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Three Essays on Mechanism Design and Institutions

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A Thesis submitted in partial fulfillment of the requirements for the degree of Doctor in Philosophy in Economics.

University of Warwick

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May 2011

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Acknowledgments

First of all, I would like to thank Asako Ohinata for her invaluable help and support throughout the past five years. Without her, it would have been close to impossible to continue through the rough years of the PhD thesis. I would also like to express my appreciation and gratitude to Kostas Koufopoulos, Abhinay Muthoo and Motty Perry. Their wise and constant guidance has contributed greatly to both my research ability and my academic knowledge. Moreover, their hard work and meticulous comments were very crucial for completing this thesis.

Furthermore, I would like to thank the following people for their insightlful comments and remarks, for which I feel grateful: Vincent Anesi, Valentina Corradi, Amrita Dhillon, Thodoris Diasakos, Bhaskar Dutta, Peter Hammond, Kevin Hasker, Christopher Hennessy, Matthew Jackson, Levent Koçkesen, Eric Maskin, Claudio Mezzetti, Philip Reny, Julio Robledo, Macchiavello Rocco, Tomas Sjöström, John Tsoukalas, Juuso Välimäki, Xiaohua Yu, Paul Youdell, Nikos Zygouras.

Finally, I would like to thank the Greek State Scholarships Foundation and the Department of Economics of the University of Warwick for their financial support in the past five years.

Declaration

I declare that any material contained in this thesis has not been submitted for a degree to any other university. I also declare that chapters one and three of this thesis are based on working papers co-authored with Dr. Kostas Koufopoulos.

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May 2011

Abstract

This thesis is concerned with both mechanism design and political economy issues. The first chapter examines the conditions under which information aggregation (through an appropriately designed mechanism) can solve hidden-types (also commonly referred as adverse selection) problems. The remaining two chapters adopt a contract theory approach in order to explain prominent institutions of many contemporary political regimes. Chapter two provides a theory on why laws, which restrict the freedom of private parties to commit on certain transactions, may actually be beneficial for increasing social surplus and promoting economic growth and how the evolution of these laws interacts with the process of economic development. Finally, chapter three examines the issue on how the separation between the legislative and the executive branch of a government can complement political competition in order to achieve an efficient provision of public goods.

Introduction

This thesis is comprised of three chapters, each one of which examines a different topic. Chapter one is related to the mechanism design literature and examines the role of information aggregation in relaxing incentive compatibility and implementing efficient allocations. Chapters two and three examine the emergence of specific legislative and judiciary institutions and their implications for social welfare.

More specifically, chapter one is concerned with a general economy, where agents have private information about their types. Types can be multi-dimensional and potentially interdependent. It is shown that, if the interim distribution of types is common knowledge (the exact number of agents for each type), then a mechanism exists, which is consistent with truthful revelation of private information and which implements firstbest allocations of resources as the unique Bayes-Nash equilibrium.

The result requires weak restrictions on preferences (Local Non-Common Indifference Property) and on the Pareto correspondence (Anonymity) and it is robust for small enough noise around the interim distribution. This is useful in understanding the power of information aggregation in alleviating incentive constraints and is particularly pertinent to games with large populations, in which case the interim distribution of types approaches the ex-ante distribution.

Chapter two relates the design of contract law to the process of development. In this chapter, contract law defines which private agreements are enforceable (i.e. are binding and enforced by courts) and which are not. Specifically, it considers an economy where agents face a hold-up problem (moral hazard in teams). The resulting timeinconsistency problem leads to inefficiently low levels of effort and trading among agents. The solution to this problem requires a social contract which meets two conditions: (i) an economy-wide delegate (judge) responsible for the enforcement of the social contract and (ii) a set of non-enforceable private contracts (regulation).

However, because this mechanism is costly, its effectiveness depends on the aggregate production of the economy. To capture the interaction between contract enforcement and development, a multi-period economy is introduced and it is shown that, in the early stages of development, the mechanism is infeasible. The appearance of enforcement institutions and regulation is delayed for the later stages. At this point of time, the hold-up problem is solved and this spurs economic growth further. Finally, the relationship between economic development and the evolution of contract law may be non-monotonic, which may explain why empirical studies fail to find a robust relationship between the two.

Finally, chapter three provides a political game where agents decide whether to become legislators or politicians. Legislators determine the political institutions constraining politicians' behavior and politicians compete for gaining the power to make decisions about the level of the public good.

The following results are derived: (i) Political competition is a necessary but not a sufficient condition for the elimination of political rents. (ii) Agents utilize the separation of powers in order to endogenously select institutions which restrict the power of politicians. (iii) In conjunction with political competition, these institutions implement the Lindahl allocation in the economy as a sub-game perfect Nash equilibrium of the political game. iv) As a consequence of the previous result, political rents are zero in equilibrium, in the sense that the winning politician does not extract part of the social surplus because of his power.

1 Chapter One: Information Aggregation and Adverse Selection

1.1 Introduction

As first shown by the papers of Akerlof (1970), Spence (1973) and Rothschild and Stiglitz (1976), hidden-types (adverse selection) problems can have significant consequences in terms of efficiency on economic outcomes¹. More specifically, incentive compatibility constraints limit the set of feasible allocations that can be attained. How are these restrictions relaxed as more information becomes common knowledge? And what is the minimum additional information required for achieving first-best efficiency? These are some of the questions that have emerged in the attempt to better understand the effects of information aggregation on efficiency. Indeed, some early papers by McAfee (1992), Armstrong (1999) and Casella (2002) already point out towards this direction.

In this chapter, it is claimed that if the number of agents with the same type is known for all types in a population (what we call the interim distribution of types), then it is possible, under fairly general conditions, to implement first-best allocations. More precisely, an economy with asymmetric information and finite agents is considered, each one of whom has private information about his type. It is also assumed that (i) the interim-distribution of types is common knowledge, (ii) preferences satisfy the Local Non-Common Indifference Property and (iii) the social choice set satisfies Anonymity².

¹The title of this chapter may be slightly misleading. Adverse selection is, of course, the outcome that may be generated in private information environments. The true source of the problem is the hidden information. Despite the fact that we examine a hidden-types economy, it is shown that in the equilibrium of this mechanism, individuals reveal their information truthfully and they receive first-best allocations based on that. Therefore, adverse selection problems never arise as an equilibrium of this game. So, the main claim is that information aggregation, under certain conditions, can eliminate the possibility of adverse selection outcomes.

²Since we are considering an economy of incomplete information, different realizations of types, which are consistent with the same interim-distribution, result in different desirable allocations. There-

Given these general conditions, it is shown that it is possible to construct a mechanism which has a unique Bayes-Nash equilibrium, where all agents reveal their type truthfully and they receive a first-best allocation.

This result has two interpretations. On one hand, one may consider economic applications with a finite number of agents, where, in addition to the private information that each individual has, there is knowledge about how many agents have each type. This additional information could come from a positive or negative information shock. For example, a retail store has received pre-paid orders from its customers, has already the goods in stock and is ready to make the deliveries. However, the records on the orders get destroyed due to an accident and the store's manager does not know who made each order. What is he to do? Can he induce the customers to reveal the orders they have made truthfully without them making unreasonable claims or receiving orders that were meant for other customers? It turns out that this is possible, as long as the manager posts a list with all the orders made and gives to each customer a basket of goods, which depends on how many other agents have claimed to have ordered it.

On the other hand, one can interpret this result as an application of the law of large numbers. If the ex-ante probability distribution is known, then, for sufficiently large populations, one can obtain a quite accurate estimate of the aggregate number of agents who have a specific type and, based on this information, he can address adverse selection problems. An example of this case would be insurance companies, which have data on million of cases, collected over decades, and know with very high accuracy the probability of certain accidents taking place and how personal characteristics affect these probabilities. While the main result is originally stated for the case where the interim distribution is known with perfect precision, subsequently it is proved that it

fore, the term Social Choice Set instead of the term Social Choice Rule or Correspondence is used, which usually refers to complete information environments. See also Jackson (1991) and Palfrey and Srivastava (1989).

holds for the case where it is known with a small noise.

This formulation is general enough to accommodate both interpretations and the intuition behind the result is common. If the interim-distribution is known, then one can aggregate the messages that all agents are sending out and uncover any misreport(s), even if the identity of the liar is not known. As a consequence, appropriately designed punishments for lying can induce agents to reveal their information truthfully.

We talk about appropriately designed punishments, because one of the features of this mechanism is that punishments must not be too extreme. If the punishment from detecting a lie is too severe, then some agents may deliberately lie about their type in order to force other agents to also do so. The lies cancel out in terms of the aggregate information and the former agents "steal" the allocations of the latter, who are forced to lie under the fear of the extreme punishments. This can lead to coordination failures and multiplicity of equilibria. Therefore, uniqueness of the equilibrium requires a careful construction of the allocations when lies are detected. It is shown that such punishments exist when the indifference curves of different types are not locally identical, meaning that in the neighborhood of any allocation one can find other allocations such that each type prefers one of these over the rest.

It should also be pointed out that this result is derived for a general hidden-types environment. Types can be multi-dimensional, valuations can be independent or interdependent and the joint probability distribution over type-profiles allows for correlation across types or dependencies on the identity of the agents (different agents may face different probability distributions over types). The only restriction imposed on the notion of (Pareto) efficiency is Anonymity. Anonymity requires that the allocation, which an agent receives, depends only on his type (and possibly on the interim-distribution) but not on his identity. It is a reasonable assumption which is satisfied by the majority of social choice sets. For instance, in many mechanism design papers, a mechanism is efficient if it implements the utilitarian social choice set, which satisfies our definition of Anonymity³.

The Walrasian correspondence is another example of a well-known social choice set which satisfies Anonymity. The issues of the existence of equilibrium and its welfare properties in economies with adverse selection have been analyzed by many papers in the context of the Walrasian mechanism⁴. It has been shown that the equilibrium, if it exists, is inefficient. Since the usual justification for competitive behavior is the large number of agents in both sides of markets (indeed, most of these papers assume a continuum of agents), one can apply the mechanism in this chapter in order to implement the full-information competitive equilibrium allocations in the examined economies.

Moreover, it should be pointed out that the assumption of the interim distribution of types being common knowledge is needed because we consider general social choice sets. If one focuses on the implementation of specific allocations on the Pareto frontier so that allocations depend only on ones type, one can implement the first-best as a unique equilibrium even if agents have heterogeneous beliefs or no information at all about the interim distribution⁵. The mechanism of this chapter can still implement the desirable allocations truthfully, given that the social planner knows the interim distribution. This is because, as becomes clear in section 1.4, players' best-response correspondences depend on their beliefs about how many misreports will be detected by the mechanism and not on their ability to detect other agents' lies. For instance, this formulation fits the example of the store manager provided earlier. The manager does not have to post the list of orders. It is sufficient that agents know that he knows

them.

³See for example the papers by Mezzetti (2004), Jackson and Sonnenschein (2007).

⁴Examples include Prescott and Townsend (1984), Gale (1992 and 1996), Dubey and Geanakoplos (2002), Dubey, Geanakoplos and Shubik (2005), Bisin and Gottardi (2006), Rustichini and Siconolfi (2008).

⁵E.g. the Walrasian correspondence in the Rothschild-Stiglitz model.

We also provide necessary and sufficient conditions for full implementation when the interim-distribution is common knowledge and examples of well known economies with adverse selection, where the mechanism can be implemented. Finally, the issue of robustness to small perturbations regarding the knowledge of the interim-distribution and the issue of participation constraints are examined.

1.2 Related Literature

This chapter is most closely related to papers that use information aggregation to implement first-best allocations in economies with asymmetric information. Thus, in terms of spirit and research questions, Jackson and Sonnenschein (2007) is the paper closest to this research. They consider a specific set of agents, who play multiple copies of the same game at the same time and their types are independently distributed across games. They allow for mechanisms, which "budget" the number of times that an agent claims to be of a certain type. If the number of parallel games becomes very large, then all the Bayes-Nash equilibria of these mechanisms converge to first-best allocations.

This model differs from that of Jackson and Sonnenschein in four dimensions: (i) It does not require multiple games to be played at the same time but a stronger assumption on what is common knowledge is imposed (or, in certain cases, what is known by the central planner). (ii) It allows for interdependent values, while they consider an independent values setting. (iii) It allows for a more general joint probability over type profiles, since types can be independently or interdependently distributed in this formulation, and apart from preferences, types may concern other individual characteristics as well (productivity parameters, proneness to accidents, etc.). iv) It also allows for a more general social choice set. In terms of results, if values are interdependent (but still independently distributed), the Jackson-Sonnenschein mechanism may have multiple equilibria in the limit, while we prove the uniqueness of the equilibrium under small perturbations.

McLean and Postlewaite (2002, 2004) also consider efficient mechanisms in economies with interdependent values. The state of the world is unknown to all agents, but each individual receives a noisy private signal about the state. They show that when signals are sufficiently correlated with the state of the world and each agent has small informational size (in the sense that his signal does not contain additional information about the state of the world when the signals of all the other agents are taken into account), then their mechanism implements allocations arbitrarily close to first-best allocations.

There are two main differences between their setting and the one in this chapter. First, in the model of McLean and Postlewaite when private signals are perfectly correlated with the state of the world all agents learn not only their own type but also the type of all other agents. That is, in the limit, the framework of McLean and Postlewaite is one of complete information. In contrast, in the setting of this chapter agents can, at most, know the interim distribution of types (when the signal is perfect)⁶. Second, McLean and Postlewaite implement allocations arbitrarily close to first-best while the mechanism in this chapter achieves the implementation of the exact first-best allocations even when agents face a slight uncertainty about the interim-distribution, i.e. when private signals are slightly noisy.

The chapter is also related to the auctions literature with interdependent types. In this context, Crémer and McLean (1985) and Perry and Reny (2002, 2005), show the existence of efficient auctions when types are interdependent. Crémer and McLean, however, require large transfers which may violate ex-post feasibility. Also, Perry and Reny require the single crossing property on preferences which is a stronger restriction than ours. The general framework presented here can encompass auction design prob-

⁶In a sense, in this model agents receive private signals as well, but one can think of them as perfect signals about the interim distribution. As it has already been mentioned, a small noise about the precision of these signals does not alter the results.

lems as well. Furthermore, the main focus is the uniqueness of the equilibrium, an issue which is not studied in these papers.

It is also noteworthy that in the framework of auction design the papers by Maskin (1992), Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001) show, in increasing generality, that efficiency and incentive compatibility can not be simultaneously satisfied if the single crossing condition is violated or if signals are multidimensional. In that respect, the additional information allows us to overcome this impossibility and implement efficient outcomes, even if conditions, which are necessary in the standard mechanism design literature for implementation, are violated.

Rustichini, Satterthwaite and Williams (1994) show that the inefficiency of trade between buyers and sellers of a good, who are privately informed about their preferences, rapidly decreases with the number of agents involved in the two sides of the market and in the limit it reaches zero. Effectively, the paper examines the issue of convergence to the competitive equilibrium as the number of agents increases. However, their model is limited to private values problems and hence it can be seen as a special case of our formulation.

More recently, the papers by Mezzetti (2004) and Ausubel (2004),(2006) examine the issues of efficient implementation under interdependent valuations and independently distributed types. However, they also assume that agents' preferences are quasi-linear with respect to the transfers they receive, whereas in the model below utility may not be transferable. Moreover, the mechanisms proposed in these papers may generate multiple equilibria (in most of which truth-telling is violated), while we are interested in a mechanism which has a unique truth-telling equilibrium.

1.3 The Economy

The economy consists of a finite set I of agents, with I standing for the aggregate number of agents as well. Θ is the set of potential types (so ϑ_i is the type of a single agent *i*). The vector $\boldsymbol{\theta}$ contains I elements and is a type-profile, a realization of a type for each agent. Each agent has private information about his own type, but does not know the types of the other agents. Φ is the ex-ante cumulative distribution function over the set of all possible type-profiles $\boldsymbol{\Theta}$, with $\Phi(\boldsymbol{\theta})$ the ex-ante probability that the type-profile $\boldsymbol{\theta}$ is realized.

S is the set of all states. Each state s is a complete description of the world, including the economic characteristics of each agent. This means that the state describes agents' features, such as preferences, productivity, individual endowments or any other economically pertinent information. The probability distribution over states Π is a function of the type-profile $\boldsymbol{\theta}$. Therefore, $\pi(s|\boldsymbol{\theta})$ is the probability of state s arising, conditional on the type-profile $\boldsymbol{\theta}$.

 β is an unordered collection of I realizations of types (potentially the same types for some realizations). One interpretation is that β is the distribution of types that have been realized. Given a β , the exact number of agents who have a specific type is known for all types. We slightly abuse terminology by calling β the interim distribution of types⁷. $\Theta(\beta)$ is the set of all type-profiles consistent with the interim distribution β , while $\Theta(\beta)$ is the set of types consistent with β . In other words, $\Theta(\beta)$ is the set of all vectors which match each agent to a type and which result from β , while $\Theta(\beta)$ is the set of types which are realized in β .

The above elements characterize the economy: $E = \{I, \Theta, \Phi, S, \Pi, \beta\}$. It is assumed

⁷A more accurate definition of the interim distribution is the percentage of realizations of each type over the entire population, namely the collection of numbers $I(\vartheta) = \lambda_{\beta}(\vartheta)/I$, where $\lambda_{\beta}(\vartheta)$ is the number of agents who have type ϑ given the collection β . However, since the collection β already contains this information and for notational simplification, the misnomer of interim-distribution for β is retained.

that E is common knowledge. Given E, let A(E) (or simply A) be the set of all feasible allocations, with elements $a \in A \subseteq R_+^{I \times S \times L}$, with $L \times S \ge 2$. L can be interpreted as the number of commodities in the economy. Each a is an S-tuple of feasible statedependent allocations. In other words, the collection of feasible allocations may depend on the state of the world. Furthermore, it is assumed that preferences are represented by expected utility functions:

$$U_{i}(a) = \sum_{\boldsymbol{\theta}_{-i}} \left[\sum_{s \in S} u_{i}(a,s) \ \pi\left(s | \vartheta_{i}, \boldsymbol{\theta}_{-i}\right) \right] \phi(\boldsymbol{\theta}_{-i} | \vartheta_{i}, \beta) \ , \quad \boldsymbol{\theta}_{-i} \in \boldsymbol{\Theta}_{-i}(\beta | \vartheta_{i})$$

 $U_i(a)$ is the expected utility to agent *i* when he receives allocation *a*, with $u_i(a, s)$ the decision-outcome payoff in state s (preferences may be state-dependent) and $\boldsymbol{\theta}_{-i}$ is a type-profile for all agents, excluding *i*, which is consistent with the interim-distribution of types β^8 .

The formulation of the economy allows for modeling a wide variety of economic situations. Types may or may not be independently distributed, and the characteristics of agents may or may not depend on the types of other agents. Hence, both adverse-selection problems with independent or inter-dependent valuations can be seen as special cases of this formulation.

1.4 Implementation of First Best Allocations

1.4.1 Implementation

In this subsection it is shown that the conditions specified in section 1.3 are sufficient for the implementation of truthful strategies. Full implementation (i.e. the uniqueness of the truthful equilibrium) requires additional conditions, which are specified in

⁸Therefore, the standard six axioms for expected utility representation are implicitly required: Completeness, Transitivity, Local Non-Satiation, Convexity, Continuity and Independence of Irrelevant Alternatives.

subsections 1.4.2 and 1.4.3. The main idea is simple. The knowledge of the interim distribution of types allows the construction of a direct mechanism, which provides allocations conditional on the message profile being consistent with the interim distribution or not. If the message profile is different from the interim distribution, this is considered as an indication of lying by some agent, in which case the mechanism provides a "punishment" allocation. As a result, an agent reveals his information truthfully, if all other agents reveal their information truthfully as well.

Let $a^* = (a_1^*, a_2^*, ..., a_i^*, ..., a_I^*)$ be a Pareto efficient allocation of the economy. a_i represents an individual allocation, namely it is a vector a state-contigent allocations for agent *i*. Let a^m be an individual allocation such that $a_{ls}^m = \min\{a_{ils}^*\}$ for every $i \in I$ and for each state-contingent commodity ls. By construction, $I \times a^m$ is feasible. Consider the direct mechanism $M_0(g, a)$, $g: M \to A$, in which agents state their type. $\lambda_\beta(\vartheta)$ is the number of agents with type ϑ according to the interim distribution β and $\lambda_{\mathbf{m}}(\vartheta)$ is the number of agents who report type ϑ . Agents receive allocations according to the following message profiles:

- If $\lambda_{\beta}(\vartheta) = \lambda_{\mathbf{m}}(\vartheta)$, $\forall \ \vartheta \in \Theta(\beta)$, then $a_i = a_i^*$.
- If $\lambda_{\beta}(\vartheta) \neq \lambda_{\mathbf{m}}(\vartheta)$ for at least one $\vartheta \in \Theta(\beta)$, then $a_i = a^m$.

Claim 1: M_0 has a truthful equilibrium.

Proof: Suppose I - 1 agents report truthfully. By Local Non-Satiation, $U_i(a_i^*) \ge U_i(a^m)$. Therefore, it is a best-response for agent *i* to report truthfully as well.

This demonstrates that the interim-distribution is sufficient for truthful implementation under the standard conditions on preferences in general economic environments. In fact, implementation of the truthful equilibrium is possible even when there is a single state contingent commodity. Hence, the implementation of first-best allocations is possible in the most well-known models of adverse selection (Akerlof (1970), Spence (1973), Rothschild-Stiglitz (1976)) if one makes the additional assumption that the interimdistribution is known.

Even though this is a strong assumption, in subsection 1.4.6, it is shown that as the number of agents increases, the interim-distribution converges to the ex-ante distribution of types. Hence, the standard assumptions of the literature are sufficient for implementation of first-best allocations when the number of agents is sufficiently large.

1.4.2 Full Implementation

In this section we provide sufficient conditions for full implementation. Three assumptions additional to section 1.3 are made. A series of Lemmata, which are used in the proof of the main Proposition, are presented and we provide the main claim of the chapter: if the interim-distribution of types is common knowledge, preferences satisfy the Local Non-Common Indifference Property (LNCIP) and the social choice set satisfies Pareto efficiency and Anonymity, then a mechanism exists that fully implements it. The additional assumptions required for this result are the following.

Assumption 1: The Social Choice Sets satisfy Anonymity.

Definition 1: A Social Choice Set satisfies Anonymity if, for every social choice function in the set, each agent's assigned allocation depends only on his type and the interim-distribution of types: $a_i^* = a(\vartheta_i, \beta)$.

Under Anonymity, agents who have identical types receive identical allocations. There-

fore, an agent's identity per-se has no impact on the agent's final allocation. As a result, for any interim-distribution of types there is a unique collection of allocations to be assigned to agents. The order of the allocations does depend on the type-profile θ , but the collection of individual allocations is the same for all type-profiles consistent with the same interim-distribution.

It is also noteworthy that Anonymity is a desirable property for a social choice rule. In most cases of interest, economists are concerned with the economic characteristics of agents and not with their identity. Therefore, it is reasonable to assume that if the distribution of these characteristics remains unchanged, so does the distribution of the economically desirable outcomes. It is also a property satisfied by many commonly used social choice rules, like the Walrasian correspondence and the utilitarian social welfare function.

Assumption 2: Preferences satisfy the Local Non-Common Indifference Property (LNCIP).

This is a requirement that the intersection of the indifference planes around any individual allocation of any two agents with different types is of at least one dimension lower than the dimensions of the indifference planes themselves. In other words, if the indifference planes are n-dimensional (e.g. three-dimensional surfaces), the intersection around any allocation a_i is (n-1)-dimensional (e.g. curves). Formally:

Definition 2: Let $C_{i\epsilon}(a) = \{c \in A : U_i(c|\vartheta_i, \theta_{-i}) = U_i(a|\vartheta_i, \theta_{-i}), \|c - a\| < \epsilon\}.$ The **Local Non-Common Indifference Property** is satisfied if $\forall i \in I, \forall a \in A$ and $\forall j \in I, \ \vartheta_j \neq \vartheta_i$, there exists $\overline{\epsilon}_{ij} > 0 : dim (C_{i\epsilon}(a) \cap C_{j\epsilon}(a)) \leq L \times S - 1, \ \forall \epsilon < \overline{\epsilon}_{ij}.$

Figure 1: Indifference Curves satisfying LNCIP

LNCIP is a weaker restriction than the Single-Crossing Property (SCP) which is usually used in the literature. For example, any pair of indifference curves that has finitely many intersections satisfies the LNCIP but it violates the SCP. Also, LNCIP allows for tangent indifference planes (as long as the tangent parts "miss" at least one dimension compared to the indifference planes), while the SCP does not. On the other hand, if SCP is satisfied then LNCIP is also satisfied⁹. Figure 1 provides two diagrams, which illustrate the LNCIP and distinguish it from the SCP.

Finally, we denote by $\underline{A}(a_i)$ the set of individual allocations strictly less than a_i : $\underline{A}(a_i) = \{c_i \in A : c_{ils} \leq a_{ils}, \forall ls\}$. $L_i(a_i)$ is the lower contour-set for an agent *i* given some individual allocation a_i .

Assumption 3: If for ϑ , ϑ' holds that $a_{\vartheta}^* \succ_{\vartheta'} a_{\vartheta'}^*$ and $a_{\vartheta'}^* \succ_{\vartheta} c$, $\forall c \in \underline{A}(a_{\vartheta}^*) \cap L_{\vartheta'}(a_{\vartheta'}^*)$, then $\lambda_{\beta}(\vartheta') \ge \lambda_{\beta}(\vartheta)$, where ϑ and ϑ' are different types.

⁹Note that we could alternatively characterize this restriction on preferences in terms of the axiomatic approach. Apart from the standard axioms (Completeness, Transitivity, Local Non-Satiation, Convexity, Continuity and Independence of Irrelevant Alternatives), the Axiom of Local Non-Common Indifference would be required. In this case, the only difference from the definition provided above is the definition of $C_{i\epsilon}(a) = \{c \in A : c \sim_i a, ||c-a|| < \epsilon\}$.

Assumption 3 ensures feasibility off-the-equilibrium-path and is discussed in more detail after the presentation of Lemma 4. Below, we provide three results which hold for any Pareto efficient allocation. The combination of these results shows that every allocation on the Pareto frontier of an economy generates a "social ranking" among the agents of the economy, such that agents of "lower ranks" envy the allocations of "higher ranks". By exploiting the common knowledge of this ranking, due to the common knowledge of the interim-distribution and the efficiency of the allocation, a mechanism is constructed, which has a unique equilibrium and in which agents reveal their private information truthfully.

Lemma 1: Let PF(E) be the Pareto Frontier of economy E. Then, for every allocation a on the Pareto Frontier, there exists at least one agent $i \in I$, who does not envy the allocation of any other agent: $U_i(a_i) \ge U_i(a_j), \forall j \in I$.

Proof: See Appendix A

Lemma 2: For every allocation a on the Pareto Frontier, there exists at least one agent $i \in I$, whose allocation is not envied by any other agent: $U_j(a_j) \ge U_j(a_i), \forall j \in I$.

Proof: See Appendix A

Corollary 1: If $a \in PF(E)$, then Lemma 1 and 2 hold for any subset of I. Namely, let $\check{I} \subseteq I$ and let $\check{A} = \{a_i : i \in \check{I}\}$. Then, if $a \in PF(E)$, Lemma 1 and 2 hold for \check{I} with regard to \check{A} as well.

Proof: See Appendix A

Lemma 1 and 2 provide two necessary conditions for Pareto efficiency. If these conditions are violated, then an allocation can not be Pareto efficient. However, they are not sufficient. One can easily find examples, where these conditions hold but the allocation is not on the Pareto frontier of the economy. Most importantly for our purposes, they imply that any Pareto efficient allocation exhibits a social ranking between groups of agents who envy and groups who are envied.

Let $\operatorname{Rank}(\mathbf{K}) = \{i \in I : U_i(a_i) \ge U_i(a_j), \forall j \in I\}$, be the set of agents who do not envy the allocation of any other agent. By Lemma 1, we know that this set is nonempty. Then, by removing this set of agents from the set I and applying Corollary 1, one can define $\operatorname{Rank}(\mathbf{K}-\mathbf{1}) = \{i \in I - Rank(K) : U_i(a_i) \ge U_i(a_j), \forall j \in I - Rank(K)\}$. By iteration, we define K groups, $1 \le K \le I$, such that the agents in each one of them do not envy any of the agents in their own group or groups with lower rank, but they envy the allocation of some agent(s) in groups with higher rank¹⁰. We will also refer to group $\operatorname{Rank}(\mathbf{K})$ as the group with the highest rank and group $\operatorname{Rank}(\mathbf{1})$ as the group with the lowest rank. Some additional results required for the proof come from the LNCIP and are provided in Lemma 3 and Lemma 4.

Lemma 3: If the LNCIP holds, then around the neighborhood of any individual allocation a_i , there exists a set of allocations such that each agent of a certain type prefers a particular allocation over the rest.

Proof: See Appendix A

¹⁰One extreme case is when an allocation exhibits no-envy, in which case Rank(K) contains the whole set of agents and Lemma 1 and 2 apply for all (egalitarian allocations). The other extreme case is when each rank-group contains a single agent, in which case the agents form a complete hierarchy, from the one who is envied by all the other agents to the one who is not envied by anyone else.

In effect, Lemma 3 states that it is possible to find incentive compatible allocations for any type in the neighborhood of any allocation, which implies that it is possible to satisfy no-envy, at least in a local sense.

