be impossible to negotiate a lower repurchase price because bondholders require a premium in anticipation of the investment. Finally, we show that a firm with severe financial constraints will be better off if it allocates cash directly to the investment instead of first undertaking a debt repurchase.

The analysis here also provides insights on debt repurchase timing. It is clear that buying back debt is costly and at least partially irreversible. Because the expected gain from a repurchase increases with the risk of default, managers have an incentive to postpone the repurchase until a date closer to debt maturity. Therefore, the option to buy back debt must be kept "alive" by increasing cash reserves instead of immediately reducing debt. It is therefore important, going forward, to recognize that shareholders may intentionally engage in simultaneous borrowing and saving to increase the value of the repurchase option.

This paper is related to the literature on debt restructuring and debt exchanges. Since the seminal contributions of Froot (1989), Bulow and Rogoff (1991), Bulow, Rogoff, and

Dornbusch (1988), and Gertner and Scharfstein (1991), who focused on implications of the sovereign and corporate debt exchanges prevalent in the 1980s, ours is the first formal study to address the current phenomenon of corporate debt repurchases.[3] Gertner and Scharfstein (1991) show, in particular, that offering new senior securities (cash paid to debtholders is one example of such a security) in exchange for distressed junior debt is beneficial to shareholders. However, Gertner and Scharfstein (1991) do not discuss the price, optimal timing, and the determinants of debt repurchases, which are the focus of our paper. Froot (1989), Bulow and Rogoff (1991), Bulow, Rogoff, and Dornbusch (1988), and others study open-market *sovereign* debt repurchases in the presence of the debt overhang problem. The major difference between corporate and sovereign debt buybacks is that, in the latter, cash and assets cannot be meaningfully pledged (see Bulow, 1992, for details).

Our work is also related to the strategic debt service literature (Mella-Barral and Perraudin, 1997, Hart and Moore, 1998). Firms facing financial distress can act strategically and force concessions from debtholders. However, whereas strategic debt service deals with bargaining after default, when cash effectively already belongs to creditors, we discuss repurchases by a solvent firm. For this reason, some of the predictions in our model are contrary to those in the debt renegotiation literature. For example, we show that the dispersion of debtholders that is commonly seen as an impediment to renegotiations actually helps to reduce leverage and the probability of bankruptcy in debt repurchases.

Our results have connections to the literature that investigates investment, debt, and the propensity to save in financially constrained firms. For example, Acharya, Almeida, and Campello (2007) describe the intuitive trade-off between saving cash and repurchasing risky debt when investment opportunities are positively correlated with cash flows and debt can be purchased at the market price. We extend their results by laying out the conditions that determine the repurchase price. Finally, our study is related to the growing literature

[3] In related empirical studies that focus on debt exchanges and repurchases, the propensity for debt reduction has been linked to the proportion of public and bank debt, debt seniority, maturity, and the value of growth options. James (1996) offers a comprehensive overview of this literature. Kruse, Nohel, and Todd (2009) provide recent evidence that shareholders benefit when a firm repurchases debt.

that examines the role of cash holdings within dynamic models and sheds light on the large observed cash accumulation.[4] We show that saving cash can be beneficial when the firm anticipates future debt repurchases.

2. Institutional Background

There are three main mechanisms for buying back corporate debt: open-market repurchase, tender offer, and privately negotiated repurchase.[5] An open-market repurchase, which includes repurchases in private markets by institutional buyers, is executed over a period of time and allows for potentially different prices for each bond sold back to the firm. A tender offer is typically conducted by offering a single price to all bondholders. Repurchases are conducted using cash savings, proceeds from the sale of assets or proceeds from senior security issuance collateralized by these assets (Gertner and Sharfstein, 1991). In this paper, we do not discuss debt-for-equity exchanges, which have different implications.

An open market repurchase is an easy way for an issuer to buy back relatively small amounts of debt. Other than complying with the anti-fraud provisions of the federal security laws, these transactions are not normally subject to review by the U.S. Securities and Exchange Commission (SEC).[6] However, it is difficult to repurchase large amounts in a limited time on the open market. Also, this mechanism does not permit the issuer to amend the covenants of the bonds because the issuer or the affiliates are not entitled to vote for the purpose of giving consents under the indenture.

Tender offers can include a fixed premium over the current trading price and allow the repurchase of larger amounts. Importantly, tender offers may include additional incentives for bond investors, which all but guarantee a successful repurchase. To motivate the holders

[4] For example, Morellec and Nikolov (2009) and Hugonnier, Malamud, and Morellec (2011) link cash holdings to investment, competition, and a desire for liquidity. In Riddick and Whited (2009), saving policy trades off tax penalties and the reduction in expected future financing costs.

[5] Debt repurchases may also be conducted as auctions. For example, Hovnanian Enterprises Inc. used a modified Dutch auction with base bid prices ranging from \$480 to \$750 per \$1,000 of the face value. The company eventually paid \$223 million to buy back \$578 million of debt in February and April of 2009.

[6] However, issuers may face greater regulation by the SEC if they proceed with very large repurchases through these transactions. See, e.g., the May 2009 Pepper Hamilton LLP note "Corporate and Securities Law Update."

of bonds to tender without offering a large premium and to avoid the need to comply with all of the existing contractual requirements, companies also solicit "exit consents" with their offer, in which case the holders of the securities are asked to consent to amendments to the security as a condition of their acceptance of the offer (Kaplan and Truesdell, 2008). If the consent solicitation is successful, any holders who refuse to accept the offer would continue to hold their old securities, which are stripped of protective covenants and made effectively junior to the new security.

An additional advantage of conducting a tender offer with "exit consents" is the ability to remove existing covenants that restrict the borrower's future actions (Mann and Powers, 2007). Having removed these covenants, the company may gain more flexibility in investment and financing decisions. For example, a firm may be able to increase capital expenditures, make an acquisition, increase dividends, liquidate assets, transfer money to subsidiaries, change the financial reporting procedure, alter collateral, consolidate assets, merge with another company, change lines of business, or modify its bylaws (Roberts and Sufi, 2009, and King and Mauer, 2000).

However, there are two serious difficulties that companies must overcome. First, tender offers for publicly traded debt require compliance with the Trust Indenture Act of 1939, section 316(b), which prohibits debtholders from changing the principal of debt without the debtholders' unanimous consent. It is designed, in particular, to prevent the company from exploiting minority bondholders. Managers can (and do) avoid this restriction by buying back a portion of debt on the open market or by combining cash repurchases with exchanges for other securities (see, e.g., Brudney, 1992, Gertner and Scharfstein, 1991, and Shuster, 2007).[7] They can also avoid having their repurchase classified as a tender offer by soliciting a limited number of holders, repurchasing over a fairly long period of time, and/or purchasing on different terms from each holder.

[7] Shuster (2007) gives examples of the provisions, which were originally designed to remove small percentages of abstaining bondholders in otherwise fully consensual agreement, but can be used to satisfy the requirements of section 316(b) without agreement of the majority of bondholders.

Second, whenever debt is repurchased below its face value, the firm is subject to a tax on the COD income. Unless an exception applies, such as insolvency or bankruptcy at the time of the repurchase, shareholders must recognize the COD income upon satisfaction of its indebtedness for less than the amount due under the obligation. The COD income is usually the difference between the amount due under contract and the amount paid.[8] Firms facing COD may find that the additional tax partially offsets the benefits of buying back debt at a low price. However, the recently enacted American Recovery and Reinvestment Act allows deferring the COD tax costs for up to 11 years, effectively making debt repurchases more attractive.[9]

3. Model of Debt Repurchase

In this section, we lay out the basic single-date model in the frictionless case with a single bondholder. Later we relax some of the assumptions of this framework and analyze how different financing frictions affect debt repurchases.