Lemma 4: Suppose $a^* \in PF(E)$ and Assumptions 1 and 2 hold. $\forall \vartheta, \vartheta' \in \Theta(\beta)$ there exist some feasible individual allocations $\{a_1(\vartheta, \vartheta'), a_2(\vartheta, \vartheta')\}$, such that, if $\operatorname{Rank}(\vartheta) > \operatorname{Rank}(\vartheta')$, then $a_1(\vartheta, \vartheta') \succ_{\vartheta} a^*_{\vartheta'} \succeq_{\theta} a_2(\vartheta, \vartheta')$, $a^*_{\vartheta'} \succeq_{\vartheta'} a_2(\vartheta, \vartheta') \succ_{\vartheta'} a_1(\vartheta, \vartheta')$.

Proof: Because a Pareto efficient allocation is feasible by definition, any allocation $c \in \underline{A}(a^*_{\vartheta}) \cup \underline{A}(a^*_{\vartheta'})$ is feasible. Also, due to the Pareto efficiency of a^* and the fact that ϑ' envies the first-best allocation of ϑ , $L_{\vartheta'}(a^*_{\vartheta'}) \cap \underline{A}(a^*_{\vartheta}) \neq \emptyset$. Take an individual allocation c inside this intersection and arbitrarily close to (and below) the indifference plane of ϑ' that passes through $\underline{A}(a^*_{\vartheta'})$. Therefore, $a^*_{\vartheta'} \succ_{\vartheta'} c$. There are two possible sub-cases to consider (the case of indifference is being ignored because it always possible to move c slightly so that it falls under the following two cases).

Case a): $c \succ_{\vartheta} a_{\vartheta'}^*$. In this case, let $a_1(\vartheta, \vartheta') = c$ and $a_2(\vartheta, \vartheta') = a_{\vartheta'}^*$ and this completes the proof. $\lambda_{\beta}(\vartheta)$ allocations c and $\lambda_{\beta}(\vartheta')$ allocations $a_{\vartheta'}^*$ are feasible on aggregate.

Case b): $a_{\vartheta'}^* \succ_{\vartheta} c$. In this case, by LNCIP, it is possible to find an allocation d very close to $a_{\vartheta'}^*$ such that: $d \succ_{\vartheta} a_{\vartheta'}^*$ and $c \succ_{\vartheta'} d$. Because c is in the interior of $\underline{A}(a_{\vartheta}^*)$, it is always possible to find such points (one can define distance ϵ and make sure that $B_{\epsilon}(c) \cap U_{\vartheta'}(a_{\vartheta'}^*) \neq \emptyset$, while $B_{\epsilon}(d) \cap U_{\vartheta'}(a_{\vartheta'}^*) = \emptyset$, where $B_{\epsilon}(c)$ is the open ball with radius ϵ around c). Therefore, let $a_1(\vartheta, \vartheta') = d$ and $a_2(\vartheta, \vartheta') = c$. $\lambda_{\beta}(\vartheta)$ allocations dand $\lambda_{\beta}(\vartheta')$ allocations c are feasible on aggregate.

Lemma 4, provides pairs of feasible and incentive compatible allocations for any pair

of types ϑ, ϑ' which are of different rank. However, feasibility is ensured under the implicit assumption that the number of agents is equal across types. If this is not the case, then additional restrictions on the interim distribution are required. This is the role of Assumption 3. Specifically, the set $\{a_1(\vartheta, \vartheta'), a_2(\vartheta, \vartheta')\}$ in Lemma 4 is feasible by construction whenever $\lambda_{\beta}(\vartheta) = \lambda_{\beta}(\vartheta')$. In case a) $c \succ_{\vartheta} a_{\vartheta'}^*$, it is always possible to find the desired allocations for any number of agents of the two types, since the allocation which ensures incentive compatibility c is in the interior of $\underline{A}(a_{\vartheta}^*)$.

Case b) $a_{\vartheta'}^* \succ_{\vartheta} c$, however, is problematic if $\lambda_{\beta}(\vartheta) > \lambda_{\beta}(\vartheta')$. In this case it is feasible to provide $\lambda_{\beta}(\vartheta)$ allocations of type c and $\lambda_{\beta}(\vartheta')$ allocations of type d. If the ϑ -type agents are much more than the ϑ' -type agents, then there may be too few allocations d in order to ensure that $a(\vartheta, \vartheta') \succ_{\vartheta} a_{\vartheta'}^{*11}$. Assumption 3 rules out those cases by imposing restrictions on the number of agents who are envied. This is a joint restriction on preferences and the interim-distribution.

Lemmas 3 and 4, along with the knowledge of the "social ranking" of the allocations, allows us to construct a mechanism which makes it a dominant strategy for agents of higher rank to report their type truthfully. The main idea is that, if the number of agents, who report a specific type is higher than the number who have this type, according to the interim distribution, then they all receive an allocation, which the "true" types prefer to the first-best allocations of the misreporting types, but the other types do not prefer. This acts as an effective punishment for lies by those who envy allocations of other types. Hence, one can use iterated elimination of dominated strategies to prove the uniqueness of the proposed equilibrium. The proof of Proposition 1 constructs this argument formally.

Proposition 1: Assume that the economy E, described in section 1.3, satisfies As-

¹¹The last condition is required for ensuring incentive compatibility. See also 4.3 for the necessity of this condition for full implementation.

sumptions 2 and 3. Then, for every allocation $a^* \in PF(E)$, which satisfies Assumption 1, there exists a mechanism, for which a^* is the unique Bayes-Nash equilibrium allocation and agents report their private information truthfully.

Proof: The proof is done by construction. Let $a^* \in PF(E)$, which satisfies Anonymity, and let $a^*(\boldsymbol{\theta})$ be the first-best allocation which is to be implemented for each type-profile, with individual allocations $a_i = a_i^*(\vartheta_i, \beta), \forall i \in I$. Also, let $\hat{a}_{\vartheta}(a, \epsilon)$ denote an individual allocation in the ϵ -neighborhood of allocation a which is incentive compatible for type ϑ , in the sense of Lemma 3, and let $a_1(\vartheta, \vartheta'), a_2(\vartheta, \vartheta')$ be individual allocations as constructed by Lemma 4. Recall that $\lambda_{\beta}(\vartheta)$ and $\lambda_{\mathbf{m}}(\vartheta)$ is the number of agents of type ϑ according to the interim distribution β and the received messages \mathbf{m} , respectively, and a^m is the minimum allocation, as defined in 4.1.

Each agent reports his type m_i and a final allocation is received according to the following mechanism $M_1(g, a)$:

- (i) If $\mathbf{m} \in \Theta(\beta)$, then $a_i(m_i, \mathbf{m}_{-i}) = a^*(m_i, \beta), \forall i \in I$.
- (ii) If m is such that for two types, (ϑ, ϑ'), the number of reported agents is different from number of agents in the interim-distribution by one, specifically λ_m(ϑ) = λ_β(ϑ) + 1, λ_m(ϑ') = λ_β(ϑ') − 1, then:
 - If Rank(ϑ) = Rank(ϑ'), agents who reported types ϑ, ϑ' choose an allocation from the set {a^{*}(ϑ, β) ε, a^{*}(ϑ', β) ε}. ε is strictly positive for all state-contingent commodities and it is sufficiently small so that a^{*}(ϑ, β) ε ≻_ϑ a^{*}(ϑ', β) and a^{*}(ϑ', β) ε ≻_{ϑ'} a^{*}(ϑ, β).
 - If Rank(ϑ) > Rank(ϑ'), agents who reported types ϑ, ϑ' choose an allocation from the set {a₁(ϑ, ϑ'), a₂(ϑ, ϑ')}.

- If $Rank(\vartheta) < Rank(\vartheta')$, agents who report type ϑ' receive allocation $a_{\vartheta'}^*$ and agents who report type ϑ receive allocation $\frac{\lambda_{\beta}(\vartheta)}{\lambda_{\mathbf{m}}(\vartheta)}a_{\vartheta}^*$.
- For all $m_k \neq \{\vartheta, \vartheta'\}, a_k(m_k, \mathbf{m}_{-i}) = a^*(m_k, \beta).$

(iii) For any other case, $a_i(\vartheta, \mathbf{m}_{-i}) = \hat{a}_{\vartheta}(a_m, \epsilon), \, \forall \vartheta \in \Theta(\beta).$

Under the mechanism above, it is a strictly dominant strategy for all agents with types of rank(K) to report their type truthfully. To see this consider the different beliefs of an agent of rank(K) (say *i*) about the messages that other agents will send. If *i* believes that all other agents will report their type truthfully, then the best-response for him is to report truthfully. This is because $a^*(\vartheta_i, \beta) \succ_i a^*(\vartheta_i, \beta) - \epsilon$, in the case he reports another type of the same rank, and $a^*(\vartheta_i, \beta) \succ_i \frac{\lambda_{\beta}(\vartheta')}{\lambda_{\mathbf{m}}(\vartheta')}a^*(\vartheta', \beta)$, in case he reports a type of lower rank.

If *i* believes that only one other agent will misreport, then *i* still prefers to report his type truthfully, irrespectively of the rank of the other agent. Say that *i* believes that *j* is of the same rank as him but of different type and that *j* will misrepresent her preferences as being of type ϑ_i . If *i* reports that he is of type ϑ_j , then the two lies will cover each other and *i* will receive $a^*(\vartheta_j, \beta)$. But if he chooses to report ϑ_i , then $\lambda_{\mathbf{m}}(\vartheta_j) = \lambda_{\beta}(\vartheta_j) - 1$ and $\lambda_{\mathbf{m}}(\vartheta_i) = \lambda_{\beta}(\vartheta_i) + 1$. In the latter case, *i* chooses one allocation from $\{a^*(\vartheta, \beta) - \epsilon, a^*(\vartheta', \beta) - \epsilon\}$. Since $a^*(\vartheta, \beta) - \epsilon$ is constructed to be strictly preferred by *i* to $a^*(\vartheta_j, \beta)$, *i* strictly prefers to report truthfully.

The same argument holds if *i* believes that *j* is of type ϑ_j , which is of lower rank than *K*, and that *j* will report ϑ_i . Note that, by the construction of the set $\{a_1(\vartheta_i, \vartheta_j), a_2(\vartheta_i, \vartheta_j)\}$ (see also Lemma 4), there are $\lambda_\beta(\vartheta_i) + \lambda_\beta(\vartheta_j)$ individual allocations that are feasible. If one of the two allocations is requested more times than it is feasible, then, in the game induced by *i*'s report: assign first the allocations in excess supply to the agents who request them and then assign the rest of the agents randomly to the remaining allocations. This ensures that there are no coordination failures and all agents choose their most preferred allocation. Also, note that in both cases where *i* believes that *j* misreports, *i* strictly prefers to report truthfully than to send any other message $\vartheta \neq \{\vartheta_i, \vartheta_j\}$, because, in the latter case, *i* receives $\hat{a}_{\vartheta}(a_m, \epsilon)$, which makes him strictly worse-off.

In the case where *i* believes that multiple misrepresentations will take place, either in types of rank(K), or in other ranks, then, irrespectively of his message, $\mathbf{m} \neq \Theta(\beta)$ (if all representations but one cancel out then we go back to the analysis of the previous cases). This means that his message, alone, can not hide the fact that some agent(s) misrepresents(misrepresent) her(their) type(s). His best response remains to report truthfully: $U_i(\hat{a}_{\vartheta_i}(a_m, \epsilon)) > U_i(\hat{a}_{\vartheta'}(a_m, \epsilon)), \forall \vartheta' \neq \vartheta_i$, by construction (recall that $I \times a_m$ is feasible). We conclude that, under all possible beliefs, *i* strictly prefers to report truthfully.

Given this, it is a best response for an agent of rank(K-1) to report his type truthfully as well. Say that agent *i*, who is of rank(K-1), envies the allocation of some type ϑ_j of rank(K). Of course, if *i* believes that some agent of type ϑ_j will report as being of type ϑ_i , then the best response for *i* is $m_i = \vartheta_j$, but, as we showed, this cannot be an equilibrium¹². Hence, if *i* believes that all agents will report truthfully, he prefers to report truthfully as well. If he believes that only one agent of the same or lower rank will misreport their types as his own, he will still prefer to reveal his type truthfully, for the same type of reasoning as in the case of an agent of rank(K). Finally, if he believes that many agents will misreport their types, he still prefers to receive an incentive compatible allocation (by construction) than misrepresenting his own type. Therefore, given that rank(K) agents report truthfully, agents of rank(K-1) also report truthfully.

By induction, we conclude that for an agent of $Rank(\kappa)$, if all agents of higher rank

¹²This argument also makes clear that the mechanism is not one of dominant strategy implementation, as only rank(K) individuals have dominant strategies.

are expected to report truthfully their types, his best-response is to report truthfully, irrespectively of the actions of agents of the same or lower rank. Since it is a dominant strategy for rank(K) agents to report truthfully, then, by iterated elimination of strictly dominated strategies, the only possible equilibrium is when all agents report truthfully. Therefore, the unique Bayes-Nash equilibrium of the mechanism is for all agents to reveal their type and to receive the allocation $a_i^*(\vartheta_i, \beta), \forall i \in I$.

The result depends crucially on the fact that the rank of types is known. This is due to the interim-distribution being common knowledge. On the other hand, Anonymity ensures that agents do not gain any strategic benefit from their personal identity. For instance, even if β is common knowledge, if different type-profiles result in different ranks between types, then it may not be a dominant strategy for any agent to reveal his type truthfully. As one's rank, in this case, also depends on the realized types of the other agents, there may be situations where an agent misreports his type in order to force someone to misreport as well. This may cause multiplicity of equilibria. In other words, if Anonymity fails, implementation is still possible, but full implementation may fail.

The LNCIP is also required for the uniqueness of the equilibrium, as it allows for agents to strictly improve their payoff if they report truthfully. Once again, if LNCIP is violated, then one can still construct mechanisms which implement the first-best allocations, but the uniqueness of the equilibrium may not be possible. Therefore, the common knowledge of the interim-distribution, Anonymity and LNCIP (along with Assumption 3) are jointly sufficient conditions for full implementation of first-best allocations, but they are not necessary.

We would also like to comment on the advantages of this mechanism in comparison to the existing literature (see for example, Maskin, 1999, Jackson, 1991). First, the mechanism holds even with two agents (or even in the degenerate case of one agent). Second, the required message space is minimal, since agents send messages only about their own type. Third, it does not require any ad-hoc game, which has no equilibrium in pure strategies (like an integer game), in order to rule out undesirable equilibria. This is achieved by "enticing" some of the misreporting agents to report truthfully, whenever there are multiple misrepresentations. Fourth, full implementation is also achieved if the equilibrium concept is changed to iterated elimination of strictly dominated strategies, which is, in fact, the solution concept used in the proof of Proposition 1. Therefore, the mechanism is not limited only to Bayesian implementation.

Finally, Assumptions 1,2 and 3 are relatively weak and there are many cases of interest that comply with them. To demonstrate this, in 4.4, some well-known examples of economies with hidden types (and the solutions that this framework provides) are provided. But first, let us characterize the problem by providing necessary and sufficient conditions for full implementation when the interim-distribution is common knowledge.

1.4.3 Full Implementation: Necessary and Sufficient Conditions

Condition 1: Suppose $a^* \in PF(E)$. $\forall \vartheta, \vartheta' \in \Theta(\beta)$ such that $a^*_{\vartheta} \succ_{\vartheta'} a^*_{\vartheta'}$, $\exists a(\vartheta, \vartheta') \in A$ such that: (i) $a_{\vartheta}(\vartheta, \vartheta') \succ_{\vartheta} a^*_{\vartheta'}$, and (ii) $a^*_{\vartheta'} \succ_{\vartheta'} a_{\vartheta}(\vartheta, \theta')$.

Proposition 2: Condition 1 is necessary for full implementation.

Proof: Full implementation of a^* requires that $g(\mathbf{m}) = a^*$ if $m_i = \vartheta_i$, $\forall i \in I$ and that the strategy profile $m_i = \vartheta_i$, $\forall i \in I$ is the unique Bayes-Nash equilibrium. Consider any direct mechanism M(g, a), which specifies some allocation $a(\mathbf{m}) \neq a^*$, whenever \mathbf{m} is such that $\lambda_{\mathbf{m}}(\vartheta'') \neq \lambda_{\beta}(\vartheta'')$ for some $\vartheta'' \in \Theta(\beta)$ (whenever this is the case, then, by common knowledge of the interim-distribution, it follows that $m_i \neq \vartheta_i$ for some $i \in I$). Suppose that, apart from i (of type ϑ) and j (of type ϑ'), incentive compatibility is satisfied for all other agents and that they report truthfully (this is done in order to check the necessity of the condition).

Because Condition 1 is violated, then either part (i) or part (ii) of the condition is violated (or both). This means that at least one of the following will hold: (i) $a(m_i = \vartheta, m_j = \vartheta, \mathbf{m}_{-i,j}) \succ_j a_{\vartheta'}^*$, (ii) $a_{\vartheta'}^* \succ_i a(m_i = \vartheta, m_j = \vartheta, \mathbf{m}_{-i,j})$. In case (i), truthful reporting is not equilibrium, because, if everyone else reports truthfully, j's best-response is $m_j = \vartheta$ (incentive compatibility is violated for j). In case (ii), there may be multiple equilibria because, if the truthful equilibrium exists, then so does another equilibrium, where i reports type ϑ' and j reports type ϑ . To see this, notice that if i believes that j is of type ϑ' and that $m_j = \vartheta$, then his best-response is $m_i = \vartheta'$, in which case it is also a best-response for j to report $m_j = \vartheta$. Finally, in the case where both parts of Condition 1 are violated, then there can be no truthful equilibrium (as j strictly prefers to report ϑ , if everyone else reports truthfully), while an untruthful equilibrium may exist, where i reports j's type and vice versa. In all cases, full implementation is impossible.

Condition 1 is similar in spirit to Bayesian Monotonicity, which is necessary for full implementation in economies with incomplete information (Jackson, 1991). In our case, full implementation is possible, if there is a feasible allocation through which some agent (i) "signals" cases of misreport. As a result, not all efficient allocations are fully implementable when the interim-distribution is common knowledge. However, Condition 1 holds whenever the number of agents of lower-rank are less or equal to the number of agents of higher ranks. Assumption 3 in section 1.4.2 made this restriction clear. On the other hand, Condition 1 is weaker than Assumption 3, and may hold in cases where this assumption is violated. Note that Condition 1 is also sufficient for full implementation if one allows for mechanisms with games that do not have an equilibrium in pure strategies (for example integer games, as in Maskin (1999) or modulo games, as in Jackson (1991))¹³. This is because one can rule out undesirable equilibria with multiple misrepresentations of types (sub-case (iii) in the mechanism of Proposition 1) by making agents to play such a game, whenever the message-profile differs from the interim-distribution by more than one message. However, if one restricts attention to mechanisms where agents send only messages about their types, the following condition is also required.

Condition 2: Suppose $a^* \in PF(E)$. There exists allocation $a \in A$, such that $a^*_{\vartheta} \succ_{\vartheta} a_{\vartheta}$ and $a_{\vartheta} \succ_{\vartheta} a_{\vartheta'} \forall \vartheta, \vartheta' \in \Theta(\beta)$.

Condition 2 ensures that whenever there are more than one misrepresentations of types, it is a best-response for one of the "liars" to deviate and report truthfully, while it is not a best-response to deviate from truth-telling. It becomes apparent that Assumption 3 and the LNCIP (Lemma 4) satisfy Condition 1, while LNCIP (Lemma 3) also satisfies Condition 2. Jointly, Condition 1 and 2 are necessary and sufficient for full implementation for this restricted set of mechanisms when the interim-distribution is common knowledge¹⁴.

1.4.4 Examples

Spence (1973)

The Spence economy consists of two types. Group I has low productivity \underline{a} and is a proportion q_1 of the population. Group II has high productivity \overline{a} and is a proportion

 $^{^{13}}$ See Appendix A for the proof. It is omitted here, since it is similar to the proof of Proposition 1. 14 See Appendix A for the proof

 $1 - q_1$ of the population. Acquiring y units of education costs y/\underline{a} for Group I and y/\overline{a} for Group II. Productivity parameters are private information and firms hire workers according to a wage schedule, based on verifiable educational attainment. The payoff for an individual is the value of his wage minus the educational cost and for a firm the productivity parameter minus the wage.

Spence argues that agents will acquire education (which does not increase productivity in his model) in order to signal their productivity to firms. In equilibrium, the wage schedules are such that high productivity workers acquire some educational and credibly signal their type, while low productivity workers acquire no education, and firms correctly infer that they are low productivity. The education acquired by Group II is a deadweight loss, necessary for signaling their abilities.

Assume that the total population N is common knowledge. Then Nq_1 is the total number of agents of Group I and $N(1 - q_1)$ is the total number of agents of Group II. Based on this, the following mechanism can separate types without any agent incurring educational costs in equilibrium:

Let all workers report their type. If the number of agents who report Group I and II is Nq_1 and $N(1 - q_1)$, respectively, then agents who report Group I receive wage $w_{G_I} = \underline{a}$ and those who report Group II, receive wage $w_{G_{II}} = \overline{a}$. Otherwise, those who report Group I receive $w_{G_I} = \underline{a}$ and those who report Group II, are asked to undertake one additional time-period of education and receive $w_{G_{II}} = \underline{a} + \frac{1}{\overline{a}} + \epsilon$, with $\frac{\overline{a}-\underline{a}}{\overline{a}\underline{a}} > \epsilon > 0$.

The above mechanism fully implements the first-best allocations in this economy. First, consider the strategies of an \overline{a} -type, who has already acquired education y. It is clear that, irrespectively of the reports of the other agents, it is a dominant strategy for her to report \overline{a} , since $\overline{a} > \underline{a}$ and $\underline{a} + \epsilon > \underline{a}$. Then, it is a best-response for an \underline{a} -type to report truthfully as well. This is because $\underline{a} > \underline{a} + \epsilon + \frac{1}{\overline{a}} - \frac{1}{\underline{a}}$. Hence, all agents report truthfully in equilibrium. Anticipating the outcome of the job-market, all agents acquire zero education before reporting their types. Figure 2 provides the graphical representation.



Figure 2: Spence, 1973

Rotschild-Stiglitz (1976)

Consider the following, slightly modified, version of the Rothschild-Stiglitz economy. There is a finite number of N risk-averse agents and one risk-neutral entrepreneur. There is one commodity. Agents have a stochastic endowment with two possible states w_H and w_L , with $w_H > w_L$. The entrepreneur has an endowment w_E , which is subject to no risk. An agent's utility function depends on her consumption on both individual states: $U(c_L, c_H)$. There are two types of agents. K of them are of type 1 and face a high probability of suffering from the low endowment state: p_H . The remaining L = N - Kare of type 2 and have a low probability of w_L : $p_L < p_H$. Types are private information, but the rest characteristics of the economy are common knowledge. Finally, assume that w_E is large enough so that, even if all other agents suffer from the low-endowment state,
they can still be fully insured by the entrepreneur's wealth.

Assuming that the other side has full bargaining power and hence the entrepreneur makes no profits from her services, the following mechanism can be utilized in order to implement first-best allocations (see also Figure 3). All agents report their type. If the message-profile matches the interim-distribution then each agents receives the insurance contract that corresponds to her message (C_1^{FB} and C_2^{FB} are the state-contingent allocations resulting from the first-best insurance contracts for 1 and 2 respectively). Otherwise, agents who report type 1, receive an insurance contract which results to allocation A_1 , while agents who report type 2, receive A_2 .



Figure 3: Rothschild-Stiglitz, 1976

Notice that, by construction, $A_2 \succ_2 C_1^{FB} \succ_2 A_1$ and $C_1^{FB} \succ_1 A_1 \succ_1 A_2$. Also, providing any combination of these individual allocations to the agents of the economy is feasible, since they all lie in the interior of $\underline{A}(C_2^{FB})$. Therefore, Condition 1, is satisfied. It is easy to check that it is a dominant strategy for type 2 to report truthfully. Given this, it is a best-response for any agent of type 1 to report truthfully, as well. Therefore, the proposed mechanism has a unique Bayes-Nash equilibrium, which is truthful.

1.4.5 Robustness to Small Perturbations

So far it has been assumed that the interim-distribution of types is commonly known with perfect precision. This is a very strong assumption, and hence one would like to make sure that small relaxations of it would not change the results dramatically. As it turns out, if there is a sufficiently small noise about β , then the main claim still holds.

Let Γ be the set of all possible interim-distributions that can be generated by Θ . By definition, $\bigcup_{\gamma \in \Gamma} \Theta(\beta) = \Theta$. Suppose, now, that there is a small noise about the probability of the interim-distribution. Agents have a probability distribution over the set of interim-distributions. With probability $1 - \sum_{\gamma \in \Gamma} \epsilon_{\gamma}$, the interim-distribution β will be realized, while ϵ_{γ} is the probability that some other interim-distribution γ will be realized, with $\epsilon_{\gamma} > 0, \forall \gamma \in \Gamma$.

We maintain the assumption that each agent knows his own type with certainty but has no information about the other agents' type. The expected utility of agent i has to be modified in order to include the uncertainty over the interim distribution:

$$U_{i}(a) = (1 - \sum_{\gamma \in \Gamma} \epsilon_{\gamma}) \sum_{\boldsymbol{\theta}_{-i} \in \boldsymbol{\Theta}_{-i}(\beta | \vartheta_{i})} \left[\sum_{s \in S} u_{i}(a, s) \ \pi \left(s | \vartheta_{i}, \boldsymbol{\theta}_{-i} \right) \right] \phi(\boldsymbol{\theta}_{-i} | \vartheta_{i}, \beta)$$
$$+ \sum_{\gamma \in \Gamma} \epsilon_{\gamma} \left[\sum_{\boldsymbol{\theta}_{-i} \in \boldsymbol{\Theta}_{-i}(\gamma | \vartheta_{i})} \left[\sum_{s \in S} u_{i}(a, s) \ \pi \left(s | \vartheta_{i}, \boldsymbol{\theta}_{-i} \right) \right] \phi(\boldsymbol{\theta}_{-i} | \vartheta_{i}, \gamma) \right]$$

It is also assumed that for each $\gamma \in \Gamma$ and for every ϑ_i corresponds an individual allocation $a_i^*(\vartheta_i, \gamma)$ such that any I-collection of individual allocations is consistent with γ , Pareto optimal and satisfies Anonymity. In other words, for every γ there is a set of Pareto-optimal allocations to be implemented, each one corresponding to a specific realization of a type-profile $\boldsymbol{\theta}$ consistent with γ and Anonymity.

In the case of uncertainty about the interim distribution, the rank of each agent is also uncertain, as different γ may correspond to different sets of realized types and different ranks. The problem then would be one similar to the problem when the Anonymity property is violated. However, if this uncertainty is sufficiently small, the equilibrium strategies of agents will not change. To see this, consider an agent *i* who has the highest rank under β (and potentially other ranks for other γ 's). If he knows that β is the interim distribution with certainty, then under the mechanism presented in 4.2, he would strictly prefer to report his type truthfully than report any other type:

$$U_i(\vartheta_i, \mathbf{m}_{-i}|\beta) > U_i(\vartheta', \mathbf{m}_{-i}|\beta), \,\forall \vartheta' \neq \vartheta_i \in \Theta, \,\forall \mathbf{m}_{-i} \in M$$

Adding a small uncertainty about the interim distribution means that his expected utility by reporting his type truthfully becomes:

$$U_i(\theta_i, \mathbf{m}_{-i}) = (1 - \sum_{\gamma \in \Gamma} \epsilon_{\gamma}) U_i(\vartheta_i, \mathbf{m}_{-i} | \beta) + \sum_{\gamma \in \Gamma} \epsilon_{\gamma} U_i(\vartheta_i, \mathbf{m}_{-i} | \gamma)$$

It is evident that, if ϵ_{γ} is sufficiently small for every γ , the expected utility of *i* approaches the expected utility under β and hence it remains a strictly dominant strategy to report his type truthfully. The argument can be repeated for any other agent *j* of different rank according to β . Given a sufficiently small vector of probabilities $\boldsymbol{\epsilon}$, *j* expects all higher-rank agents to report truthfully and his best-response is to report truthfully as well, irrespectively of the messages send by agents of the same or lower ranks. Hence, there exists some vector $\boldsymbol{\epsilon}$, with strictly positive elements, such that the equilibrium strategies under certainty over β remain the unique equilibrium strategies under uncertainty over β .

Corollary 2: If the interim distribution of types is uncertain but there is a sufficiently high probability that some distribution β will be realized, then the mechanism of Proposition 1 fully implements the first-best allocations for every interim-distribution.

Proof: It follows from the analysis above.

It is noteworthy that, due to the fact that truthful revelation of one's type is the only equilibrium action for all agents, the desirable individual allocations will be implemented for any interim distribution γ . In other words, the almost certainty about β makes agents to report their type truthfully irrespectively of the interim distribution that is eventually realized. As a consequence, agents receive first-best allocations for all realized interim-distributions. This confirms that the result is robust to small perturbations of the information structure and it is not just a construction of perfect knowledge of the interim distribution.

1.4.6 Convergence to Ex-Ante Distributions

So far we have shown the main result and that it is robust to small uncertainty about the interim distribution. We also want to show that if the number of agents becomes very large then the interim-distribution converges to the ex-ante distribution of types, in which case the informational assumptions made in this chapter converge to the standard assumptions in the adverse selection literature, i.e. agents know the ex-ante probability of each type occurring. This allows us to relate this formulation and results to large economies with adverse selection problems, and make the claim that in these economies, because the interim-distribution is effectively common knowledge, one can implement first-best allocations. Of course, this requires some restrictions on the joint probability function Φ . The easiest way is to assume that types are independently and identically distributed. This means that the probability of acquiring type ϑ , $\tau(\vartheta)$, is the same across all agents and the draws of types from the ex-ante distribution are uncorrelated. Then, by directly applying the Weak Law of Large Numbers we get:

$$\lim_{I \to \infty} \left(\frac{\lambda_{\beta_I}(\vartheta)}{I} \right) = \tau(\vartheta)$$

This is exactly the information provided by the interim-distribution: the number of agents, for whom type ϑ has realized. Hence, at the limit, the relative frequency of types in the population (interim-distribution) coincides with the ex-ante probability, if types are independently and identically distributed, and the interim distribution of types coincides with the ex-ante distribution at the limit¹⁵. Hence, the mechanism of this chapter can be applied to economies with large populations without requiring any additional information than the literature on asymmetric information. This comes at the cost of additional restrictions on the joint probability function, which, however, are common with many other papers in mechanism design.

1.4.7 Participation Constraints

A final note is required regarding the issue of participation constraints. In many important applications of adverse selection problems, agents are given the opportunity not to participate in a contract or in a mechanism if the expected utility they anticipate by entering is less than some exogenously given threshold. In this model, however, we have completely ignored any participation constraint restrictions. Fortunately, this omission does not result in loss of generality. If participation constraints are to be taken into

¹⁵Notice, however, that other formulations of the Law of Large Numbers do not require independence or identically distributed types. Therefore, the results also hold for these cases.

consideration, then this only restricts the points of the Pareto frontier that satisfy these constraints and does not alter the rest of the analysis¹⁶.

1.5 Conclusion

In this chapter we consider a general hidden-type economy and, under relatively weak conditions, we show that it is possible to construct a mechanism which has a unique Bayes-Nash equilibrium, where all agents reveal their type truthfully and they receive a first-best allocation. Our result relies on information aggregation and appropriately chosen punishments. If the interim distribution is known (perfectly or imperfectly), then one can aggregate the messages that all agents are sending out and uncover any misreport(s), even if the identity of the liar is not known.

Truth-telling, however, requires appropriately designed punishments for lying. If the punishment from detecting a lie is too severe, then some agents may deliberately lie about their type in order to force other agents to also do so. The lies cancel out in terms of the aggregate information and the former agents "steal" the allocations of the latter, who are forced to lie under the fear of the extreme punishments. This can lead to coordination failures and multiplicity of equilibria. Therefore, uniqueness of the equilibrium requires a careful construction of the allocations when lies are detected. It has been shown that such punishments exist when the indifference curves of different types are not locally identical, meaning that in the neighborhood of any allocation one can find other allocations such that each type prefers one of these over the rest.

It should also be pointed out that the assumption on the interim distribution of

¹⁶Of course, in all interesting problems, the intersection of all participation constraints with the Pareto-frontier is non-empty. Notice that, in off-the-equilibrium-path situations, the resulting allocations may violate certain participation constraints. But as long as agents decide and commit on their participation before the mechanism is played (based on the expectation of an outcome, which results from some equilibrium of the sub-game), then the uniqueness and efficiency of the equilibrium guarantees the participation of all agents.

types being common knowledge is needed because we consider general social choice sets. If we focus on the implementation of specific allocations on the Pareto frontier so that allocations depend only on one's type, one can implement the first-best as a unique equilibrium even if agents have heterogeneous beliefs or no information at all about the interim distribution. The mechanism can still implement the desirable allocation truthfully, given that the social planner knows the interim distribution. This is because players' best-response correspondences depend on their beliefs about how many misreports will be detected by the mechanism and not on their ability to detect other agents' lies. Finally, an interesting question is whether the implementation of firstbest allocations in this setting can be achieved through a decentralized mechanism. We plan to address this question in the near future.

2 Chapter Two: Contract Law and Development

2.1 Introduction

The main body of the economic literature on contract theory assumes that private agreements are enforceable by an external authority so that private contracts are binding for the contracting members¹⁷. It is also assumed that the enforcement of all agreements, which private parties are willing to commit to, increases social welfare. Because the role of enforcement institutions is obvious in this context, the related papers do not attempt to explicitly model the enforcement authority and how it achieves its goals. On the other hand, the literature on economic growth and development has placed emphasis on the importance of property rights and their enforcement, but contract law has received disproportionately less attention.

This chapter relates the emergence and evolution of contract law to the process of development. We show that the relationship between the two is reciprocal. As the process of development unfolds, contract law evolves accordingly and at the same time it generates new opportunities for economic growth and development. Furthermore, the relationship between the two may be non-monotonic and hence it is difficult to be captured by empirical studies. Through the analysis, we also rationalize two stylized facts of enforcement institutions: (i) the existence of institutional agents (such as judges or bureaucrats) who act on behalf of these institutions and are rewarded for their function, and (ii) the fact that not all types of private agreements are permitted in a society (regulation).

By contract law we mean the types of private agreements which are enforced by judicial institutions as binding contracts. If a type of private transaction is non-enforceable this means that judges will not impose the terms of the contract on the transacting

¹⁷See for example page 3 in the introduction of "Contract Theory" by Bolton and Dewatripont.

members, even if one of them breaches the initial agreement. Since most agreements made into contracts are time-inconsistent (i.e. party members do not wish to carry out their part of the initial promise when the execution time comes), non-enforceability of certain transactions effectively prohibits them from taking place.

In order to make the points described above, a simple economy is presented, which consists of multiple pairs of agents. Each pair is comprised by a producer and a consumer of a specialized good, who face a bilateral hold-up problem. The cost of the specialized good is uncertain and trade is valuable to both parts only if the low-cost state materializes, in which case the surplus generated by trade is divided between them according to their bargaining power. The probability of this event depends on the effort levels of both the consumer and the producer. The effort levels, the state of nature and the utilities of agents are non-verifiable. Because both agents bear the full marginal cost of effort exertion but receive only part of the marginal benefit, they generally exert sub-optimal effort levels. The problem could be solved by a mechanism of transfers contingent on trade, if agents could commit not to make any other private agreement for transfers of resources. However, this solution is not time-consistent and, as a result, mechanisms with exogenous full enforcement can not implement the first-best outcome.

In the first part of the chapter, a mechanism is proposed which solves this timeinconsistency problem. In this mechanism, the agents themselves decide which contracts are enforceable and which are not. That is, contract enforcement is endogenous. It turns out that the implementation of the first-best requires that some contracts be non-enforceable. Specifically, the mechanism involves a social contract between an institutional agent, who has the enforcement power in the economy, and the rest of the agents. The social contract specifies what types of private agreements are enforceable or not at each point in time and the rewards and punishments for the institutional agent, if she acts according to its clauses or not. Thus the optimal social contract specifies ex-post transfers from the trading members to the institutional agent or from the institutional agent to the trading parties, conditional on the occurrence of trade.

We show that this mechanism can induce agents to exert optimal effort levels and is renegotiation-proof if: (i) at least one agent plays the role of the institutional agent so that she has a stake to the outcomes generated by the enforcement process and (ii) some forms of private agreements are not enforceable and hence non-credible. However, the implementation of the institutional design also requires a minimum amount of resources and so it is inevitably related to the level of economic development.

Note that this is a model of perfect enforcement. That is, there are no exogenous frictions on enforcement. It is costless for the judge to obtain information on verifiable variables and there are no costs for punishing an agent who did not execute her part of the contract. Furthermore, we do not impose any exogenous restrictions on the contract space, apart from the condition that contracts should be written on verifiable variables (the natural one). However, in equilibrium, agents will choose to render some (otherwise fully enforceable) contracts as non-enforceable, in order to credibly commit themselves to exert high-effort. In other words, we abstract away from any possible enforcement friction in order to show that the contractual space is optimally chosen to be incomplete.

In the second part of the chapter a multi-period economy is introduced and the hold-up problem (and its solution) is connected to the process of economic growth and development. The agents in the economy have access to the production technology of a non-specialized or autarchic good, which exhibits no uncertainty, and to the production technology of a specialized good in pairs of buyer and seller, exactly as in the previous section. The economy starts with no initial institutions, but at the beginning of the first period agents can propose and generate institutions of government and enforcement through a social contract. Institutional agents play the dual role of the governor and judge and they are responsible for imposing taxation to agents, spend public revenues to investments (which increase the productivity of agents), enforce private agreements and provide reductions in taxation conditional on the trade of the specialized good. The reductions in taxation work as subsidies which induce agents to exert higher effort and thus increase the value of trade.

In this economic set-up, the initial level and the rate of increase of productivity in the production of the autarchic good are the main parameters which drive the evolution of institutions and growth. The most interesting case is when productivity growth is fast enough so that the economy passes from multiple stages of development. We identify the necessary conditions for this case to arise and the following results are derived: (i) The economy starts from low levels of productivity, where the trade of the specialized good is not feasible, but as productivity increases the feasibility constraints are relaxed. At a specific point in time trade becomes feasible and enforcement institutions are created. (ii) Restrictions on the set of enforceable private agreements arise endogenously, inducing trading parties to exert high effort and to increase the value of trade and this spurs further economic growth. Therefore, the causal relationship between enforcement institutions and growth goes in both directions. (iii) The change of restrictions on the set of enforceable agreements is non-monotonic to the process of economic growth.

These results are interesting because they emphasize the reciprocal relationship between contract law and growth. More importantly, the relationship between the two may be non-monotonic. Accemoglu and Johnson (2005) test whether property rights institutions or contract enforcement institutions have a positive impact on growth. While they find that property rights seem indeed to affect growth positively, contract institutions do not present a statistically significant impact. Their main hypothesis is that the lower the cost of contract enforcement, the more easily private parties can contract and hence the greater the impact of economic growth. They test for this effect by using as a proxy for enforcement costs the legal origin of contract law (whether it is common law or civil law).

While, this model can not distinguish between the two types of law origins, if one is willing to assume that enforcement costs are positively correlated with the number of transactions which are enforceable (which is justified in subsection 2.4.6, in the part where the empirical implications of the model are discussed), then one can offer an explanation to the findings of the empirical literature. With a non-monotonic relationship between the set of enforceable agreements and the level of growth, the impact of contract law on the latter can not be captured by linear specifications.

These results also show that regulation may be optimal for social welfare. Thus, limiting agents' economic freedom may have beneficial results if hold-up problems are prevalent in economic exchanges, a point which goes against the classic economic intuition that more economic freedom implies greater welfare. Apart from the example of trade of specialized goods, which we provide in this chapter, there are many other cases of economic interest, where a trade-off between ex-ante incentives and ex-post efficiency may arise. This framework can be applied in these cases to explain why certain types of regulation are imposed or why certain types of contracts are forbidden from being written.

For instance, the model could be used to explain the abolishment of slavery in the 19th century as banning certain property contracts in order to induce the accumulation of unobservable human capital. It could also be used to rationalize certain laws, which protect collective bargaining agreements between employees and employers as an effective banning of one-to-one contracts. This, in turn, increases labour wages and, apart from redistributive effects, it increases the ex-ante incentives of workers to acquire human capital. On the other hand, forbidding prenuptial agreements, which may dissolve dysfunctional partnerships in an effective way ex-post, may be a way into incentivizing parents in exerting high effort to their family affairs and children up-bringing.

We also believe that our model can be extended to generate a more complete theory of the trade-off between economic freedom and incentives (ex-post efficiency versus exante incentives) and it can also provide a model of regulation cycles. Finally, we provide a different rationale for the existence of institutional agents, such as bureaucrats and judges. Besides being the executors of authority, they also guarantee the credibility of the institutions they represent by having a stake in their functionality and by not always agreeing to change them. Thus institutions acquire persistence, which again is crucial for their credibility and the solution of hold-up problems.

2.2 Related Literature

There is an extensive literature dealing with the determinants of property rights, their value for society and development. Examples include Umbeck (1981), Skaperdas (1982), Grossman (2001) and Gonzalez (2007). On the contrary, there is little work on how enforcement institutions and contract law can affect the development process. A notable exception is Dhillon and Rigolini (2009), which relates the process of development to enforcement institutions through the functioning of commodity markets. The current framework differs from theirs in two ways. First, this chapter considers only formal institutions while their model is concerned with the co-determination of both formal and informal institutions of enforcement. Second, the process of development in their paper is exogenous and related to the reliability of the production process to generate high quality goods, while the chapter is concerned with the co-evolution of contract law and development through the changes on regulation and productivity respectively.

A different strand of literature examines the impact of limited enforcement on economic transactions. Telser (1980) is one of the first papers to model self-enforcing agreements, while Bull (1987) examines self-enforcing agreements in the context of the US labor market and Ray (2002) examines their time-structure. However, these papers are concerned with cases where contract enforcement is impossible and this is an economic restriction which agents can not overcome. Other papers, like Cooley, Marimon and Quadrini (2004) and Ellingsen and Kristiansen (2008), are concerned with the impact of limited enforceability on financial contracting. Krasa and Villamil (2000) consider the case where enforcement of the contractual agreement is a choice variable of the contracting members but it is costly and show that the costly state verification model can be seen as a reduced form of their enforcement problem. None of the above papers, however, examines the issue of enforceability from the perspective of endogenous limitations on the types of agreements that are enforced.

On the other hand, there is an extensive literature that is concerned with the issues of institutional authority. Aghion and Tirole (1997), Aghion, Alesina and Trebbi (2004), Greif and Laitin (2004), Greif et al (2008), Laffont and Martimort (1998), Sanchez and Straub (2006) are some of the papers concerned with the issues of authority in organizations or the endogenous formation of institutions. The main difference is that we focus on enforcement institutions and its relationship to the process of economic development. On top of that, in the analysis to follow we combine both the questions of how these institutions emerge and evolve and how authority is determined.

The chapter is also related to the literature regarding the hold-up problem. Since the seminal contributions by Grossman and Hart (1986) and Hart and Moore (1988, 1990), a long list of papers has been devoted to presenting the inefficiencies generated by this problem or solving it¹⁸. The chapter does not attempt to solve the most general type of hold-up. Instead, a simple example of a hold-up problem is used (which can

¹⁸See for example Aghion, Dewatripont and Rey (1994), Che and Hausch (1999), Hart and Moore (1999), Maskin and Moore (1999), Maskin and Tirole (1999a), Maskin and Tirole (1999b), Guriev and Kvassov (2005) and Evans (2008)

not be solved by the mechanisms presented in the papers above) in order to show the importance of enforcement institutions in solving time-inconsistency problems.

However, Baliga and Sjöström (2009) adopt a very similar framework to ours. In their paper they allow agents to contract with a third party and show that, with an appropriately designed mechanism, agents can solve their time-inconsistency problem. Furthermore, they show that side-contracting between the third party and one of the agents does not alter their results. They apply their framework to the hold-up problem and to the problem of moral-hazard in teams. Similar results are obtained in this chapter through the institutional agent. The main differences are two. First, the third party in this chapter has a specific type of authority in the economy (to enforce private agreements or not), which the third party in their paper does not have. Second, and most important, they assume that all private agreements are enforceable while here enforceability is treated as an endogenous variable.

Finally, the chapter is related to the literature concerned with issues of delegation. These papers examine the ability of an uninformed principal to extract information from an informed agent, who is asked to perform a task. Some examples are the papers by Holmström (1982), Fershtman, Judd and Kalai (1991), Faure-Grimaud, Laffont and Martimort (2003), Szalay (2005) and Alonso and Matouschek (2008). While in these papers delegates act on behalf of a principal, in the model of this chapter institutional agents are economy-wide delegates who act on behalf of the society. We also eschew away from issues of information extraction as it is assumed that the actions of institutional agents are fully observable by the rest of the agents.

2.3 A simple model with two agents

In this section we examine the solution to a static hold-up problem and derive the main results and intuition which are required for the analysis of the dynamic model. A plough-maker (say agent j, who is also sometimes referred as the producer) and a farmer (agent i) face a simple hold-up problem with bilateral externalities. The ploughmaker has the ability to produce one unit of plough g, which is custom-made to satisfy the farmer's requirements. This good has valuation equal to v for the farmer in terms of some numeraire commodity (think of it as the additional quantity of wheat, which the farmer can produce due to its use).

However, the cost of the plough, which is again reflected in terms of the numeraire commodity, is uncertain and depends on the state of nature. There are two states of nature, one with a high cost ($\theta_1 : k_1 = k_H$) and one with a low cost ($\theta_2 : k_2 = k_L$). The probability of the low-cost state depends on the effort level of both the producer and the farmer. The intuition for this assumption is that effort is exerted for acquiring skills relevant to their occupation. The more skilled the farmer is in cultivating the land, the higher the chances that he requires a crude, but inexpensive plough to do his job. Similarly, the more skilled the plough-maker is, the higher the chances that he can produced the required plough with minimal use of resources¹⁹.

If e_i and e_j are the effort levels exerted by them, then $f(e_i, e_j)$ is the joint probability function of the low cost state arising. There are two effort levels for each agent: $\{\overline{e}, \underline{e}\}$: $\overline{e} > \underline{e}$. The corresponding cost of effort, which is homogeneous across agents, is given by: $c_i(\overline{e}) = c_j(\overline{e}) = \overline{c} > c_i(\underline{e}) = c_j(\underline{e}) = \underline{c}$ and $f(\overline{e}, \overline{e}) > f(\overline{e}, \underline{e}) = f(\underline{e}, \overline{e}) > f(\underline{e}, \underline{e})$. The probability function f exhibits decreasing returns to scale: $f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e}) > f(\overline{e}, \underline{e}) = f(\underline{e}, \underline{$

The agents are endowed with a sufficiently large amount of the numeraire commod-

¹⁹Alternatively, the model can be formulated so that uncertainty is added on the consumption side of the economy. For instance, the valuation for the plough could be high or low and its probability can depend on the effort level of the farmer. In this case, there would be four states of nature, but the main results of the chapter would be the same if the trade generates social surplus in only one of the four states. The economic interpretation would be similar as well, especially if one thinks of goods which require some effort from the farmer's part in order to learn how to use them efficiently.

ity w (think of it as wheat or bread)²⁰. Moreover, agents are risk neutral. The utility of the plough-maker depends only on the amount of the numeraire commodity that she consumes, but the utility of the farmer also depends on good g. In terms of notation, $u_j = x_j^a - c_j$ and $u_i = x_i^a + I_g v - c_i$, where x_j^a and x_i^a are the consumption levels of the numeraire commodity by the producer and the farmer respectively and I_g is an indicator function that takes the value one if g is traded and zero otherwise. If agents decide to trade after the state of nature is realized, they divide the gains from trade according to some exogenous bargaining power²¹. Let β_i (β_j) be the bargaining power of agent i (j), with $0 < \beta_i < 1$ and $\beta_i + \beta_j = 1$. Furthermore, assume that the following inequalities hold:

$$[f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})]\beta_{\xi}(v - k_L) - (\overline{c} - \underline{c}) < 0, \forall e_{\zeta} \in \{\overline{e}, \underline{e}\}$$
(1)

$$[f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})](v - k_L) - (\overline{c} - \underline{c}) > 0, \forall e_{\zeta} \in \{\overline{e}, \underline{e}\}$$
(2)

$$k_l - k_H + 2 \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} > 0$$
(3)

In the inequalities above, ξ denotes one of the two parts of the transaction (either *i* or *j*) and ζ denotes the other part (if $\xi = i$, then $\zeta = j$ and vice versa). The timing of events is: first, agents choose effort levels, then the state of nature is determined and then agents decide whether to trade or not. Committing to trade before uncertainty is resolved is sub-optimal as it entails welfare losses in the state where the cost of *g* is

²⁰ for the purposes of this section it is sufficient that $w > v - k_L$

 $^{^{21}}$ This can be interpreted as the probability of one side to make a take-it-or-leave-it offer to the other side.

higher than v.

Inequalities (1) and (2) reflect the conditions under which the hold-up problem arises. Inequality (1) states that, independent of the effort level of the other agent, the marginal benefit of increasing the effort level for type ξ is lower than the marginal cost of doing so, while (2) states that, under the same conditions, the marginal social benefit exceeds the marginal social cost, which is equal to the private cost. In conjunction, inequalities (1) and (2) imply that it is a dominant strategy for both types of agents to exert low effort, but this is socially sub-optimal.

In fact, adding inequality (2) for different effort levels and types, one can show that $[f(\overline{e}, \overline{e}) - f(\underline{e}, \underline{e})](v - k_L) - 2(\overline{c} - \underline{c}) > 0$, which means that if both agents choose to exert high effort the aggregate social benefit is positive. In other words, the two conditions form a simple model of a hold-up problem: both agents would benefit from exerting high effort but because they receive only a part of the social surplus they generate, they do not have an incentive to do so.

Inequality (3), on the other hand, is a technical condition, which is required for making the incentive compatibility problem meaningful. As is shown in the proof of Proposition 1, when this condition holds, subsidizing trade in order to induce high effort, without any restrictions on enforceable agreements, is ineffective, because agents prefer to trade in the high cost in order to receive the subsidies and redistribute them among themselves.

Effort levels, effort costs, the state of nature and the level of utility of each agent are observable but non-verifiable. In addition, the effort levels of the agents are nontransferable, so that property rights can not solve the problem. This is a plausible assumption since in many cases of economic interest the economic surplus may depend on the actions of some individuals which can not be easily replicated by others. In our example, the plough-maker has a specific set of skills which are needed for the production of the good, which the farmer does not have and can not acquire.

If effort choices were transferable, an easy solution to this problem would be the allocation and trade of property rights on the effort decisions (see also Grossman and Hart, 1986). Also, if any of the non-verifiable variables, could be verified, even at some cost, then the agents could design mechanisms of subsidy provision or punishments in order to induce the first-best effort levels. For example, if the state of nature were costlessly verifiable, then the following mechanism would implement the first-best effort levels: the farmer and the plough-maker give out $\tau_i = f(\bar{e}, \bar{e})(1 - \beta_i)(v - k_L)$ and $\tau_j = f(\bar{e}, \bar{e})\beta_i(v - k_L)$ units of the numeraire commodity to a risk-neutral agent, who has the obligation to return to them $s_i = (1 - \beta_i)(v - k_L)$ and $s_j = \beta_i(v - k_L)$ units respectively, if the low-cost state arises. In such a case, agents receive a subsidy in the low-cost case which aligns the marginal costs and benefits of effort exertion to the social costs and benefits and therefore achieves first-best outcomes.

Of course, as the relevant literature points out (see for example Hart and Moore (1988) or Maskin and Tirole (1999b)), the problem with such a mechanism is that truthtelling about the state of nature is not incentive compatible when the states of nature are non-verifiable. If the high-cost state arises, agents have incentive to lie in order to receive and redistribute the subsidies between them. Such redistributions require binding agreements on net transfers of resources. Ex-ante, agents prefer to ban such transfers, so that the mechanism satisfies incentive compatibility and generates optimal incentives for effort provision, but ex-post agents prefer to renegotiate the mechanism and allow for the transfers to take place. Therefore, this mechanism fails to provide incentives for efficient effort exertion, because it is not renegotiation-proof and, hence, credible.

This section shows how this problem can be circumvented by the introduction of an **institutional agent**, a type of delegate, who enforces the mechanism and whose final payoff depends on the outcome of the mechanism. In this case, because the institutional agent has an incentive to block any renegotiation that reduces his expected payoff, an endogenous commitment not-to-renegotiate the mechanism arises ex-post. Furthermore, the enforceability of ex-post transfers and the incentives of the institutional agent arise endogenously, through the ex-ante **social contract** between the agents and their delegate.

For the rest of this section the timing of events as represented by Figure 4 is assumed, which is similar to the one adopted by Watson (2007). Before proceeding to the main result of the section, we provide the necessary definitions.



Figure 4: Timing of Events

Definition 1: An institutional agent ($\Sigma = 1$) is a third party whose actions and rewards are determined by the agents through a social contract. The action set of the

institutional agent is the payment of subsidies to the agents and the enforcement or not of private contracts.

Definition 2: A private contract $\pi(q, p(g), I_g)$ is any agreement between the farmer and the plough-maker. This formulation includes agreements for selling good g at a price p(g), agreements which promise a net transfer of resources q conditional on the trade and the price of good g (side contracts) and agreements for an unconditional transfer of resources q (irrespectively of trade). Q is the set of all possible contracts. Whether a subset of Q is enforceable or not depends on the social contract.

Definition 3: A social contract $S(\Sigma, \Phi(Q), \tau)$ is a contract between the farmer, the plough-maker and potentially (but not necessarily) an institutional agent, which defines ex-post transfers τ conditional on the verifiable trade of the good g and on its price $p(g)^{22}$, the inclusion of the institutional agent or not ($\Sigma = 1$ or $\Sigma = 0$, respectively) and the set of enforceable private agreements $\Phi(Q)$.

It is assumed that even if an institutional agent is not included in the social contract, the farmer and the plough-maker can still utilize the enforcement authority of the economy (which is treated as an automaton or a machine in that case). This is done so that we can contrast these results with the existing literature, which assumes that enforcement authorities exist but they are not explicitly modeled. In particular, we want to show why the incompleteness of the contractual space and the structure of incentives for the institutional agent as well, are so important for solving the hold-up problem.

However, in the analysis that follows it is implicitly assumed that the judge is punished if he violates his part of the agreement with the other two agents, and hence

 $^{^{22}}$ Ex-post transfers can be either positive (subsidization) or negative (taxation).

he is bound to execute the social contract, unless they all agree to renegotiate it²³. Also notice that the cases of exogenous enforcement with complete or incomplete set of enforceable agreements are special cases of social contracts in our framework. In section 2.4, we make the more realistic assumption that the economy starts of from a point of no institutions and we derive their emergence, evolution and structure endogenously.

Propositions 1 and 2 below show the role of contract law for solving the hold-up problem. The main intuition is that, in order to induce agents to exert high effort, subsidization of trade is required. However, once the high-cost state materializes, agents may have an incentive to conduct trade, even if it is suboptimal, in order to receive the subsidies. Stopping them from doing so requires limitations to the maximum amount of net transfers that the farmer can provide to the producer. However, these limitations, though optimal from an ex-ante point of view, are not credible ex-post. Without an institutional agent, who bears the cost of subsidies and the benefits of taxation, the agents would simply undo the regulation they set in place after the state of nature realizes. Therefore, a credible solution to this time-inconsistency problem requires a judge, who enforces regulation and who has a stake on the function of the enforcement institutions. In other words, the solution to the hold-up problem is not imposed exogenously, but arises as the equilibrium outcome of an institutional process (the social contract).

Proposition 1: Let $\overline{p} = k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})}$. Consider the social contract S^* , which defines:

(i) if trade of good g takes place, agent ξ receives subsidy (negative taxation)

 $^{^{23}}$ Of course, the important question of what ensures the compliance of the institutional agent with the social contract is omitted from the analysis. However, one way to deal with this issue is to assume an infinitely lived judge and construct a reward structure for him and trigger-strategies for the rest of the agents, such that the compliance with the social contract is self-enforced. We leave this interesting direction for future work.

$$\tau_{\xi 1} = (1 - f(\overline{e}, \overline{e})) \left(\beta_{\xi} (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) < 0.$$

(ii) if trade of good g does not takes place, agent ξ pays out taxation

 $\tau_{\xi 0} = f(\overline{e}, \overline{e}) \left(-\beta_{\xi} (v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - \overline{f(\overline{e}, \underline{e})}} \right) > 0.$

(iii) any private contract $\pi(0, \hat{p}, I_g)$ or $\pi(\hat{q}, p, I_g)$, with $\hat{p} > \overline{p}$ or $\hat{q} > \overline{p} - p$ is non-enforceable.

Then S^* implements the first best effort levels and it is renegotiation-proof.

Proposition 2: The existence of the institutional agent and the non-enforcement of private contracts contingent on trade are necessary conditions for the implementation of first-best effort levels.

The proofs of the propositions are provided in Appendix B. Given these results, one can also see the implications of this mechanism for development. Social contracts can solve the hold-up problem as long as there is sufficient production to be taxed and provided as subsidies. At the early stages of development, production is relatively low and the mechanism is infeasible. As a result, there is low effort exertion and low probability of trade. Once productivity increases sufficiently, then the mechanism becomes feasible and this implies the emergence of regulation, which is necessary to support high effort levels. In turn, the probability and the marginal value of trade increase, which encourages more investments to the know-how of the specialized good and promotes productivity growth further. Therefore, a feedback mechanism emerges between economic growth and enforcement institutions. The following section formalizes these arguments and shows why the relationship between the two may be non-monotonic.

2.4 Enforcement Institutions and Development

This section links the process of economic development with contract law. Formally, the model follows closely the notation and assumptions of the previous section. The economy lasts for T periods and consists of two groups of agents, each one of which is represented by a continuum of measure one. The group i are buyers (farmers) of the specialized good and group j are sellers (plough-makers). Each seller j can produce a specific variety of an intermediary good g, which a specific buyer i can use to generate additional production of the numeraire commodity (good a). In each period, a random matching process matches one agent from group i with one from group j and together they form an exclusive partnership²⁴. Apart from their specialization in a specific variety of g, the partnerships are identical and thus we use i and j as the notation for the representative buyer-seller party.

The structure of the production for the specialized good is the same as in section 2.3, but the gains from trade are allowed to vary over time by a scaling factor Γ_t^{25} . Γ_t is a positive coefficient, common for the whole economy, which can be interpreted as the productivity on agents' effort levels for good g. Agents can save resources (in terms of the numeraire commodity) and invest in an aggregate, non-depreciable amount of capital Z_g which increases the value of trade Γ_t . It is assumed that Γ_t is a concave function of Z_g .

In addition to the specialized good g, both agents have access to the production

 $^{^{24}}$ This means that the buyer has the know-how of using only a specific variety of the good g, which is produced by only a specific seller and the appropriate variety changes randomly every year.