3.1. The Frictionless Case

Suppose that the firm has cash C, or a liquid riskless asset of an equivalent value, or proceeds from a senior security collateralized by this asset. The existing assets of the firm generate a cash flow x, distributed according to the cumulative distribution function $F(x)$ on the non-negative support $[\underline{X}, \overline{X}]$. If cash flows can be negative, $\underline{X} < 0$, we redefine $C' = C + \min(X)$; if the firm can spend only part of available cash on debt repurchase, then C contains only this part. We assume that all of the firm's debt D (including accumulated interest at rate r) matures shortly after realization of x. Since the problem is trivial in the

[8] This difference and the associated COD tax can be non-trivial. For example, Harrah's paid about 48 cents on the dollar to repurchase $788 million of debt in the second quarter of 2009. If not for the ARR tax deferral, Harrah's would face an immediate COD tax levied on the discount of about $400 million.

[9] The act does not alter how COD income arises, but rather affects when the debtor pays tax on the income. Usually, for repurchases after December 31, 2008 and before January 1, 2011, the bondholders can elect to apply the COD over a five-year period beginning in 2014. Therefore, a firm that repurchased in 2008 will finish paying the COD tax in 2019. The interested reader can find details in, for example, the 2009 Pepper Hamilton LLP note "Stimulus package: buy back debt today, pay tax later."

case of riskless debt, we require that the firm defaults in at least some states of the world, i.e.,

$$C + \underline{X} < D \leq C + \overline{X}. \tag{1}$$

If the firm becomes bankrupt, the priority rule is observed and debtholders have first claim on the firm's assets. In the frictionless model, we assume that there are no costs associated with bankruptcy. Additionally, since our objective is to determine the impact of a firm's financial position on the incentive to increase or decrease leverage, we assume that the firm "inherits" debt and postpone the discussion of optimal leverage until Section 5. Lenders assume equal seniority; however, future debt issues are restricted to subordinate claims only and do not affect the recovered amount of the senior lender in the event of default.

We first consider debt held by a sole lender, such as a private investor, or alternatively several large lenders, who collude when negotiating the sale price of debt. We assume that the firm is restricted from paying dividends or conducting share repurchases because such a distribution of cash would result in the value transfer from the lenders to the shareholders. Provisions limiting distributions affecting debt repayments are commonly included in debt covenants (Smith and Warner, 1979). Obviously, if unlimited dividends or share repurchases are allowed before the principle amount of debt comes due, shareholders' first-best strategy entails selling all assets to maximize the payout. Shareholder-debtholder conflicts are trivially resolved in this case (see, e.g., Jensen and Meckling, 1976). Finally there are other uses for firm's cash, which we do not allow in a simple model, such as investment considered in the later sections of this paper, compensation to employees, or perks to the management.

The objective of the manager is to maximize the value of equity with respect to financing decisions. In particular, the manager considers two alternative strategies: saving amount C, or using cash to repurchase an amount of debt ΔD from the lenders. Note that the average repurchase price is $P_R = C/\Delta D$; for example $P_R = 1$ means that repurchase is made at face value. Assume that firm's cash flows are independent of the repurchase. That is, the repurchase does not generate any synergies that can increase the value of the assets and therefore lead to the bondholder hold-out problem (similar to, e.g., Shleifer and Vishny, 1986).

We derive the repurchase price restrictions from the participation conditions for equity and debt holders. Define the equity value as S_0

$$S_0 = \int_{D-C}^{\overline{X}} (x + C - D) dF(x). \tag{2}$$

Define the equity value if the firm buys back ΔD of outstanding debt using all available cash C as S_R

$$S_R = \int_{D-\Delta D}^{\overline{X}} [x - D + \Delta D] dF(x). \tag{3}$$

Similarly, define the market values of debt as, respectively, d_0 and d_R

$$d_0 = \int_{\underline{X}}^{D-C} (x + C) dF(x) + D \int_{D-C}^{\overline{X}} dF(x), \tag{4}$$

$$d_R = \int_{\underline{X}}^{D-\Delta D} x dF(x) + (D - \Delta D) \int_{D-\Delta D}^{\overline{X}} dF(x). \tag{5}$$

Note that, because of assumption (1), the initial price of debt is below face value, $P_0 = d_0/D < 1$. The following proposition links equity and debt values to the price of the repurchase.

Proposition 1 *Under the assumptions of the frictionless case, the following statements are equivalent*

$$P_R < 1,$$

$$S_R > S_0,$$

$$d_R + C < d_0. \tag{6}$$

Proof. (all proofs can be found in the Appendix). ■

The proposition says that, if the repurchase price is lower than the face value, shareholders are better off after the repurchase and the bondholders are worse off. It follows that the face value is the only price at which both sides agree to buy and sell debt. Note that the repurchase price is unique because in the frictionless case debt repurchase does not change the total value of the firm. However, we show in the next section that there is a range of acceptable prices in case when the firm's assets are subject to bankruptcy costs. The prospect of reducing bankruptcy costs makes room for negotiations between shareholders and bondholders.

3.2. Bankruptcy Costs

Here we assume that in the event of default lenders take over the firm and implement first-best policies, subject to a fraction of the firm's assets being lost during the transfer. Firm entering bankruptcy results in fixed cost, B, and proportional cost β, which is known both to shareholders and to creditors. Unlike in, e.g., Leland (1994), we recognize that safe assets may be different from risky assets and assume that cash or liquid assets are subject to cost $\beta_1 \in (0, 1)$, and other assets are subject to cost $\beta_2 \in (0, 1)$. Although not crucial for our argument, it may be reasonable to conjecture $\beta_1 < \beta_2$, meaning that safe/liquid assets are easier to transfer to new owners. Parameter β_1 can also be interpreted as the agency cost, such as the manager's ability to "burn" cash before bankruptcy.

The expected bankruptcy costs are therefore

$$BC_0 = \int_{\underline{X}}^{D-C} (\beta_2 x + \beta_1 C + B) dF(x). \tag{7}$$

The following proposition gives the upper and lower bounds for the repurchase price and shows that the repurchase price is lower in the case with bankruptcy costs than in the frictionless case.

Proposition 2 *Assume that the support of x is bounded and function $F(x)$ is continuous,*

$B > 0$, $\beta_1 > 0$, $\beta_2 > 0$, and that, except for bankruptcy costs, assumptions from the frictionless case hold. Then

$$P_R \in [P_R^{\min}, 1],$$

where the lower bound on the repurchase price, $P_R^{\min} < 1$, is the unique solution to equation (26) in the Appendix.

Intuitively, if shareholders have all bargaining power in splitting the surplus from the bankruptcy costs reduction, the lowest price, $P_R^{\min}$, is obtained. If, instead, bondholders have all bargaining power, then debt is repurchased at the face value, as in the frictionless case.

Because the bankruptcy costs decrease, firm value increases after the repurchase. From Proposition 2 and expression (7), firm value increases by

$$\Delta(BC) = C\beta_1 \int_{\underline{X}}^{D-C} dF(x) + \int_{D-\Delta D}^{D-C} (B + \beta_2 x)\, dF(x). \tag{8}$$

Bankruptcy costs decrease, intuitively, for two reasons. First, during the repurchase, cash (or safe asset) C is transferred directly to bondholders in exchange for lower debt. It matters because, if the firm subsequently defaults or becomes bankrupt, this cash or asset, which are inside the firm, would be subject to the proportional cost β_1.[10] Therefore, expected bankruptcy costs are reduced even if the probability of bankruptcy is fixed, as captured in the first term in (8).

The second effect arises because repurchases generally lead to a lower probability of bankruptcy. Because of the reduction in proportional bankruptcy costs, a lower repurchase price can be negotiated, resulting in an additional benefit in the form of lower bankruptcy risk (the second term in (8)).