²⁵So, good g yields $\Gamma_t v$ units of good a to the appropriate buyer, its cost takes values $\Gamma_t k_H$ or $\Gamma_t k_L$ and the cost of effort by an agent ξ for g in period t is $\Gamma_t c_{\xi gt}$. Also, conditions (1) and (2) hold.

technology of the numeraire commodity a, which can be thought of as a non-specialized or "autarchic" good (both terms are used interchangeably). The output of this good in each period is a linear non-stochastic function of the economy-wide productivity variable A_t and of the effort level an agent exerts for its production: $y_{\xi t} = A_t e_{\xi a t}$, where $y_{\xi t}$ is the production of good a in period t by an agent of type ξ and $e_{\xi a t}$ is the effort level exerted by her²⁶. Unlike the effort levels for the production of good g, we assume that the effort level for the production of good a is continuous and the cost of effort $c_{\xi a t}$ is a convex function of $e_{\xi a t}$ with $\frac{\partial c_{\xi a t}}{\partial e_{\xi a t}} > 0$, $\frac{\partial^2 c_{\xi a t}}{(\partial e_{\xi a t})^2} > 0$. The total cost of effort for an agent is the summation of the two efforts: $c_{\xi t} = c_{\xi a t} + c_{\xi g t}$.

The productivity variables A_t and Γ_t are common for all agents. Furthermore, their values depend on two types of cumulative, non-depreciable capital, Z_a and Z_g respectively. These capital levels can be interpreted as the technological know-how of the economy for the production of the non-specialized and the specialized good respectively²⁷. Agents can choose to save amounts of the non-specialized good and invest into the two forms of capital. However, due to the infinitesimal size each agent and the fact that productivities are common for all, private savings are zero. Hence, private savings are ignored in the analysis and consider public taxation and investment, conducted by a governor.

Let z_{at} , z_{gt} be the aggregate investment to the production process a and g respectively in period t. It is also assumed that $A_t(Z_a)$ and $\Gamma_t(Z_g)$ are concave functions of the respective capital stocks, satisfying the following conditions: A(0) > 0, $\Gamma(0) >$ 0, Inada condition: $A'(0) = \Gamma'(0) = \infty$. Assuming that the economy starts with zero capital stocks ($Z_{a0} = Z_{g0} = 0$), the first two conditions guarantee that some production is attainable even with zero capital stocks while the last two conditions guarantee that

 $^{^{26}}i$ has an increased production of good *a* if she buys the appropriate variety of the specialized good. This is additional to y_{iat} .

²⁷Alternatively, one could allow the capital levels to also depend on effort and think of the relative cost in terms of foregone production, but the mathematical formulation of the two problems is identical.

at least some investment in productivity is socially beneficial.

Agents are risk-neutral. The state-dependent utility is the summation of production of a, gains from trade, taxation and cost of effort:

$$u_{i\theta t} = y_{it} + I_g(\Gamma_t v - p) - \tau_{igt} - (c_{iat} + c_{igt})$$

$$u_{j\theta t} = y_{jt} + I_g(p - k_\theta) - \tau_{jgt} - (c_{jat} + c_{jgt})$$
$$U_{\xi} = \sum_{t=0}^T \delta^t \left(\sum_{\theta} f_{\theta}(e_{\xi gt}, e_{\zeta gt}) u_{\xi \theta t} \right)$$

In terms of the timing of events, every period is split into sub-periods which roughly follow the order of events of section 2.3. In the beginning of period zero the agents of the economy propose and vote social contracts, which establish the main institutions of government and enforcement in the economy. Thereafter, in every period t, agents decide how much effort to exert on autarchic and specialized production, the state of nature is determined, and agents decide whether to trade or not and sign private contracts. Once production (and potentially trade) has taken place, the institutional agent imposes taxation (conditional on trade) and decides whether to enforce certain private agreements. Finally, consumption takes place. Figure 5 presents the sub-stages of the game and the timing of events for every period after period zero.

The social contract S is similar to section 2.3, with three main differences: (i) Given that it is proposed in period 0, it is an exhaustive plan of all future dates. (ii) It includes investment plans z_a , z_g for increasing respective productivities A and Γ , so S takes now the form: $S(\Sigma, \Phi(Q), \tau, z_a, z_g)$. (iii) Σ can contain multiple agents. In this context, an institutional agent can be thought of as a governor and a judge at the same time (there is no separation of powers).



Figure 5: Timing of Events

As mentioned above, in the beginning of the game any agent of the economy can propose a social contract. A proposal becomes a valid social contract if all agents vote for it (unanimity requirement), even if there is only one proposal made²⁸. If no proposal achieves the unanimity requirement, then there is no government and enforcement institutions in the economy, and agents can utilize only the autarchic production technology. In this case, the game proceeds with effort exertion, production and consumption under autarchy in each period²⁹.

Also notice that the unanimity requirement for the selection of the social contract acts as a participation constraint. If the enforcement of a proposed social contract gives

 $^{^{28}}$ This means that agents have the option of not voting at all.

²⁹The state of nature is inconsequential for autarchic production and therefore it is omitted from the analysis of these sub-games

lower utility to an agent than the utility of autarchy, then the agent can block the social contract by not voting for it. Therefore, there can be no equilibrium of the game where the final expected utility for a subset of agents is below their autarchic continuation utility. For the rest of the section, when we refer to participation constraints, we imply the autarchic utility level that each agent receives in each period.

On the other hand, any proposal that generates a Pareto undominated allocation and satisfies the participation constraints of all agents can be an equilibrium of the game. This is again a result of the unanimity requirement, since it gives veto power to players³⁰. In the analysis of the following sub-sections only one of the equilibria of the game is analyzed, the equilibrium where the proposed social contract is designed so as to maximize the summation of the utilities of all non-institutional agents, given that it satisfies the participation constraint of the institutional agents³¹.

If a certain social contract is voted in the beginning period zero, the selected institutional agents do not exert effort in the production process. In other words, it is assumed that the activities of the institutional agent and production are mutually exclusive³². The rest of the agents decide on how much effort to exert in producing the generic good or investing effort for the specialized good and whether to trade or not.

Before proceeding to the analysis of the problem, it is worth mentioning that institutional agents play a dual role in this model. They are both governors and judges at the same time. They are governors because they collect taxes from the rest of the

 $^{^{30}}$ The analysis of this result is available from the author upon request.

³¹We effectively compute the social contract which arises from a Nash-bargaining procedure which allocates an equal bargaining coefficient on both types. The general set of equilibria, with unrestricted coefficients can also be derived, however, we simplify the analysis by examining only the limiting case of equal bargaining power.

³²One consequence of this assumption is that occupation in institutions generates a cost in terms of foregone production and hence a trade-off between the functionality of institutions and productive capacity. It also implies a simple solution to this trade-off: the minimization of the set of institutional agents. Relaxing this assumption can generate a more interesting trade-off and richer model predictions for institutions but it would lead us astray from the main topic of the analysis. We leave this aspect of the problem for future work.

agents and allocate them to public spending, i.e. investments to the two types of productivities. They are also judges because the enforce private agreements. This dual role seems to better fit the role of monarchs in pre-industrial economies. The potential reasons behind the separation of powers in post-industrial economies are not examined. The main focus are the questions of when do restrictions on enforcement arise and how they affect the development path of the economy.

2.4.1 Agents' Maximization Problem and Best-Response Functions

An agent of type ξ , who is occupied with production activities, chooses effort levels $e_{\xi gt}$ and $e_{\xi at}$ in each period t in order to maximize his utility, given an accepted social contract S (and the implied values of $\tau_{\xi gt}$, A_t , Γ_t). Each pair (i, j) also exchange g on the "fair" price $p^* = \Gamma_t((1 - \beta_i)v + \beta_i k_L)$, if its cost is low or if the contract law does not allow (side-)payments which violate incentive compatibility. Otherwise, they exchange good g for the price: p+q. Assume the former case³³. In this case agent ξ solves:

$$\max_{\{e_{\xi at}, e_{\xi gt}\}} A(Z_{at-1})e_{\xi at} + f(e_{igt}, e_{jgt}) \left(\Gamma(Z_{gt-1})\beta_{\xi}(v - k_L) - \tau_{\xi 1t}\right) + \left(1 - f(e_{igt}, e_{jgt}) \left(-\tau_{\xi 0t}\right)\right) - \Gamma(Z_{gt-1})c_{igt} - c_{iat}$$
(4)

The solution to problem (4) is given by:

$$e_{\xi at} : A(Z_{at-1}) = \frac{\partial c_{\xi at}}{\partial e_{\xi at}}$$
(5)

³³We will shortly show a similar condition to condition (iii) of Proposition 1, which ensures incentive compatibility and the "fair" price.

$$\begin{cases}
e_{\xi g t} = \overline{e} & \text{if } \tau_{\xi 0 t} - \tau_{\xi 1 t} \geqslant -\frac{\Gamma(Z_{g t - 1}) \left[\left(f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta}) \right) \beta_{\xi}(v - k_L) - (\overline{c} - \underline{c}) \right]}{f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})} \\
e_{\xi g t} = \underline{e} & \text{otherwise}
\end{cases} \tag{6}$$

Since the value of of $e_{\xi gt}$ also depends on the choice of effort of the trade-partner ζ , equations (5) and (6) give the best-response function of agent ξ .

As far as institutional agents are concerned, they do not directly engage in productive activities. Their utility is a function of the total taxes they collect minus the resources they invest in Z_a , Z_g . Though it is not explicitly modeled in the social contract, one can ensure the compliance of institutional agents by including certain rewards and punishments, conditional on its execution or not.

Denote by r_{σ}^+ the reward of an institutional agent for executing the social contract and by r_{σ}^- her punishment if she deviates. Her reward is equal to the aggregate taxes she collects minus the aggregate investments in physical capitals:

$$r_{\sigma}^{+} = f(e_i, e_j)(m_i \tau_{\sigma i1t} + m_j \tau_{\sigma j1t}) + (1 - f(e_i, e_j))(m_i \tau_{\sigma i0t} + m_j \tau_{\sigma j0t}) - z_{\sigma at} - z_{\sigma gt}$$

In the expression above, m_i , m_j is the total mass of agents of type *i* and *j* respectively, who are occupied in productive activities (non-institutional agents) and σ is the institutional agent, who is asked to execute the taxation plan τ_{σ} and investment plans $z_{\sigma a}$, $z_{\sigma g}$. On the other hand, if the institutional agent executes different plans in period t, say $\{\tau'_{\sigma\xi\theta t}, z'_{\sigma at}, z'_{\sigma gt}\}$, for $\xi \in \{i, j\}$, for $\theta \in \{0, 1\}$, then her utility is the aggregate units of the autarchic goods she accumulates according to her plan minus her punishment. By adding and subtracting r_{σ}^+ and by using the equation above, we get:

$$f(e_i, e_j) \left(m_i (\tau'_{\sigma i1t} - \tau_{\sigma i1t}) + m_j (\tau'_{\sigma j1t} - \tau_{\sigma j1t}) \right) + (1 - f(e_i, e_j)) \left(m_i (\tau'_{\sigma i0t} - \tau_{\sigma i0t}) \right)$$

$$+m_{j}(\tau_{\sigma j0t}'-\tau_{\sigma j0t})) - (z_{\sigma at}'-z_{\sigma at}) - (z_{\sigma gt}'-z_{\sigma gt}) + r_{\sigma}^{+} - r_{\sigma}^{-}.$$

This states that the utility of the institutional agent is the utility she would receive by executing the policy plans of the social contract, plus the extra resources of the new taxation, minus the additional expenses by the new investment plans minus the penalty defined by the social contract for deviating from the original agreement. It is clear that as long as the punishment for deviation is lower than the net additional resources, the institutional agent will choose to deviate.

In fact, the optimal choice of the institutional agent is to tax all production of the non-specialized good so as to maximize the net resources from deviation. Such a behavior can be prevented, of course, by setting the penalty of deviation sufficiently high. Lemma 2 in Appendix B provides the necessary rewards and punishments for an institutional agent, so that her participation constraint is satisfied, while maintaining her incentives for executing the social contract. For the rest of the analysis we suppress this problem from the analysis.

2.4.2 The Optimal Design of Contract Law

Lemma 1 characterizes the optimal set of institutional agents. Proposition 3 characterizes the optimal design of contract law and follows from Proposition 1. Both proofs are provided in Appendix B.

Lemma 1: The optimal social contract S^* determines that the set Σ is of measure zero. That is, the total number of institutional agents is infinitesimal compared to the aggregate population.

The intuition of Lemma 1 is simple. Since productive and government activities are

mutually exclusive and there is no limit to the span of control in governance, having a strictly positive measure of institutional agents is a social waste of resources: it reduces the productive capacity of the economy without generating any additional benefit. Therefore, the minimization of Σ is the optimal option. In other words, the optimal social contract defines that the number of institutional agents is finite, so that some institutional agents exist. However, the exact number is indeterminate, as they do not impact the aggregate economy due to the continuum-of-agents assumption. Without loss of generality, henceforth it is assumed that there is only one institutional agent in equilibrium.

Proposition 3: Let $\overline{p} = \Gamma(Z_{gt-1}) \left(k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(e_i, \overline{e}) - \overline{f}(e_i, \underline{e})} \right)$. Incentive compatibility requires that any private contract $\pi(0, \hat{p}, I_g)$ or $\pi(\hat{q}, p, I_g)$, with $\hat{p} > \overline{p}$ or $\hat{q} > \overline{p} - p$ is non-enforceable.

Proof: See Appendix B

Proposition 3 gives the necessary regulation required to support an incentive compatible subsidization scheme, which parallels that of Proposition 1. Notice that \overline{p} , which stands for the maximum transfer allowed between the two agents when trade takes place, depends on the values of Γ_t , the productivity of the specialized good, and the induced effort level e_i . These are endogenous variables which change over time and, therefore, regulation is dynamic. While the impact of Γ_t has a positive effect on \overline{p} (meaning that it relaxes the incentive compatibility condition), the evolution of e_i has a negative impact on \overline{p} , and the two effects generate a non-monotonic relationship between the optimal contract law and development. This effect is analyzed in more detail in subsections 2.4.4 and 2.4.5.

2.4.3 Unconstrained optimal taxation and investment plans

A consequence of the result stated in the Lemma 1 is that the utility of the institutional agent is infinitesimally small compared to the utility of the agents employed in productive activities and hence it can be ignored in the determination of the optimal social contract. Provided the optimal values for Σ and $\Phi(Q)$ (by Lemma 1 and Proposition 3), the rest of the variables included in the social contract are given as the solution to the problem which maximizes the expected utility of both types of agents with respect to taxation and investment plans and subject to the incentive compatibility, government budget and feasibility constraints:

$$\max \int \left(\sum_{t=0}^{T} \delta^{t} \left(\sum_{\theta} f_{\theta}(e_{\xi g t}, e_{\zeta g t}) u_{it}\right)\right) di + \int \left(\sum_{t=0}^{T} \delta^{t} \left(\sum_{\theta} f_{\theta}(e_{\xi g t}, e_{\zeta g t}) u_{jt}\right)\right) dj \Leftrightarrow$$

$$\max \quad \frac{1}{2} \sum_{t=0}^{T} \left[\delta^{t} \left[A(Z_{at-1})e_{iat}^{*} + f(e_{igt}, e_{jgt})\left(\Gamma(Z_{gt-1})\beta_{i}(v - k_{L}) - \tau_{i1t}\right) + \left(1 - f(e_{igt}, e_{jgt})\right)\left(-\tau_{i0t}\right) - c_{iat} - c_{igt}\right]\right]$$

$$+ \frac{1}{2} \sum_{t=0}^{T} \left[\delta^{t} \left[A(Z_{at-1})e_{jat}^{*} + f(e_{igt}, e_{jgt})\left(\Gamma(Z_{gt-1})\beta_{j}(v - k_{L}) - \tau_{j1t}\right) + \left(1 - f(e_{igt}, e_{jgt})\right)\left(-\tau_{j0t}\right) - c_{jat} - c_{jgt}\right]\right] \quad (7)$$

With respect to: $\{\tau, z_a, z_g\}$, and subject to:

Best Response Function of type i (8)

Best Response Function of type j (9)

$$\tau_{j0t} - \tau_{j1t} \leqslant \Gamma(Z_{gt-1})k_H - (p+q)$$
 Incentive Compatibility Constraint for type j (10)

$$z_{at} + z_{gt} \leqslant f(e_{igt}, e_{jgt})(\tau_{i1t} + \tau_{j1t}) + (1 - f(e_{igt}, e_{jgt}))(\tau_{i0t} + \tau_{j0t})$$

Government Budget Constraint (11)

$$f(e_{iqt}, e_{jqt}) \left(\tau_{i1t} + \tau_{j1t}\right) + \left(1 - f(e_{iqt}, e_{jqt})\right) \left(\tau_{i0t} + \tau_{j0t}\right) \leqslant A(Z_{at-1})(e_{iat} + e_{jat}) - f(e_{iqt}, e_{jqt})k_L$$

Feasibility Constraint (12)

Due to the risk neutrality of the utility functions of agents, the concavity of productivity functions $A(Z_a)$, $\Gamma(Z_g)$ and the Inada condition, the optimal taxation and investment plans take the form of a stopping-time problem: at the beginning of the economy, when the marginal productivity of investment is higher than the marginal value of consumption, the institutional agent taxes all income from the other agents to fund investments in productivity. At some threshold value of productivity, taxation drops and no further investments are made. Non-taxable production is consumed thereafter.

However, trade between agents and inducing high effort levels for one of the types (or both) require specific threshold level of production of the non-specialized good, which are endogenously determined by investment plan z_a . Specifically, trade can take place between *i* and *j* only if the non-specialized production of agent *i* is sufficient to cover the cost of production in the low-cost state:

$$A(Z_{at-1})e_{iat} \geqslant k_L \tag{13}$$

Inducing high effort for one type of agent requires that the aggregate production of the

non-specialized good minus the expected production costs of the specialized good are greater than the expected reduction in taxation:

$$A(Z_{at-1})\left(e_{iat} + e_{jat}\right) - f(\overline{e}, e_{\zeta})\Gamma(Z_{gt-1})k_L \ge f(\overline{e}, e_{\zeta})\left(-\frac{\Gamma(Z_{gt-1})\left[\beta^{min}\left(f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})\left(v - k_L\right) - (\overline{c} - \underline{c})\right]\right)}{f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})}\right)$$
(14)

Similarly, incentivizing both agents to exert high effort level requires:

$$A(Z_{at-1})\left(e_{iat} + e_{jat}\right) - f(\overline{e}, \overline{e})\Gamma(Z_{gt-1})k_L \ge f(\overline{e}, \overline{e})\left(-\frac{\Gamma(Z_{gt-1})\left[\beta^{max}\left(f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})\right)\left(v - k_L\right) - (\overline{c} - \underline{c})\right]}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})}\right) + f(\underline{e}, \overline{e})\left(-\frac{\Gamma(Z_{gt-1})\left[\beta^{min}\left(f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})\right)\left(v - k_L\right) - (\overline{c} - \underline{c})\right]}{f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})}\right)$$
(15)

In inequalities (14),(15), $\beta^{min} = \min\{\beta_i, \beta_j\}$ and $\beta^{max} = \max\{\beta_i, \beta_j\}$. It is easy to solve the problem when these constraints are not binding, which is the case when the initial value of productivity for the non-specialized good is sufficiently high. In this case the optimal social plan induces both agents to exert high effort level by reducing taxation in the low-cost state and increasing it in the high-cost state (which are truth-fully revealed through trade). It also increases capital stocks in period zero up to the point where the future marginal increase in production is equal to the marginal cost of production for both goods. Formally, this solution is defined by the equations below:

$$\tau_{\xi 0t} - \tau_{\xi 1t} = -\frac{\Gamma(Z_{gt-1}) \left[\beta_{\xi} \left(f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})\right) \left(v - k_{L}\right) - (\overline{c} - \underline{c})\right]}{f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})} \quad \text{for } \xi \in \{i, j\}$$

$$z_{a0}$$
 such that: $\sum_{t=1}^{T} \delta^t \frac{\partial A(Z_{a0})}{\partial Z_a} \left(e_{ia}^*(Z_{a0}) + e_{ia}^*(Z_{a0}) \right) = 1$, where $Z_{a0} = z_{a0}$

$$z_{g0}$$
 such that: $\sum_{t=1}^{T} \delta^t \frac{\partial \Gamma(Z_{g0})}{\partial Z_g} \left(f(\overline{e}, \overline{e})(v - k_L) - 2\overline{c} \right) = 1$, where $Z_{g0} = z_{g0}$

The above equations characterize the problem only for very high values of A(0). If A(0) is such that any of the feasibility constraints (13)-(15) is not satisfied at time t = 0, then the optimal investment plan z_a depends on the marginal benefits and costs of prolonging investment in future periods. The benefits come from the increased future production of the autarchic good and satisfying the minimum production level required for trade to take place or for providing subsidies³⁴. The costs come from the value of foregone consumption in the period of investment.

Due to the fact that equations (13) to (15) define threshold values, which are determined endogenously by past investment and taxation, there are multiple different cases to consider regarding optimal investment and taxation plans. Figures 11 to 13 in Appendix B provide all possible cases, but in the following subsection we examine only the most interesting of these cases. Before doing so, we define some useful threshold values, which will be used in the analysis thereafter. The following threshold values $\underline{K}(Z_{gt-1})$ and $\overline{K}(Z_{gt-1})$ represent the aggregate production of the autarchic good required for the subsidization of one or two types of agents respectively form equations (14) and (15).

³⁴Which generate extra value by increasing the volume of trade of the specialized good.
Let
$$\underline{K}(Z_{gt-1}) = f(\overline{e}, \underline{e}) \left(-\frac{\Gamma(Z_{gt-1}) \left[\beta^{min}(f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e}))(v-k_L) - (\overline{c}-\underline{c}) \right]}{f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})} \right) + f(\overline{e}, \underline{e}) k_L$$

Let
$$\overline{K}(Z_{gt-1}) = f(\overline{e}, \underline{e}) \left(-\frac{\Gamma(Z_{gt-1}) \left[\beta^{min} (f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e}))(v - k_L) - (\overline{c} - \underline{c}) \right]}{f(\overline{e}, \underline{e}) - f(\underline{e}, \underline{e})} \right)$$

 $+ f(\overline{e}, \overline{e}) \left(-\frac{\Gamma(Z_{gt-1}) \left[\beta^{max} (f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e}))(v - k_L) - (\overline{c} - \underline{c}) \right]}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) + f(\overline{e}, \underline{e}) k_L$

As before, $\beta^{min} = \min\{\beta_i, \beta_j\}$, $\beta^{max} = \max\{\beta_i, \beta_j\}$. Define by $\widetilde{Z}_a, \underline{Z}_a, \overline{Z}_a$ the minimum require physical capital of type a, such that the inequalities (13) to (15) hold respectively at time zero. Formally:

$$\widetilde{Z}_a$$
 : $A(\widetilde{Z}_a)e^*_{ia}(\widetilde{Z}_a) = k_L$

$$\underline{Z}_a : A(\underline{Z}_a) \left(e_{ia}^*(\underline{Z}_a) + e_{ja}^*(\underline{Z}_a) \right) = \underline{K}(0)$$

$$\overline{Z}_a : A(\overline{Z}_a) \left(e_{ia}^*(\overline{Z}_a) + e_{ja}^*(\overline{Z}_a) \right) = \overline{K}(0)$$

If the solutions to the above equations do not exist (and since effort exertion is an increasing function of productivity), there are two possible cases to consider for each equation. Either the required value of Z_a is so low that it violates the non-negativity constraint for the capital stock or the limit $\lim_{Z_a\to\infty} A(Z_a)(e_{iat} + e_{jat})$ is lower than the

required threshold.

The first case is the case where A(0) is sufficiently high so that feasibility constraints for trade and effort exertion are not binding and, therefore, the analysis is the same as of the unconstrained problem provided above. In the second case, the respective feasibility constraint is always binding, which implies that either trade will never take place, irrespectively of the capital stock of the economy, or inducing high effort for at least one or both agents is not attainable. However, the second class of results can also arise in the case where the required capital stock for trade (or inducing high effort) is attainable, but the cost of foregone consumption is too high and such an investment plan is not optimal. Therefore, in terms of economic consequences, nothing is lost by restricting attention to the cases where the critical values \tilde{Z}_a , \underline{Z}_a and \overline{Z}_a exist and are non-negative.

The importance of these thresholds is that one can examine the optimal investment and taxation plans when one of these constraints is more difficult to satisfy than the others. Notice that because $\underline{K}(Z_{gt})$ is always greater than $\overline{K}(Z_{gt})$, then $\overline{Z}_a > \underline{Z}_a$, which means that feasibility constraint (15) is always more difficult to satisfy than (14). Furthermore, for each one of these critical values and given some plan z_a , which utilizes all non-specialized production in each period for investments in productivity and subsidizing effort levels, there exists a point t in time such that the accumulated capital stock Z_a reaches the respective critical value. Define \tilde{t} , \underline{t} and \overline{t} as the respective points in time and assume that each one of them is less than T. Under these assumption and results the following case of interest is examined.

2.4.4 An Economy with non-Monotonic Regulation

We now analyze the case where the three feasibility constraints (13)-(15) are binding at time zero. We derive the required conditions on parameter values so that the economy

passes through the different stages of economic development (no-trade, trade with low effort levels, trade with high effort levels). We also find the conditions for regulation to change non-monotonically over these stages. Enforcement institutions are required at the point where productivity in good a is high enough to support trade, but contract law becomes relevant only when subsidization of trade for at least one type is feasible. In turn, the increase in the trade of good g, generated by regulation, spurs further economic growth by increasing its marginal value and this leads to further investments in the productivity of this good. Therefore, we claim that the interaction between markets and institutions goes in both directions³⁵.

As before, there are different combinations of assumptions that can give similar qualitative results in our model³⁶. We examine the conditions that present the most detailed interpretation of economic development and enforcement institutions evolution. The main requirement is that the continuation value of investment in productivity of goods a and g is greater than the cost of foregone consumption in all three threshold points $(\tilde{t}, \underline{t}, \bar{t})$. This generates optimal investment paths for five different stages of development: $[0, \tilde{t}], [\tilde{t}, \underline{t}], [\underline{t}, \overline{t}], [\bar{t}, t^*]$ and $[t^*, T]$. These are described below:

Stage 1: [0, t]

Assume that $\widetilde{Z}_a < \underline{Z}_a < \overline{Z}_a$. This implies that the most difficult to satisfy constraint is the subsidization of both types of agents. Define \widetilde{t} as the time period which satisfies the following condition and growth path: $\sum_{t=0}^{\widetilde{t}} z_{at} = \widetilde{Z}$, $z_{at} = A(Z_{t-1})(e_{iat-1}^* + e_{jat-1}^*)$ and assume that $\frac{\partial A(\widetilde{Z}_a)}{\partial Z_a}[e_{ia}^*(\widetilde{Z}_a) + e_{ja}^*(\widetilde{Z}_a)]\left(\sum_{t=\widetilde{t}}^T \delta^{t-\widetilde{t}}\right) > 1$. The last assumption means that the net marginal benefit of investment z_a at the critical value \widetilde{Z}_a is positive, so that investment in the productivity of good a must continue beyond this threshold. Therefore, z_{at} follows the

³⁵The paper by Dhillon and Rigolini is also important in that respect, as the determination of prices and the use of formal and informal channels of enforcement arise endogenously. The main focus is on the impact of the design of contract law.

³⁶See Appendix B.

state-independent growth path $z_{at} = A(Z_{t-1})(e_{iat-1}^* + e_{jat-1}^*)$ up to time \tilde{t} . Call this investment plan $IP_{\tilde{t}}^*$. During stage 1, productivity for good a is low and the autarchic production of type i is not sufficient for covering the cost of production of good g in any state. Trade does not take place, but production of a is taxed away and invested in increasing Z_a .

Stage 2: $[\tilde{t}, \underline{t}]$ Define \underline{t} such that: $\widetilde{Z}_a + \sum_{t=\tilde{t}+1}^{\underline{t}} z_{at} = \underline{Z}_a$

 z_{at} , z_{gt} are defined by the following investment paths:

$$z_{gt} = A(Z_{at-1})(e_{iat-1}^* + e_{jat-1}^*) - \Gamma(Z_{gt})f(\underline{e},\underline{e})k_L , \ Z_{gt} = Z_{gt-1} + z_{gt} \quad \text{and} \quad z_{at} = 0$$

if
$$\frac{\partial A(Z_{at-1})}{\partial Z_a} \left[e_{ia}^*(Z_{at-1}) + e_{ja}^*(Z_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) < \frac{\partial \Gamma(Z_{gt-1})}{\partial Z_g} \left[f(\underline{e},\underline{e})(v-k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right)$$

otherwise, $z_{at} > 0$ and z_{at} , z_{gt} are such the marginal returns are equalized:

$$\frac{\partial A(Z_{at-1})}{\partial Z_a} \left[e_{ia}^*(Z_{at-1}) + e_{ja}^*(Z_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) = \frac{\partial \Gamma(Z_{gt-1})}{\partial Z_g} \left[f(\underline{e}, \underline{e})(v - k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right) ,$$

$$z_{at} + z_{gt} = A(Z_{at-1})(e_{iat-1}^* + e_{jat-1}^*) - \Gamma(Z_{gt-1})f(\underline{e}, \underline{e})k_L$$

Call the optimal investment path above $IP_{\underline{t}}^*$. At time \tilde{t} there is enough production of good a by agents of type i, so that trade arises in the economy, but, since $\underline{Z}_a > \widetilde{Z}_a$, there is not enough production for subsidizing effort exertion. This constraint holds until the physical capital reaches the critical value \underline{Z}_a , which happens in period \underline{t} . Also at the same time (\tilde{t}) enforcement institutions emerge but no restrictions on enforceability are required.

On the other hand, after \tilde{t} , due to the Inada conditions, investment takes place in productivity Γ and the optimal investment plan must divide available production between z_a and z_b . The optimal rule is to invest production to the productivity of good g only, until the marginal returns of production are equalized between goods a and g. Thereafter, investment is divided between the two goods so as to maintain the equality of marginal returns. This is represented by the first (the "if") and second (the "otherwise") part respectively of IP_t^* .

Stage 3: $[\underline{t}, \overline{t}]$

The main logic and intuition proceeds in the same way as in stage 2. Define \overline{t} such that: $\underline{Z}_a + \sum_{t=t+1}^{\overline{t}} z_{at} = \overline{Z}_a$. Furthermore assume that:

$$\begin{split} \frac{\partial A(\underline{Z}_{a})}{\partial Z_{a}} \left[e_{ia}^{*}(\underline{Z}_{a}) + e_{ja}^{*}(\underline{Z}_{a}) \right] \left(\sum_{s=\underline{t}}^{T} \delta^{s-\underline{t}} \right) > 1 \\ \frac{\partial \Gamma(\underline{Z}_{gt})}{\partial Z_{g}} \left[f(\underline{e},\underline{e})(v-k_{L}) - 2\underline{c} \right] \left(\sum_{s=\underline{t}}^{T} \delta^{s-\underline{t}} \right) > 1 \\ \sum_{t=\underline{t}}^{T} \delta^{t} \left[A(Z_{at-1}) \left(e_{ia}^{*}(Z_{at-1}) + e_{ja}^{*}(Z_{at-1}) \right) + \Gamma(Z_{gt-1}) \left(f(\overline{e},\underline{e})v - \overline{c} - \underline{c} \right) \right] > \\ \sum_{t=\underline{t}}^{T} \delta^{t} \left[A(\hat{Z}_{at-1}) \left(\hat{e}_{ia}^{*}(\hat{Z}_{at-1}) + e_{ja}^{*}(\hat{Z}_{at-1}) \right) + \Gamma(\hat{Z}_{gt-1}) \left(f(\underline{e},\underline{e})v - \overline{c} - \underline{c} \right) \right] > \end{split}$$

The first two conditions ensure that it is optimal to invest both in the productivity of good a and g beyond time period \underline{t} , at which point there is enough production of good a so that one of the two agents can be subsidized to exert high effort. Notice that the marginal benefit of one of the agents exerting high effort is the same irrespectively of who is subsidized, due to the symmetry of the probability function in terms of effort levels. However, the subsidy required for inducing high effort is lower for the agent with the lower bargaining power. This is the reason why we have defined \underline{Z}_a to depend on β^{min} . Let us assume that type ξ is the one with the low bargaining power.

At time \underline{t} , it is optimal for type ξ to receive subsidy conditional on trade, if the overall social surplus from the increased probability of trade and the slower increase in

productivity is greater than the social surplus under the lower probability of trade but the faster increase in productivity. This trade-off between the probability of trade and productivity growth comes from the fact that both must be funded from the taxation imposed on agents and they both face ultimately the same feasibility constraint. The third condition ensures that paying out subsidies to one type of agents Pareto dominates not paying subsidies at all. The investment path with partial subsidies $\{z_a, z_g\}$ and without any subsidies $\{\hat{z}_a, \hat{z}_g\}$, are given below:

$$\begin{aligned} z_{gt} &= A(Z_{at-1})(e^*_{iat-1} + e^*_{jat-1}) - \underline{K}(Z_{gt}) \quad , \quad Z_{gt} = Z_{gt-1} + z_{gt}, \quad z_{at} = 0 \ , \\ \text{if} \quad \frac{\partial A(Z_{at-1})}{\partial Z_a} \left[e^*_{ia}(Z_{at-1}) + e^*_{ja}(Z_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) < \frac{\partial \Gamma(Z_{gt-1})}{\partial Z_g} \left[f(\underline{e}, \underline{e})(v - k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right) \end{aligned}$$

otherwise, $z_{at} > 0$ and z_{at} , z_{gt} are such the marginal returns are equalized: $\frac{\partial A(Z_{at-1})}{\partial Z_a} \left[e_{ia}^*(Z_{at-1}) + e_{ja}^*(Z_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) = \frac{\partial \Gamma(Z_{gt-1})}{\partial Z_g} \left[f(\overline{e}, \underline{e})(v - k_L) - \underline{c} - \overline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right),$ $z_{at} + z_{gt} = A(Z_{at-1})(e_{iat-1}^* + e_{jat-1}^*) - \underline{K}(Z_{gt-1})$

$$\hat{z}_{gt} = A(\hat{Z}_{at-1})(e^*_{iat-1} + e^*_{jat-1}) - \Gamma(\hat{Z}_{gt})f(\underline{e},\underline{e})k_L \quad , \quad \hat{Z}_{gt} = \hat{Z}_{gt-1} + \hat{z}_{gt}, \ \hat{z}_{at} = 0$$

$$\text{if} \quad \frac{\partial A(\hat{Z}_{at-1})}{\partial Z_a} \left[e^*_{ia}(\hat{Z}_{at-1}) + e^*_{ja}(\hat{Z}_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) < \frac{\partial \Gamma(\hat{Z}_{gt-1})}{\partial Z_g} \left[f(\underline{e},\underline{e})(v-k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right)$$

otherwise, $\hat{z}_{at} > 0$ and \hat{z}_{at} , \hat{z}_{gt} are such the marginal returns are equalized: $\frac{\partial A(\hat{z}_{at-1})}{\partial Z_a} \left[e_{ia}^*(\hat{Z}_{at-1}) + e_{ja}^*(\hat{Z}_{at-1}) \right] \left(\sum_{s=t}^T \delta^{s-t} \right) = \frac{\partial \Gamma(\hat{Z}_{gt-1})}{\partial Z_g} \left[f(\underline{e}, \underline{e})(v - k_L) - 2\underline{c} \right] \left(\sum_{s=t}^T \delta^{s-t} \right)$ $\hat{z}_{at} + \hat{z}_{gt} = A(\hat{Z}_{at-1})(e_{iat-1}^* + e_{jat-1}^*) - \Gamma(\hat{Z}_{gt-1})f(\underline{e}, \underline{e})k_L$

Therefore, the optimal investment plan for $[\underline{t}, \overline{t}]$ (call it $IP_{\overline{t}}^*$) is characterized by the three inequalities and the investment plan $\{z_a, z_g\}$ above³⁷.

³⁷See also Appendix B for the possible cases when some of the above inequalities are violated.

Stage 4: $[\overline{t}, t^*]$

Similarly to stage 3, define t^* such that:

$$\frac{\partial A(Z_a)}{\partial Z_a} \left[e_{iat}^*(Z_a) + e_{jat}^*(Z_a) \right] \left(\sum_{s=t^*}^T \delta^{s-t^*} \right) = \frac{\partial \Gamma(Z_{gt^*})}{\partial Z_g} \left[f(\overline{e}, \overline{e})(v - k_L) - 2\overline{c} \right] \left(\sum_{s=t^*}^T \delta^{s-t^*} \right) = 1$$

Also assume that:

$$\begin{aligned} \frac{\partial A(\overline{Z}_{a})}{\partial Z_{a}} \left[e_{ia}^{*}(\overline{Z}_{a}) + e_{ja}^{*}(\overline{Z}_{a}) \right] \left(\sum_{s=\overline{t}}^{T} \delta^{s-\overline{t}} \right) > 1 \\ \frac{\partial \Gamma(Z_{g\overline{t}})}{\partial Z_{g}} \left[f(\overline{e}, \underline{e})(v - k_{L}) - \overline{c} - \underline{c} \right] \left(\sum_{s=\overline{t}}^{T} \delta^{s-\overline{t}} \right) > 1 \\ \sum_{t=\overline{t}}^{T} \delta^{t} \left[A(\check{Z}_{at-1}) \left(e_{ia}^{*}(\check{Z}_{at-1}) + e_{ja}^{*}(\check{Z}_{at-1}) \right) + \Gamma(\check{Z}_{gt-1}) \left(f(\overline{e}, \overline{e})v - 2\overline{c} \right) \right] > \\ \sum_{t=\overline{t}}^{T} \delta^{t} \left[A(Z_{at-1}) \left(e_{ia}^{*}(Z_{at-1}) + e_{ja}^{*}(Z_{at-1}) \right) + \Gamma(Z_{gt-1}) \left(f(\overline{e}, \underline{e})v - \overline{c} - \underline{c} \right) \right] \end{aligned}$$

These conditions are equivalent to the conditions specified for periods $[\underline{t}, \overline{t}]$. The first two ensure that investment in productivity of good a and g are optimal after $t = \overline{t}$. At this point of time, there is adequate production of the autarchic good so that subsidizing the effort levels of both types is feasible. As before, subsidizing both agents is optimal if it generates greater overall utility than subsidizing only one agent, which, given the conditions specified thus far, is better than no subsidization for any type. The required assumption for this result is the third condition. Call the optimal investment plan until t^* as $IP_{t^*}^*$. $IP_{t^*}^*$ incorporates all the previous optimal investment plans up to \overline{t} plus the investment plan { $\check{z}_{at}, \check{z}_{gt}$ } thereafter. This is given below:

$$\check{z}_{gt} = A(\check{Z}_{at-1})(e^*_{iat-1} + e^*_{jat-1}) - \overline{K}(\check{Z}_{gt}), \ \check{Z}_{gt} = \check{Z}_{gt-1} + \check{z}_{gt}, \ \check{z}_{at} = 0$$

$$\text{if } \frac{\partial A(\check{Z}_{at-1})}{\partial Z_a} \left[e_{ia}^*(\check{Z}_{at-1}) + e_{ja}^*(\check{Z}_{at-1}) \right] \left(\sum_{s=\bar{t}}^T \delta^{s-\bar{t}} \right) < \frac{\partial \Gamma(\check{Z}_{gt-1})}{\partial Z_g} \left[f(\bar{e},\bar{e})(v-k_L) - 2\bar{c} \right] \left(\sum_{s=\bar{t}}^T \delta^{s-\bar{t}} \right)$$

otherwise, $\check{z}_{at} > 0$ and \check{z}_{at} , \check{z}_{gt} are such marginal returns are equalized:

$$\frac{\partial A(\check{Z}_{at-1})}{\partial Z_a} \left[e_{iat}^*(\check{Z}_{at-1}) + e_{jat}^*(\check{Z}_{at}) \right] \left(\sum_{s=\bar{t}}^T \delta^{s-\bar{t}} \right) = \frac{\partial \Gamma(\check{Z}_{gt-1})}{\partial Z_g} \left[f(\bar{e},\bar{e})(v-k_L) - 2\bar{c} \right] \left(\sum_{s=\bar{t}}^T \delta^{s-\bar{t}} \right),$$
$$\check{z}_{at} + \check{z}_{gt} = A(\check{Z}_{at-1})(e_{iat-1}^* + e_{jat-1}^*) - \overline{K}(\check{Z}_{gt-1})$$

The plan $\{z_{at}, z_{gt}\}$ follows the same growth path and is subject to the same feasibility constraints as the optimal plan for the periods $[\underline{t}, \overline{t}]$. Of course, by the third inequality above, it is suboptimal to $\{\check{z}_{at}, \check{z}_{gt}\}$, and is not part of the equilibrium path (See also Appendix B for the cases where the inequalities above do not hold).

Stage 5: $[t^*, T]$

After t^* investment falls to zero, since the marginal utility increase by additional investment in either types of physical capital is lower than the marginal cost of foregone consumption. In this case, agents receive subsidies whenever they trade, pay taxes whenever they do not trade and consume the remainder of the production. The evolution of regulation depends on the investment path for Z_g and the optimally induced effort levels. Recall that the maximum permitted transfers are given by $\overline{p} = \Gamma(Z_{gt}) \left(k_H + \beta_j (v - k_L) - \frac{\overline{c} - c}{f(e_i, \overline{e}) - f(e_i, \underline{e})}\right).$

Figures 6 and 7 give the graphical representation of the economy according to the conditions above. A first note is that they are drawn as if time is continuous. While the model is one of discrete time, the figures can be closely approximated if T takes sufficiently high values. Figure 6 depicts the equilibrium path of capital stocks Z_a and Z_g and Figure 7 depicts the equilibrium path of contract law, as represented by



Figure 6: Capital-Accumulation Paths

the maximum enforceable transfer \overline{p} . These graphs also present the different stages of economic development.

To recapitulate, during $[0, \tilde{t}]$ productivity Z_a is very low and trade is infeasible. There is no need for enforcement institutions or regulation and taxation is used in increasing Z_a . This stage reflects a rather primitive stage of economic organization.

In stage $[\tilde{t}+1, \underline{t}]$ trade becomes feasible and enforcement institutions are required to support it by making private agreements enforceable. However, providing tax-breaks to agents in order to incentivize high effort remains infeasible and hence regulation is still not required. Any agreement on exchanging specialized goods is enforceable. On the other hand, public spending can be used for increasing both types of capital stock,



Figure 7: Maximum enforceable transfers

but, since the marginal increase in productivity of g is much higher than the marginal productivity of good a, taxation flows only to the former. This continues until effective marginal returns are equalized, which depends on parameter values.

In periods $[\underline{t} + 1, \overline{t}]$, Z_a exceeds the threshold value \underline{Z}_a and partial (one-type) subsidization of trade takes place. At this point of development, limitations on the set of enforceable transfers are set in order to make tax-breaks effective in inducing high effort. Thus the probability of trade increases for each group. On average, trade and its marginal value increase as well, which induces greater investment in Z_g . This is shown in the diagram by the kink in the slope of Z_g at period \underline{t} . Higher investment in Z_g also means that the optimal stopping time for investment t^* is delayed. Moreover, as Γ_t increases, \overline{p} also increases, which means that regulation is initially severe but it is subsequently relaxed. This is an important stage of the development process, where one can see the interaction between contract law, specialization and productivity growth and could be loosely interpreted as a stage akin to merchantilism before industrialization³⁸.

The next stage, $[\bar{t}+1, t^*]$, is the stage where the economy develops fully its productive capacity. Investments in Z_a and Z_g reach their peak in t^* and tax-breaks are given to both types. The feedback effect between contract law and productivity is repeated. Furthermore, as Figure 7 depicts, if $f(\underline{e}, \overline{e}) - f(\underline{e}, \underline{e}) > f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})$, then contract law tightens again (\overline{p} decreases). This gives rise to a non-monotonic pattern of regulation over time, which may explain why it is difficult in empirical studies to find a robust effect between contractual institutions and growth.

Finally, the remaining periods of the economy, $[t^*+1, T]$, represent a fully developed economy. Capital stocks and trade have reached their optimum level and taxation is used solely for inducing high effort levels. The remainder of production is used for consumption purposes.

2.4.5 The non-monotonicity between enforcement costs and development

As mentioned in the introduction, one of the purposes of the chapter is to show that enforcement costs may not be monotonically related to economic growth and, therefore, the empirical specification by Acemoglu and Johnson (2005) may not be the appropriate one for testing the relationship between contract law and development. As was shown in the previous subsection, it is possible that the evolution of regulation is non-monotonic over the process of economic development. That is, there may be periods where regulation is relaxed and the contractual space expands, while other periods where the

 $^{^{38}}$ It is also noteworthy to mention that Landes (1998) provides evidence that, despite common beliefs, the medieval period experienced a rapid increase in the productivity of the agricultural sector. This is in line with the argument made here that the initial growth in productivity of the non-specialized sector precedes specialization and the period of severe regulation.

contractual space shrinks (as shown in Figure 7).

If this is the case, and if one makes the additional (and plausible) assumption, that contract enforcement costs are positively correlated with the set of enforceable agreements, then it follows that contract enforcement costs may not be positively correlated with growth. Of course, in this model, for theoretical reasons, it was implicitly assumed that contract enforcement costs are zero. This helped to contrast the results of this chapter, where the limits on contractual space arise endogenously, with the results of the previous literature, where, due to exogenous enforcement costs, the contractual space is incomplete. But one can modify the model to include strictly positive enforcement costs.

Consider the following modification of the model. Assume that in the economy there are two categories of ploughs. Those which are described by our model and which suffer from a hold-up problem in their production, and another category, which is unaffected by any hold-up considerations. However, the second variety of ploughs (call them the class-two ploughs), varies in terms of its value for the farmer, while it has a constant cost k_2 . Say that, for class-two, v is distributed uniformly between $[0, \bar{v}]$, with $\bar{v} > \max{\bar{p}_t}$. Furthermore, a plough of class-two may turn out to be defective after its sale, in which case the farmer can ask for a refund, if he proves his case in the court. This is necessary, if his output in terms of wheat can be verified by the court but not the seller (as is usually the case).

In this economy, as long as the class-two category is not too large a fraction of trade (so that the ex-post inefficiencies generated by banning some of these exchanges are not too large), it is still optimal to not allow some contracts from being written in equilibrium (for optimal incentives in the production of class-one ploughs). Moreover, the dynamic evolution of contract law is still represented by Figure 7 (under the relevant conditions) and the aggregate transactions for ploughs of class-two are positively correlated with \overline{p}_t . As a consequence, the number of cases brought to courts and aggregate enforcement costs are positively correlated with the design of contract law (\overline{p}_t) as well. Of course, the plausibility of the result depends on the credibility of the assumptions.

But still the assumption (the positive correlation between contract-enforcement costs and the set of enforceable agreements) can be justified in terms of intuition. This is because one expects that the greater the aggregate number of transactions in an economy, the greater the number of cases that end up in courts and hence the higher the aggregate enforcement costs (and the costs per case, due to the dis-economies of scale exhibited by bureaucracies). And the aggregate number of transactions is expected to be positively correlated with the size of the set of permissible transactions. Of course, these hypotheses, though plausible, remain to be tested empirically as well.

2.4.6 Discussion

As mentioned before, the institutional agent stands for a governor or a monarch, who is bound by the agreement she has made with the rest of the citizens in period zero. She agent plays a double role in this economy. The standard economic role implied by most models of public economics is the role of the government, which collects taxes and allocates public expenditures in economic activities where the market mechanism fails to provide efficient outcomes. In this model, investment in the production technology is a public good, due to the assumption of the continuum of agents and the assumption that productivity parameters are economy-wide.

The second role of the institutional agent is her role as an enforcement authority for private agreements. As was have shown, in this setup, it may not always be optimal for the enforcement authority to guarantee the conduct of trade, unlike the previous literature. Because agents are perfectly rational, they will never honor their part of agreement when they know that it will not be enforced on them. Hence making some private agreements non-enforceable is equivalent to effectively forbidding them. So one can interpret the endogenously determined set of non-enforceable private agreements as laws of banning some types of transactions and this can have a positive effect on social welfare.

Overall, the analysis of this subsection presents an economy which moves through all the stages of development. It starts from a point where production is limited to activities who do not suffer from hold-up problems, it moves to a period where trade begins and enforcement authorities are necessary and finally it ends up in a stage with limitations on enforceable agreements (regulation). Furthermore, the resources spent for incentivizing agents to exert high effort is proportional to the productivity parameter Γ_t and hence as the value of trade grows so do the optimal subsidies. This can be interpreted as an endogenous growth of enforcement institutions, at least in terms of the resources devoted to their cause. Moreover, the exertion of high effort increases the marginal value of trade and, as a consequence, it increases the threshold value for the capital stock Z_g . This implies that the overall investments made in the productivity of trade are higher and this generates a positive feedback loop from economic growth to contract law and back to economic growth. In other words, the model provides a formalization of the argument that growth and law are inexorably entangled and the development of one has consequences for the other.

2.5 Conclusion

The main purpose of the chapter is to relate the process of economic development with the emergence and evolution of enforcement institutions and contract law. We demonstrate the importance of restrictions on enforceable agreements for increasing social welfare and growth. In equilibrium, the relationship between economic growth and the intensity of regulation may be non-monotonic and this may explain why empirical studies do not find a positive correlation between enforcement institutions and development. Finally, the model derives the design of enforcement institutions and the centralization of authority as equilibria phenomena.

The model of this chapter can be extended in various directions in order to explain different phenomena related to the process of economic development, institutional design and regulation. Notice that in the models of section 2.3 and 2.4, there is no loss of efficiency. Despite the fact that regulation is necessary, all socially valuable transactions take place in equilibrium. So regulation does not impose any cost from an efficiency point view, which goes against standard economic theory and intuition. However, one can extend the model to incorporate multiple sources of trade among agents. In this case, regulation generates social value by underpinning the mechanism, which induces optimal incentives, but destroys social value by forbidding some welfare enhancing transactions. The trade-off between incentives and economic freedom is even more clear in the extended model. We believe that one can use this more elaborate model of the design of optimal regulation in order to explain the differences in regulatory regimes that we observe between economies.

Furthermore, the model can be modified in order to generate a theory of regulation cycles. In the current form of the model, the set of enforceable agreements changes whenever the number of agents, who need to be incentivized to exert high effort, changes. But in a model with a stream of new traded good being introduced at different stages of economic development, the introduction of a new good is associated with a tightening of regulatory restrictions, which are subsequently relaxed. This cycle is then repeated every time a new good is introduced. Thus, we can have a model of regulation cycles which are related to the process of innovation and technological advance.

Similarly, the model can be used to relate the role of enforcement institutions with

the degree of specialization in the economy and the organizational complexity by assuming that new production processes require the cooperation of a larger set of individuals who face a multi-agent version of the hold-up problem. This model is able to generate regulation cycles and relate the degree of specialization to the existence of appropriate institutional restrictions at the same time.

Another assumption that can be relaxed is the assumption of the mutual exclusivity between productive and governmental activities. By allowing institutional agents to have a certain span of control, which they have to allocate between their institutional and their economic role, one can provide a more rigorous analysis of how and under what conditions centralization of power emerges. This can provide us with a more complete theory of institutional size and how it may vary along the development path of an economy.

For all the above reasons, we believe that this model is an interesting contribution to the literature, which highlights the importance of enforcement institutions to the process of development.

3 Chapter Three: Separation of Powers, Political Competition and Efficient Provision of Public Goods

3.1 Introduction

Voting games on public goods usually have two undesirable features: (i) non-existence of equilibrium when policy platforms are multi-dimensional, (ii) inefficiently low provision of public goods when the equilibrium exists (Jackson and Moselle, 2002). Citizencandidate models (Osborne-Slivinski, 1996, Besley-Coate, 1997) solve the problems of existence and efficiency of Nash equilibria in voting games, but generate a different concern. The elected politician is free to choose any allocation of resources he prefers and hence, in any equilibrium of these games, the social position of an individual (citizen or executive) matters for his payoff. Therefore, in a sense, the equilibria of these games generate excessive "rents" for the elected politician, which can be captured by the difference between their payoffs as citizens and as elected politicians.

The purpose of this chapter is to show that this negative side-effect of the citizencandidate models can be solved by the use of appropriately designed institutional restrictions in economies with public goods and complete information (which is the natural setting for these voting games). We show how agents may reach agreement on the type of political institutions selected and how these institutions lead to efficient social choices with zero political rents in equilibrium.

The institutions that arise endogenously from this political game is the utilization of the separation of powers by agents (some of them choose to become politicians, while others choose to become legislators to set the constitution) and the constitution (a set of restrictions on the voting behavior of citizens and politicians). Therefore, institutional arrangements on collective decisions become a necessary prerequisite for efficiency in this case.

More specifically, we present an economy with one private and one public good and we use a five-stage game, where all agents start as citizens. At stage one each citizen decides on whether to become a politician or a legislator (but not both) or to remain as a citizen. At stage two, legislators set the constitution of the economy, which defines restrictions on political competition, and at stage three politician propose platforms. At stage four, agents vote and at stage five the elected politician imposes taxation and produces the level of the public good according to his proposal and constitutional restrictions.

It is shown that the pure-strategy sub-game perfect equilibria of this political game implement the Lindahl allocation of the economy, which implies that none of the agents has sufficient power to achieve his most preferred outcome (and essentially becoming a social dictator). Thus, political rents are zero, in the sense that, in equilibrium, the utility of an agent is not dependent on whether he is a politician or not. To the best of our knowledge, no paper in the voting literature so far has implemented efficient allocations implying zero political rents as Nash equilibria. In this chapter, this outcome is due to political competition in conjunction with appropriate political institutions, and hence this highlights the importance of these two factors in eliminating political rents and achieving efficiency.

The following assumptions are crucial for the results: (i) An agent can become either a politician or a legislator, but not both (Separation of Powers), (ii) the rules set by legislators apply equally for all agents, conditional on their characteristics, namely preferences and endowments (No-discrimination Principle).

In section 3.3, we start with a very simple model. We show why both political competition and political institutions are necessary conditions for the implementation of Lindahl allocations when political parties (or politicians) are exogenous. The economy consists of 2 agents and 2 goods, one private and one public. Political parties are selfish entities which make proposals over the allocation of resources in order to extract as much of the social surplus as possible. Agents vote for their most preferred proposal and the party which wins the election becomes the government and implements its policy.

The actions of the parties and agents may be restricted by the Constitution, which in this section is an exogenously imposed set of restrictions. The constitution determines the dimension of commitment to political proposals and the maximum amount of taxation, which a government can levy on citizens. A particular form of the constitution is considered, which specifies that political proposals are committing only to the level of the public good but not taxation levels and the maximum taxation on a citizen must be such that his marginal willingness-to-pay for the proposed level of the public good is not violated³⁹.

Using the above constitutional rule, three different cases are examined. The first case assumes that the constitution limits taxation, but there is a single candidate politician. In this case, it is shown that the party acts as a social dictator and reaps as much political rents as possible, given the limitation it faces. In the second case, we allow for free entry of political parties, but the maximum taxation restriction is removed from the constitution. In this case, it is shown that, despite the presence of political competition, parties still earn political rents. In fact, because the taxation restriction is removed, parties face weaker restrictions than the social dictator of the previous case and they may earn strictly higher rents than him.

In the third case, we allow for both political competition and the maximum taxation restriction to apply in the economy. It is shown that under these conditions the equi-

³⁹We explain this definition of the maximum-taxation constraint more thoroughly in section 3.3. It essentially implies that the taxation imposed by the government on an agent can not reduce his utility below the utility he would have received if he were on his offer curve for the specific level of the public good implemented.

libria of the game are the Lindahl allocation of the economy and prove that political rents to parties are zero. We, thus, establish the necessity of both types of checks and balances over the power of government for efficiency.

In section 3.4 we move one step further and show how political institutions emerge endogenously, by extending the political game to the five-stage game described earlier. More specifically, at the first stage of the game agents decide what type of political power they want to hold from the two types available: legislative and executive power. Given theses choices, agents are distinguished into three classes, namely legislators, politicians and citizens. Therefore, we introduce separation of powers as a potential institutional control on the power of politicians, and agents in the economy choose whether to utilize it or not. Legislators determine the constitution of the economy, which is the set of political institutions that restrict voting behavior and political actions. Specifically, legislators are allowed to determine how committing political proposals will be and what is the constraint on maximum taxation. The rest of the political game, then follows the game in section 3.3.

The extended game has multiple sub-game perfect Nash equilibria, all of which implement the Lindahl allocation of the economy. Under any preference profile, legislators decide that politicians will be committed to the level of the public good they announce but not to the taxation level. Instead they set an upper bound to the level of taxation politicians can impose, namely the maximum-taxation constraint of section 3.3. With these restrictions in place, and because of the free entry of candidates in the political arena, politicians can not extract social surplus by simply being in power. In other words, political rents are zero⁴⁰.

Therefore, the contribution of this chapter is threefold: First, it contributes to the

⁴⁰There are also other equilibria of the game, which hold only for specific preference profiles, but the equilibrium allocation remains the same. The only difference is in terms of the constitutional restrictions that arise in these equilibria.

citizen-candidate models by showing how institutional restrictions and the separation of powers can facilitate political competition in achieving zero-political rents. Second, it shows that this requires that political proposals be only partially committing (committing only to the level of the public good but not taxation). Third, it shows how the required institutions can emerge endogenously by the actions of the agents themselves.

As most voting models, we assume that information is perfect and so agents' preferences and endowments are common knowledge. Hence, taxation can be conditioned on these observable characteristics (preferences and endowments). We understand that the assumption of full information may be restrictive and gives rise to taxation rules that are not empirically observed. However, this assumption serves two purposes: (i) It allows us to build a benchmark model which shows how political competition along with certain institutional restrictions (arising from the separation of powers) leads to efficient equilibria with zero political rents. (ii) It facilitates the comparison of the results with those of other related voting models. We intend to consider the case where preferences and endowments are private information in the near future.

3.2 Related Literature

The model closest to this chapter is the citizen-candidate model, pioneered by Osborne and Slivinski (1996). In their paper, each agent (citizen) in the economy decides whether to become a candidate politician or not and then citizens vote for electing one of the politicians under different electoral rules. The winner of the election chooses his most preferred policy. The authors show that the number of candidates at the second stage depends on the cost of running the campaign and the potential benefits of winning. They also show that the plurality rule generates more candidates than an electoral rule based on runoffs.

Besley and Coate (1997) introduce the citizen-candidate framework into a multi-

dimensional policy setting and examine the implications of the model for the efficiency of the final allocations. They also present an application of their model in economies with public goods. They show that an equilibrium of the game always exists, even though the policy space is multi-dimensional, and that the resulting allocations are Pareto efficient.