Overall, we predict that the average debt repurchase price is lower when expected bank-

[10] Cash is subject to bankruptcy costs, even if the firm can eventually restructure and exit the bankruptcy. For example, LoPuchki and Doherty (2010) estimate that only direct legal fees on all assets including cash can be as high as 2%.

ruptcy costs are higher. Additionally, keeping bankruptcy costs parameters fixed, the repurchase price increases with relative bargaining power of bondholders.

3.3. Multiple Bondholders

We model a single-date same-seniority ("pari passu") debt repurchase from a group of identical bondholders, each holding the same small share of debt. When the firm has outstanding debt of different seniorities, the argument extends to the most senior debt. Sometimes, in addition to the senior debt, the companies also attempt to buy back their junior debt. For example, the 2009 Royal Bank of Scotland tender debt repurchase offer included subordinated notes. However, understanding repurchase offers for junior debt is complicated because they lead to an additional conflict between the different *classes* of the bondholders.

Additionally, we assume in this section that revolving credit facilities and other high-priority obligations are repaid before the price for senior debt can be negotiated, debtholders are fully rational and attentive, and there are no bankruptcy costs or other financing frictions.

Consider first a tender offer, when a fixed price is offered to everyone who sells their bonds. If all bondholders tender simultaneously, they are served sequentially in random order until the full amount allocated for this purpose is spent. There is usually no minimum subscription requirement for the offer. It is intuitive that the tender offer equilibrium is contingent on how the offer price, P, compares to the pre- and post-repurchase prices. For example, if the tender offer price is high, the bondholders will participate because the expected post-repurchase price, P_R, is going to be lower. If the tender offer price is low, the bondholders will all abstain because the debt price without the repurchase, P_0, is higher. As the first step in formalizing this intuition, we define a "fixed-point" price, P_F, at which the post-repurchase price remains exactly the same as the offer price.

Lemma 1 *Suppose debt is repurchased through the tender offer from multiple bondholders:*

(i) there is a unique fixed-point tender offer price P_F*, such that* $P_F \equiv P = P_R$*,*

(ii) if $P > P_F$ *then* $P_R < P$*.*

(ii) $P_F < P_0$.

The Lemma defines the fixed-point price and states that it is strictly lower than the pre-repurchase price. According to the Lemma, repurchasing debt at any price above the fixed-point price (including the pre-repurchase price) will decrease the value of the bonds for the remaining bondholders.

Next, we discuss possible equilibria. First, we consider the case when the tender offer price is high, above the pre-repurchase and the fixed-point price, $P \geq P_0 > P_F$. From Lemma 1, the bondholders who do not tender receive a strictly smaller post-repurchase price, $P_R < P_0$. Therefore, there is a unique equilibrium in this case: the firm offers a price equal to or just above P_0, all bondholders tender, and a fraction $C/(PD)$ of them are served randomly until all cash C is spent.[11]

Second, consider a tender offer price between the pre-repurchase price and the fixed-point price, $P_0 > P > P_F$. The equilibrium in this region depends on the beliefs about the number of bondholders participating in the repurchase.

Proposition 3 *Suppose the tender offer price* $P \in (P_F, P_0)$. *If every bondholder has a uniform belief* j *about the fraction of bondholders who will participate in the offer, then:*

1. for $j \geq j^*$, *all bondholders tender, and the tender offer is successful.*

2. for $j < j^*$, *all bondholders abstain from the tender offer, and the offer fails.*

The threshold belief $j^* \in (0, C/(PD))$ *is given as a unique solution to the equation (32) in the Appendix.*

The proposition gives the threshold belief regarding the fraction of tendering bondholders, which can trigger the "bank run" (Diamond and Dybvig, 1983). For example, if the belief about the success of the offer is highly optimistic, i.e., $j \to 1$, then it implies $P_R < P$, and the offer is successful as nontendering bondholders are expected to be worse off. In contrast,

[11] A parallel result to this case can be found in Dhillon, Noe, and Ramirez (2001), who show that shareholders can successfully tender callable debt from multiple bondholders. In their model, the offer price must be above the threat-point call price.

$j \to 0$ implies that $P_R > P$, and the offer fails. Following Diamond and Dybvig (1983), we treat belief j as exogenous.

Finally, any tender offer price, which is equal to or below the fixed-point price, trivially leads to the repurchase failure. By the definition of the fixed point, for any $P \leq P_F$ and any belief j, the post-repurchase price is expected to increase, $P_R \geq P$, and therefore every bondholder will abstain from tendering.

Intuition for the *open-market* debt repurchases is similar to the tender offer case. An important difference, however, is that bondholders may receive different prices for their holdings, depending on the relative timing of the sale. As we have argued, the price for the remaining debt will decrease with each repurchase at the price above the fixed point, including the market price. Therefore, bondholders have a strong incentive to participate, and those who sell first will receive the best deal. At first, this may appear counterintuitive because a debt reduction would seem to make the remaining debt safer. Instead, a repurchase consumes cash inside the firm, making the remaining debt riskier. We do not formally define the equilibrium for the case of open market repurchases as it requires modeling heterogeneity among bondholders and building a sequential game for the stages of the repurchase.

There are two other important points that we would like to bring to light in conjunction with the case of multiple bondholders. First, we have assumed throughout that each investor holds an identical small fraction of debt and sells it entirely to the firm. Such continuum of homogeneous investors is a sufficient condition for our results, but not a necessary one. For example, when the creditor composition involves both large and small investors, debt will first be repurchased from the small investors. These investors can sell their entire debt holding in response to the offer and do not need to internalize the consequences of the repurchase on the outstanding debt.

Second, the news of the incoming tender offer, including information on the size of the offer and its outcome, may alter the market prices for both debt and equity. Specifically, anticipation of the repurchase at the price below the face value can result in the market value of debt lower than the initial price. Recall that the initial price, P_0 is defined as the expected payoff to bondholders if the repurchase is not anticipated, or if it is not expected to be successful. The Appendix provides the expression for the market price with the adjustment for the repurchase, which may be different from the initial price P_0. It is important, however, that the equilibrium does not depend on the true market price. It depends only on the relation between the tender offer price, the price if the repurchase fails, P_0, and the fixed-point price P_F.

The main insight from our study of the dispersed creditors case—debt held by multiple bondholders can be repurchased at a lower cost—contrasts sharply with the predictions of the literature on debt renegotiation and strategic debt service (e.g., Hart and Moore, 1998; Mella-Barral and Perraudin, 1997). Like in this literature, shareholders in our model are able to force concessions from debtholders. However, the strategic debt service deals with bargaining after default, when cash effectively already belongs to creditors and the negotiation is purely targeted to reduce bankruptcy costs. For this reason, small bondholders in their models, who can either abstain from negotiations or demand a premium, free-riding on large bondholders, make debt renegotiation impractical. The distinction must be made, because existing literature on the topic often draws conclusions on the basis of the debt renegotiation theory. For example, Mann and Powers (2007) argue that tender offers are easier to complete in firms with more concentrated debt ownership.