Despite the similar structure of political competition between the above papers and the one presented here, there are some major differences as well. In both models (Osborne and Slivinski, Besley and Coate), the (lack of) commitment to political proposals is exogenously imposed, while in our case it emerges endogenously. In other words, the case they consider, namely that politicians implement their most preferred policy when they are in power, corresponds to the case in our model where legislators decide that political proposals are not committing to any dimension. Moreover, it is shown that if commitment is endogenous this case will never be chosen (that is, in our model, this case is off the equilibrium path.). It is also implicitly assumed that political entry is costless, while the assumption in these papers is that each citizen must pay some cost to become a candidate.

As a result, the properties of the equilibrium allocations in the two types of games differ substantially. The main difference is that in case examined here, politicians do not implement their most preferred policy. In fact, the equilibrium allocation does not depend on the identity of the politician and as a result, as long as there are at least two candidates, there are no incentives for strategic entry. A second implication of this is that, in this model, political rents are zero in equilibrium, in the sense that, given a specific equilibrium allocation, the utility of an agent is not dependent on whether he is a politician or not. In other words, in equilibrium, becoming a politician does not provide additional benefit to a citizen. Obviously, in the political game of Osborne-Slivinski or Besley-Coate this does not apply, as the equilibrium utility level of an agent depends critically on his social identity (citizen or politician).

There is an extensive literature on voting games with simultaneous proposals and multi-dimensional policy space. The main finding of these papers is that, if the proposing members are free to make any type of offer, then the corresponding voting games have generally no equilibrium. The theoretical literature has tried to overcome this problem by examining restrictions on preferences that would make them compatible with a notion of political equilibrium. It is not in our intentions to provide a comprehensive list of these articles. Some of the most noteworthy contributions are related with the work of Sen (1964, 1966) and Inada (1964), but they restrict their analysis to triplets of preferences. Kramer (1973) provides a general characterization of necessary conditions in order for social welfare functions to be consistent with Arrow's assumptions and shows how restrictive these requirements can be. Plott (1967) provides a different notion of political equilibrium and demonstrates how general preferences violate the conditions required to satisfy it under a simple majority rule. Subsequently, Slutsky (1979) generalizes this result for any type of majority rules, including unanimity.

The chapter is also related to the model adopted by Baron and Ferejohn (1989). They adopt a sequential bargaining approach for the sharing of a private good, which is essentially a generalization of the sequential bargaining game by Rubinstein (1982). Each agent in their model has a positive probability of being a proposer and if his allocation is objected by a majority of the agents, the bargaining process moves to the next round. The authors show that when the time discount factor is less than one there is a sub-game perfect equilibrium, where the first individual to propose makes an offer which the majority accepts. It is a general feature of their model that the first proposer has superior bargaining position compared to the rest so that some bargaining rents will accrue to him. On the contrary, in this chapter it is shown that political rents are zero in equilibrium.

Nevertheless, many authors, following their seminal work, have demonstrated how social choices can be implemented through the mechanism of a sequential bargaining game⁴¹. Jackson and Moselle (2002) extend Baron and Ferejohn's model to the case where the economy contains public goods (alternatively, an ideological dimension). They show that, if there is a sufficiently high cost of delay, then the offer of the first proposing legislator will be approved and will contain a decision in both dimensions. The offer will trade part of the potential private good distribution gains for a compromise in the public good dimension and under this procedure there is a wide set of potential equilibrium proposals. The main difference between our model and Jackson and Moselle is that the sequential approach generates allocations where the final quantity of the public good does not fully reflect the associated externalities and therefore it is under-produced. In contrast, the equilibrium outcome of our model implies the elimination of political rents and the efficiency of proposals, irrespectively of party identities.

More recently, Dávila, Eeckhout and Martinelli (2006) have proposed a similar sequential bargaining mechanism for the distribution of a private and a public good between two individuals. They find that as the cost of delay vanishes the equilibria of the game converge to the Lindahl allocations and so the inefficiency generated by sequential bargaining disappears. In our game, though, the efficiency result of the proposals remains even if one were to assume strictly positive costs of delay. Also, it is not clear whether their result holds for more than two agents, whereas our result holds for any number of players greater or equal to three. Furthermore, the equilibrium outcome of our game is exactly the Lindahl allocation.

Finally, the chapter is related to the literature of political competition as a driving force for eliminating political rents. Stigler (1972) was among the first to point out

 $^{^{41}\}mathrm{See}$ for example Merlo and Wilson, 1995 and Banks and Duggan, 2000

the similarities that exist between political and market competition. In a similar way that competition among producers reduces their ability to earn abnormal returns, competition among candidates or political parties reduces the magnitude of opportunistic behavior and the adoption of socially undesirable policies. Wittman (1989) pushes the argument one step further, by presenting many features of the modern representative democracies as institutional designs of monitoring and control over the actions of politicians. Despite the existence of informational constraints on their actions or the bargaining power nested in their authorities, institutions, like political parties, elections or the structure of the legislative bodies, create a variety of reputation and competition considerations that prevent politicians from extensive abuse of their positions. Wittman's conclusion is that one should not expect the inefficiencies of the political system in democracies to be greater than the failures of competitive markets.

Though the analysis does not consider such a general set of institutional designs it is in line with the political efficiency argument. The main difference is that this chapter is explicitly concerned with the issue of the provision of the public good and the role of political competition in solving it, while the aforementioned research agenda is centered around the elimination of political rents, whatever form they may take.

3.3 Description of the economic environment and the mechanism

Consider an economy with 2 agents and 2 goods. Good 1 is a private good while good 2 is a public good. Let e_1 and e_2 be the endowments of the private good for agents 1 and 2 respectively. The public good is produced through a linear production function F(z) = mz, where z stands for the aggregate quantity of the private good used as an input and m is a scaling coefficient (technological constant).

Agent $i = \{1, 2\}$ has a well defined ordering of preferences which can be represented

by a continuous, non-decreasing, strictly quasi-concave utility function $u_i(x_i, y)$, where x_i represents the consumption of private good for agent i and y represents the quantity of the public good produced. We assume that $u_i \to 0$, as $x_i \to 0$ or $y \to 0$. Furthermore, for every agent, the demand for the public good is strictly increasing as its relative price decreases. This means that the offer curve of each individual is strictly increasing (see also Figure 8), which is the case when the income effect is not strong (negative) enough to overcome the (positive) substitution effect. Besides simplifying the analysis, this assumption is made in order to restrict the attention to economies with a unique Lindahl allocation⁴².

As a benchmark case we define the allocation outcome generated by competitive markets. Each agent places an order for a certain quantity of the public good to firms so as to maximize his utility given his endowment and the order of the other agent. Firms, facing conditions of free entry, buy inputs from agents and try to maximize their profits. Assume that k is the number of firms operating in the economy, where k is a large number. Assume, without loss of generality, that the equilibrium allocation of resources under free markets is unique and is given by: $a^{fm} = \{x_1^{fm}, x_2^{fm}, y^{fm}\}$. The resulting utility level for agent 1 and 2 is $v_1^{fm} = u_1(x_1^{fm}, y^{fm})$ and $v_2^{fm} = u_2(x_2^{fm}, y^{fm})$ respectively: $\mathbf{v}^{fm} = \{v_1^{fm}, v_2^{fm}\}^{43}$.

Because of the nature of good 2, a^{fm} is not Pareto efficient. There exists a feasible re-allocation of resources that can make at least one of the agents better-off without

⁴²The uniqueness of the Lindahl allocation, in turn, is required in order to ensure the existence of equilibrium in our game, as we also note later on.

⁴³Formally, agents maximize their utility with respect to the quantity of the public good they privately demand (y_i) : $\max_{y_i} u_i(e_1 - py_i, \sum_i y_i)$, and firms maximize their profit: $\max_{y^f} (p - \frac{1}{m})y^f$, where p is the price of the public good in terms of the private. One can also formulate the problem in game-theoretic terms by assuming that each agent has access to the production technology of the public good and chooses how much to produce as a best-response to the choice of the other agent: $\max_{y_i} u_i(e_1 - \frac{y_i}{m}, \sum_i y_i)$. It is easy to verify that the two formulations give the same final allocations. See also Bergstrom, Blume and Varian (1986) for the definition and the characterization of the Nash equilibrium of the above game. They also establish the uniqueness of the equilibrium under very weak assumptions.

making the other worse-off. This can be achieved through a centralized decision making process, which takes into account the consumption externalities. However, at the same time, we allow each agent to veto any centralization process, in which case it is assumed that it is effectively blocked and agents resort to competitive markets for allocating resources. Therefore, under the assumption of veto power, v^{fm} is an effective outside option, which determines the individual participation constraints on any centralized allocation scheme. Even though the ability to veto centralized processes does not change the results (political rents are defined in terms of the competitive equilibrium utility levels instead of the no-private consumption outcome that would be produced under absolute dictatorship), we include it for checking their robustness to the existence of participation constraints or not.

First, we highlight the importance of political competition for the efficient provision of the public good. In order to make the source of political rents as transparent as possible, we initially take the institutional constraints and political parties as exogenous (we will relax these assumptions in the subsequent subsection). Consider the following centralized decision making mechanism manifested into a voting game dictated by the rules of a Constitution. The players of the mechanism are political parties (or alternatively politicians) and the 2 agents. A political party is an exogenous entity which makes offers of prospective quantities of the public good to agents and tries to be elected as government. Parties exhibit risk neutrality and their utility is the probability to win the election in the voting game times the rents they receive from their offers: $V^p = p_{win}r^{p-44}$. Agents play the double role of being the consumers of the final allocations produced in the economy and voters, who decide which party will become the government.

The Constitution is a exogenous political institution which puts restrictions on the

⁴⁴None of the results is affected by the degree of risk aversion of political parties.

action set of parties and voters. More specifically, it specifies the types of political proposals that parties can make, the way agents vote and how a government is elected to implement its proposed allocation. Agents vote for the party whose proposal provides the greatest level of utility for them (sincere voters). If agents are indifferent between two proposals, then it is assumed that they vote arbitrarily for one of them, say the proposal of the party with the lowest index⁴⁵. The party which receives the majority of votes, wins the election. Ties are again solved arbitrarily, say for the party with the lowest index⁴⁶.

Party proposals consist of only one element: the quantity of the public good to be produced (y^p) . Let $PR^p = \{y^p\}$ denote the political proposal of party p. If a party is elected into power, then it will be called to implement the level of the public good it proposed before the election. Note, however, that, while the party has committed itself to the quantity of the public good, it has not committed itself to the taxation levels that will be imposed on agents. The only constraint, which we assume that is imposed on the government by the Constitution, is that the taxation each individual will pay can not exceed the taxation that the same agent would have paid for the proposed level of the public good if he were on his offer curve. This is equivalent to saying that, given a specific proportion of aggregate taxation that an agent pays, the maximum taxation possible is one that gives the agent the same utility level as the one he would have obtained when the proposed level of the public good was an optimal choice for the agent. For example, the maximum taxation possible for agent i for the proposed level y in Figure 8 (page 96) is equal to t_i . For the rest of the chapter, this institutional restriction is called the maximum willingness-to-pay constraint or the maximum-taxation constraint.

⁴⁵Even though the assumption of sincere-voting is quite restrictive, it is without loss of generality. We will come back to this point and explain it in more detail in the analysis that follows.

⁴⁶Of course, any other assumption about who wins the election under a tie would work equally well.

However, on its own, this restriction is not sufficient to eliminate political rents, as is shown for the case of a single party. A party that faces only this constraint can find levels of the public good for which the aggregate willingness-to-pay exceeds the required expenditure. That is, the presence of political competition is also necessary for the elimination of rents. On the other hand, if there are more than one parties, but the Constitution does not impose the maximum willingness-to-pay constraint, then political parties can still earn political rents, despite the presence of political competition. Therefore, some form of institutional restrictions are also necessary for the efficient provision of public goods.

In order to show that political competition and institutional constraints are both necessary requirements for the efficient provision of the public good in this economy, we present the equilibrium of the game under 3 different conditions: (i) when the Constitution restricts party proposals and imposes the maximum-taxation constraint, but there is only one party in the economy, (ii) when there are two parties in the economy, but the maximum-taxation constraint is not in place (the only constraint that applies is the standard participation constraint) and finally (iii) when both conditions (multiple parties and the maximum-taxation constraint) are satisfied.

Case I

Consider, first, the case when there is only one party, which has secured the control of the government and acts as a dictator. This provides a base of comparison for political competition. The party's objective is to maximize its rents given the constitutional constraint on policies, and hence it tries to find the level of the public good, for which the summation of agents net valuation is the highest. More formally, the party's maximization problem can be described as:

$$\max_{y} r^{p}(y) = \sum_{i} t_{i}(y) - \frac{y}{m}$$

subject to

$$t_i(y) = \frac{y s_i^m(y)}{m}$$
$$s_i^m(y) = \left\{ s_i | \frac{\frac{\partial u_i}{\partial y}}{\frac{\partial u_i}{\partial x_i}} = \frac{s_i}{m} \right\} \ , \forall i \in \{1, 2\}$$

The party's problem is straightforward. It needs to choose a level of the public good such that both agents would like to contribute a share of their endowment as big as possible, so that political rents are maximized. The rents come from the fact that, at the proposed level of the public good, aggregate taxation will be higher than the required resources for its production, so that the difference is received by the party. Below we show that these rents are positive⁴⁷.

Proposition 1: Under the assumptions made above on agents' preferences, the maximization problem of the party has at least one solution with strictly positive rents.

Proof: The party's maximization problem can be rewritten as:

⁴⁷Note that the formulation above does not include agents' participation constraints. It is easy to show that the main result of Proposition 1 (namely that political rents are positive) holds when participation constraints are included. The main intuition is that the allocation generated by competitive markets is inefficient and, therefore, the political party can still find an allocation that generates strictly positive political rents, even when some participation constraints are binding. The results are available by the authors upon request.

$$\max_{y} r^{p}(y) = \left(\sum_{i} \left(\frac{y s_{i}^{m}(y)}{m}\right) - \frac{y}{m}\right) \Leftrightarrow$$
$$\max_{y} r^{p}(y) = \frac{y}{m} \left(\sum_{i} s_{i}^{m}(y) - 1\right)$$

The First Order Condition for this problem is given by:

$$\frac{\partial r_p(y)}{\partial y} = 0 \Leftrightarrow \frac{\partial}{\partial y} \left[\frac{y}{m} \left(\sum_i s_i^m(y) - 1 \right) \right] = 0 \Leftrightarrow$$

$$\frac{1}{m} \left(\sum_i s_i^m(y) - 1 \right) + \frac{y}{m} \left(\sum_i \frac{\partial s_i^m(y)}{\partial y} \right) = 0 \Leftrightarrow$$

$$\sum_i s_i^m(y) = 1 - y \sum_i \frac{\partial s_i^m(y)}{\partial y} \tag{16}$$

The left-hand side of equation (16) is the marginal benefit to the party by an increase in the level of the public good, while the right-hand side reflects the marginal cost. Also, notice that $s_i^m(y)$ is a continuous, strictly decreasing function of y. Because of the assumptions of non-satiation and strict quasi-concavity of the utility functions, for every level of expenditure sharing s_i there exists a unique level of the public good y, such that agent i maximizes his utility. Furthermore, by assumption, as s_i decreases the demand for the public good strictly increases. In other words, the offer curves for both agents are strictly decreasing functions of s_i . Essentially, $s_i^m(y)$ is the inverse function of the offer curve and hence it is also a decreasing function of y: $\frac{\partial s_i^m(y)}{\partial y} < 0$.

First, notice that as $y \to 0$, the left-hand side of equation (16) goes to 2, as both individuals are willing to shoulder the full burden of taxation for low levels of the public good. At the same time, the right-hand side of equation (16) is equal to 1 $(\partial s_i^m(y)/\partial y) = 0$ for very small values of y), which means that the difference of the left-hand side minus the right-hand side is positive⁴⁸. On the other hand, as $y \to \infty$, the left-hand side tends to 0, as individuals are willing to provide an infinitesimally small part of their endowment for very high levels of the public good. At the same time, because $\partial s_i^m(y)/\partial y < 0 \Rightarrow -y \sum_i \partial s_i^m(y)/\partial y > 0$ for large values of y, and hence the right-hand side is greater than 1. This means that, as $y \to \infty$, the difference of the left-hand side minus the right-hand side is negative. Since both sides are continuous functions of y, there exists at least one level of the public good y^* such that the two sides are equal.

Second, because $\frac{\partial s_i^m(y)}{\partial y} < 0$, $-y^* \sum_i \frac{\partial s_i^m(y)}{\partial y}|_{y=y^*} > 0$, so that at any solution of the party's problem it holds that: $\sum_i s_i^m(y^*) > 1$. This means that the shares of expenditure that agents are willing to provide for the public good exceed the required expenditure and therefore political rents are strictly positive.

The intuition for this result is simple. When only one party is allowed to operate in the economy it knows that it has full bargaining power over the population since its offers will go unchallenged, so long as both agents are willing to forgo a part of their endowment for the proposed level of the public good. It therefore becomes a social

⁴⁸Recall the assumption made earlier that individuals have access to competitive firms, which can produce the public good instead of the government. As a result, the maximum proposed share of public expenditure $(s_i^m(y))$ will not exceed one. If it did, the agent would be better off by producing the good on her own, by ordering it by a firm. In other words the maximum value of $s_i^m(y)$ is 1 and for very small values of y, $\partial s_i^m(y)/\partial y = 0$.



Figure 8: Offer curve for individual i

dictator, using its power to provide allocations that maximize its rents. Because the marginal utility of the public good is higher than the marginal rate of transformation for both agents when its quantity is very low, proposals associated with positive political rents are easy to find. Of course, all such proposals are socially inefficient, since they imply excessive supply of resources into the production process and consequently waste (because politicians are exogenous entities, political rents are deadweight loss for the society.).

Case II

The main elements of the game are the same as in the first case. However, we assume that there are two parties in the economy and the maximum-taxation constraint does not hold⁴⁹. This means that parties are free to choose any taxation level after being elected in government, as long as the participation constraints are satisfied. In order

 $^{^{49}}$ It is straightforward to generalize for cases with more than two parties as the same reasoning applies. Essentially, the only requirement for political competition is free entry of parties in the political contest.

to be more explicit, the structure of the game is presented below:

- Stage 1: Each party makes an offer on the level of the public good and it is committed to it.
- Stage 2: Each agent decides which party to vote and the election takes place. The party which receives the majority of votes wins the election. In case of draw, party 1 is arbitrarily chosen to implement its proposal.

Stage 3: The elected party takes over power and implements its proposal.

The removal of the maximum-taxation constraint has an important implication for the equilibrium outcome. Because the party in power is not constrained over the level of taxation, political competition is rendered powerless. No matter what promises parties make at the first stage for the level of the public good, the government will impose such a high level of taxation on each agent, so that he is indifferent between the market and the governmental allocation of resources. This happens because there is no effective commitment to taxation levels after the election has taken place.

Agents, anticipating this, understand that all proposals imply the same utility level for them, irrespectively of their promise over the quantity of the public good. Therefore, they are indifferent between voting for one party or the other and vote arbitrarily for one (we restrict our attention to pure-strategies). Political parties, of course, anticipate this as they realize that their commitment to the level of the public good does not affect agents' voting behavior at the subsequent stage. Since the probability of winning the election (which is either zero or one, depending on the pure-strategies of agents when they are indifferent about party proposals) is independent of its proposal for any party, the best choice for them is to commit to the level of the public good that maximizes their rents after the election and simultaneously satisfies the participation constraints of agents. In this case, parties are acting effectively as social dictators. Proposition 2 summarizes the result.

Proposition 2: The equilibrium outcome of the 2-agent, 2-party game, without the maximum-taxation constraint enforced by the Constitution, implies strictly positive political rents for the party that is elected in government.

Proof: At stage 3, whichever party is elected will impose the maximum taxation possible. Given that there is no commitment to the level of taxation at stage one by a party's proposal and that there is no constitutional restriction, the maximum taxation is the one that makes each individual indifferent between the allocation he would obtain by competitive markets and the one implemented by the government.

At stage 2, agents are indifferent between party proposals, as all of them imply the same utility level for each individual. Therefore, their vote can not affect the final outcome of the game and they vote arbitrarily for one party. At stage 1, parties realize that their political offer has no impact on the voting behavior of agents. Their best response is to set the level of the public good so as to maximize their political rents. Formally, each party solves the following problem:

$$\max_{y} r^{p}(y) = \sum_{i} t_{i}(y) - \frac{y}{m}$$

subject to

$$t_i(y) = \{t_i | u_i(e_i - t_i, y) = v_i^{fm})\}, \ \forall i \in \{1, 2\}$$

From the First Order Condition we get that:

$$\sum_{i} \frac{\partial t_i(y)}{\partial y} = \frac{1}{m} \tag{17}$$

This is a simple cost-benefit equation. It states that the party should offer a level of the public good such that for the last unit of it, the marginal benefit of the extra taxation is equal to the marginal cost of the extra resources required for its production. Let \hat{y} denote this level of the public good. Notice that \hat{y} is unique. This is due to the strict quasi-concavity of agents' utility functions, which implies that $\frac{\partial t_i(y)}{\partial y} > 0$ and $\frac{\partial^2 t_i(y)}{(\partial y)^2} < 0$. This means that the first partial derivative of $t_i(y)$ is a strictly decreasing continuous function and hence the left-hand side of equation (17) is also a strictly decreasing continuous function. Hence, the level of the public good that satisfies (17) is unique. Also, from the total derivative of the participation constraint notice that:

$$\frac{dt_i}{dy} = \frac{\frac{\partial u_i}{\partial y}}{\frac{\partial u_i}{\partial x_i}} \Rightarrow \sum_i \left(\frac{\frac{\partial u_i}{\partial y}}{\frac{\partial u_i}{\partial x_i}}\right)_{y=\widehat{y}} = \frac{1}{m}$$

This implies that the summation of the ratio of marginal utilities is greater than the marginal rate of transformation for all $y < \hat{y}$:

$$\sum_{i} \left(\frac{\frac{\partial u_i}{\partial y}}{\frac{\partial u_i}{\partial x_i}}\right)_{y < \widehat{y}} > \frac{1}{m} \Rightarrow \int_{0}^{\widehat{y}} \sum_{i} \left(\frac{\frac{\partial u_i}{\partial y}}{\frac{\partial u_i}{\partial x_i}}\right) dy > \int_{0}^{\widehat{y}} \frac{1}{m} dy \Rightarrow$$

$$\sum_{i} \int_{0}^{y} \left(\frac{dt_{i}}{dy}\right) dy > \frac{\widehat{y}}{m} \Rightarrow \sum_{i} t_{i} - \frac{\widehat{y}}{m} > 0$$

The last inequality above states that political rents are strictly positive for the party that proposes \hat{y} . Now, since agents vote arbitrarily at stage two (because they are indifferent on which party to vote), it might be the case that they always vote for
one of the two parties, say party one. The other party anticipates this and may propose any level of the public good (since it expects to lose the election). But the winning party is not indifferent, as it maximizes its rents by proposing \hat{y} . If, on the other hand, both parties receive a strictly positive probability of winning the election (say one agent votes for party one and the other for two), then both parties will propose \hat{y} in equilibrium. In other words, the game has multiple sub-game perfect equilibria in terms of strategies, but the equilibrium level of public good is unique and it implies strictly positive political rents for the elected party. This completes the proof of proposition 2.

This shows that political competition on its own is not a sufficient condition for the elimination of political rents. Institutional restrictions are also necessary, a point that will be emphasized in the next case. In fact, without the maximum-taxation constraint, political parties can implement perfect price discrimination at the third stage of the game, so that the political rents for the ruling party will be at least as large as the ones of the social dictator in case I, under any combination of individual preferences and endowments. This is because political competition is powerless if there are no restrictions on the maximum level of taxation and as a result parties face one less constraint than the sole party of the previous case. Once the maximum-taxation constraint is reinstated, however, political competition leads to efficiency, as shown below.

Case III

The primitives of the economy and the political game remain the same as in the previous case, with the difference that the two parties in the economy face the maximum-taxation constraint. An immediate consequence of competition is that parties can not secure election victory by simply satisfying agents' willingness-to-pay, as was the case with a single party. In fact political rents will be zero in equilibrium, irrespectively of the offer that will pass.

Proposition 3: The political game as described above, with 2 agents, 2 parties and the Constitution as described in the previous section, has a unique sub-game perfect Nash equilibrium. Both parties propose the level of the public good that corresponds to the Lindahl allocation of the economy. Both agents are indifferent and vote arbitrarily for one. At the third stage, the party which receives most votes becomes the government, otherwise party 1 is selected to implement the common proposal.

Proof: Note that, because both individuals have strictly increasing offer curves, they intersect at most once. This means that there is a unique Lindahl allocation in the economy. Let y^L, s_1^L, s_2^L be the quantity of the public good and the respective expenditure shares associated with the Lindahl allocation of this economy. By definition, $s_1^L + s_2^L = 1$.

At the last stage of the game, the party that wins the election maximizes its rents given the commitment it has undertaken at stage 1 regarding the level of the public good. The implication of this is that agents will be asked to contribute their maximum willingness-to-pay at stage 3. If a party has offered y^L , then it can not extract any political rents after election, since the maximum willingness-to-pay of the agents is exactly the same as the expenditure required for the public good. To see that, recall from the previous section that $s_i^m(y)$ (the maximum willingness-to-pay of agent *i*) is a decreasing function of *y* and that the Lindahl allocation is defined as a sharing of the public good expenditure such that both agents agree on the demanded quantity. This means that $s_1^m(y^L) + s_2^m(y^L) = 1$, while for $y < y^L$: $s_1^m(y) + s_2^m(y) > 1$ and for $y > y^L$: $s_1^m(y) + s_2^m(y) < 1$.

If a party ever offered $y^p > y^L$, then agents would anticipate that such a level of the

public good can not be implemented without violating their maximum willingness-topay and hence they would not vote for the corresponding party. On the other hand, if party p offers $y^p < y^L$, then agents, as noted in the previous paragraph, anticipate strictly positive political rents for the party. Furthermore, both agents would be strictly better-off by an offer with a greater level of the public good. This is because levels of y closer to the Lindahl allocation correspond to points on the offer curves with higher utility (See also Figure 8).

Therefore, if party p offers $y^p = y^L$, then the other party will lose the election with certainty if it makes any other offer. If party p offers $y^p < y^L$, then the other party can win the election with certainty by offering a quantity of the public good slightly greater. Finally, any offer $y^p > y^L$ is not credible, and party q can win with certainty by making any offer with $y^q \leq y^{L50}$. As a result, the unique sub-game perfect equilibrium involves both parties proposing $y = y^L$. The rest of the proposition follows immediately.



Figure 9: Voting equilibrium under political monopoly and competition

⁵⁰Given the enforcement of the maximum-taxation constraint, any level of the public good, that is greater than the Lindahl, means that the summation of private taxation, that can be levied on the citizens, is less than the resources required to produce it. Hence any party that makes such a proposal, if it is voted on power, will have to either accept the infeasibility of the proposal and implement a lower level of public good (in this sense the proposed allocation is not credible) or to pay out the difference by its own wealth (negative rents, in which case the proposal is clearly not a best-response for the party). Therefore, there can be no equilibrium of the game where the implemented level of the public good exceeds y^L .

The main intuition of the proposition is that, when competition is allowed, then parties can not maximize their political rents without taking into account the offers of their contestants. Since agents anticipate that parties can commit to the level of the public good, but not to the tax level, they will vote the proposal which minimizes rents. Note that the Lindahl allocation is the only credible allocation on the Pareto frontier. Political contesters understand this and make efficient offers. The resulting equilibrium of the game is represented diagrammatically in Figure 9. The level of the public good y^m corresponds to the choice that a monopolistic party would do. Such a level implies strictly positive political rents for the government, as the summation of the maximum willingness-to-pay of the two individuals exceeds one. On the other hand, y^L is the level of the public good that is obtained under conditions of political competition and it corresponds to the level of the public good under the Lindahl allocation L.

One can also see now why the assumption of sincere voting is not crucial for the result. In the case where political proposals commit parties only to the level of the public good, agents' expected utility is an increasing function of the proposed public good levels. Therefore, all agents would like to vote for the party which offers the highest level of the public good. Even though coordination failures may arise (indeed, without the sincere voting assumption, one can find equilibria where all agents vote for a party with a dominated platform), one can easily dismiss them. For instance, instead of simultaneous voting, consider the modified game where agents vote publicly and sequentially. This eliminates any type of coordination failure and allows all agents to vote only for the party that offers the highest utility to all of them⁵¹.

Proposition 3 seems to hold because of the way the maximum-taxation constraint is constructed. In the following section we extend the political game and allow agents

⁵¹Of course one may point out that sequential, public voting almost never occurs in contemporary democracies. However, explaining this fact would require an environment with multiple public goods and would add additional complications to the model, which would derail us from our initial purpose. We leave these and some other considerations for future work.

to create the Constitution and to make proposals for the allocation of resources in the economy. We thus allow the required conditions for the efficient provision of the public good to arise endogenously.

3.4 Separation of Powers and Endogenous Political Institutions

In the previous section we showed the importance of both political competition and institutional restrictions for the efficient provision of the public good in the economy. Most elements of the political game, however, were exogenously imposed and it would seem as if the results are derived by assuming the partial commitment of parties to their proposal and the maximum-taxation constraint.

In this section it is shown how these elements of the institutional environment can arise endogenously. Most importantly, we show that separation of powers is an important institution for imposing checks and balances on a government.

Consider an economy with n agents, where $n \ge 3$. As in the previous section, there is one private and one public good. Each agent has an endowment e_i of the private good and a utility function $u_i(x_i, y)$, which satisfies the same assumptions as before. Let the production function of the public good be also the same as before: F(z) = mz. Once again, let \mathbf{v}^{fm} be the vector of utilities that the agents of the economy receive, if the public good is provided by a decentralized mechanism (competitive markets). Of course, such an allocation is suboptimal. Finally, note that the assumptions made on preferences mean that the economy has a unique Lindahl allocation.

Consider the following political game. At stage 1, agents decide what type of political power to hold. There are two types of power-holders: (i) legislators and (ii) politicians. Legislators decide the institutional arrangements (the Constitution) of the economy. Politicians, participate in the election by making proposals over the level of the public good to be produced. Once in government they implement their policy. Agents decide whether they want to become legislators or politicians or neither. However, an agent can not become both. If no agent becomes a legislator, then no Constitution is set and the politician elected in government has unlimited power (i.e. non-commitment of political proposals and non-existence of the taxation constraint is the status quo). On the other hand, if no agent becomes a politician, then no centralized decision is made and competitive markets decide the level of the public good to be produced (i.e. the allocation a^{fm} is the status-quo).

At stage 2, legislators decide on the form of the Constitution. Specifically, they decide on two different institutions of political competition: (i) which elements of a political proposal are committing if the respective politician rises to power, and (ii) whether the government will face the maximum-taxation constraint (as defined in the previous section) or not. In other words, legislators choose the institutional constraints for political parties and the government. Each legislator simultaneously makes a proposal on these two issues and according to a given choice rule, one of the proposals is chosen to be the Constitution of this economy.

The choice rule used for deciding the Constitution is inconsequential for the final outcome of the political game, as is shown later. For reasons of expositional clarity, it is assumed that the legislative proposal which is made by the majority of legislators becomes the Constitution. It is also assumed that the Constitution is binding for politicians. If any of its clauses is violated by the government or other agents, then the centralized decision making process breaks down and agents allocate resources through competitive markets (a^{fm}) .

In terms of the decisions, which the legislators make on the Constitution, the following assumptions are made. First, legislators choose whether political offers are committing to the level of the public good only, to the level of taxation only, to both or to neither. Second, legislators can choose whether to impose the maximum-taxation constraint or not. But, the maximum-taxation constraint is anonymous. It holds for either all agents in the economy or none. In other words, if an upper bound on taxation is set, it can not be the case that some agents in the economy enjoy this privilege while others are heavily taxed by the government. We call this condition the Anonymity of the Taxation Constraint.

If there is no restriction on the maximum level of taxation and no commitment to taxation during the election, then the government faces only one form of constraint: the participation constraint of agents in the economy, which implies a lower bound to the utility level agents can receive by the centralized allocation, equal to \mathbf{v}^{fm} . Effectively, we allow any agent to block the formation of any centralized decision making mechanism, which gives him lower utility than the one he receives under competitive markets.

The rest of the stages are similar to the ones in the previous section. At stage 3, those, who have become politicians, make proposals over the quantity of the public good and the level of taxation. At stage 4, each individual in the economy votes for one proposal and the proposal that receives most votes wins⁵². At stage 5, the politician who made the successful proposal, receives the power to levy taxation and implement the allocation of resources, given the restrictions of the Constitution.

Notice that some sub-games of the game above may not have an equilibrium in pure strategies. For example, if legislators choose that political proposals are committing to both the public good and individual taxation, then, irrespectively of whether the maximum-taxation constraint holds or not, there is no equilibrium in pure strategies. For all these sub-games, we avoid issues of non-existence of equilibrium by adopting a sequential bargaining approach. Specifically, it is assumed that there are T stages of bargaining, $T \in N$. At every stage, each political receives an equal probability of

 $^{^{52}}$ One can show that the type of election rule does not affect the main results. We will discuss this in more detail later.

being chosen to make a proposal. If the proposal wins the majority of votes then the politician wins the election. Otherwise, the procedure moves to the next stage. If the final proposal is not passed then the agents return to the status-quo allocation (a^{fm}) . Finally, all agents have the same discount factor δ .

We solve the game and derive its results by backward induction. Since there are four different types of commitment to political proposals and two different options on the maximum-taxation constraint, it is convenient to conduct the analysis in terms of the sub-games which result from the eight different Constitutions that legislators can set. Below, we analyze each in turn, by focusing on the sub-game equilibrium payoff of non-politicians, since this is crucial for the decisions of legislators at stage two. Also note that in all the cases analyzed below it is assumed that at stage one at least two agents have decided to become politicians and one agent is a legislator (the rest of possible cases are examined subsequently).

Sub-game 1: No commitment to political proposals, no taxation constraint

Suppose that at stage two of the game legislators have chosen to impose no constraints on politicians. This means that political proposals are not committing to any dimension and that there is no maximum-taxation constraint. Recall that, given the assumptions made about the structure of the game, this is equivalent to the sub-game where no citizen decides to become a legislator. It is also equivalent to the structure of the games in the citizen-candidate literature.

At stage five, the elected politician chooses the policy that maximizes his utility given the participation constraints of agents. This generally implies positive political rents, in the sense that the elected politician receives higher utility than the level of utility he receives either under the Lindahl allocation or the competitive markets allocation. Formally, politician p solves the following problem:

$$\max_{y,\mathbf{t}} u_p\left(\sum_i t_i - \frac{y}{m}, y\right)$$

subject to
$$u_i(e_i - t_i, y) \ge v_i^{fm}$$
 for $i \ne p$

Apart from the politician, all the other agents lie on their participation constraint. Otherwise, the politician would tax away all their private endowment and they would consume only the public good (recall that $u_i \to 0$, as $x_i \to 0$). Hence, $u_i = v_i^{fm}, \forall i \neq p$.

Given this, at stage four, all non-politicians are indifferent on whom to vote, since their utility levels are not affected by the politician in power, and they vote arbitrarily for some agent (as in section 3.3, only pure strategies are considered). At stage three, politicians also anticipate voters' actions and they are also indifferent on which political platform to propose, as they do not affect the election result. Hence, they arbitrarily propose some policy. However, the main point is that all non-politicians expect to receive $u_i = v_i^{fm}$ from this sub-game.

Sub-game 2: Commitment to the level of the public good, no commitment to taxation, no taxation constraint

At stage five the elected politician is committed to the level of the public good he proposed at stage three. However, we obtain the same result as in sub-game 1. This is due to the non-commitment to taxation and the lack of the maximum-taxation constraint, which allows the politician to tax each agent's private endowment until his participation constraint becomes binding. Hence, non-politicians are indifferent at stage four on whom to vote and politicians' best-response at stage three is to propose the level of the public good that maximizes their utility given the non-commitment to taxation at stage five.

Sub-game 3: Commitment to taxation, no commitment to the level of the public good, no taxation constraint

Since political proposals are committing to individual taxation and not to the public good, politicians' preferences matter for voting behavior. This is because, given the same taxation proposals from politicians, voters prefer the politician who has the strongest preferences for the public good and hence will produce more of it. However, this sub-game may not have an equilibrium in pure strategies and it is analyzed in terms of the sequential voting procedure described earlier. For instance, if all politicians have the same preferences, then any proposal by one politician can be countered by a proposal which slightly decreases taxation on all voters and increases proposed taxation slightly for the politician who made the original offer.

In terms of the sequential voting procedure, at the last stage of political offers, stage T, the politician who makes the offer faces no competition and hence he will propose the allocation that maximizes his utility given the participation constraint of all other agents. Let $P^{max} = \{\mathbf{t}_P^{max}, y_P^{max}\}$, be this proposal. As before, it is easy to check that all participation constraints are binding under P^{max} . Also, let v_p^{max} be the utility level that the elected politician p receives under P^{max} . Let also E be the total number of votes required for passing a proposal: $\frac{n+1}{2} \leq E \leq n$. As a result, the expected utility of a politician p at the beginning of stage T is equal to: $E_T(u_p) = \frac{1}{K}v_p^{max} - \frac{K-1}{K}v_p^{fm}$, where K is the total number of politicians. Note also that the expected utility of a non-politician i at the beginning of stage T is v_i^{fm} .

At the bargaining stage T - 1, the chosen politician q can win the approval of nonpoliticians by offering $Q = \mathbf{t}_Q, y$ such that $u_i(t_{Q,i}, y_Q^{max}) = \delta v_i^{fm}$. If the number of nonpoliticians is greater than the election threshold, $n - K \ge E$, then politician q secures election by proposal Q. Otherwise proposal Q must be such that $u_i(t_{Q,i}, y_Q^{max}) = \delta v_i^{fm}$ for the n - K non-politicians and $u_p(t_{Q,p}, y_Q^{max}) = \delta E_T(u_p)$ for E - (n - K) politicians. If such a proposal Q is not feasible then politician q can not receive adequate support for any of his offers and the game moves to stage T.

The same reasoning applies to any bargaining stage $t \leq T$. This means that in any sub-game perfect equilibrium of the sub-game with commitment to taxation and no maximum-taxation constraint, the maximum equilibrium payoff of non-politicians, $\delta^t v_i^{fm}$ for $\forall i \in N - P$, is strictly less v_i^L . Again, the main result of the analysis is that the expected utility of non-politicians is strictly less than the utility they receive under the Lindahl allocation.

Sub-game 4: Commitment to taxation and to the level of the public good, no taxation constraint

Due to the commitment of political proposals to both dimensions, the sub-game has no equilibrium in pure strategies and it is analyzed in terms of the sequential voting. The analysis is identical to the one in sub-game three. At each stage of the bargaining procedure the randomly chosen politician tries to win the minimum amount of votes require in order to secure the election. This, however, implies that non-politicians receive the discounted value of their participation constraints.

Sub-game 5: No commitment to political proposals, taxation constraint

Political proposals are not committing and hence they are not credible. At the last stage of the game, the politician maximizes his utility subject to the participation and the maximum-taxation constraints. This means that if the politician chooses to produce the level of the public good y, he will impose the maximum taxation possible to each agent for that level of the public good and place agents on their respective offer curves (as long as their participation constraints are not violated). Also, recall that, due to the assumptions on preferences, the utility level of an agent along his offer curve is strictly increasing and that participation constraints are not binding if the Lindahl allocation is provided.

Politician p can not produce any level of the public good above the Lindahl allocation, because this either violates the maximum-taxation constraint for some agent or it is not feasible. If the politician chooses the Lindahl level of the public good it will also impose taxation consistent with the Lindahl allocation, so that agent i receives the final utility that corresponds to the Lindahl equilibrium v_i^L . This is because the summation of the maximum willingness-to-pay of all agents is exactly equal to the inputs needed to produce the the public good at the Lindahl allocation and, hence, any other taxation scheme violates the maximum-taxation constraint for at least one agent.

If the politician in power reduces the level of the public good below the Lindahl, then his utility may increase because the maximum willingness-to-pay of the agents relaxes and he can extract political rents for private consumption, but it also may decrease because the level of the public good is reduced. Adopting the same notation as before, $t_i^m(y) = \frac{ys_i^m(y)}{m}$ is the maximum taxation which can be imposed on agent *i* for the level of the public good *y*, where $s_i^m(y)$ is the maximum willingness-to-pay of agent *i*. Given that the budget constraint of the politician is given by $x_p + y/m \leq e_p + \sum_{i \neq p} t_i^m(y)$, the overall effect in his utility by a small reduction of the public good below the Lindahl level is given by:

$$-\frac{du_p}{dy}|_{y=y_L} = -\frac{\partial u_p}{\partial x_p} \left[-\frac{s_p^m(y_L)}{m} + \frac{y_L}{m} \sum_{i \neq p} \frac{\partial s_i^m(y)}{\partial y}|_{y=y_L} \right] - \frac{\partial u_p}{\partial y}$$

In the sum above, the first term is positive (both terms inside the brackets are nega-

tive) and reflects the marginal increase in utility due to the increase in the consumption of the private good, while the second term is negative and reflects the marginal decrease in utility due to the decrease of the public good. If the first term is greater than the second in absolute values, then the politician prefers to decrease the level of the public good below the Lindahl. In this case, agents final utility decreases as they move along their offer curves to lower levels of the public good. If the second term is greater than the first, then the politician imposes the Lindahl allocation of the economy.

At stage 4 agents anticipate this behavior by the elected politician. Therefore, they vote for the politician who will choose the highest level of the public good at stage 5, and this voting behavior is independent of any political proposal. As a consequence, any combination of political proposals at stage 3 is an equilibrium of this sub-game and agents, except for the preferred politician, receive at most the utility levels of the Lindahl allocation (v^L) .

Sub-game 6: Commitment to the level of the public good, no commitment to taxation, taxation constraint

This is effectively Case III of section 3.3. Since political proposals are committing to the level of the public good only and the maximum-taxation constraint holds, politicians compete on who will offer the highest level of the public good. As a consequence, in the equilibrium of this sub-game, non-politicians will end up receiving v_i^L .

Sub-game 7: Commitment to taxation, no commitment to the level of the public good, taxation constraint

Since this is one of the sub-games that may have no equilibrium in pure strategies, the

analysis follows closely the one for sub-game 3. The only difference is that politicians can not tax agents more than their maximum willingness-to-pay due to the maximumtaxation constraint. Let $P^{mw} = \{\mathbf{t}_p^{mw}, y_p^{mw}\}$ be the policy that maximizes the utility of politician p under the maximum taxation and individual participation constraints. As was shown for the case where there is no commitment to any dimension of political proposals but the maximum-taxation constraint is imposed (see also sub-game 5), $y^{mw} \leq y^L$ and therefore for any agent i other than the politician it holds that $u_i(t_i^{mw}, y^{mw}) \leq u_i(t_i^L, y^L)$.

Therefore, at the last stage of proposals, stage T, if politician p is chosen, he makes the offer $P^{mw} = {\mathbf{t}_p^{mw}, y^{mw}}$ and agents vote for it. Then, by backward induction and by using the same reasoning as in sub-game three, at every stage $t \leq T$ the chosen politician makes a proposal which gives to the rest of the agents the maximum between their continuation value of the game and their participation constraint. In all possible cases, the maximum utility level for non-politicians does not exceed v_L^i .

Sub-game 8: Commitment to taxation and the level of the public good, taxation constraint

The analysis of sub-game 7 also implies in this case.

Stage 2:

It is clear that the critical stage is stage 2, at which legislators decide the Constitution, given that they know the identity of all politicians. Using the analysis that preceded, we conclude that it is a weakly dominant strategy for legislators to set the constitutional rules of sub-game 6. This is because these rules ensure that the Lindahl allocation will be implemented by political competition irrespectively of the identity of the politicians⁵³.

To see this, recall that in sub-games 1 to 4, legislator l expects to receive utility equal to v_l^{fm} , in other words, his participation constraint is binding. In sub-game 5, the legislator's expected utility is dependent on the preferences of the elected politician and his utility is at most v_l^L , only if the politicians' most preferred level of the public good (given constitutional restrictions) is y^L (which, in turn, requires specific restrictions on the politicians' preferences). In sub-games 7 and 8, the legislators expected utility depends on the identity of all politicians, since they all have a chance of being elected at stage one of bargaining, and his expected utility is at most v_l^L , only if all politicians' most preferred allocation is the Lindahl.

In contrast, in the equilibrium of sub-game 6, l expects to receive v_l^L , irrespectively of the preferences of politicians. Notice also that it is a weakly dominant strategy for every legislator to set the constitutional rules of sub-game 6, irrespectively of his preferences. Therefore, in any sub-game perfect equilibrium of sub-stage two in which there is at least one legislator and two politicians, the Lindahl allocation will be the equilibrium outcome. If, furthermore, none of the politicians' preferences satisfy the condition described at sub-stage 5 (i.e. the restrictions on preferences that make the Lindhal allocation the most preferred one by a politician), then the unique optimal action for all legislators is to make political proposals committing to the level of the public good only and to impose the maximum-taxation constraint.

Notice also that these equilibria are independent of the election rule for either the Constitution (the way legislators decide on the political constraints) or the politician (the way voters decide on who will be the elected politician).

⁵³It remains to be shown that political competition will indeed be part of the equilibrium path at stage one.

Stage 1: Political Entry

Finally, at stage one, the entry of citizens on political competition and the legislation authority is analyzed. Clearly, it is a best-response for at least one agent to become a politician. Since a^{fm} is generally below the Pareto frontier, if one agent becomes a politician then, under any constitution, he can strictly improve his utility by proposing his most preferred allocation given constitutional restrictions.

We consider different constitutional cases in order to examine whether a second agent prefers to become a politician. Assume that there is at least one citizen who decides to become a legislator at stage one. If agent p decides to become a politician and his preferences satisfy the condition of sub-game 5, then legislators will be indifferent between setting the rules of sub-game 5 or 6 at stage two. Given that, any citizen at stage one will be indifferent between becoming a politician or a legislator and remaining as a citizen, as the final outcome will be the Lindahl allocation, irrespectively of how many other agents become politicians.

If, however, p does not satisfy the condition of sub-game 5 (which is generally the case) then there are two cases to consider. If there is no legislator, then there is no commitment to political proposals, any non-politician i expects to receive v_i^L and therefore he is indifferent on whom to vote. If another agent p' decides to become a politician, then he may win the election if he is favored by the arbitrary strategies of voters when they are indifferent on whom to vote or by the election rule in case of a tie. If such p' exists then it is a best-response for him to become a politician and hence it can not be an equilibrium of the game p to be the only politician.

If there is no other agent p' who is favored over p, then all other agents are indiffer-

ent on whether to enter political competition or not. However, this may still not be an equilibrium of the game. If p preferences are such that, under the maximum-taxation constraint, his most preferred policy implies strictly greater utility than v_l^L for some agent l, then this agent's best-response is to become a legislator instead of remaining a citizen in order to impose the maximum-taxation constraint at stage two. In other words, there is an equilibrium of the game such that agent p becomes the only politician and all the other agents remain citizens, but it requires restrictions on the set of preferences for p and specific pure-strategies for voting when voters are indifferent on whom to vote (for example, always vote for agent 1 whenever his proposal is equivalent to any other politicians').

First, notice that the above equilibrium of the game is equivalent to the equilibria generated by the citizen-candidate models. However, unlike these models, the positive-political-rents equilibrium holds only for specific preference profiles and only if one considers specific voting strategies for the breaking. Hence it is not a general equilibrium of the game. Notice also that, whenever the conditions for this equilibrium are fulfilled there are also other equilibria of the game with zero political rents (which are derived shortly). Finally, this equilibrium is not robust if there is an infinitesimally small but positive probability that agents would vote for another candidate (since that agent's best-response would be to enter the political contest in order to reap the infinitesimally small expected political-rents).

On the other hand, if there is at least one legislator, then, by becoming a politician, another agent p' ensures that it is the best response for the legislator to choose the constitutional restrictions of sub-game 6 and hence ensures a minimum payoff of $v_{p'}^L$. In other words, apart from some special cases described above, it is not an equilibrium of the game for only one agent to become a politician. Therefore, the set of politicians is greater or equal to two.

Stage 1: Entry of Legislators

We now come to examine if any citizen decides to become a legislator. First, suppose that there are no legislators, a number of citizens greater or equal to one and at least two politicians. Then this is clearly not an equilibrium outcome, since at least one of the agents can improve his utility by becoming a legislator, imposing the constitutional restrictions of sub-game 6 and receiving a final payoff of $v_i^L > v_i^{fm}$. This means that the only potential equilibrium with no legislators, apart from a special case examined above, is the one where all agents decide to become politicians.

We have also examined the case where there is no legislator and only one politician and under which conditions it may turn out be an equilibrium of the game. The remaining alternatives to consider are the following cases: (i)all agents become politicians and (ii) there is at least one politician and at least one legislator.

Suppose that all N agents decide to become politicians. If voting is sincere, then each agent votes for his own proposal and agent 1 is elected (recall that in the case of ties, the politician with the lowest index is elected). This can not be an equilibrium of the game, as at least one other politician can increase his utility by becoming a legislator instead and ensuring a payoff of v_i^L for himself. If voting is not sincere, then under any pure strategy profile, a specific politician will be chosen (for example, all politicians vote for p). This can not be an equilibrium either by the same argument as above. Hence, there is no pure-strategy equilibrium of the game with all agents deciding to become politicians⁵⁴.

Consider the case where there is at least one legislator and only one politician. This

⁵⁴Things get a little more complicated under mixed-strategy equilibria, but one can show that if the number of agents is sufficiently large, then there is at least one agent who prefers to become a legislator

is an equilibrium of the game only if p's preferences satisfy the condition of sub-game 5. If this condition is satisfied then legislators set the maximum-taxation constraint and this is sufficient for ensuring that the final policy implements the Lindahl allocation (as was shown in sub-game 5). The rest of the agents of the economy are indifferent whether they become politicians or not and hence there is no profitable deviation. Once again, this is a special type of equilibrium that holds for specific preference profiles.

However, the following class of equilibria holds for all possible preference profiles⁵⁵. Suppose that at least two agents decide at stage one to become politicians and at least one to become legislator. Then the best-response of legislators at stage two is to impose the maximum-taxation constraint and make political proposals committing to the level of the public good only (this is a best-response for legislators for all preference profiles, unlike the previous case examined above). As a consequence and due to the analysis in sub-game 6, all agents receive the level of utility corresponding to the Lindahl allocation.

This is a sub-game prefect equilibrium of the game, because no one can unilaterally deviate and become better off. If the number of politicians is greater than two and the number of legislators in greater than one, a unilateral deviation by any agent does not affect equilibrium institutions or political proposals. If, on the other hand, there are only two politicians, none of them wants to exit political competition as in the best of cases his final utility will remain unchanged and in the worst it will strictly decrease. Likewise, if there is only one legislator there is no other strategy for him that can strictly increase his utility and therefore it is a best-response for him to remain a legislator. However, apart from at most three agents, the rest of the citizens are indifferent on what social role to choose and hence any distribution of agents between politicians, legislators and citizens, which is consistent with the above results, is an equilibrium of the game.

 $^{^{55}}$ Because the conditions for the other "special-case" equilibria are mutually exclusive, there are preference profiles for which the general class of equilibria is the only class of equilibria of the game

Proposition 4, below, summarizes the analysis so far for the class of equilibria which are independent of the preference profile of the economy.

Proposition 4: The game of section 3.4 has multiple sub-game perfect pure-strategy equilibria. However, the following class of equilibria is the only class of equilibria that holds for all possible preference profiles of the game: At least two agents decide to become politicians, at least one becomes a legislator, political proposals are committing to the level of the public good only and the maximum-taxation constraint holds. The equilibrium allocation is the Lindahl allocation.

First, the following assumptions are crucial for the results: (i) the Anonymity of the Taxation Constraint, and (ii) the restriction that an agent can hold only one power. Other assumptions can be relaxed without affecting the equilibria of the game. For instance, the choice rule through which legislators decide the Constitution plays no role, since in equilibrium, all legislators agree on the desirable set of restrictions. It was used only for the facilitation of the analysis.

Second, in this game, almost all political institutions required for the implementation of an efficient allocation of resources, arise endogenously. Legislators decide what type of restrictions to set to voters and politicians. Proposals are also made endogenously by politicians. The anonymity condition and the separation of powers are the only institutions which are not created by agents. However, as far as separation of powers is concerned, it should be noted that agents have the choice between utilizing this institution or not. Since both types of power are used in equilibrium, it makes sense to say that separation of powers emerges endogenously.

3.5 Conclusion

Centralized decision making is very helpful for the solution of the free riding problem, but, without any set of restrictions on the authority that implements it, inefficiencies, in the form of political rents, arise. This chapter shows why political competition is a necessary but not sufficient condition for political efficiency. Other forms of institutional restrictions, like restrictions to maximum taxation, are required for aligning political incentives with societal interests, so that voting games achieve equilibria, which otherwise they would not have. It is also worth noting that the focus the analysis is on public goods, because private goods do not exhibit externalities and therefore, if centralized decisions fail to provide efficient outcomes, this is not crucial for societal welfare. Competitive markets could be used, instead, to allocate resources. In other words, the reason why we examine the role of political institutions is exactly because they impact the efficiency of social decisions when they are needed the most: to solve problems which involve public goods.

This chapter takes the analysis one step further, by asking whether and how the required political institutions can emerge endogenously. The answer given to this question is to the affirmative. In the extended political game of section 3.4, it was shown how separation of power can arise endogenously and how legislators select appropriate institutions in order to limit the extractive powers of politicians. Thus, the point made is that whenever collective decisions may increase societal welfare, agents have an incentive to devise and agree upon appropriate political institutions so that the decision process does not break down. In fact, because the equilibrium outcomes of the game presented coincide with the Lindahl allocations of the economy, one can say that agents have the incentive to devise appropriate institutions so that they limit the rents of politicians.

There is a variety of dimensions which the game can be extended to, while retaining

its power, and most of these dimensions were discussed in the preceding sections. The next step is to generalize the game for economies with multiple public goods and find if additional political institutions are necessary for achieving the same set of equilibria. We also intend to examine how these institutions can emerge by the actions of the agents. Another question of interest is whether the results can be extended to economies with asymmetric information. In this case, what are the political incentives for selecting a specific mechanism and through which institutions do agents align political interests with their own? These questions are left for future research.

Appendix A: Appendix for Chapter One

Lemma 1: Let PF(E) be the Pareto Frontier of economy E. Then, for every allocation a on the Pareto Frontier, there exists at least one agent $i \in I$, who does not envy the allocation of any other agent: $U_i(a_i) \ge U_i(a_j), \forall j \in I$.

Proof: Suppose that the claim does not hold. Then, all agents envy at least one other agent: $\forall a_i \exists j \in I, j \neq i : U_i(a_j) > U_i(a_i)$. But, since this holds for all agents, then there exists at least one reassignment of individual allocations among the I agents such that some of them are made strictly better-off and the rest remain as well-off as under a.

In order to find one such reassignment, use the following algorithm. Pick an arbitrary $i \in I$ and let $\overline{i} = \{j \in I : U_i(a_j) > U_i(a_i)\}$, be the set of agents whom i envies. Reassign a_j , for some $j \in \overline{i}$ to i. If $i \in \overline{j}$, then reassign a_i to j and stop the reassignment. If $i \notin \overline{j}$, then reassign some a_h , $h \in \overline{j}$ to j and then proceed to agent h. Continue until you reach some agent k, such that either $i \in \overline{k}$ or there exists some $l \in \overline{k}$, whose allocation a_l has already being reassigned. In the first case, reassign to k allocation a_i and stop the reassignments. In the latter case, ignore all reassignments preceding agent l (these agents retain their original allocations), reassign to l the allocation a_k and stop the reassignments.

Since the set of agents is finite and all agents envy at least one allocation, after at most I reassignments, the algorithm above will end-up in some agent, whose allocation has already been reassigned, or the first agent, where reassignment started. In this case, a reassignment of allocations has been found, which makes some agents in I better-off (from agent l until agent k) while the rest remain equally well-off. This constitutes a

Pareto improvement and violates the initial assumption that $a \in PF(E)$.

Lemma 2: For every allocation a on the Pareto Frontier, there exists at least one agent $i \in I$, whose allocation is not envied by any other agent: $U_j(a_j) \ge U_j(a_i), \forall j \in I$.

Proof: The proof is similar to the proof of Lemma 1. Suppose that the claim does not hold. Then, all agents are envied by at least one other agent: $\forall a_i \exists j \in I, j \neq$ $i : U_j(a_i) > U_j(a_j)$. But, this implies that there exists at least one reassignment of individual allocations among the I agents such that some of them are made strictly better-off and the rest remain as well-off as under a.

In order to find one such reassignment, use the following algorithm. Pick an arbitrary $i \in I$ and reassign a_i to one of the agents in the set $\underline{i} = \{j \in I : U_j(a_i) > U_j(a_j)\}$. Then reassign a_j . If $i \in \underline{j}$, then reassign a_j to i and stop the reassignment. If $i \notin \underline{j}$, then reassign a_j to some arbitrary $h \in \underline{j}$ and repeat the reassignment. Continue until you reach some agent k, such that there exists some $l \in \underline{k}$, whose allocation a_l has already being reassigned. Ignore all reassignments preceding agent l (these agents retain their original allocations), reassign to l the allocation a_k and stop the reassignments.

Since the set of agents is finite and all allocations are envied by at least one agent, after at most I reassignments, the algorithm above will end-up in some agent whose allocation has already been reassigned. In this case, we have found a reassignment of allocations which makes some agents in I better-off while the rest remain equally welloff. This constitutes a Pareto improvement and violates the initial assumption that $a \in PF(E)$. **Corollary 1:** If $a \in PF(E)$, then Lemma 1 and 2 hold for any subset of I. Namely, let $\check{I} \subseteq I$ and let $\check{A} = \{a_i : i \in \check{I}\}$. Then, if $a \in PF(E)$, Lemma 1 and 2 hold for \check{I} with regards to \check{A} as well.

Proof: Take any subset of agents \check{I} of the set I. Suppose that Lemma 1 and 2 do not hold over the set \check{A} , which is the set of individual allocations of the agents in \check{I} . Then, it is possible to find a reassignment of allocations between the agents in \check{I} , such that some of them will be made better-off while the rest remain as well-off. But that is a Pareto-improvement for some agents in I, which contradicts the assumption that $a \in PF(E)$.

Lemma 3: If the LNCIP holds, then around the neighborhood of any individual allocation a_i , there exists a set of allocations such that each agent of a certain type prefers a particular allocation over the rest.