3.4. Tax and Transaction Costs

As we have argued, the discounted repurchases are beneficial to the shareholders; however these benefits are likely to be reduced by transaction costs and tax. First, we discuss the tax implications of repurchasing debt versus saving cash. As is standard in the literature (see, e.g., Auerbach, 2001), we track the after-tax payoff to shareholders under the two alternatives. If the firm saves C for one period, the after-tax dividend to shareholders is

$$\text{Payoff}_{\text{Save}} = C\left(1 + r\left(1 - T_c\right)\right)\left(1 - T_d\right), \tag{9}$$

assuming that tax T_c is levied on corporate income and cash distributions are subject to further tax at the rate T_d. Alternatively, if the firm repurchases a portion of its debt, ΔD, the after-tax dividend

$$\begin{aligned} \text{Payoff}_{\text{Rep}} &= \Delta D \left(1 + r\left(1 - T_c\right)\right)\left(1 - T_d\right) \\ &\quad - T_{COD} \max(\Delta D - C, 0)(1 - T_d), \end{aligned} \tag{10}$$

where the second term is an additional tax on the COD income if debt is repurchased at a discount. From (9) and (10), we compute the debt repurchase tax advantage over saving as

$$\begin{aligned} \text{Adv}_{\text{Rep}} &= \left(\Delta D - C\right)\left(1 + r\right) - \left(\Delta D - C\right) r T_C \\ &\quad - T_{COD} \max(\Delta D - C, 0). \end{aligned} \tag{11}$$

The direct benefit of repurchasing debt at a discount (first term) is reduced by a higher corporate tax due to the lower debt-net-of-cash (second term) and also a higher COD tax (third term). We compare this expression to our base model and conclude that corporate and COD tax reduce the ex-post benefits from the repurchase.

Transaction costs, trivially, can also reduce the repurchase incentives, and therefore must be considered against benefits of the repurchase. Firms incur significant direct and indirect costs when conducting debt repurchases, including premia paid in the tender offers and open market purchases. Costs associated with amending the contracts as well as attorneys' fees can also be significant (see, e.g., Roberts and Sufi, 2009). Finally, large indirect fees commonly appear, which take the form of time and effort spent by both borrowers and lenders on understanding the implications of the transaction, and obtaining approval or waivers in case of syndicated loans.

Overall, we find that bankruptcy costs and dispersed debt ownership, two assumptions that are common to U.S. firms, result in a lower repurchase price. With moderate transaction costs, debt repurchases are therefore beneficial to equity. However, despite the advantage of immediate repurchase, we show in the next section that treating the repurchase as an option and delaying its exercise results in higher expected profits.

4. Multi-Period Extension

We previously adopted the assumption that debt must be repurchased on a single date. This section extends the previous analysis by studying the intertemporal debt/cash policy in a two-period model. Such model allows us to understand what determines the optimal timing of debt repurchase and in particular the shareholder's incentives to delay the repurchase.

4.1. Frictionless case

Assume there are three dates, $t = 0, 1, 2$, and values are denominated in date $t = 0$ dollars. The firm's total profit at the end date $t = 2$ is equal to the sum of the independently distributed profits from the first and the second period, $x_1 + x_2$, where $x_{1,2} \in [\underline{X}, \overline{X}]$. At $t = 1$, the information about x_1 becomes available, and at date $t = 2$, the information about x_2 becomes available.

Equity maximizes the expected payoff with respect to saving/debt reduction decisions. Since the model now extends beyond a single period, we need to adjust the subscript notation accordingly. Assume that the initial face value of debt is D_0 and that it can be reduced to D_1 (before profit x_1 is revealed). At the next date, D_1 can be further reduced to D_2 (before x_2 is revealed). Similarly, we denote the cash changes due to the first and second repurchases as $C_0 - C_1$ and $C_1 - C_2$. Observing from Proposition 1 that at $t = 2$ shareholders benefits from buying the maximum amount, we set $C_2 = 0$.

In absence of intermediate dividends, the objective function of the shareholders is the expected value of the payoff at the last date $t = 2$

$$\max_{(C_1)} V_0 = \int_{\underline{X}}^{\overline{X}} \left[\int_{D_2(C_1)-x_1}^{\overline{X}} (x_1 + x_2 - D_2(C_1)) dF(x_2) \right] dF(x_1), \tag{12}$$

where $F(x_1)$ and $F(x_2)$ are the cumulative distribution functions for x_1 and x_2. With a minor abuse of terms, the derivative of this function, $\partial V_0/\partial C_1$, can be interpreted as "propensity to save" or "propensity not to reduce debt," used in prior literature. To maximize equity value, the manager minimizes the final-period debt value with respect to the repurchase policy.

Lemma 2 *In the frictionless case timing of the repurchase is irrelevant.*

It is straightforward to see from (12) why in the absence of frictions equity value is independent of repurchase timing. In this case, the price equals to the face value, regardless of the time of the repurchase, and cash simply cancels an equal amount of debt, $D_2 = D_0 - C_0$. It is important to recognize that the irrelevancy result exists in the frictionless case because the firm never "regrets" undertaking repurchases at the first date. Below we show that the timing matters outside of frictionless case.

4.2. The option to delay debt repurchases

Suppose the firm can repurchase on the open market at the current market price. We show that the shareholders are better off repurchasing later. This is because the future price is uncertain, and the value function is convex in the repurchase price. Therefore, by invoking the Jensen's inequality, we immediately obtain the following result.

Proposition 4 *Suppose debt is repurchased at price P_M^1 at date $t = 1$, and at price $P_M^2(x_1)$ at date $t = 2$, such that*

$$P_M^1 = \int_{x_1} P_M^2(x_1) dF(x_1). \tag{13}$$

Then it is optimal to delay repurchase.

The proposition shows that a debt repurchase presents a valuable option to shareholders; the value of this option is higher if the exercise can be delayed.

If debt repurchase is associated with additional transaction costs, it may become optimal to abandon the repurchase when debt price becomes too high. Transaction costs effectively increase the cost of the repurchase and therefore the firm repurchases selectively. We delegate details for this case to the Appendix. We show, in particular, that a proportional linear fee levied on the total transaction amount forces the firm to repurchase only if the first-date profit does not exceed a particular trigger value x_1^*. Otherwise, the firm will optimally abandon the repurchase and avoids paying the transaction fee. Therefore waiting until $t = 2$ to learn about the realization of the first-period profitability leads to a higher firm value. A similar intuition applies to the fixed costs, with exception that the optimal strategy depends on the volume of the repurchase.

Based on the two-date model in this section we conclude that companies, including those that would benefit from buying back debt using the first opportunity, are better off delaying the repurchase. At the end, we may not observe as many repurchases in the data as predicted by the simple one-period model because the option to buy back debt may expire unexercised. Our hypothesis that the firm benefits from saving cash for a future repurchase contributes to the literature on the determinants of cash holdings.[12]

5. Optimal Leverage

As discussed earlier, discounted debt repurchases may result in ex-post wealth transfers from bondholders to shareholders. In this section, we study how repurchases affect the ex-ante firm value—sum of initial equity and debt values—in order to derive optimal leverage, expected tax, and optimal debt structure.

5.1. Repurchases, Debt Capacity, and Firm Value.

We cast the classical trade-off intuition in our model and discuss the optimal leverage. Following previous work on capital structure (e.g., Leland (1994)), we assume that the firm trades tax benefits of debt with bankruptcy costs. Since we know from the previous sections

[12] Examination of the optimal cash holding policy appears in several recent studies. For example, Foley, Hartzell, Titman, and Twite (2007), Opler, Pinkowtz, Stulz, and Williamson (1999), and Faulkender and Wang (2006) point to a variety of the problems originating from holding excessive cash. On the theory side, DeAngelo, DeAngelo, and Whited (2009) argue that carrying cash is costly, while Dasgupta, Noe, and Wang (2009) predict that cash holdings have a beneficial effect by relaxing the inter-temporal financing constraints.

that buying back debt at face value leaves the total firm value unchanged, we focus only on the market-price repurchases. The following proposition demonstrates, using for simplicity the uniform distribution for the profit x, that both optimal leverage and firm value increase with repurchases.