Proof: Recall that $C_{i\epsilon}(a) = \{c \in A : U_i(c|\vartheta_i, \boldsymbol{\theta}_{-i}) = U_i(a|\vartheta_i, \boldsymbol{\theta}_{-i}), \|c-a\| < \epsilon\}.$ Also, define $L_j(a_i)$ to be the lower-contour set of agent j associated with allocation a_i : $L_j(a_i) = \{c \in A : U_j(c|\vartheta_j, \boldsymbol{\theta}_{-j}) < U_j(a_i|\vartheta_j, \boldsymbol{\theta}_{-j})\}$ and $V_j(a_i)$ to be the upper-contour set: $V_j(a_i) = \{c \in A : U_j(c|\vartheta_j, \boldsymbol{\theta}_{-j}) > U_j(a_i|\vartheta_j, \boldsymbol{\theta}_{-j})\}.$

H is a $L \times S - 1$ hyper-plane, which passes through a_i , and is perpendicular to the MRS of some type's indifference curve, which also passes through a_i . **H** splits the space of allocations in two sub-spaces, A_1 and A_2 . In each of these sub-spaces, and due to the LNCIP, there exists some $\bar{\epsilon} > 0$ such that for every $\epsilon < \bar{\epsilon}$, within the open ball $B_{\epsilon}(a_i)$, the upper contour set of a type is a subset of the upper contour set of some other type (see also the picture below).



Figure 10: LNCIP and Local Incentive Compatibility

Say that agent k is the type with the smallest upper contour set within ball $B_{\epsilon}(a_i)$ and subspace A_1 : $V_k(a_i) \bigcap B_{\epsilon}(a_i) \bigcap A_1 \subset V_l(a_i) \bigcap B_{\epsilon}(a_i) \bigcap A_1, \forall l \in \Theta$. Then, there exists some allocation $b \in B_{\epsilon}(a_i)$ such that a_i is strictly preferred to b by agents of type k, but the agents of all other types strictly prefer b to a_i : $b \in L_k(a_i)$ and $b \in V_l(a_i), \forall l \in \Theta$.

Likewise, there exists allocation c, which does not belong in the two smallest upper contour sets within $B_{\epsilon}(a_i)$ but it is within all the other upper contour sets, which means that a_i is strictly preferred by type k to b and c, b is strictly preferred by the type with the second smallest contour set to a_i and c and all the other types prefer c to a_i and b. By induction, one can construct $\Theta - 1$ allocations in the ϵ -neighborhood of a_i , such that the agents of one type strictly prefer one allocation over all the other.

Proposition 3: In the space of mechanisms, which permit sub-games with no equilibrium in pure strategies, Condition 1 is sufficient for full implementation.

Proof: Suppose that $G_I(P, A)$ is a simultaneous move game $G : P \to A$ with I players, and assume that G has no Nash equilibrium in pure strategies (examples include Jackson (1991) and Maskin (1999)). Also, arbitrarily restrict the payoffs of G_I such that the maximum possible payoff for any type is lower than if he were to receive the first-best allocation of any other type⁵⁶. Let $R_i(p_{-i}, G)$ be the best-response correspondence of agent i if game G_I is played. Finally, suppose that Condition 1 is satisfied and that the interim distribution of types, β , is common knowledge. The mechanism below fully implements any Pareto efficient allocation which satisfies Anonymity.

Each agent reports his type m_i and a final allocation is received according to the following mechanism M(g, a):

- (i) If $\mathbf{m} \in \Theta(\beta)$, then $a_i(m_i, \mathbf{m}_{-i}) = a^*(m_i, \beta), \forall i \in I$.
- (ii) If m is such that for two types, (ϑ, ϑ'), the number of reported agents is different from number of agents in the interim-distribution by one, specifically λ_m(ϑ) = λ_β(ϑ) + 1, λ_m(ϑ') = λ_β(ϑ') − 1, then:
 - If Rank(ϑ) = Rank(ϑ'), agents who reported types ϑ, ϑ' choose an allocation from the set {a^{*}_ϑ ϵ, a^{*}_{ϑ'} ϵ}. ϵ is strictly positive for all state-contingent commodities and it is sufficiently small so that a^{*}_ϑ ϵ ≻_ϑ a^{*}_{ϑ'} and a^{*}_{ϑ'} ϵ ≻_{ϑ'} a^{*}_ϑ.
 - If $Rank(\vartheta) > Rank(\vartheta')$, agents who reported types ϑ, ϑ' choose an allocation from the set $\{a_{\vartheta}(\vartheta, \vartheta'), a_{\vartheta'}(\vartheta, \vartheta')\}$. $a(\vartheta, \vartheta')$ satisfies Condition 1.
 - If $Rank(\vartheta) < Rank(\vartheta')$, agents who report type ϑ' receive allocation $a_{\vartheta'}^*$ and agents who report type ϑ receive allocation $\frac{\lambda_{\beta}(\vartheta)}{\lambda_{\mathbf{m}}(\vartheta)}a_{\vartheta}^*$.
 - For all $m_k \neq \{\vartheta, \vartheta'\}$, $a_k(m_k, \mathbf{m}_{-i}) = a^*(m_k, \beta)$.

 $^{^{56}\}mathrm{An}$ easy way to do this is to multiple all payoffs of G_I with an arbitrarily small but positive number.

(iii) For any other case, the mechanism induces the game G_I .

If more than one misreport is detected, M induces G_I , which has no equilibrium⁵⁷. Therefore, there can be no equilibrium of the mechanism where agents believe that more than two misreports will be detected. Conditional on that, it is a strictly dominant strategy for the agents of the highest rank to report truthfully their type. To see this, take agent i of type ϑ and suppose that his rank is K. Agent i's only possible equilibrium beliefs are that: either (i) all other agents will report truthfully or (ii) one other agent will misreport or (iii) there will be multiple misreports but they will cover each other (e.g. type ϑ_k reporting as type ϑ_l and vice versa) apart from one. Case (ii) and (iii) are strategically equivalent for i as his response induces the same allocation.

If *i* believes that all other agents will report their type truthfully then his best response is to report truthfully as well. Otherwise, he receives either the allocation $a_{\vartheta}^* - \epsilon$, if he misreports his type as of another type with equal rank, or the allocation $\frac{\lambda_{\beta}(\vartheta')}{\lambda_{\mathbf{m}}(\vartheta)}a_{\vartheta'}^*$, where ϑ' is of lower rank than *i*. Clearly, *i* strictly prefers a_{ϑ}^* to the above allocations and his best response is to report his type truthfully.

If, on the other hand, *i* believes that an agent (say *j* of type ϑ') of rank(K) will misreport his type to ϑ , then by reporting truthfully he receives $a_{\vartheta}^* - \epsilon$, while by reporting type ϑ' he receives $a_{\vartheta'}^*$. By construction, $a_{\vartheta}^* - \epsilon \succ_{\vartheta} a_{\vartheta'}^*$. If *i* reports any other type then his payoff will be even less due to the restrictions on the payoffs of G_I . Hence, *i*'s best response is to report truthfully.

If *i* believes that an agent (say *m* of type ϑ'') of a lower rank will misreport his type to ϑ , then a similar argument goes through. Reporting truthfully is strictly preferred to reporting any other type, since $a_{\vartheta}(\vartheta, \vartheta'') \succ_{\vartheta} a_{\vartheta''}^*$. Finally, if *i* believes that some agent *m* of type ϑ_m will misreport his type to ϑ_n , then *i* prefers reporting truthfully

⁵⁷More than one misreport detected means that either $\lambda_{\beta}(\vartheta) \neq \lambda_{\mathbf{m}}(\vartheta)$ for more than two types or that $\lambda_{\mathbf{m}}(\vartheta) - \lambda_{\beta}(\vartheta) \ge 2$ and $\lambda_{\mathbf{m}}(\vartheta') - \lambda_{\beta}(\vartheta') \le 2$ for some types ϑ, ϑ' .

and receiving a_{ϑ}^* to reporting untruthfully and receiving some payoff induced by G_I .

Hence, for all beliefs that can be consistent with equilibrium, all agents of rank K strictly prefer to report their type truthfully. Given this and by following the same reasoning, agents of rank(K-1) strictly prefer to report truthfully as well. By induction and iterated elimination of strictly dominated strategies, we conclude that all ranks will report truthfully and hence the unique Bayes-Nash equilibrium of the mechanism is for all agents to report their type truthfully.

Proposition 4: Condition 1 and 2 are jointly sufficient for full implementation.

Proof: Suppose that Condition 1 and 2 are satisfied and that the interim distribution of types, β , is common knowledge. The mechanism below fully implements any Pareto efficient allocation which satisfies Anonymity. Each agent reports his type m_i and a final allocation is received according to the following mechanism M(g, a):

(i) If
$$\mathbf{m} \in \Theta(\beta)$$
, then $a_i(m_i, \mathbf{m}_{-i}) = a^*(m_i, \beta), \forall i \in I$.

- (ii) If m is such that for two types, (ϑ, ϑ'), the number of reported agents is different from number of agents in the interim-distribution by one, specifically λ_m(ϑ) = λ_β(ϑ) + 1, λ_m(ϑ') = λ_β(ϑ') − 1, then:
 - If Rank(ϑ) = Rank(ϑ'), agents who reported types ϑ, ϑ' choose an allocation from the set {a^{*}_ϑ − ϵ, a^{*}_{ϑ'} − ϵ}. ϵ is strictly positive for all state-contingent commodities and it is sufficiently small so that a^{*}_ϑ−ϵ ≻_ϑ a^{*}_{ϑ'} and a^{*}_{ϑ'}−ϵ ≻_{ϑ'} a^{*}_ϑ.
 - If $Rank(\vartheta) > Rank(\vartheta')$, agents who reported types ϑ, ϑ' choose an allocation from the set $\{a_{\vartheta}(\vartheta, \vartheta'), a_{\vartheta'}(\vartheta, \vartheta')\}$. $a(\vartheta, \vartheta')$ satisfies Condition 1.

- If $Rank(\vartheta) < Rank(\vartheta')$, agents who report type ϑ' receive allocation $a_{\vartheta'}^*$ and agents who report type ϑ receive allocation $\frac{\lambda_{\beta}(\vartheta)}{\lambda_{\mathbf{m}}(\vartheta)}a_{\vartheta}^*$.
- For all $m_k \neq \{\vartheta, \vartheta'\}, a_k(m_k, \mathbf{m}_{-i}) = a^*(m_k, \beta).$

(iii) For any other case, an allocation \tilde{a} , which satisfies Condition 2, is implemented.

The mechanism above is identical to the mechanism of Proposition 3, with the only exception that, if more than one misreport is detected, then instead of inducing a game without an equilibrium, the mechanism provides an allocation which is constructed according to Condition 2. By construction of \tilde{a} , all types prefer to report truthfully if they believe that many misreports will be detected.

Therefore, even if a rank(K)-agent believes that there will be several detections of misreports, he still prefers to report truthfully. He also prefers to report truthfully than reporting any other type, if he believes that there is only one misreport $(a_{\vartheta}(\vartheta, \vartheta'') \succ_{\vartheta} a^*_{\vartheta''} \succ_{\vartheta} \tilde{a}_{\vartheta'})$. Since his best-response remains the same for all other beliefs, this means that any agent of rank(K) has a strictly dominant strategy to report truthfully. Therefore, by following the same reasoning as in the proof of Proposition 3 and by iterated elimination of strictly dominated strategies, we conclude that the mechanism has a unique Bayes-Nash equilibrium, at which all agents report their type truthfully. \blacksquare

Appendix B: Appendix for Chapter Two

Proposition 1: Let $\overline{p} = k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})}$. Consider the social contract S^* , which defines:

(i) if trade of good g takes place, agent ξ receives subsidy (negative taxation)

$$\tau_{\xi 1} = (1 - f(\overline{e}, \overline{e})) \left(\beta_{\xi} (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) < 0.$$

(ii) if trade of good g does not takes place, agent ξ pays out taxation

 $\tau_{\xi 0} = f(\overline{e}, \overline{e}) \left(-\beta_{\xi} (v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\overline{e}, \underline{e})} \right) > 0.$

(iii) any private contract $\pi(0, \hat{p}, I_g)$ or $\pi(\hat{q}, p, I_g)$, with $\hat{p} > \overline{p}$ or $\hat{q} > \overline{p} - p$ is nonenforceable.

Then S^* implements the first best effort levels and it is renegotiation-proof.

Proof of Proposition 1: Without any subsidization of effort, the farmer and the producer will trade the good only in the low-cost state at the expected price $p = (1 - \beta_i)v - \beta_i k_L$. The expected utility from trade is $v - p = \beta_i (v - k_L)$ for the farmer and $p - k_L = (1 - \beta_i)(v - k_L)$ for the plough-maker.

Agent ξ exerts high effort in the production of good g if the expected utility of high effort is higher than the expected utility of low effort. Assuming that trade takes place at the expected price and given the beliefs of ξ for the effort level of ζ :

$$e_{\xi} = \overline{e} \Leftrightarrow f(\overline{e}, e_{\zeta}) \left(\beta_{\xi}(v - k_L) - \tau_{\xi 1}\right) + \left(1 - f(\overline{e}, e_{\zeta})\right) \left(-\tau_{\xi 0}\right) - \overline{c} \ge f(\underline{e}, e_{\zeta}) \left(\beta_{\xi}(v - k_L) - \tau_{\xi 1}\right) + \left(1 - f(\underline{e}, e_{\zeta})\right) \left(-\tau_{\xi 0}\right) - \underline{c} \Leftrightarrow$$

 $\tau_{\xi 0} - \tau_{\xi 1} \ge -\beta_{\xi} (v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, e_{\zeta}) - f(\underline{e}, e_{\zeta})}$

Since we require both agents to exert high effort⁵⁸: $\tau_{\xi 0} - \tau_{\xi 1} \ge -\beta_{\xi}(v - k_L) + \frac{\overline{c-c}}{f(\overline{c},\overline{c}) - f(\underline{c},\overline{c})}$. This expression gives the minimum required wedge between taxation in the two states in order to induce high effort for ξ . Notice that the right hand side of the inequality is positive due to the condition imposed by inequality (1) in section 2.2. On the other hand, incentive compatibility requires that the producer of the good, who suffers from the high production costs in state H, does not want to trade in that state. The farmer may be willing to offer him an increased price in order to cover the high production cost, in which case incentive compatibility is impossible to satisfy without restrictions on the transfers that the two agents can do. Indeed, if the farmer pays an expected price \hat{p} , incentive compatibility for the producer requires(notice that in this case, since we are referring to the producer, $\xi = j$):

$$\hat{p} - k_H - \tau_{j1} \leqslant -\tau_{j0} \Leftrightarrow \tau_{j0} - \tau_{j1} \leqslant k_H - \hat{p}$$

Combining the two inequalities above, the combination of incentive compatibility and effort exertion conditions, gives:

$$-\beta_{j}(v-k_{L}) + \frac{\overline{c}-\underline{c}}{f(\overline{e},\overline{e})-f(\underline{e},\overline{e})} \leqslant k_{H} - \hat{p} \Leftrightarrow$$
$$\hat{p} \leqslant k_{H} + \beta_{j}(v-k_{L}) - \frac{\overline{c}-\underline{c}}{f(\overline{e},\overline{e}) - f(\underline{e},\overline{e})}$$
(18)

Any net transfer of resources higher than the right hand side of (18) will lead to violation of incentive compatibility and trade in the high cost state, which then leads to low-effort exertion. This is because agents know that they will trade and receive the

⁵⁸One may worry that coordination failures are possible if both agents believe that the other agent will exert low effort level and if $f(\overline{e},\overline{e}) - f(\underline{e},\overline{e}) > f(\overline{e},\underline{e}) - f(\underline{e},\underline{e})$. In this case one can define the difference between taxation levels in terms of the $min\{f(\overline{e},\overline{e}) - f(\underline{e},\overline{e}), f(\overline{e},\underline{e}) - f(\underline{e},\underline{e})\}$. This ensures that it is a strictly dominant strategy for each agent to exert high effort.

subsidies in any state of the world. Let $\overline{p} = k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})}$. First, note that \overline{p} is greater than the fair price p:

$$\overline{p} > p \Leftrightarrow k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})} > (1 - \beta_i)v + \beta_i k_L \Leftrightarrow k_H - k_L > \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})}$$

The inequality above holds, due to the assumptions of the model and inequality (1):

$$k_H - k_L > v - k_L > \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})}$$

Since $\overline{p} > p$, the producer will not accept to trade in the high cost state without receiving an additional payment above the "fair" price. So violation of incentive compatibility from one side requires a higher exchange price or a side contract, actions which themselves are verifiable. Second, the farmer is willing to pay a price as high, because the utility of trading under this price is greater than the utility of not-trading:

$$v - \overline{p} - \tau_{i1} > \tau_{i0} \Leftrightarrow v - k_H - \beta_j (v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})} - (\tau_{i0} - \tau_{i1}) > 0 \Leftrightarrow$$

$$k_L - k_H + 2 \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})} > 0$$
, which holds due to inequality (3) in chapter 2.

Therefore, inducing high-effort exertion and incentive compatibility, require that any transfer $\hat{p} > \overline{p}$ or any side-payment $\hat{q} > \overline{p} - p$ is non-enforceable. Required taxation conditional on trade is given by:

$$\tau_{\xi 0} - \tau_{\xi 1} \ge -\beta_{\xi} (v - k_L) + \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - f(\underline{e}, \overline{e})}$$
(19)

$$f(\overline{e},\overline{e})\tau_{\xi 1} + (1 - f(\overline{e},\overline{e}))\tau_{\xi 0} = 0$$
(20)

This system provides the required taxation levels as given by the proposition. We turn now to the issue of renegotiation-proofness. Consider first the ex-post renegotiation. If the low-cost state arises then the farmer and the producer can enforce the trade and receive the subsidies by making the same trade agreement as the one they would have made in the absence of the social contract. This provides them the maximum expost payoff. Therefore, any renegotiation of the social contract can not increase their payoffs and agents have no incentive to renegotiate it with the institutional agent. If the high-cost state arises, then the two agents can increase their pay-offs if they modify the social contract so that private contracts are enforceable. In such a case, net transfers of resources between the two agents are allowed and they can extract the subsidies by trading at the pre-specified price p, even though this is suboptimal. Since it is the institutional agent who bears the true cost of this sub-optimal transaction, both i and jwould agree on it. However, the institutional agent anticipates the renegotiation of the social contract in the bad state. Furthermore, any credible reward the agents are willing to provide to him in order to agree to renegotiate does not cover for the losses by paying out the subsidies. Therefore, the institutional agent does not agree to renegotiate the social contract in period 4.

A similar type of argument shows that the institutional agent does not agree to renegotiate the social contract in the interim stage. \blacksquare

Proposition 2 below provides the necessary conditions for the implementation of firstbest effort levels. It is proven for a more general environment, where effort levels are continuous and hence inequalities (1)-(2) are not necessary. Condition (3) is substituted for $v - k_L > k_H - v$, but the interpretation and role is the same. Due to the Revelation Principle, we restrict attention to direct mechanisms S, where agents send messages about the state of the world and the mechanism determines if the good is traded or not, at what price and if subsidies are given.

Proposition 2: The existence of the institutional agent and the non-enforcement of private contracts contingent on trade are necessary conditions for the implementation of first-best effort levels.

Proof of Proposition 2: First, we prove that any renegotiation-proof mechanism S that does not include an institutional agent ($\Sigma = \emptyset$) can not implement the first-best effort levels. Consider any direct mechanism $S(., \Sigma = \emptyset)$, and assume that the mechanism infers that the cost is low if the agents send the messages $\hat{\mathbf{m}}$. Implementation of the first-best effort levels requires that trade takes place and appropriate subsidies are provided only in the low-cost state. Therefore, in terms of ex-post utility, the combination of incentive compatibility and efficiency requires:

$$u_i(\hat{m}_i, \hat{m}_j | k_L) = v - k_L \geqslant u_i(m'_i, \hat{m}_j | k_L), \forall m'_i$$

$$\tag{21}$$

$$u_j(\hat{m}_j, \hat{m}_i | k_L) = v - k_L \geqslant u_j(m'_j, \hat{m}_i | k_L), \forall m'_j$$

$$\tag{22}$$

 $u_{\xi}(\widetilde{m}_{\xi}, \hat{m}_{\zeta}|k_H) = 0 \ge u_{\xi}(\hat{m}_{\xi}, \hat{m}_{\zeta}|k_H), \text{ for some } \widetilde{m}_{\xi} \neq \hat{m}_{\xi} \text{ and for some } \xi \in \{i, j\}$ (23)

$$u_{\zeta}(m_{\zeta}, \widetilde{m}_{\xi} | k_H) = 0 , \forall m_{\zeta} and \zeta \neq \xi$$
(24)

Inequalities (21) and (22) imply that the aggregate subsidy given in the low-cost state is equal to the surplus: $\sum_{\xi} s_{\xi} = v - k_L$. Inequalities (23) and (24) require that at least one of the agents has the incentive not to trade if the high-cost state arises, while the other does not benefit from the non occurrence of trade.

If the mechanism $S(., \Sigma = \emptyset)$ allows for private contracts, contingent on messages (the equivalent of private contracts conditional on trade), (23) can not be satisfied. This is because $\sum_{\xi} s_{\xi} = v - k_L > k_H - v$ and therefore there is always a private contract $\pi(\hat{m}_i, q, \emptyset)$ such that $u_{\xi}(\hat{m}_{\xi}, \hat{m}_{\zeta}|k_H) + q > 0$ for both agents. Without loss of generality, assume that $S(., \Sigma = \emptyset)$ specifies a price p (not necessarily equal to $(1 - \beta_i)v + \beta_i k_L)$, if g is traded, and subsidies $\{s_i, s_j\}$, which satisfy (21) and (22). Then $u_i(\hat{m}_i, \hat{m}_j|k_H) = v - p + s_i$, $u_j(\hat{m}_j, \hat{m}_i|k_H) = p - k_H + s_j$ and $u_i(\hat{m}_i, \hat{m}_j|k_H) + u_j(\hat{m}_j, \hat{m}_i|k_H) = v - k_H + \sum_{\xi} s_{\xi} = (v - k_L) - (k_H - v) > 0$. Therefore, there exists a net-transfer q and a private contract $\pi(\hat{m}_{\xi}, p, \emptyset)$ such that $u_i(\hat{m}_i, \hat{m}_j|k_H) + q > 0$ and $u_j(\hat{m}_j, \hat{m}_i|k_H) - q > 0$, which violates the incentive compatibility-cum-efficiency condition.

If, on the other-hand, $S(., \Sigma = \emptyset)$ does not allow for the enforcement of private contracts, then there are two possibilities. Either inequality (23) can not be satisfied, in which case the result of the proposition holds, or (23) is satisfied. In this case, however, M is not ex-post renegotiation proof, because there exists another mechanism $S'(., \Sigma = \emptyset)$, which allows for private contract enforcement and makes both agents better off, as shown above. This completes the first part of the proposition, namely that the existence of the institutional agent is a necessary condition for the implementation of the first-best effort levels.

The necessity of the non-enforcement of the private contracts follows from the first part of the proof. Even if the mechanism specifies an institutional agent, inequalities (21)-(24) must still be satisfied, and with enforceable agreements of the form $\pi(\mathbf{m}, q, \emptyset)$, we already showed how (23) is violated.

Lemma 1: The optimal social contract S^* determines that the set Σ is of measure zero. That is, the total number of institutional agents is infinitesimal compared to the aggregate population.
Proof: Formally, $S^* = \arg \max_{S} \{m_{i(S)}U_i + m_{j(S)}U_j + m_{\Sigma S}r_{\sigma}^+\}$, where $m_{i(S)}$ is the mass of agents of type *i* who engage in productive activities according to *S*, $m_{j(S)}$ is the mass of type *j* agents, who engage in productive activities, and $m_{\Sigma(S)}$ is the mass of institutional agents set by S.

We show that any contract S, which assigns strictly positive mass to the set of institutional agents can not be socially optimal and will be not chosen. To see this, consider the social contract $S(\Sigma, \Phi(Q), \tau, z_a, z_g)$, with $m_{\Sigma} > 0$. Let S' be another social contract which specifies exactly the same taxation and investment schemes as S ($\tau' = \tau$, $z'_a = z_a$, $z'_g = z_g$), has the same set of enforceable contracts $\Phi(Q)$, but specifies a smaller set of institutional agents Σ' , subset of Σ ($\Sigma' \subset \Sigma \Rightarrow m_{\Sigma'} < m_{\Sigma}$). S' also provides the same rewards **r** as S to all agent in the set Σ , even though some of the agents in this set are not institutional agents according to S'. In other words, S' keeps exactly the same vector of transfers of S but shrinks the set of institutional agents.

Then S' provides a Pareto improvement over S. This is because all agents who are occupied in productive activities under S continue to pay the same taxation and receive the same benefits from productivity investments in S', so their utility is unchanged. Also the institutional agents in the set Σ' are equally well-off as in S. But the agents in the set $\Sigma - \Sigma'$ are made better off because they receive the same reward as before, plus they are occupied in productive activities under S' and receive all the marginal benefits of their own production: S' is a Pareto improvement over S. Hence, any social contract with positive measure of institutional agents can not be part of the equilibrium strategies of the game.

Lemma 2: The optimal social contract specifies the following rewards and punishments for the institutional agent: $\{r_{\sigma t}^+, r_{\sigma t}^-\} = \{(A(Z_{at})e_{\sigma a}^* - c_{ia0}(e_{\sigma a}^*)), \frac{1}{2}A(Z_{at})(e_{iat}^* + e_{jat}^*)\},\$ with $e_{\xi at}^*$ being the optimal effort level for type ξ in period t. **Proof:** Suppose that some social contract S is proposed in period t. The reward $r_{\sigma t}^+$ for the proposed institutional agent σ in period t must be at least as large as the discounted value of production of the good a that σ can produce in period t. Otherwise, she can vote against the proposal and receive her autarchic production. Therefore, in order to achieve the agreement of the institutional agent, the optimal social contract must set $r_{\sigma t}^+$ equal to $(A(Z_{at})e_{\sigma a}^* - c_{ia0}(e_{\sigma a}^*))$, where $e_{\sigma a}^*$ is such that $\frac{\partial c_{\sigma a0}}{\partial e_{\sigma a}} = A(Z_{at})$. On the other hand, the optimal social contract minimizes the resources allocated to non-productive activities, hence r_{σ}^+ can not be greater. These two conditions imply that $r_{\sigma t}^+ = \delta^t (A(Z_{at})e_{\sigma a}^* - c_{ia0}(e_{\sigma a}^*))$, where $e_{\sigma a}^* : \frac{\partial c_{\sigma a0}}{\partial e_{\sigma a}} = A(Z_{at})$.

 r_{σ}^{-} must be set sufficiently high, so as to prevent the institutional agent from not complying with the contract. Since the greatest amount of resources that the governor can extract from the economy is the aggregate production of the autarchic good, setting $r_{\sigma t}^{-}$ equal to $\frac{1}{2}A(Z_{at})(e_{iat}^{*}+e_{jat}^{*})$ is sufficient for making the governor to comply with her role in period t. The above mean that the rewards and punishments must increase with the increase in the productivity of the economy for the non-specialized good and the aggregate exerted effort level in its production. This generates the optimal punishment plan $\{\mathbf{r}_{\sigma}^{+}, \mathbf{r}_{\sigma}^{-}\}$, which is a vector with elements $r_{\sigma t}^{+}(A(Z_{at}))$ and $r_{\sigma t}^{-}(A(Z_{at}))$ for $t \in$ [0, T].

Proposition 3: Let $\overline{p} = \Gamma(Z_{gt-1}) \left(k_H + \beta_j (v - k_L) - \frac{\overline{c} - \underline{c}}{f(\overline{e}, \overline{e}) - \overline{f(\overline{e}, \underline{e})}} \right)$. Incentive compatibility requires that any private contract $\pi(\hat{p}, I_g)$ or $\pi(\hat{q}, p, I_g)$, with $\hat{p} > \overline{p}$ or $\hat{q} > \overline{p} - p$ is non-enforceable.

Proof: The proof follows from the proof of Proposition 1 in Appendix B. Incentive compatibility for type j requires that she prefers not to trade in the high cost state

than to trade, which implies that $p + q - \Gamma_t k_H - \tau_{j1t} \leq -\tau_{j0t}$. At the same time, the difference in taxation conditional on trade must be sufficiently high so that agent j prefers to exert high effort. The condition is given by equation (6) in subsection 2.4.1. Combining the two conditions gives the condition that $p + q \leq \overline{p}$.

The figures in the following three pages present diagrammatically all the possible cases of investment plans. We do not derive the conditions under which these paths arise, but the required analysis is similar to subsection 2.4.4. The cases depend on the size of the three critical values \tilde{Z}_a , \underline{Z}_a and \overline{Z}_a . In each case, the optimal investment path is described in terms of: (i) its duration (as provided by the endogenous thresholds $\tilde{t}, \underline{t}, \overline{t}$ and t^*), (ii) for which goods is the productivity increased through investments in the respective capital stocks, and (iii) for how many types of agents is effort exertion subsidized. Recall that if only one type of agent receives a taxation-break, then this is the type with the lowest bargaining power.





Figure 11: Case I

Investment in productivity Investment in productivity Investment in productivity Of good a until period: Of good a until period: Of good a until period: t_1 : $\bar{t} \leq t_1$ t_1 : $t \leq t_1$ t_1 : $\bar{t} \leq t_1$ Optimal investment in Optimal investment in Optimal investment in productivity of good g. productivity of good g. productivity of good g. Inducement of high effort Inducement of high effort Inducement of high effort for both agents. only for one agent. only for one agent. t_1 Investment in productivity of good a until period: $t_1 : \widetilde{t} \leq t_1 < t$ Optimal investment in productivity of good g. No inducement of high effort. t Investment in productivity of good a until period: $t_1: t_1 < \widetilde{t}$ No investment in productivity of good g. No inducement of high effort. 0

Case II: $\underline{Z_a} < \widetilde{Z}_a < \overline{Z_a}$

Figure 12: Case II





Figure 13: Case III

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