Proposition 5 *Suppose x is distributed uniformly on $[\underline{X}, \overline{X}]$, and shareholders have an option to repurchase debt with cash C at the market price, then the optimal amount of debt issued at $t = 0$ is*

$$D^* = \frac{rT_C}{\beta_2} \left(\bar{X} - \underline{X}\right) \frac{d_0}{d_0 - C} - B \left(\frac{d_0}{d_0 - C}\right)^2, \tag{14}$$

the ex-ante firm value is given by

$$\begin{aligned} V^* \;=\; & \underbrace{\int_{\underline{X}}^{\overline{X}} x\left(1 - T_C\right) dF(x)}_{\textit{after-tax asset value}} + \underbrace{D^* r \left(1 - \frac{C}{d_0}\right) T_C}_{\textit{tax shield}} \\ & + C - \underbrace{\beta_2 \int_{\underline{X}}^{D^* - \frac{CD^*}{d_0}} x dF(x)}_{\textit{bankruptcy cost on assets}} - \underbrace{B \int_{\underline{X}}^{D^* - \frac{CD^*}{d_0}} dF(x)}_{\textit{fixed costs of bankruptcy}}, \end{aligned} \tag{15}$$

both D^* and V^* increase with the amount of repurchase.

Proof. see the Appendix ■

The firm value and leverage are higher when repurchases are allowed because the bankruptcy cost and the probability of bankruptcy is lower. Note that (14) must be treated as an implicit equation, because d_0 can also depend on the optimal debt.

Our result is directly comparable to the classic dynamic capital structure literature, where leverage is higher because "the firm has an option to lower the leverage ratio in the future, (and is) more aggressive initially in order to increase current debt benefits" (Goldstein, Ju, and Leland, 2001). We conclude that discounted debt repurchases are beneficial to shareholders. Contrary to the initial intuition that exploiting bondholders during the process leads to an agency problem, allowing debt repurchases can actually increase the ex-ante firm value. This is because the shareholders gain more than bondholders loose. The option to repurchase reduces the instances of defaults and increases debt capacity. Finally, note that the proposition only determines optimal leverage given cash holding; that is, cash C is "inherited" from the firm's past activities and is not jointly determined with the optimal debt.

5.2. Implications of Debt Repurchases on the Optimal Creditor Structure and Debt Contract Design.

As we argue above, debt repurchases positively affect capital structure flexibility and therefore increase the total firm value. Therefore, debt creditor structure, contract features, and covenants should not prohibit or complicate future repurchases.

First, this concerns the explicit restrictions on buying, redeeming, or exchanging debt at prices below par, which is present in some debt covenants. Second, debt conversion options can also have implications on the firm's ability to repurchase debt. They contain equity part and therefore necessitate an additional SEC approval prior to the repurchase. Third, the option to repurchase debt is directly affected by seniority structure. Our base model gives results for same seniority for all bondholders, based on the observation that the repurchase offer is typically made for a single class of senior debt. However, firms commonly carry several tranches of debt with a slightly different seniority for each separately sold debt fraction, makes repurchases more difficult and reducing shareholder value. Finally, the optimal creditor structure, in particular distribution of debt among creditors, can also affect repurchases. The more dispersed is debt ownership, the easier it is to restructure through a tender offer or an open market debt repurchase. Additionally, our model implies that publicly traded debt is the easiest to repurchase, as compared to privately held debt or bank debt.

6. Debt Repurchases and Investment

In this section, we follow earlier literature and investigate whether repurchases can mitigate investment inefficiencies caused by excessive leverage, such as debt overhang or underinvestment. As we argued earlier, debt repurchases can reduce firm risk only if the repurchase price is low. Therefore, we anticipate that the ability to mitigate the debt overhang problem also depends on the low repurchase price. To demonstrate and extend this point, we introduce capital investment into the existing model of risky debt and study how buying back debt affects investment incentives.

6.1. The Debt Overhang Problem

Following the literature,[13] we consider the situation when a firm is plagued by a debt overhang problem. The problem manifests itself in prohibitively high cost of external equity for firms with risky debt, leading to insufficient capital expenditures and high post-investment "marginal q" (Myers, 1977, and Hennessy, 2004). For example, Myers (1977) demonstrates that such firms forgo positive NPV opportunities since undertaking investment increases the value of debt and decreases the value of the firm's equity. Starting from this observation, it is natural to conclude that, all else equal, firms can increase investment by reducing leverage. However, debt reduction through the repurchase is not all else equal because it also decreases firm's safe assets.

To model investment in a simple form, we assume that shareholders can invest amount I, expecting the payoff $x(I)$. The effect of investment on the cash flows is modeled through the cumulative distribution function $G(x|I)$ on the domain $[\underline{X}, \overline{X}]$. Specifically, since investment must positively affect future profits, we assume that the payoff from larger investment first-

[13] For example, Bulow and Rogoff (1991) show that the buyback of sovereign debt is a giveaway to creditors because the relief from debt overhang is expected to increase the market value of debt.

order stochastically dominates the payoff from the smaller investment.[14] That is,

$$\frac{\partial G(x|I)}{\partial I} < 0 \text{ for } \forall\ x \in [\underline{X}, \overline{X}]. \tag{17}$$

Finally, to make the problem nontrivial, we assume that investment must be financed externally and the financing is subject to cost $\phi(.)$. With these assumptions, we derive the optimal investment, which maximizes firm value net of costs of investment:

$$\max_I \left[\int_{D-C}^{\overline{X}} (x + C - D) dG(x|I) - \phi(I) \right]. \tag{18}$$

It can be simplified with integration by parts as

$$\max_I \left[(\overline{X} + C - D) - \int_{D-C}^{\overline{X}} G(x|I) dx - \phi(I) \right]. \tag{19}$$

The optimal investment obtains from the first-order condition

$$\int_{D-C}^{\overline{X}} \underbrace{\left(-\frac{dG(x|I)}{dI} \right)}_{\text{positive}} dx = \frac{d\phi(I)}{dI}, \tag{20}$$

and can be interpreted as the marginal value of investment equal to the marginal cost at the optimum. Because of the debt overhang problem, the investment is below the first-best level. First-best is defined by the same expression as (20), but with the integral limit equal to $\underline{X}$ instead of $D - C > \underline{X}$. The second-order condition holds under additional regulatory conditions shown in Appendix. Based on (20) we can now discuss how debt repurchases can affect debt overhang.

6.2. Repurchase Price and Debt Overhang

[14] A simple example for this investment is the linear shift in the probability distribution of the payoff, corresponding to a constant positive return $R > 1$

$$G(x|I) = F(x - RI), \tag{16}$$

where $F(x)$ is the CDF of the payoff distribution without investment.

First, notice that the optimal investment is a function of debt net of cash only. Therefore, as we conjectured, investment incentives are unchanged with the dollar-for-dollar repurchases. Intuitively, the debt overhang problem persists because safe assets are reduced by the same amount as debt. Moreover, it follows from (20) that the optimal investment strictly decreases in $D - C$. Therefore repurchases at the price below the face value will positively affect the optimal investment.

Second, we ask if pre-investment debt repurchase can potentially be done at a price below the face value. As we discussed in Section 3, a low repurchase price may be obtained, e.g., when debt is repurchased through a tender offer or in the open market. However, the prospect of valuable investment increases the repurchase price. This is because market will internalize the benefits of the investment and bondholders can demand the premium. We provide additional details in the Appendix.

For firms with concentrated debt ownership, there is another possibility. They can attempt to negotiate a low price as a concession from the bondholders by promising to secure debt with investment once the repurchase is completed. We show in Appendix that the set of investment options supporting this case is limited. Intuitively, to induce investment, which secures the risky part of their claim, the bondholders must make an equivalent concession of the safe part of their claim. At the same time, the high-profit investment assumption will contradict our initial premise that the firm suffers from debt overhang.

Third, debt overhang can also be mitigated if cash or assets C is simply used to cover a part of investment cost instead of repurchasing debt. To illustrate this, consider a firm deciding to allocate one dollar to the cost of repurchase or to the cost of investment. The condition for this tradeoff is that the marginal value from the repurchase is equal to the marginal cost of financing investment

$$\int_{D}^{\overline{X}} \left(-\frac{dG(x|I)}{dI} \right) dx = \phi'(I - C). \tag{21}$$

Obviously, for the firms with high marginal costs of external financing the optimal solution is to allocate at least some of the cash directly to investment.

In summary, the assertion that buying back debt can mitigate debt overhang relies on the firm's ability to negotiate a low repurchase price and also depends on costs of external financing. When debt is repurchased at the face value, the risk of firm's levered assets is unchanged. At the same time, the discounted repurchase may not be feasible.

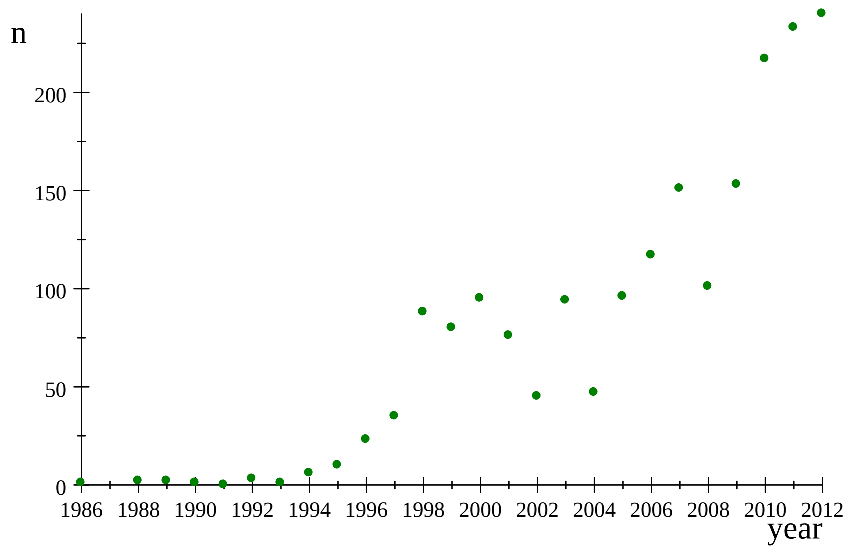
7. Conclusion

When managers are confronted with a choice between saving cash and repurchasing debt, they face a trade-off between costs and benefits of the repurchase. This paper provides a theoretical guidance for these decisions. We find that firms that can buy back debt at a discounted price benefit from the repurchase and also benefit more if they delay the repurchase. Simultaneous saving and borrowing creates an opportunity to buy back debt conditional on a lower price in the future, or scrap the repurchase plan otherwise. Our findings have implications for security design and pricing of debt contracts.

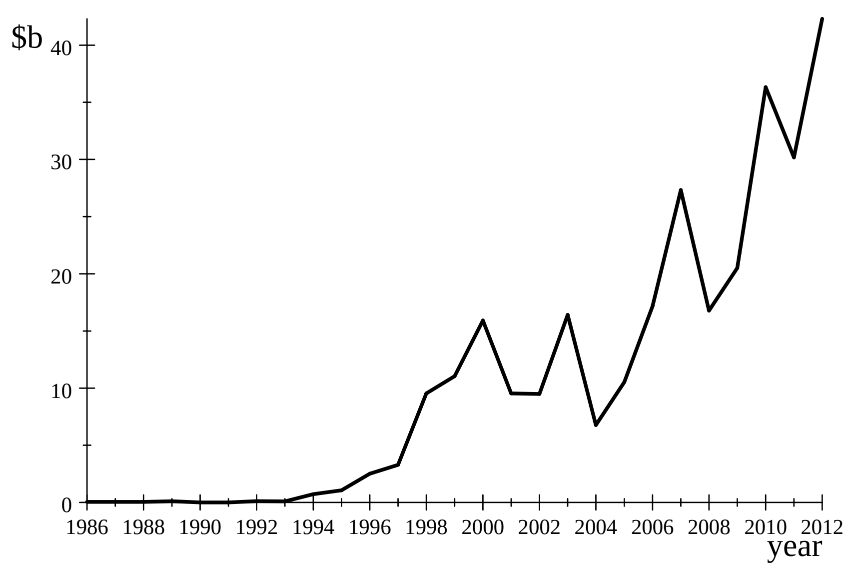
Our theory produces novel empirical hypotheses. First, discounted debt repurchases result in a value transfer from bondholders to shareholders, and therefore should increase the value of equity and decrease the value of debt. The size of the value transfer, and therefore the magnitude of the price reaction, is expected to be larger with the repurchase discount. A similar contrasting prediction for the bond and share prices was developed and verified in the stock share repurchase literature; see, e.g., Maxwell and Stephens (2003). Second, the repurchase price must be lower when the expected bankruptcy costs are higher, or when debt is dispersedly held and can be repurchased in the open market. Third, we expect firms to simultaneously carry cash and risky debt. This hypothesis finds some support in the existing studies. For example, Bates, Kahle, and Stulz (2009) state that "the average firm can pay back all of its debt obligations with its cash holdings." Finally, we predict lower market values for the firms that are unable to utilize debt buybacks for restructuring, for example if debt covenants prohibit repurchases.

Figure I. 1986-2012 U.S. Debt Repurchases.

Data are from the Fixed Income Securities Database (FISD) over 1986-2012. We include only "Tender Offer" (code 'T') or "Issues Repurchase" (code 'IRP') transactions with corporate bonds. The repeated repurchases by the same company are treated as separate. Total volume of repurchases is computed as the repurchase price, equal to the averaged-over-year "action price" in FISD, multiplied by the number of shares repurchased in this transaction, and summed over all transactions for this year. We dropped three observations, for which the "action price" likely contains a recording error (e.g., equal to zero).



(A) Annual Number of Repurchases n for years 1986-2011. For 2012, we plot projection based on the data available before May 8th.



(B) Annual Volume of Repurchases for 1986-2011. For 2012, we plot projection based on the data available before May 8th.

Appendix A. Repurchase Price Derivation

Proof of Proposition 1

To show that $S_R > S_0 \Leftrightarrow P_R < 1$, we define the function of $G(y)$

$$G(y) = \int_{D-y}^{\overline{X}} [x + y - D] dF(x),\ y \in [0, D]. \tag{22}$$

Function $G(y)$ increases in the argument,

$$\begin{aligned} G'(y) &= (D-y)f(D-y) + \int_{D-y}^{\overline{X}} dF(x) - (D-y)f(D-y) \\ &= \int_{D-y}^{\overline{X}} dF(x) > 0, \end{aligned} \tag{23}$$

and therefore $P_R < 1$, or alternatively $C < \Delta D$, implies $G(C) < G(\Delta D)$

$$\int_{D-C}^{\overline{X}} (x + C - D) dF(x) < \int_{D-\Delta D}^{\overline{X}} [x + \Delta D - D] dF(x), \tag{24}$$

which is the same as $S_R > S_0$, using notation (2) and (3) in the main text. The last claim in the Proposition for the debt value can be easily checked using expressions (4)-(6) in the main text.

Proof of Proposition 2

The lower bound on the repurchase price obtains when the bondholders' participation condition binds:

$$d_0 \leq d_0^R + C. \tag{25}$$

Setting it to equality and using (4) and (5), with $\beta_{1,2} > 0$, we obtain the implicit expression for the minimum repurchase price $P_R^{\min}$

$$\int_{D-C/P_R^{\min}}^{D-C}(x-D+C/P_R^{\min})dF(x)+\int_{D-C}^{\overline{X}}(C/P_R^{\min}-C)dF(x)= \tag{26}$$
$$\left[\int_{D-C/P_R^{\min}}^{D-C}(B+\beta_2 x)\,dF(x)+\beta_1\int_{\underline{X}}^{D-C}CdF(x)\right].$$

Since $F(x)$ is continuous on $\left[\underline{X},\overline{X}\right]$, the left-hand side of this equation is a continuously decreasing function for $P_R^{\min}\in[C/D,1]$, and has a minimum of zero at $P_R^{\min}=1$. Additionally, since the right-hand side of the equation is strictly positive, there is a *unique* $P_R^{\min}<1$. That is, the lower bound on the repurchase price is below face value. The upper bound to the repurchase price obtains when the shareholders' participation constraint binds. From Proposition 1, $P_R^{\max}=1$. Finally, note that after the repurchase, the bankruptcy costs decrease to

$$BC_R=\int_{\underline{X}}^{D-\Delta D}(B+\beta_2 x)\,dF(x), \tag{27}$$

which is used to derive (8) in the text.

Proof of Lemma 1: Fixed-Point Price.

Suppose the tender offer repurchase price is equal to the post-repurchase price, $P_F\equiv P=P_R$. Note that, in this case, the face value of the debt after repurchase is reduced to $(D-C/P_F)$. Therefore, using (5), P_F can be solved from the following equation

$$d_R/\left(D-C/P_F\right)\equiv\int_{\underline{X}}^{D-C/P_F}x/(D-C/P_F)dF(x)+\int_{D-C/P_F}^{\overline{X}}dF(x)=P_F. \tag{28}$$

This equation has a unique solution for P_F, which is between C/D and the pre-repurchase price, $P_0>C/D$. It follows from considering the $P_F=P_0$, and $P_F\to(C/D)+$, and using the fact that function (28) is continuous in between.

Specifically, suppose $P_F=P_0$, we show that the left-hand side of (28) is smaller than the right-hand side. This is because, from Proposition 1, we have

$$d_R < d_0 - C, \tag{29}$$

or

$$d_R / (D - C/P_0) < (d_0 - C) / (D - C/P_0) = P_0. \tag{30}$$

Now suppose $P_F \to (C/D)+$, then the left-hand side of (28) approaches one, which is higher than P_F.

Proof of Proposition 3: Tender Offer Equilibria.

The threshold belief j^* is defined as the fraction of participating bondholders, at which the post-repurchase price is exactly equal to the tender offer price:

$$P_R(j^*) = P, \tag{31}$$

which is

$$\frac{1}{D - j^* D} \int_{\underline{X}}^{D - j^* D} x dF(x) + \int_{D - j^* D}^{\overline{X}} dF(x) = P. \tag{32}$$

The left-hand side is monotonically decreasing in j^*, therefore the solution for j^* is unique for $\forall P \in (P_F, P_0)$. In particular, $j^* = 0$ for $P = P_0$ and $j^* = C/(PD)$ for $P = P_F$. Therefore, for $j > j^*$, $P_R(j) < P$, and the offer is successful; for $j < j^*$, $P_R(j) > P$, and the bondholders will choose to abstain.

Derivation of the Market Price after the Repurchase Announcement

To support the discussion in Section 3.3 (Multiple bondholders), we prove the following: (i) the market price of debt reacts negatively to the news of the discounted repurchase; and

(ii) the market price is lower when the tender offer price is lower.

The market price is the weighted average of the price that paid in the tender offer and the price of the bonds after the repurchase. Since (C/DP) of the bonds are repurchased and $(1 - C/DP)$ remain outstanding, we have

$$P_{EX}(P) = \frac{C}{DP}P + [1 - \frac{C}{DP}]P_R(P). \tag{33}$$

Note that for $P = 1$ (repurchase at the face value), the market price is unaffected by the repurchase announcement,

$$P_{EX}(1) = P_0. \tag{34}$$

Finally, we can show that

$$\frac{dP_{EX}(P)}{dP} = \frac{\partial d_R}{\partial D_R}\frac{dD_R}{dP} > 0,$$

and therefore the market price decreases more if the offer price is lower.

Appendix B. Repurchase Timing

Proof of Lemma 2

This Lemma concerns repurchase timing in the *frictionless case*. Using $P_R^{\max} = 1$ from Proposition 1, and therefore letting in (12)

$$\begin{aligned} C_1 &= C_0 + D_1 - D_0, \text{ and} \\ D_2 &= D_1 - C_1, \end{aligned} \tag{35}$$

we find that (12) is independent of C_1, and therefore timing of the repurchase is irrelevant in the frictionless case.

Proof of Proposition 4

The proposition assumes that bonds are sold at the *market price* $P_M^1 \equiv d_0/D_0$ at $t = 1$ and $P_M^2(x_1) \equiv d_1(x_1)/D_1$ at $t = 2$, where

$$\begin{aligned} d_1(x_1) \quad = \quad & \int_{x_1+\widetilde{x_2}+C_1 \geq D_1} D_1 dF(x_2) + \\ & \int_{x_1+\widetilde{x_2}+C_1 < D_1} (x_1 + \widetilde{x}_2 + C_1)\, dF(x_2). \end{aligned} \tag{36}$$

Then the budget conditions are

$$\begin{aligned} C_1 \quad &= \quad C_0 + P_M^1(D_0 - D_1), \\ D_2 \quad &= \quad D_1 - C_1/P_M^2(x_1). \end{aligned} \tag{37}$$

To show that it is optimal to repurchase at $t = 2$, we compare shareholders' value at date $t = 0$, $S(C_1 = 0)$ and $S(C_1 = C_0)$ under two cases $C_1 = 0$ (use all cash to repurchase at $t = 0$) and $C_1 = C_0$ (use all cash to repurchase at $t = 1$). We show that $S(C_1 = 0) < S(C_1 = C_0)$, by applying the Jensen's inequality twice to get apply the Jensen's inequality twice:

$$\begin{aligned} S(C_1 \quad = \quad 0) &= \int_{\underline{X}}^{\overline{X}} \int_{D_0 - C_0/P_M^1 - x_1}^{\overline{X}} (x_1 + x_2 - \big(D_0 - C_0/P_M^1 - x_1\big)) dF(x_2) dF(x_1) \\ &\leq \int_{\underline{X}}^{\overline{X}} \left[\int_{E_{x_1}\left[D_0 - C_0/P_M^2\right] - x_1}^{\overline{X}} (x_1 + x_2 - E_{x_1}\left[D_0 - C_0/P_M^2\right]) dF(x_2) \right] dF(x_1) \\ &\leq \int_{\underline{X}}^{\overline{X}} \left[\int_{D_0 - C_0/P_M^2 - x_1}^{\overline{X}} (x_1 + x_2 - D_0 - C_0/P_M^2) dF(x_2) \right] dF(x_1) = S(C_1 = C_0), \end{aligned} \tag{38}$$

and the result follows.

Optimal Timing with Transaction Costs

The model introduces transaction costs as a proportional fee γ, levied on the total transaction amount. We consider only open market repurchases and make the following claims: (i) the firm repurchases at $t = 2$ only if $x_1 < x_1^*$, for some threshold $x_1^* \in \left[\underline{X}, \overline{X}\right]$ and (ii) the propensity to delay the repurchase increases in γ.

Note that, from Proposition 1, shareholders benefit from a repurchase when $D_1 - D_2 < C_1 - C_2$. Therefore, from (42),

$$(1-\gamma)D_1 = d_1(x_1^*), \tag{39}$$

which proves our first claim.[15]

Second, for an interior x_1^*, we can rewrite (12), as a sum of two separate terms reflecting value when the repurchase is optimal (the first term) and when it is not (the second term):

$$\begin{aligned} \max_{(C_1,\, C_2)} S &= \int_{\underline{X}}^{x_1^*} \int_{D_2 - x_1}^{\overline{X}} (x_1 + x_2 - D_2) dF(x_2) dF(x_1) \\ &+ \int_{x_1^*}^{\overline{X}} \int_{D_1 - C_1 - x_1}^{\overline{X}} (x_1 + x_2 + C_1 - D_1) dF(x_2) dF(x_1). \end{aligned} \tag{40}$$

Equity maximization is subject to the budget constraints for the repurchase at $t = 1$

$$(1-\gamma)(C_0 - C_1) = P_M^1 \left(D_0 - D_1\right), \tag{41}$$

and $t = 2$

$$(1-\gamma)(C_1) = P_M^2(x_1) \left(D_1 - D_2\right), \tag{42}$$

where we used $C_2 = 0$, by Proposition 1, since $t = 2$ is the final date.

[15] Note that benefits per dollar used in the repurchase are measured by the difference between the face value and the market value, $1 - d_1(x_1^*)/D_1$, and that the cost of the repurchase per dollar is given by γ, therefore the threshold x_1^* defines the point at which the benefit exactly offsets the cost. Similar intuition applies to the case with nonlinear transaction costs.

The first derivative of (40) with respect to C_1 (the propensity to save) produces

$$\frac{\partial S}{\partial C_1} = \int_{x_1^*}^{\overline{X}} (1 - \frac{\partial D_1}{\partial C_1}) \int_{D_1 - C_1 - x_1}^{\overline{X}} dF(x_2) dF(x_1) - \int_{\underline{X}}^{x_1^*} \frac{\partial D_2}{\partial C_1} \int_{D_2 - x_1}^{\overline{X}} dF(x_2) dF(x_1), \tag{43}$$

which increases in γ because $\frac{\partial D_1}{\partial C_1}$ decreases in γ from (41), and because $\frac{\partial D_2}{\partial C_1}$ increases in γ from (42). Therefore

$$\frac{\partial S}{\partial C_1}|(\gamma > 0) > \frac{\partial S}{\partial C_1}|(\gamma = 0) = 0, \tag{44}$$

where the last equality follows from Lemma 1. This proves the second claim.

Appendix C. Optimal Debt.

Proof of Proposition 5

Omitting the distribution tax, we can write the value of equity as

$$S_0 = \int_{D-C}^{\overline{X}} (x + C - D) dF(x) - \int_{\underline{X}}^{\overline{X}} (x + r(C - D)) T_C dF(x), \tag{45}$$

where the second term is the expected value of tax payments. The market value of debt is

$$d_0 = \int_{\underline{X}}^{D-C} (x + C) dF(x) + D \int_{D-C}^{\overline{X}} dF(x) - \int_{\underline{X}}^{D-C} (\beta_2 x + \beta_1 C - B) dF(x), \tag{46}$$

where the last term is the expected value of bankruptcy costs. Summing (45) and (46) produces firm value without repurchases.

$$\begin{aligned} V &= \underbrace{\int_{\underline{X}}^{\overline{X}} (x + C)(1 - T_C) dF(x)}_{\text{after-tax asset value}} + \underbrace{r(D - C) T_C}_{\text{tax shield}} \\ &\quad - \underbrace{\beta_1 C \int_{\underline{X}}^{D-C} dF(x)}_{\text{bankruptcy costs (on cash)}} - \underbrace{\beta_2 \int_{\underline{X}}^{D-C} (x + B) dF(x)}_{\text{bankruptcy costs}}. \end{aligned} \tag{47}$$

The optimal debt D^* is directly obtained from the first-order condition. For example, if x is distributed uniformly on $[\underline{X}, \overline{X}]$, then we have

$$D^* = \frac{r}{\beta_2} T_C \left(\overline{X} - \underline{X}\right) + \frac{\beta_2 - \beta_1}{\beta_2} C - \frac{B}{\beta_2}. \tag{48}$$

Now consider the case with discounted debt repurchases. Suppose that debt is repurchased at the price $d_0 < D$. Bondholders compute expected value of debt taking into account the anticipated repurchase

$$d_R = C + (1 - \beta_2) \int_{\underline{X}}^{D_R} x dF(x) - B \int_{\underline{X}}^{D_R} dF(x) + D_R \int_{D_R}^{\overline{X}} dF(x),$$

where, from the budget condition, the remaining debt after the repurchase

$$D_R = D - \frac{CD}{d_0}, \tag{49}$$

and d_0 is given by (46).

The value of equity is

$$S_R = \int_{D_R}^{\overline{X}} (x - D_R) dF(x) - \int_{\underline{X}}^{\overline{X}} (x - rD_R) T_C dF(x). \tag{50}$$

The sum of the value of debt and equity values produces (15) in the main text. The F.O.C. condition of (15) with respect to D^* yields the optimal level of debt in (14). It then follows directly that D^* and the firm value increase with the amount of the repurchase.

Appendix D. Incentives to Invest and the Debt Overhang Problem.

Investment in Single Bondholder Case

Note that to ensure that the solution for optimal I exists, we must impose two regulatory conditions:

$$\lim_{I\to\infty} \frac{\partial \left[\int_{\underline{X}}^{\overline{X}} x dG(x|I)\right]}{\partial I} \leq 1 \text{ and } \frac{\partial^2 \left[\int_{\underline{X}}^{\overline{X}} x dG(x|I)\right]}{\partial I^2} < 0.$$

The repurchase price is determined by the bondholders' participation constraint, where d_0 and d_R, the market values before and after the debt repurchase respectively, are influenced by investment:

$$d_R + C \geq d_0. \tag{51}$$

Substituting d_0 and d_R, we obtain

$$\underbrace{\int_{\underline{X}}^{D_R} x dG(x|I_R(D_R)) + D_R \int_{D_R}^{\overline{X}} dG(x|I_R(D_R))}_{\text{post-repurchase debt value}} \geq \tag{52}$$

$$\underbrace{\int_{\underline{X}}^{D_0 - C} x dG(x|I_0) dx + (D_0 - C) \int_{D_0 - C}^{\overline{X}} dG(x|I_0) dx}_{\text{debt value, if debt was repurchased at its face value}},$$

where $I_0(D_0)$ is optimal investment before repurchase, and $I_R(D_R)$ is optimal investment after the repurchase.

To alleviate debt overhang, we have shown that it is necessary that

$$D_R \leq D_0 - C.$$

To achieve higher debt value with a lower face value, the investment opportunity should increase the firm value when it is below D_R, i.e., the risky part of debt, to compensate the debt holder's forgiveness of safe part of the claim.

Investment in Dispersed Bondholder Case

Suppose the firm could invest a larger amount, $I_R > I_0$, and increase the firm value, after completing the tender offer. First, note that, by assumption, equity cannot undertake investment right away because of debt overhang

$$\int_D (x - D)dG(x|I_0) - (I_0 - C) > \int_D (x - D)dG(x|I_R) - (I_R - C). \tag{53}$$

Second, if repurchasing and investing is optimal, the shareholder participation constraint must be satisfied

$$\int_{D-\Delta D} [x - (D - \Delta D)]dG(x|I_R) - I_R \geq \int_D (x - D)dG(x|I_0) - (I_0 - C). \tag{54}$$

Combining these two conditions we obtain the highest repurchase price

$$P_R = \frac{C}{\Delta D} \leq \int_{D-\Delta D}^{\overline{X}} dG(x|I_R) + \frac{\int_{D-\Delta D}^{D} [x - D]dG(x|I_R)}{\Delta D}, \tag{55}$$

where the first term can be interpreted as a probability that the firm does not default; the second term is strictly less than zero. It is easy to show that the post-repurchase, post-investment price of the remaining bonds is higher than P_R

$$P_{post} = \frac{\int_{\underline{X}}^{D-\Delta D} dG(x|I_R) + \int_{D-\Delta D}^{\overline{X}} (D - \Delta D)dG(x|I_R)}{D - \Delta D} > P_R. \tag{56}$$

Since the expected post-repurchase price is higher than the offer price, we conclude that the bondholders may not participate in the repurchase.

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